Appendix
Methodology

- We use the Log-Periodic Power Law Singularity (LPPLS) model to hunt for the distinct fingerprint of Financial Bubbles. Basic assumptions of the model are:
  1. During the growth phase of a positive (negative) bubble, the price rises (falls) faster than exponentially. Therefore the logarithm of the price rises faster than linearly.
  2. There are accelerating log-periodic oscillations around the super-exponential price evolution that symbolize increases in volatility towards the end of the bubble.
  3. At the end of the bubble, the so-called critical time $t_c$, a finite time singularity occurs after which the bubble bursts.

- Together, these effects encompass irrational imitation and herding phenomena amongst market participants that lead to blow-up and instability of asset prices.
Bubble Regimes

- **Positive bubble**: (the pressure builds up, generally in multiple stages)
- **Positive anti-bubble**: (the pressure is progressively released, generally in multiple stages)
- **Negative bubble**: (pressure towards panic = herding in bearish phase)
- **Negative anti-bubble**: (negative pressure released, progressively)
Mathematically, the simplest version of the Log-Periodic Power Law Singularity (LPPLS) model that describes the expected trajectory of the logarithmic price in a bubble is given as:

\[ E[\ln(P(t))] = A + B(t_c - t)^m + (t_c - t)^m [C_1 \cos(\omega \ln(t_c - t)) + C_2 \sin(\omega \ln(t_c - t))] \]

The seven parameters describing the model dynamics are:

- \(A\) The finite peak (valley) log-price at the time \(t_c\) when the positive (negative) bubble ends.
- \(m\) The power law exponent.
- \(B\) The power law intensity.
- \(C_1, C_2\) Magnitude coefficients of the log-periodic accelerating oscillations.
- \(\omega\) The log-periodic angular frequency of the log-periodic oscillations.
- \(t_c\) The critical time at which the bubble ends.

The set of seven model parameters is obtained by fitting the LPPLS formula to the price time series via a combination of Ordinary Least Squares and nonlinear optimization. The resulting values of the fit parameters reveal whether an asset is in a bubble state. Furthermore, the central parameter of interest, the critical time \(t_c\), may warn of an imminent crash.
LPPLS Analysis of Price Time Series

- In order to avoid overfitting and to continuously collect information about price dynamics, we scan asset log-price trajectories for super-exponential price dynamics by sequentially fitting the LPPLS model in different time windows to the underlying price series. The procedure is illustrated in the plot.

- For a fixed fit window end time, \( t_2 \), we select different window start times \( t_1 \) and fit the LPPLS model in each of the resulting windows. This gives one set of calibrated LPPLS parameters per fit window. In our monthly report, \( t_2 \), the time of analysis is always the start of the month, i.e., the report date.

- We employ the methodology in [1] to determine the starting time of a bubble, and the DS LPPLS Confidence Indicator defined in [2] to measure the maturity of a bubble. The DS LPPLS Confidence Indicator quantifies the presence of super-exponential price dynamics obtained over various differently sized time windows. A high value of the indicator signals that LPPLS signatures were detected on many timescales.

- To measure the reliability of the results, we employ k-means clustering to our LPPLS calibrations [3] to measure the similarities of the fit parameter sets obtained from different time window sizes, and to find possible future scenarios for the ending of a bubble. We report the mean critical times \( \mu_{t_c} \) and standard deviations \( \sigma_{t_c} \) of the largest cluster. Furthermore, as complement to the Confidence Indicator, we report the associated scenario probability of the biggest cluster, defined as the number of members in the largest cluster divided by the total number of fits.

Result Presentation

- We present the monthly results of our bubble analysis in the form of a table such as the example given below.

- In each table, we separately list assets that are in a positive, respectively, negative bubble state. Furthermore, the table is divided into two sections, bubble data and cluster analysis.

- The first section provides asset and estimated bubble characteristics (size and duration), as well as the value of the confidence indicator. We rank assets according to their geometric average of the absolute of bubble size and confidence indicator. In this way, we incorporate the bubble size into the ranking.

- In the table section cluster analysis, the prediction data of the two most probable bubble burst scenarios are presented.

<table>
<thead>
<tr>
<th>Bubble Data</th>
<th>Cluster Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Bubble Size $bs$ [%]</td>
</tr>
<tr>
<td>Positive Bubbles</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>iBoxx GEMX Kenya Index</td>
</tr>
<tr>
<td>Negative Bubbles</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Result Presentation

- For each asset class, we also supply the confidence indicator time series for the bubble assets listed in the tables. The plot shows the cumulative return (left y-scale, in %) of the analyzed price trajectory (blue) since the beginning of the plot time range. We also plot the time series of the positive (green) and negative (red) DS LPPLS Confidence indicators (right y-scale). The indicator time series are calculated by repetitively applying the procedure described on the slide ‘The DS LPPLS Confidence Indicator’ over moving window end times $t_2$. Furthermore, if, at the last analyzed time, a non-zero indicator value results, i.e. the asset is presently in a bubble state, we outline the time interval for the positive (green shaded) or negative (red shaded) bubble from its beginning to present.
Real Effective Exchange Rate Indices

- 98 Real Effective Exchange Rate (REER) Indices for different currencies are investigated for bubble characteristics.

- The (here CPI-weighted) REER Indices are a measure for the trading competitiveness of the corresponding country.

- In contrast to single currency cross rates, the REER is a rather absolute measure of the domestic currency value because it is calculated versus a selection of other currencies.

- This has the advantage that, unlike with the methodologies that were used in previous reports, positive and negative bubbles in the value of the currency can clearly be distinguished, as visible in the table above.
Single Stocks
Quadrants 1 and 2 – strong positive bubble signals with strong (respectively weak) fundamentals
Single Stocks
Quadrants 3 and 4 – strong negative bubble signals with weak (respectively strong) fundamentals