Importance of positive feedbacks and overconfidence in a self-fulfilling Ising model of financial markets

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Utility theory

$$\sum_i p_i\, u(w_i) > \sum_i q_i\, u(w_i)$$

Von Neumann and Morgenstern

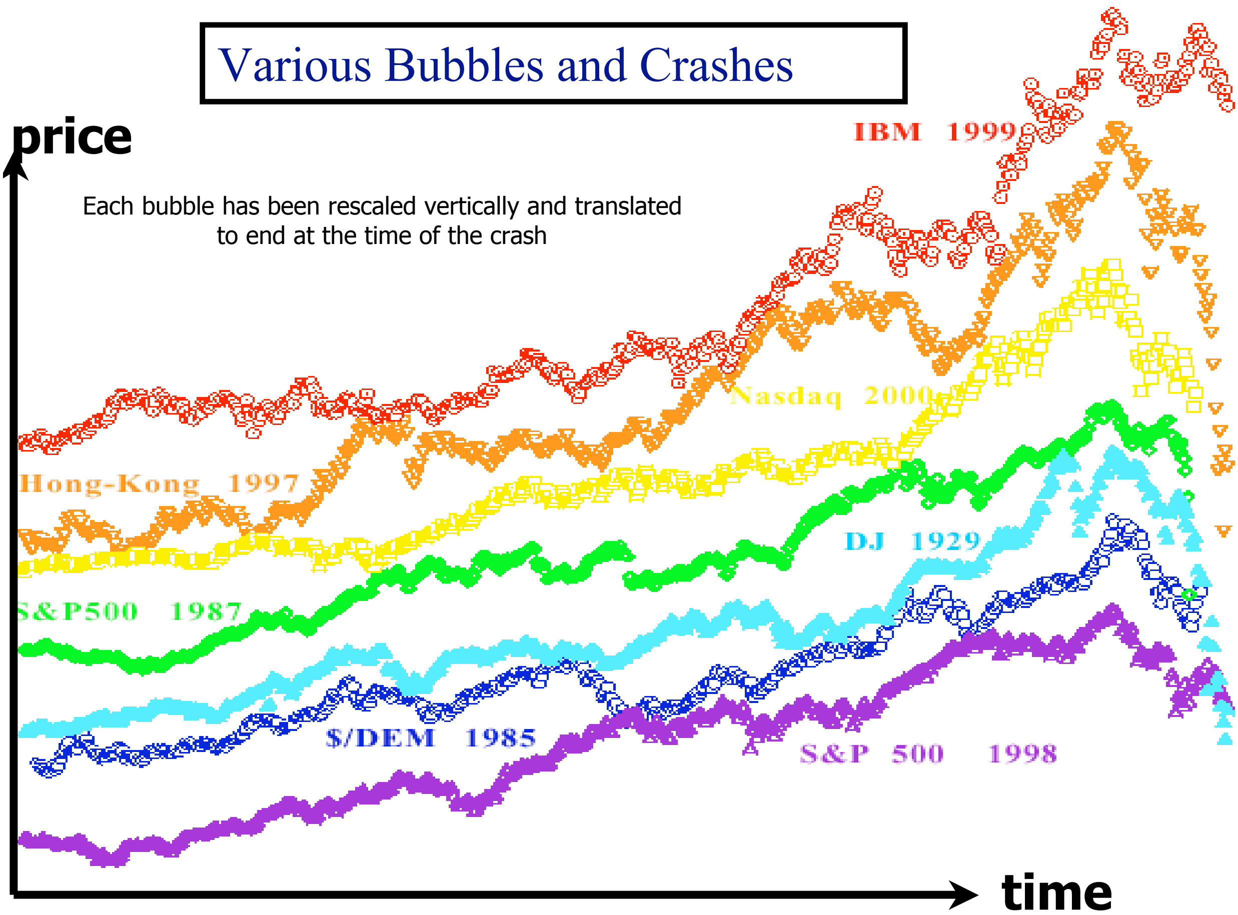
Behavioral Finance: one person

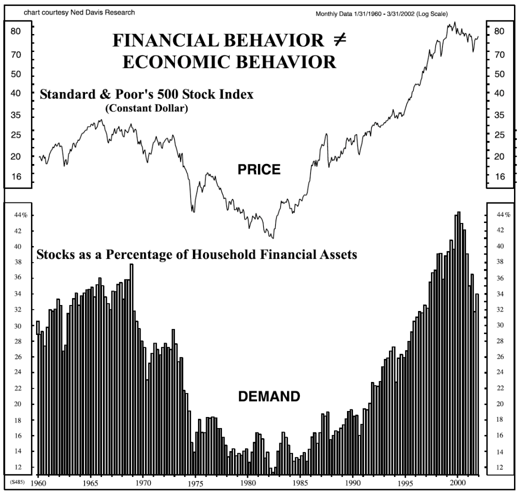
- Fear and Greed
- Over-confidence
- Anchoring
- Law of small numbers (gambler's fallacy)
- Representativeness (=>weight recent past too heavily)
- Availability and rational inattention
- Allais' paradox: relative reference level
- Subjective probabilities
- Procedure Utility

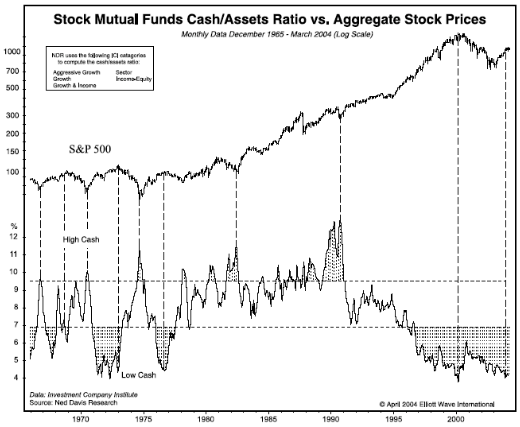
$$\sum_i \pi(p_i) v(\Delta w_i) > \sum_i \pi(q_i) v(\Delta w_i)$$

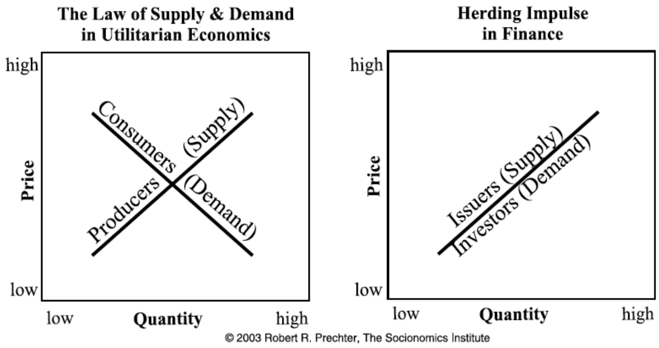
Kahneman and Tversky


THIS WAY TO THE
Bull Market!
Almost there. . . 300 feet. . . Don't Miss The Fun!
I'm a trader.
Hey this Koolaid tastes great!
Must own Yahoo.
Don't be negative.
Stoopid bears.
Take a deep breath!
I'm flying mommy!
El Cliffo


Various Bubbles and Crashes
price
Each bubble has been rescaled vertically and translated
to end at the time of the crash
IBM 1999
Hong-Kong 1997
Nasdaq 2000
S&P500 1987
DJ 1929
$/DEM 1985
S&P 500 1998
time







Imitation

-Imitation is considered an efficient mechanism of social learning.

- Experiments in developmental psychology suggest that infants use imitation to get to know persons, possibly applying a ‘like-me’ test (‘persons which I can imitate and which imitate me’).

- Imitation is among the most complex forms of learning. It is found in highly socially living species which show, from a human observer point of view, ‘intelligent’ behavior and signs for the evolution of traditions and culture (humans and chimpanzees, whales and dolphins, parrots).

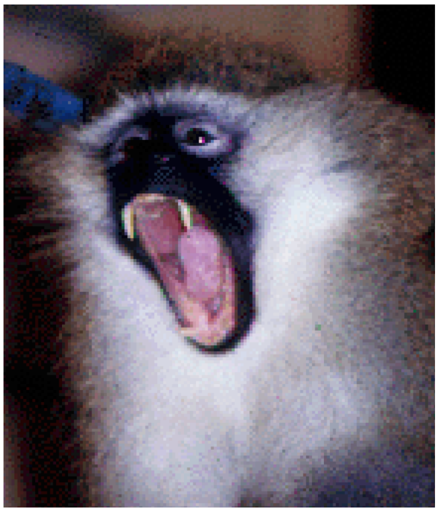
- In non-natural agents as robots, tool for easing the programming of complex tasks or endowing groups of robots with the ability to share skills without the intervention of a programmer. Imitation plays an important role in the more general context of interaction and collaboration between software agents and human users.

OBSERVATIONAL LEARNING

For evolutionary fears, monkeys and people learn by watching what other animals and people do (not by doing themselves and learning from the consequences).

Hands-on learning may not always be the best! THE APE AND THE SUSHI MASTER (Frans de Waal's book): in Japan, apprentic sushi cooks spend three years just watching the sushi master prepare sushi. When the apprentice finally prepares his first sushi, he does a good job of it. ("The watching of skilled models firmly plants action sequences in the Head that come in handy, sometimes much later, when the same taskes need to be carried out." The ape and the sushi Master: cultural reflections of a primatologist (New York: Basic Books, 2001)

Temple Grandin and C. Johnson,
Animals in translation (Scribner, New York, 2005)



VERVET MONKEY

FEARS ARE CONTAGIOUS

Psychologist S. Mineka's experiments with monkeys and snakes :
lots of phobias and fears are CONTAGIOUS

Monkeys in the wild are terrified by snakes

Monkeys in the lab are not worried by snakes

Dr. Mineka taught a lab monkey to be just a terrified of snakes as any monkey living in the wild. When Dr. Mineka exposes her fearless monkeys to wild-reared monkeys acting afraid of snakes, the lab monkeys instantly got scared themselves, and they stayed scared for life. The lab-monkeys learned the same level of fear as the demonstrator-monkey. If the demonstrator-monkey was scared but not panicked, the observer-monkey became scared but not panicked.

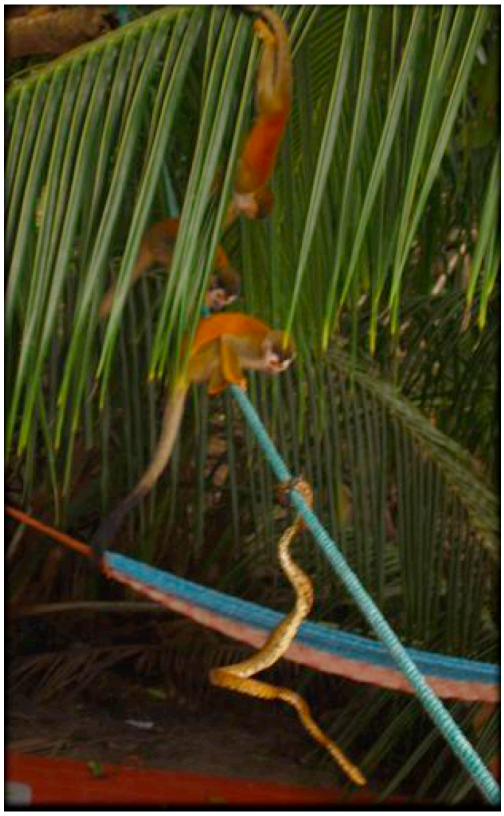
It is impossible to teach a monkey to be afraid of a flower by the same technique! (video tape of a flower followed by a monkey acting terrified).

Fear of snake is SEMI-INNATE: monkeys are born ready to fear snakes at the first hint of trouble (prepared stimulus)

One can protect an animal from developing fear: If Dr. Mineka first exposed a lab-reared monkey to another lab-reared monkey NOT acting afraid of a snake, that gave him "immunity": after that, if he saw a wild-reared monkey acting scared of a snake, he did NOT develop snake fear himself. He held to his first lesson.

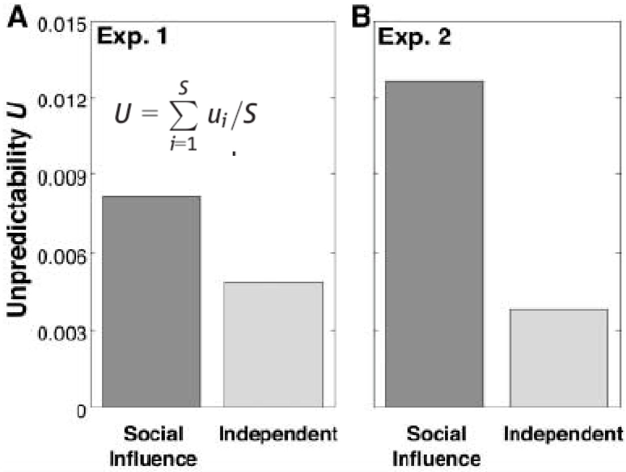
Temple Grandin and C. Johnson,
Animals in translation (Scribner, New York, 2005)

Red squirrel monkeys and six-foot Costa Rica snake



CURIOSITY-SUSCEPTIBIILTY "THEOREM"
Or CURIOSITY-FEAR THEOREM

Unpredictability of success $u_i = \sum_{j=1}^{W} \sum_{k=j+1}^{W} |m_{i,j} - m_{i,k}| / \binom{W}{2}$



Random order of popularity   Ranked popularity

With a little help from my friends. When making choices, individuals are influenced by what others think is best, making the final outcome unpredictable.

Popular songs became more popular and unpopular songs became less popular when individuals influenced one another.

The structure of social action—that is, the pattern and strength of social influence—in and of itself is of considerable importance for explaining the social phenomena we observe.

M. J. Salganik, P. S. Dodds, D. J. Watts, *Science* **311**, 854 (2006)

Why do we have a big brain?

- Epiphenomenal hypothesis: large brains are unavoidable consequences of a large body
- Developmental hypothesis: maternal energy constraints determine energy capacity for fetal brain growth (frugivory=richer diet)
- Ecological hypothesis: brain evolved to process information of ecological relevance (frugivory, home range navigation, extractive foraging)
- Social hypothesis: brain size constrains size of social network (group size) (memory on relationships, social skills)

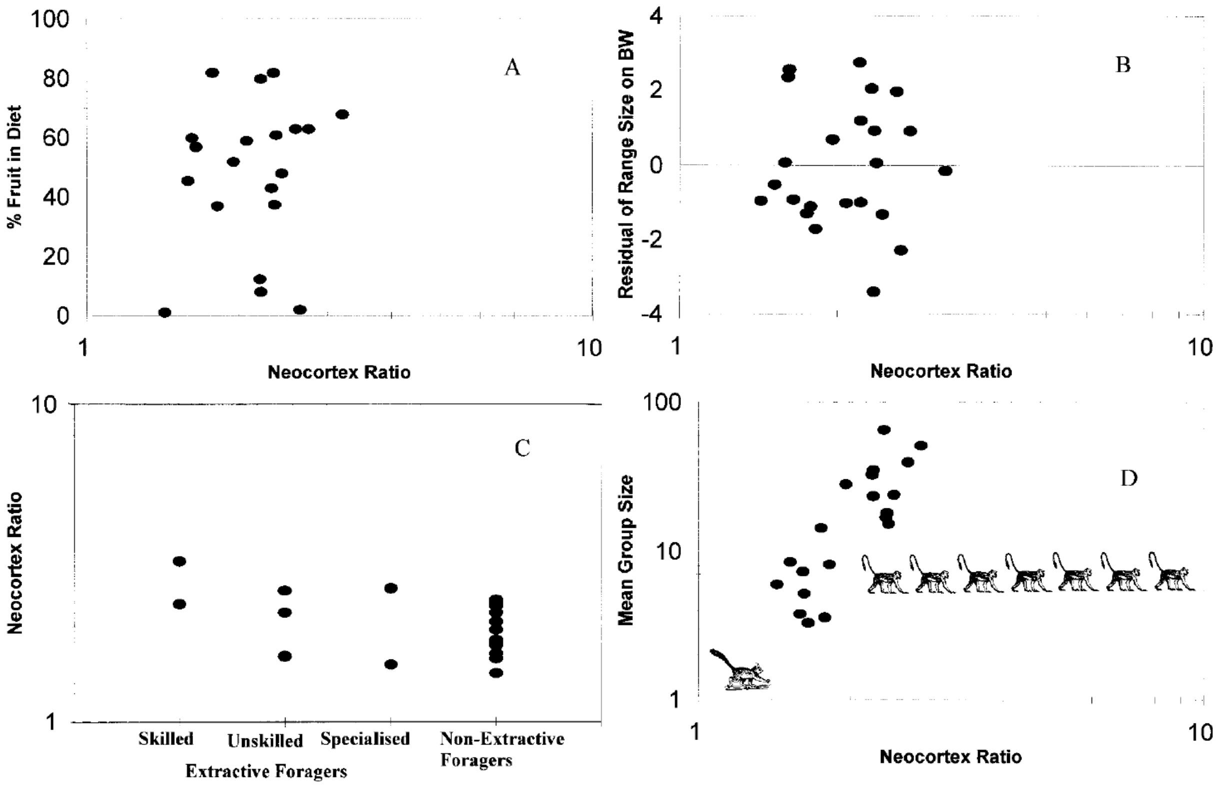


Figure 2. Relative neocortex size in anthropoid primates plotted against (a) percentage of fruit in the diet, (b) mean home-range size scaled as the residual of range size regressed on body weight (after Dunbar[24]), (c) types of extractive foraging (after Gibson[4]), and (d) mean group size. ((a), (b), and (d) are redrawn from Dunbar[24], Figures 6, 2 and 1, respectively; (c) is from Dunbar,[35] Figure 2.)

Dunbar, R.I.M., The social brain hypothesis. Evolutionary Anthropology 6, 178-190 (1998).

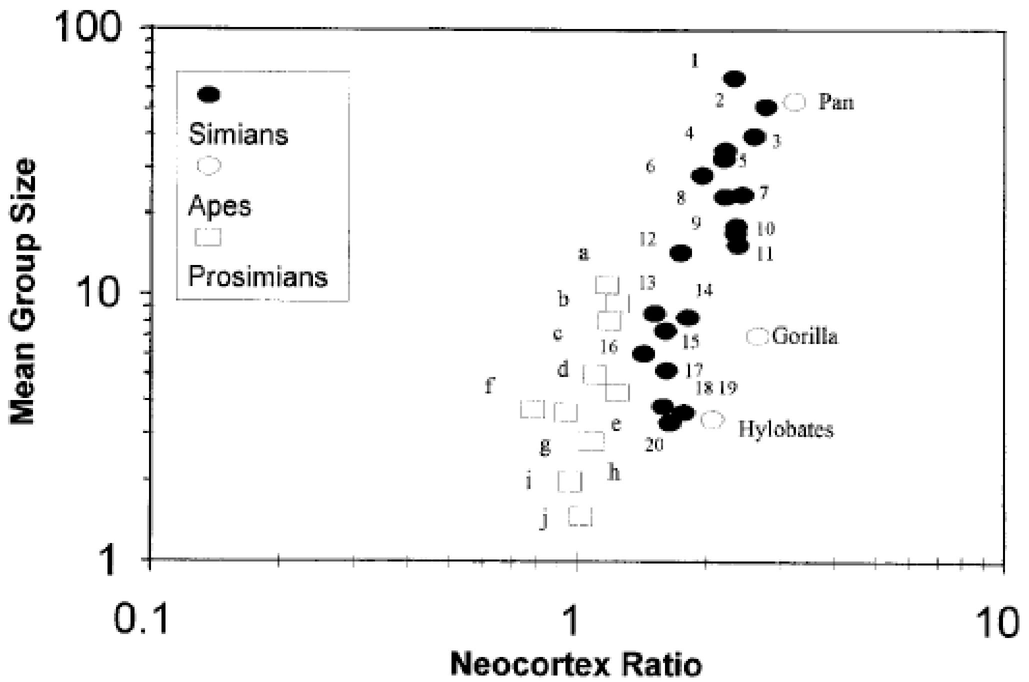
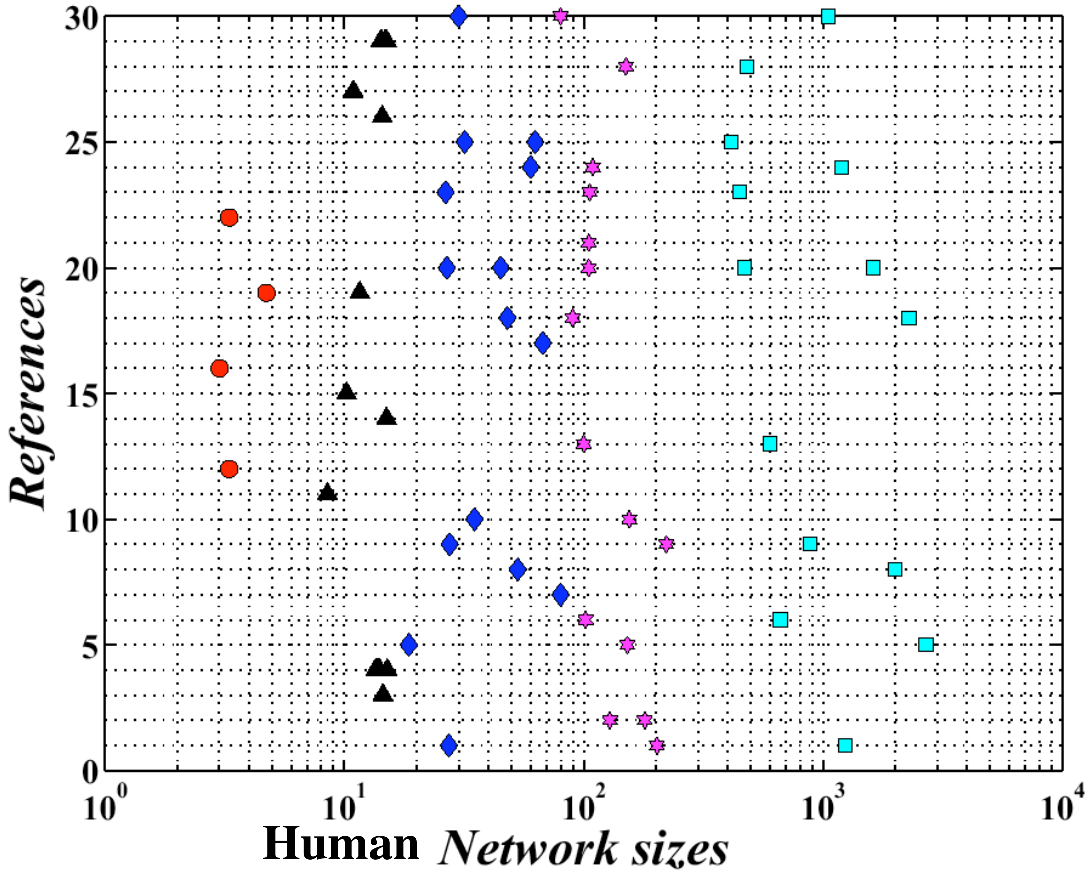
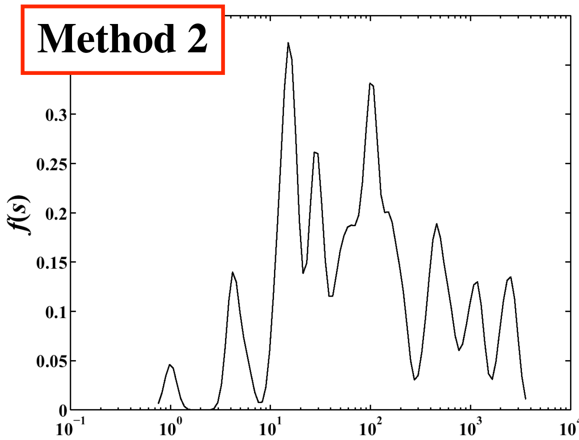


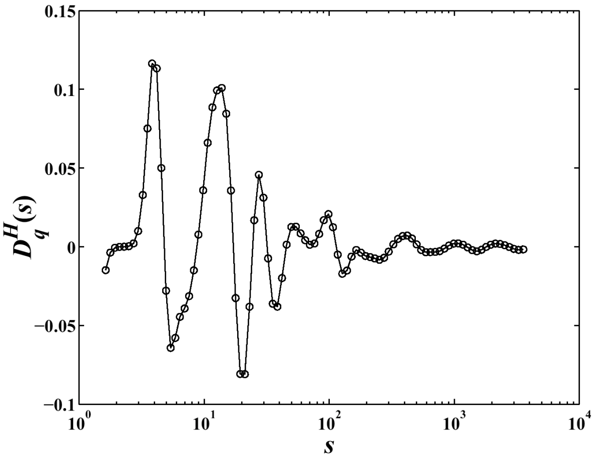
Figure 3. Mean group size plotted against neocortex ratio for individual genera, shown separately for prosimian, simian, and hominoid primates. Prosimian group size data, from Dunbar and Joffe,[25] include species for which neocortex ratio is estimated from total brain volume. Anthropoid data are from Dunbar.[24] Simians: 1, *Miopithecus;* 2, *Papio;* 3, *Macaca;* 4, *Procolobus;* 5, *Saimiri;* 6, *Erythrocebus;* 7, *Cercopithecus;* 8, *Lagothrix;* 9, *Cebus;* 10, *Ateles;* 11, *Cercocebus;* 12, *Nasalis;* 13, *Callicebus;* 14, *Alouatta;* 15, *Callimico;* 16, *Cebuella;* 17, *Saguinus;* 18, *Aotus;* 19, *Pithecia;* 20, *Callicebus.* Prosimians: a, *Lemur;* b, *Varecia;* c, *Eulemur;* d, *Propithecus;* e, *Indri;* f, *Microcebus;* g, *Galago;* h, *Hapalemur;* i, *Avahi;* j, *Perodictus.*

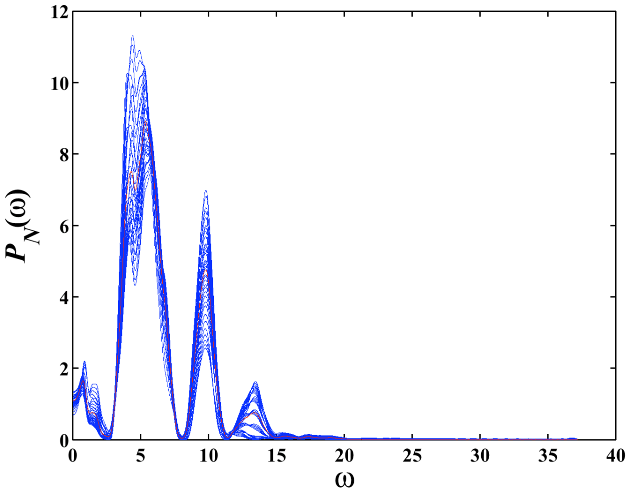
circle (support clique), triangle (sympathy group), diamond (bands), stars (cognitive groups),



Method 2







Preferred scaling ratio close to 3

Discrete Hierarchical Organization of Social Group Sizes

W.-X. Zhou[1], D. Sornette[1,2,3], R.I.M. Dunbar[4] and R. Hill[5]

Proc. Nat. Acad. Sci. (USA) 272, 439-444 (2005)

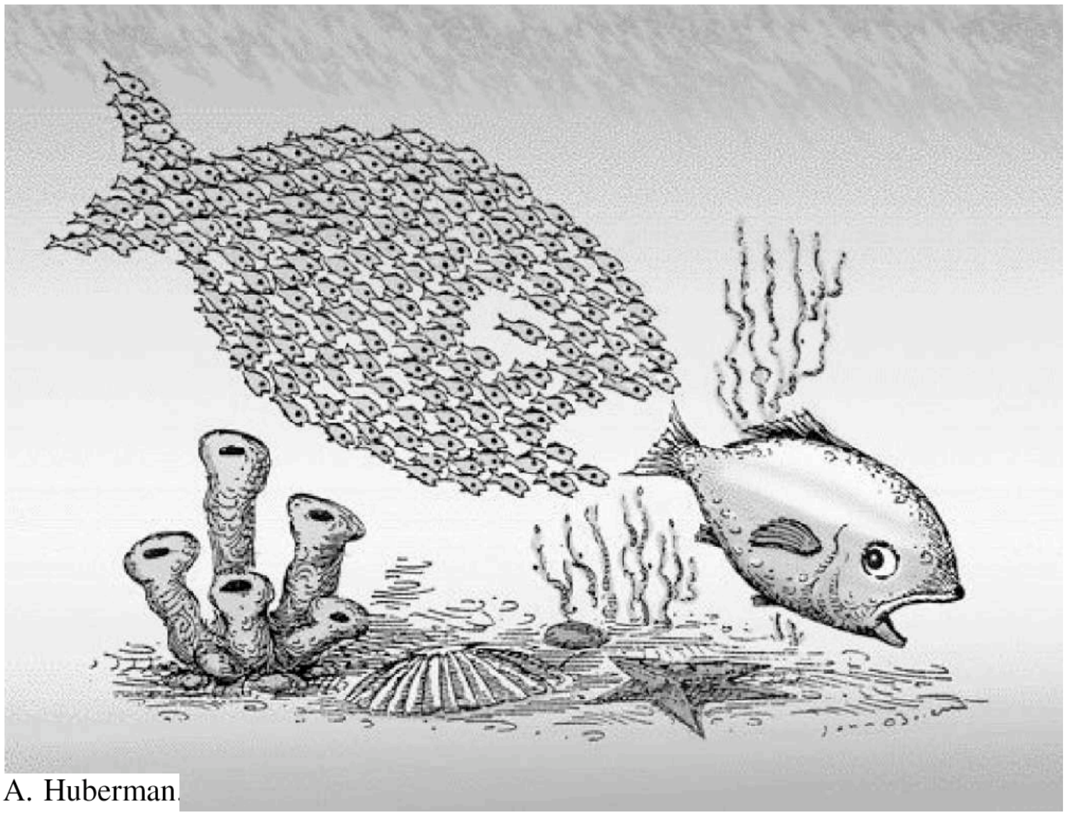
A real-life example of a hierarchical network

- Sections (squads): 10-12 soldiers
- Platoons (of 3 sections, ≈ 35 soldiers)
- Companies (3-4 platoons, ≈ 120-150 soldiers)
- Battalions (3-4 companies plus support units, ≈ 550-800)
- Regiments (or brigades) (3 battalions plus support,2500+)
- Divisions (3 regiments)
- Corps (2-3 divisions)
- Armies
- Country

. . . apes seem to be good psychologists in that they are good at reading minds, whereas monkeys are good ethologists in that they are good at reading behavior . . .

Fair trade. Capuchin monkeys refuse to cooperate when they see a comrade receive a better reward for the same task.

Collective behavior



Courtesy of B. A. Huberman

Optimal strategy obtained under limited information

Equation showing optimal imitation solution of decision in absence of intrinsic information and in the presence of information coming from actions of connected "neighbors"

$$s_i(t+1) = \mathrm{sign}\left(K \sum_{j \in N_i} s_j + \varepsilon_i\right)$$

This equation gives rise to critical transition=bubbles and crashes

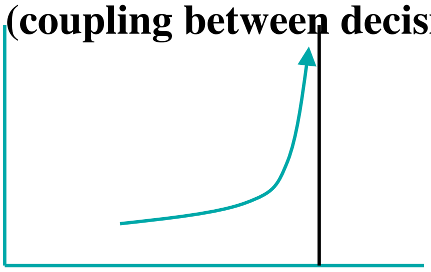
+ random dynamics of imitation strength

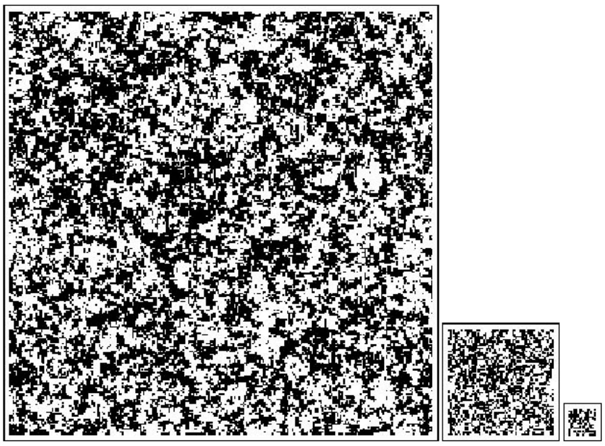
-Crash = coordinated sell-off of a large number of investors
-single cluster of connected investors to set the market off-balance
-Crash if 1) large cluster s>s*   and 2) active

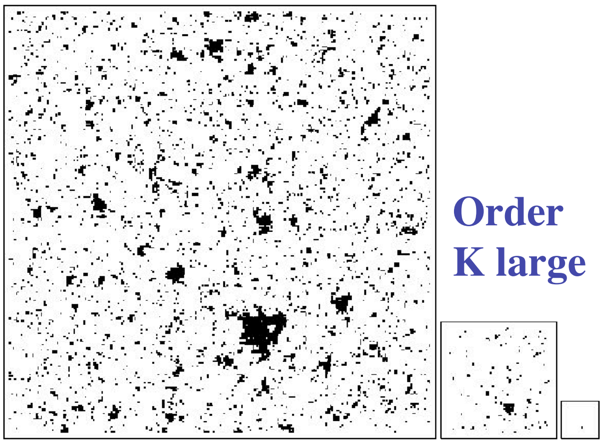
-Proba(1) = n(s)
-Proba(2) ~ s^a with $1 < a < 2$ (coupling between decisions)

Proba(crash) ~ $\sum_{s>s^*}$ n(s) s^a

If a=2, $\sum_{s>s^*}$ n(s) s^2 ~ $|K-K_c|^{-\gamma}$





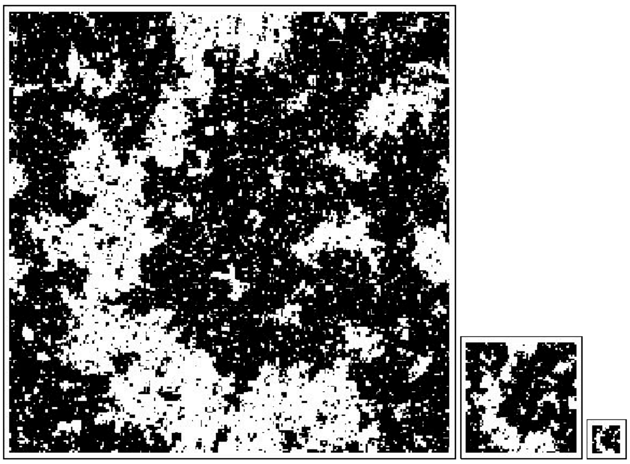


Order
K large

Disorder : K small

Renormalization group:
Organization of the description scale by scale

Critical:
K=critical value



Rational Expectation Bubbles and Crashes

Martingale hypothesis ("no free lunch"):

$$\text{for all} \quad t' > t \qquad \mathrm{E}_t[p(t')] = p(t)$$

If crashes are depletions of bubbles:

$$dp = \mu(t)\, p(t)\, dt - \kappa[p(t) - p_1]dj$$

Martingale gives

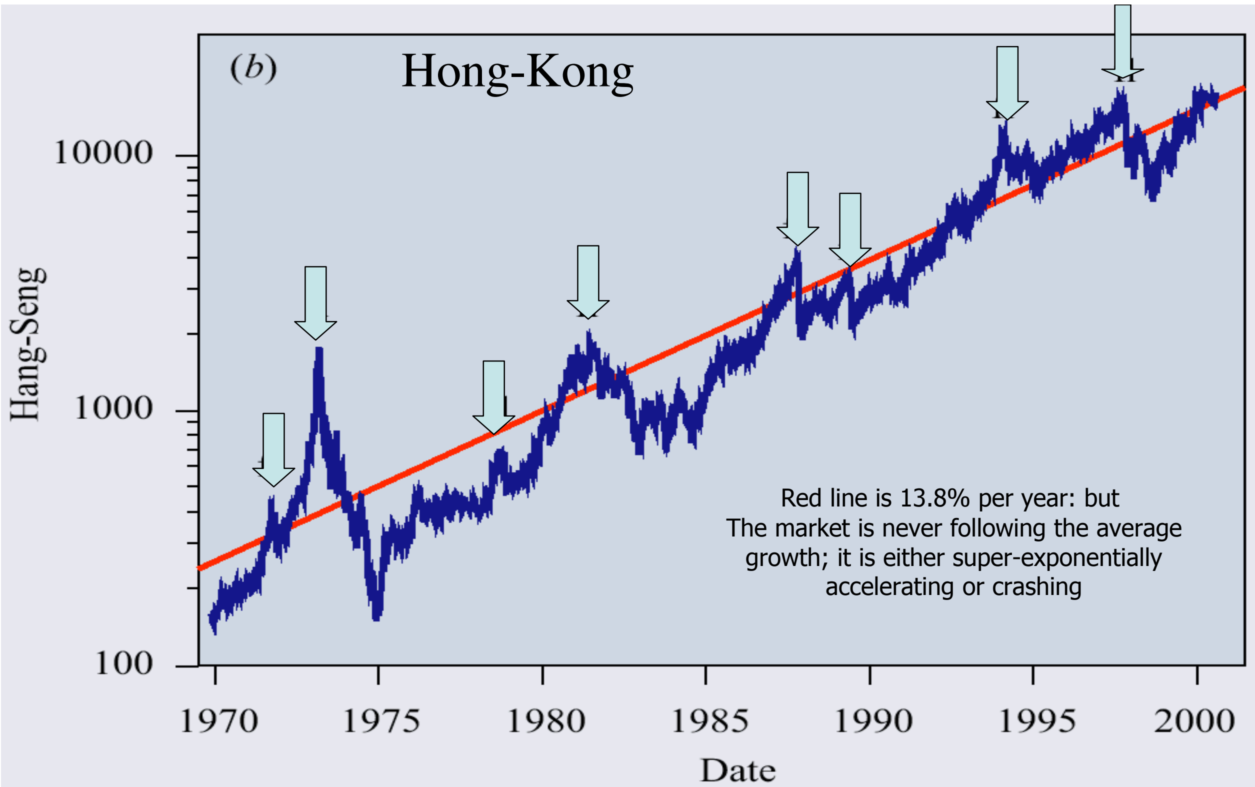
h(t)=E[dj]

$$\mu(t)p(t) = \kappa[p(t) - p_1]h(t) \ ,$$

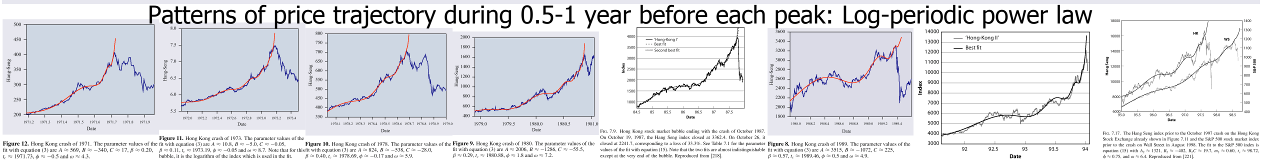
i.e., if crash hazard rate $h(t)$ increases, so must the return (bounded rationality)

A. Johansen, D. Sornette and O. Ledoit
Predicting Financial Crashes using discrete scale invariance, Journal of Risk 1 (4), 4, 5-32 (1999)

A. Johansen, O. Ledoit and D. Sornette, Crashes as critical points, International Journal of Theoretical and Applied Finance 3 (2), 219-255 (2000)



Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law



Importance of Positive Feedbacks and Over-confidence in a Self-Fulfilling Ising Model of Financial Markets

$$s_i(t) = \mathrm{sign}\left[\sum_{j\in\mathcal{N}} K_{ij}(t)\mathrm{E}[s_j](t) + \sigma_i(t)G(t) + \epsilon_i(t)\right]$$

Imitation — News — Private information

$$K_{ij}(t) = b_{ij} + \alpha_i K_{ij}(t-1) + \beta r(t-1)G(t-1)$$

(generalizes Carlos Pedro Gonçalves, who generalized Johansen-Ledoit-Sornette)

β: propensity to be influenced by the felling of others

1. **β<0: rational agents**

- **β>0: over-confident agents**

Didier Sornette and Wei-Xing Zhou, in press in Physica A (2006) (http://arxiv.org/abs/cond-mat/0503607)

News: $G(t) = \begin{cases} 1 & \text{if } I(t) > 0, \\ -1 & \text{if } I(t) \leqslant 0. \end{cases}$

Price: $p(t) = p(t-1)\exp[r(t)],$

$$r(t) = \frac{\sum_{i \in \mathcal{N}} s_i(t)}{\lambda N}$$

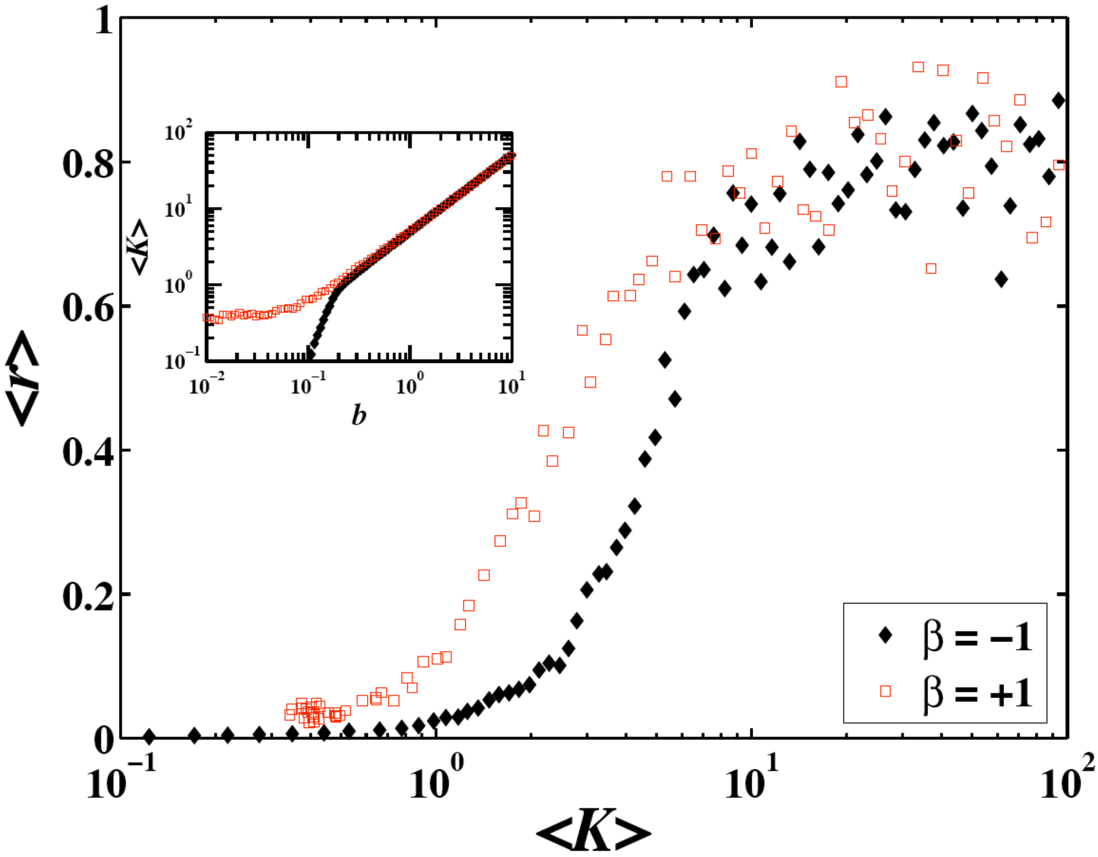
(1) the agents make decisions based on a combination of three ingredients: imitation, news and private information

(2) they are boundedly rational

(3) traders are heterogeneous (K_{ij} and σ_i);

(4) The propensity to imitate and herd is evolving adaptively as an interpretation that the agents make of past successes of the news to predict the direction of the market.

Illustration of the existence of an Ising-like phase transition, as a function of the control parameter b_{max} for both regimes β = -1 and β=1



$\alpha_i = 0$ *corresponding to the absence of memory of the coefficients* K_{ij}*'s*

$$K_{ij}(t) = K_i(t) = b_i + \beta r(t-1)G(t-1)$$

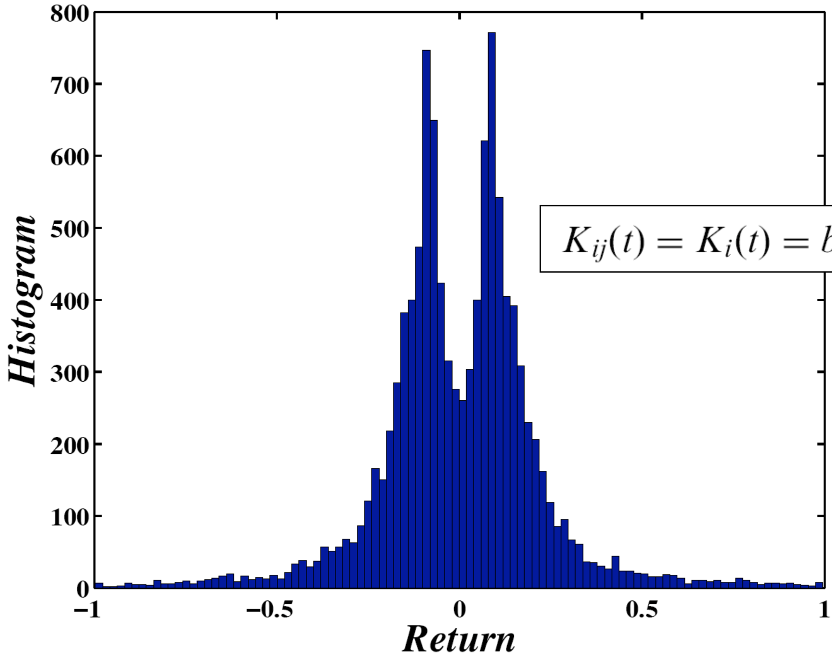


Fig. 1. Density distribution of returns r_1 for a realization of the artificial stock market model formulated by Gonçalves (2003) generated using $b_{\max} = 0.22 \sim 0.24$, $\sigma_{\max} = 0.14 \sim 0.15$ and $CV = 0.8 \sim 0.9$ as recommended by this author. The time series of returns have been kindly provided by Gonçalves. Our own simulations reproduce the same results.

News predicts the next return $\rightarrow$ *decrease of imitation:* $\beta = -1$

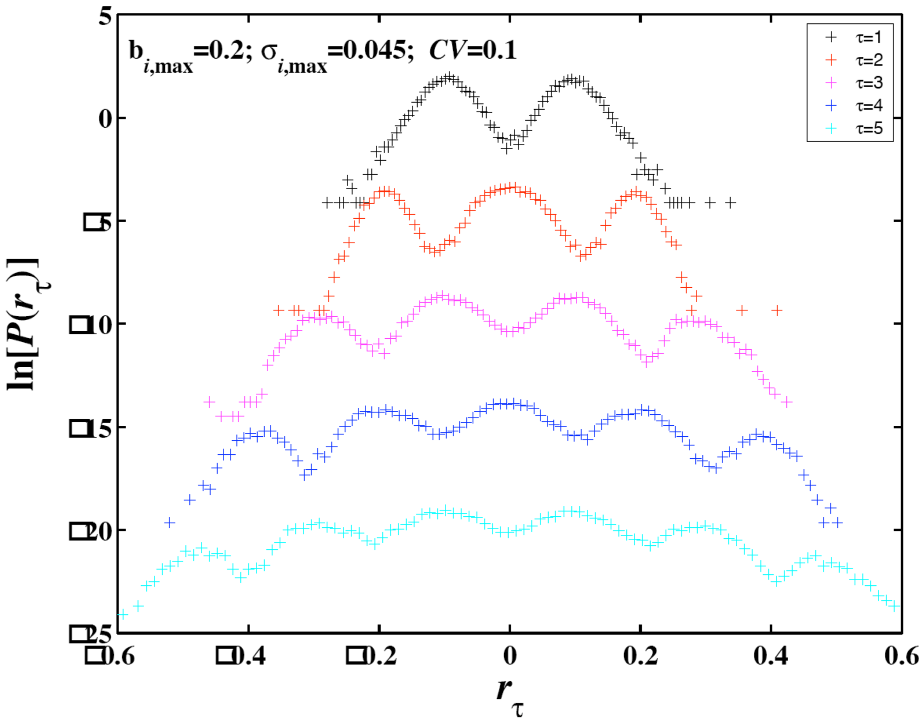


Fig. 2. A typical example of the multimodal distribution for $b_{\max} = 0.2$, $\sigma_{\max} = 0.045$, and $CV = 0.1$.

Case $\beta = +1$ ("over-confident" agents)

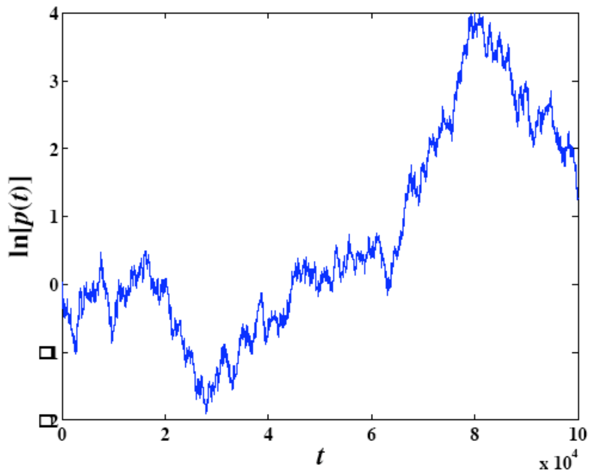


Fig. 3. A realization of the logarithm of the price over 10^5 time steps generated using $\alpha = 0.2$, $b_{\max} = 0.3$, $\sigma_{\max} = 0.03$ and $CV = 0.1$ of the generalized artificial stock market model defined by (1), (4) and (10).

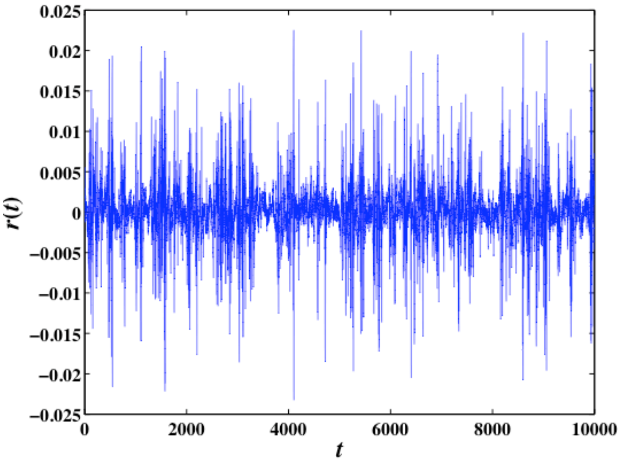
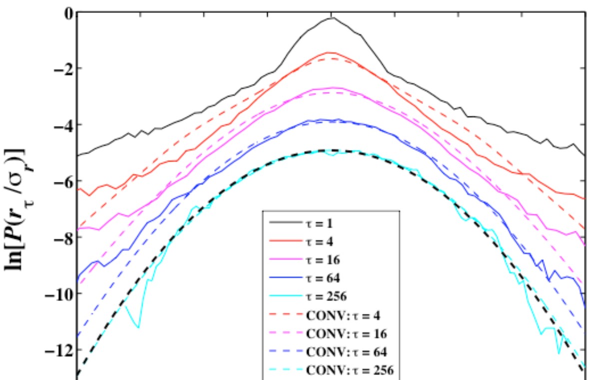
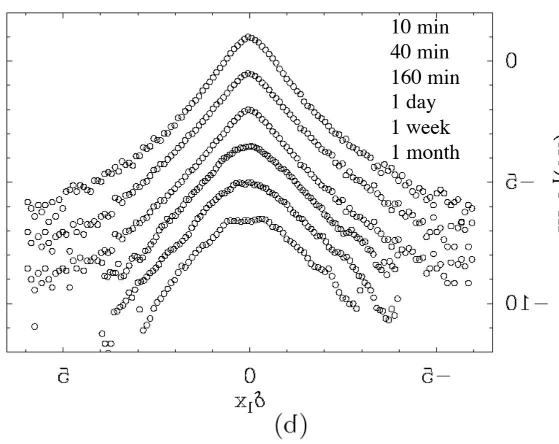
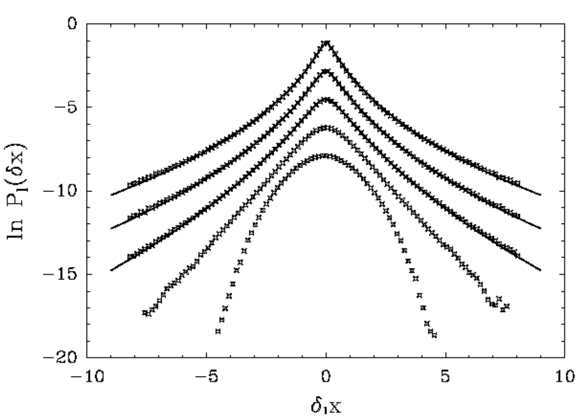


Fig. 4. Time series of the log-returns of the price shown in Fig. 3.

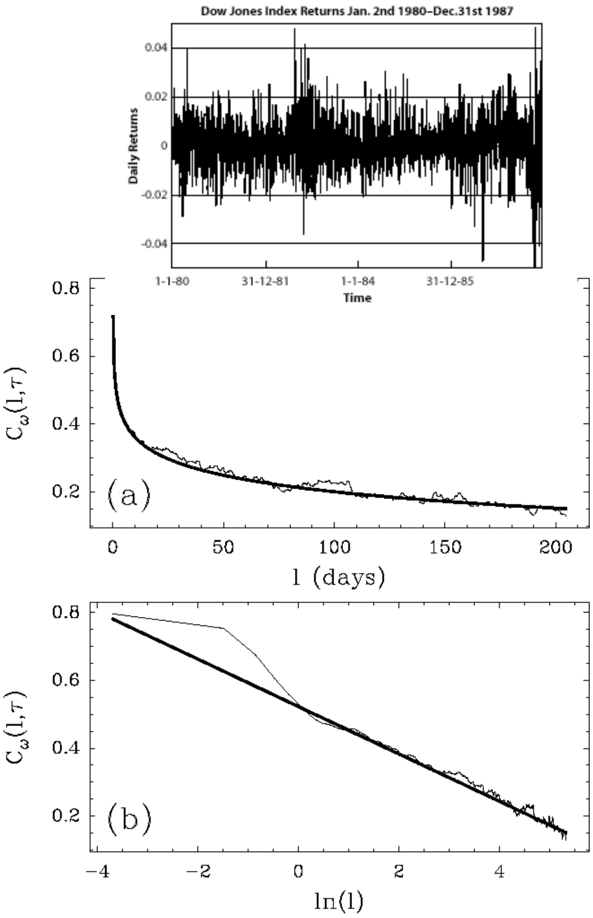
Fig. 5. (Color online) Empirical (solid lines) and theoretical (dashed thin lines) probability distribution density (in logarithmic scales) of log-returns at different time scales τ of the price time series shown in Fig. 3. The log-returns r_τ are normalized by their corresponding standard deviations σ_τ. The pdf curves are translated vertically for clarity. The thick dashed line is the Gaussian pdf.







Multifractal random walk



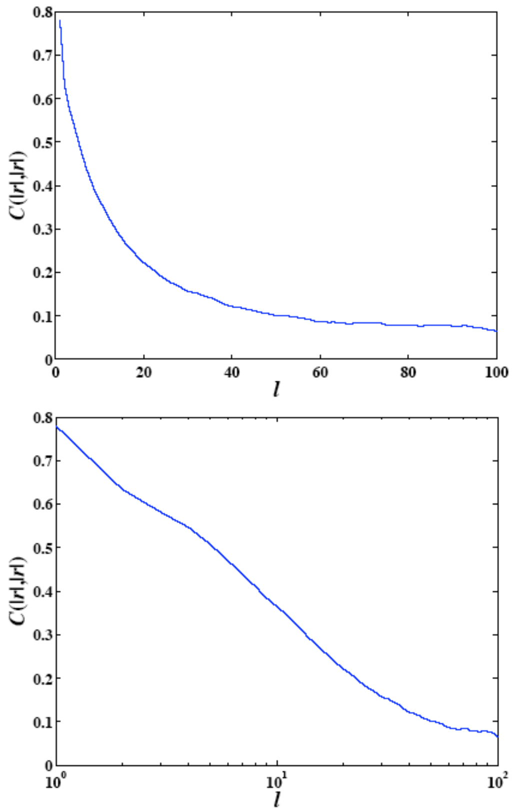


Fig. 7. Autocorrelation function of the absolute value of log-returns of the realization shown in Fig. 3. The top panel show the correlation in linear-linear scale. The bottom panel plots the correlation function as a function of the logarithm of the time lag, as suggested by the multifractal random walk model (see text).

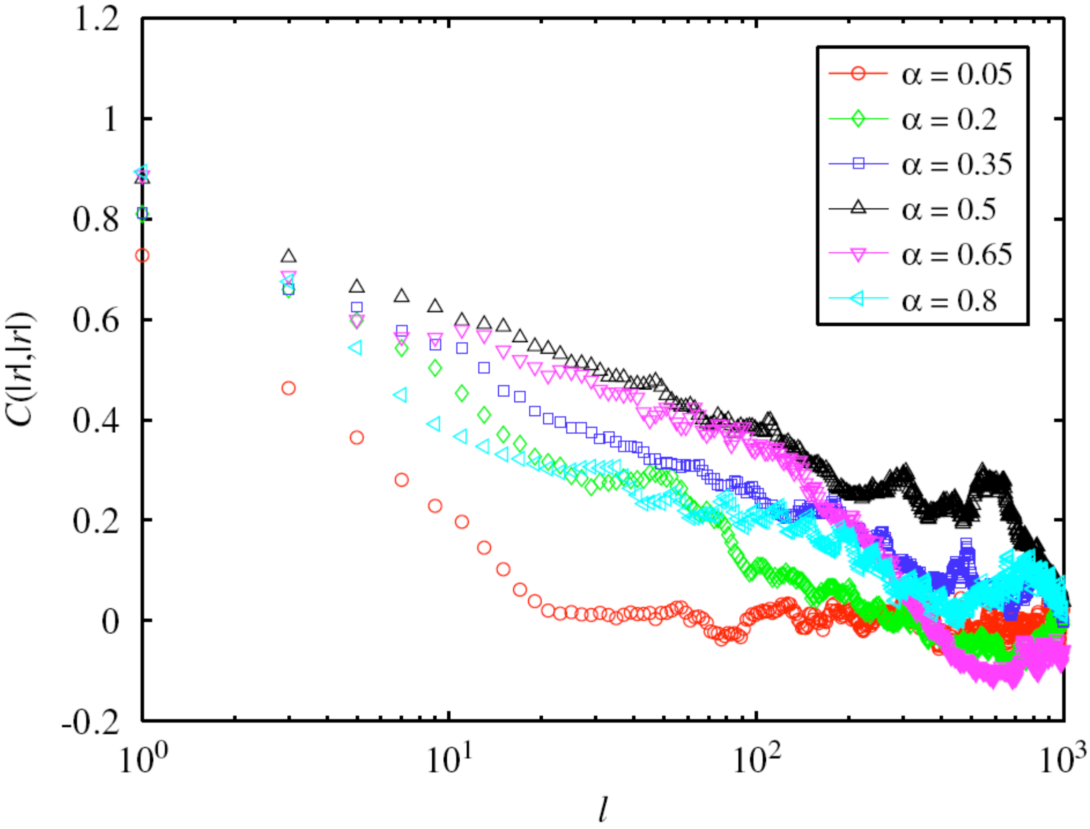


Fig. 8. The impact of α on the auto-correlation of the absolute values of the returns and of the returns.

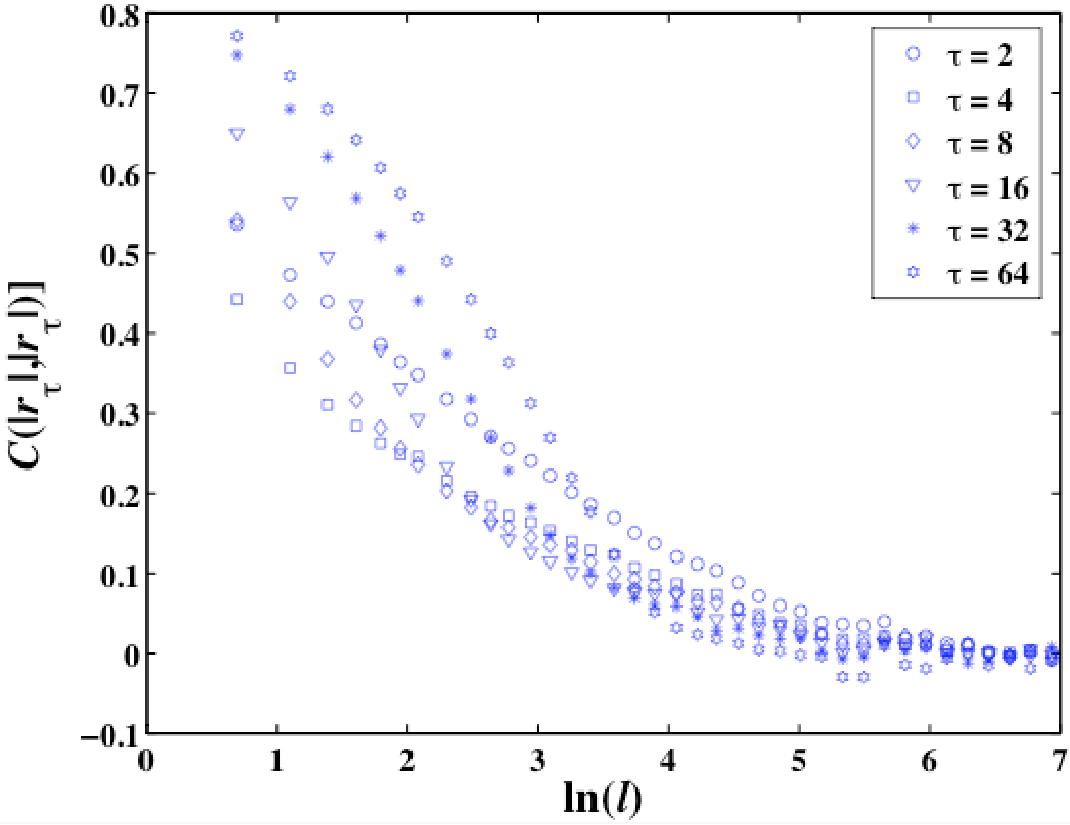


Fig. 9. Scaling of the autocorrelation functions of $|r_\tau(t)|$ for different time scales τ of the realization shown in Fig. 3.

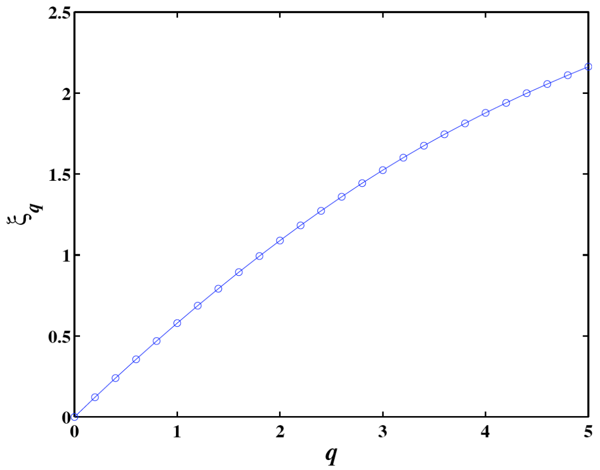


Fig. 11. Dependence of the scaling exponents ξ_q defined in (14) as a function of the order q of the structure functions $M_q(\tau) \sim \tau^{\xi_q}$. The concavity of ξ_q as a function q is the hallmark of multifractality.

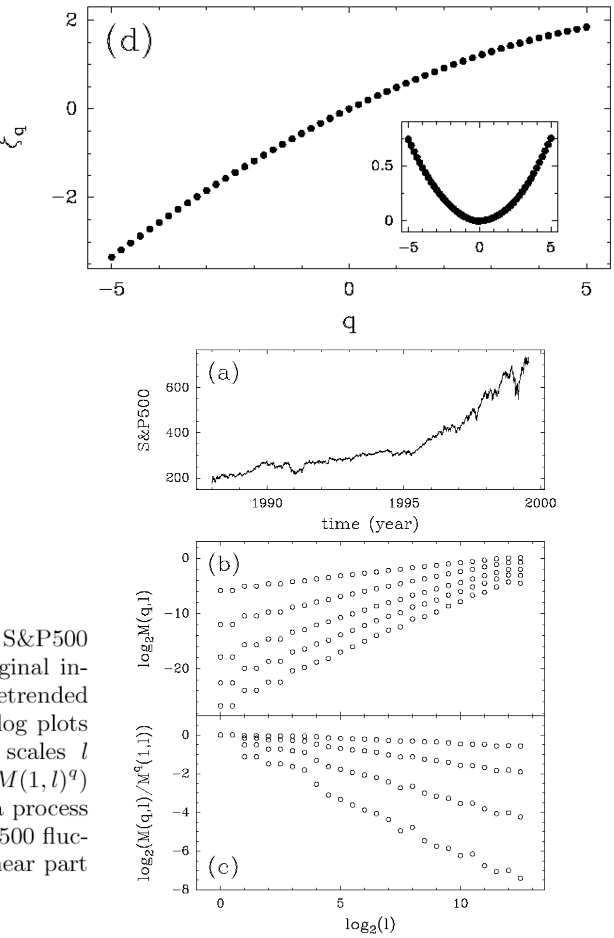
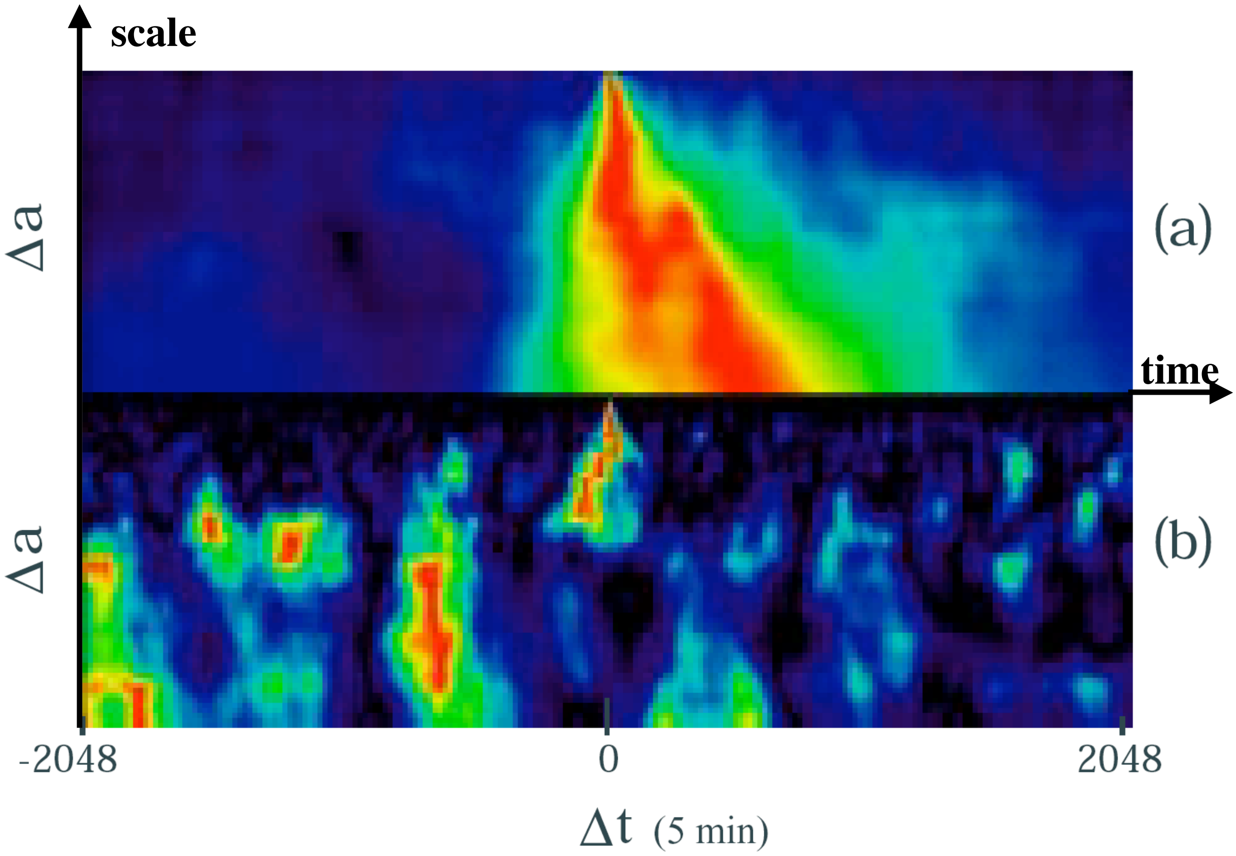


Fig. 1. Multifractal analysis of the intraday future S&P500 index over the period 1988–1999. (a) Plot of the original index time-series. The analyzed time-series is the detrended and de-seasonalized logarithm of this series. (b) Log-log plots of $M(q,l)$ *versus* l for $q = 1,2,3,4,5$. The time scales l range from 10 minutes to 1 year. (c) $\log_2(M(q,l)/M(1,l)^q)$ for $q = 2,3,4,5$. Such plots should be horizontal for a process that is not multifractal. (d) ζ_q spectrum for the S&P 500 fluctuations. The plot in the inset is the parabolic nonlinear part of ζ_q.

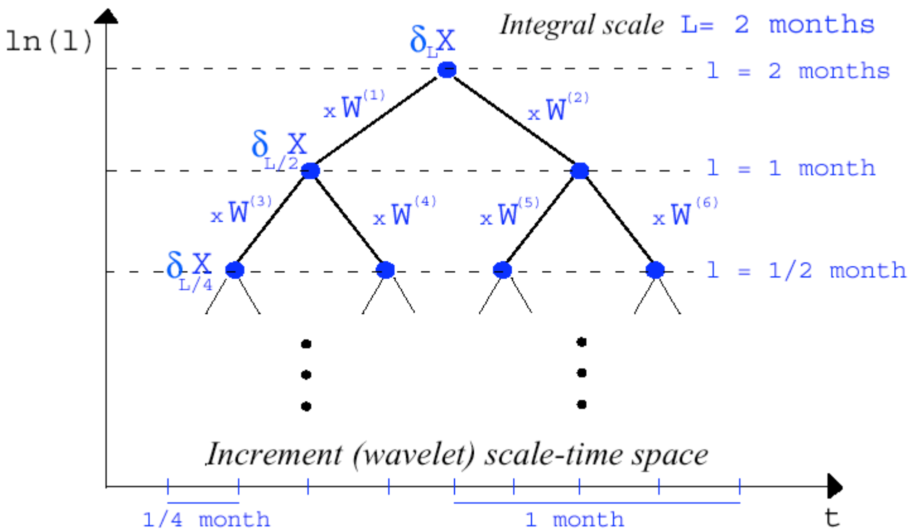


A. Arneodo, J.-F. Muzy and D. Sornette, Direct causal cascade in the stock market, European Physical Journal B 2, 277-282 (1998)

The multiplicative cascade paradigm

$$\boxed{\delta_{\lambda l} X(\lambda t) = \lambda^H \delta_l X(t) = W_\lambda \delta_l X(t)}$$

- $\mathcal{W}$-cascades (wavelet cascade)



The Multifractal Random Walk (MRW) model

$$\boxed{r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)}}$$

$$\mu_{\Delta t} = \frac{1}{2}\ln(\sigma^2 \Delta t) - C_{\Delta t}(0)$$

$$C_{\Delta t}(\tau) = \mathrm{Cov}[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln\left(\frac{T}{|\tau| + e^{-3/2}\Delta t}\right)$$

$$\boxed{\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^{t} d\tau\ \eta(\tau)\ K_{\Delta t}(t-\tau)}$$

$\omega_{\Delta t}(t)$ is Gaussian with mean $\mu_{\Delta t}$ and variance $V_{\Delta t} = \int_0^\infty d\tau\ K_{\Delta t}^2(\tau) = \lambda^2 \ln\left(\frac{Te^{3/2}}{\Delta t}\right)$

$$C_{\Delta t}(\tau) = \int_0^\infty dt\ K_{\Delta t}(t) K_{\Delta t}(t + |\tau|)$$

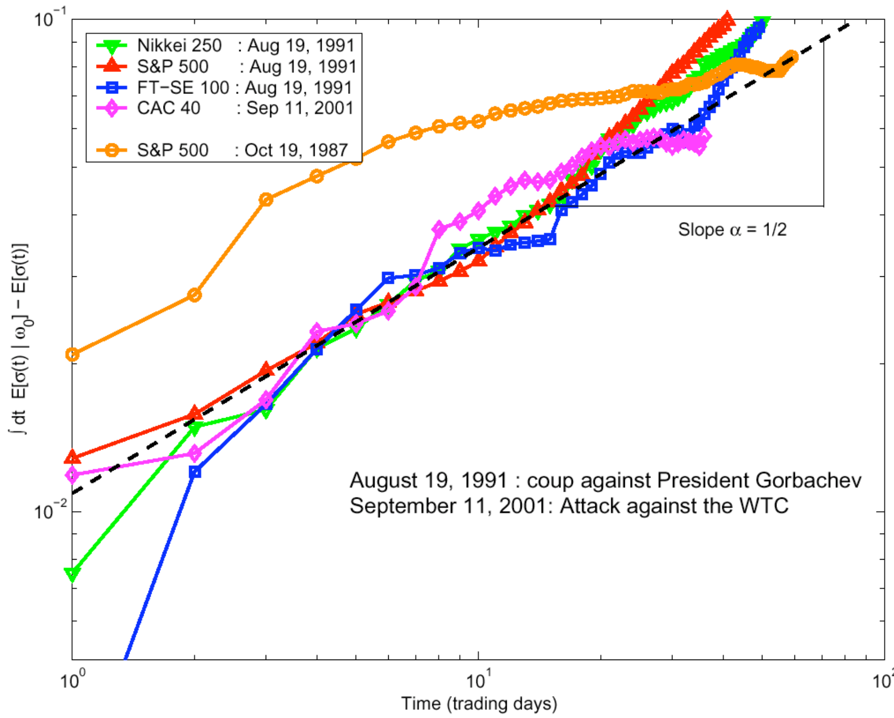
$$\hat{K}_{\Delta t}(f)^2 = \hat{C}_{\Delta t}(f) = 2\lambda^2\ f^{-1}\left[\int_0^{Tf} \frac{\sin(t)}{t} dt + O\left(f\Delta t \ln(f\Delta t)\right)\right]$$

$$\boxed{K_{\Delta t}(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for } \Delta t << \tau << T}$$

D. Sornette, Y. Malevergne and J.F. Muzy, Volatility fingerprints of large shocks: Endogeneous versus exogeneous, Risk 16 (2), 67-71 (2003)((http://arXiv.org/abs/cond-mat/0204626)

Linear response to an external shock

$$\mathrm{E}_{\mathrm{exo}}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



D. Sornette, Y. Malevergne and J.F. Muzy
Volatility fingerprints of large shocks: Endogeneous versus exogeneous,
Risk Magazine
(http://arXiv.org/abs/cond-mat/0204626)

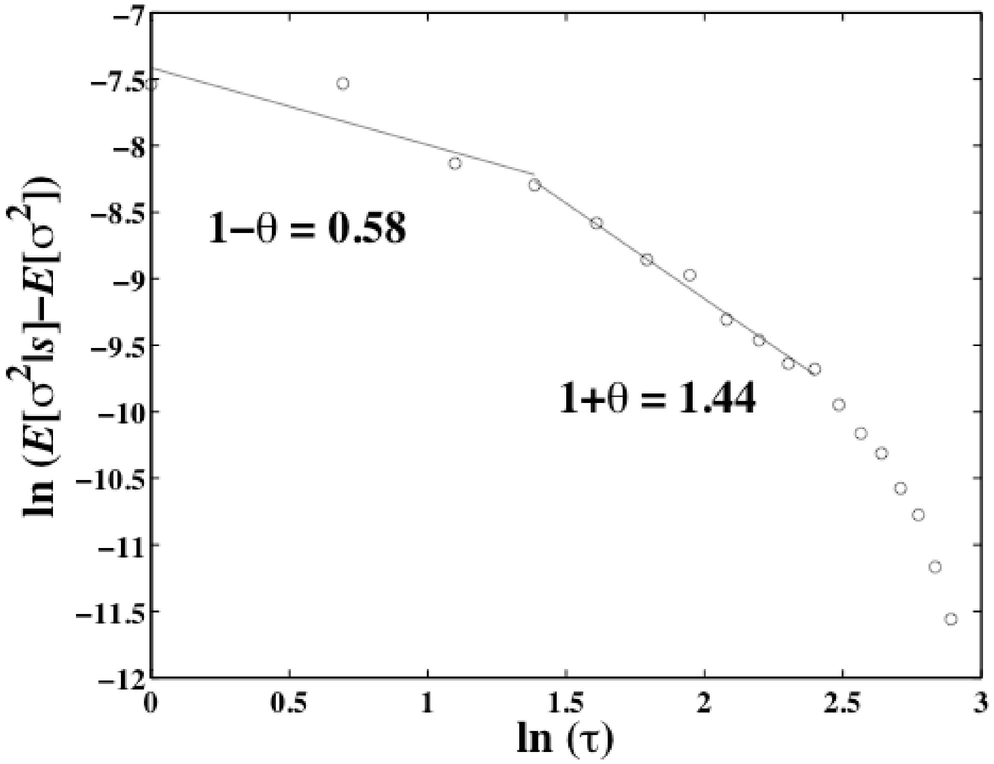


Fig. 14. Relaxation of superposed excess volatility after exogenous shocks obtained by imposing a very large news $G(t_s)$ for $\Delta t = 1$.

"Conditional response" to an endogeneous shock

$$\begin{aligned} \mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] &= \overline{\sigma^2(t)} \exp\left[2(\omega_0 - \mu) \cdot \frac{C(t)}{C(0)} - 2\frac{C^2(t)}{C(0)}\right] \\ &= \overline{\sigma^2(t)} \left(\frac{T}{t}\right)^{\alpha(s)+\beta(t)} \end{aligned}$$

Interplay between
-long memory
-exponential

where

$$\begin{aligned} \alpha(s) &= \frac{2s}{\ln\left(\frac{Te^{3/2}}{\Delta t}\right)}, \\ \beta(t) &= 2\lambda^2 \frac{\ln(t/\Delta t)}{\ln(Te^{3/2}/\Delta t)} \end{aligned}$$

Within the range $\Delta t < t << \Delta t e^{\frac{|s|}{\lambda^2}}$, $\beta(t) << \alpha(s)$

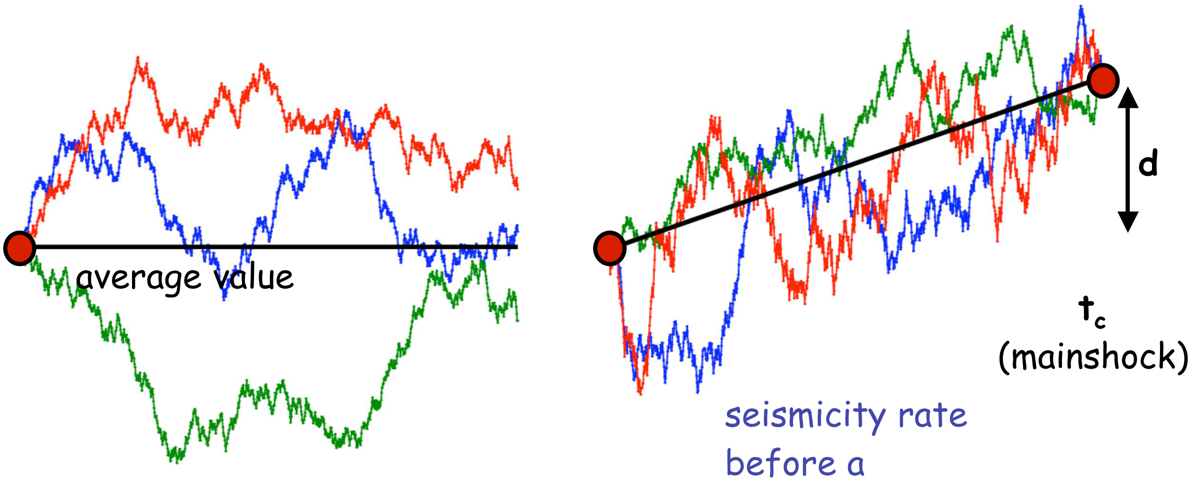
$$\boxed{\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}}$$

Inverse Omori law, conditional foreshock

Analogy Brownian motions / seismicity rate in the ETAS model

without conditioning: stationary process, average=0

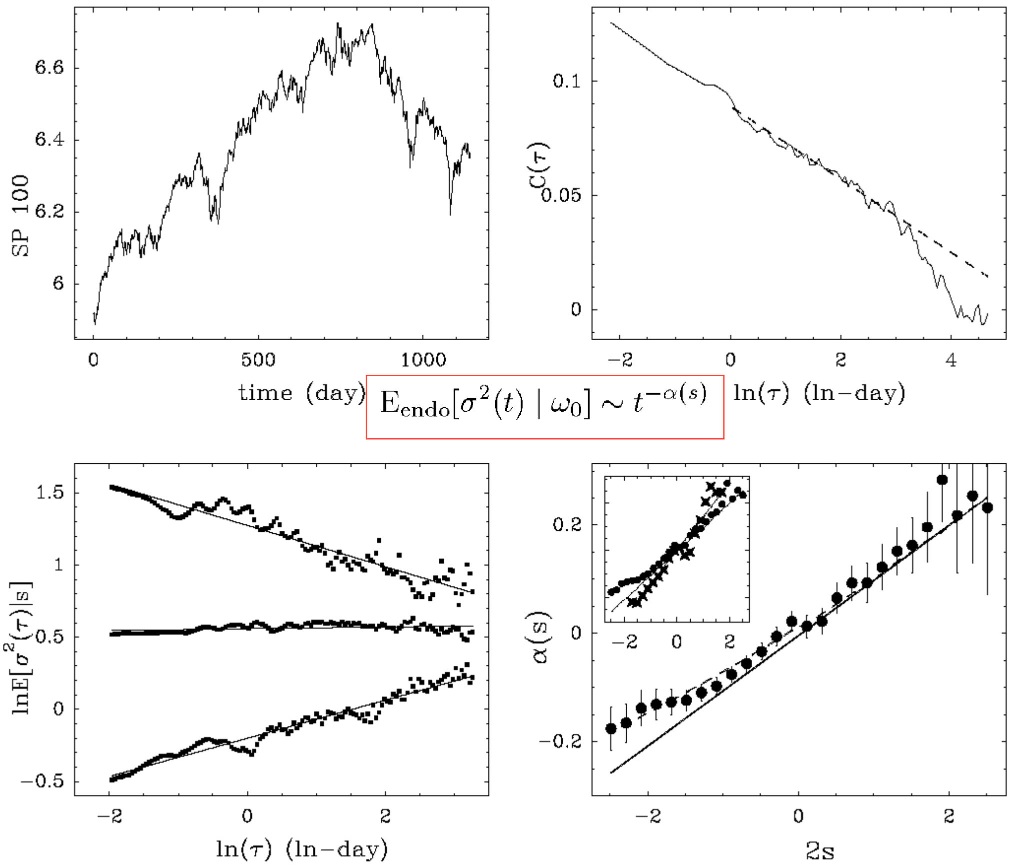
conditioning to a large value **W(t_c)=d** : non-stationary process, average # 0



stationary seismicity rate

seismicity rate before a mainshock, inverse Omori law

Real Data and Multifractal Random Walk model



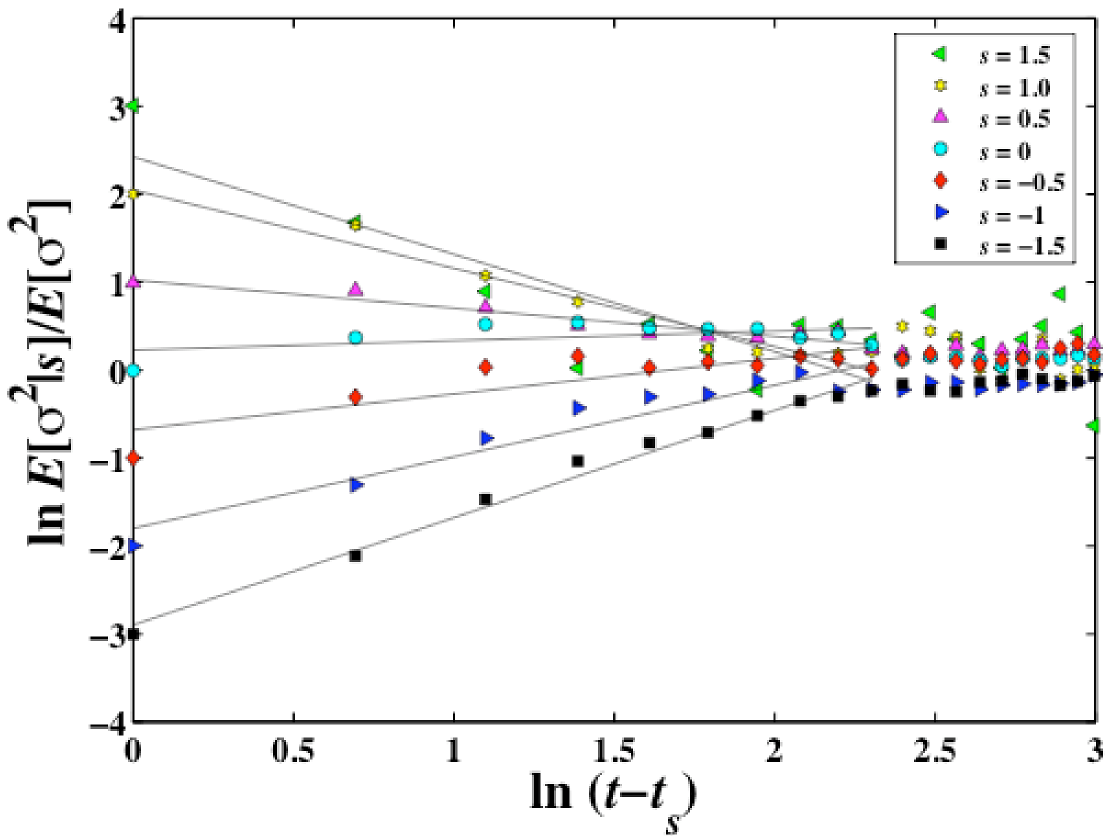


Fig. 12. Average normalized conditional volatility $\sigma^2_{\Delta t}(t)/E[\sigma^2]$ as a function of the time $t - t_s$ from the local burst of volatility at time t_s for different log-amplitudes s in double logarithmic coordinates.

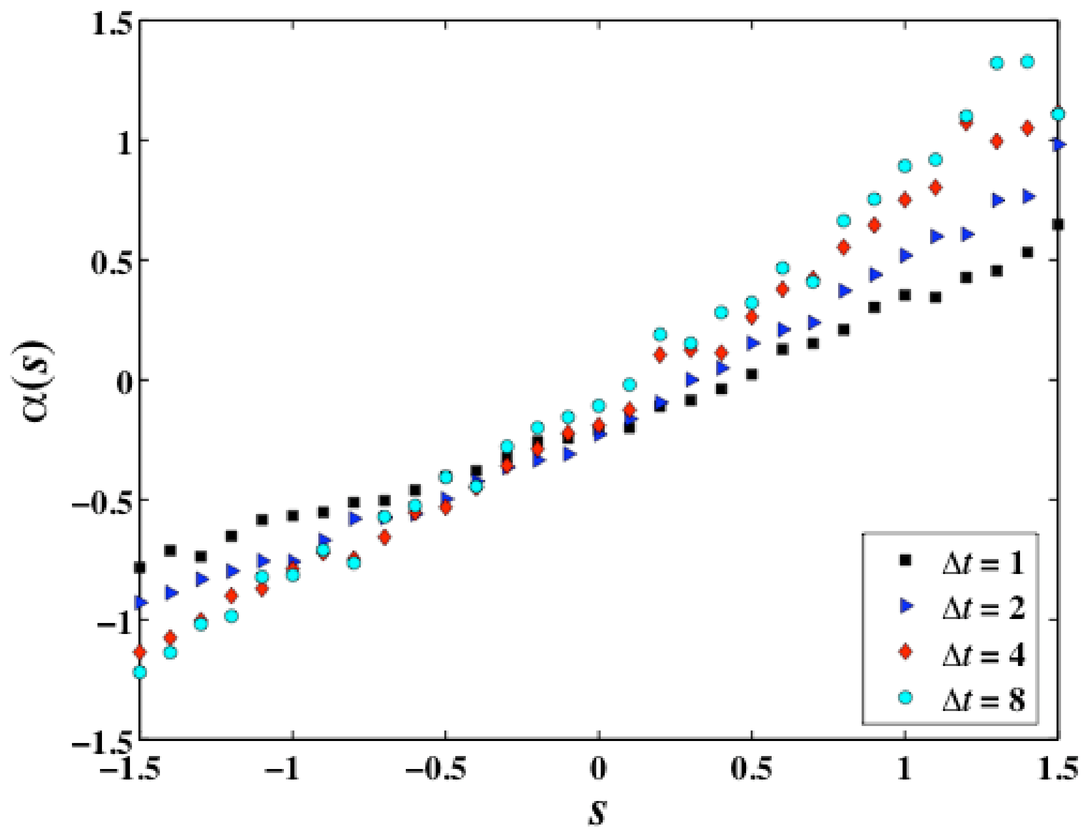
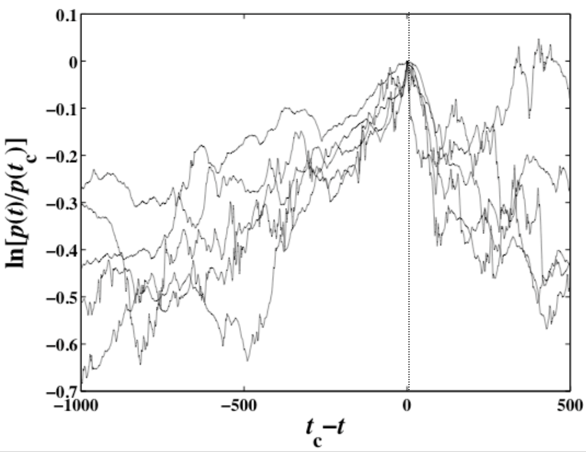


Fig. 13. Exponent $\alpha(s)$ of the conditional volatility response as a function of the endogenous shock amplitude S for $\Delta t = 1, 2, 4$, and 8.

Bubbles and crashes

Fig. 15. Five price trajectories showing bubbles preceding crashes that occur at the shifted time 0. The five time series have been translated so that the time of their crash is placed at the origin $t = 0$.



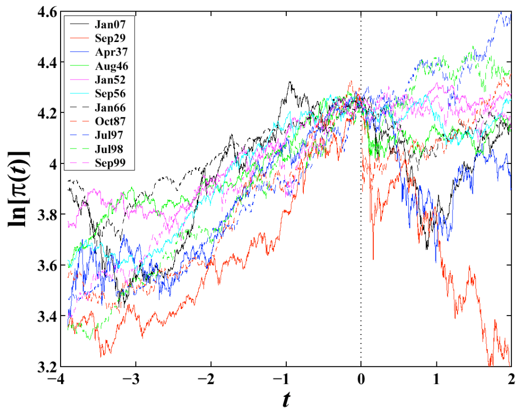


Figure 4: (Color online) Superposed epoch analysis of the 11 time intervals, each of 6 years long, of the DJIA index centered on the time of the maxima of the 11 predictor peaks above $AI = 0.3$ of the alarm index shown in Fig. 3.

D. Sornette and W.-X. Zhou
Predictability of Large Future Changes in major financial indices,
International Journal of Forecasting 22, 153-168 (2006)

All stylized facts are reproduced when

•The system operates close to the Ising critical point (large susceptibility and anomalous volatility: Shiller's paradox)

•Agents over-interpret or mis-attribute the origin of price changes

No feedback of the price on the decision making process

INFORMATION: normal people's high level of general intelligence makes them too smart for their own good.

In 1909, a broker using the pseudonym Don Guyon wrote a small book called One-Way Pockets. He was utterly mystified as to why, after a full cycle of rise and fall after which stocks were valued just where they were at the start, all his clients lost money. His answer, in a nutshell, is herding. His clients felt fearful at the start of bull markets and so traded in and out constantly. At the market's peak, they felt confidently bullish and held much more stock "for the long run,"

Rats beat humans:

The rats and the humans had to look at a TV screen and press the lever anytime a dot appeared in the top half of the screen. The experimenters did not tell the human subjects that's' what they were supposed to do; they had to figure it out for themselves the same way the rats did. The experiment was set up so that 70% of the time the dot was in the top of the screen. Since there was no punishment for a wrong response, the smartest strategy was just to push the bar 100% of the time. That way, you get the reward 70% of the time, even though you have not clue of what is the pattern.

That's what the rats did.

But the humans never figured this out!
They kept trying to come up with a rule, so sometimes they pressed the bar and sometimes they would not, trying to figure it out. Some of them thought they had come up with a rule. But they were of course deluded and their performance was much less than the rats.

People makes STORIES! Normal people have an "interpreter" in their left brain that takes all the random, contradictory details of whatever they are doing or remembering at the moment, and smoothes everything in one coherent story. If there are details that do not fit, they are edited out or revised!

Temple Grandin and C. Johnson, Animals in translation (Scribner, New York, 2005)