Financial, Real-Estate Bubble, Derivative Bubbles

Finite-Singularity Models

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"The budget should be balanced, the Treasury should be refilled, public debt should be reduced, the arrogance of officialdom should be tempered and controlled, and the assistance to foreign lands should be curtailed lest Rome become bankrupt. People must again learn to work instead of living on public assistance."

Cicero - 55 BC

long-term Collaborators:

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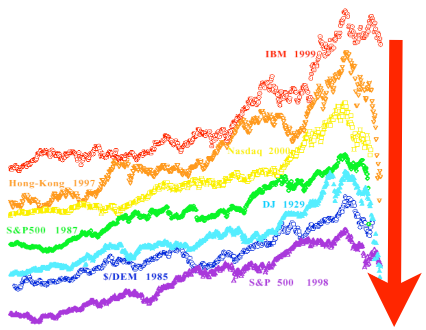
Y. Malevergne (Univ. Lyon, France)
V: Pïsarenko (Acad. Sci. Moscow, Russia)
W.-X. Zhou (ECUST, Shanghai)



Collaborators at ETH Zurich:

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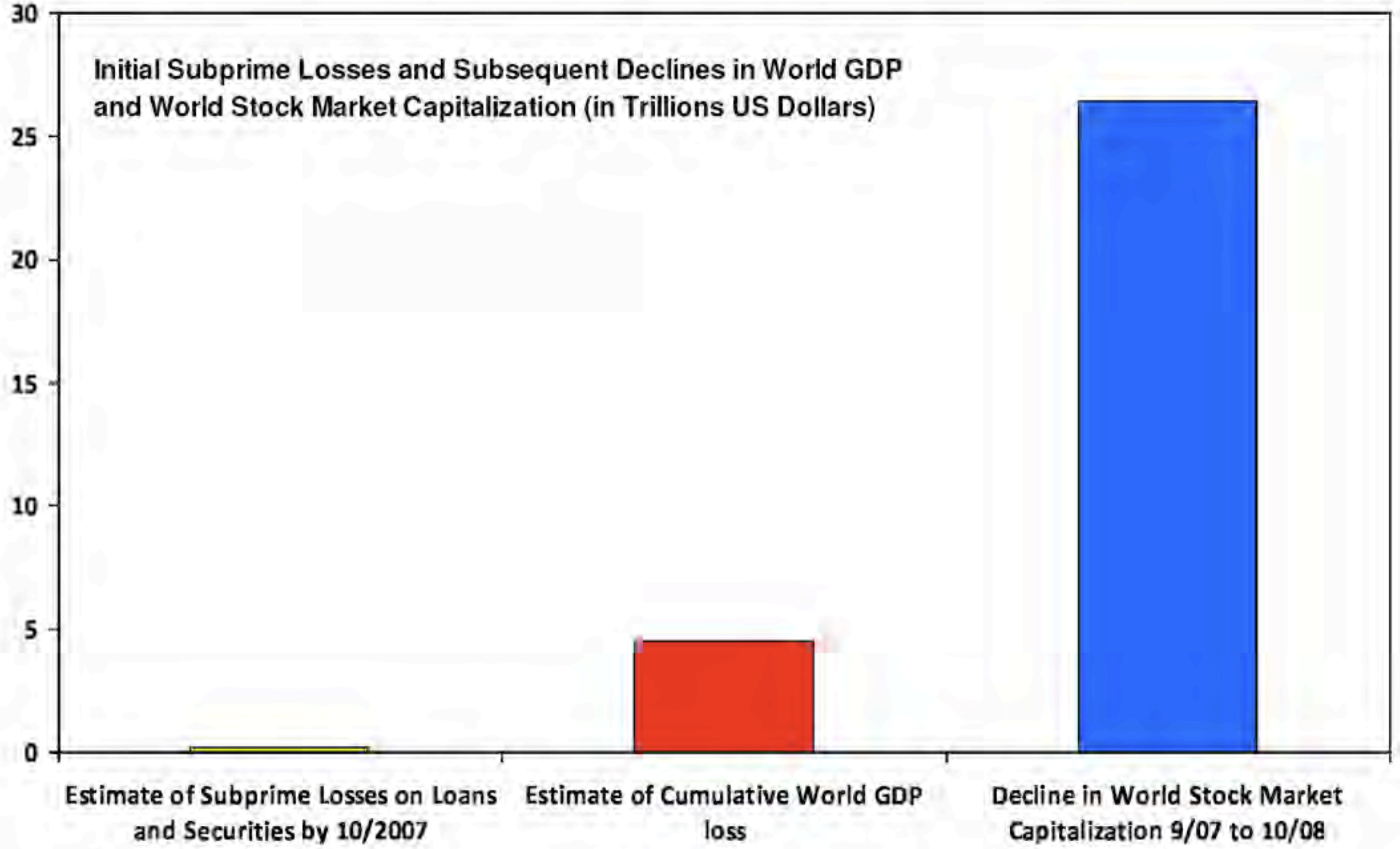
and T. Kaizoji (Tokyo)

- The financial instability hypothesis
- What are financial bubbles?
- Different models (social interactions, herding, news, value vs noise trading...)
- Finite-time singular (FTS) models
- Hypothesis 1: bubbles can be diagnosed in real time
- Hypothesis 2: the termination of bubbles can be determined probabilistically in advance
- The Financial Crisis Observatory at ETH Zurich and the Financial Bubble Experiment

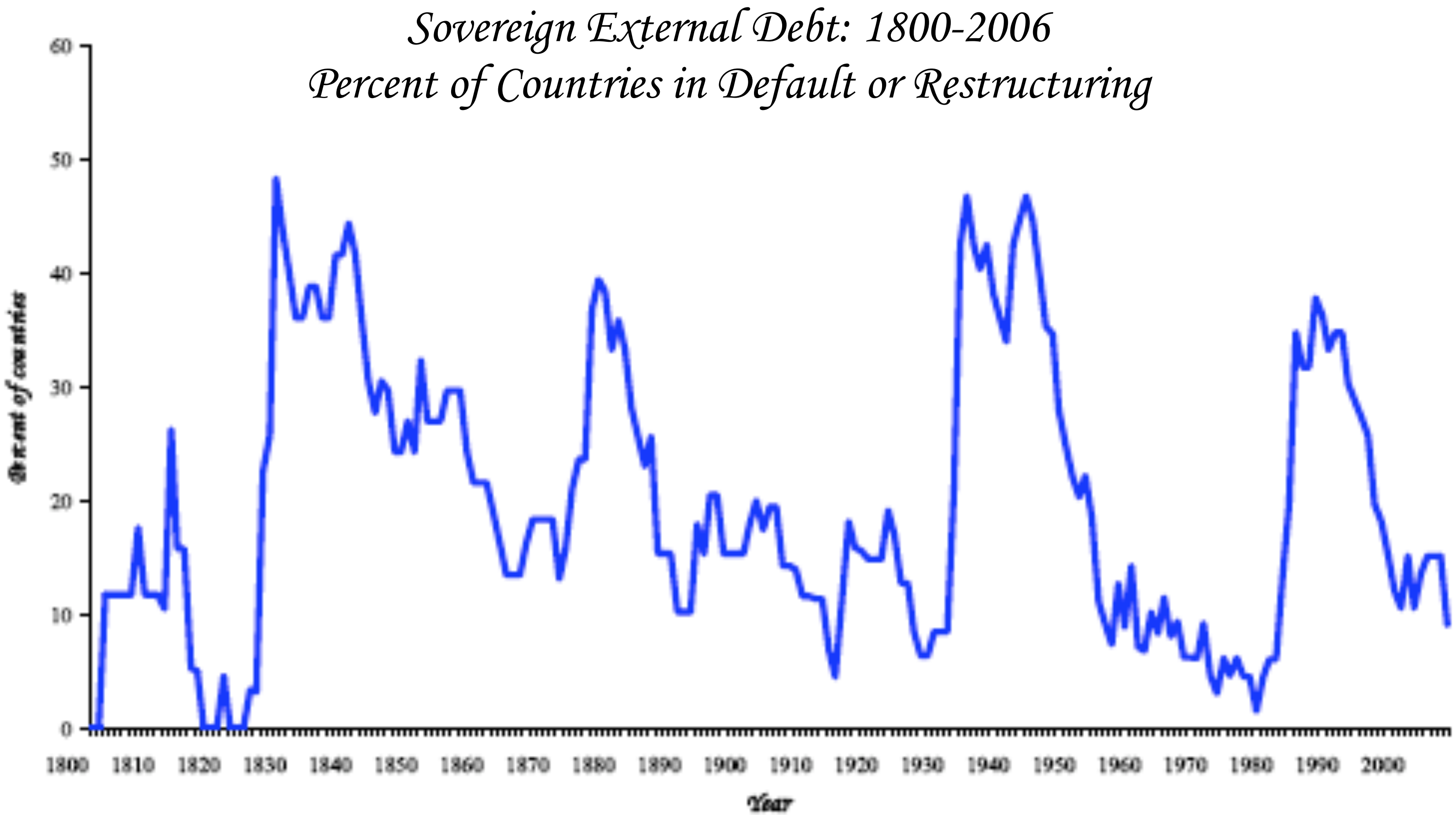
The Paradox of the 2007-20XX Crisis

(trillions of US$)

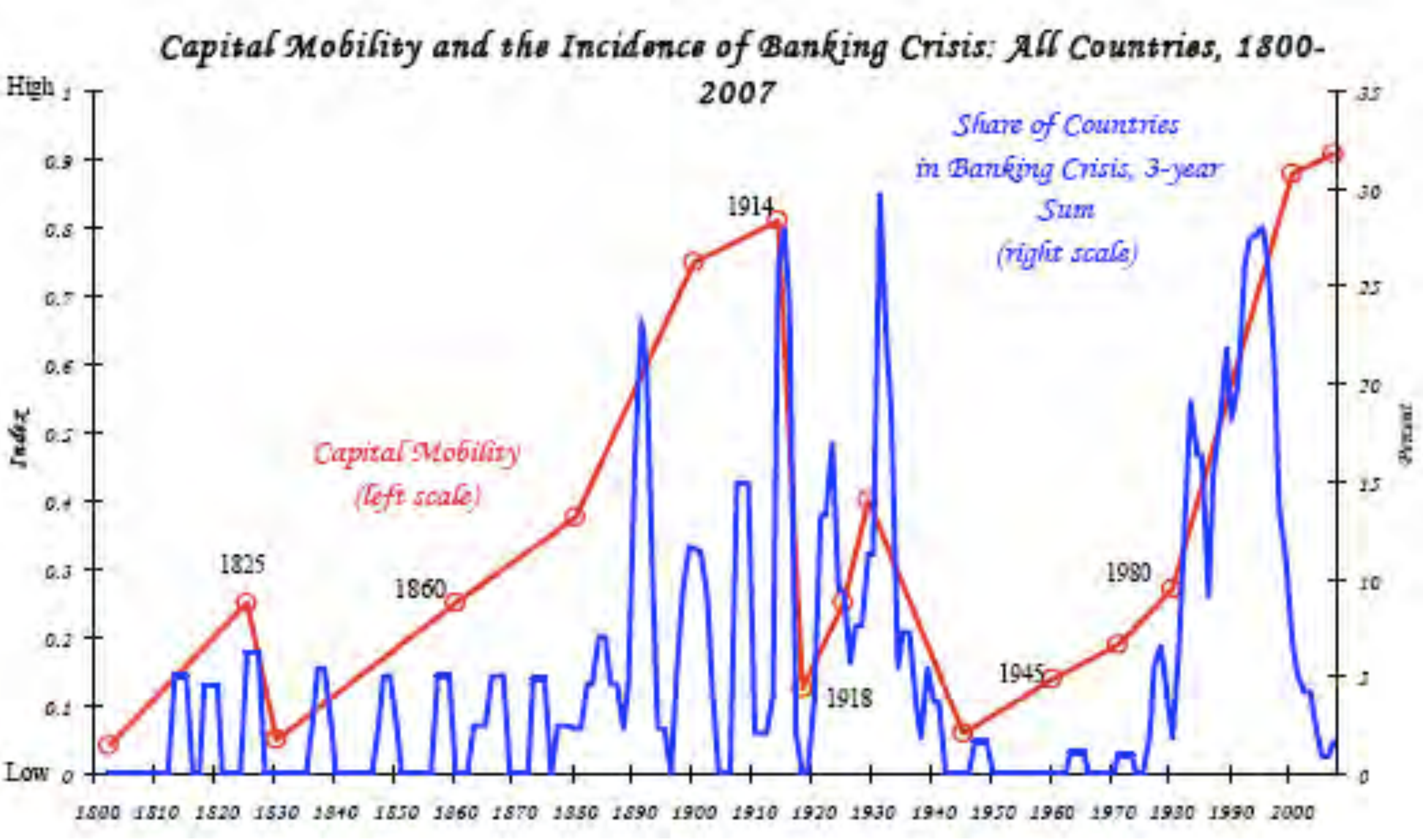


Source: IMF Global Financial Stability Report; World Economic Outlook November update and estimates; World Federation of Exchanges.

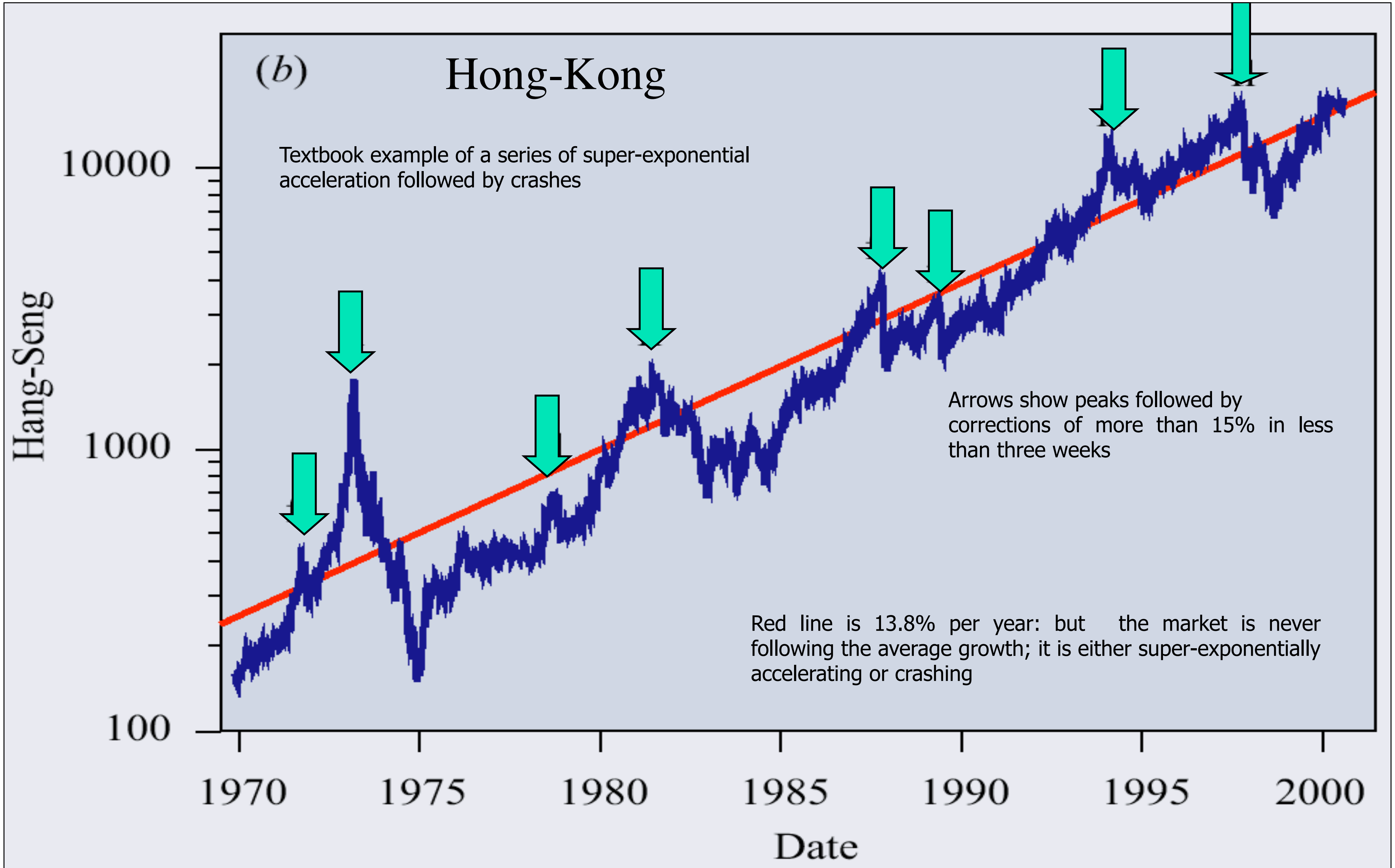
Crises frequently emanate from the financial centers with transmission through interest rate shocks and commodity price collapses. Thus, the recent US sub-prime financial crisis is hardly unique.



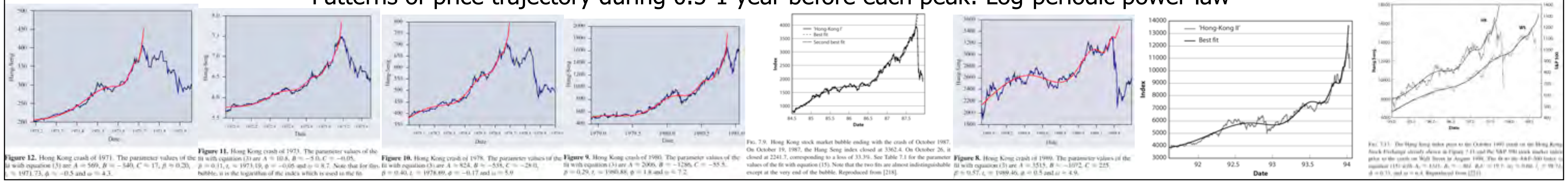
This Time is Different: A Panoramic View of Eight Centuries of Financial Crises
Carmen M. Reinhart and Kenneth S. Rogoff, NBER Working Paper No. 13882, March 2008



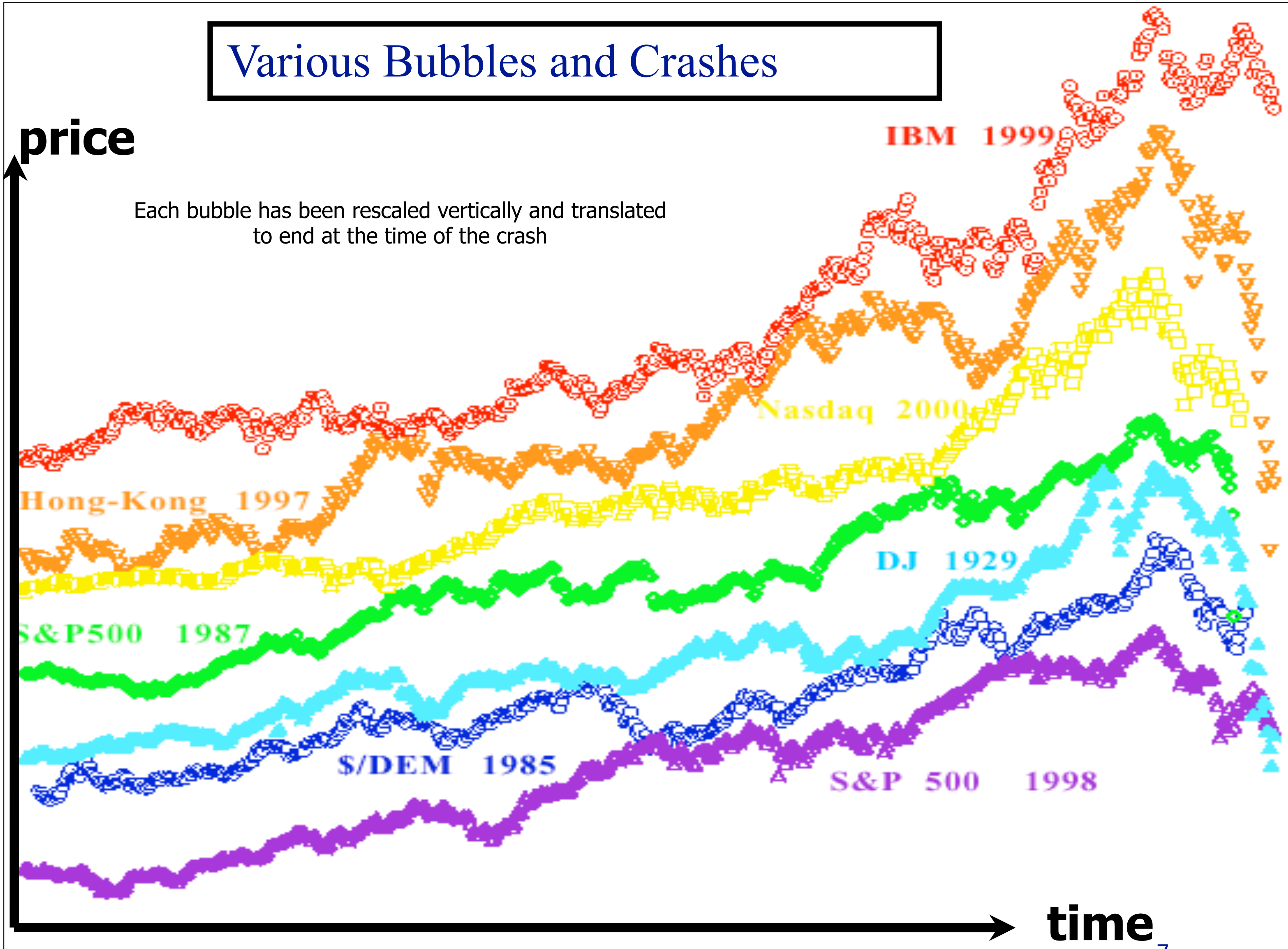
Sources: Bordo et al. (2001), Caprio et al. (2005), Kaminsky and Reinhart (1999), Obstfeld and Taylor (2004), and Carmen M. Reinhart and Kenneth S. Rogoff,



Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law



Various Bubbles and Crashes



Financial Instability Hypothesis
(Minsky, 1974)

"A fundamental characteristic of our economy is that the financial system swings between robustness and fragility and these swings are an integral part of the process that generates business cycles."

- Hedge-finance: in-flow - out-flow positive over all periods
- Speculative finance: in-flow - out-flow negative near term and expected to turn positive long-term
- Ponzi finance: in-flow - out-flow negative until the very last period at which a big gain compensates for all the previous losses.

A 15y History of the 2008- crisis

- The ITC “new economy” bubble (1995-2000)
- Slaving of the Fed monetary policy to the stock market descent (2000-2003)
- Real-estate bubbles (2003-2006)
- MBS, CDOs bubble (2004-2007) and stock market bubble (2004-2007)
- Commodities and Oil bubbles (2006-2008) Consequences (deep loss of trust, systemic instability)
- Solution?

What is a bubble?

Academic Literature: No consensus on what is a bubble...

Ex: Refet S. Gürkaynak, Econometric Tests of Asset Price Bubbles: Taking Stock.

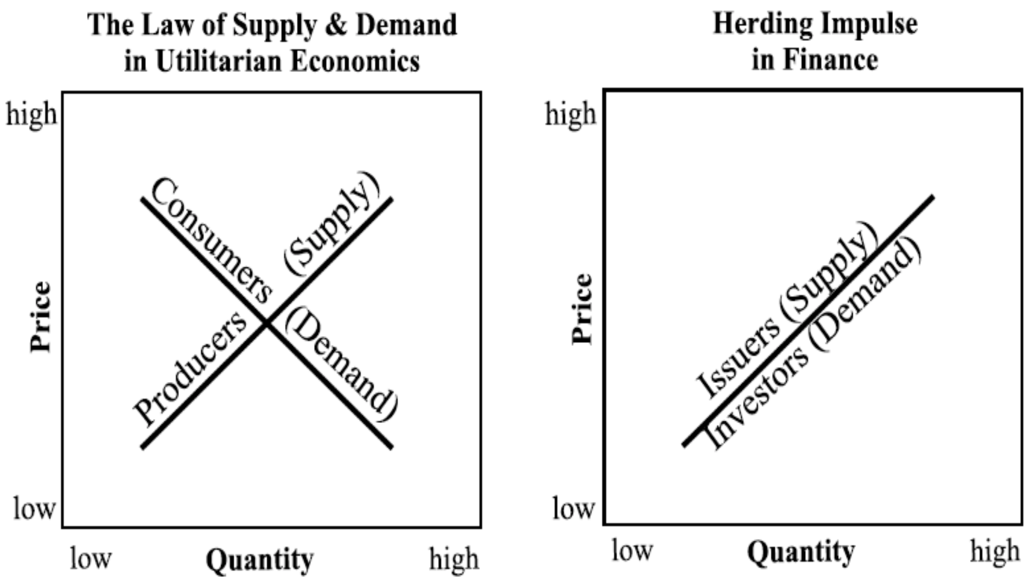
Can asset price bubbles be detected? This survey of econometric tests of asset price bubbles shows that, despite recent advances, econometric detection of asset price bubbles cannot be achieved with a satisfactory degree of certainty. For each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble. We are still unable to distinguish bubbles from time-varying or regime-switching fundamentals, while many small sample econometrics problems of bubble tests remain unresolved.

Professional Literature: we do not know... only after the crash

The Fed: A. Greenspan (Aug., 30, 2002):

"We, at the Federal Reserve…recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence… Moreover, it was far from obvious that bubbles, even if identified early, could be preempted short of the Central Bank inducing a substantial contraction in economic activity, the very outcome we would be seeking to avoid."

What is a bubble?



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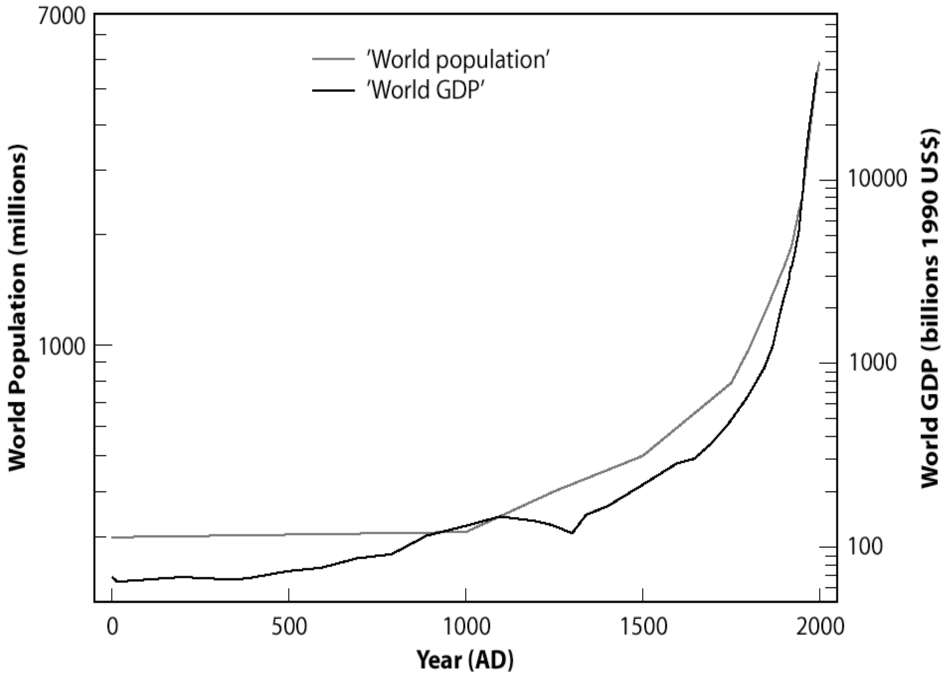
Positive feedbacks

$$\boxed{\frac{dp}{dt} = cp^d}$$

$$p(t) = \left(\frac{c}{m}\right)^{-m} (t_c - t)^{-m}$$

$$m = 1/(d-1) > 0 \text{ and } t_c = t_0 + m p_0^{1-d}/c.$$



Our proposition:

Faster than exponential

transient unsustainable

growth of price

Finite-time Singularity

- Planet formation in solar system by run-away accretion of planetesimals
- PDE's: Euler equations of inviscid fluids and relationship with turbulence
- PDE's of General Relativity coupled to a mass field leading to the formation of black holes
- Zakharov-equation of beam-driven Langmuir turbulence in plasma
- rupture and material failure
- Earthquakes (ex: slip-velocity Ruina-Dieterich friction law and accelerating creep)
- Models of micro-organisms chemotaxis, aggregating to form fruiting bodies
- Surface instability spikes (Mullins-Sekerka), jets from a singular surface, fluid drop snap-off
- Euler's disk (rotating coin)
- Stock market crashes...

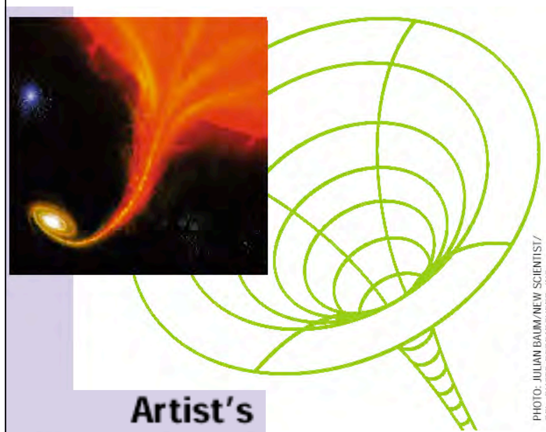


PHOTO: JULIAN BAUM/NEW SCIENTIST/ SPL, PHOTO RESEARCHERS, INC.

Artist's illustration of matter from a red giant star being pulled toward a black hole.

Mechanisms for positive feedbacks in the stock market

- Technical and rational mechanisms
  1. Option hedging
  2. Insurance portfolio strategies
  3. Trend following investment strategies
  4. Asymmetric information on hedging strategies

- Behavioral mechanisms:
  1. Breakdown of "psychological Galilean invariance"
  2. Imitation(many persons)
     a) It is rational to imitate
     b) It is the highest cognitive task to imitate
     c) We mostly learn by imitation
     d) The concept of "CONVENTION" (Orléan)

Thy Neighbor's Portfolio: Word-of-Mouth Effects in the Holdings and Trades of Money Managers

THE JOURNAL OF FINANCE • VOL. LX, NO. 6 • DECEMBER 2005

HARRISON HONG, JEFFREY D. KUBIK, and JEREMY C. STEIN*

A mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. This pattern shows up even when the fund manager and the stock in question are located far apart, so it is distinct from anything having to do with local preference. The evidence can be interpreted in terms of an epidemic model in which investors spread information about stocks to one another by word of mouth.

A fundamental observation about human society is that people who communicate regularly with one another think similarly. There is at any place and in any time a *Zeitgeist*, a spirit of the times.... Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations. (pp. 148, 155)

Shiller (2000)

Humans Appear Hardwired To Learn By 'Over-Imitation'

ScienceDaily (Dec. 6, 2007) — Children learn by imitating adults--so much so that they will rethink how an object works if they observe an adult taking unnecessary steps when using that object, according to a new Yale study.

Universal Bubble and Crash Scenario

Displacement

↓

Credit creation

↓

Euphoria

↓

Critical stage / Financial distress

↓

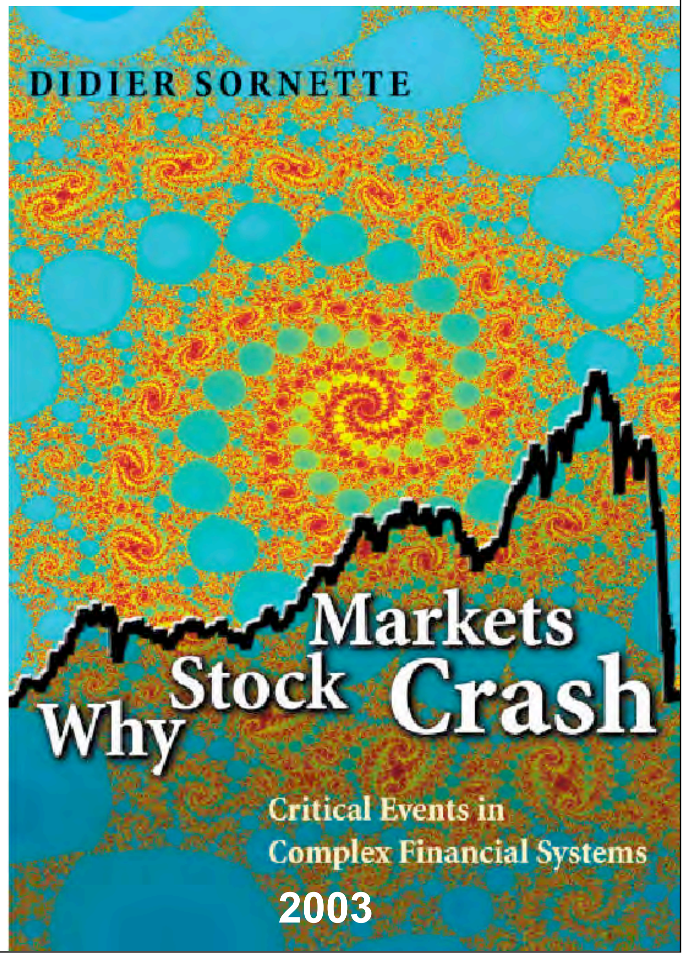
Revulsion

Charles Kindleberger, Manias, Panics and Crashes (1978)

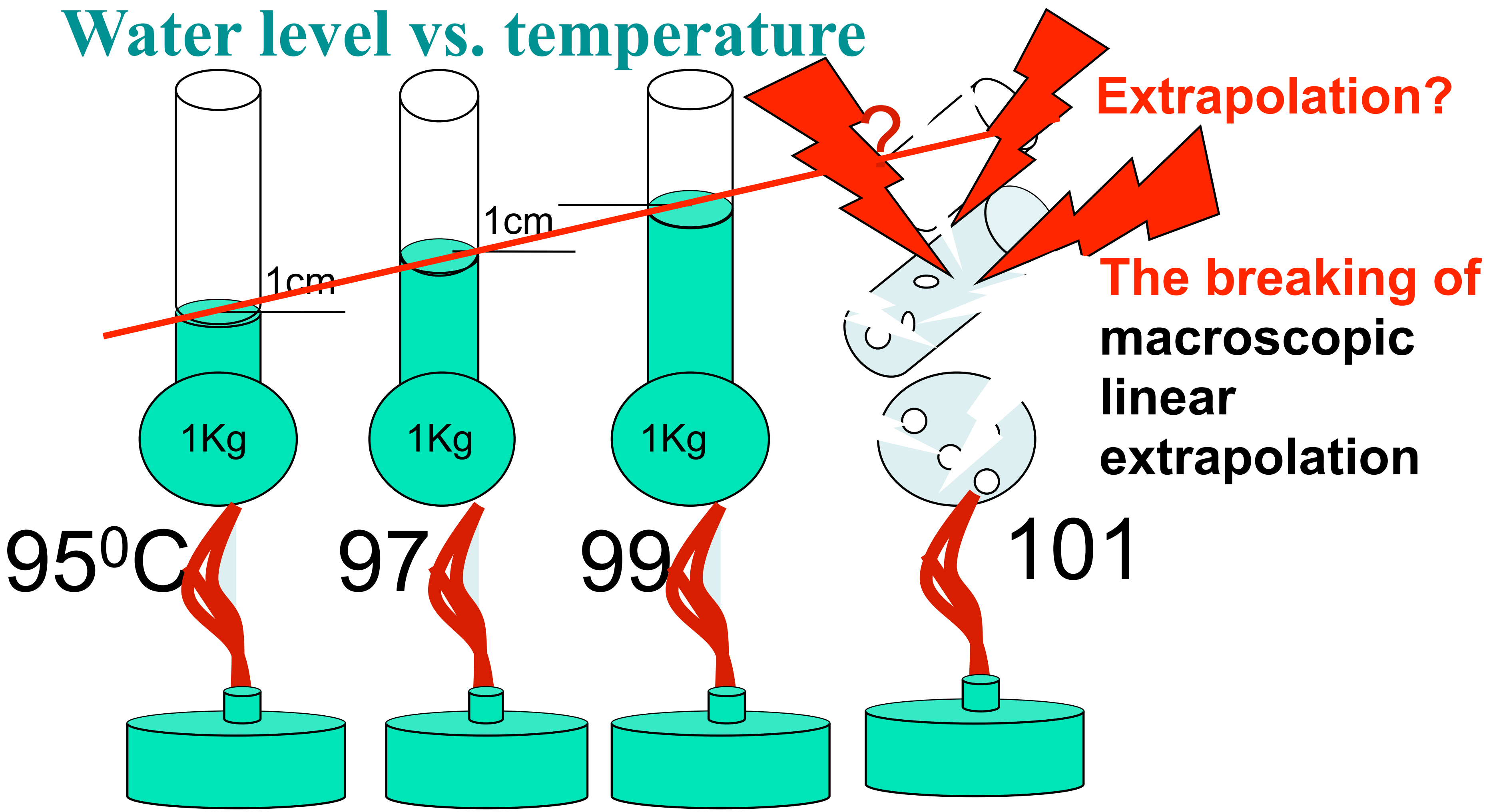
Didier Sornette, Why stock markets crash (2003)

Many bubbles and crashes

- Hong-Kong crashes: 1987, 1994, 1997 and many others
- October 1997 mini-crash
- August 1998
- Slow crash of spring 1962
- Latin-american crashes
- Asian market crashes
- Russian crashes
- Individual companies



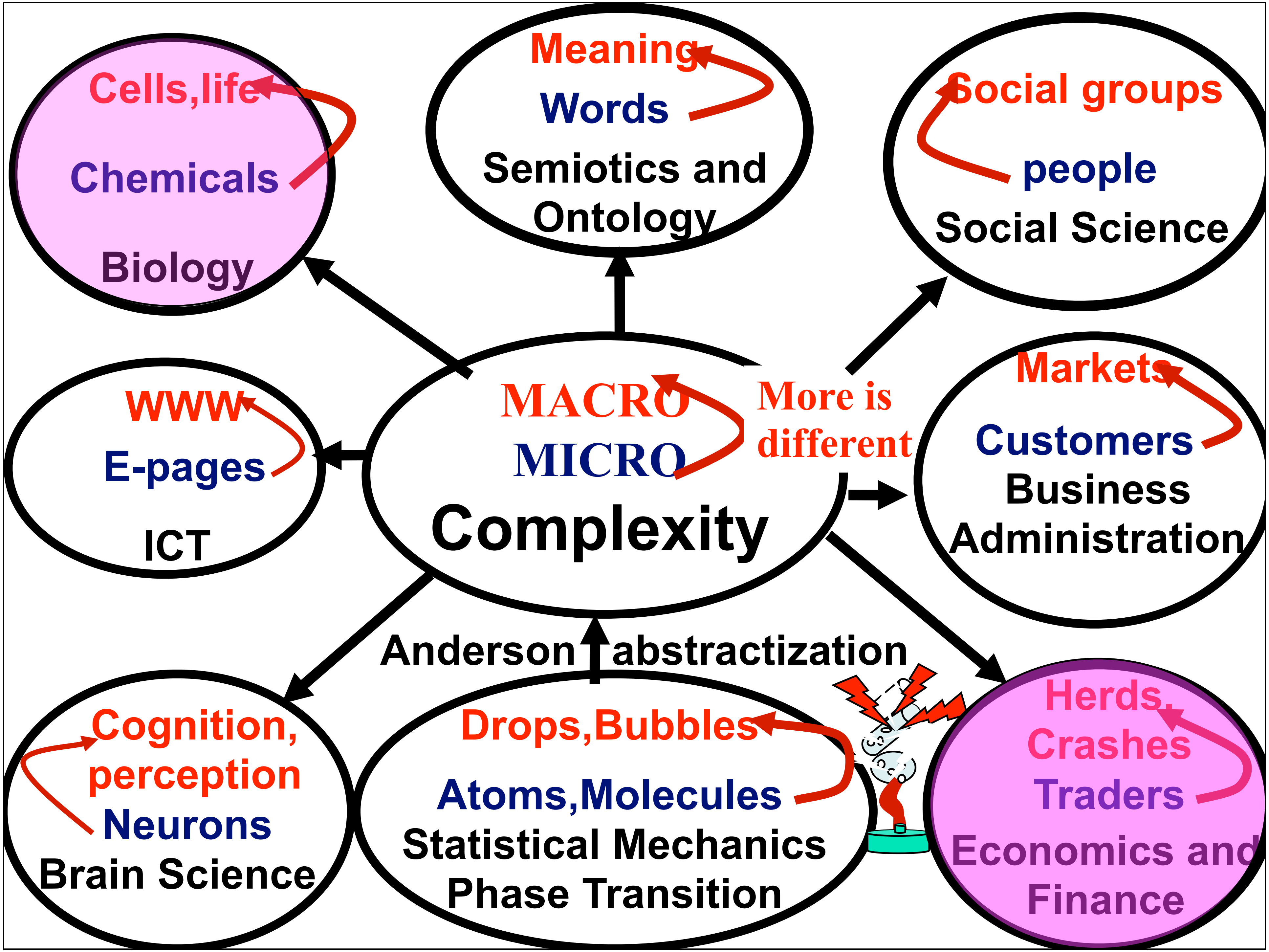
Simplest Example of a "More is Different" Transition



BOILING PHASE TRANSITION

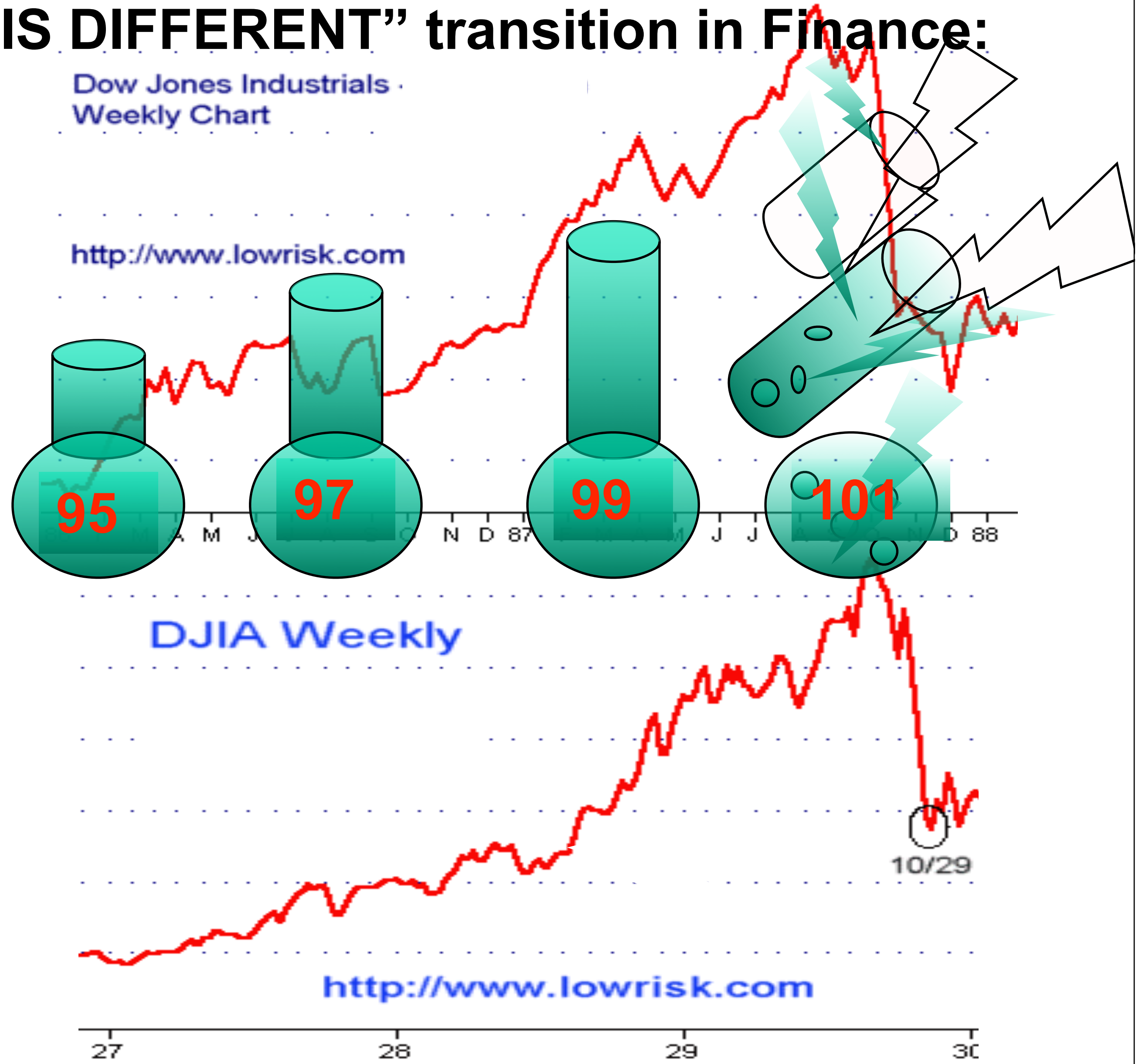
More is different: a single molecule does not boil at 100C⁰

(S. Solomon)


Cells,life
Chemicals
Biology
Meaning
Words
Semiotics and Ontology
Social groups
people
Social Science
WWW
E-pages
ICT
MACRO
MICRO
Complexity
More is different
Markets
Customers
Business Administration
Anderson abstractization
Cognition, perception
Neurons
Brain Science
Drops,Bubbles
Atoms,Molecules
Statistical Mechanics
Phase Transition
Herds, Crashes
Traders
Economics and Finance

Example of "MORE IS DIFFERENT" transition in Finance:

Instead of
Water Level:
-economic index
(Dow-Jones etc…)



Crash = result of collective behavior of individual traders

(S. Solomon)

Rational Expectation Bubbles and Crashes (Blanchard-Watson)

Martingale hypothesis ("no free lunch"):

$$\text{for all} \quad t' > t \qquad \mathrm{E}_t[p(t')] = p(t)$$

If crashes are depletions of bubbles:

$$dp = \mu(t)\, p(t)\, dt - \kappa[p(t) - p_1]dj$$

Martingale gives

$$\mu(t)p(t) = \kappa[p(t) - p_1]h(t) \ ,$$

i.e., if crash hazard rate $h(t)$ increases, so must the return (bounded rationality)

A. Johansen, D. Sornette and O. Ledoit
Predicting Financial Crashes using discrete scale invariance, Journal of Risk, vol. 1, number 4, 5-32 (1999)

A. Johansen, O. Ledoit and D. Sornette, Crashes as critical points, International Journal of Theoretical and Applied Finance Vol. 3, No. 2 219-255 (2000)

Optimal strategy obtained under limited information

Equation showing optimal imitation solution of decision in absence of intrinsic information and in the presence of information coming from actions of connected "neighbors"

$$s_i(t+1) = \mathrm{sign}\left(K \sum_{j \in N_i} s_j + \varepsilon_i \right)$$

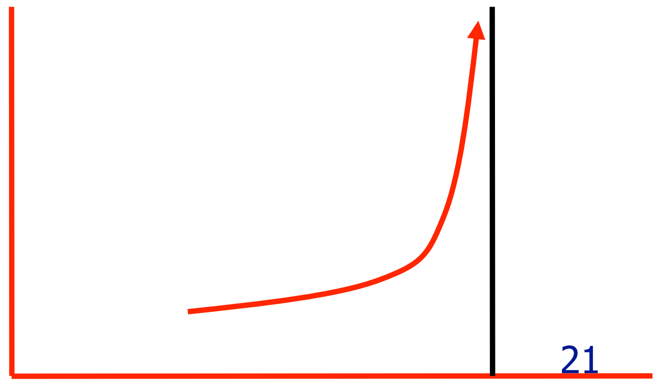
This equation gives rise to critical transition=bubbles and crashes

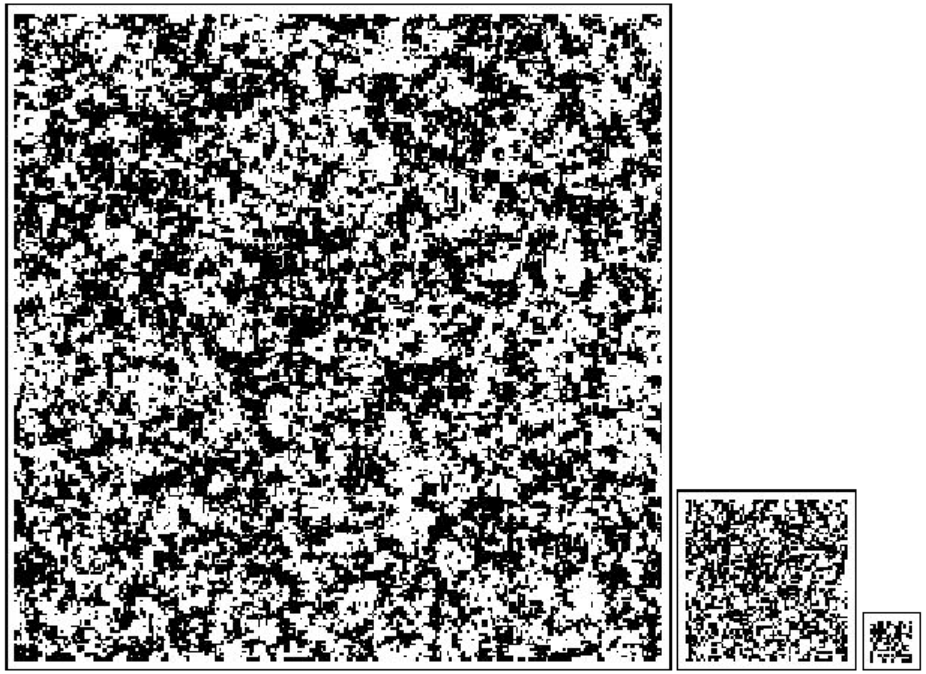
-Crash = coordinated sell-off of a large number of investors
-single cluster of connected investors to set the market off-balance
-Crash if 1) large cluster s>s*   and 2) active

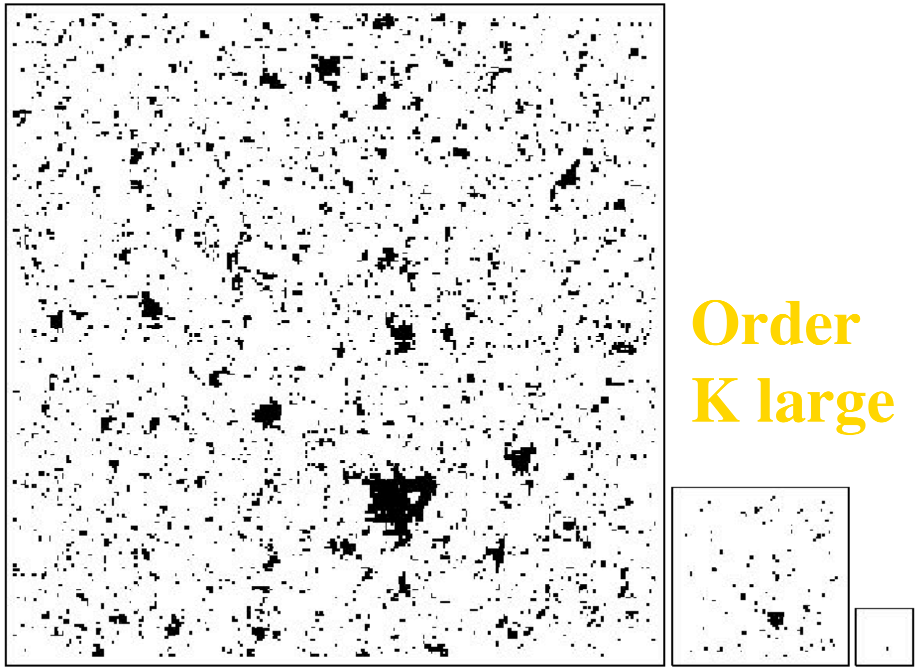
-Proba(crash) = n(s)
-Proba(active cluster) ~ s^a with $1 < a < 2$ (coupling between decisions)

Proba(crash) ~ $\sum_{s>s^*} n(s)\, s^a$

If a=2, $\sum_{s>s^*} n(s)\, s^2 \sim |K-K_c|^{-\gamma}$





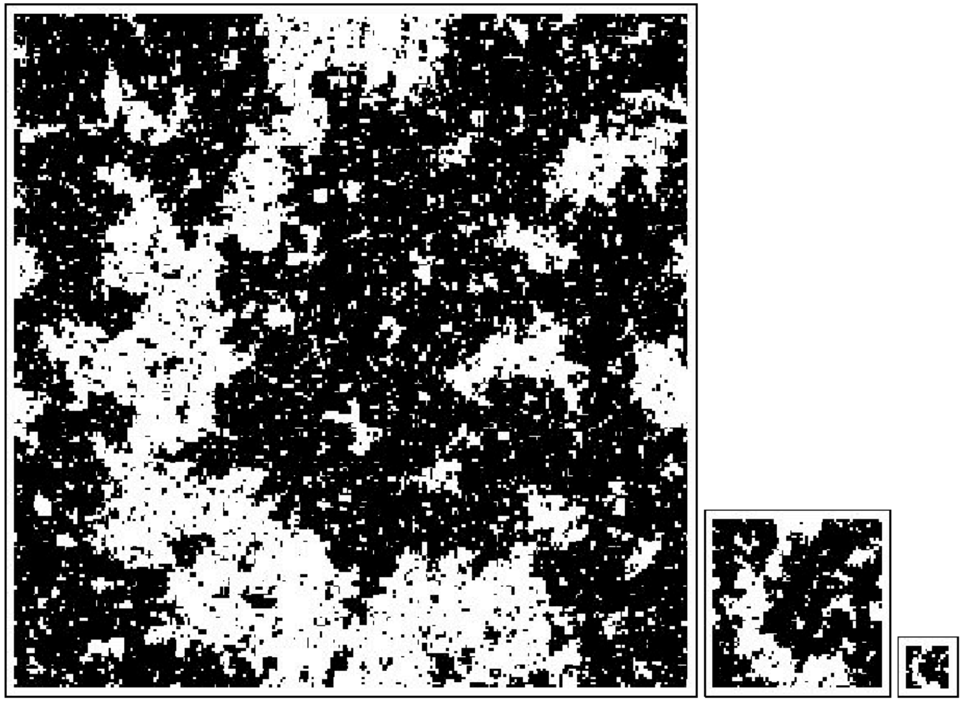


Order
K large

Disorder : K small

Renormalization group:
Organization of the description scale by scale

Critical:
K=critical value



Bubble with stochastic finite-time singularity due to positive feedbacks

$$\frac{dB(t)}{B(t)} = \mu dt + \sigma dW_t - \kappa dj$$

$$\begin{aligned} \mu(B)B &= \frac{m}{2B}[B\sigma(B)]^2 + \mu_0[B(t)/B_0]^m \\ \sigma(B)B &= \sigma_0[B(t)/B_0]^m \ , \end{aligned}$$

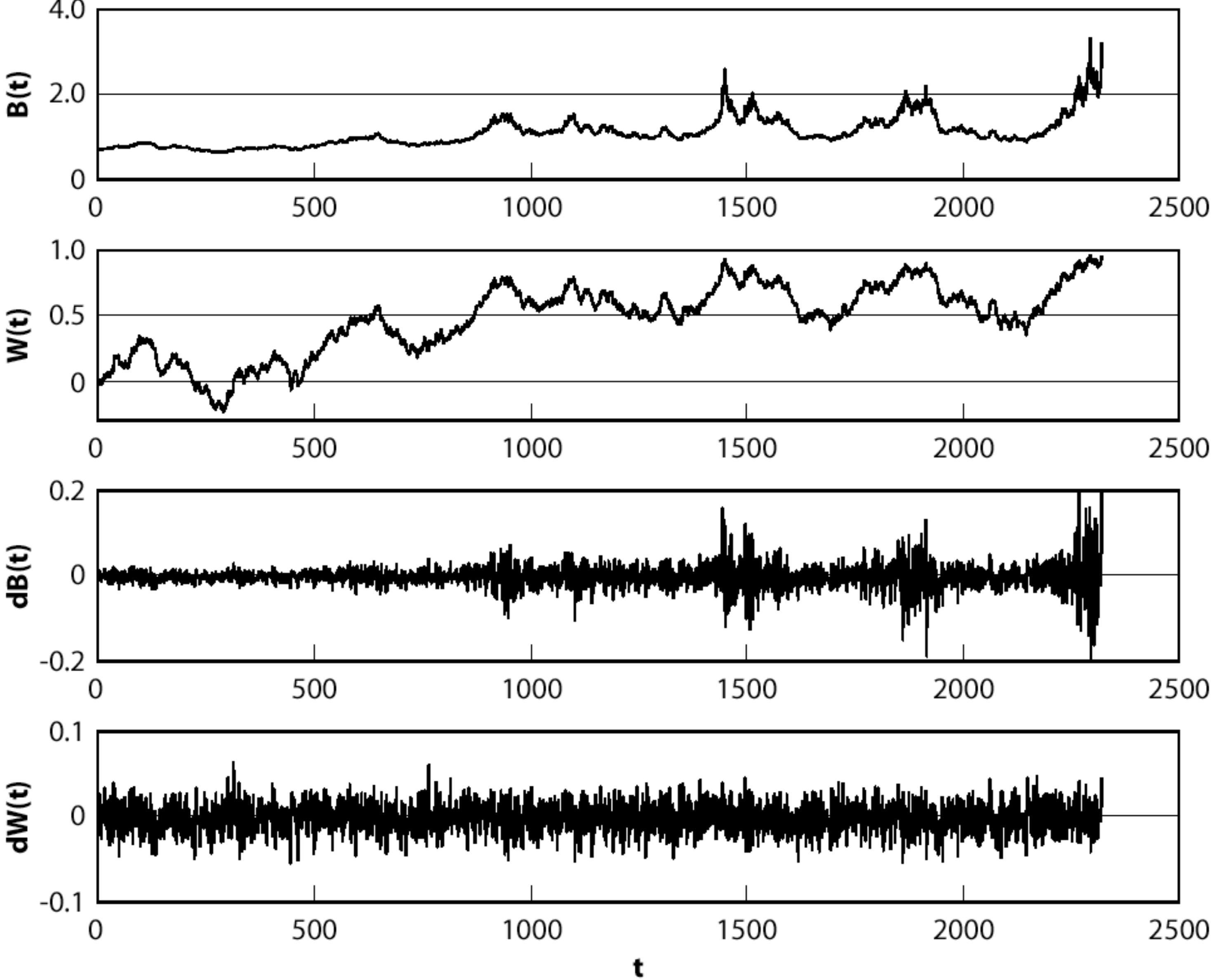
$$\frac{dB}{dt} = (a\mu_0 + b\eta)\ B^m - \kappa B dj \qquad h(t) = \frac{\mu(B(t))}{\langle\kappa\rangle}$$

$$B(t) = \alpha^\alpha \frac{1}{\left(\mu_0[t_c - t] - \frac{\sigma_0}{B_0^m} W(t)\right)^\alpha} \ , \quad \text{where } \alpha \equiv \frac{1}{m-1}$$

Stochastic finite-time singularity

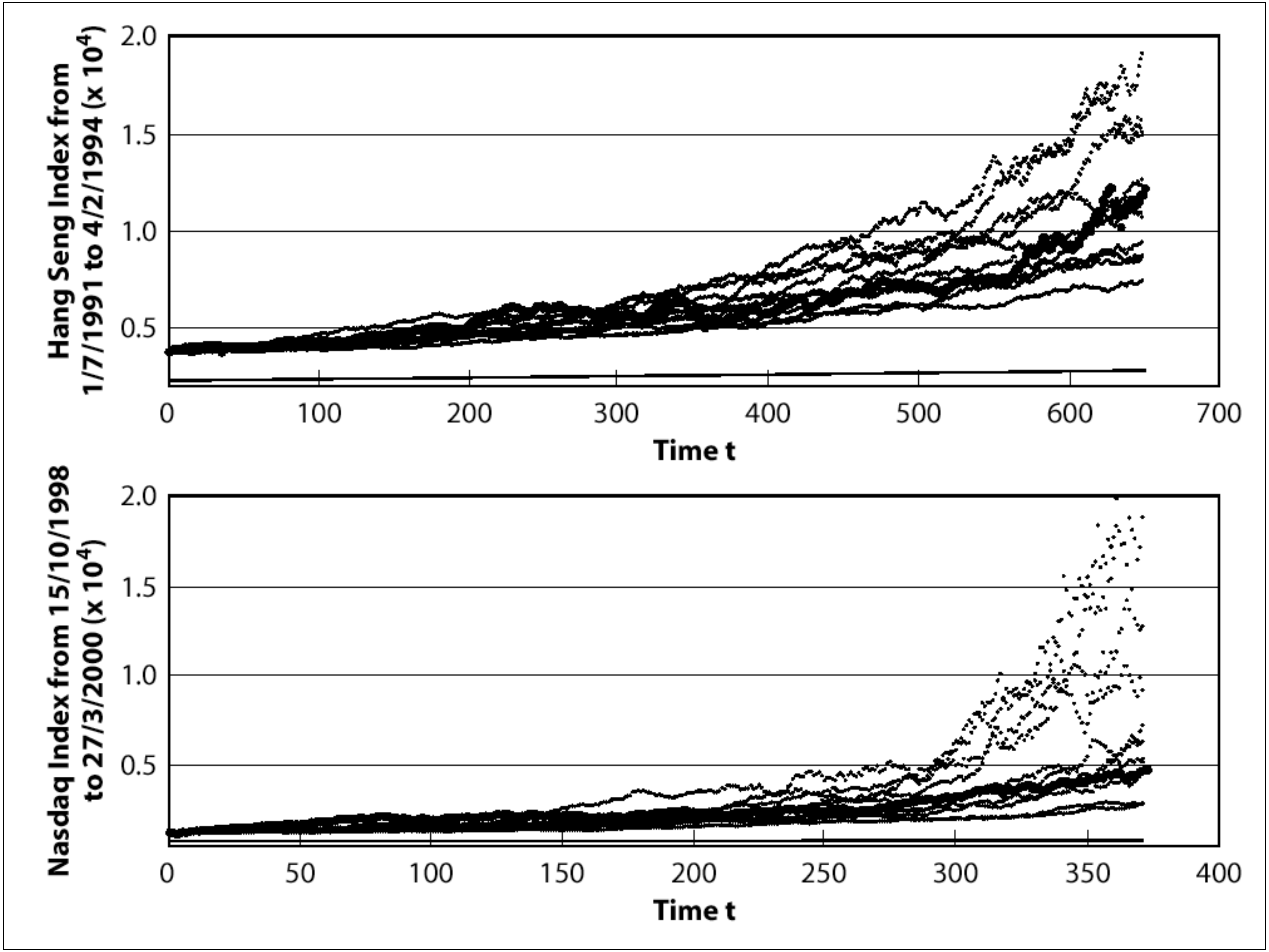
Nonlinear Super-Exponential Rational Model of Speculative Financial Bubbles

$$B(t) = \alpha^{\alpha} \frac{1}{(\mu_0[t_c - t] - (\sigma_0/B_0^m)W(t))^{\alpha}}$$



The price drives the crash hazard rate.

D. Sornette and J.V. Andersen
A Nonlinear Super-Exponential Rational Model of Speculative Financial Bubbles,
Int. J. Mod. Phys. C 13 (2), 171-188 (2002)



Hang Seng Index from 1/7/1991 to 4/2/1994 (x 10^4)
2.0
1.5
1.0
0.5
0
100
200
300
400
500
600
700
Time t
Nasdaq Index from 15/10/1998 to 27/3/2000 (x 10^4)
2.0
1.5
1.0
0.5
0
50
100
150
200
250
300
350
400
Time t

$$B(t) = \alpha^{\alpha} \frac{1}{(\mu_0[t_c - t] - (\sigma_0/B_0^m)W(t))^{\alpha}} \quad \text{where } \alpha \equiv 1/m - 1$$

Contains two ingredients:

(1) growth faster than exponential

(2) growth of volatility

$$\text{limit } 1/\alpha \to 0 \ (m \to 1)$$

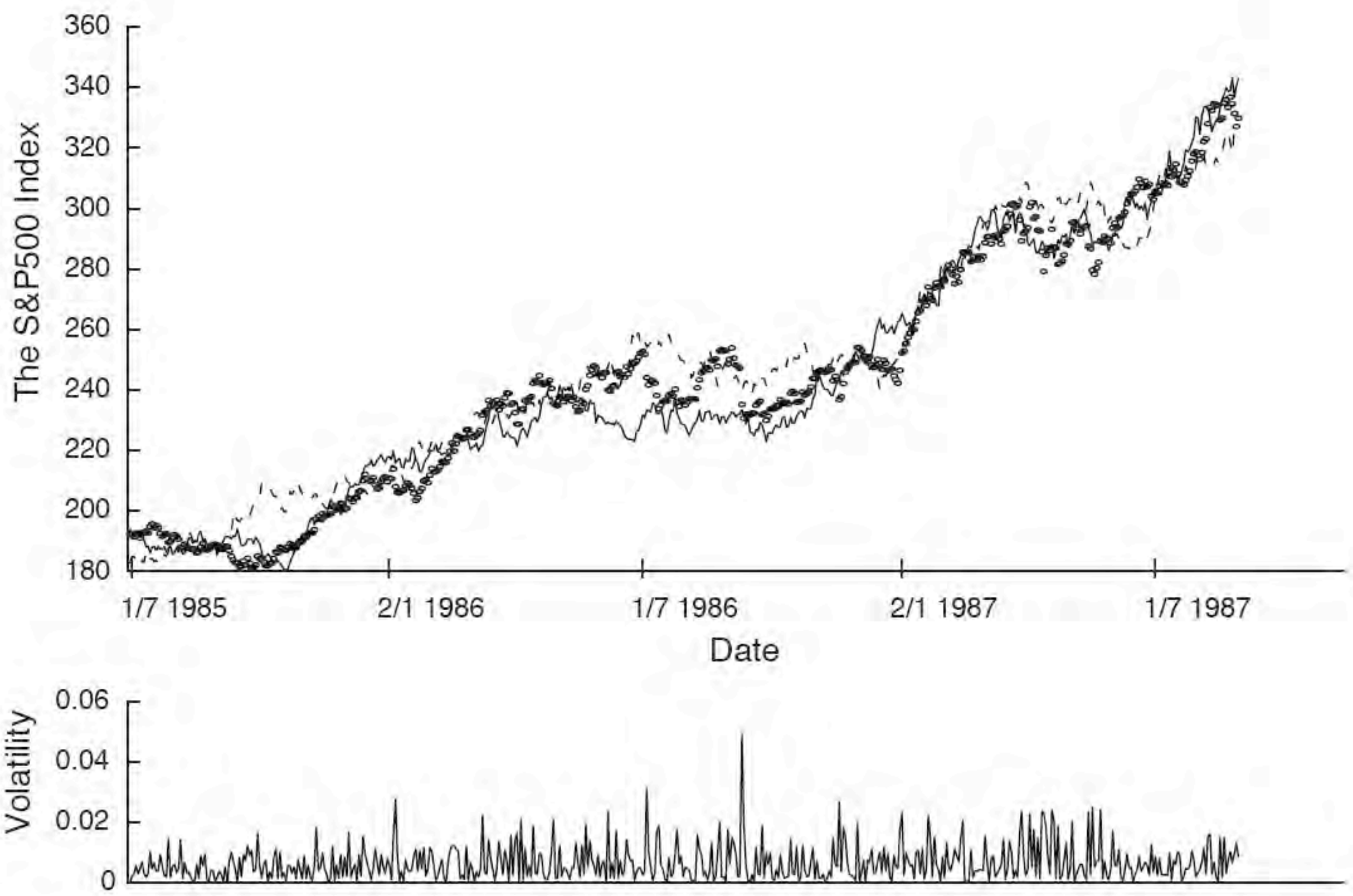
$$B_{BS}(t) = \exp(\mu_0 t + \sigma_0 W(t))$$ **Standard Geometric random walk**

Wilks' test of embedded hypotheses

Test of the existence of both ingredients

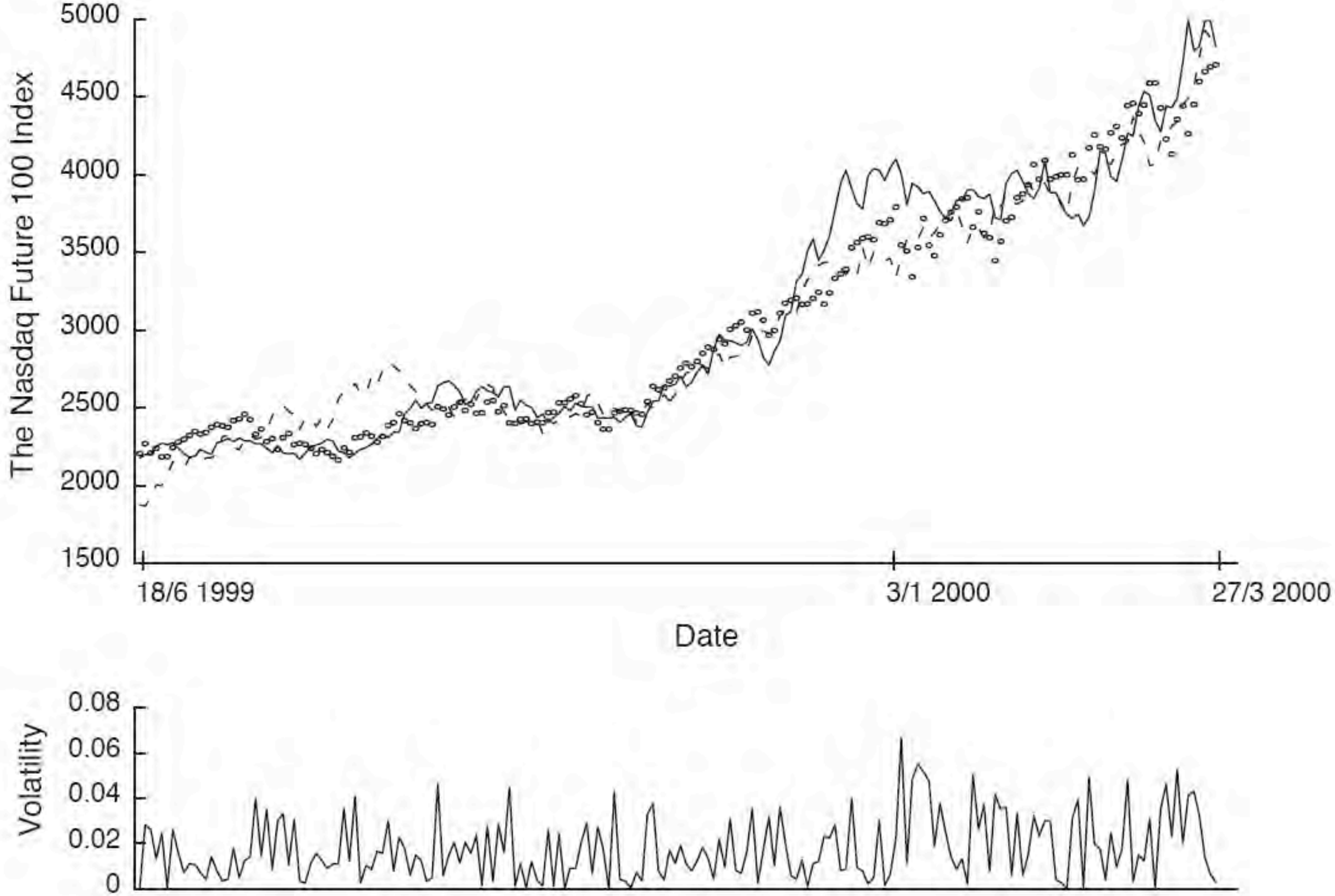
J.V. Andersen, D. Sornette / Physica A 337 (2004) 565–585

Example of a "fearful" super-exponential bubble



S&P 500 index from 1/7 1985 to 31/8 1987						
Percentage of 100 searches $P_{SB} = 0.70$, $P_{BS} = 0.28$						
Parameters of the curves in the figure	F	r	σ	μ	α	T_c
SB	110.64	2.0×10^{-5}	4.59×10^{-5}	7.36×10^{-4}	3.0	3390.54
BS	185.06	2.0×10^{-4}	7.81×10^{-5}	9.65×10^{-4}	–	–
T=183.71 Prob.(SB) > 0.9999						

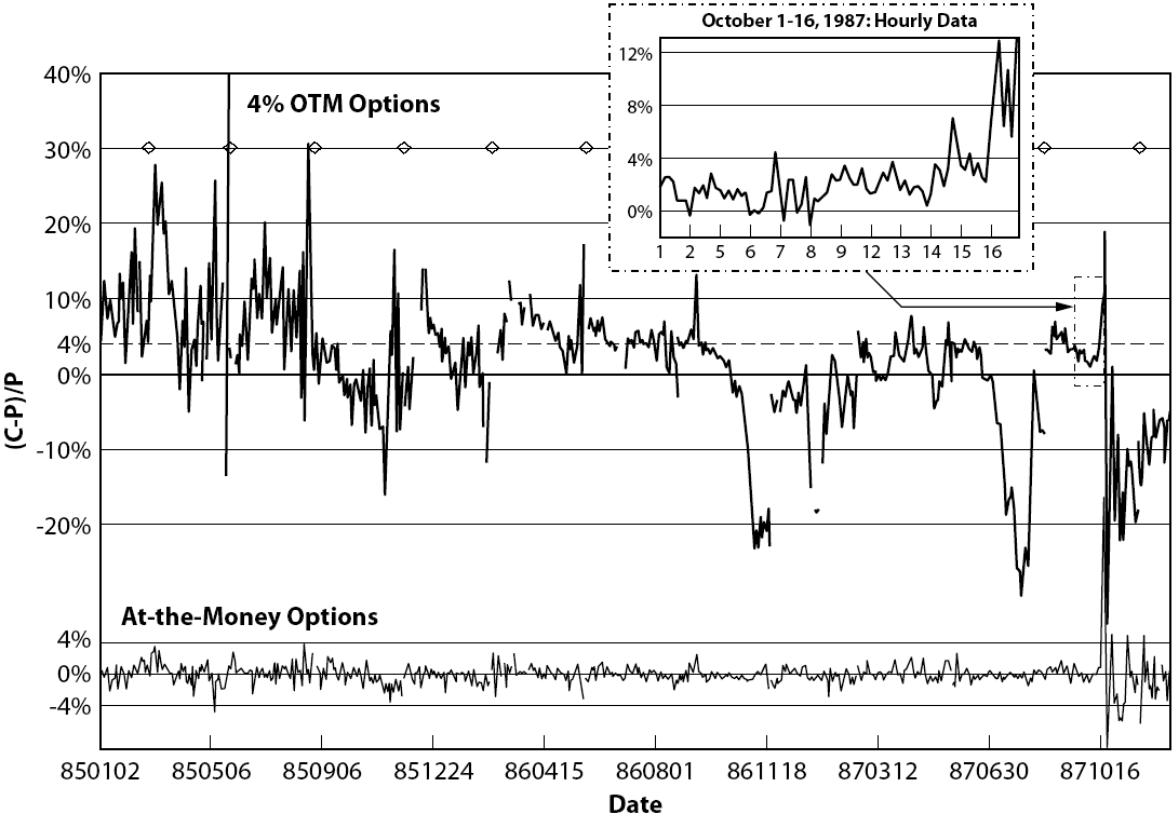
Example of a "fearless" super-exponential bubble



Nasdaq future 100 index from 18/6 1999 to 27/3 2000						
Percentage of 100 searches $P_{SB} = 0.03$, $P_{BS} = 0.97$						
Parameters of the curves in the figure	F	r	σ	μ	α	T_c
SB	1262.08	2.0×10^{-5}	8.89×10^{-5}	1.55×10^{-3}	1.0	368.34
BS	1845.20	1.60×10^{-4}	5.10×10^{-4}	3.73×10^{-3}	–	–
$T = -45.30$ Prob.(BS) > 0.9999						

THE CRASH OF OCTOBER 1987

Intermittent anticipation of the crash reflected in out-of-the-money option prices



Percentage deviation (C-P)/P of call from put prices (skewness premium) for options at-the-money and 4% out-of-the-money, over 1985–87. The percentage deviation (C-P)/P is a measure of the asymmetry between the perceived distribution of future large upward moves compared to large downward moves of the S&P 500 index. Deviations above (below) 0% indicate optimism (fear) for a bullish market (of large potential drops). The inset shows the same quantity (C-P)/P calculated hourly during October 1987 prior to the crash: ironically, the market forgot its "fears" close to the crash.

The Wall Street Journal on August 26, 1987, the day after the 1987 market peak: "In a market like this, every story is a positive one. Any news is good news. It's pretty much taken for granted now that the market is going to go up."

Bates, D. S. (1991). Journal of Finance 46, 1009–1044.

Renormalization Group approach

$$\boxed{f(K)=g(K)+\frac{1}{\mu}f[R(K)]}$$

(Derrida, Eckmann, Erzan, 1983)

$$f(K)=\sum_{n=0}^{\infty}\frac{1}{\mu^n}g[R^{(n)}(K)],$$

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{\mu^n}g[\gamma^n x]$$

$$f_W=\sum_{n=0}^{\infty}b^n\cos[a^n\pi x],$$

$$f(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\hat{f}(s)x^{-s}ds,$$

Inverse Mellin transform

$$f(x)=f_s(x)+f_r(x)$$

$$\boxed{f_s(x)=\sum_{n=0}^{\infty}A_n x^{-s_n}}$$

$$f_r(x)=\sum_{n=0}^{\infty}B_n x^n$$

$$A_n=\frac{\hat{g}(s_n)}{\ln\gamma}$$

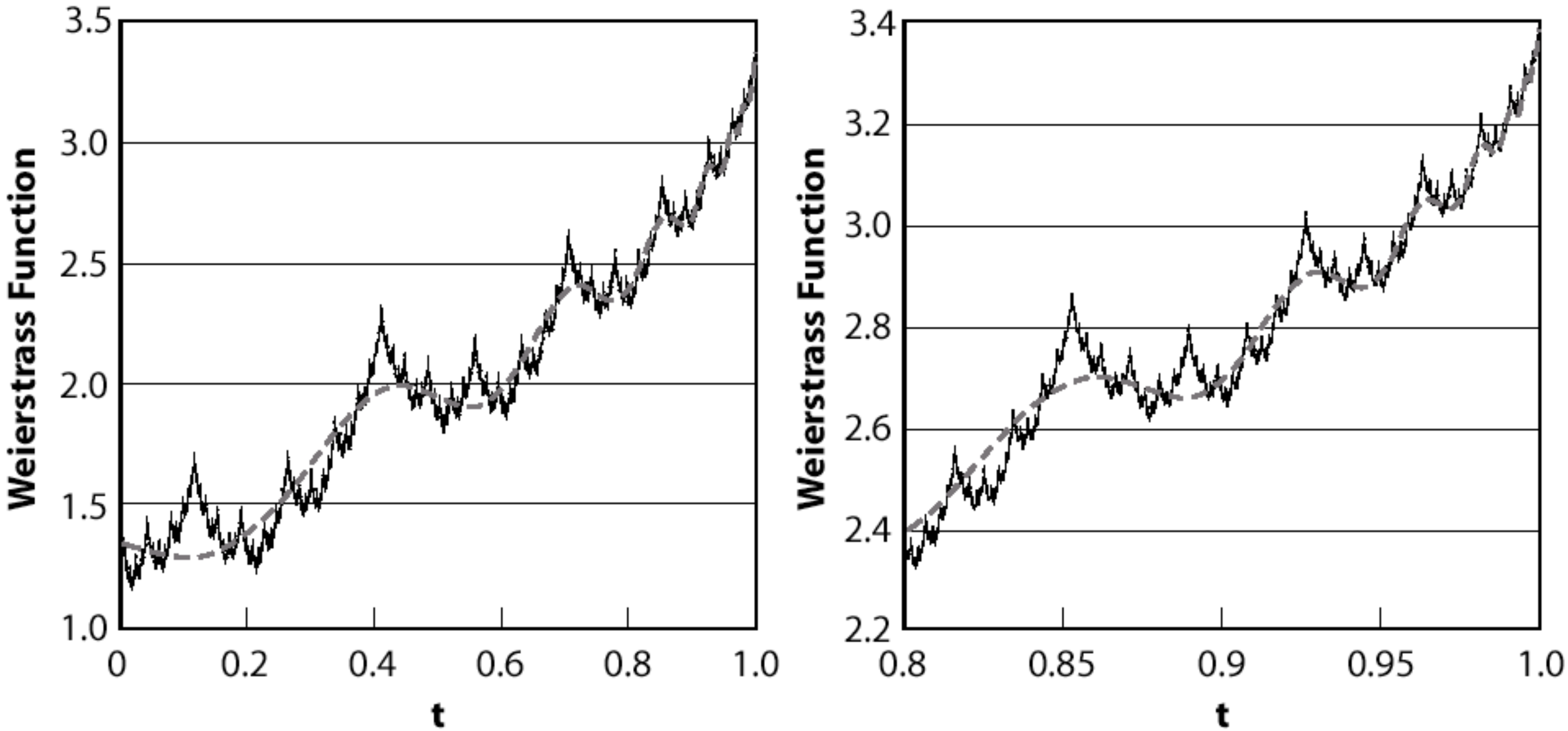
$$s_n=-m+i\frac{2\pi}{\ln\gamma}n$$

$$=-m+i\omega n$$

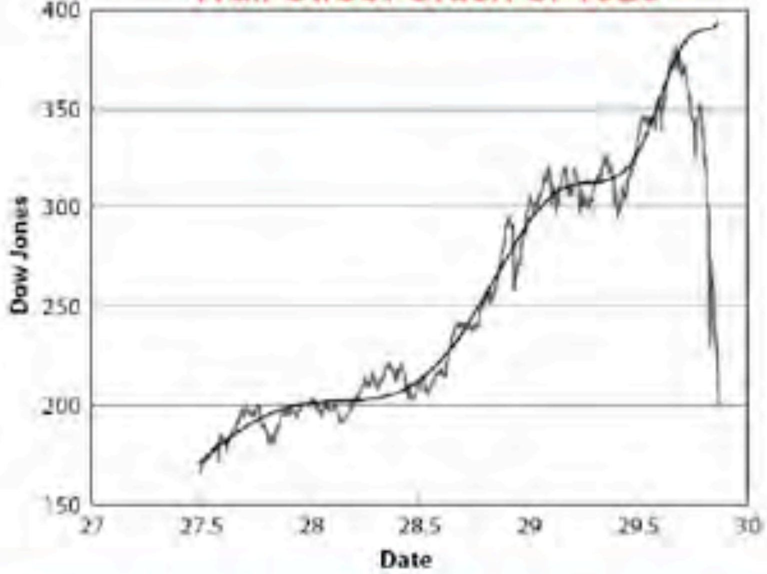
S. Gluzman and D. Sornette, Log-periodic route to fractal functions, art. no. 036142, Phys. Rev. E V6503 N3 PT2A:U418-U436 (2002)

Fractal function

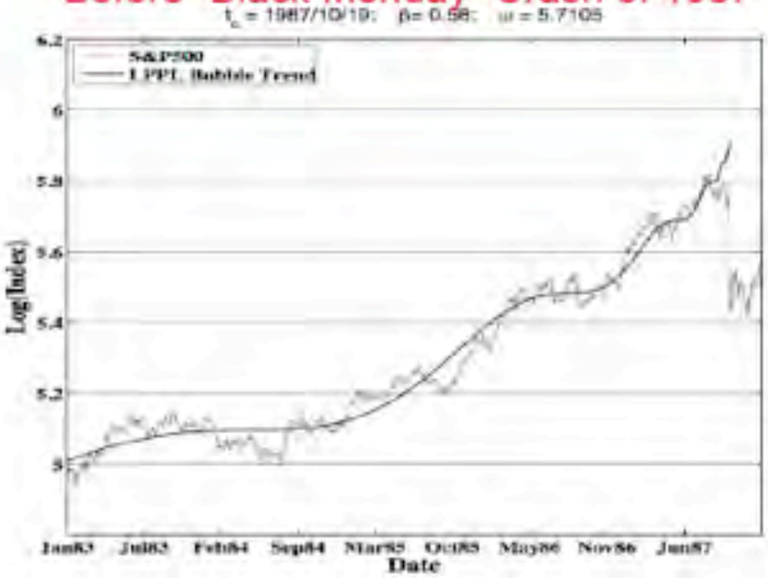
(Weierstrass)



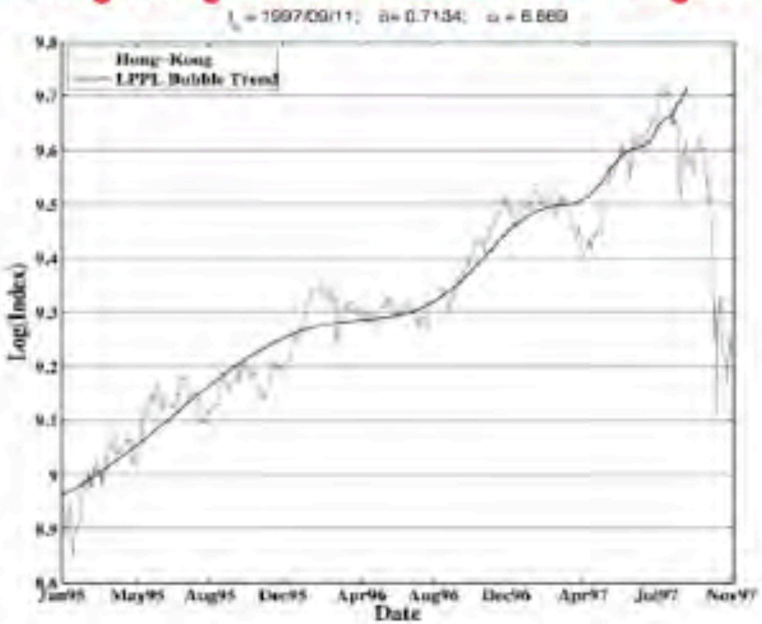
Wall Street Crash of 1929



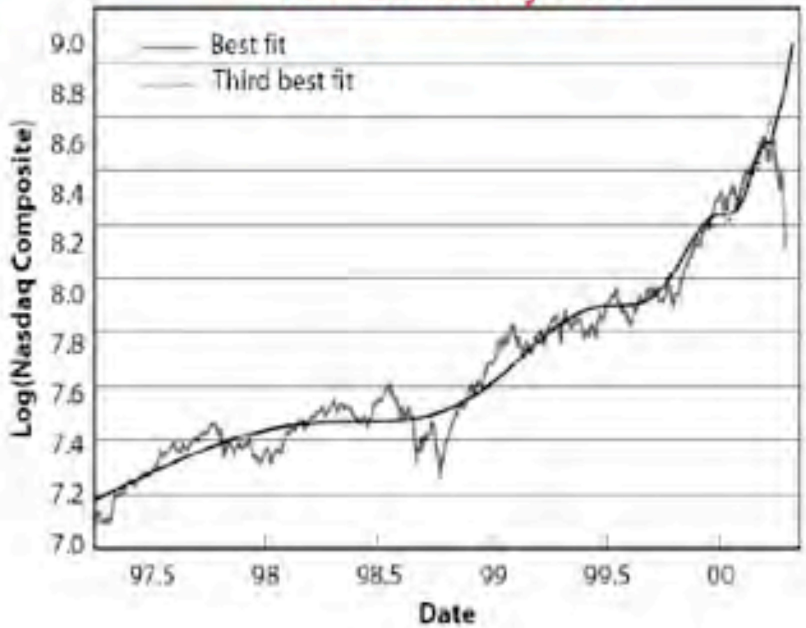
Before "Black Monday" Crash of 1987



Hong-Kong bubble ended in mid-August 1997



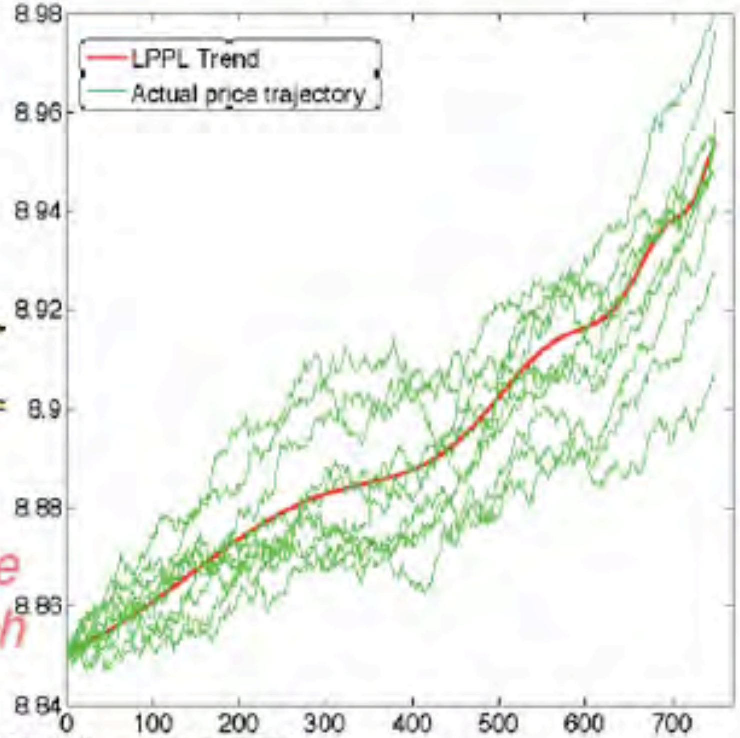
"New Economy": ICT 2000



- Empirical test on LPPL: nonparametric test; nonlinear least-square fits of price (focusing on price)
- Directly test LPPL on return is infeasible:
  - Signal is one-order-of magnitude smaller than background noise.
  - Lack of long time series since bubble is a transient structure.
- Chang and Feigenbaum(2006): Bayesian method to return time series
  - Without considering crash probabilities, BS model outperforms the JLS model.

JLS model=deterministic trajectory
+ random walk

Variance ever increasing!
Price strays father and farther away
from LPPL trend



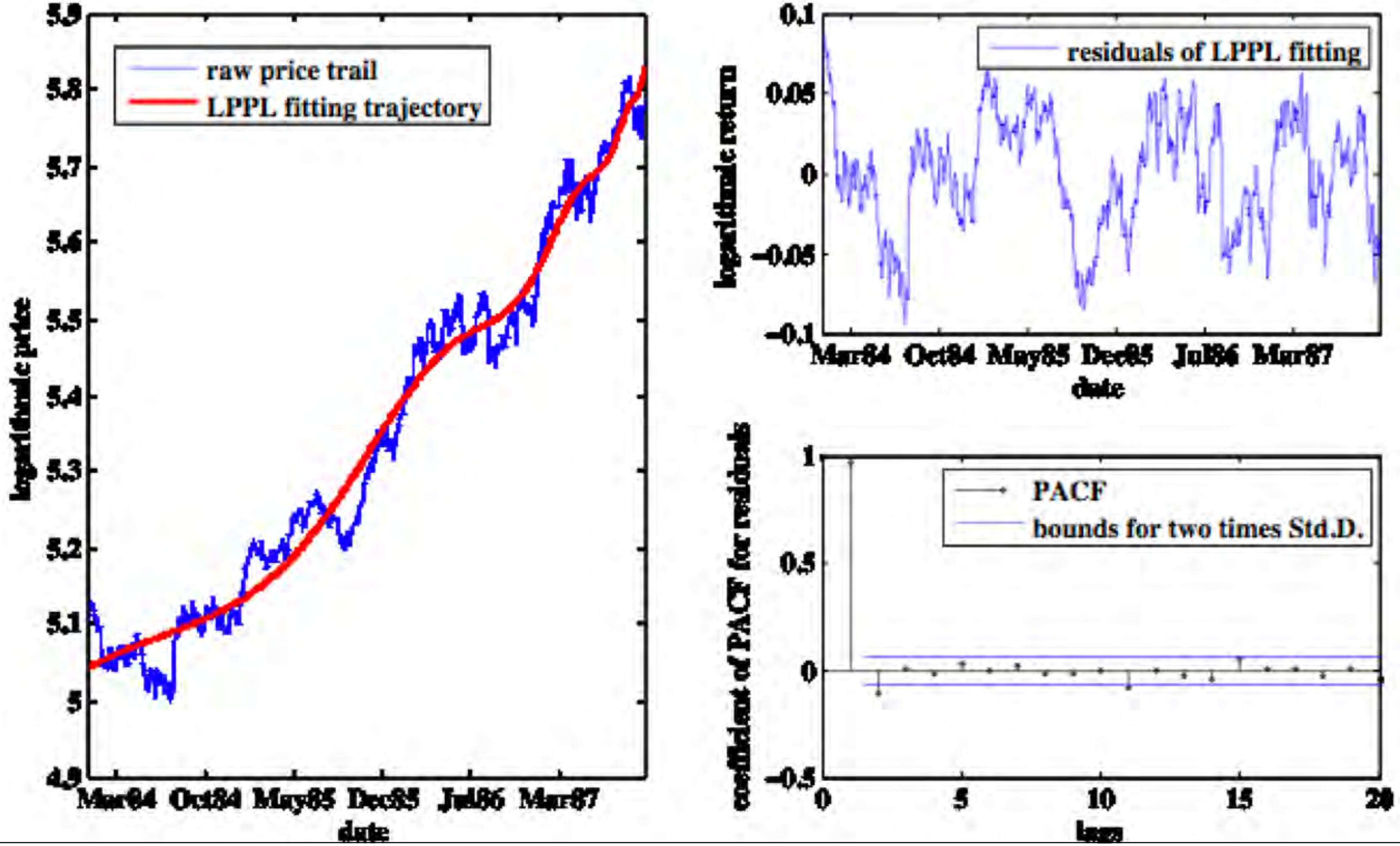
To be consistent, actual price series should be co-integrated with LPPL nonlinear trend, which means the residuals ought to be a stationary process ! Granger and Newbold(1974) and Phillips(1986)

A Consistent Model of 'Explosive' Financial Bubbles With Mean-Reversing Residuals

L. Lin, R. E. Ren and D. Sornette (2009)

http://papers.ssrn.com/abstract=1407574

$$\frac{dI}{I} = [r + \rho\Sigma + \kappa h(t)\,]dt - \alpha\rho_Y Y\,dt + (\sigma_Y + \sigma_W)dW$$



First model

Volatility Confined LPPL= deterministic component + *Ornstein-Uhlenbeck process*

LPPL fitting

Stationary
Mean reversal

- first model: based on Rational Expectation (RE) condition
  - Original price process: $$\frac{dp}{p} = \mu(t)dt + \sigma_Y dY + \sigma_W dW - kdj$$ $$dY = -\alpha Y dt + dW$$
  - Stochastic Discount Factor: $$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \rho_Y dY - \rho_W dW$$
  - Under no-arbitrage condition:

Belief updating

$$\mu(t) = \text{LPPL component} + \alpha(\sigma_Y\text{-}\rho_Y)Y_t$$

$$r_{i+1} = \ln p_{t_{i+1}} - \ln p_{t_i} \sim N(\Delta H_{t_{i+1},t_i} - \alpha(\ln p_{t_i} - H_{t_i}), \sigma_u^2(t_{i+1} - t_i))$$

$$H_{t_i} = A - B(t_c - t_i)^\beta [1 + \frac{C}{\sqrt{1 + (\frac{\omega}{\beta})^2}} \cos(\omega \ln(t_c - t_i) + \phi)]$$

Second model

The second model: based on concept of **Behavioral Stochastic Discount Factor(BSDF)** with critical behavior

- Shefrin(2006) introduce BSDF to characterize market sentiment. Critical behavior can be attributed to one kind of market sentiment.
- BSDF: $$\frac{d\Lambda_t^{ST}}{\Lambda_t^{ST}} = -[r-a]dt - bdj + \rho_Y dY + \rho_W dW$$
- Objective price process: $$\frac{dp}{p} = \mu(t)dt + \sigma_Y dY + \sigma_W dW$$ $$dY = -\alpha Y dt + dW$$

Phase transition

- The solution is same as the first model.
- Crash breaks the no-arbitrage conditions. Absence of basic securities cause price continuously falling. This vicious circles results form positive feedback.

- Evaluation of GARCH process to test for type I error (false positive)
  - Stylized features of LPPL
  - False positive rate: less than 0.2%
  - Unit-roots test shows: most of residual series can not reject non-stationary hypothesis.

$B > 0;\ 0.1 \le \beta \le 0.9$

$6 \le \omega \le 13;\ |C| < 1$

Ornstein – Uhlenbeck Residuals

- Test on S&P500 index for nearly 60 years

start of window	end of window	reject H_0 for residuals	type of sliding step
May. 7, 1984	Apr. 24, 1987	Yes	I
Jun. 12, 1984	Jun. 1, 1987	Yes	I & II
Jun. 18, 1984	Jul. 7, 1987	Yes	I
Mar. 15, 1991	Feb. 16, 1994	Yes	I & II
Mar. 25, 1994	Mar. 13, 1997	Yes	I
May. 3, 1994	Apr. 18, 1997	Yes	I & II
Jun. 8, 1994	May. 23, 1997	Yes	I
Jul. 14, 1994	Jun. 30, 1997	Yes	I & II
Sep. 23, 1994	Sep. 10, 1997	Yes	I & II
Oct. 28, 1994	Oct. 15, 1997	Yes	I
Apr. 28, 1995	Apr. 11, 1998	Yes	I & II
Jun. 5, 1995	May. 15, 1998	Yes	I
Jun. 11, 1995	Jun. 21, 1998	Yes	I & II
Sep. 16, 1996	Sep. 30, 1999	Yes	I & II

Unit roots test:

$$H_0:\ \phi=1;\ H_1:\ |\phi|<1$$

$$\Delta y_t = (\phi-1)y_t + \varepsilon_t$$

Specific Test focusing on classic crash of October 87

- Test for the validity of the LPPL conditions and unit-root tests on residuals in windows all ending on Sep. 30, 1987 with different starting dates for the S&P500 US index. The smallest window size is 750 days. P_{LPPL} is the percentage of windows that obey the LPPL conditions in all the test windows. $P_{StationaryResi.|LPPL}$ is the probability that the null unit-root tests for non-stationarity are rejected for the residuals, conditional on the fact that the LPPL conditions are met. The unit-root tests are also the Phillips-Perron and Dickey-Fuller tests (both produce the same results) with significance level of 0.001.

start of window	number of samples	number of series satisfy LPPL condition	P_{LPPL}	$P_{\text{StationaryResi.}\|\text{LPPL}}$
Jan. 2, 1980	242	43	17.78%	100%**
Jan. 3, 1983	90	43	47.48%	100%**
Sep. 1, 1983	57	42	73.68%	100%**
Dec. 1, 1983	44	43	97.73%	100%**
Mar. 1, 1984	32	32	100%	100%**

Bayesian approach

S&P500 1987 and Hong-Kong 1997

- Bayesian Factor
  - $B(\text{model\_1}, \text{model\_2}) = \dfrac{\text{Marginal Likelihood (model\_1)}}{\text{Marginal Likelihood (model\_2)}}$
  - Model_1: Volatility Confined LPPL
  - Prior probability
  - Model_2: Black–Scholes model
- Calculation Results

$$\mathscr{L}_{\mathrm{LPPL}}(2.5\% - 97.5\%) = 3173.546 - 3176.983$$

$$\mathscr{L}_{\mathrm{BS}}(2.5\% - 97.5\%) = 3169.808 - 3170.097$$

LPPL outperform BS here

$$\begin{aligned}
\mu &\sim N(0.0003, (0.01)^2) \\
\tau &\sim \Gamma(1.0, 10^5) \\
\alpha &\sim \Gamma(1.0, 0.05) \\
A &\sim N(6, 0.05) \\
B &\sim \Gamma(1, 0.01) \\
C &\sim U(0, 1) \\
\beta &\sim B(40, 30) \\
\omega &\sim \Gamma(16, 0.4) \\
\phi &\sim U(0, 2\pi) \\
t_c - t_N &\sim \Gamma(1, 30)
\end{aligned}$$

Diagnostics of Rational Expectation Financial Bubbles with Stochastic Mean-Reverting Termination Times

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ETH Zurich and School of Economics and Management, Beihang University, 100191 Beijing, China

http://arxiv.org/abs/0911.1921

First model: finite-time singularity in the price dynamics with stochastic critical time

$$\mathrm{d}p = \mu p^m (1 + \delta(p,t))\mathrm{d}t + \sigma p^m \mathrm{d}W$$

$$\delta(p,t) = \alpha\, \tilde{t}_c(t) + \frac{\sigma^2}{2\mu} m\, [p(t)]^{m-1} \qquad (3)$$

$$\mathrm{d}\tilde{t}_c = -\alpha \tilde{t}_c \mathrm{d}t + (\sigma/\mu)\mathrm{d}W \qquad (4)$$

Proposition 1. *Provided that $\delta(p,t)$ follows the process (3) with (4), the solution of equation* (1) *can be written under a form similar to (2) as follows,*

$$p(t) = K(\widetilde{T}_c - t)^{-\beta}\ ,$$

with

$$\beta = \frac{1}{m-1}\ , \quad K = \left(\frac{\beta}{\mu}\right)^{\beta}\ , \quad T_c = \frac{\beta}{\mu} p_0^{-\frac{1}{\beta}}\ , \quad \widetilde{T}_c = T_c + \tilde{t}_c\ .$$

Let us denote $t_i > 0$ the time at which the i's arbitrageur has entered the market.

Being aware of the price dynamics, at each instant t, the rational arbitrageur forms a belief quantified by her hazard rate $h_i(t)$, of the probability that a crash might occur in the next instant, conditional on the fact that it has not yet happened.

She estimates the probability $1 - \Pi_i(t)$ that the crash will not happen until time t.

The arbitrageur forms a belief of the crash hazard rate which is of the same form, that is,

$$h_i(t) = \frac{\pi_i(t)}{1 - \Pi_i(t)} \propto (T_{c,i} - t)^{-\beta_i} \tag{8}$$

critical time for the end of the bubble that the arbitrageur i has estimated when entering the market.

Occurrence of the market collapse is posited to be triggered when a sufficiently large number of arbitrageurs have exited the market, leading to a large price movement, amplified by the herding of noise traders.

Optimal exit time: $\max_t \ \mathbb{E}^i[(1 - \Pi_i(t)) \cdot \mathrm{d}p - \pi_i(t)\mathrm{d}t \cdot \kappa p]$

Solution: $(1 - \Pi_i(t))\mathbb{E}^i(\mu p^m(1 + \delta(p,t))) = \pi_i(t)\mathbb{E}^i(\kappa p)$

Proposition 2. *Given a population of heterogeneous arbitrageurs, which form their expectation of the crash hazard rate according to (8) with heterogeneous anticipated critical times $T_{c,i}$ and exponents β_i reflecting their different views on the riskiness of the market, a given arbitrageur i decides to exit the market at the date t_i^{ex} which is the solution of*

$$\frac{\mathbb{E}^i[dp(t_i^{ex})]}{\mathbb{E}^i[\kappa p(t_i^{ex})]} = h_i(t_i^{ex}) \propto (T_{i,c} - t_i^{ex})^{-\beta_i} \ . \tag{11}$$

Since $\mathbb{E}^i(p) \neq \mathbb{E}^j(p)$ and $h_i(t) \neq h_j(t)$, we have $t_i^{ex} \neq t_j^{ex}$. Notwithstanding the fact that the presence of the bubble is common knowledge among all rational arbitrageurs, the absence of synchronization of their market exit allows the bubble to persist and run its course up to a time close to its expected value (7).

This synchronization problem is analogous to that identified by Abreu and Brunnermeier (2003), with the important difference that we emphasize that the lack of synchronization results from the heterogeneous beliefs concerning the critical end tc of the bubble.

The uncertainty in Abreu and Brunnermeier (2003) comes from heterogeneous awareness of the BEGINNING of the bubble.

Construction of alarms

$$\widetilde{T}_{c,i}(t) = \frac{1}{K}\frac{1}{[p(t)]^{1/\beta}} + t, \qquad t = t_i - 749, \cdots, t_i$$

$$T_{c,i} = \frac{1}{750}\sum_{t=1}^{750}\widetilde{T}_{c,i}(t) \qquad \tilde{t}_{c,i}(t) = \widetilde{T}_{c,i}(t) - T_{c,i}$$

Bubble diagnostic if

(i) $\beta^* > 0$ such that $m > 1$ (the signature of a positive feedback in our model) and

(ii) $T_{c,i} - t_i < 750$, i.e., the estimated termination time of the bubble is not too distant

(iii) Dickey – Fuller unit – root test is rejected at 99.5% significance level

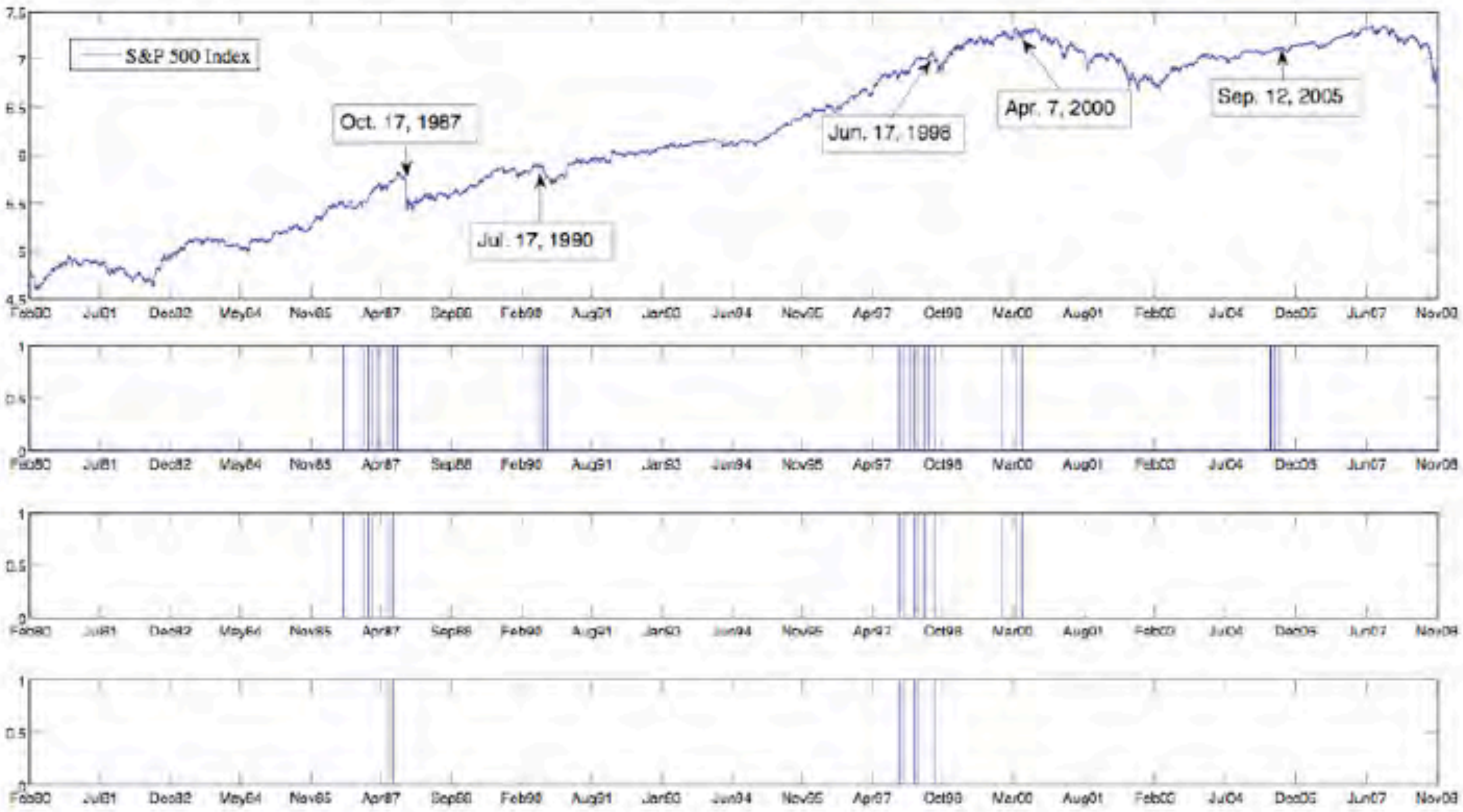


Figure 1: Logarithm of the historical S&P500 stock index and corresponding alarms shown in the three lower panels as vertical lines indicating the ends of the windows of 750 trading days in which our procedure using the first bubble model of section 2 flags a diagnostic for the presence of bubble. The three lower panels corresponds to alarms for which $T_{c,i} - t_i < 750$, $T_{c,i} - t_i < 500$ and $T_{c,i} - t_i < 250$, from top to bottom. By definition, the set of alarms of the lowest panel is included in the set of alarms of the middle panel which is itself included in the set of alarms of the upper panel. The exponents m found for the upper panel corresponding to $T_{c,i} - t_i < 750$ have a mean of 2.76 with a standard deviation of 0.33.

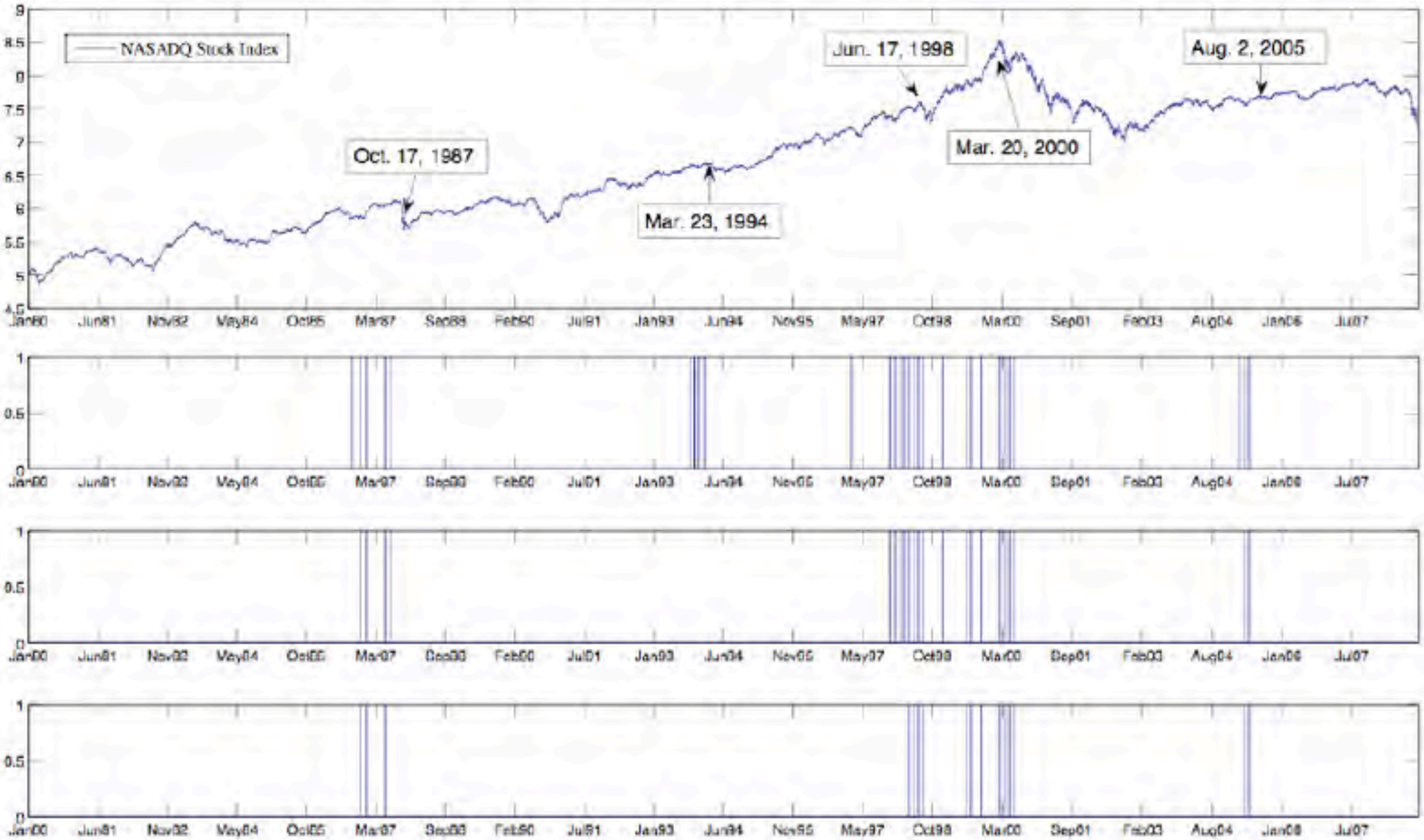


Figure 2: Same as Fig.1 for the Nasdaq Composite index. The exponents m found for the upper panel corresponding to $T_{c,i} - t_i < 750$ have a mean of 2.85 with a standard deviation of 0.23.

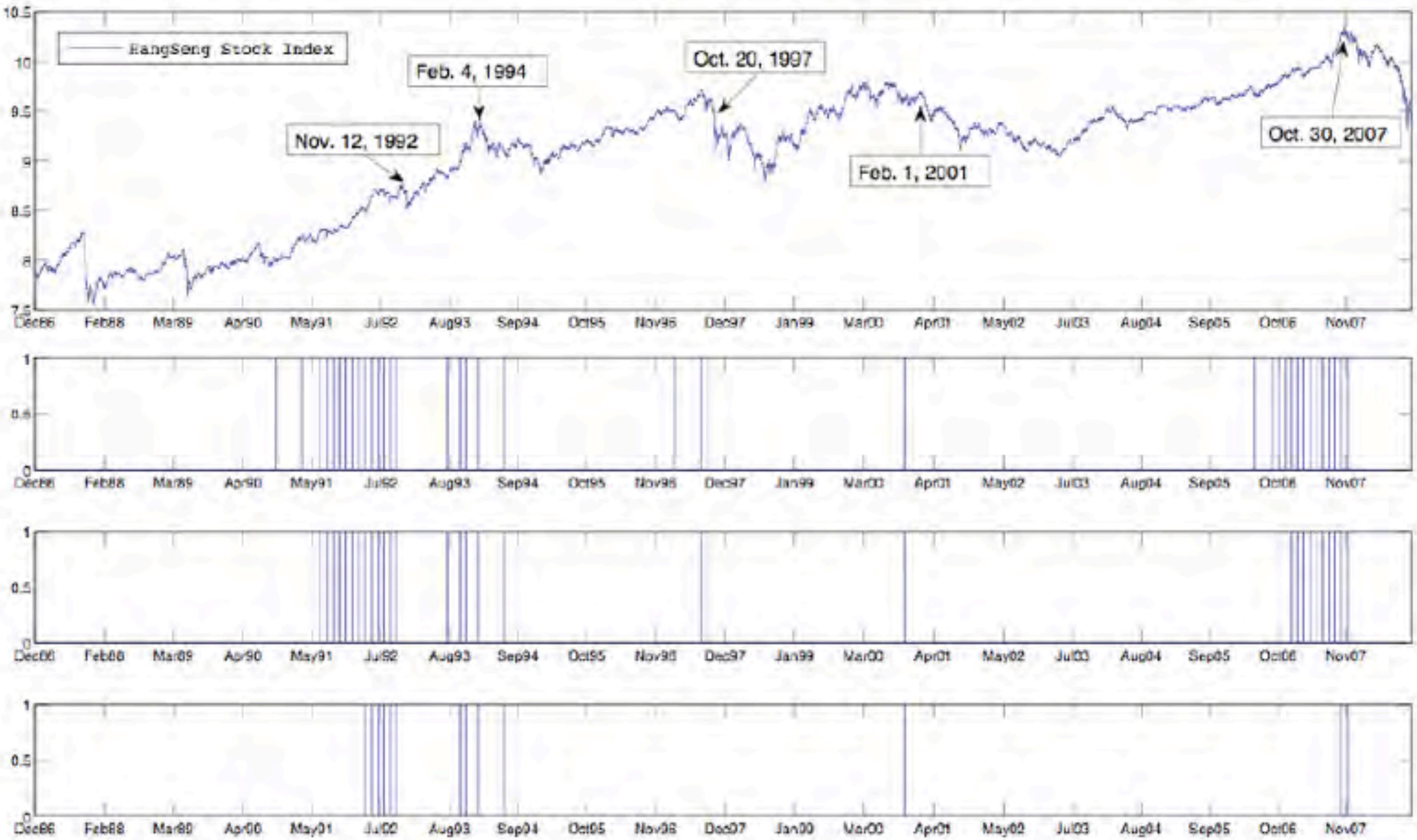


Figure 3: Same as Fig.1 for the Heng Seng index of Hong Kong. The exponents m found for the upper panel corresponding to $T_{c,i} - t_i < 750$ have a mean of 2.84 with a standard deviation of 0.22.

Second model: finite-time singularity in the momentum price dynamics with stochastic critical time

$$y(t) = \ln p(t)$$

$$\mathrm{d}y = x(1+\gamma(x,t))\mathrm{d}t + (\sigma/\mu)x\mathrm{d}W$$
$$\mathrm{d}x = \mu x^m(1+\delta(x,t))\mathrm{d}t + \sigma x^m\mathrm{d}W$$

Deterministic limit: $$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \mu\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^m$$

$$y(t) = A - B(T_c - t)^{1-\beta}$$

$\beta = \frac{1}{m-1}$, $T_c = (\beta/\mu)(\frac{\mathrm{d}p}{\mathrm{d}t}\big|_{t=t_0})^{-\frac{1}{\beta}}$, $B = \frac{1}{1-\beta}(\mu/\beta)^{-\beta}$ and $A = p(T_c)$.

We postulate the following specific processes

$$\gamma(x,t) = \alpha\tilde{t}_c(t) + \frac{\sigma^2}{2\mu}[x(t)]^{m-1}$$
$$\delta(x,t) = \alpha\tilde{t}_c(t) + \frac{\sigma^2}{2\mu}m[x(t)]^{m-1}$$

$$\mathrm{d}\tilde{t}_c = -\alpha\tilde{t}_c\mathrm{d}t + (\sigma/\mu)\mathrm{d}W$$

Proposition 3. *Provided that $\gamma(x,t)$ and $\delta(p,t)$ follow the processes given respectively by (19) and (20), then the solution of (15,16) for the log-price $y(t) = \ln p(t)$ can be written under a form similar to expression (18) as follows,*

$$y(t) = A - B(T_c + \tilde{t}_c(t) - t)^{1-\beta}$$

where

$$\beta = \frac{1}{m-1}\ , \quad T_c = \frac{\beta}{\mu}x_0^{1/\beta}, \quad x_0 := x(t=0)\ , \quad B = \frac{1}{1-\beta}(\beta/\mu)^{\beta}\ ,$$

and A is a constant.

Construction of alarms

$$\widetilde{T}_{c,i}(t) = t_i + \left(\frac{A - \ln p(t)}{B}\right)^{\frac{1}{1-\beta}} , \qquad t = t_i - 899, \cdots, t_i \ .$$

$$T_{c,i} = \frac{1}{750}\sum_{t=1}^{750} \widetilde{T}_{c,i}(t) \qquad\qquad \tilde{t}_{c,i}(t) = \widetilde{T}_{c,i}(t) - T_{c,i}$$

Bubble diagnostic if

(i) $0 < \beta^* < 1$ such that $m > 2$ (the signature of a positive feedback in the momentum price dynamics model) and

(ii) $-25 \leq T_{c,i} - t_i \leq 50$, such that the estimated termination time of the bubble is close to the right side of the time window.

(iii) We further refine the filtering by considering three levels of significance quantified by the value of the exponent m: level 1 ($m > 2$), level 2 ($m > 2.5$) and level 3 ($m > 3$).

(iv) Dickey – Fuller unit – root test is rejected at 99.5% significance level

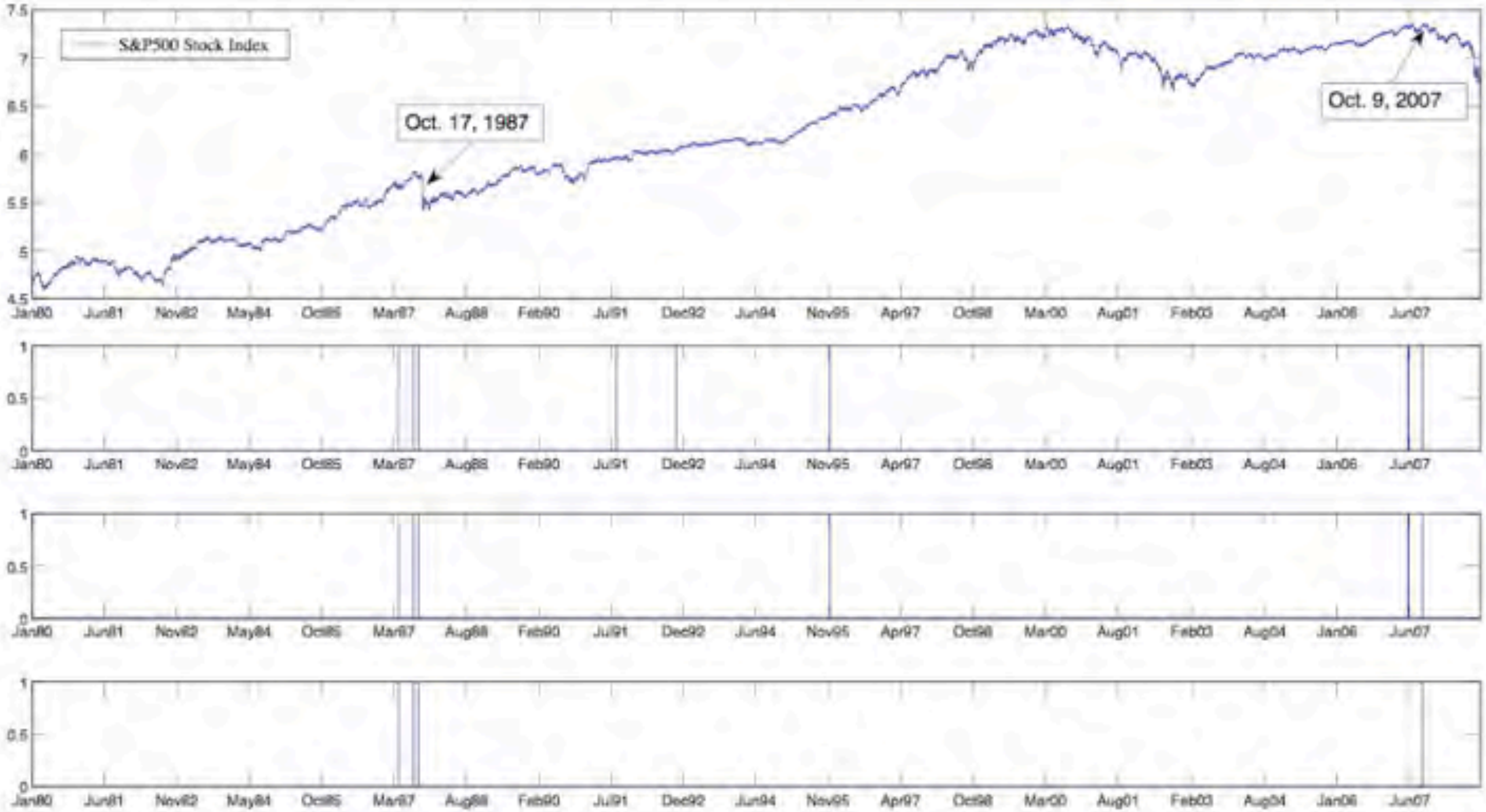


Figure 4: Logarithm of the historical S&P500 stock index and corresponding alarms shown in the three lower panels as vertical lines indicating the ends of the windows of 900 trading days in which our procedure using the second bubble model of section 3 flags a diagnostic for the presence of bubble. The three lower panels corresponds to alarms for which $m > 2$, $m > 2.5$ and $m > 3$, from top to bottom. By definition, the set of alarms of the lowest panel is included in the set of alarms of the middle panel which is itself included in the set of alarms of the upper panel.

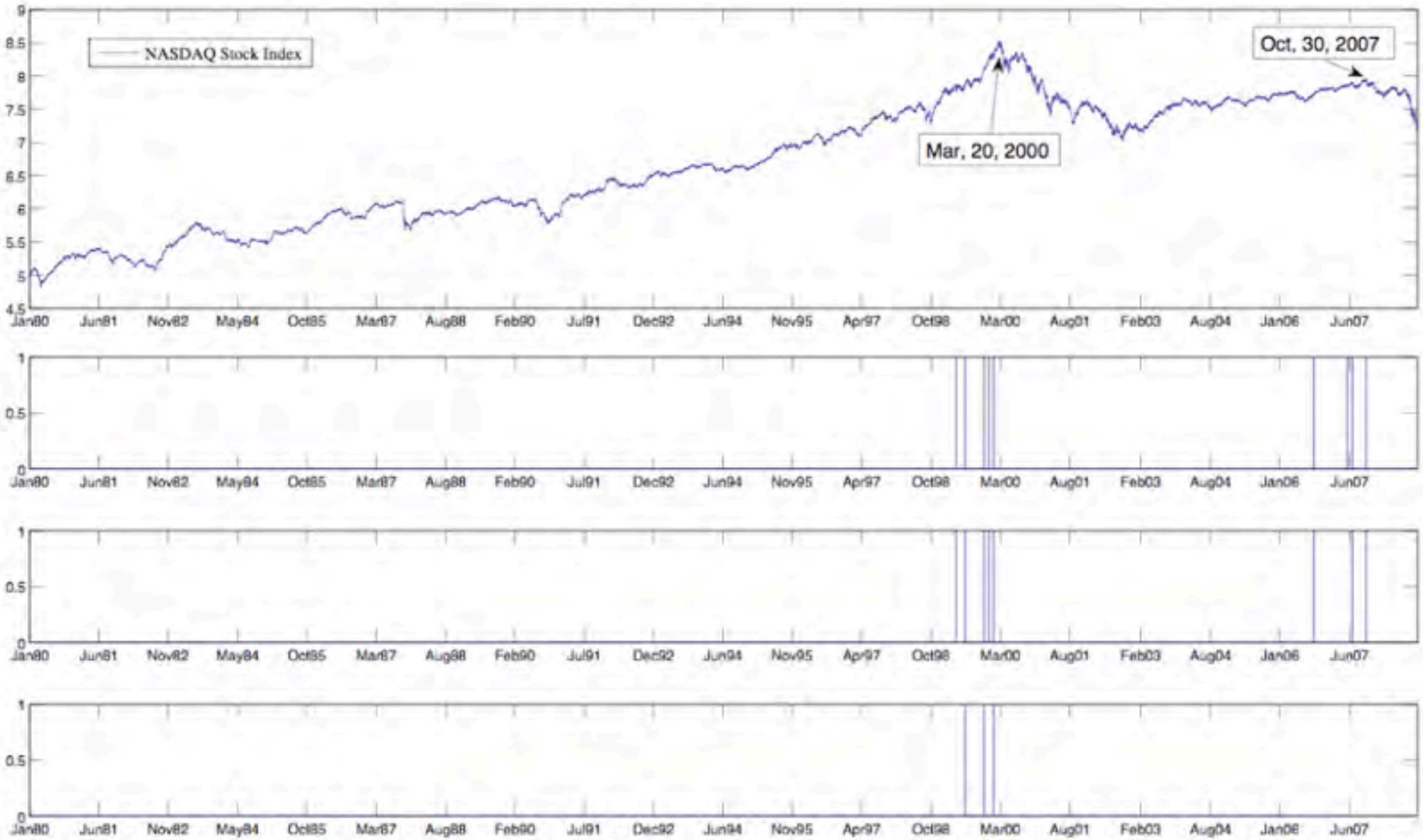


Figure 5: Same as Fig.4 for the Nasdaq Composite index.

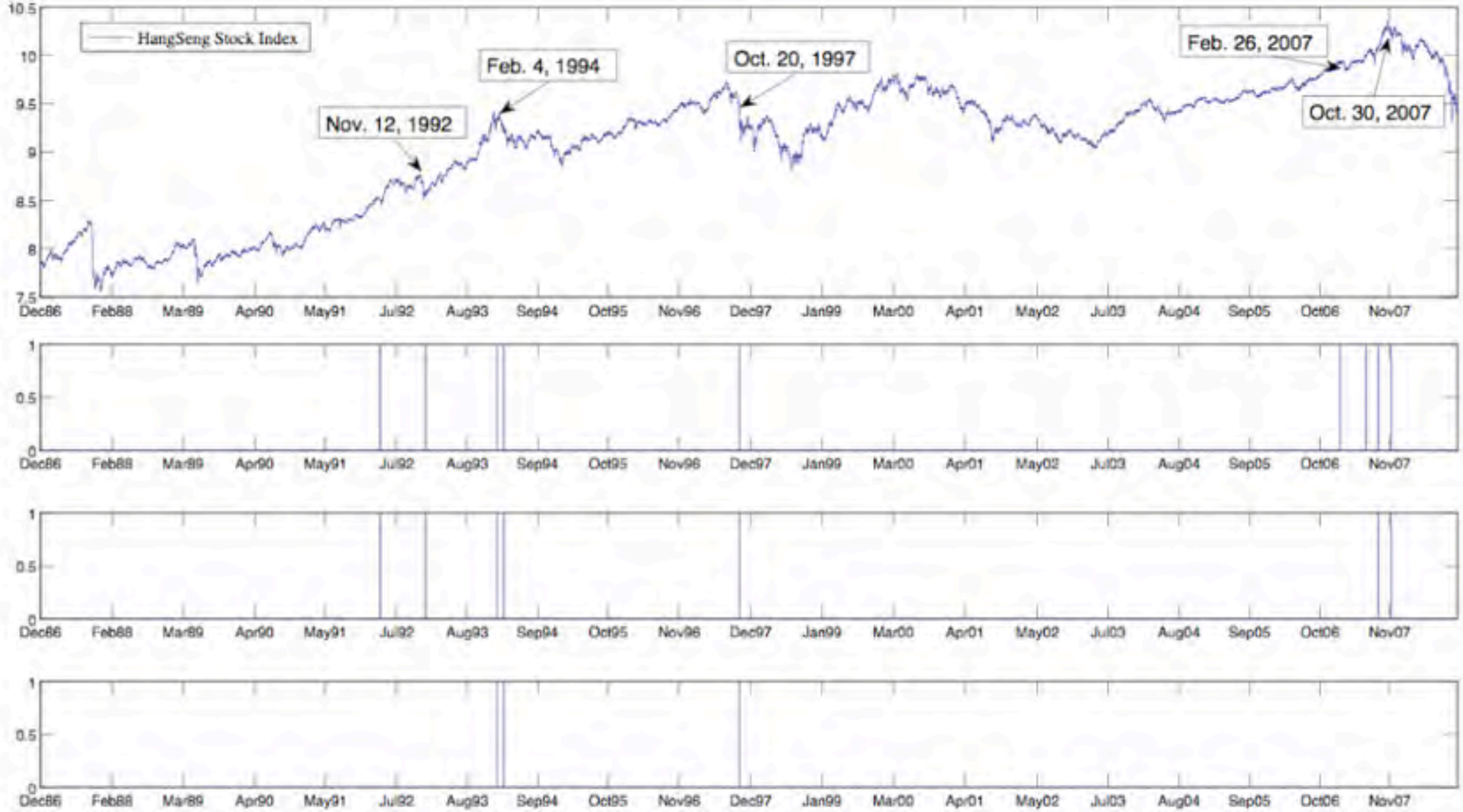


Figure 6: Same as Fig.4 for the Heng Seng index of Hong Kong.

The Financial Bubble Experiment

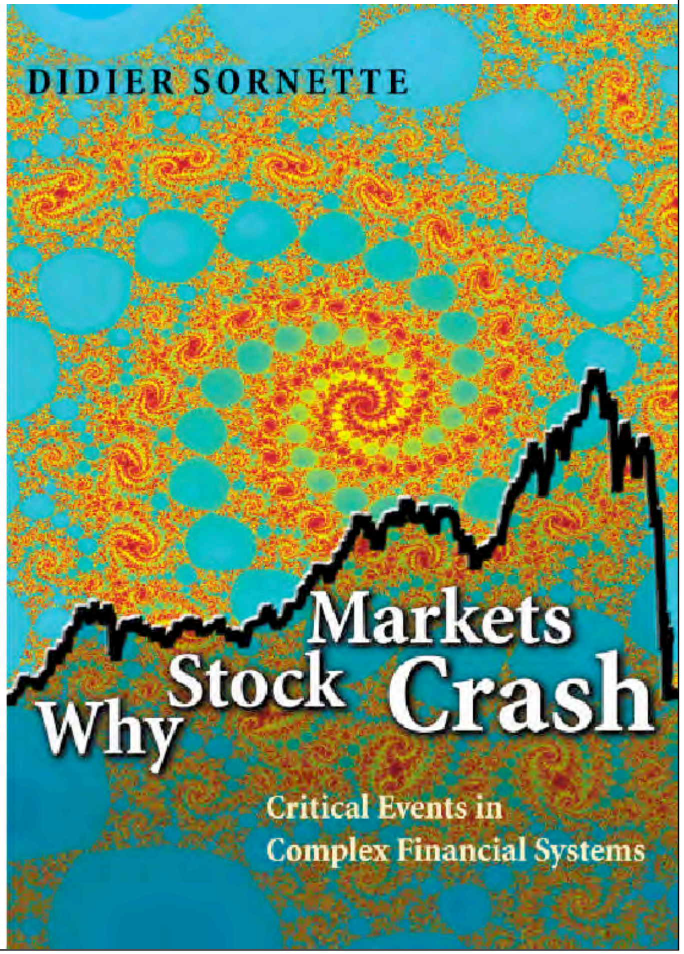
advanced diagnostics and forecasts of bubble terminations

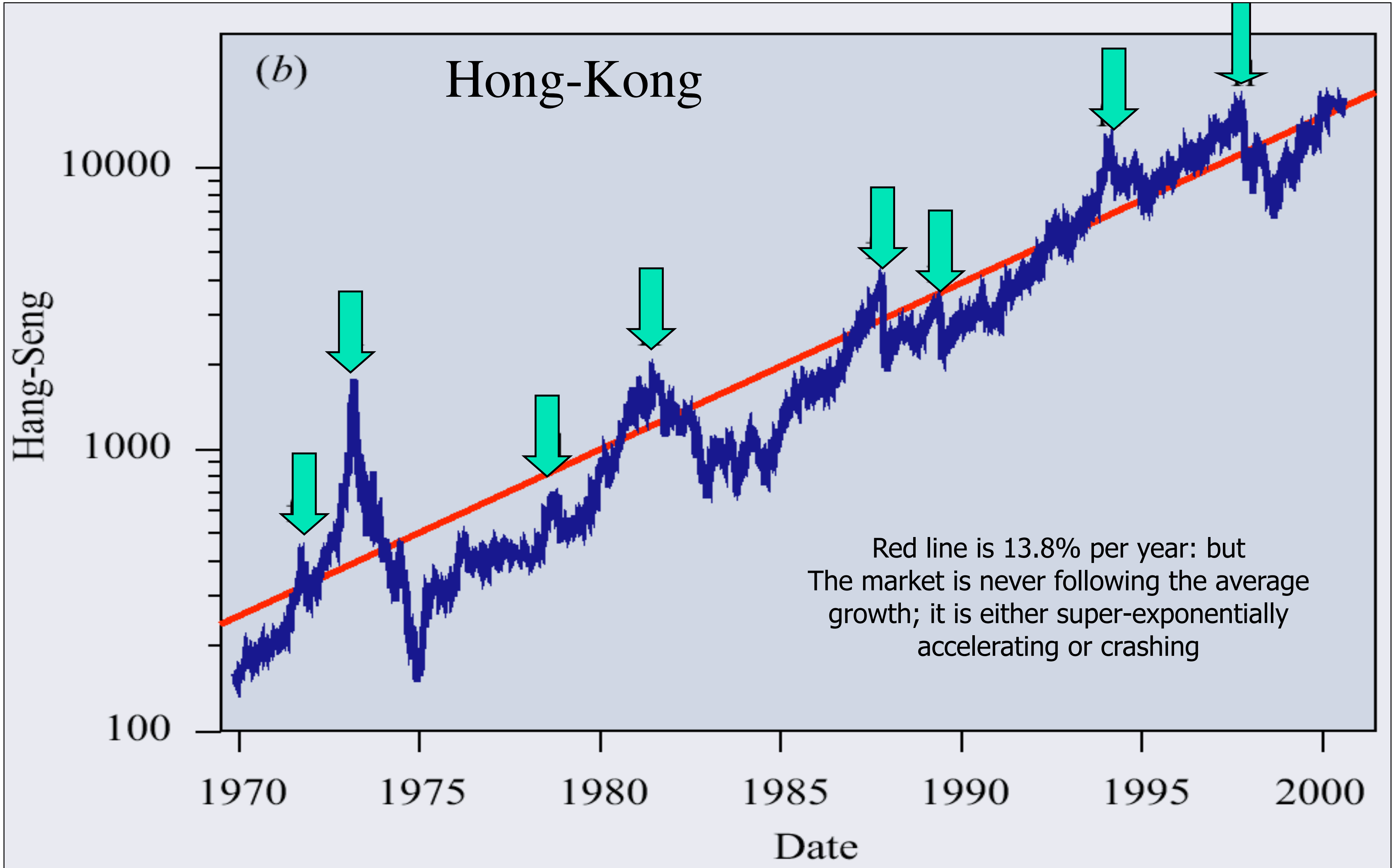
- ***Hypothesis H1*: financial (and other) bubbles can be diagnosed in real-time before they end.**

- ***Hypothesis H2*: The termination of financial (and other) bubbles can be bracketed using probabilistic forecasts, with a reliability better than chance (which remains to be quantified).**

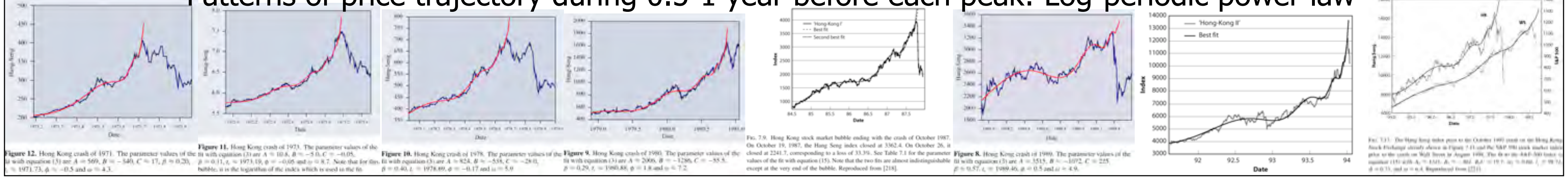
Many other bubbles and crashes

- Hong-Kong crashes: 1987, 1994, 1997 and many others
- October 1997 mini-crash
- August 1998
- Slow crash of spring 1962
- Latin-american crashes
- Asian market crashes
- Russian crashes
- Individual companies



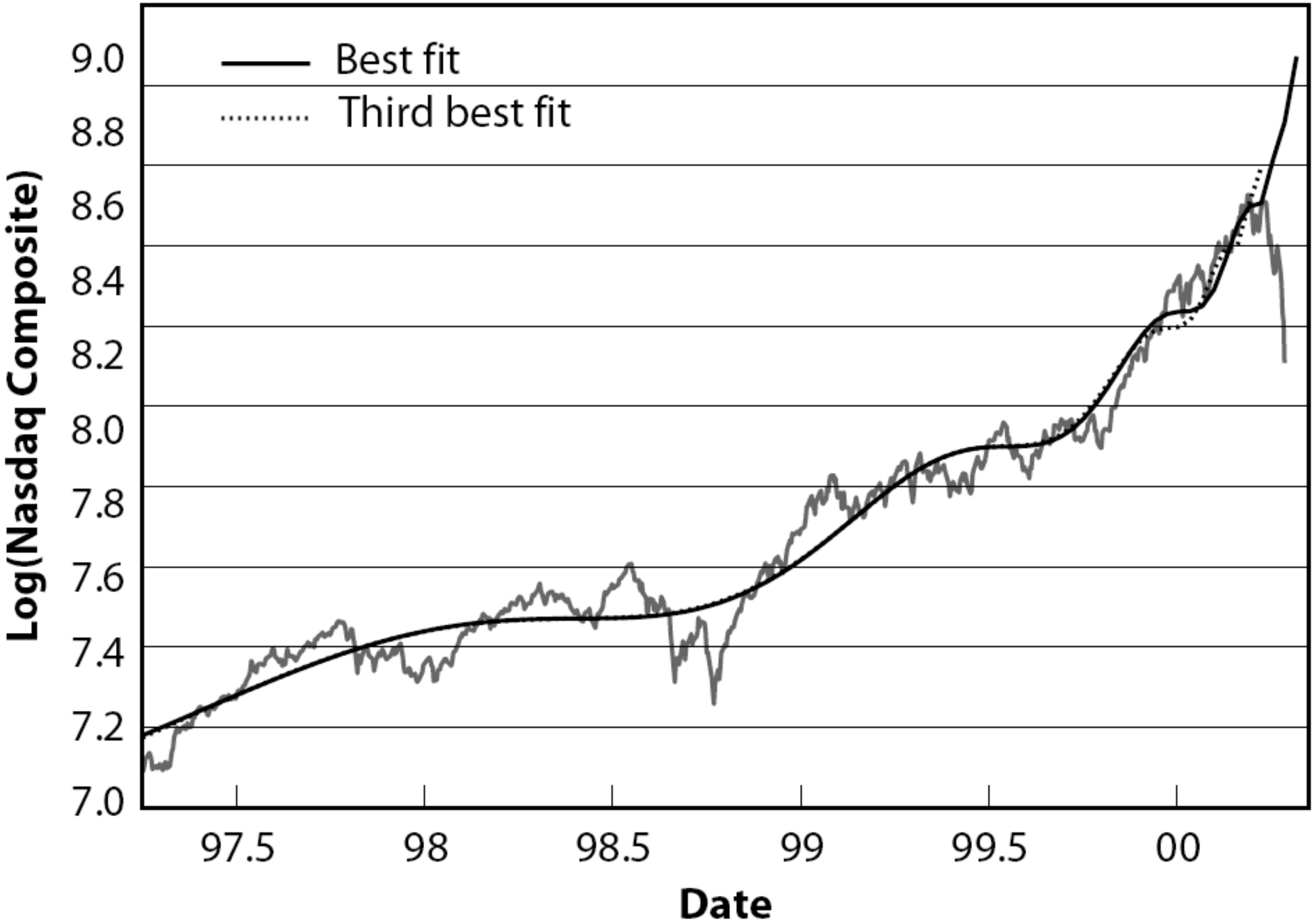


Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law



THE NASDAQ CRASH OF APRIL 2000

"New Economy": ICT



Real-estate in the UK

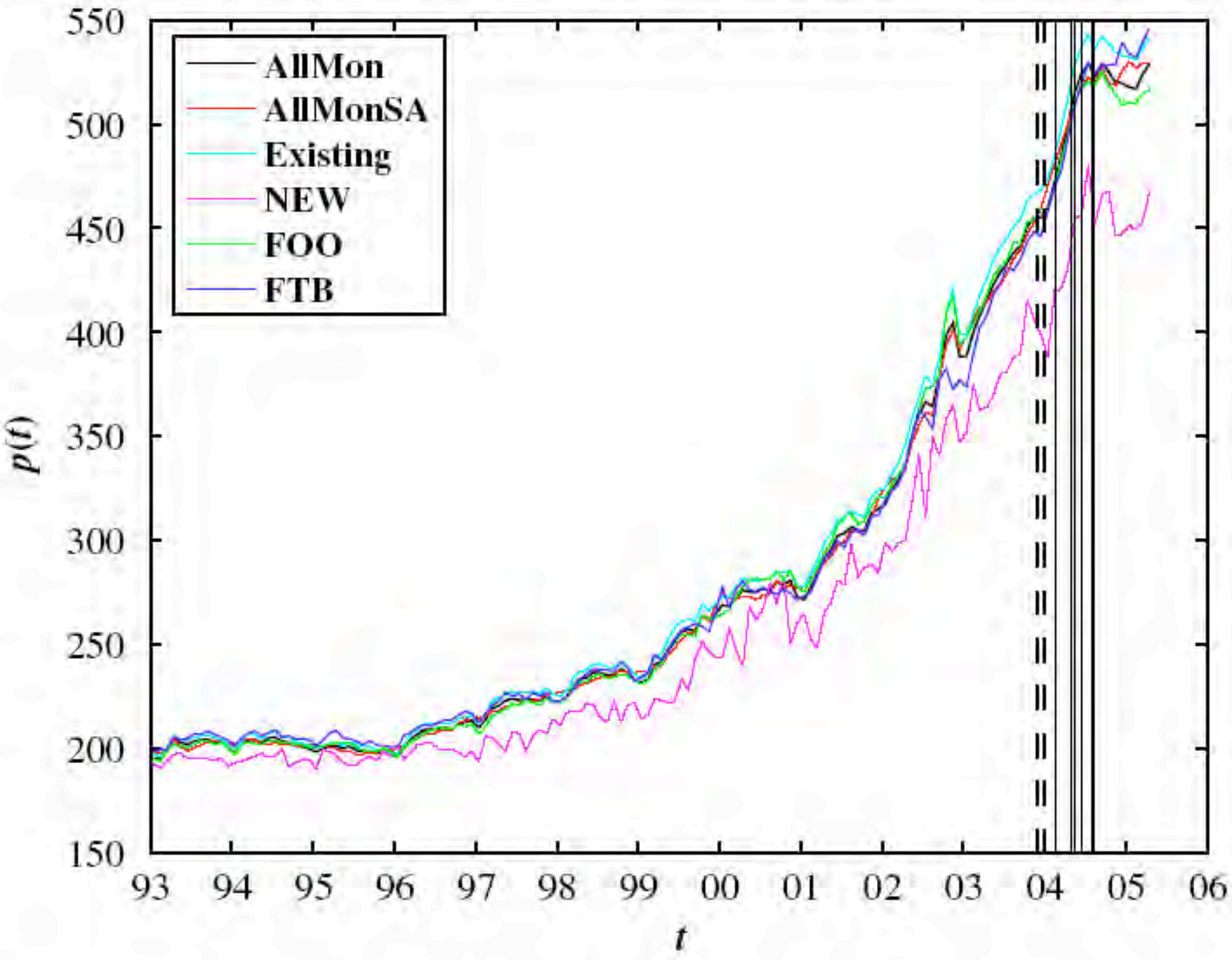


Fig. 1. (Color online) Plot of the UK Halifax house price indices from 1993 to April 2005 (the latest available quote at the time of writing). The two groups of vertical lines correspond to the two predicted turning points reported in Tables 2 and 3 of [1]: end of 2003 and mid-2004. The former (resp. later) was based on the use of formula (2) (resp. (3)). These predictions were performed in February 2003.

W.-X. Zhou, D. Sornette, 2000–2003 real estate bubble in the UK but not in the USA, Physica A 329 (2003) 249–263.

Real-estate in the USA

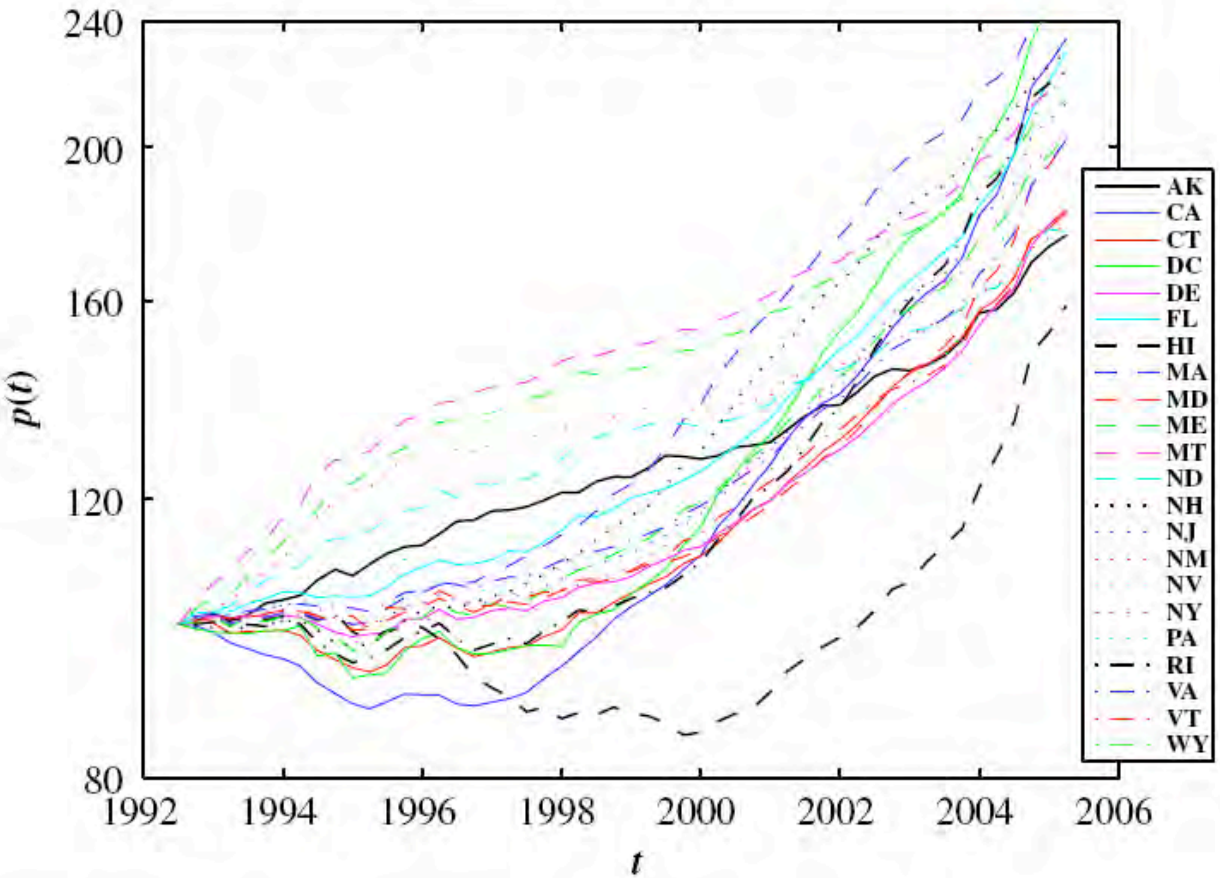
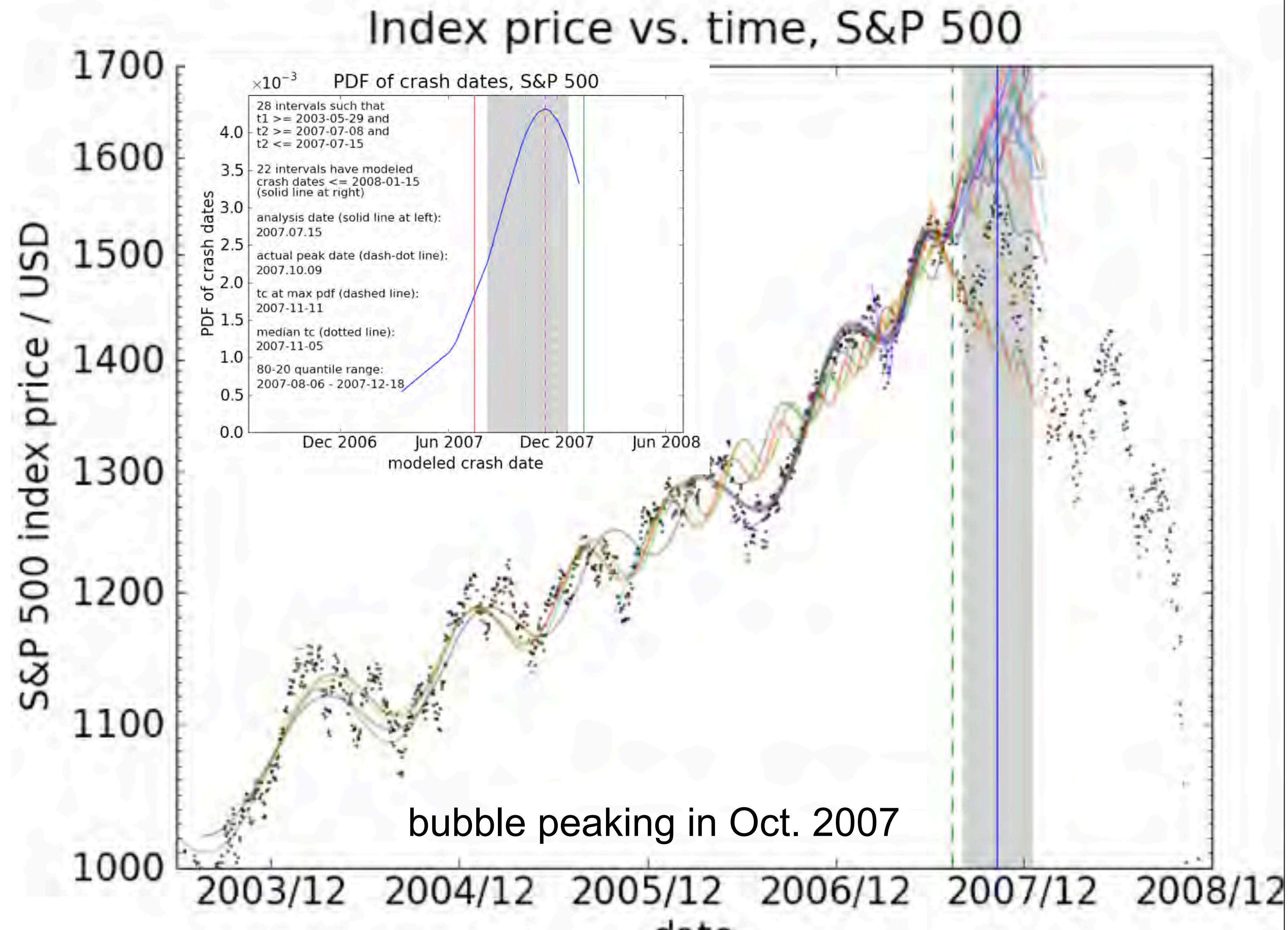


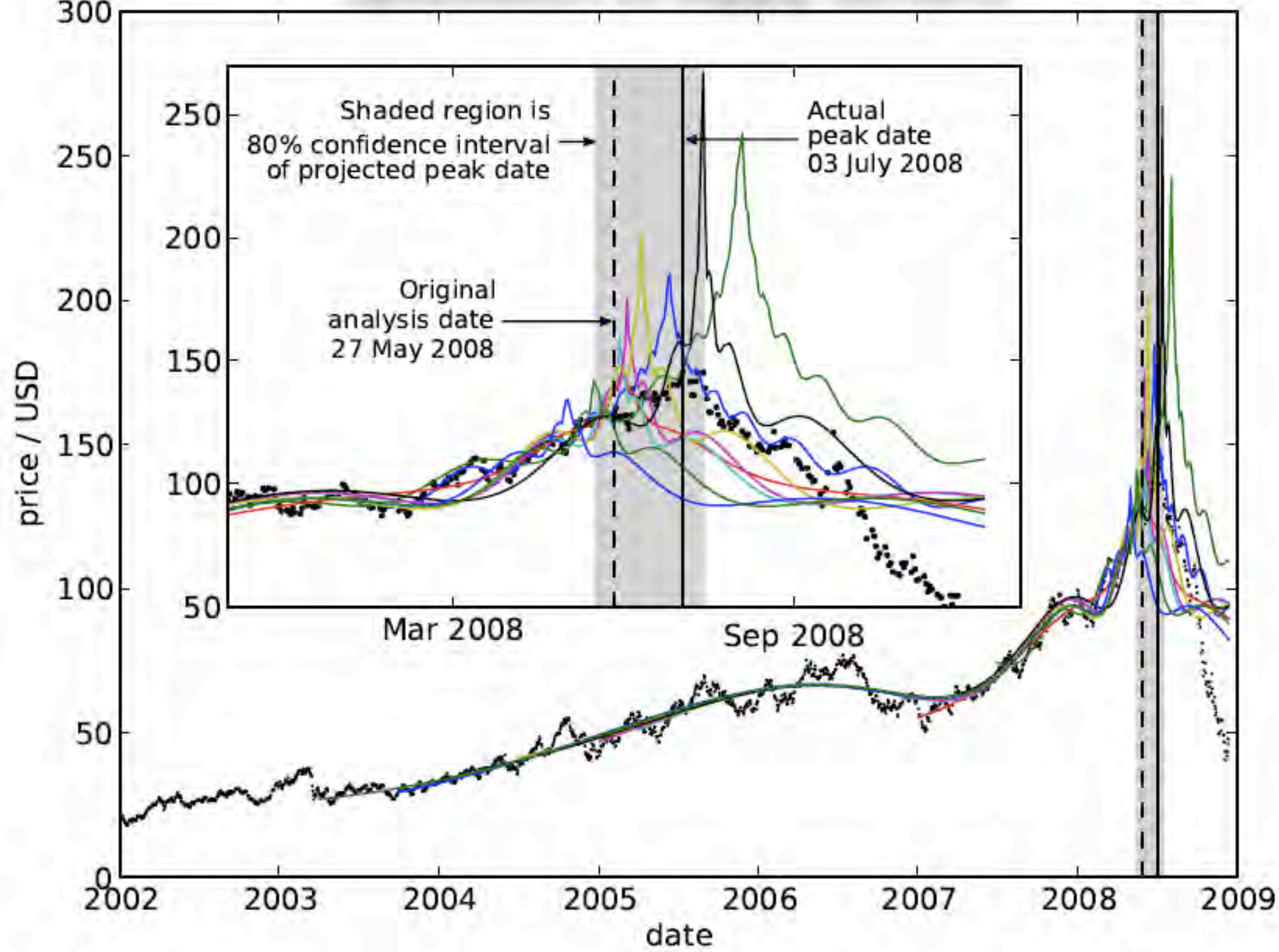
Fig. 5. (Color online) Quarterly average HPI in the 21 states and in the District of Columbia (DC) exhibiting a clear upward faster-than-exponential growth. For better representation, we have normalized the house price indices for the second quarter of 1992 to 100 in all 22 cases. The corresponding states are given in the legend.

W.-X. Zhou, D. Sornette / Physica A 361 (2006) 297–308



Source: R. Woodard (FCO, ETH Zurich)

2006-2008 Oil bubble
Speculation vs supply-demand



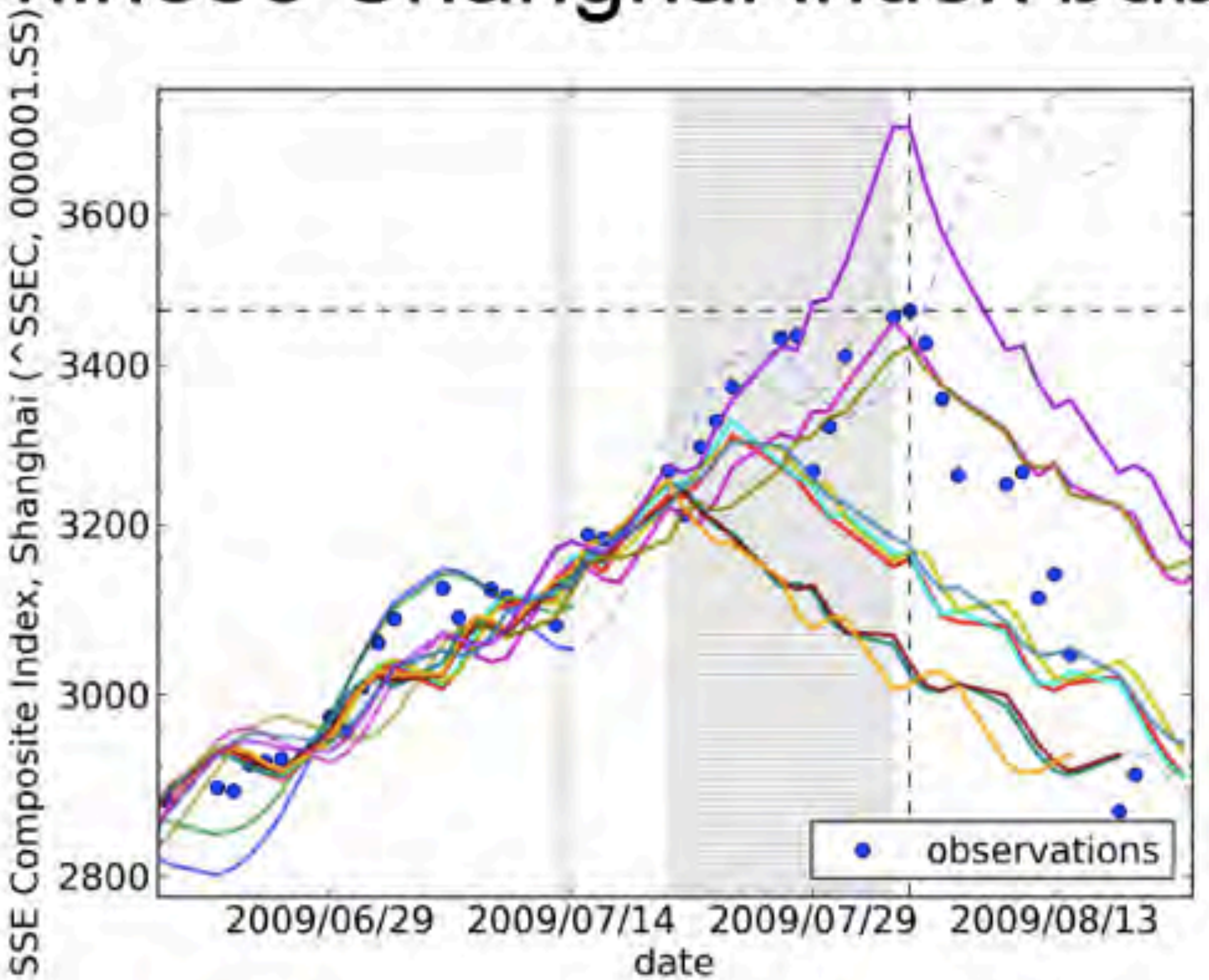
D. Sornette, R. Woodard and W.-X. Zhou, The 2006-2008 Oil Bubble and Beyond, Physica A 388, 1571-1576 (2009) (arXiv.org/abs/0806.1170)

Typical result of the calibration of the simple LPPL model to the oil price in US$ in shrinking windows with starting dates t_{start} moving up towards the common last date t_{last} = May 27, 2008.

Source: R. Woodard (FCO, ETH Zurich)

10 July 2009

Successful forecast of end of Chinese Shanghai index bubble



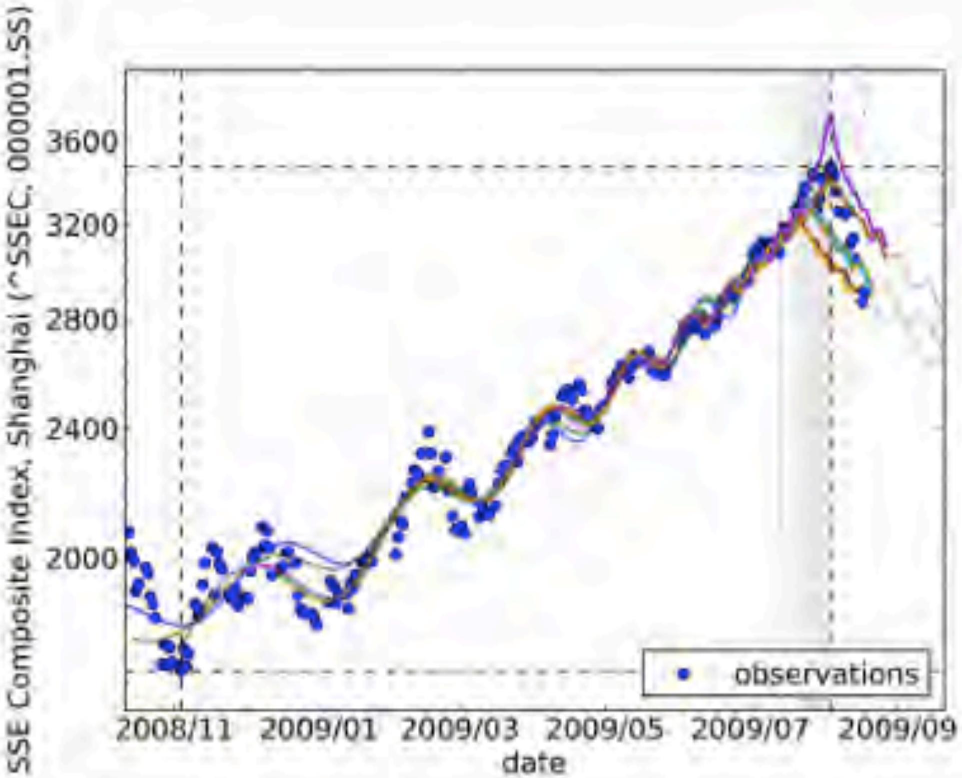
The Chinese Equity Bubble: Ready to Burst,

K. Bastiaensen, P. Cauwels, D. Sornette, R. Woodard and W.-X. Zhou, July 10, 2009 (http://arxiv.org/abs/0907.1827)

FCO@ETH: Towards operational science of financial instabilities

Didier Sornette, Maxim Fedorovsky, Stefan Riemann, Hilary Woodard, Ryan Woodard, Wanfeng Yan, Wei-Xing Zhou

- Main mission:
  - Identify bubbles
- Theory:
  - Positive feedback
- Deliverables
  - Weekly global bubble scan
  - Research, papers
  - Public forecasts
  - Digital timestamps



www.er.ethz.ch/fco

Financial Crisis Observatory

The Financial Crisis Observatory (FCO) is a scientific platform aimed at testing and quantifying rigorously, in a systematic way and on a large scale the hypothesis that financial markets exhibit a degree of inefficiency and a potential for predictability, especially during regimes when bubbles develop.

Entrepreneurial Risks

Reports
Internal reports

FCO RSS Feed
RSS

Financial Bubble Experiment

1 Nov, 2009

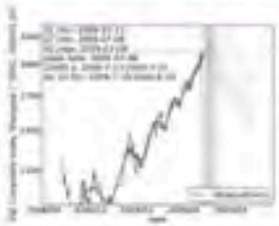
We introduce a new experiment involving the forecasts of the end of bubbles in financial time series using techniques developed over the past 15 years. The majority of forecasts that we have made in the past have been published after we found them to be successful. That is, we have predicted certain bubbles to end and then have written about the post-mortem analysis. In this new experiment, we propose a new method of delivering our forecasts where the results are revealed only after the predicted event has passed but where the original date when we produced these same results can be publically, digitally authenticated. More information can be found in the first delivery of the Financial Bubble Experiment.

Highlighted Papers

D. Sornette, Dragon-Kings, Black Swans and the Prediction of Crises, in press in the International Journal of Terraspace Science and Engineering (2009), (http://arXiv.org/abs/0907.4290)
Featured on the FT blog "Dragon-king of the outlier events"

Didier Sornette and Ryan Woodard, Financial Bubbles, Real Estate bubbles, Derivative Bubbles, and the Financial and Economic Crisis (2009), (http://arxiv.org/abs/0905.0220), to appear in the Proceedings of APFA7 (Applications of Physics in Financial Analysis). This conference series, organized by Misako Takayasu and Tsutomu Watanabe, focuses on the analysis of large-scale Economic data, (http://www.thic-apfa7.com/en/htm/index.html)

Past analysis and forecasts



CHINESE EQUITY (10 July 2009)
Amid the current financial crisis, there has been one equity index beating all others: the Shanghai Composite. Our analysis of this main Chinese equity index shows clear signatures of a bubble build up and we go on to predict its most likely crash date: **July 17-27, 2009** (20%/80% quantile confidence interval). See full analysis and results in this paper.

CDS (19 February 2009)
Our analysis has been performed on data kindly provided by Amjed Younis of Fortis on 19 February 2009. It consists of 3 data sets: credit default swaps (CDS); German bond futures prices; and spread evolution of several key euro zone sovereigns. The date

Weekly scan of (soon-to-be) all major global stock exchanges and indexes.

FCO Report - US - P bubbles - 21 October

Academic Portfolio Tracking

- Allow non-integer number of shares
- Use daily adjusted closing prices
- No transaction fees

[FCO] [FCO Reports]

SP500	SP1000
Weekly reports	Weekly reports
Historical appearances sorted by:	Historical appearances sorted by:
• Stock • Sector • Crashes/Rallies	• Stock • Sector • Crashes/Rallies
Portfolio transactions:	Portfolio transactions:

Ticker info			Last t1 (1 July 2009)				Qualified fits					Indicators					News	ER
Ticker	Sector	num. qual. fits	<ret>	<ret> over risk	fraction rets > 0	max. dd index obs. fit	<ret>	<ret> over risk	fraction rets > 0	max. dd index obs. fit	max. dd t1	Bull fit obs	Vol	Crash	Bull Vol Crash	20 Q Crash 80 Q	Bloomberg Google Yahoo	Recommends
AAN	Consumer Discretionary	4	-0.129	-5.076	0.000		-0.042	-0.526	0.136		2008-04-01	0.188 0.656	0.086	86	DVC -VC	2009-12-12 2010-01-14 2010-04-20	B G Y	Abstain
ASEI	Industrials	7	-0.043	-1.309	0.000		-0.010	-0.100	0.481		2008-04-01	0.448 0.580	0.106	59	-VC -VC	2009-11-13 2009-12-18 2010-02-17	B G Y	Abstain
B	Industrials	3	0.072	1.522	1.000		0.007	0.043	0.424		2008-01-01	0.315 0.357	0.188	21	DHC DHC	2009-10-28 2009-11-10 2009-12-04	B G Y	Abstain And Watch Crash Risk
BRKL	Financials	3	-0.064	-1.157	0.091		-0.002	-0.021	0.545		2008-04-01	0.487 0.884	0.080	57	-C U-C	2009-11-10 2009-12-16 2010-03-02	B G Y	Abstain
BWLD	Consumer Discretionary	10	0.278	1.897	1.000		0.003	0.012	0.382		2007-01-01	0.450 0.154	0.228	148	-H- DH-	2009-11-21 2010-03-17 2010-06-18	B G Y	Abstain
[illegible]	Financials	7	[illegible]	[illegible]	1.000		[illegible]	[illegible]	0.545		[illegible]	0.609	[illegible]	[illegible]	-HC	2009-10-28 [illegible]	B G Y	Abstain

METHODOLOGY OF THE FINANCIAL BUBBLE EXPERIMENT

- We choose a series of dates with a fixed periodicity on which we will reveal our forecasts (1 May 2010 + 6 months periodicity)
- Continuous research of +30'000 global financial time series.
- Confident forecast => summarize it in a simple .pdf document
- We do not make this document public.
- We make its digital fingerprint public (MD5 hash algorithm and SHA-2 hash) => three strings of letters and numbers that are unique to this file.
- First version of our "meta" document (description of our theory and methods, the MD5 and SHA-2 hashes of our first forecast and the date (1 May 2010) on which we will make the first original .pdf document public)
- Upload to http://arxiv.org. It makes public the MD5 and SHA-2 hashes of our first forecasts + independent timestamp 'v1' (version 1) (trusted third party)
- Next confident forecast => new secret .pdf file and public SHA-2 hash in 'v2' on http://arxiv.org.
- We continue this protocol until 1 May 2010 at which time we upload our final version of the master document and publish all .pdf forecast files + our summary and analysis of the forecasts.

The Financial Bubble Experiment: advanced diagnostics and forecasts of bubble terminations

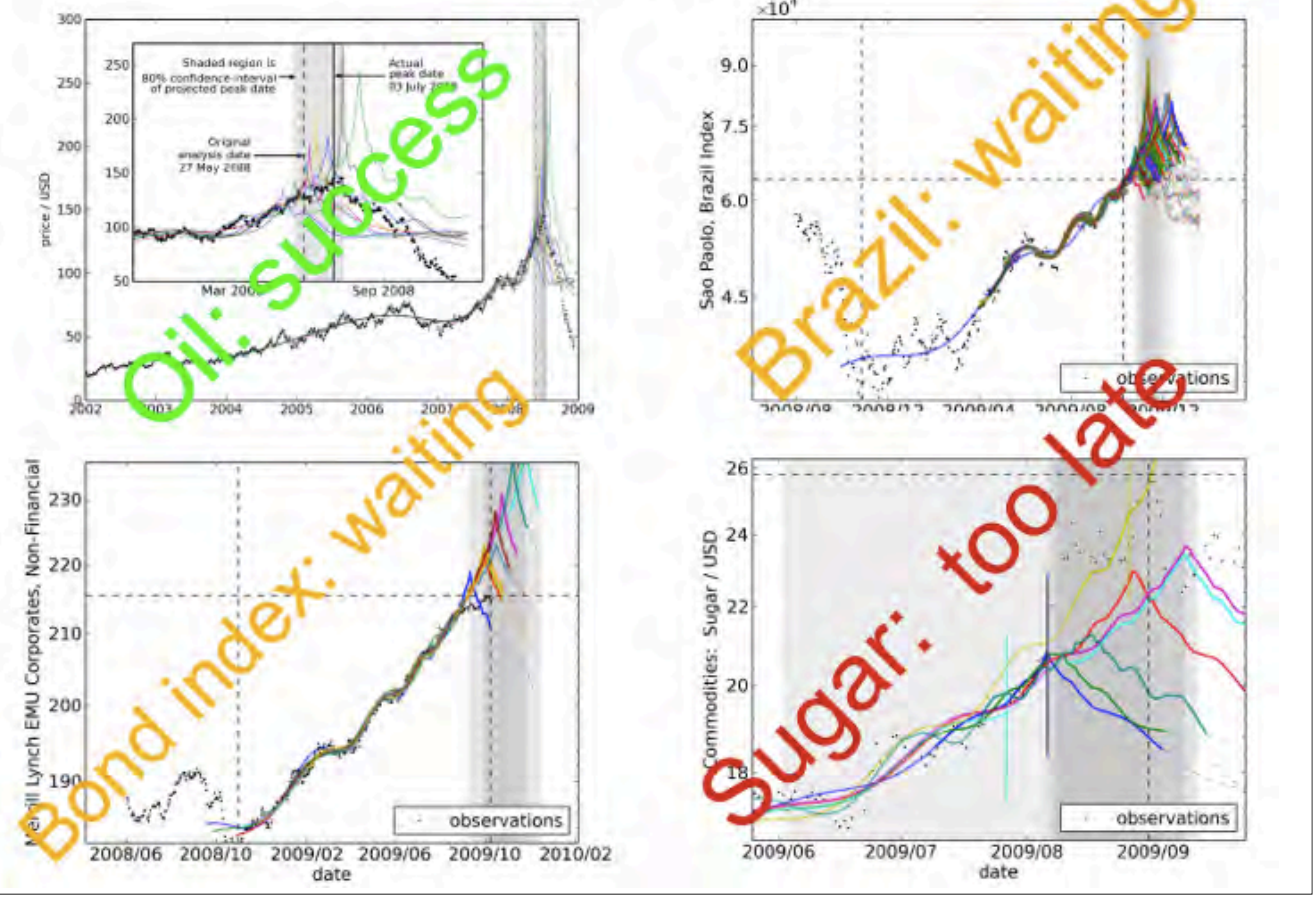
The Financial Crisis Observatory*
Department of Management, Technology and Economics, ETH Zurich, Kreuzplatz 5, CH-8032 Zurich, Switzerland
(Dated: November 2, 2009)

Publication date	MD5SUM SHA256SUM SHA512SUM
2009-11-02	6d9479eb2849115a12c219cfa902990e d7ad5c9531166917ba97f871fb61bd1f6290b4b4ce54e3ba0c26b42e2661dc06 808bbfaddbca3db8d0f55d74cabedf5201ecd70340f86e27dfac589ce682144f52f6fc4b3ff1ac75231038d86dae58bd320e7fb17ef321b4bc61a19e88071039
2009-11-02	5d375b742a9955d4aeea1bd5c7220b2b 5a9c395b9ab1d2014729ac5ff3bb22a352e14096fa43c59836ea0d4ae0e3b453 e7ef9150b4738253f4021b0600eff1cd455b2671e421b788b9268b518439b56699994b3f8b395742bdc7622b5536034e74ade86e0a46bff71ed5ff9a293f809f
2009-11-02	fd85000d0ce3231892ef1257d2f7ab1e d3f3d504d85d50eb3dc0fe2c3042746db2f010509f4d1717370d14012972e86f 91a8fa82b7f08deea2df2a1f7cef266f5aa155bb0c047f65b14315f7229d92976cc7b30453453fb8ecd0350783907c83652192d32ba90fb1cce128385832e63a

TABLE I: Checksums of Financial Bubble Experiment forecast documents.

arXiv:0911.0454v1 [q-fin.CP] 2 Nov 2009

Oil, Brazil, bonds, sugar



Thursday, November 05, 2009

Forecasting financial crashes: the ultimate experiment begins

If a new technique for predicting crashes really works, a bold new experiment will measure how well.

Technology Review PUBLISHED BY MIT

Is it really possible to predict the end of financial bubbles? Didier Sornette at the Swiss Federal Institute of Technology in Zurich thinks so and has set up the Financial Crisis Observatory at ETH to study the idea.

We've looked at his extraordinary predictions before. Earlier this year, he identified a bubble in the Shanghai Composite Index and much to this blog's surprise, forecast its end with remarkable accuracy.

Final remarks

1-All proposals will fail if we do not have better science and better metrics to monitor and diagnose (ex: biology, medicine, astronomy, chemistry, physics, evolution, and so on)

2-Leverage as a system variable versus the illusion of control by monetary policy, risk management, and all that

3-Need to make endogenous policy makers and regulators ("creationist" view of government role, illusion of control and law of unintended consequences of regulations)

4-Fundamental interplay between system instability and growth; the positive side of (some) bubbles

5-Time to reassess goals (growth vs sustainability vs happiness). In the end, endogenous co-evolution of culture, society and economy

KEY CHALLENGE: genuine trans-disciplinarity by
TRAINING in 2-3 disciplines + CHANGE OF CULTURE

Why bubbles are not arbitraged away?

1. limits to arbitrage caused by noise traders (DeLong et, 1990)
2. limits to arbitrage caused by synchronization risk (Abreu and Brunnermeier, 2002 and 2003)
3. short-sale constraints (many papers)
4. lack of close substitutes for hedging (many papers)
5. heterogenous beliefs (many papers)
6. lack of higher-order mutual knowledge (Allen, Morris and Postlewaite, 1993)
7. delegated investments (Allen and Gorton, 1993)
8. psychological biases (observed in many experiments)
9. positive feedback bubbles

Further Reading

T. Kaizoji and D. Sornette, Market Bubbles and Crashes, in press in the Encyclopedia of Quantitative Finance (Wiley, 2008)
(preprint at http://arxiv.org/abs/0812.2449)

D. Sornette and R. Woodard Financial Bubbles, Real Estate bubbles, Derivative Bubbles, and the Financial and Economic Crisis
(preprint at http://arxiv.org/abs/0905.0220) will appear in the Proceedings of APFA7 (Applications of Physics in Financial Analysis, http://www.thic-apfa7.com/en/htm/index.html)

D. Sornette, Dragon-Kings, Black Swans and the Prediction of Crises,
in press in the International Journal of Terraspace Science and Engineering (2009)
(http://arXiv.org/abs/0907.4290)

Didier Sornette, Why Stock Markets Crash
(Critical Events in Complex Financial Systems)
Princeton University Press, January 2003

Y. Malevergne and D. Sornette, Extreme Financial Risks (From Dependence to Risk Management) (Springer, Heidelberg, 2006).