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Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

D MTEC Departement Management, Technology, and Economics

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Entrepreneurial Risks

FUTURE RESILIENT SYSTEMS 未来风险管理科学

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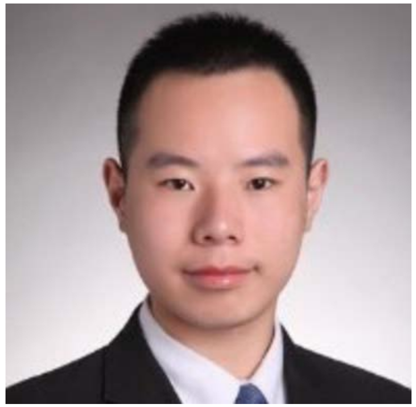
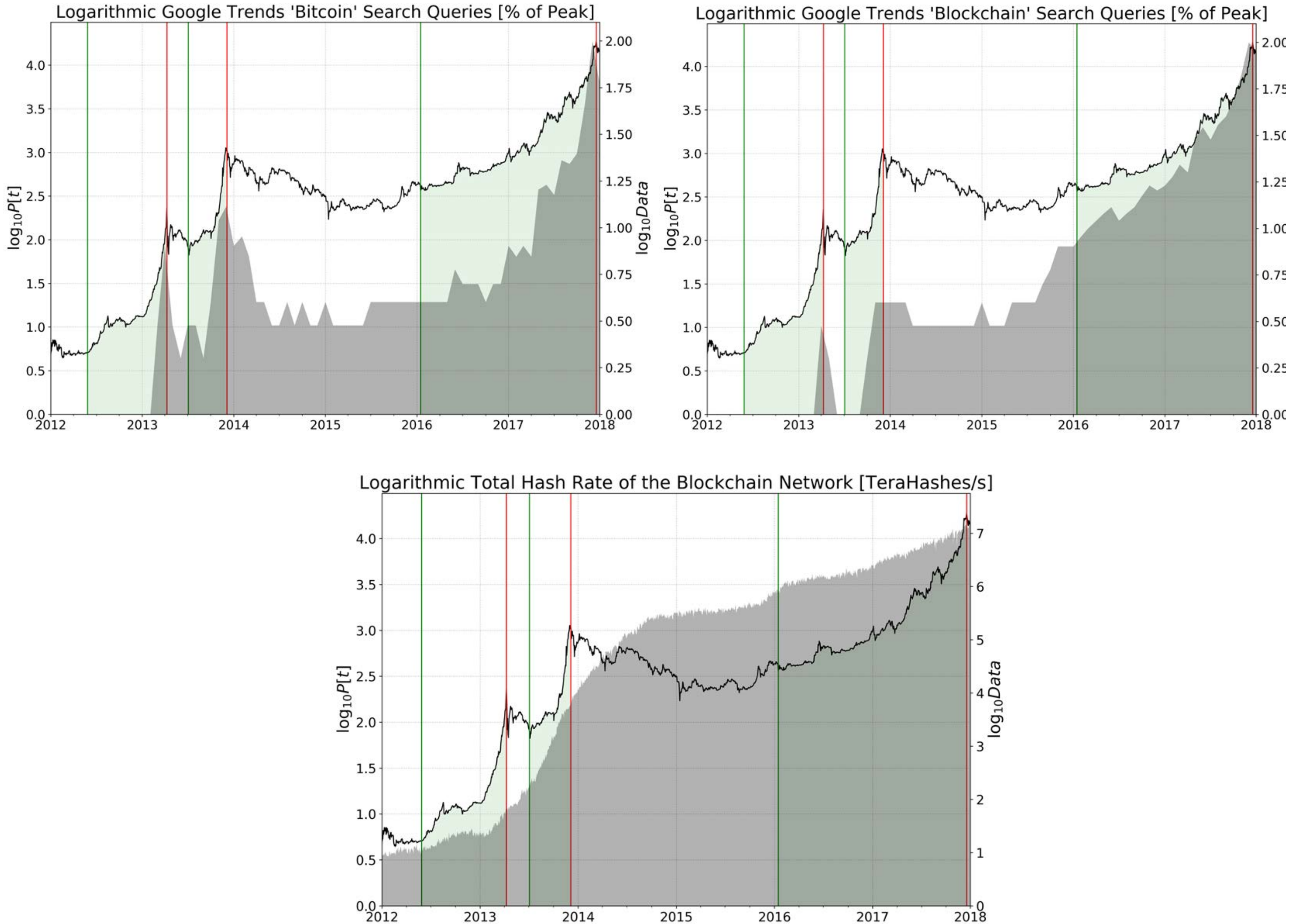
Quantification of the crypto-currency ecosystem:

Generalised Zipf law of capitalisations
and
the hierarchy of bitcoin bubbles

Jan-Christian Gerlach, **Didier Sornette**, Dr. Spencer Wheatley and Ke Wu

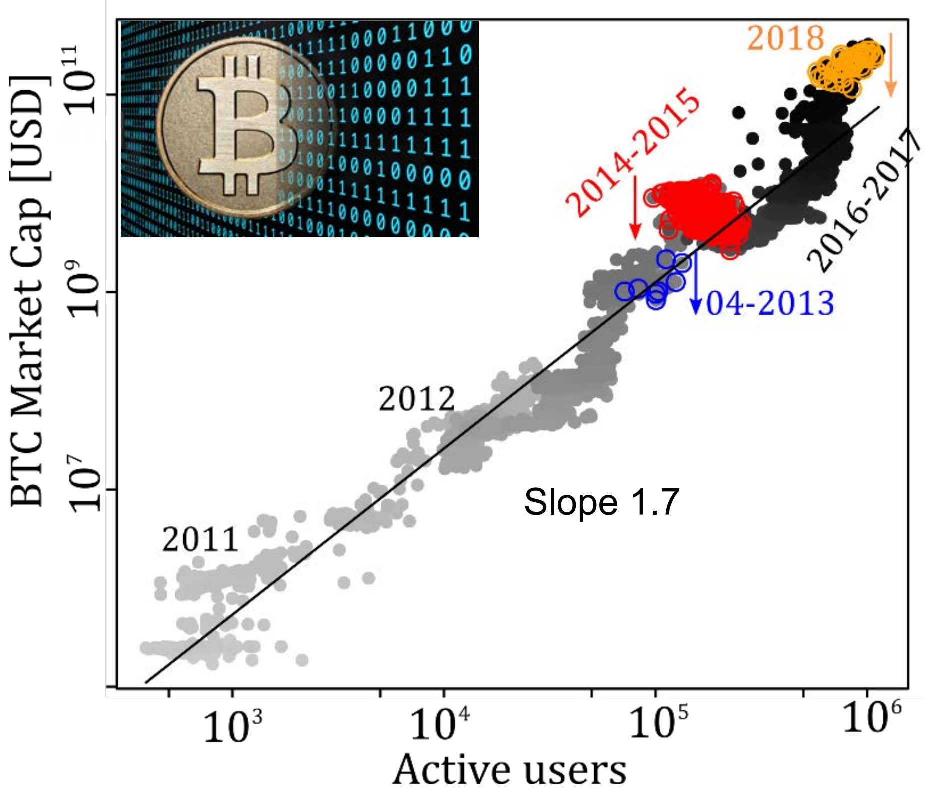


www.er.ethz.ch



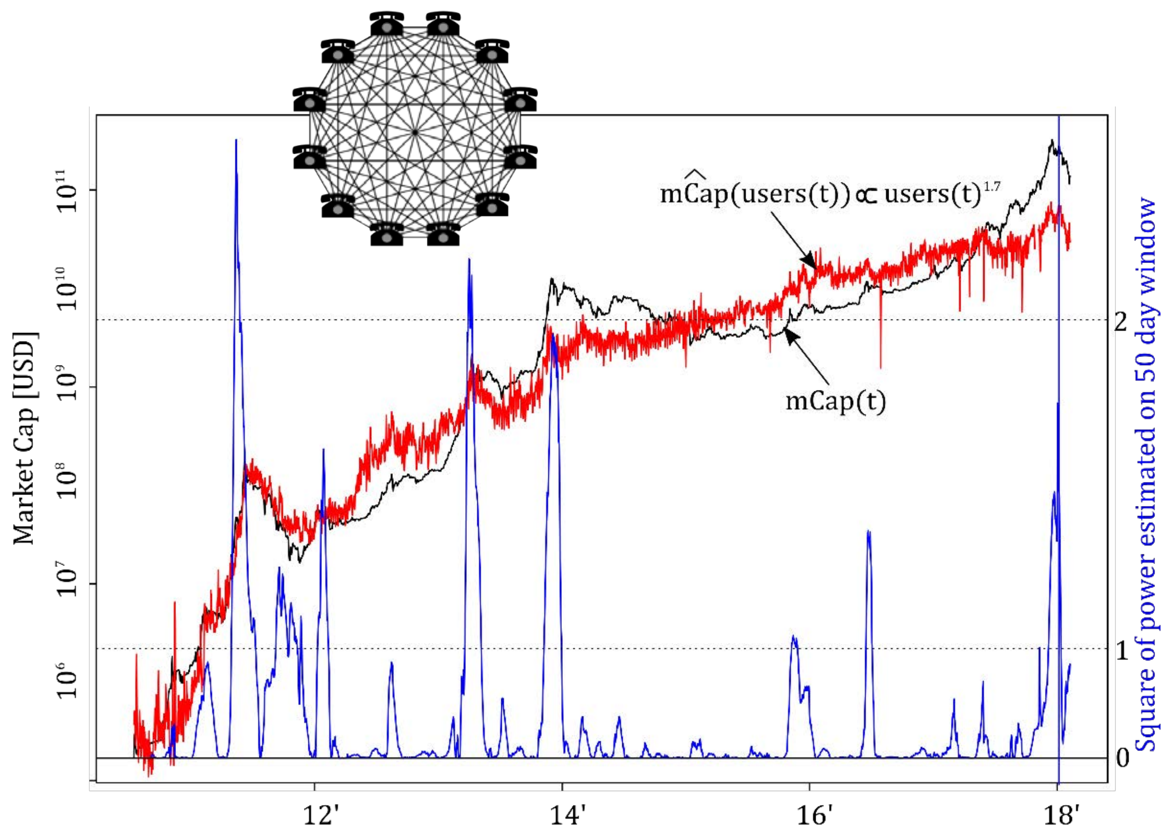
Logarithmic Google Trends 'Bitcoin' Search Queries [% of Peak]
$\log_{10}P[t]$
$\log_{10}Data$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2012
2013
2014
2015
2016
2017
2018
Logarithmic Google Trends 'Blockchain' Search Queries [% of Peak]
Logarithmic Total Hash Rate of the Blockchain Network [TeraHashes/s]
0
1
2
3
4
5
6
7

Valuation: "Network Effects" & Metcalfe's Law



Bitcoin mcap versus active users*, grey to black over time, with major drawdowns indicated from 2013, 2014-2015, and early 2018. Metalfe regression fit with power 1.7.

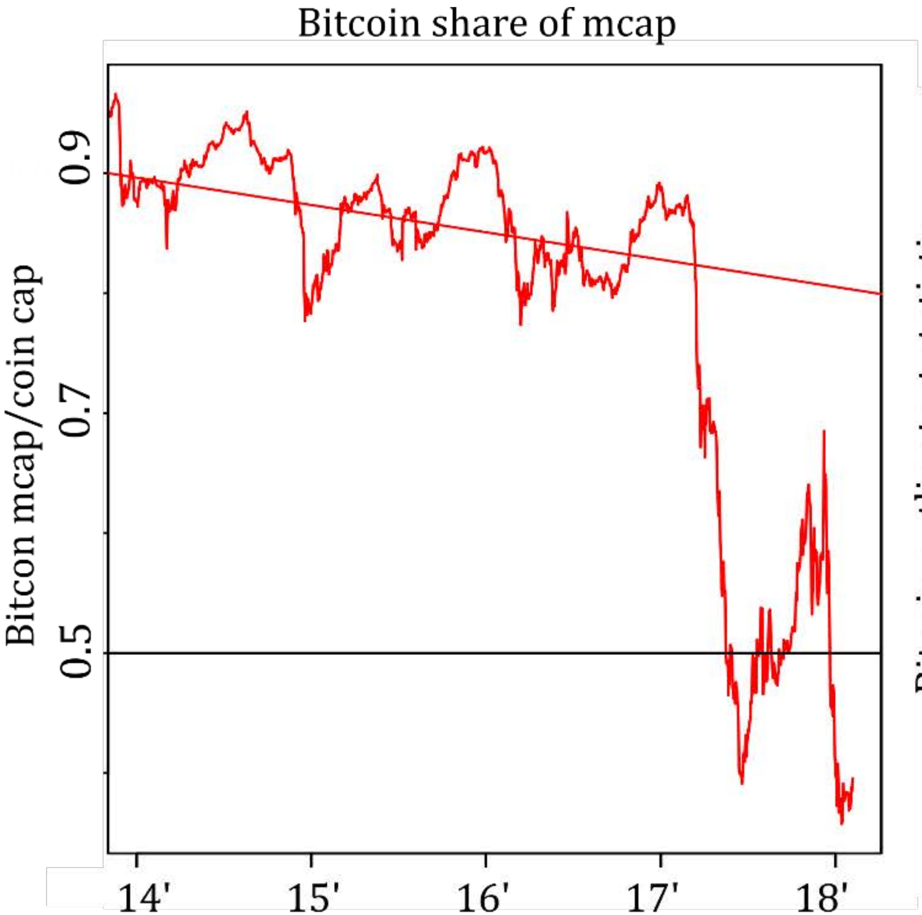
*Unique addresses making transactions. https://bitinfocharts.com/

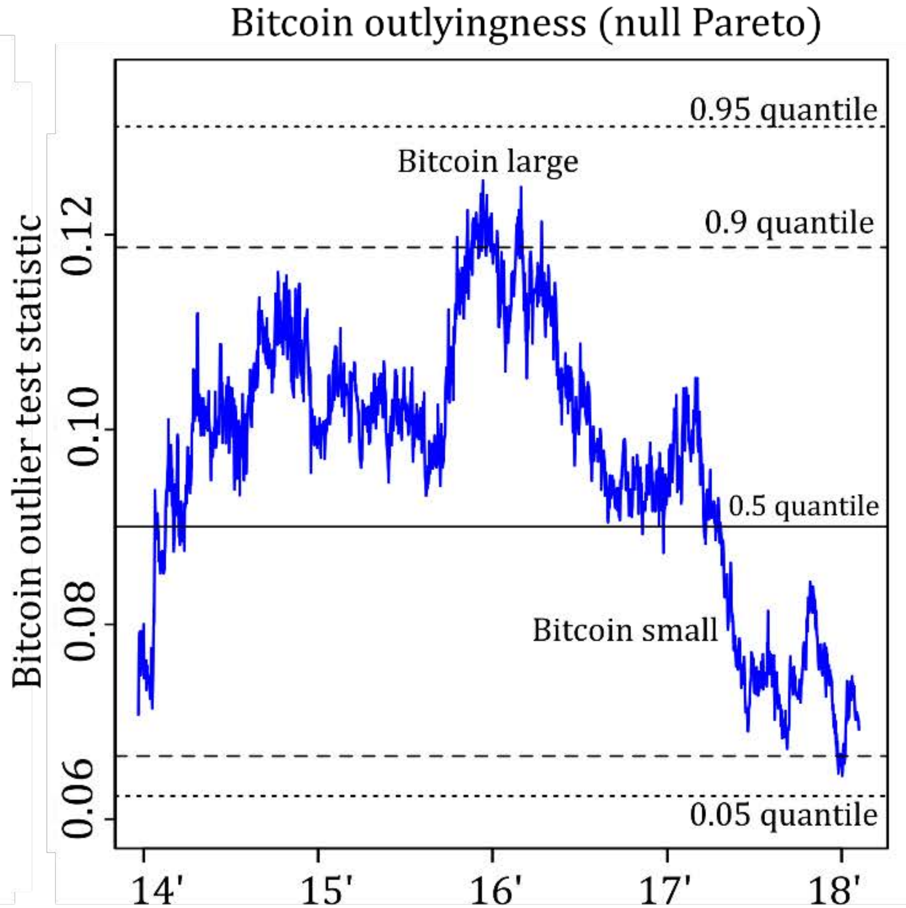


Bitcoin mcap (black), mcap implied by active users to power 1.7 (red), and square of the power estimated on 50 day moving window.

- **Metcalfe's law:** value of a telecom network is proportional to the square of the number of connected users (n^2). Prescribed for valuation of cryptos.
- # Bitcoin users unknown. At least 15 Mil users on Coinbase. Consider active users*, having growth rate about 0.0012 per day (from 10k in 2014 to 50-100k now).
- Regression gives power of 1.7, less than Metcalfe's value
- Power often zero, but bursts of high values (≥2) drive growth spurts.
- Model indicates current price of $5-10 thousand per Bitcoin, on the range of cost of mining. Assumes continued user growth.
- Misvaluation? Ethereum has similar number of active users, with faster growth, but mcap scales with power 1.3, hence lower price.
- Number of users grows exponentially, but we observe superexponential price behaviour…

Bitcoin dominance/maximalism: can there be only one?





Bitcoin mcap divided by top 100 coins mcap in stationary exponential (transformed) sample[1] Null quantiles given.

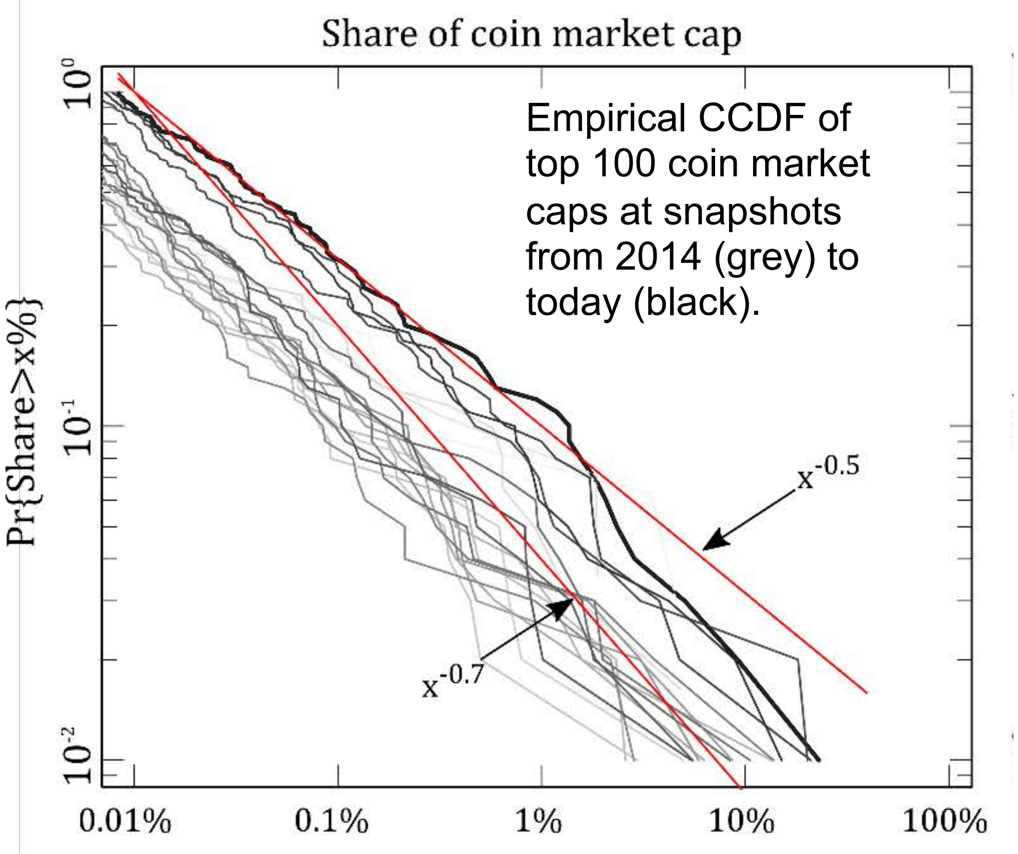
1. Wheatley, S. and Sornette, D. Multiple outlier detection in samples with exponential & pareto tails: Redeeming the inward approach & detecting dragon kings, 15–28. Geneva: Swiss Finance Institute Research Paper.

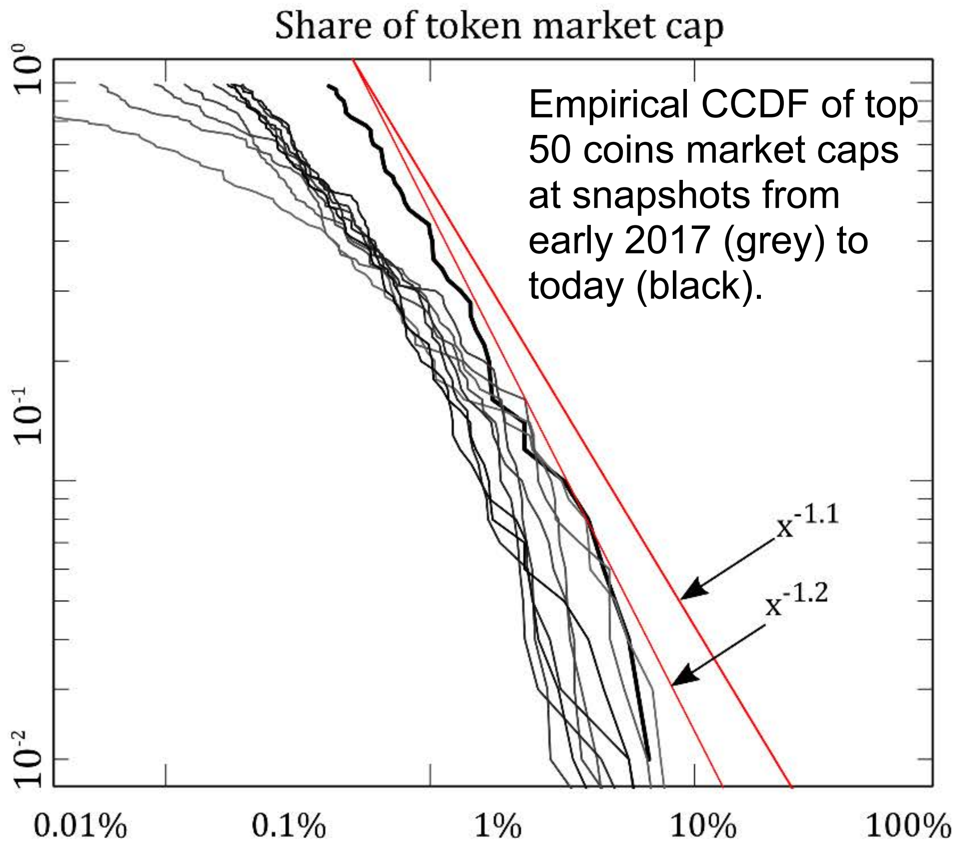
- (By now the answer is largely apparent)
- Is/was Bitcoin an outlier?
- Bitcoin dominated mcap, but is now < 40%.
- Assuming Pareto mcap distribution: BTC was somewhat beyond distribution, but now is somewhat too small

- Proportional growth* / pref. attachment "network effect" → rich get richer and first mover advantage
- Considering varying coin fitness indicates that BTC will soon be overtaken by next generation coins. "fit get rich"**

*Simon, Herbert A., and Charles P. Bonini. "The size distribution of business firms." *The American economic review*48.4 (1958): 607-617.
**Bianconni G, Barab´asi A (2001). “Competition and Multiscaling in Evolving Networks.” Europhysics Letters, 54, 436.
Caldarelli, Guido, et al. "Scale-free networks from varying vertex intrinsic fitness." *Physical review letters* 89.25 (2002): 258702.

Coin & Token share of mcap distribution



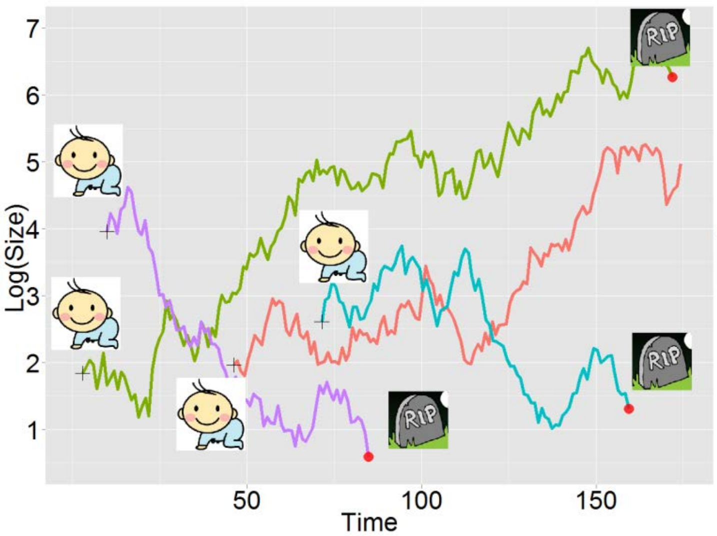


- Coins and tokens are different: Coins much more heavy tailed
  - Only 25 tokens at 01-2017; now more than 400
  - 75% of tokens are on the ETH network

- Lognormal versus Pareto tail:
  - Coin market cap: top 275 (out of >500 coins), lognormal not superior to Pareto (at p=0.05 level)*
  - Token market cap: Evolving towards Zipf law: For the top 50 tokens the lognormal is not superior to the Pareto.

Malevergne, Yannick, Vladilen Pisarenko, and Didier Sornette. "Testing the Pareto against the lognormal distributions with the uniformly most powerful unbiased test applied to the distribution of cities." *Physical Review E* 83.3 (2011): 036111.

Birth + Proportional Growth + Stochastic Death



1) Growth of intensity of birth (i.e. $\nu(t) = a\exp(dt)$)
2) Growth of average initial size (i.e. $S_0(t) = b\exp(c_0 t)$)
3) Proportional growth (i.e. GBM: $dS(t) = rS(t) + \sigma S(t)dW(t)$)
4) Death-hazard rate (i.e. $Q(t) = \exp(-ht)$)
=>induces **5 key parameters d, c_0, r, σ, h**

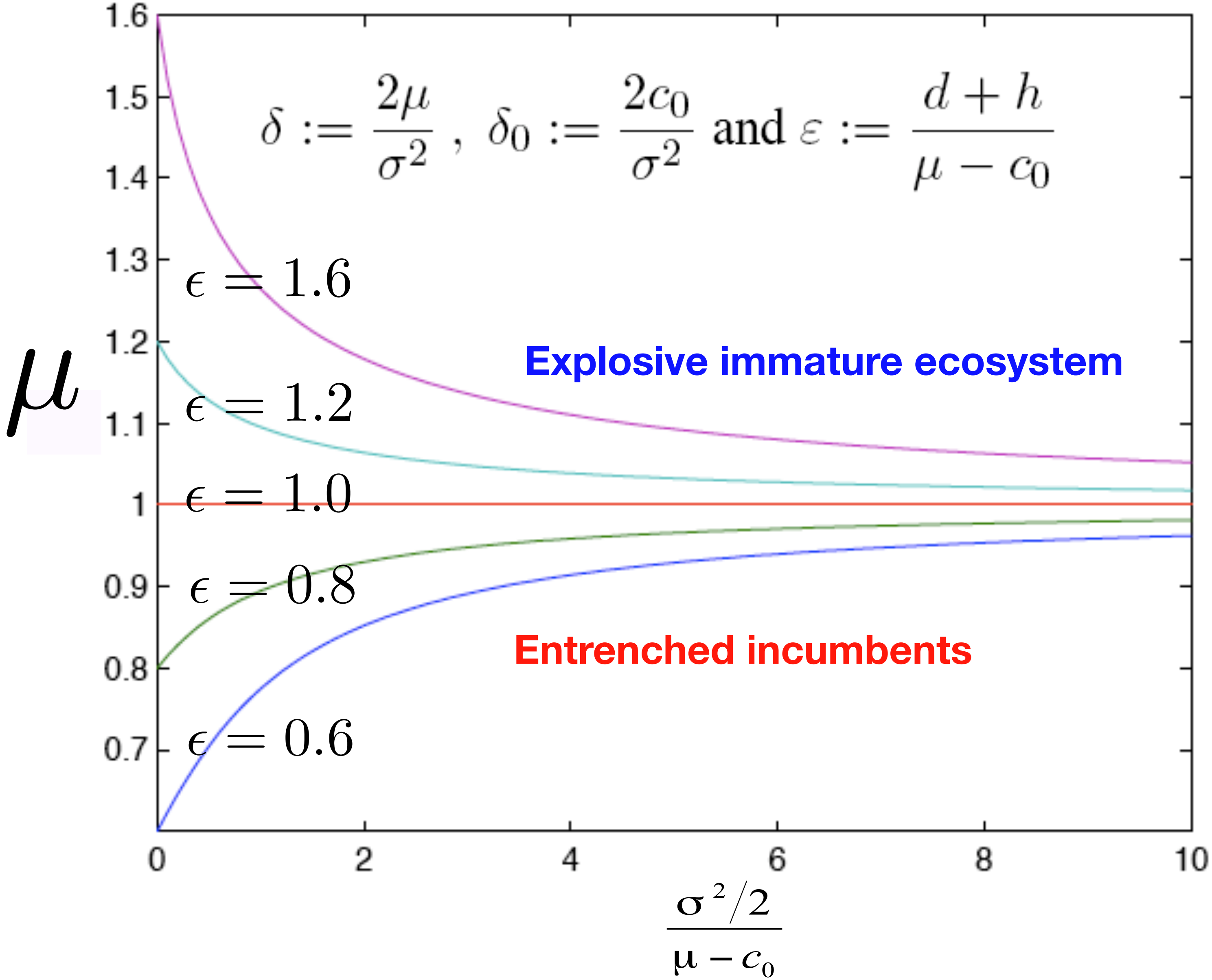
Under mild conditions we could predict that, asymptotically, the process generates a power-law distribution with tail index

$$pdf(S)dS \sim \frac{1}{S^{1+\mu}}dS$$

$$\mu := \frac{(1-2\frac{r-c_0}{\sigma^2}) + \sqrt{(1-2\frac{r-c_0}{\sigma^2})^2 + 8\frac{d+h}{\sigma^2}}}{2}$$ for time t larger than $t - t^* \geq [(r - \frac{\sigma^2}{2} - c_0)^2 + 2\sigma^2(d+h)]^{-\frac{1}{2}}$

Y. Malevergne, A. Saichev and D. Sornette, Zipfs law and maximum sustainable growth, Journal of Economic Dynamics and Control 37 (6), 1195-1212 (2013)
Saichev A, Malevergne Y, Sornette D, Zipf's law and beyond, Lecture Notes in Economics and Mathematical System 632, Springer, (2009).

$$\mu := \frac{1}{2}\left[(1-\delta+\delta_0)+\sqrt{(1-\delta+\delta_0)^2+4(\delta-\delta_0)\varepsilon}\right]$$



Verification of Gibrat's rule – proportional growth

- **Both the growth of coins and tokens are proportional (Fig 1).**

Over a time interval Δt=1 day, the average growth rate $\left\langle\frac{\Delta MC}{MC}\right\rangle$ should be given by

$$\left\langle\frac{\Delta MC}{MC}\right\rangle = r \times \Delta t,$$

And the standard deviation of the growth rate should follow

$$\sigma\left(\frac{\Delta MC}{MC}\right) = \sigma \times \sqrt{\Delta t}$$

- **Coin grow much faster than tokens (2 times!)**
- **Volatilities of coins and tokens are similar: 24.6%!**

Verification of Gibrat's rule – proportional growth

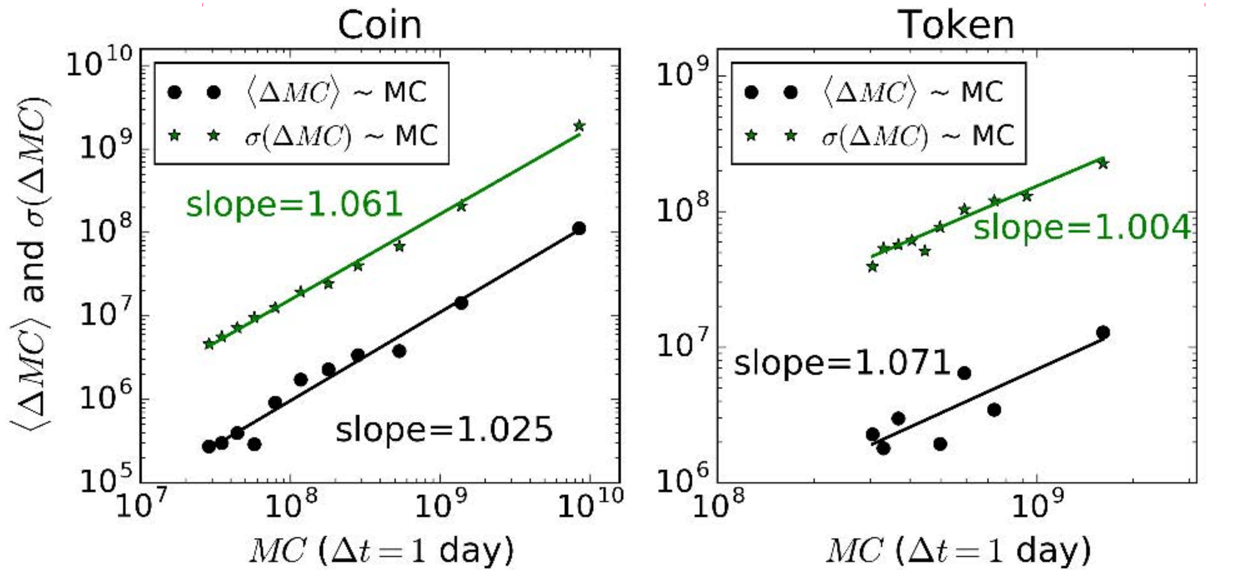


Fig 1. Test of Gibrat's law of proportional growth for market cap MC of coins and tokens until Feb 7, 2018 with 1 year window. Only positive points are shown. The left panel shows the test for coins and the right panel for tokens. The black circles are the mean of the increments (i.e., $\langle \Delta MC \rangle$) versus its current market cap. The green starts are the standard deviation of the increments (i.e.,$\sigma(\Delta MC)$) versus its current market cap. In both panels, the lines show the OLS fit to the data points.

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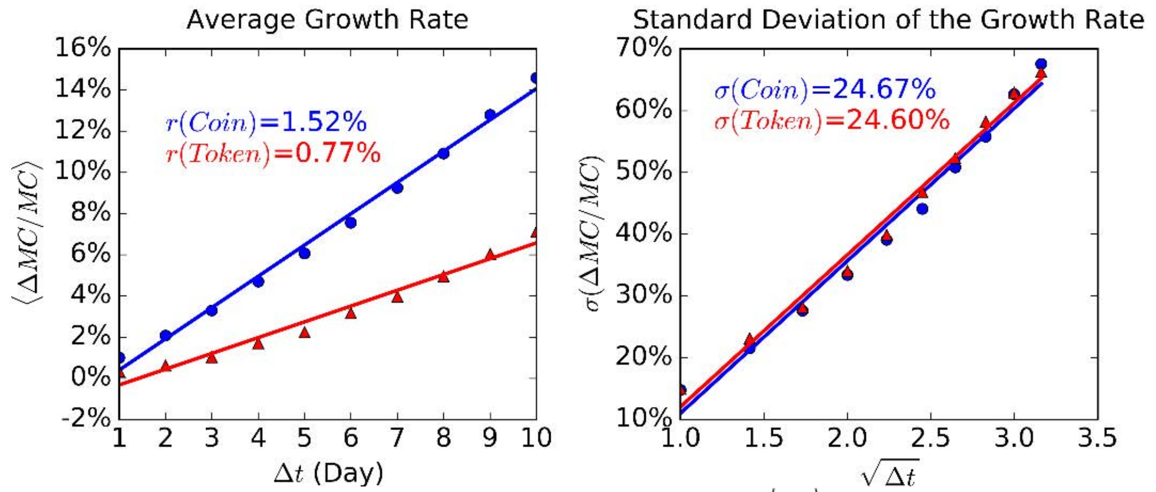


Fig 2. Left panel is the relationship between the average growth rate $\left\langle \frac{\Delta MC}{MC} \right\rangle$ versus the time interval Δt for coins (blue circle) and tokens (red angels) respectively. Right panel depicts the standard deviation of the growth rate $\sigma\left(\frac{\Delta MC}{MC}\right)$ versus the standard deviation of the time interval $\sqrt{\Delta t}$ for coins and tokens respectively.

Birth Size

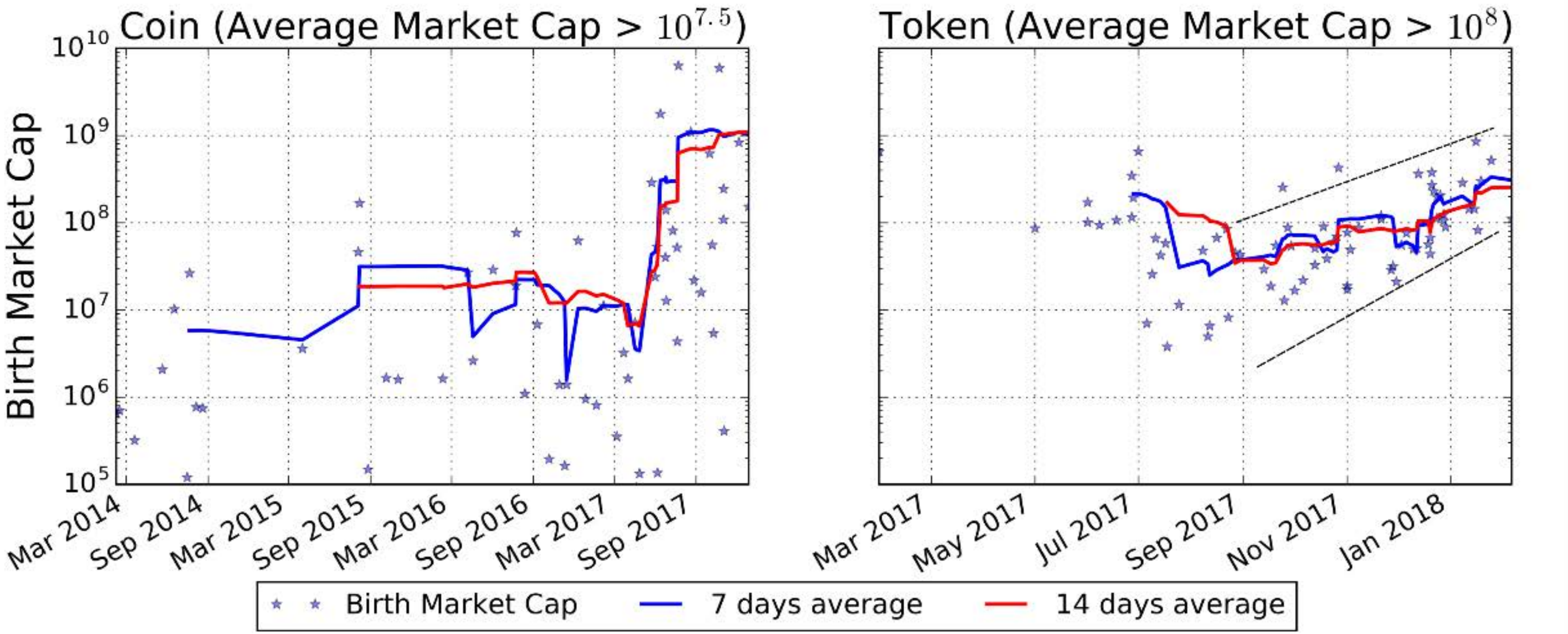


Fig 3. The birth market cap of coins (left panel) and tokens (right panel). Only large coins (average market cap over life time > $10^{7.5}$) and tokens (average market cap over life time > 10^{8}) are shown. Blue (resp. red) line is 7 (resp. 14) days moving average.

Birth Size

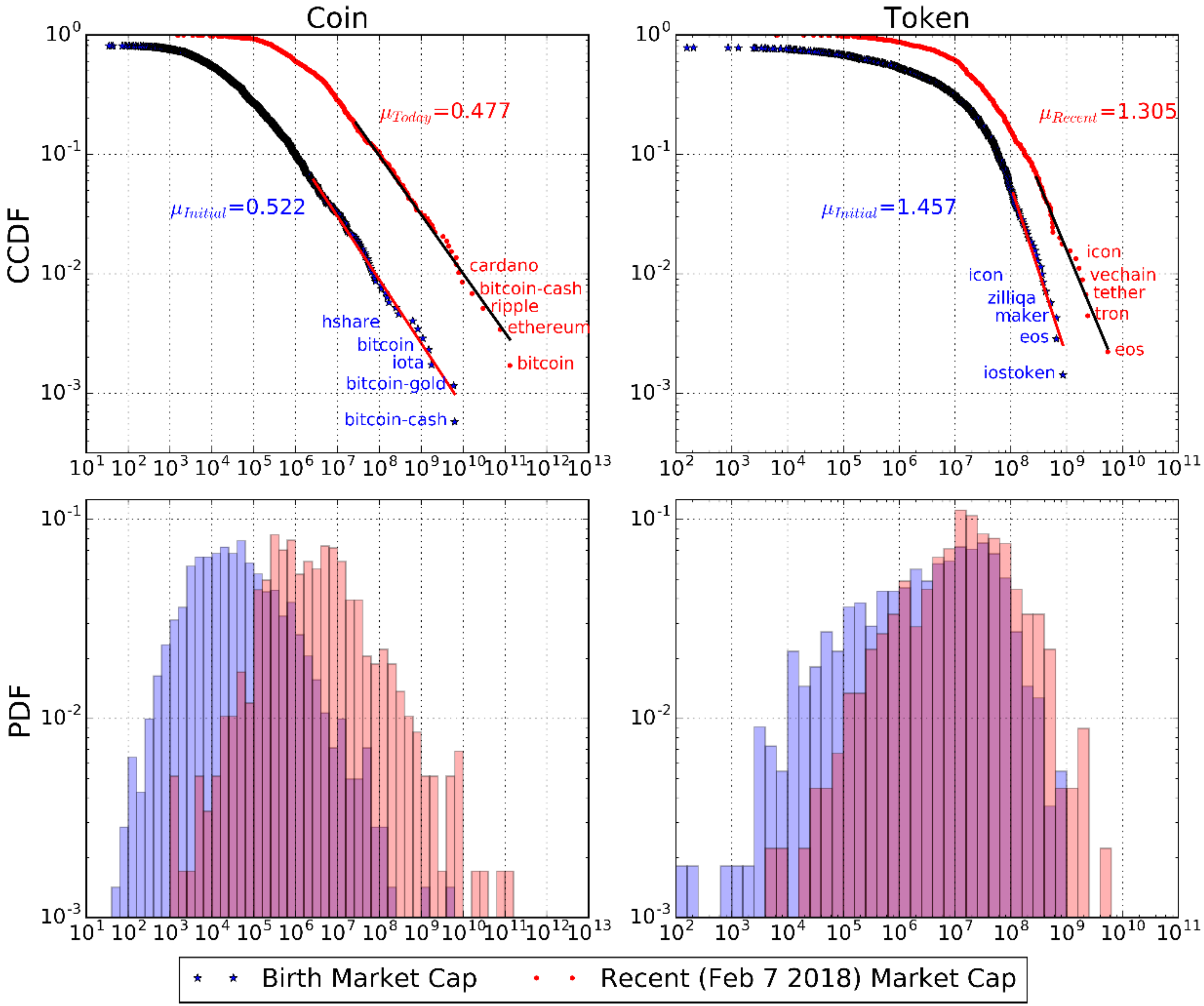


Fig 4. Comparison between the distributions of birth market cap and the recent market cap, for coins (left panel) and tokens (right panel) respectively. The black stars are the birth market cap and the red dots are the market cap on Feb 7 2018. The upper panel depicts the complementary cumulative distribution function (CCDF) of the birth and recent market cap. Lower panel shows the pdf of the birth (blue) and recent (red) market cap. The largest five coins/tokens are indicated in the upper panel.

Birth Size

- For both coins and tokens, the distribution of birth market cap is thinner than the current distribution.
- For large coins and tokens:
  - The birth size of coins has two stable regimes shifted around May 2017 => $c_0 = 0$
  - The birth size of tokens is gradually growing, so

$$c_0 \approx 1.06\% > 0$$

- The number of tokens has been growing significantly since May 2017, while the number of coin's birth is relatively stable with periodicity.
- For large coins and tokens:
  - Coin's birth intensity is stable => $d = 0$
  - Token's birth intensity is growing => $d \approx 1.14\% > 0$

Birth Intensity

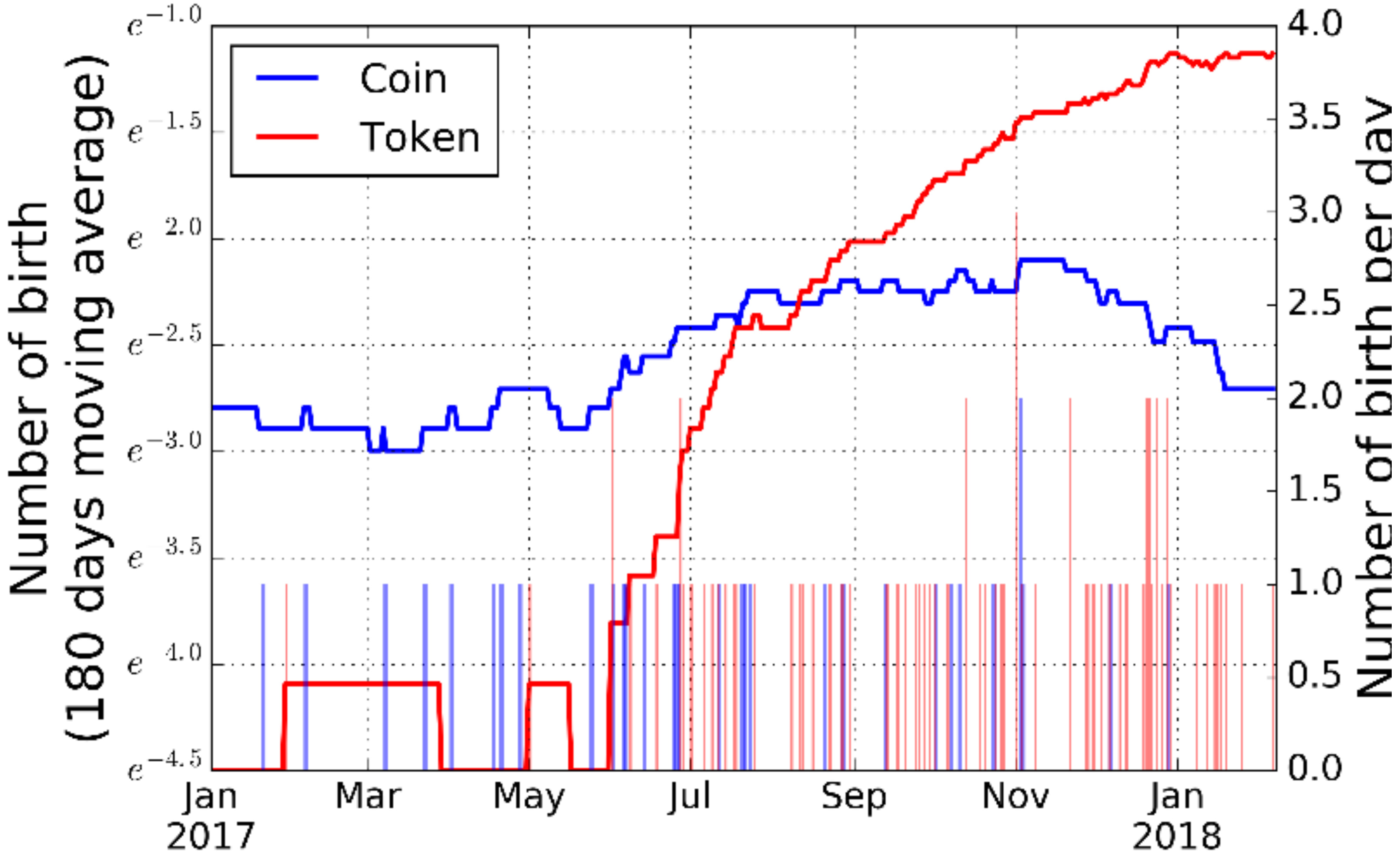


Fig 5. Number of birth of coins (blue) and tokens (red), smoothed by 180 days moving average. Only large coins (average market cap over life time > $10^{7.5}$) and tokens (average market cap over life time > 10^{8}) are shown. Number of birth per day is shown in blue (coin) and red (token) bars against the right y-axis.

Birth Intensity

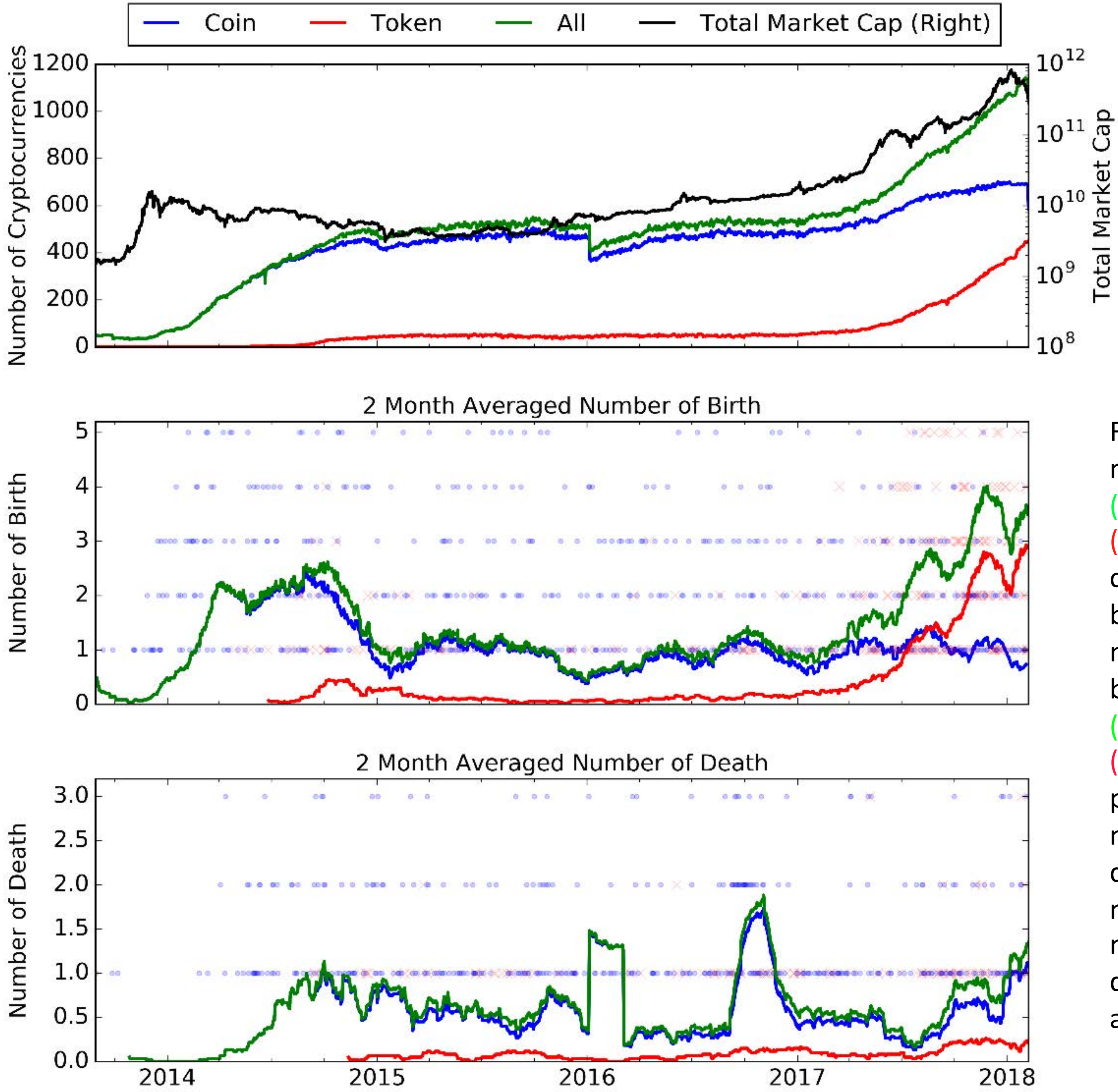


Fig 6. The upper panel is the number of all cryptocurrencies (green), coins (blue), and tokens (red). The total market cap of all cryptocurrencies is plotted in black against the right y-axis. The middle panel plots the number of birth for all cryptocurrencies (green), coins (blue), and tokens (red) respectively. The lower panel is the corresponding number of death. Both birth and death rate are smoothed by 2 month moving average. The number of birth and death per day is plotted in blue dot (coin) and red cross (token).

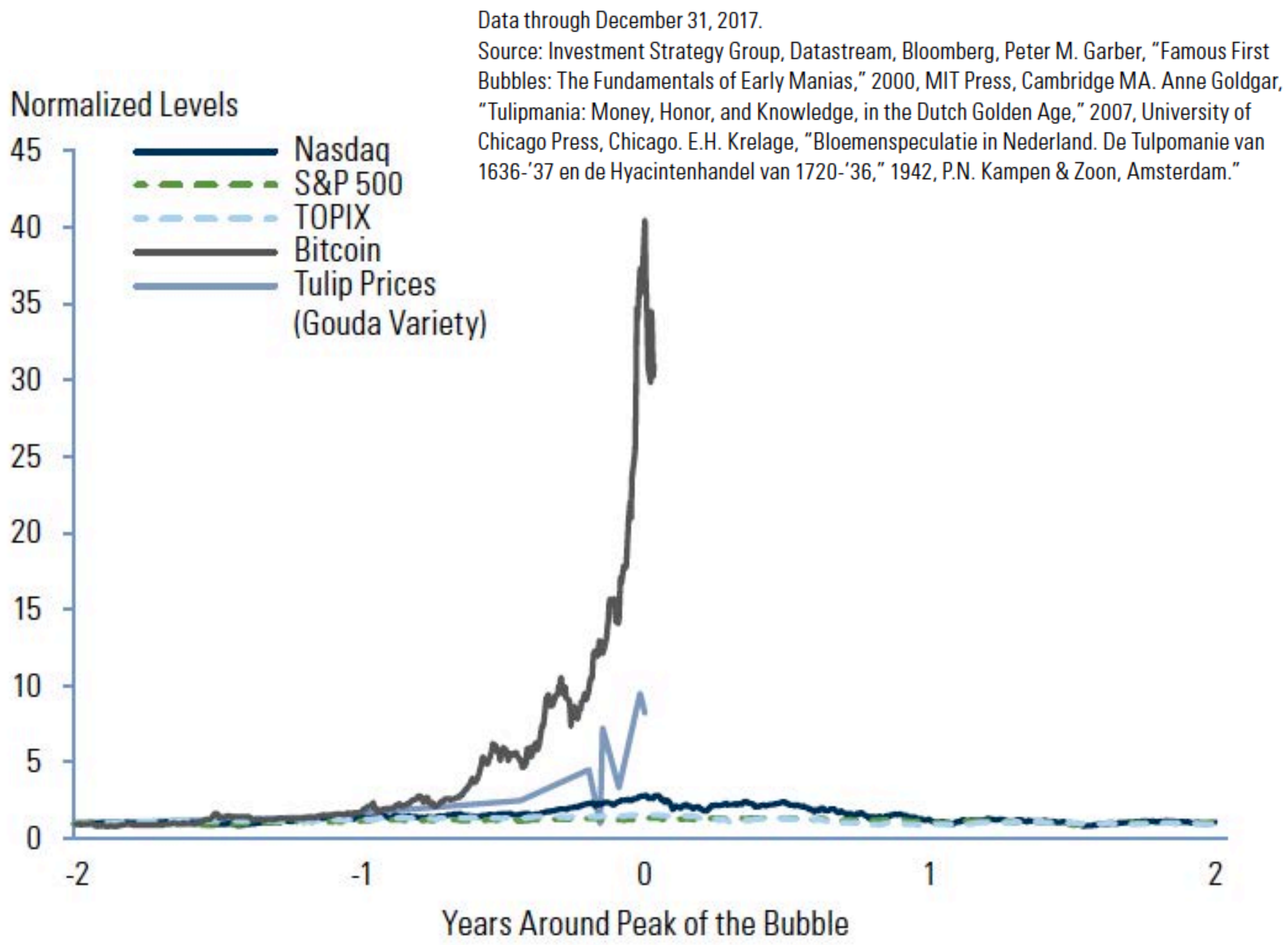
Theoretical Prediction vs empirical values

	Coin	Token
Growth rate of market cap r	1.52%	0.77%
Growth volatility σ	24.67%	24.6%
Exit hazard rate h	0	0
Growth rate of birth size c_0	0	1.06%
Growth of the birth intensity d	0	1.14%
Empirical tail exponent μ_{MLE}	0.48	1.31
Theoretical tail exponent μ_{TH}	0.50	1.36

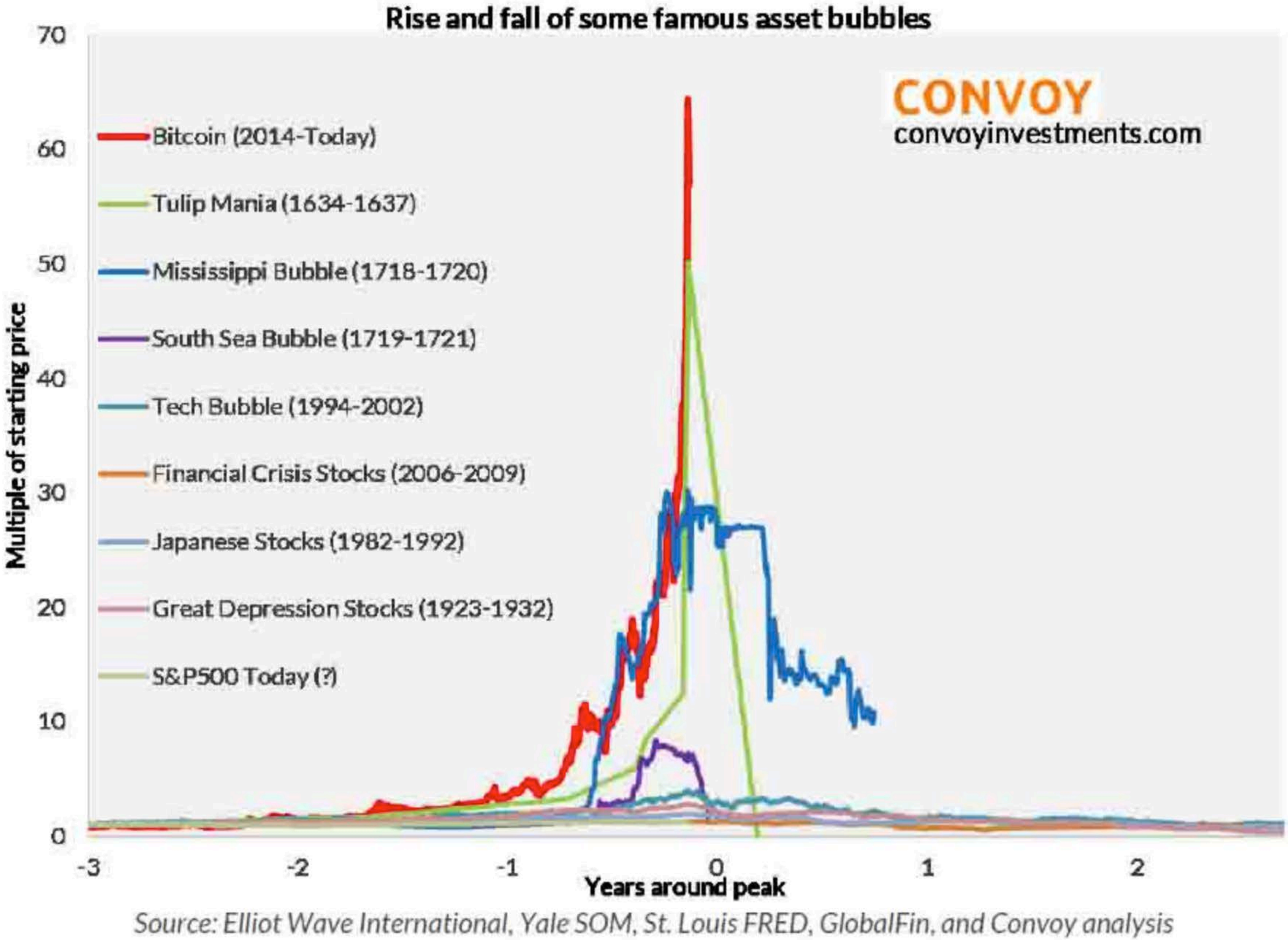
- For both large coins and tokens, there were less than 3 dead, so we consider the exit hazard rate as 0 for both coins and tokens.
- The theoretical prediction for the tail exponent is very close to the empirical result.

Implications

- **The crypto world is similar to many other systems that can be explained by proportional growth + stochastic birth and death.**
- **Four parameters matter!**
  - growth rate of market cap r
  - exit hazard rate h
  - growth rate of birth size c_0
  - growth of the birth intensity d
- **Coin $\mu < 1 : r - h > d + c_0$** **Entrenched incumbents**
  - Higher growth rate of market cap => More larger coins => fatter tail
  - More serious development of existing coins (Ethereum foundation) => Large coins are less likely to die
  - More mature community => stable birth size and lower growth of the birth intensity
- **Token $\mu \geq 1 : r - h \leq d + c_0$** **Explosive immature ecosystem**
  - Immature community => More small tokens (ICOs) => thinner tail
  - Young community => growing birth size and the birth intensity



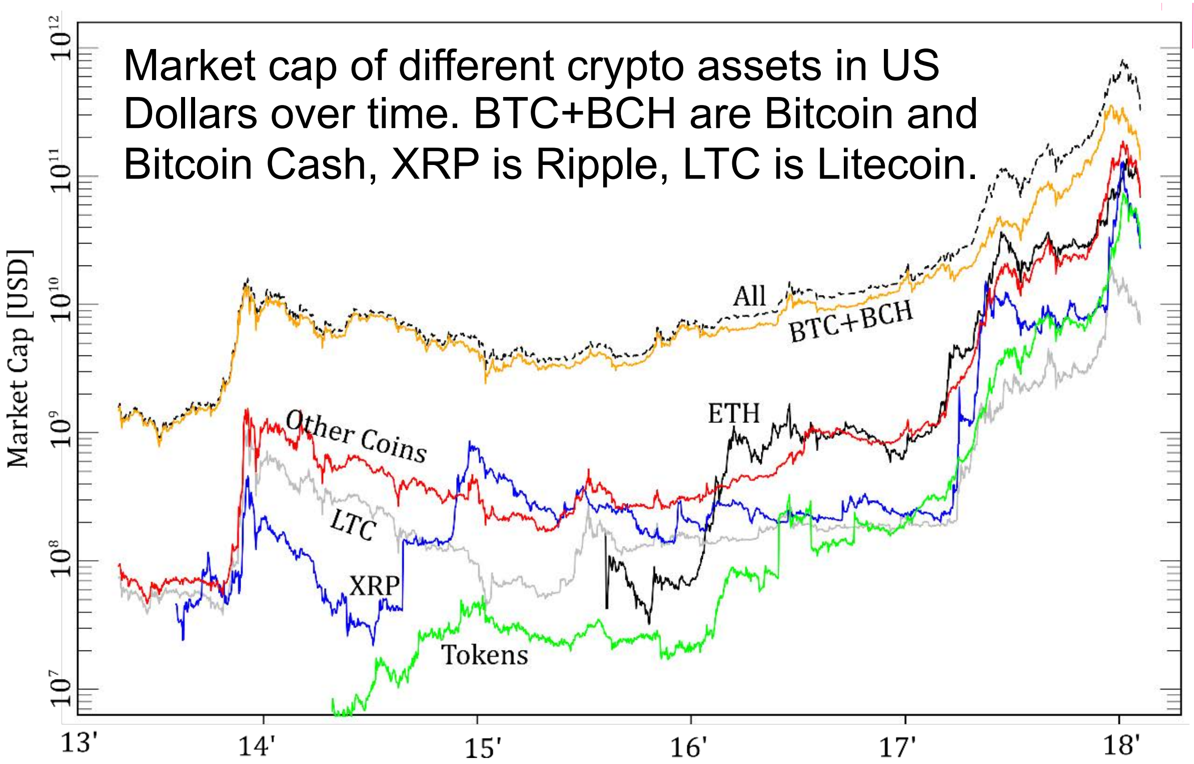
(Goldman Sachs, Investment Strategy Group, Jan. 2018)


Rise and fall of some famous asset bubbles
70
60
50
40
30
20
10
0
Multiple of starting price
CONVOY
convoyinvestments.com
Bitcoin (2014-Today)
Tulip Mania (1634-1637)
Mississippi Bubble (1718-1720)
South Sea Bubble (1719-1721)
Tech Bubble (1994-2002)
Financial Crisis Stocks (2006-2009)
Japanese Stocks (1982-1992)
Great Depression Stocks (1923-1932)
S&P500 Today (?)
-3
-2
-1
0
1
2
Years around peak
Source: Elliot Wave International, Yale SOM, St. Louis FRED, GlobalFin, and Convoy analysis

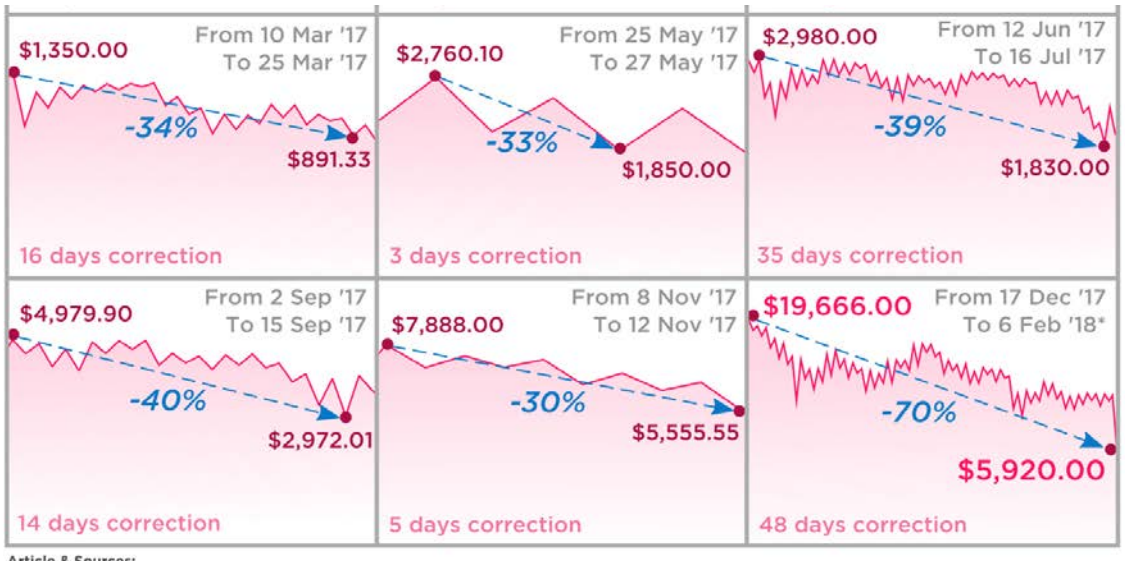
(13 Dec 2017)

Market Background: 4 steps forward and 1 step back?

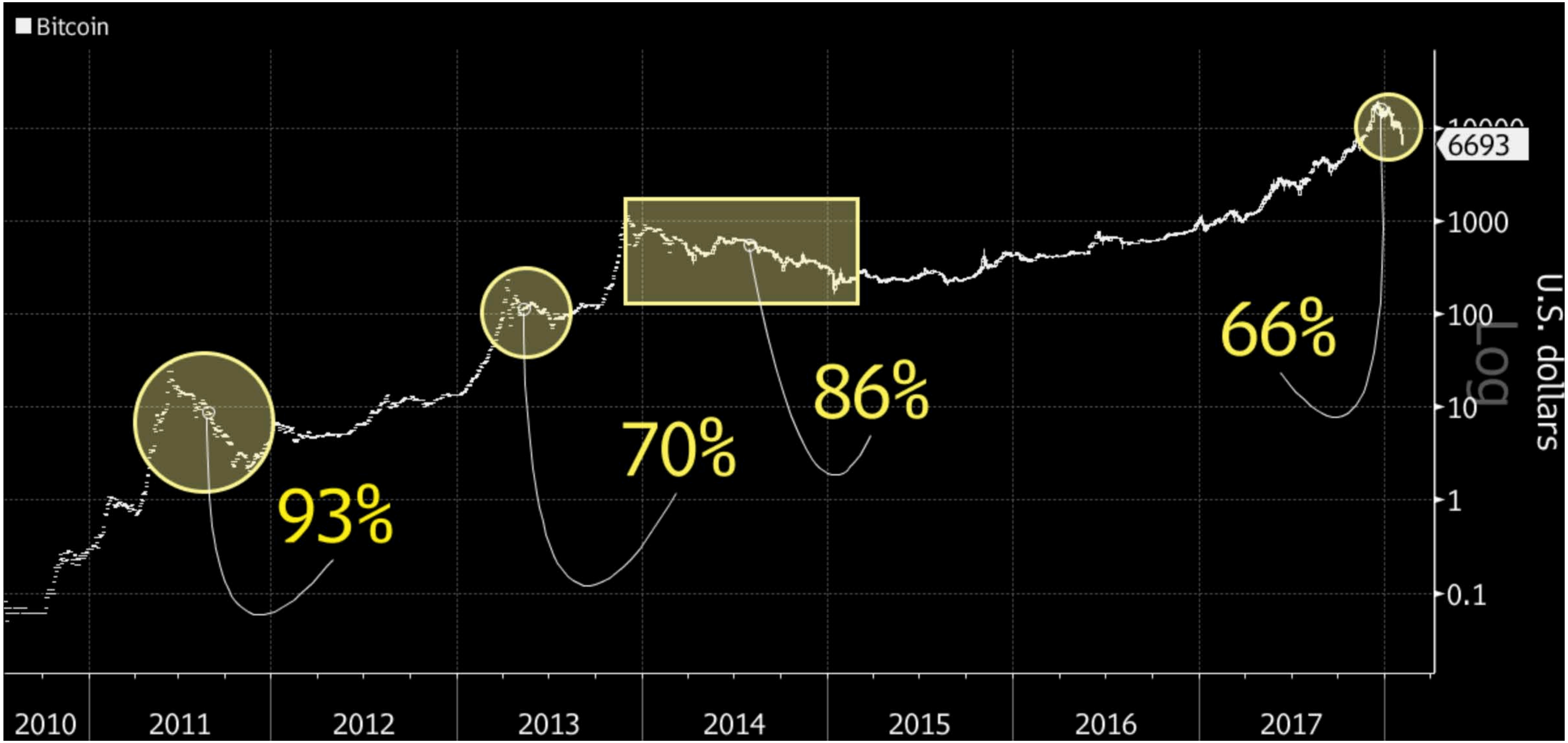
- "Altcoins" have been around since early:
  - Pre-2016: Ripple, Litecoin, and others together similar.
  - Since 2016: ETH and other ‘alts’ surged
  - Tokens still less than 10% of market, growing rapidly.
- Mcap ATH ~$800Bil, near mcap of Apple Inc.
- When BTC crashes, the market (traded in and assoc. with BTC) tends to crash as well.
- Current BTC crash large but <u>so far</u> not exceptional.
  - BTC History: about ten rapid drawdowns >20%, and a few longer term corrections of >50% .
  - 2016-2017: repeated months of growth and partial corrections.



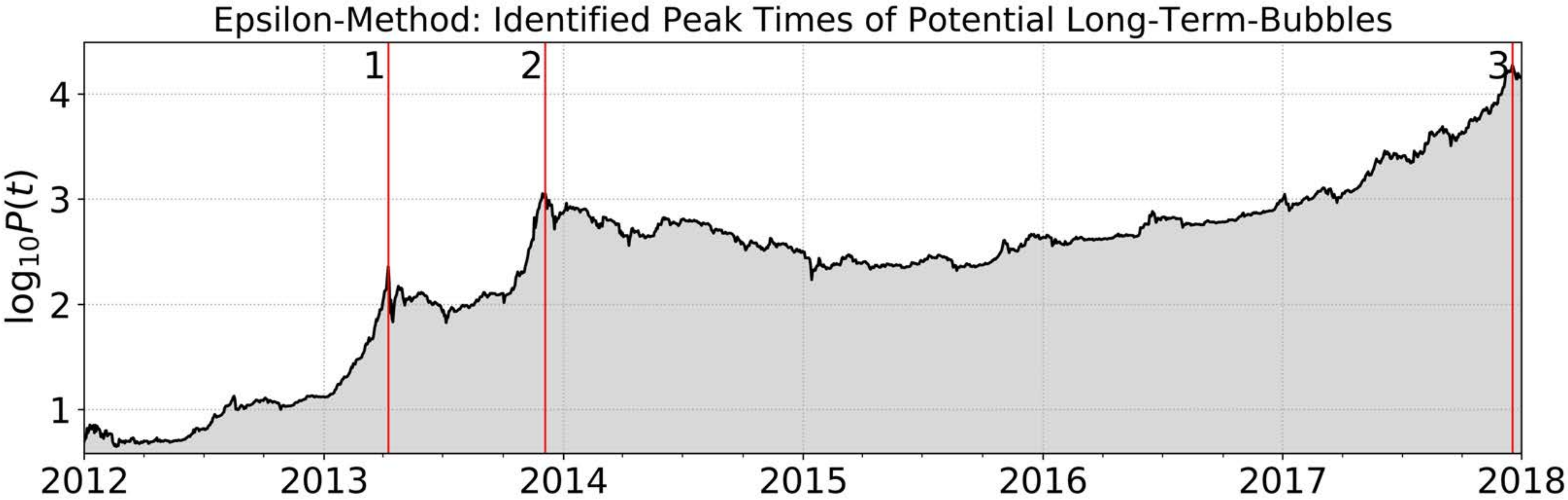
Market cap of different crypto assets in US Dollars over time. BTC+BCH are Bitcoin and Bitcoin Cash, XRP is Ripple, LTC is Litecoin.

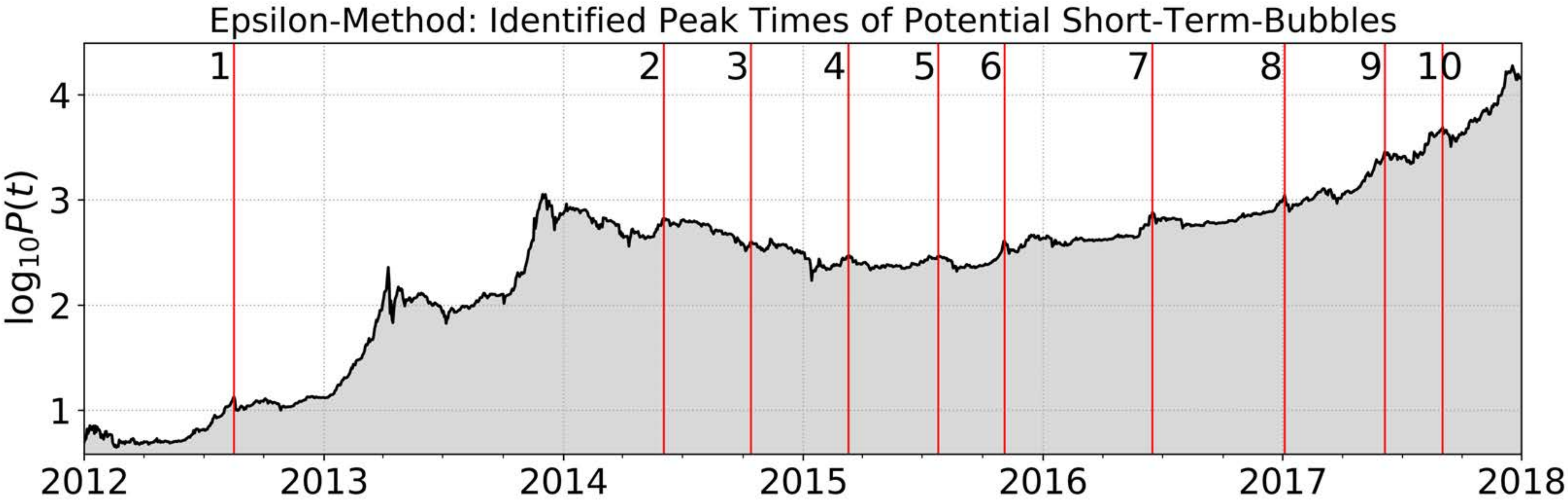


Bitcoin drawdowns in 2017. Source: https://howmuch.net/articles/bitcoin-all-major-crashes



(Bloomberg, 5 Feb. 2018)



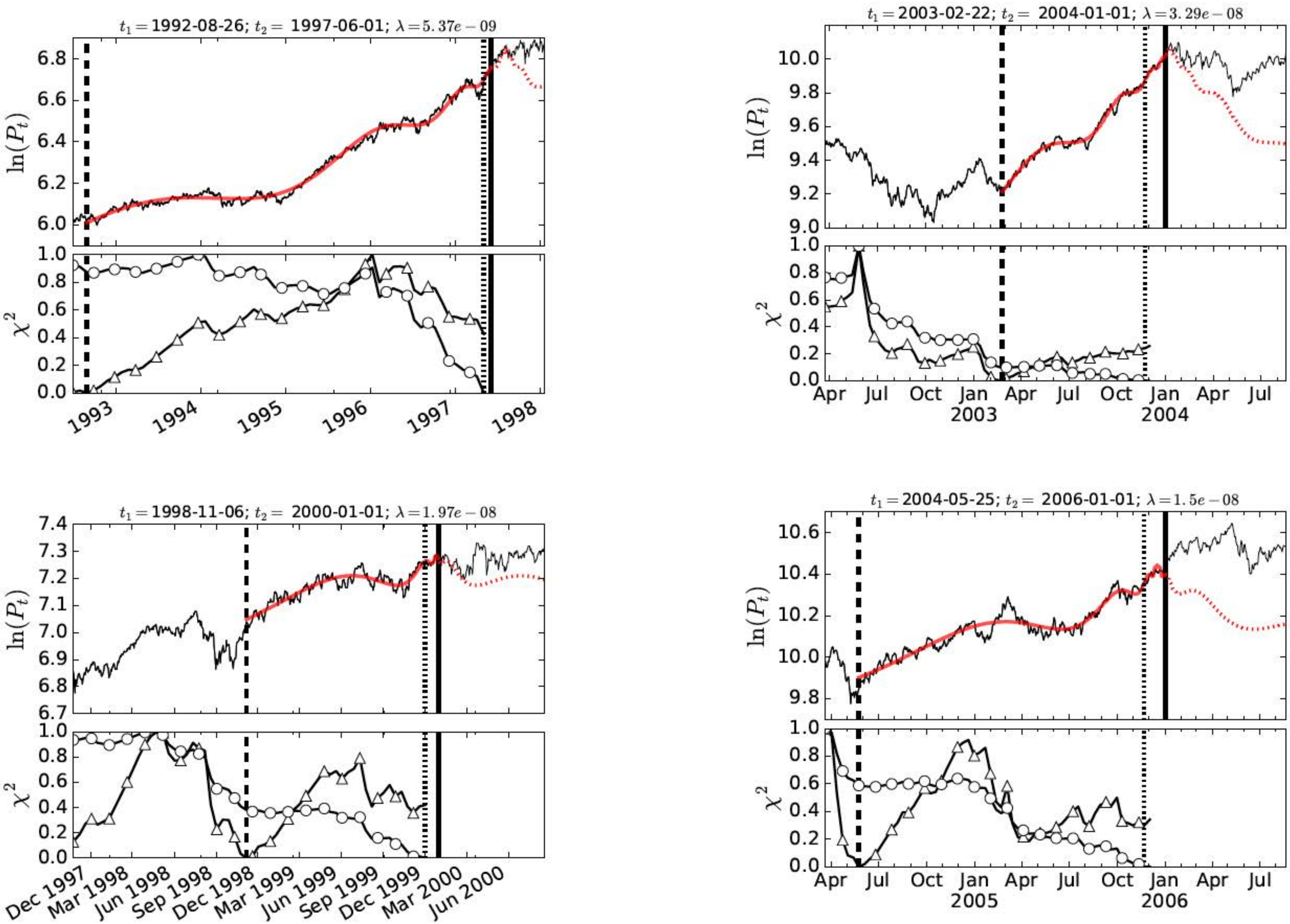


A peak is identified as the end time of a price drawup. The epsilon metric identifies the end of drawup (drawdown) phases as the points when, during a run / decline of the price, the price moves in opposite direction exceeding a certain tolerance . The tolerance is chosen as where is a pre-set, fixed multiplier, and is the moving window volatility estimated over a window of length reaching back into the past from the present time.

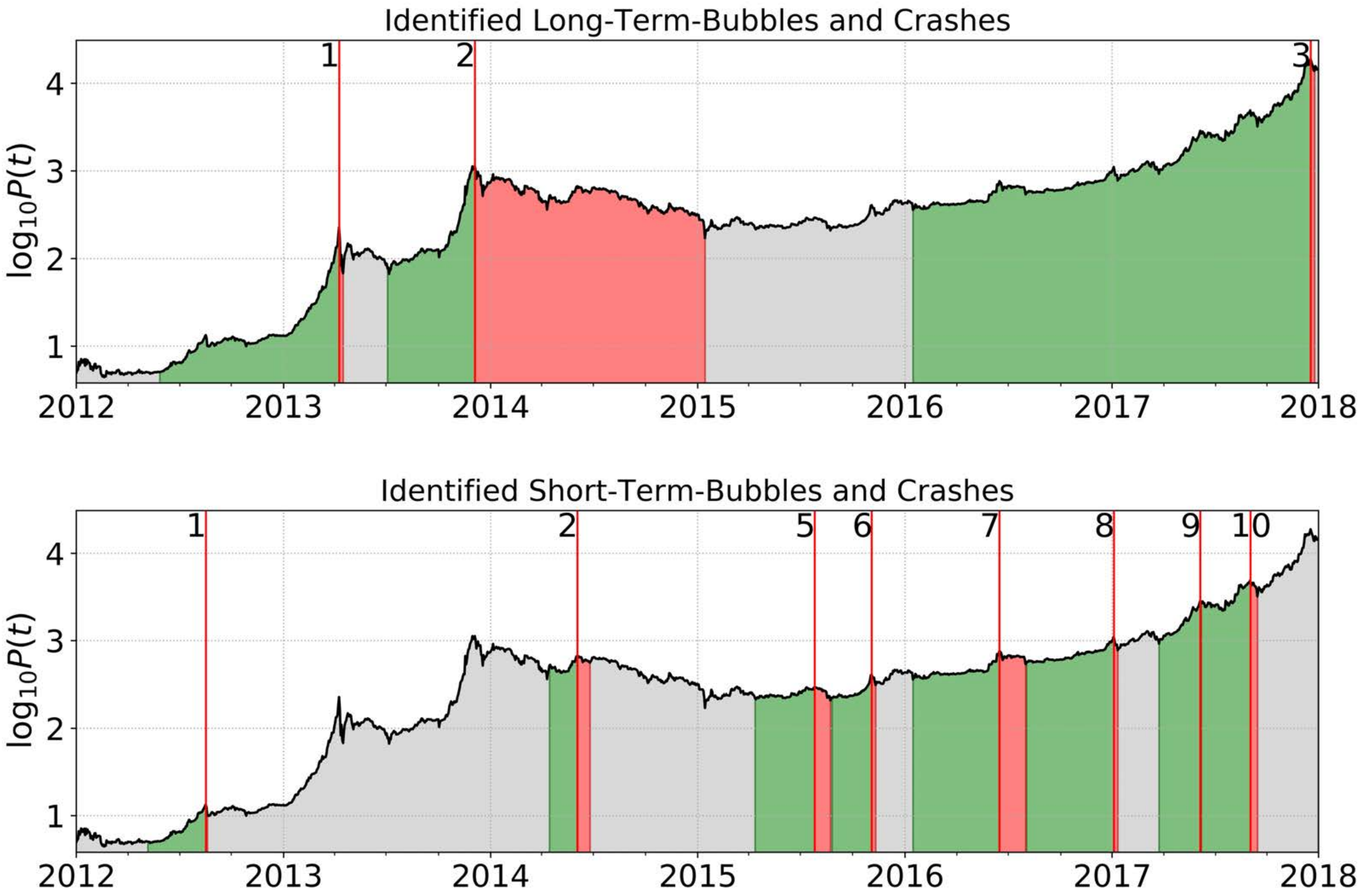
Johansen, A. and D. Sornette, Large Stock Market Price Drawdowns Are Outliers, Journal of Risk 4(2), 69-110, Winter 2001/02

Bubble Start Times with the Lagrange Regularisation Approach

The Lagrange technique determines bubble start times as the times corresponding to the fit window size for which the detrended average SSE of all fit results calculated at the same t2 (here at the peak times) is minimized.



Guilherme Demos and Didier Sornette, Lagrange regularisation approach to compare nested data sets and determine objectively financial bubbles' inceptions, Computational Statistics (2017) (http://ssrn.com/abstract=3007070)


Identified Long-Term-Bubbles and Crashes
log10P(t)
1
2
3
4
1
2
3
2012
2013
2014
2015
2016
2017
2018
Identified Short-Term-Bubbles and Crashes
log10P(t)
1
2
3
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1
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7
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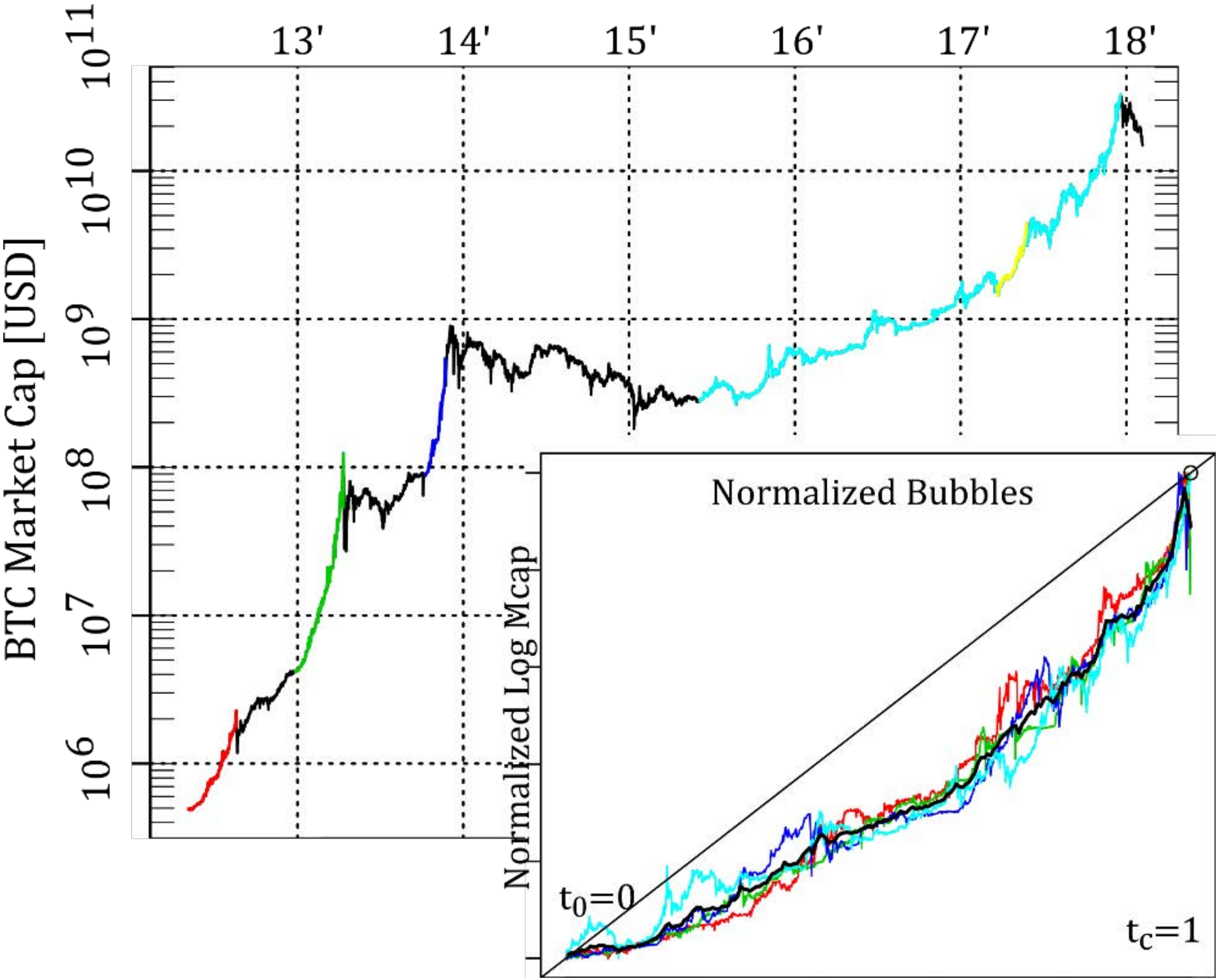
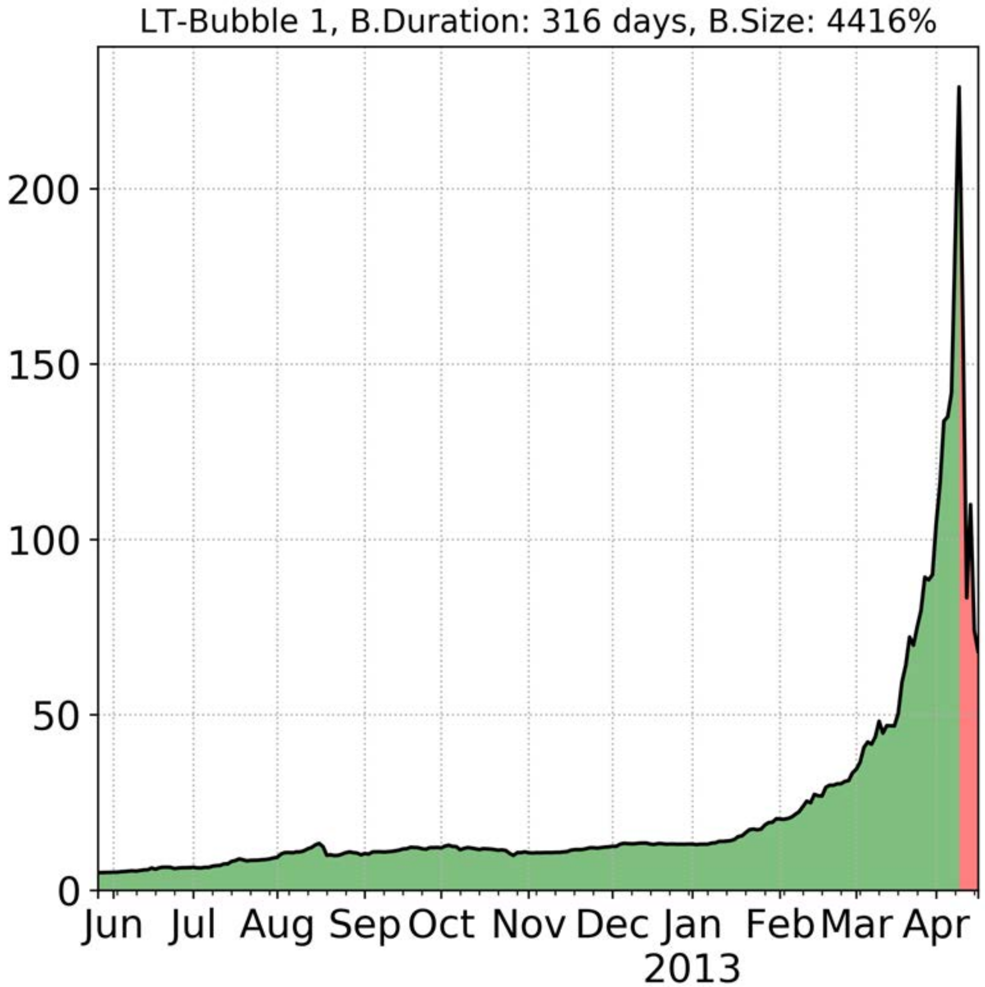
9
10
2012
2013
2014
2015
2016
2017
2018

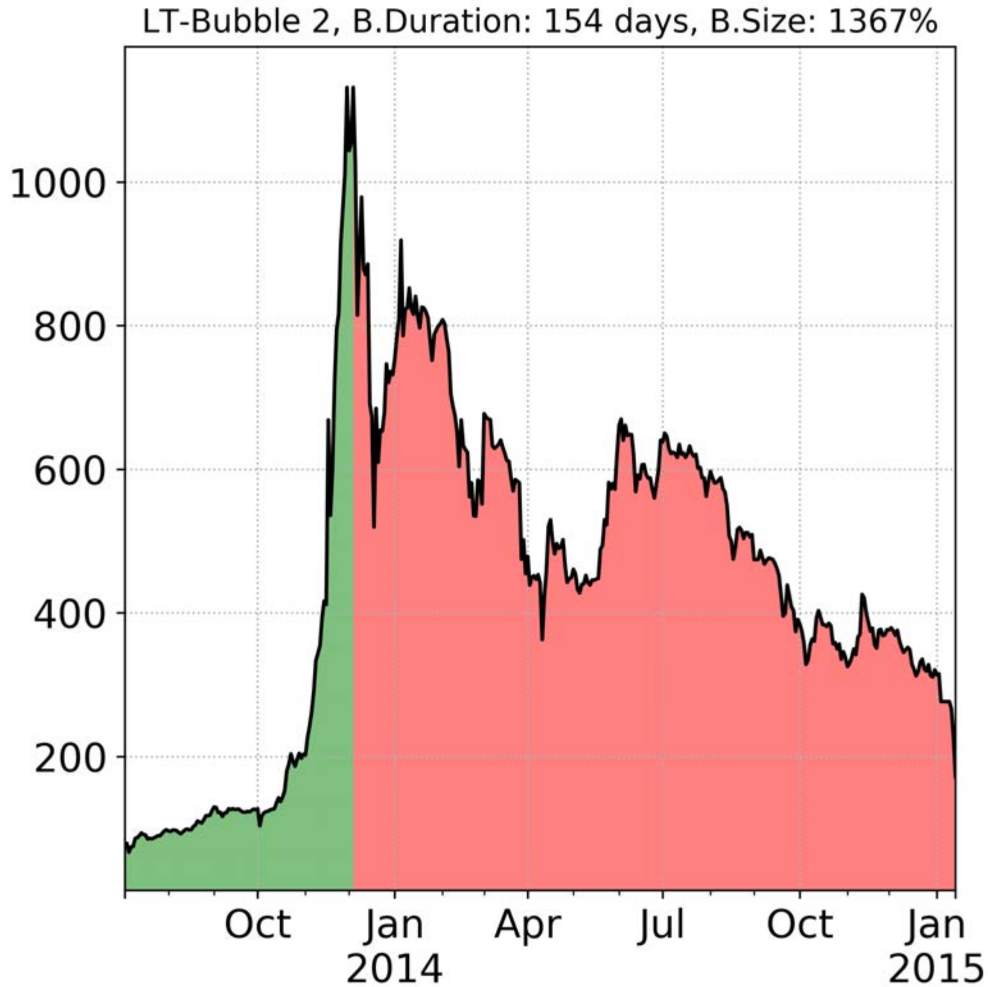
Are market instabilities predictable?



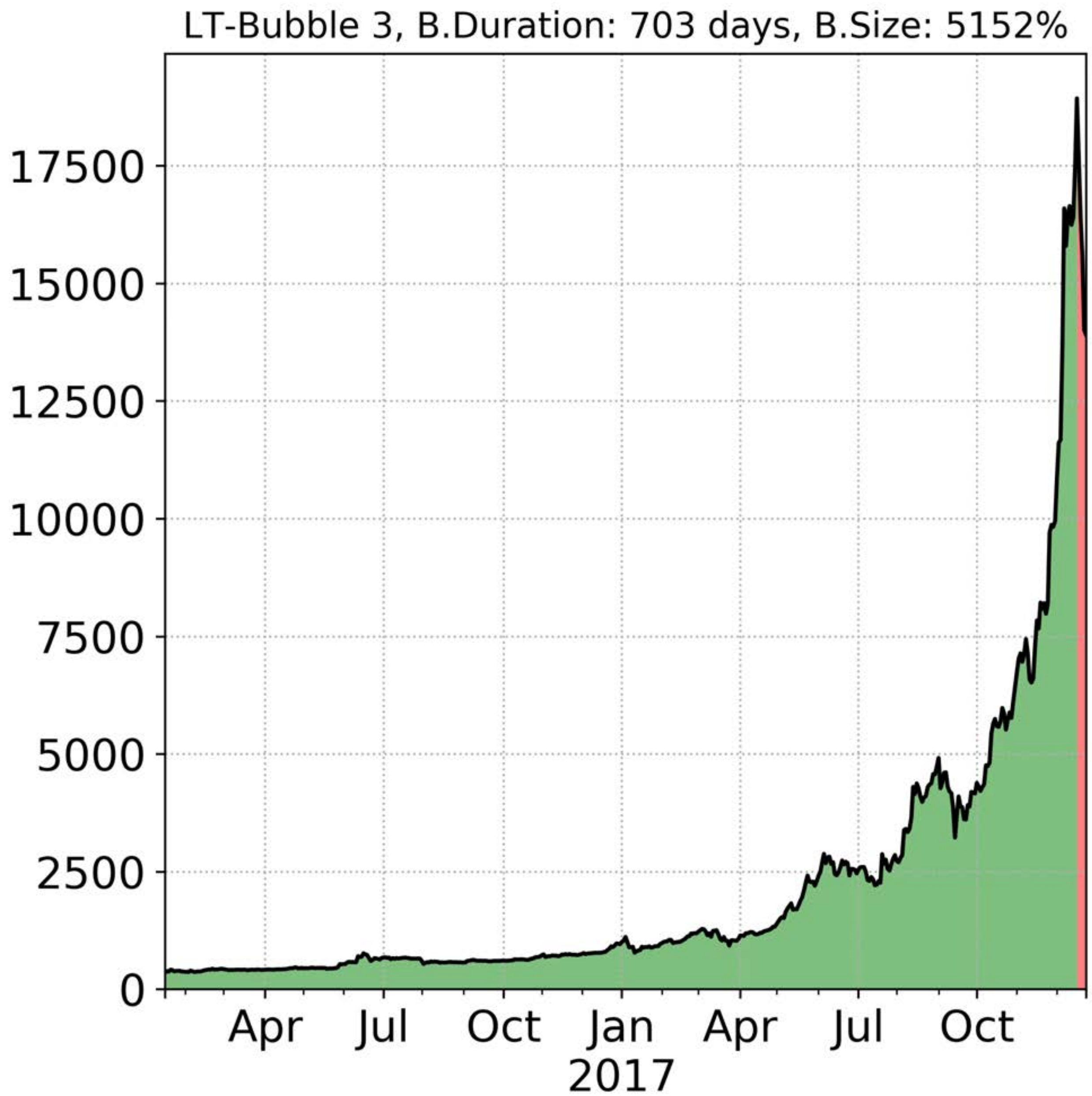
Bitcoin market cap, long bubbles indicated, and normalized to equal length and height in inset panel.

Long-Term Bubbles

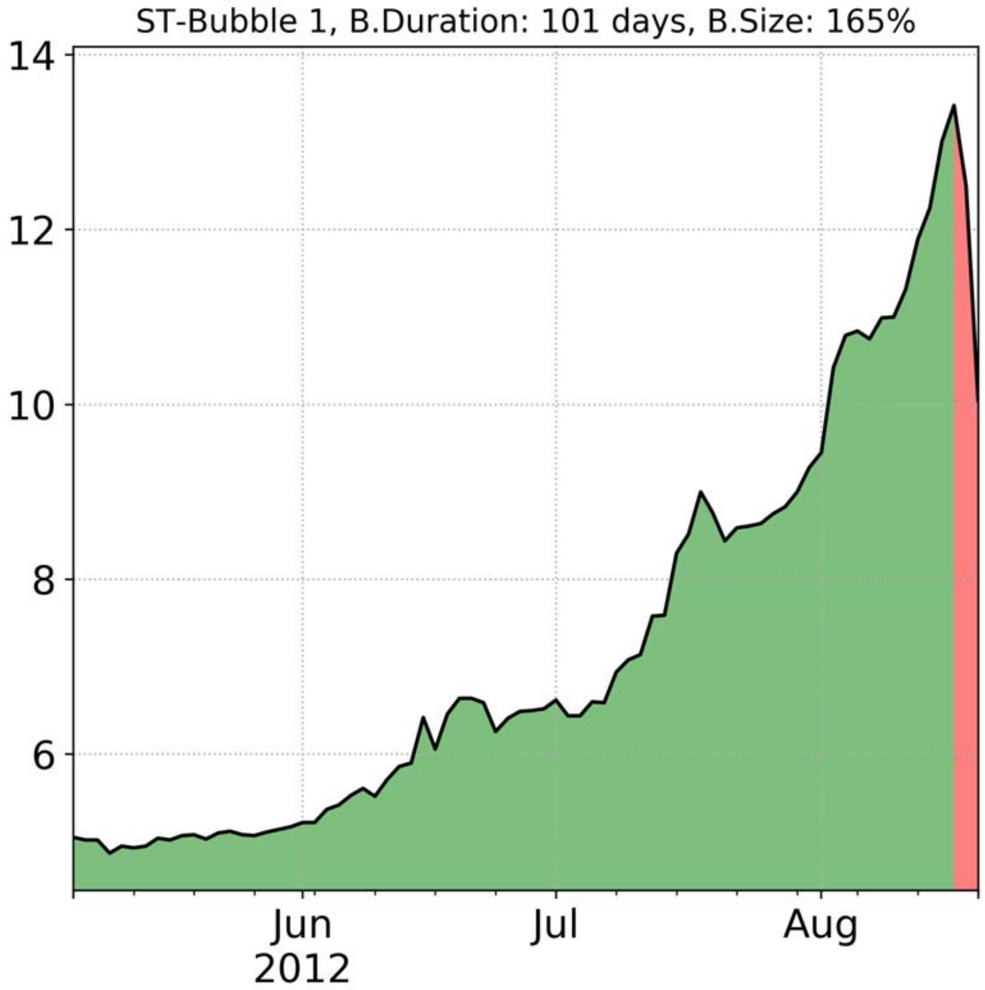


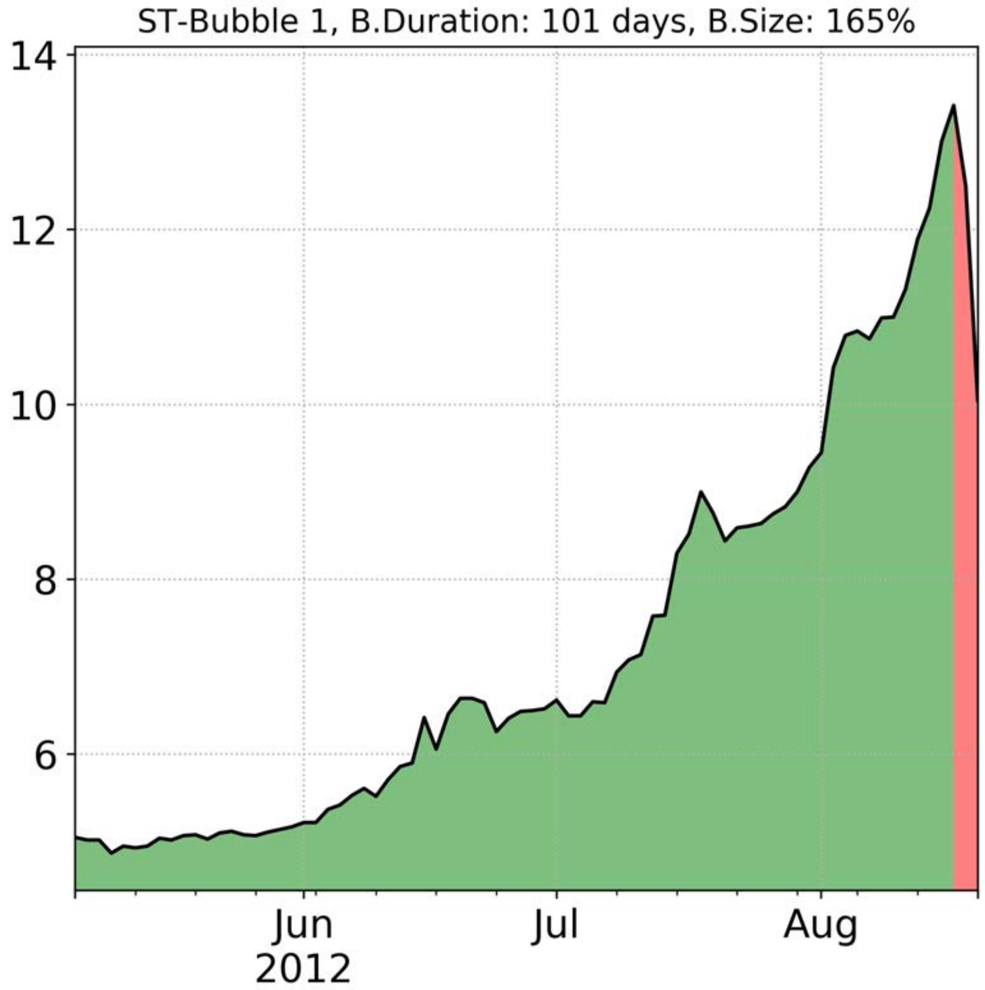


Long-Term Bubbles



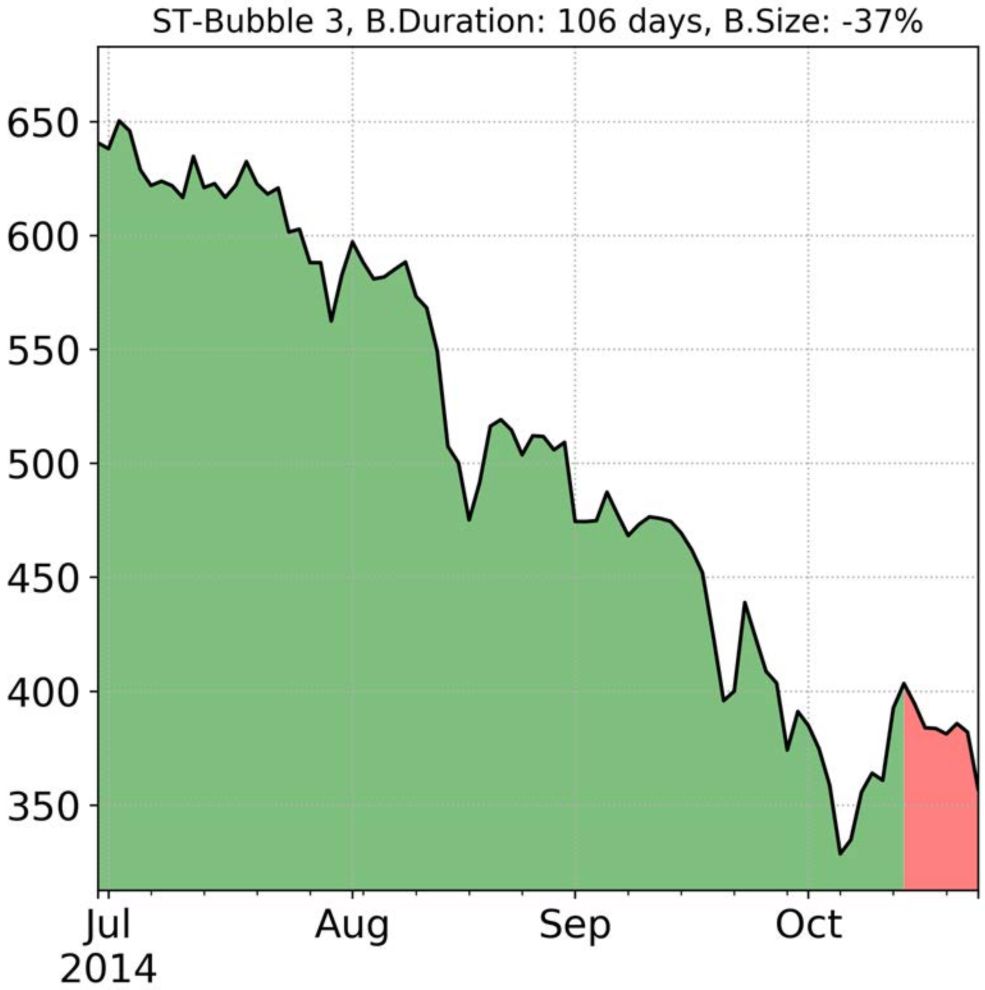
Short-Term Bubbles

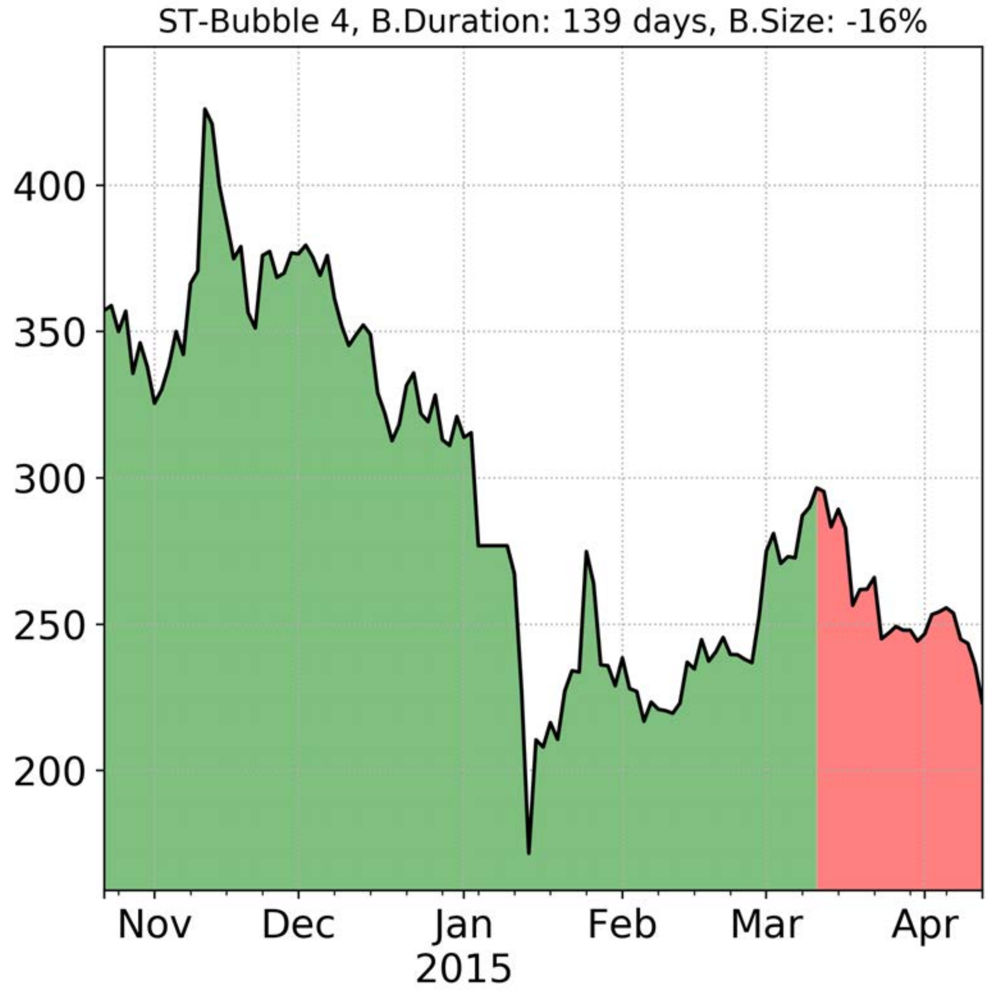




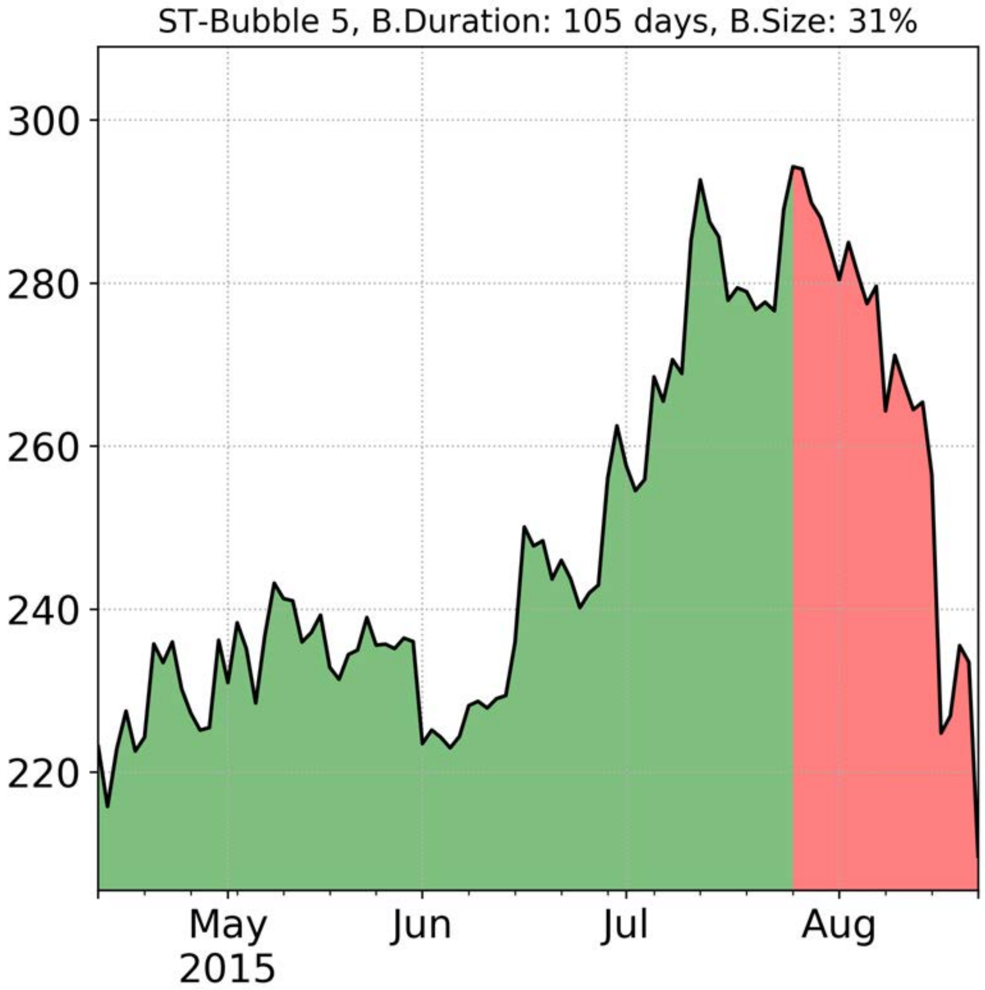
Short-Term Bubbles

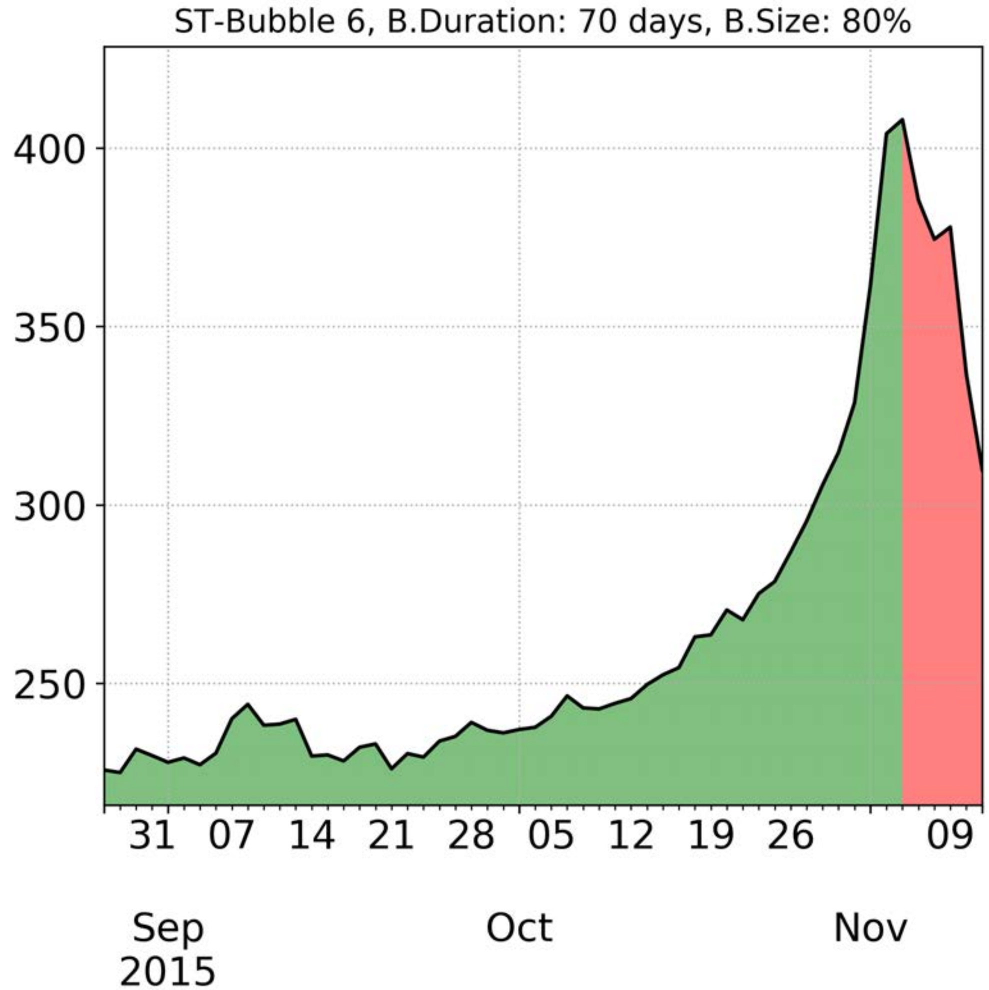
negative bubbles



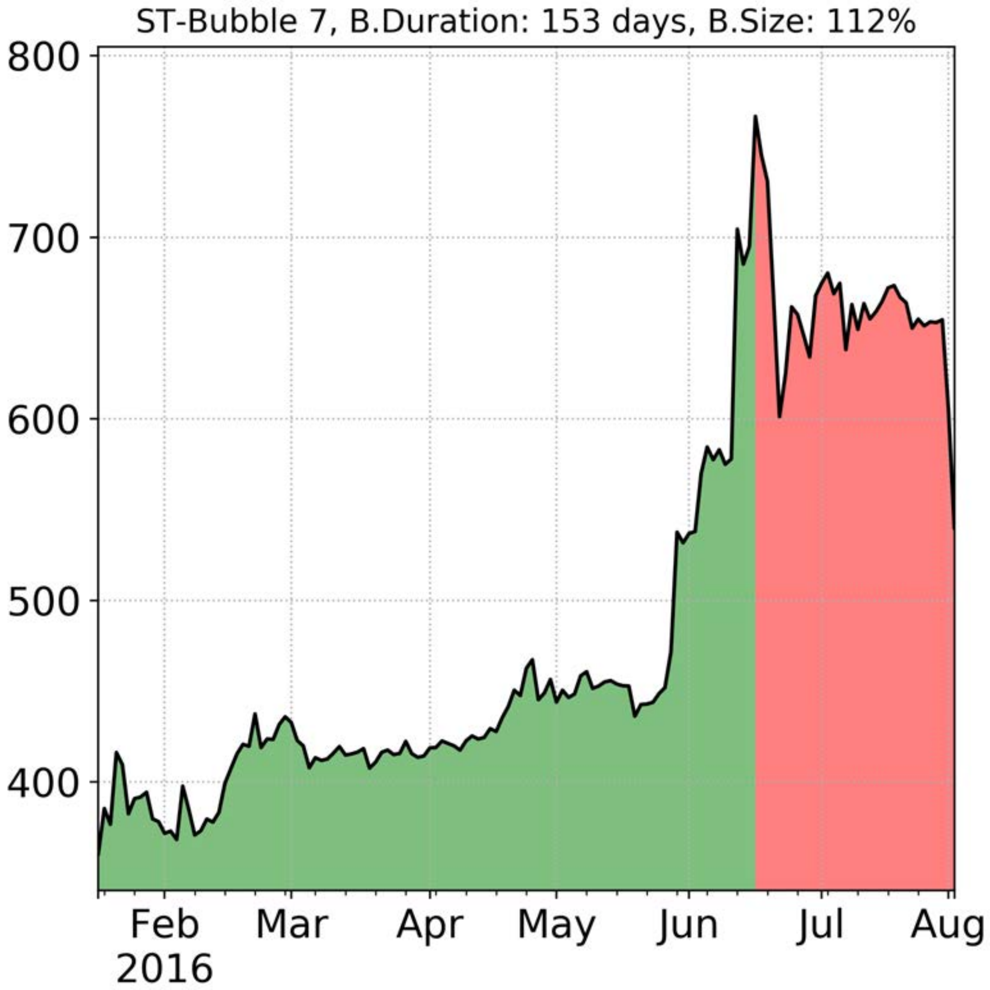


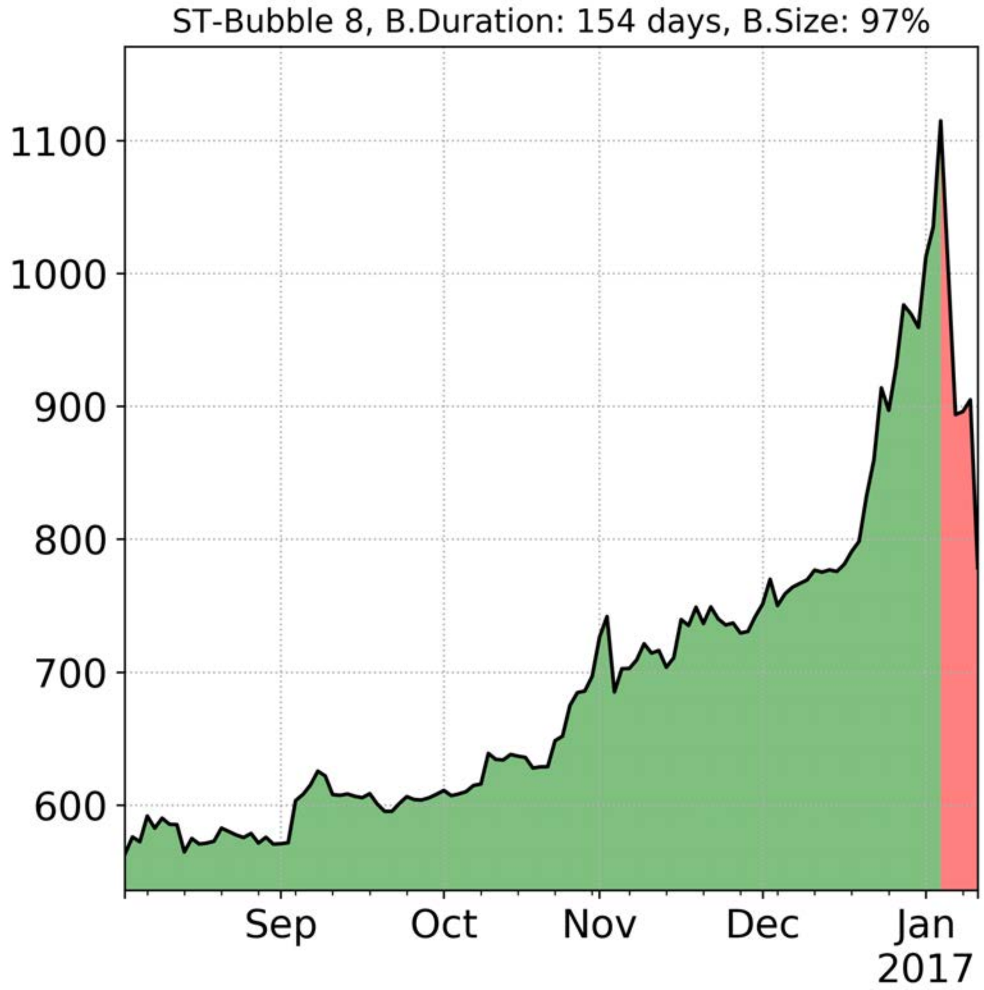
Short-Term Bubbles



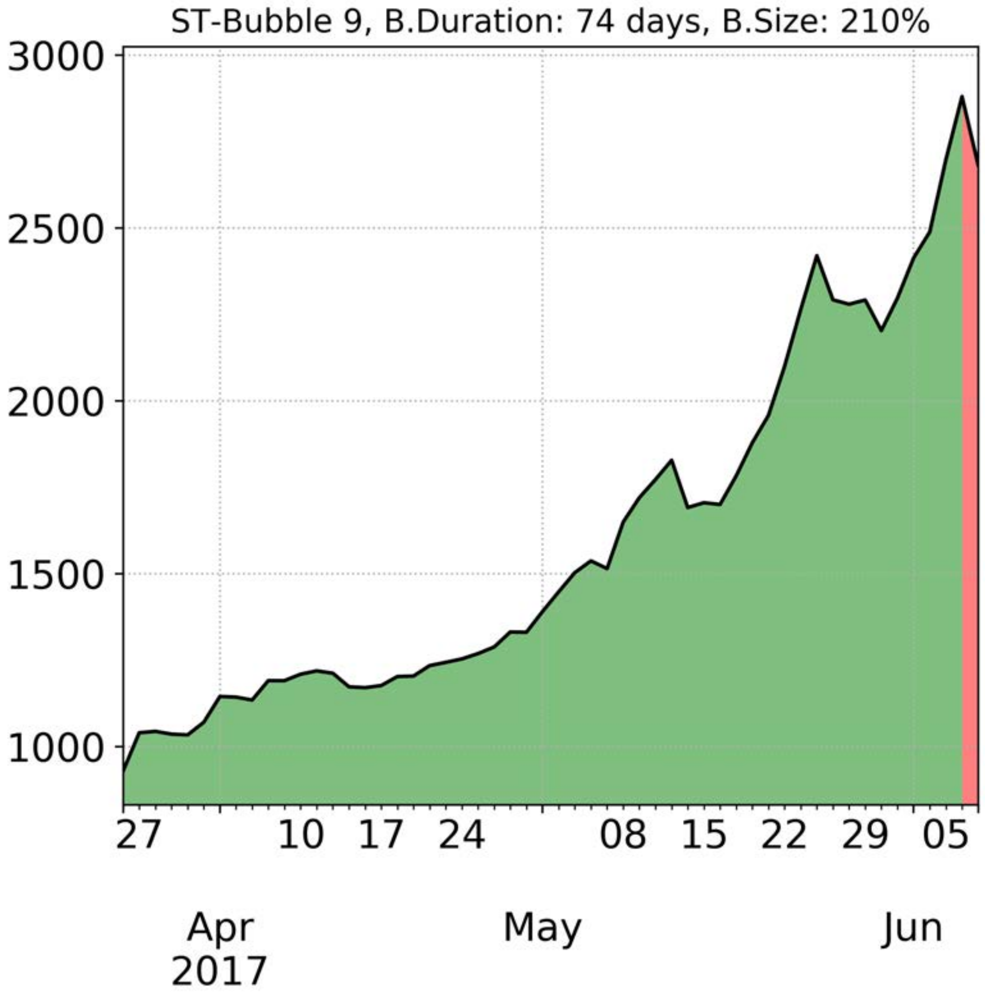


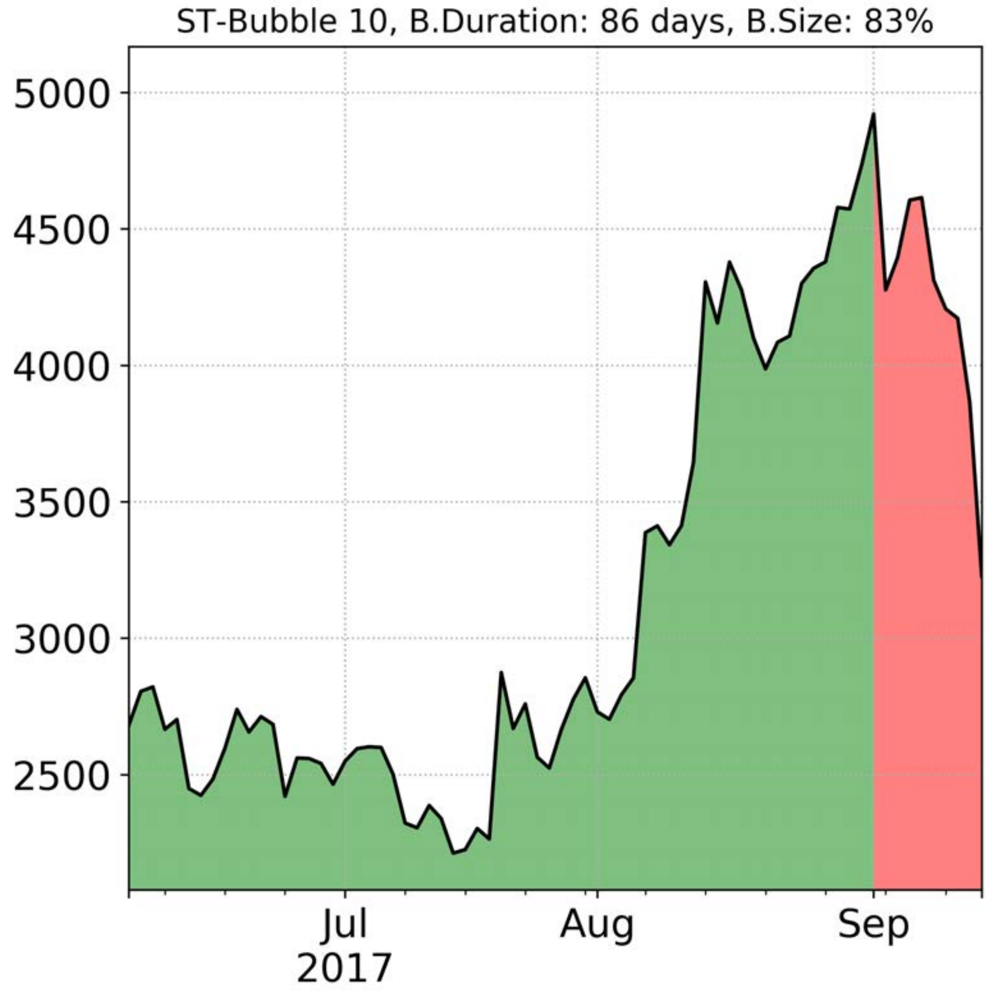
Short-Term Bubbles





Short-Term Bubbles





Methodology for diagnosing bubbles

The Log-Periodic Power Law (LPPL) model

- Positive feedbacks of higher return anticipation
  - Super exponential price
  - Power law “Finite-time singularity”
- Positive feedback of negative spirals of crash expectation
  - Accelerating large-scale financial volatility
  - Log-periodic discrete scale-invariant patterns

Expectation component of the price dynamics:

$$\ln(P) = \overset{①}{\boxed{A + B\,(t_c - t)^m}} + \overset{②}{\boxed{C\,(t_c - t)^m}} \cos\Big(\overset{③}{\boxed{\omega \ln[t_c - t]}} - \phi\Big)$$

A. Johansen, D. Sornette and O. Ledoit
Predicting Financial Crashes using discrete scale invariance,
Journal of Risk, vol. 1, number 4, 5-32 (1999)

A. Johansen, O. Ledoit and D. Sornette, Crashes as critical points,
International Journal of Theoretical and Applied Finance Vol. 3, No. 2
219-255 (2000)

DS LPPL Confidence Multiscale Indicator

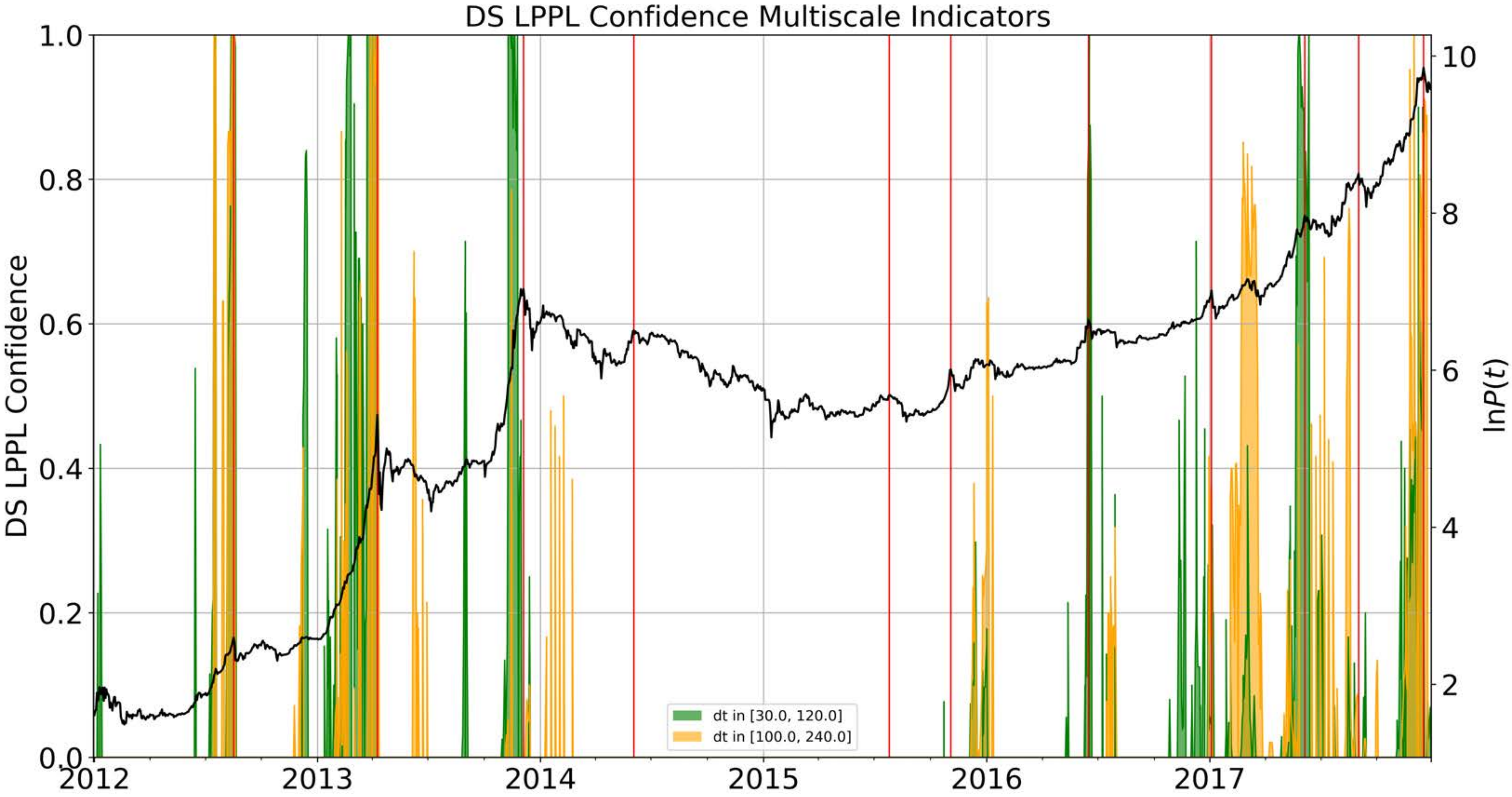
DS LPPL Confidence Indicator:

The Confidence indicator quantifies the amount of valid or 'qualified' fits of the total number of fits performed at each t2. We tag a fit as qualified when its fit parameters fulfil the following filter criteria:
ω in [4,25],
B<0,
D>0.5,
if $|\frac{C}{B}|$>0.05, OSC>2.5.
with D being the Damping $D = \frac{m|B|}{\omega|C|}$ and OSC being the number of log-periodic oscillations in the fit window $[t_1, t_2]$, OSC=$\frac{\omega}{2\pi}\ln\left(\frac{|t_c - t_1|}{|t_c - t_2|}\right)$.

Conventionally, we then calculate the indicator value at each t2 as the fraction of the number of qualified fits at this t2 divided by the total number of fits, i.e. here 691. This results in a series of values between [0,1] indicating the amount of identified super-exponential price dynamics signals, i.e. bubble activity, over time.

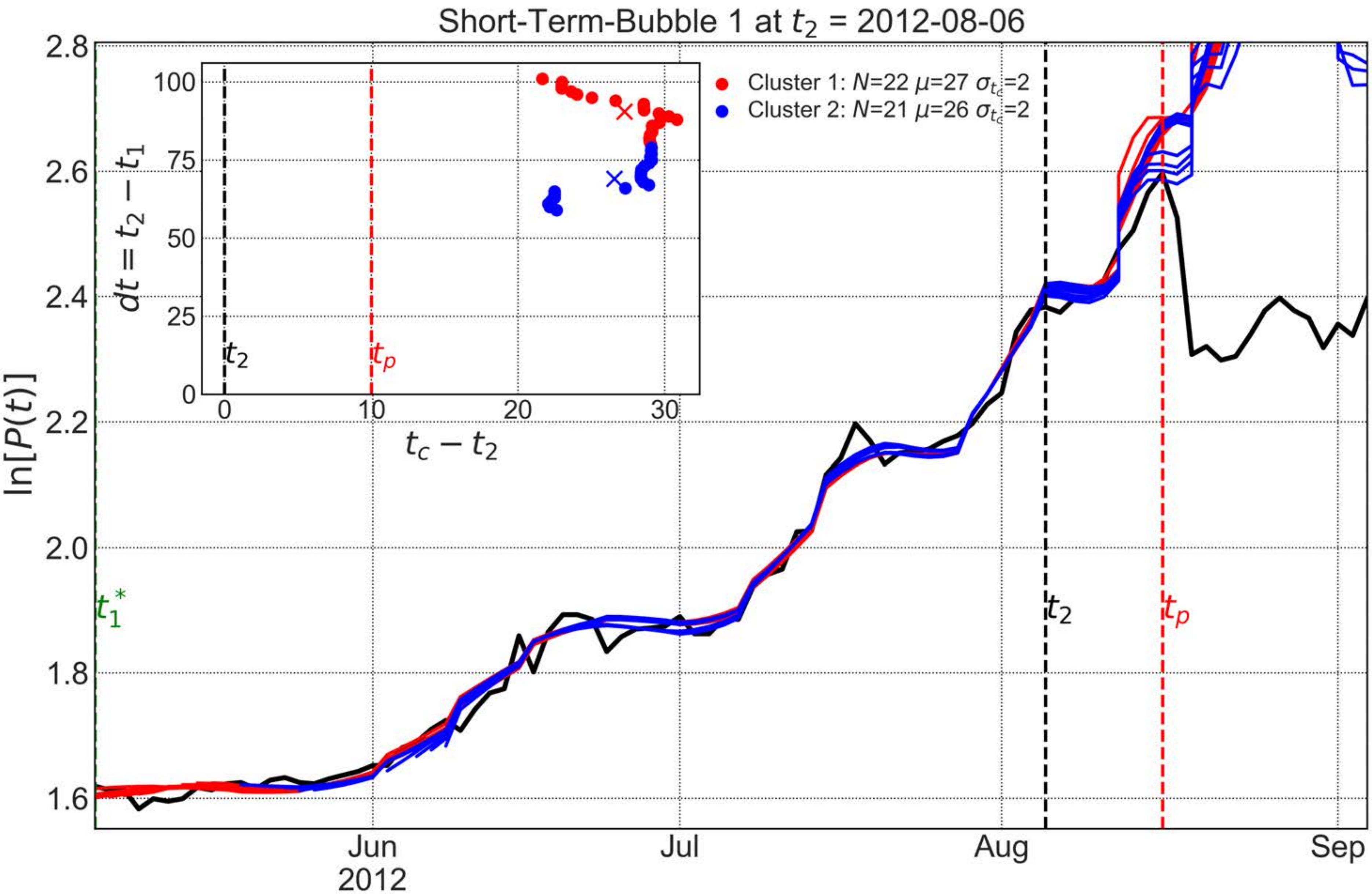
As bubbles generally have a multiscale character, i.e. they we can identify them on different timescales corresponding to ranges of the fit window size dt, we are interested in splitting up the fit results according to the dt-ranges [30,120], [100,240] and [200,720] and calculating separately the indicator values for these ranges. The methodology to identify qualified fits remains the same, however the total number of possible fits per indicator changes according to the covered ranges. Here, as divisors for the ranges above we then have 91, 141 and 521.

DS LPPL Confidence Multiscale Indicator

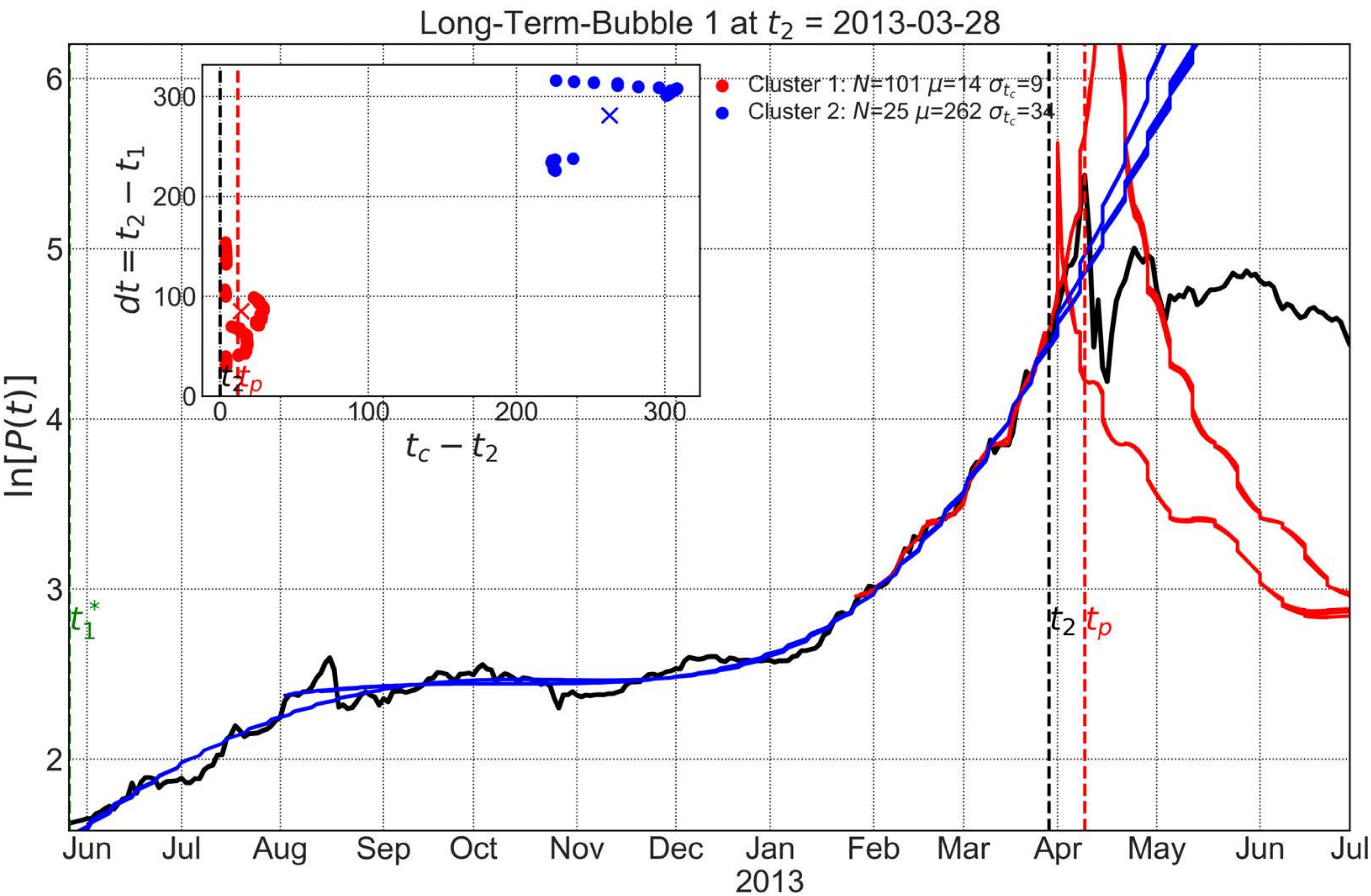


k-Means Clusters: Short-Term Bubbles

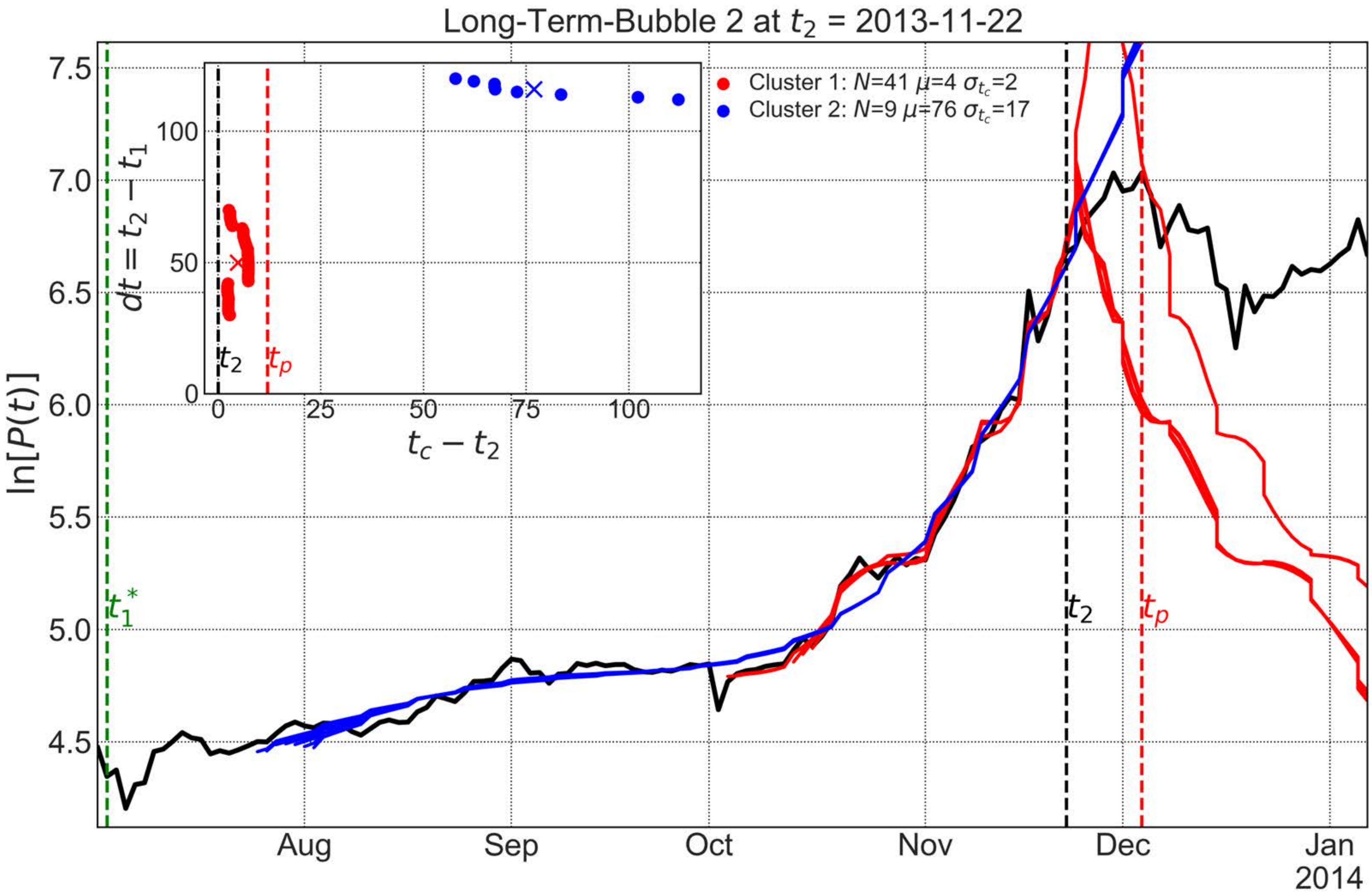
Chosen analysis dates were set to ten business days in advance to peak times. Using k-means clustering, we analyse the fit results at the resulting analysis dates by clustering them for the value of tc that they predict versus the fit window size dt. We select the optimal number of clusters according to the Silhouette Method. Crosses in the inset plot indicate the mean of the corresponding clusters. Additionally, we provide the (horizontal) standard deviation of the predicted value of tc.



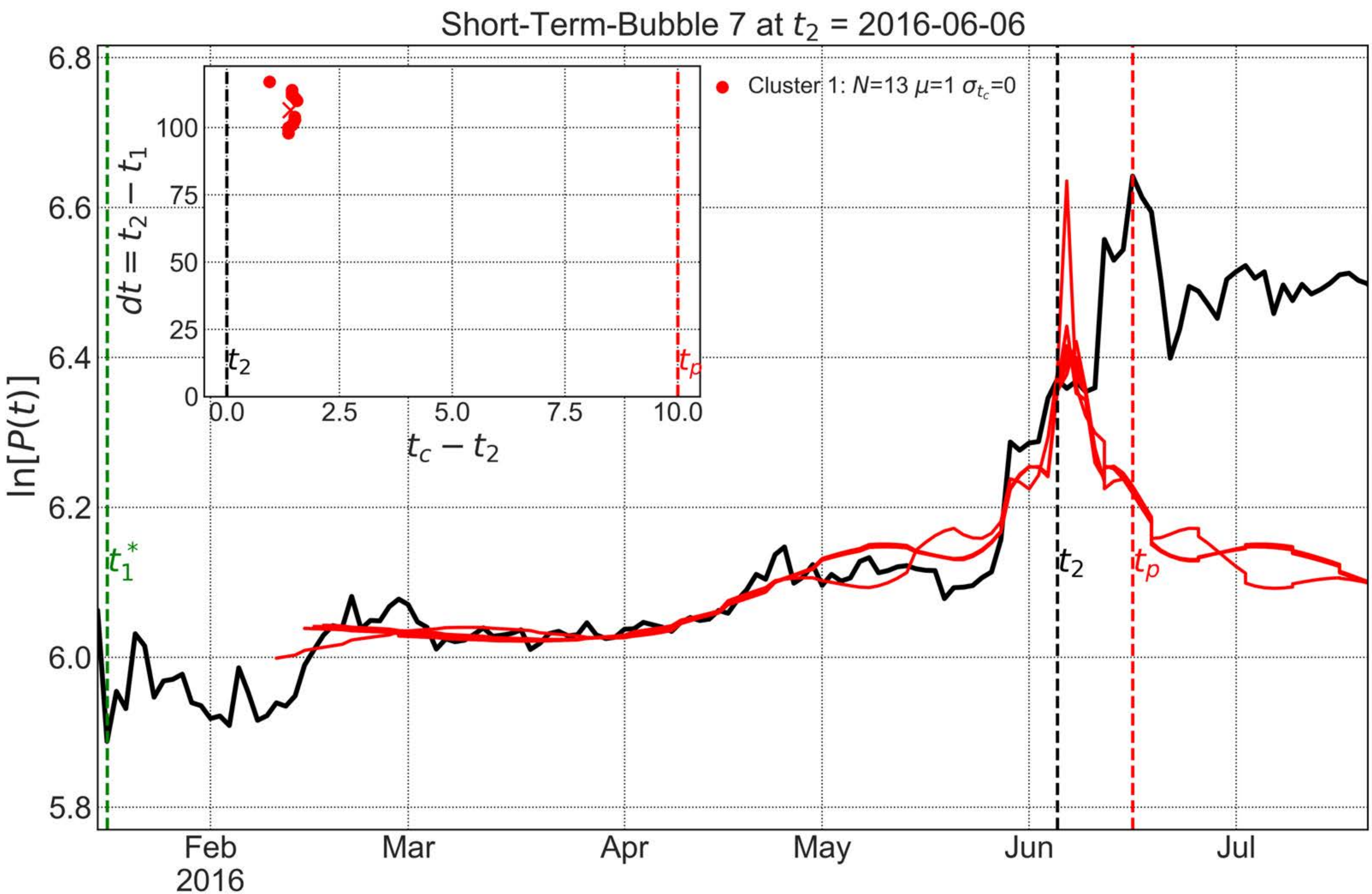
k-Means Clusters: Long-Term Bubbles



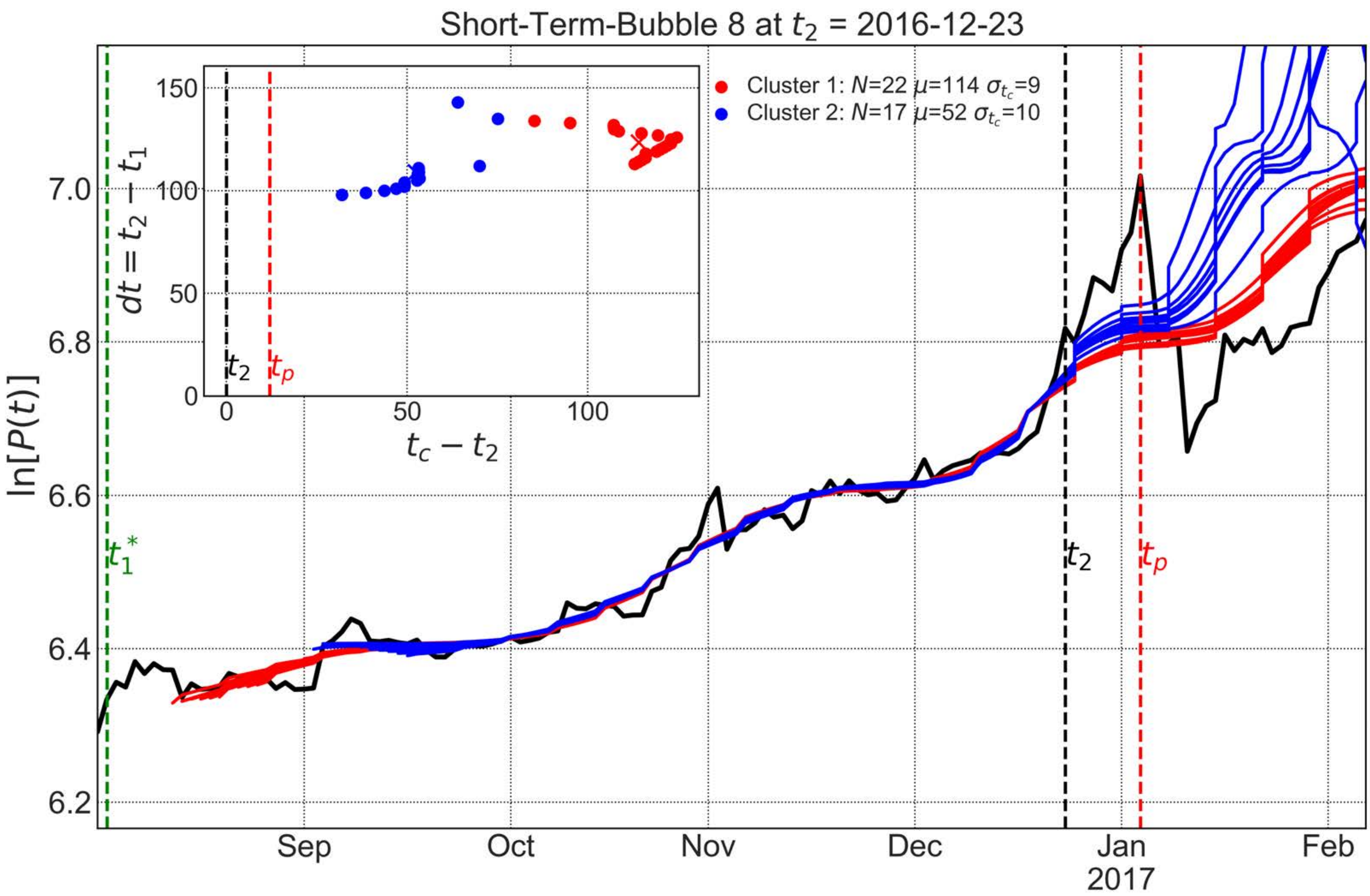
k-Means Clusters: Long-Term Bubbles



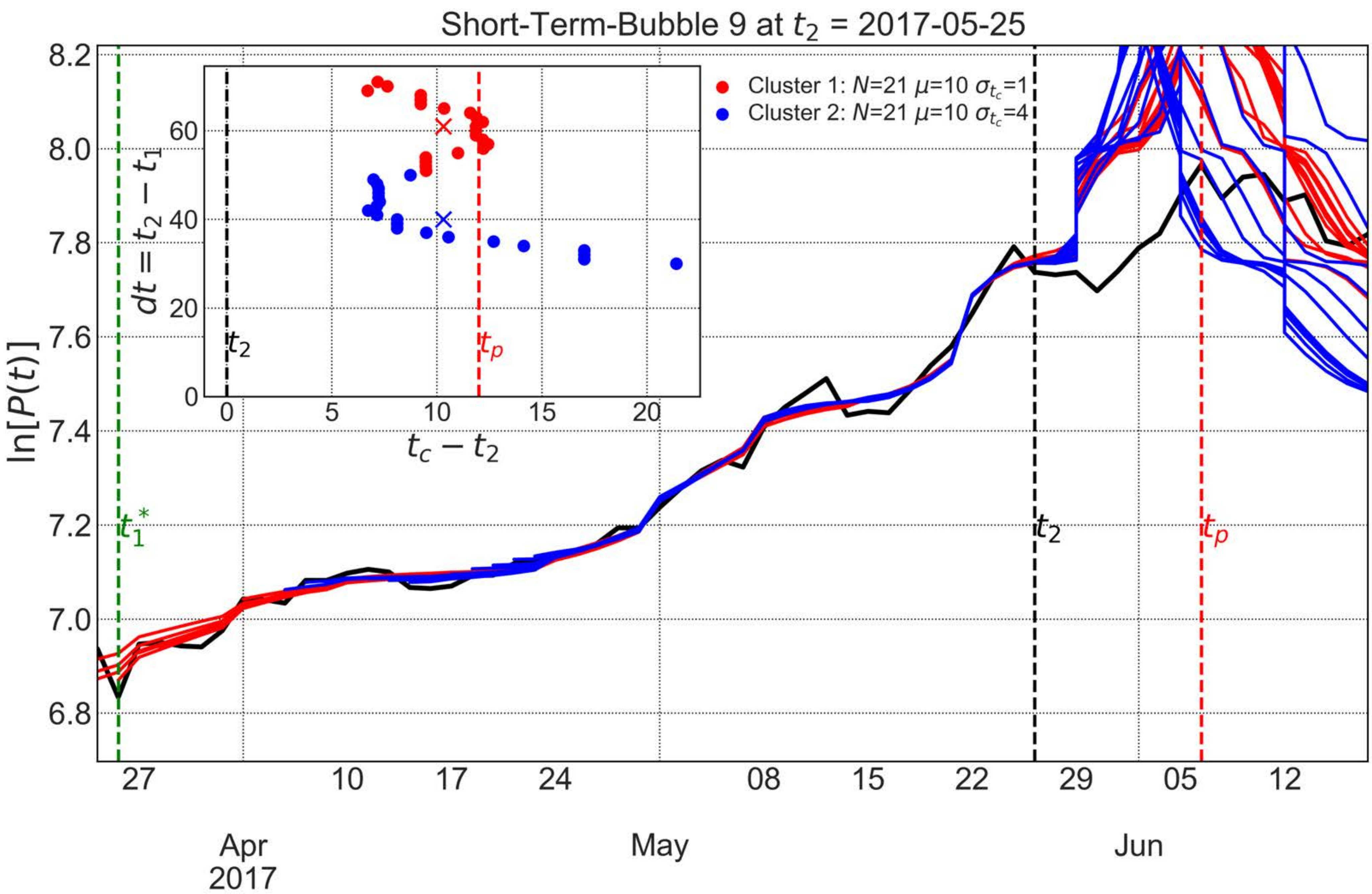
k-Means Clusters: Short-Term Bubbles



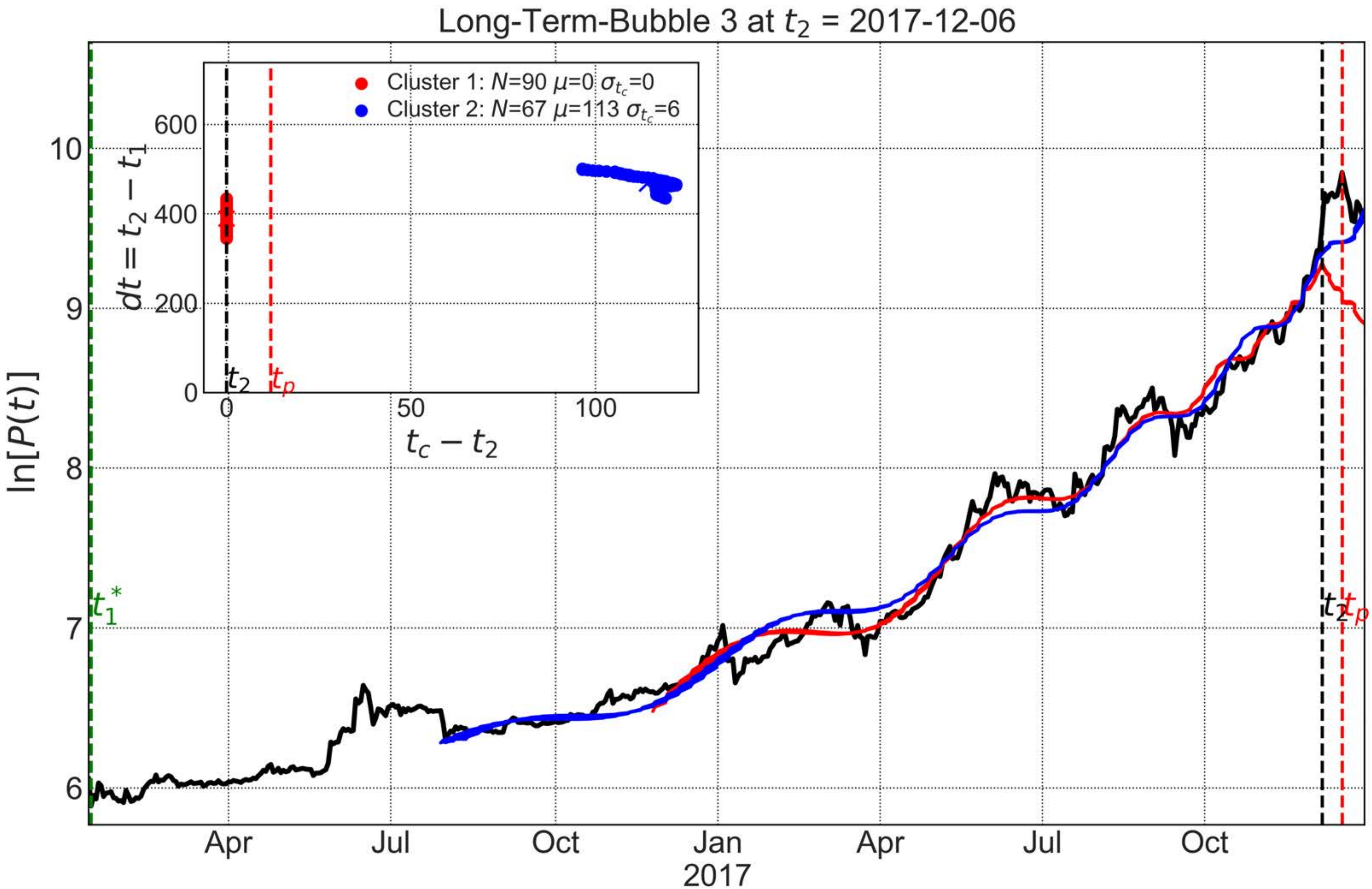
k-Means Clusters: Short-Term Bubbles



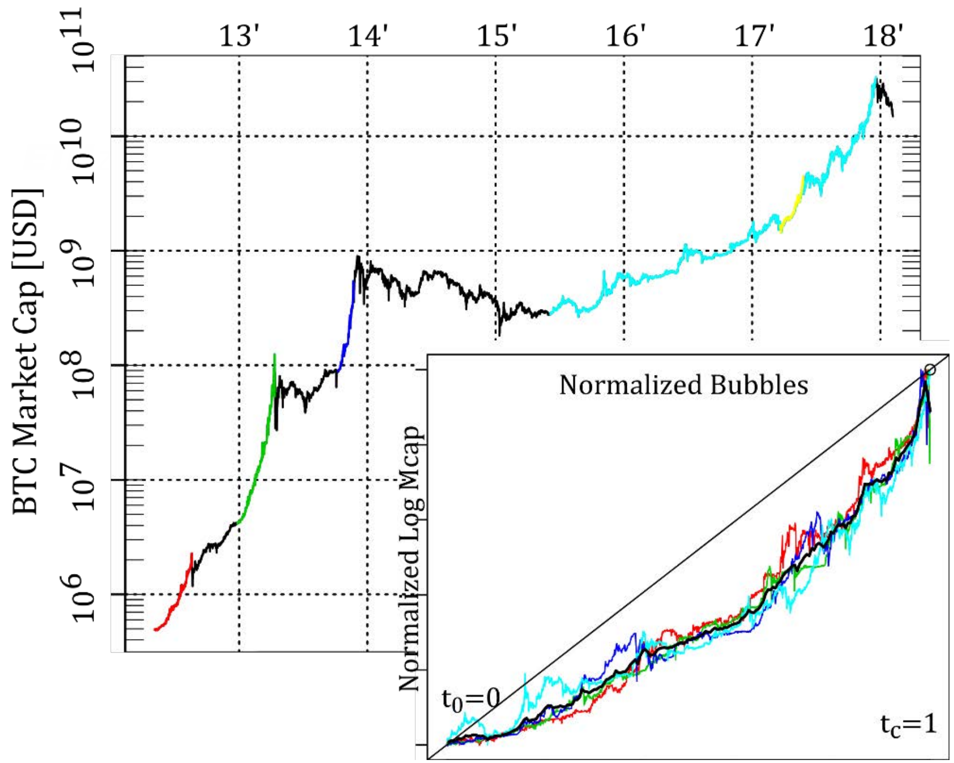
k-Means Clusters: Short-Term Bubbles



k-Means Clusters: Long-Term Bubbles

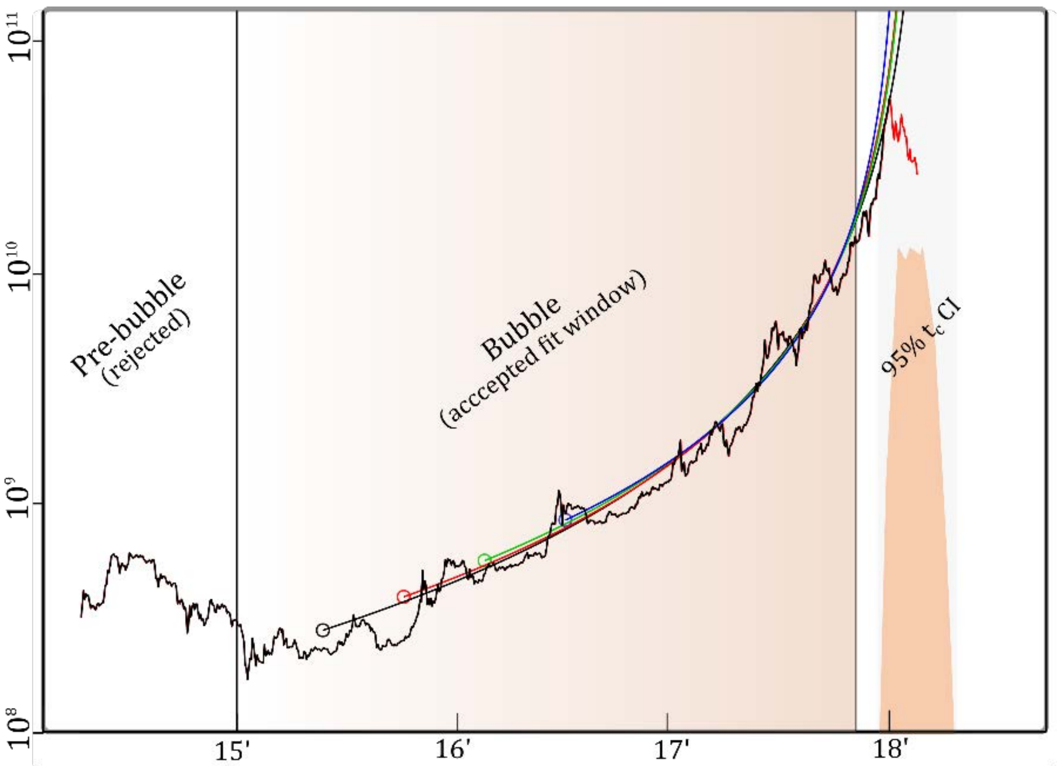


Are market instabilities predictable?



Bitcoin market cap, long bubbles indicated, and normalized to equal length and height in inset panel.

2015-2018 Bitcoin bubble. Accepted Bitcoin log-price regressions, 2 months prior to the turning point and 95% confidence interval and distribution for critical time.

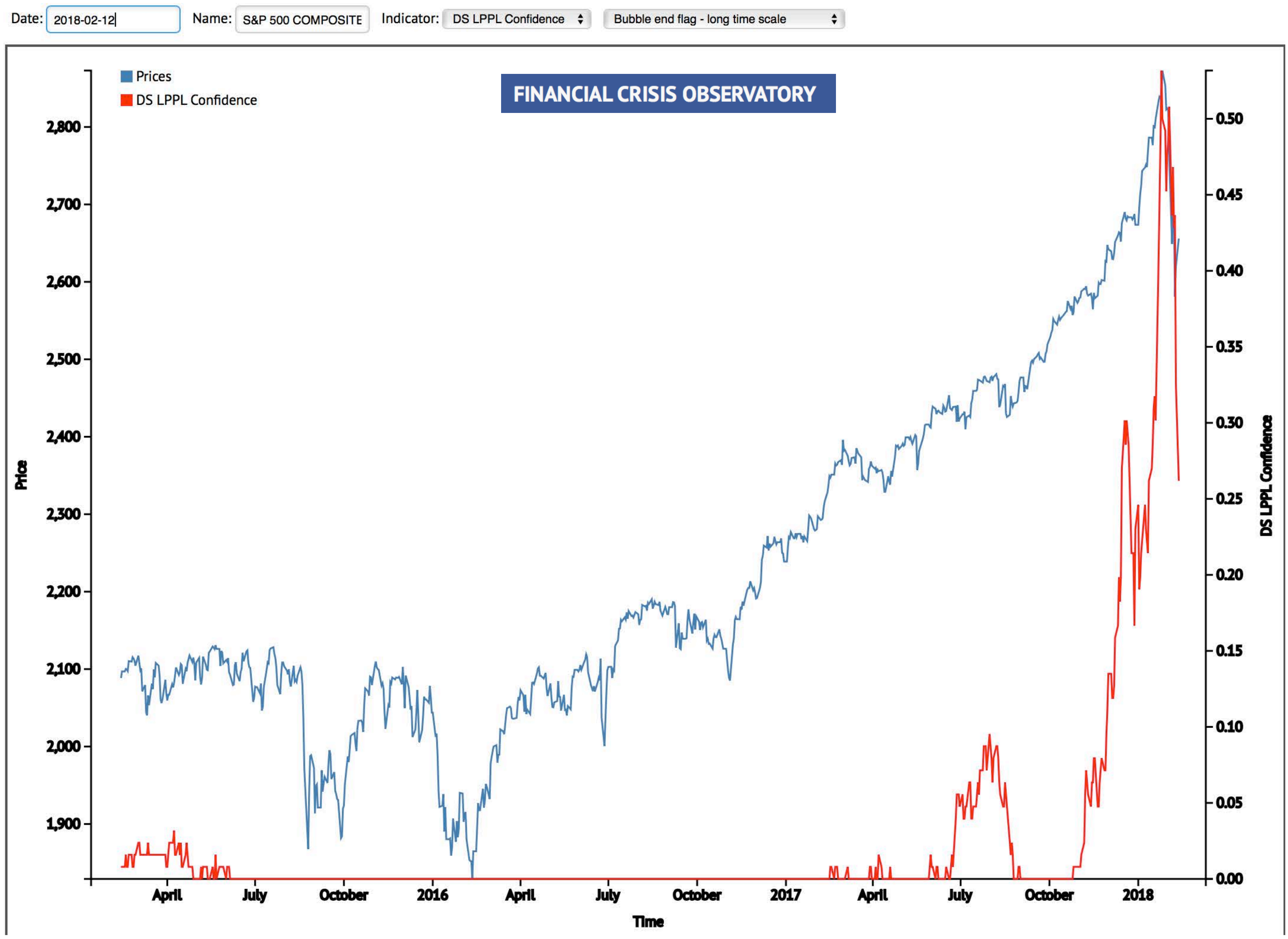


- Speculative bubbles in Bitcoin: local levels beyond what the market can sustain. Universal?
- Don’t look at the straw that breaks the camel’s back, but the heavy load it is already carrying!
- Propose: Power model for faster than exponential growth with finite time singularity at t_c
- For the log price: $\ln(p_i)=a-b(t_c-t_i)^m+\varepsilon_i$, $m>0$, $\ln(p_c)=a$
- In the JLS* model, this implies an exploding crash hazard rate proportional to: $(t_c-t_i)^{m-1}$, $m-1<0$
- In Nov 2017, bracket crash in early 2018; becomes tighter as t_c approached → useful early warning!

*Johansen, D. Sornette, O. Ledoit, Predicting Financial Crashes Using Discrete Scale Invariance, Journal of Risk 1 (4) (1999) 5–32.
*D. Sornette, A. Johansen, Significance of log-periodic precursors to financial crashes, Quantitative Finance 1 (4) (2001) 452–471.

No Coincidence Bitcoin And P/E Multiples Peaked The Same Day

source: Morgan Stanley and https://www.zerohedge.com/news/2018-02-12/morgan-stanley-no-coincidence-bitcoin-and-pe-multiples-peaked-same-day


Date: 2018-02-12
Name: S&P 500 COMPOSITE
Indicator: DS LPPL Confidence
Bubble end flag - long time scale
Prices
DS LPPL Confidence
FINANCIAL CRISIS OBSERVATORY
Price
2,800
2,700
2,600
2,500
2,400
2,300
2,200
2,100
2,000
1,900
DS LPPL Confidence
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
April
July
October
2016
April
July
October
2017
April
July
October
2018
Time

http://tasmania.ethz.ch/pubfco/fco.html

Illusions and lottery economy:
market paper growth, economic growth and crypto-currencies

1945-1970: reconstruction boom and consumerism

1971-1980: Bretton Woods system termination and oil shocks / inflation shocks

1981-2007: Illusion of the "perpetual money machine" and virtual financial wealth

2008-2020s: New era of pseudo growth fueled by QEs and other Central Banks+Treasuries actions

- -very low interest rate for a very long time (decades)
- -net erosion even in the presence of apparent low (disguised) inflation
- -reassessment of expectation for the social and retirement liabilities
- -a turbulent future with many transient bubbles
- -need to capture value and be contrarian => exploit herding and fear

2020s-20xx: Interconnection of many systemic risks