ETH
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Parallels between Earthquakes, Financial crashes and epileptic seizures

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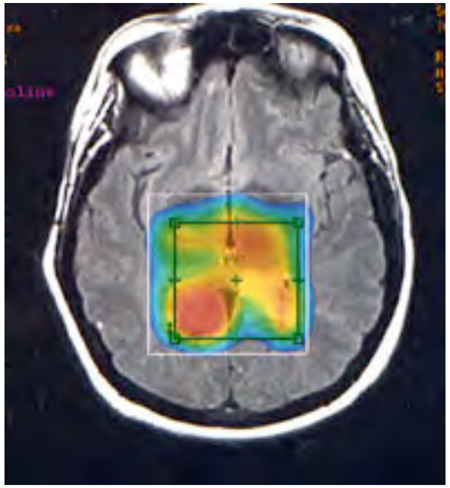
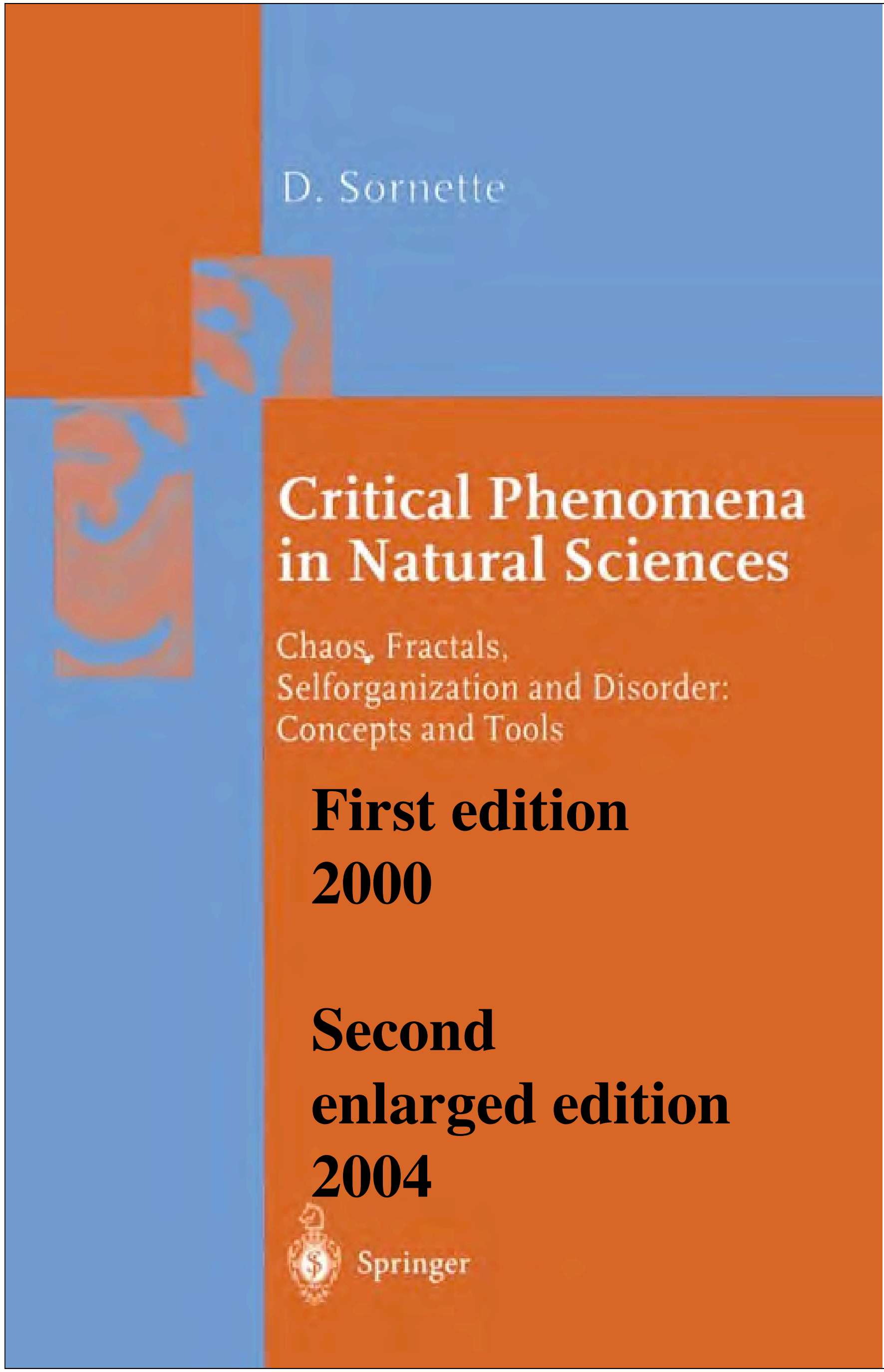
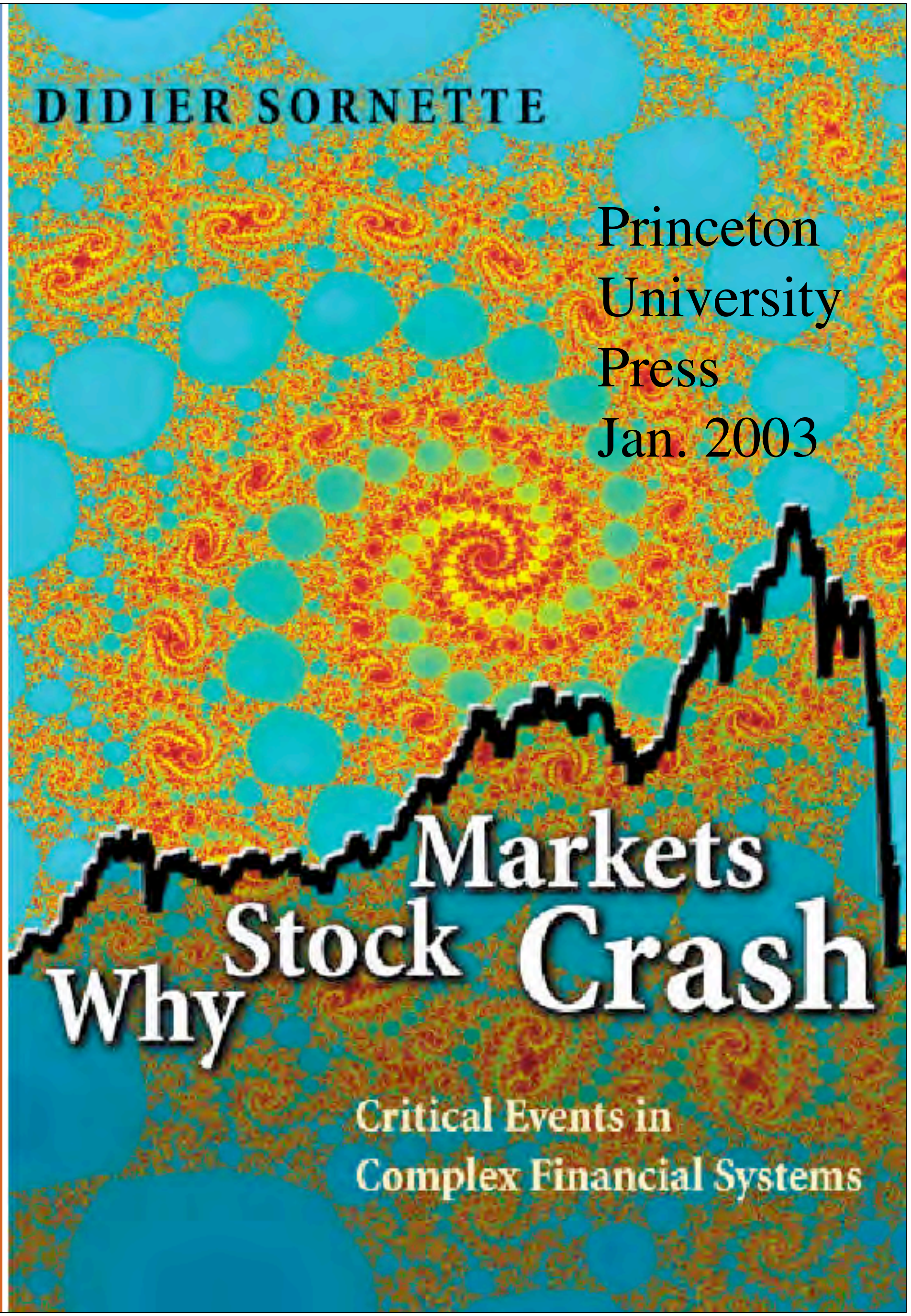
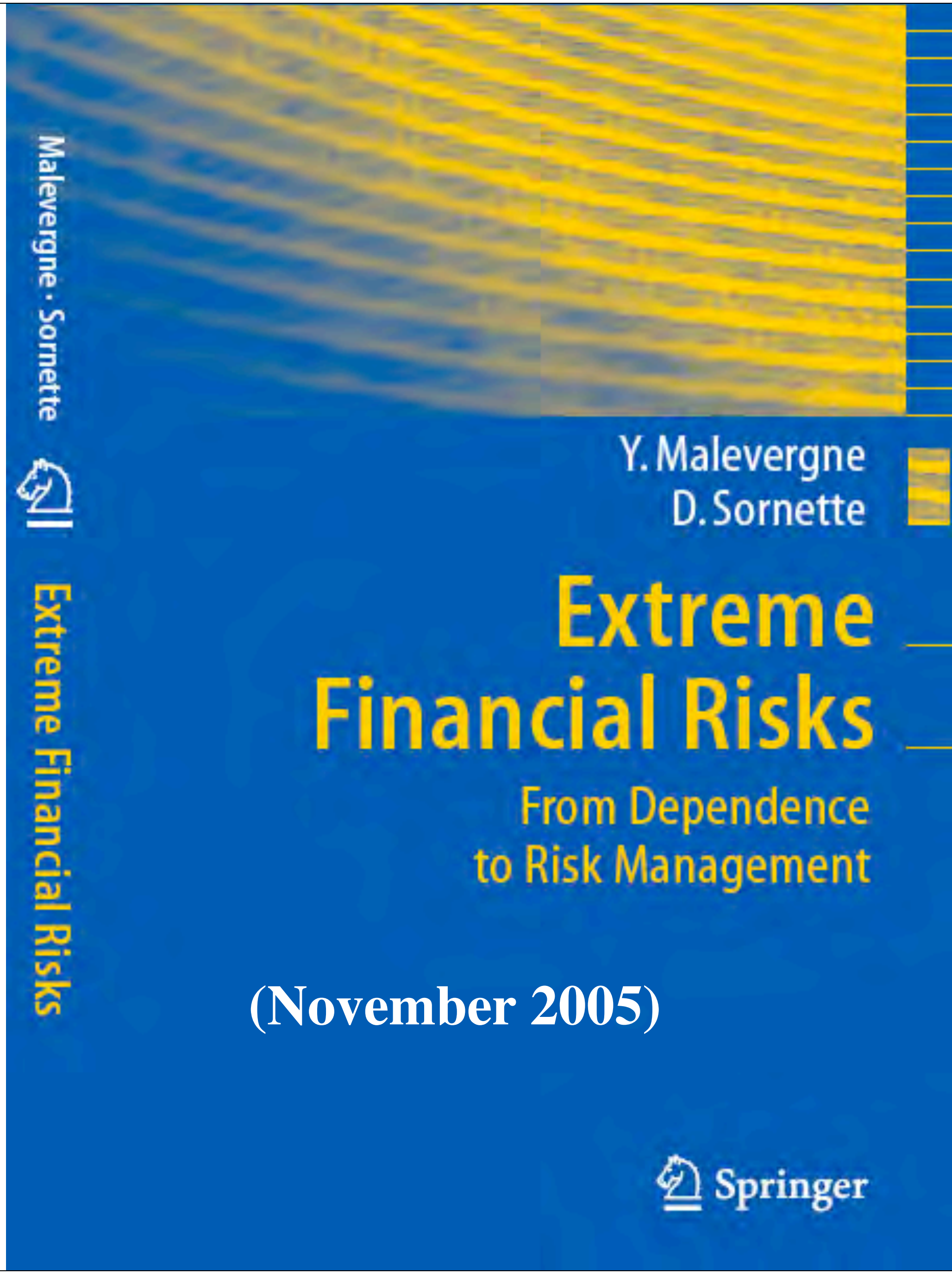
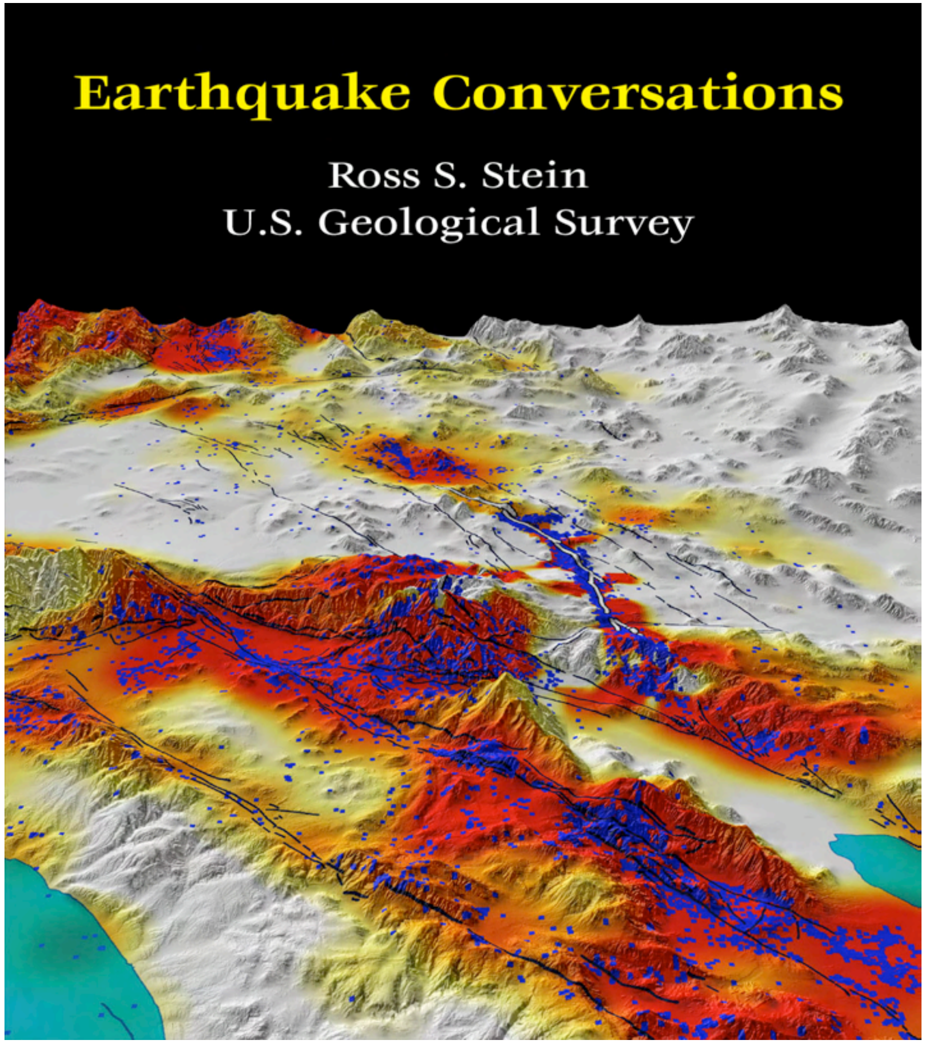


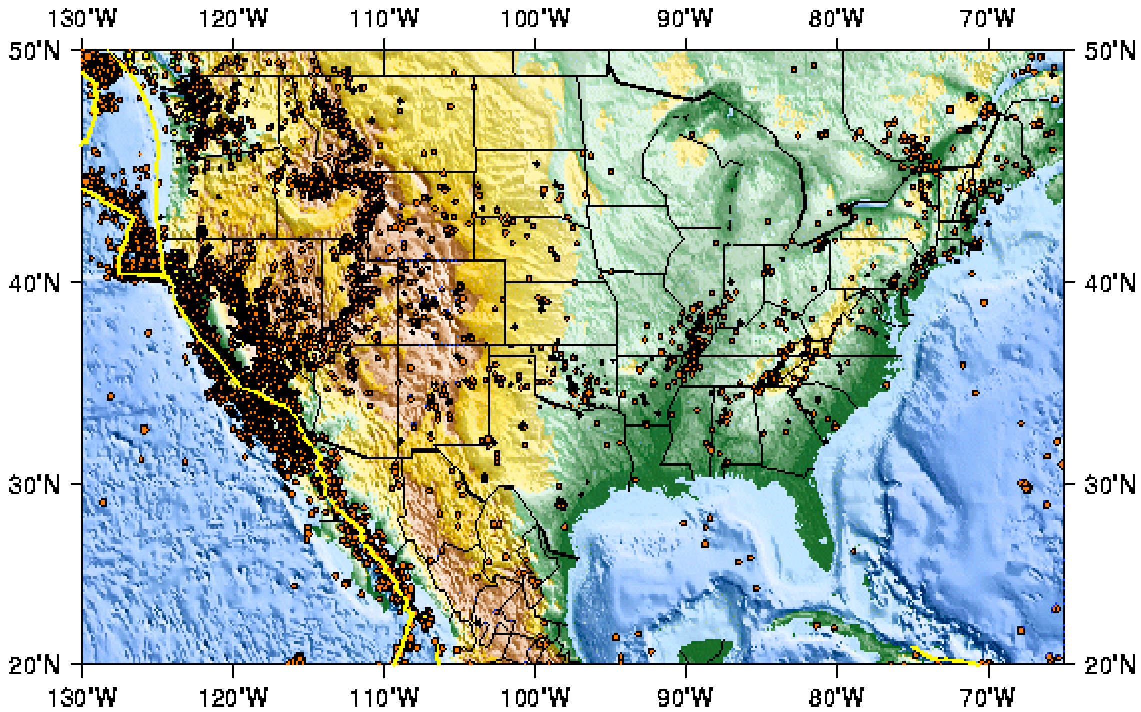

D. Sornette
Critical Phenomena in Natural Sciences
Chaos, Fractals, Selforganization and Disorder: Concepts and Tools
First edition 2000
Second enlarged edition 2004
Springer


DIDIER SORNETTE
Princeton University Press Jan. 2003
Why Stock Markets Crash
Critical Events in Complex Financial Systems


Malevergne · Sornette
Extreme Financial Risks
Y. Malevergne
D. Sornette
Extreme
Financial Risks
From Dependence
to Risk Management
(November 2005)
Springer



Epidemic processes by word-of-mouth, sentiment, convention...


130°W
120°W
110°W
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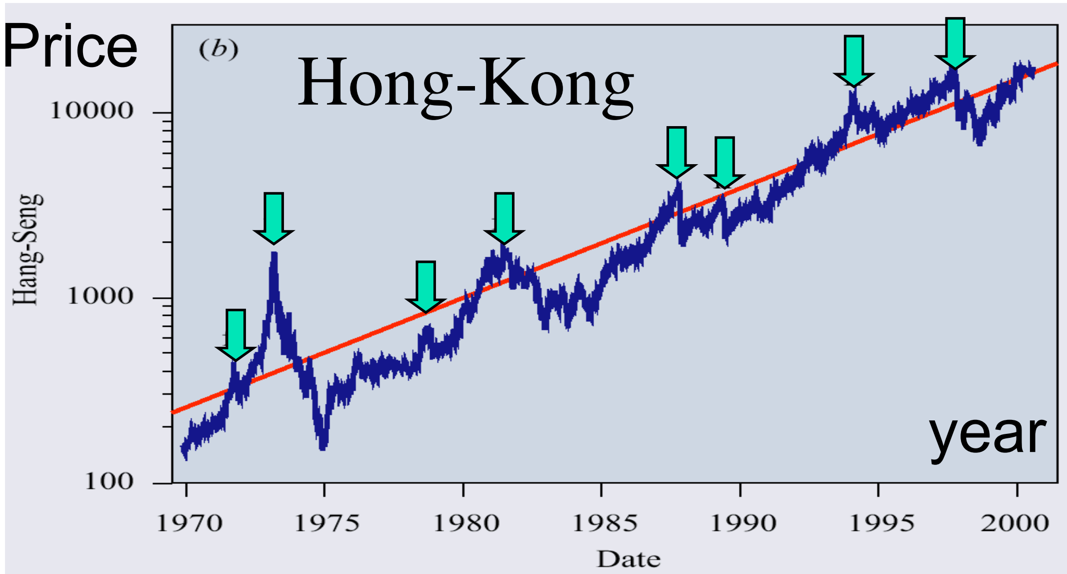
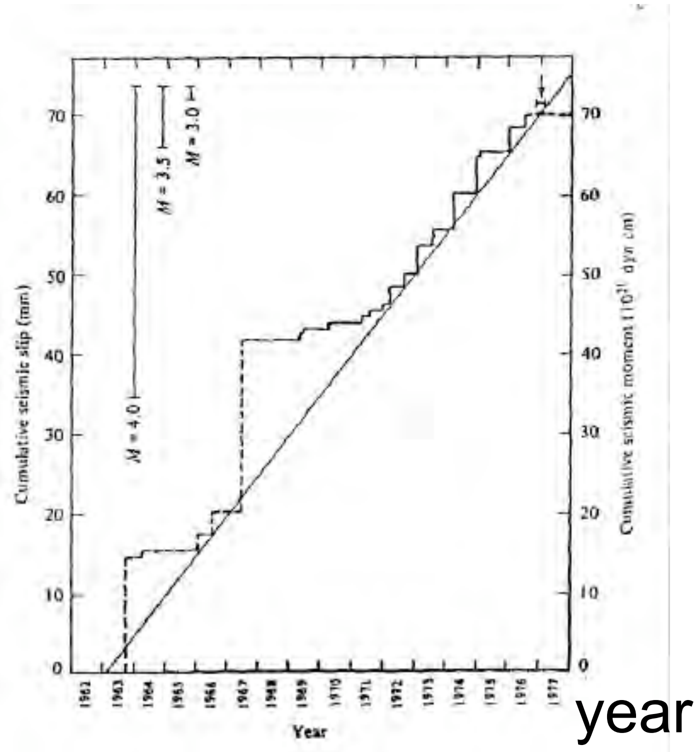
90°W
80°W
70°W
50°N
40°N
30°N
20°N



Red line is 13.8% per year: but The market is never following the average growth; it is either super-exponentially accelerating or crashing

Cumulative Slip



Cumulative moment and seismic slip in a zone of the Calaveras fault (1962-77)

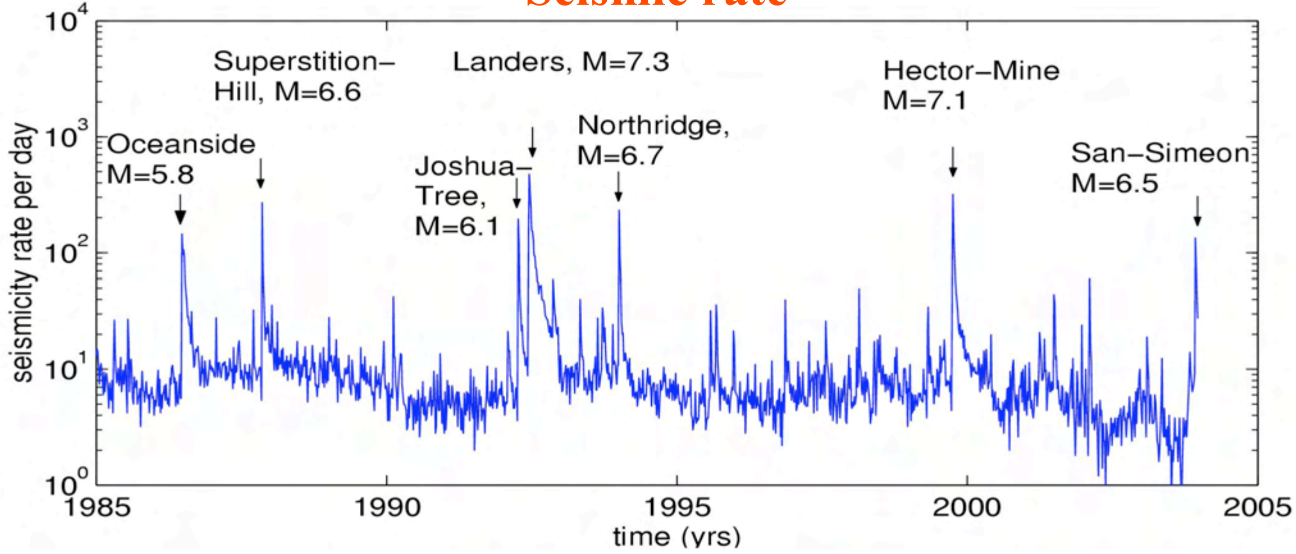
Statistical laws of seismicity

- Gutenberg-Richter law: $\sim 1/E^{1+\beta}$ (with $\beta \approx 2/3$)
- Omori law $\sim 1/t^p$ (with $p \approx 1$ for large earthquakes)
- Productivity law $\sim E^a$ (with $a \approx 2/3$)
- PDF of fault lengths $\sim 1/L^2$
- Fractal/multifractal structure of fault networks $\zeta(q)$, $f(\alpha)$
- PDF of seismic stress sources $\sim 1/s^{2+\delta}$ (with $\delta \geq 0$)
- Distribution of inter-earthquake times
- Distribution of seismic rates

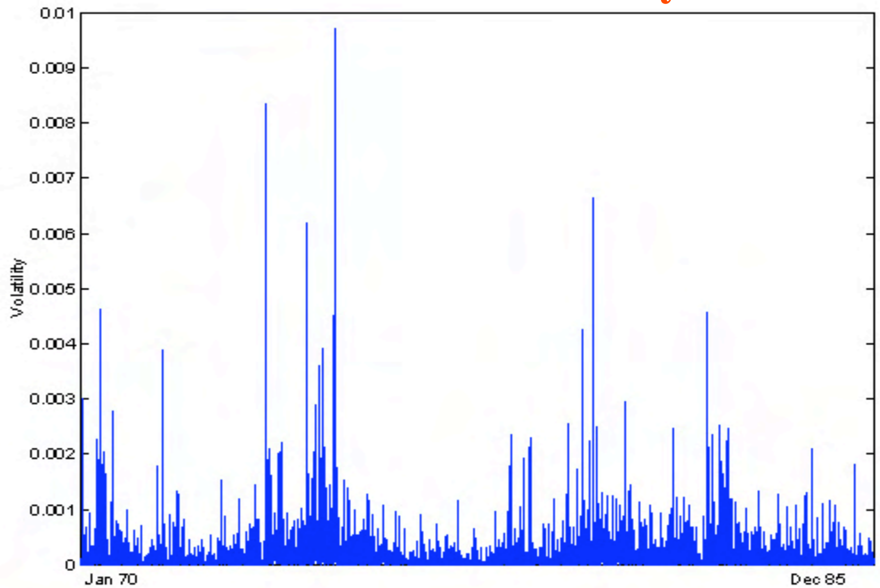
Stylized facts of financial markets

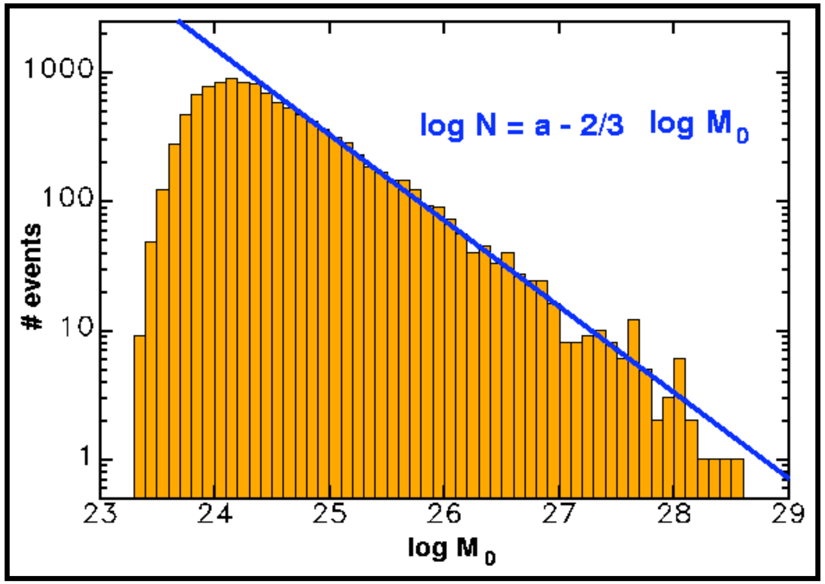
- Heavy-tail pdf of returns
- Omori law and Long-memory of volatility
- Price impact function Price ~ V^{β} with β=0.2-0.6
- Pareto distribution of wealth
- Multifractal structure of returns
- PDF of news' sizes?
- Distribution of inter-shock times
- Distribution of limit order sizes
- "Leverage" effect

Seismic rate

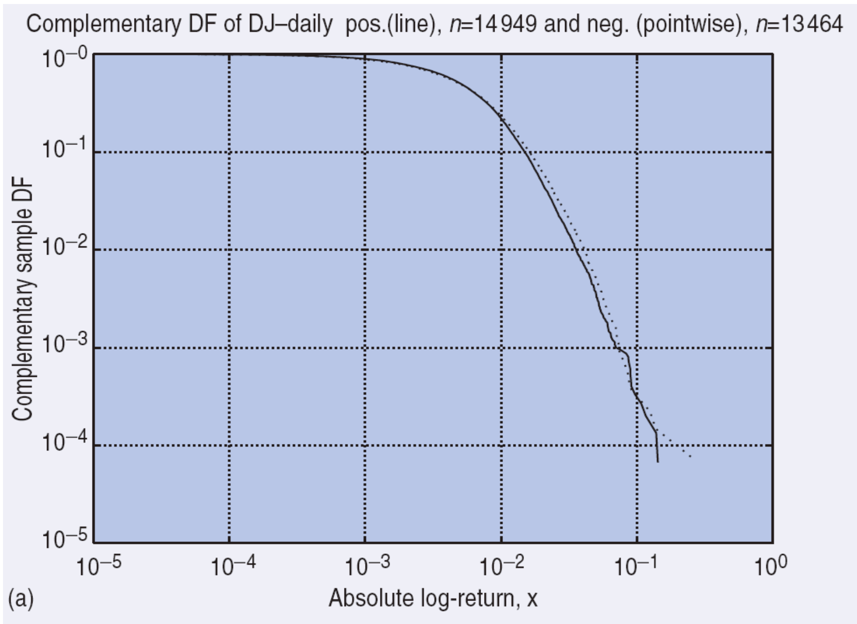


Financial Volatility

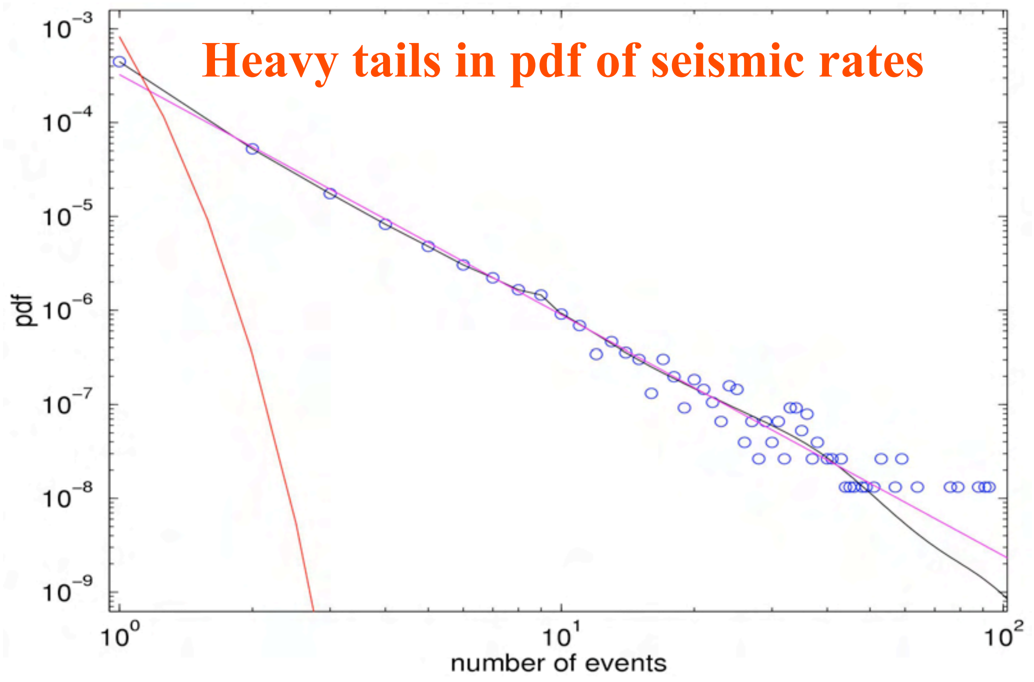


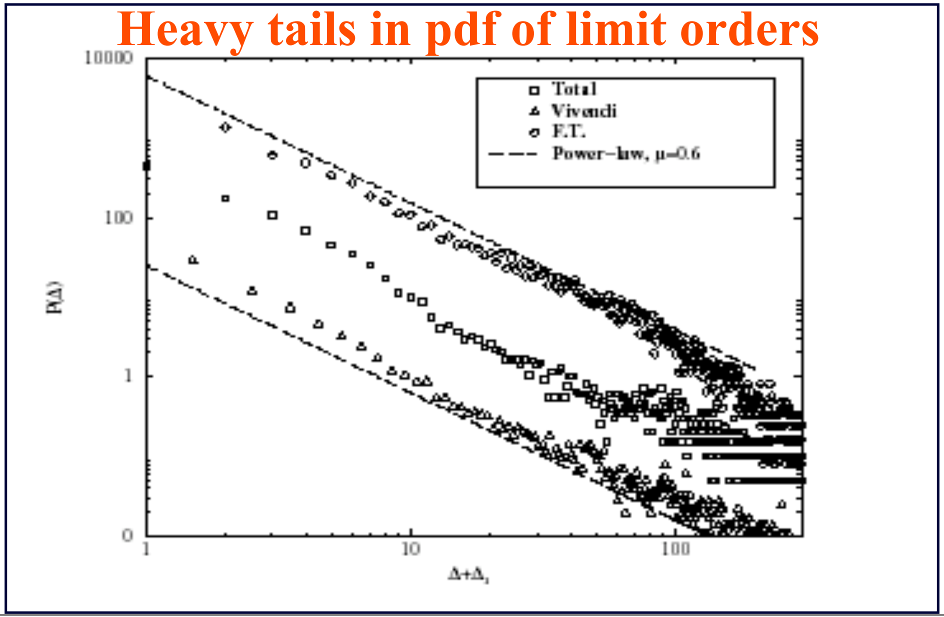


Heavy tails in pdf of earthquakes b=2/3



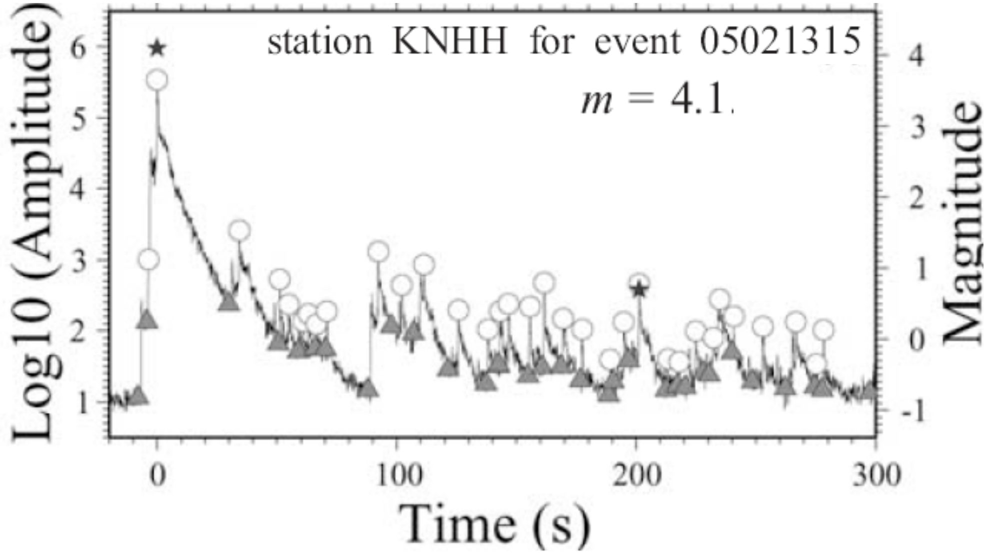
Heavy-tails of price changes b=3

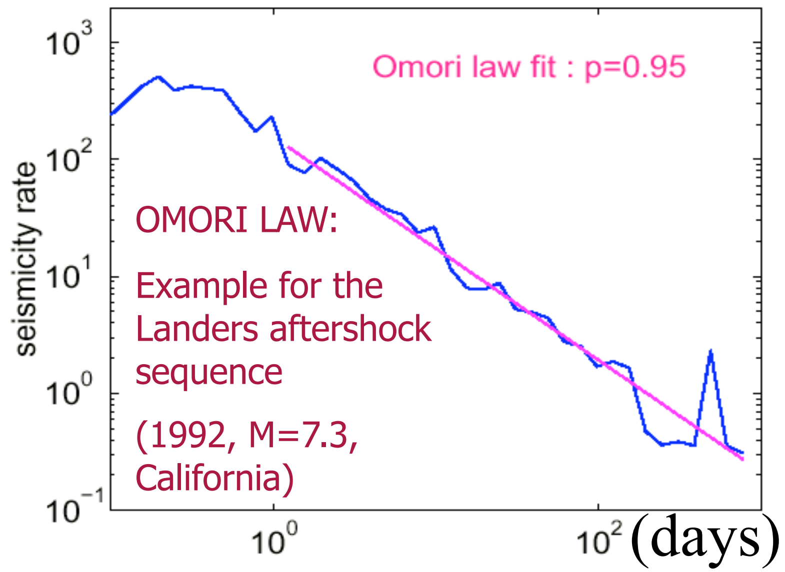

Heavy tails in pdf of seismic rates
pdf
number of events

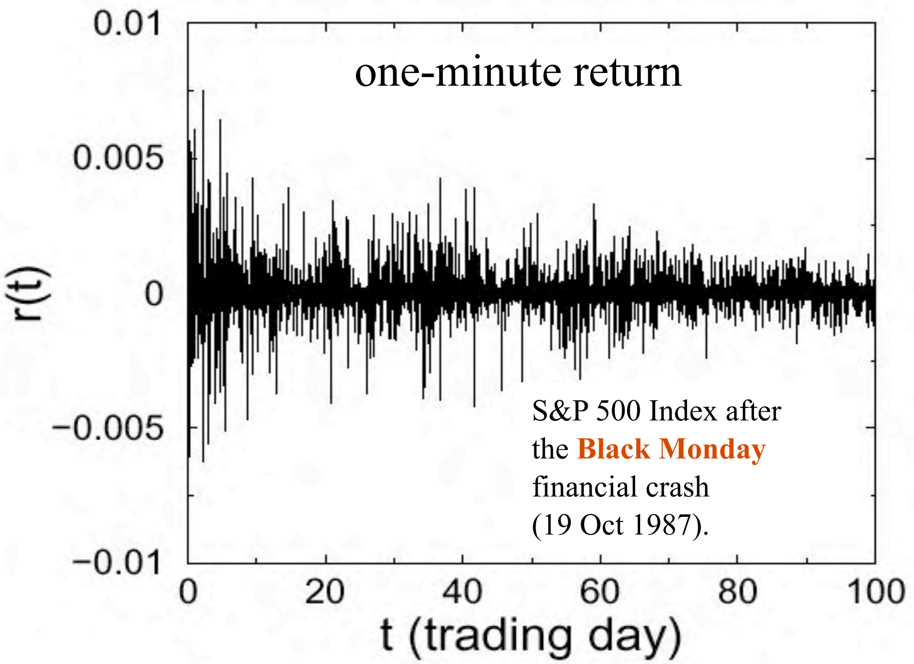

Heavy tails in pdf of limit orders
Total
Vivendi
F.T.
Power-law, μ=0.6
P(Δ)
Δ+Δ

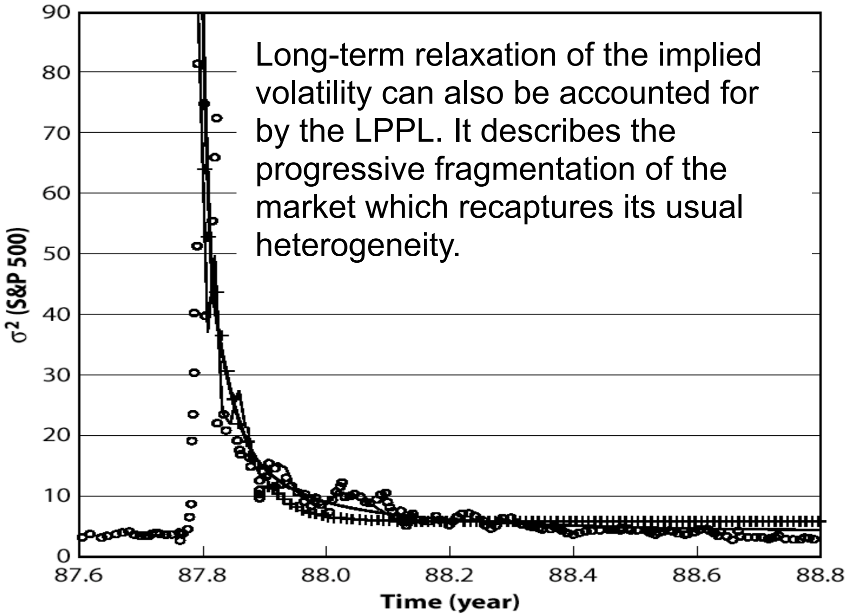
Peng et al.

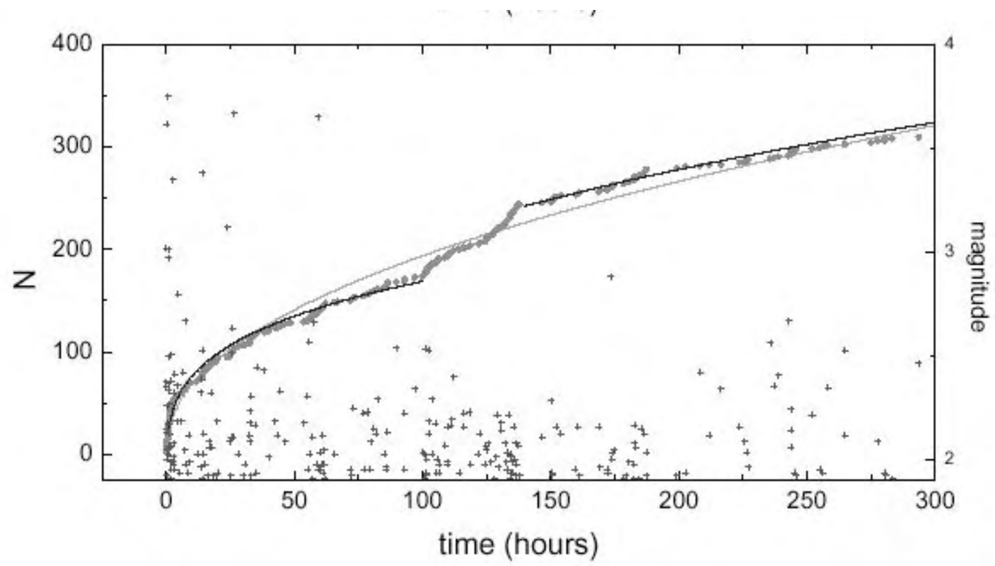
JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 112, B03306, doi:10.1029/2006JB004386, 2007







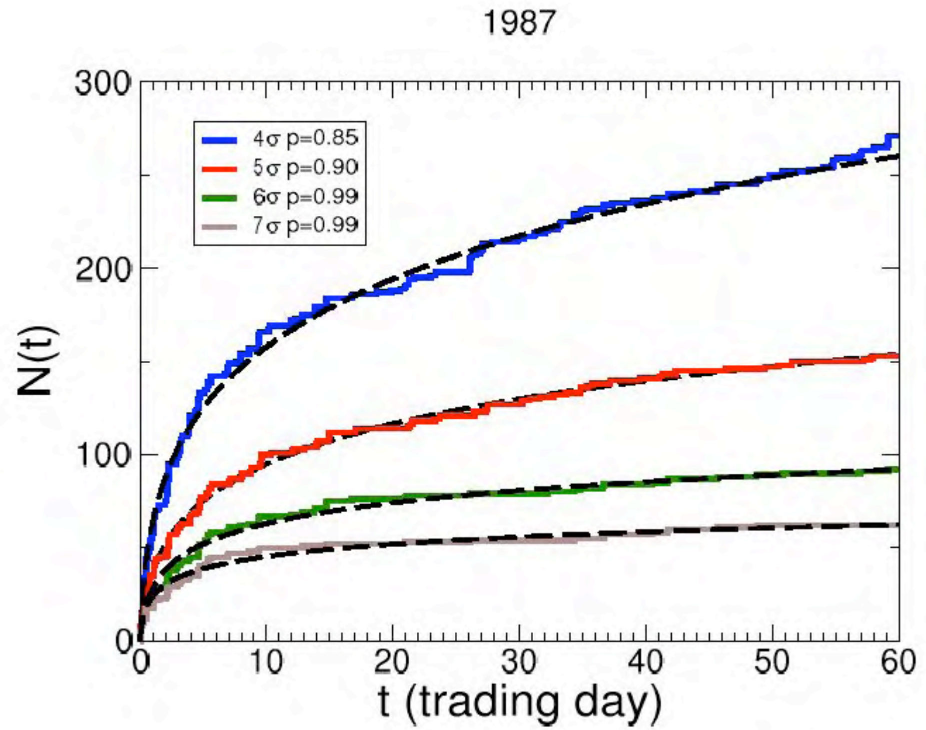




Cumulative number of aftershocks in the earthquake occurring in eastern Pyrenees on February 18, 1996 (from Moreno *et al.*, J. of Geophys. Res., **106 B4**, 6609-6619 (2001))

$$n(t) \propto t^{-p}; \qquad N(t) = \int_0^t n(s)ds$$

N(t)=K[(t+τ)$^{1-p}$- τ^{1-p}]/(1-p)



Oct. 1987 crash: Cumulative number of S&P500 index returns exceeding a given threshold **nσ**

[†]Lillo and Mantegna, PRE **68**, 016119 (2003)

Critical earthquakes? Critical crashes?

Sornette and Sammis [1995]

RENORMALIZATION GROUP THEORY OF EARTHQUAKES

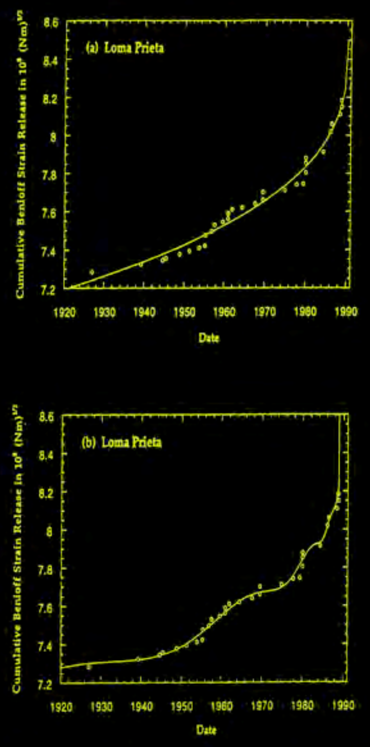
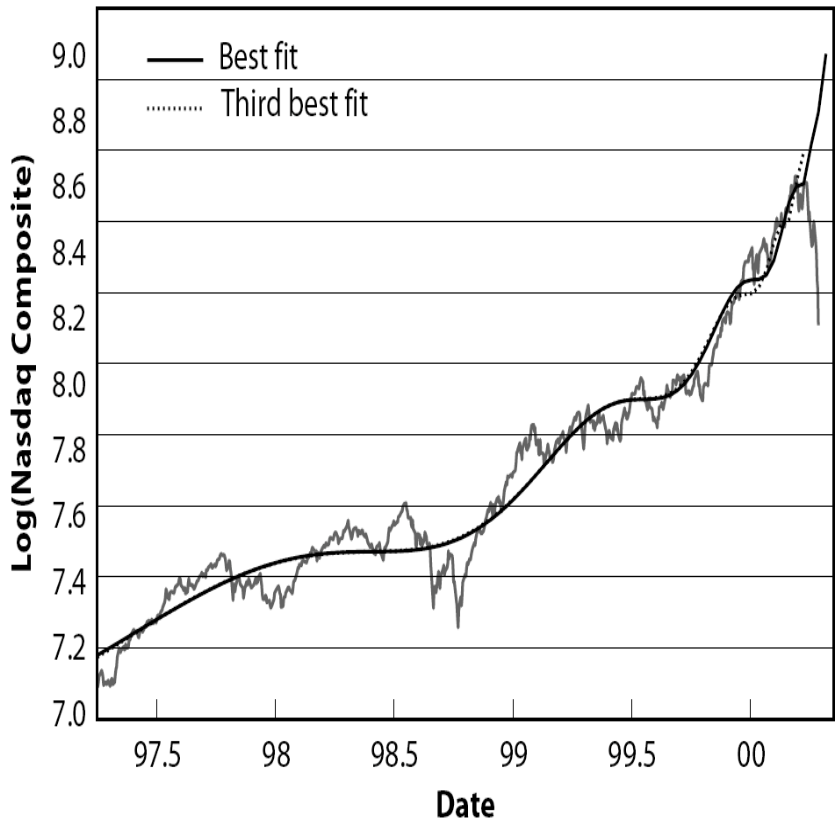
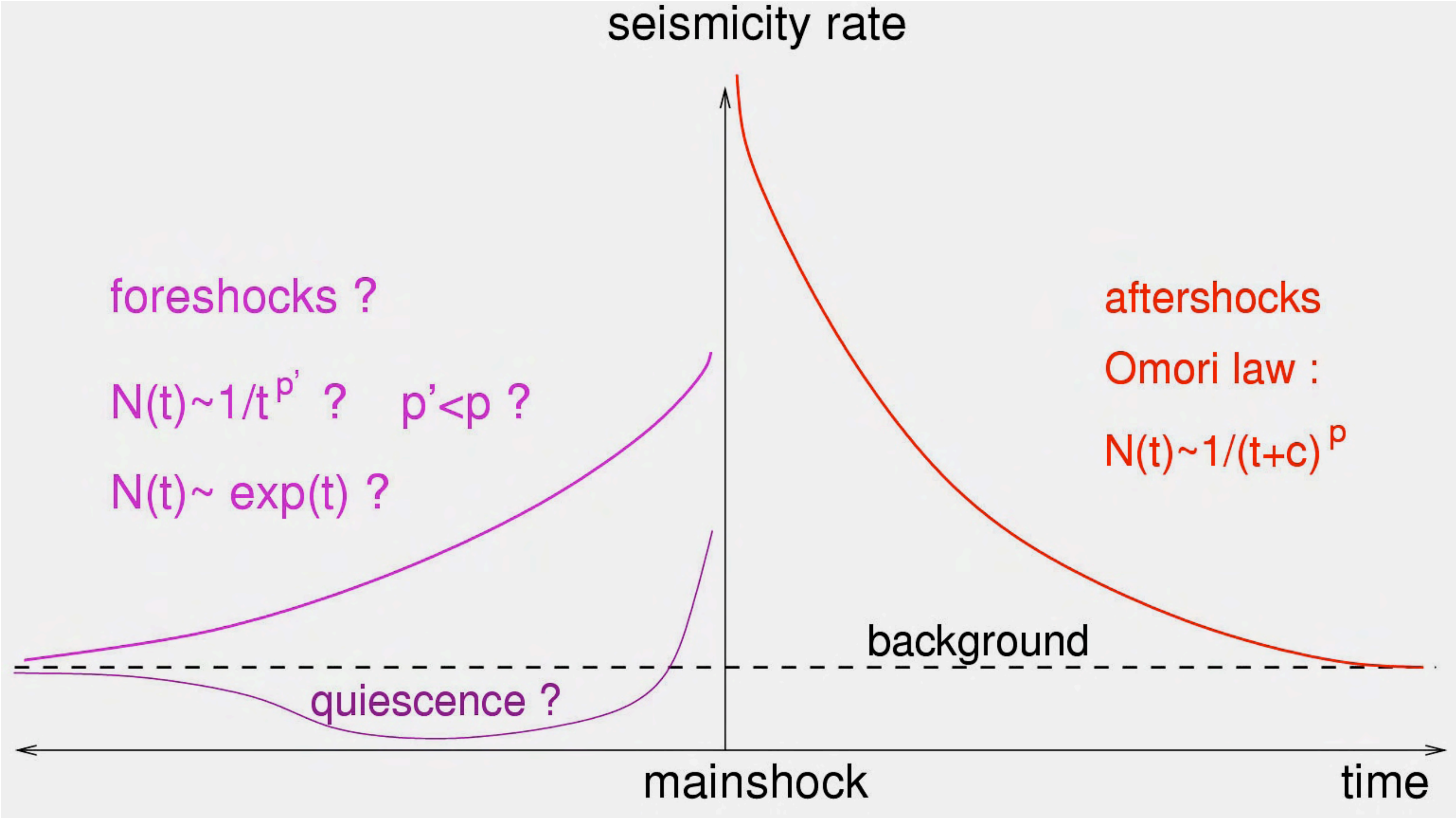


Fig. 1. — Cumulative Benioff strain released by magnitude 5 and greater earthquakes in the San Francisco Bay area prior to the 1989 Loma Prieta eaerthquake (from Ref. [32]). In (a), the data have been fit to the powerlaw equation (2) as in Bufe and Varnes [32]. In (b), the data have been fit to equation (8) which includes the first order correction to scaling. Parameters of both fits are given in Table I.

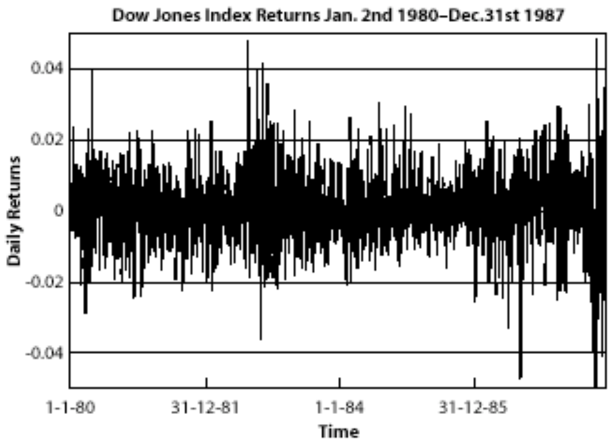
THE NASDAQ CRASH OF APRIL 2000

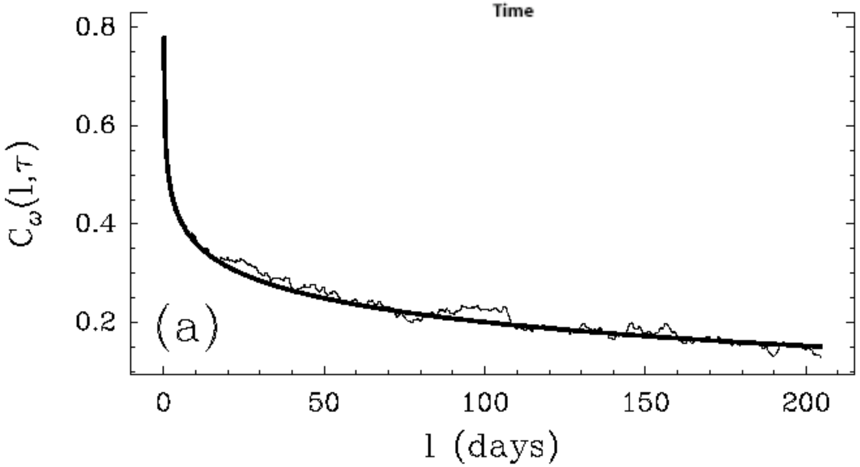


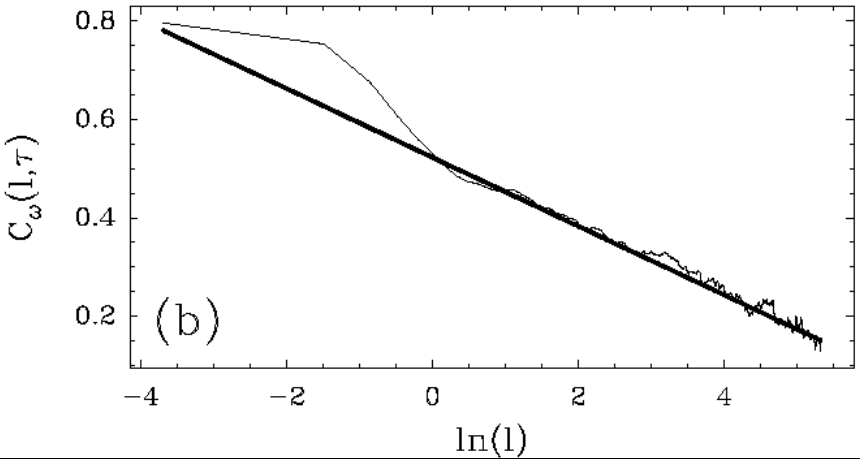
Generic multifractality
Endogenous versus Exogenous responses

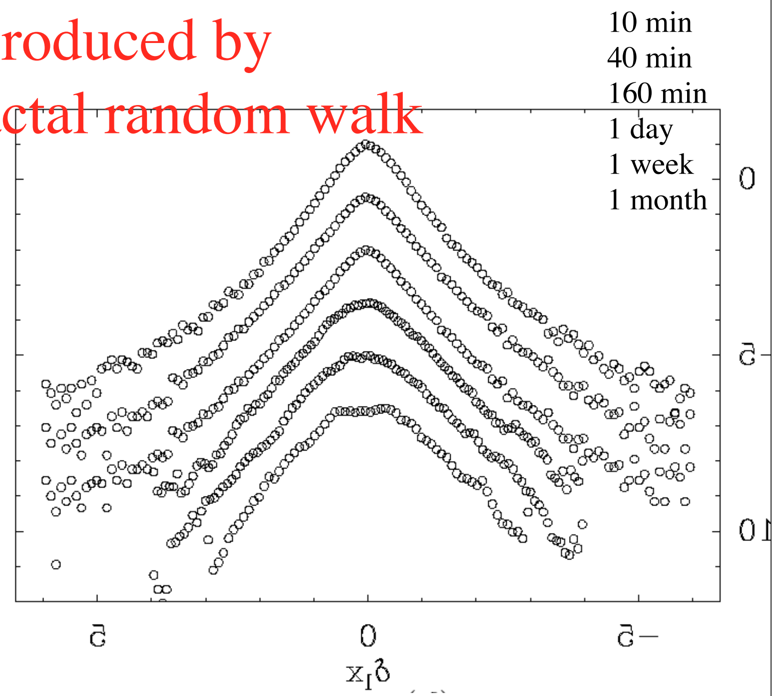


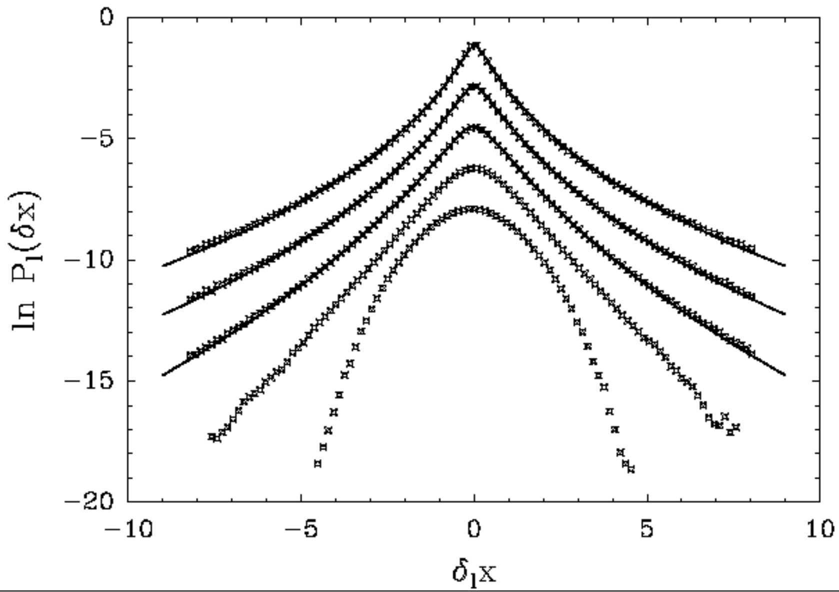
Stylized facts in financial markets well-reproduced by Multifractal random walk



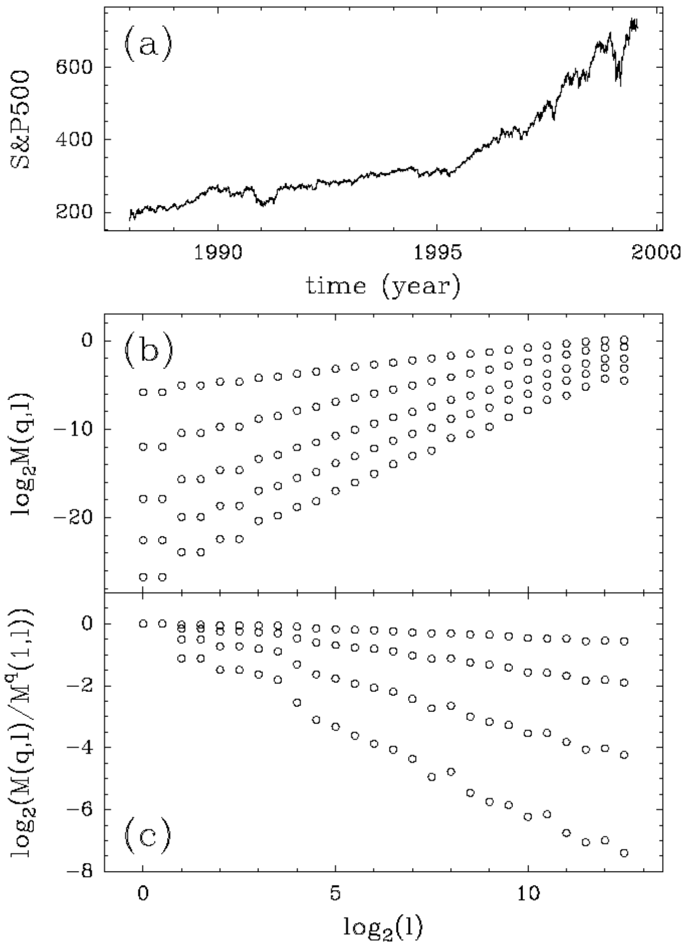






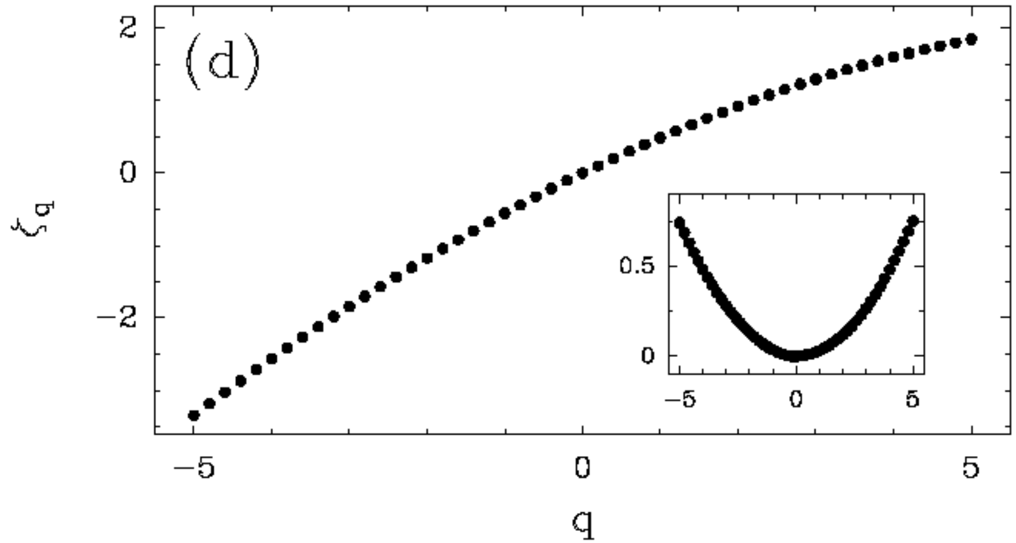


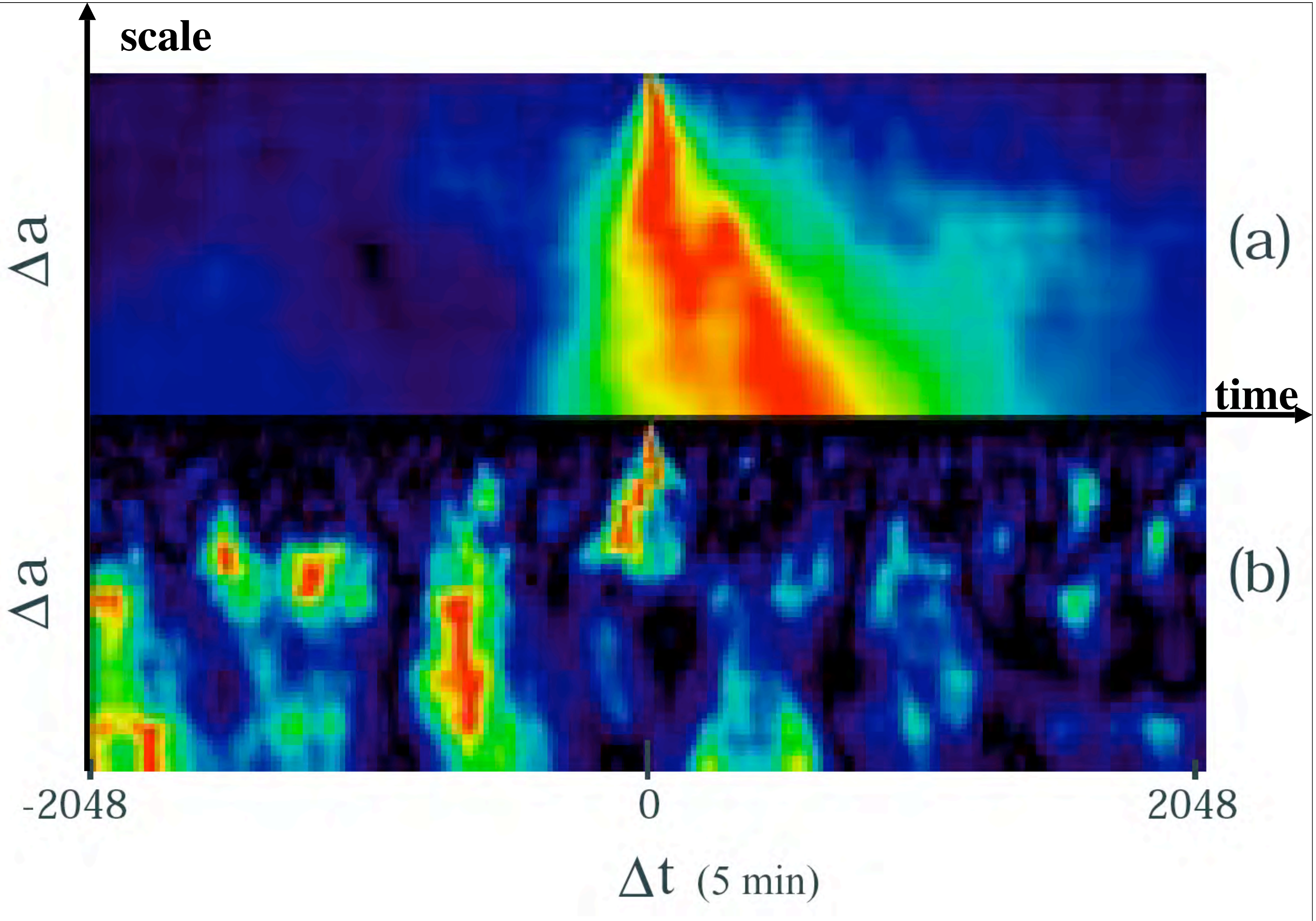
Multifractality: $$\langle [\delta_\tau X(t)]^q \rangle = a(q)\,\tau^{\zeta(q)}, \quad \text{for } \tau < T.$$



$$\zeta(q) = \left(1 + \frac{\lambda^2}{2}\right)q - \frac{\lambda^2}{2}q^2$$

$\lambda^2 = -\zeta''(0)$ is the so-called *intermittency coefficient*.



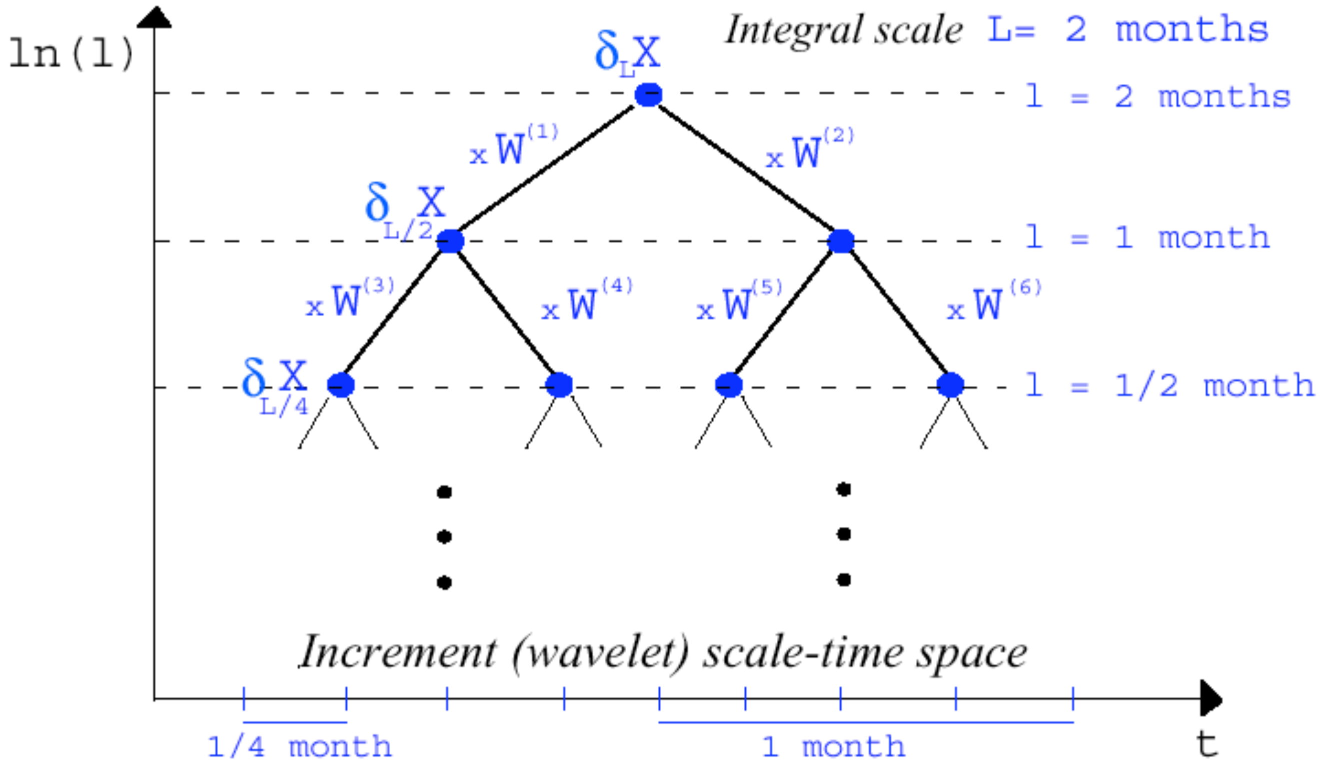


A. Arneodo, J.-F. Muzy and D. Sornette, Direct causal cascade in the stock market, European Physical Journal B 2, 277-282 (1998)

The multiplicative cascade paradigm

$$\delta_{\lambda l} X(\lambda t) = \lambda^H \delta_l X(t) = W_\lambda \delta_l X(t)$$

- $\mathcal{W}$-cascades (wavelet cascade)



The Multifractal Randow Walk (MRW) model

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)}$$

$$\mu_{\Delta t} = \frac{1}{2} \ln(\sigma^2 \Delta t) - C_{\Delta t}(0)$$

$$C_{\Delta t}(\tau) = \mathrm{Cov}[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln\left(\frac{T}{|\tau| + e^{-3/2}\Delta t}\right)$$

$$\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^{t} d\tau\ \eta(\tau)\ K_{\Delta t}(t-\tau)$$

$\omega_{\Delta t}(t)$ is Gaussian with mean $\mu_{\Delta t}$ and variance $V_{\Delta t} = \int_0^\infty d\tau\ K^2_{\Delta t}(\tau) = \lambda^2 \ln\left(\frac{Te^{3/2}}{\Delta t}\right)$

$$C_{\Delta t}(\tau) = \int_0^\infty dt\ K_{\Delta t}(t) K_{\Delta t}(t + |\tau|)$$

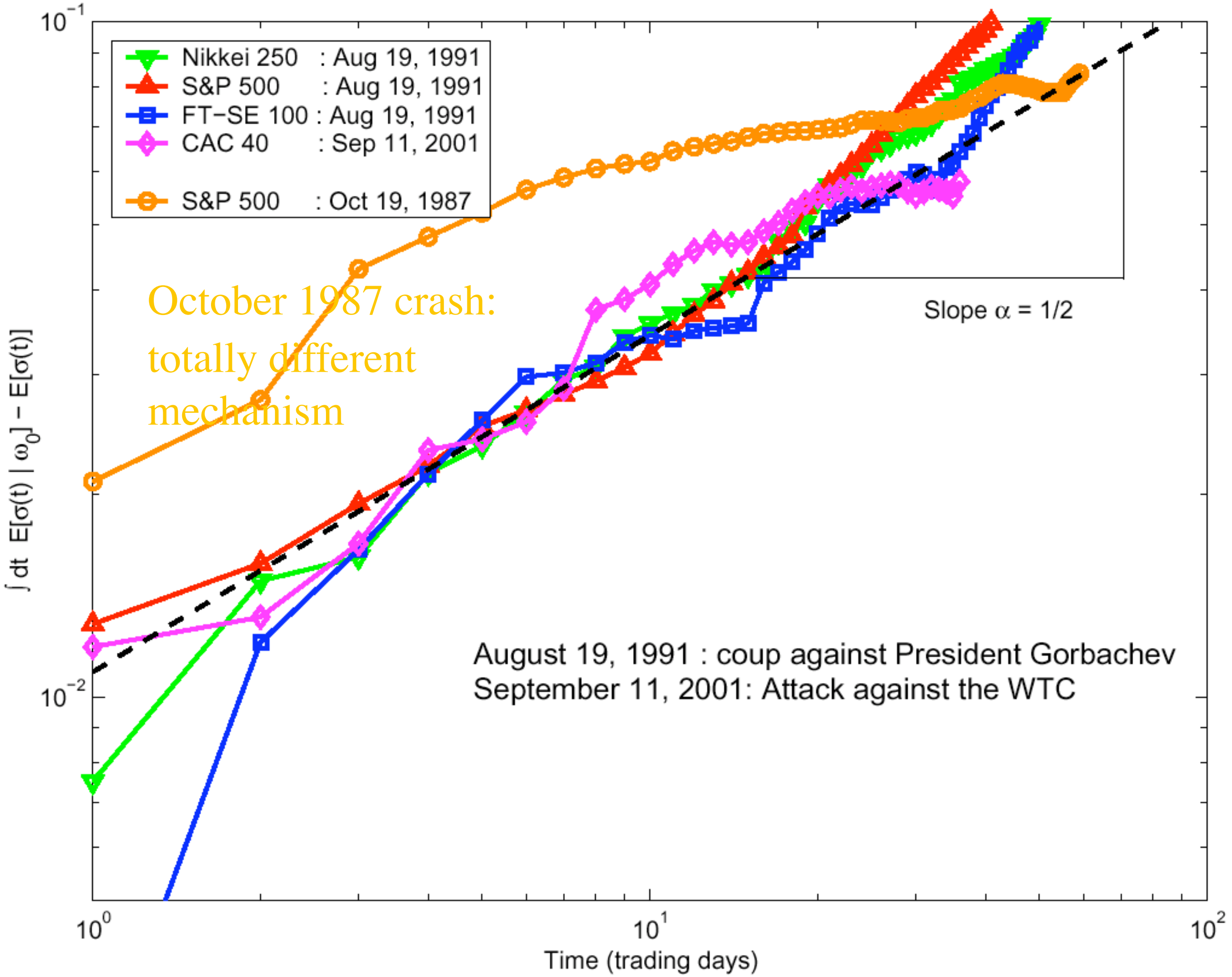
$$\hat{K}_{\Delta t}(f)^2 = \hat{C}_{\Delta t}(f) = 2\lambda^2\ f^{-1} \left[\int_0^{Tf} \frac{\sin(t)}{t} dt + O\left(f \Delta t \ln(f \Delta t)\right)\right]$$

$$K_{\Delta t}(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for } \Delta t << \tau << T$$

Linear response to an external shock

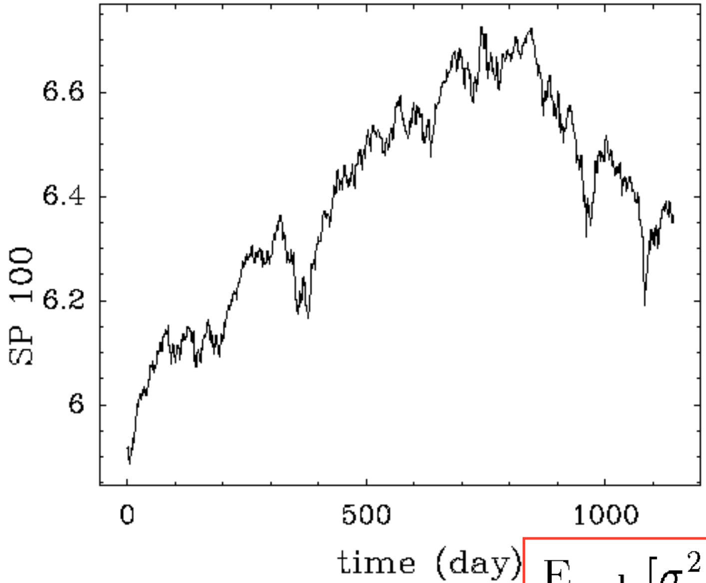
(Multifractal Random Walk model)

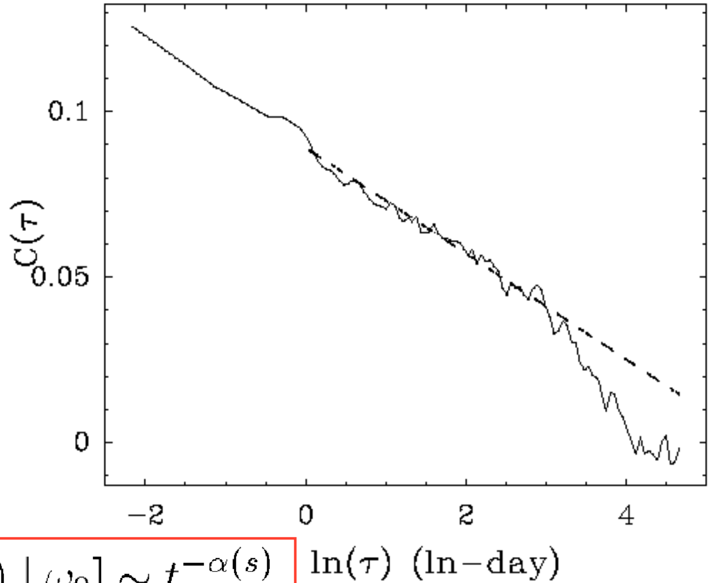
$$E_{exo}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



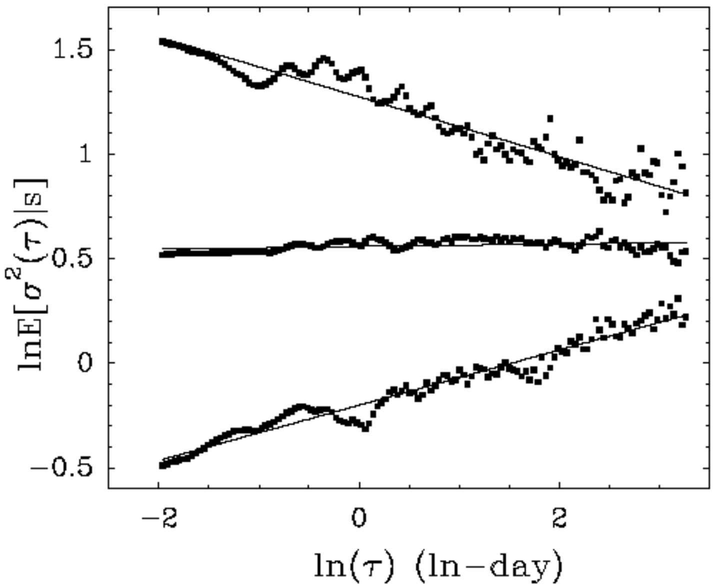
D. Sornette, Y. Malevergne and J.F. Muzy. Risk 16 (2), 67-71 (2003)

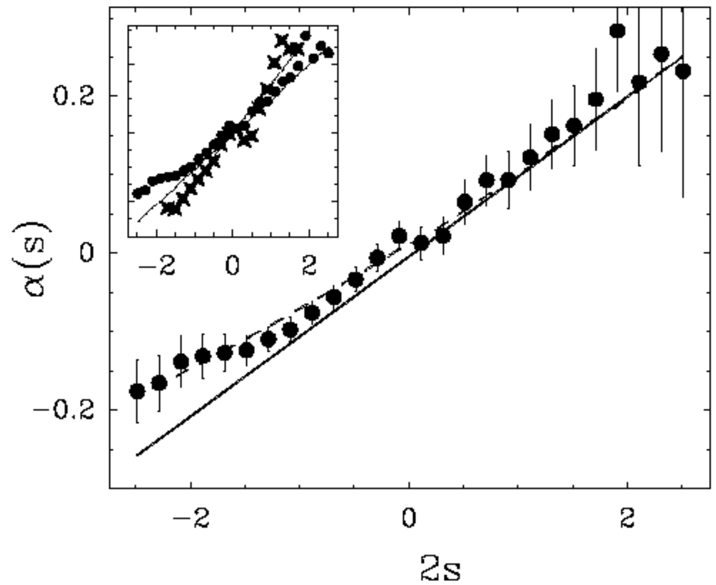
Endogenous shocks and Multifractal Random Walk model





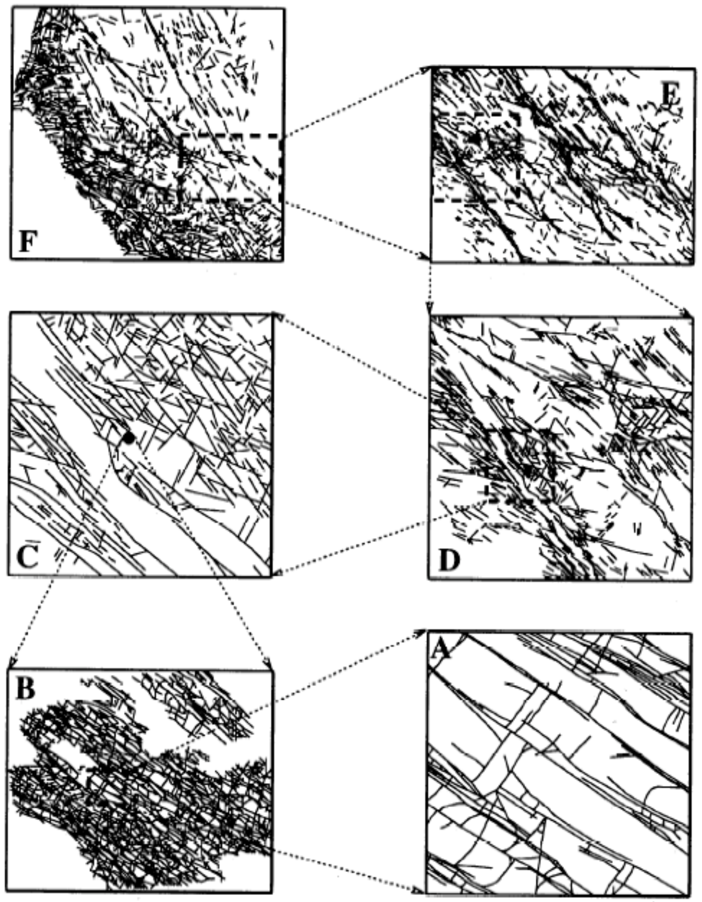
$$E_{\text{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$



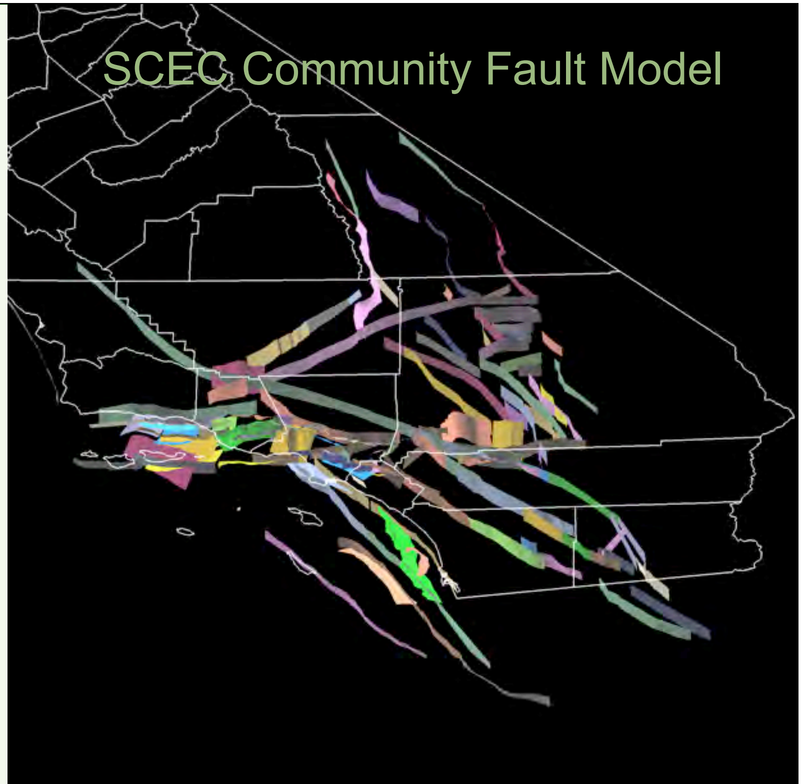


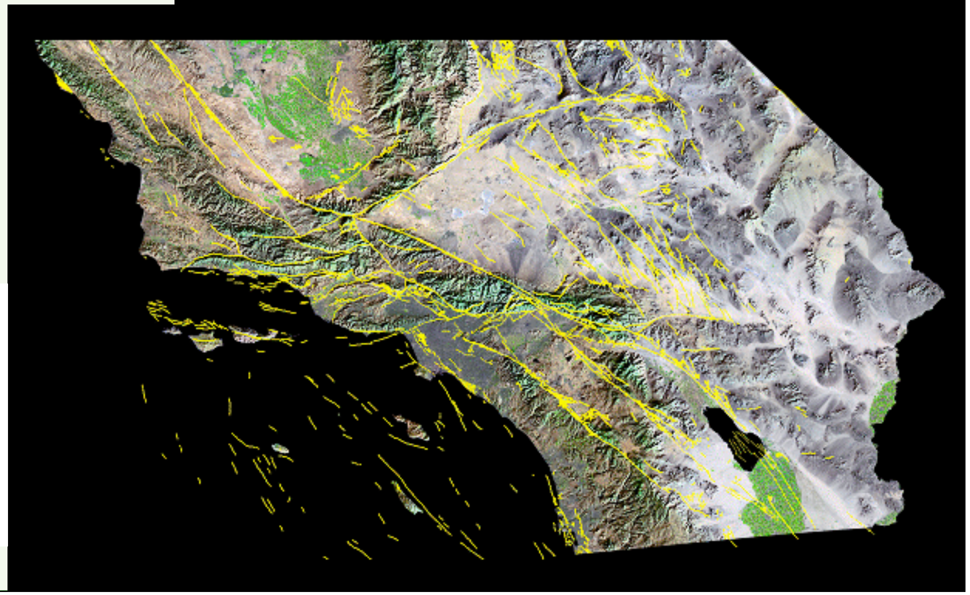
Hierarchical geometry of faulting

Ouillon, Castaing, Sornette (JGR 1996)



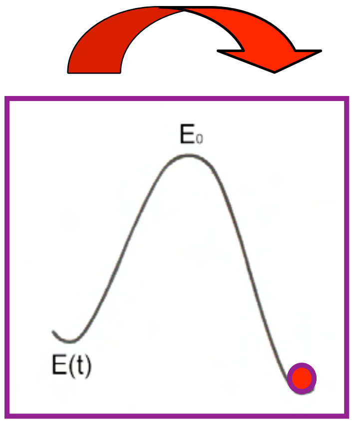
Map A: linear size=10 m, orig. scale=1:1
Map B: linear size=60 m, orig. scale=1:220
Map C: linear size=11 km, orig. scale=1:62,500
Map D: linear size=45 km, orig. scale=1:125,000
Map E: linear size=150 km, orig. scale=1:250,000
Map F: linear size=400 km, orig. scale=1:1,000,000





The physical model : thermal activation driven by stress

Before the shock



After the shock

Energy barrier = E_0-E(t)

Arrhenius law for the activation rate: $$\lambda(t) = \lambda_0 \exp\left(-\frac{E_0 - E(t)}{kT}\right)$$

stress barrier = $\sigma_0 - \sigma(t)$

$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V\right)$$

Compatible with state-and-rate friction, stress corrosion, …

- $\lambda(t)$: instantaneous rate
- λ_0 ~ average nucleation rate
- σ_0 : material strength
- $\sigma(t)$: applied stress
- V : activation volume
- T : temperature
- k : Boltzmann constant

Experiments by Zhurkov   Int. J. Fract. Mech. **1**, 311 (1965)

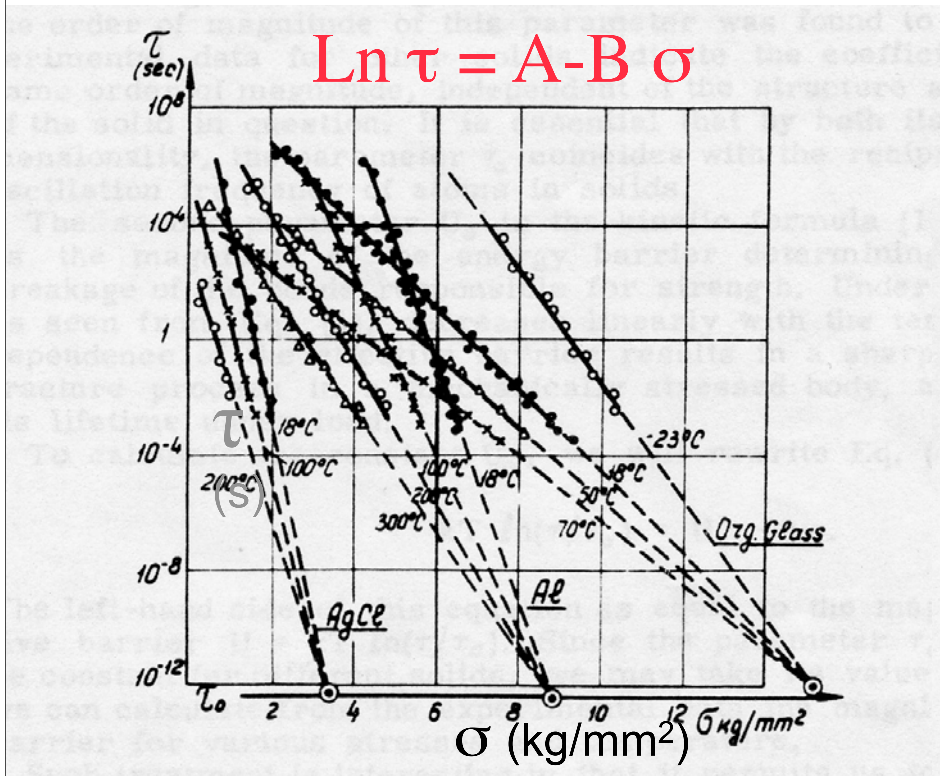


Fig. 5. Time and temperature dependence of the lifetime of solids on stress.
1. Silver chloride (Reference 4)
2. Aluminum (Reference 5)
3. Plexiglas (Reference 6)

$$\tau = \tau_o \exp\left(\frac{U}{k_B T}\right)$$

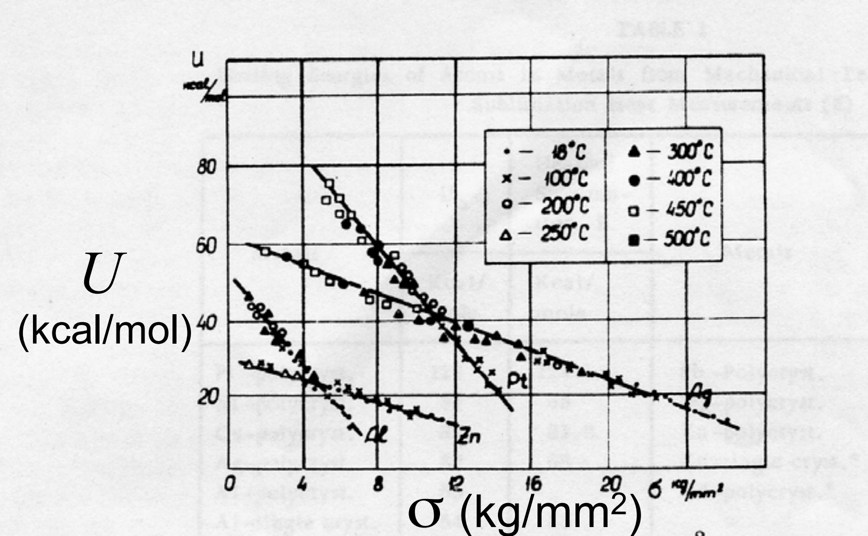


Fig. 6. Effective barrier U kkal/mol vs. tensile stress σ kg/mm² for polycrystall

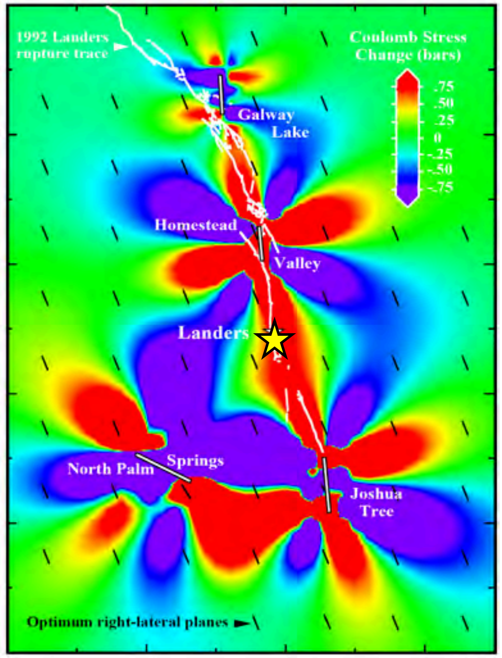
Empirical energy barrier

$$U = U_o - \alpha\sigma$$

où U_o : énergie de sublimation

A possible mechanism : <u>**thermal activated process**</u>

Taking account of history and boundary conditions



$$\lambda(\vec{r},t) = \lambda_0' \exp\left(\frac{\sigma(\vec{r},t)}{kT} V\right)$$

Stress is assumed to be a scalar for the sake of simplicity

$$\Sigma(\vec{r},t) = \Sigma_{\text{far field}}(\vec{r},t) + \int_{-\infty}^{t} \int dN[d\vec{r}\,' \times d\tau] \Delta\sigma(\vec{r}\,',\tau) g(\vec{r}-\vec{r}\,', t-\tau)$$

local stress

tectonic loading

Stress fluctuations induced by all past events in the system

D. Sornette and G. Ouillon, Multifractal Scaling of Thermally-Activated Rupture Processes, Phys. Rev. Lett. 94, 038501 (2005)
G. Ouillon and D. Sornette, Magnitude-Dependent Omori Law: Theory and Empirical Study, J. Geophys. Res., 110, B04306, doi:10.1029/2004JB003311 (2005).

$$\Sigma(\vec{r},t) = \Sigma_{\text{far field}}(\vec{r},t) + \int_{-\infty}^{t} \int dN[d\vec{r}\,' \times d\tau]\Delta\sigma(\vec{r}\,',\tau)g(\vec{r}-\vec{r}\,',t-\tau)$$

$$g(\vec{r},t) = f(\vec{r}) \times h(t)$$

The **rheology is viscoplastic**, with a relaxation function featuring a very large relaxation time t_M :

$$h(t) = \frac{h_0}{(t+t_1)^{1+\theta}} \exp\left(-\frac{t}{\tau_M}\right)$$

At each location, **stress fluctuations** due to previous events are distributed as:

$$P(\sigma)\,d\sigma \approx \frac{C}{(\sigma+\sigma_0)^{1+\mu}}\,d\sigma$$

Theoretical predictions using tail covariance (Ide-Sornette, 2001)

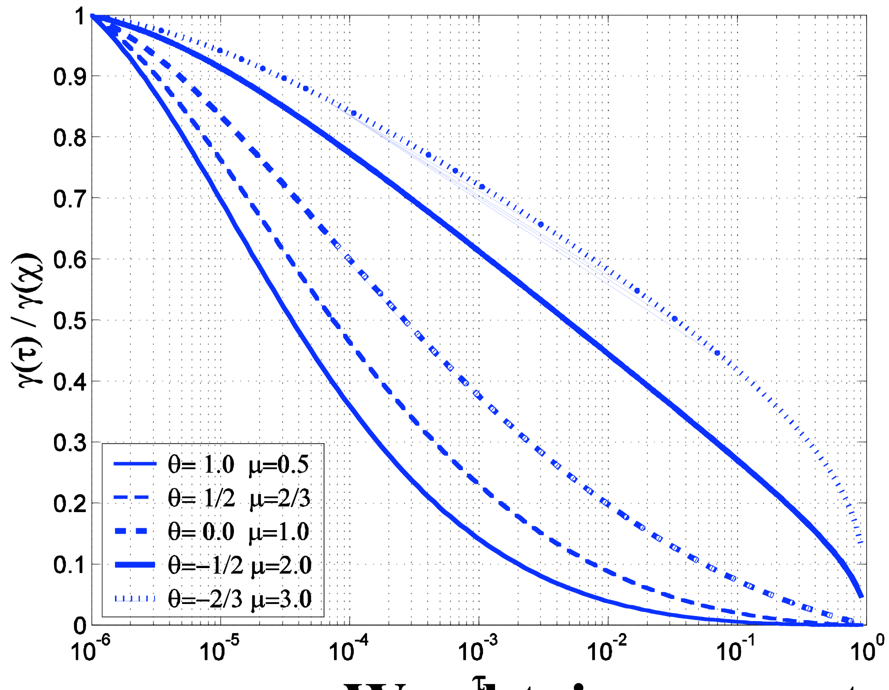
$$\Pr[\lambda(t) > \lambda_q | \lambda_M] = \Pr[e^{\beta\omega(t)} > \frac{\lambda_q}{\lambda_{\rm tec}} | \omega_M] = \Pr[\omega(t) > (1/\beta) \ln\left(\frac{\lambda_q}{\lambda_{\rm tec}}\right) | \omega_M]$$

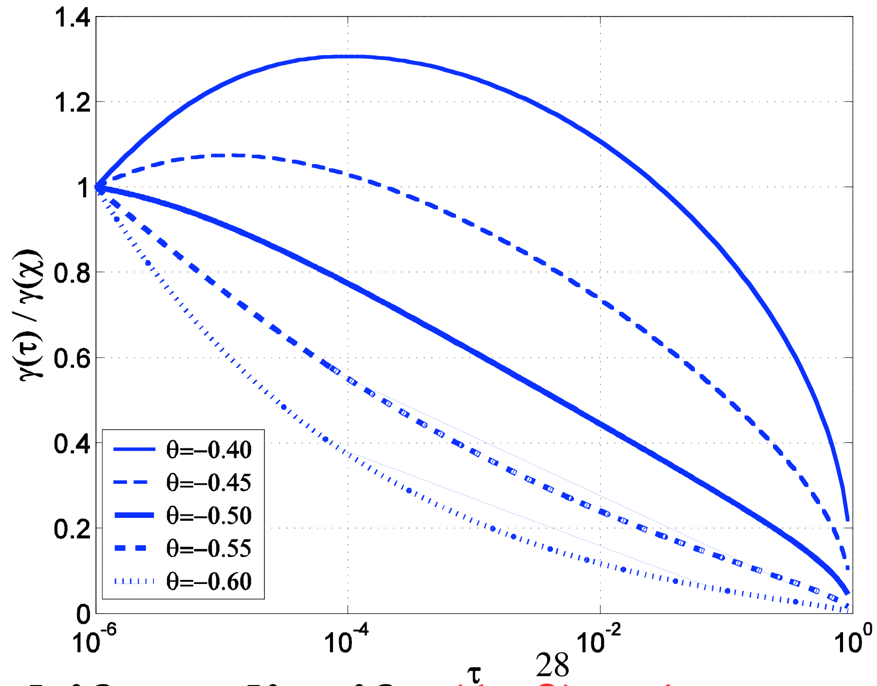
$$\lambda_q(t) = A_q \; \lambda_{\rm tec} \; e^{\beta\gamma(t)\omega_M}$$

$$\gamma(t) = \frac{h_0^2}{\Delta t^{2/\mu}} \left(\frac{1}{t^{2m-1}} \int_{c/t}^{\frac{T+c}{t}-1} dy \frac{1}{(y+1)^m} \; \frac{1}{y^m} \right)^{\frac{2}{\mu}}$$

$$m = (1+\theta)\mu/2.$$

Since $\gamma(t) \sim \ln(t)$ and $\omega_m \sim M$, we obtain $p(M) = a\,M + b$





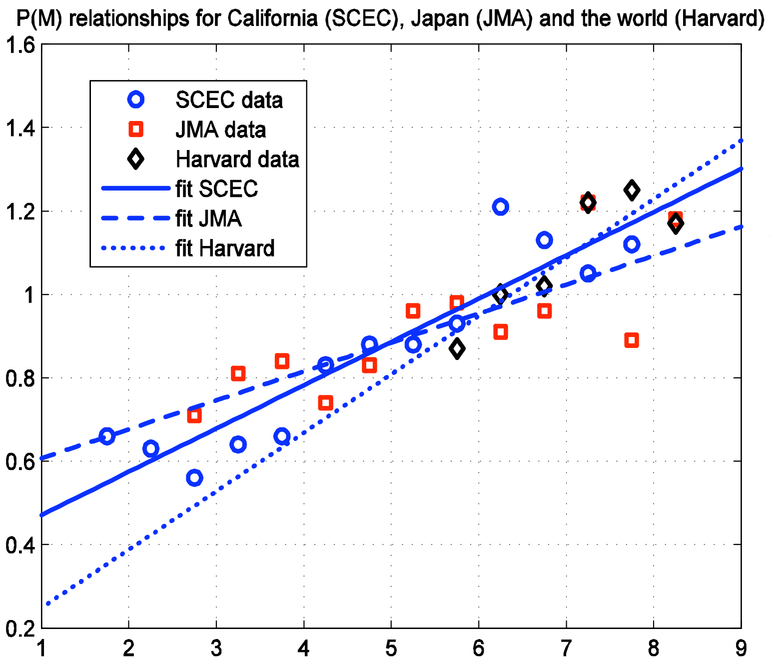
We obtain an exact multifractality if $\mu(1+\theta) \sim 1$

p(M) = aM + b

We processed three catalogs, that we pre-processed to check for their completude and its evolution with time.
We then computed stacked aftershocks time series, sorting them within intervals of 0.5 magnitude amplitudes.

We clearly observed a linear dependence of *p* with magnitude M.

Statistical tests have been performed using a bootstrap strategy, and we were able to show that all slopes were significantly different from 0, and that all linear relationships were significantly different from each other.



Ribeiro et al, 2006

For Southern California (SCEC catalog):

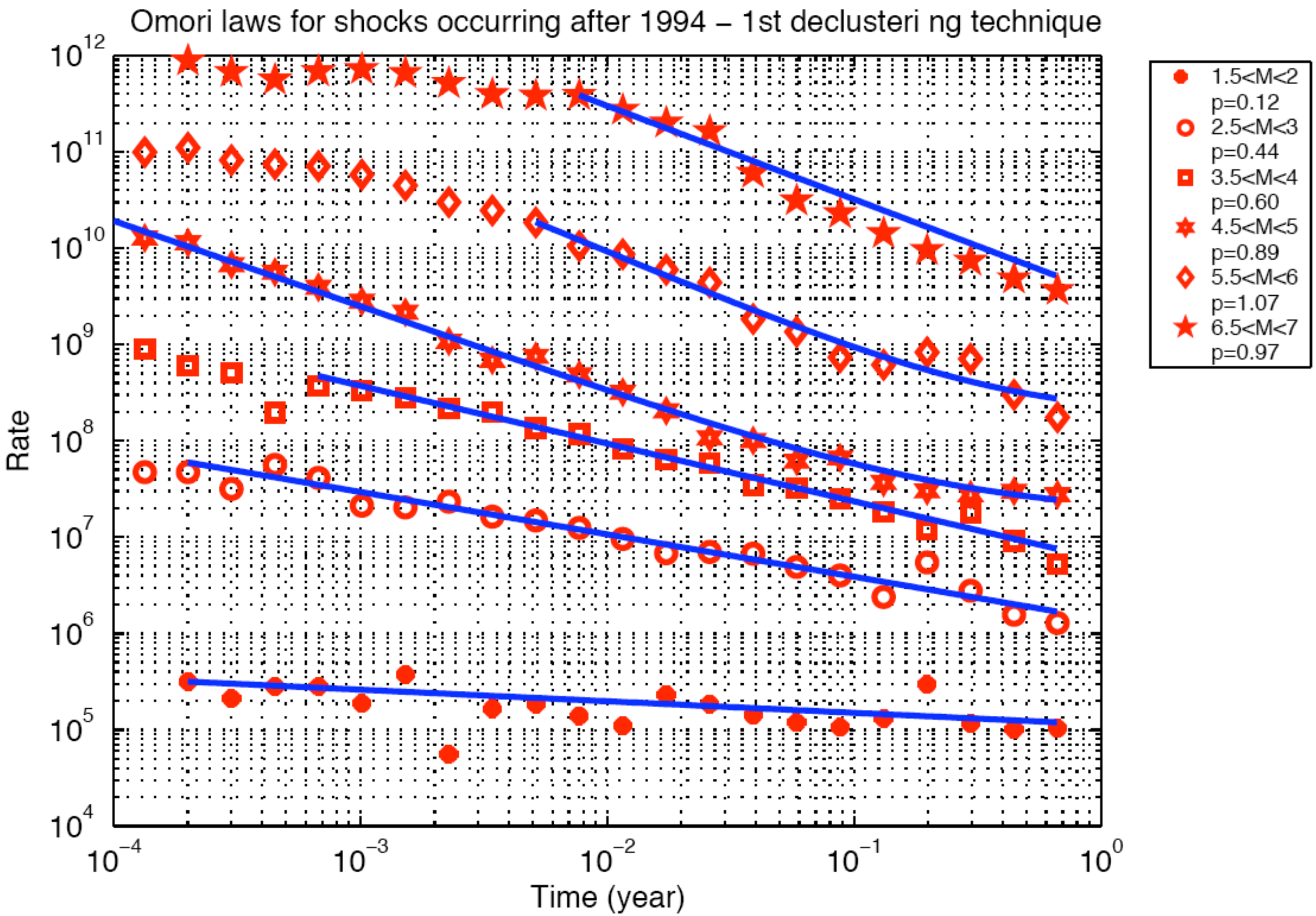
$$p(M) = 0.10M + 0.37$$

For Japan (JMA catalog):

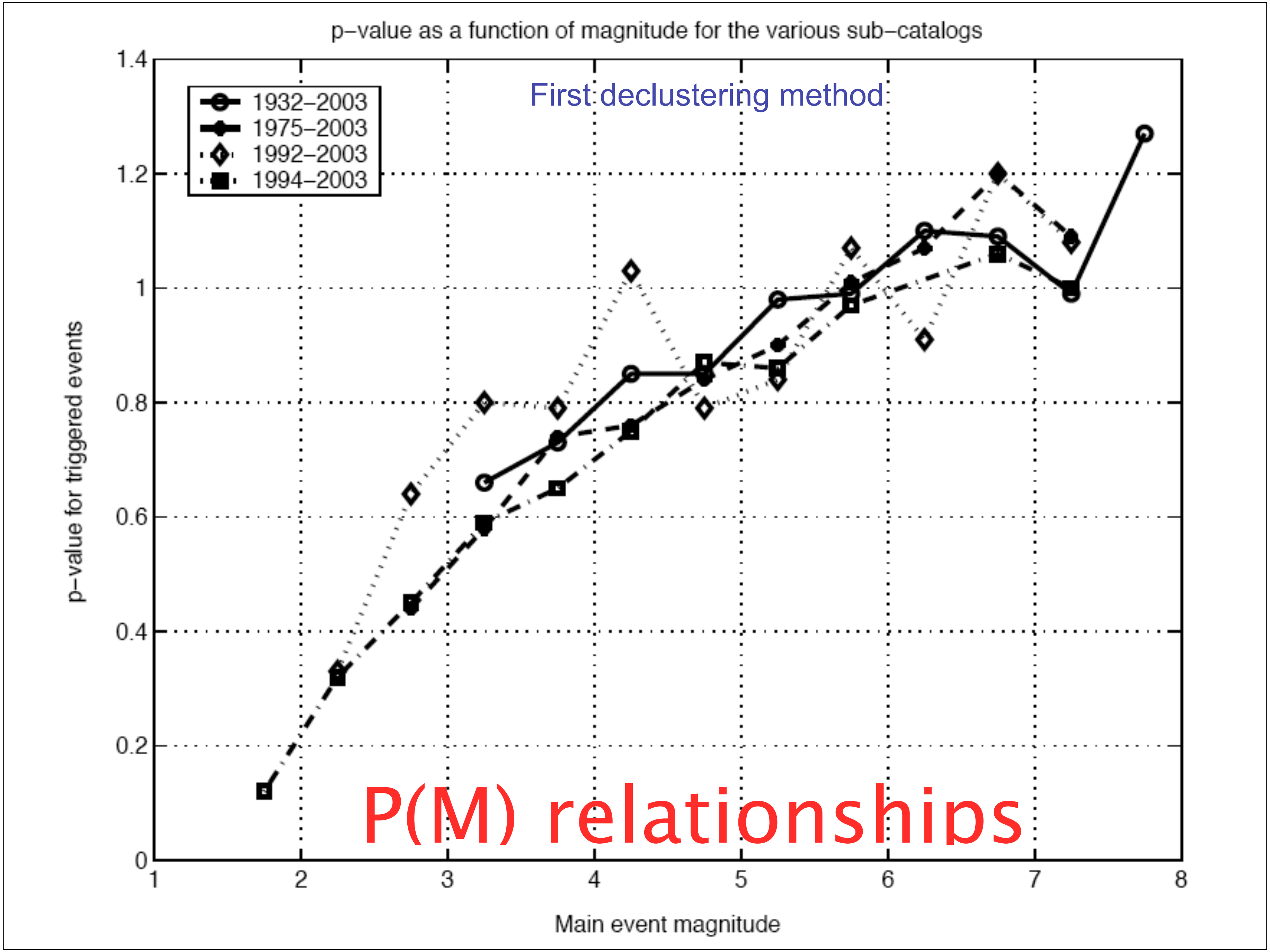
$$p(M) = 0.07M + 0.54$$

For the World (Harvard catalog):

$$p(M) = 0.14M + 0.11$$



D. Sornette and G. Ouillon, Multifractal Scaling of Thermally-Activated Rupture Processes, Phys. Rev. Lett. 94, 038501 (2005)

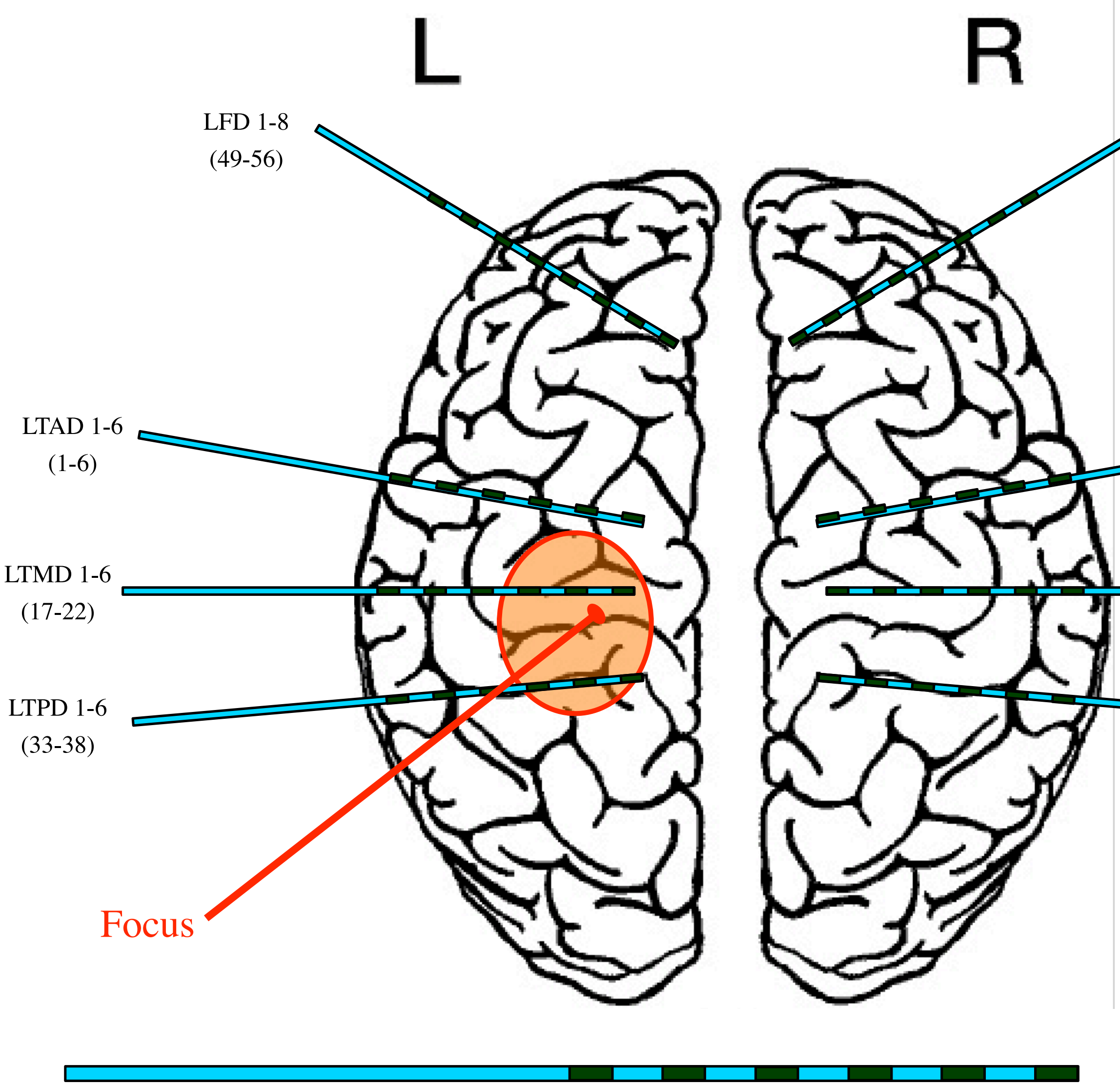


p–value as a function of magnitude for the various sub–catalogs
First declustering method
1932–2003
1975–2003
1992–2003
1994–2003
p–value for triggered events
Main event magnitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1
2
3
4
5
6
7
8
P(M) relationships

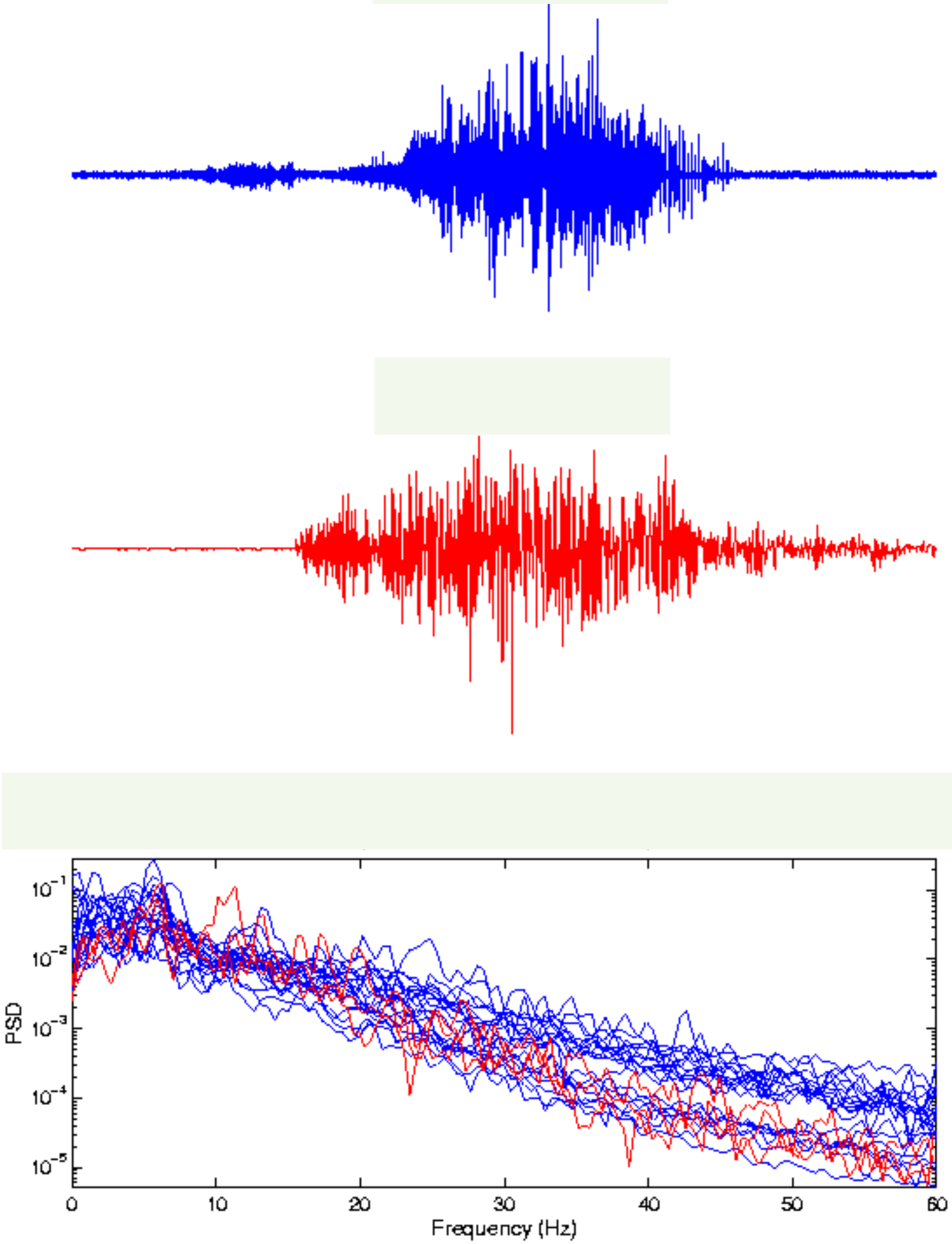
Epileptic Seizures – Quakes of the Brain?

with Ivan Osorio – KUMC & FHS
Mark G. Frei - FHS
John Milton -The Claremont Colleges

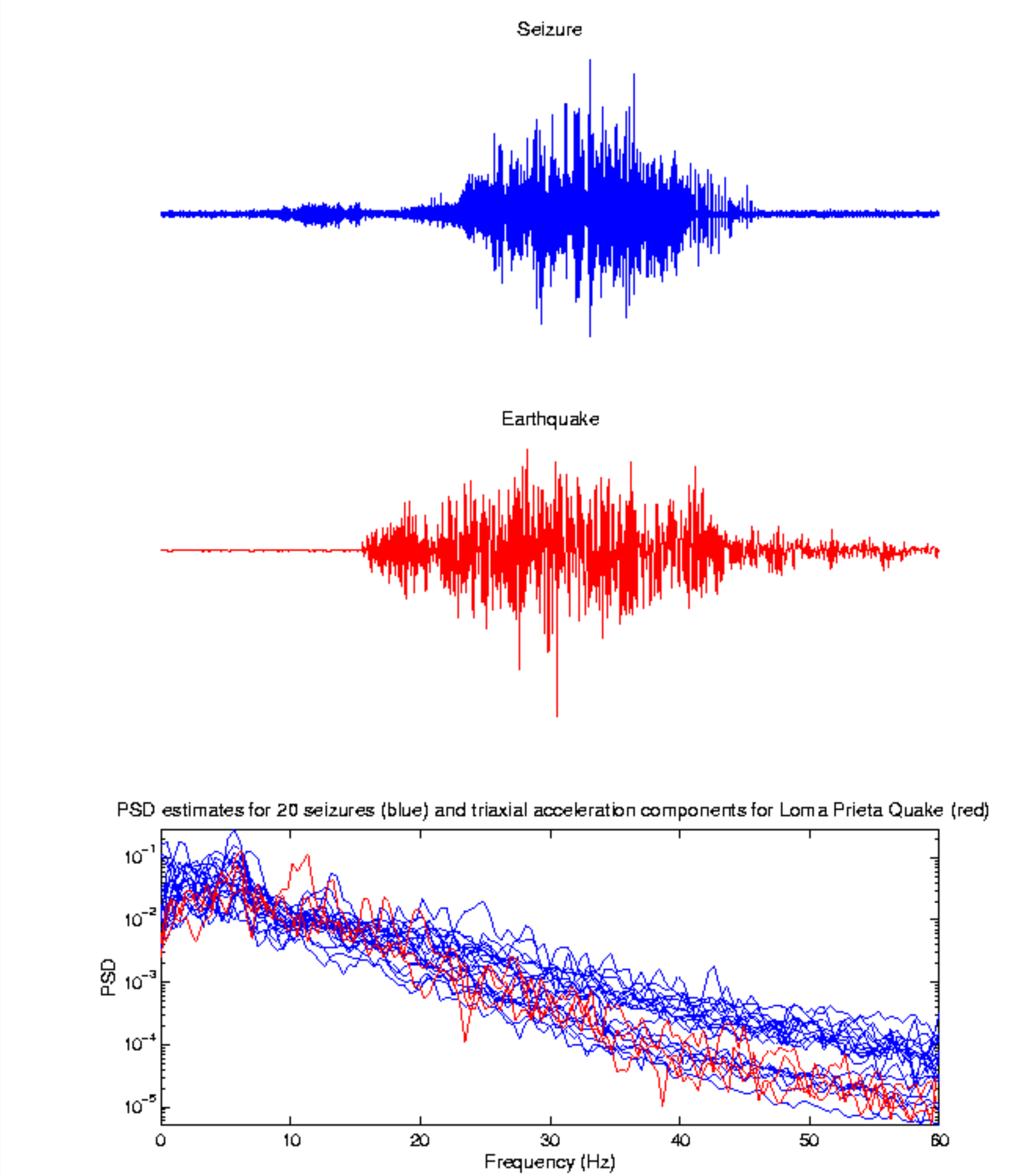
(arxiv.org/abs/0712.3929)



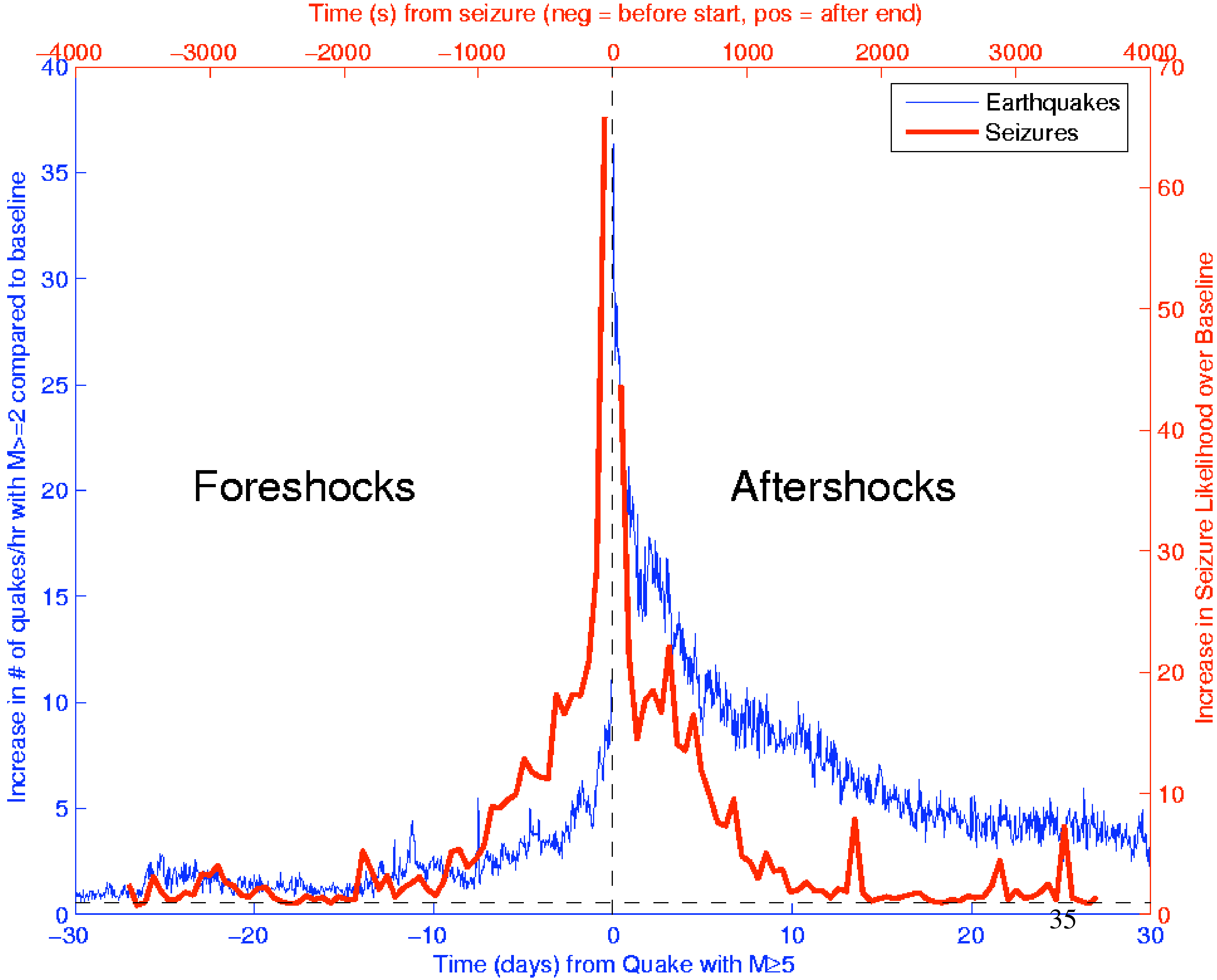
Key: L=Left
R=Right
A=Anterior
M=Mesial
P=Posterior
D=Depth
T=Temporal

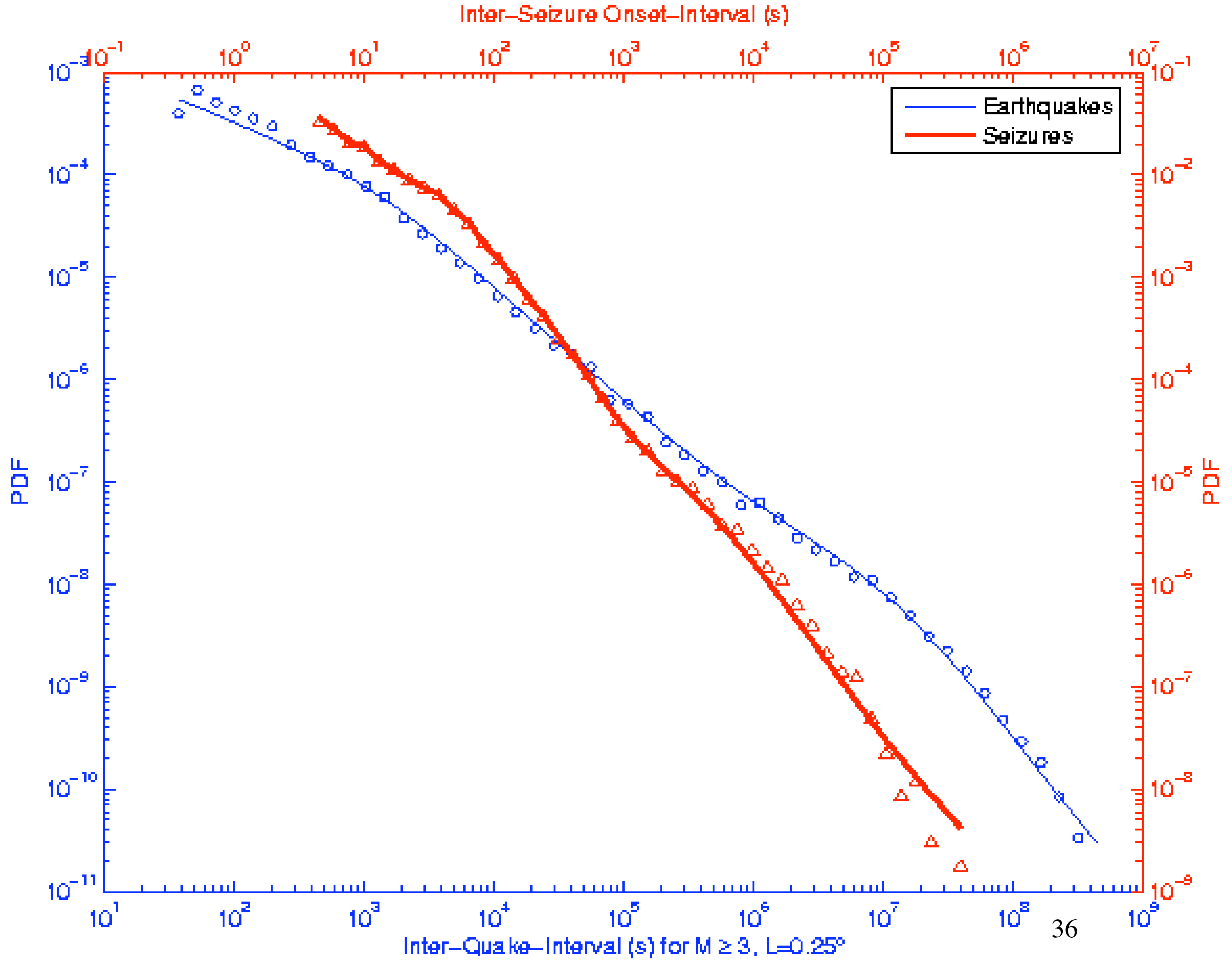


PSD
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0
10
20
30
40
50
60
Frequency (Hz)


Seizure
Earthquake
PSD estimates for 20 seizures (blue) and triaxial acceleration components for Loma Prieta Quake (red)
PSD
Frequency (Hz)
0
10
20
30
40
50
60

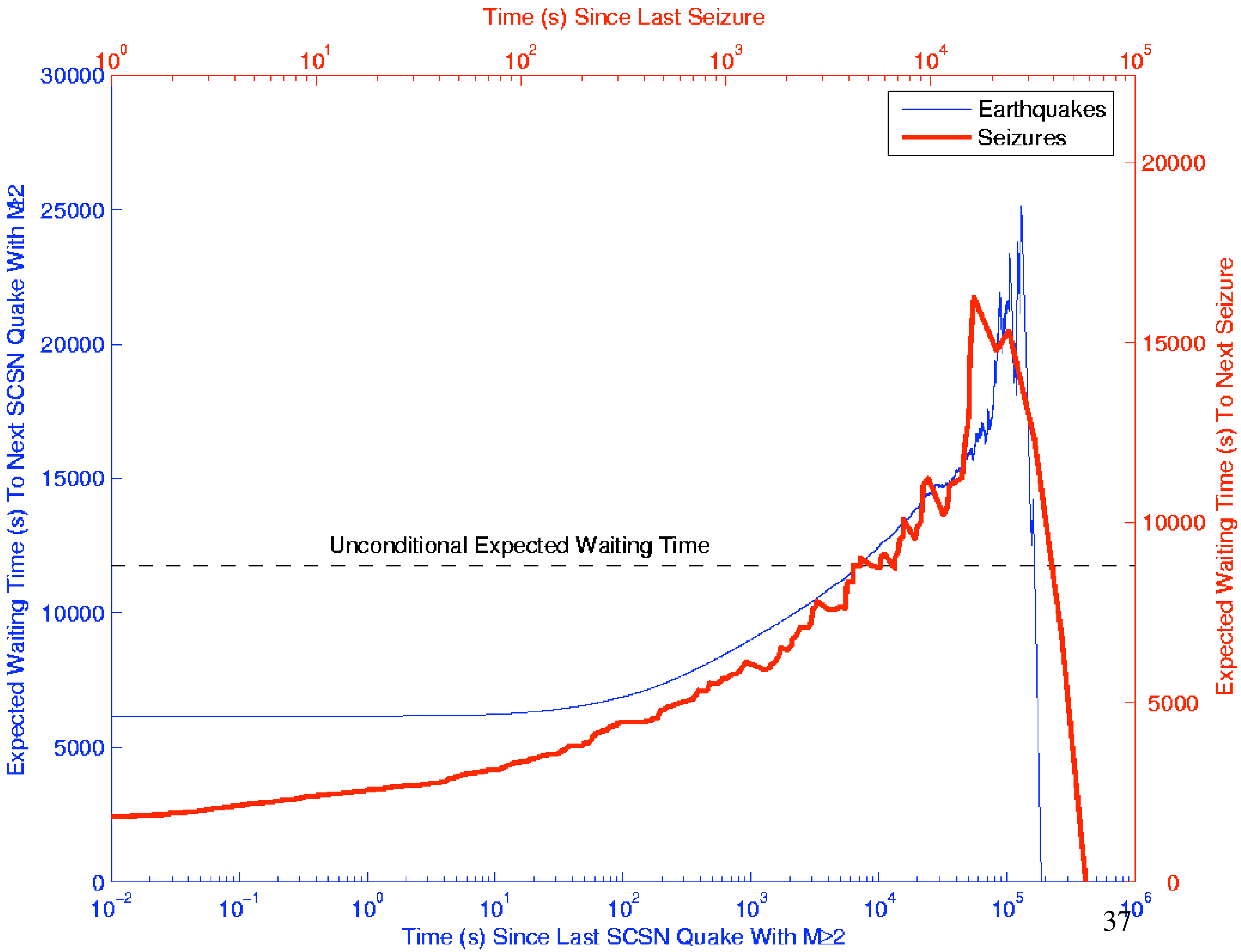
Omori law: Direct and Inverse



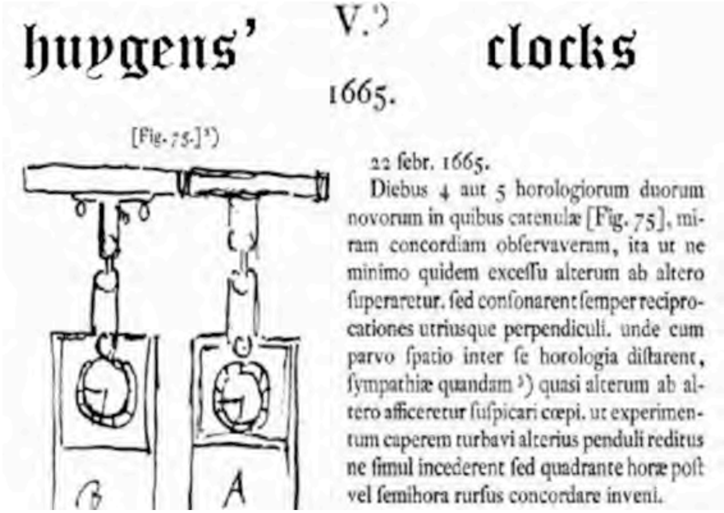


Inter–Seizure Onset–Interval (s)
Earthquakes
Seizures
PDF
PDF
Inter–Quake–Interval (s) for M ≥ 3, L=0.25°

The longer it has been since the last event,
the longer it will be since the next one! (Sornette&Knopoff, 1997)



SYNCHRONISATION AND COLLECTIVE EFFECTS IN EXTENDED STOCHASTIC SYSTEMS


Huygens' clocks
V.[1]
1665.
[Fig. 75.][2])
22 febr. 1665.
Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriuſque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3]) quaſi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

Fireflies

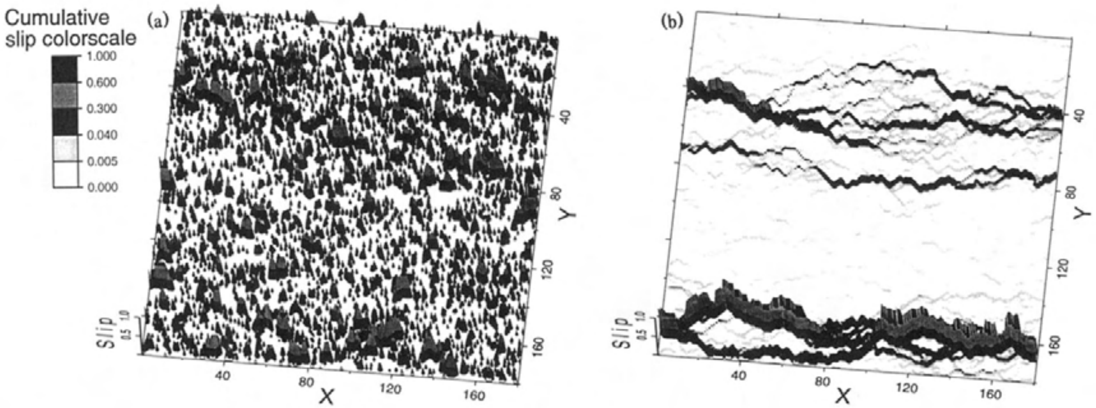
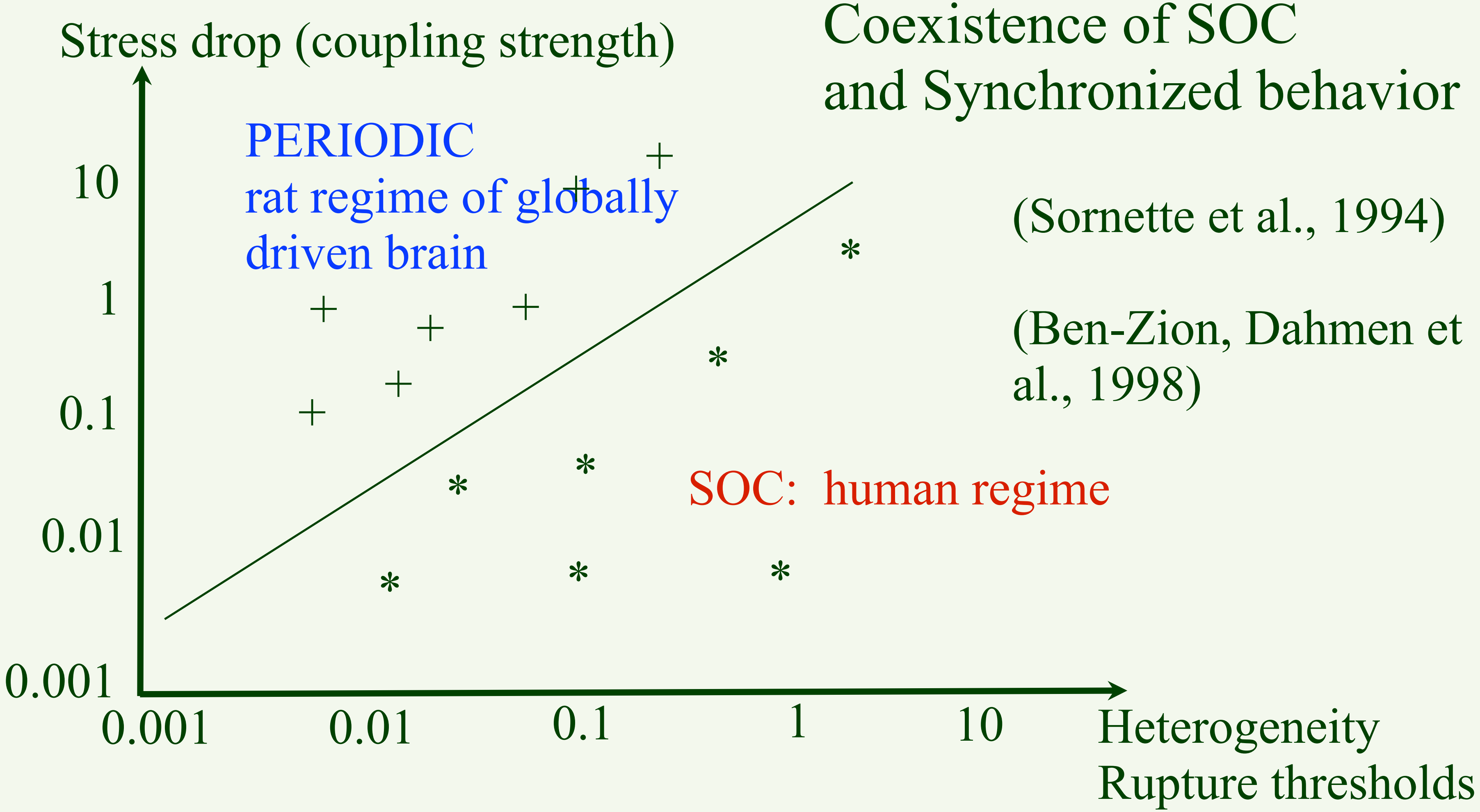


FIG. 1. Evolution of the cumulative earthquake slip, represented along the vertical axis in the white to black color code shown above the picture, at two different times: (a) early time and (b) long time, in a system of size $L=90$ by $L=90$, where $\Delta\sigma=1.9$ and $\beta=0.1$.

Miltenberger et al. (1993)

Coexistence of SOC and Synchronized behavior

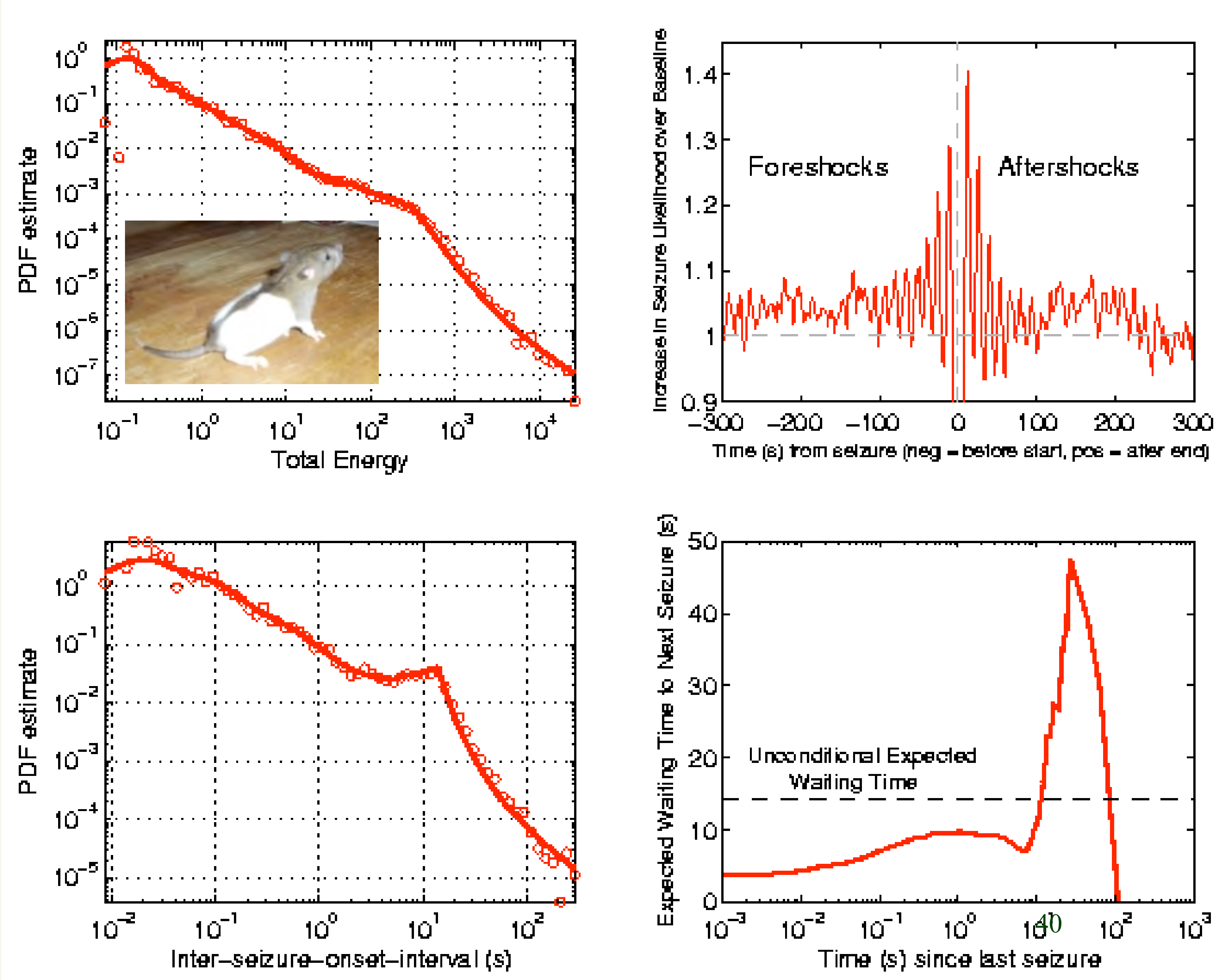


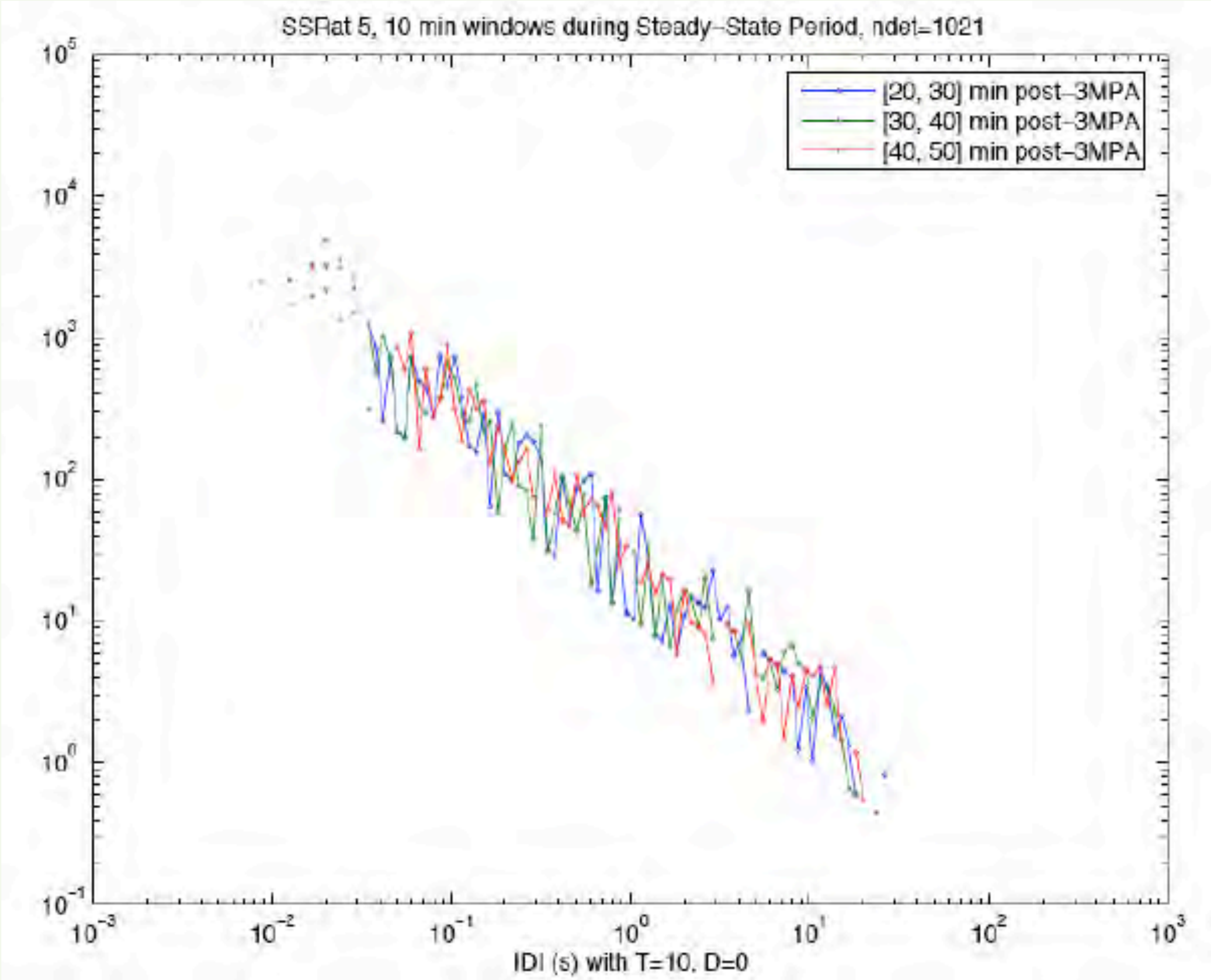
(Sornette et al., 1994)

(Ben-Zion, Dahmen et al., 1998)

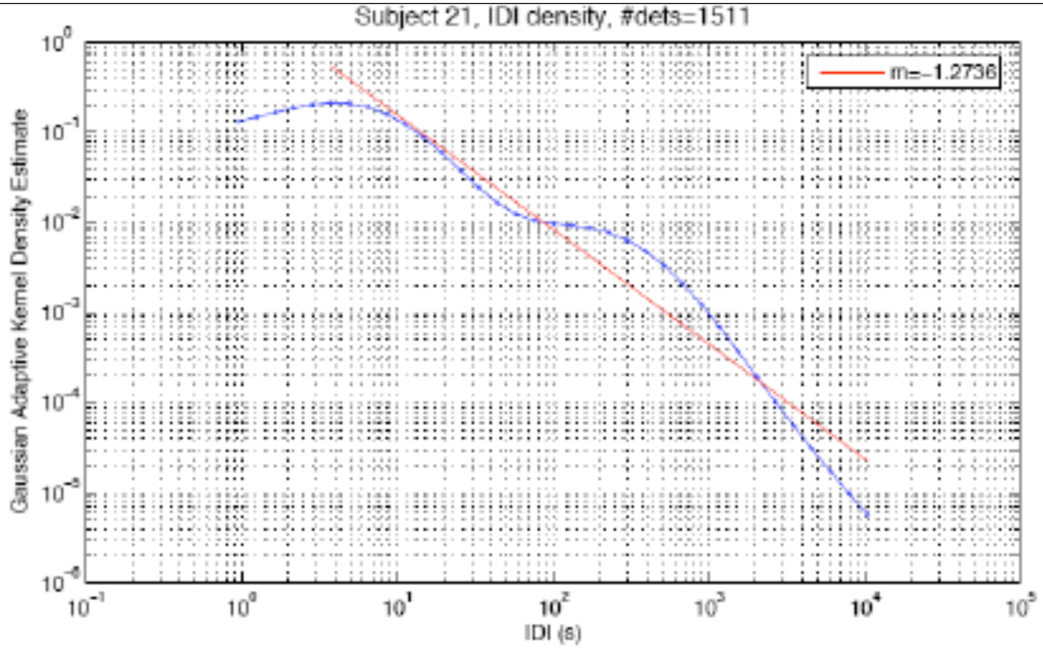
“Phase diagram” for the model in the space (heterogeneity, stress drop).
Crosses (+) correspond to systems which exhibit a periodic time evolution.
Stars * corresponds to systems that are self-organized critical, with a
Gutenberg-Richter earthquake size distribution and fault localization whose
geometry is well-described by the geometry of random directed polymers.

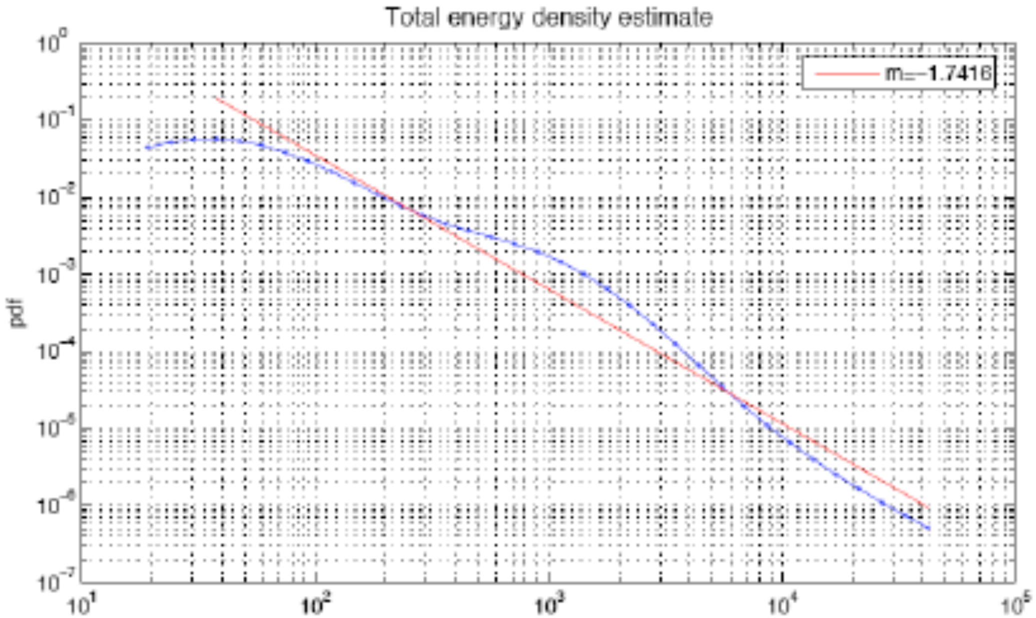
19 rats treated intravenously (2) with the convulsant 3-mercapto-proprionic acid (3-MPA)





Distribution of inter-seizure time intervals for rat 5, demonstrating a pure power law, which is characteristic of the SOC state. This scale-free distribution should be contrasted with the pdf's obtained for the other rats, which are marked by a strong shoulder associated with a characteristic time scale, which reveals the periodicregime.



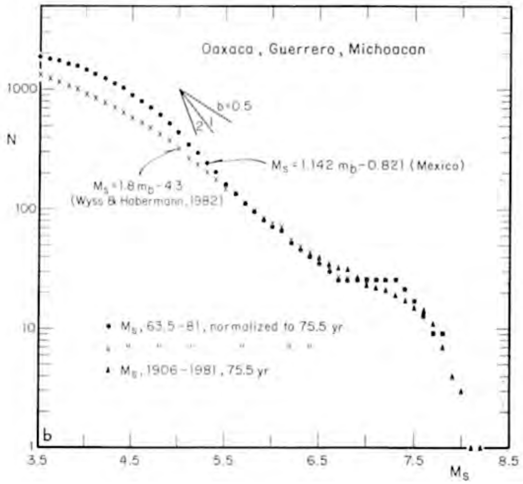


The pdf's of the seizure energies and of the inter-seizure waiting times for subject 21.

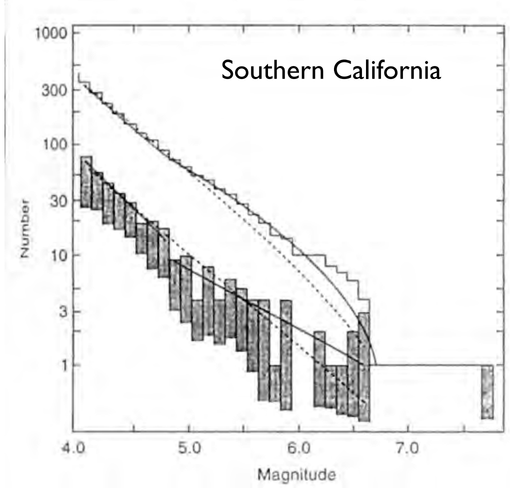
Note the shoulder in each distribution, demonstrating the presence of a characteristic size and time scale, qualifying the periodic regime.

Complex magnitude distributions

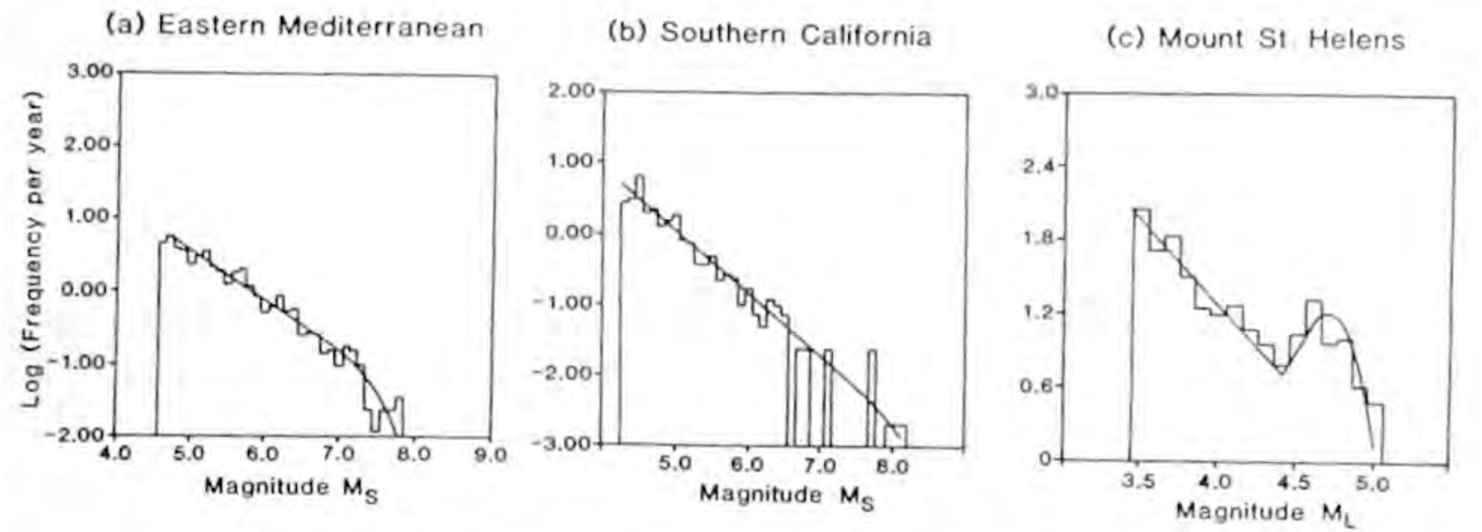
Characteristic earthquakes?



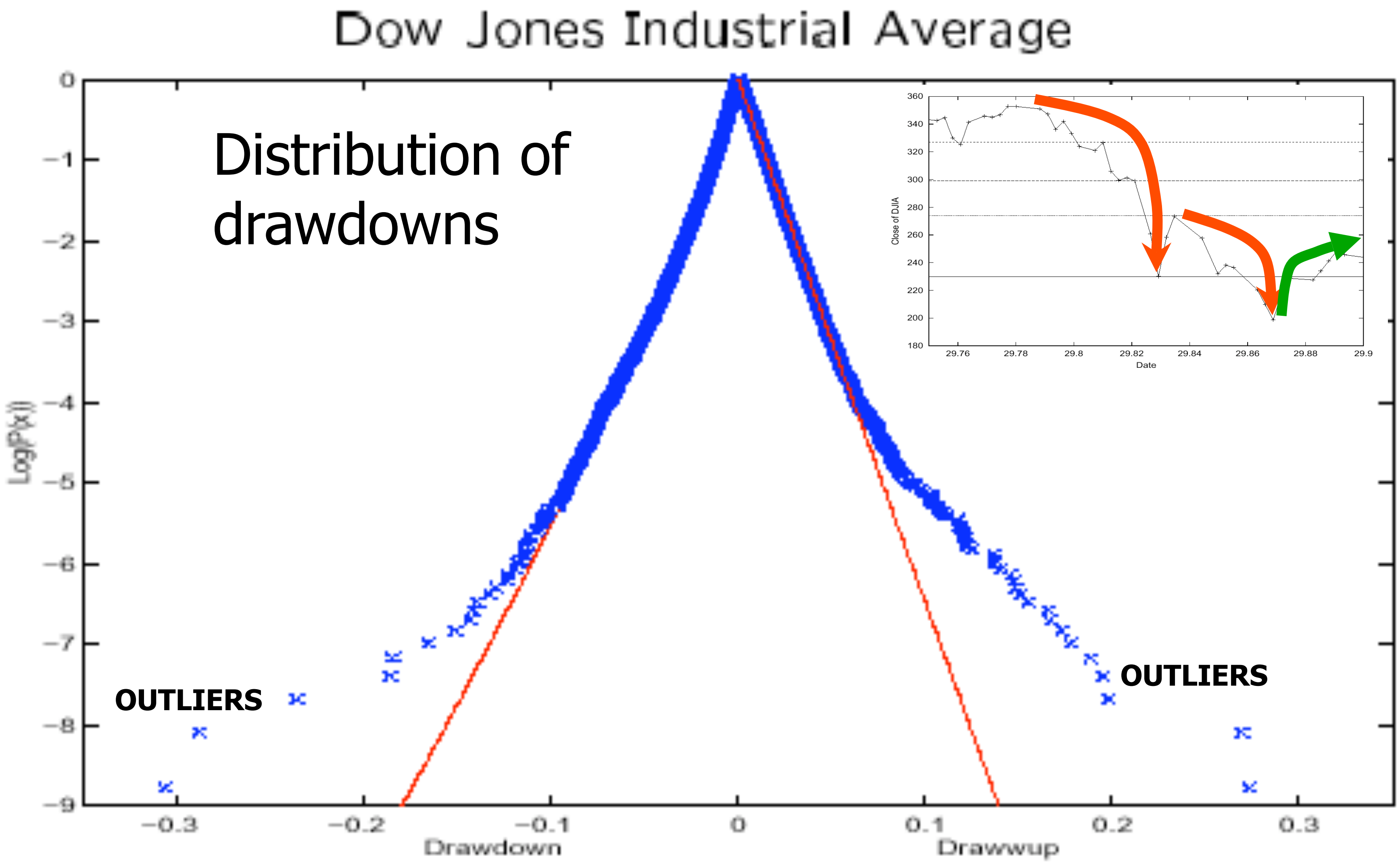
Singh, et. al., 1983, BSSA 73, 1779-1796



Knopoff, 2000, PNAS 97, 11880-11884



Main, 1995, BSSA 85, 1299-1308



A. Johansen and D. Sornette, Stock market crashes are outliers, European Physical Journal B 1, 141-143 (1998)

A. Johansen and D. Sornette, Large Stock Market Price Drawdowns Are Outliers, Journal of Risk 4(2), 69-110, Winter 2001/02

Securitization of credit risks: is it the next "systemic collapse"?

- Securitization of credit risks leads to smaller risks
- But more inter-connected

  $\Rightarrow$ global risk?

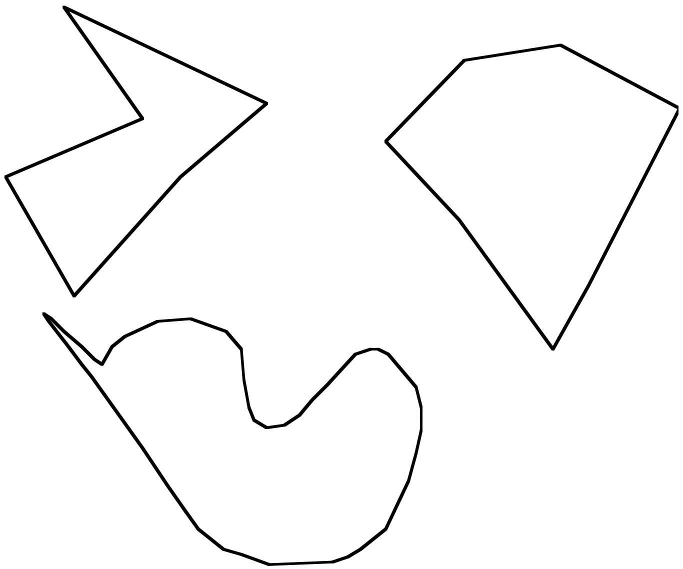
CDS and CDO: form of insurance contracts linked to underlying debt that protects the buyer in case of default.

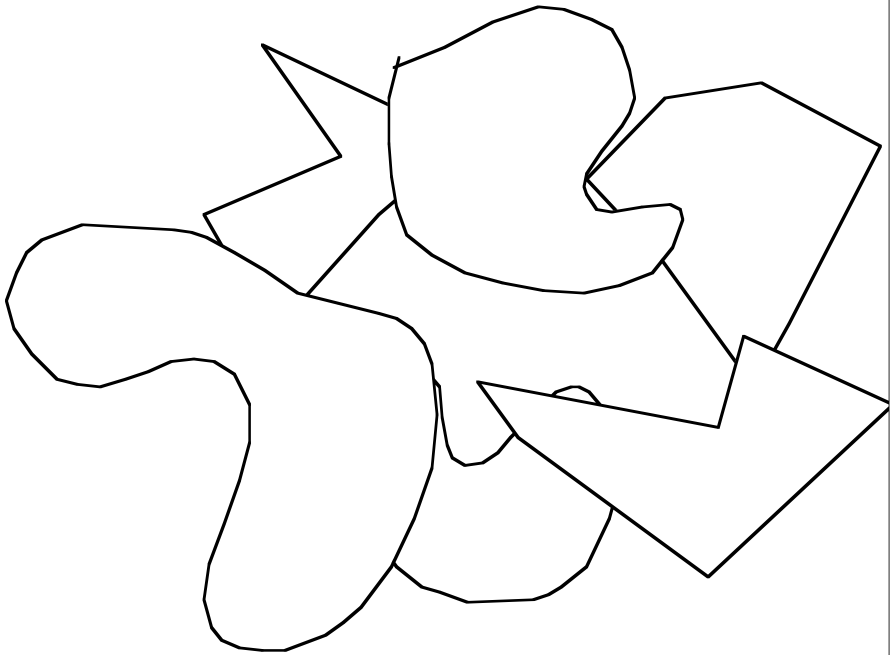
The market has almost doubled in size every year for the past five years, reaching $20 trillion in notional amounts outstanding last June 2007, according to the Bank for International Settlements.

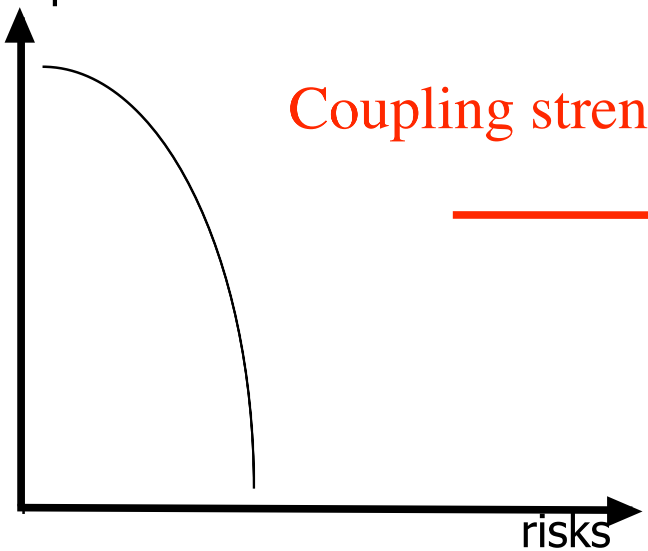
Bundling of indexes of CDSs together and slicing them into tranches, based on riskiness and return. The most toxic tranche at the bottom exposes the holder to the first 3% of losses but also gives him a large portion of the returns. At the top, the risks and returns are much smaller-unless there is a systemic failure.

Separation of financial and credit risks

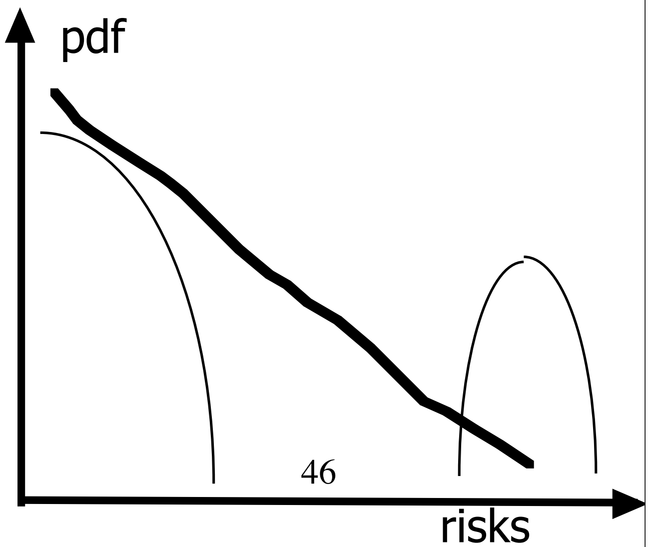
Securitization leads to larger inter-connectivity

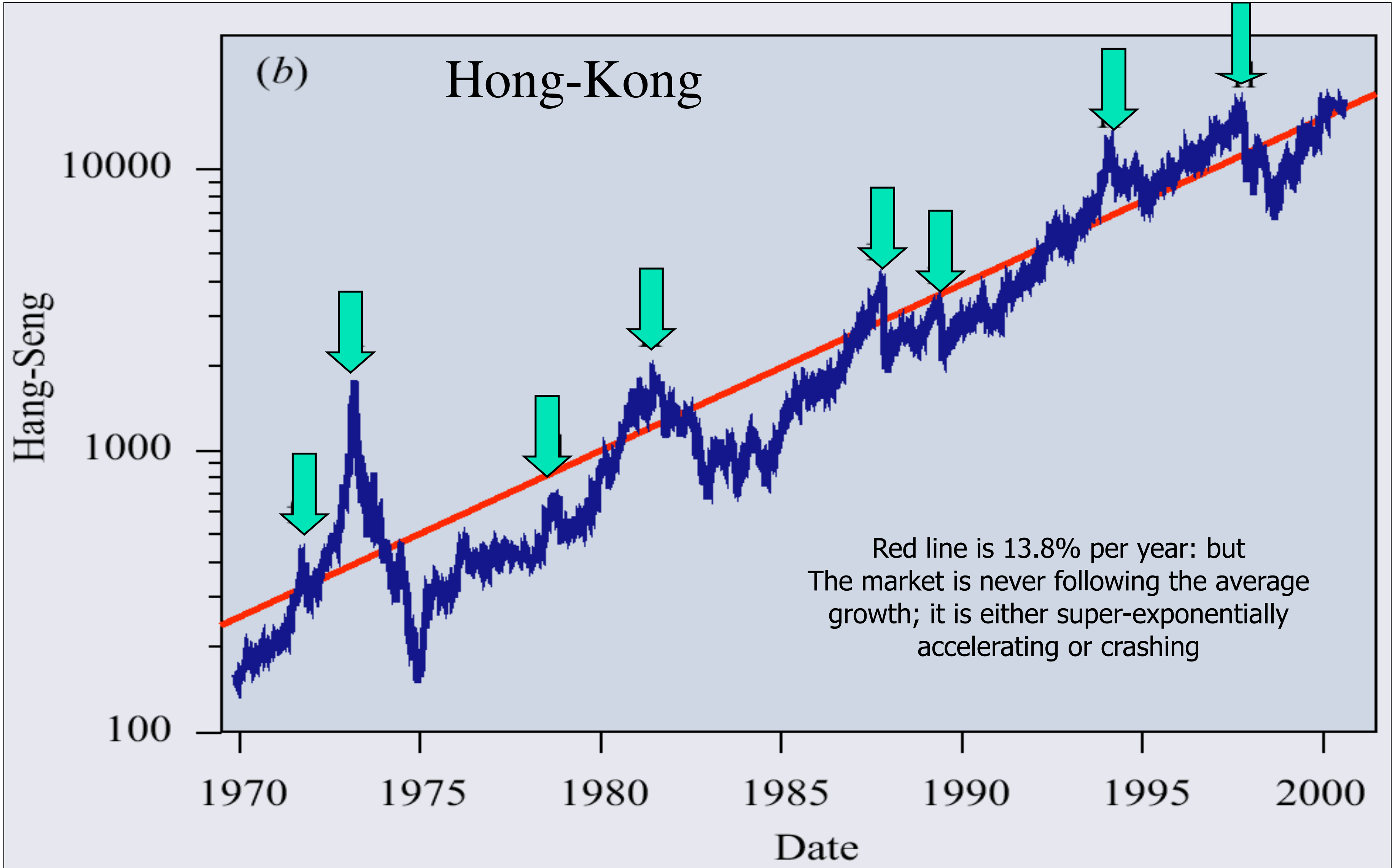




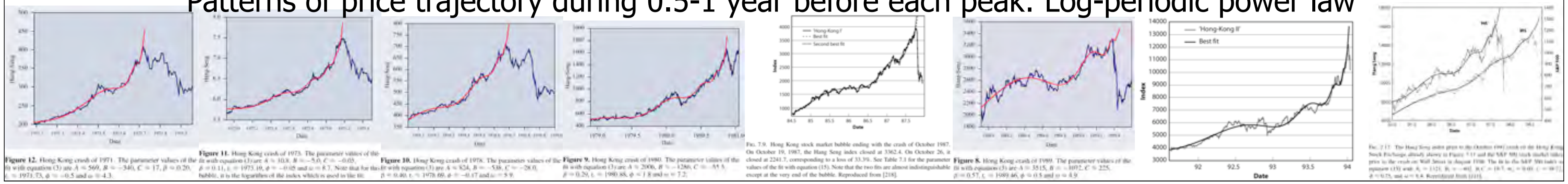


Coupling strength increases





Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law



What is the cause of the crash?

- ✓ Proximate causes: many possibilities
- ✓ Fundamental cause: maturation towards an instability

An instability is characterized by

- large or diverging susceptibility to external perturbations or influences
- exponential growth of random perturbations leading to a change of regime, or selection of a new attractor of the dynamics.