Parallels between
Earthquakes, Financial crashes and epileptic seizures

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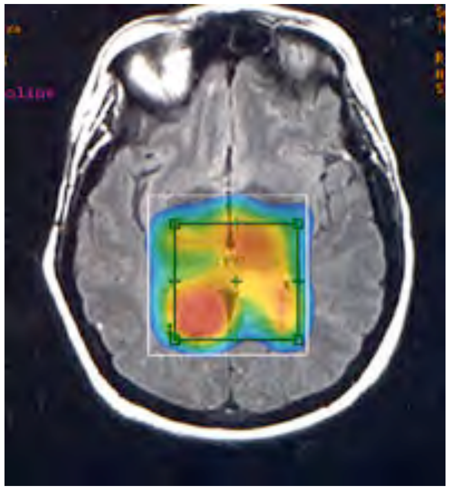

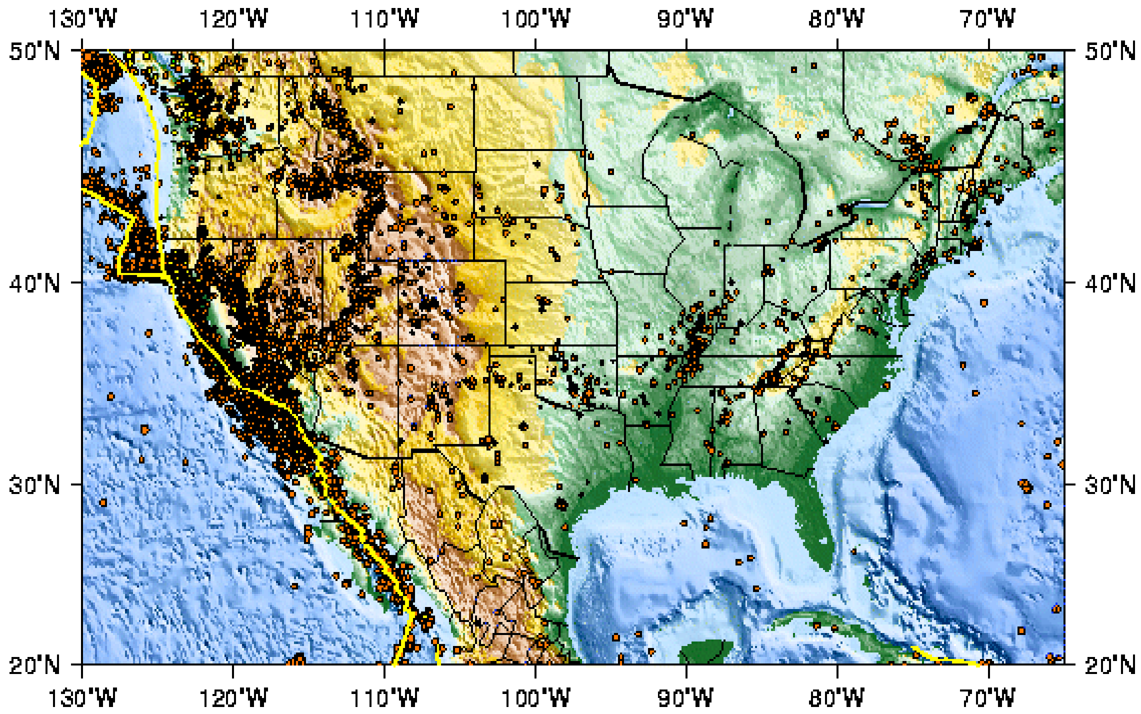
130˚W
120˚W
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80˚W
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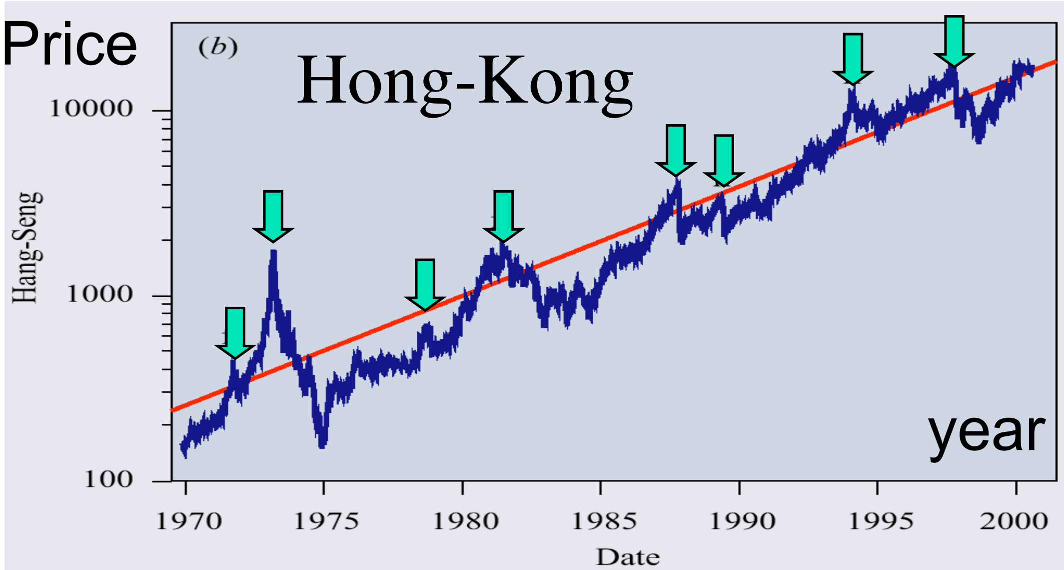
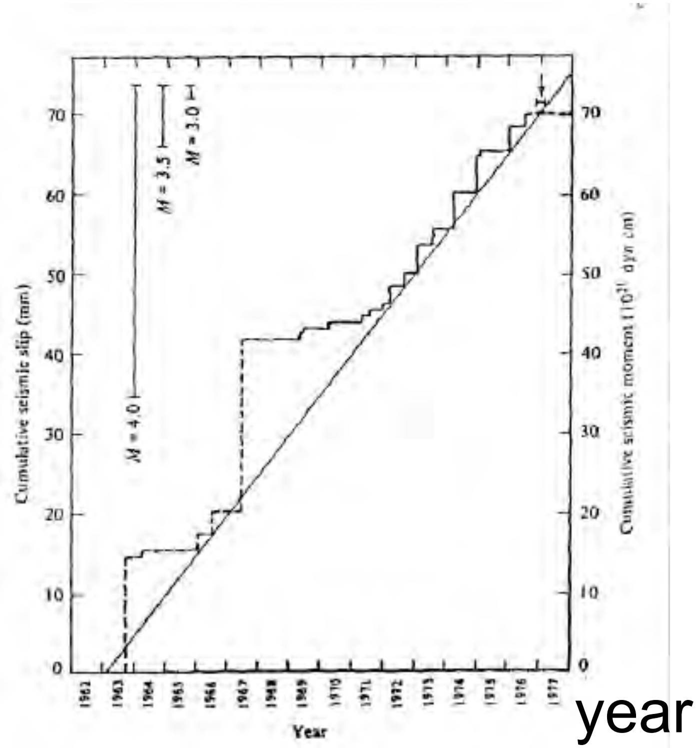
20˚N



Red line is 13.8% per year: but The market is never following the average growth; it is either super-exponentially accelerating or crashing

Cumulative Slip



Cumulative moment and seismic slip in a zone of the Calaveras fault (1962-77)

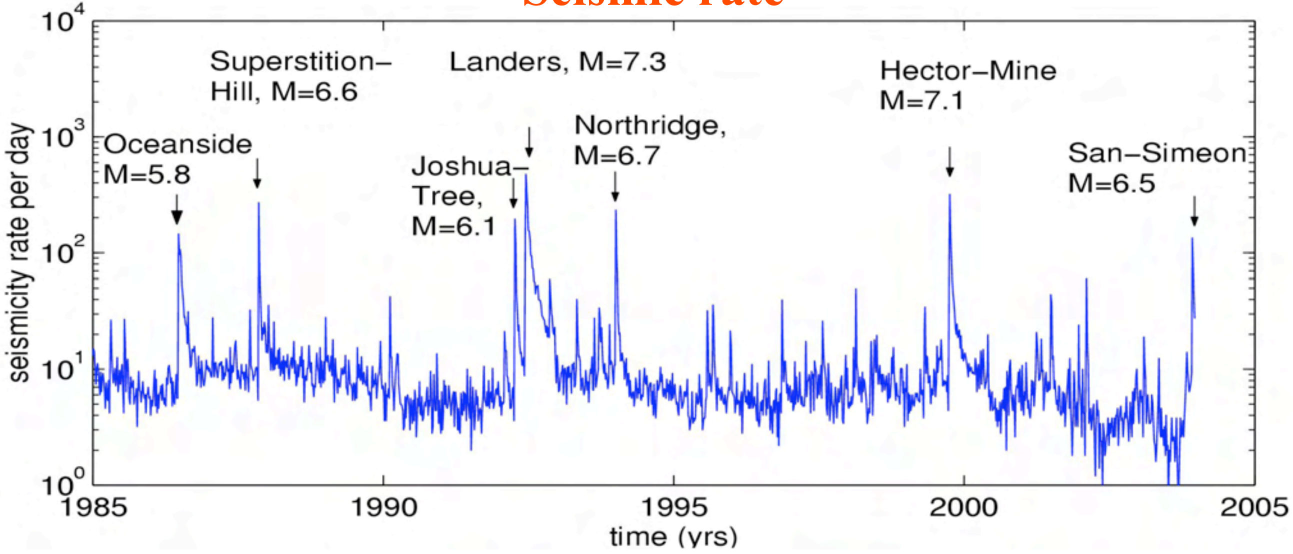
Statistical laws of seismicity

- Gutenberg-Richter law: $\sim 1/E^{1+\beta}$ (with $\beta \approx 2/3$)
- Omori law $\sim 1/t^p$ (with $p \approx 1$ for large earthquakes)
- Productivity law $\sim E^a$ (with $a \approx 2/3$)
- PDF of fault lengths $\sim 1/L^2$
- Fractal/multifractal structure of fault networks $\zeta(q)$, $f(\alpha)$
- PDF of seismic stress sources $\sim 1/s^{2+\delta}$ (with $\delta \geq 0$)
- Distribution of inter-earthquake times
- Distribution of seismic rates

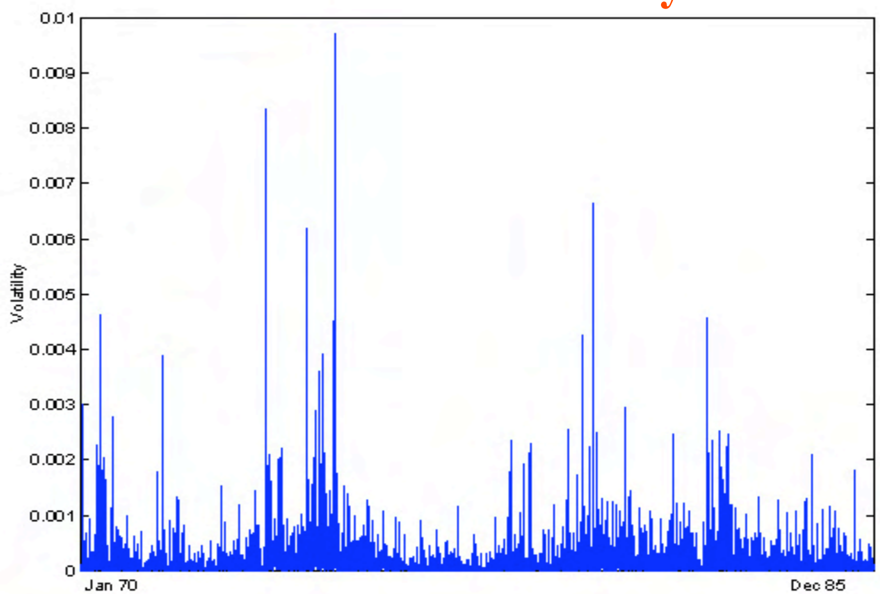
Stylized facts of financial markets

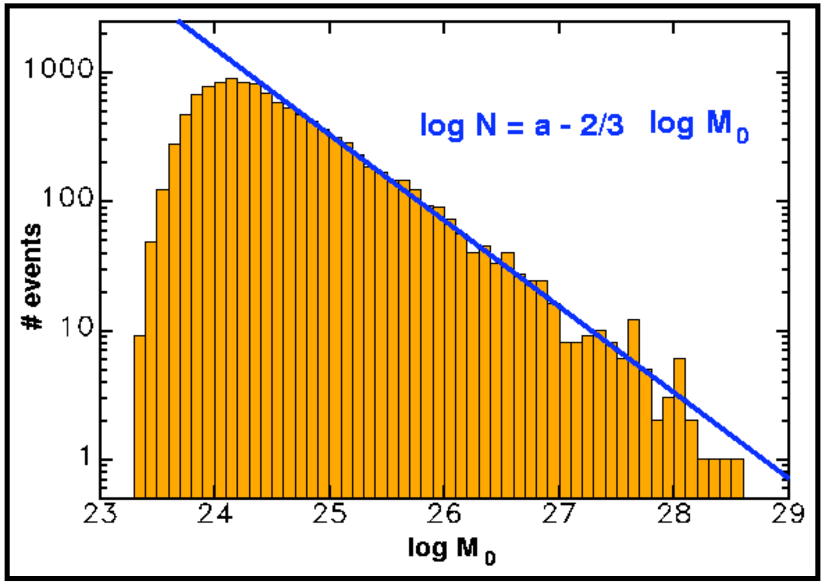
- Heavy-tail pdf of returns
- Omori law and Long-memory of volatility
- Price impact function Price ~ V^{β} with β=0.2-0.6
- Pareto distribution of wealth
- Multifractal structure of returns
- PDF of news' sizes?
- Distribution of inter-shock times
- Distribution of limit order sizes
- "Leverage" effect

Seismic rate

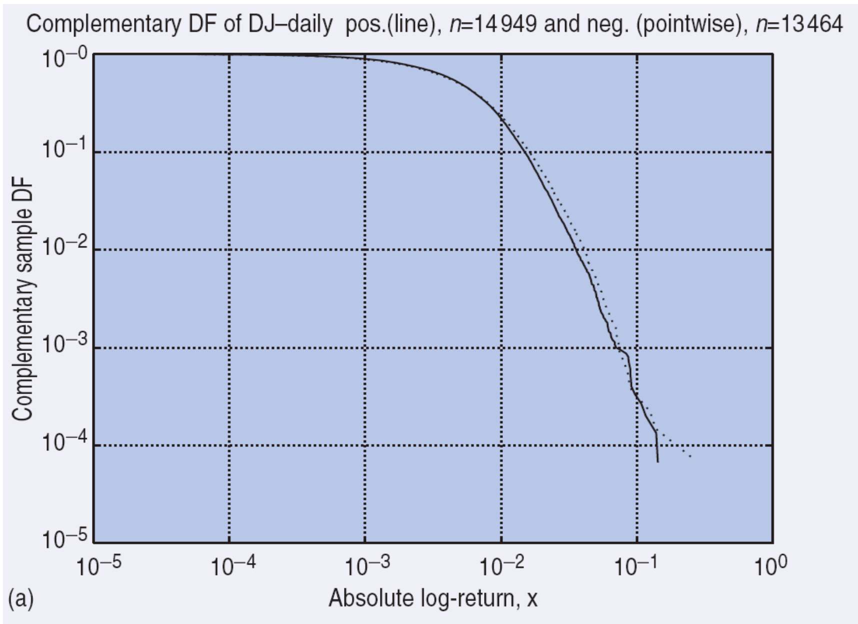


Financial Volatility

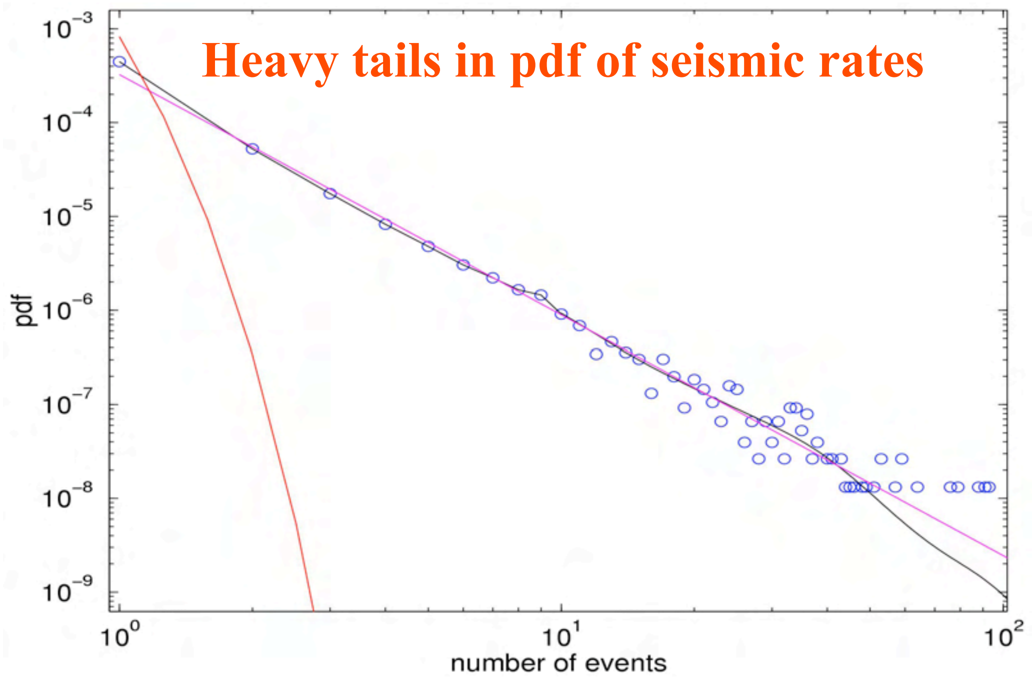


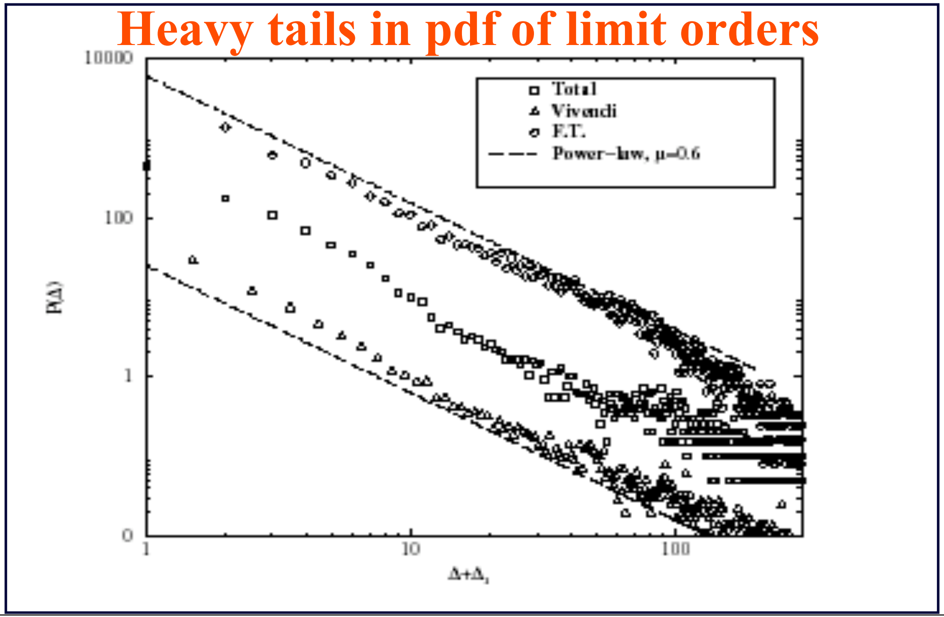


Heavy tails in pdf of earthquakes b=2/3



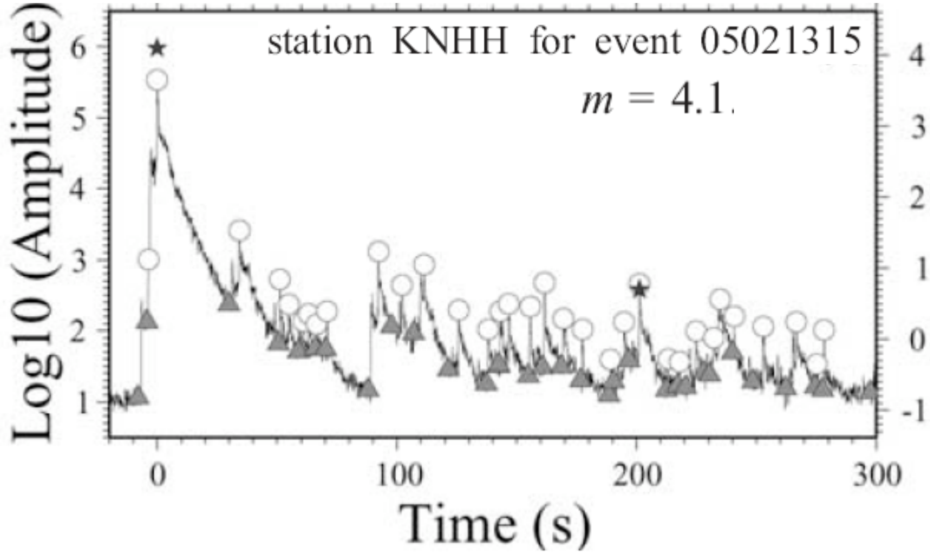
Heavy-tails of price changes b=3

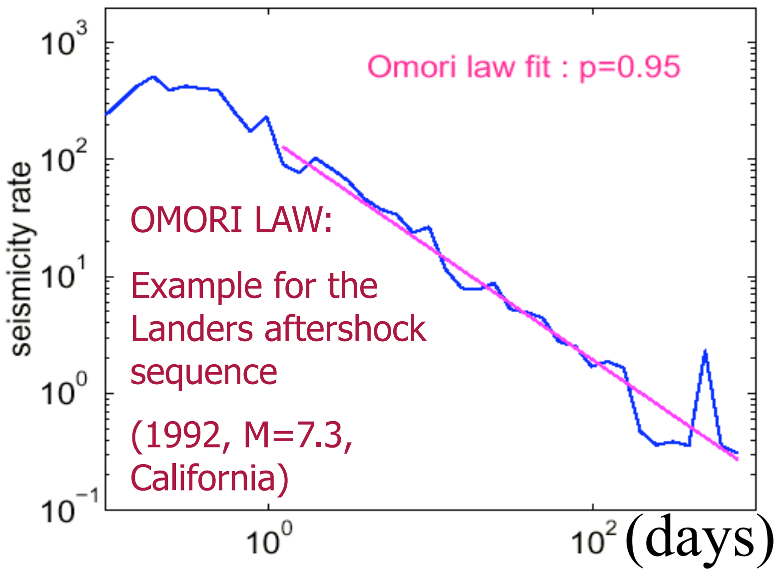

Heavy tails in pdf of seismic rates
pdf
number of events

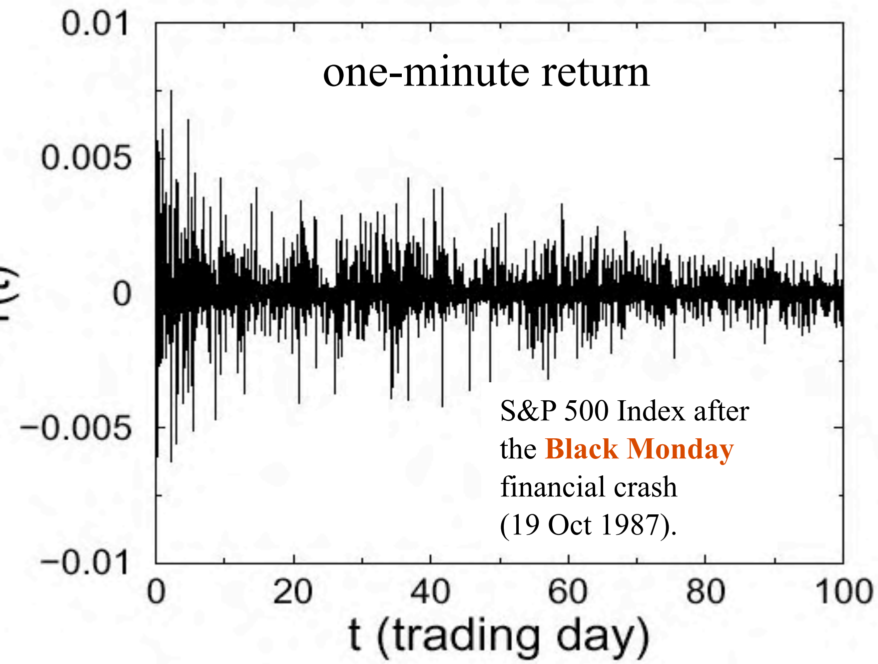

Heavy tails in pdf of limit orders
Total
Vivendi
F.T.
Power-law, μ=0.6
P(Δ)
Δ+Δ

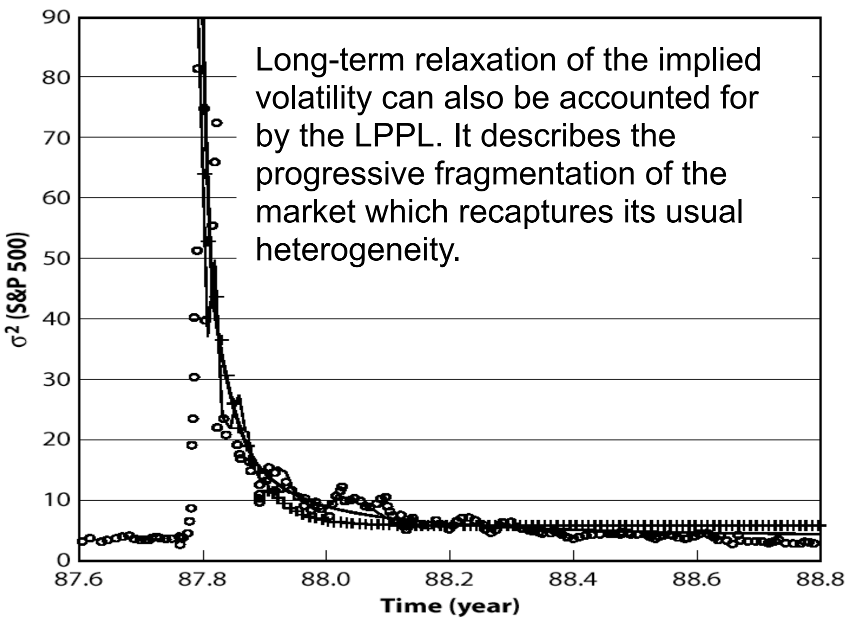
Peng et al.

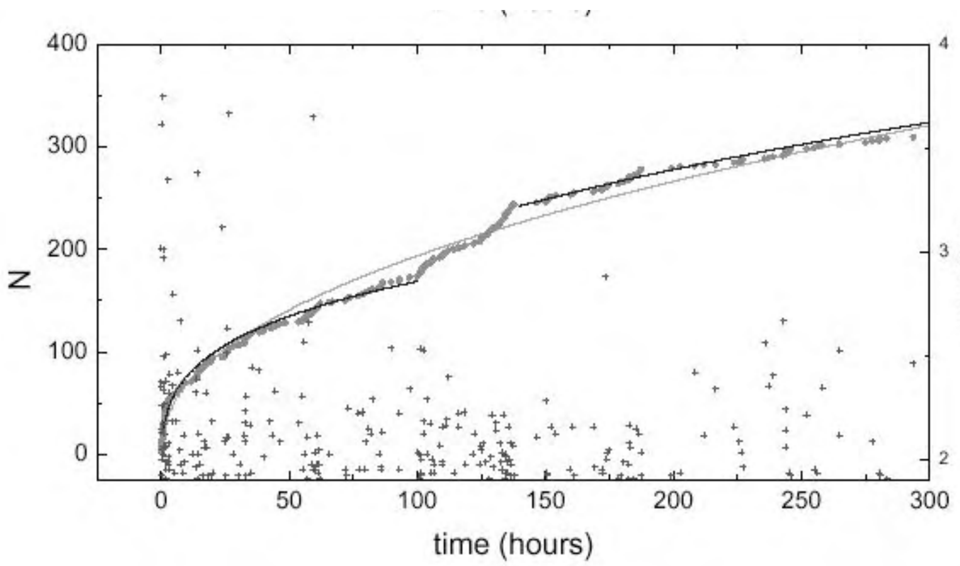
JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 112, B03306, doi:10.1029/2006JB004386, 2007







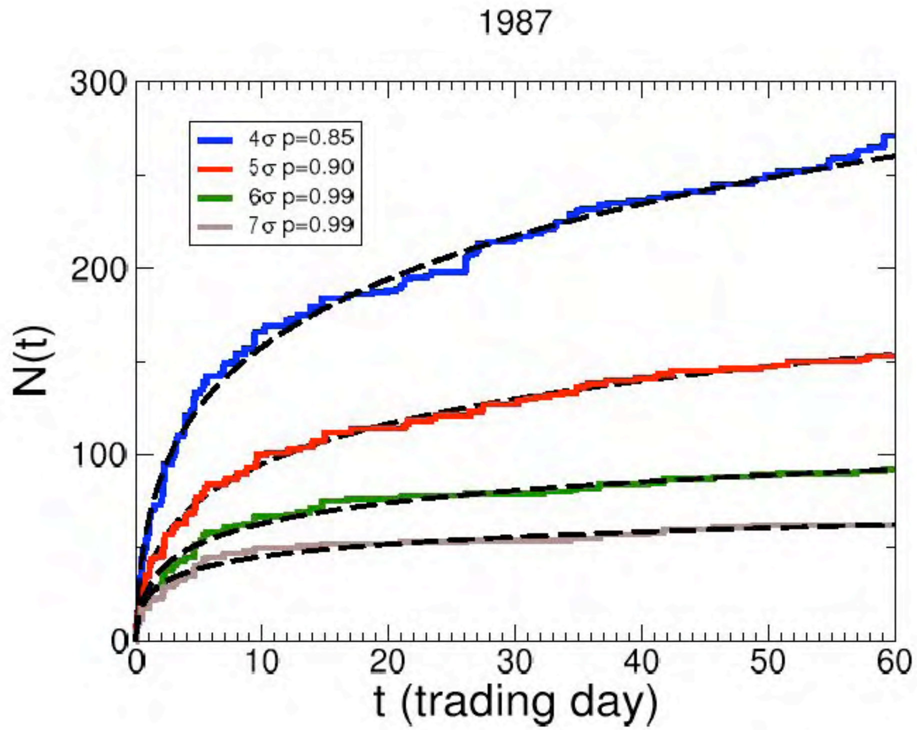




Cumulative number of aftershocks in the earthquake occurring in eastern Pyrenees on February 18, 1996 (from Moreno *et al*., J. of Geophys. Res., **106 B4**, 6609-6619 (2001))

$$n(t) \propto t^{-p}; \qquad N(t) = \int_0^t n(s)ds$$

N(t)=K[(t+τ)$^{1-p}$- τ^{1-p}]/(1-p)

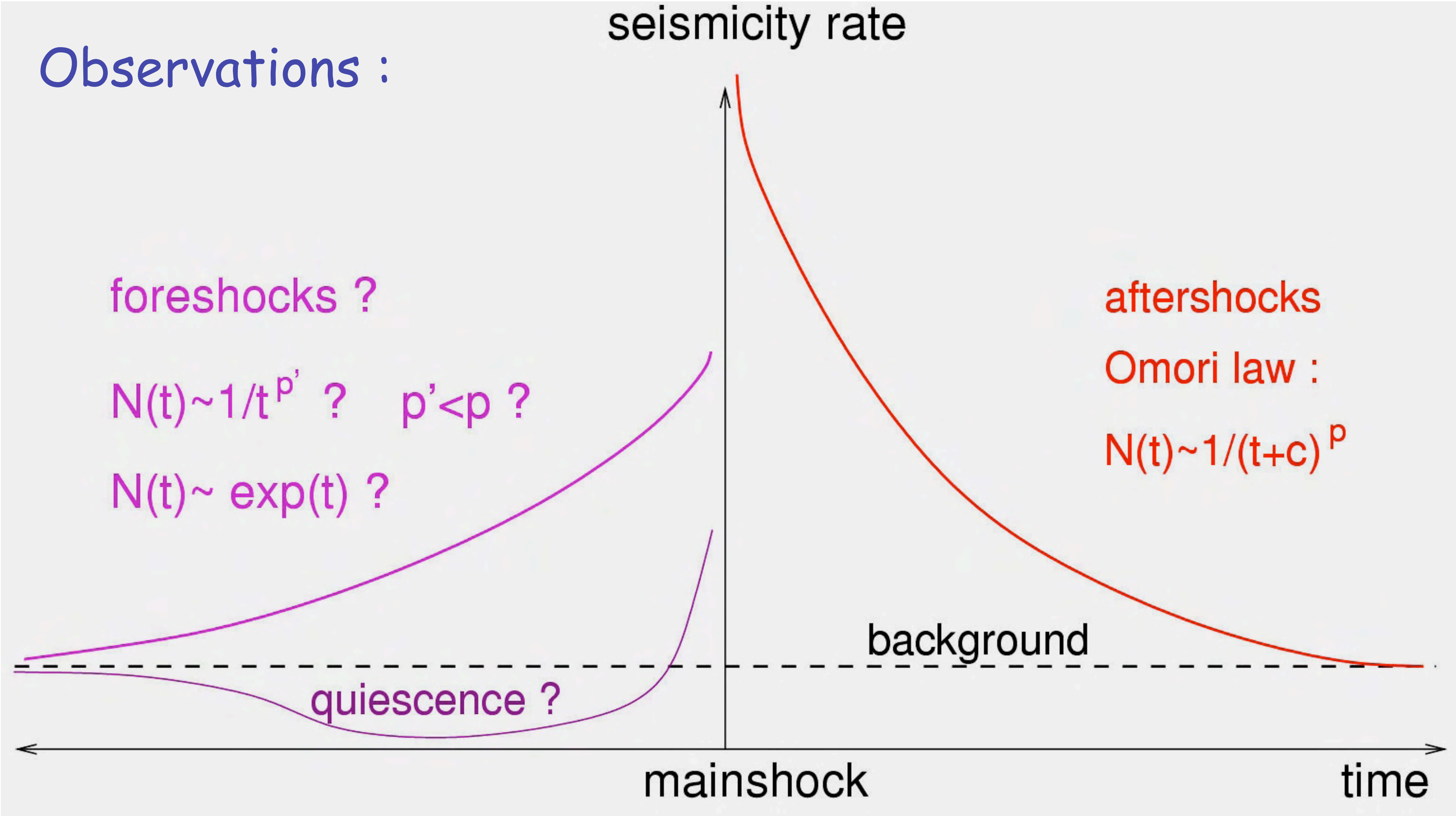


Oct. 1987 crash: Cumulative number of S&P500 index returns exceeding a given threshold **nσ**

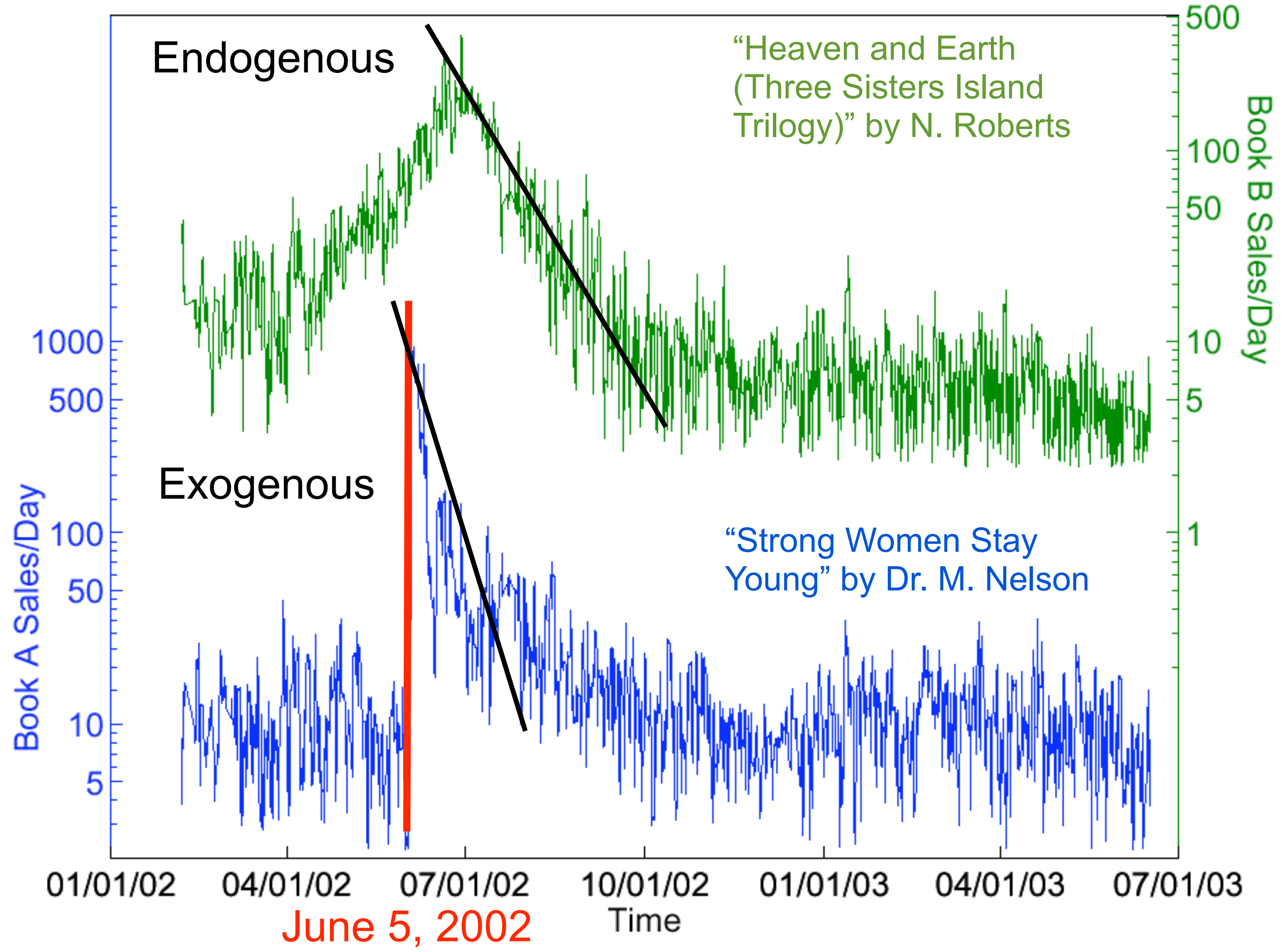
†Lillo and Mantegna, PRE **68**, 016119 (2003)

Endogenous versus Exogenous Origins of Crises

Foreshocks - Aftershocks



Book sales dynamics



June 4, 2002:
New York Times article crediting the “groundbreaking research” of Dr. Nelson

D. Sornette et al., Phys. Rev. Letts. 93 (22), 228701 (2004)

Theory: Endogenous vs Exogenous Response

- Mean field treatment: ensemble averages, rather than individual behavior :

EXO ENDO

$$S(t) \equiv \langle \lambda(t) \rangle = \eta(t) + n \int_{-\infty}^{t} \phi(t-\tau) S(\tau)\, d\tau$$

(where n is the branching ratio of the network)

- One can then solve this equation for an exogenous shock $\eta(t) = \delta(t)$:
- For $\phi \sim 1/t^{1+\theta}$

$$S_{exo}(t) \equiv K(t) : \frac{1}{(t-t_c)^{1-\theta}} \quad \text{with} \quad 0 < \theta < 1$$

Theoretical predictions

- The tests are about the slopes of the response functions, conditional on the class of peak determined by the slope of the growth AT CRITICALITY n=1

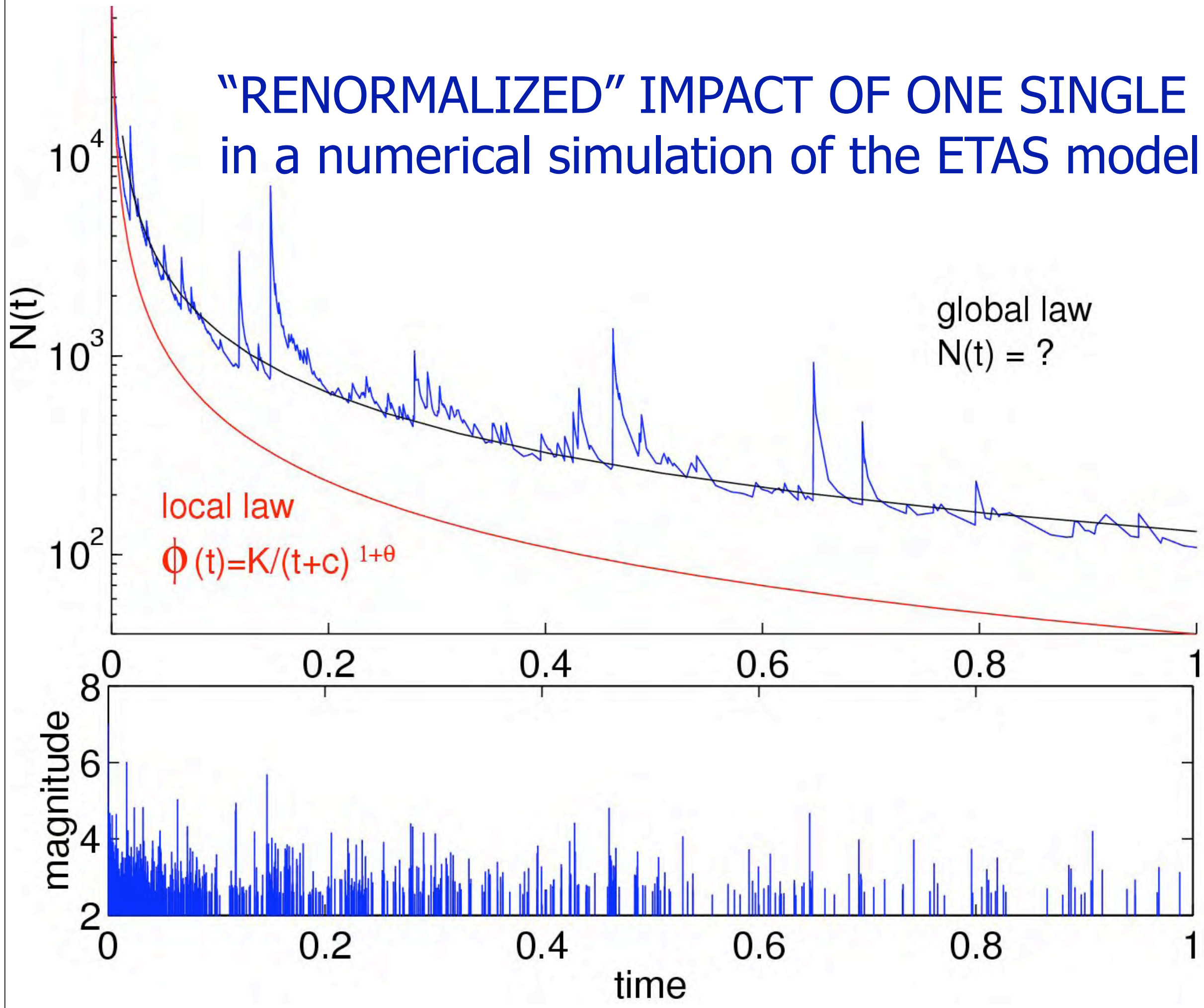
	Endogenous	Exogenous
Foreshock (or growth)	$S(t) \propto \frac{1}{\lvert t \rvert^{1-2\theta}}$	Abrupt peak
Aftershock (or decay)	$S(t) \propto \frac{1}{t^{1-2\theta}}$	$S(t) \propto \frac{1}{t^{1-\theta}}$

Non-critical: $S(t) \propto \frac{1}{t^{1+\theta}}$

Hawkes ETAS model and numerical simulations

The impact of cascades of generations

"RENORMALIZED" IMPACT OF ONE SINGLE PIECE OF INFORMATION
in a numerical simulation of the ETAS model



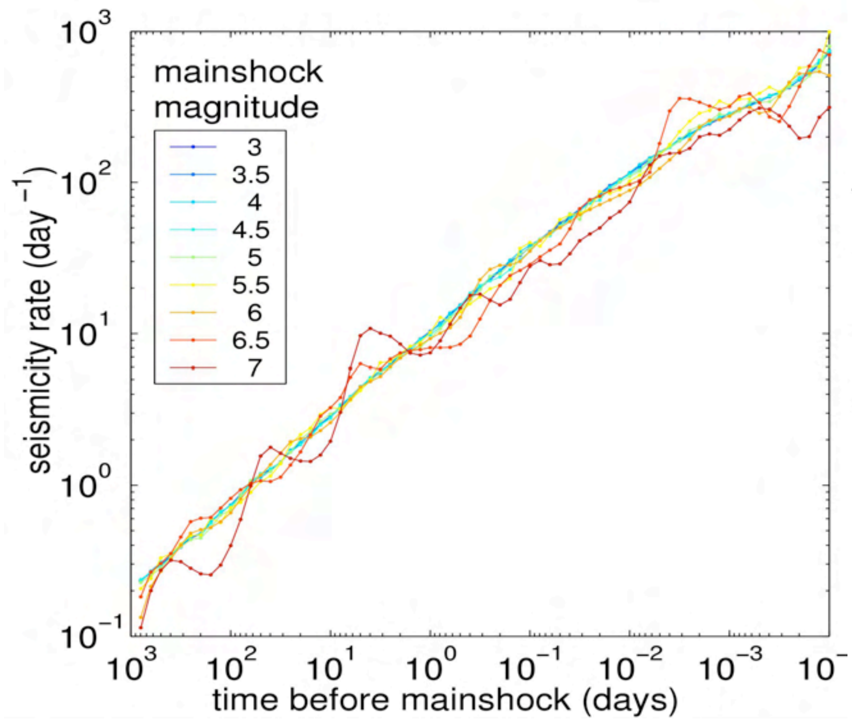
For $\phi \sim 1/t^{1+\theta}$ $\quad S_{exo}(t) \equiv K(t) : \dfrac{1}{(t-t_c)^{1-\theta}}$ with $0 < \theta < 1$

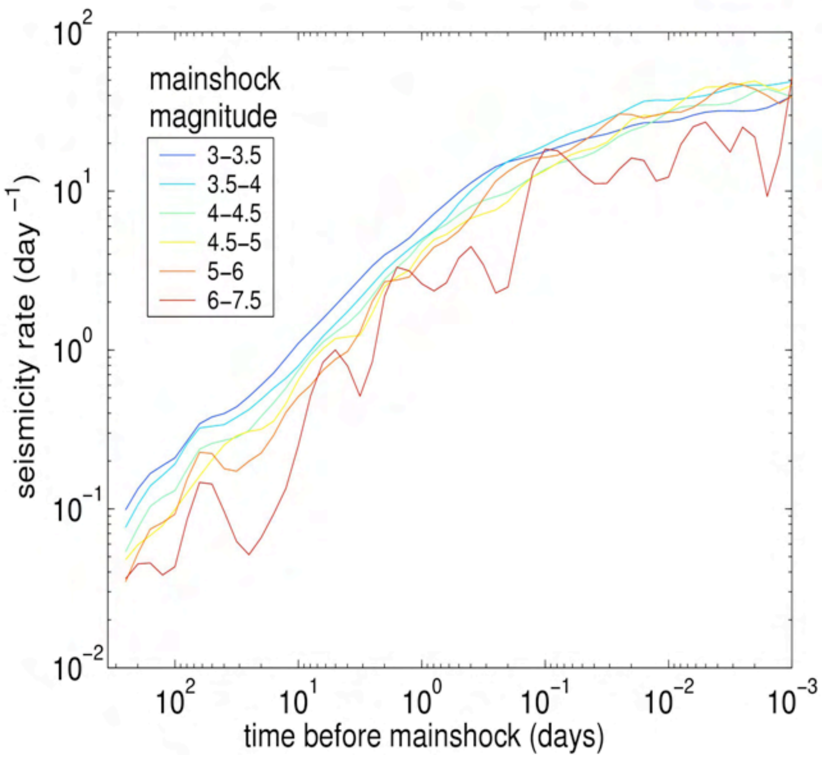
[Helmstetter and Sornette, GRL, 2003a]

Foreshocks – Inverse Omori law

Increase of the AVERAGE seismicity rate before an EQ ('mainshock')

Synthetic catalog, ETAS model California seismicity (SCEC 1932-2002)

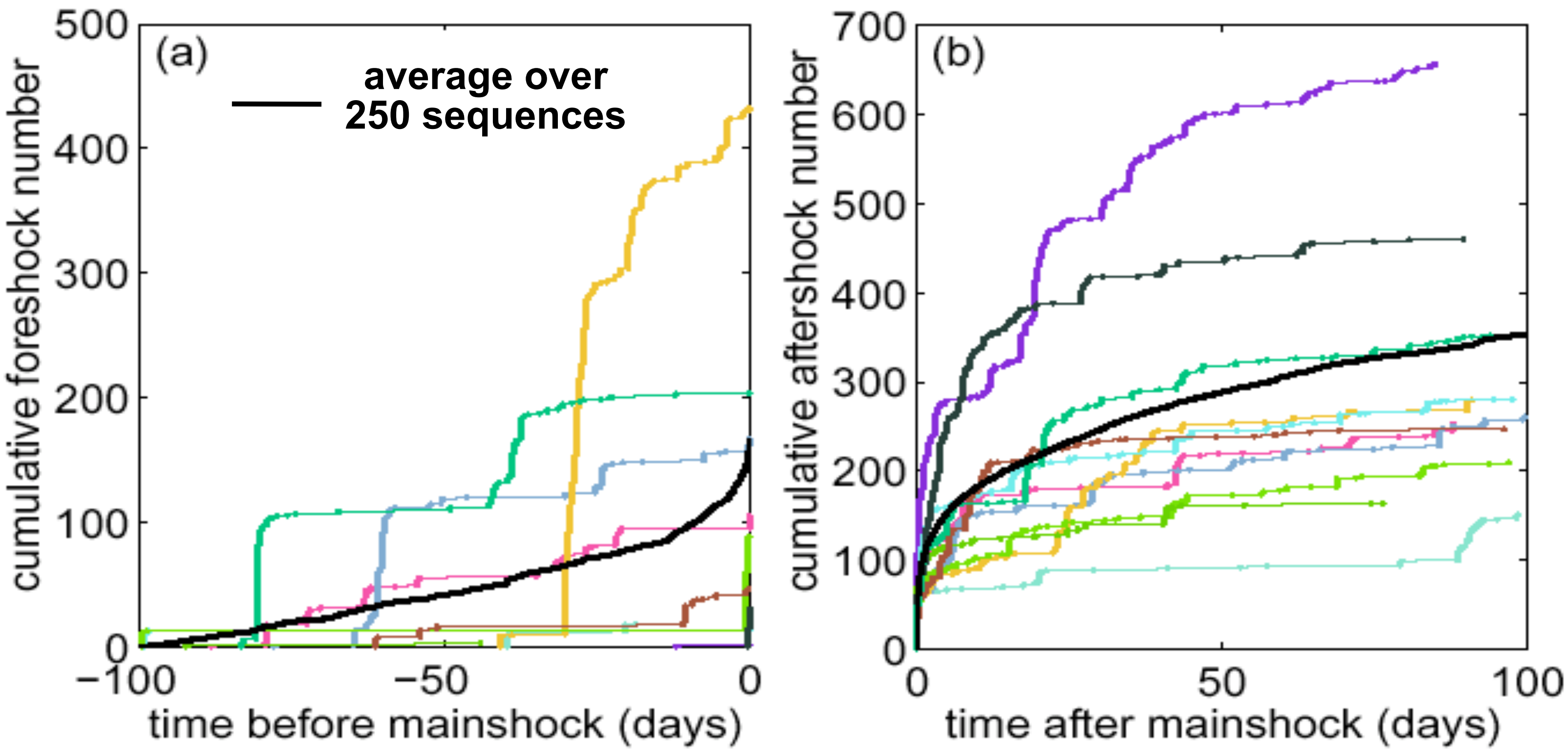




[Helmstetter, Sornette and Grasso, JGR 2003; Helmstetter and Sornette, JGR 2003]

Typical aftershock and foreshock sequences

numerical simul. ETAS : foreshocks aftershocks

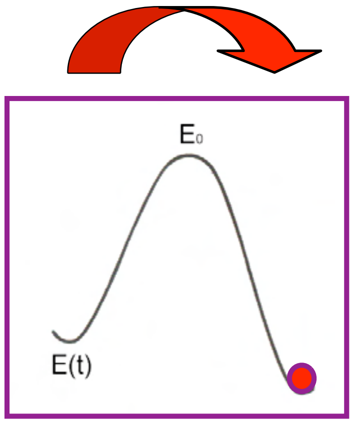


- large fluctuations of the seismic activity before a mainshock
- The inverse Omori law is observed only by averaging over many sequences

The physical model : thermal activation driven by stress

3

Before the shock



After the shock

Energy barrier = E_0-E(t)

Arrhenius law for the activation rate:

$$\lambda(t) = \lambda_0 \exp\left(-\frac{E_0 - E(t)}{kT}\right)$$

stress barrier = $\sigma_0 - \sigma(t)$

$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V\right)$$

Compatible with state-and-rate friction, stress corrosion, …

$\lambda(t)$: instantaneous rate

λ_0 ~ average nucleation rate

σ_0 : material strength

$\sigma(t)$: applied stress

V : activation volume

T : temperature

k : Boltzmann constant

Experiments by Zhurkov Int. J. Fract. Mech. **1**, 311 (1965)

$$\ln \tau = A - B\,\sigma$$

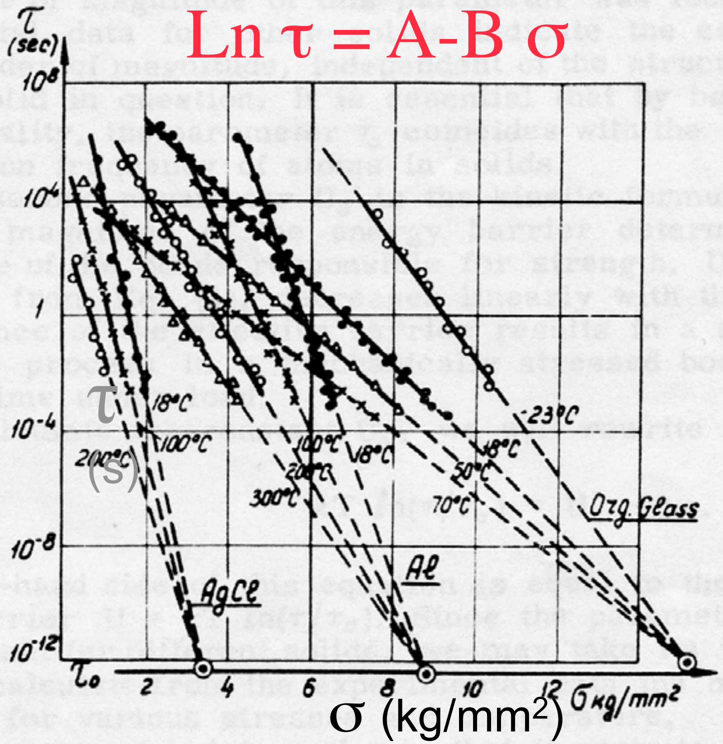


Fig. 5. Time and temperature dependence of the lifetime of solids on stress.
1. Silver chloride (Reference 4)
2. Aluminum (Reference 5)
3. Plexiglas (Reference 6)

$$\tau = \tau_o \exp\left(\frac{U}{k_B T}\right)$$

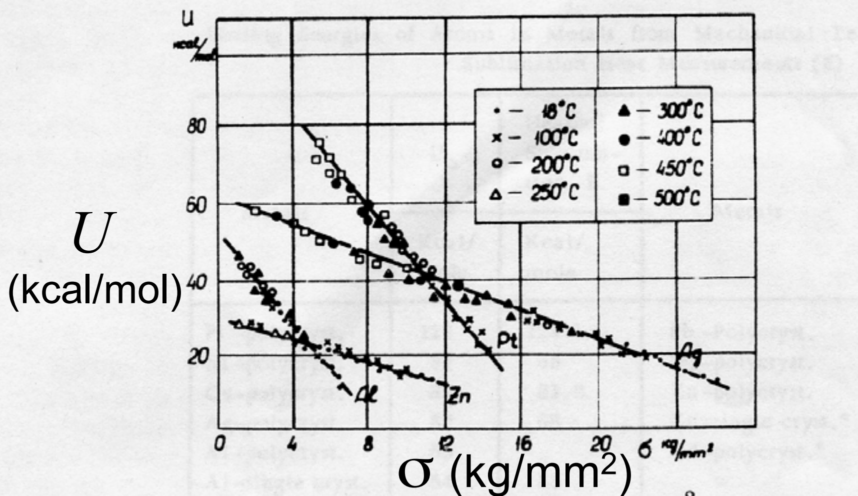


Fig. 6. Effective barrier U kkal/mol vs. tensile stress σ kg/mm² for polycrystall

Empirical energy barrier

$$U = U_o - \alpha\sigma$$

où U_o : énergie de sublimation

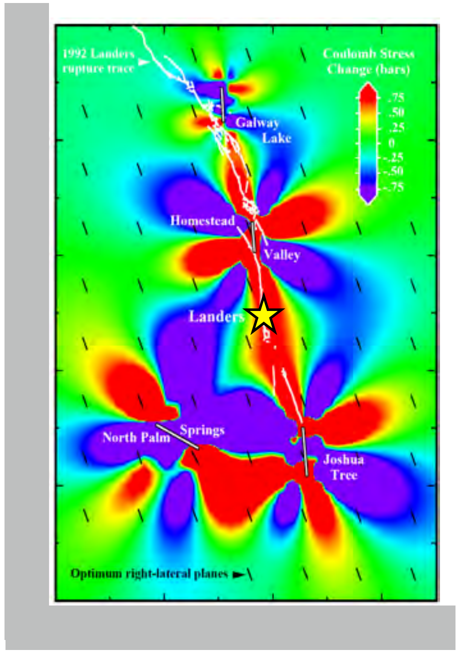
A possible mechanism : <u>thermal activated process</u>

Taking account of history and boundary conditions

4

$$\lambda_0' = \lambda_0 \exp\left(-\frac{\sigma_0}{kT} V \right)$$

$$\lambda(\vec{r},t) = \lambda_0' \exp\left(\frac{\sigma(\vec{r},t)}{kT} V \right)$$



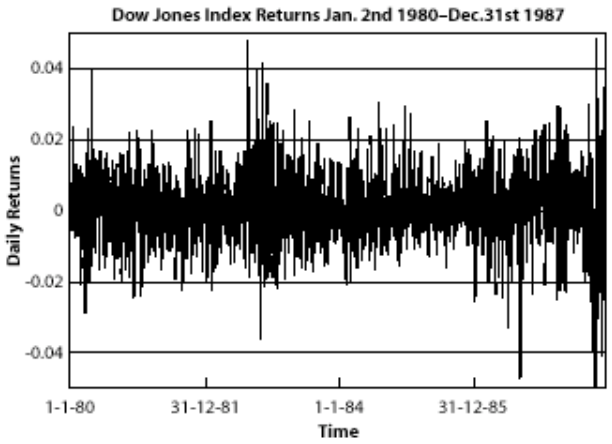
Stress is assumed to be a scalar for the sake of simplicity

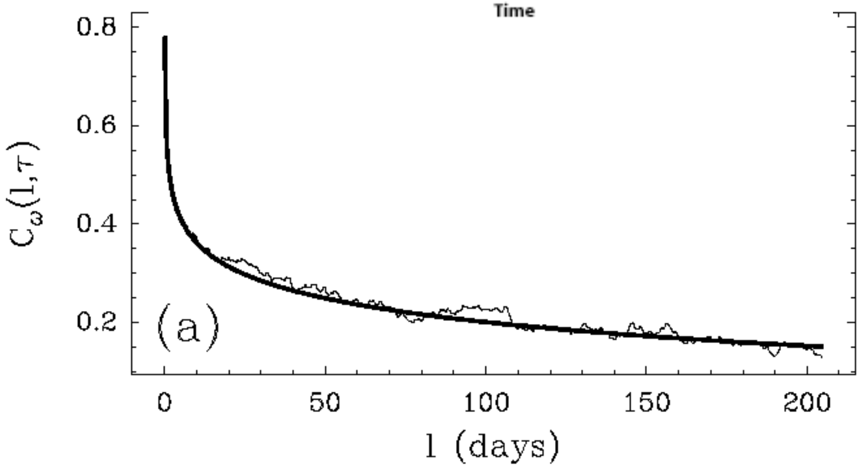
$$\sigma(\vec{r},t) = \sigma(\vec{r})_{far\ field} + \int_{-\infty}^{t} \int_{space} dN[d\vec{\rho} \times d\tau] \Delta\sigma(\vec{\rho},\tau) G(\vec{r}-\vec{\rho}, t-\tau)$$

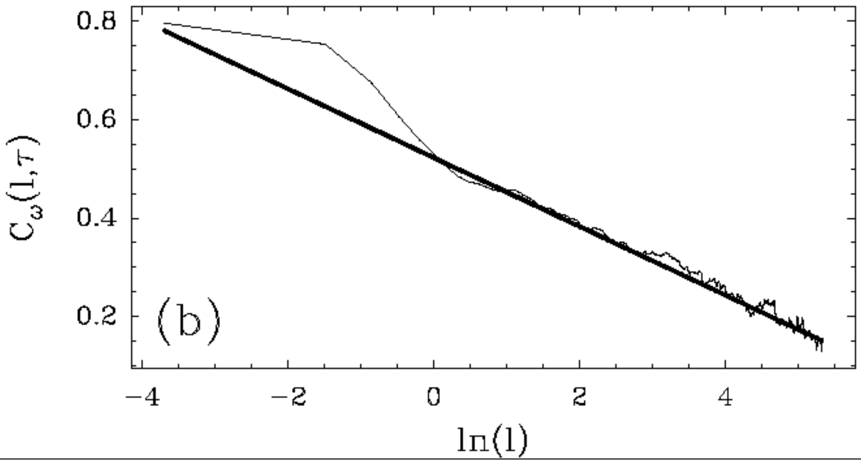
local stress — tectonic loading — **Stress fluctuations induced by all past events in the system**

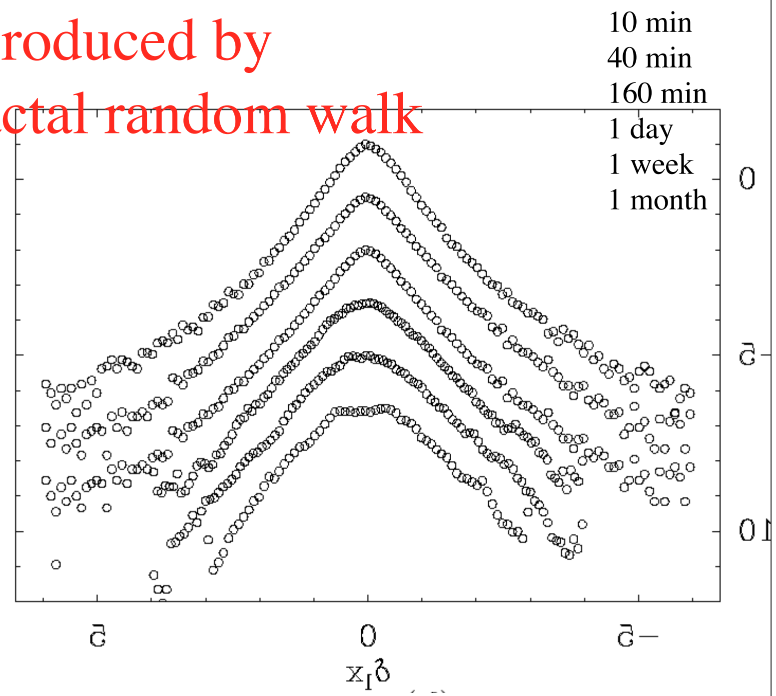
D. Sornette and G. Ouillon, Multifractal Scaling of Thermally-Activated Rupture Processes, Phys. Rev. Lett. 94, 038501 (2005)
G. Ouillon and D. Sornette, Magnitude-Dependent Omori Law: Theory and Empirical Study, J. Geophys. Res., 110, B04306, doi:10.1029/2004JB003311 (2005).

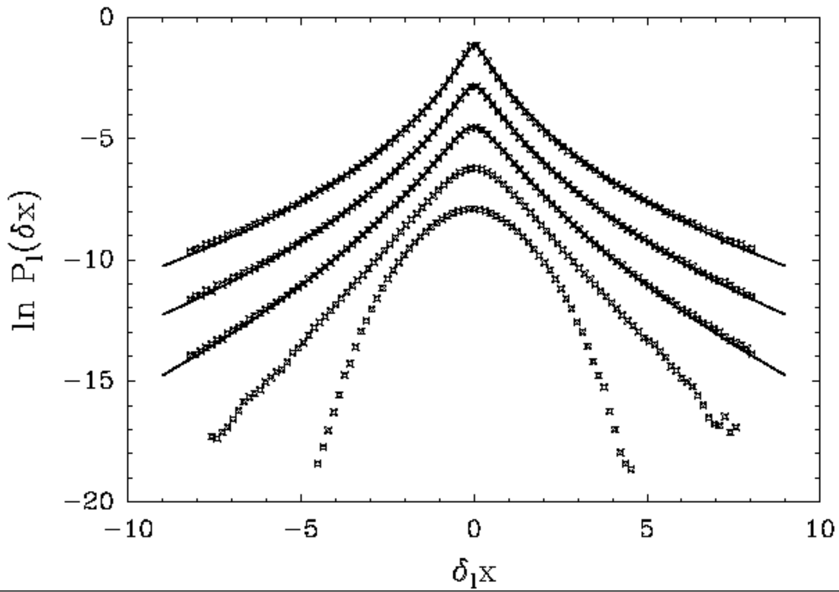
Stylized facts in financial markets well-reproduced by Multifractal random walk











Generic multifractality in exponentials of long memory processes

$$\delta_\tau X(t) = \int_{t-\tau}^{t} \mu(t')dt', \quad \text{with } \mu(t) = \kappa e^{\omega(t)},$$

$$\omega(t) = \int_{-\infty}^{t} dW(t')h(t-t'),$$

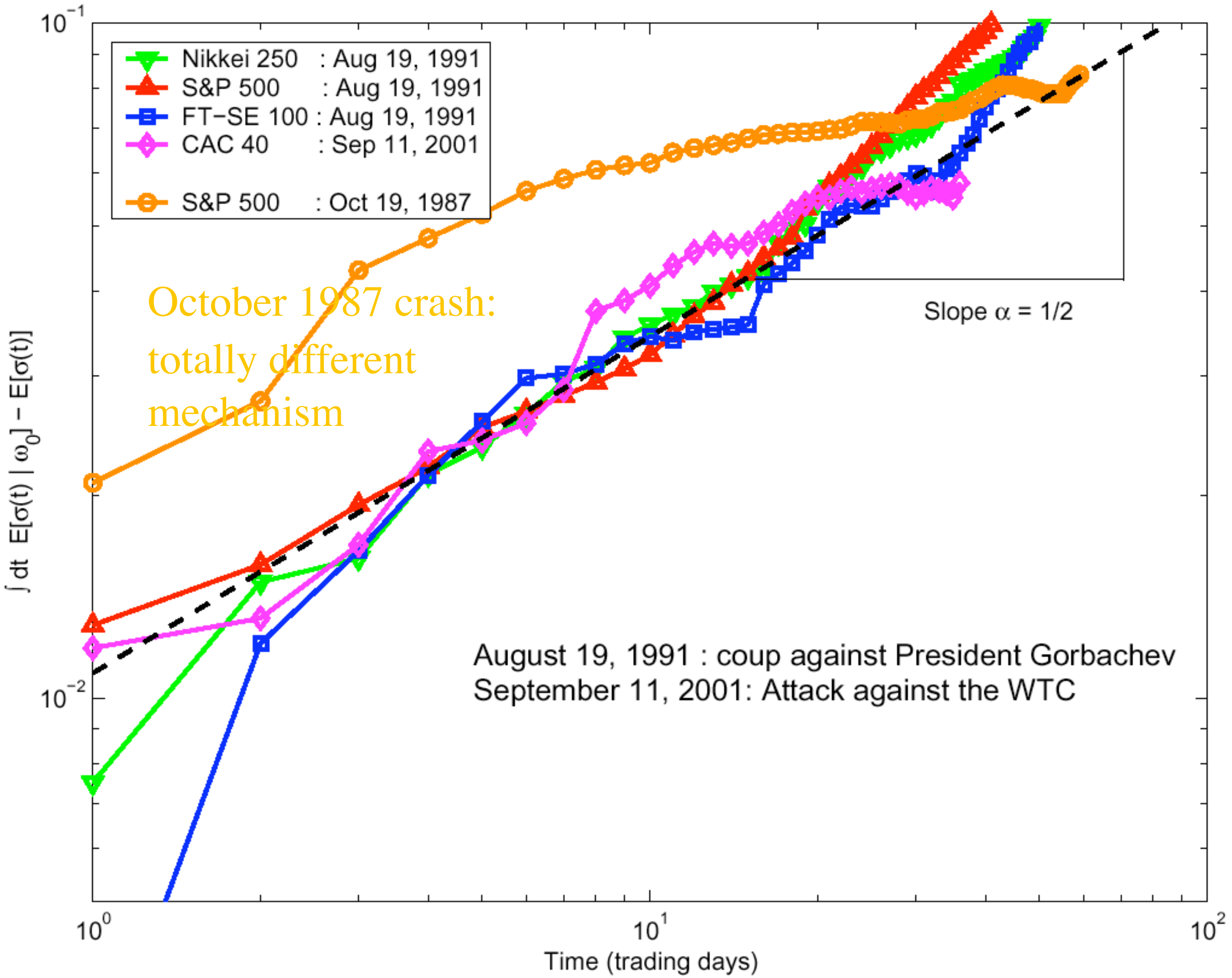
$$h(t) = \frac{h_0}{(1+x)^{\varphi+1/2}} H(t), \quad x = t/\ell,$$

A. Saichev and D. Sornette, Generic Multifractality in Exponentials of Long Memory Processes, Physical Review E 74, 011111 (2006)

Linear response to an external shock

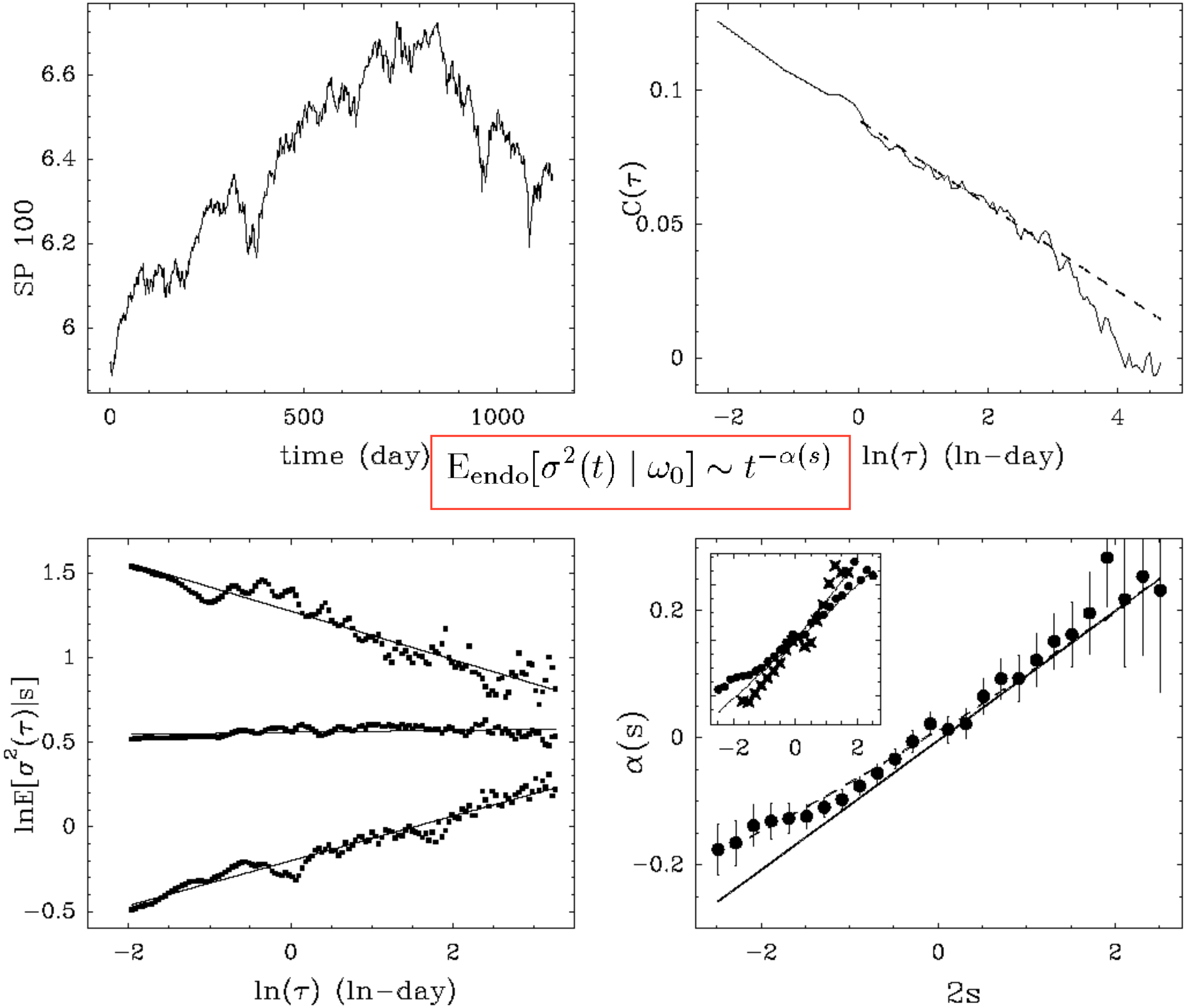
(Multifractal Random Walk model)

$$\mathrm{E}_{\mathrm{exo}}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



D. Sornette, Y. Malevergne and J.F. Muzy. Risk 16 (2), 67-71 (2003)

Endogenous shocks and Multifractal Random Walk model

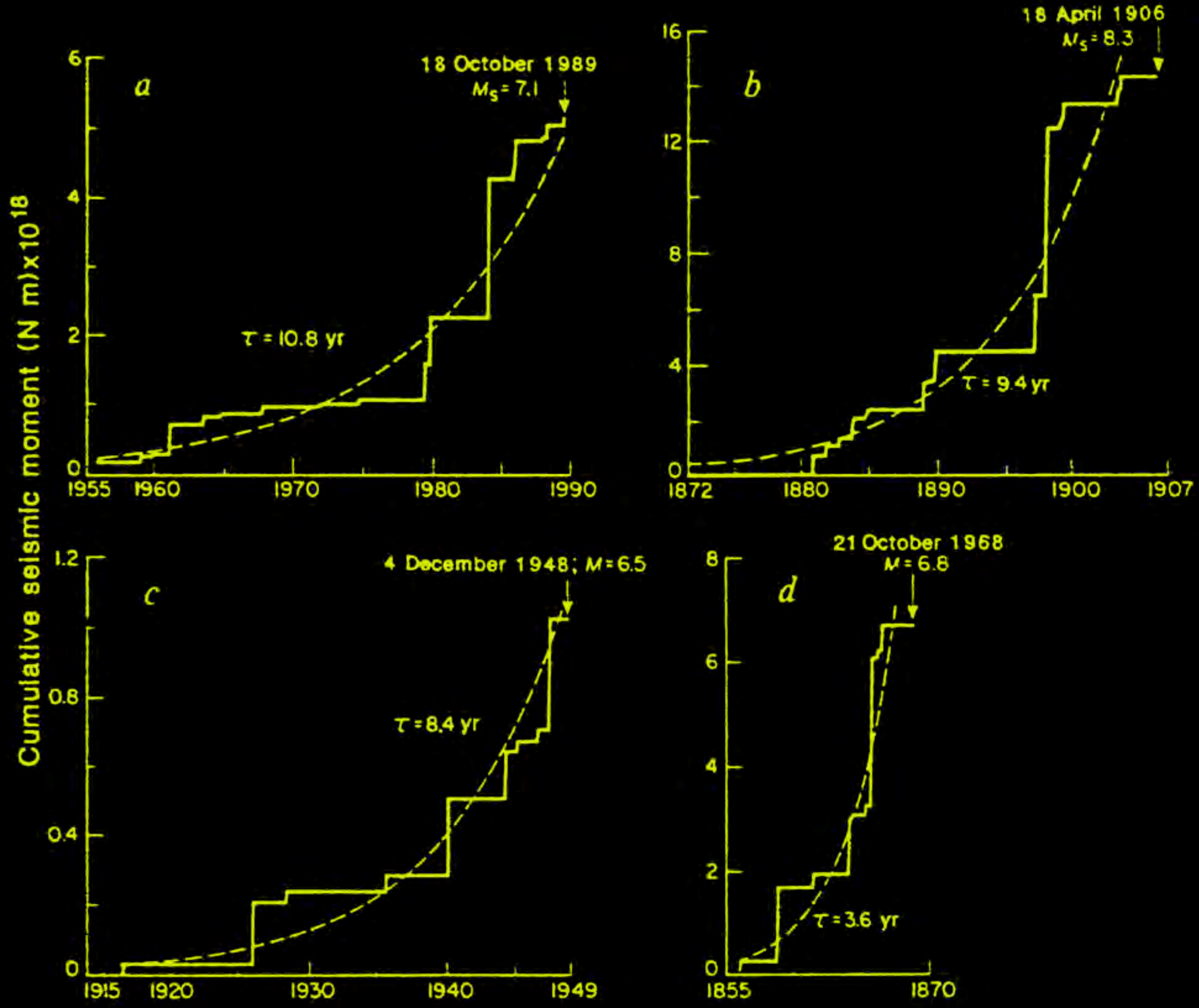


$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

Critical earthquakes?

Critical crashes?

Accelerating Moment Release Before Four Earthquakes in the San Francisco Bay Region



From *Sykes and Jaumé* [1990]

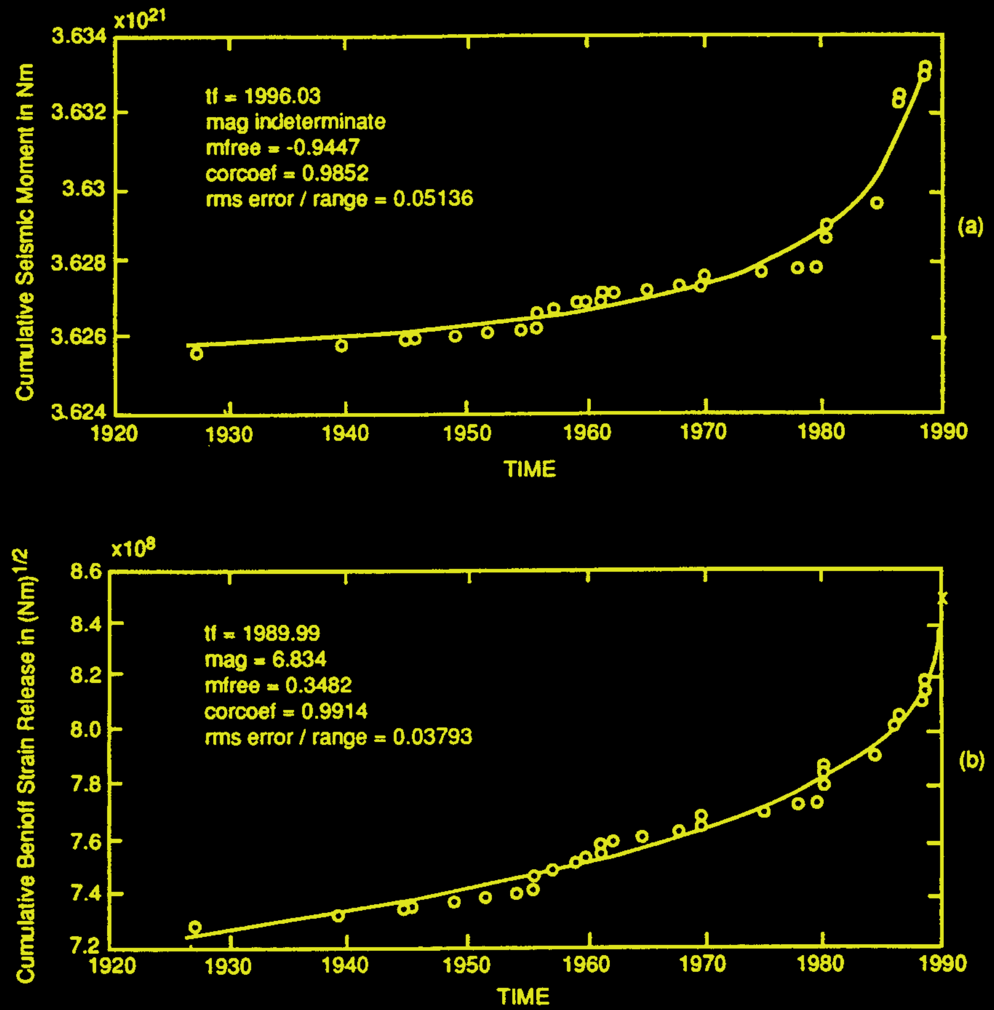


Fig. 3. Cumulative (from 1855) prestep values of seismic release parameters (Ω), (*a*) seismic moment and (*b*) Benioff strain release for northern California earthquakes of magnitude 5 or greater for the period 1927–1988. The lines are best fit solutions for m and t_f in equation (8). No solution was obtained for the cumulative event count data of Figure 2*c*.

Sornette and Sammis [1995]

RENORMALIZATION GROUP THEORY OF EARTHQUAKES

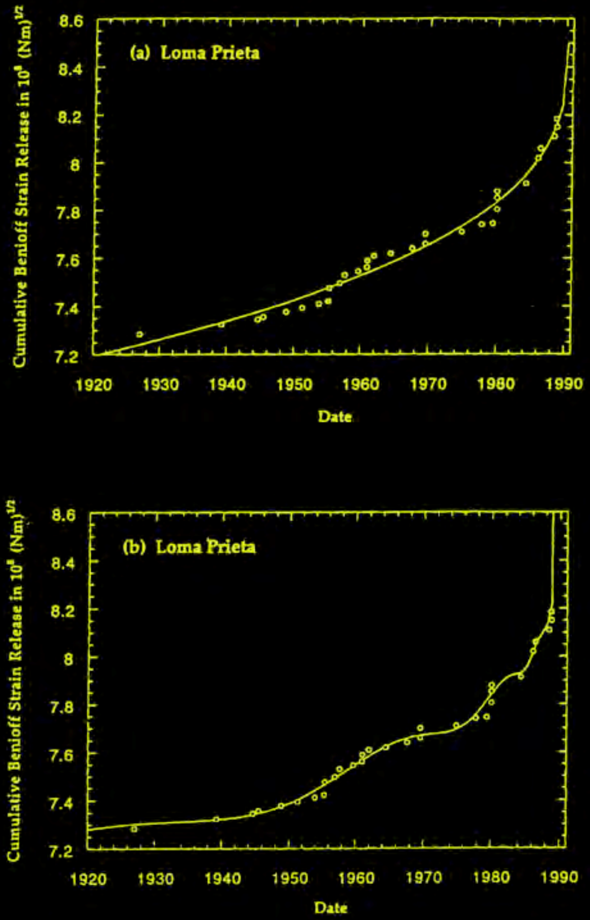


Fig. 1. — Cumulative Benioff strain released by magnitude 5 and greater earthquakes in the San Francisco Bay area prior to the 1989 Loma Prieta eaerthquake (from Ref. [32]). In (a), the data have been fit to the powerlaw equation (2) as in Bufe and Varnes [32]. In (b), the data have been fit to equation (8) which includes the first order correction to scaling. Parameters of both fits are given in Table I.

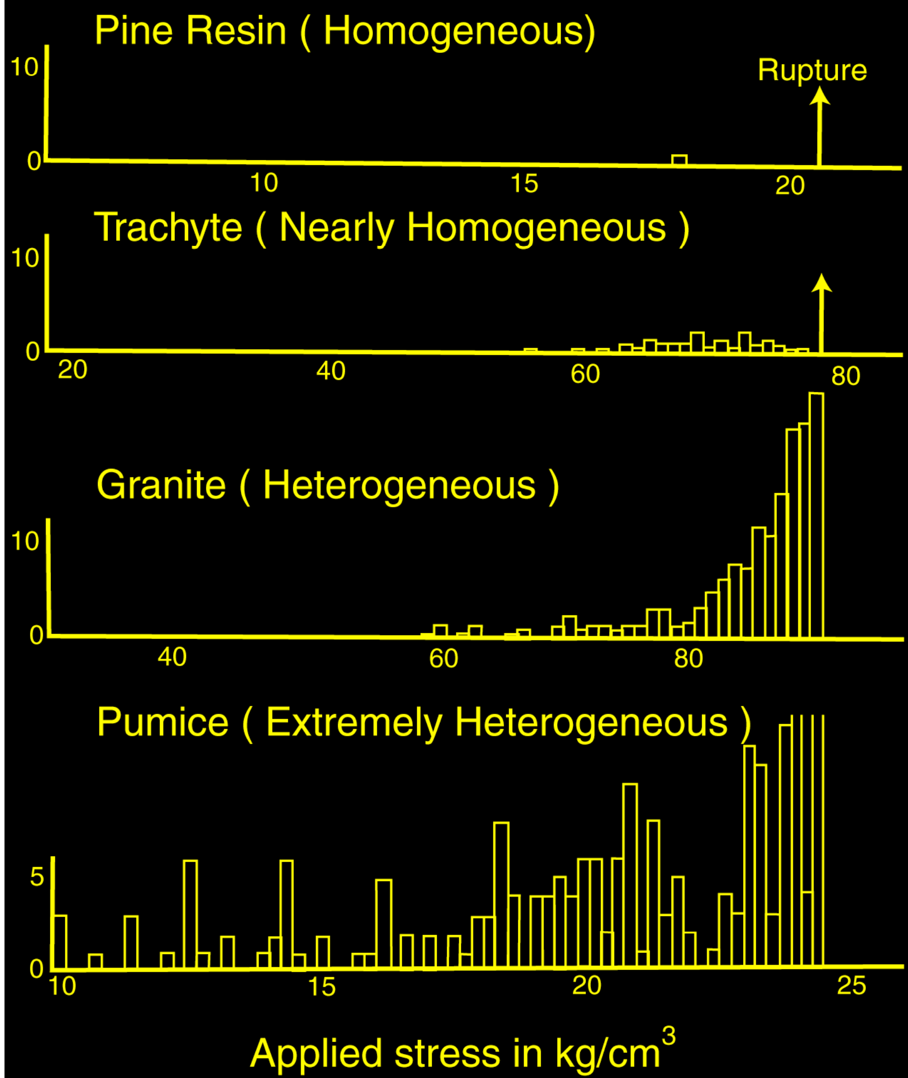
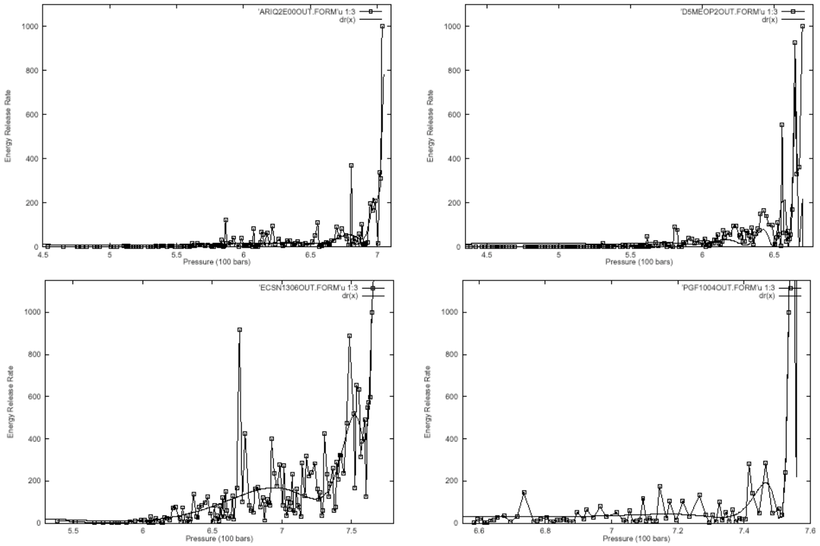
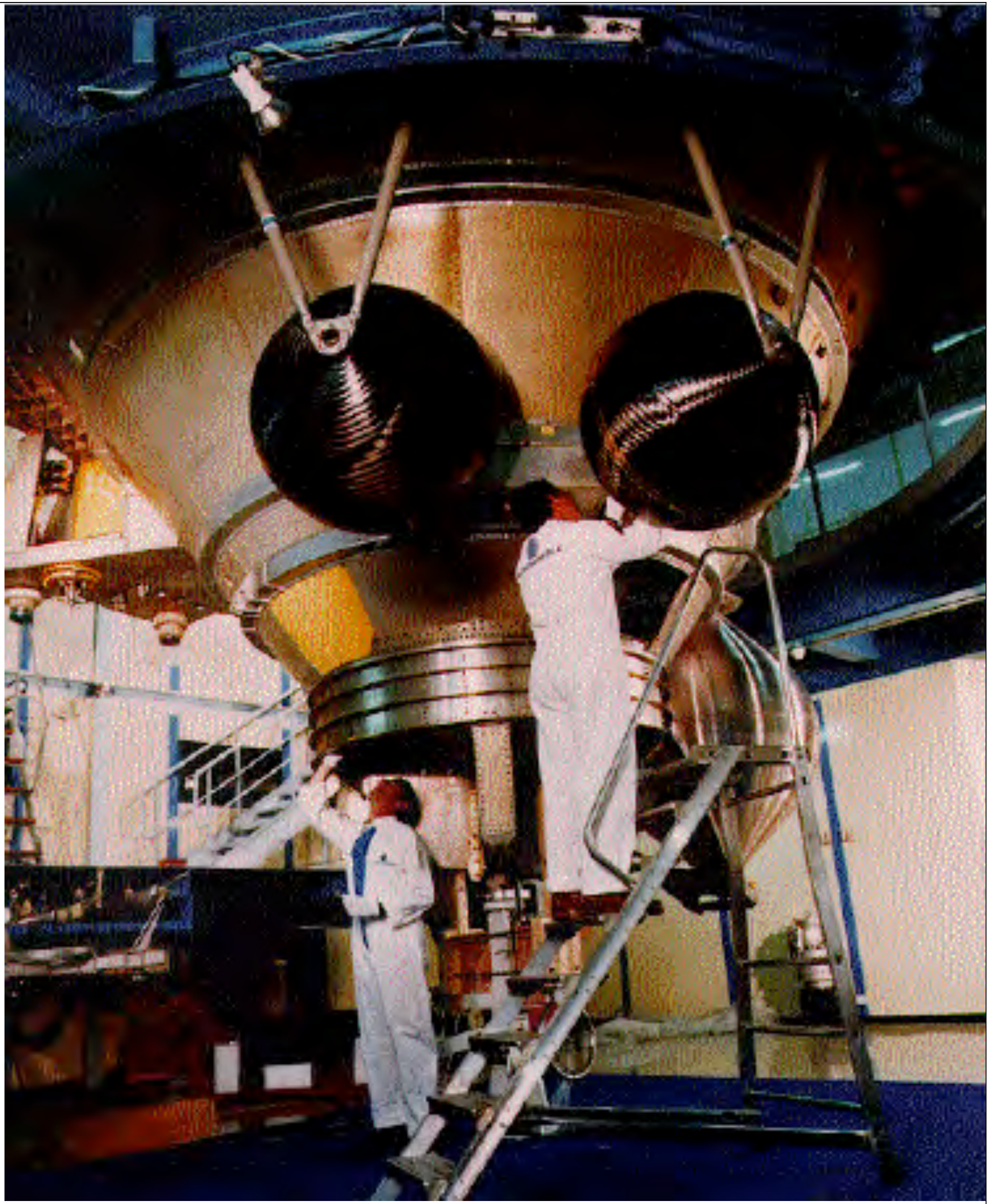


Fig. 4. Frequency of elastic shocks under increasing stresses in materials with different heterogeneity. From Mogi [1962]

Critical ruptures





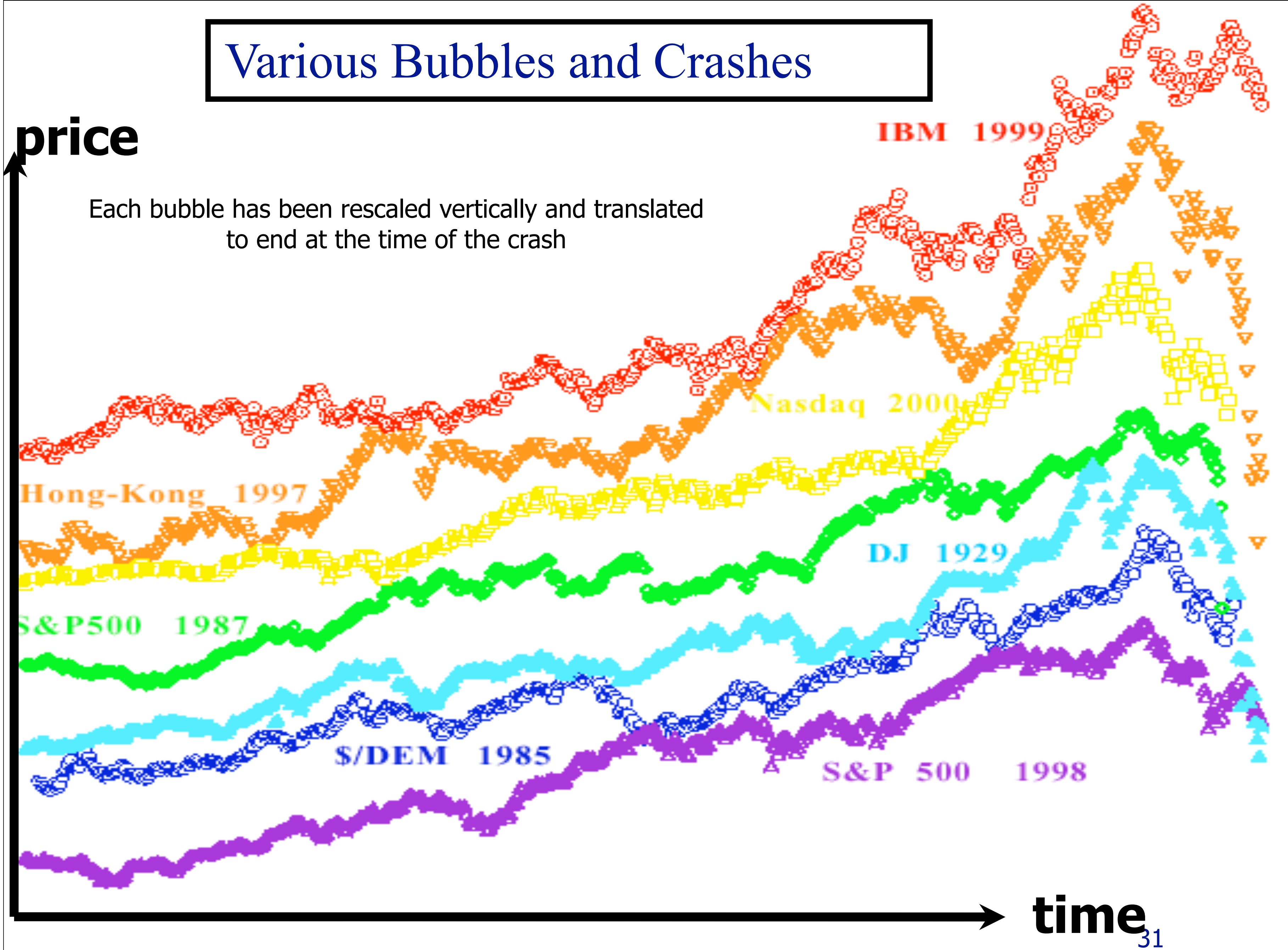
Our prediction system is now used in the industrial phase as the standard testing procedure.

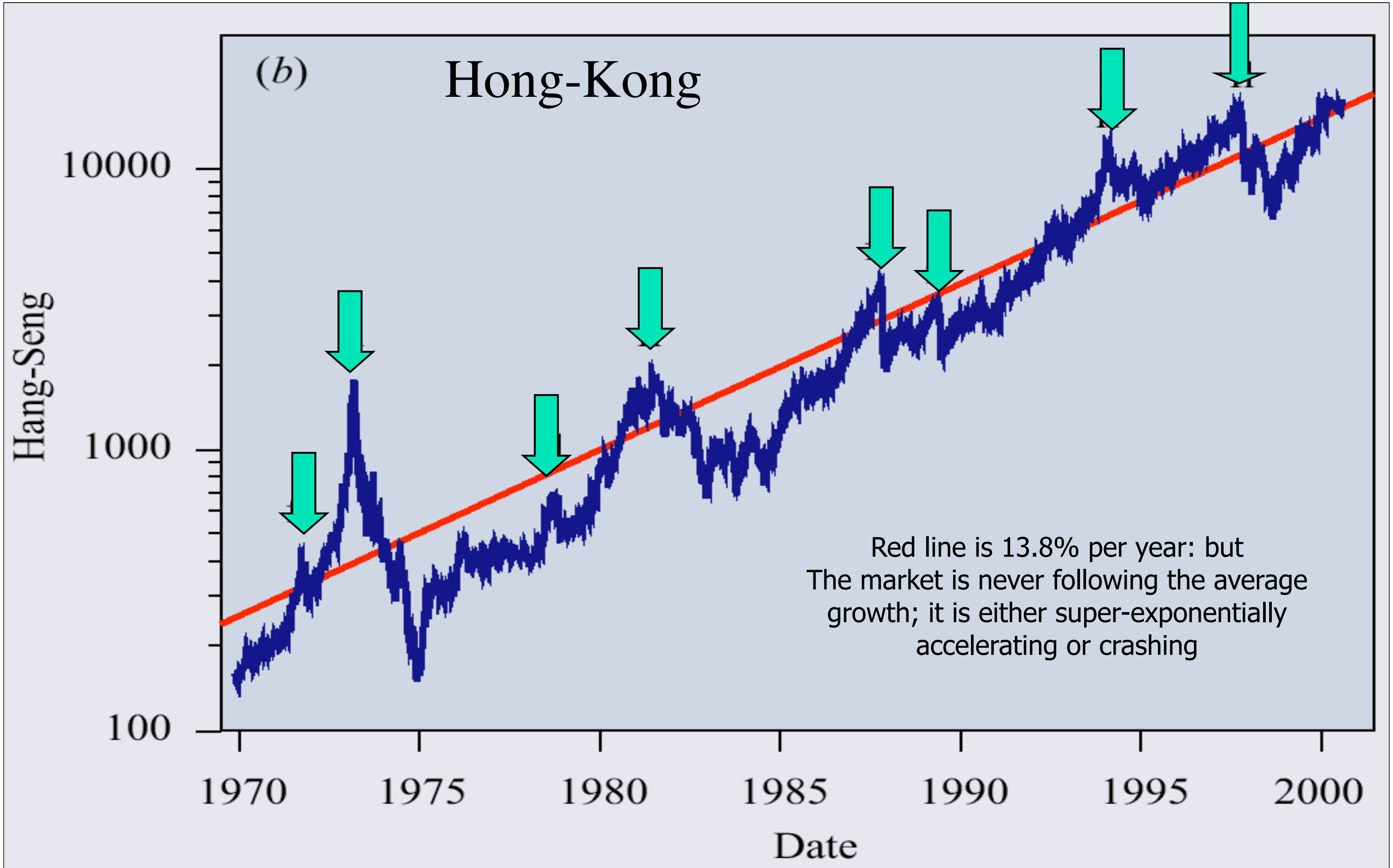
EADS LAUNCH VEHICLES

J.-C. Anifrani, C. Le Floc'h, D. Sornette and B. Souillard
"Universal Log-periodic correction to renormalization group scaling for rupture stress prediction from acoustic emissions", J.Phys.I France 5, n°6, 631-638 (1995)

Various Bubbles and Crashes

Each bubble has been rescaled vertically and translated to end at the time of the crash



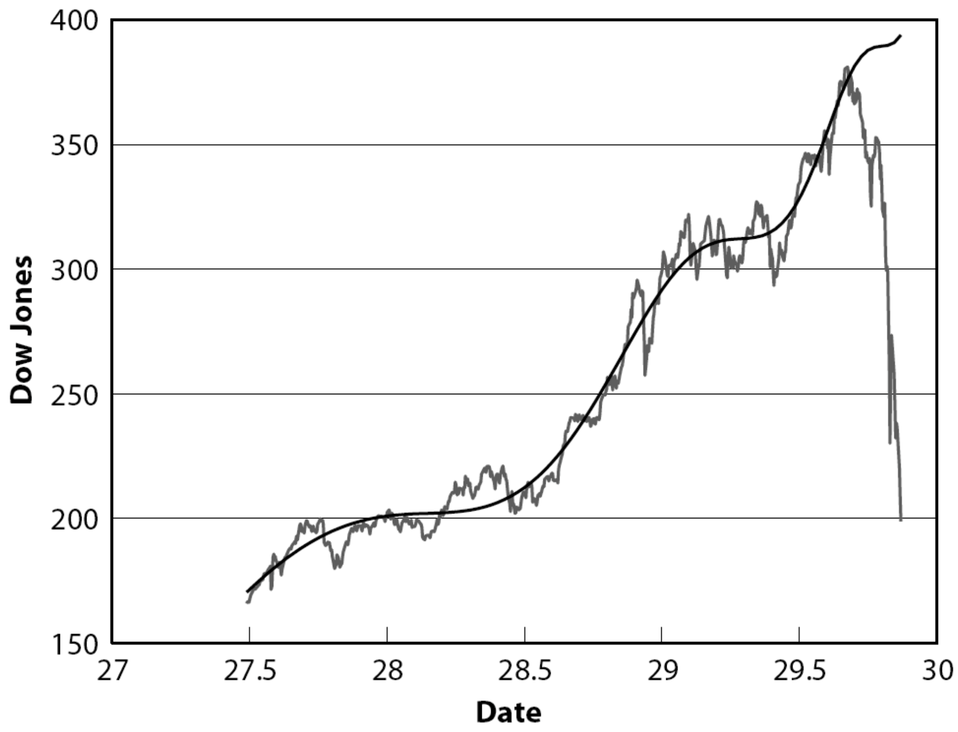


Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law

THE CRASH OF OCTOBER 1929

Stock market crashes are often unforeseen for most people, especially economists. “In a few months, I expect to see the stock market much higher than today.” Irving Fisher, famous economist and professor of economics at Yale University,14 days before Wall Street crashed on Black Tuesday, October 29, 1929.

“A severe depression such as 1920–21 is outside the range of probability. We are not facing a protracted liquidation.” This was the analysis offered days after the crash by the Harvard Economic Society to its subscribers… It closed its doors in 1932.

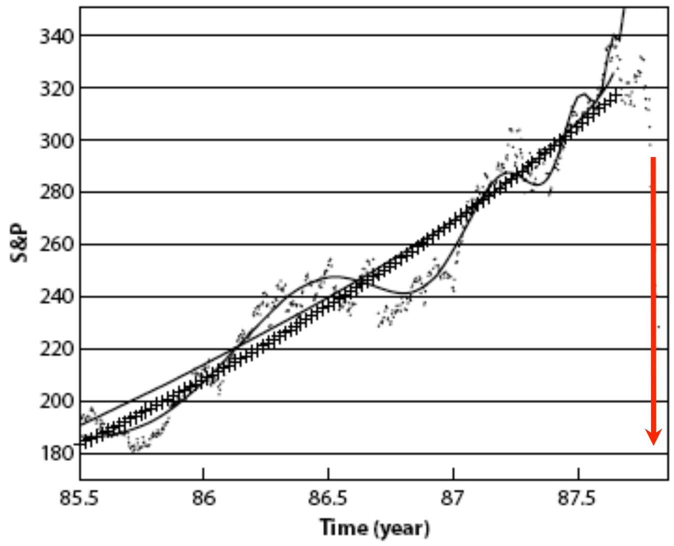


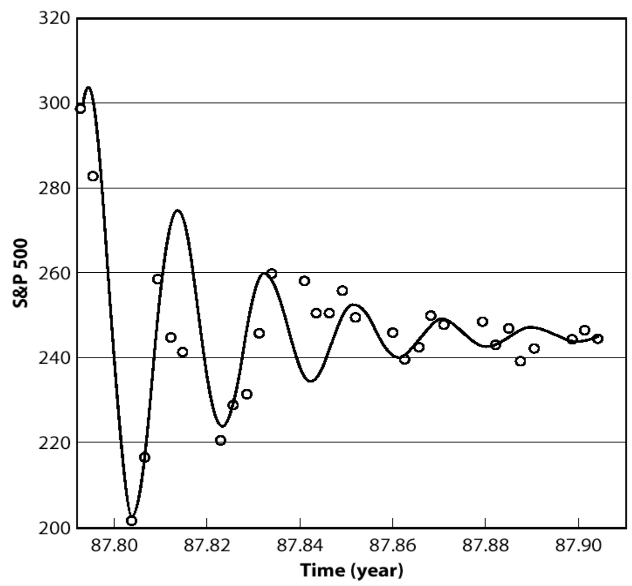
The DJIA prior to the October 1929 crash on Wall Street.

THE CRASH OF OCTOBER 1987

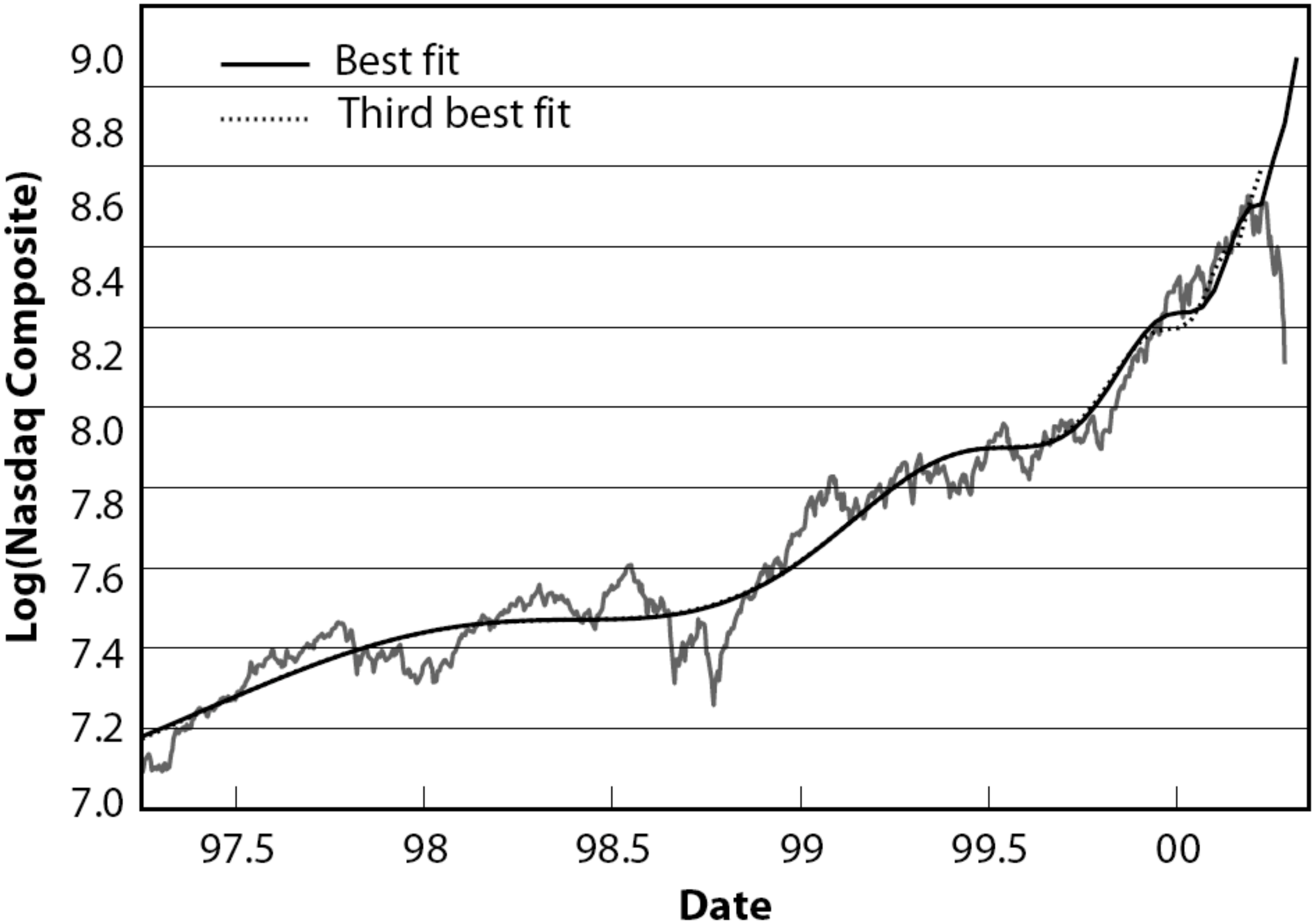
Proximate explanations after the fact!

- Computer trading
- Derivatives
- Illiquidity
- Trade and budget deficits
- Over-valuation
- The auction system
- Off-market and off-hours trading
- Floor brokers
- Forward market effect
- Different investor styles





THE NASDAQ CRASH OF APRIL 2000



Foreign capital inflow

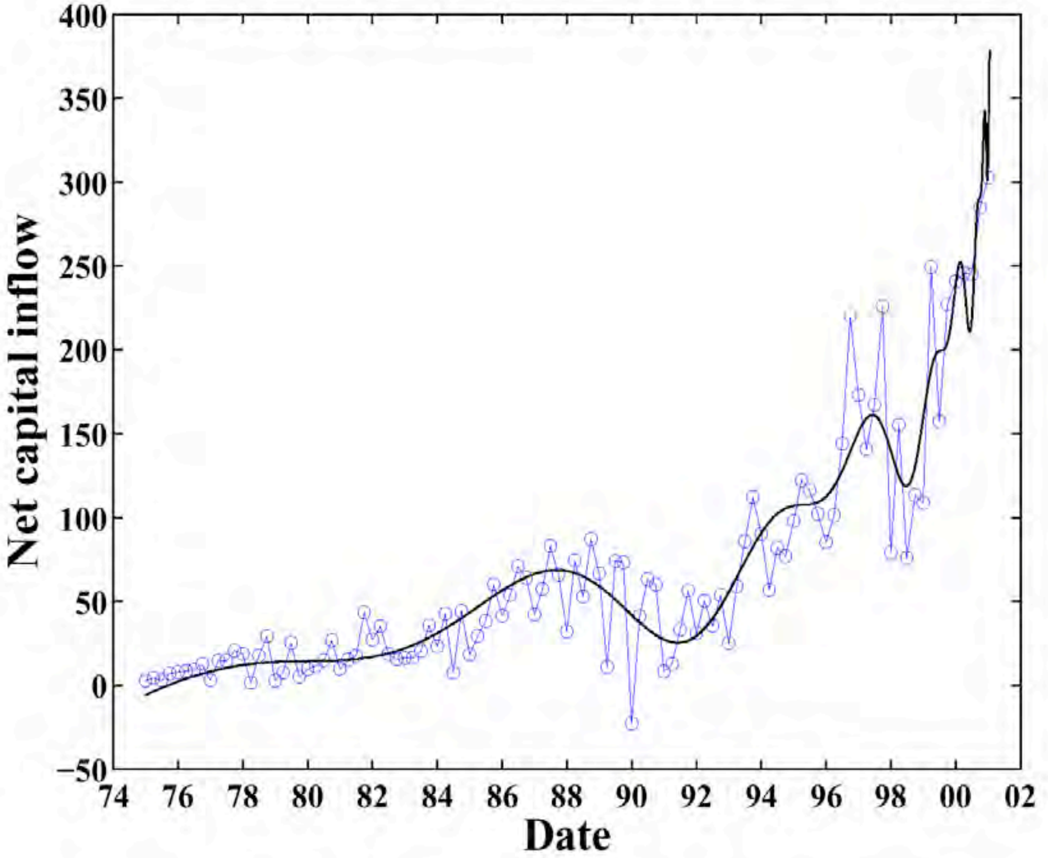
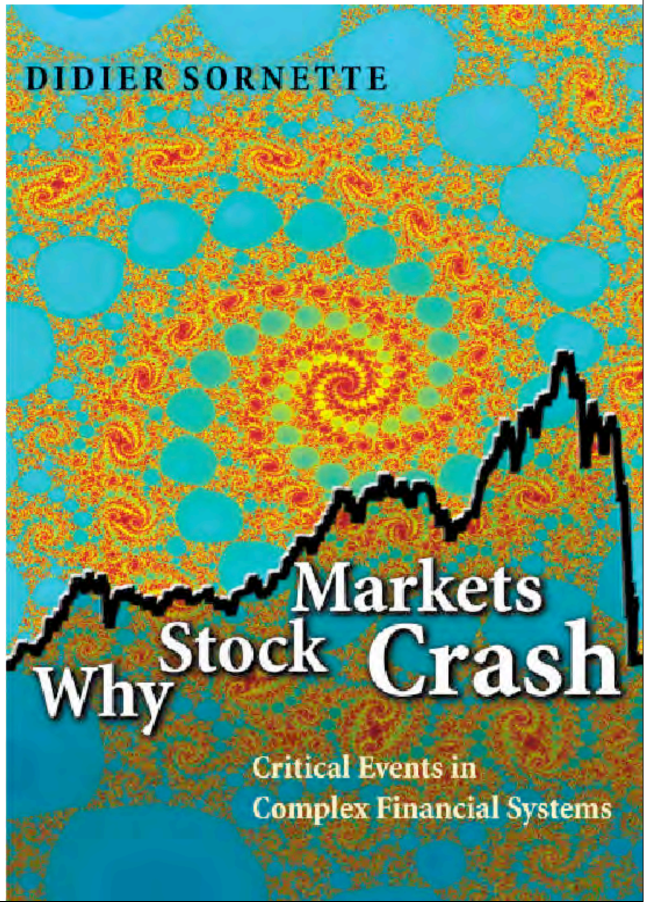


Fig. 2. Fit of the time evolution of the foreign net capital inflow $I(t)$ in the USA from 1975 till the first quarter of 2001 when it reached its maximum, by a second-order Weierstrass-type function given by expression (1). The predicted critical time is $t_c = 2001/03/12$, the power-law exponent is $m = 0.01$, and the angular log-frequency is $\omega = 4.9$. The fitted linear parameters are $A = 7355$, $B = -6719$, $C_1 = 21.5$ and $C_2 = 16.2$. The r.m.s. of the residuals of the fit is 22.810.

Many other bubbles and crashes

- Hong-Kong crashes: 1987, 1994, 1997 and many others
- October 1997 mini-crash
- August 1998
- Slow crash of spring 1962
- Latin-american crashes
- Asian market crashes
- Russian crashes
- Individual companies



What is the cause of the crash?

- ✓ Proximate causes: many possibilities

- ✓ Fundamental cause: maturation towards an instability

An instability is characterized by

- large or diverging susceptibility to external perturbations or influences
- exponential growth of random perturbations leading to a change of regime, or selection of a new attractor of the dynamics.

Thomas Robert Malthus (1766–1834)

1798

autocatalitic proliferation: $\frac{dx}{dt} = a \cdot x$

with *a* =birth rate - death rate

exponential solution: $X(t) = X(0)e^{a\,t}$

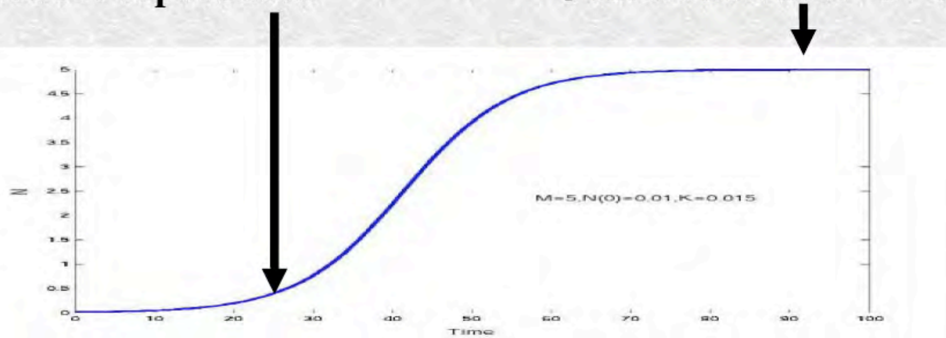
contemporary estimations= doubling of the population every 30yrs

Pierre François Verhulst (1804-1849)

way out exponential explosion:

$dX/dt = a\,X - c\,X^2$ **1838**

Solution: exponential ==========→saturation at X= a / c



For humans data at the time could not discriminate between:

1. exponential growth of Malthus
2. logistic growth of Verhulst

But data fit on animal population: sheep in Tasmania

- exponential in the first 20 years after their introduction and completely saturated after about half a century. ==> Verhulst

Positive feedbacks and finite-time singularity

Conjecture: Many systems exhibit transient FTS as "ghost-like" solutions that the system follows for a while before being attenuated.
Analogous to exponential sensitivity to initial condition with reinjection → chaos **but** here FTS blow-up.

$$\frac{dp}{dt} = rp(t)[K - p(t)],$$

$$\frac{dp}{dt} = r[p(t)]^{1+\delta},$$

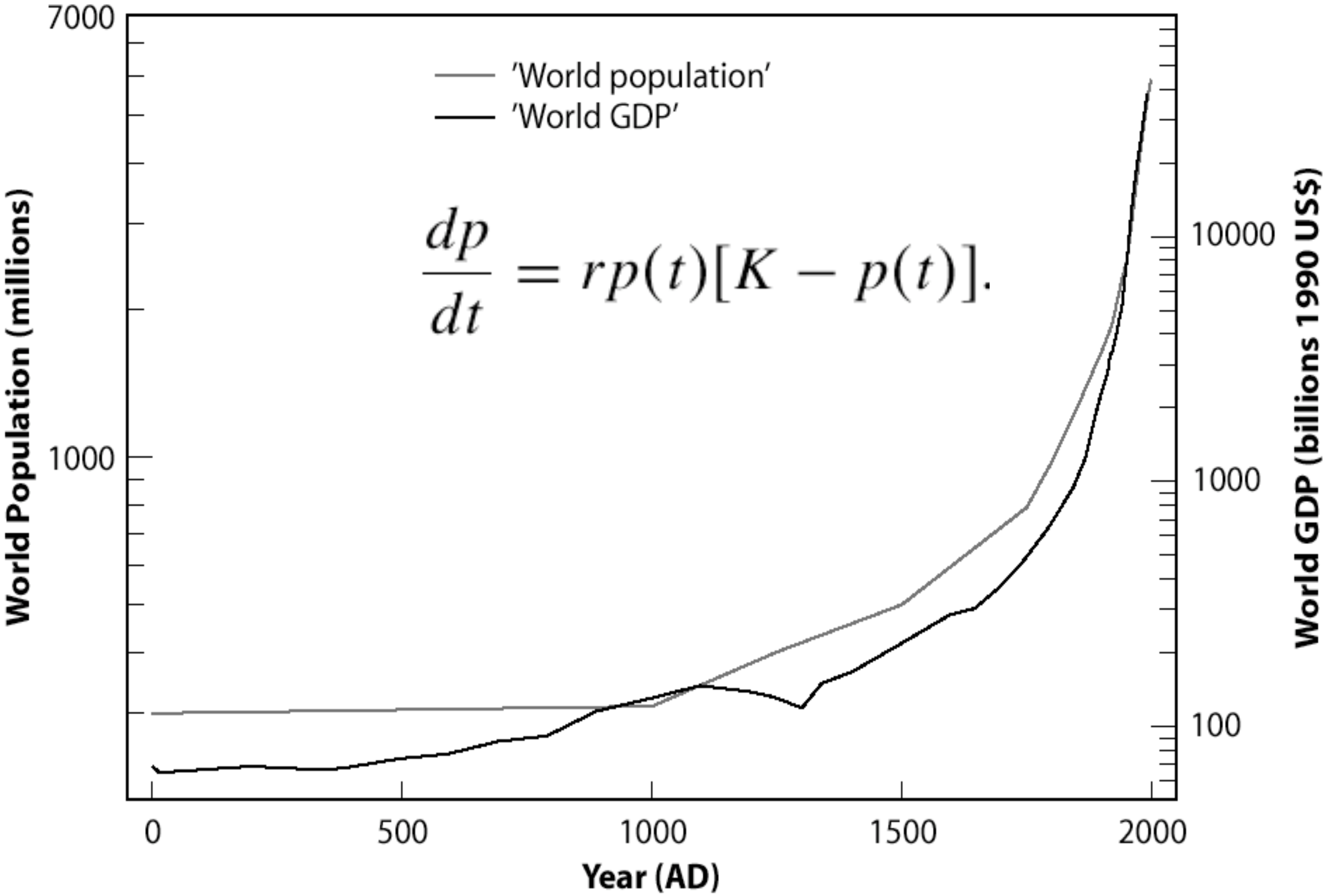
with $K \propto p^{\delta}$

$$p(t) \propto (t_c - t)^z, \text{ with } z = -\frac{1}{\delta} \text{ and } t \text{ close to } t_c.$$

Multi-dimensional generalization: multi-variate positive feedbacks

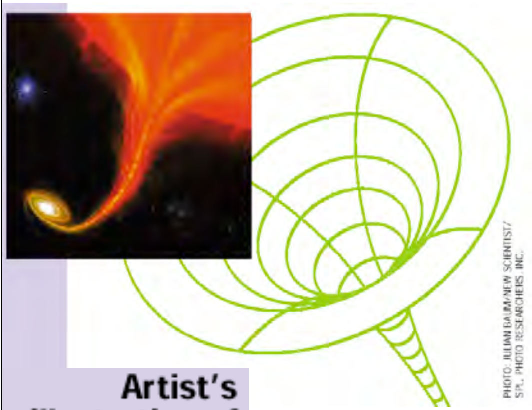
Super-exponential growth of world population

by positive feedbacks



Finite-time Singularity

- Planet formation in solar system by run-away accretion of planetesimals
- PDE's: Euler equations of inviscid fluids and relationship with turbulence
- PDE's of General Relativity coupled to a mass field leading to the formation of black holes
- Zakharov-equation of beam-driven Langmuir turbulence in plasma
- rupture and material failure
- Earthquakes (ex: slip-velocity Ruina-Dieterich friction law and accelerating creep)
- Models of micro-organisms chemotaxis, aggregating to form fruiting bodies
- Surface instability spikes (Mullins-Sekerka), jets from a singular surface, fluid drop snap-off
- Euler's disk (rotating coin)
- Stock market crashes...



Artist's illustration of matter from a red giant star being pulled toward a black hole.

Mechanisms for positive feedbacks in the stock market

- Technical and rational mechanisms
  1. Option hedging
  2. Insurance portfolio strategies
  3. Trend following investment strategies
  4. Asymmetric information on hedging strategies

- Behavioral mechanisms:
  1. Breakdown of "psychological Galilean invariance"
  2. Imitation(many persons)
     a) It is rational to imitate
     b) It is the highest cognitive task to imitate
     c) We mostly learn by imitation
     d) The concept of "CONVENTION" (Orléan)

Kal


JUST A NORMAL DAY AT THE NATION'S MOST IMPORTANT FINANCIAL INSTITUTION...
I'VE GOT A STOCK HERE THAT COULD REALLY EXCEL
REALLY EXCEL?
EXCEL?
SELL?
SELL?
SELL
SE
SELL!
SELL!
KAL
SELL!
SELL!
ELL!
SELL!
THIS IS MADNESS! I CAN'T TAKE ANYMORE
GOOD BYE!
GOOD BYE?
BYE?
Buy?
BUY!
Buy!
BUY!!
Buy!
Buy
BUY!
I'VE GOT A STOCK HERE..
CARTOONISTS & WRITERS SYNDICATE http://CartoonWeb.com

Thy Neighbor's Portfolio: Word-of-Mouth Effects in the Holdings and Trades of Money Managers

THE JOURNAL OF FINANCE • VOL. LX, NO. 6 • DECEMBER 2005

HARRISON HONG, JEFFREY D. KUBIK, and JEREMY C. STEIN*

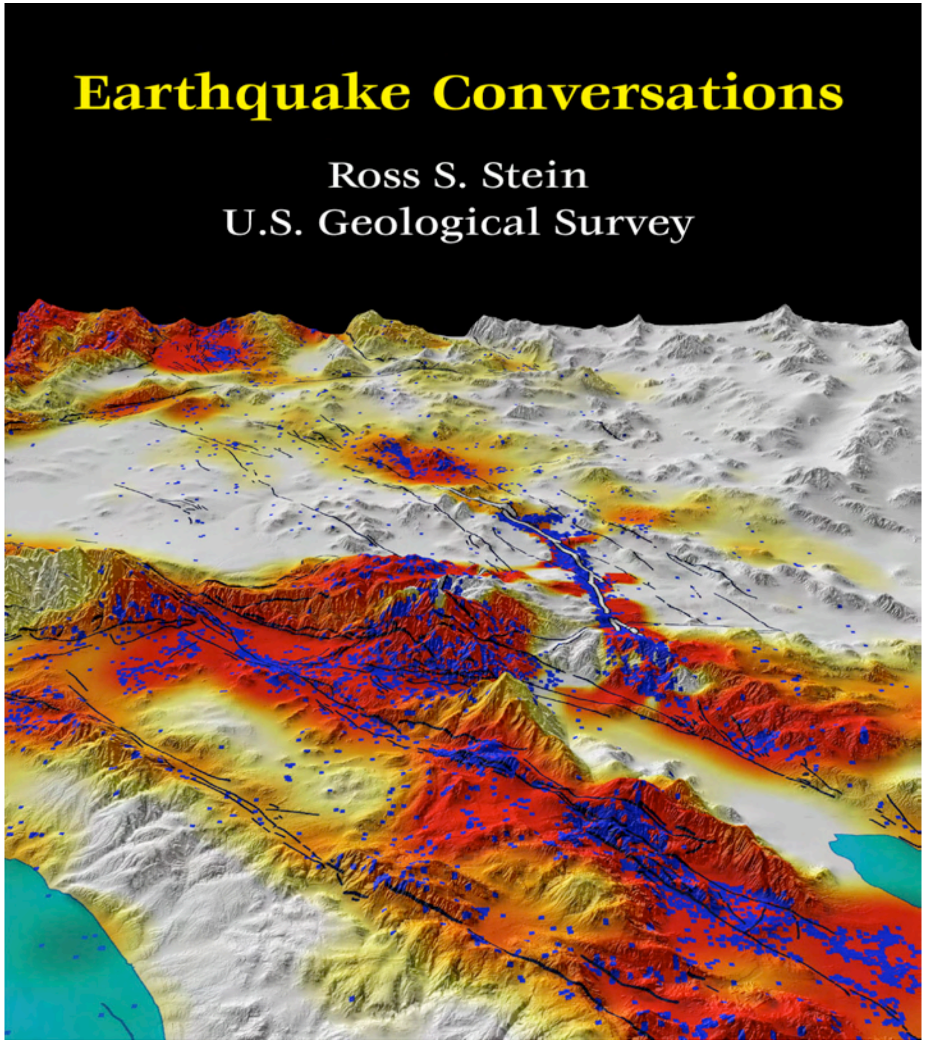
A mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. This pattern shows up even when the fund manager and the stock in question are located far apart, so it is distinct from anything having to do with local preference. The evidence can be interpreted in terms of an epidemic model in which investors spread information about stocks to one another by word of mouth.

Humans Appear Hardwired To Learn By 'Over-Imitation'

ScienceDaily (Dec. 6, 2007) — Children learn by imitating adults--so much so that they will rethink how an object works if they observe an adult taking unnecessary steps when using that object, according to a new Yale study.

A fundamental observation about human society is that people who communicate regularly with one another think similarly. There is at any place and in any time a *Zeitgeist*, a spirit of the times. . . . Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations. (pp. 148, 155)

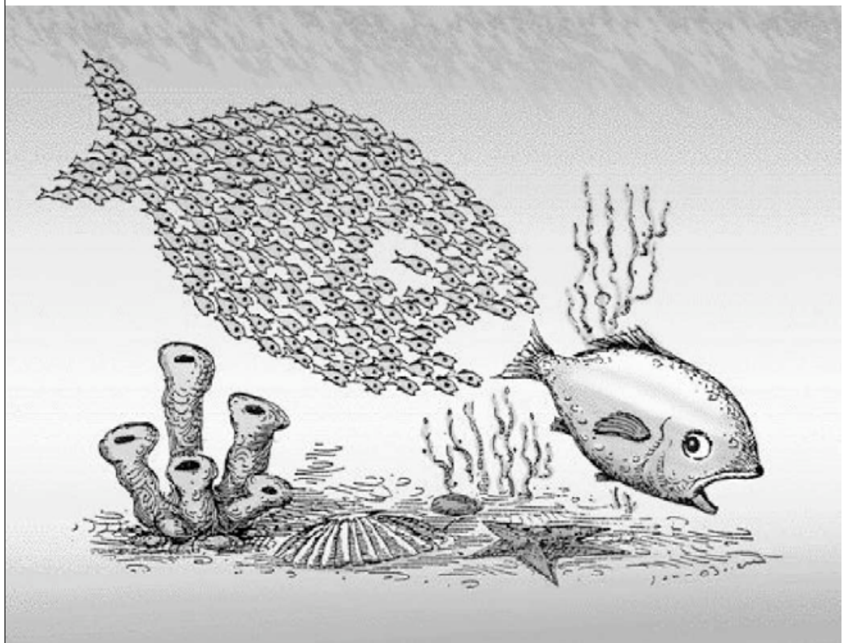
Shiller (2000)

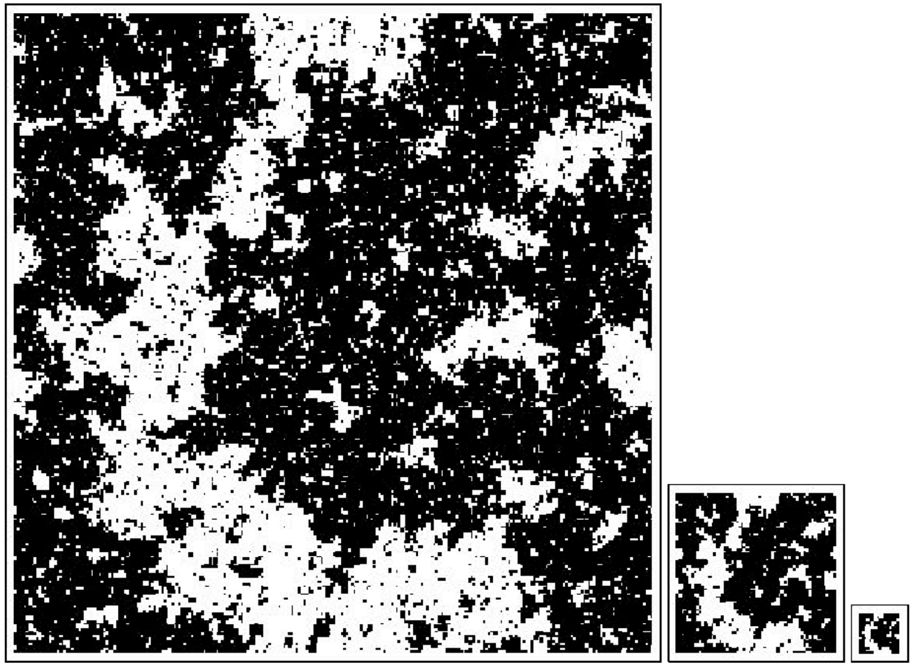


Epidemic processes by word-of-mouth, sentiment, convention...

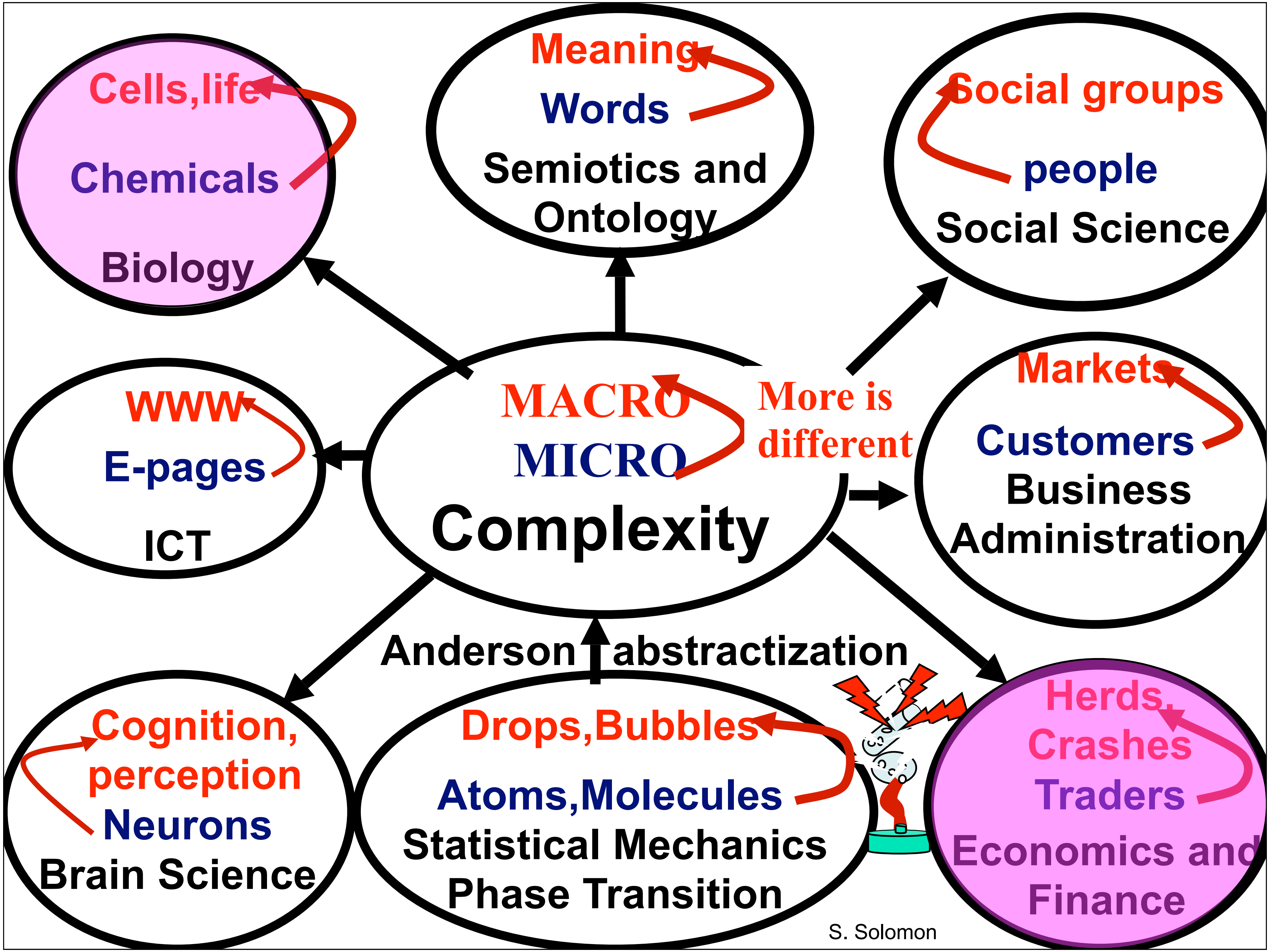
Emergence: bottom-up vs top-down

- ✓A system can display properties not present in its components.
- ✓Emergent behaviors are not obvious from components alone.
- ✓No contradiction with mechanism, rather, emergent properties of mechanistic parts are far richer than previously imagined.
- ✓Top-down = "direct cascade"; bottom-up = "inverse cascade"
- ✓Understanding emergence is the central topic of complex systems.





Agent-based models

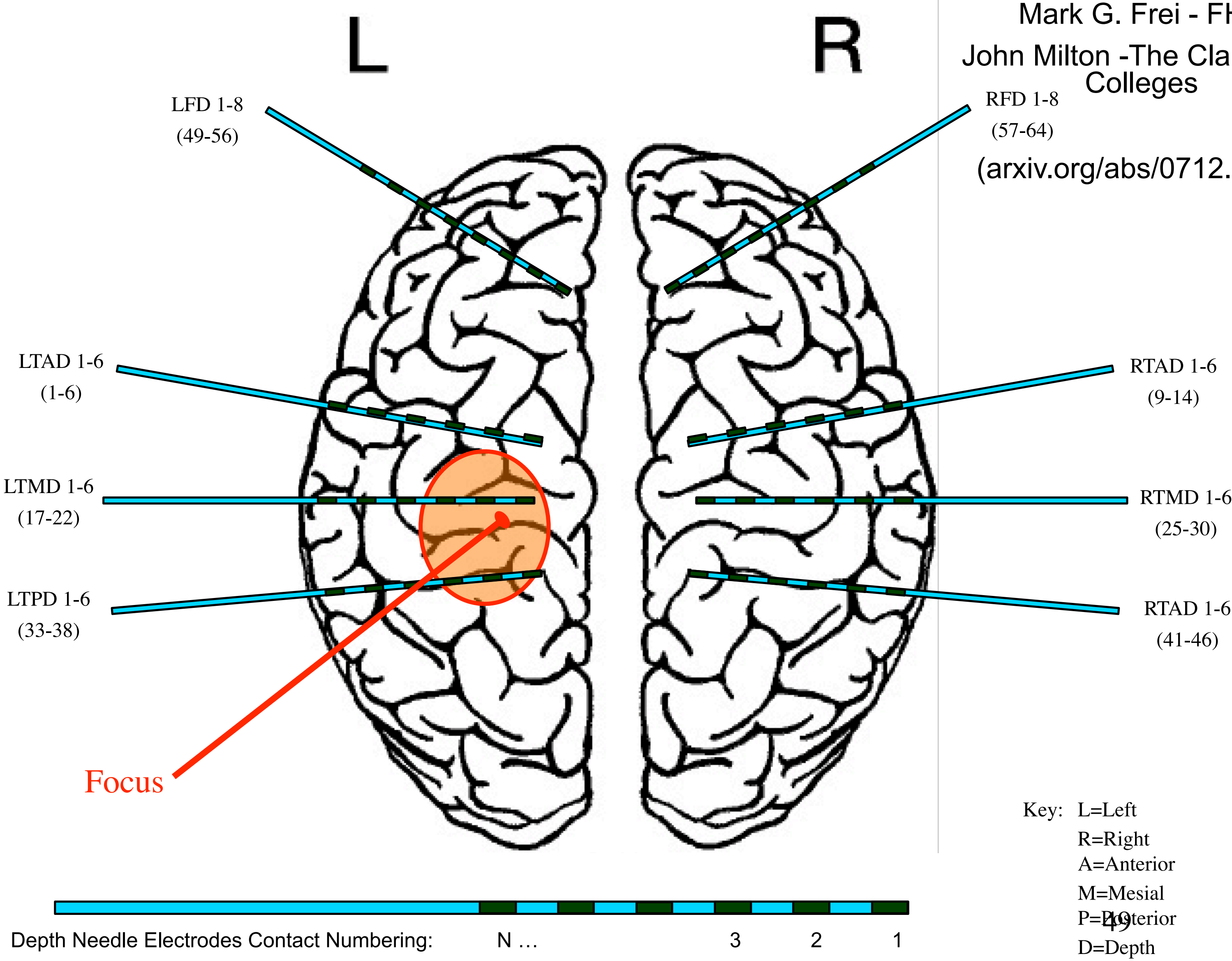


Cells,life
Chemicals
Biology
Meaning
Words
Semiotics and
Ontology
Social groups
people
Social Science
WWW
E-pages
ICT
MACRO
MICRO
Complexity
More is
different
Markets
Customers
Business
Administration
Anderson
abstractization
Cognition,
perception
Neurons
Brain Science
Drops,Bubbles
Atoms,Molecules
Statistical Mechanics
Phase Transition
Herds,
Crashes
Traders
Economics and
Finance
S. Solomon

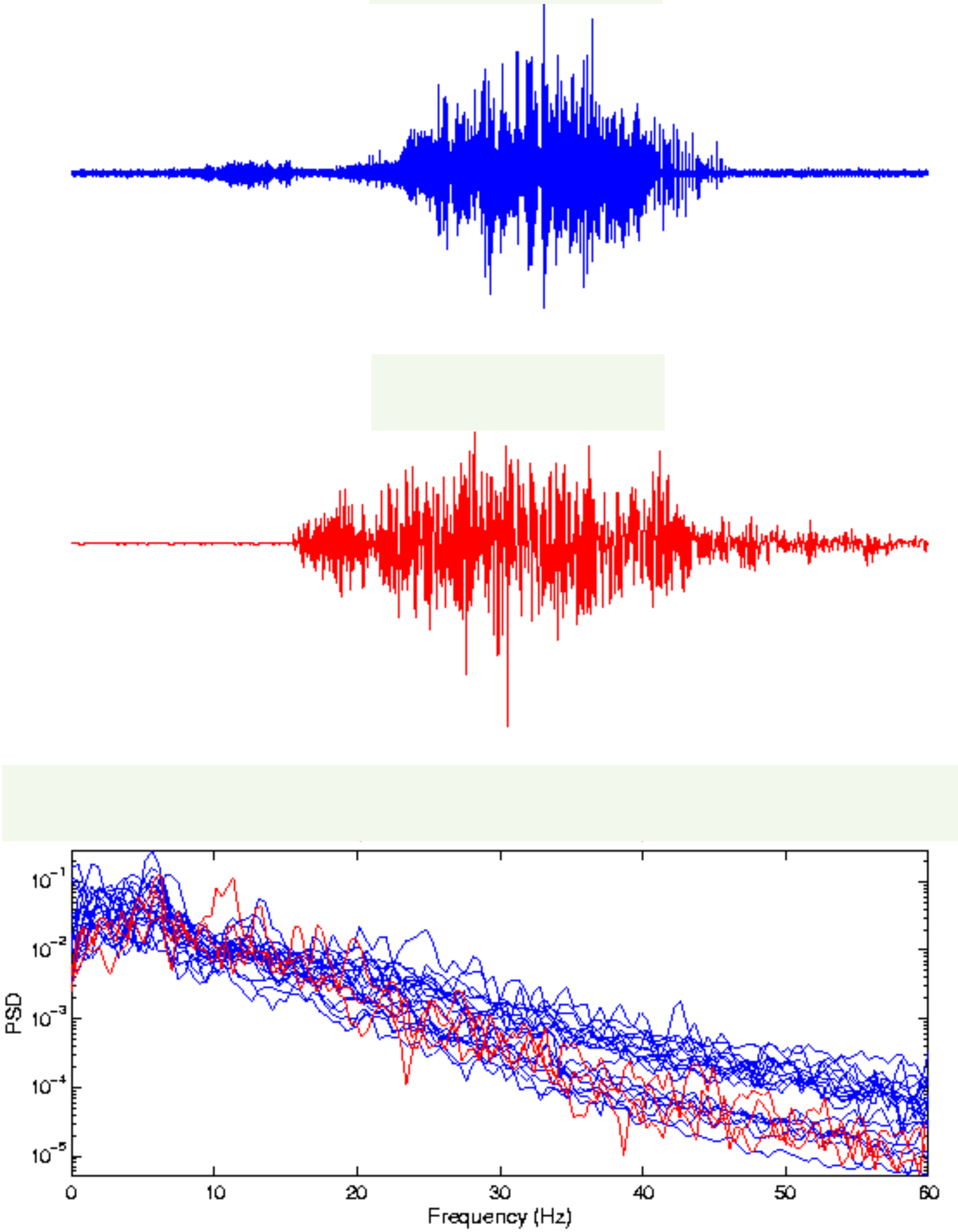
Epileptic Seizures – Quakes of the Brain?

with Ivan Osorio – KUMC & FHS
Mark G. Frei - FHS
John Milton -The Claremont Colleges

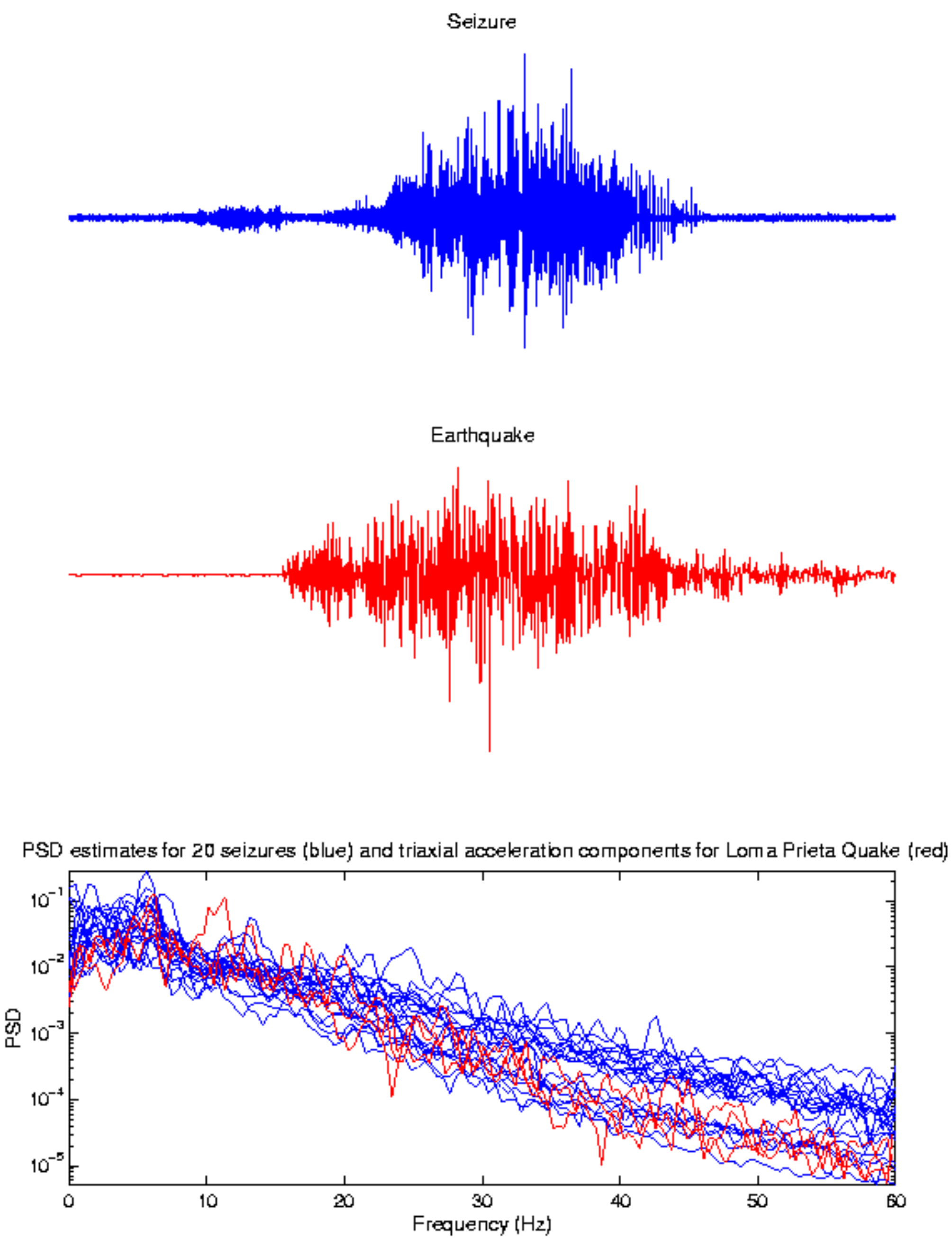
(arxiv.org/abs/0712.3929)



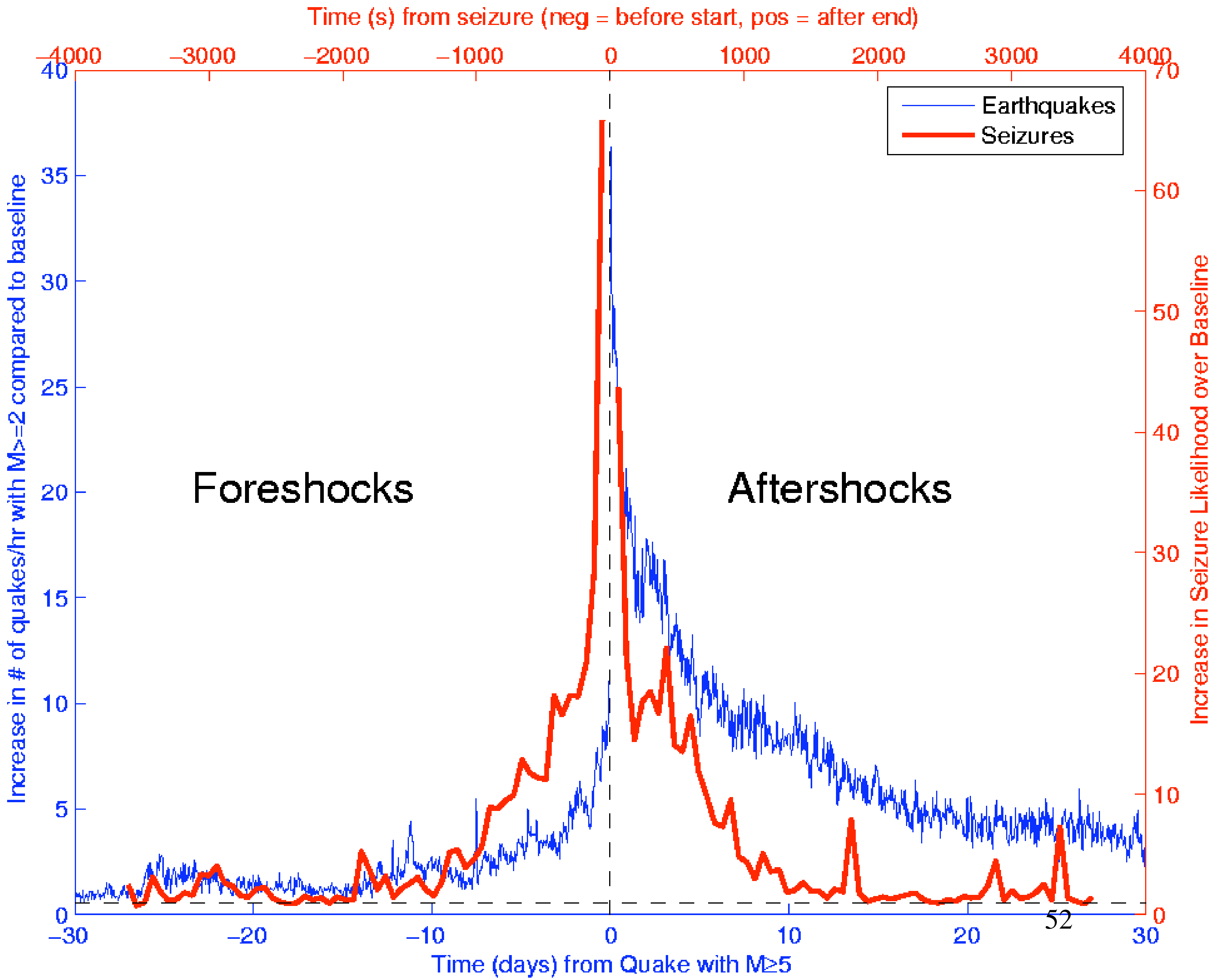
Key: L=Left
R=Right
A=Anterior
M=Mesial
P=Posterior
D=Depth
T=Temporal

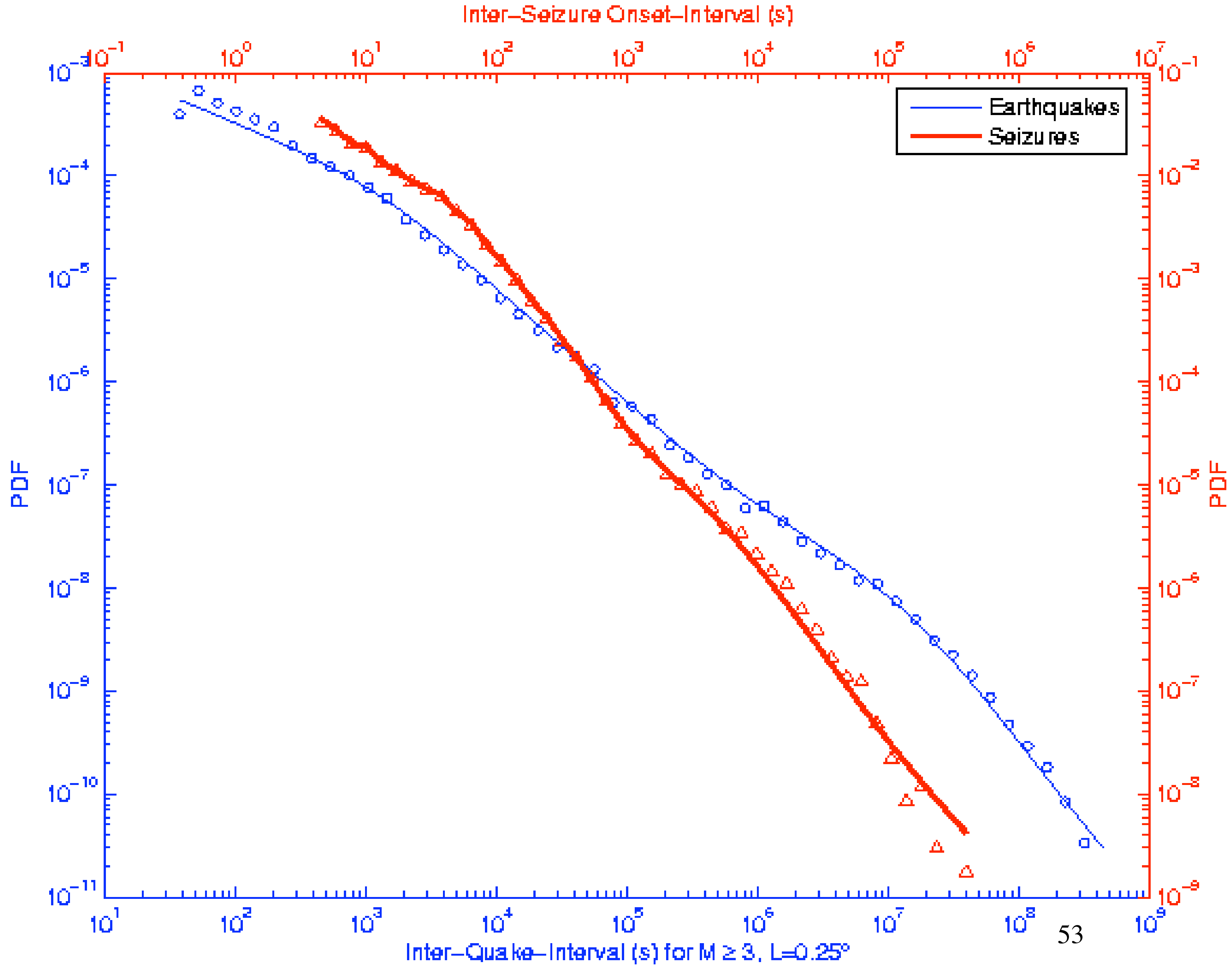


PSD
10^-1
10^-2
10^-3
10^-4
10^-5
0
10
20
30
40
50
60
Frequency (Hz)

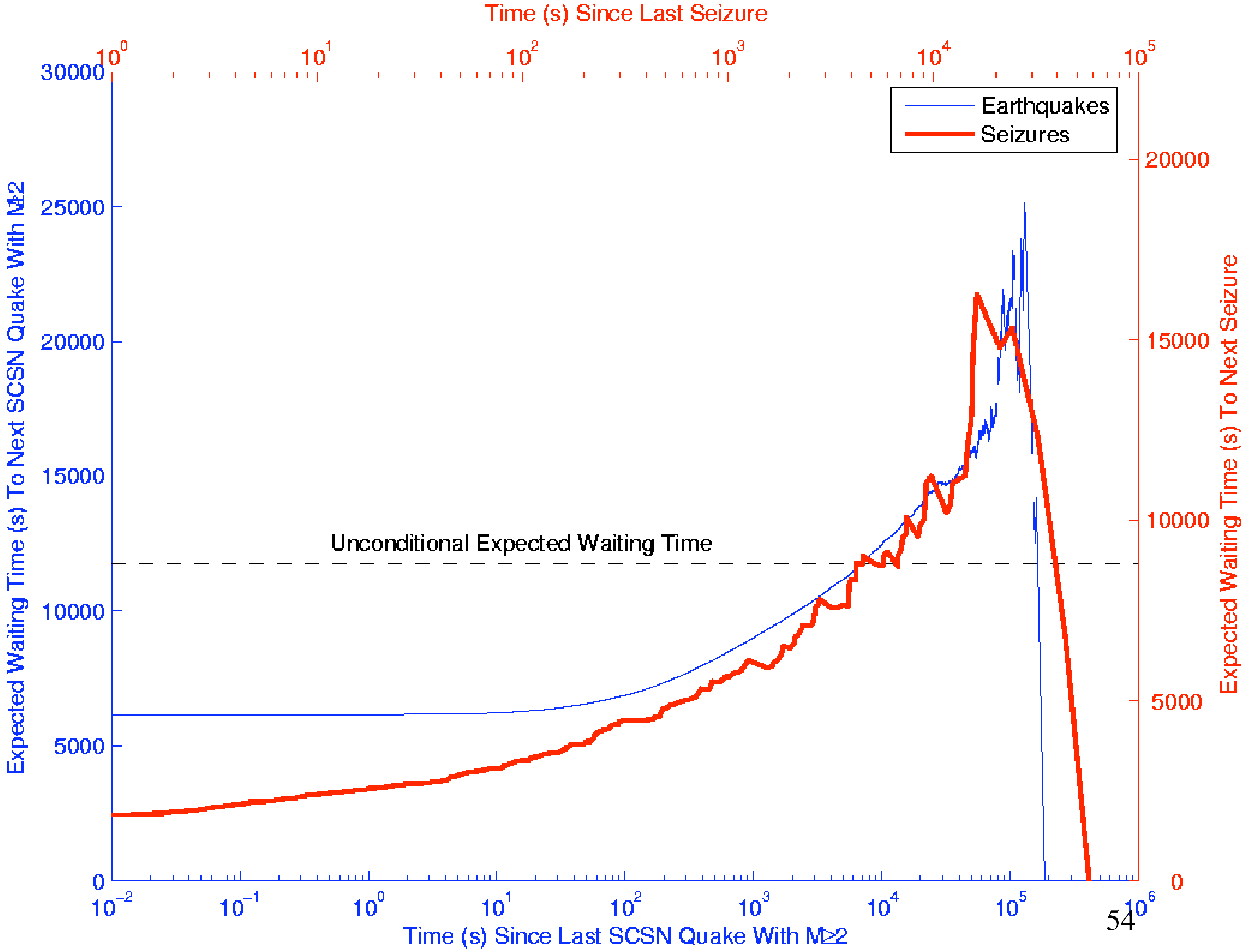

Seizure
Earthquake
PSD estimates for 20 seizures (blue) and triaxial acceleration components for Loma Prieta Quake (red)
PSD
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10
20
30
40
50
60
Frequency (Hz)

Omori law: Direct and Inverse

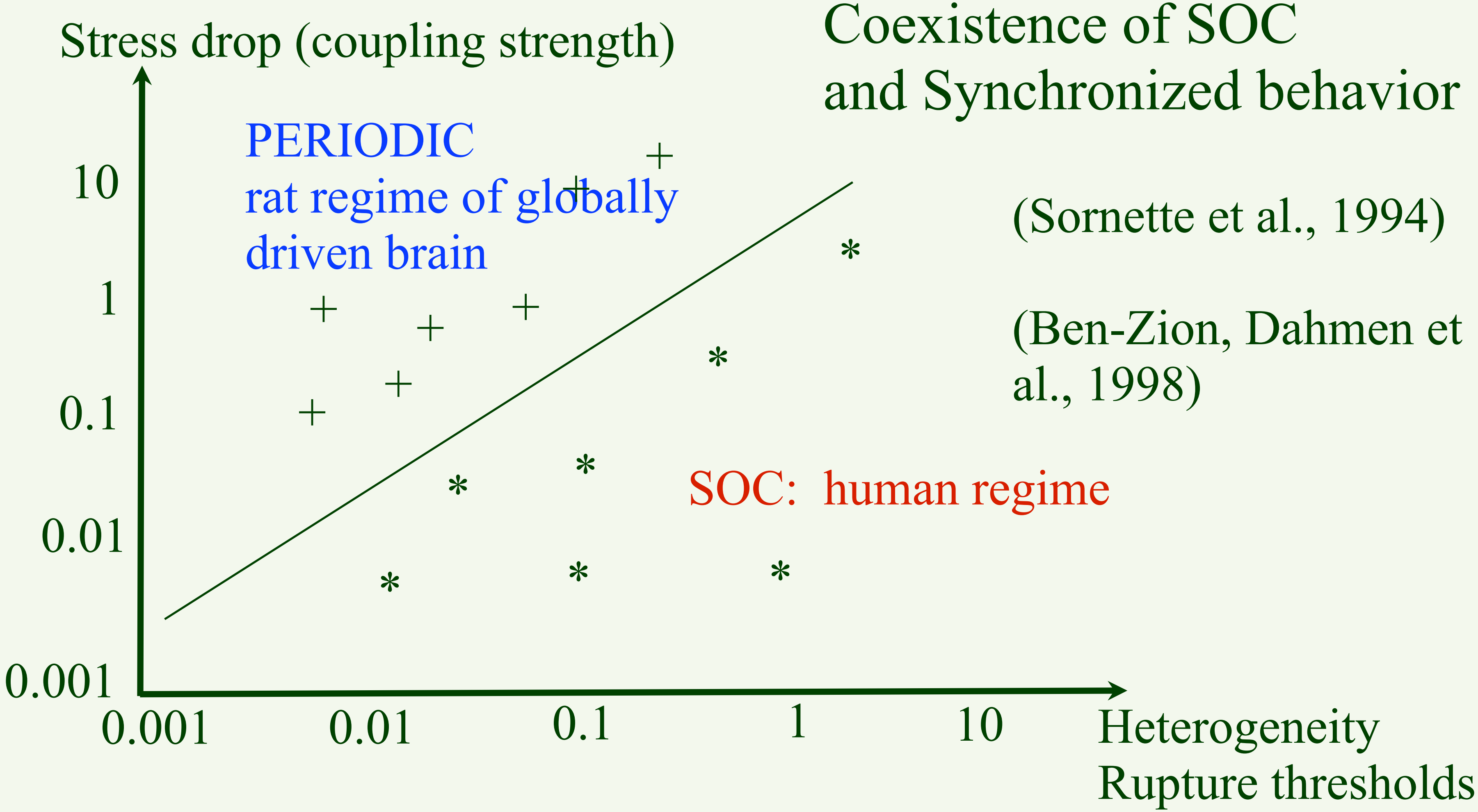



Inter–Seizure Onset–Interval (s)
Earthquakes
Seizures
PDF
PDF
Inter–Quake–Interval (s) for M ≥ 3, L=0.25°

The longer it has been since the last event, the longer it will be since the next one! (Sornette&Knopoff, 1997)



Coexistence of SOC and Synchronized behavior

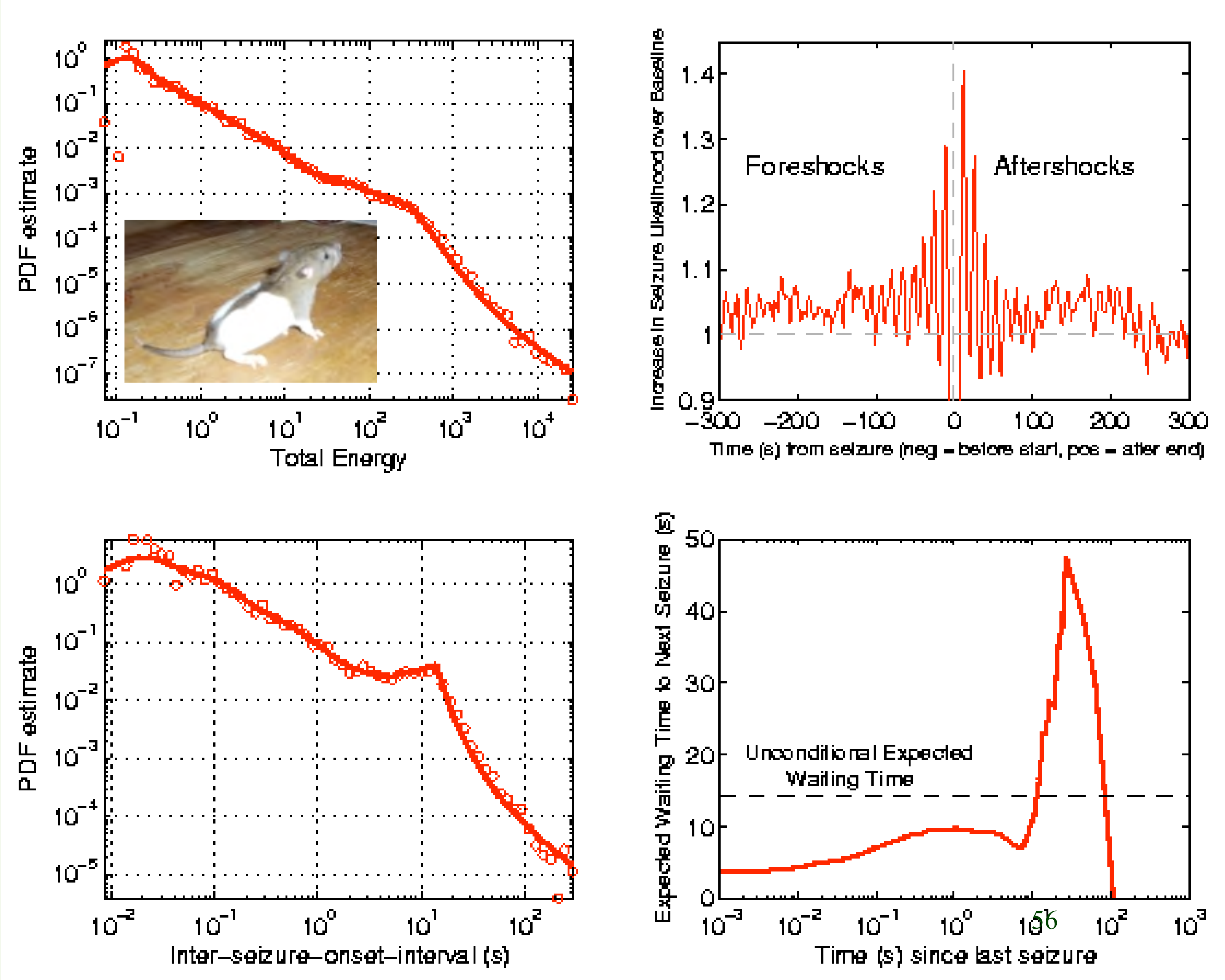


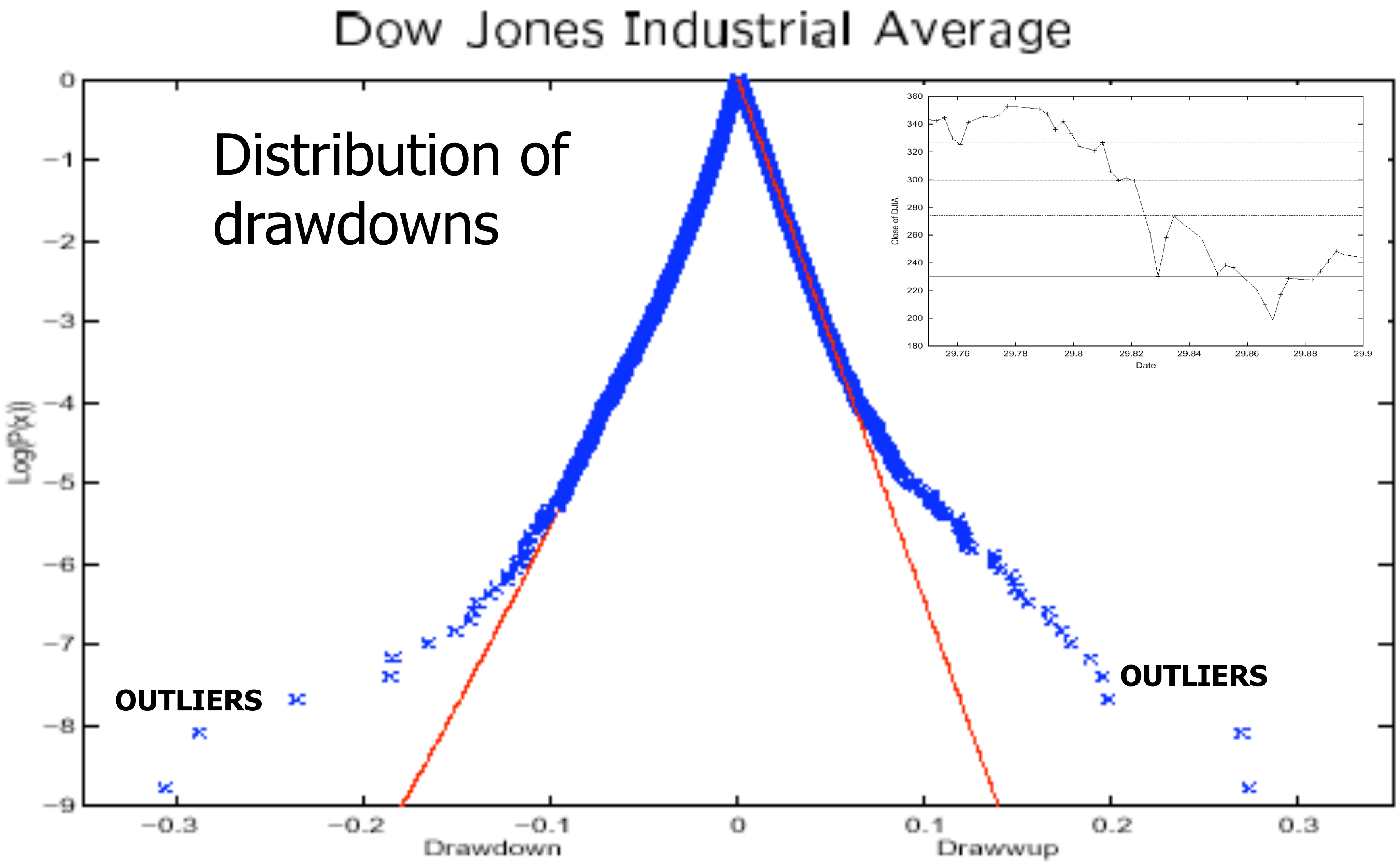
(Sornette et al., 1994)

(Ben-Zion, Dahmen et al., 1998)

“Phase diagram” for the model in the space (heterogeneity, stress drop). Crosses (+) correspond to systems which exhibit a periodic time evolution. Stars * corresponds to systems that are self-organized critical, with a Gutenberg-Richter earthquake size distribution and fault localization whose geometry is well-described by the geometry of random directed polymers.

19 rats treated intravenously (2) with the convulsant 3-mercapto-proprionic acid (3-MPA)



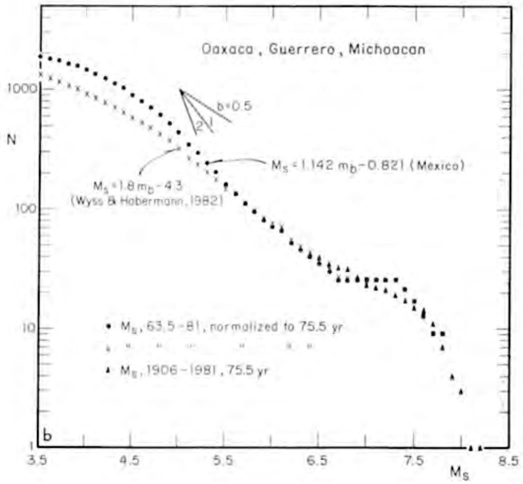


A. Johansen and D. Sornette, Stock market crashes are outliers,
European Physical Journal B 1, 141-143 (1998)

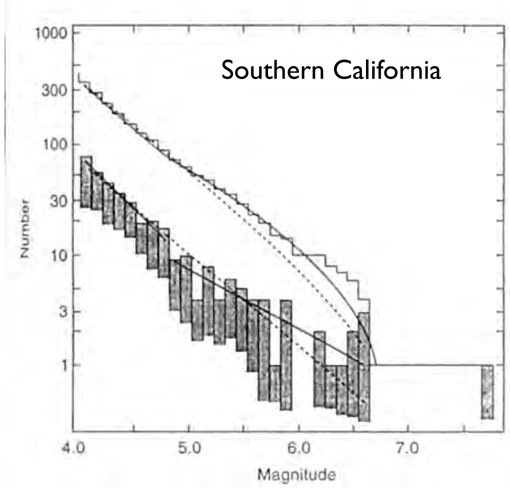
A. Johansen and D. Sornette, Large Stock Market Price Drawdowns Are Outliers,
Journal of Risk 4(2), 69-110, Winter 2001/02

Complex magnitude distributions

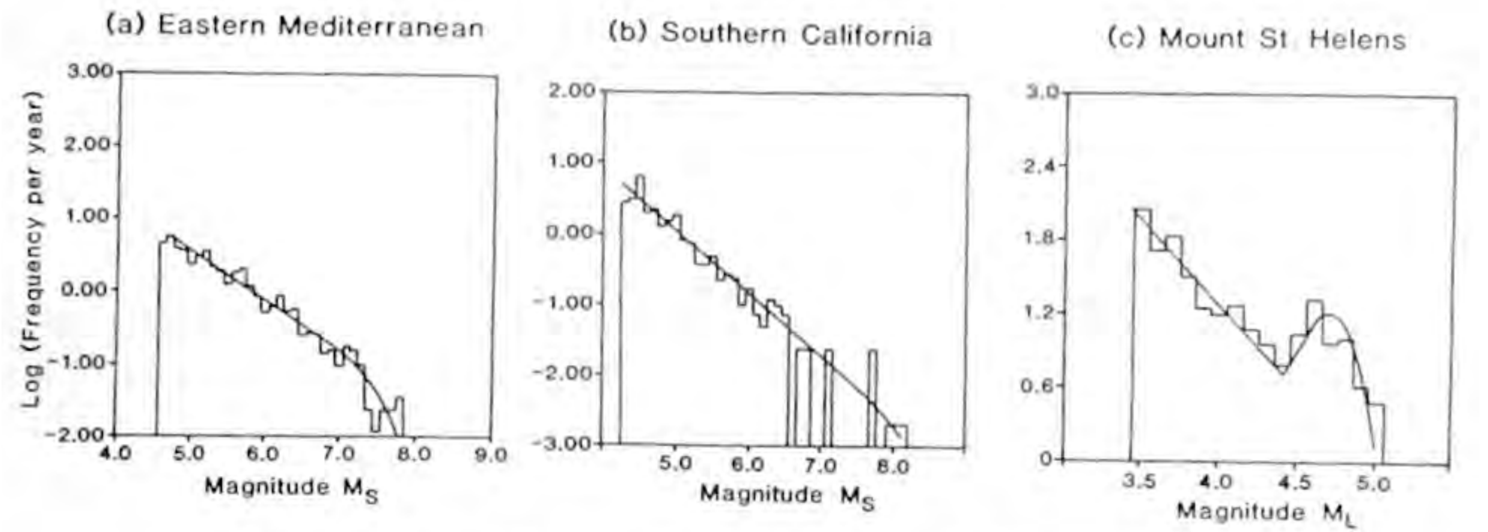
Characteristic earthquakes?



Singh, et. al., 1983, BSSA 73, 1779-1796



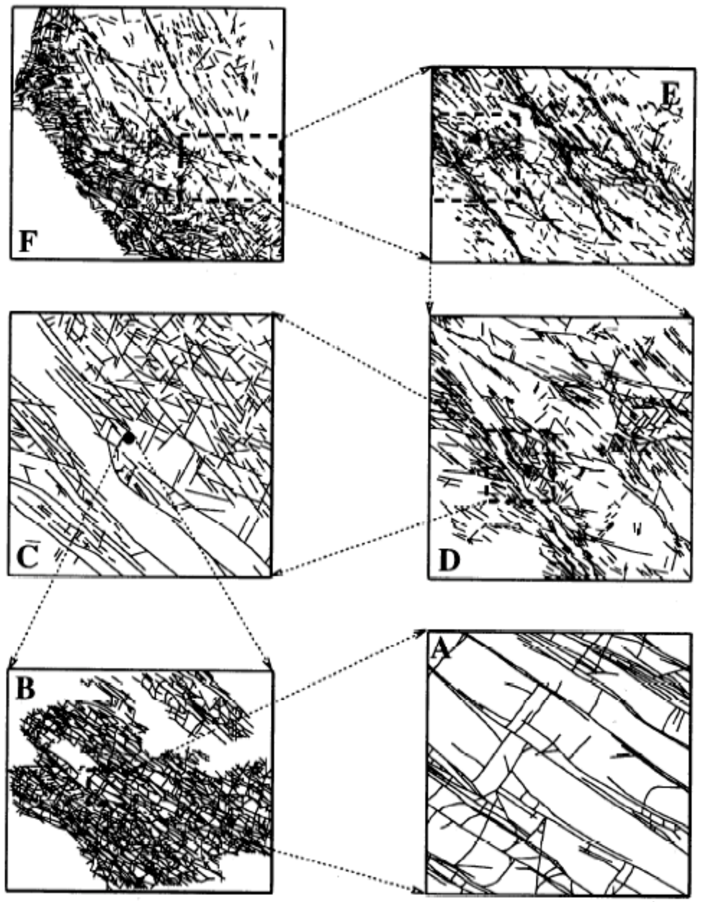
Knopoff, 2000, PNAS 97, 11880-11884



Main, 1995, BSSA 85, 1299-1308

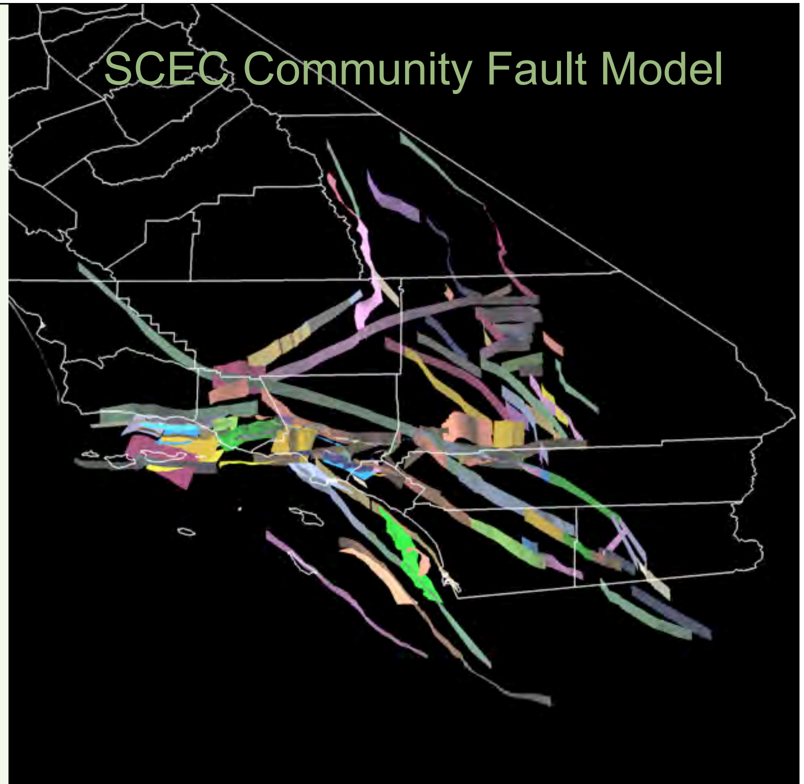
Hierarchical geometry of faulting

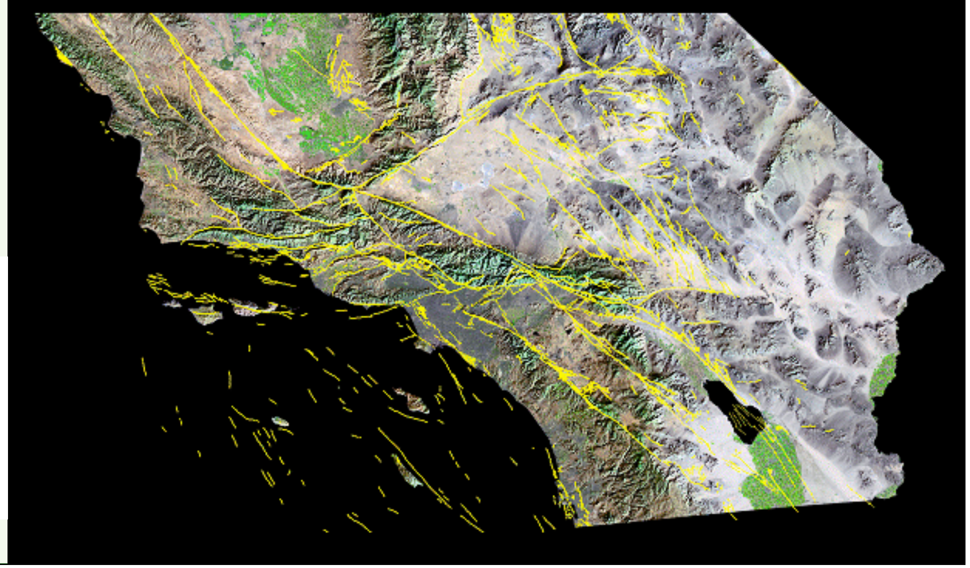
Ouillon, Castaing, Sornette (JGR 1996)

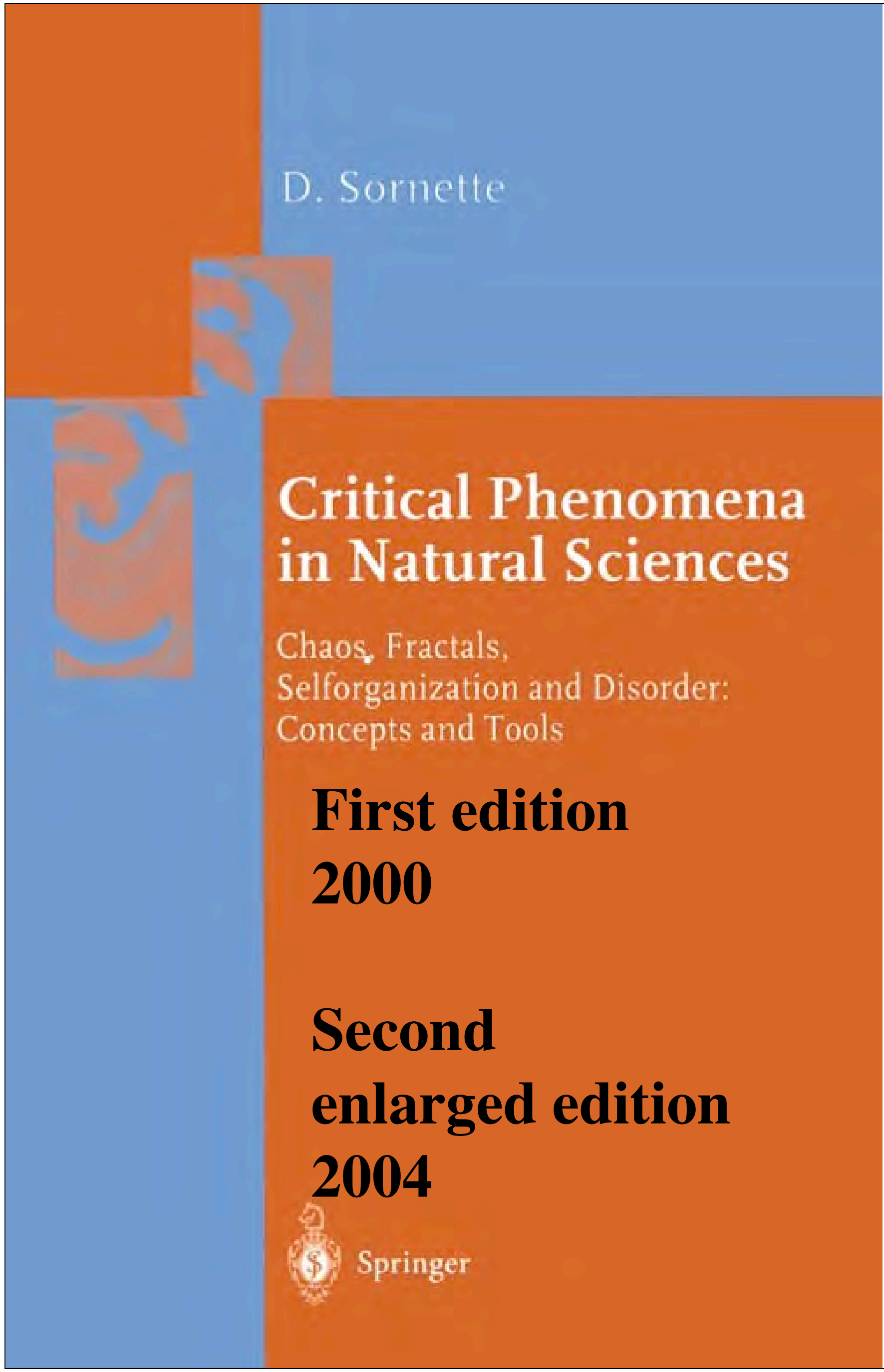


Map A: linear size=10 m, orig. scale=1:1
Map B: linear size=60 m, orig. scale=1:220
Map C: linear size=11 km, orig. scale=1:62,500
Map D: linear size=45 km, orig. scale=1:125,000
Map E: linear size=150 km, orig. scale=1:250,000
Map F: linear size=400 km, orig. scale=1:1,000,000

SCEC Community Fault Model

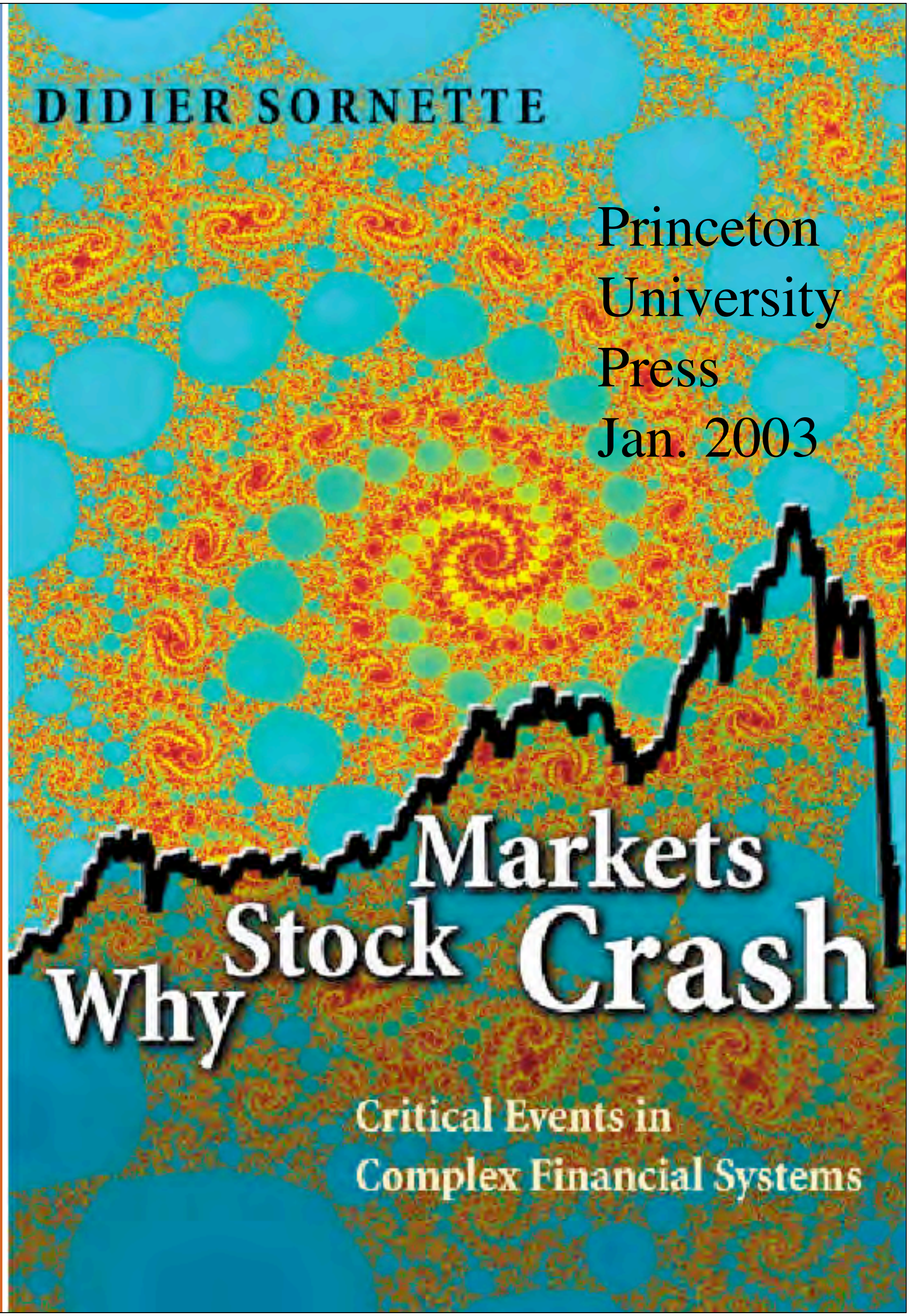




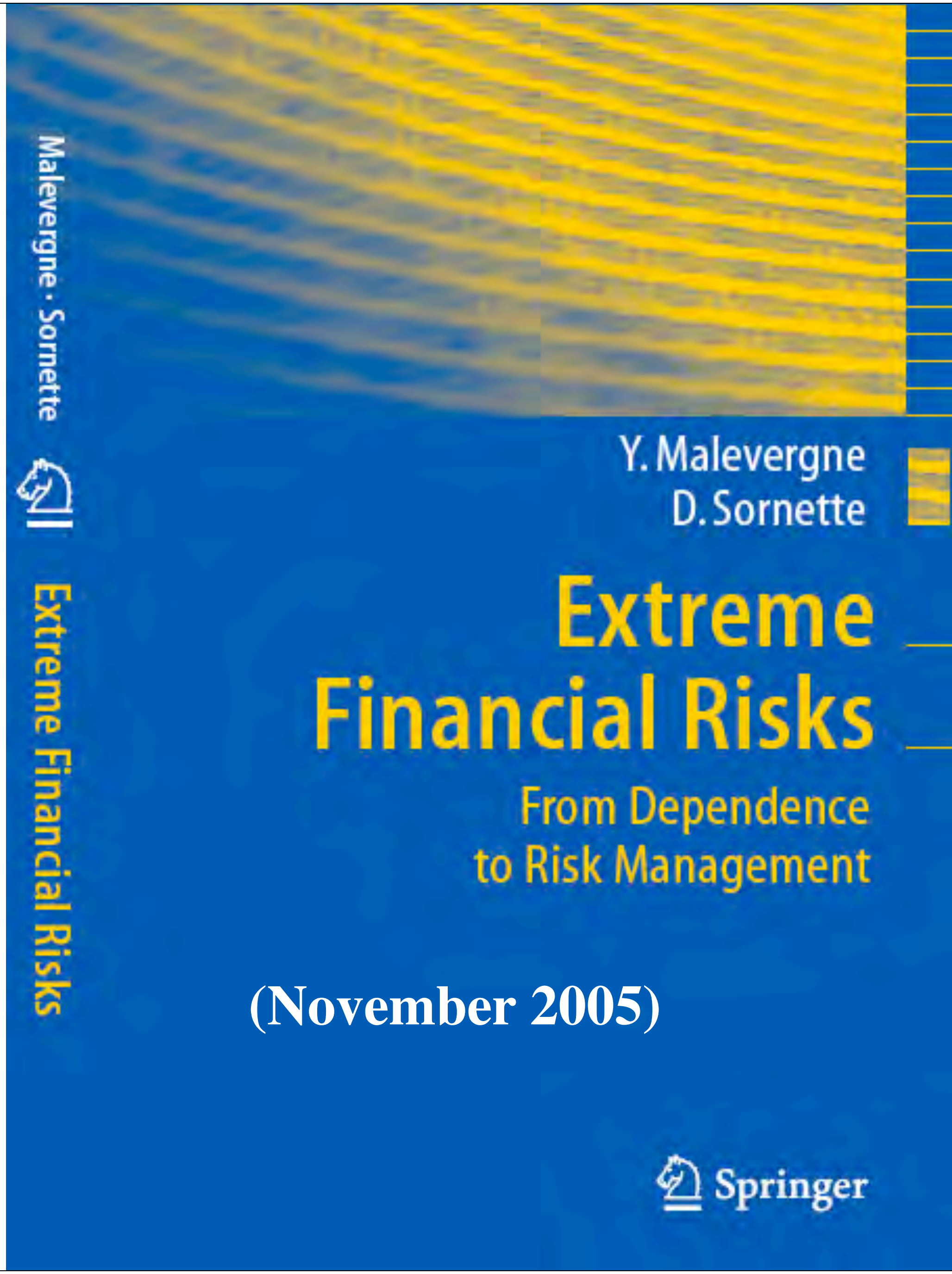


First edition 2000

Second enlarged edition 2004



Princeton University Press Jan. 2003



(November 2005)