# ASYMMETRIC ELASTICITY IN A CRACKED CRUST

## New Mechanism for Long-Range Elastic Interactions

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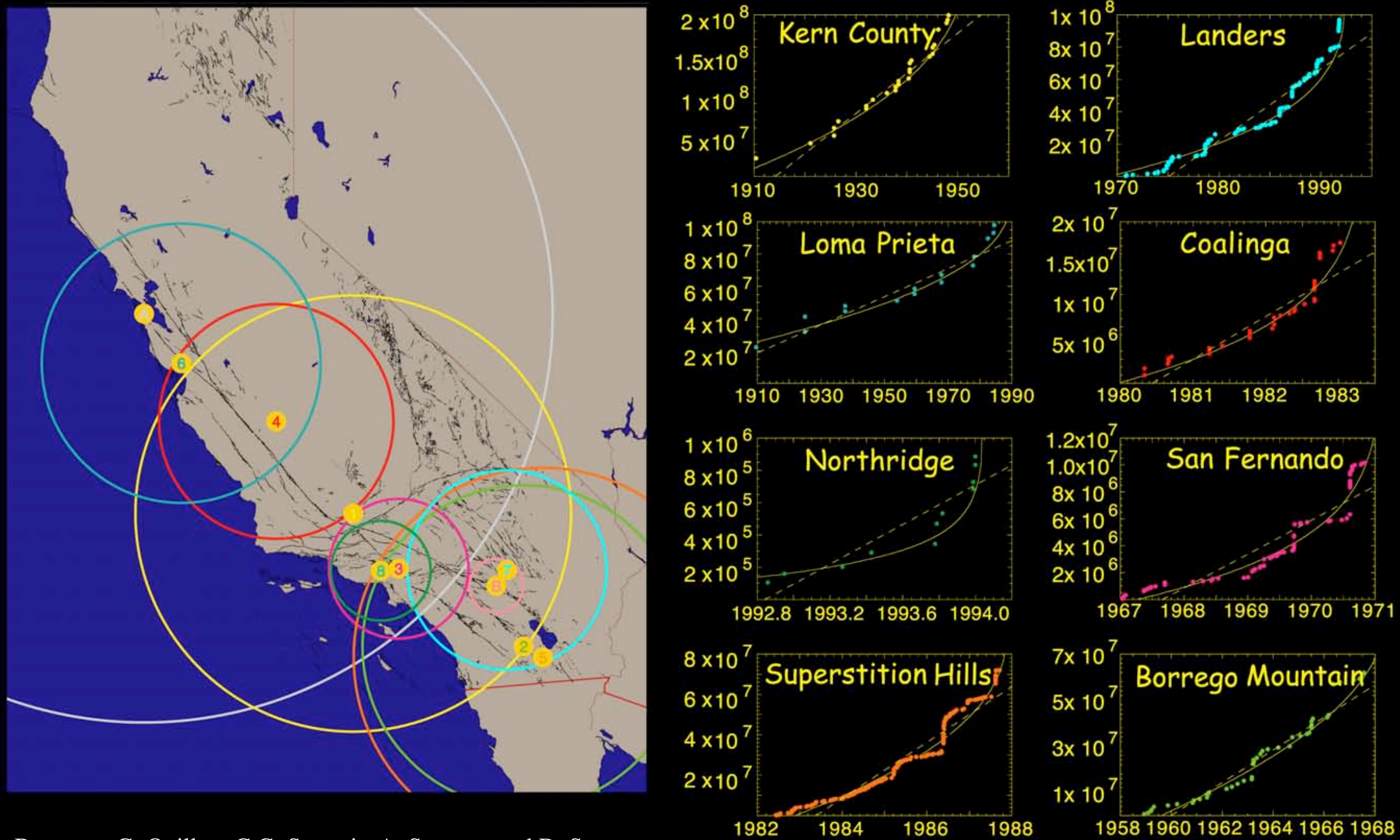
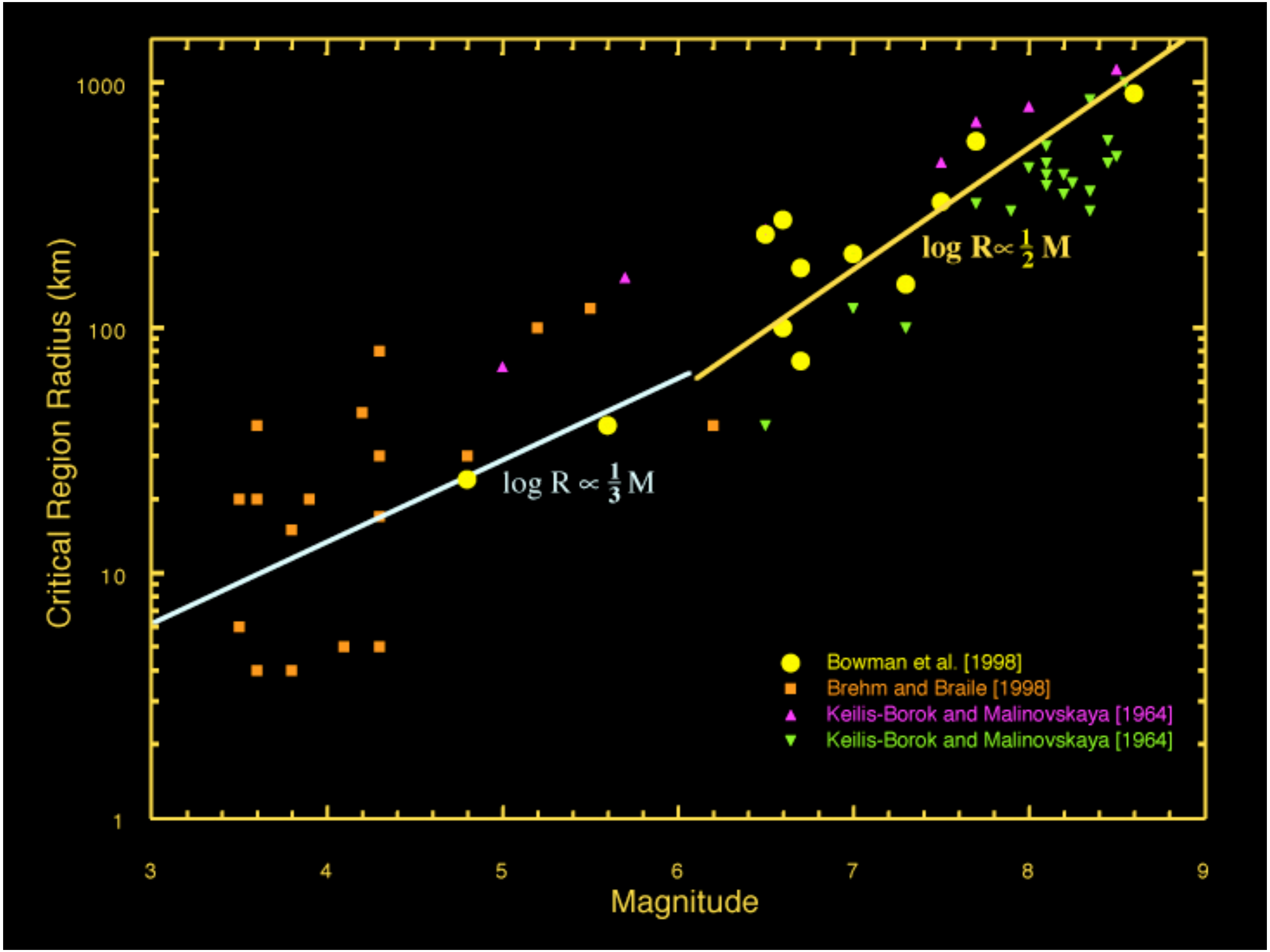
# All California Earthquakes M≥6.5
# 1950-1995

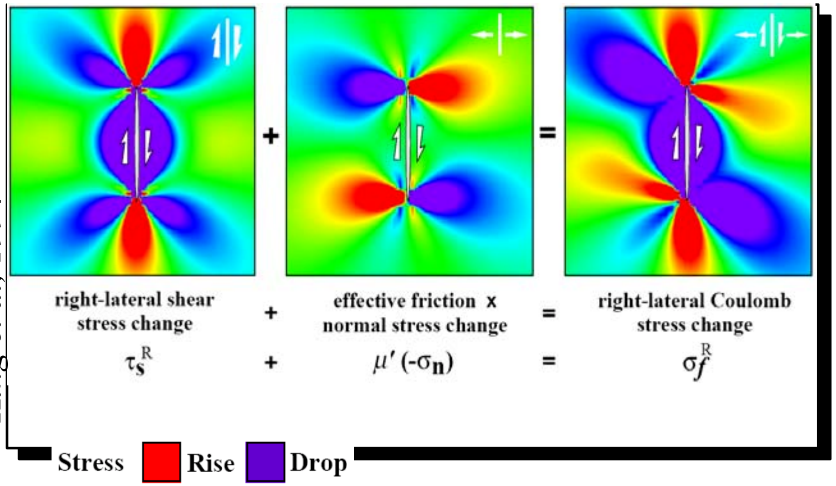



D.D. Bowman, G. Ouillon, C.G. Sammis, A. Sornette and D. Sornette
An Observational test of the critical earthquake concept, J.Geophys. Res. 103, (NB10), 24359-24372 (1998).



Critical Region Radius (km)
1000
100
10
1
3
4
5
6
7
8
9
Magnitude
log R ∝ ½ M
log R ∝ ⅓ M
Bowman et al. [1998]
Brehm and Braile [1998]
Keilis-Borok and Malinovskaya [1964]
Keilis-Borok and Malinovskaya [1964]

# Linear elastic stress transfer

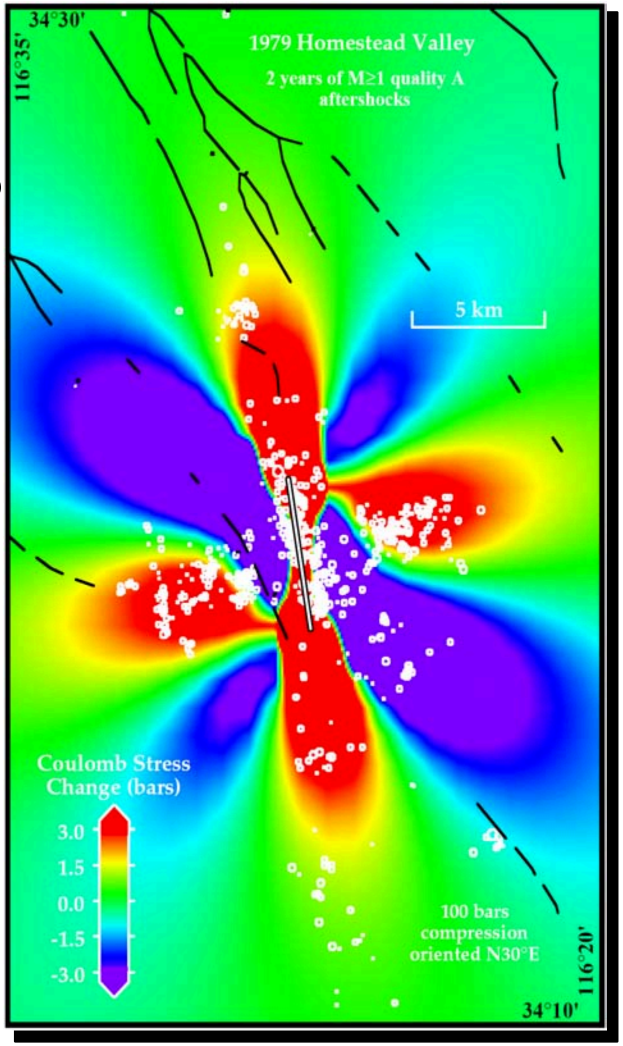



*King et al, 1994*

Rock is usually modeled using an elastic rheology. Static stress patterns have a diffuse structure.

An earthquake is modeled using boundary elements, with a uniform stress drop on its fault plane.

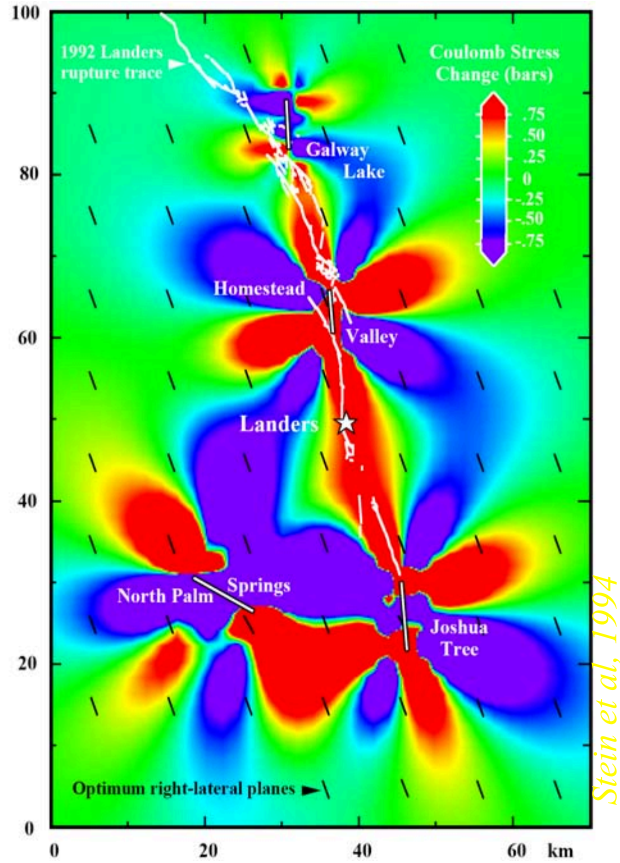
*King et al, 1994*



# Linear elastic stress transfer

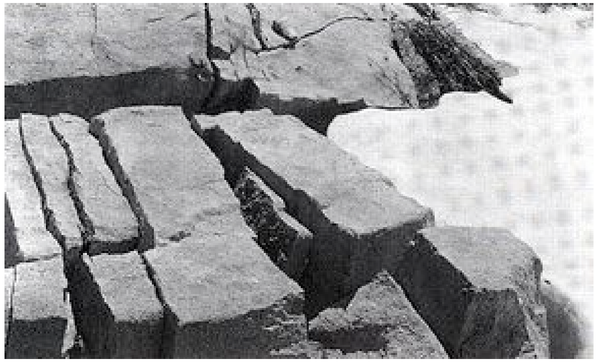
Linear elasticity implies that stress fluctuations of different sources just add up.

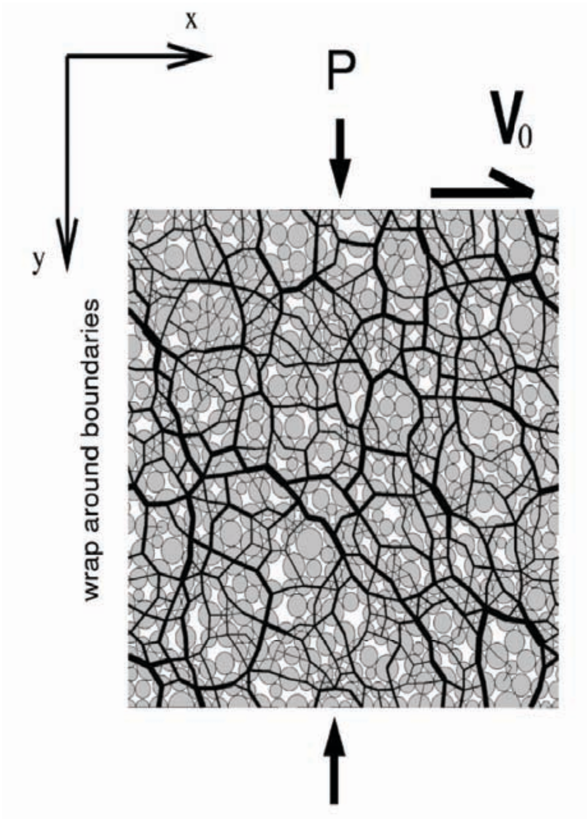
This ensures very fast and efficient computations of the resulting stress field, that can be compared with subsequent seismicity.



# Stress transfer in granular media

Rock is fractured and fragmented at every scale, characterized by a discrete, blocky structure.

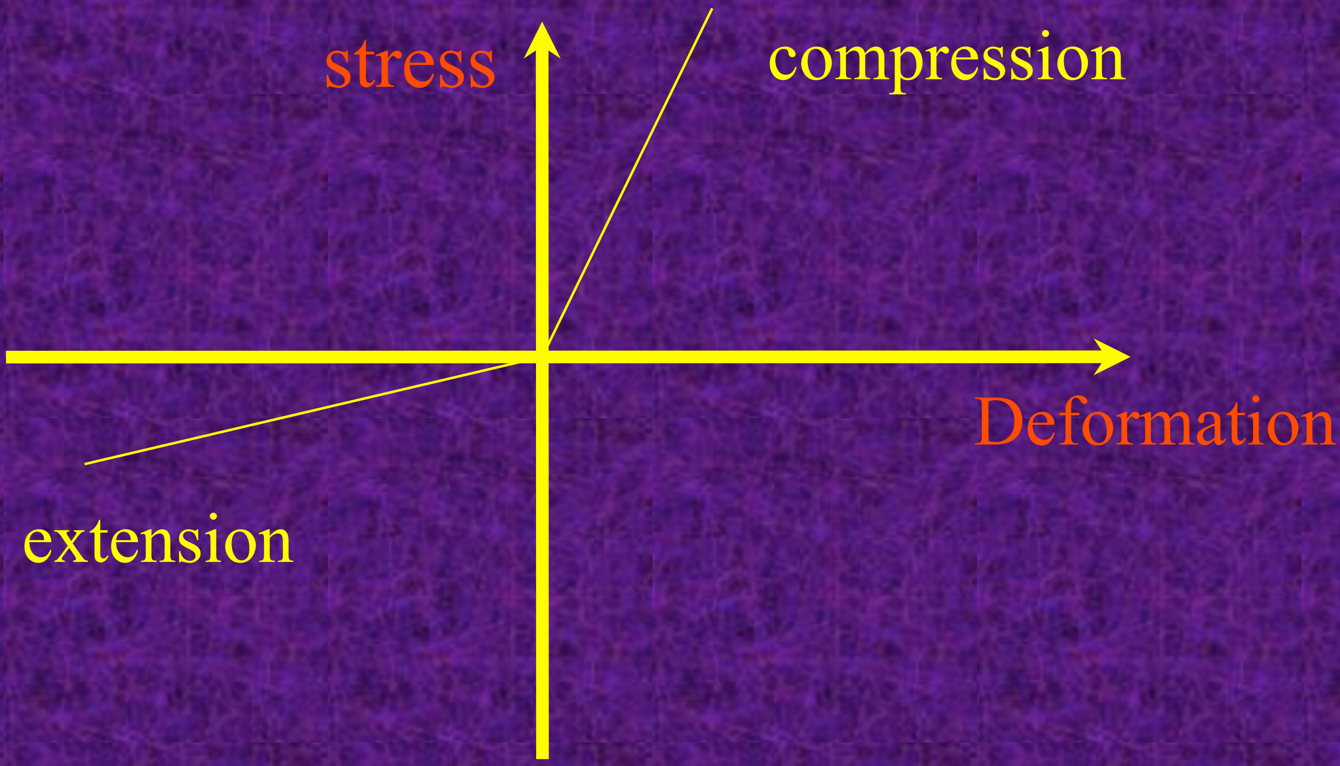


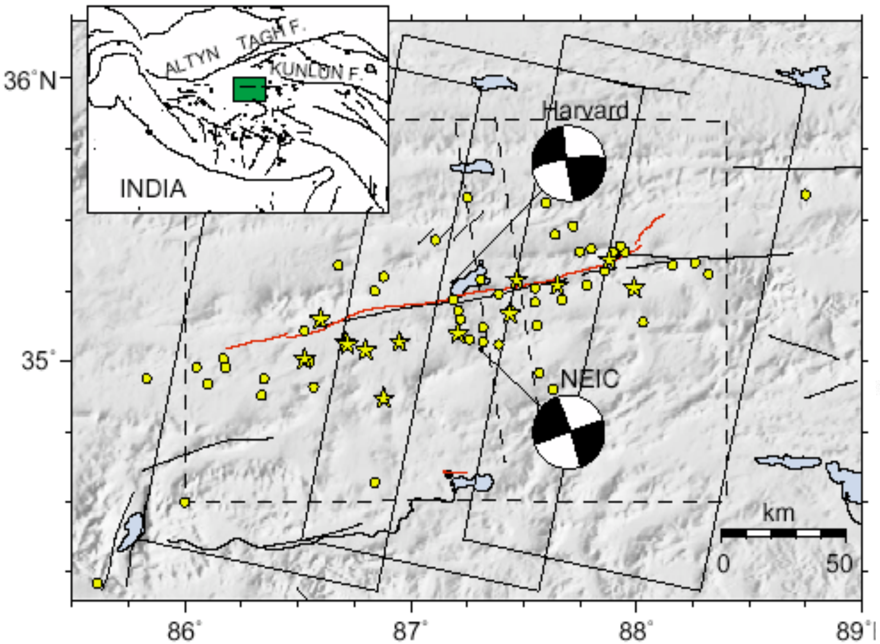


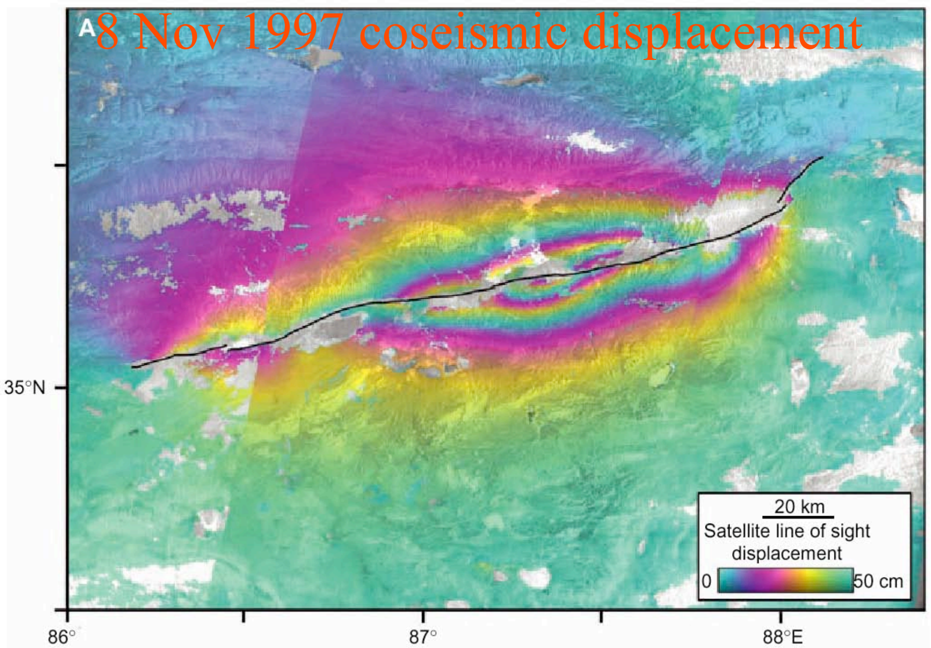

Granular media offer a nonlinear rheology, due to a vanishing tensile strength and a small amount of contacts among grains.
This results in long-range propagation of forces along very narrow corridors.

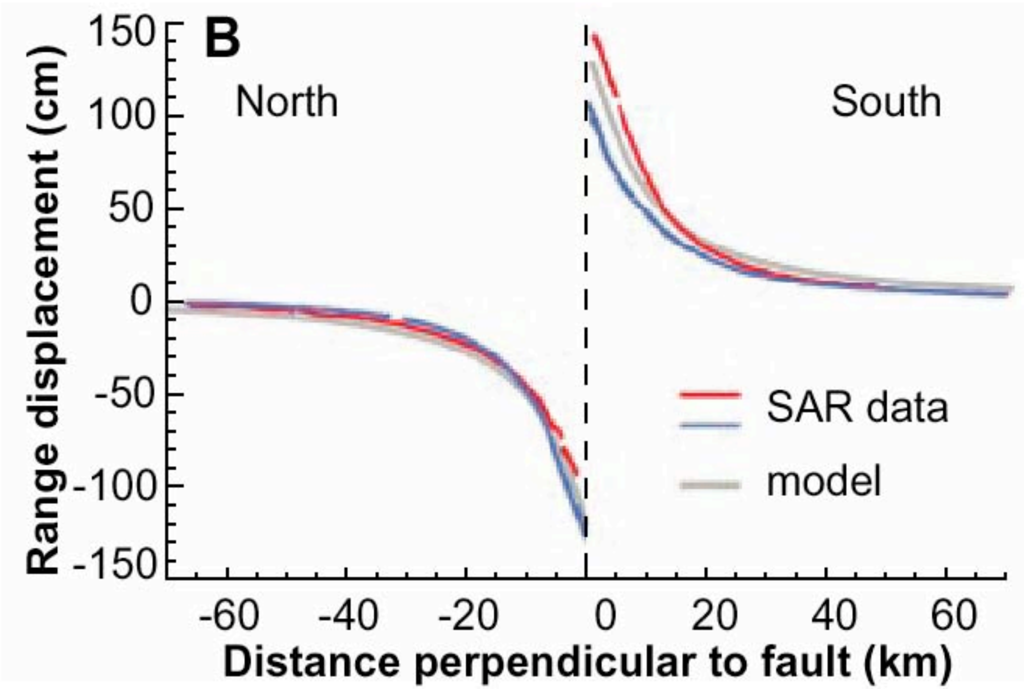
## A simple and plausible nonlinear elastic model for the Earth's crust

We now consider that the crust is a nonlinear elastic medium characterized by an asymmetric response to compressive versus extensive perturbations around the lithostatic state. This nonlinearity stems from the fact that the crust is crisscrossed by cracks, joints and faults at many different scales filled with drained fluid in contact with delocalized reservoirs at pressure close (or at) lithostatic pressure.






**Evidence of Nonlinear Elasticity of the Crust from the Mw7.6 Manyi (Tibet) Earthquake**

G. Peltzer, F. Crampé and G. King
Science, 286, 272 (1999)

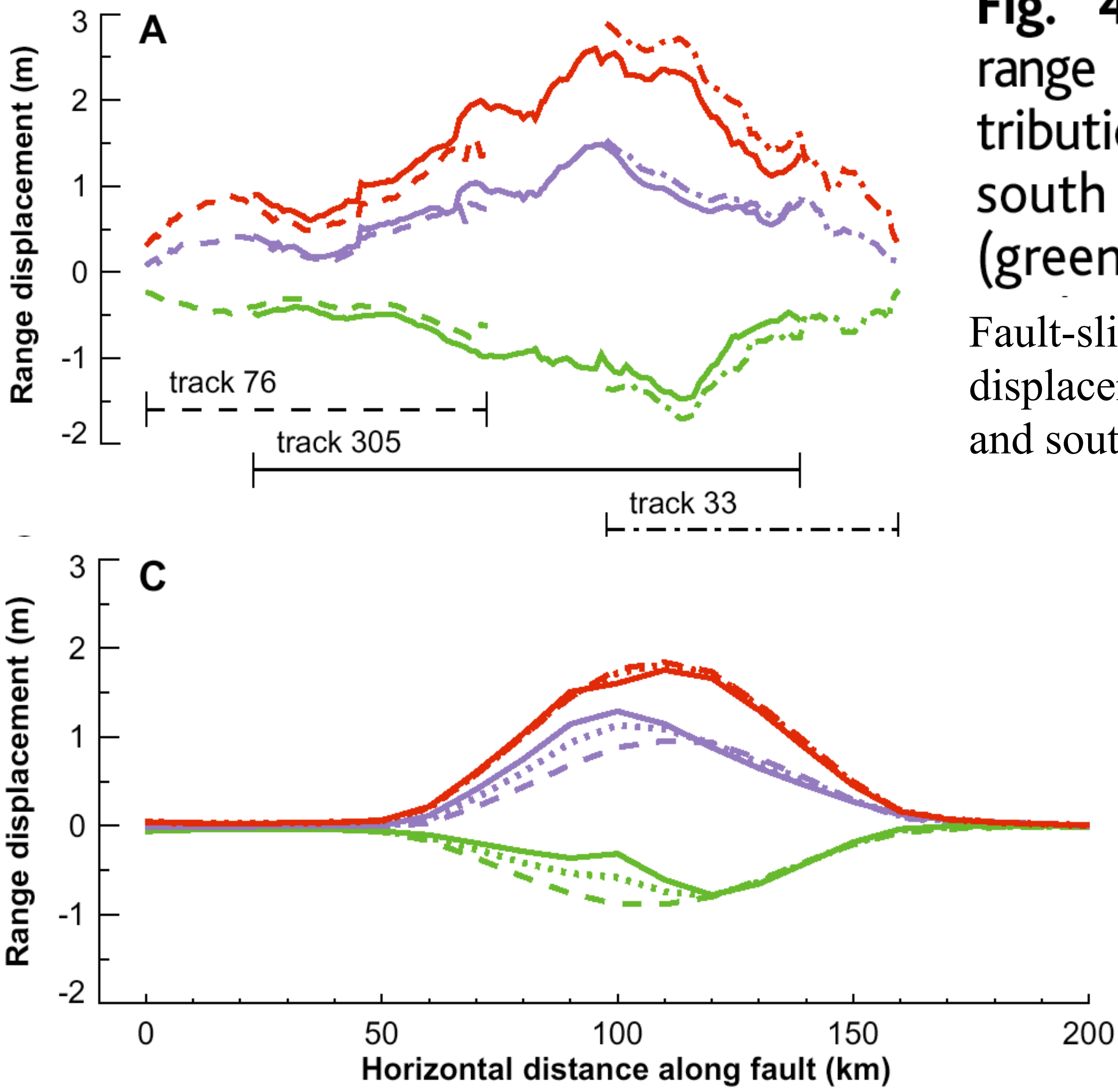



**Fig. 4. (A)** Observed range displacement distribution profiles along south (blue) and north (green) sides of fault.

Fault-slip distribution (relative displacement between north and south sides shown in red

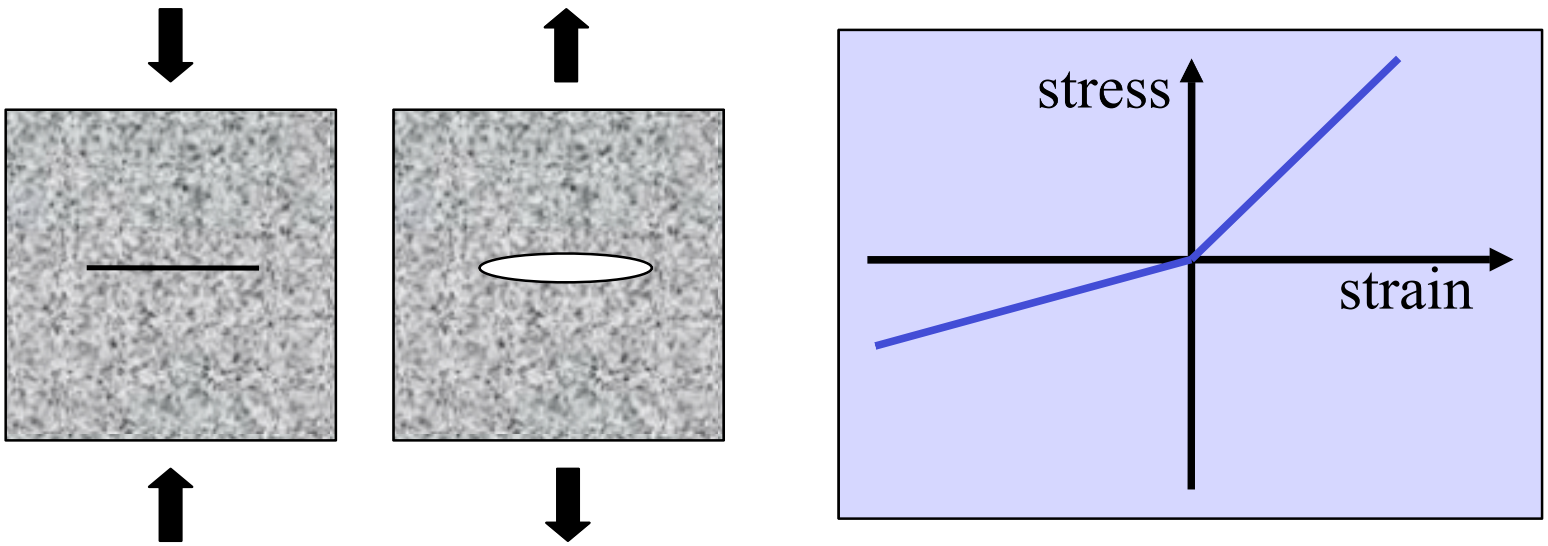
To begin with the study of such a system (which would imply fluid circulation when the medium is perturbed) we will consider a 2D medium containing many small cracks, uniformly distributed in space, filled with a non-viscous, infinitely compressible fluid (like air). The medium is drained, initially free of any stress on its boundaries, and boundary displacements are imposed to be zero on those boundaries. We then impose a stress or strain perturbation at the center of the plate in order to simulate the static perturbation due to an earthquake. We then compute the displacement and stress fields within the plate to check the influence of the loss of compression/tension symmetry at the microscopical scale on the macroscopic structure of the elastic fields.

# A micromechanical model

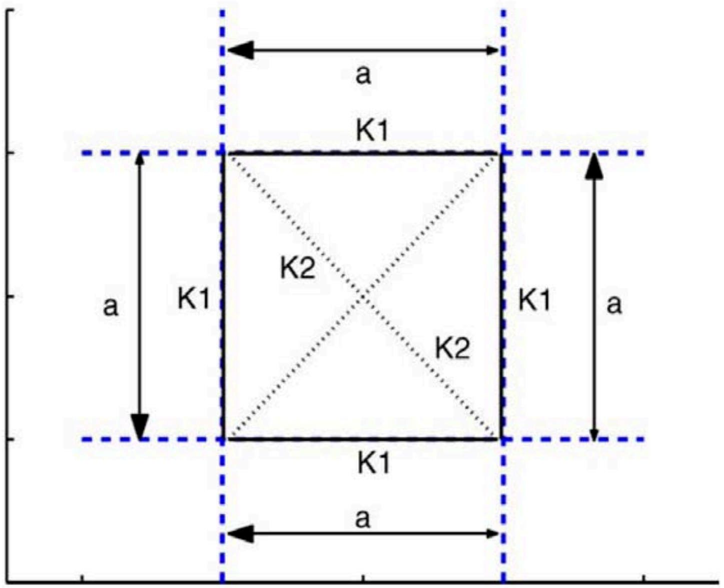
Solving this model using methods like finite elements would prove very difficult as there is indeed a complex feedback loop between the strain tensor and the rock rigidity tensor. We thus chose to use a simple central-force spring model in 2D. We consider a square plate of size $L$=2000 km discretized into a regular grid of mesh size $a$=10 km. The Figure below shows the mechanical structure of an elementary cell: each node is connected to its nearest neighbours by springs of stiffness $K_1$, and to its next-nearest neighbours by (diagonal) springs of stiffness $K_2=K_1/2$ (this condition ensuring elastic isotropy when the medium is a symmetric one). We then define an asymmetry parameter $\alpha < 1$, such that if any spring of stiffness, say, $K$ is subjected to a net tensile strain, then its stiffness drops down to $\alpha K$ (while it remains equal to $K$ if the spring suffers a net compressive strain). This new parameter (which will be allowed to have any value from 0 to 1) mimics the effects of cracks within the springs.

# Nonlinear elasticity

We propose an intermediate rheology based on the concept of asymmetric elasticity: under compressive stress states, cracks are closing and rock stiffness is large; under tensile stress states, cracks are opening and rock stiffness is small.



# Central-force springs microscopic model




Square plate - 200x200 nodes.

Square cells with 4 corner nodes.

a = 10km

2 sets of central-force springs:

- horizontal/vertical : $K_1$

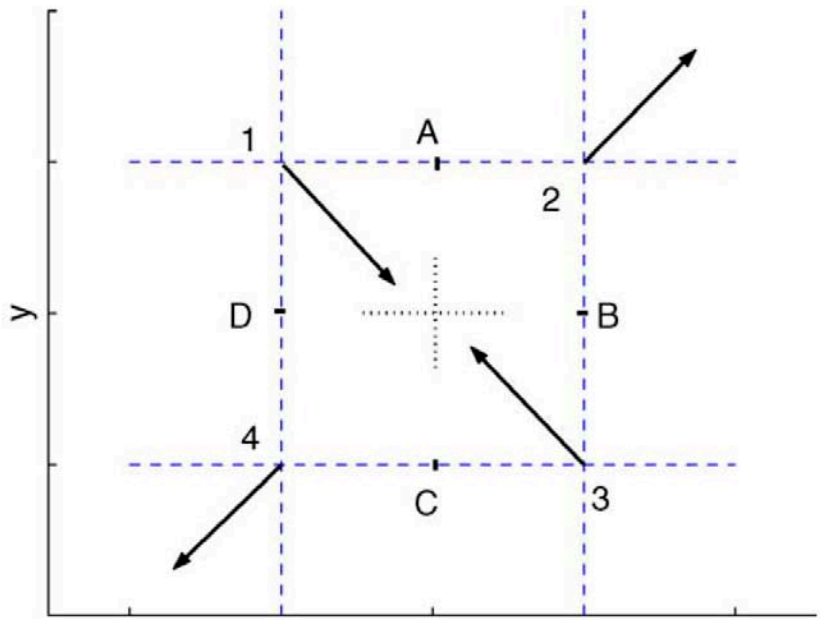
- diagonal : $K_2$

Isotropy is achieved only if $K_2=K_1/2$

For each spring,

$$K_{tensile} = \alpha . K_{compressive} \qquad 0 < \alpha < 1$$

# Pointwise earthquake mechanical model




The central cell is loaded according to:

• a pure shear stress
arrows = imposed forces
crack model

• a pure shear strain
arrows = imposed displacements
dislocation model

Both loading conditions are equivalent in the linear elastic case.

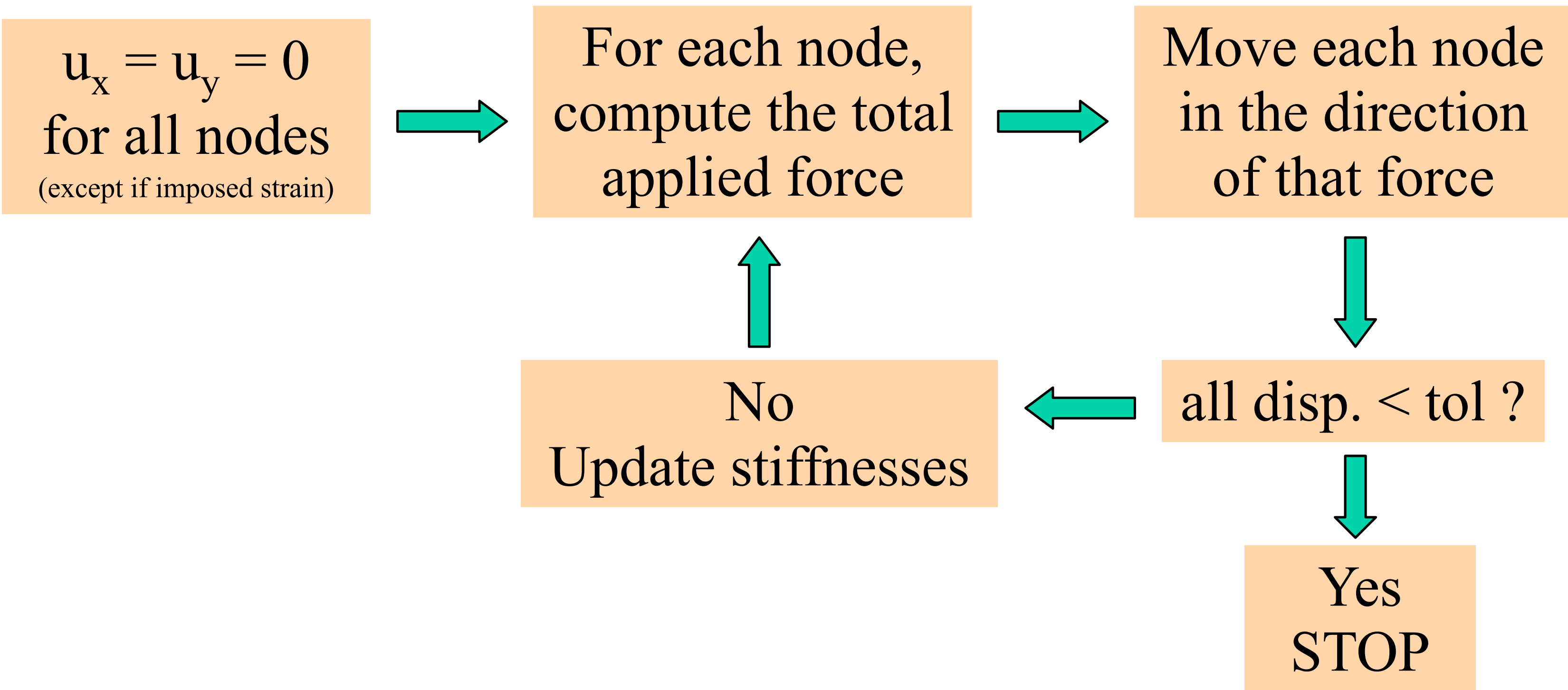
The corresponding fault is horizontal/dextral or vertical/sinistral.

# Numerical scheme for central force springs

The stiffness of each spring depends on the solution => iterative method

Boundary nodes:
$u_x = u_y = 0$

Source nodes:
$(u_x,u_y)$ or $(F_x,F_y)$ imposed

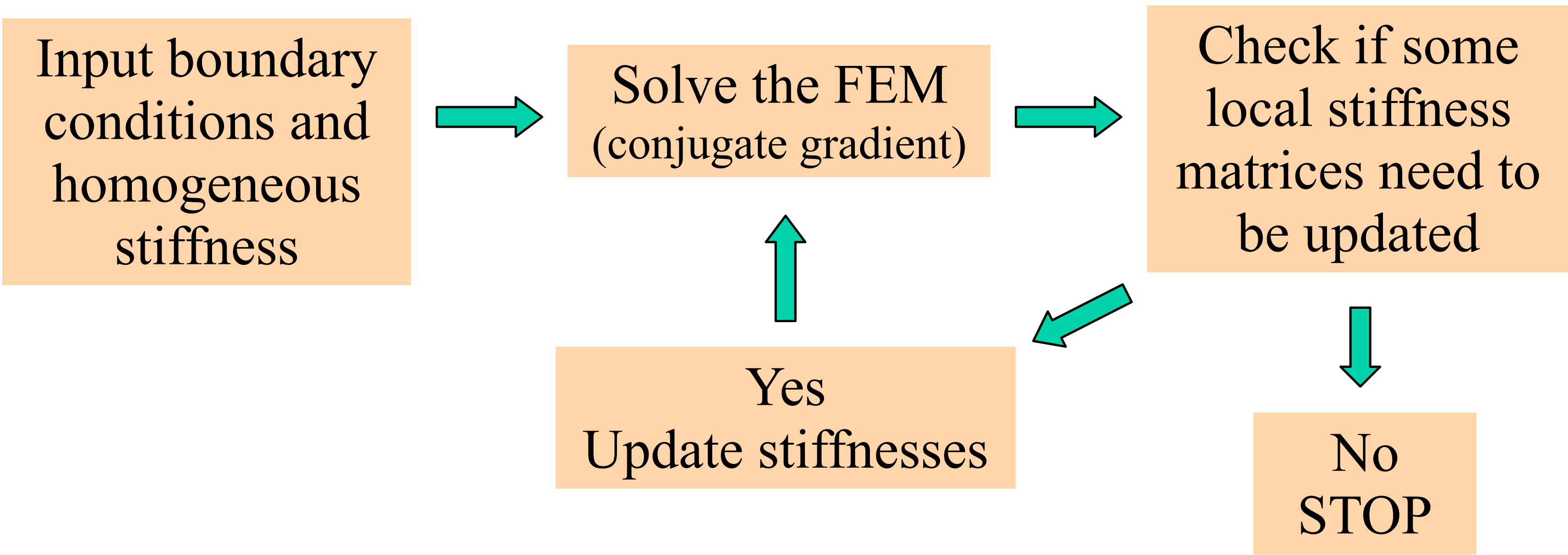


# Isotropic Finite Elements Model

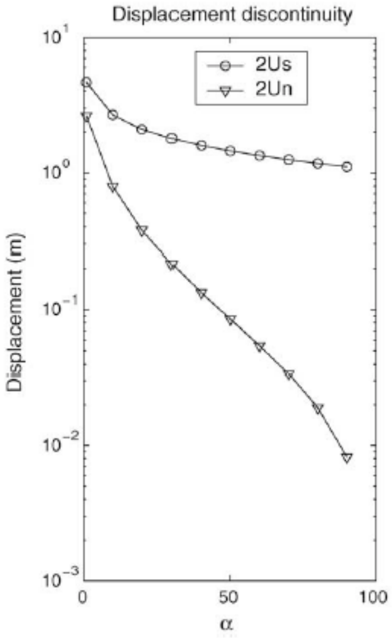
Four-nodes square elements with isotropic stiffness matrix K.

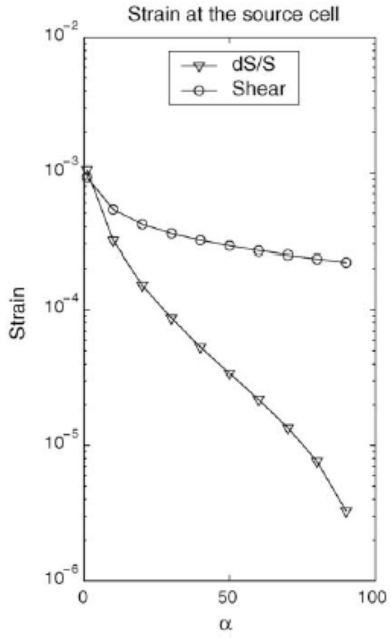
The stiffness criterion is now isotropic at the cell scale:

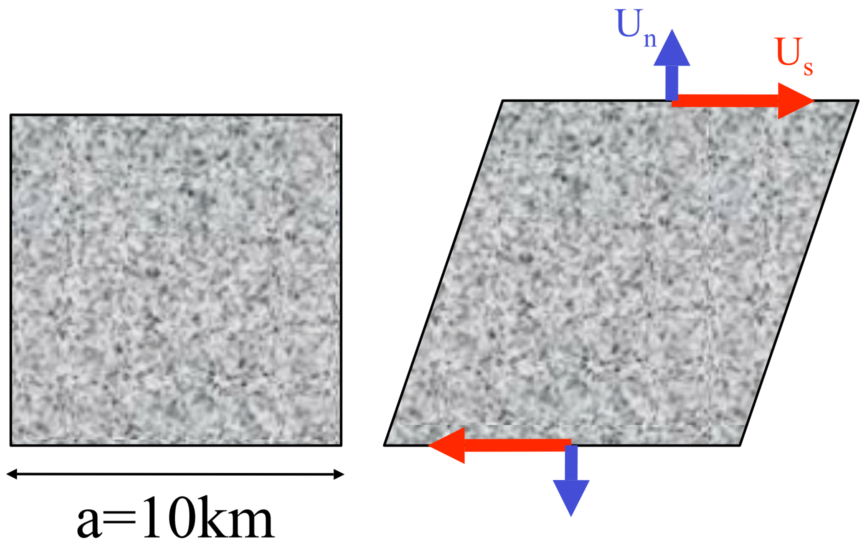
$$\Delta V>0 \qquad K \Rightarrow \alpha K \qquad 0 < \alpha < 0$$



# Strain at the source – central force spring model
# Crack model

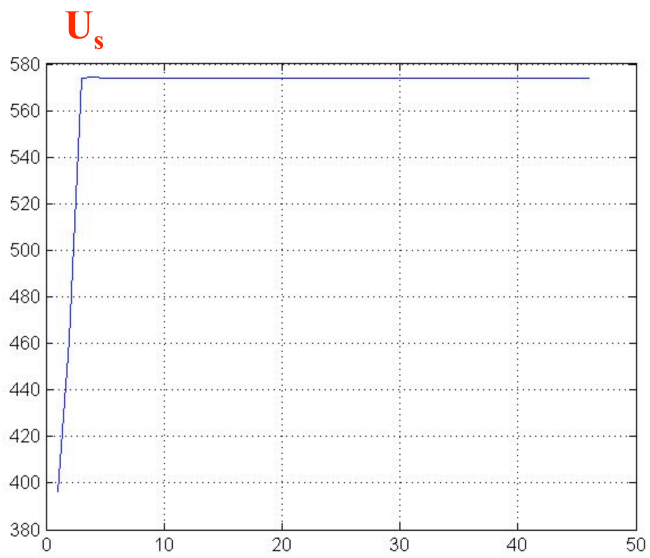


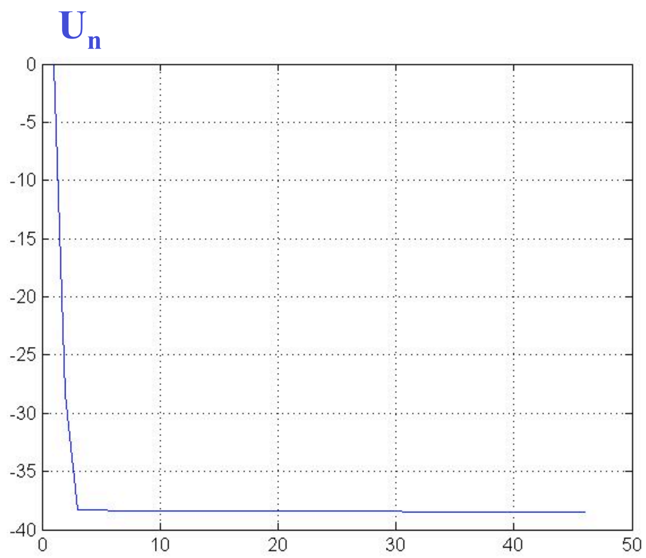




The volumetric strain has a negative sign in the FEM case.

# Convergence of the FEM

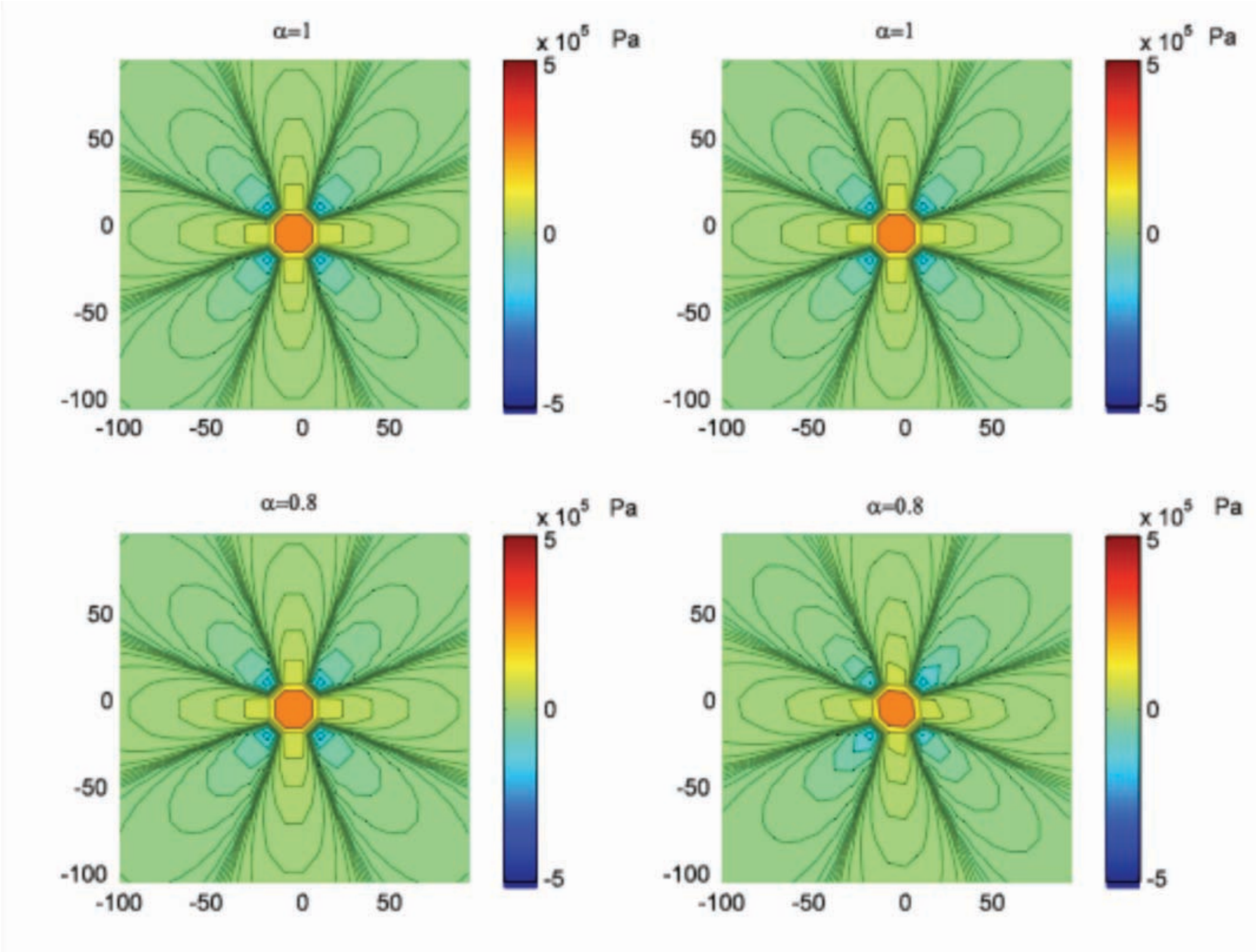



Iteration #



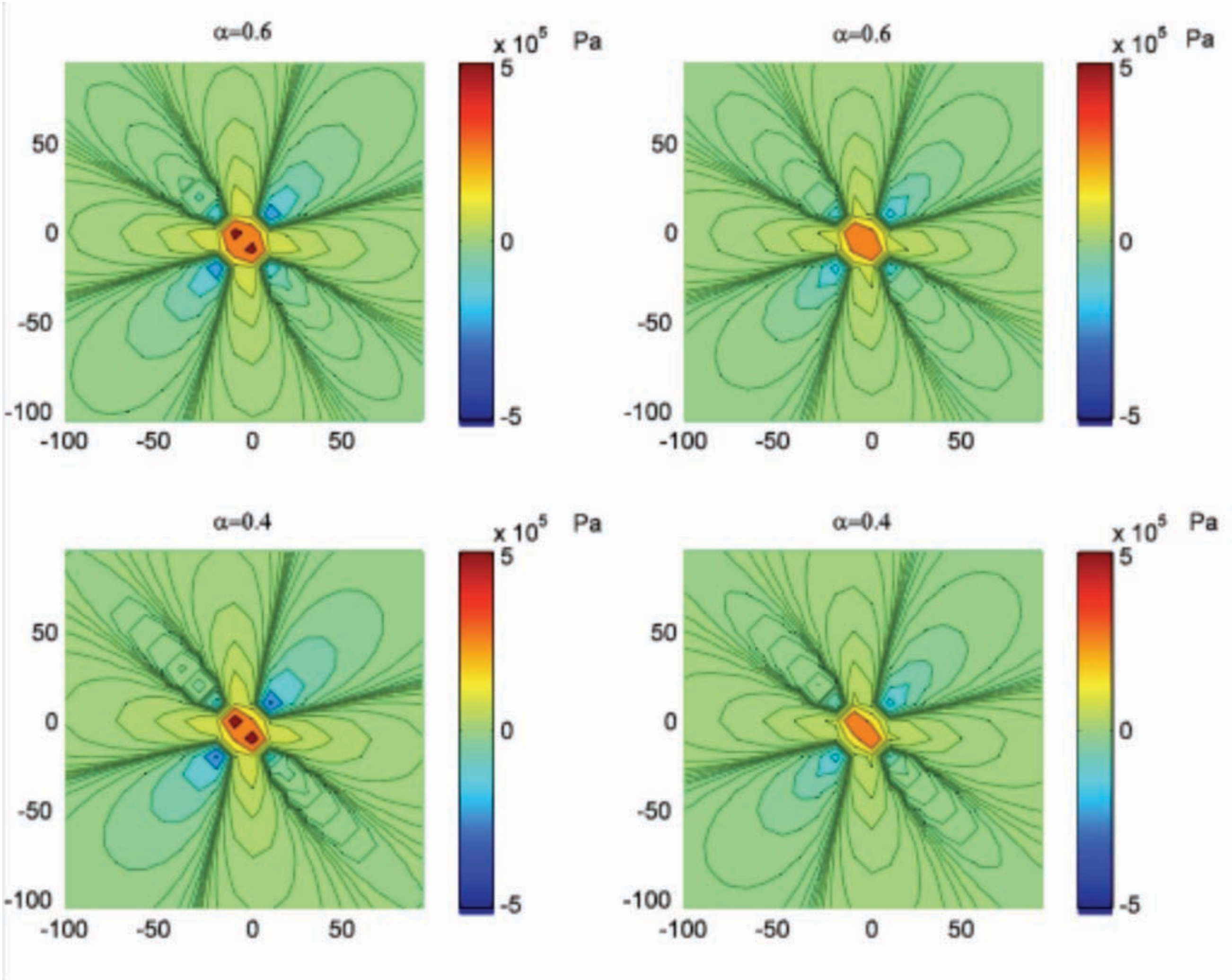

Iteration #

# $S_{xy}$ pattern with spring model



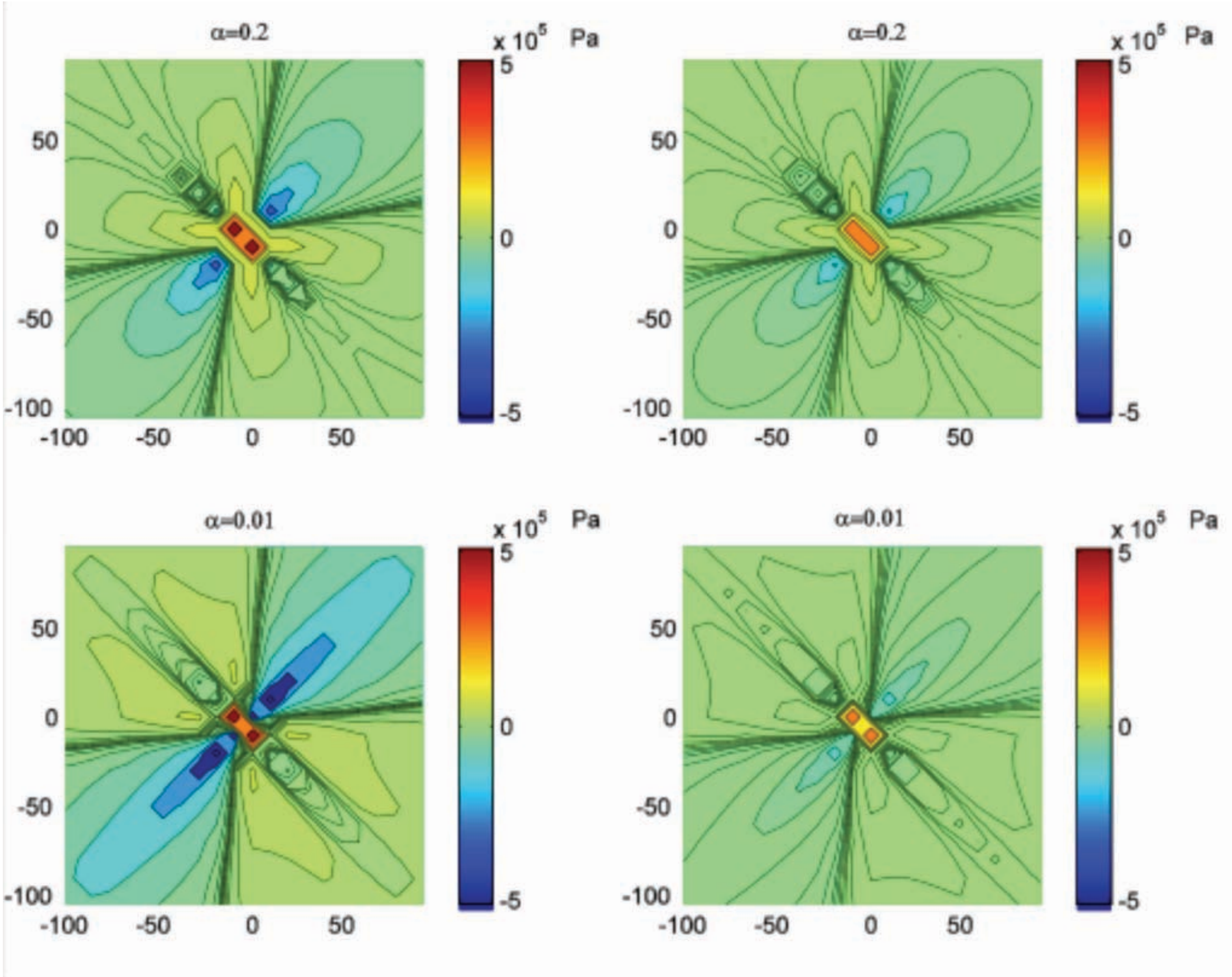

crack dislocation

# $S_{xy}$ pattern with spring model



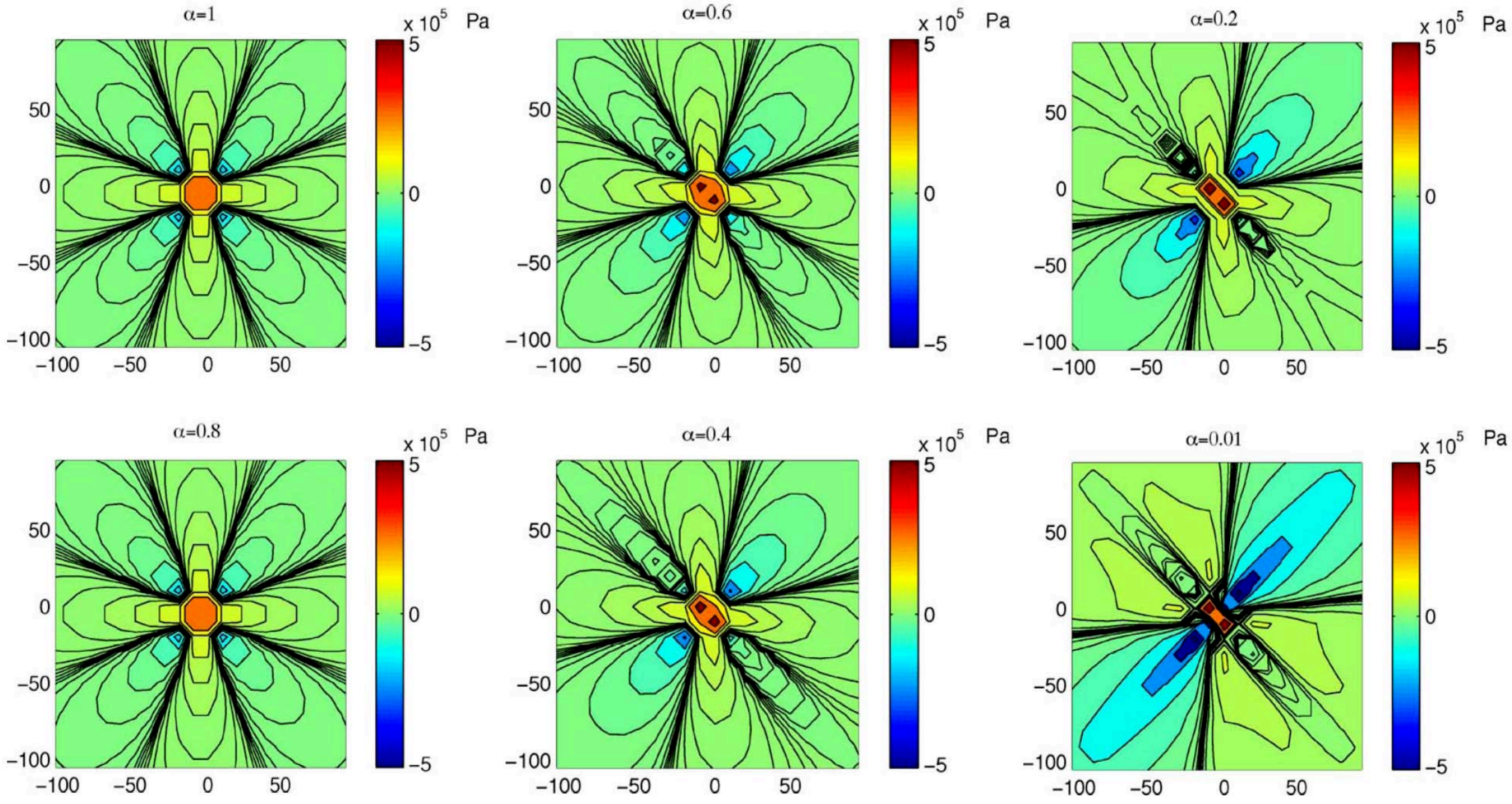

crack dislocation

# $S_{xy}$ pattern with spring model




crack dislocation

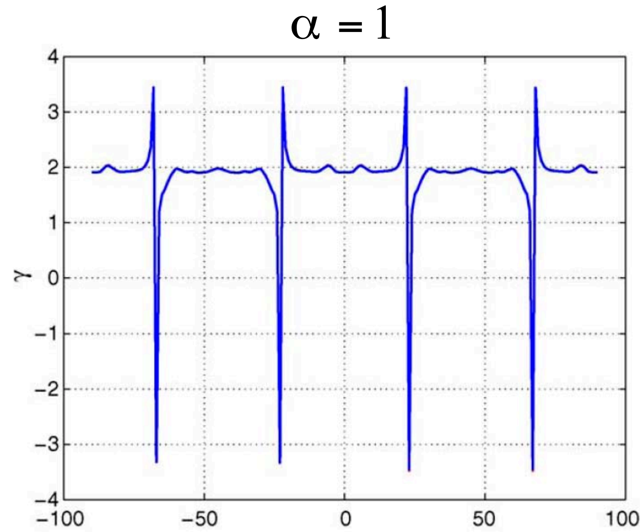
# $S_{xy}$ pattern with spring model - crack

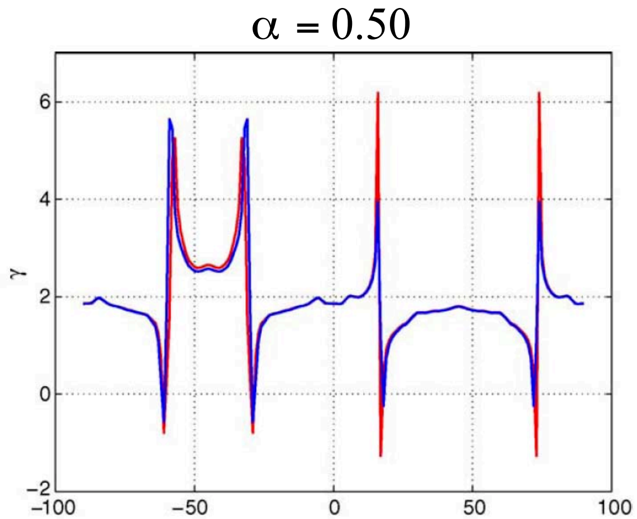


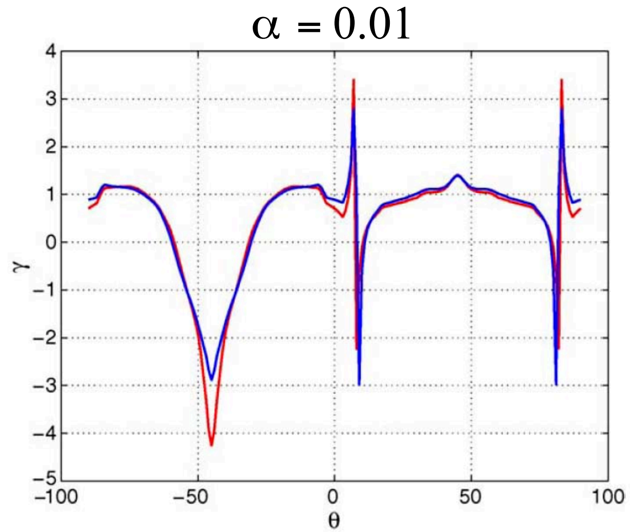
# Spatial decay of $S_{xy}$ with spring model

$$S_{xy}(r,\theta) \propto r^{-\gamma(\theta)}$$

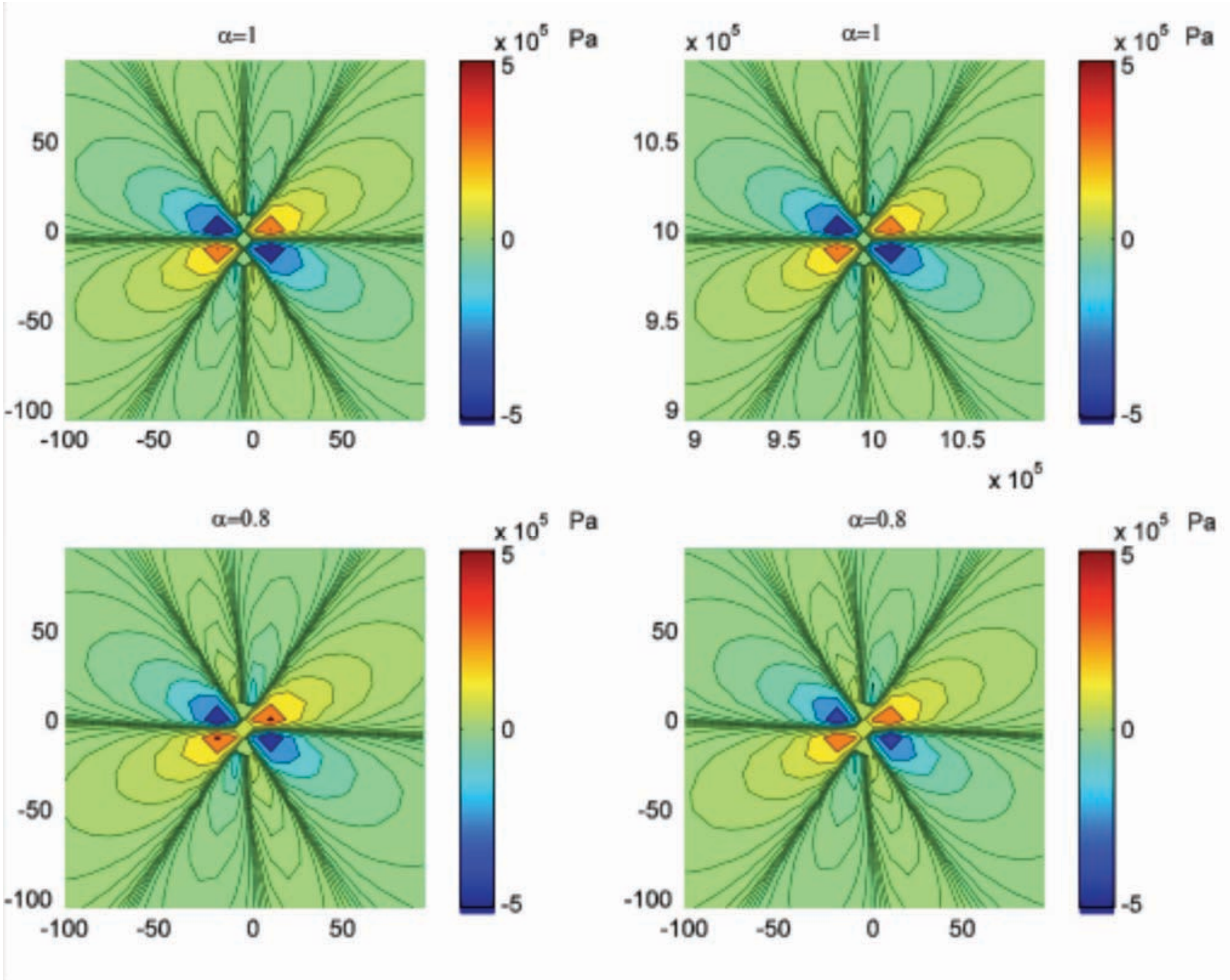
*blue: disloc*
*red:crack*



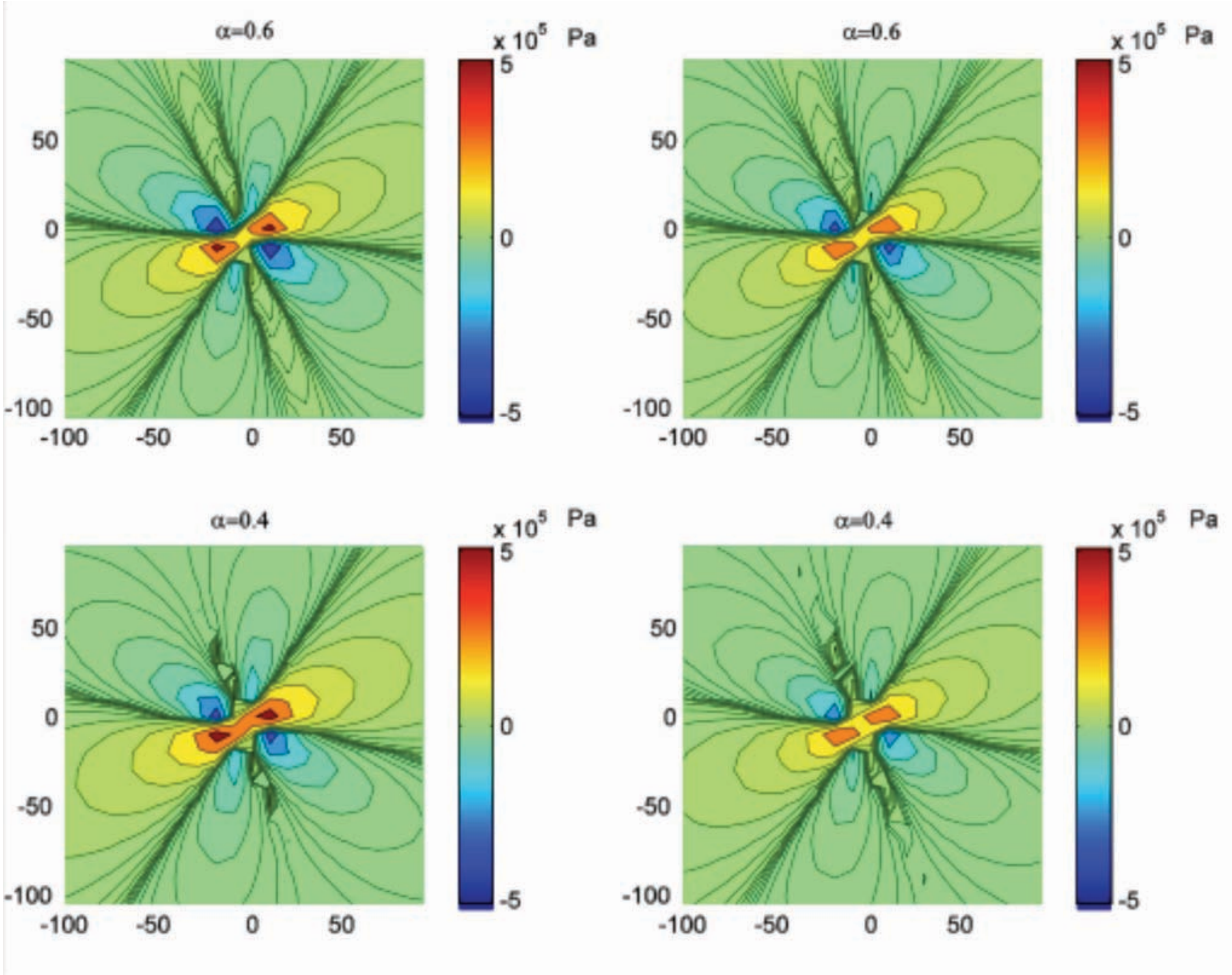



# $S_{xx}$ pattern with spring model



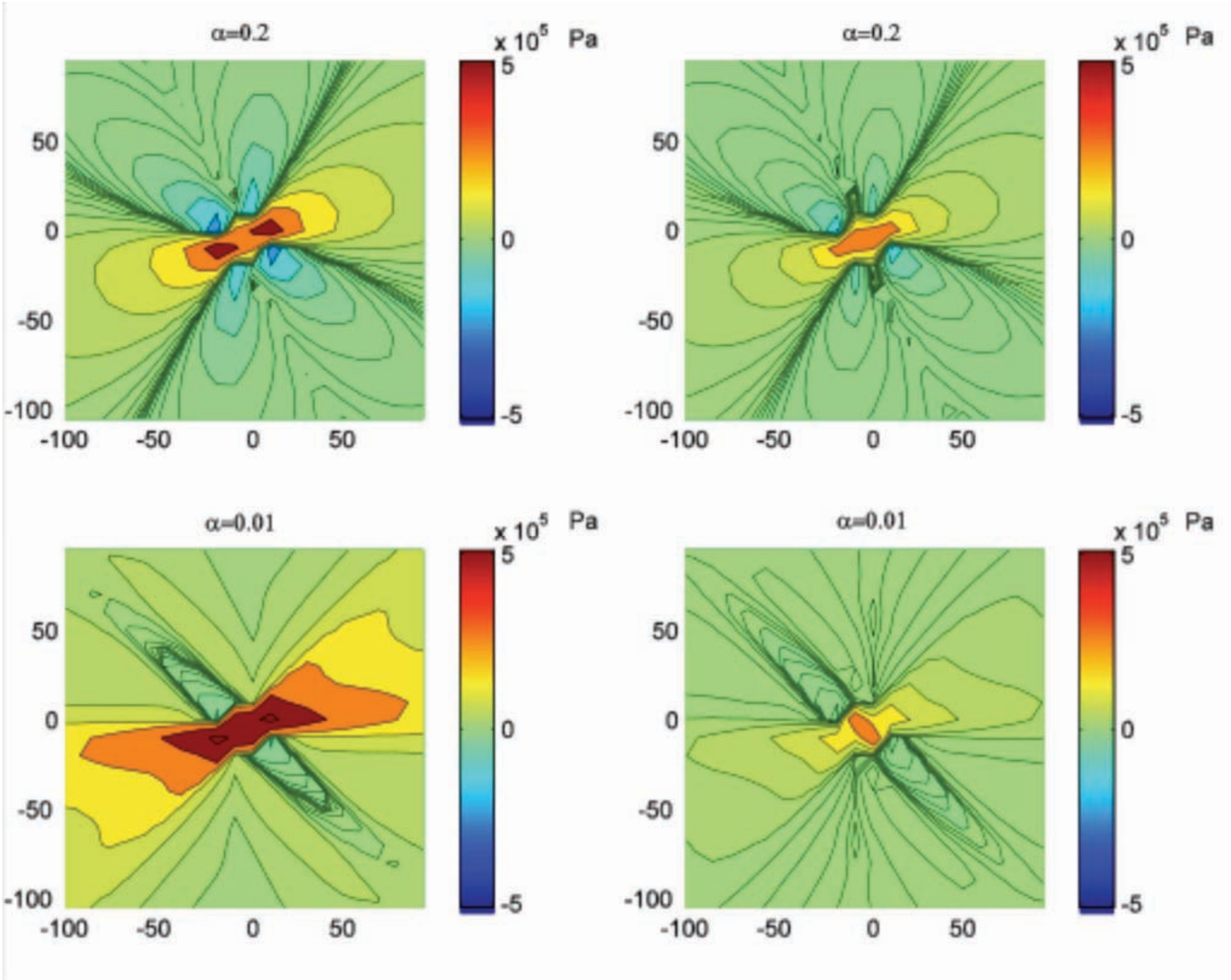

crack dislocation

# $S_{xx}$ pattern with spring model



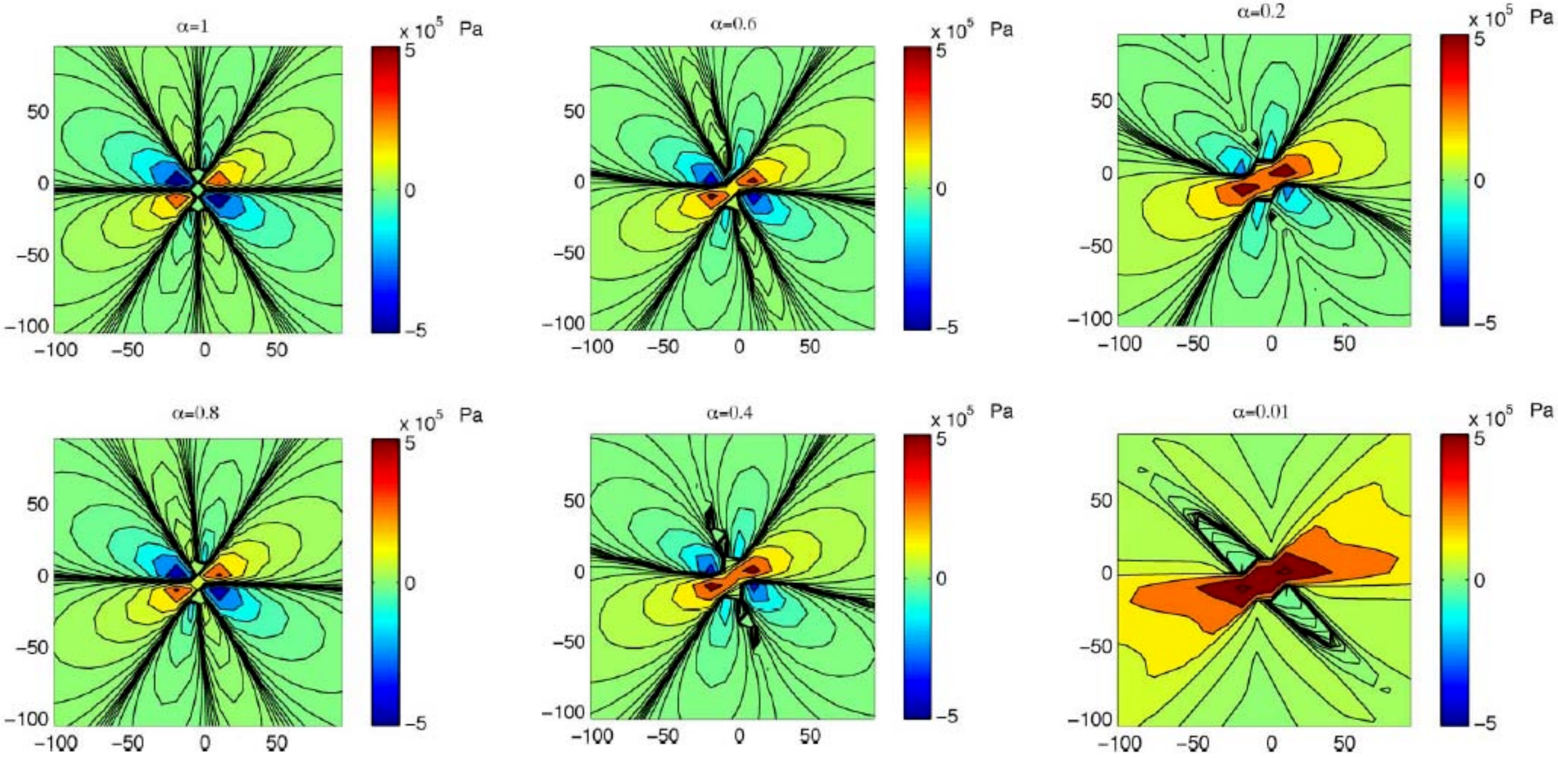

crack dislocation

# $S_{xx}$ pattern with spring model

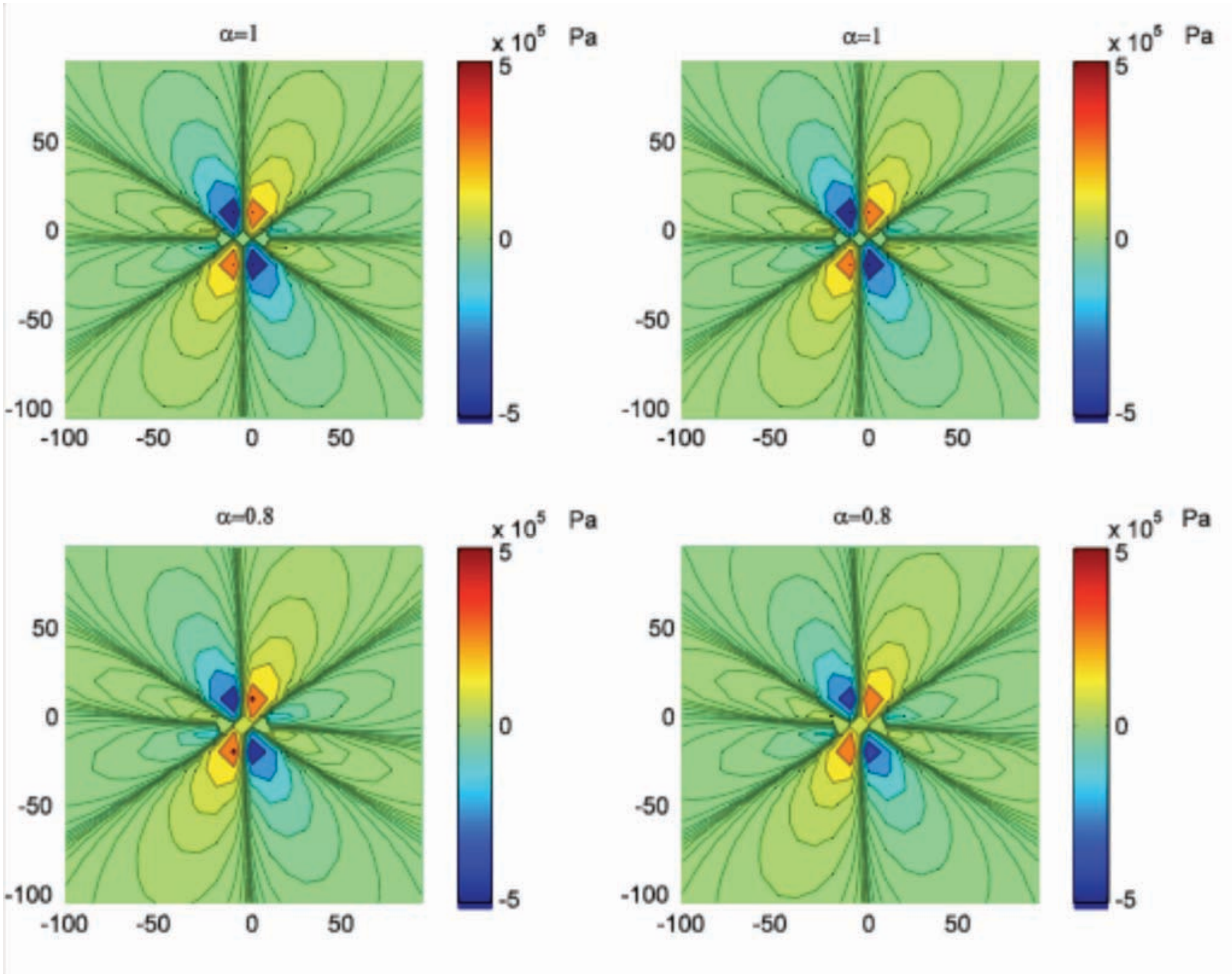



crack dislocation

# $S_{xx}$ pattern with spring model - crack

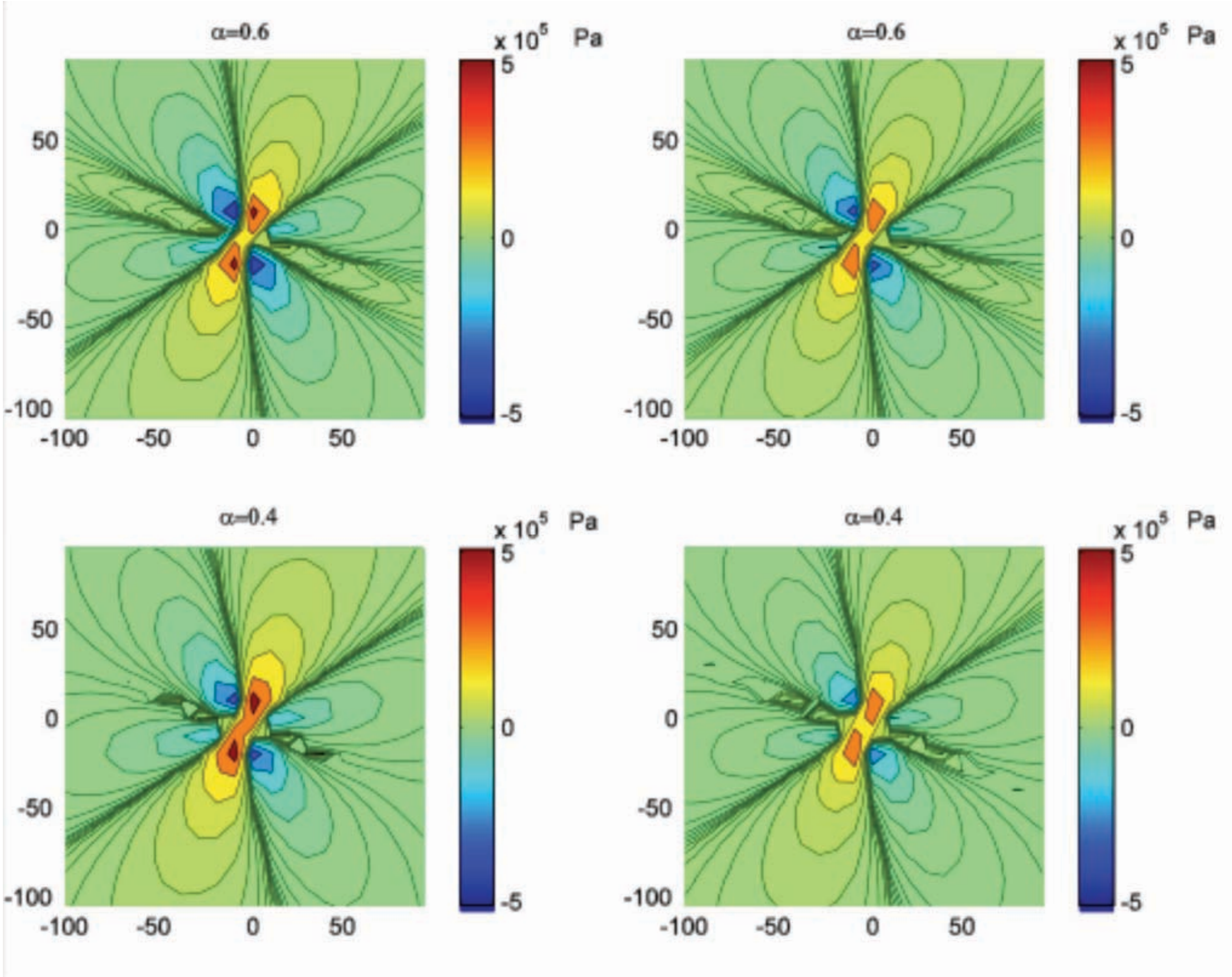


# $S_{yy}$ pattern with spring model



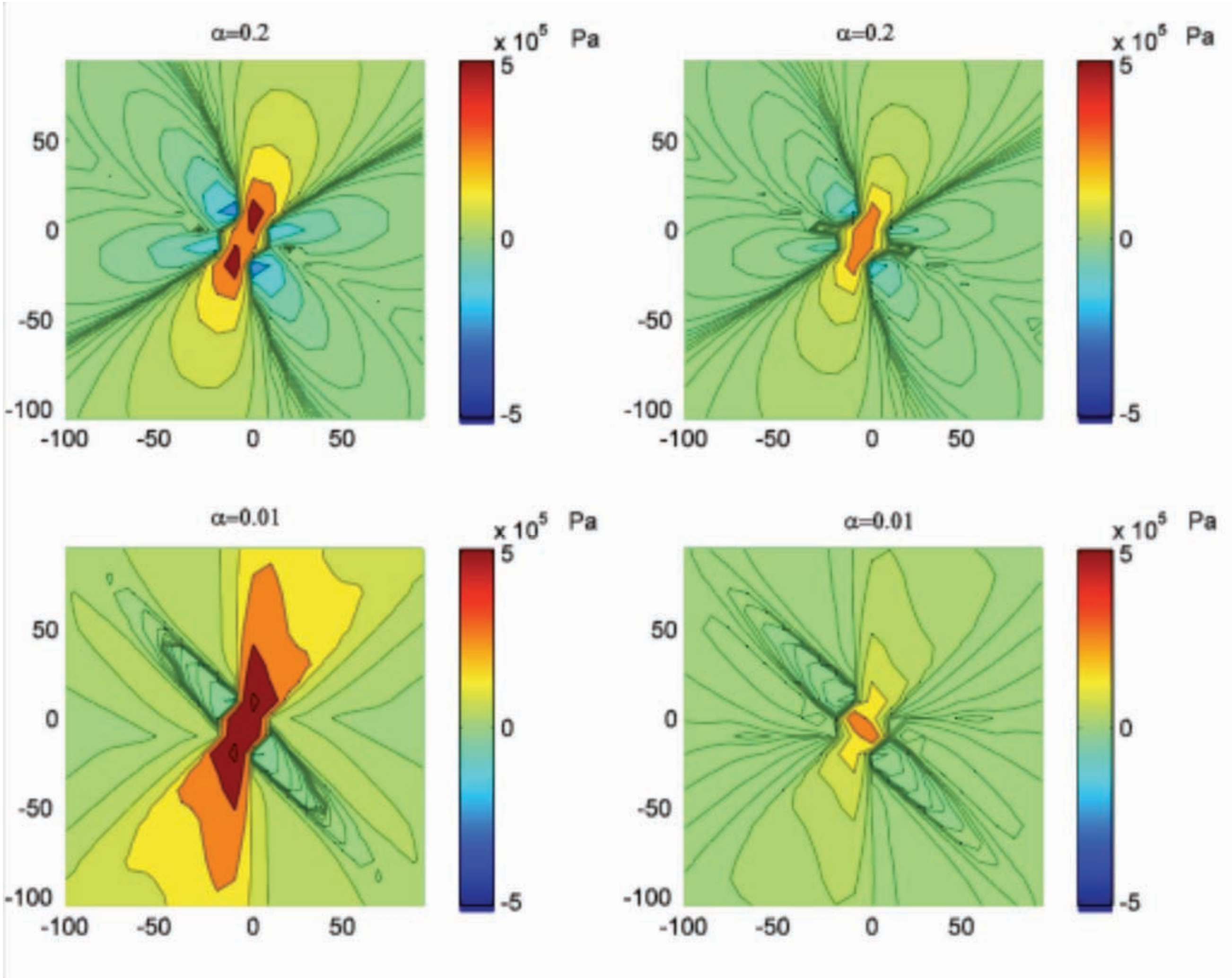

crack dislocation

# $S_{xx}$ pattern with spring model




crack dislocation

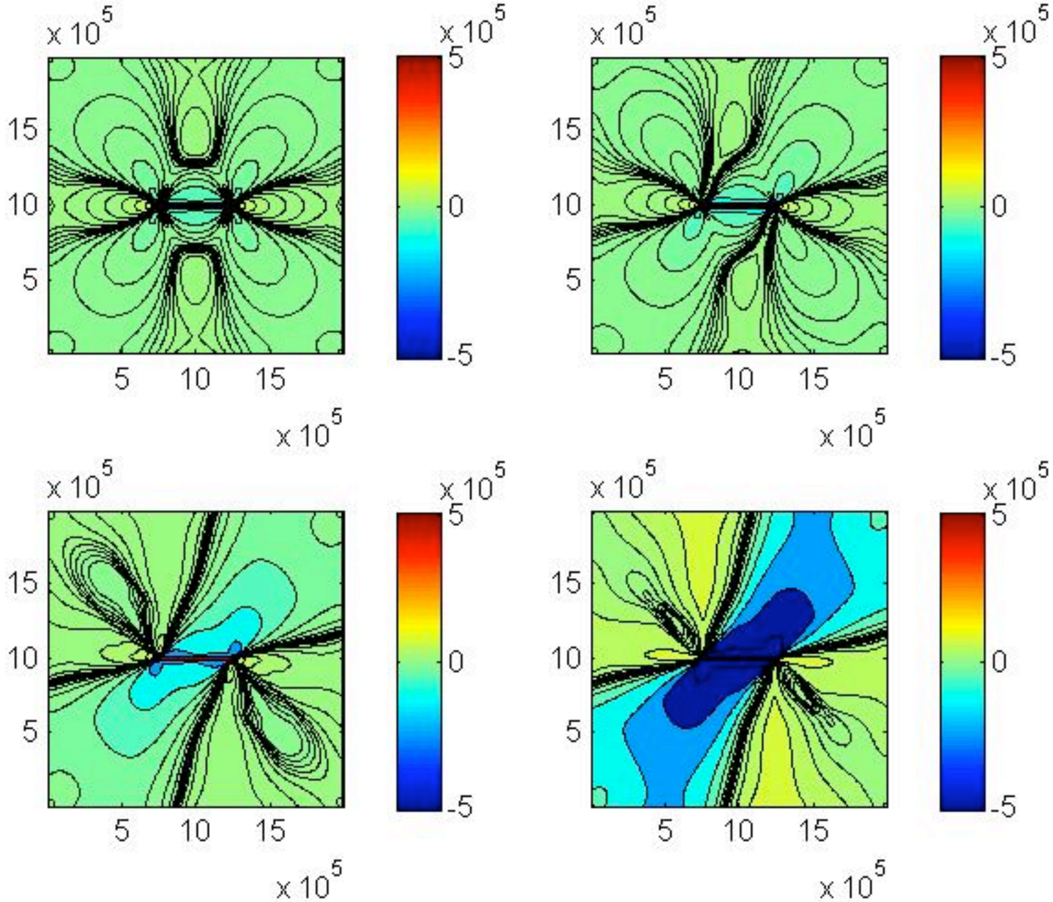
# $S_{xx}$ pattern with spring model




crack dislocation

# The finite fault problem: spring crack model

Horizontal/dextral fault spanning 50 cells – computation of $S_{xy}$




$\alpha = 1$

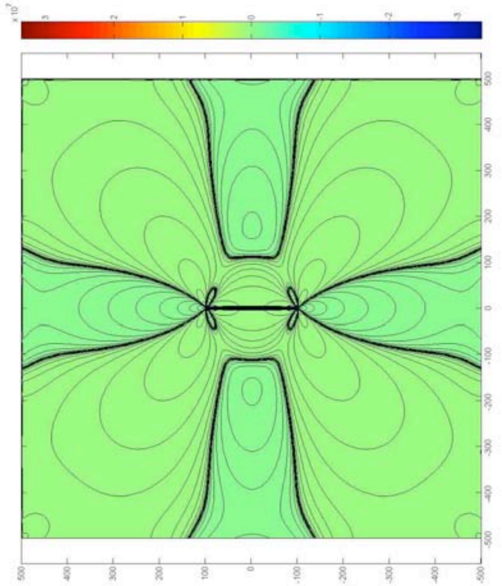
$\alpha = 0{,}50$

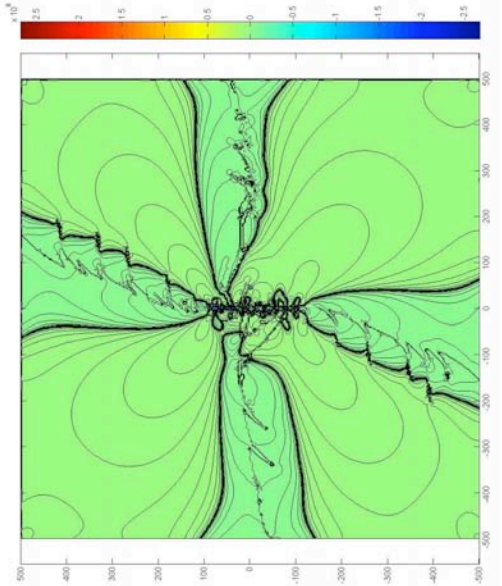
$\alpha = 0.10$

$\alpha = 0.01$

# The finite fault problem : FEM approach - $S_{xy}$

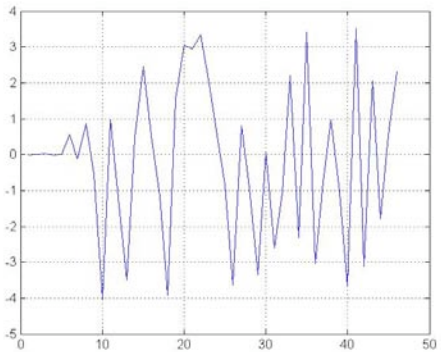
$\alpha = 1$





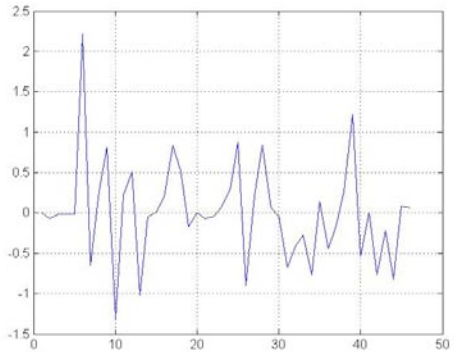
$\alpha = 0.10$

$U_s$

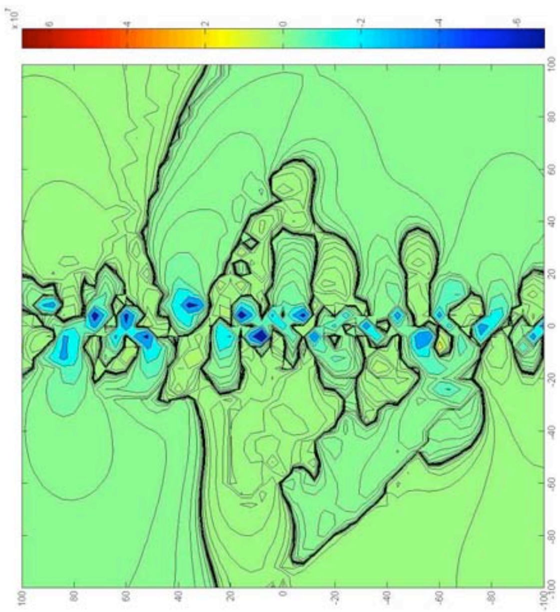


$\alpha = 0.10$

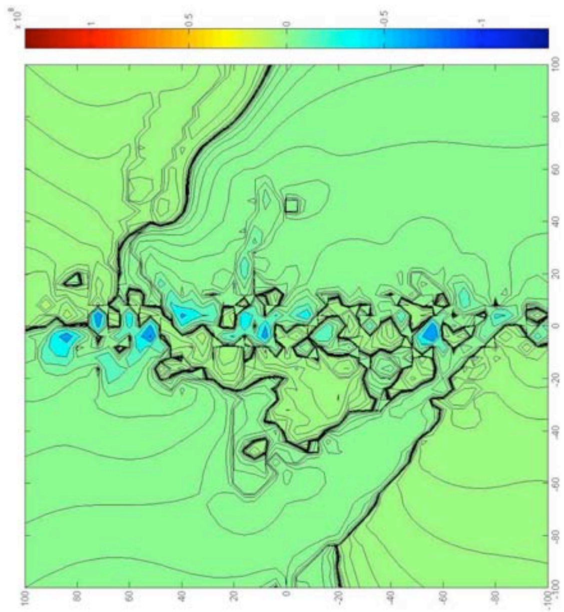
$U_n$



# The finite fault problem : FEM approach



$S_{xx}$



$S_{yy}$

This spatial structure varies from one iteration to the other.

# Comments on the effect of asymmetry

The idea of mechanical asymmetry and/or feedback between local damage and stress decay is not new, but it is the first time that it is implemented in a real 2D plane problem that could be applied to seismotectonics, after improving our present model up to an asymmetric poroelastic model, taking account fluid circulation.

The existence of elastic asymmetry in the crust has been proposed on the basis of the Manyi ($M_w = 7.6$) earthquake using SAR interferometry data [5] to explain asymmetric displacement patterns. However, the authors didn't consider the possibility that the asymmetry could also modify the long-range decay of the stress field.

The important question is now to check if this model also holds at seismogenic depth. In that case, seismic triggering of an event by another one could occur, or be inhibited, at very large distances in cases where $\gamma$ is low. Testing this hypothesis on real data is not simple, as this rheology is not linear, so that the cumulative effect of successive events is not the sum of individual effects. Stress field evolution may then have much sharper transitions in space and time than predicted by models of linear elasticity, a behavior reminiscent of the mechanics of granular media (which can be seen as the limit $\alpha=0$). The consequence is that, to compare with the standard stress transfer mechanism, we need to know in details the state of stress within the crust prior to an event to map predicted stress transfer lobes onto subsequent seismic activity.

Moreover, the complete asymmetric poroelastic solution will depend on time: at short times, the solution will be close to a standard linear elastic solution, whereas at long times it will be close to the nonlinear solution. We thus still need more computations to estimate relaxation times in such a model.

# Implications for earthquake prediction

Proving or refuting this model is of prime importance for understanding the spatial and temporal patterns of earthquakes, including precursory activity. In a recent numerical work, [D. Wheatherley, Mora P. and Xia M., preprint (2003)] studied the progressive damage on a fault plane before its macroscopic rupture. They employ cellular automaton techniques to simulate tectonic loading, rupture events and strain/stress redistribution. The Green function for redistribution is taken to vary as $r^{-p}$, where $p$ is a parameter.

They note that if $p<2$, large events are preceded by a clear acceleration of energy release by the system, which reminds of the critical earthquake model. If $p>2$, the trend of energy release before a large event is linear, and the large event is unpredictable (using that trend criterion).

Their model doesn't map exactly onto ours, but we can expect that if the damage/asymmetry parameter $\alpha$ is under a certain threshold (still to be determined), then $\gamma$ (which is roughly equivalent to their $p$) would be low enough for the critical earthquake scenario to occur (reminding that our $\gamma$ is also a function of $\theta$), so that large events would be predictable. In the other hands, if there are some zones within which $\alpha$ is too large, then large events would be unpredictable.

If the predictability of large events from seismic time-series relies on the exponent of the Green function of stress transfer, then we have pointed out a very simple physical mechanism allowing to tune that exponent.

Large scale fluctuations of fluid pressure from lithostatic (so that cracks can open or close according to arbitrarily small stress fluctuations) to infra-lithostatic (cracks remain always closed) could then explain why in some cases large events are preceeded by strain energy release acceleration, while the opposite holds in other cases.

[1] V.I. Keilis-Borok and L.N. Malinovskaya, J. Geophys. Res., 69, 3019 (1964)

[2] D.D. Bowman, G. Ouillon, C.G. Sammis, A. Sornette and D. Sornette, J. Geophys. Res., 103, 24359-24372 (1998)

[3] L. Knopoff, T. Levshina, V.I. Keilis-Borok and C. Mattoni, J. Geophys. Res., 101, 5779 (1996)

[4] E. Roeloffs and E. Quilty, Pure Appl. Geophys., 149, 21 (1997)

[5] G. Peltzer, F. Crampé and G. King, Science, 286, 272 (1999)

[6] D. Wheatherley, Mora P. and Xia M., preprint (2003)

-G. Ouillon and D. Sornette, Search for Direct Stress Correlation Signatures of the Critical Earthquake Model, in press in Geophysical Journal International (http://arXiv.org/abs/cond-mat/0208498)

-G. Ouillon and D. Sornette, Long-range Static Directional Stress Transfer in a Cracked, Nonlinear Elastic Crust Elsevier FGCS (Future Generation Computer Systems) http://www.elsevier.com/locate/future (http://arXiv.org/abs/physics/0304054)