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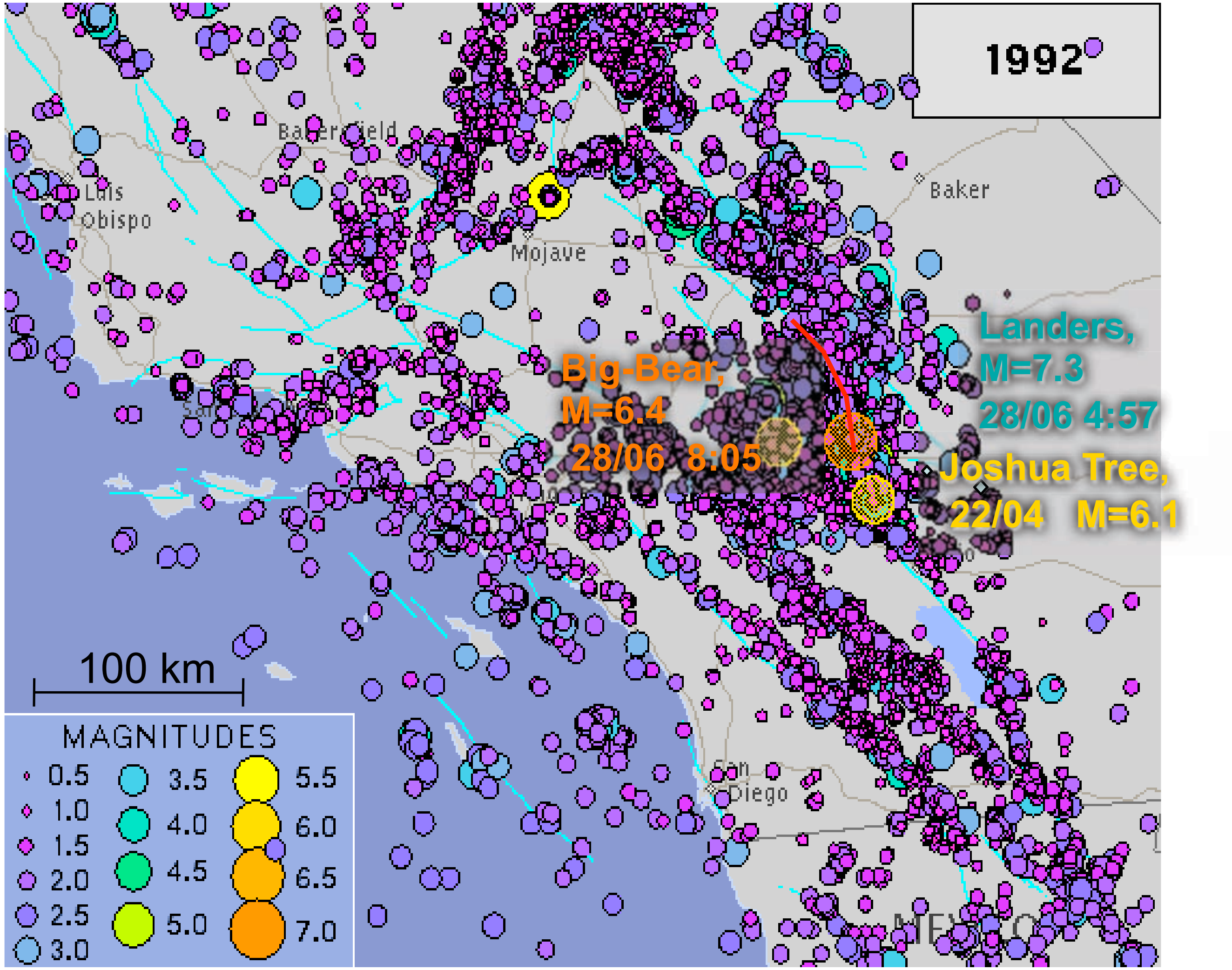
Multifractal Omori-Utsu Law for Earthquake Triggering

Theory and empirical tests

Spatial and temporal organization of seismicity in Californie

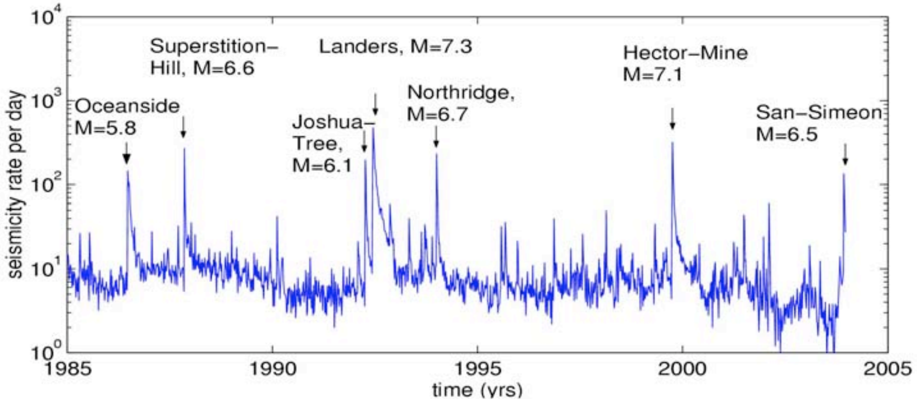
Landers
28 june 1992
M=7.3



Aftershocks Time Series

Earthquake catalogs appear as a succession of bursts of activity – each event, whatever its magnitude, is followed by a decay of activity.

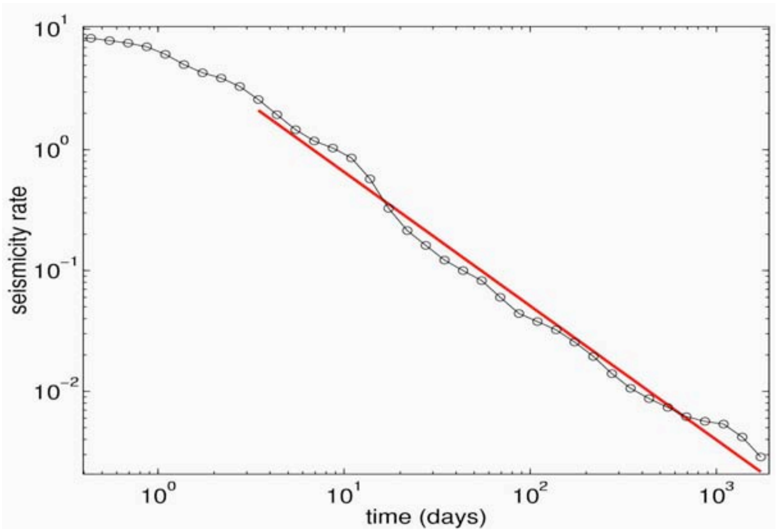
Events occurring during this relaxation phase are usually refered to as aftershocks or triggered seismicity.



The time decay of triggered seismicity rate is measurable after large events, as their number of aftershocks is large enough.

This time decay is known as the Omori-Utsu law (1894):

$$N(t)\,dt \sim t^{-p}\,dt$$



The exponent p is close to 1 for most sequences.

Each event thus defines a **mathematical singularity.**

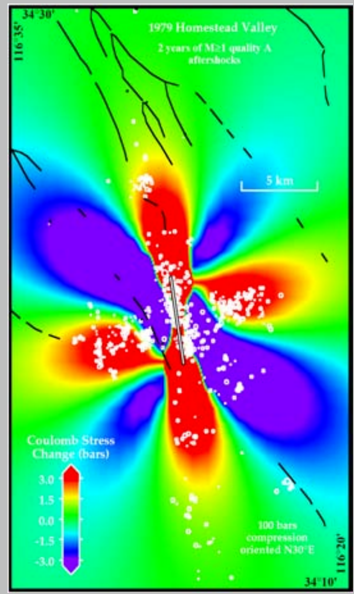
Single body approach

1

Single body approach

1

Stein et al, 1994



Spatial correlations between Coulomb stress and aftershocks but no account of the stress fluctuations due to aftershocks.

Single body approach

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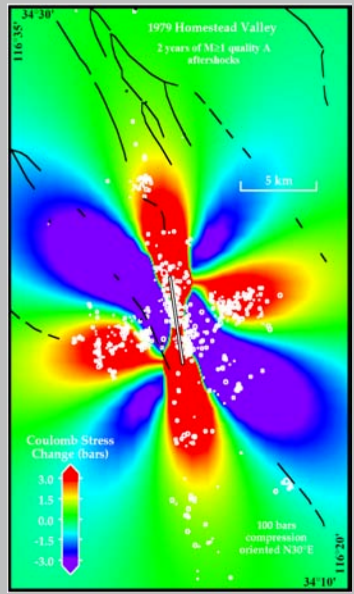

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Earthquake nucleation activated by static stress

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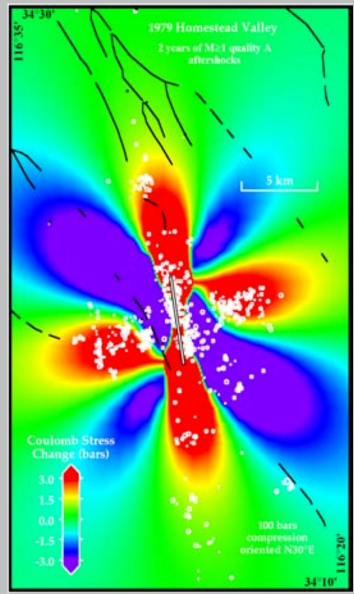

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State and rate friction

Dieterich (1994)

Single body approach

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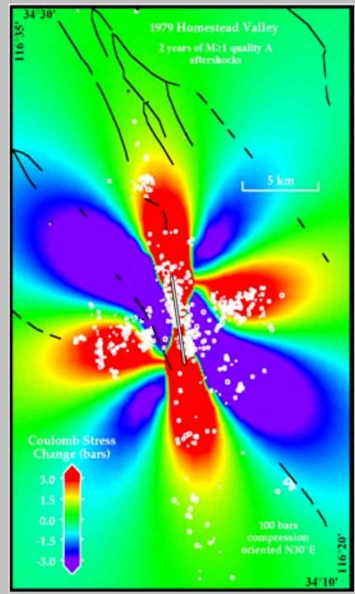

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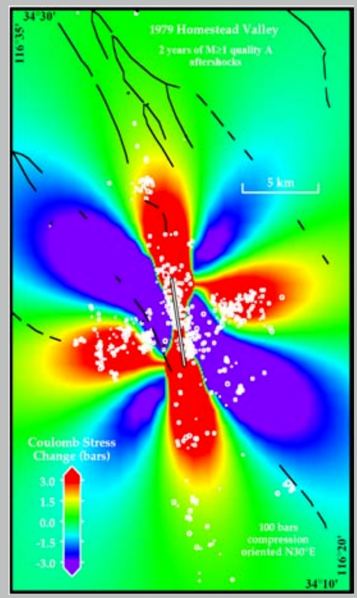

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Time dependence

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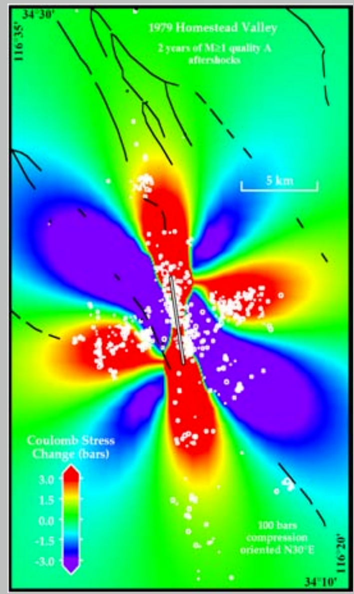

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Earthquake nucleation activated by static stress

Time Dependence

State and rate friction
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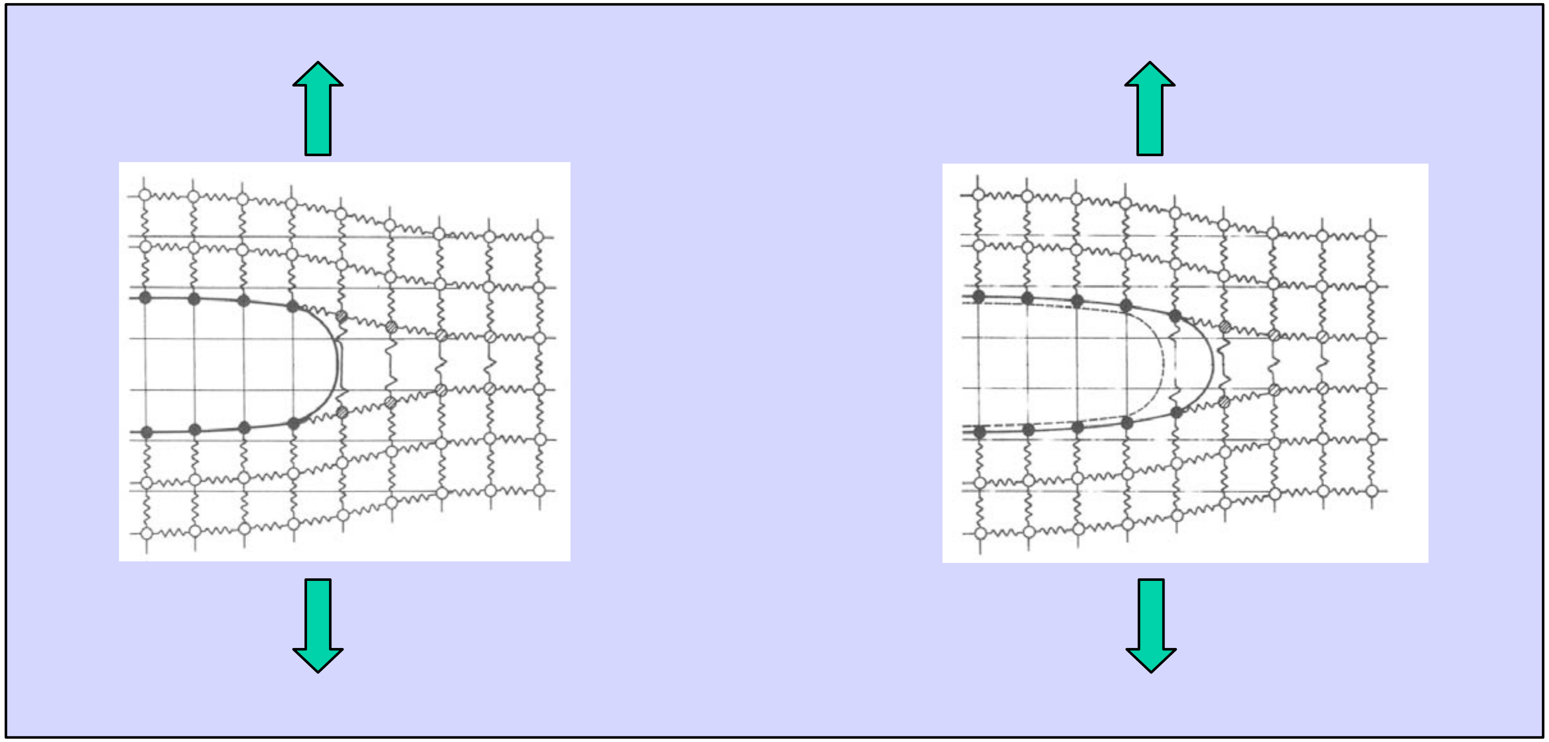
Stress corrosion
Yamashita and Knopoff (1987)

Omori law with p=1 independent of *M*

Slow crack growth due to stress corrosion

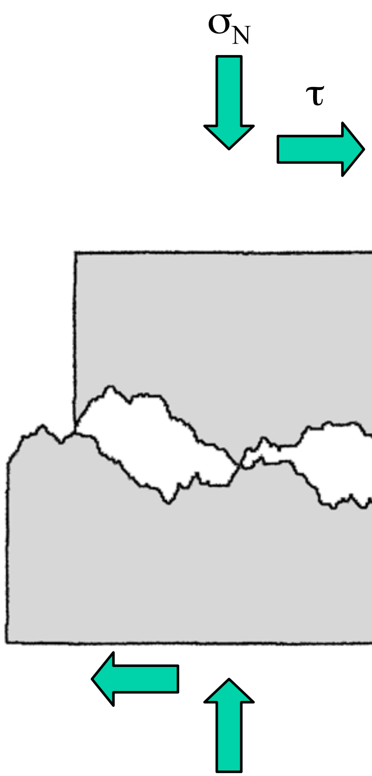


$$\frac{dL}{dt} \sim L^n \qquad L < L_c$$

$$\frac{dL}{dt} = V_d \qquad L \geq L_c$$

Thermally activated process driven by stress

State and rate friction law



$$\tau = \mu \sigma_N$$

$$\mu = \mu_0 + A \ln\left(\frac{v}{v_0}\right) + B \ln\left(\frac{\theta}{\theta_0}\right)$$

Thermally activated process driven by stress

The need of a manybody approach

2

The need of a manybody approach

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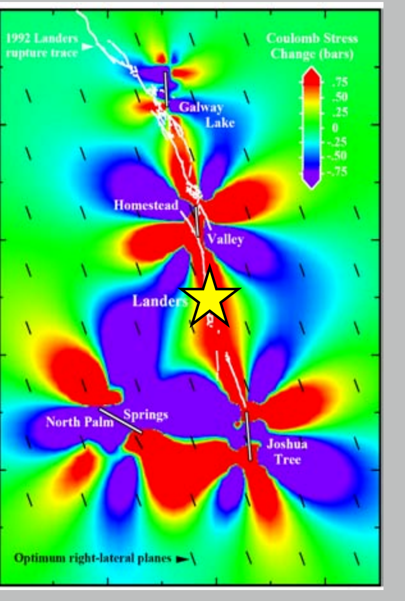

Complex previous history with known or unknown rupture parameters

Stein et al, 1994

The need of a manybody approach

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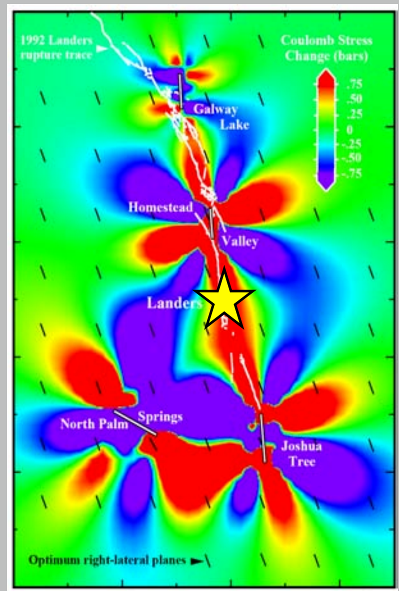

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Time dependence

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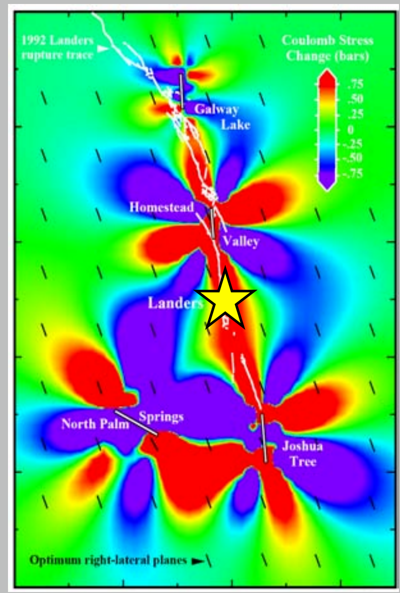

Stein et al, 1994

Complex previous history with known or unknown rupture parameters

Earthquake nucleation activated by static stress

Time dependence

State and rate friction

Stress corrosion

Omori law with **p=???**

The physical model : thermal activation driven by stress

3

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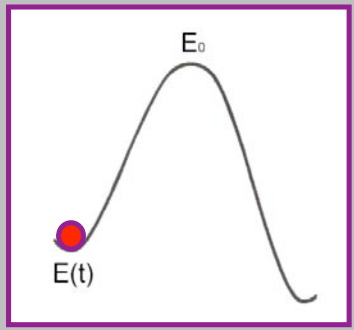
3

Before the shock



The physical model : thermal activation driven by stress

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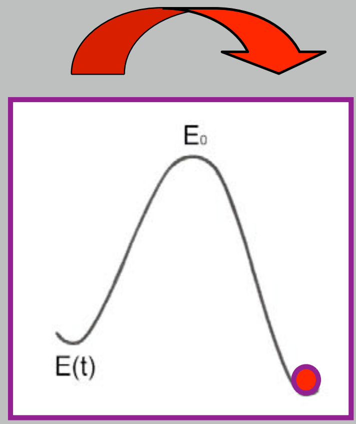

After the shock

Energy barrier = E_0-E(t)

The physical model : thermal activation driven by stress

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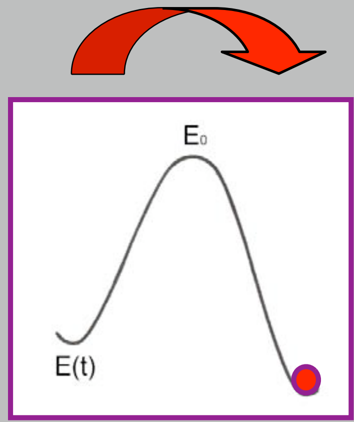

After the shock

Energy barrier = E_0-E(t)

Arrhenius law for the activation rate: $\lambda(t) = \lambda_0 \exp\left(-\frac{E_0 - E(t)}{kT}\right)$

The physical model : thermal activation driven by stress

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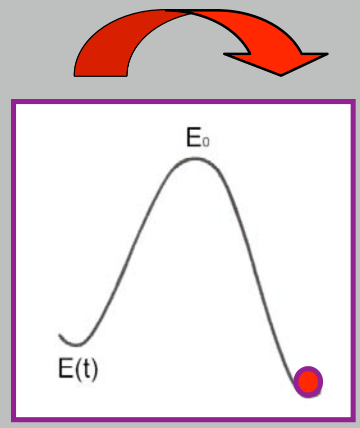

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stress barrier = $\sigma_0 - \sigma(t)$

$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V\right)$$

The physical model : thermal activation driven by stress

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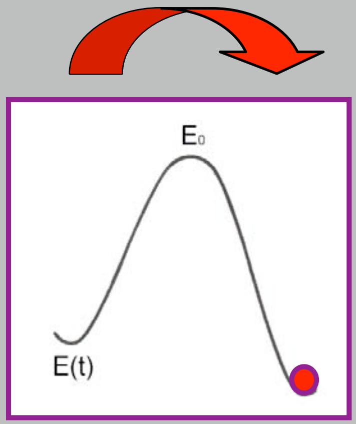

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stress barrier = $\sigma_0 - \sigma(t)$

$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V\right)$$

Compatible with state-and-rate friction, stress corrosion, …

$\lambda(t)$: instantaneous rate

λ_0 ~ average nucleation rate

σ_0 : material strength

$\sigma(t)$: applied stress

V : activation volume

T : temperature

k : Boltzmann constant

Experiments by Zhurkov   Int. J. Fract. Mech. **1**, 311 (1965)

$$\ln \tau = A - B\,\sigma$$

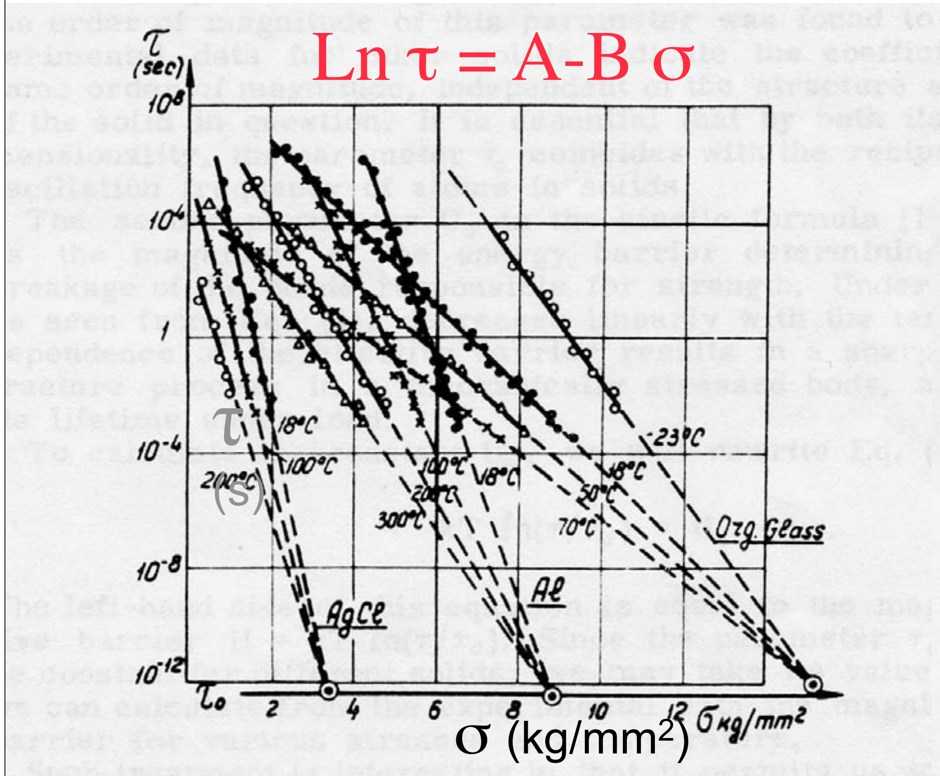

Fig. 5. Time and temperature dependence of the lifetime of solids on stress.
1. Silver chloride (Reference 4)
2. Aluminum (Reference 5)
3. Plexiglas (Reference 6)

$$\tau = \tau_o \exp\left(\frac{U}{k_B T}\right)$$

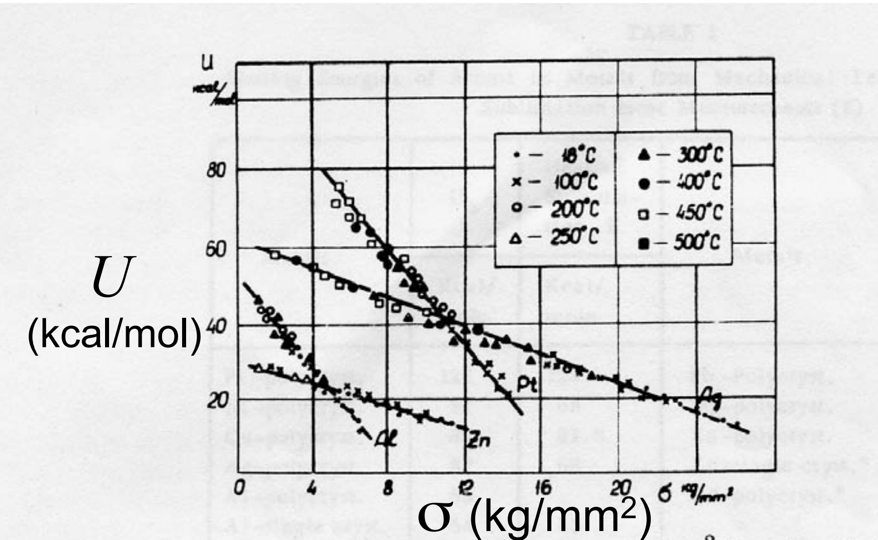

Fig. 6. Effective barrier U kkal/mol vs. tensile stress σ kg/mm² for polycrystall

Empirical energy barrier

$$U = U_o - \alpha\sigma$$

où U_o : énergie de sublimation

A possible mechanism : <u>thermal activated process</u>

Taking account of history and boundary conditions

4

$$\lambda_0' = \lambda_0 \exp\left(-\frac{\sigma_0}{kT} V \right)$$

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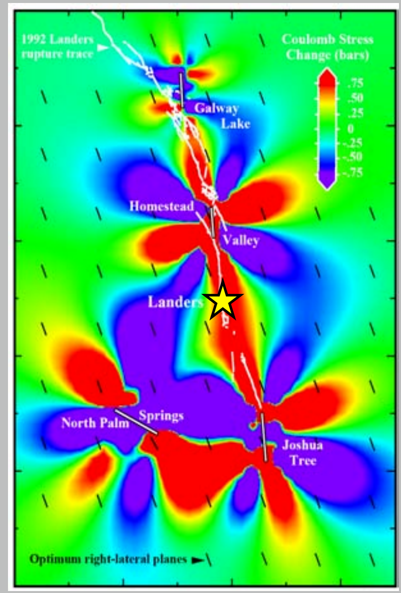

Stress is assumed to be a scalar for the sake of simplicity

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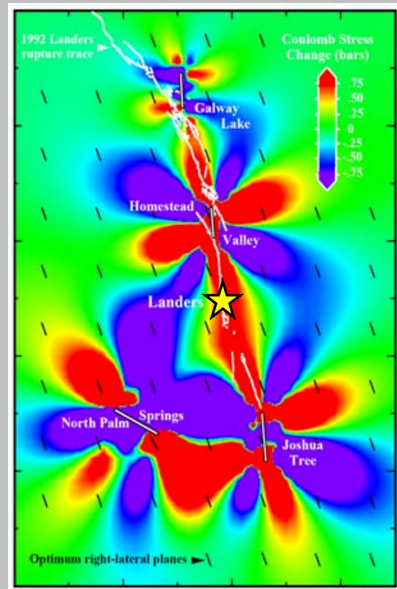

$$\lambda(\vec{r},t) = \lambda_0' \exp\left(\frac{\sigma(\vec{r},t)}{kT}V\right)$$

Stress is assumed to be a scalar for the sake of simplicity

$$\sigma(\vec{r},t) = \sigma(\vec{r})_{far\ field} + \int_{-\infty}^{t} \int_{space} dN\left[d\vec{\rho} \times d\tau\right] \Delta\sigma(\vec{\rho},\tau)\, G(\vec{r}-\vec{\rho},t-\tau)$$

local stress

Taking account of history and boundary conditions

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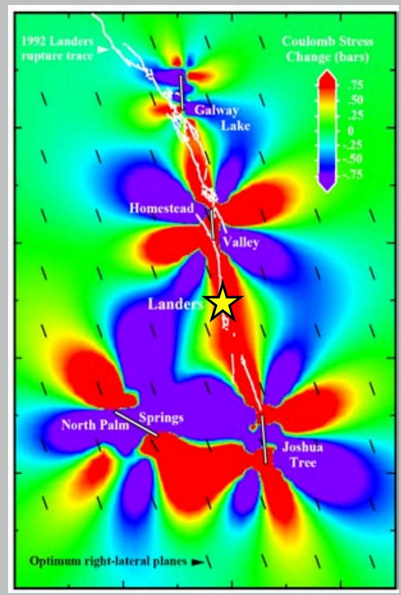

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local stress

tectonic loading

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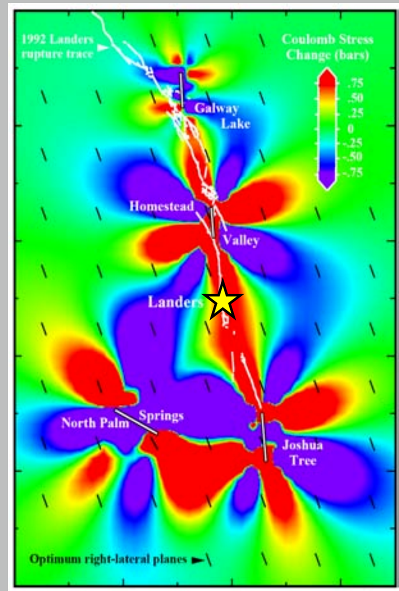

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local stress — tectonic loading — **Stress fluctuations induced by all past events in the system**

Taking account of history and boundary conditions

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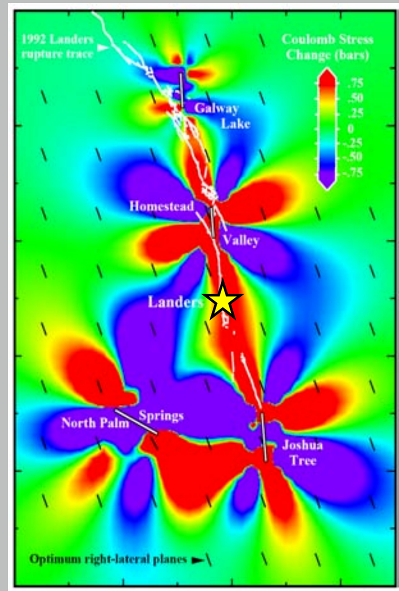
$$\lambda_0' = \lambda_0 \exp\left(-\frac{\sigma_0}{kT}V\right)$$



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local stress

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Taking account of history and boundary conditions

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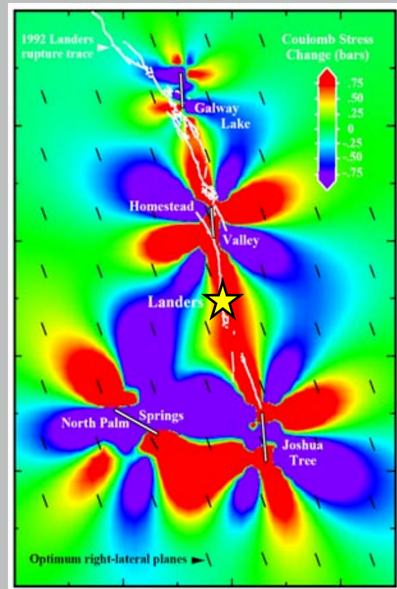

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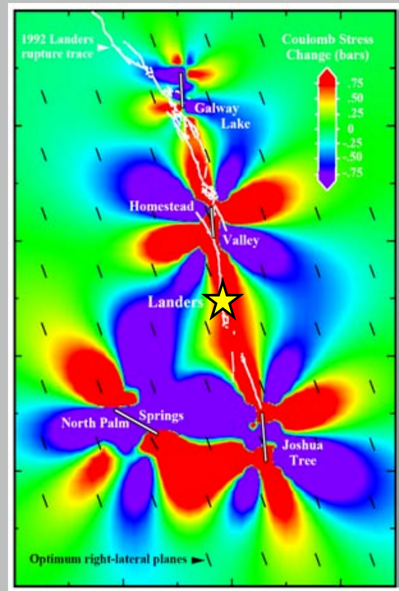

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local stress

tectonic loading

Time and space distribution of past sources

Taking account of history and boundary conditions

4

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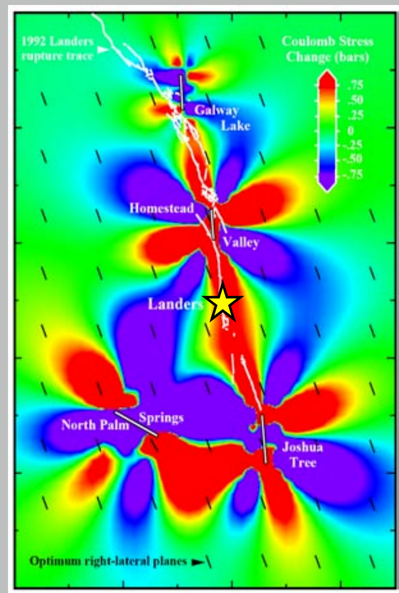

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local stress

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Time and space distribution of past sources

Stress fluctuations at sources

Taking account of history and boundary conditions

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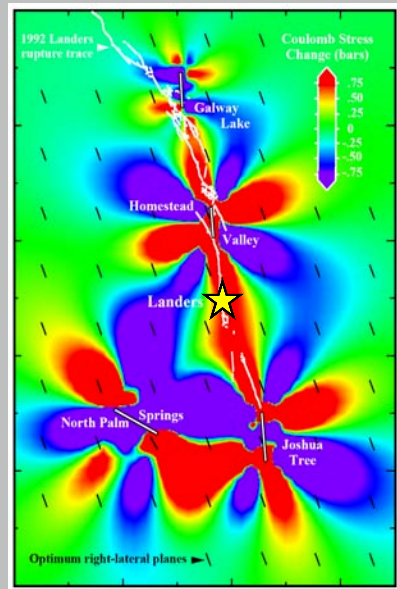

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local stress | tectonic loading | Time and space distribution of past shocks | Stress fluctuations at sources | **Green function for stress transfer**

A few working hypotheses

5

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Every shock is activated by stress and temperature according to Arrhenius law

A few working hypotheses

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Every shock of magnitude M triggers instantaneously 10^{qM} aftershocks (ETAS-like productivity law)

A few working hypotheses

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Every shock of magnitude M triggers instantaneously 10^{qM} aftershocks (ETAS-like productivity law)

Separation of variables : $G(\vec{r},t) = f(\vec{r}) \times h(t)$

A few working hypotheses

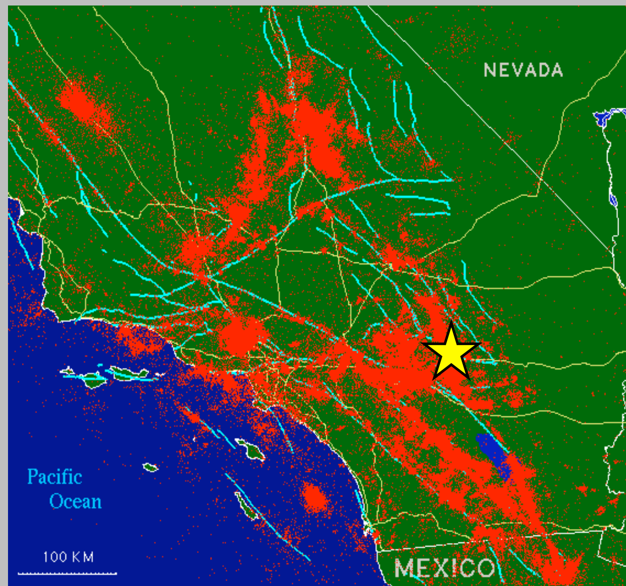
5

Every shock is activated by stress and temperature according to Arrhenius law

Every shock of magnitude M triggers instantaneously 10^{qM} other events

Separation of variables : $G(\vec{r},t) = f(\vec{r}) \times h(t)$

Stress fluctuations at ★ depend on the location of events (red dots), their rupture geometry, and on the spatial decay of the Green function. Most of these parameters are unknown, and some events even not recorded at all. Those fluctuations are thus considered as realizations of a random variable.



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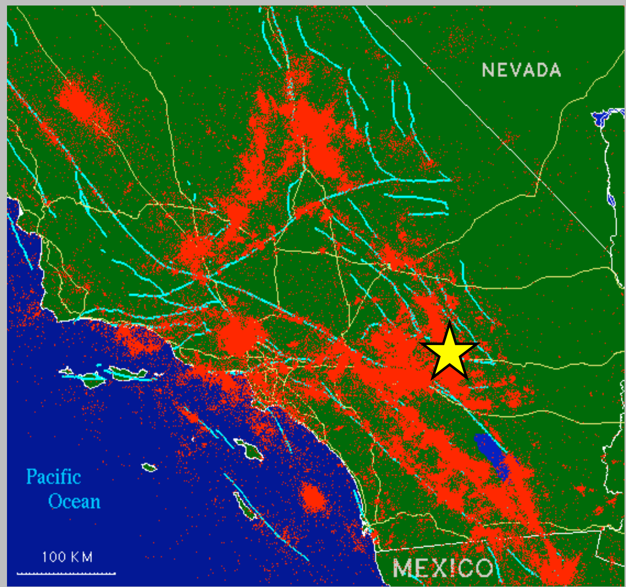
Separation of variables : $G(\vec{r},t) = f(\vec{r}) \times h(t)$

Stress fluctuations at location ☆ due to previous events :

$$P(\sigma_{fluc})\, d\sigma_{fluc} \approx \frac{C}{(\sigma_{fluc} + \sigma_{f1})^{+\mu}}\, d\sigma_{fluc}$$

Exponent μ depends on (and encapsulates) the spatial structure of the fault pattern, the GR law, as well as *f(r)*.



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Elastoviscoplastic rheology

$$h(t) = \frac{h_0}{\left(t + t_1\right)^{1+\theta}} \exp\left(-\frac{t}{\tau_M}\right)$$

Maxwell time

τ_M >> time scale of observations

$h(t)$: dislocations motion and unresolved seismicity

Relaxation after a magnitude M event

6

Relaxation after a magnitude M event

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We re-write in discrete form after spatial averaging :

$$\lambda(t) = \lambda_{tec} \exp\left[\frac{V}{kT} \sum_{past} \sigma_{fluc}(t_i)h(t - t_i)\right]$$

cf Ouillon and Sornette, JGR, 2005

λ_{tec} is the average seismicity rate, modulated by a time-varying activation term. The formulation is thus non-linear.

Relaxation after a magnitude M event

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$$P(\sigma_{fluc})\,d\sigma_{fluc} \approx \frac{C}{\left(\sigma_{fluc}+\sigma_{f1}\right)^{1+\mu}}\,d\sigma_{fluc}$$

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Relaxation after a magnitude M event

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λ_{tec} is the average seismicity rate, modulated by a time-varying activation term. The formulation is thus non-linear.

$$if \quad \mu(1+\theta) = 1$$

$$\lambda(t) \propto t^{-p(M)} \qquad p(M) = aM + b$$

$$P(\sigma_{fluc})\, d\sigma_{fluc} \approx \frac{C}{(\sigma_{fluc} + \sigma_{f1})^{1+\mu}}\, d\sigma_{fluc}$$

$$h(t) = \frac{h_0}{(t + t_1)^{1+\theta}} \exp\left(-\frac{t}{\tau_M}\right)$$

Power-law relaxation rate of aftershocks increases with the size of energy fluctuations => **multifractality**

Multifractal **S**tress **A**ctivation model

Theoretical predictions using tail covariance (Ide-Sornette, 2001)

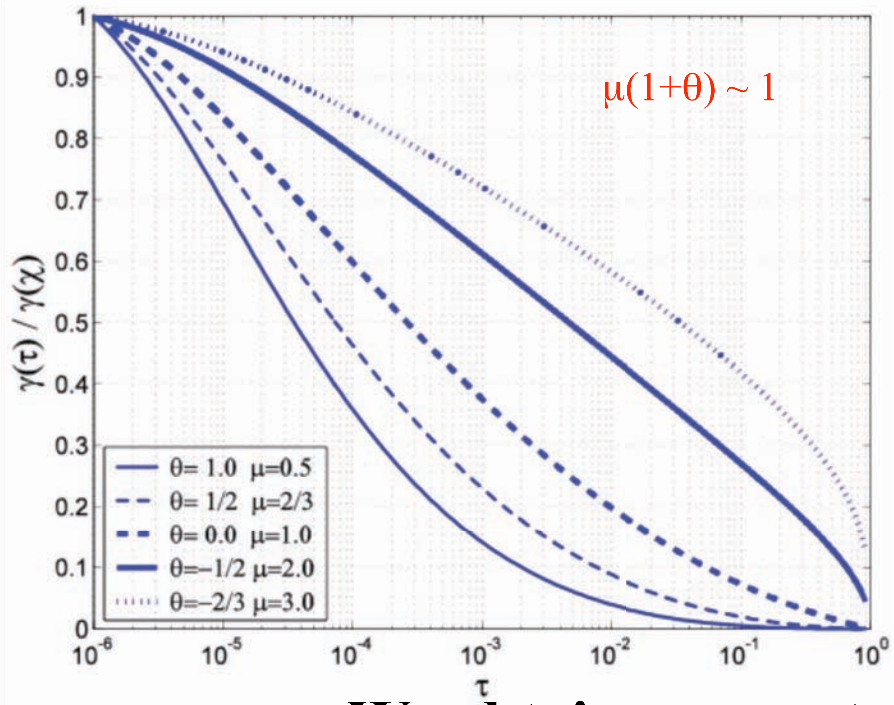
$$\Pr[\lambda(t) > \lambda_q | \lambda_M] = \Pr[e^{\beta\omega(t)} > \frac{\lambda_q}{\lambda_{\rm tec}} | \omega_M] = \Pr[\omega(t) > (1/\beta) \ln\left(\frac{\lambda_q}{\lambda_{\rm tec}}\right) | \omega_M]$$

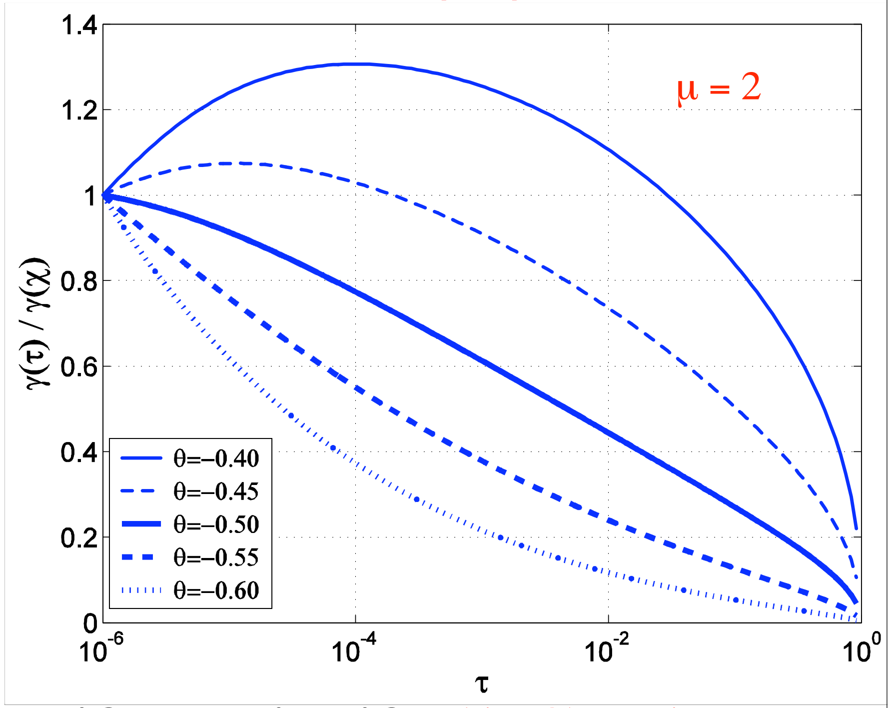
$$\lambda_q(t) = A_q \ \lambda_{\rm tec} \ e^{\beta\gamma(t)\omega_M}$$

$$\gamma(t) = \frac{h_0^2}{\Delta t^{2/\mu}} \left(\frac{1}{t^{2m-1}} \int_{c/t}^{\frac{T+c}{t}-1} dy \frac{1}{(y+1)^m} \frac{1}{y^m} \right)^{\frac{2}{\mu}}$$

$$m = (1+\theta)\mu/2.$$

Since $\gamma(t) \sim \ln(t)$ and $\omega_m \sim M$, we obtain $p(M) = a M + b$





We obtain an exact multifractality if $\mu(1+\theta) \sim 1$

Building aftershocks time series

8

Building aftershocks time series

8

Select all events within a given magnitude range [M_1;M_2].

Building aftershocks time series

Select all events within a given magnitude range **[M_1;M_2]**.

Define a spatial window (*R*=2*L*)

Building aftershocks time series

Select all events within a given magnitude range **[M_1;M_2]**.

Define a spatial window (***R=2L***)

Define a time window (*T=1 year*)

Building aftershocks time series

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Select all events within a given magnitude range **[M_1;M_2]**.

Define a spatial window (***R=2L***)

Define a time window (***T=1 year***)

Consider all events within [0,R]x[0,T] as aftershocks

Building aftershocks time series

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Select all events within a given magnitude range **[M_1;M_2]**.

Define a spatial window (***R=2L***)

Define a time window (***T=1 year***)

Consider all events within [0,R]x[0,T] as aftershocks

If the starting event is the aftershock of a larger event, remove it and its aftershocks

Building aftershocks time series

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Define a time window (***T=1 year***)

Consider all events within [0,R]x[0,T] as triggered events

If the starting event is the aftershock of a larger event, remove it and its aftershocks

Stack all individual aftershocks series

Building aftershocks time series

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Time distribution of aftershocks

$$N(t) = A \times t^{-p} + B(t)$$

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Omori law

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Stack all individual aftershocks series

Time distribution of aftershocks

background

$$N(t) = A \times t^{-p} + B(t)$$

Omori law

The Scaling Function Analysis (SFA)

9

The Scaling Function Analysis (SFA)

9

The background term is regular

$$B(t) = b_0 + b_1 \times t + b_2 \times t^2 + \ldots + b_n \times t^n$$

The Scaling Function Analysis (SFA)

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Scaling Function

$$\Psi\left(\frac{t}{s}\right) = \left(a_0 + a_1 \times \left(\frac{t}{s}\right) + a_2 \times \left(\frac{t}{s}\right)^2 + \ldots + a_m \times \left(\frac{t}{s}\right)^m\right) \times \exp\left(-5\left(\frac{t}{s}\right)^2\right)$$

Time scale s is chosen by the user

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Time scale s is chosen by the user

All a_i's and m are chosen such that

$$\int_0^{\infty} \Psi\left(\frac{t}{s}\right) B(t)\,dt = 0$$

(depending only on n)

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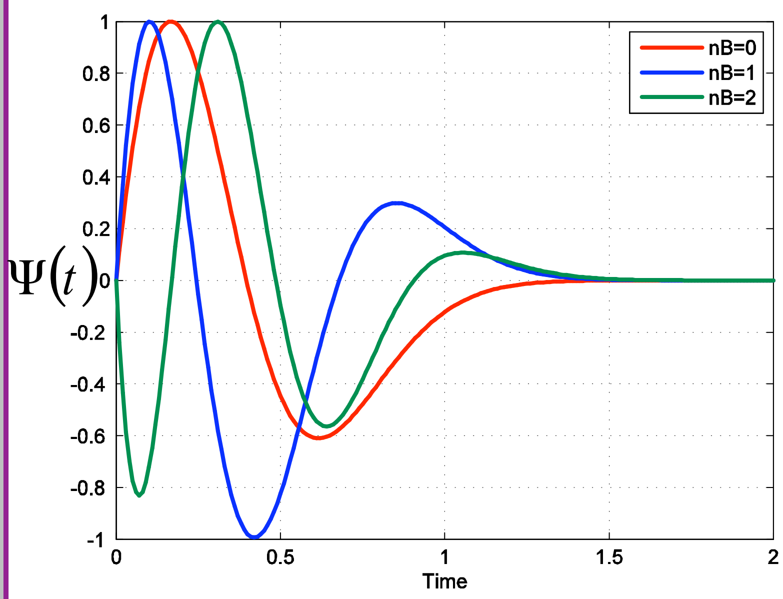

The Scaling Function Analysis (SFA)

10

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The Scaling Function Analysis (SFA)

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Scaling Function Analysis Coefficient

$$C(s) = \int_0^\infty \Psi\left(\frac{t}{s}\right) N(t)\,dt = s^{1-p} A \int_0^\infty \Psi(t) \times t^{-p}\,dt + \int_0^\infty \Psi\left(\frac{t}{s}\right) B(t)\,dt$$

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$$C(s) \propto s^{1-p}$$

An example on a real catalog

11

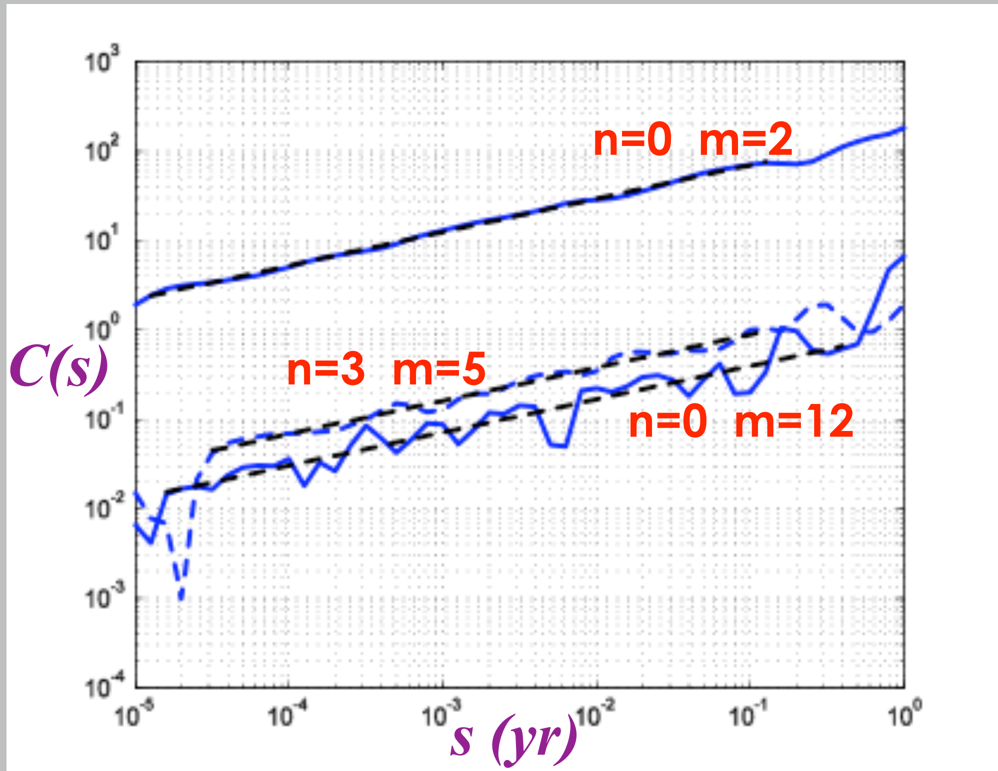
An example on a real catalog

11

Ouillon et al, 2007 submitted to GJI

3 different scaling functions yield power law scaling with the same value of p.

SCEC – M[2.5;3.0] – **p=0.63**

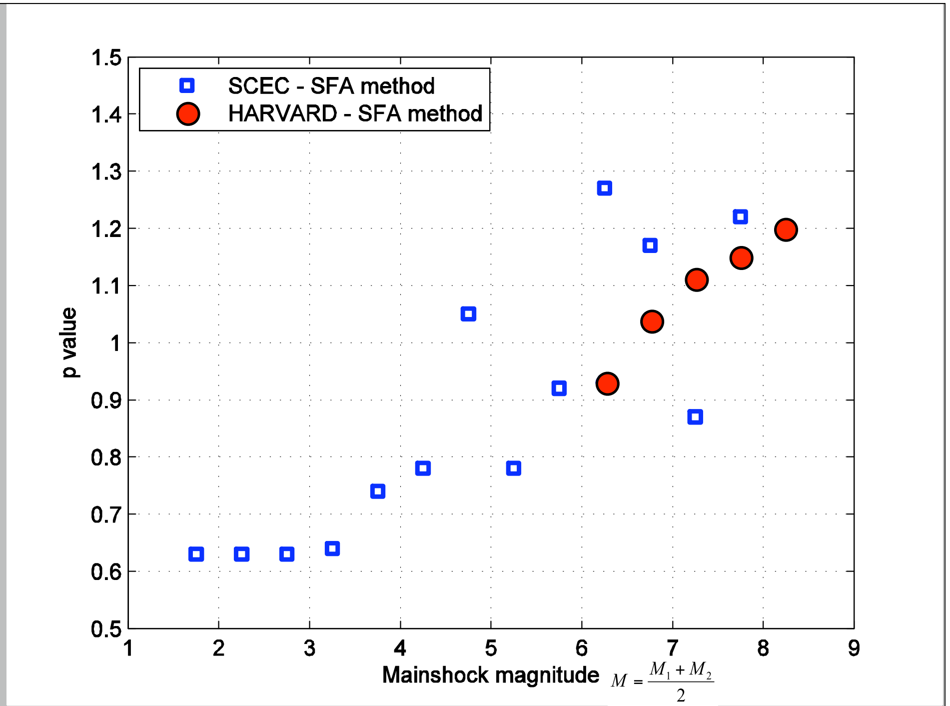


Results on real catalogs

12

Results on real catalogs

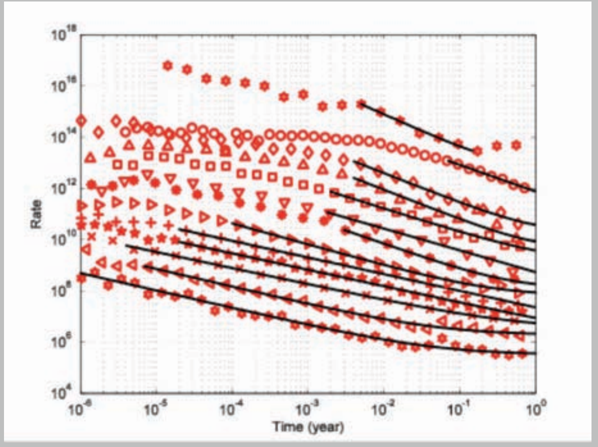
12



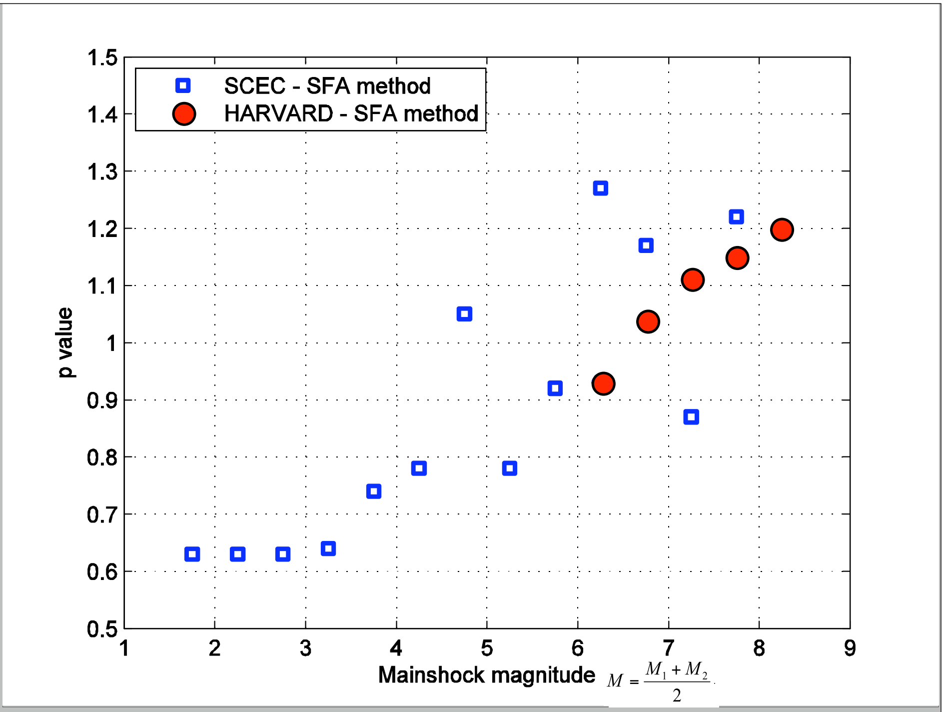
Results on real catalogs

12

Construction of standard bined time series and least squares fits.



$$N(t) = A \times t^{-p} + b_0$$



Results on real catalogs

12

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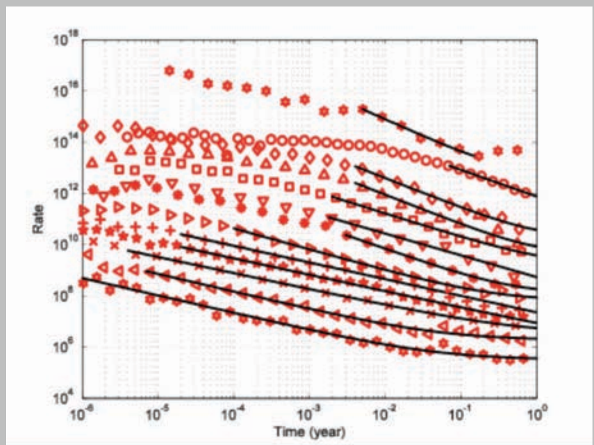

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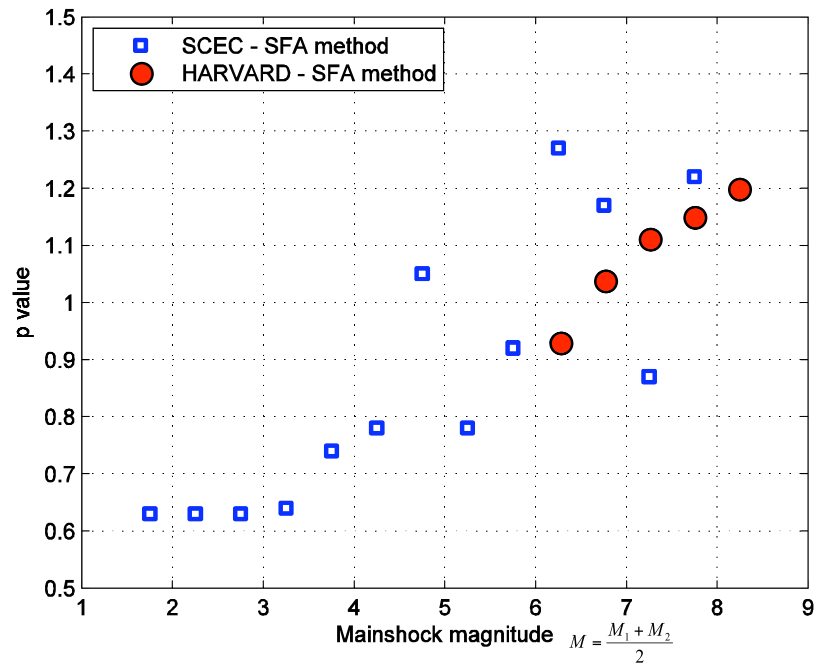
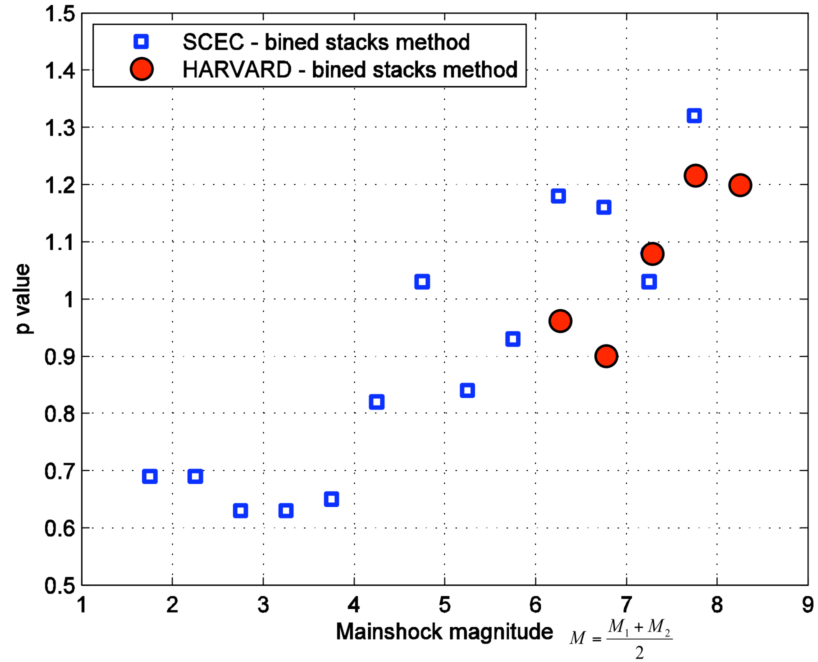
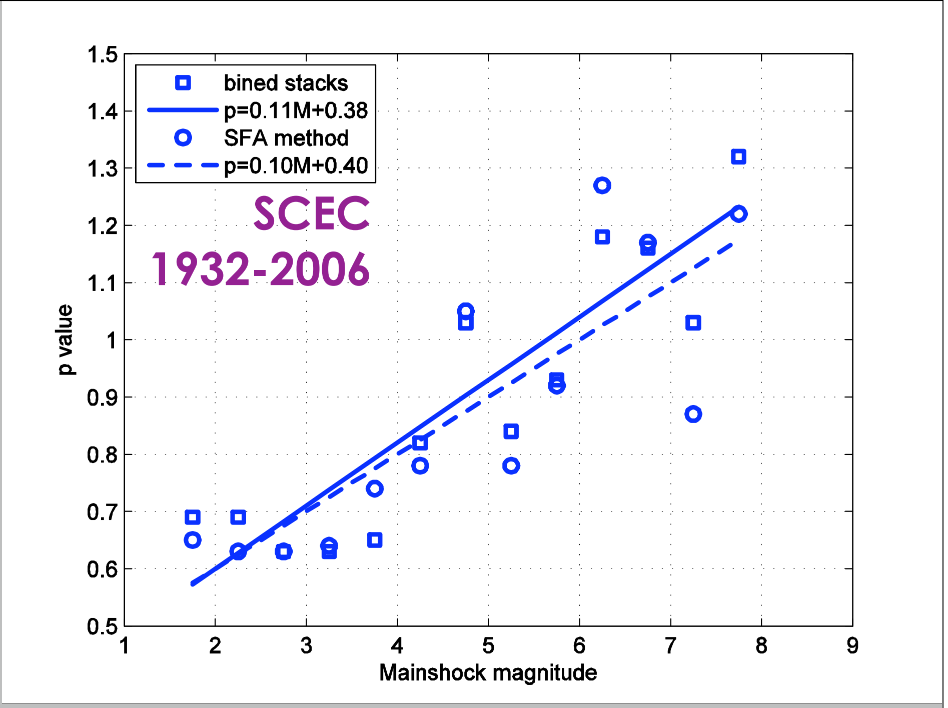




Results on real catalogs

13

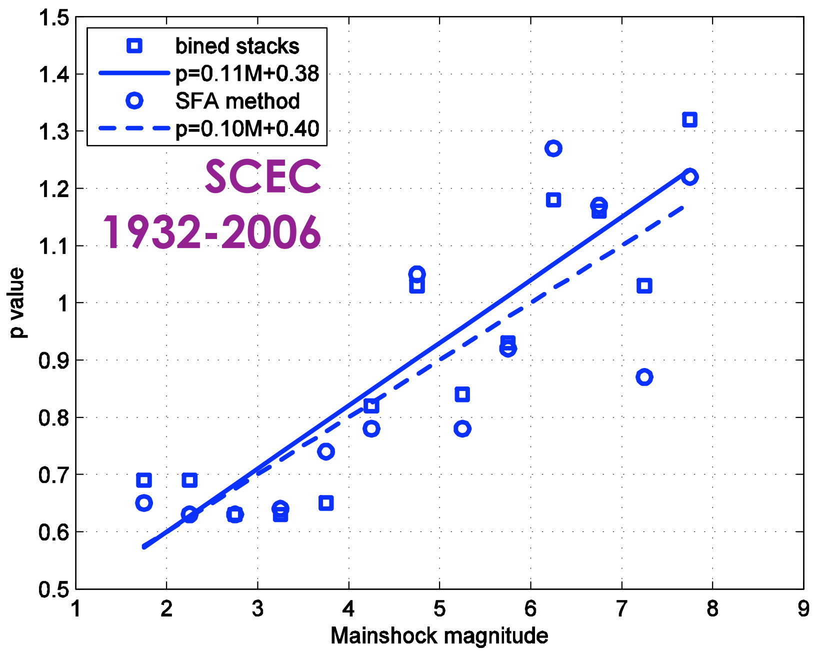


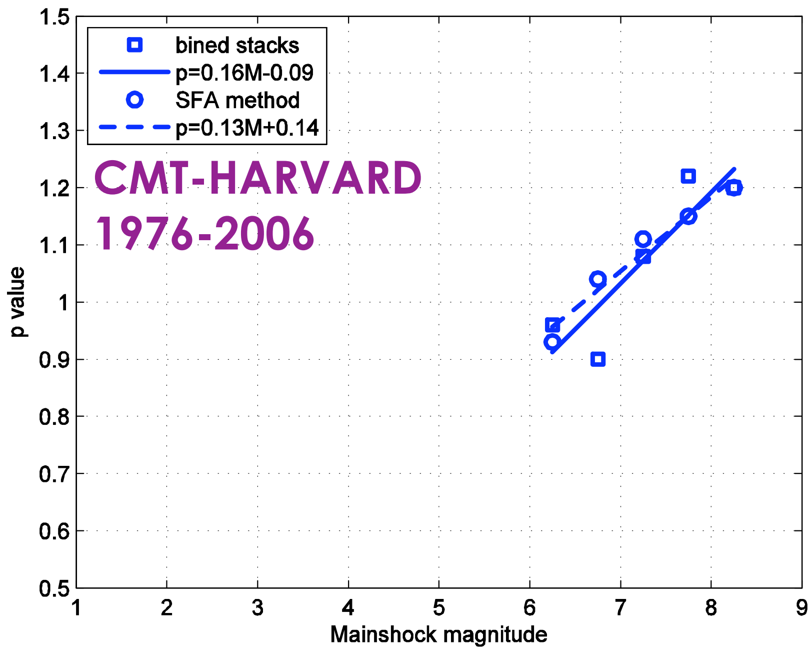
SFA : **p(M)=0.11M+0.38**

Bins : **p(M)=0.10M+0.40**

Results on real catalogs

13



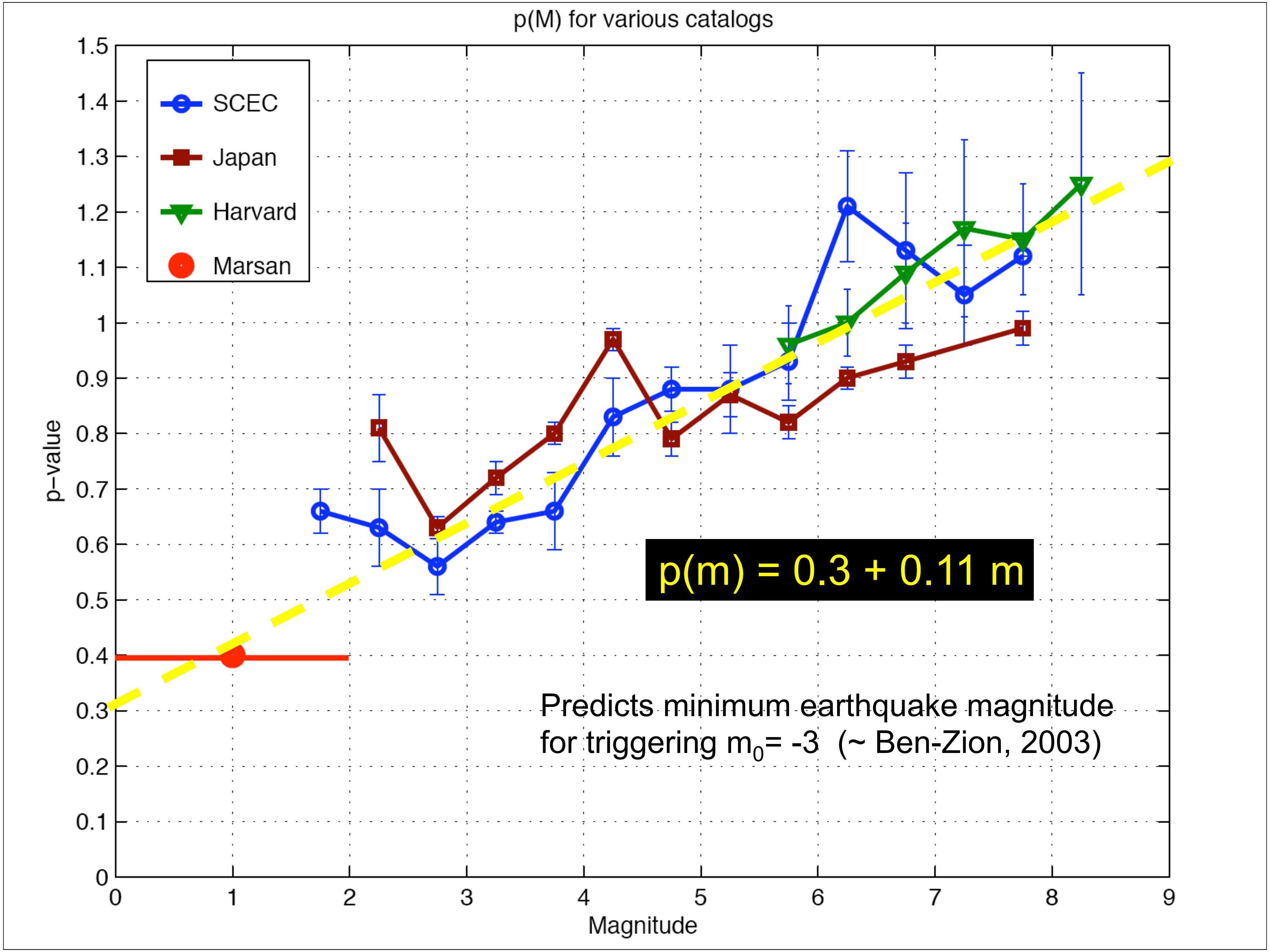


SFA : **p(M)=0.11M+0.38**

Bins : **p(M)=0.10M+0.40**

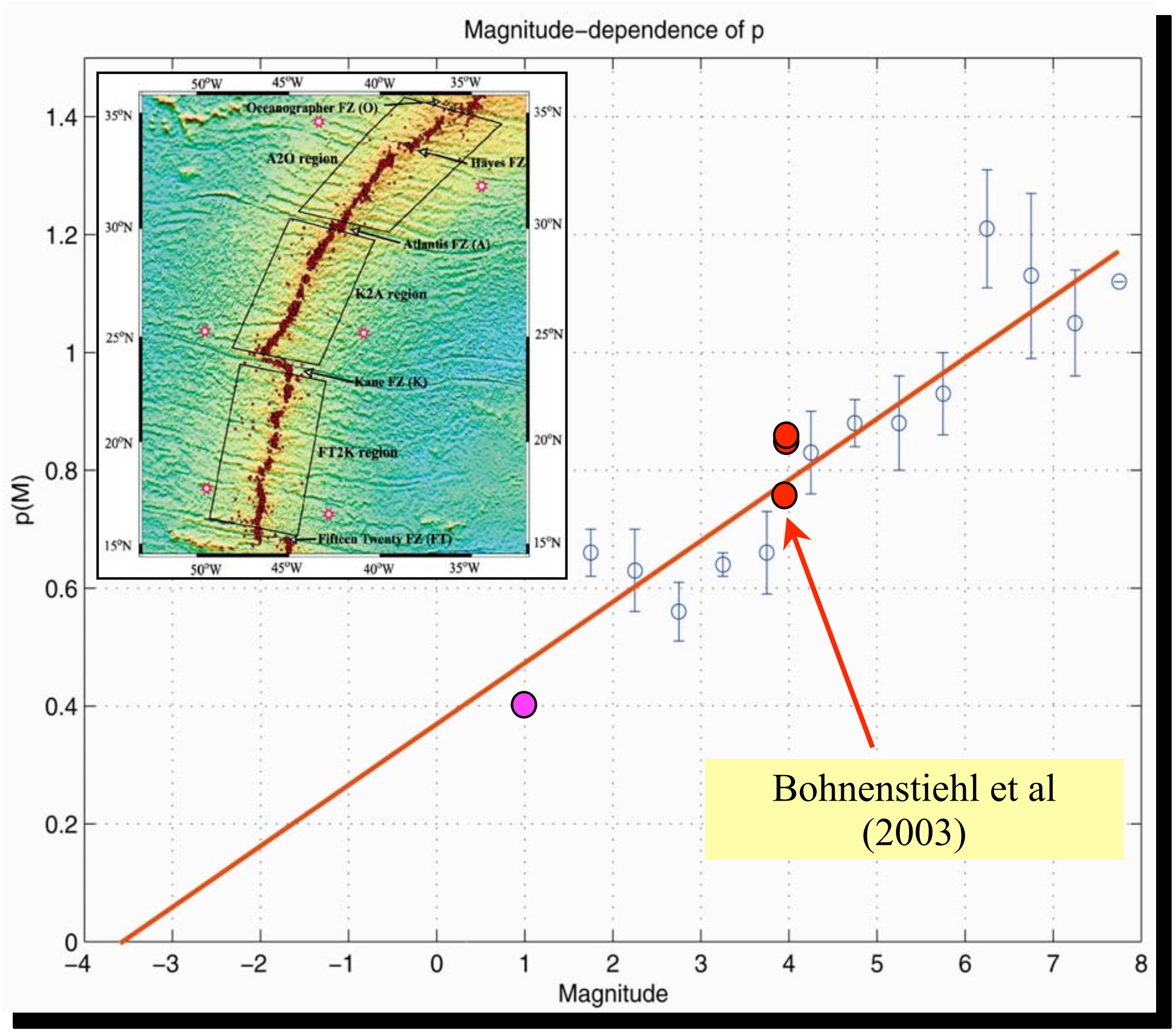
SFA : **p(M)=0.16M-0.09**

Bins : **p(M)=0.13M+0.14**

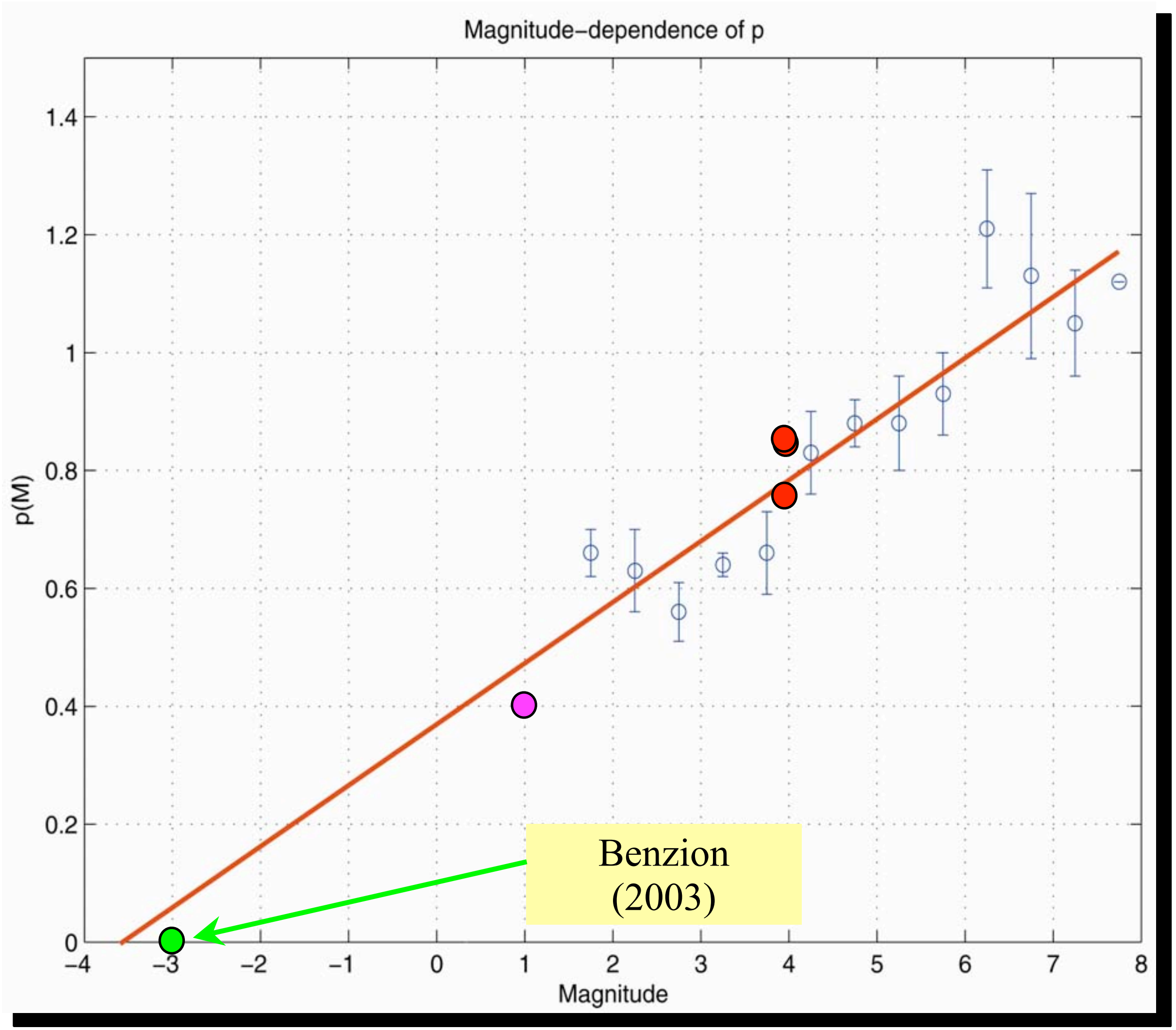


p(M) for various catalogs
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
p−value
SCEC
Japan
Harvard
Marsan
p(m) = 0.3 + 0.11 m
Predicts minimum earthquake magnitude
for triggering m0= -3 (~ Ben-Zion, 2003)
0
1
2
3
4
5
6
7
8
9
Magnitude

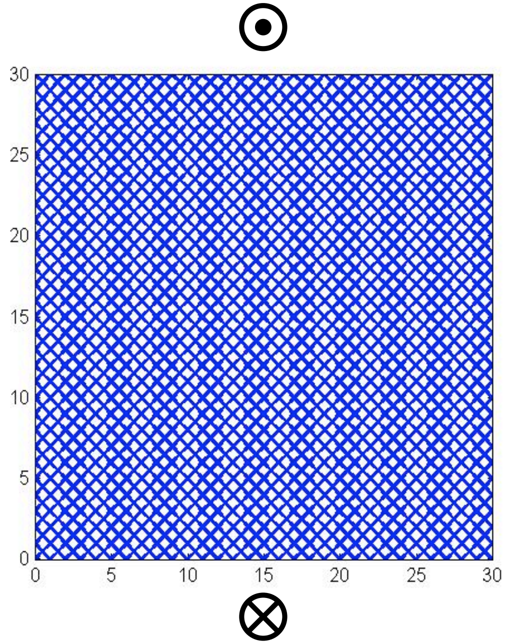
Seismicity on the medio-atlantic rift

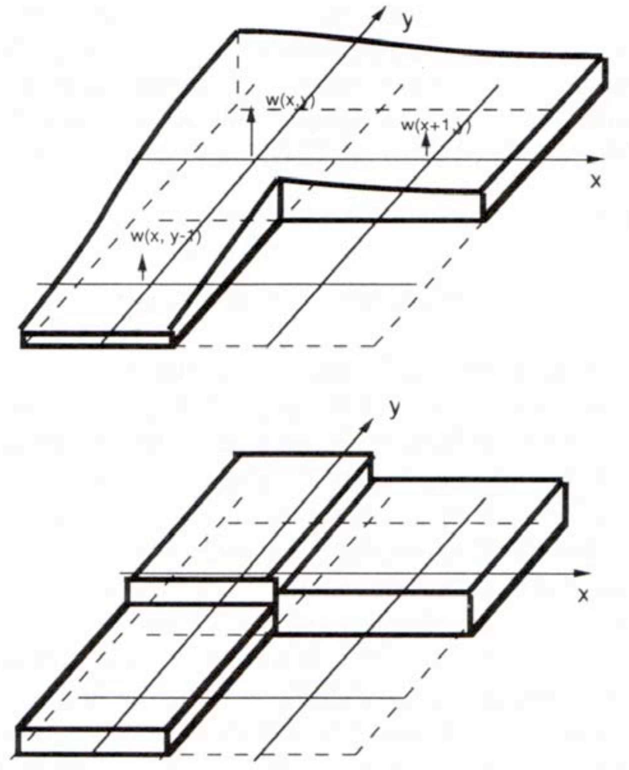


A new universal law?



Numerical Simulations of thermally activated earthquake on on a fault network





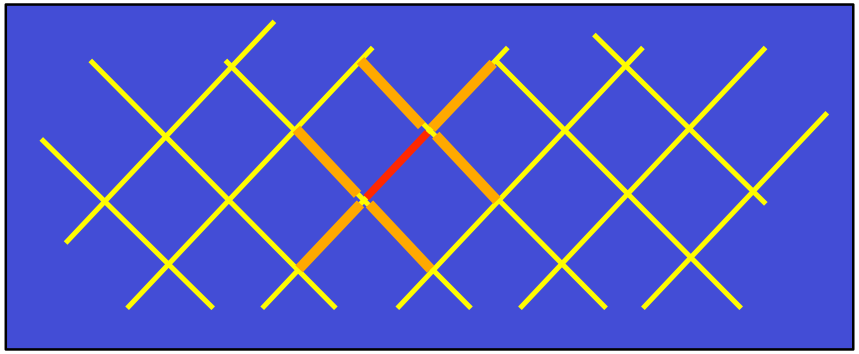
$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V \right)$$

Boundary conditions and rupture rules

The upper boundary moves at constant velocity => stress increases linearly with time within the plate.

Rupture is thermally activated on each fault segment => we predict the time and location of occurrence of the next earthquake using a thinning approach.

The ruptured element slips irreversibly and radiates a dynamic stress on its immediate neighbours.

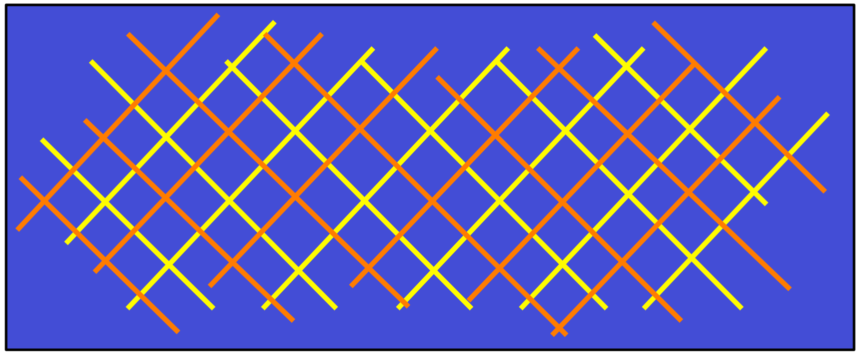


One of those neighbours may rupture due to this dynamic stress => rupture propagation

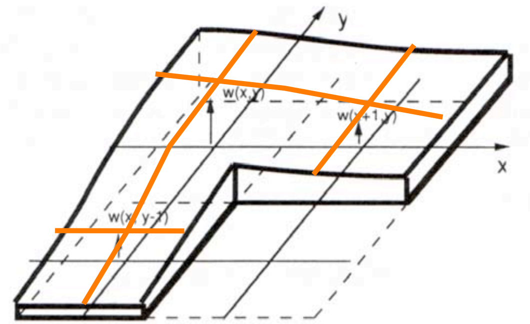
When the rupture stops, we compute the equilibrium static stress field. If a segment is subjected to a too high stress, it ruptures and may continue rupture propagation.

When a rupture definitely stops, we predict the time and location of the next rupture.

Computation of the static stress field using an electric analog



We define 2 networks:
- the fault network (yellow)
- an electric resistance dual network (orange)

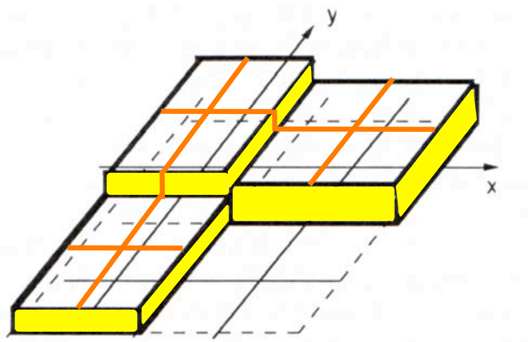


Each node k has a potential V_k (⇔ displacement)

Linearly increasing potential is maintained at the top of the plate

Each link j has an electrical resistance R_j

Within each link, we have an intensity I_j (⇔ stress)



Computation of the static stress field using an electric analog

To solve the problem we:

- Express voltages U_j on links as a function of end nodes potentials V_k

$$\vec{U} = M_{UV}\vec{V} + \vec{V}_0$$

The vector V_0 represents the loading conditions

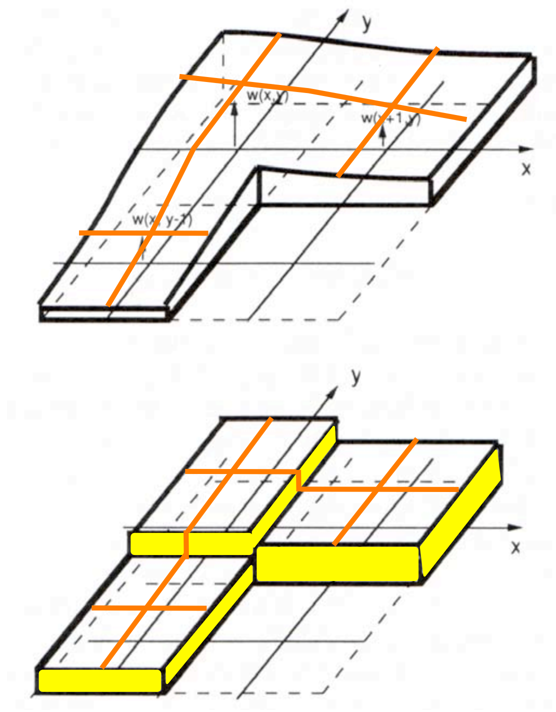
- Express relationships between I_i and U_i.

$$\vec{I} = M_{IU}\vec{U} - \vec{I}_c$$

The vector I_c represents sources equivalent to the cumulative plastic displacement on each fault.

- Use Kirchoff law at each node

$$\vec{O} = M_{OI}\vec{I} = M_{UV}^{T} I$$



Computation of the static stress field using an electric analog

(1) $\vec{U} = M_{UV}\vec{V} + \vec{V}_0$

(2) $\vec{I} = M_{IU}\vec{U} - \vec{I}_c$

(3) $\vec{O} = M_{OI}\vec{I} = M_{UV}^T\vec{I}$

(2) & (3) $M_{UV}^T M_{IU}\vec{U} = M_{UV}^T\vec{I}_c$

& (1) $M_{UV}^T M_{IU} M_{UV}\vec{V} = M_{UV}^T\vec{I}_c - M_{UV}^T M_{IU}\vec{V}_0$

We solve for displacements V by a conjugate gradient approach.

General algorithm

1) Build the fault network and define boundary conditions (constant loading rate).

2) Define the strength of each fault segment with

$$P(s_{th}) = \frac{1}{s_{th}\sqrt{2\pi\Delta\sigma^2}} \exp\left[-\frac{(\ln s_{th})^2}{2\Delta\sigma^2}\right]$$

3) Compute the stress map and translate it into a nucleation rate map (exponential activation).

4) Use a thinning method to choose time and location of the next event.

5) Transfer dynamic stress to neighbors and test them for rupture – propagate rupture until the dynamic rupture criterion fails.

6) Impose a stress drop on each failed element and compute the new static stress map.

7) If stress on an element is larger than its s_{th} then continue rupture (and use again the dynamic rupture criterion).

8) When all segments are stable, start the loading again and go to step 3)

The thinning procedure

1) Last event occurred at t_i

2) At time t the nucleation rate on each segment is

$$\lambda(x,t) = \lambda(x,t_i)\exp(\mu(t - t_i))$$

where μ is the loading rate.

3) We sum all rates over the plate to obtain the total nucleation rate at time t, λ(t)

4) Find a constant C such that C> $\lambda(t_{i+1})$; as t_{i+1} is unknown, we choose a large C

5) Starting from t_i we generate an event using a Poisson process with rate C. Its time of occurrence is t'

6) Generate a random variable U uniformly distributed in [0,1]

7) If CU< λ(t') then $t_{i+1}=t'$, otherwise $t_i=t'$ and go to 5)

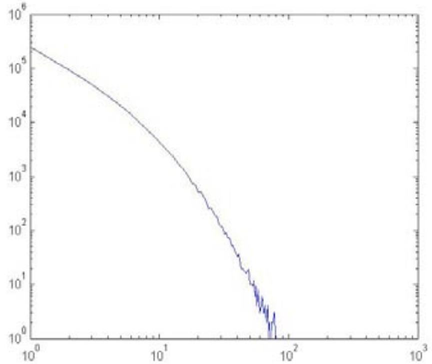
8) Generate the spatial location of the event

9) Start the rupture routine

Effect of disorder – no dynamic stress transfer

Size distribution of events

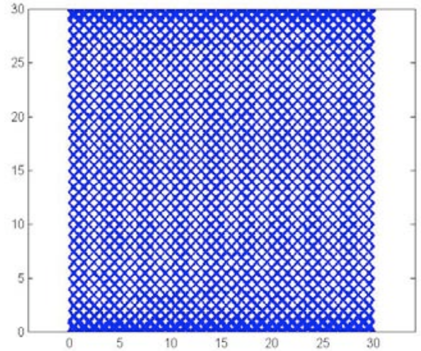
Final geometry of fault pattern

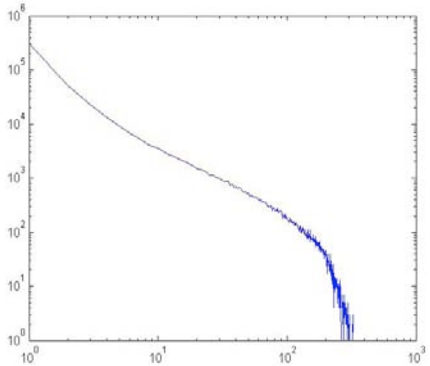


0

No disorder

No dynamic stress

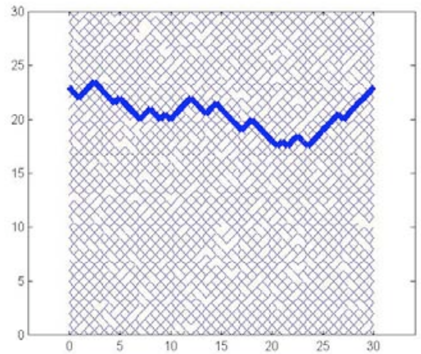




1

50% disorder

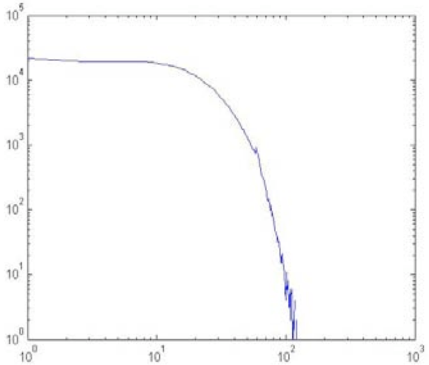
No dynamic stress



Effect of disorder – with dynamic stress transfer

Size distribution of events

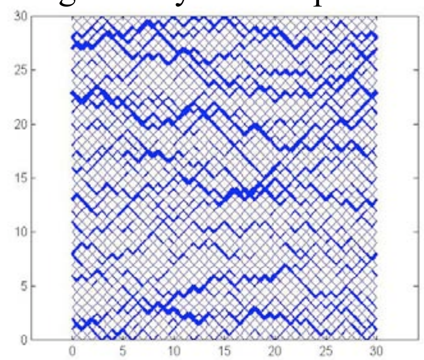
Final geometry of fault pattern

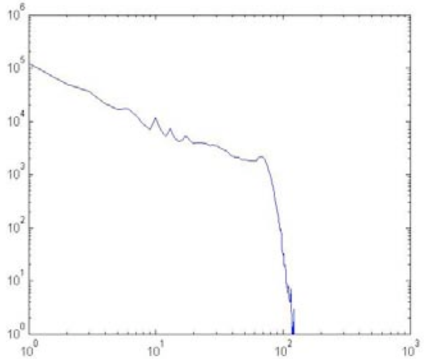


3

10% disorder

Dynamic
stress

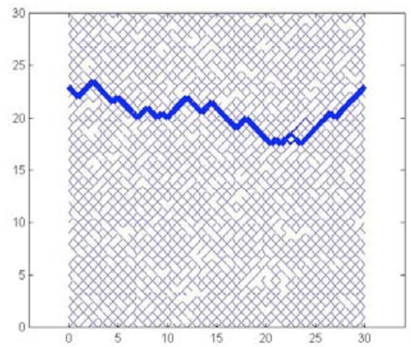


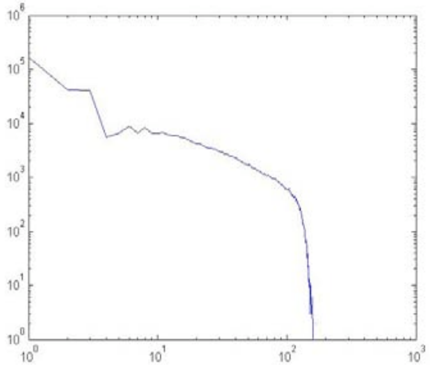


2

50% disorder

Dynamic
stress

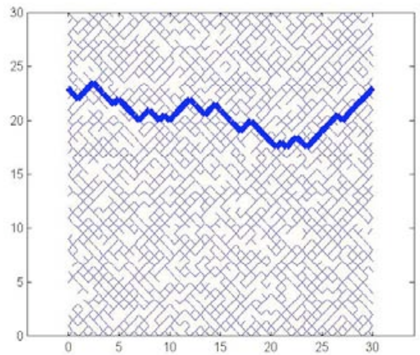




4

90% disorder

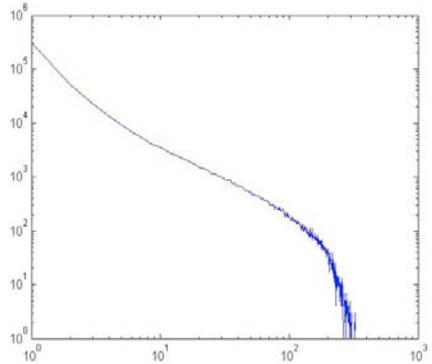
Dynamic stress



Effect of dynamic stress transfer

Size distribution of events

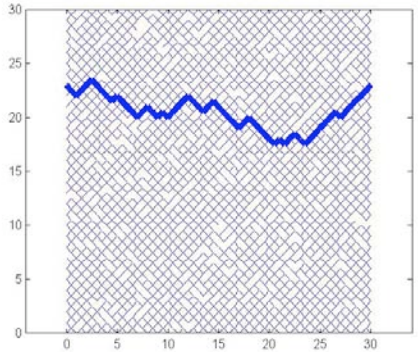
Final geometry of fault pattern

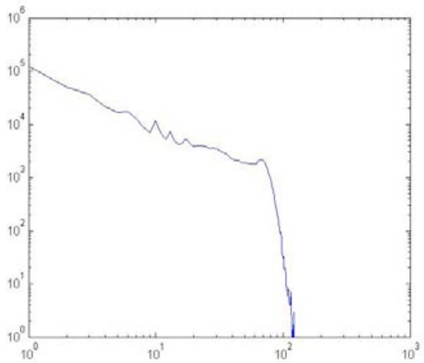


1

50% disorder

No dynamic stress

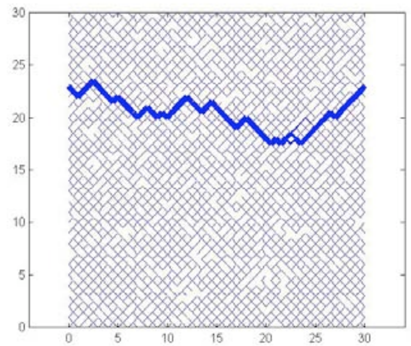




2

50% disorder

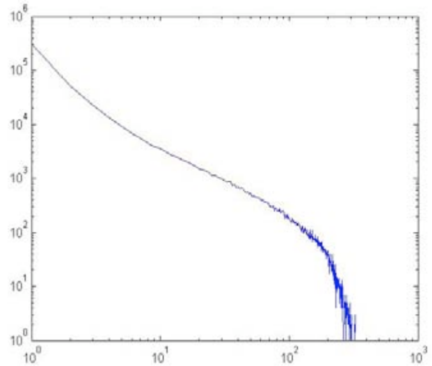
Dynamic stress



Effect of temperature – no dynamic stress transfer

Size distribution of events

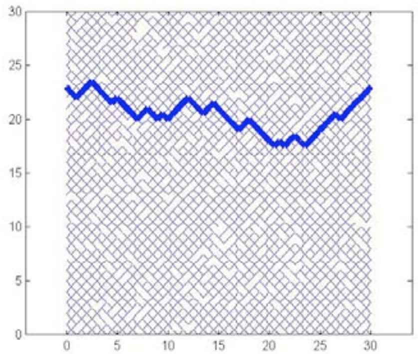
Final geometry of fault pattern

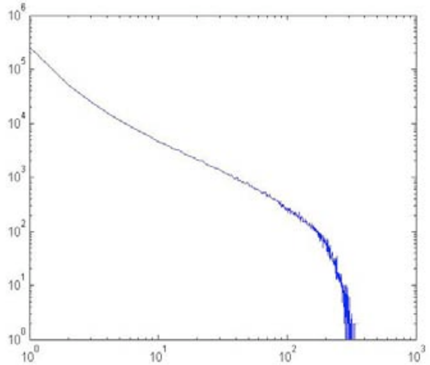


1

50% disorder

No dynamic stress



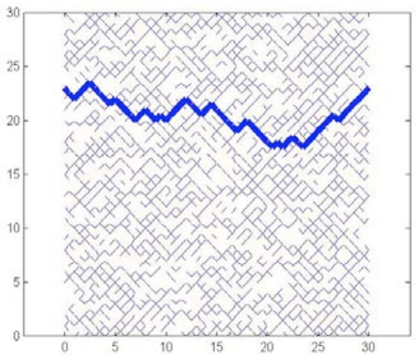


7

50% disorder

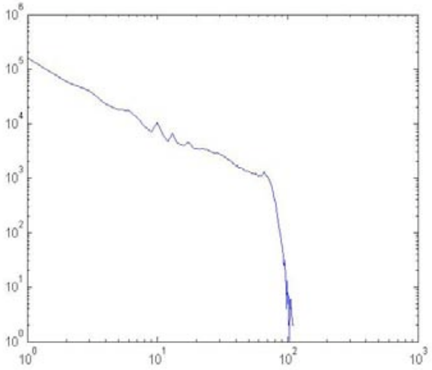
No dynamic stress

Low temperature

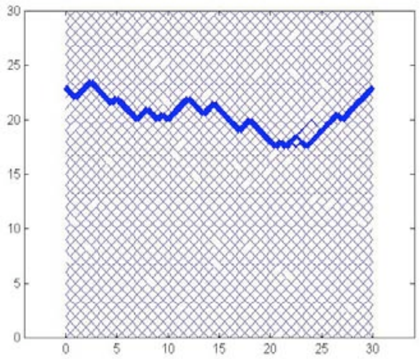


Effect of temperature – dynamic stress transfer

Size distribution of events



Final geometry of fault pattern

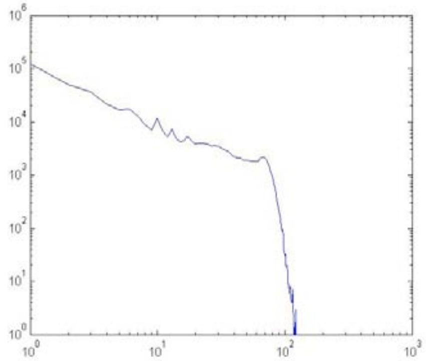


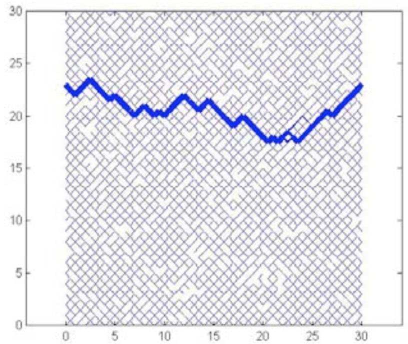
5

50% disorder

Dynamic stress

High temperature





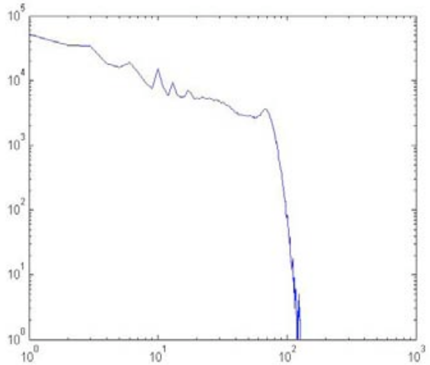
2

50% disorder

Dynamic stress

Size distribution of events

Final geometry of fault pattern

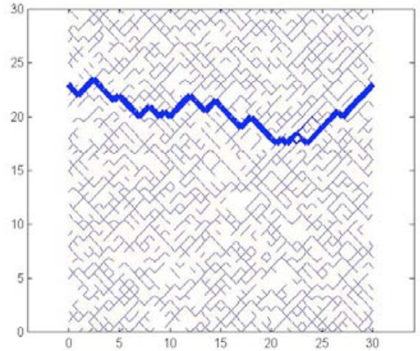


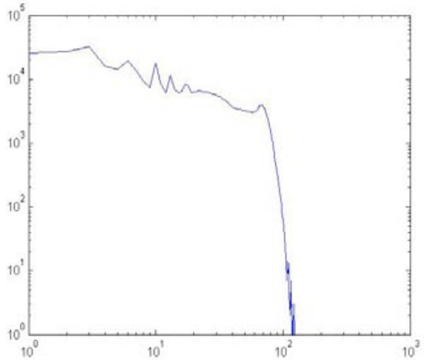
6

50% disorder

Dynamic stress

Low temperature



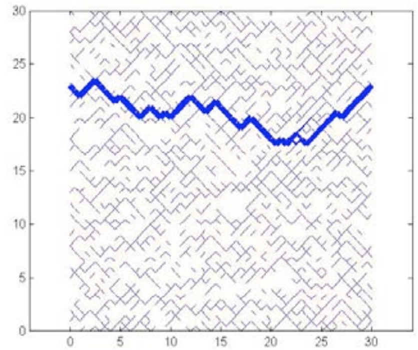


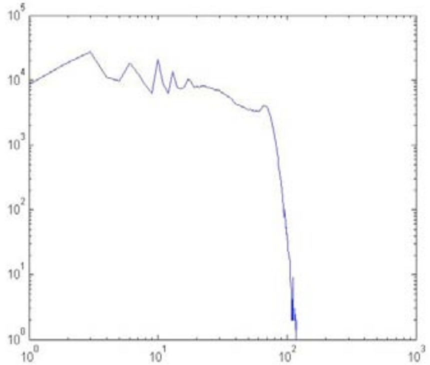
8

50% disorder

Dynamic stress

Very low temperature



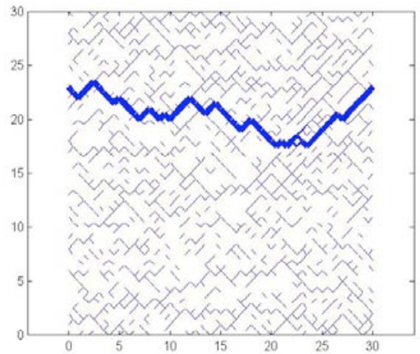


9

50% disorder

Dynamic stress

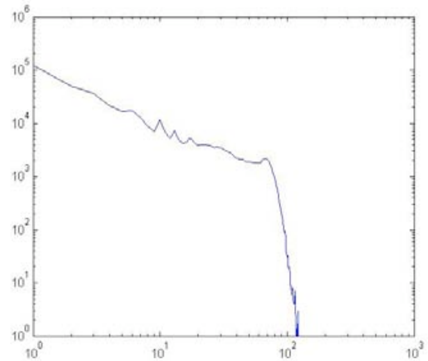
Ultra-low temperature



Effect of strain rate – dynamic stress transfer

Size distribution of events

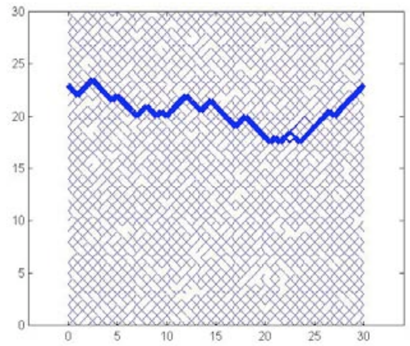
Final geometry of fault pattern

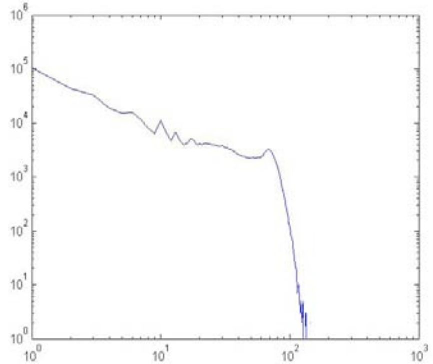


2

50% disorder

Dynamic stress



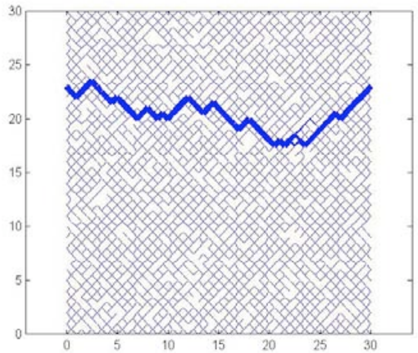


10

50% disorder

Dynamic stress

High strain rate



Preliminary results

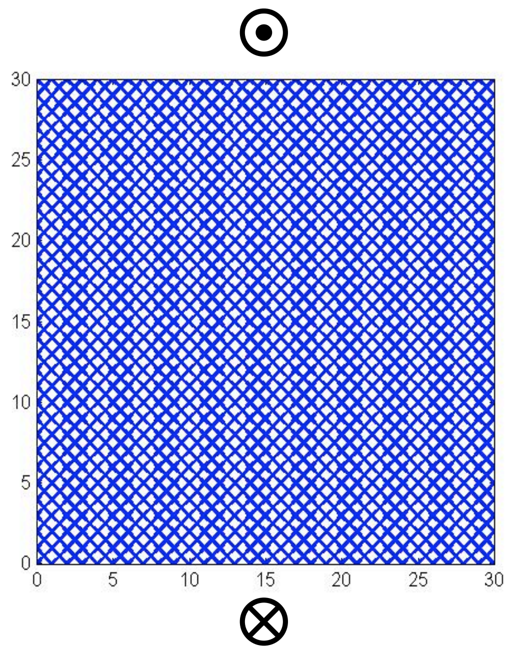
Strain localization is promoted by frozen disorder – little influence of temperature or dynamic stress loading.

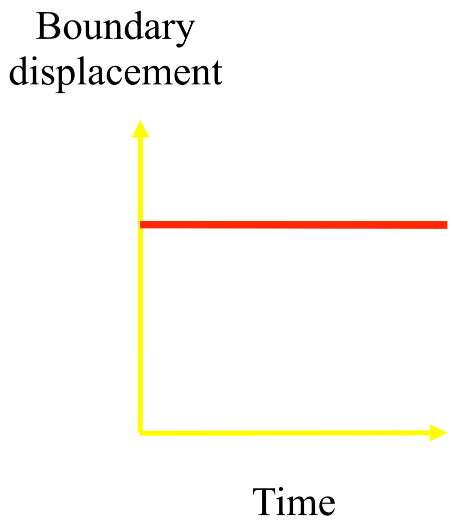
Rupture length distribution: a Gutenberg-Richter law is promoted by :

- disorder
- dynamic stress distribution

While temperature only weakly influences this distribution.

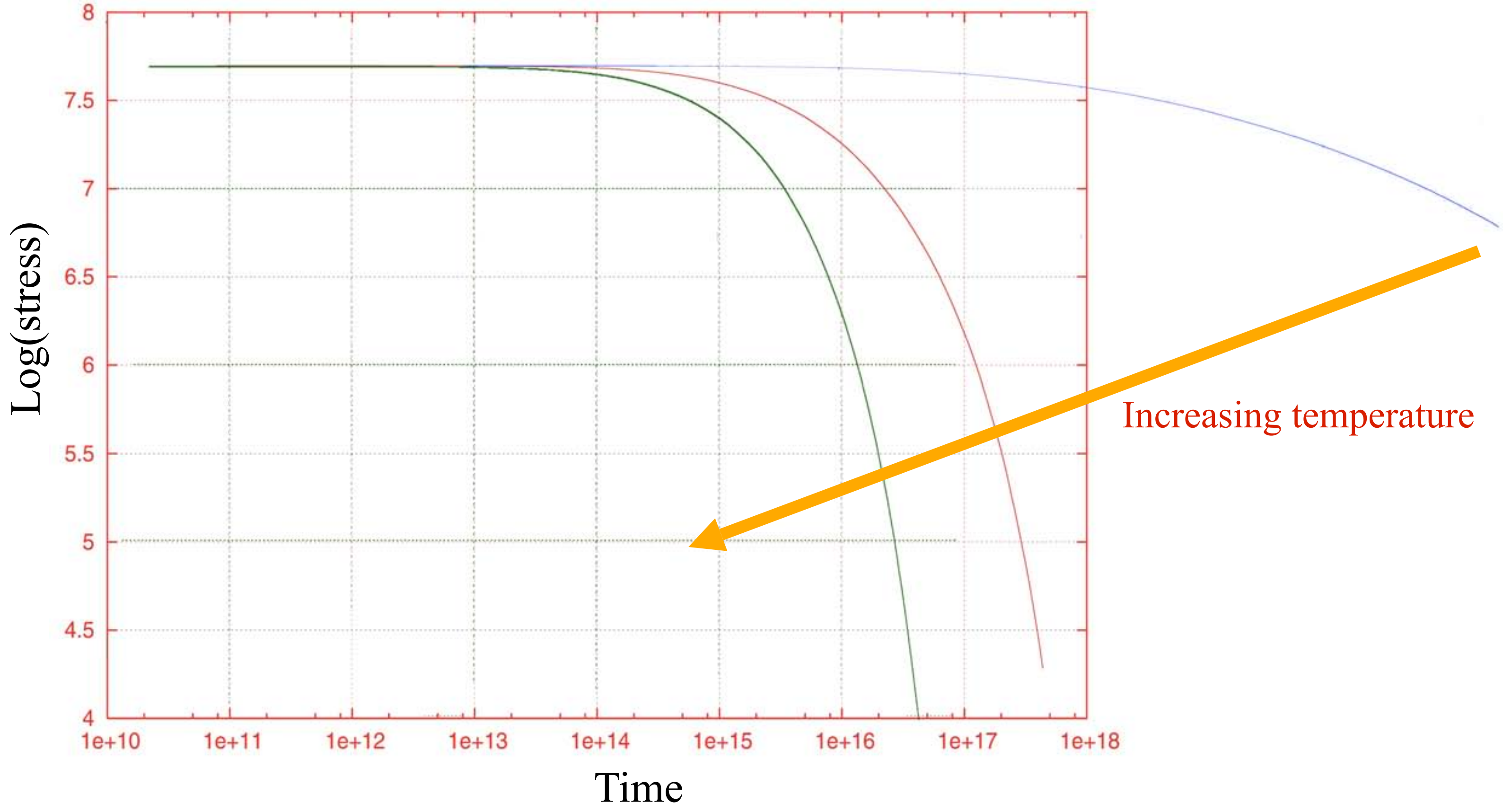
Relaxation of a stress step





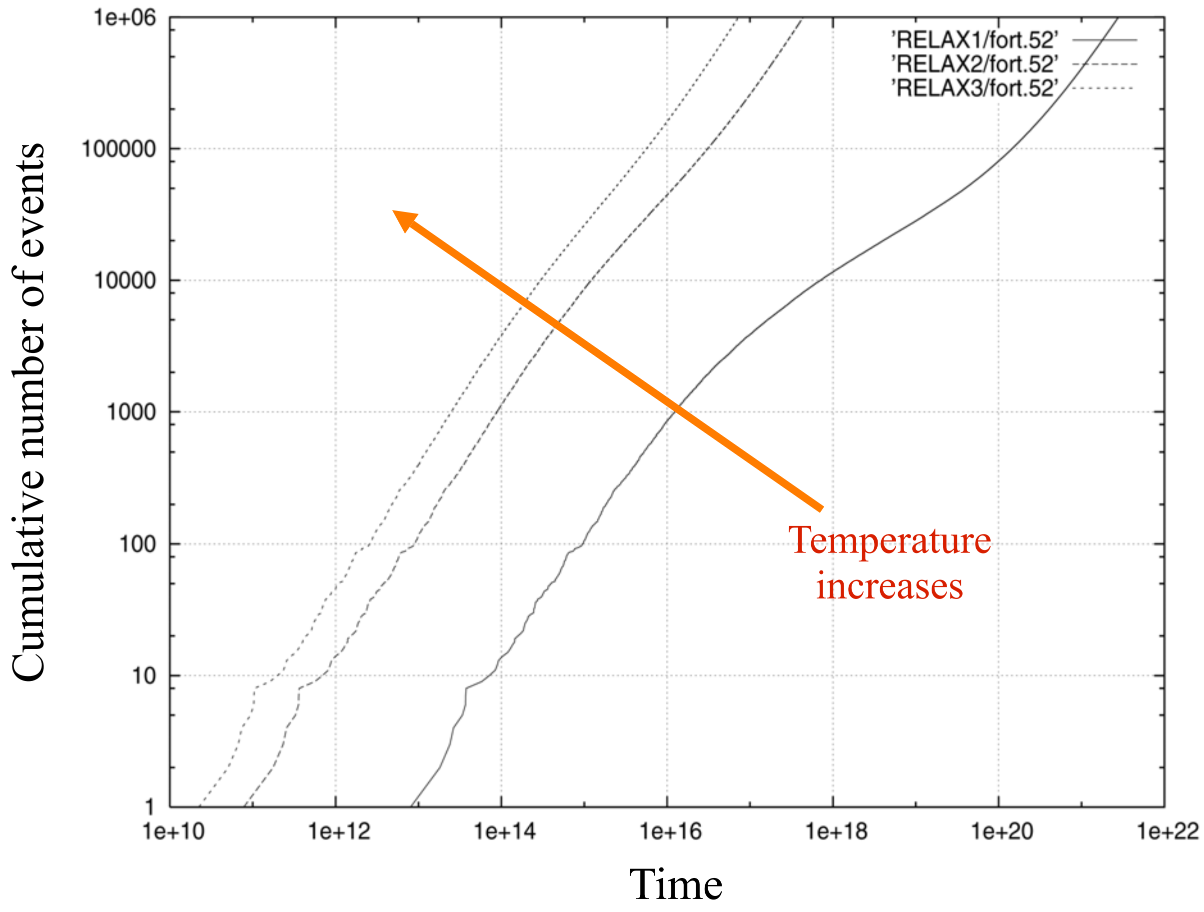
$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V \right)$$

Average stress relaxation function



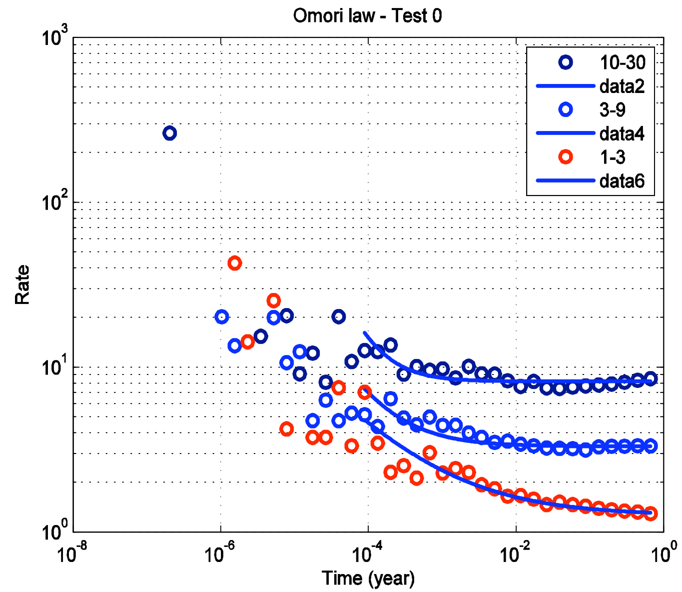
Stress Relaxation is nearly exponential but slower

Seismicity relaxation function



$N(t)dt \sim t^{-p}$ $p \sim 0.1$

Aftershock sequences



Each dataset corresponds to a different rupture length of the mainshock

Aftershocks occur due to thermal activation

But a larger system is needed to provide series with more events

Summary

14

Mechanical model taking account of interactions between all events

Summary

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Summary

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Mechanical model taking account of interactions between all events

Seismicity rate depends exponentially on applied stress

Summary

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Seismicity rate depends exponentially on applied stress

Stress fluctuations are distributed as power laws (μ)

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Mechanical model taking account of interactions between all events

Seismicity rate depends exponentially on applied stress

Stress fluctuations are distributed as power laws (μ)

Stress fluctuations decay with time as power laws (θ)

Summary

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Mechanical model taking account of interactions between all events

Seismicity rate depends exponentially on applied stress

Stress fluctuations are distributed as power laws (μ)

Stress fluctuations decay with time as power laws (θ)

$\mu(1+\theta)=1$ **=> p(M)=aM+b in agreement with empirical observations**

Summary

14

Mechanical model taking account of interactions between all events

Seismicity rate depends exponentially on applied stress

Stress fluctuations are distributed as power laws (μ)

Stress fluctuations decay with time as power laws (θ)

$\mu(1+\theta)=1$ => $p(M)=aM+b$ in agreement with empirical observations

This model is the only one that is able to predict the multifractal nature of seismicity

Summary

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Mechanical model taking account of interactions between all events

Seismicity rate depends exponentially on applied stress

Stress fluctuations are distributed as power laws (μ)

Stress fluctuations decay with time as power laws (θ)

$\mu(1+\theta)=1$ => $p(M)=aM+b$ in agreement with empirical observations

This model is the only one that is able to predict the multifractal nature of seismicity

Multifractality stems from the spatio-temporal self-organization of the fault pattern ($\mu(1+\theta)=1$)