The Illusion of control
in Minority and Parrondo games

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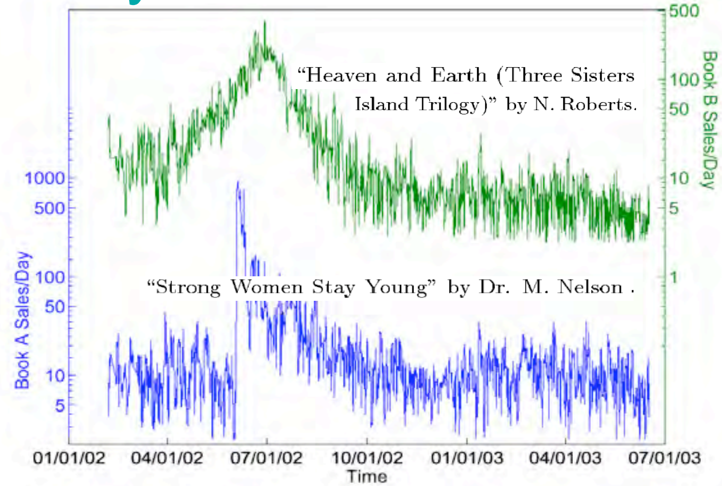
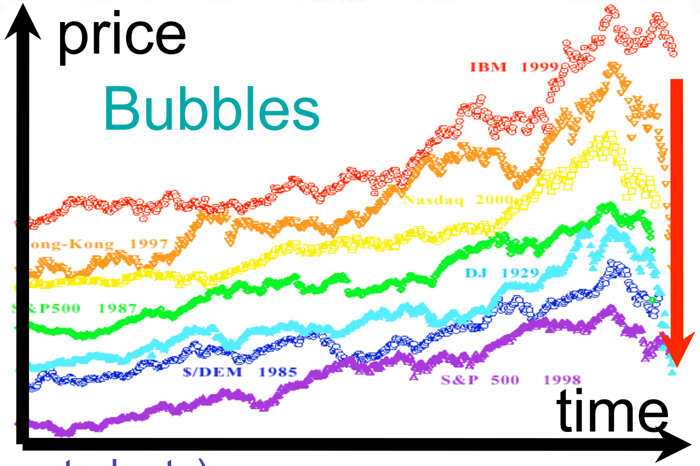
- Collective dynamics and organization of social agents (Commercial sales,YouTube, Open source softwares, Cyber risks)
- Agent-based models of bubbles and crashes, credit risks, systemic risks
- Prediction of complex systems, stock markets, social systems
- Asset pricing, hedge-funds, risk factors…
- Human cooperation for sustainability
- Natural and biological hazards (earthquakes, landslides, epidemics, critical illnesses…)

Dynamics of success





(2-3 guest-professors, 5 foreign associate professors, 1 post-docs, 1 senior researcher, 9 PhD students, 4-6 Master students)

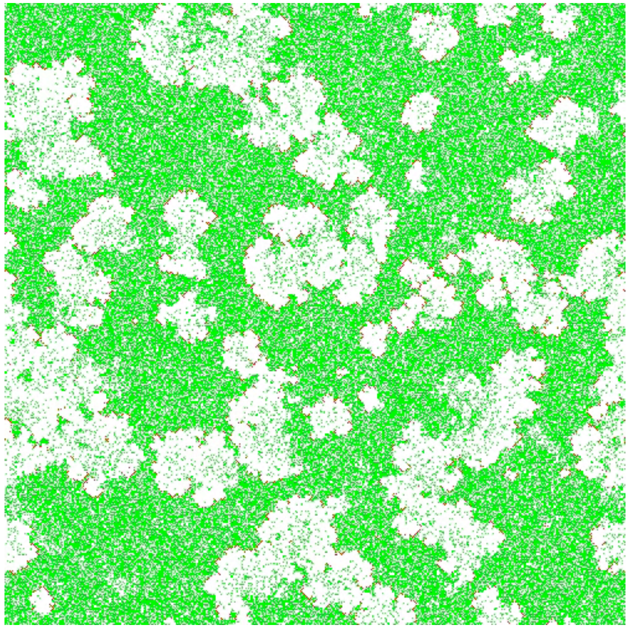
Why are there still?

- forest fires
- traffic jams
- financial bubbles and crashes
- actively managed funds
- economic recessions
- conflicts, wars
- ...

FIRE CONTROL

The primary response from government has been to initiation aggressive fire suppression and management in an attempt to eliminate fire from native lands. In spite of these aggressive fire suppression efforts large wildfires continue to consume vast acreages of chaparral in Southern California.

Minnich (1983, 1997) comparing the chaparral fire regimes in southern California and Baja California found that in Baja California numerous small fire events fragment stands into a fine mixture of age classes, a process which appears to help preclude large fires. While the pattern of large fires in Southern California appears to be an artifact of suppression.



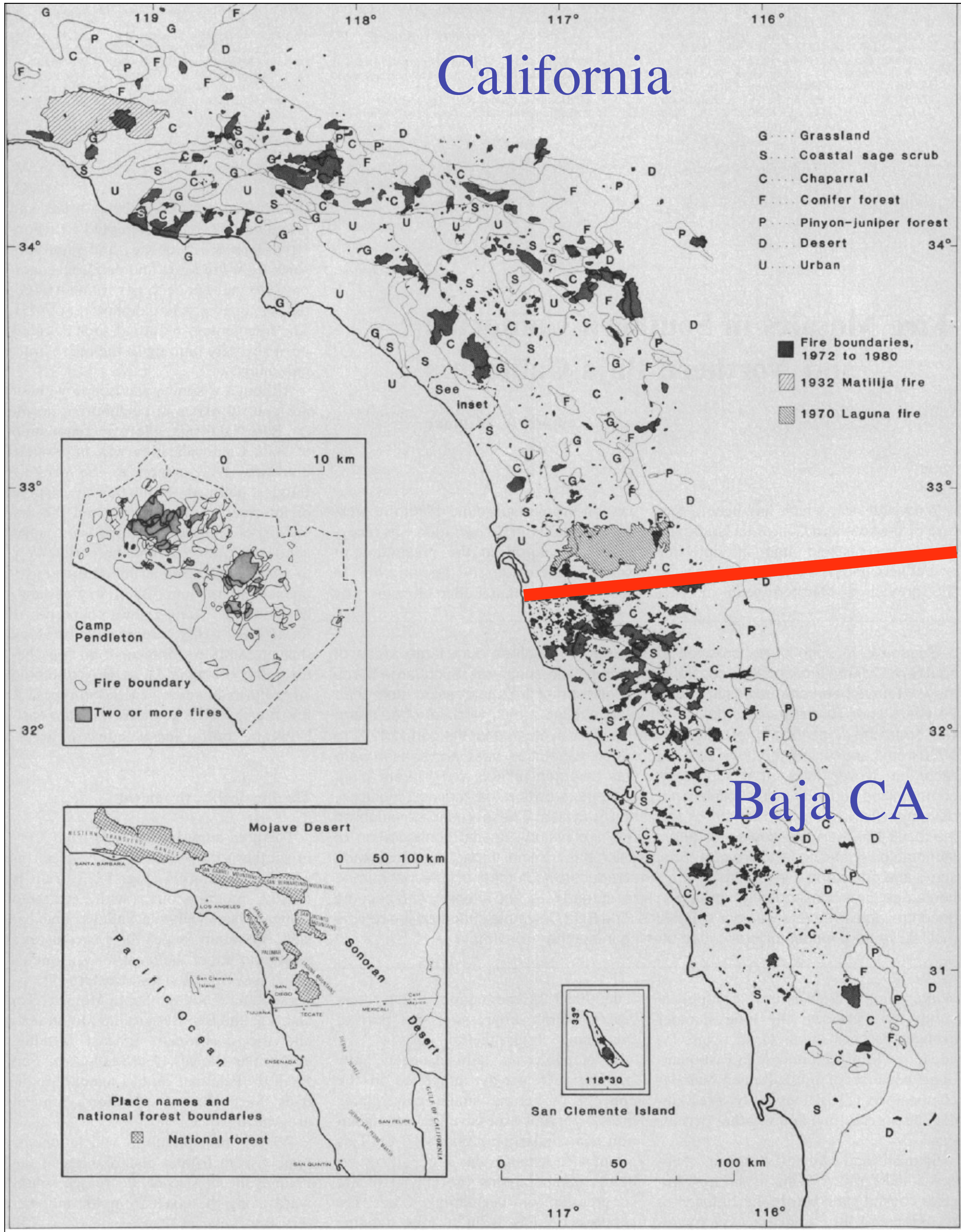


Fig. 1. Wild-land fires in southern California and northern Baja California, 1972 to 1980, with vegetation noted. A broad gradient of increasing fire area northward in Baja California shifts to a pattern of infrequent small to very large fires north of the border. Divergences in fire size between the two countries are most evident in chaparral. Fire data was mapped from Landsat imagery; vegetation was mapped from aerial photography (33).

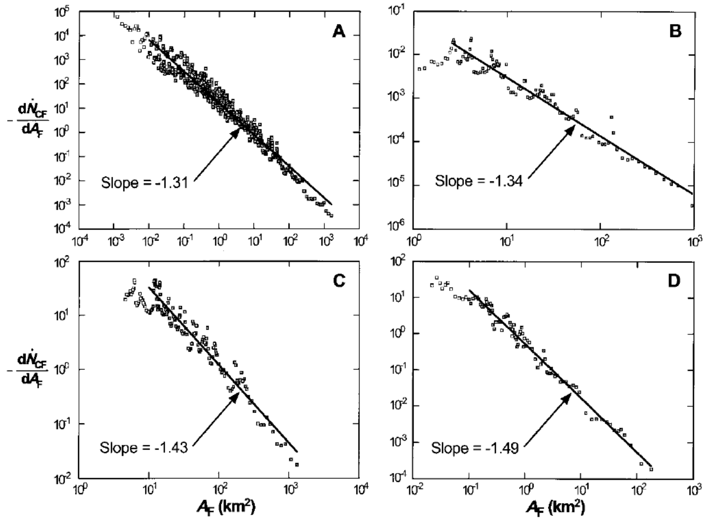
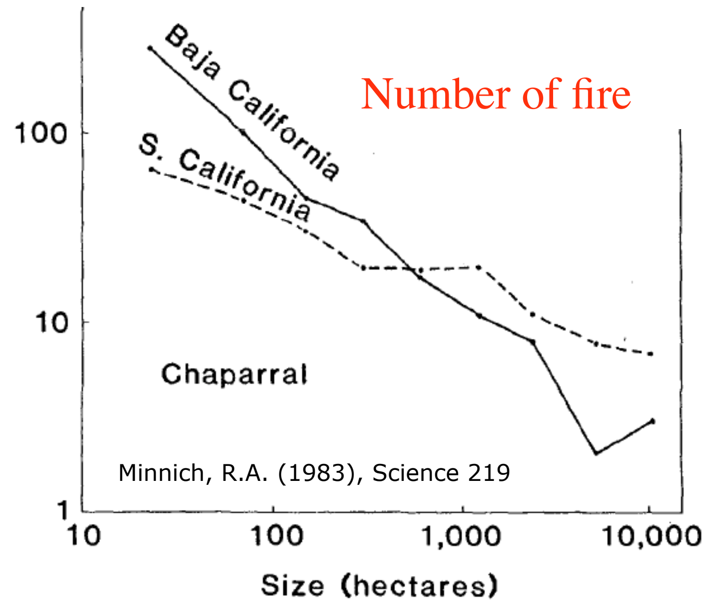


Fig. 2. Noncumulative frequency-area distributions for actual forest fires and wildfires in the United States and Australia: (**A**) 4284 fires on U.S. Fish and Wildlife Service lands (1986–1995) (*9*), (**B**) 120 fires in the western United States (1150–1960) (*10*), (**C**) 164 fires in Alaskan boreal forests (1990–1991) (*11*), and (**D**) 298 fires in the ACT (1926–1991) (*12*). For each data set, the noncumulative number of fires per year ($-d\dot{N}_{CF}/dA_F$) with area (A_F) is given as a function of A_F (*13*). In each case, a reasonably good correlation over many decades of A_F is obtained by using the power-law relation (Eq. 1) with α = 1.31 to 1.49; $-\alpha$ is the slope of the best-fit line in log-log space and is shown for each data set.

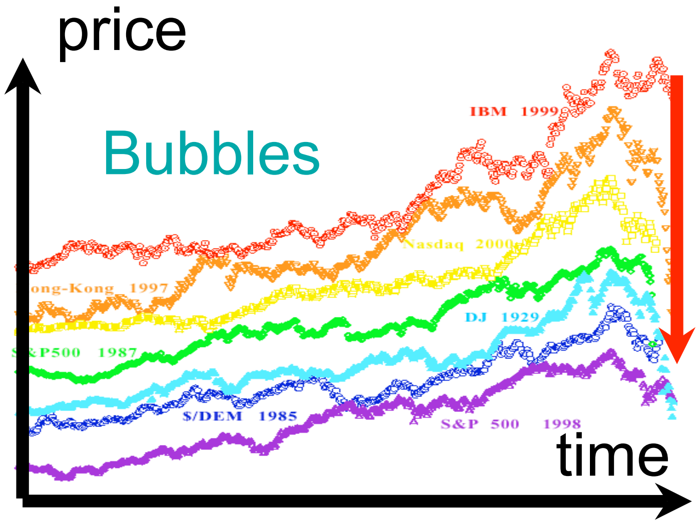
Malamud et al., Science 281 (1998)



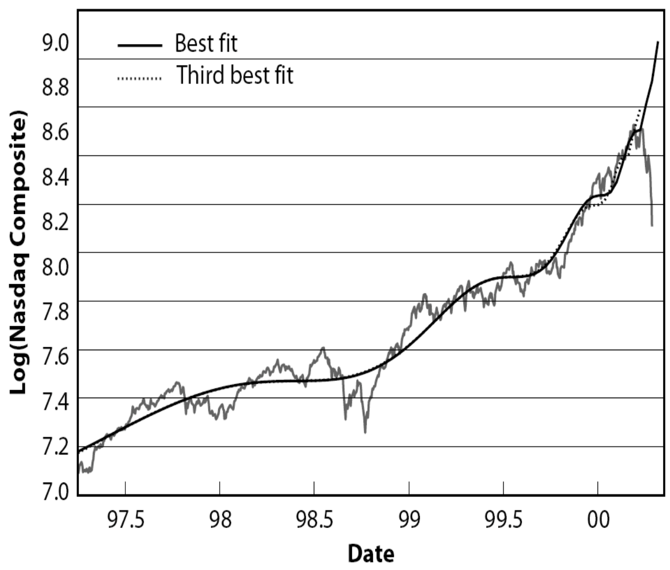
FINANCIAL BUBBLES AND CRASHES

The Fed: A. Greenspan (Aug. 30, 2002):

"We, at the Federal Reserve… recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence… Moreover, it was far from obvious that bubbles, even if identified early, could be preempted short of the Central Bank inducing a substantial contraction in economic activity, the very outcome we would be seeking to avoid."



THE NASDAQ CRASH OF APRIL 2000



✓After a full cycle of rise and fall after which stocks were valued just where they were at the start, all his clients lost money (Don Guyon, 1909).

✓Many academic works suggest that most managers underperform "buy-and-hold" strategy; persistence of winners is very rare, etc.

✓Most funds consistently fail to overperform random strategies (dart throwing).

THE ILLUSION OF CONTROL

✓Individuals appear hard-wired to over-attribute success to skill, and to underestimate the role of chance, when both are in fact present.

[Langer, E. J., The Illusion of Control, Journal of Personality and Social Psychology 32 (2), 311-328 (1975)]

Rats beat humans in simple games

People makes STORIES!

Normal people have an "interpreter" in their left brain that takes all the random, contradictory details of whatever they are doing or remembering at the moment, and smoothes everything in one coherent story. If there are details that do not fit, they are edited out or revised!

(T. Grandin and C. Johnson, Animals in translation (Scribner, New York, 2005)

I. Message

- Control and Optimization often yields perverse results… (in economic policy-making:

“Law of Unintended Consequences”)

- …but not always: When and why?
- Attempt to formally characterize conditions that yield perverse outcomes under optimization

II. Overview: THMG

A. Time-Horizon MG (THMG): Pro/Con

B. In general, agents underperform strategies for "reasonable" **t** (no impact)

C. Agent performance declines with Hamming distance d_H between their strategies

D. Agent evolution: $d_H \rightarrow 0$

E. "Counteradaptive" agents perform best

J.B. Satinover and D. Sornette,
"Illusion of Control" in Minority and Parrondo Games,
Eur. Phys. J. B 60, 369-384 (2007)

III. Overview: Parrondo Games

A. 1^0 effect: two losing games win if alternated
B. History-dependent games
C. Attempt to optimize this effect inverts it
D. Shown in unusual multi-player setting
E. Here in natural single-player setting

J.B. Satinover and D. Sornette, Illusion of Control in a Brownian Game, Physica A 386, 339-344 (2007)

IV. Overview: Other

A. Control in the MAJG and $G
B. Persistence/Anti-persistence in TH games
C. Cycle decomposition of TH games
D. Cycle predictor for real-world 1D series

J.B. Satinover and D. Sornette, Illusory versus Genuine Control in Agent-Based Games, Eur. Phys. J. B. (under revision) (http://arXiv.org/abs/0802.4165)

J.B. Satinover and D. Sornette, Cycles, determinism and persistence in agent-based games and financial time-series, Quantitative Finance (under revision) (http://arXiv.org/abs/0805.0428)

The illusion of control: Minority game

Example of strategy

signal (μ)	*prediction*
000	0
001	0
010	1
011	0
100	1
101	0
110	1
111	0

Total action of agents $A^{\mu}(t) \equiv \sum_i a_i^{\mu}(t)$

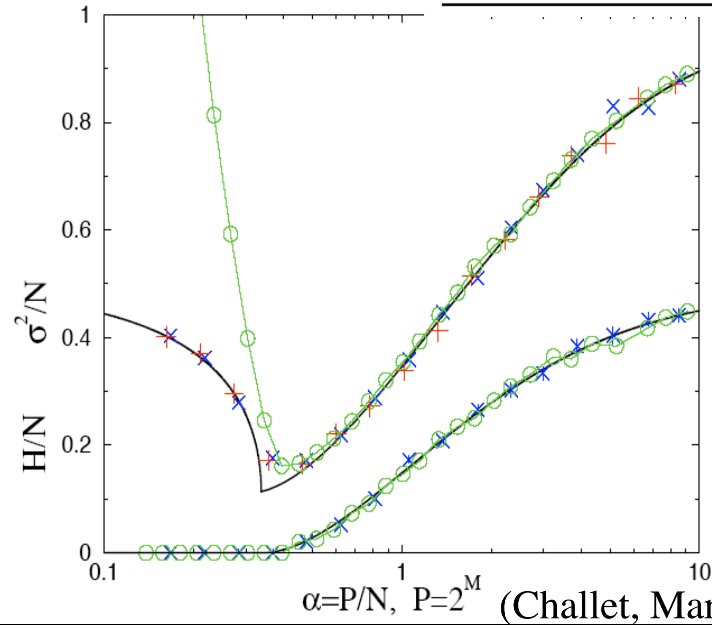
Parameters: m, s, **τ**, N

MG payoff of strategy i :

$$g_i(t) = -a_i(t) A(t)$$

✓Inductive reasoning

✓Minority mechanism



(Challet, Marsili, Zhang)

THMG Markov Chain

(n.b.: structure of A_D = 1^0 determ. cycle; compare later to choose worst)

$$A(\mu_t) = A_D(\mu_t) + A_U(\mu_t)$$

$$A_U(\mu_t) = \left\{ \left(1 - \left[\hat{\mathbf{a}}^T_{(Mod[\mu_t - 1, 4]+1)} \otimes_\delta \hat{\mathbf{a}}^T_{(Mod[\mu_t - 1, 4]+1)} \right] \right) \circ (\vec{s}_{\mu_t} \otimes_\delta \vec{s}_{\mu_t}) \circ \hat{\mathbf{\Omega}} \right\}_{(Mod[\mu_t - 1, 2^m]+1)}$$

$$A_D(\mu_t) = \left(\sum_{r=1}^{8} \left\{ \left[(1 - \mathrm{sgn}\,[\vec{s}_{\mu_t} \ominus \vec{s}_{\mu_t}]) \circ \hat{\mathbf{\Psi}} \right] \bullet \hat{\mathbf{a}} \right\}_r \right)_{(Mod[\mu_t - 1, 2^m]+1)}$$

Markov chain formalism on the Time-Horizon-Minority-Game

Average gain per time step:

$$\left\langle \Delta W_{Agent} \right\rangle = -\frac{1}{N}\left|\vec{A}_D\right| \cdot \vec{\mu}$$

$$\left\langle \Delta W_{Strategy} \right\rangle = \frac{1}{2N}\left(\hat{\mathbf{s}}_\mu \cdot \vec{\kappa}\right) \cdot \vec{\mu}$$

μ is a $(m+\tau)$-bit "path history" (sequence of 1-bit states)

$\vec{\mu}$ is the normalized steady-state probability vector for the history-dependent $(m+\tau)\times(m+\tau)$ transition matrix $\hat{\mathbf{T}}$,

where a given element $T_{\mu_t,\mu_{t-1}}$ represents the transition probability that μ_{t-1} will be followed by μ_t

$\vec{A}_D$ is a $2^{(m+\tau)}$-element vector listing the particular sum of decided values of A(t) associated with each path-history

$\hat{\mathbf{s}}_\mu$ is the table of points accumulated by each strategy for each path-history

$\vec{\kappa}$ is a $2^{(m+\tau)}$-element vector listing the total number of times each strategy is represented in the collection of N agents.

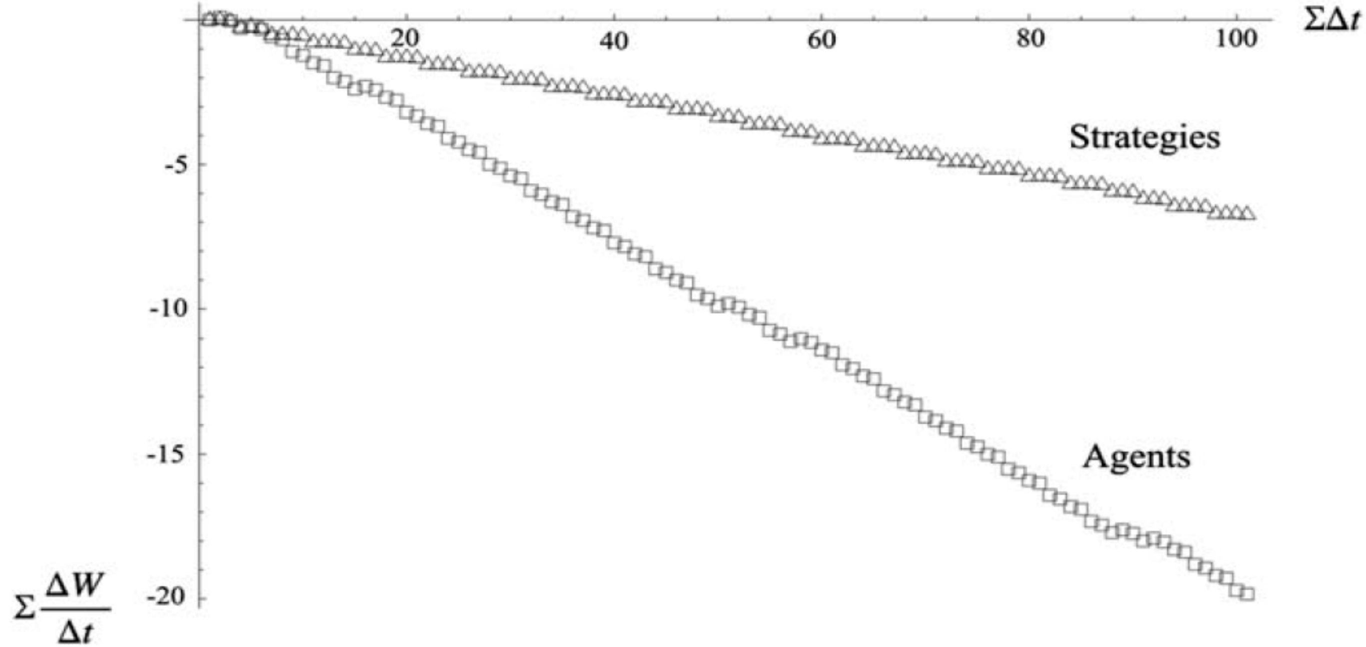
$\hat{\mathbf{T}}$ may be derived from $\vec{A}_D$, $\hat{\mathbf{s}}_\mu$ and $\vec{N}_U$, the number of undecided agents associated with each path history. Thus agents' mean gain is determined by the non-stochastic contribution to A(t) weighted by the probability of the possible path histories.

The illusion of control: Minority game

Parameters: m, s, τ, N

$$\overline{\left\langle \Delta W_{Agent} \right\rangle} \leq \overline{\left\langle \Delta W_{Strategy} \right\rangle}$$

All *N, m, S and* τ



Mean Strategy versus Agent Cumulative Change in Wealth in the THMG. $\{m, S, N\} = \{2, 2, 31\}$; 100 time steps

The illusion of control: Minority game

Impact effect (nb of non-optimizing agents):

For m=2, S=2, τ=1 and N=31, and 2500 random initializations and n optimizing agents,

$\langle \Delta W_{\text{Non-Opt}} \rangle - \langle \Delta W_{\text{Agent}} \rangle =$ (2.380, 2.270, 2.289, 2.275, 2.145, 2.060, 2.039, 1.994, 1.836, 1.964) $\times 10^{-3}$
for n=1, 2, …, 10.

Dependence on Hamming distance between strategies:

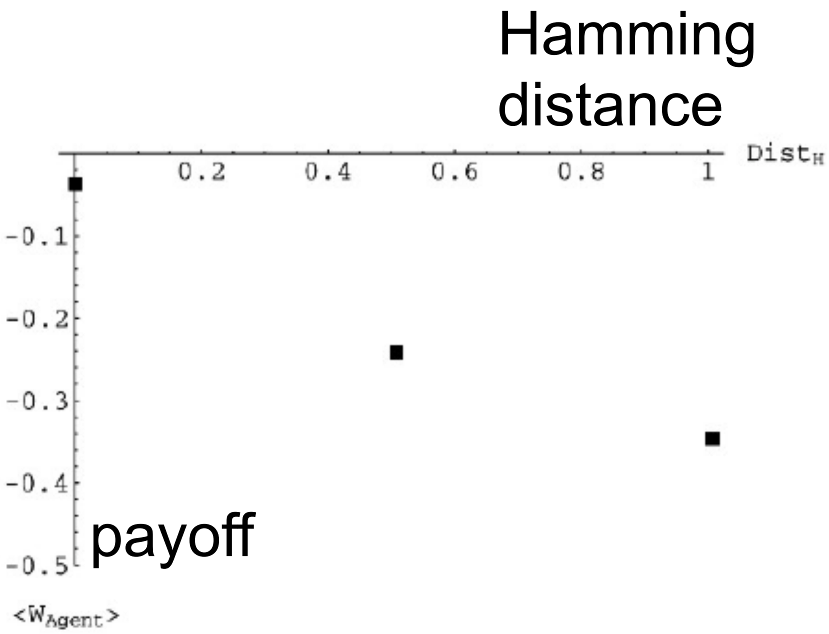
- The average payoff per time step is a decreasing function of d_H.

R. D'Hulst, R. and G. J. Rodgers, Physica A 270, 514-525 (1999).
Y. Li, R. Riolo, and R. S. Savit, Physica A, 276, 234-264 (2000) and ibid, 276, 265-283 (2000)

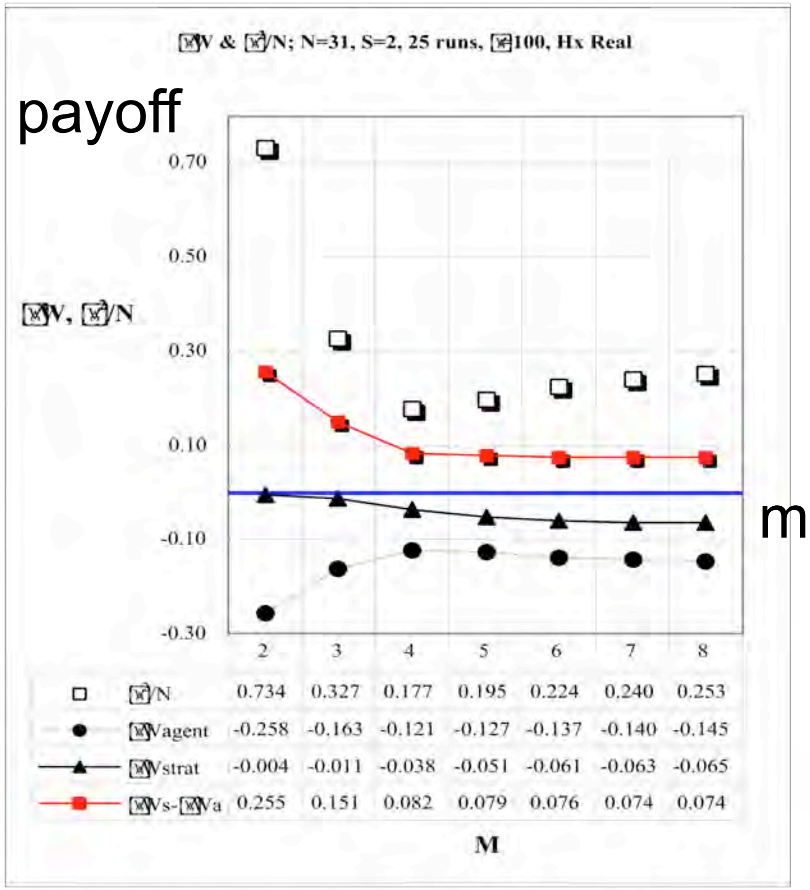
- Performance increases as evolution selects smaller d_H: when learning is introduced, the system learns to rid itself of the illusory optimization method that has been hampering it.

Is the illusion-of-control so powerful that inverting the optimization rule could yield equally unanticipated and opposite results? YES!

The illusion of control: Minority game



Agent wealth as a function of Hamming distance between strategy pairs in agents for the example simulation.



	2	3	4	5	6	7	8
/N	0.734	0.327	0.177	0.195	0.224	0.240	0.253
Vagent	-0.258	-0.163	-0.121	-0.127	-0.137	-0.140	-0.145
Vstrat	-0.004	-0.011	-0.038	-0.051	-0.061	-0.063	-0.065
Vs-Va	0.255	0.151	0.082	0.079	0.076	0.074	0.074

Average wealth variation per time step for different agents. In red are shown the wealth variations of the three among the 31 agents which use counteradaptive ("C", choose worst) strategy selection. The usual underperformance of agents compared to individual strategies when using standard selection rule ("S", choose best) is shown in the blue dots.

The illusion of control: Minority game

The illusion-of-control effect in MG results from the fact that a strategy that has performed well in the past becomes crowded out in the future due the minority mechanism: performing well in the recent past, there is a larger probability for a strategy to be chosen by an increasing number of agents, which inevitably leads to its demise.

This argument in fact also applies to all the strategies which belong to the same reduced set.

The crowding mechanism operates from the fact that a significant number of agents have at least one strategy in the same reduced subset among the 2^m reduced strategy subsets. Optimizing agents tend on average to adapt to the past but not the present. They choose an action a(t) which is on average out-of-phase with the collective action A(t).

In contrast, non-optimizing agents average over all the regimes for which their strategy may be good and bad, and do not face the crowding-out effect.

The crowding-out effect also explains simply why anti-optimizing agents over-perform: choosing their worst strategy ensures that it will be the least used by other agents in the next time step, which implies that they will be in the minority.

The crowding mechanism also predicts that the smaller the parameter $2^m/N$, the larger the illusion-of-control effect. Indeed, for large values of $2^m/N$, it becomes more and more probable that agents have their strategies in different reduced strategy classes, so that a strategy which is best for an agent tells nothing about the strategies used by the other agents.

The illusion of control: Minority game

•Profound clash between optimization and minority payoff

•Generalization to first-entry games:

With reinforcement learning, pure equilibria involve considerable coordination on asymmetric outcomes where some agents enter and some stay out.

Learning with stochastic fictitious plays leads to symmetric equilibria in which agents randomize over the entry decisions.

There may even exist asymmetric mixed equilibria, where some agents adopt pure strategies while others play mixed strategies.

Consider the situation where agents use a boundless recursion scheme to learn and optimize their strategy so that the equilibrium corresponds to the fully symmetric mixed strategies where agents randomize their choice at each time step with unbiased coin tosses.

Consider a MG game with N agents total, N^R of which employ such a fully random symmetric choice. The remaining $N^S = N - N^R$ "special" agents (with $N^R >> N^S$) will all be one of three possible types: agents with S fixed strategies that choose their best (respectively worst) performing strategy to make the decision at the next step (referred to above as anti-optimizing) and agents with a single fixed strategy.

Our simulations confirm that these three types of agents indeed under-perform on average the optimal fully symmetric purely random mixed strategies of the N^R agents.

Pure random strategies are obtained as optimal, given the fully rational fully informed nature of the competing agents. The particular results are sensitive to which strategies are available to the special agents and to their proportion. Their under-performance in general requires averaging over all possible strategies and S-tuples of strategies.

III.Parrondo Games

(Physica A, 386,1:339-344)

A. 1^0 effect: 2 losing games win if alternated
B. Capital-dependent → History-dependent
C. Attempt to optimize this effect inverts it
D. Shown in unusual multi-player setting
E. Here (ref.) in natural single-player setting
F. Choose worst partially restores PE

III.Parrondo Games

(Physica A, 386,1:339-344)

game-theoretic equivalent to directional drift of Brownian particles in a time-varying “ratchet”-shaped potential

Consider $N > 1$ s-state Markov games G_i, $i \in \{1, 2, \ldots, N\}$,

and their N $s \times s$ transition matrices $\hat{\mathbf{M}}^{(i)}$

vector of s conditional winning probabilities as $\vec{\mathbf{p}}^{(i)} = \{p_1^{(i)}, p_2^{(i)}, \ldots, p_s^{(i)}\}$

their steady-state probability vectors as $\vec{\Pi}^{(i)}\{\pi_1^{(i)}, \pi_2^{(i)}, \ldots, \pi_s^{(i)}\}$

steady-state probability of winning $P_{win}^{(i)} = \vec{\mathbf{p}}^{(i)} \cdot \vec{\Pi}^{(i)}.$

III.Parrondo Games

Capital: X(t)

$$\vec{Y}(t) = \begin{pmatrix} X(t) - X(t-1) \\ X(t-1) - X(t-2) \end{pmatrix}$$

four states $\{(-1,-1), (-1,+1), (+1,-1)$

Transition matrices of games A and B

$$\mathbf{A} = \begin{pmatrix} 0.505 & 0 & 0.505 & 0 \\ 0.495 & 0 & 0.495 & 0 \\ 0 & 0.505 & 0 & 0.505 \\ 0 & 0.495 & 0 & 0.495 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0.105 & 0 & 0.755 & 0 \\ 0.895 & 0 & 0.245 & 0 \\ 0 & 0.755 & 0 & 0.305 \\ 0 & 0.245 & 0 & 0.695 \end{pmatrix}$$

steady state probabilities for the two independent games: $\mathbf{B}\vec{\pi}_{st} = \vec{\pi}_{st}$

$$\vec{\pi}_{st}^{(A)} = \begin{pmatrix} 0.255 \\ 0.250 \\ 0.250 \\ 0.245 \end{pmatrix}; \vec{\pi}_{st}^{(B)} = \begin{pmatrix} 0.231 \\ 0.274 \\ 0.274 \\ 0.220 \end{pmatrix}$$

$$P_{win}(A) = \vec{\pi}_{st}^{(A)} \bullet \vec{\pi}^{(A)} = 0.495$$

$$P_{win}(B) = \vec{\pi}_{st}^{(B)} \bullet \vec{\pi}^{(B)} = 0.494.$$

Alternationg A and B randomly

$$\boxed{P_{win}\left(\tfrac{1}{2}A, \tfrac{1}{2}B\right) = \vec{\pi}_{st}^{(\frac{1}{2}A, \frac{1}{2}B)} \bullet \vec{\pi}^{(\frac{1}{2}A, \frac{1}{2}B)} = 0.501.}$$

III.Parrondo Games

Under optimization ("choose best") 8 X 8 transition matrix:

$$m_j = \tfrac{1}{2}\left\{\pi_{\alpha(j)}^{(A)}\left[1+\pi_{\beta(j)}^{(A)}-\pi_{\beta(j)}^{(B)}\right]+\pi_{\alpha(j)}^{(B)}\left[1-\pi_{\beta(j)}^{(A)}+\pi_{\beta(j)}^{(A)}\right]\right\}$$

j=1,2,...,8

$$P_{win}^{best(A,B)} = 0.496$$

Under "choose worst":

$$m_j = \tfrac{1}{2}\left\{\pi_{\alpha(j)}^{(A)}\left[1-\pi_{\beta(j)}^{(A)}+\pi_{\beta(j)}^{(B)}\right]+\pi_{\alpha(j)}^{(B)}\left[1+\pi_{\beta(j)}^{(A)}-\pi_{\beta(j)}^{(A)}\right]\right\}$$

j=1,2,...,8

$$P_{win}^{worst(A,B)} = 0.507$$

$$\alpha\left(j\right)= Mod\left[j-1,4\right]+1,\ \ \beta\left[j\right]=\tfrac{1}{2}\left(j-Mod\left[j-1,2\right]+1\right)$$

IV. Control In the TH-MAJG, -$G

$$g_i^{min}\left(t\right) = -a_i\left(t\right)A\left(t\right)$$ or $$g_i^{min}\left(t\right) = -Sgn\left[a_i\left(t\right)A\left(t\right)\right]$$

$$g_i^{maj}\left(t\right) = +a_i\left(t\right)A\left(t\right)$$ or $$g_i^{maj}\left(t\right) = +Sgn\left[a_i\left(t\right)A\left(t\right)\right]$$

$$g_i^{\$}\left(t\right) = +a_i\left(t-1\right)A\left(t\right)$$ or $$g_i^{min}\left(t\right) = -Sgn\left[a_i\left(t-1\right)A\left(t\right)\right]$$

IV. Control In the TH-MAJG, -$G

$$A_D^{min}(\mu_t) =$$

$$\left(\sum_{r=1}^{8}\left\{\left(1 - Sgn\left[\vec{s}_{\mu_t}^{\,min}\,\check{z}\;\vec{s}_{\mu_t}^{\,min}\right]\right)\circ\hat{\varnothing}^{+} + \left(1 + Sgn\left[\vec{s}_{\mu_t}^{\,min}\,\check{z}\;\vec{s}_{\mu_t}^{\,min}\right]\right)\circ\hat{\varnothing}^{-}\,\right]\bullet\hat{a}\right\}_r\right)_{\left(\mathrm{Mod}\left[\mu_t-1,2^m\right]+1\right)}$$

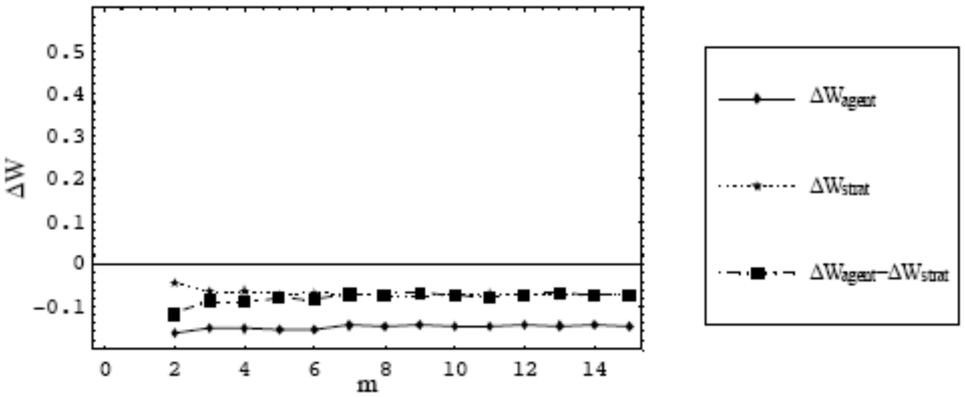
$$A_D^{maj}(\mu_t) =$$

$$\left(\sum_{r=1}^{8}\left\{\left(1 - Sgn\left[\vec{s}_{\mu_t}^{\,maj}\,\check{z}\;\vec{s}_{\mu_t}^{\,maj}\right]\right)\circ\hat{\varnothing}^{+} + \left(1 + Sgn\left[\vec{s}_{\mu_t}^{\,maj}\,\check{z}\;\vec{s}_{\mu_t}^{\,maj}\right]\right)\circ\hat{\varnothing}^{-}\,\right]\bullet\left(-\hat{a}\right)\right\}_r\right)_{\left(\mathrm{Mod}\left[\mu_t-1,2^m\right]+1\right)}$$

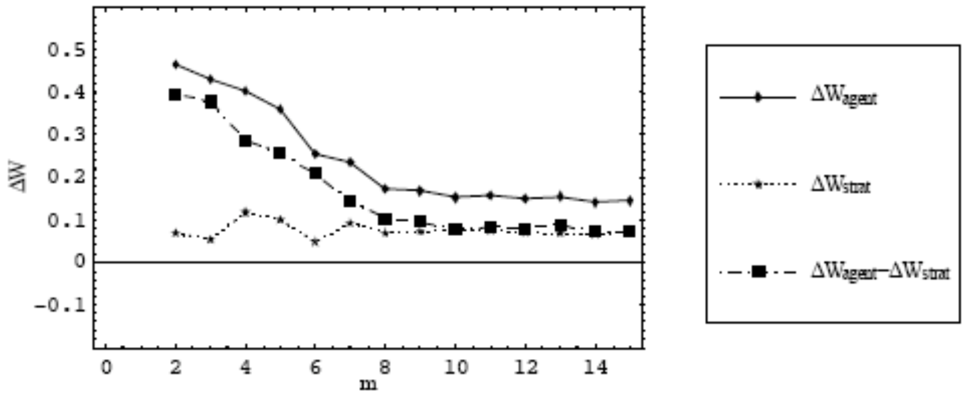
$$A_D^{\$}(\mu_t) =$$

$$\left(\sum_{r=1}^{8}\left\{\left(1 - Sgn\left[\vec{s}_{\mu_t}^{\,\$}\,\check{z}\;\vec{s}_{\mu_t}^{\,\$}\right]\right)\circ\hat{\varnothing}^{+} + \left(1 + Sgn\left[\vec{s}_{\mu_t}^{\,\$}\,\check{z}\;\vec{s}_{\mu_t}^{\,\$}\right]\right)\circ\hat{\varnothing}^{-}\,\right]\bullet\left(-\hat{a}\right)\right\}_r\right)_{\left(\mathrm{Mod}\left[\mu_{t-1}-1,2^m\right]+1\right)}$$

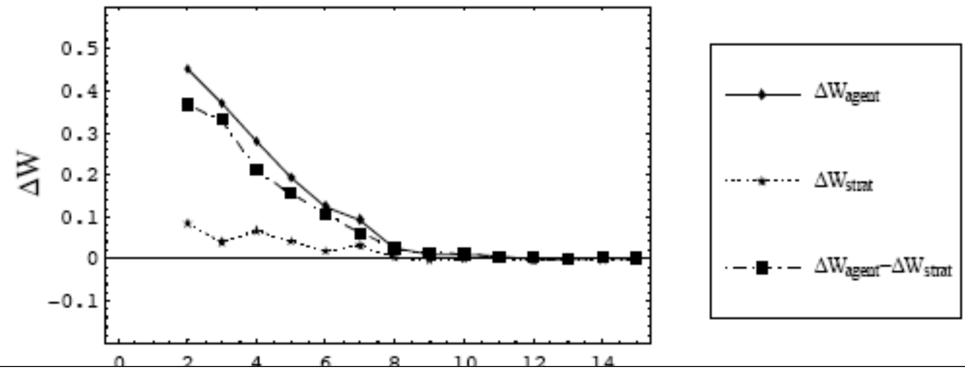
Control v. Illusion in Min.Game



Control v. Illusion in Maj.Game



Control v. Illusion in $Game



The illusion of control

Definition of "Illusion of control" in set-ups a priori defined to emphasize the importance of optimization:

Low entropy (more informative) strategies

under-perform

high-entropy (random) strategies

Other examples where uncertainty and risks can be amplified by attempt to manage and control:

-control algorithm with optimal parameter optimization based on past observations generate power law PDF of fluctuations

-quality control

-management strategy during times of crises (distressed firms…)

How can we falsify the value of control and management?