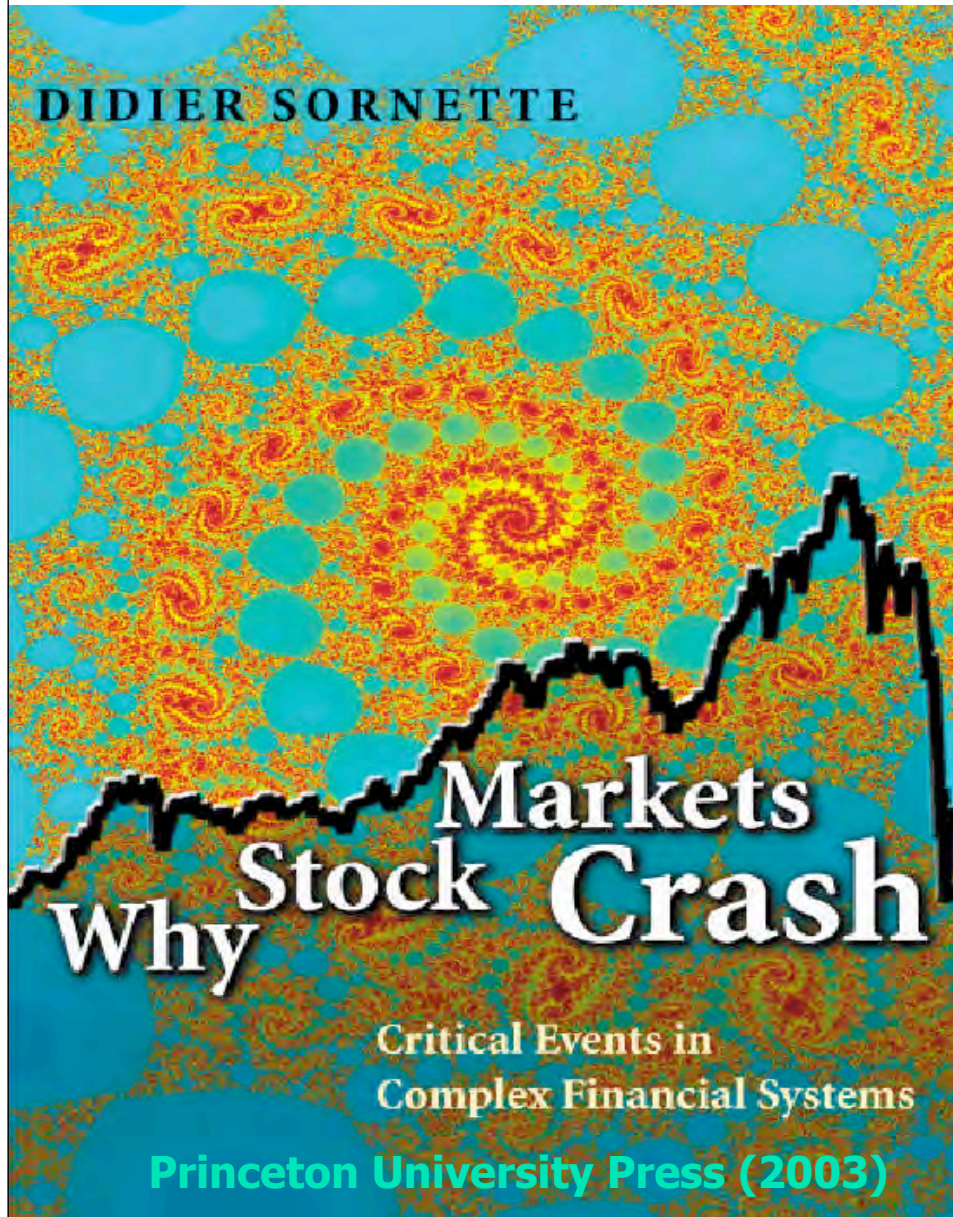


# Physics and Financial Economics (1776-2009)



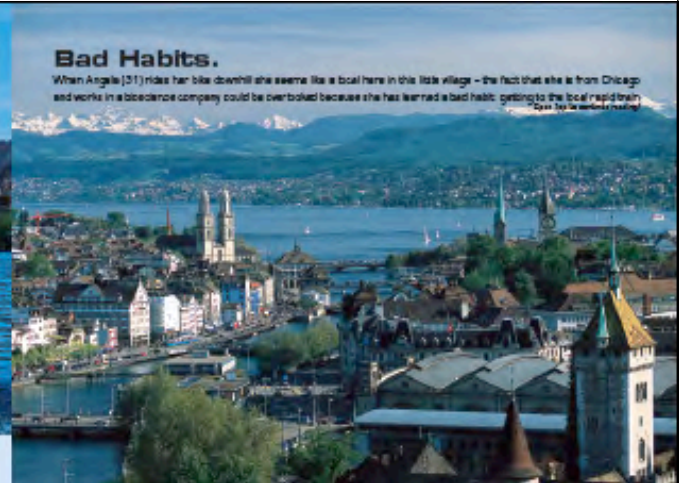
Chair of Entrepreneurial Risks

Department of Management,  
Technology and Economics,  
ETH Zurich, Switzerland

Member of the Swiss Finance  
Institute

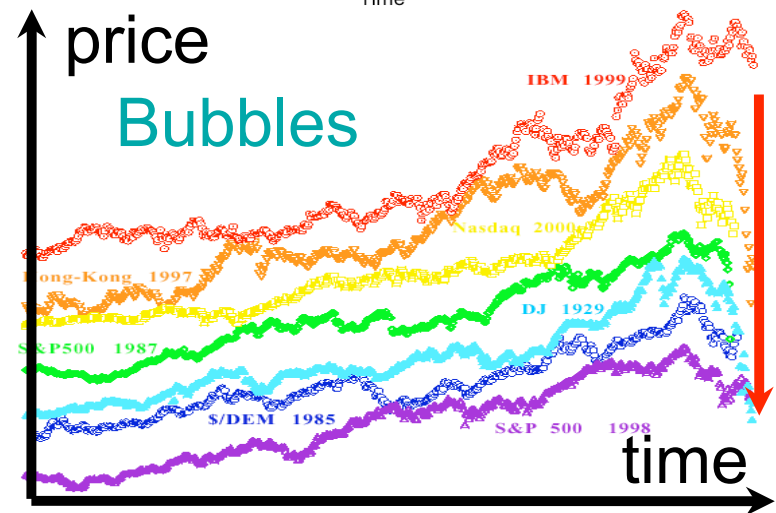
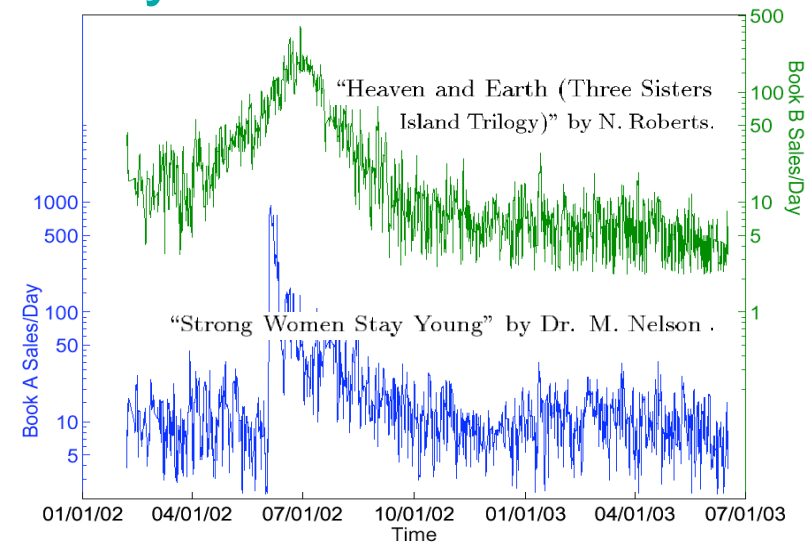
co-founder of the Competence  
Center for Coping with Crises in  
Socio-Economic Systems, ETH  
Zurich (<http://www.ccss.ethz.ch/>)

This is ETH Zurich → 12 700 students → 350 professors  
→ 3600 other teaching and research staff → 2 campuses  
→ 21 Nobel Prizes → 136 labs → 21% international students  
→ 90 nationalities → 36 languages



- Collective dynamics and organization of social agents (Commercial sales, YouTube, Open source softwares, Cyber risks)
- Agent-based models of bubbles and crashes, credit risks, systemic risks
- Prediction of complex systems, stock markets, social systems
- Asset pricing, hedge-funds, risk factors...
- Human cooperation for sustainability
- Natural and biological hazards (earthquakes, landslides, epidemics, critical illnesses...)

## Dynamics of success



(3 guest-professors, 5 foreign associate professors,  
3 post-docs, 2 senior researcher, 12 PhD students, 4-6 Master students)

# ECONOMICS

Adam Smith “Inquiry into the Nature and Causes of the Wealth of Nations” (1776)

•Francis Edgeworth and Alfred Marshall (1890) develop the concept of **equilibrium**

•“everything in the economy affects everything else”

•Vilfredo Pareto (1897): power law distribution of incomes

•Louis Bachelier (1900): random walk model of Paris stock market and solution of diffusion equation

•Benoit Mandelbrot (1963) proposes heavy-tailed distributions (Levy stable laws) for the pdf of cotton returns

•initially supported by Merton Miller, Eugene Fama, and Richard Roll (Chicago), Paul Samuelson (MIT), and Thomas Sargent (Carnegie Mellon), but opposition from Paul Cootner and Clive Granger;

•distributions of returns are becoming closer to the Gaussian law at timescales larger than one month.

# PHYSICS

•Isaac Newton Philosophiae Naturalis Principia Mathematica (1687) [(**novel at the time**) **notion of causative forces**]

•Clerk Maxwell and Ludwig Boltzmann (1871-1875): **equilibrium** in gases

•mean-field theory or self-consistent effective medium methods

•distribution of event sizes (earthquakes, avalanches, landslides, storms, forest fires, solar flares, commercial sales, war sizes, ...)

•Einstein (1905): theory of Brownian motion

•Benoit Mandelbrot (1977): Fractals

•Much of the efforts in the econophysics (1993-2000s) refined the Levy hypothesis into

$$\text{pdf}(r) \sim 1/r^3 \quad 4$$



## ECONOMICS

Adam Smith (1776)  
“Inquiry into the Nature  
and Causes of the Wealth  
of Nations”



## PHYSICS

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Philosophiae Naturalis  
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mean-field theory or self-consistent effective medium methods



Equilibrium

## ECONOMICS

Vilfredo Pareto (1897)  
power law distribution of  
incomes



## PHYSICS

distribution of event sizes  
(earthquakes, avalanches,  
landslides, storms, forest  
fires, solar flares,  
commercial sales, war  
sizes, ...)

Non-Gaussian heavy tailed distributions

## ECONOMICS

Louis Bachelier (1900)  
random walk model of  
Paris stock market and  
solution of diffusion  
equation



## PHYSICS

Einstein (1905)  
theory of Brownian motion

Theory of random walks and diffusion equation



## ECONOMICS

Benoit Mandelbrot (1963)  
Heavy-tailed distributions  
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distributions of returns are  
becoming closer to the  
Gaussian law at timescales  
larger than one month.

self-similarity

## PHYSICS

Benoit Mandelbrot (1977)  
Fractals

Much of the efforts of  
econophysicists (1993-2000s)  
refined the Levy hypothesis  
into

$$\text{pdf}(r) \sim 1/r^3$$

## ECONOMICS

- Paul Krugman (1996)  
“Self-organizing economy”
- Brian Arthur (1992)  
Induction, out-of-equilibrium
- Santa Fe Institute (1994-...)

## PHYSICS

- P. W. Anderson (1957)  
“More is different” (1972)
- Out-of-Equilibrium
- frozen heterogeneity  
(spinglasses, glasses, proteins)



Out-of-equilibrium, frozen heterogeneity,  
Self-organization, phase transitions ....

# A partial lists of achievements of Econophysics

- scaling laws, “universality”
- agent-based models, induction, evolutionary models [1, 9, 11, 21],
- minority games [8],
- option theory for incomplete markets [4, 6],
- “string theory” of interest rate curves [5, 38],
- theory of Zipf law and its economic consequences [12, 13, 27],
- theory of large price fluctuations [14],
- theory of bubbles and crashes [17, 22, 40],
- random matrix theory applied to covariance of returns [20, 36, 37],
- methods and models of dependence between financial assets [25, 43].

# FOUR EXAMPLES

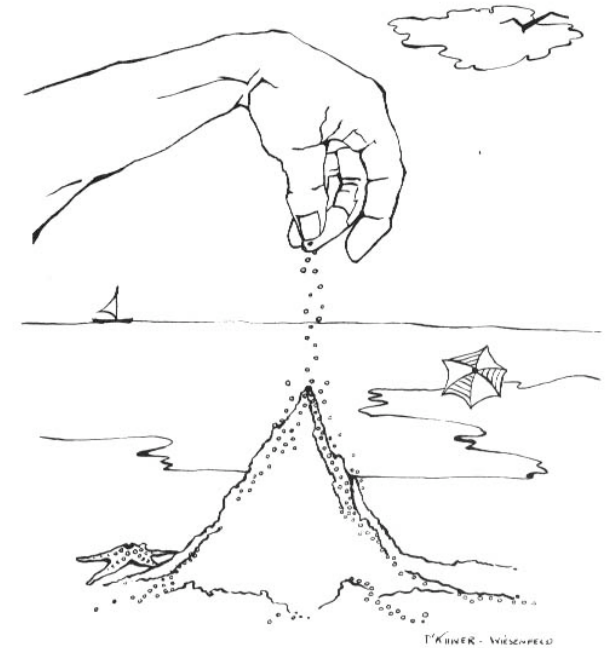
(i) the **fluctuation-susceptibility theorem** transforms into a remarkable classification of financial volatility shocks (endogenous versus exogenous),

(ii) the **Ising model of phase transitions** can be generalized to model the stylized facts of financial markets,

(iii) the concepts of collective phenomena and phase transitions (with **spontaneous symmetry breaking**) help understand financial bubbles and their following crashes,

(iv) the mathematics of quantum physics provides a new **quantum decision theory** solving the known paradoxes.

- **Self-organization?**  
**Extreme events are just part of the tail of power law distribution due to “self-organized criticality”?**  
**(endogenous)**

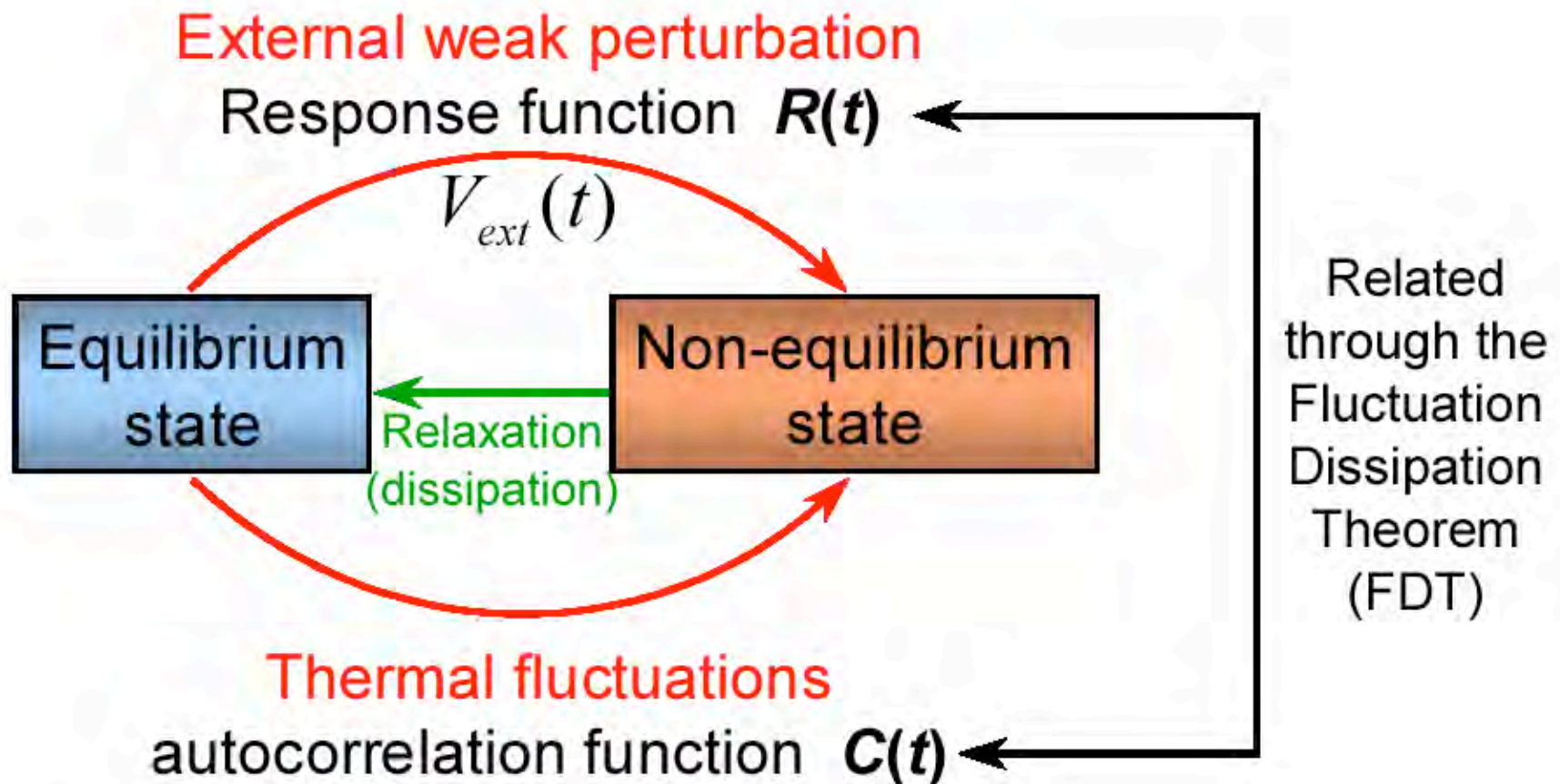


Artwork by Elaine Wiesenfeld  
(from Bak, How Nature Works)

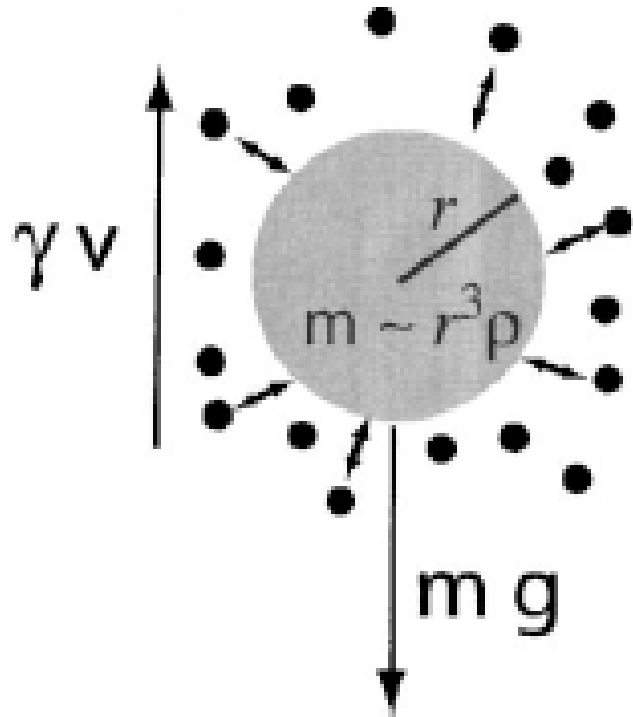
- **“Catastrophism”**: extreme events require extreme causes that lie outside the system  
**(exogenous)**
  
- **A mixture? How would it work?**

# Guidelines from Physics: perturb and study the response

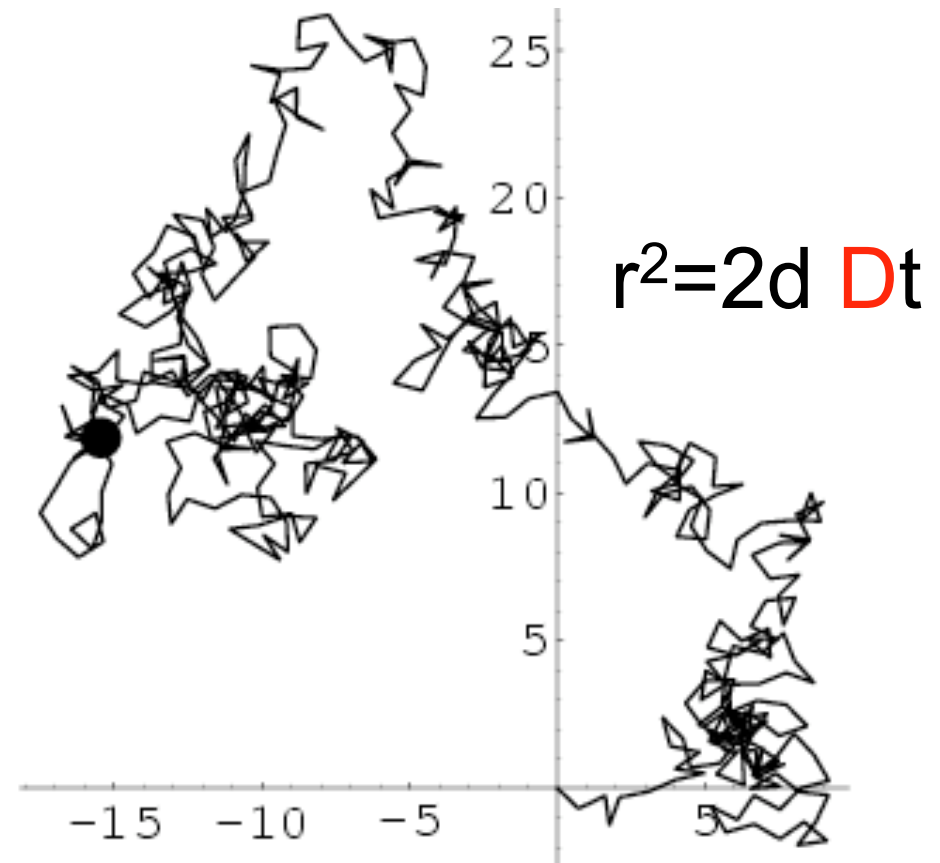
## Response Theory



**EXO:** Drag resistance  
under an external force



**ENDO:** Random walk

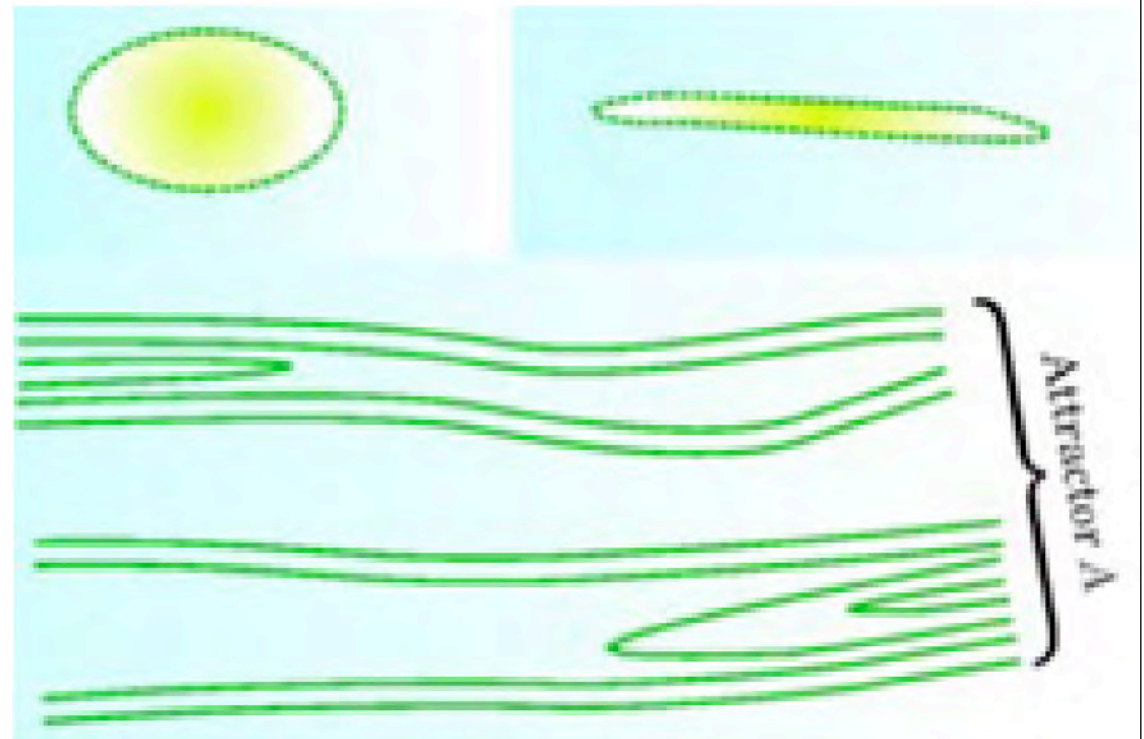


$$D = k_B T / \gamma$$

(Einstein, 1905)

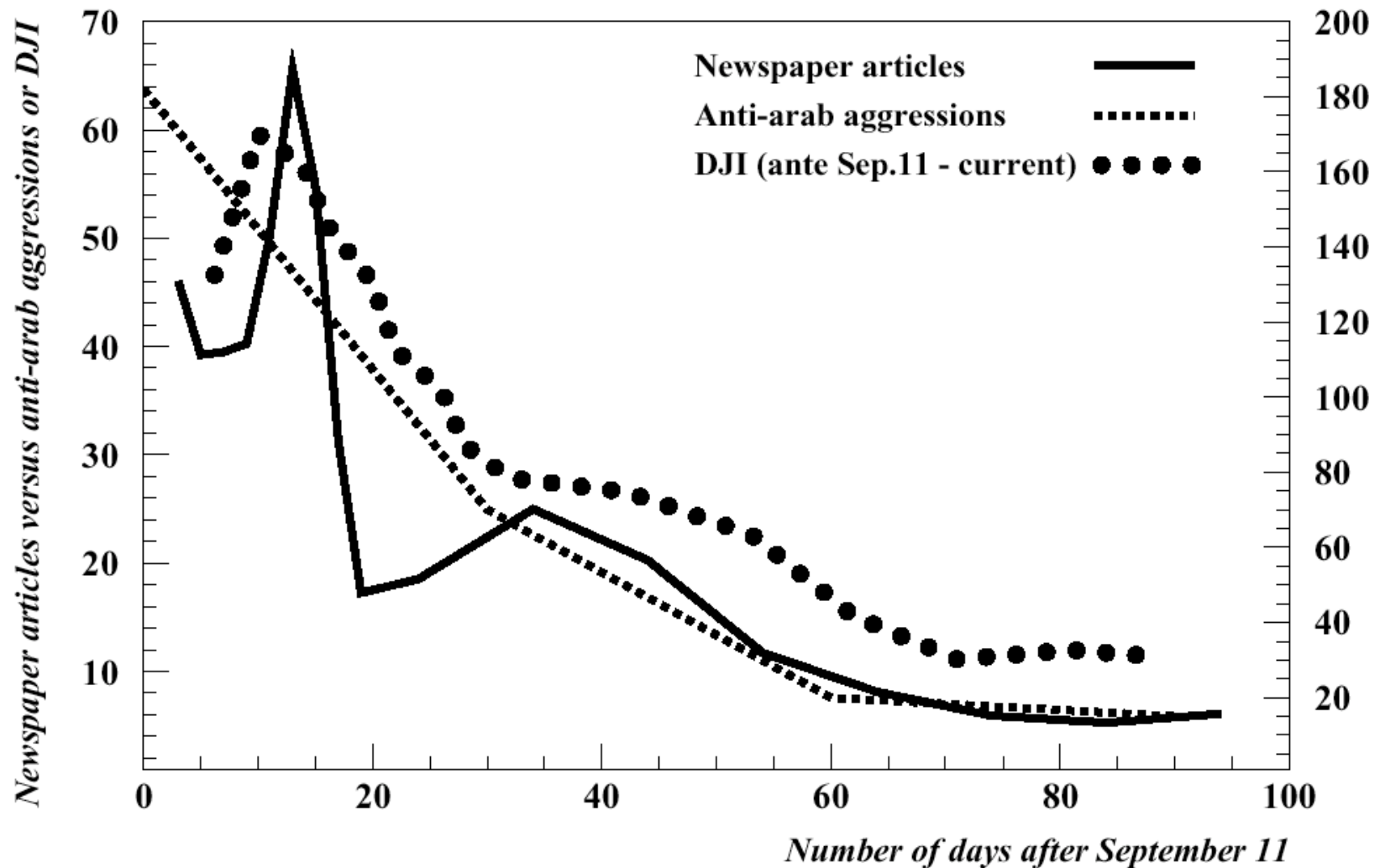
## Fluctuation-dissipation theorem far from equilibrium is not expected to hold

- Externally imposed perturbations may be different from spontaneous fluctuations (external fluctuations lie outside the complex attractor)
- Attractor of dynamics may exhibit bifurcations





# The method of critical events in economics and social sciences

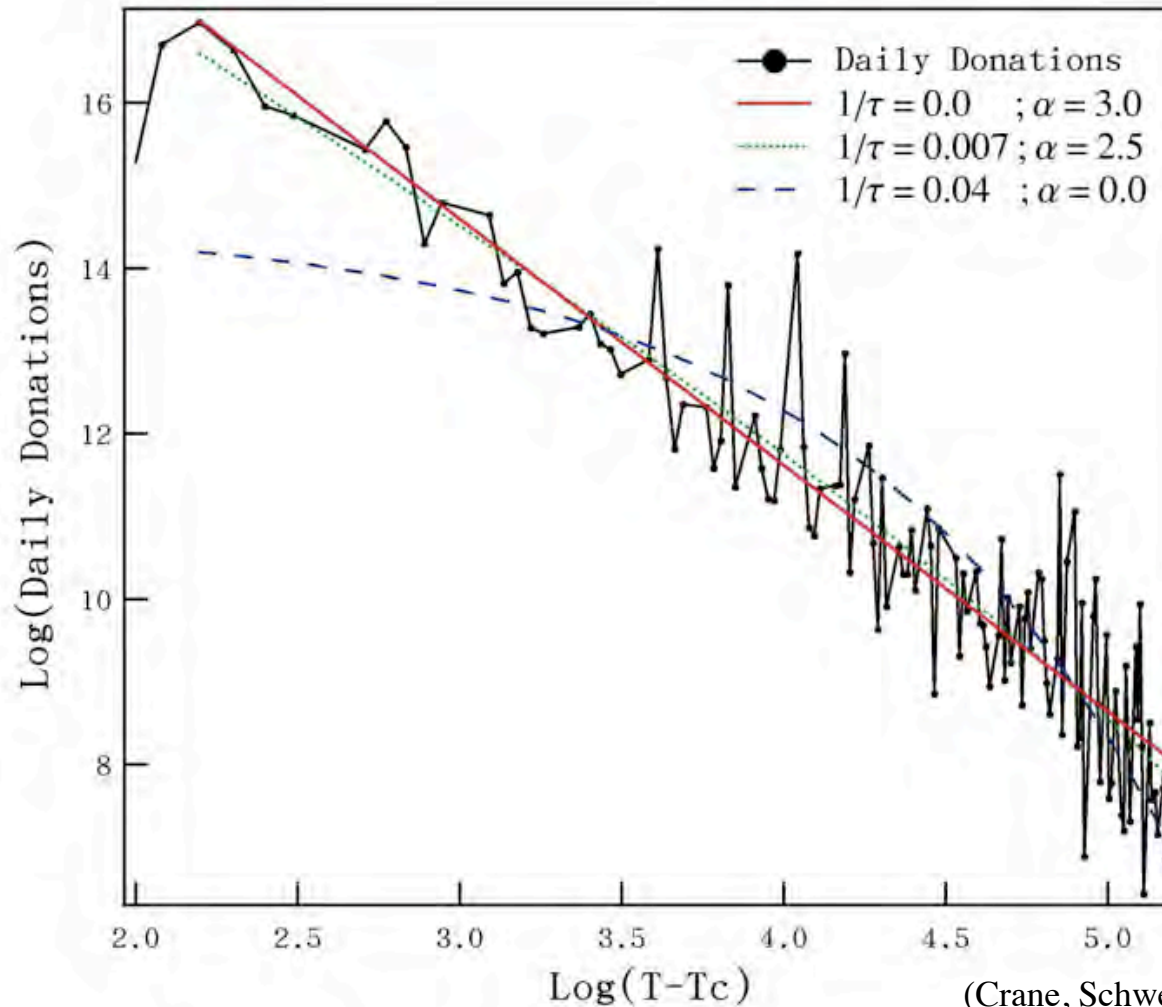


**Fig.2: Relaxation curves after the shock of September 11.** The solid line curve is the same as in Fig.1 but over a larger time interval; the broken line (scale on the right-hand side) shows the number of anti-arab aggressions in California in the three months after September 11; the dotted line shows the changes in the level of the Dow Jones Index with respect to its pre-Sep.11 level as given by the difference DJI(pre-9/11)-DJI(current). The tails of all three curves are well-approximated by power laws  $\sim 1/t^\alpha$ , with exponents  $\alpha$  comprised between -1.4 and -2.2:  $\alpha_1 = -1.8 \pm 0.7$  (newspaper articles),  $\alpha_2 = -1.4 \pm 0.5$  (anti-arab aggressions) and  $\alpha_3 = -2.2 \pm 1.6$  (DJI).

(Roehner and Sornette, 2004)



## Daily number of donations following the Asian Tsunami of December 26, 2004



## FINANCIAL SHOCKS

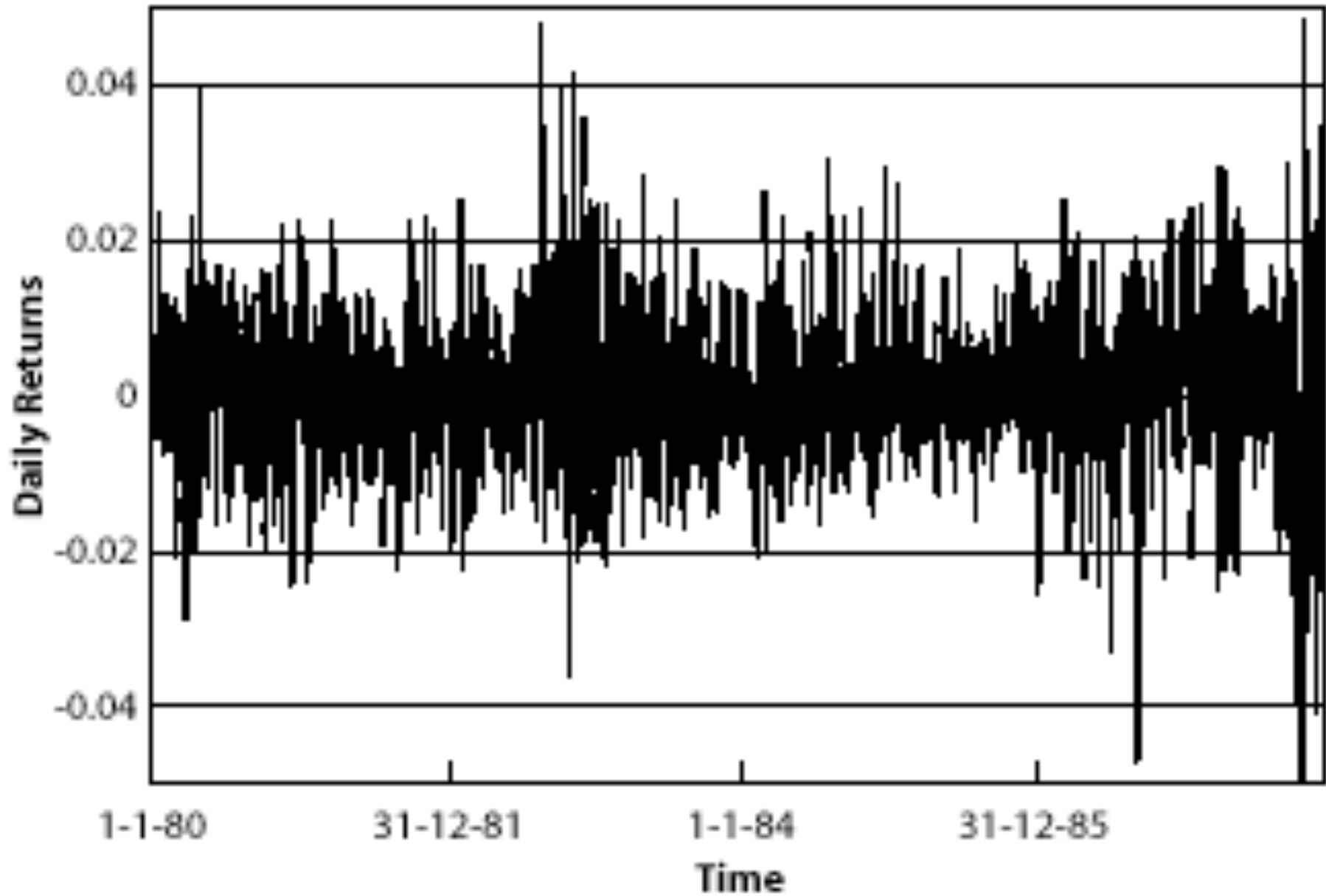


FIG. 1.8. Top panel: Time series of daily closes and volume of the Lucent Technology stock over a one-year period around the large drop of January 6, 2000. The time of the crash can be seen clearly as coinciding with the peak in volume (bottom panel). Taken from <http://finance.yahoo.com/>.

(Sornette, 2003)

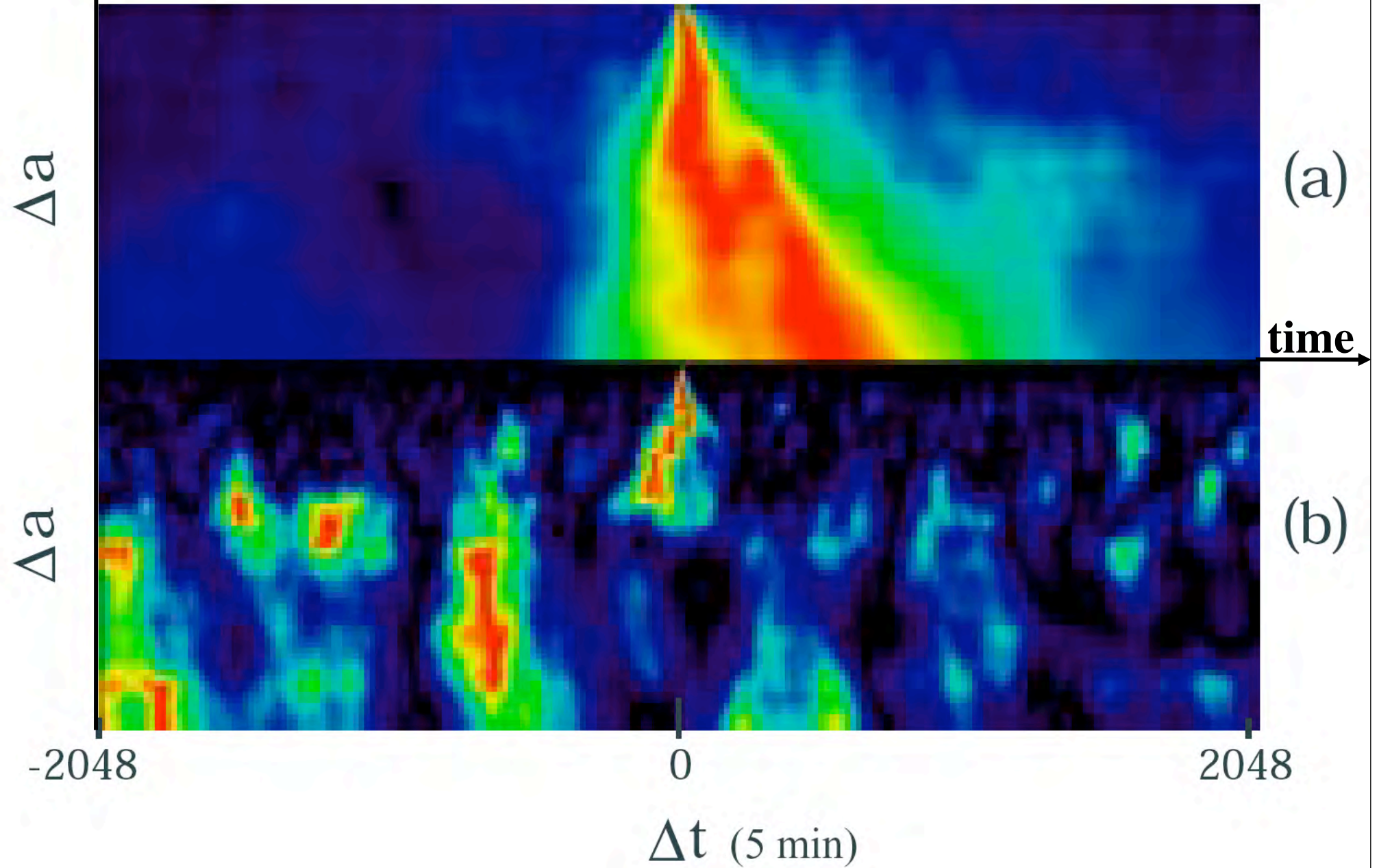
# Volatility

Dow Jones Index Returns Jan. 2nd 1980–Dec.31st 1987



scale

# DIRECT CAUSAL HIERARCHICAL CASCADE

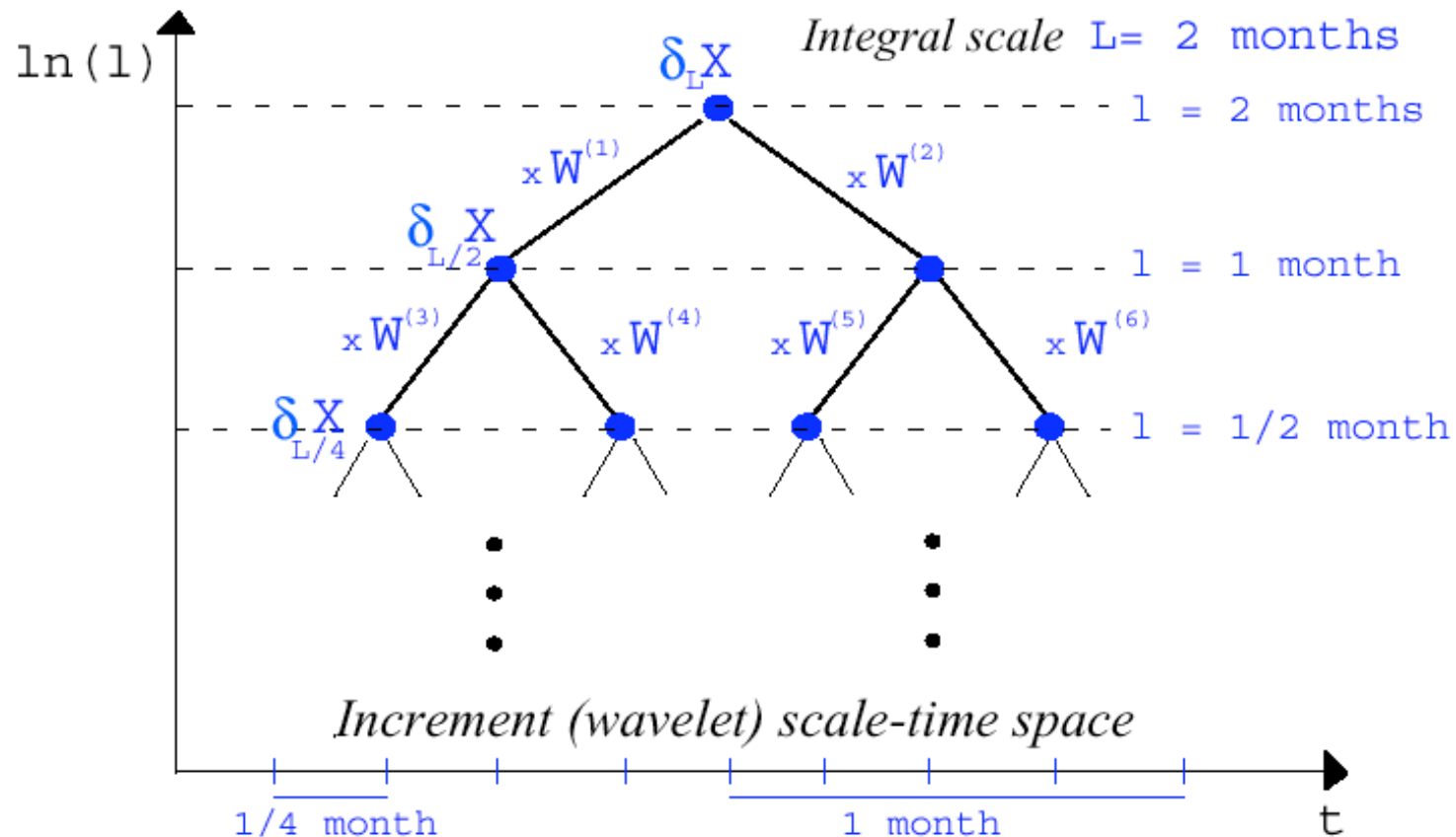


A. Arneodo, J.-F. Muzy and D. Sornette, Direct causal cascade in the stock market, European Physical Journal B 2, 277-282 (1998)

# The multiplicative cascade paradigm

$$\delta_{\lambda l} X(\lambda t) = \lambda^H \delta_l X(t) = W_\lambda \delta_l X(t)$$

- $W$ -cascades (wavelet cascade)



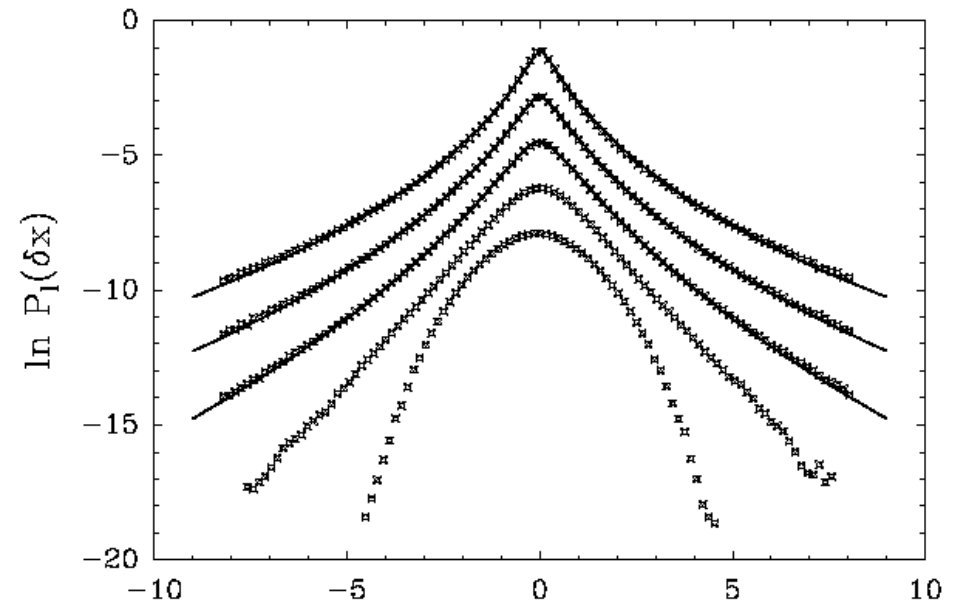
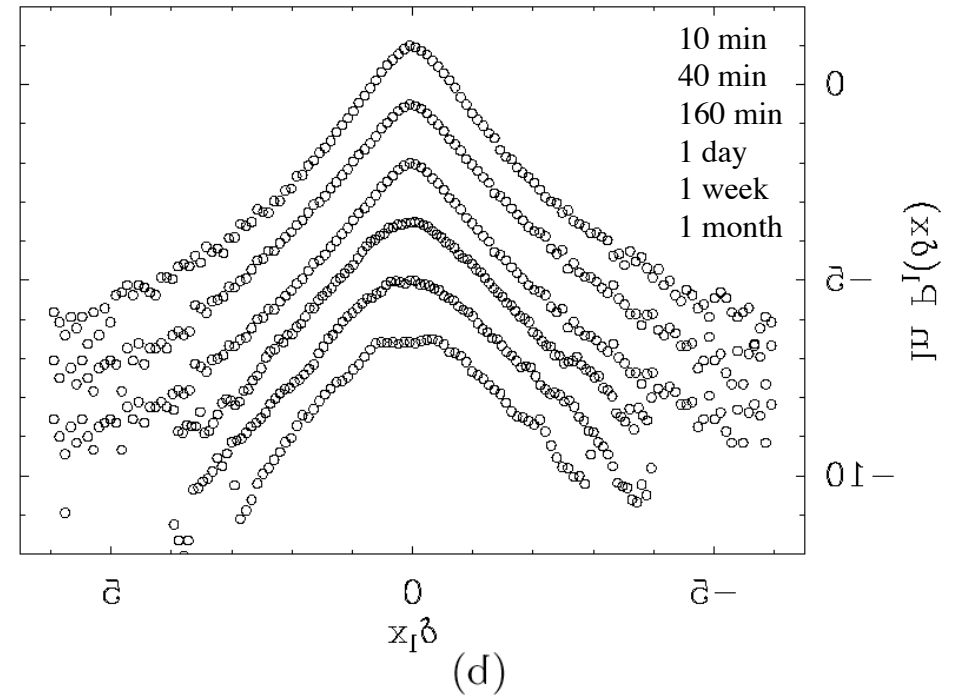
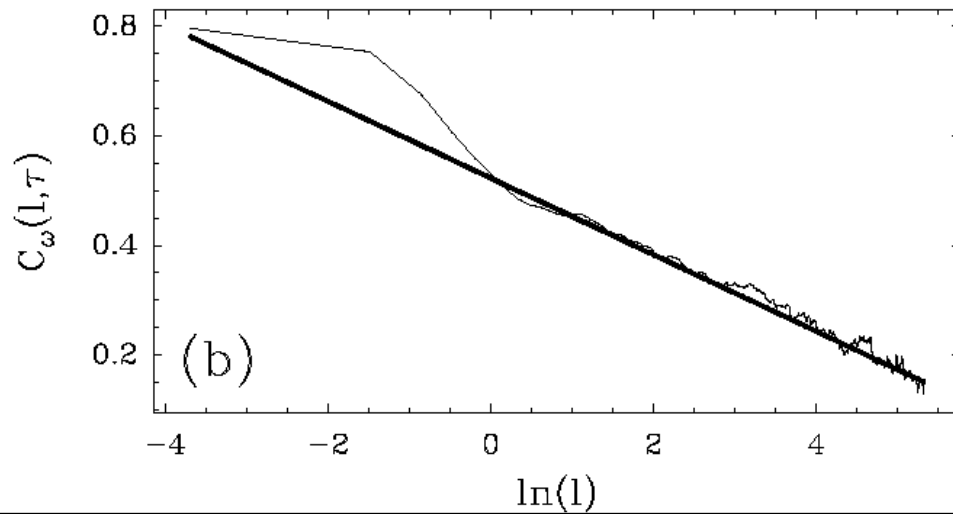
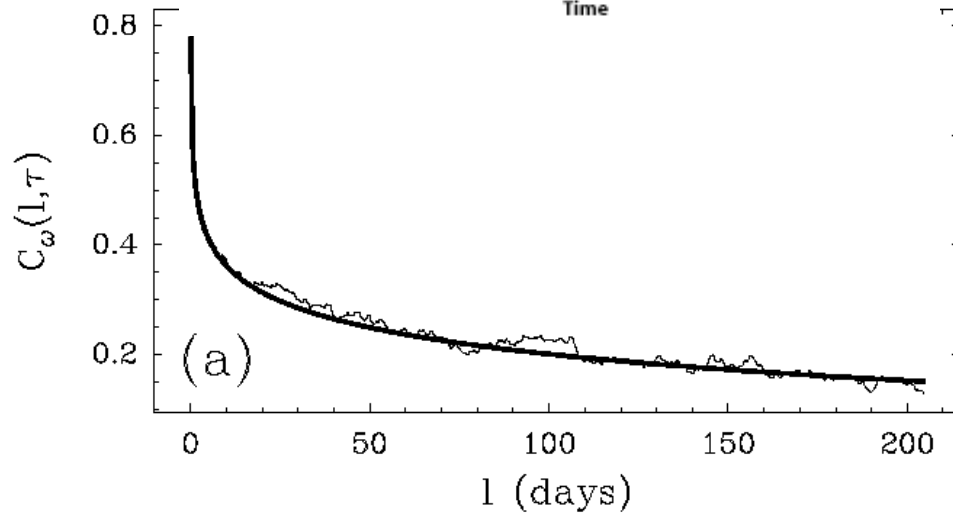
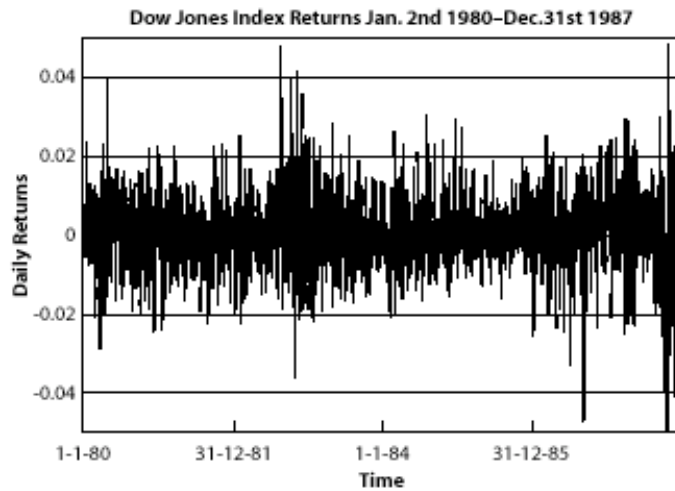
# The Multifractal Random Walk (MRW) model

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)}$$

$$\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^t d\tau \eta(\tau) K_{\Delta t}(t - \tau)$$

$$K_{\Delta t}(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for } \Delta t \ll \tau \ll T$$

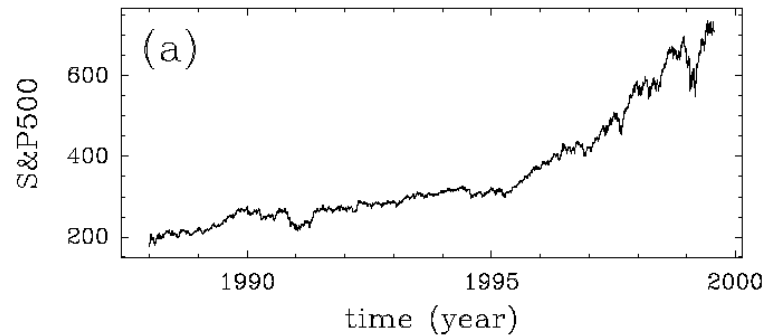
# Stylized facts in financial markets



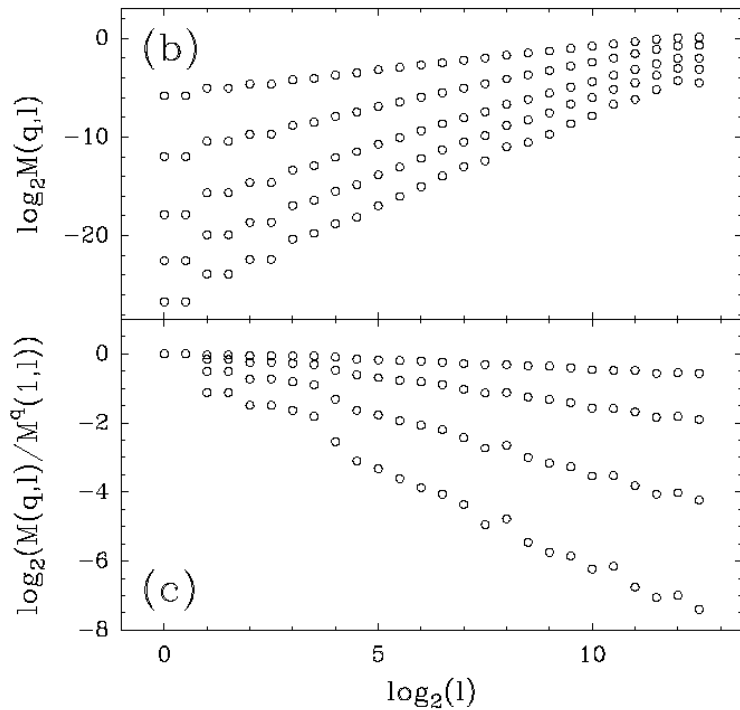
Multifractal random walk



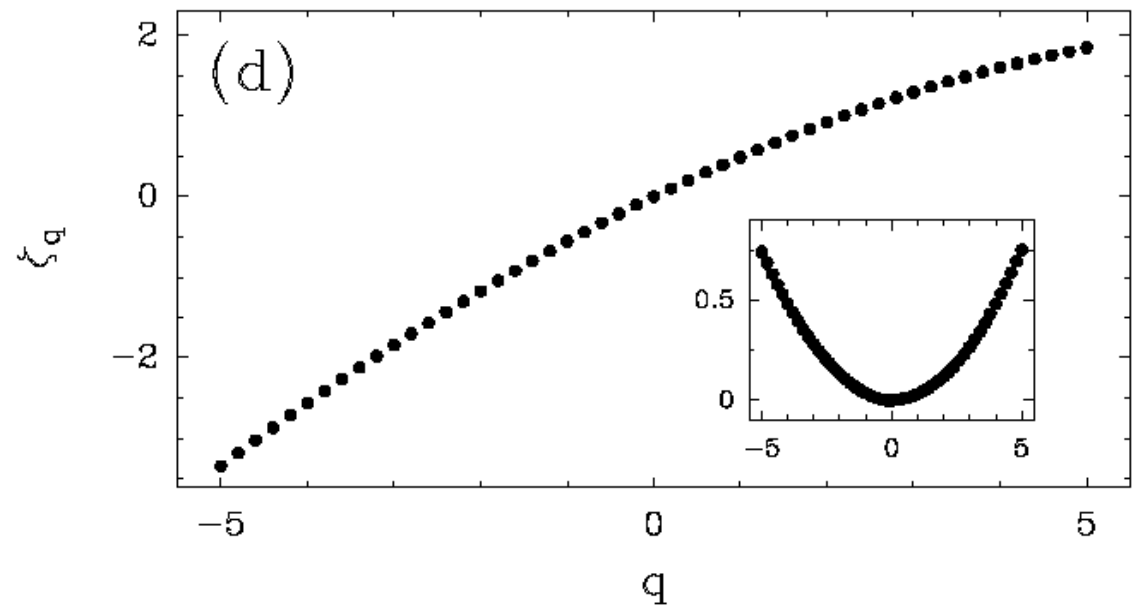
Multifractality:  $\langle [\delta_\tau X(t)]^q \rangle = a(q) \tau^{\zeta(q)}$ , for  $\tau < T$ .



$$\zeta(q) = \left(1 + \frac{\lambda^2}{2}\right)q - \frac{\lambda^2}{2}q^2$$

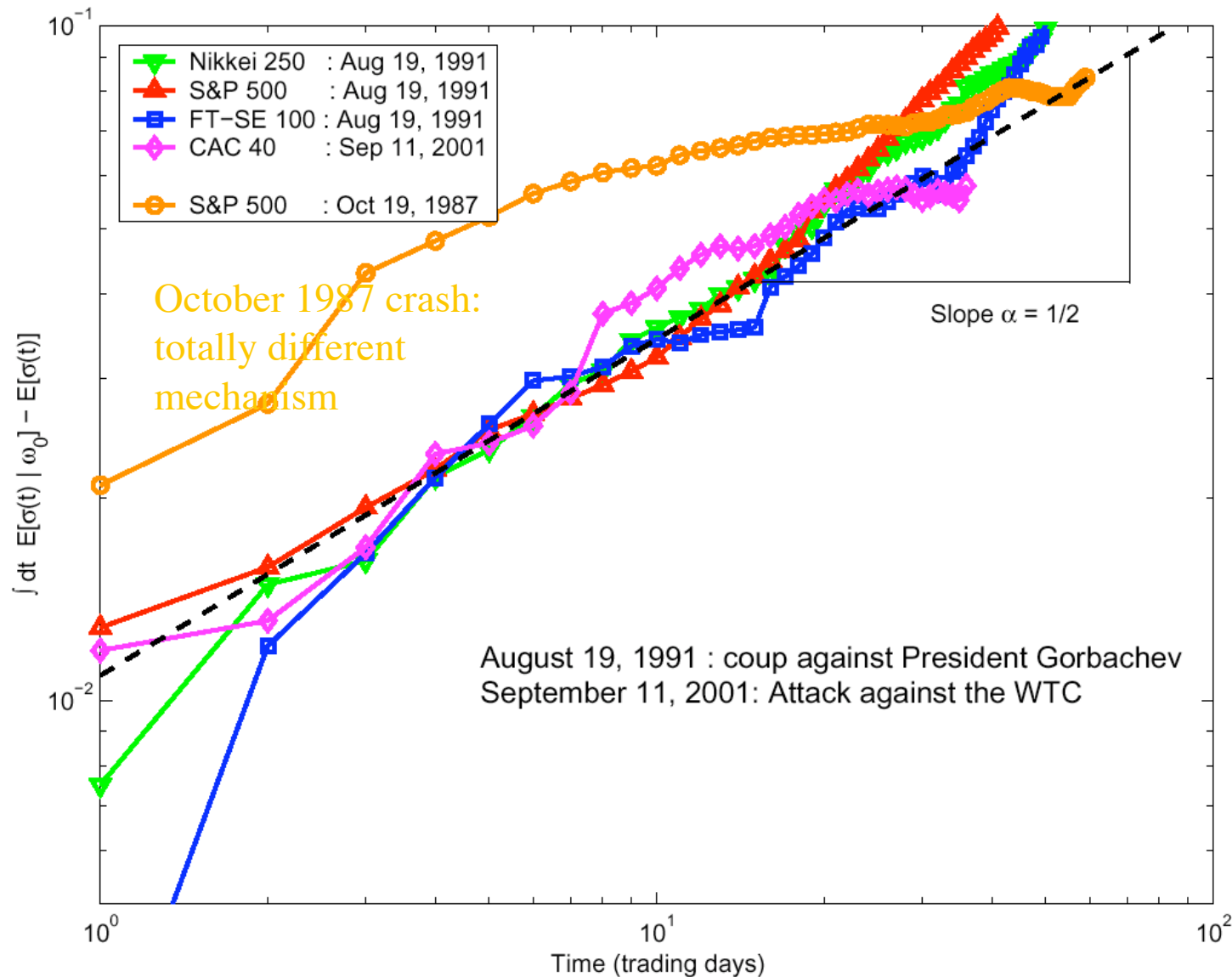


$\lambda^2 = -\zeta''(0)$  is the so-called *intermittency coefficient*.



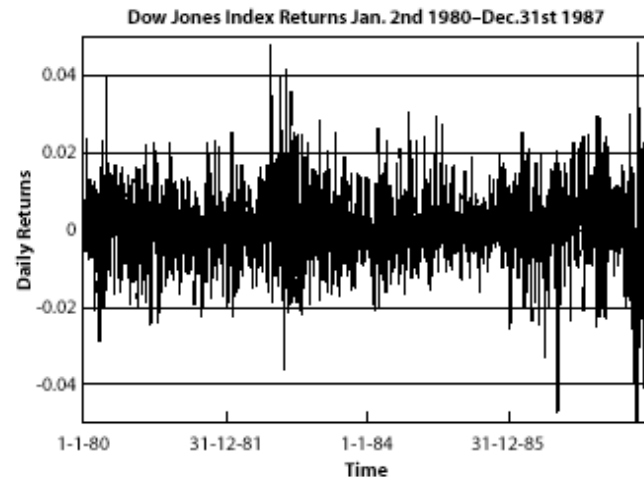
# Linear response to an external shock (Multifractal Random Walk model)

$$E_{\text{exo}}[\sigma^2(t) | \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



D. Sornette, Y. Malevergne and J.F. Muzy Volatility fingerprints of large shocks: Endogeneous versus exogeneous, Risk Magazine (<http://arXiv.org/abs/cond-mat/0204626>)

# “Conditional response” to an endogeneous shock



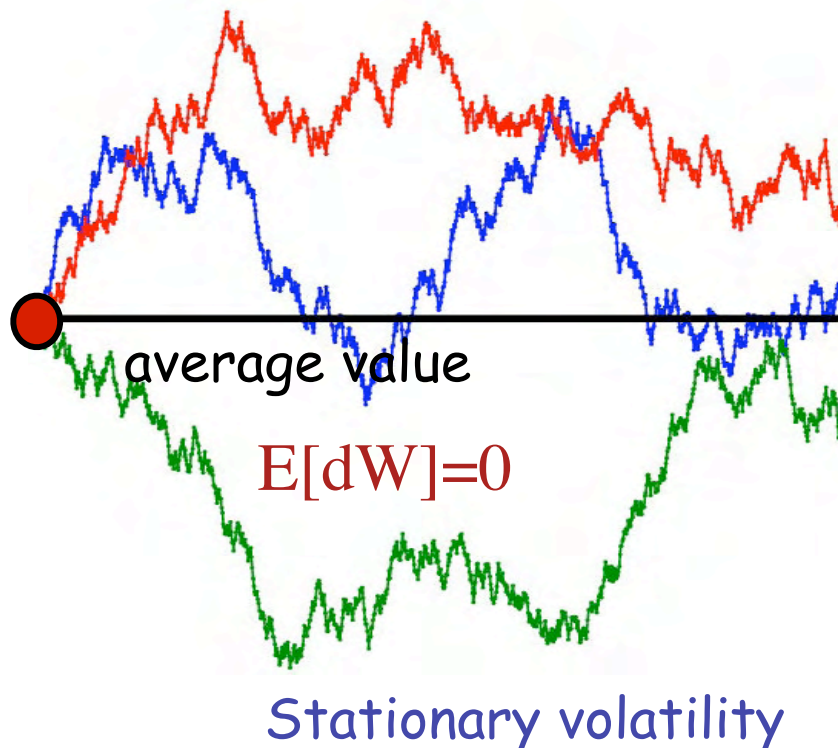
Interplay between  
-long memory  
-exponential

$$E_{\text{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

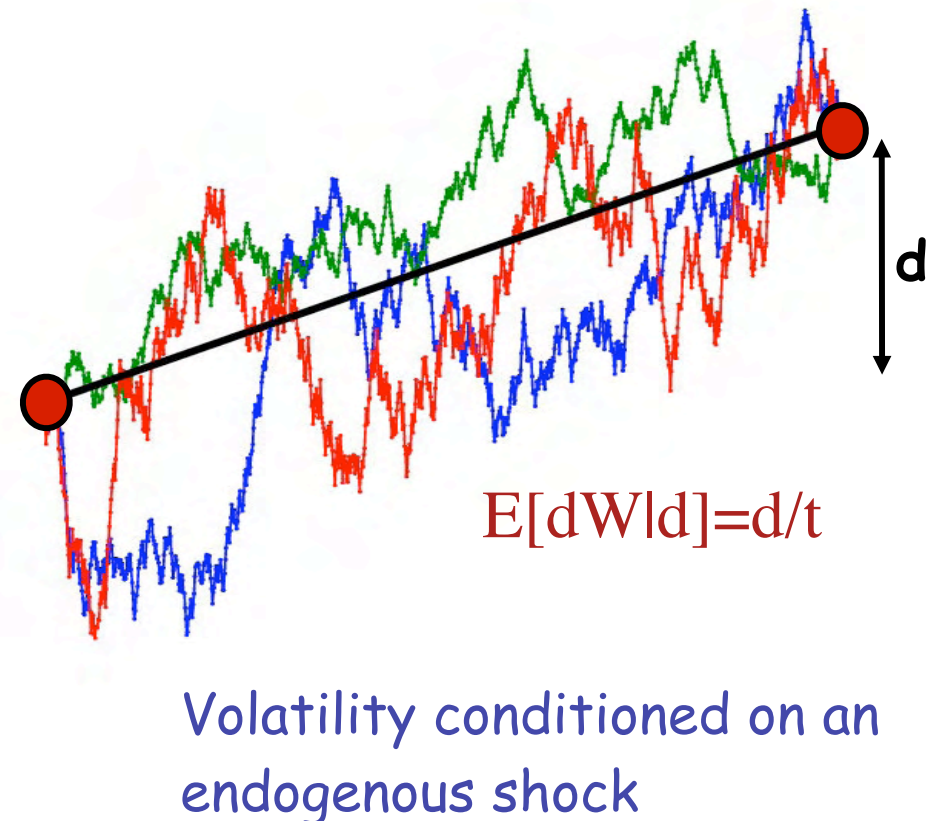
where  $\alpha(s) = \frac{2s}{\ln\left(\frac{T e^{3/2}}{\Delta t}\right)}$

# Analogy with Brownian motion

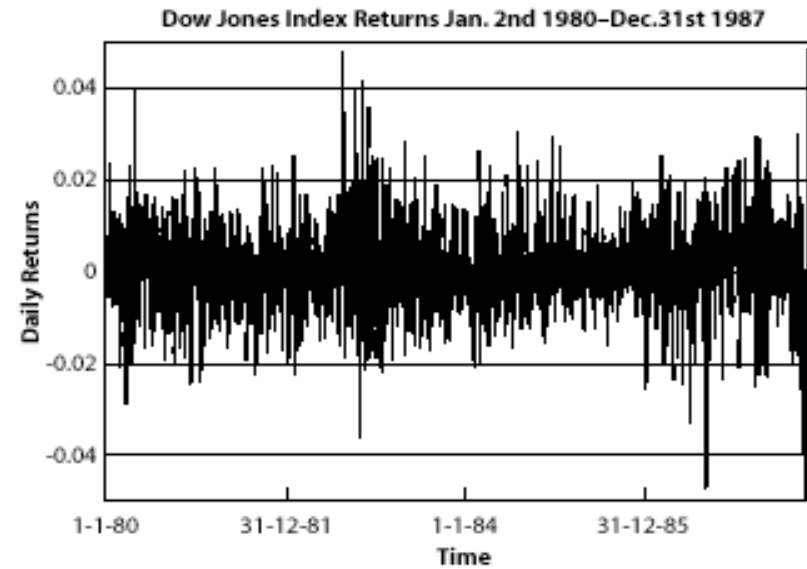
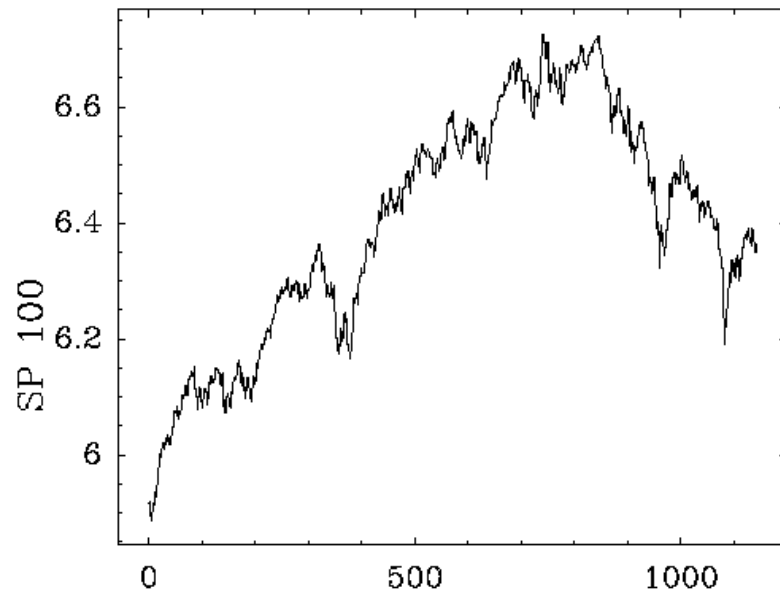
without conditioning:  
stationary process, average=0



conditioning to a large value  $W(t)=d$  :  
non-stationary process, average  $\neq 0$

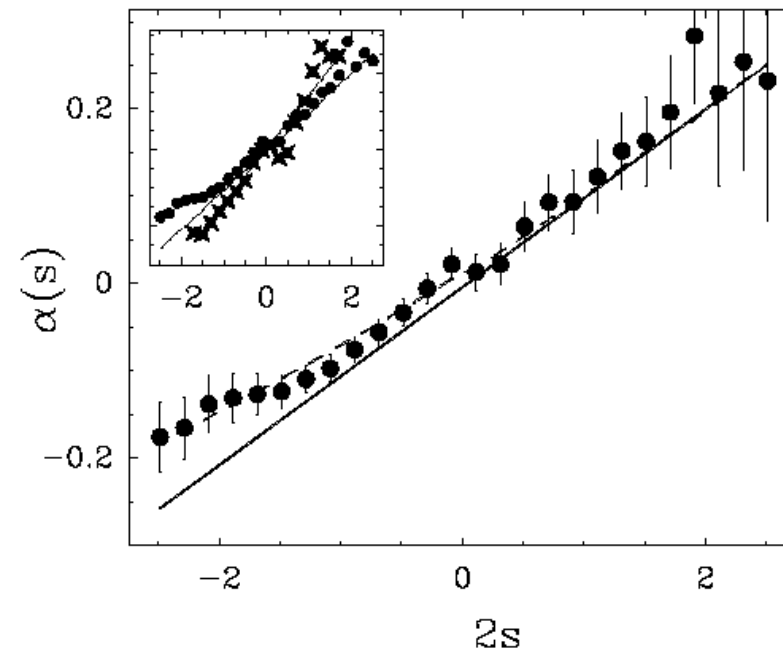
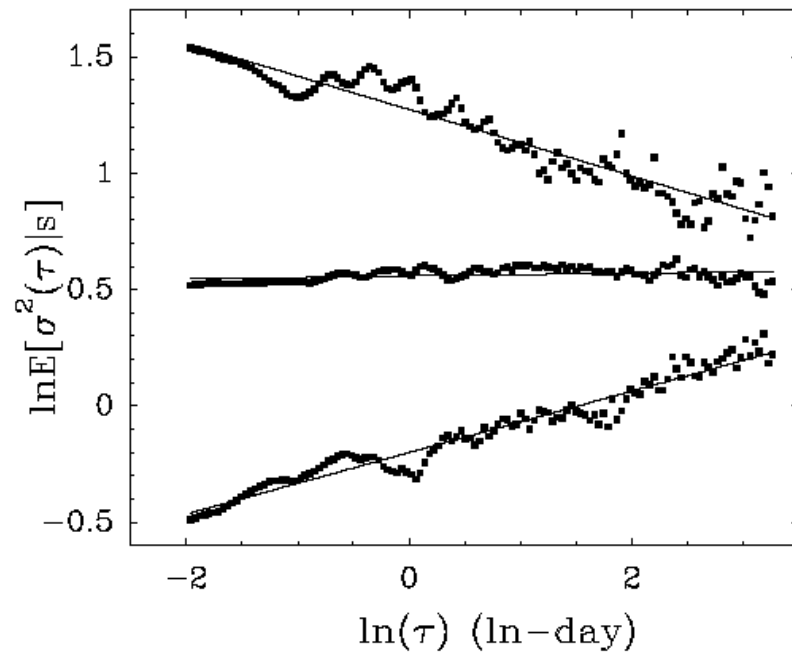


# Real Data and Multifractal Random Walk model

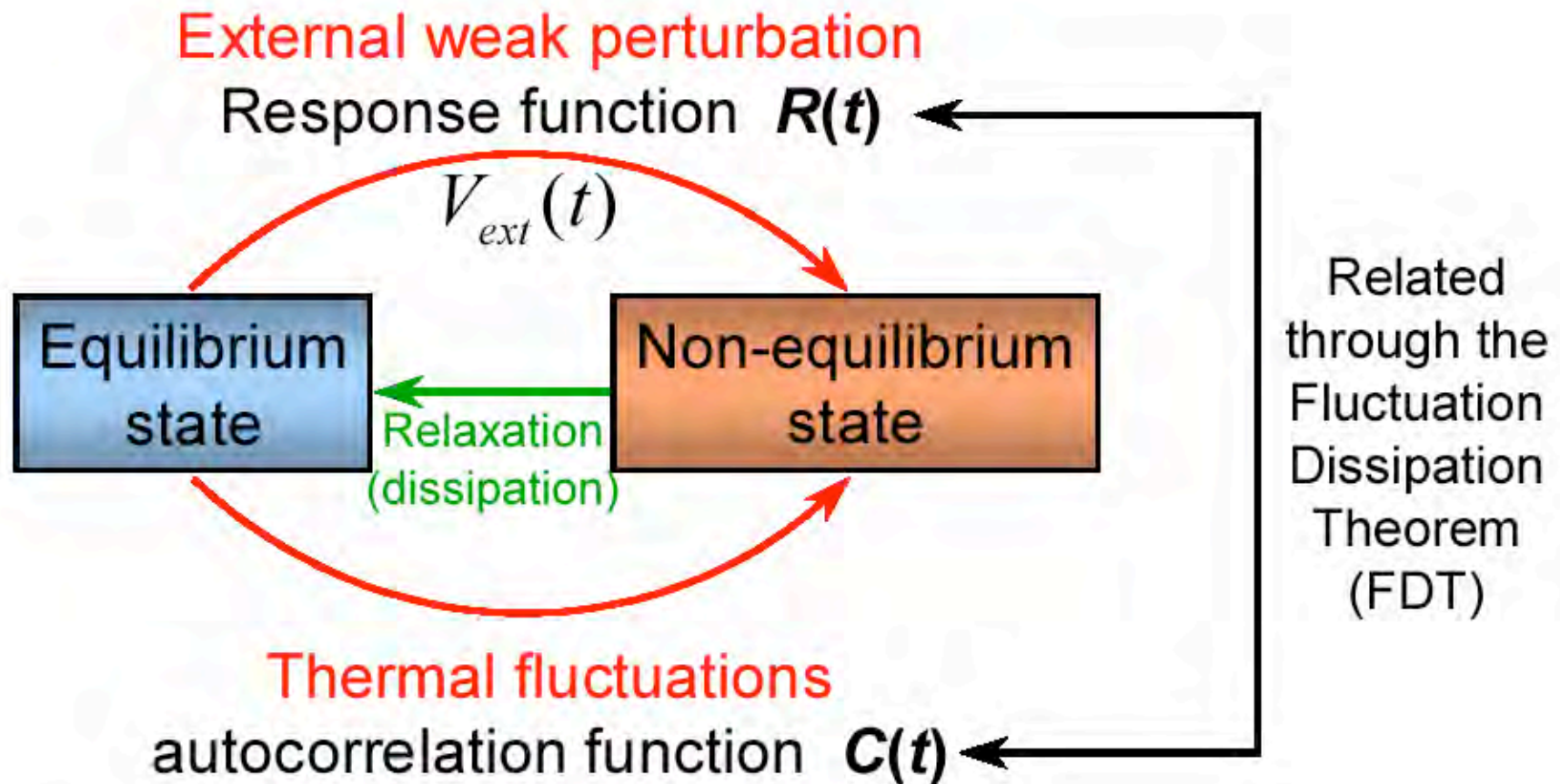


time (day)

$$E_{\text{endo}}[\sigma^2(t) | \omega_0] \sim t^{-\alpha(s)}$$



# Response Theory



# Endogenous versus Exogenous

## Extinctions

- meteorite at the Cretaceous/Tertiary KT boundary
- volcanic eruptions (Deccan traps)
- self-organized critical events

## Financial crashes

- external shock
- self-organized instability

## Immune system

- external viral or bacterial attack
- “ internal” (dis-)organization

## Brain (learning)

- external inputs
- internal self-organization and reinforcements (role of sleep)

## Aviation industry recession

- September 11, 2001
- structural endogenous problems

## Recovery after wars?

- internally generated (civil wars)
- externally generated

## Discoveries

- serendipity
- maturation

## Volatility bursts in financial time series

- external shock
- cumulative effect of “small” news

## Earthquakes

- tectonic driving
- triggering

## Parturition

- mother/foetus triggered?
- mother-foetus complex?

## Commercial success and sales

- Ads
- epidemic network

## Social unrests

- triggering factors
- rotting of social tissue

# FOUR EXAMPLES

(i) the fluctuation-susceptibility theorem transforms into a remarkable classification of financial volatility shocks (endogenous versus exogenous),

(ii) the **Ising model of phase transitions** can be generalized to model the stylized facts of financial markets,

(iii) the concepts of collective phenomena and phase transitions (with spontaneous symmetry breaking) help understand financial bubbles and their following crashes,

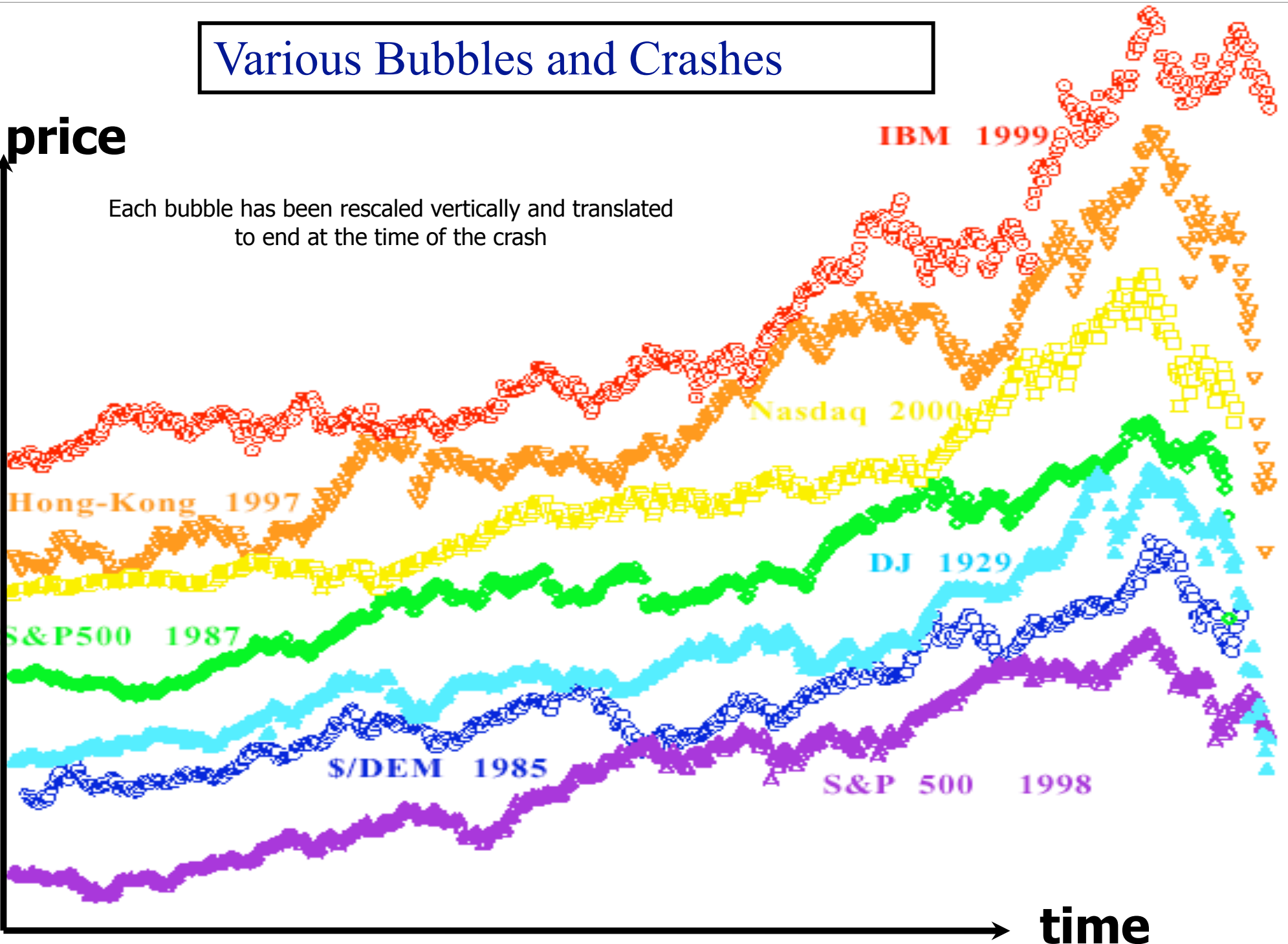
(iv) the mathematics of quantum physics provides a new quantum decision theory solving the known paradoxes.



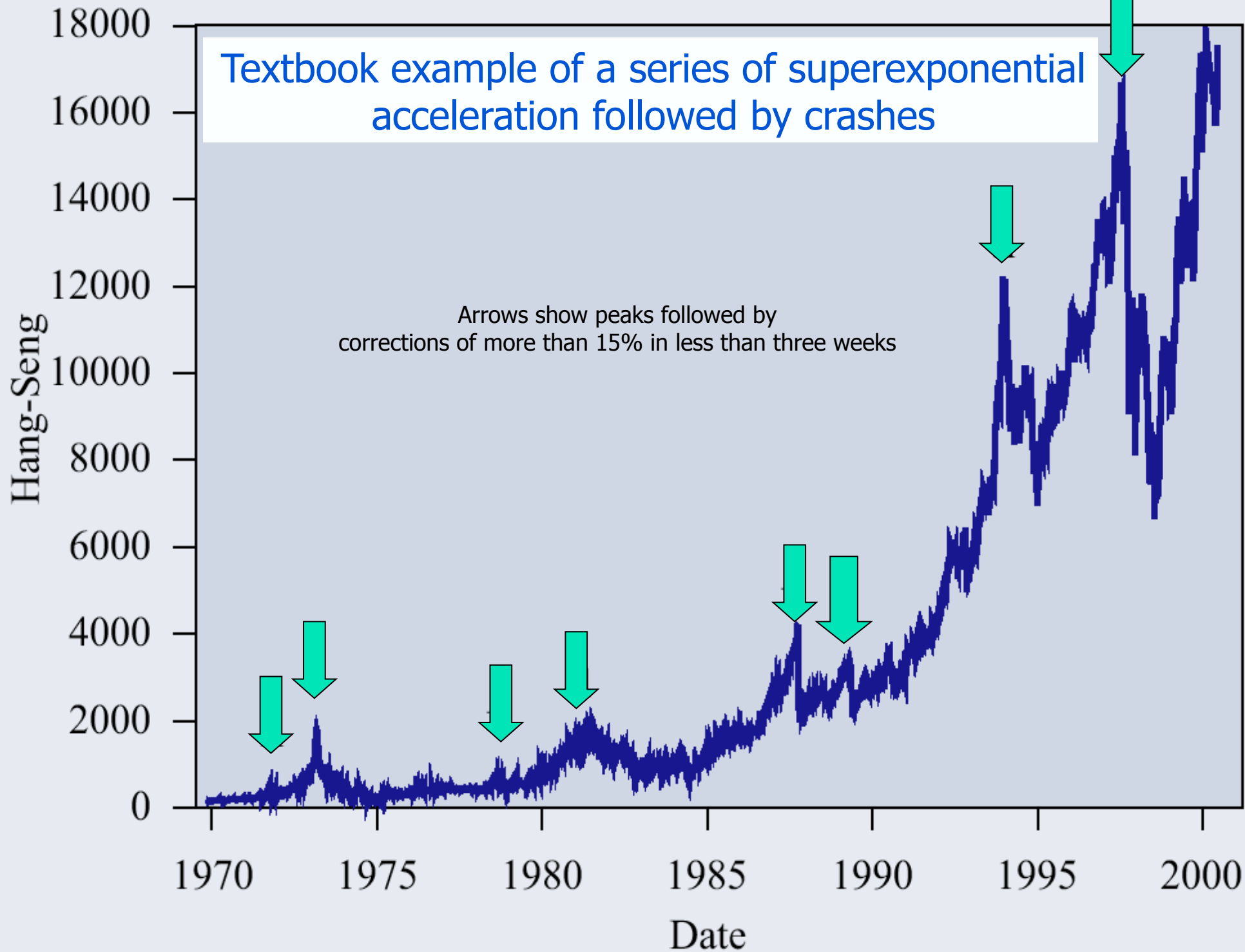
# Various Bubbles and Crashes

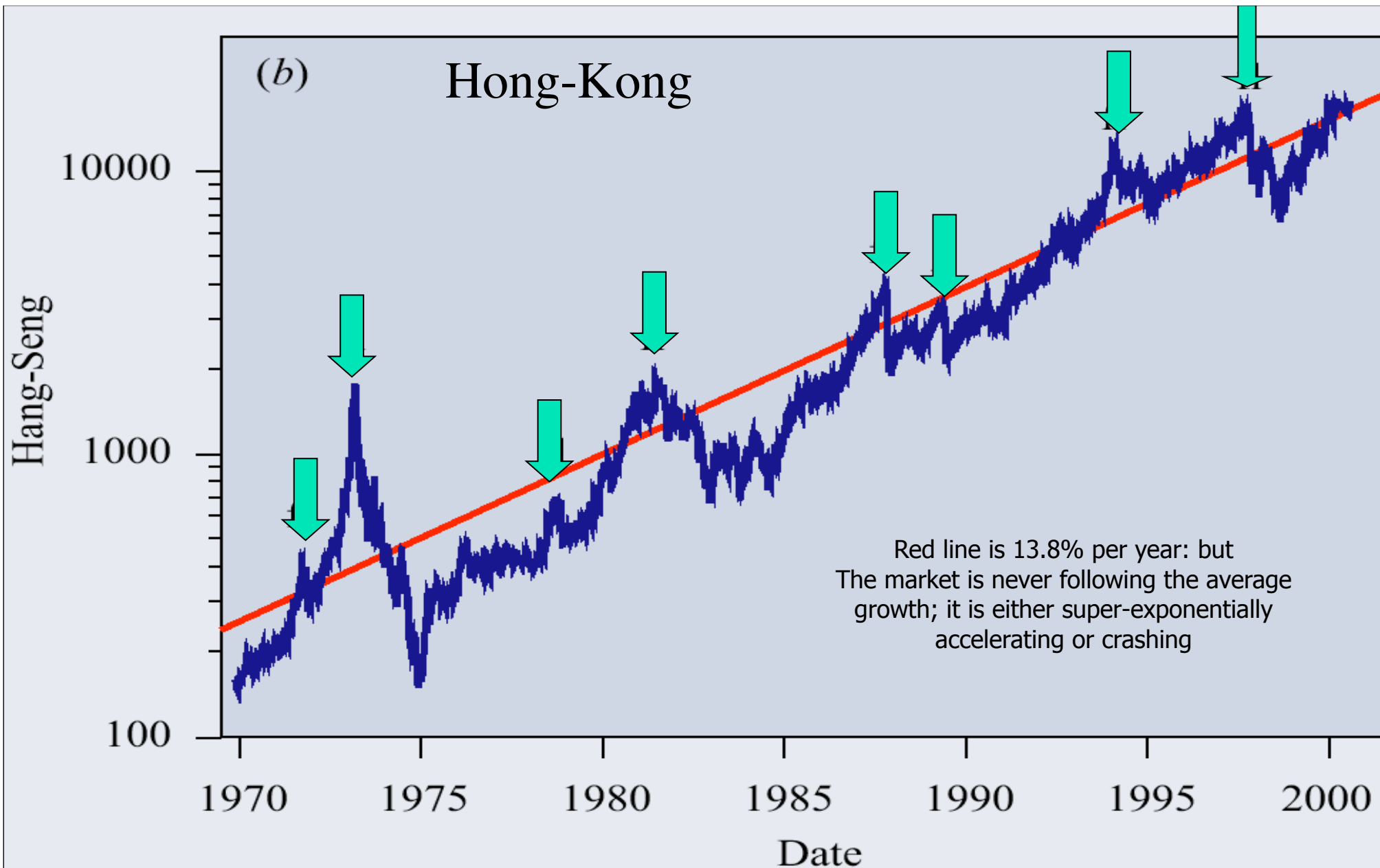
price

Each bubble has been rescaled vertically and translated to end at the time of the crash



time





Patterns of price trajectory during 0.5-1 year before each peak: Log-periodic power law

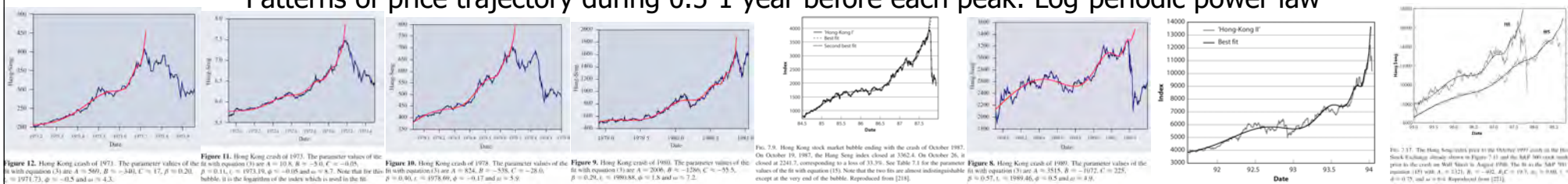


Figure 12. Hong Kong crash of 1973. The parameter values of the fit with equation (1) are  $A = 569$ ,  $\beta = -348$ ,  $C = 17$ ,  $\mu = 0.20$ ,  $\nu = 0.14$ ,  $\xi = 0.073$ ,  $\eta = -0.08$  and  $\omega = 8.7$ . Note that the fit (red) is to the logarithm of the index which is used in the fit.  
 Figure 11. Hong Kong crash of 1975. The parameter values of the fit with equation (1) are  $A = 10.8$ ,  $\beta = -5.0$ ,  $C = -0.05$ ,  $\nu = 0.14$ ,  $\xi = 0.073$ ,  $\eta = -0.08$  and  $\omega = 8.7$ . Note that the fit (red) is to the logarithm of the index which is used in the fit.  
 Figure 10. Hong Kong crash of 1979. The parameter values of the fit with equation (1) are  $A = 854$ ,  $\beta = -538$ ,  $C = -28.6$ ,  $\nu = 0.30$ ,  $\xi = 0.078$ ,  $\eta = -0.17$  and  $\omega = 5.9$ .  
 Figure 9. Hong Kong crash of 1980. The parameter values of the fit with equation (1) are  $A = 2036$ ,  $\beta = -1256$ ,  $C = -55.5$ ,  $\nu = 0.28$ ,  $\xi = 0.09038$ ,  $\eta = 1.8$  and  $\omega = 7.2$ , except at the very end of the bubble. Reproduced from [21].  
 Figure 8. Hong Kong crash of 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 7. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 6. Hong Kong crash of 1989. The parameter values of the fit with equation (1) are  $A = 3515$ ,  $\beta = -1072$ ,  $C = 225$ ,  $\nu = 0.57$ ,  $\xi = 0.09038$ ,  $\eta = 1.8$  and  $\omega = 8.6$ .  
 Figure 5. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 4. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 3. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 2. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.  
 Figure 1. The Hong Kong market bubble ending with the crash of October 1987. On October 19, 1987, the Hang Seng index closed at 3362.4. On October 26, it closed at 2243.7, corresponding to a loss of 33.9%. See Table 7.1 for the parameter values of the fit with equation (1). Note that the two fits are almost indistinguishable in this plot.

# Complex Systems

-positive feedbacks

-non sustainable regimes

-rupture

Thomas Robert Malthus (1766–1834)



1798

autocatalytic proliferation:  $\frac{dx}{dt} = a \cdot x$

with  $a$  = birth rate - death rate

exponential solution:  $X(t) = X(0)e^{a t}$

contemporary estimations = doubling of the population every 30yrs

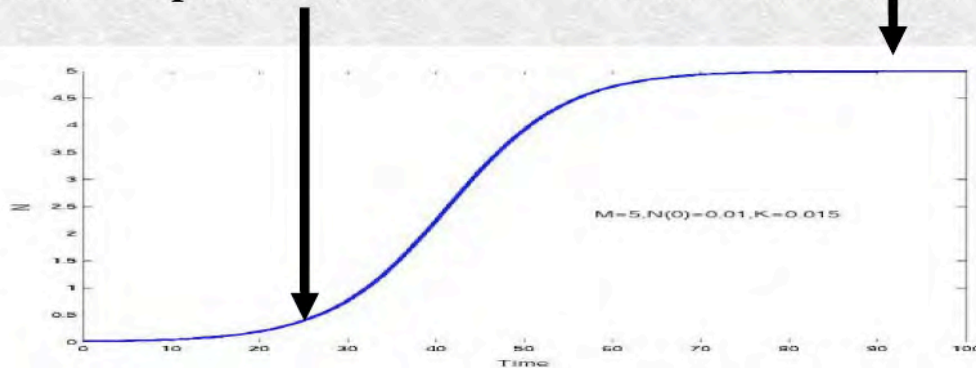
Pierre Franois Verhulst (1804-1849)



way out exponential explosion:

$$dX/dt = a X - c X^2 \quad 1838$$

Solution: exponential =====> saturation at  $X = a / c$



For humans data at the time could not discriminate between:

1. exponential growth of Malthus
2. logistic growth of Verhulst

But data fit on animal population: sheep in Tasmania

- exponential in the first 20 years after their introduction and completely saturated after about half a century. ==> Verhulst

## Positive feedbacks and finite-time singularity

**Conjecture:** Many systems exhibit transient FTS as “ghost-like” solutions that the system follows for a while before being attenuated.

Analogous to exponential sensitivity to initial condition with reinjection  $\rightarrow$  chaos **but** here FTS blow-up.

$$\frac{dp}{dt} = rp(t)[K - p(t)]$$

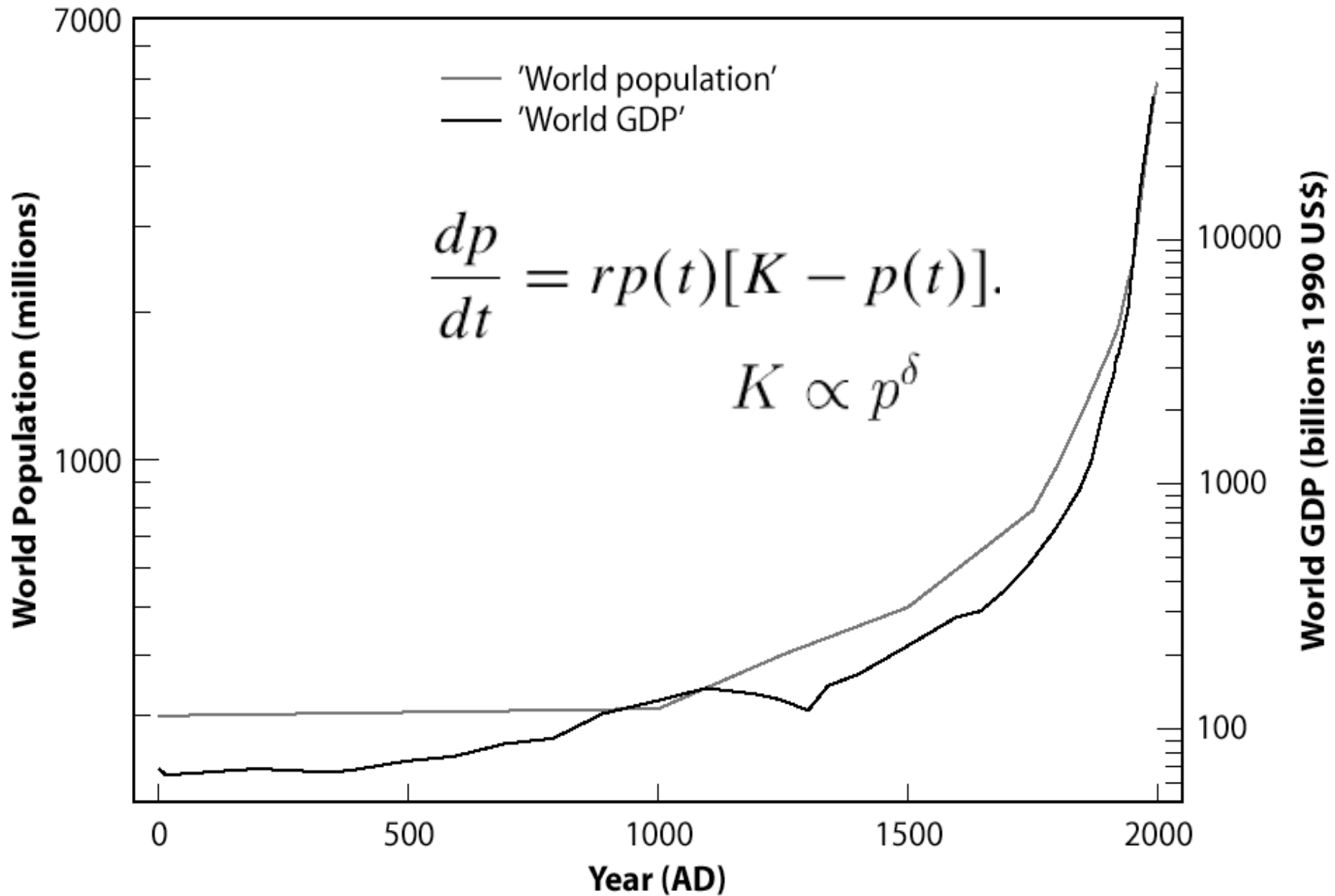
$$\frac{dp}{dt} = r[p(t)]^{1+\delta}$$

with  $K \propto p^\delta$

$$p(t) \propto (t_c - t)^z, \text{ with } z = -\frac{1}{\delta} \text{ and } t \text{ close to } t_c.$$

Multi-dimensional generalization: multi-variate positive feedbacks

# Super-exponential growth



## Faster than exponential growth

Suppose **GROWTH RATE** doubles when **POPULATION** doubles

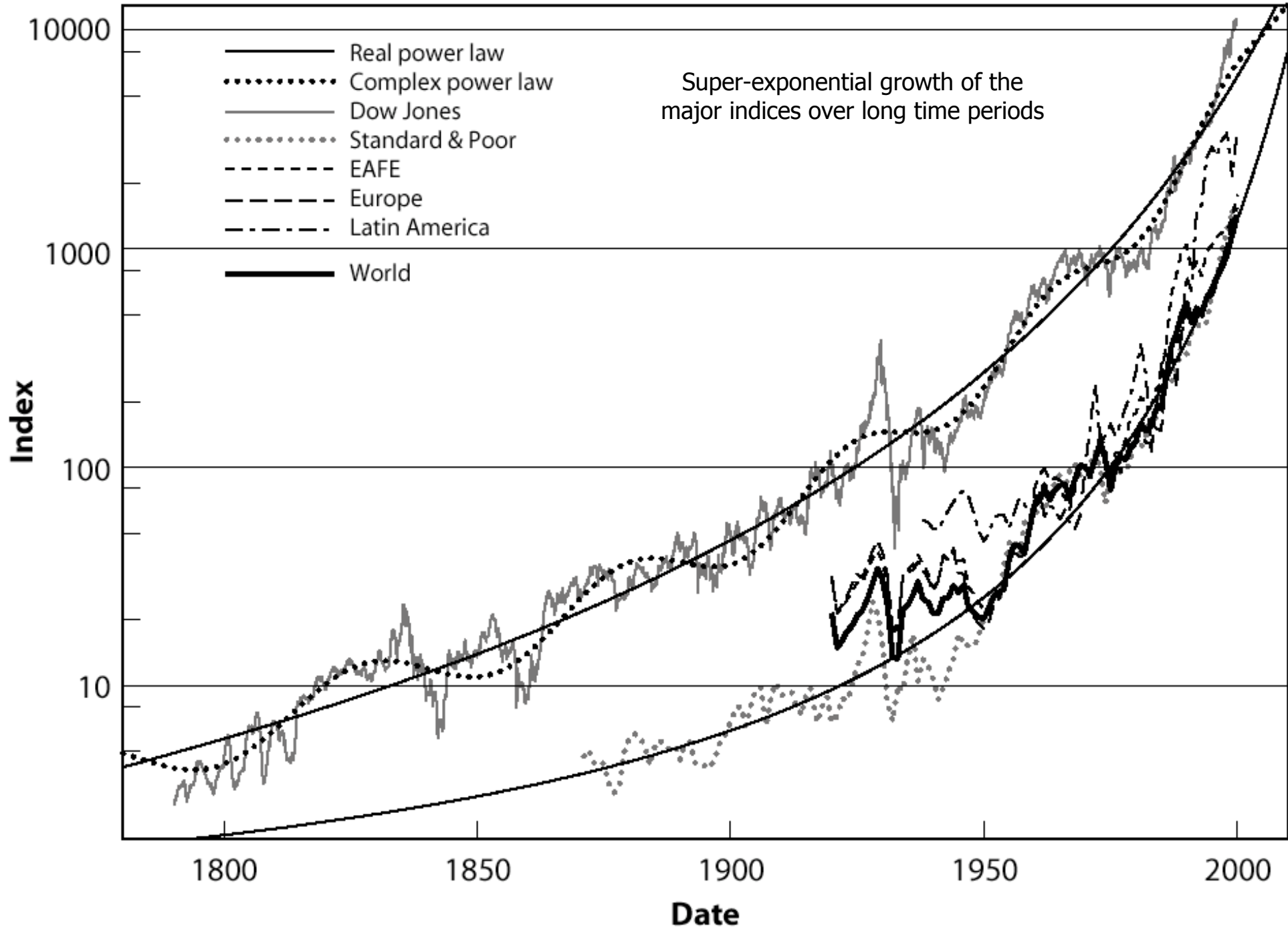
<b>POPULATION</b>	<b>GROWTH RATE</b>	<b>DOUBLING TIME</b>
<input type="checkbox"/> 1000	<input type="checkbox"/> 1%	<input type="checkbox"/> 69y
<input type="checkbox"/> 2000	<input type="checkbox"/> 2%	<input type="checkbox"/> 69/2y
<input type="checkbox"/> 4000	<input type="checkbox"/> 4%	<input type="checkbox"/> 69/4y
<input type="checkbox"/> ...	<input type="checkbox"/> ...	<input type="checkbox"/> ...
<input type="checkbox"/> $2^n \times 1000$	<input type="checkbox"/> $2^n \%$	<input type="checkbox"/> $69/2^n$ y

**Population diverges in finite time**

$$69 + 69/2 + 69/4 + 69/8 \dots = 69 \times (1 + 1/2 + 1/4 + 1/8 + \dots) = 69 \times 2 = 138y$$

**Zeno paradox**





# Finite-time Singularity



Artist's illustration of matter from a red giant star being pulled toward a black hole.

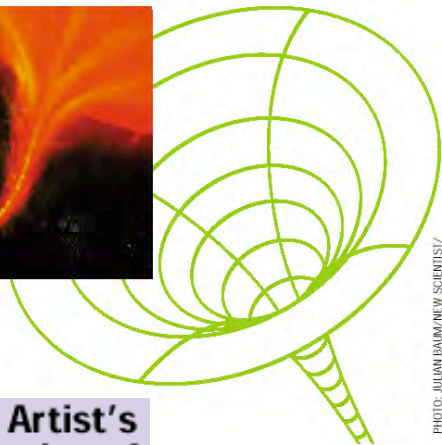


PHOTO: JULIAN DAMIAN/NEW SCIENTIST/  
SPL PHOTO RESEARCHERS, INC.

- Planet formation in solar system by run-away accretion of planetesimals
- PDE's: Euler equations of inviscid fluids and relationship with turbulence
- PDE's of General Relativity coupled to a mass field leading to the formation of black holes
- Zakharov-equation of beam-driven Langmuir turbulence in plasma
- rupture and material failure
- Earthquakes (ex: slip-velocity Ruina-Dieterich friction law and accelerating creep)
- Models of micro-organisms chemotaxis, aggregating to form fruiting bodies
- Surface instability spikes (Mullins-Sekerka), jets from a singular surface, fluid drop snap-off
- Euler's disk (rotating coin)
- Stock market crashes...

# Mechanisms for positive feedbacks in the stock market

- **Technical and rational mechanisms**
  1. Option hedging
  2. Insurance portfolio strategies
  3. Trend following investment strategies
  4. Asymmetric information on hedging strategies
- **Behavioral mechanisms:**
  1. Breakdown of “psychological Galilean invariance”
  2. Imitation(many persons)
    - a) It is rational to imitate
    - b) It is the highest cognitive task to imitate
    - c) We mostly learn by imitation
    - d) The concept of “CONVENTION” (Orléan)

# THIS WAY TO THE Bull Market!

Almost there... 300 feet... Don't Miss The Fun!

I'm a  
trader.

Hey this Koolaid  
tastes great!

Must own  
Yahoo!

Don't be negative.

Stoopid bears.

Take a  
deep  
breath!

El Cliffo

I'm flying  
mommy!



# Imitation

## Humans Appear Hardwired To Learn By 'Over-Imitation'

*ScienceDaily* (Dec. 6, 2007) — Children learn by imitating adults--so much so that they will rethink how an object works if they observe an adult taking unnecessary steps when using that object.



-Imitation is considered an efficient mechanism of social learning.

- Experiments in developmental psychology suggest that infants use imitation to get to know persons, possibly applying a ‘like-me’ test (‘persons which I can imitate and which imitate me’).
- Imitation is among the most complex forms of learning. It is found in highly socially living species which show, from a human observer point of view, ‘intelligent’ behavior and signs for the evolution of traditions and culture (humans and chimpanzees, whales and dolphins, parrots).
- In non-natural agents as robots, tool for easing the programming of complex tasks or endowing groups of robots with the ability to share skills without the intervention of a programmer. Imitation plays an important role in the more general context of interaction and collaboration between software agents and human users.

# Thy Neighbor's Portfolio: Word-of-Mouth Effects in the Holdings and Trades of Money Managers

HARRISON HONG, JEFFREY D. KUBIK, and JEREMY C. STEIN\*

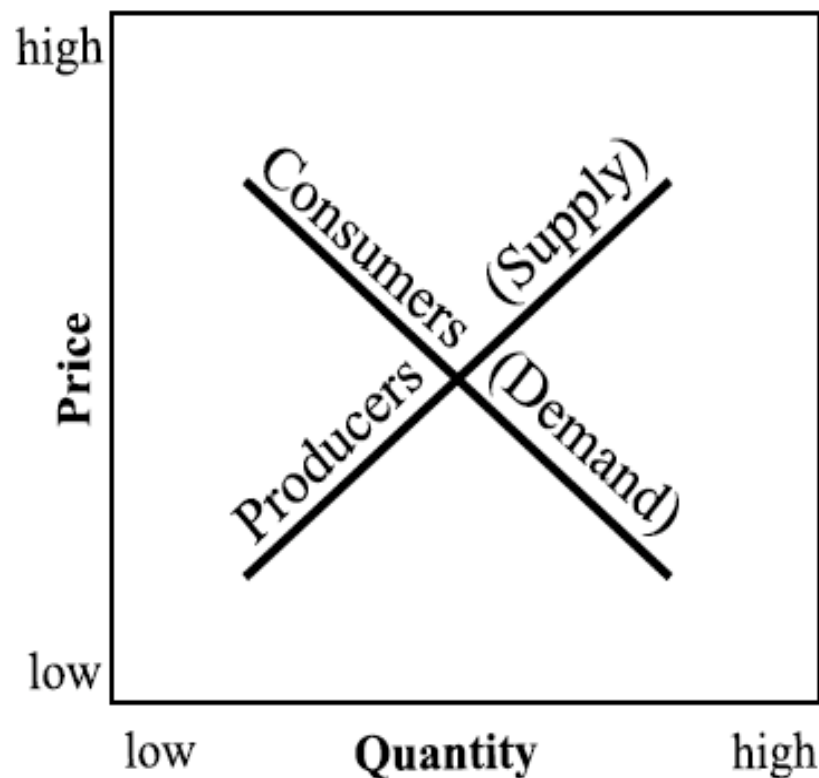
A mutual fund manager is more likely to buy (or sell) a particular stock in any quarter if other managers in the same city are buying (or selling) that same stock. This pattern shows up even when the fund manager and the stock in question are located far apart, so it is distinct from anything having to do with local preference. The evidence can be interpreted in terms of an epidemic model in which investors spread information about stocks to one another by word of mouth.

THE JOURNAL OF FINANCE • VOL. LX, NO. 6 • DECEMBER 2005

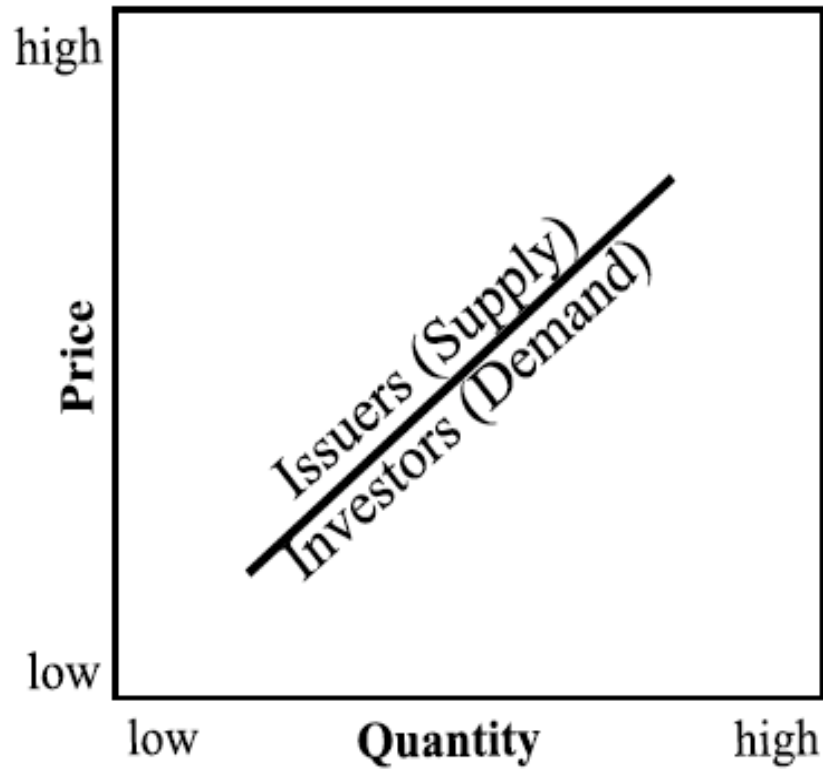
A fundamental observation about human society is that people who communicate regularly with one another think similarly. There is at any place and in any time a Zeitgeist, a spirit of the times. . . . Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations. (pp. 148, 155)

Shiller (2000)

### The Law of Supply & Demand in Utilitarian Economics

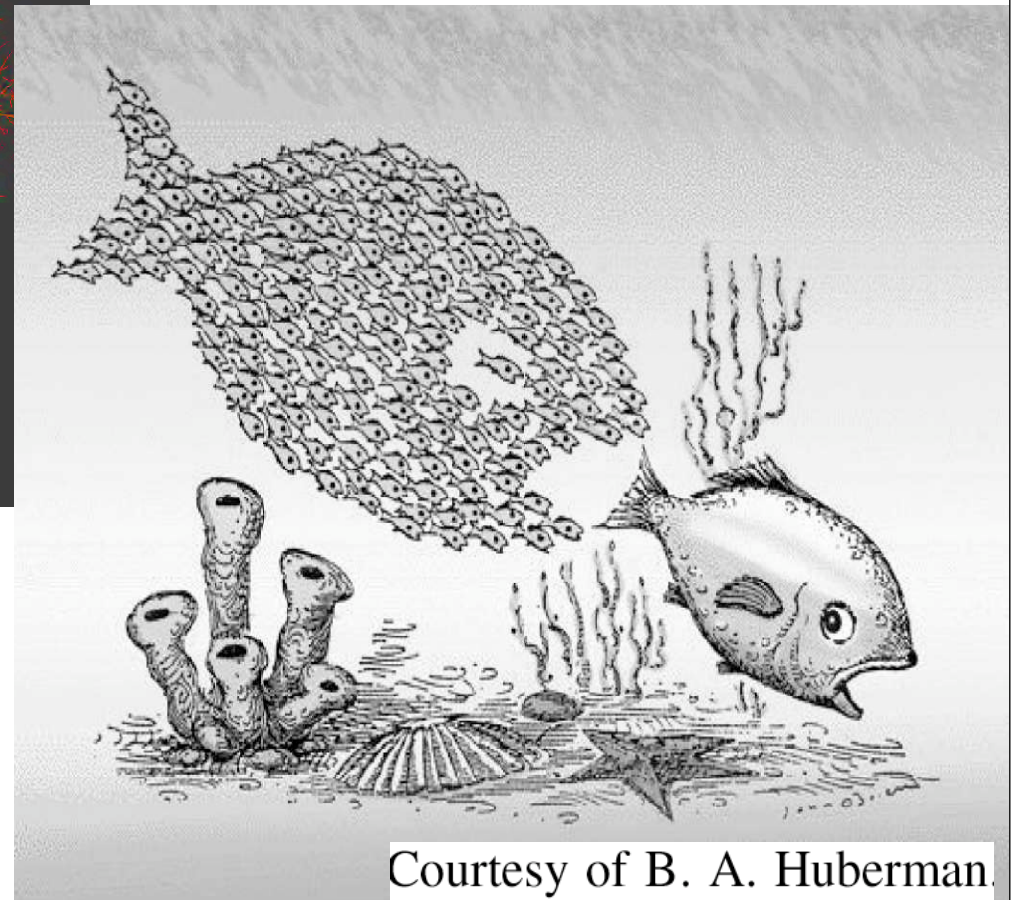
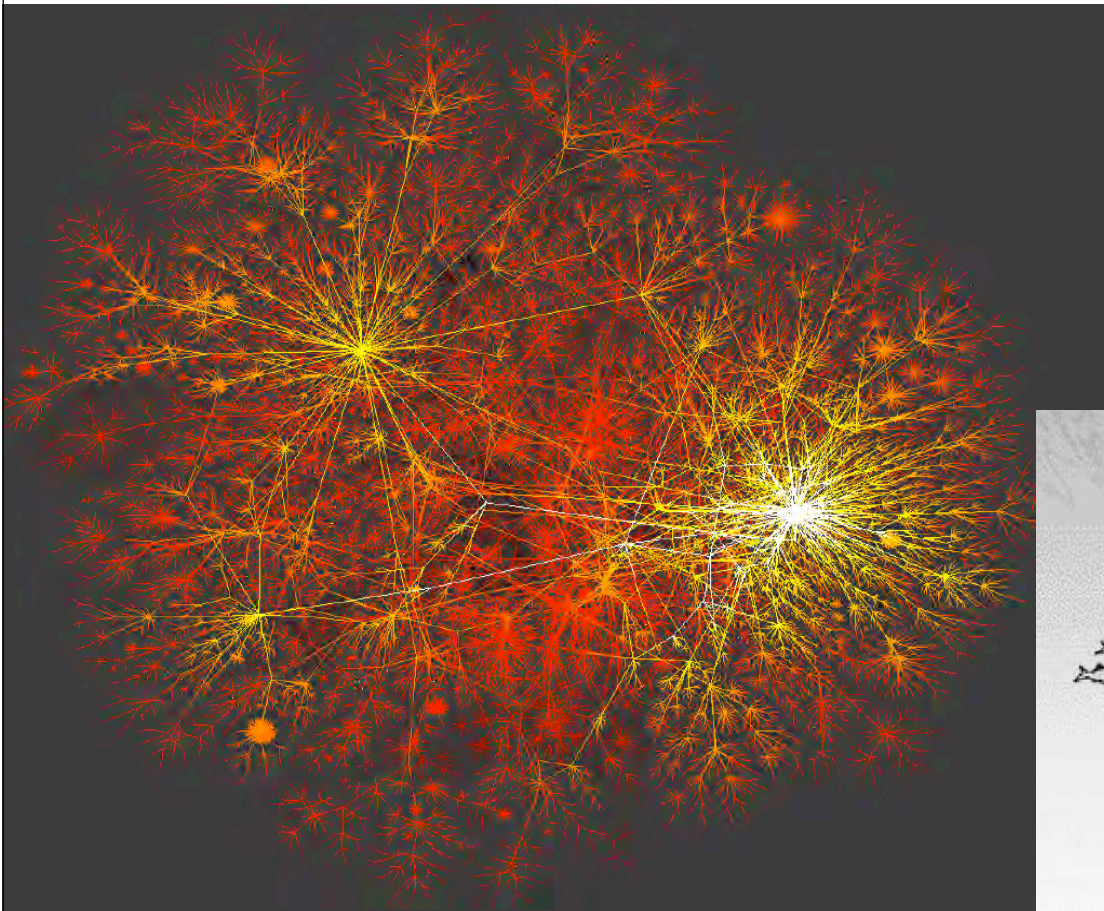


### Herding Impulse in Finance



© 2003 Robert R. Prechter, The Socionomics Institute

# Network effects and Collective behavior

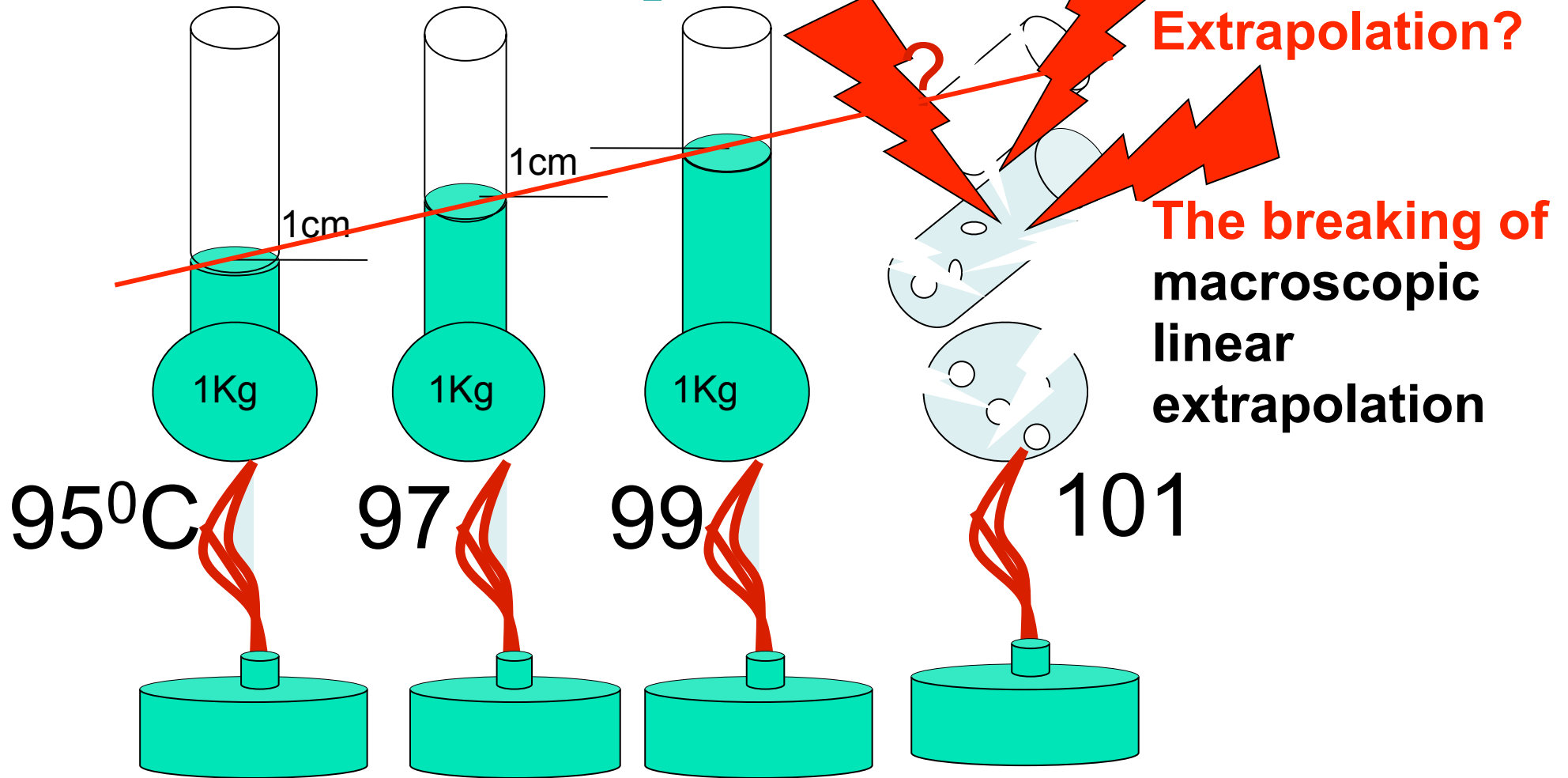


Courtesy of B. A. Huberman.



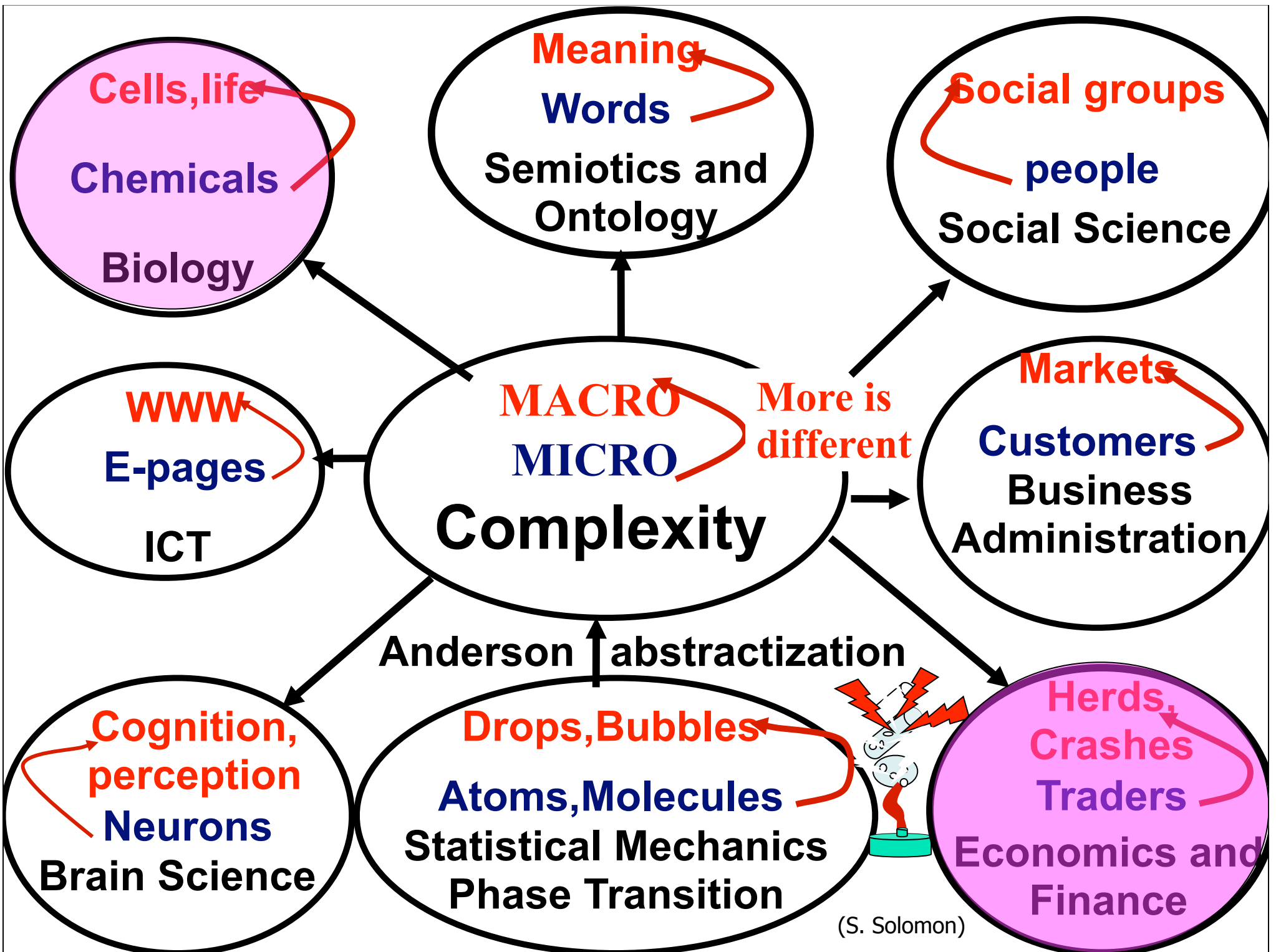
# Simplest Example of a “More is Different” Transition

## Water level vs. temperature



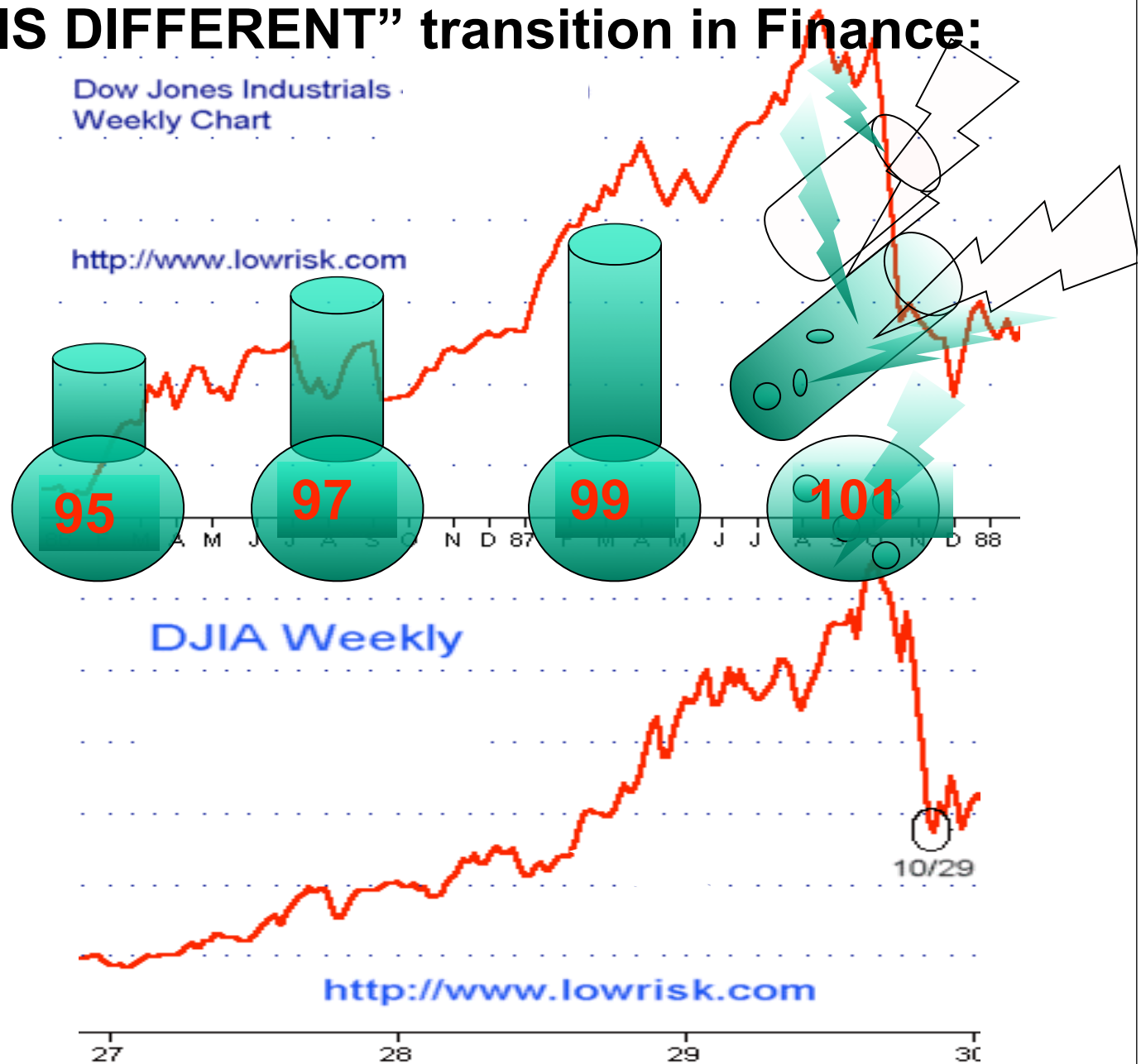
## BOILING PHASE TRANSITION

**More is different: a single molecule does not boil at 100C<sup>0</sup>**



# Example of “MORE IS DIFFERENT” transition in Finance:

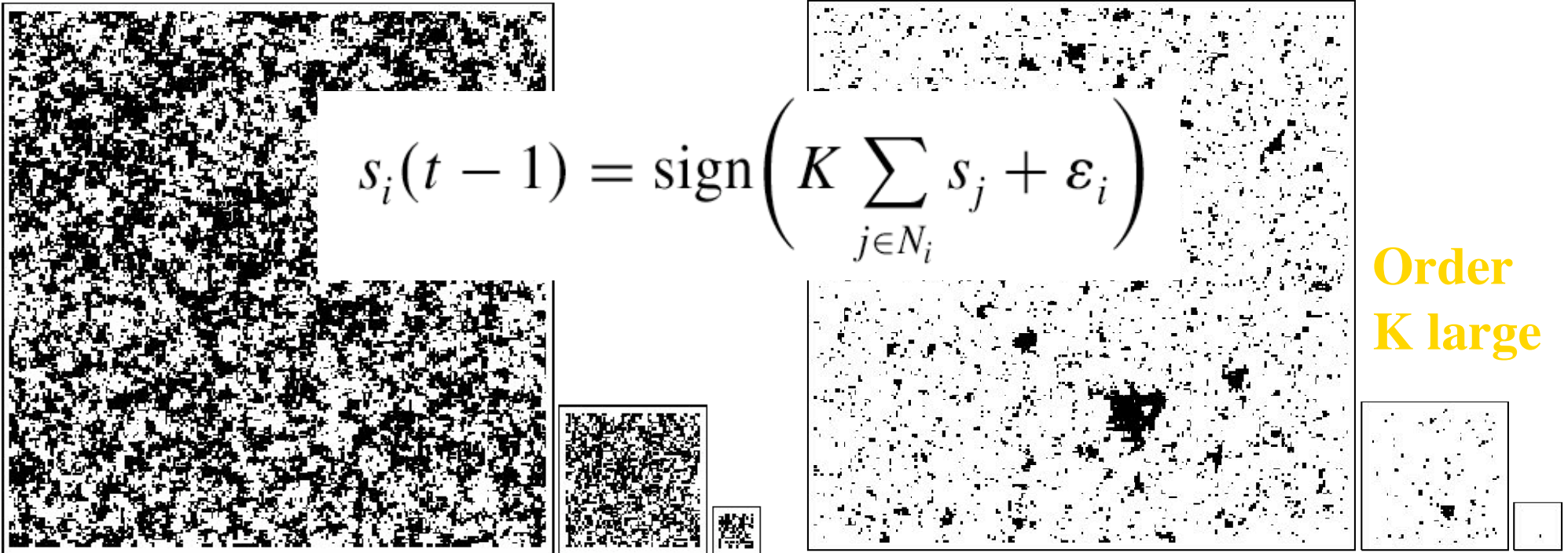
Instead of  
Water Level:  
-economic index  
(Dow-Jones etc...)



**Crash = result of collective behavior of individual traders**

$$s_i(t - 1) = \text{sign} \left( K \sum_{j \in N_i} s_j + \varepsilon_i \right)$$

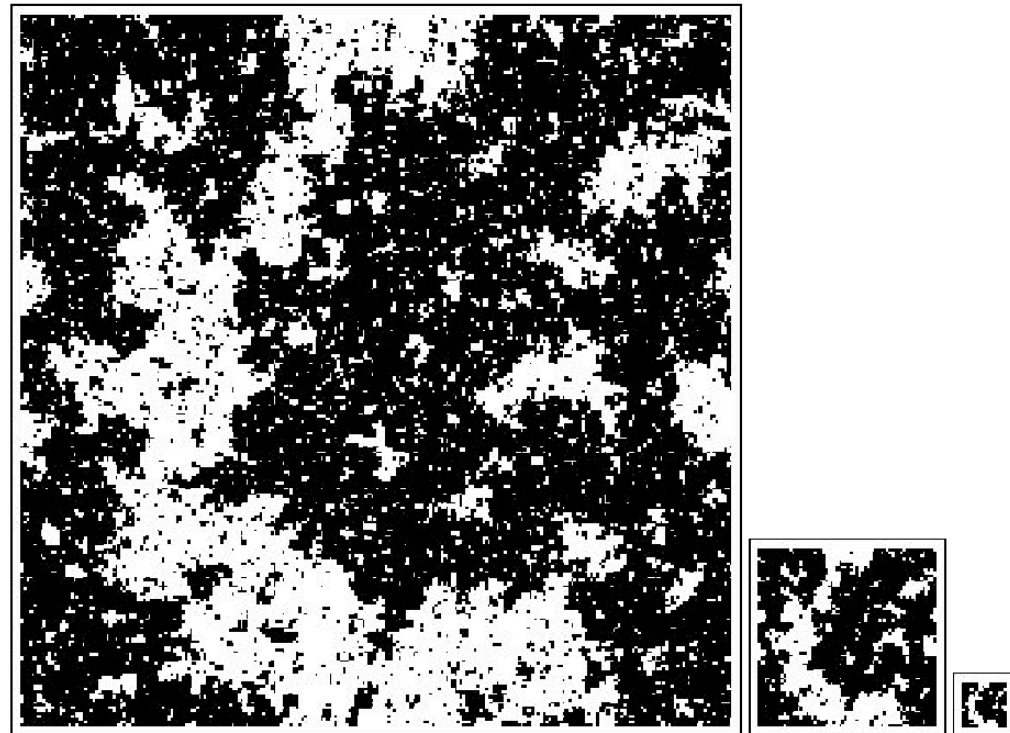
Order  
K large



Disorder : K small

Renormalization group:  
Organization of the  
description scale by scale

Critical:  
K=critical  
value



# Importance of Positive Feedbacks and Over-confidence in a Self-Fulfilling Ising Model of Financial Markets

$$s_i(t) = \text{sign} \left[ \sum_{j \in \mathcal{N}} K_{ij}(t) E[s_j](t) + \sigma_i(t) G(t) + \epsilon_i(t) \right]$$

**Imitation**                      **News**                      **Private information**

$$K_{ij}(t) = b_{ij} + \alpha_i K_{ij}(t-1) + \beta r(t-1) G(t-1)$$

(generalizes Carlos Pedro Gonçalves, who generalized Johansen-Ledoit-Sornette)

$\beta$ : propensity to be influenced by the felling of others

1.  $\beta < 0$ : **rational agents**

•  $\beta > 0$ : **over-confident agents**

# Price clearing condition

If  $info_i(t) > info_{threshold}^i$  :  $s_i(t) = +1$

$$a_i(t) = 0.02 \cdot cash_i(t-1) / price(t-1)$$

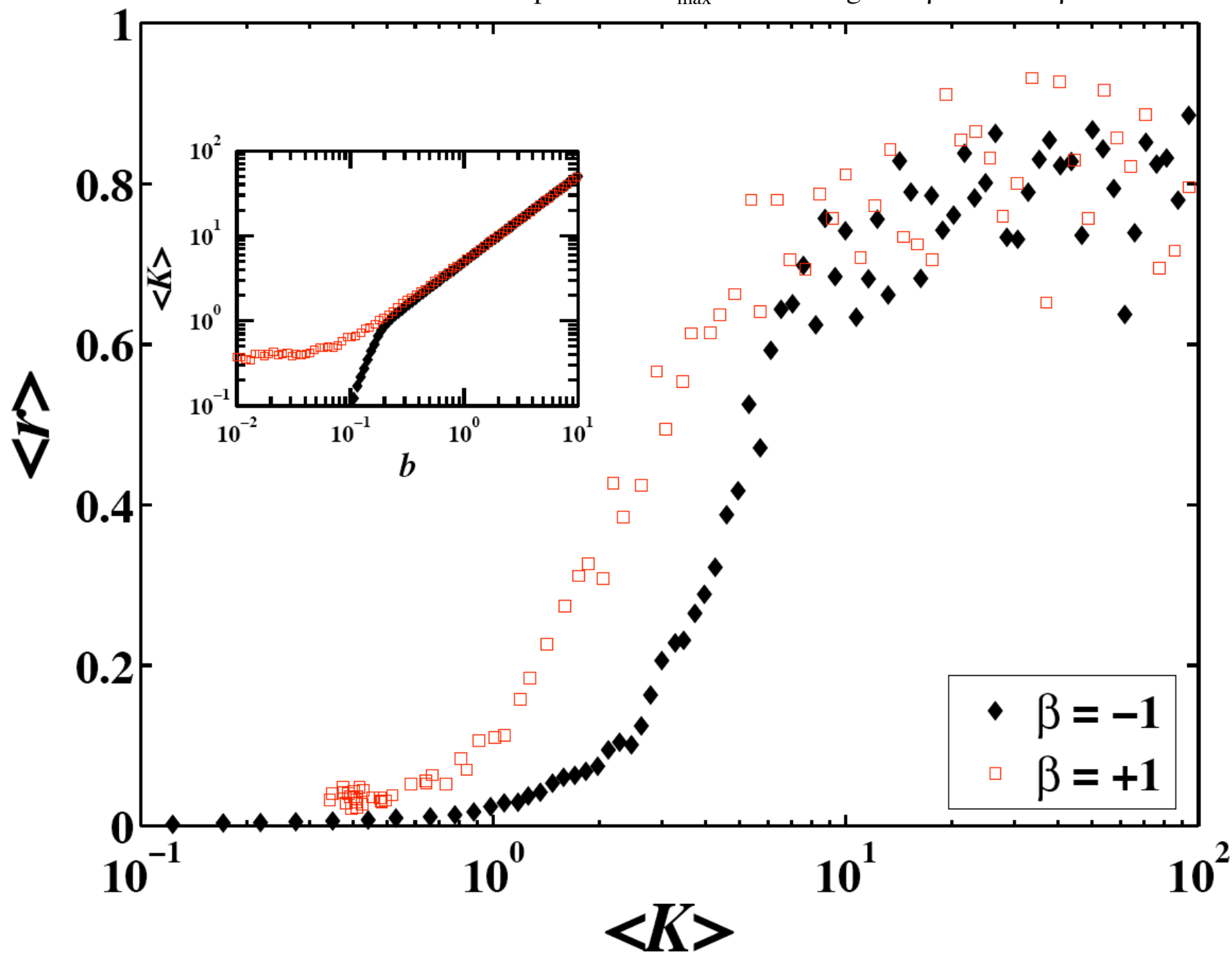
If  $info_i(t) < info_{threshold}^i$  :  $s_i(t) = -1$

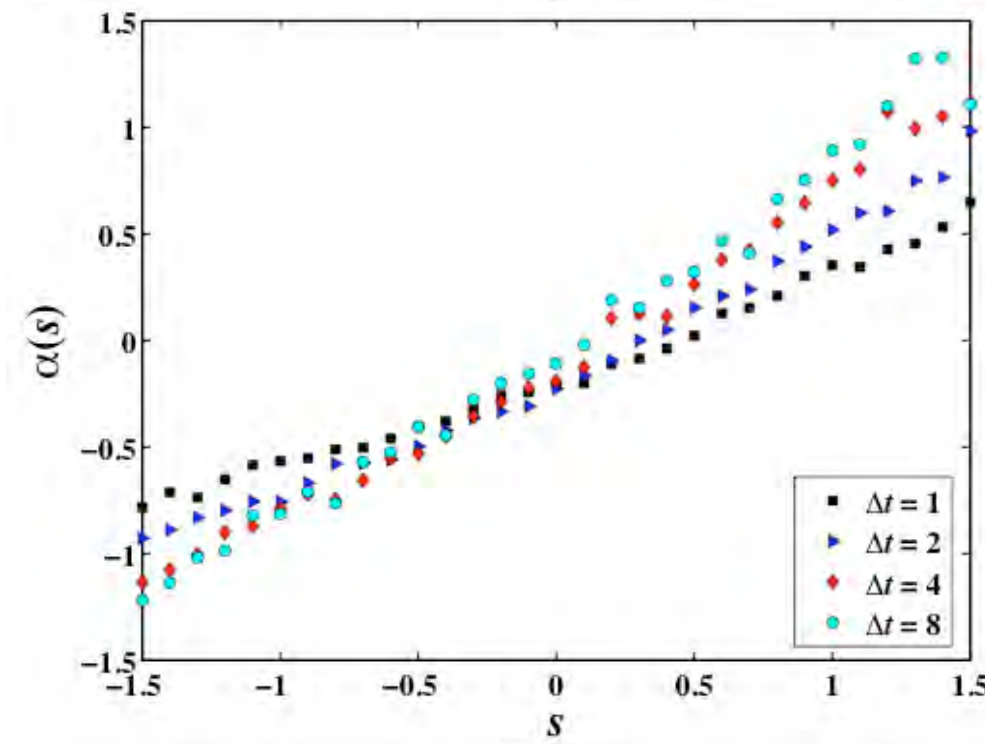
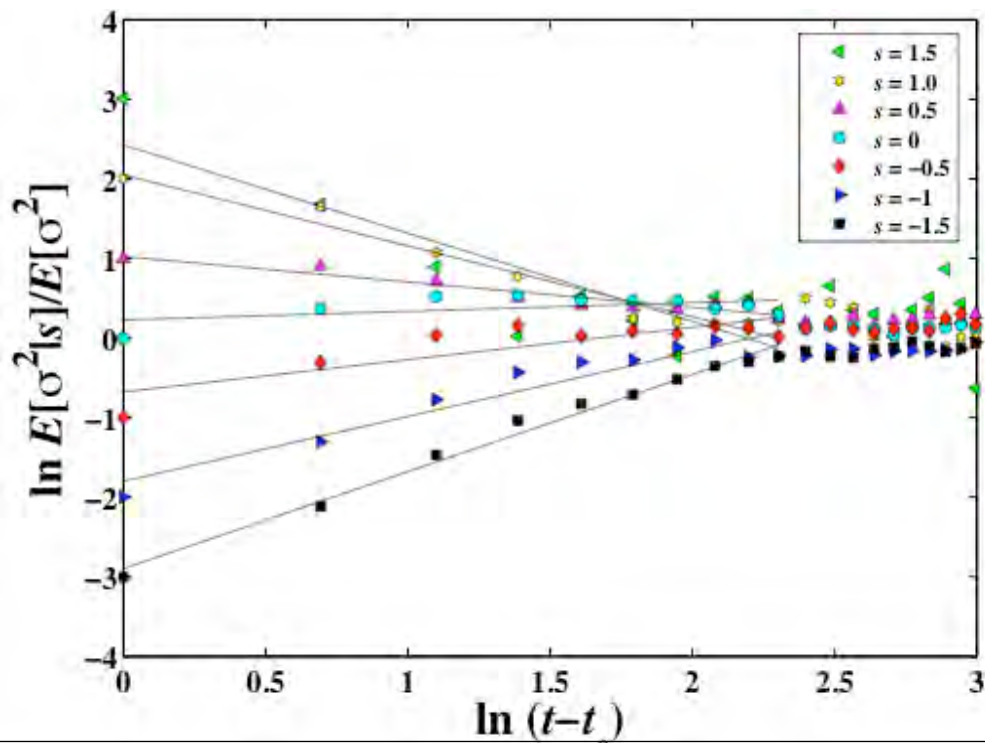
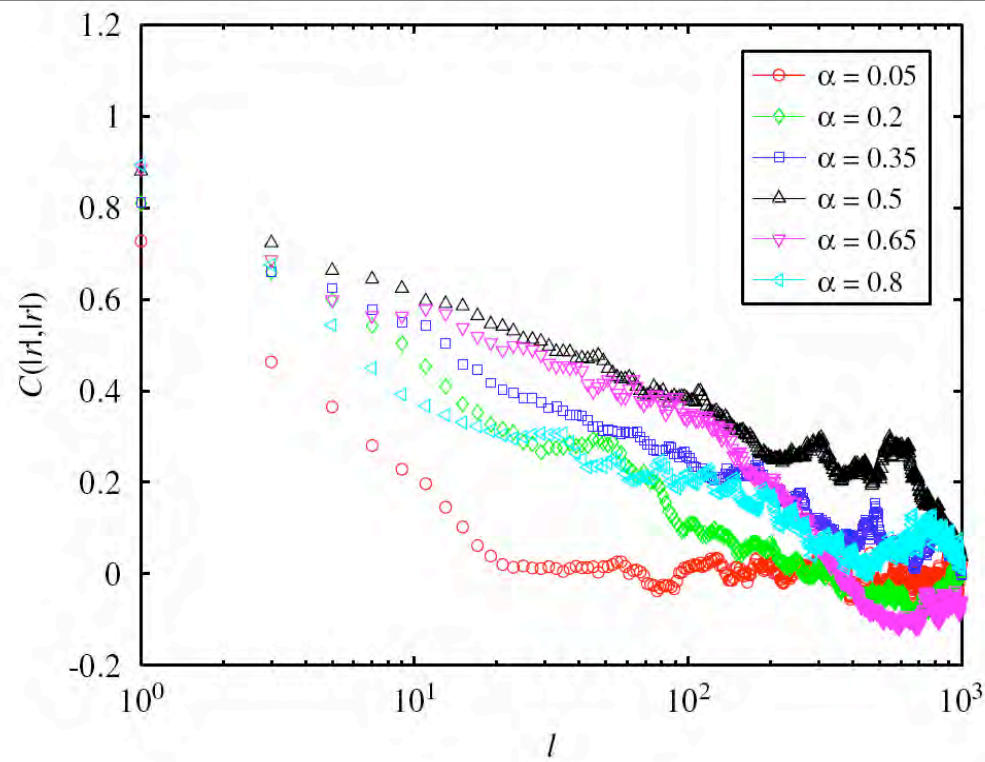
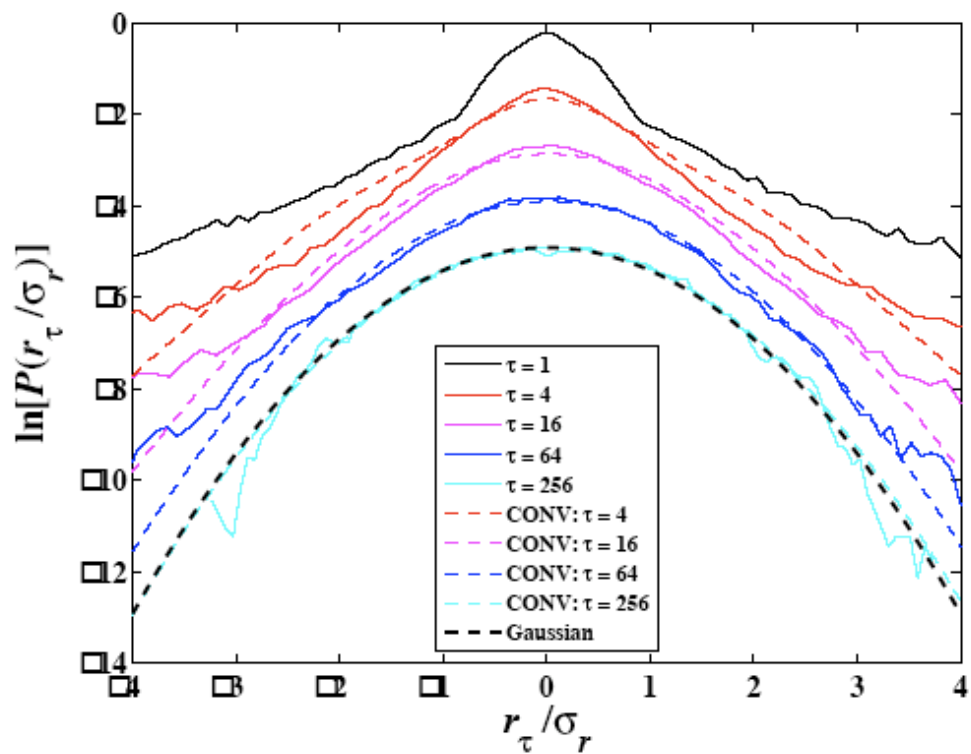
$$a_i(t) = 0.02 \cdot stocks_i(t-1)$$

$$r(t) = \frac{1}{\lambda \cdot N} \sum_{i=1}^N s_i(t) \cdot a_i(t)$$

$$\log(price(t)) = \log(price(t-1)) + r(t)$$

Illustration of the existence of an Ising-like phase transition, as a function of the control parameter  $b_{\max}$  for both regimes  $\beta = -1$  and  $\beta=1$







# Bubbles and crashes

Fig. 15. Five price trajectories showing bubbles preceding crashes that occur at the shifted time 0. The five time series have been translated so that the time of their crash is placed at the origin  $t = 0$ .

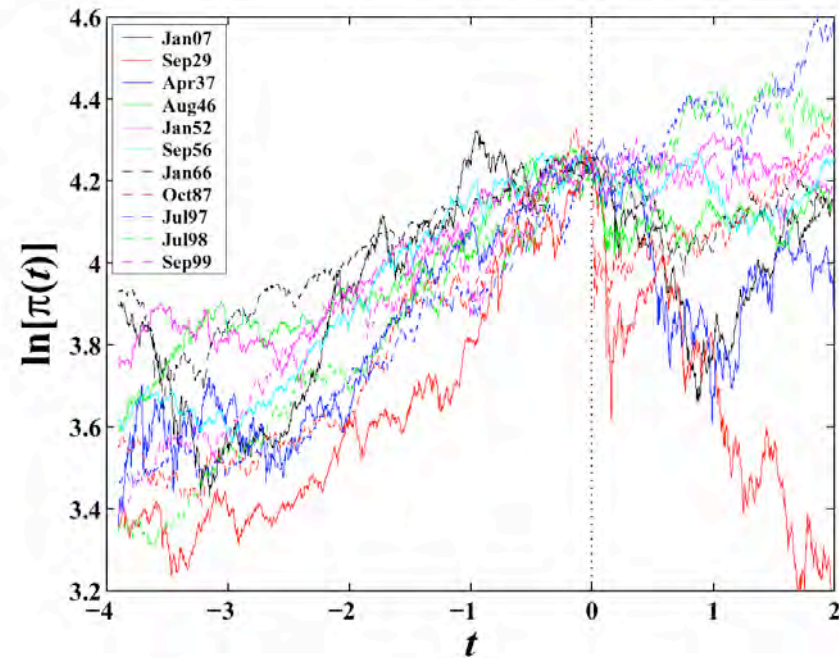
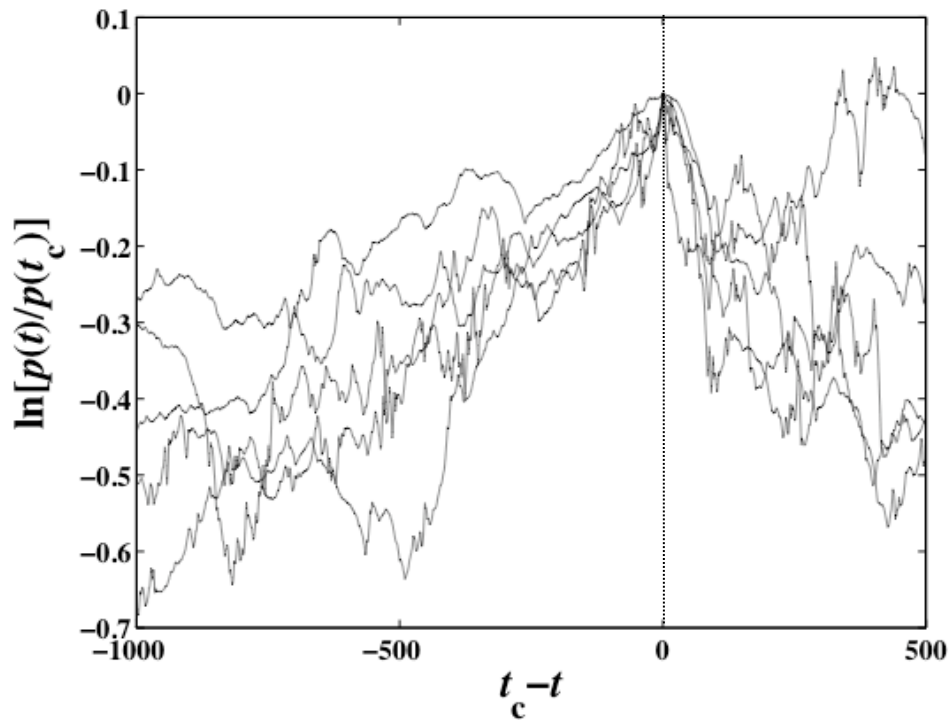


Figure 4: (Color online) Superposed epoch analysis of the 11 time intervals, each of 6 years long, of the DJIA index centered on the time of the maxima of the 11 predictor peaks above  $AI = 0.3$  of the alarm index shown in Fig. 3.

D. Sornette and W.-X. Zhou

Predictability of Large Future Changes in major financial indices,  
International Journal of Forecasting 22, 153-168 (2006)

# Aggregation of information

The aggregated information of agent  $i$  is:

$$info_i(t) = \underbrace{c_{1i} \cdot \sum_{j=1}^J k_{ij} E_i[s_j(t)]}_{\text{imitation term}} + \underbrace{c_{2i} \cdot u(t) \cdot news(t)}_{\text{news term}} + \underbrace{c_{3i} \cdot \epsilon_i(t)}_{\text{idiosyncratic term}}$$

$$c_{1/2/3} \sim UD \in [0, c_{1/2/3max}] \quad \text{personal susceptibility to the different sources of information}$$

$$\epsilon_i \sim N(0,1)$$

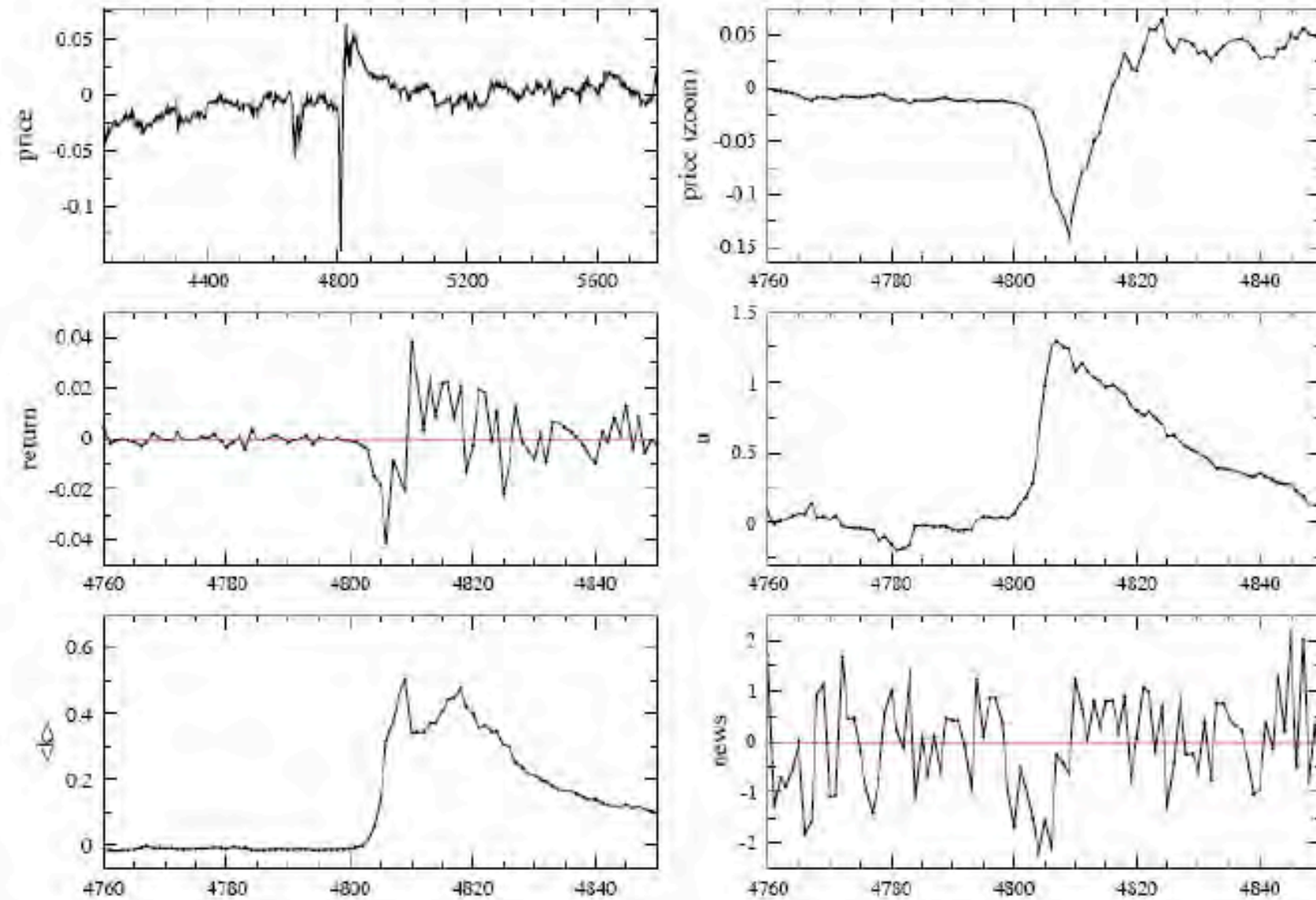
Gaussian noise

$$news(t) \sim N(0,1)$$

$$k_{ij}(t) = \alpha \cdot k_{ij}(t-1) + r(t-1) \cdot E_i[s_j(t-2)] \cdot \frac{1-\alpha}{\sigma_r} \quad \text{past neighbour } j \text{ performance}$$

$$u(t) = \alpha \cdot u(t-1) + r(t-1) \cdot news(t-2) \cdot \frac{1-\alpha}{\sigma_r} \quad \text{past news performance}$$

# News impact



Impact of the news to some values, generated with  $C_1 = C_2 = C_3 = 1.0$ .

# FOUR EXAMPLES

(i) the fluctuation-susceptibility theorem transforms into a remarkable classification of financial volatility shocks (endogenous versus exogenous),

(ii) the Ising model of phase transitions can be generalized to model the stylized facts of financial markets,

(iii) the concepts of collective phenomena and phase transitions (with spontaneous symmetry breaking) help understand **financial bubbles** and their following **crashes**,

(iv) the mathematics of quantum physics provides a new quantum decision theory solving the known paradoxes.

# DISCRETE HIERARCHY OF THE AGENT NETWORK

**Presentation of three different mechanisms leading to discrete scale invariance, discrete hierarchies and log-periodic signatures**

- ❑ Co-evolution of brain size and group size  
(Why do we have a big Brain?)
  
- ❑ Interplay between **nonlinear positive and negative feedbacks** and **inertia**
  
- ❑ Discrete scale invariance  
Complex fractal dimension  
Log-periodicity

# FRACTALS

1)  $d \in \mathbb{N}$  Euclid (ca. 325-270 BC)

2)  $d \in \mathbb{R}$  Mandelbrot (1960-1980)  
(Weierstrass, Hausdorff, Holder, ...)

3)  $d \in \mathbb{C}$

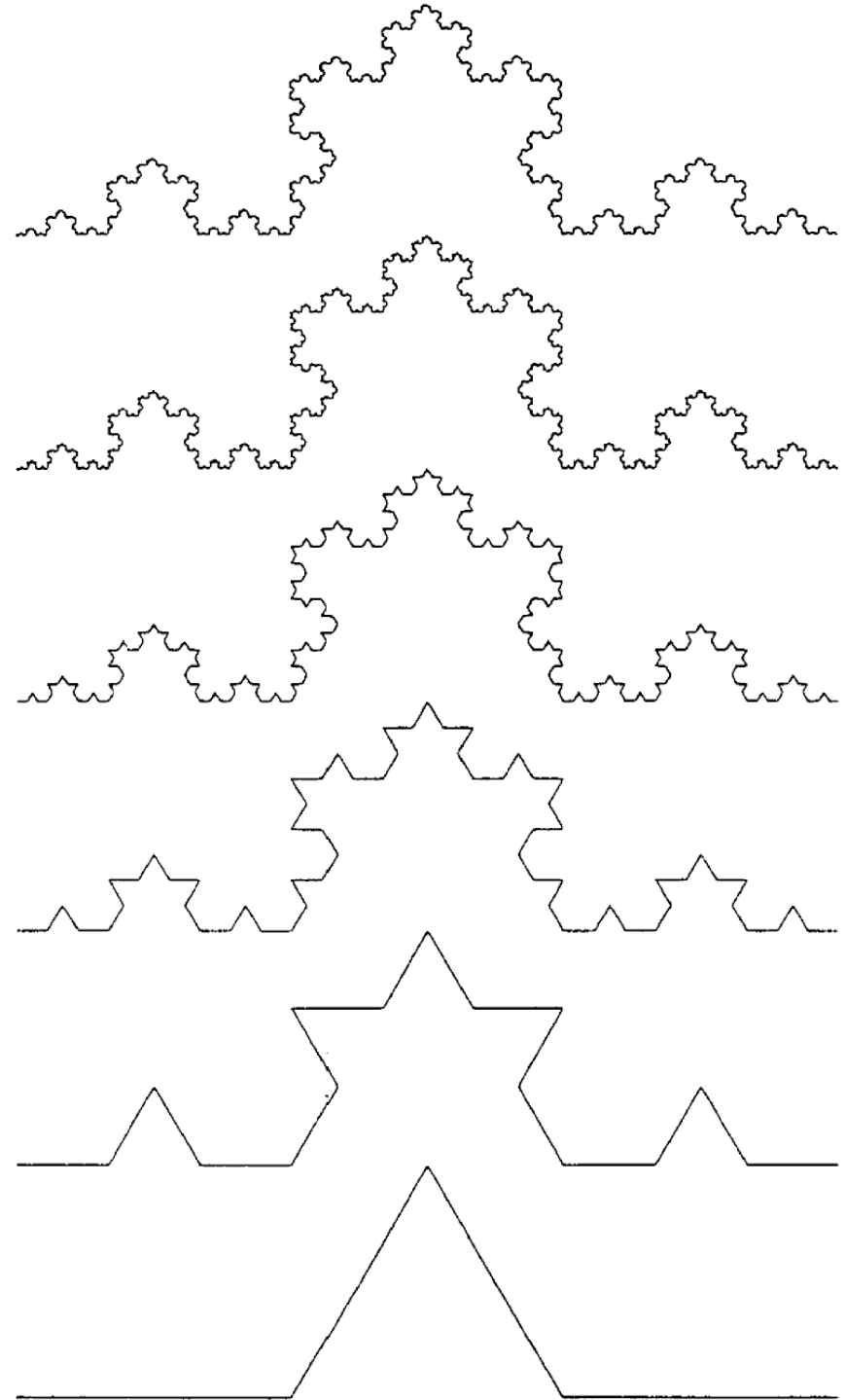
**Discrete scale invariance**

**Complex fractal dimension**

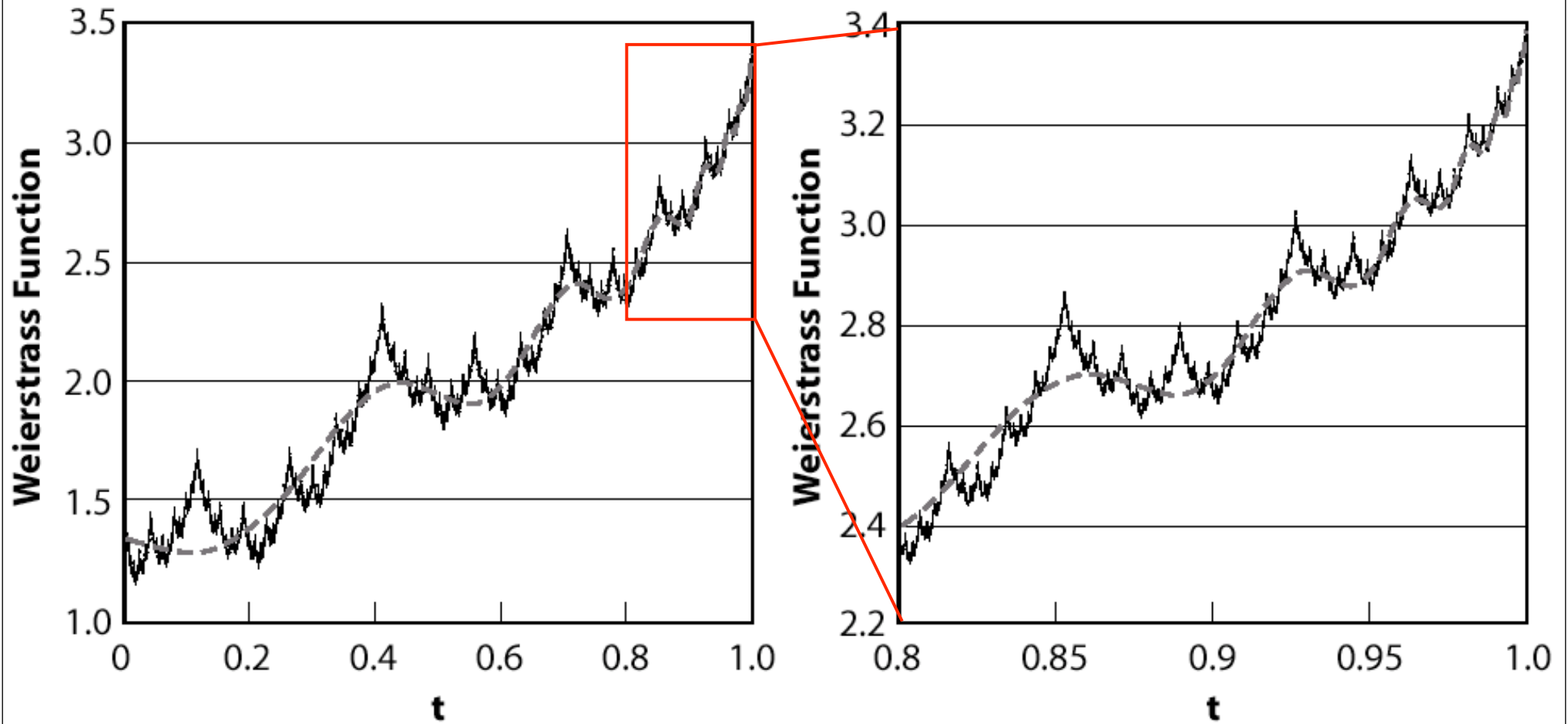
**Log-periodicity**

Preferred scaling ratio is **3**

$$D(n) = \ln 4 / \ln 3 + i 2\pi n / \ln 3$$

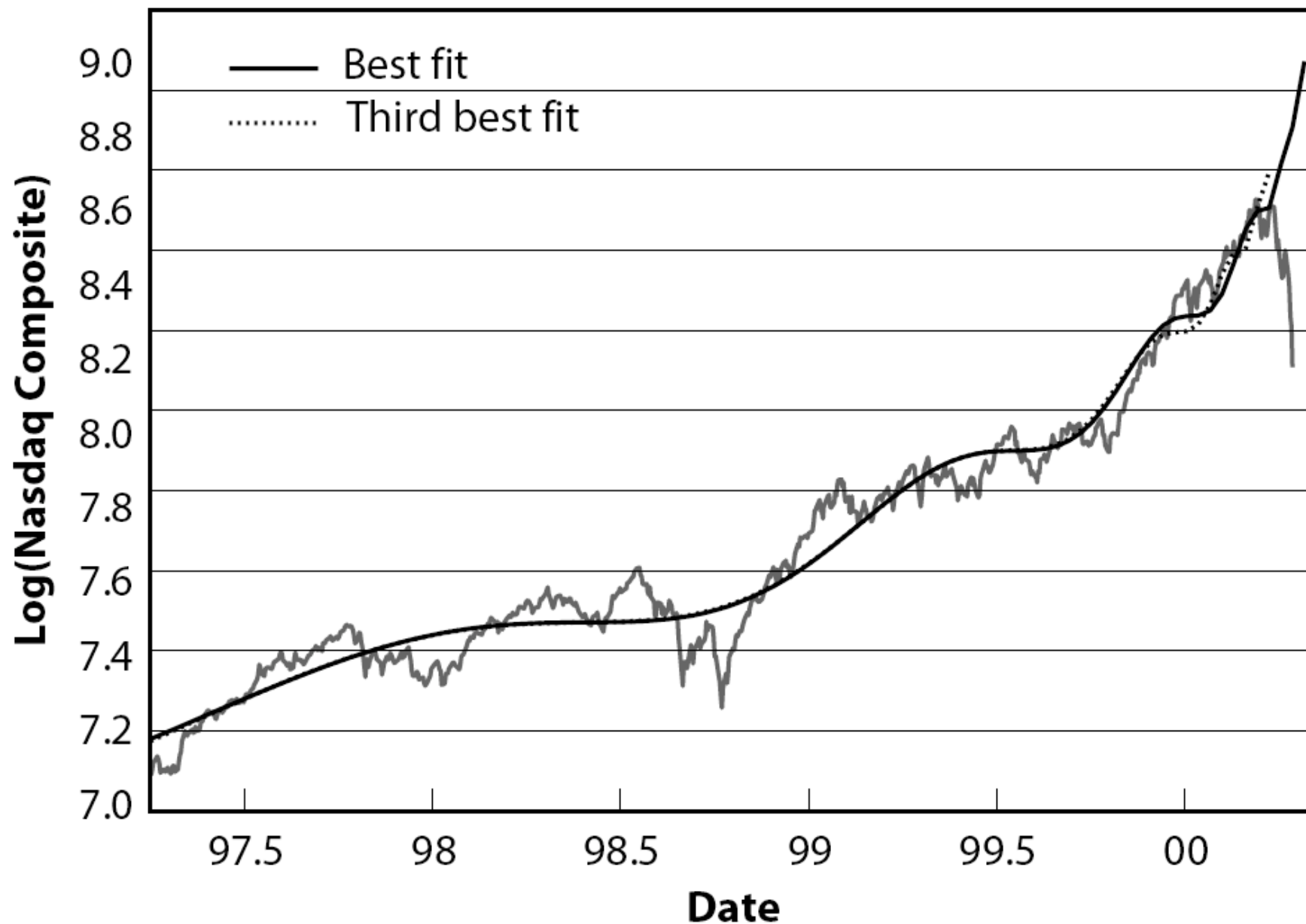


# Fractal function (Weierstrass)



# THE NASDAQ CRASH OF APRIL 2000

“New Economy”: ICT





# Real-estate in the UK

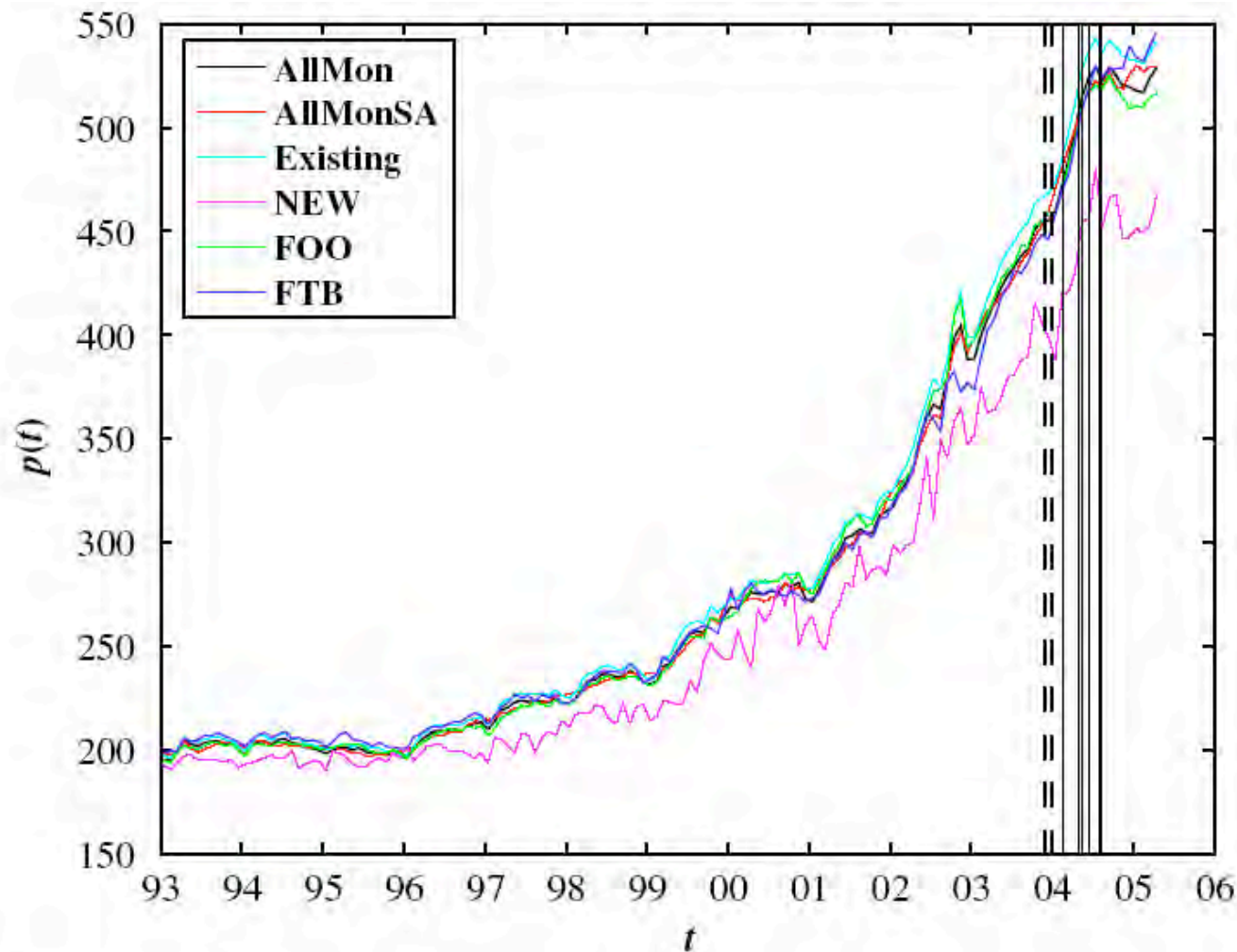


Fig. 1. (Color online) Plot of the UK Halifax house price indices from 1993 to April 2005 (the latest available quote at the time of writing). The two groups of vertical lines correspond to the two predicted turning points reported in Tables 2 and 3 of [1]: end of 2003 and mid-2004. The former (resp. later) was based on the use of formula (2) (resp. (3)). These predictions were performed in February 2003.

# Real-estate in the USA

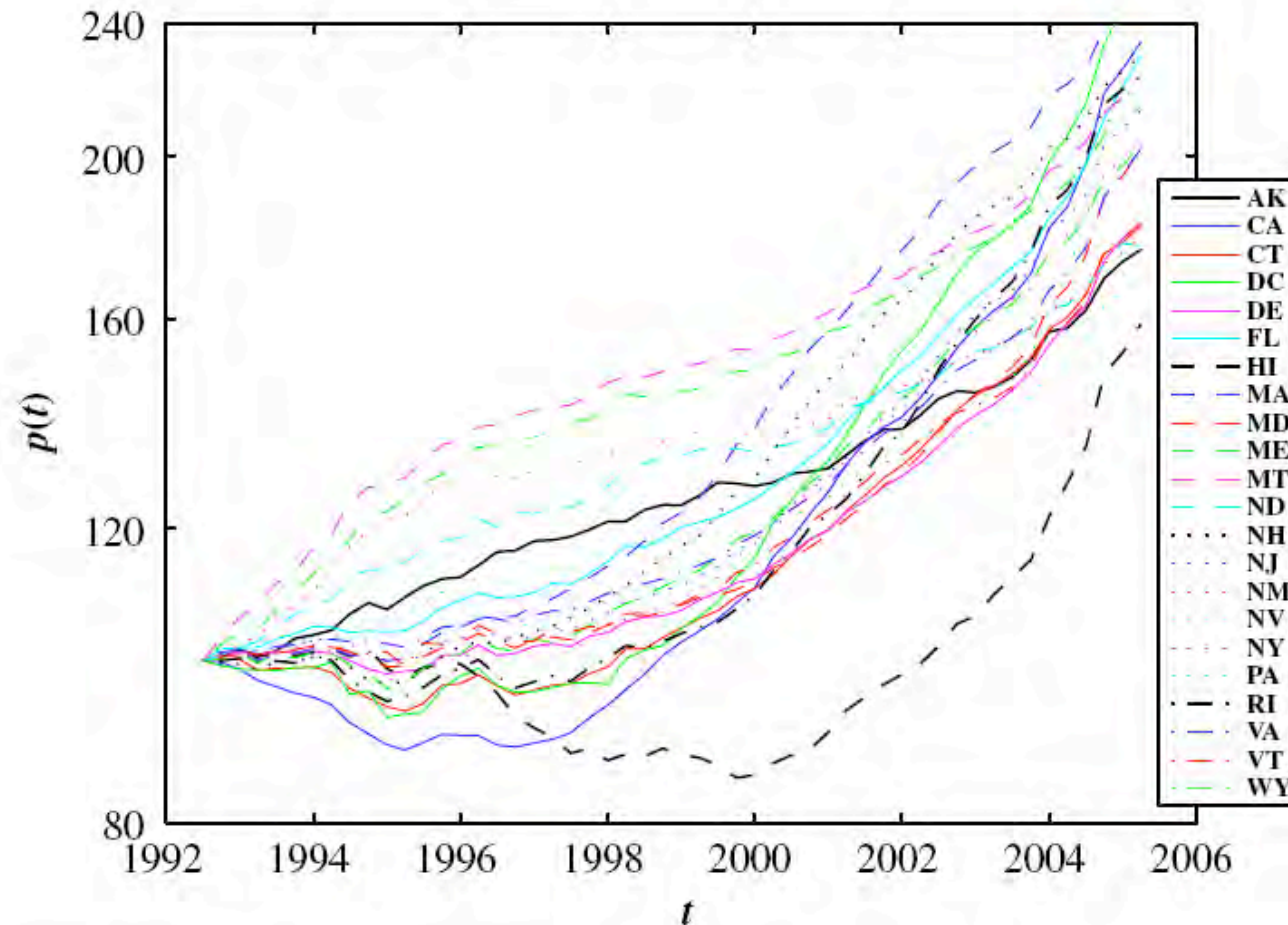
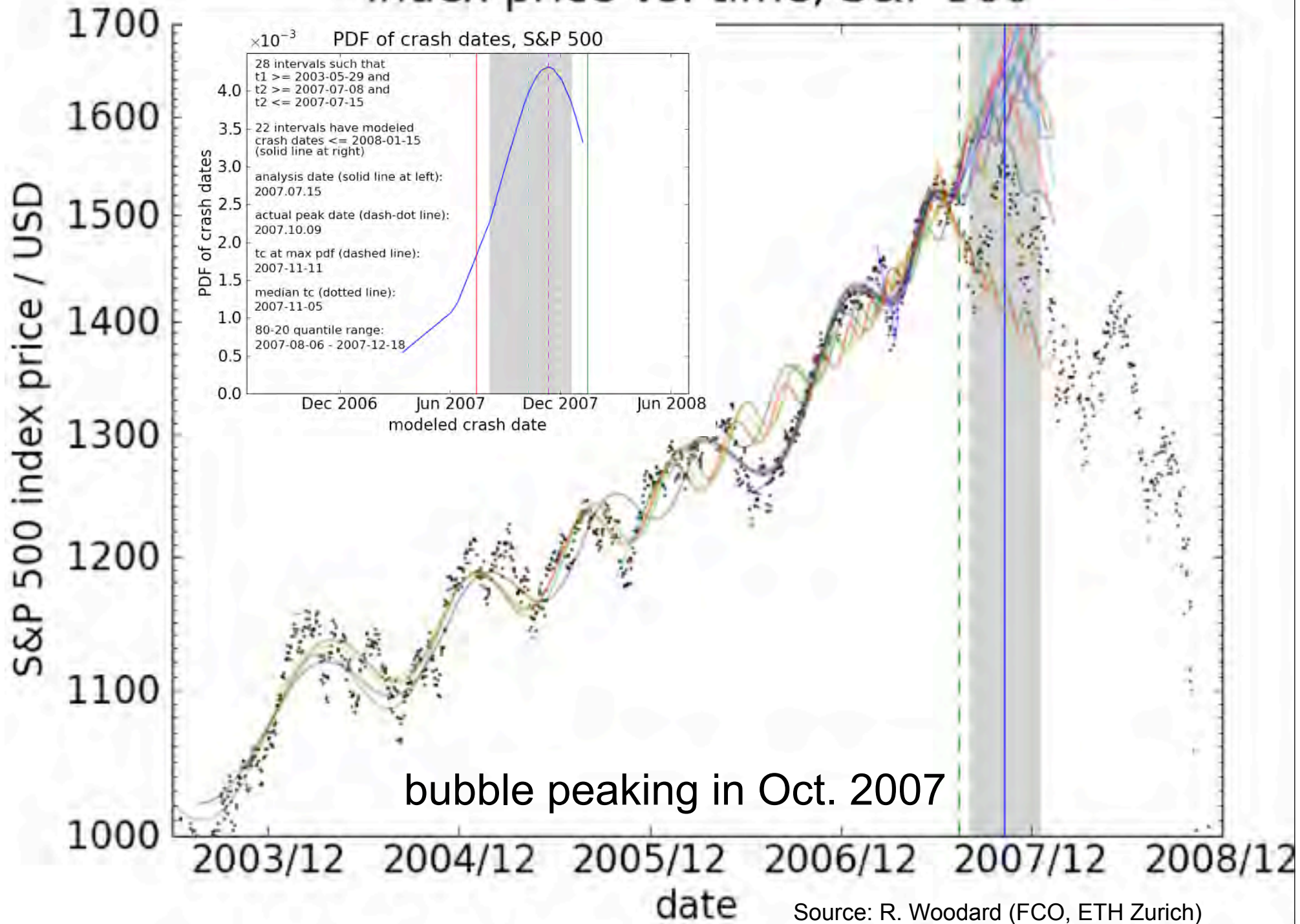


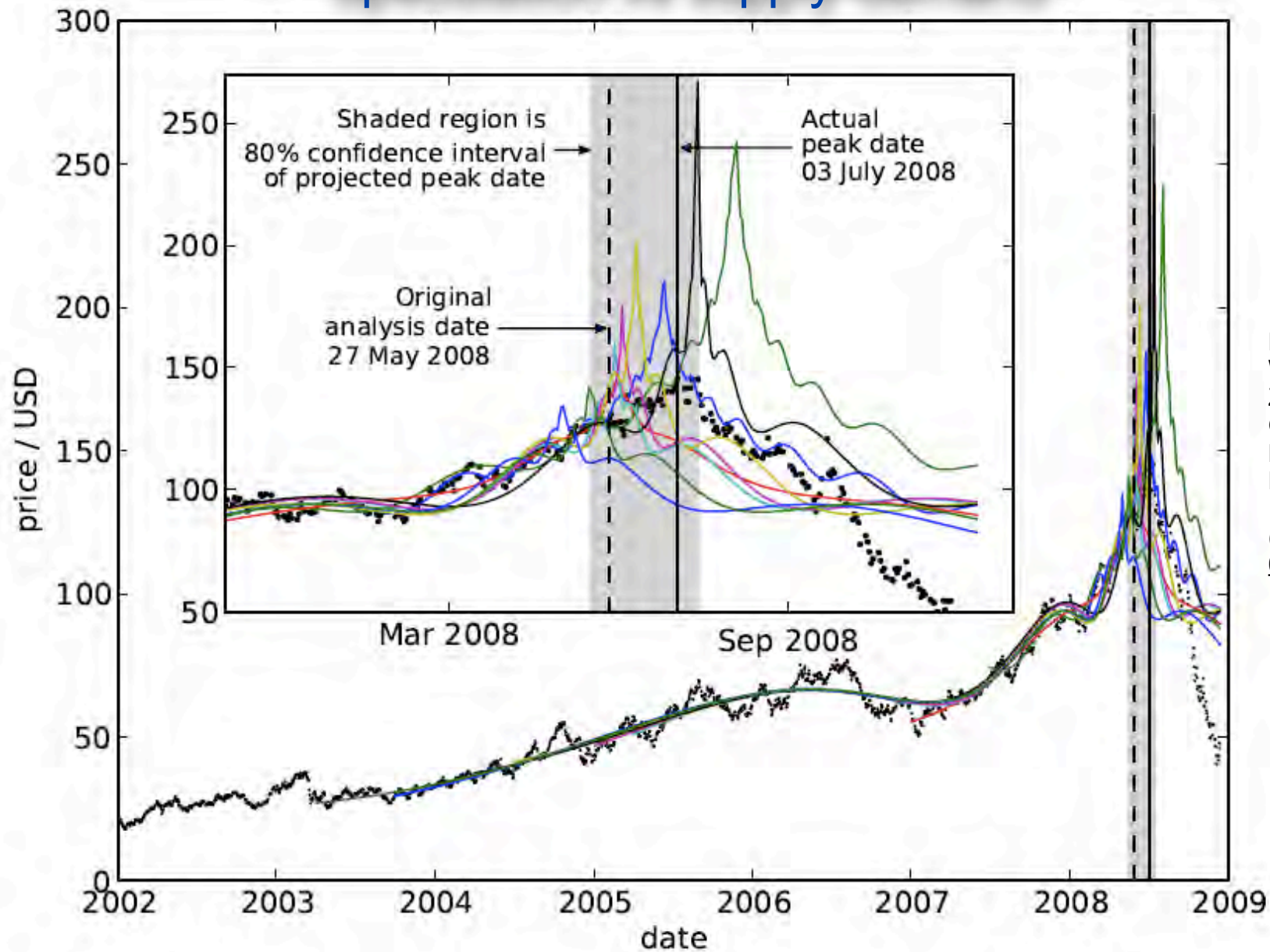
Fig. 5. (Color online) Quarterly average HPI in the 21 states and in the District of Columbia (DC) exhibiting a clear upward faster-than-exponential growth. For better representation, we have normalized the house price indices for the second quarter of 1992 to 100 in all 22 cases. The corresponding states are given in the legend.

# Index price vs. time, S&P 500



# 2006-2008 Oil bubble

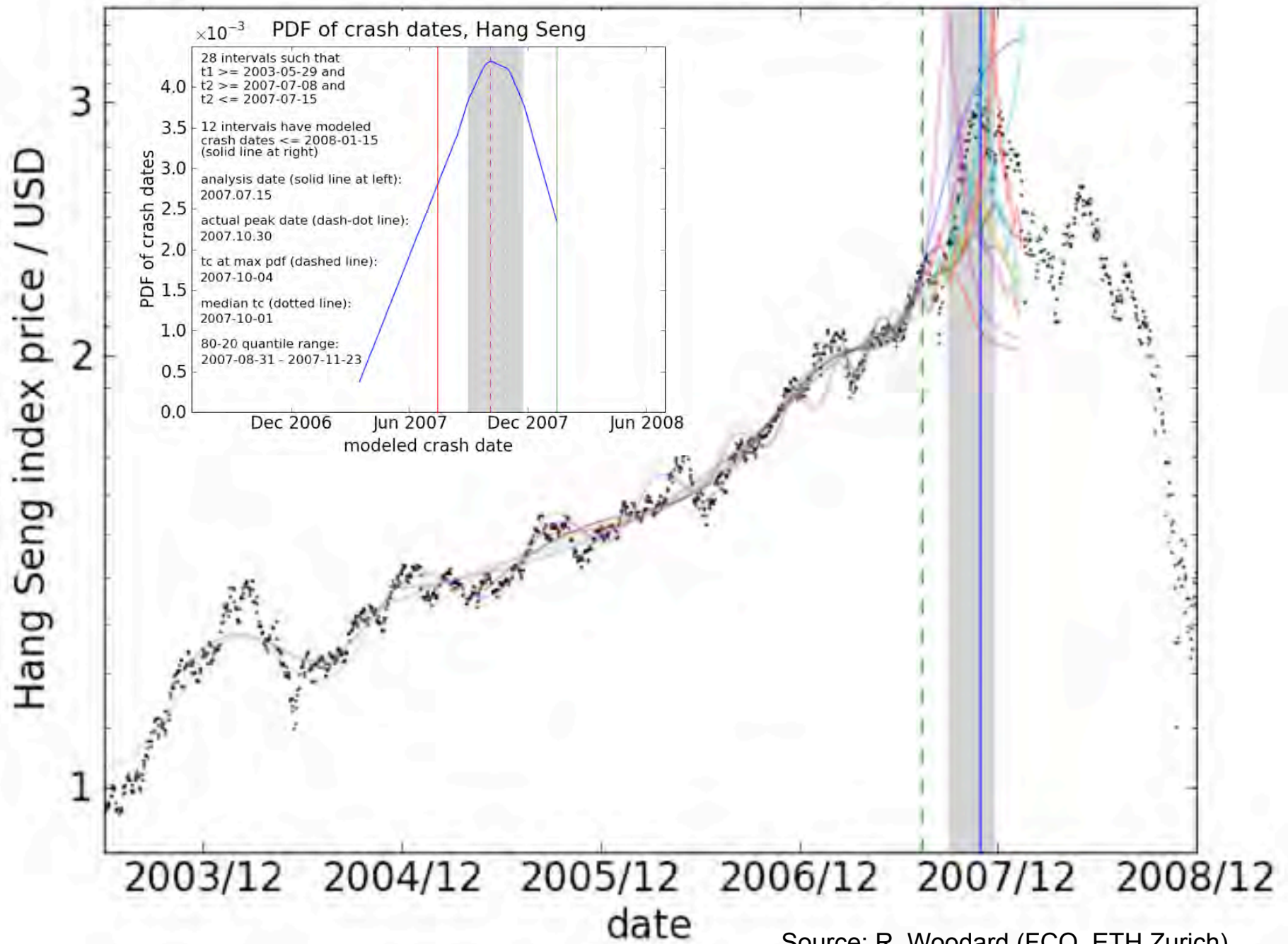
## Speculation vs supply-demand



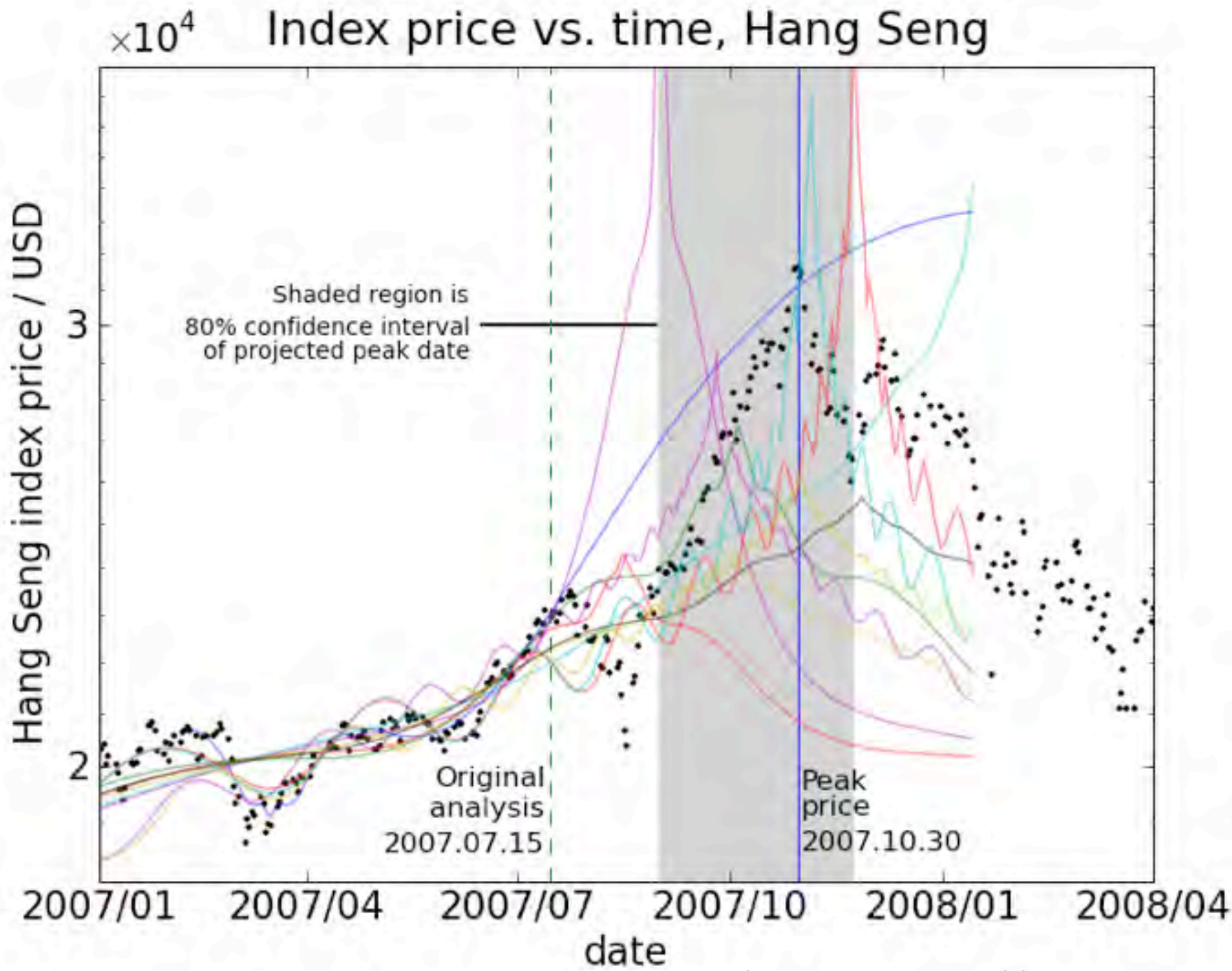
D. Sornette, R. Woodard and W.-X. Zhou, The 2006-2008 Oil Bubble and Beyond, *Physica A* 388, 1571-1576 (2009) ([arXiv.org/abs/0806.1170](http://arXiv.org/abs/0806.1170))

Typical result of the calibration of the simple LPPL model to the oil price in US\$ in shrinking windows with starting dates  $t_{start}$  moving up towards the common last date  $t_{last} = \text{May 27, 2008}$ .

# Index price vs. time, Hang Seng



Source: R. Woodard (FCO, ETH Zurich)



Source: R. Woodard (FCO, ETH Zurich)

# FOUR EXAMPLES

(i) the fluctuation-susceptibility theorem transforms into a remarkable classification of financial volatility shocks (endogenous versus exogenous),

(ii) the Ising model of phase transitions can be generalized to model the stylized facts of financial markets,

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(iv) the mathematics of quantum physics provides a new quantum decision theory solving the known paradoxes.

# 1. Classical Decision Theory

1.1. Notations and definitions

1.2. Typical paradoxes

1.3. Absence of solution

# 2. Quantum Decision Theory

2.1. Definitions and axioms

2.2. General properties

2.3. Solution of paradoxes



# Partial list of problems with standard Utility theory

- **Allais paradox** (Compatibility violation: Several choices are not compatible with utility theory)
- **Ellsberg paradox** (uncertainty aversion)
- **Kahneman-Tversky paradox** (invariance violation)
- **Rabin paradox** (payoff size effects)
- **Disjunction effect** (violation of the sure-thing principle)
- **Conjunction fallacy** (violation of probability theory)

# Save utility theory ?

*Non-expected utility functionals.*

For a lottery  $L = \{x_n, p(x_n)\}$

Instead of expected utility  $U(L)$ , utility functionals

$$F(L) = F[x_n, p(x_n), u(x_n)]$$

Minimal requirements: Risk aversion

Between two lotteries  $L_1$  and  $L_2$ , with the same mean

$$\bar{x}(L_1) = \bar{x}(L_2)$$

the lottery  $L_1$  is preferred to  $L_2$  ( $L_1 > L_2$ ) if  $\Delta^2(L_1) < \Delta^2(L_2)$ .

Then  $F(L_1) > F(L_2)$ .

**Safra and Segal (2008):** Non-expected utility functionals do not remove paradoxes!

## What to do?



1. Realistic problems are complicated, consisting of many parts.
2. Different parts of a problem interact and interfere with each other.
3. Several thoughts of mind can be intricately interconnected (entangled).

Life is complex!

# Quantum Decision Theory

- Novel approach to decision making is developed based on a complex Hilbert space over a lattice of composite prospects.
- Risk of loss and uncertainty are taken into account.
- Paradoxes of classical decision theory are explained.
- Good quantitative agreement with empirical data.
- Conjunction fallacy is a sufficient condition for disjunction effect.

V.I. Yukalov and D. Sornette (2009)

*Quantum Decision Theory*, arXiv.org.0802.3597 (2008);

*Mathematical Basis of Quantum Decision Theory*, [ssrn.com/abstract=1263853](https://ssrn.com/abstract=1263853)

# A partial lists of achievements of Econophysics

- “universality”
- agent-based models, induction, evolutionary models [1, 9, 11, 21],
- option theory for incomplete markets [4, 6],
- interest rate curves [5, 38],
- minority games [8],
- theory of Zipf law and its economic consequences [12, 13, 27],
- theory of large price fluctuations [14],
- theory of bubbles and crashes [17, 22, 40],
- random matrix theory applied to covariance of returns [20, 36, 37],
- methods and models of dependence between financial assets [25, 43].

At present: most exciting progresses at the boundary between economics and the biological, cognitive, and behavioral sciences.

Physics has still a role to play as a unifying framework full of concepts and tools to deal with the complex.

The modeling skills of physicists explain their impressive number in investment and financial institutions (data-driven approach coupled with a pragmatic sense of theorizing)

**KEY CHALLENGE:**  
true trans-disciplinarity by “marriage”