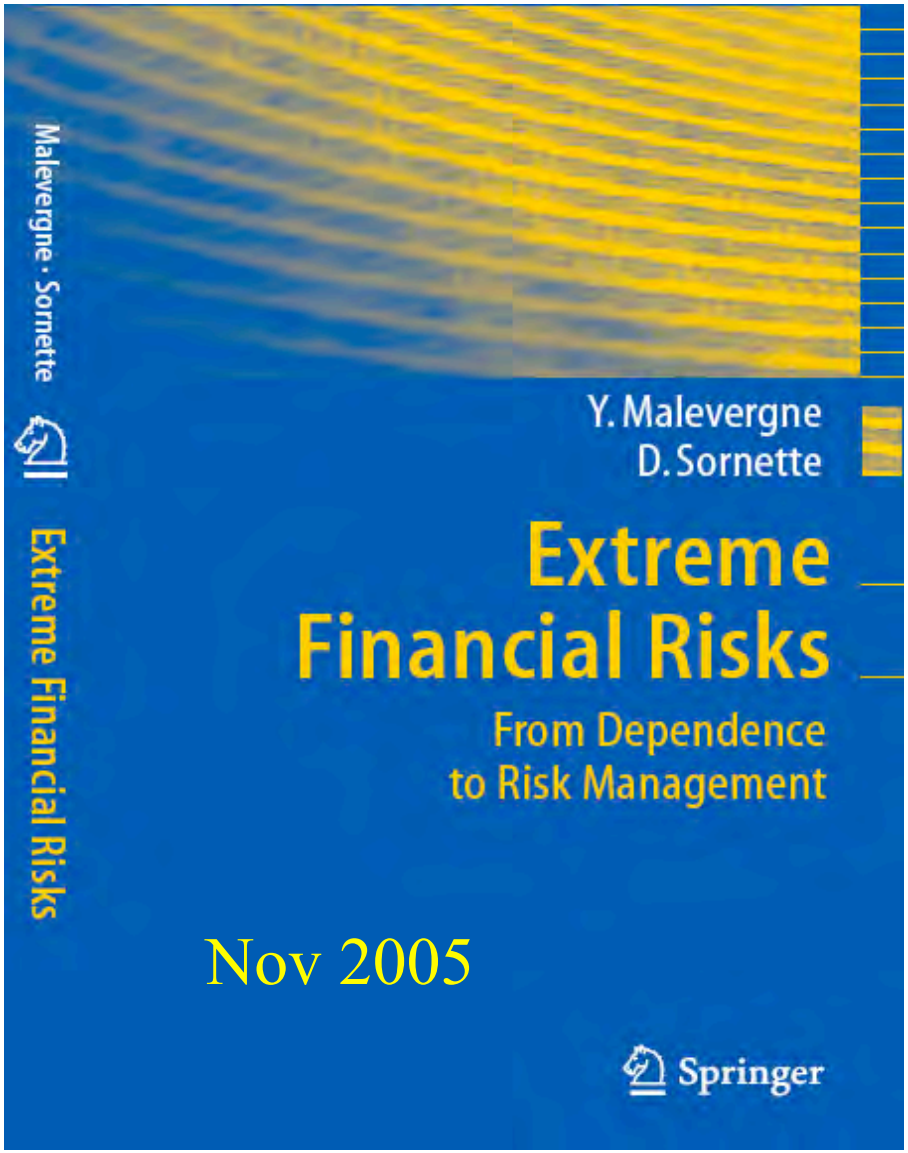
Copulas, Higher-Moments and Tail Risks

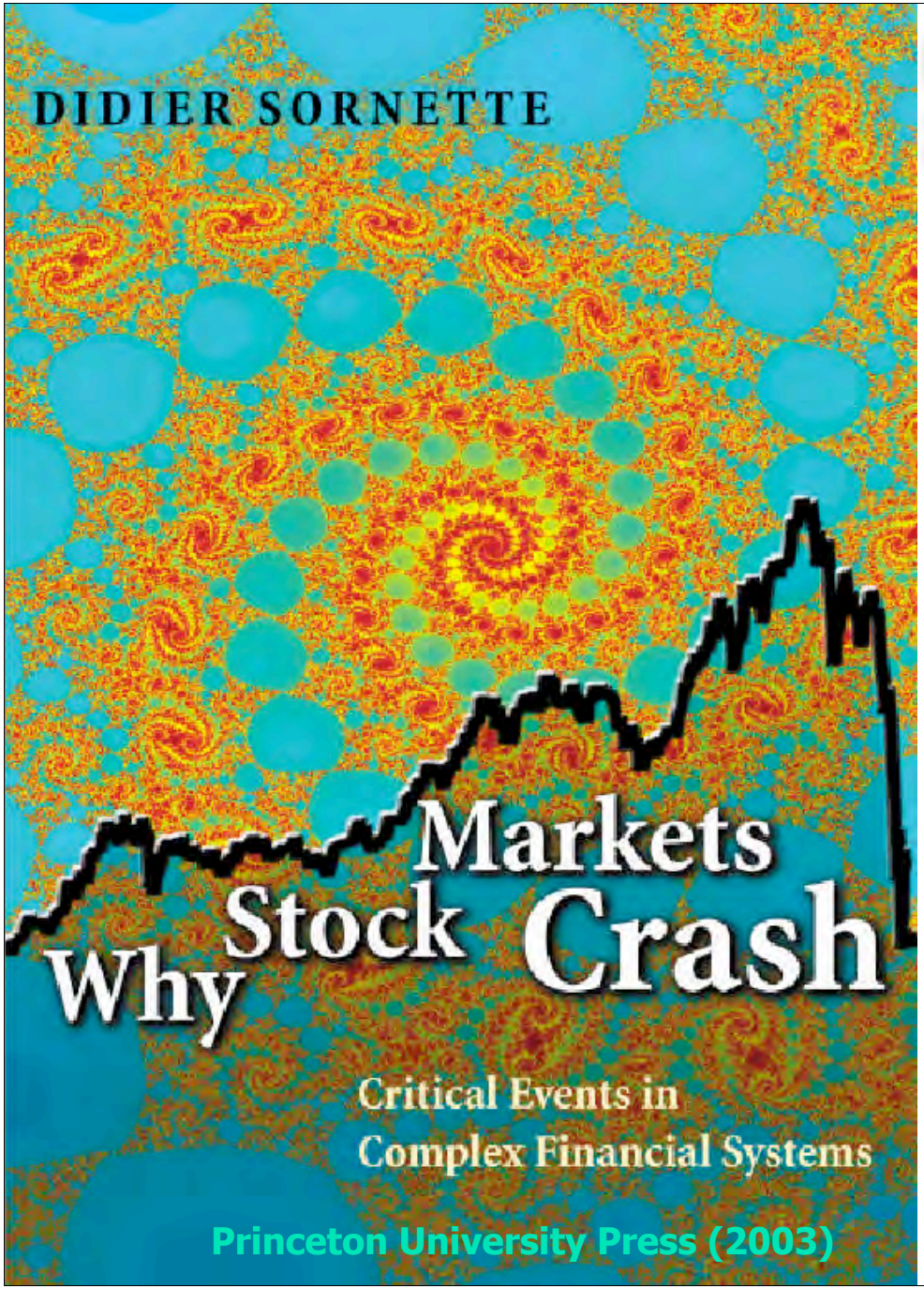


ETH-Zurich
Chair of Entrepreneurial Risks
Department of Management, Technology and Economics (D-MTEC)
Zurich, Switzerland
http://www.mtec.ethz.ch/

Optimal "orthogonal" decomposition of multivariate risks in terms of

-marginal distributions

-intrinsic dependence



EXTREMES AND THE 2008 FINANCIAL CRISIS

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Member of the Swiss Finance Institute

co-founder of the Competence Center for Coping with Crises in Socio-Economic Systems, ETH Zurich (http://www.ccss.ethz.ch/)

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J. Andersen (CNRS, France)
D. Darcet (Insight Research)
K. Ide (UCLA)
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Y. Malevergne (Univ. Lyon, France)
V: Pïsarenko (Acad. Sci. Moscow, Russia)
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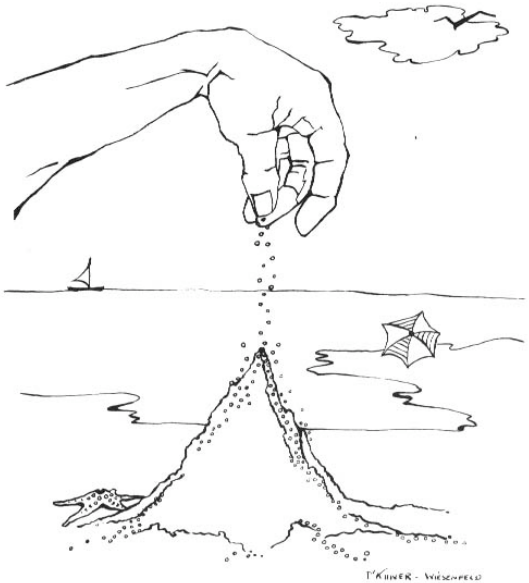
A. Saichev (ETH Zurich and Nizhny Novgorod)
R. Woodard and H. Woodard (ETH Zurich)
W. Yan (ETH Zurich)
A. Huesler (ETH Zurich)
M. Fedorovsky (ETH Zurich)
S. Reimann (ETH Zurich)

- **Self-organization?**

Extreme events are just part of the tail of power law distribution due to "self-organized criticality"? (endogenous)



Artwork by Elaine Wiesenfeld
(from Bak, How Nature Works)

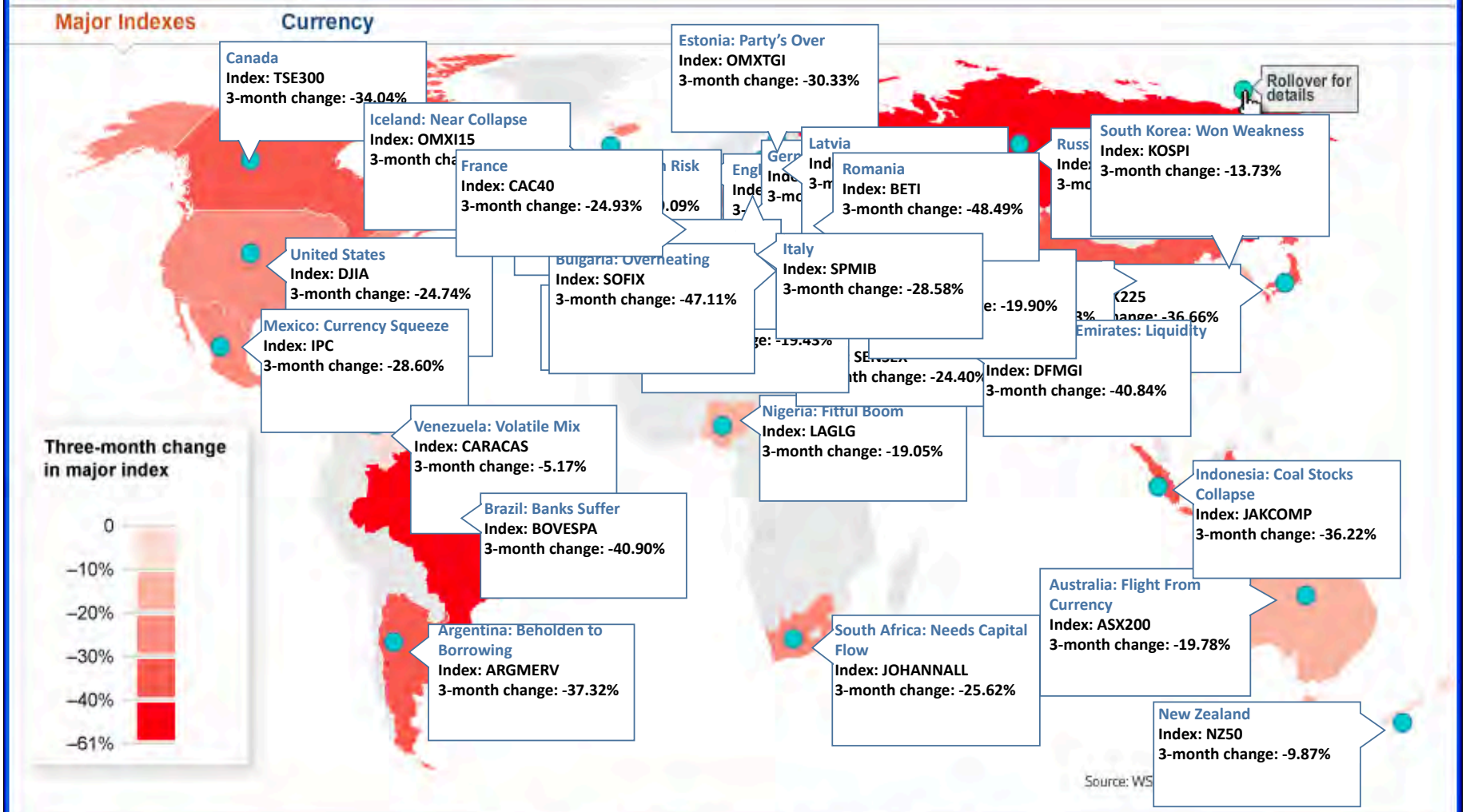
- **"Catastrophism": extreme events require extreme causes that lie outside the system (exogenous)**

- **A mixture? How would it work?**

THE WALL STREET JOURNAL.

Tumbling Stocks, Plunging Currencies

In the tightly interwoven global financial system, countries large and small have been affected by the dramatic slow-down in economic growth. **Click on a country below to see how its major stock index and its currency have fared in the last three months.**



THE WALL STREET JOURNAL.

Tumbling Stocks, Plunging Currencies

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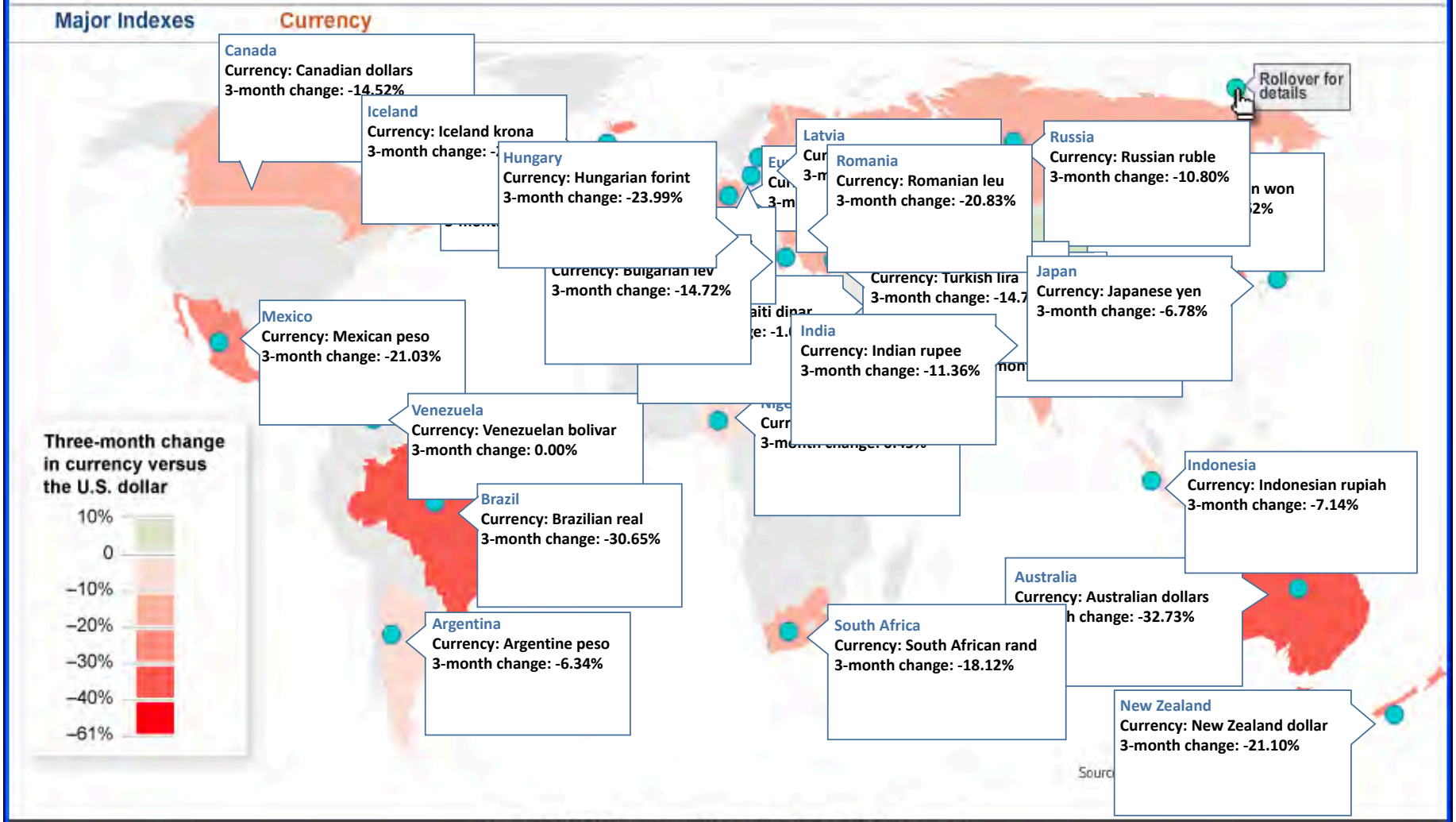
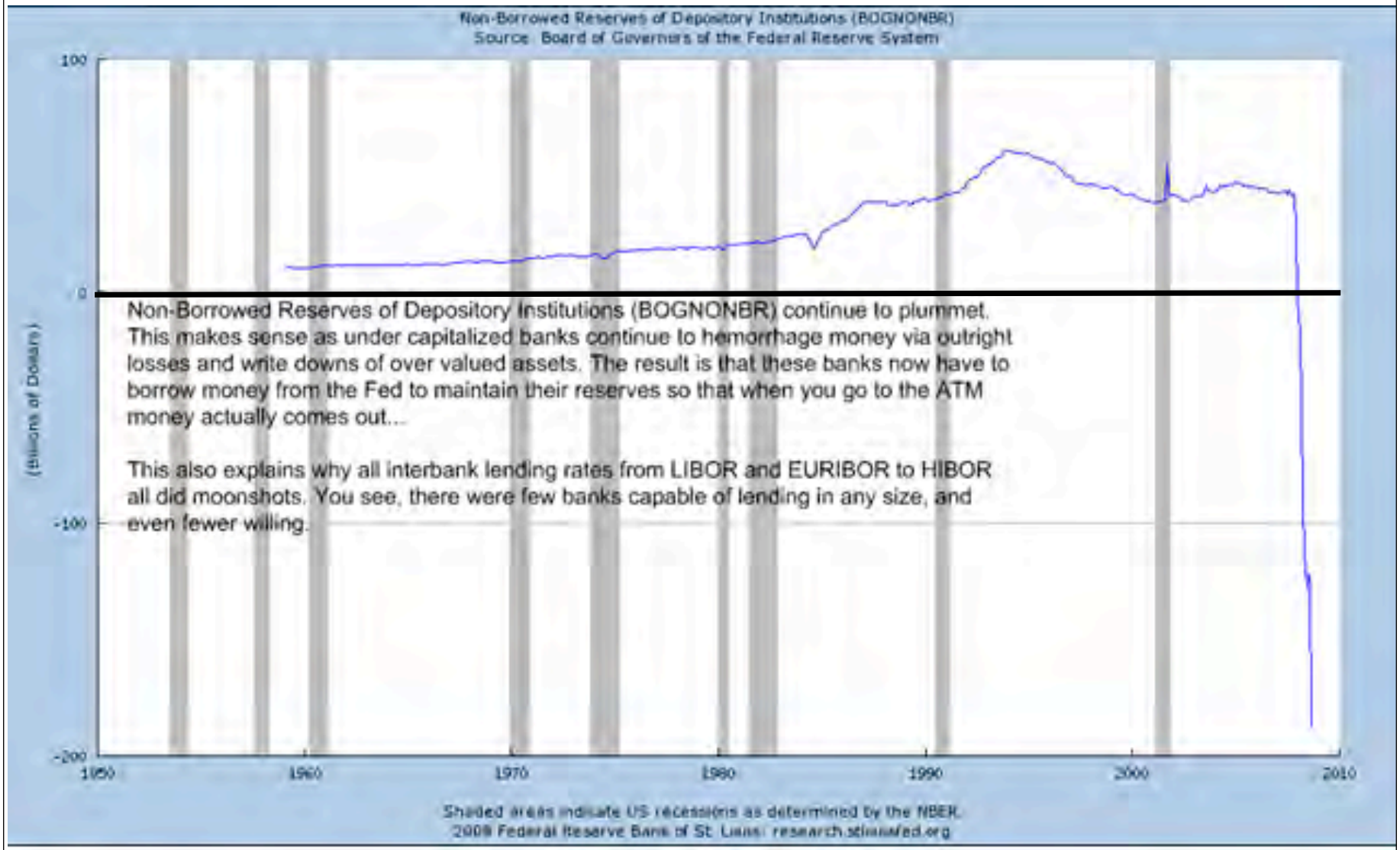
In the tightly interwoven global financial system, countries large and small have been affected by the dramatic slow-down in economic growth. **Click on a country below to see how its major stock index and its currency have fared in the last three months.**



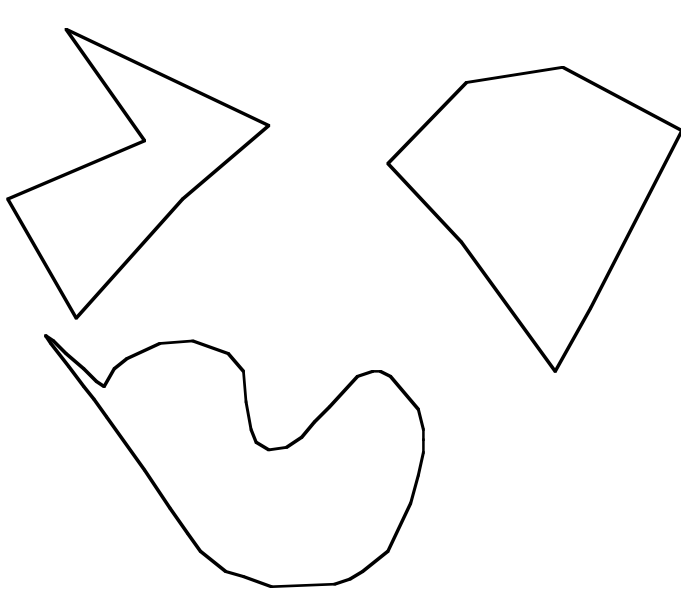


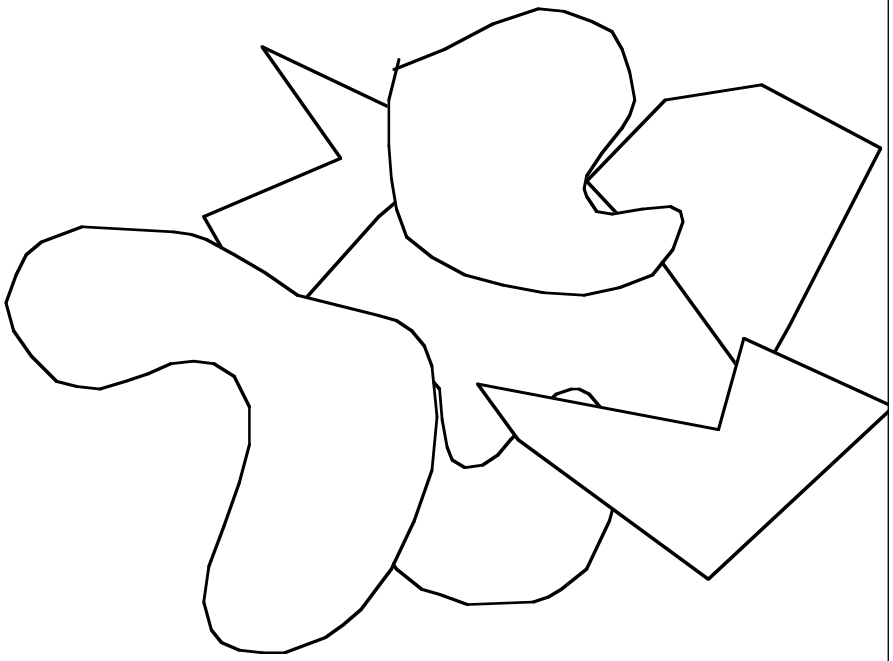
Non-Borrowed Reserves of Depository Institutions (BOGNONBR) continue to plummet. This makes sense as under capitalized banks continue to hemorrhage money via outright losses and write downs of over valued assets. The result is that these banks now have to borrow money from the Fed to maintain their reserves so that when you go to the ATM money actually comes out...

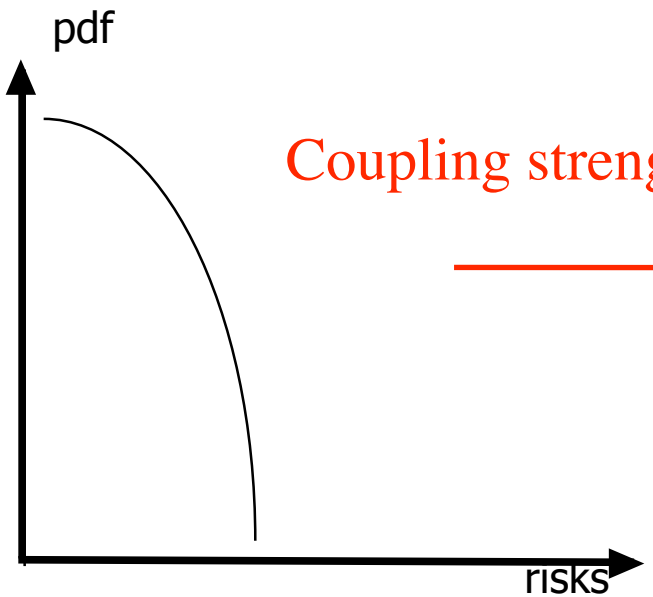
This also explains why all interbank lending rates from LIBOR and EURIBOR to HIBOR all did moonshots. You see, there were few banks capable of lending in any size, and even fewer willing.

Separation of financial and credit risks

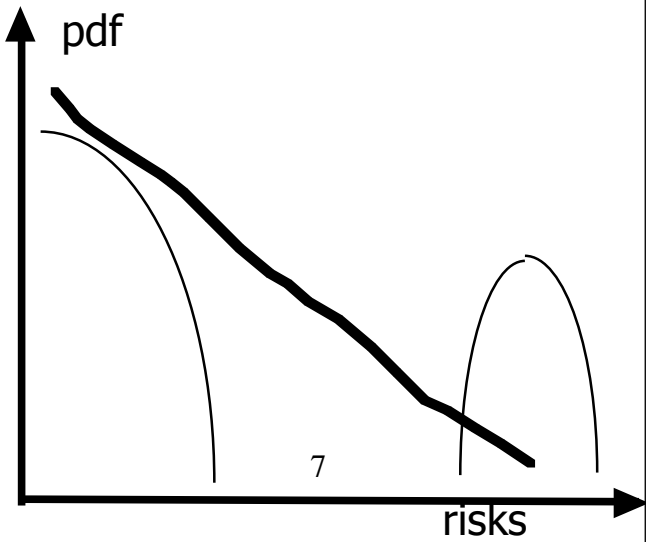
Securitization leads to larger inter-connectivity





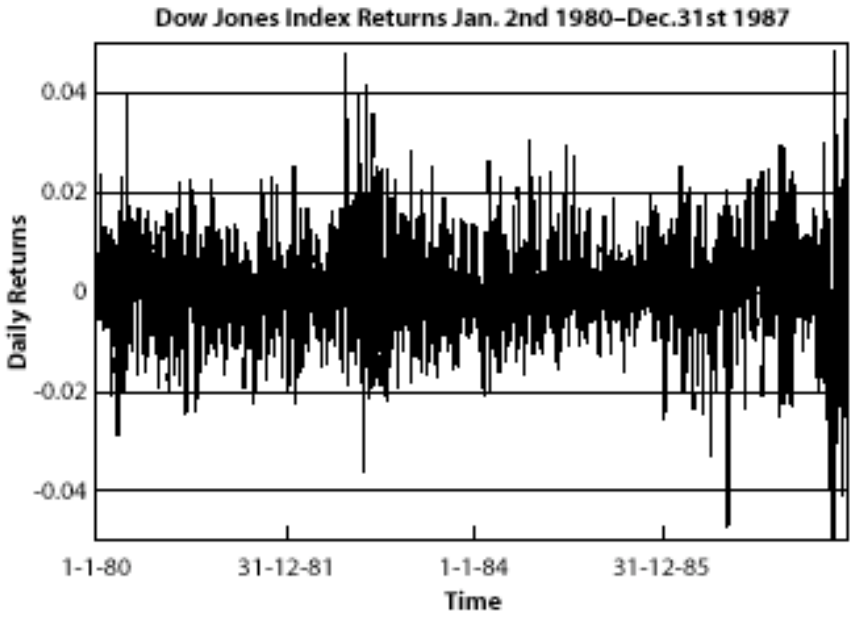


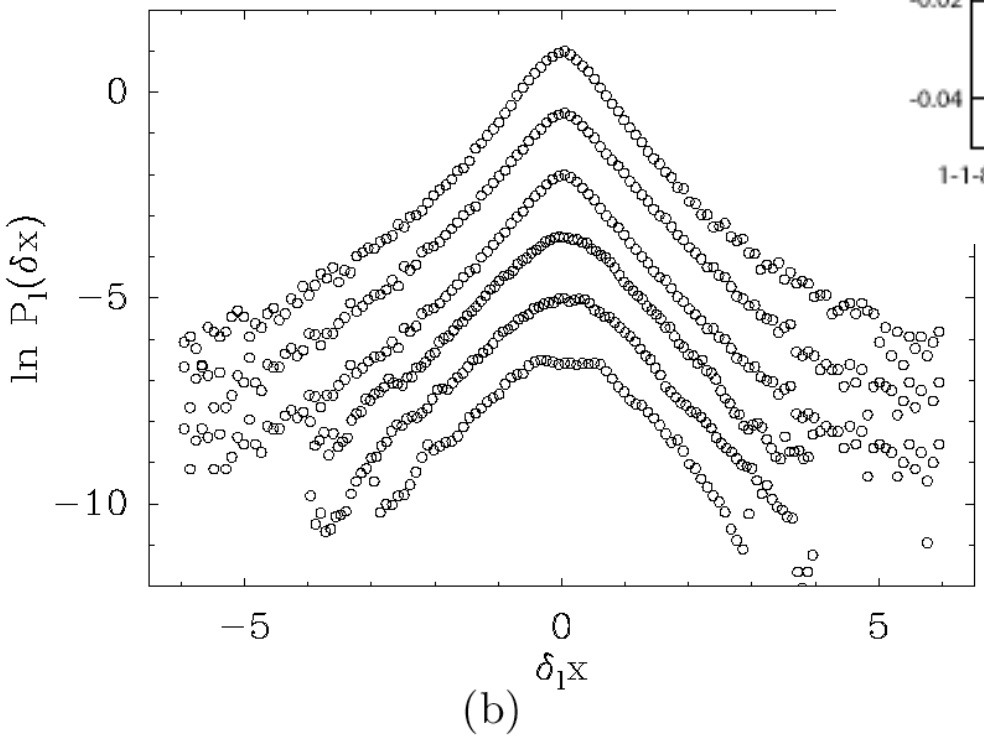
Coupling strength increases



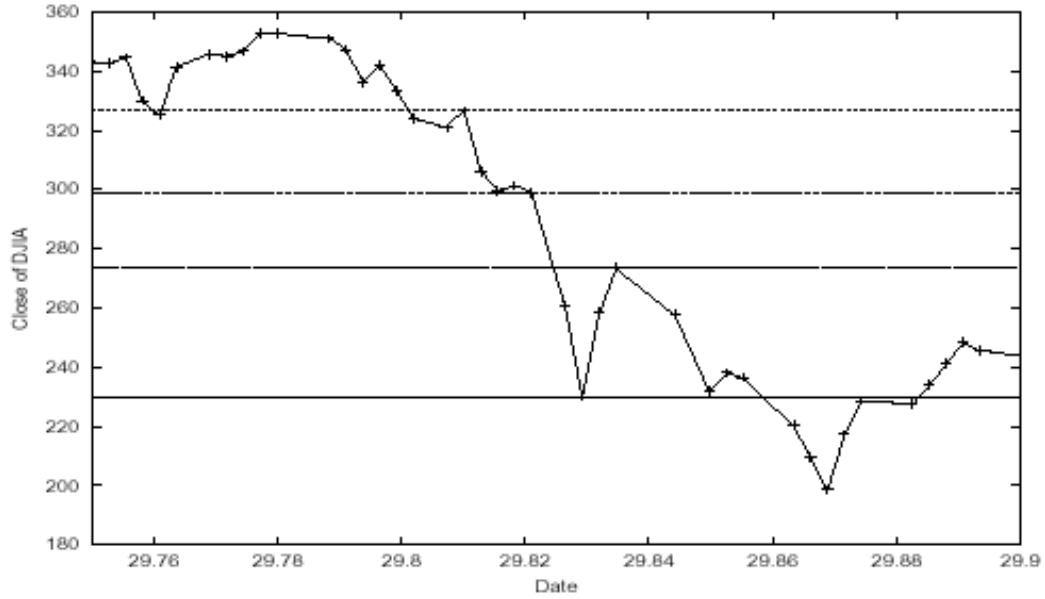
THE CONCEPT OF "Kings"

Traditional emphasis on Daily returns do not reveal any anomalous events

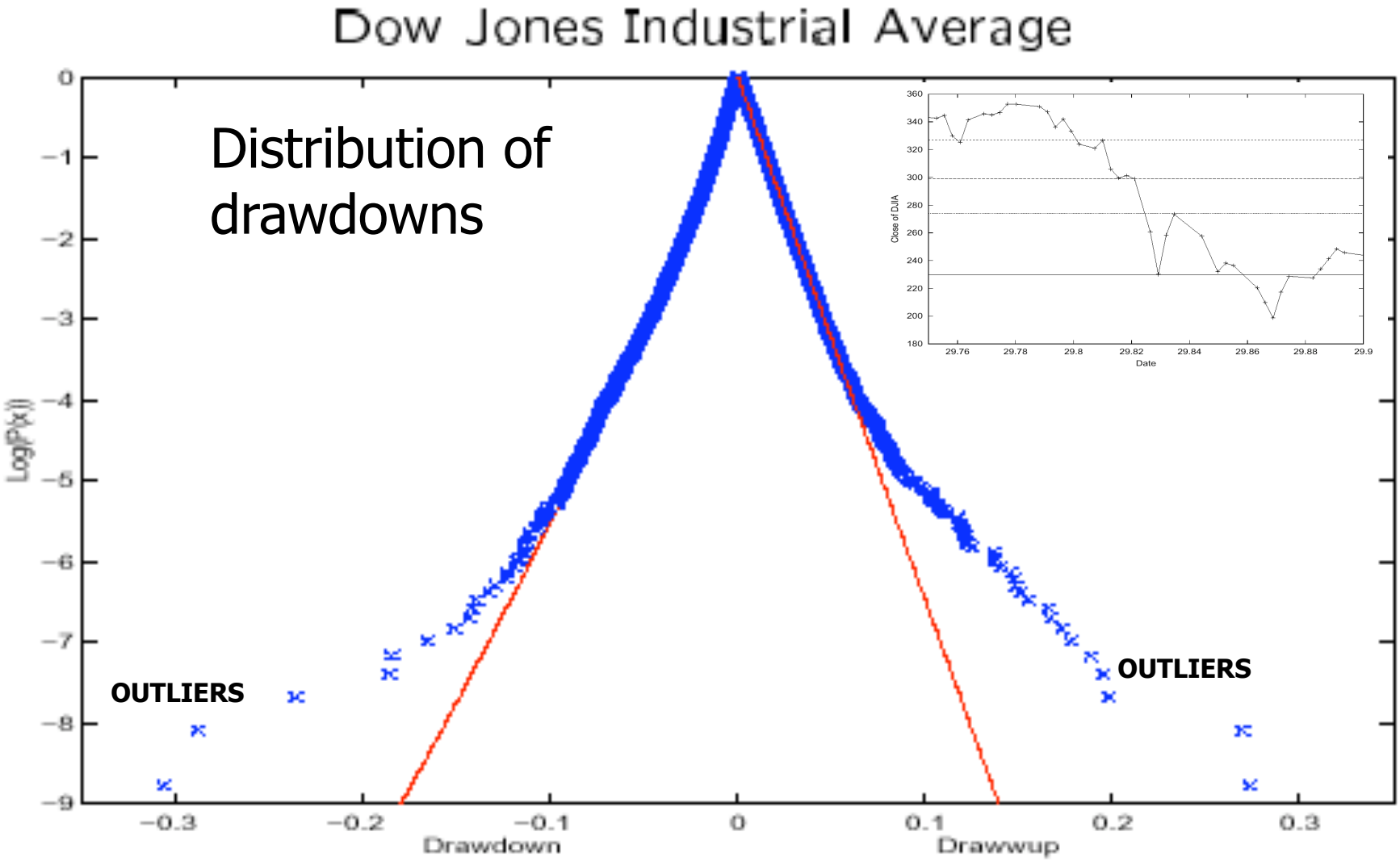




"Drawdowns/Drawups" nonlinear measure

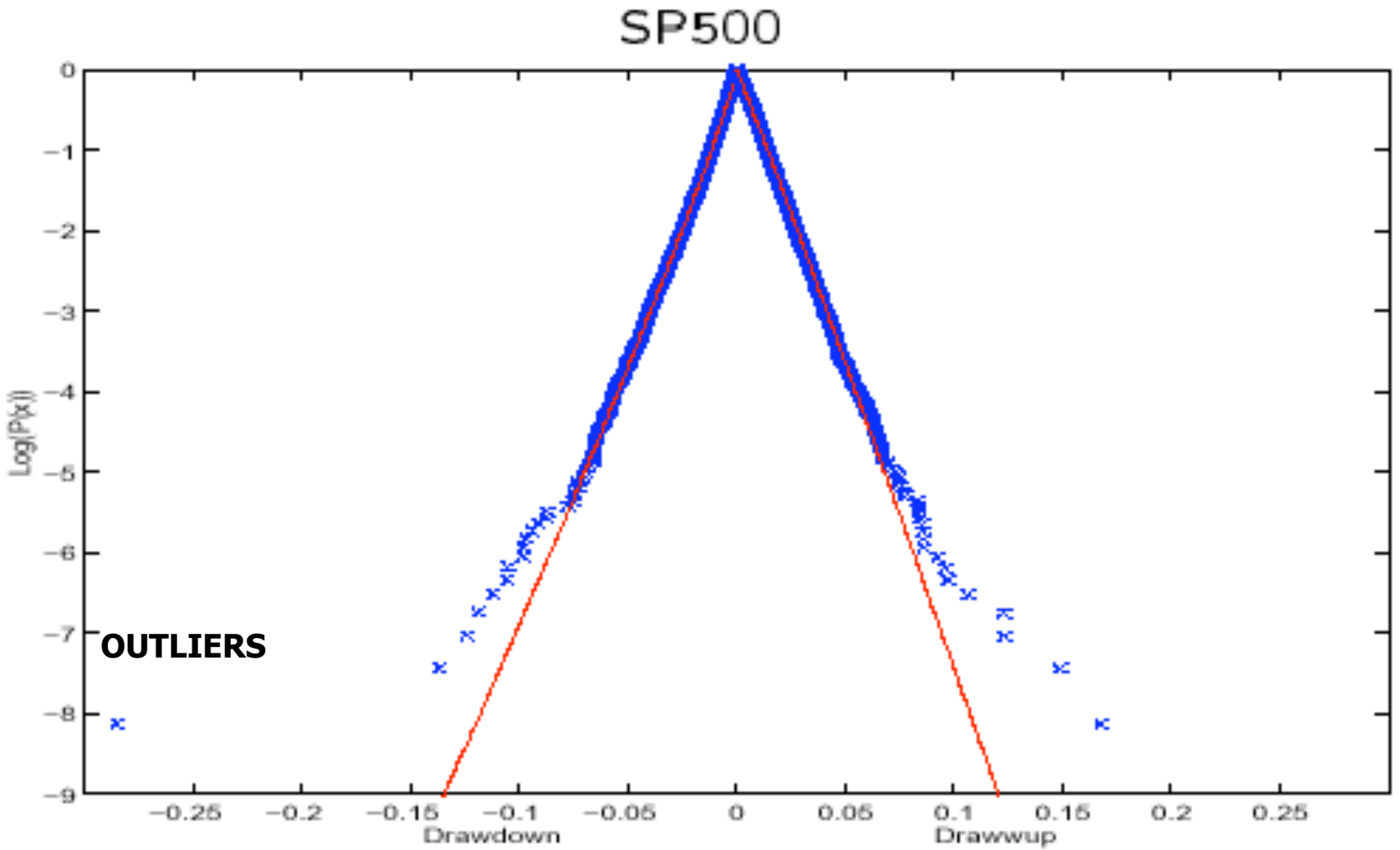


- "Elastic" time horizon determined by market dynamics.
- Worst case scenario (risk management).
- Amplification of extreme market dynamics through "filtering".



A. Johansen and D. Sornette, Stock market crashes are outliers,
European Physical Journal B 1, 141-143 (1998)

A. Johansen and D. Sornette, Large Stock Market Price Drawdowns Are Outliers,
Journal of Risk 4(2), 69-110, Winter 2001/02


SP500
Log(P(x))
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
OUTLIERS
−0.25
−0.2
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
0.2
0.25
Drawdown
Drawwup

Thirty Major US companies

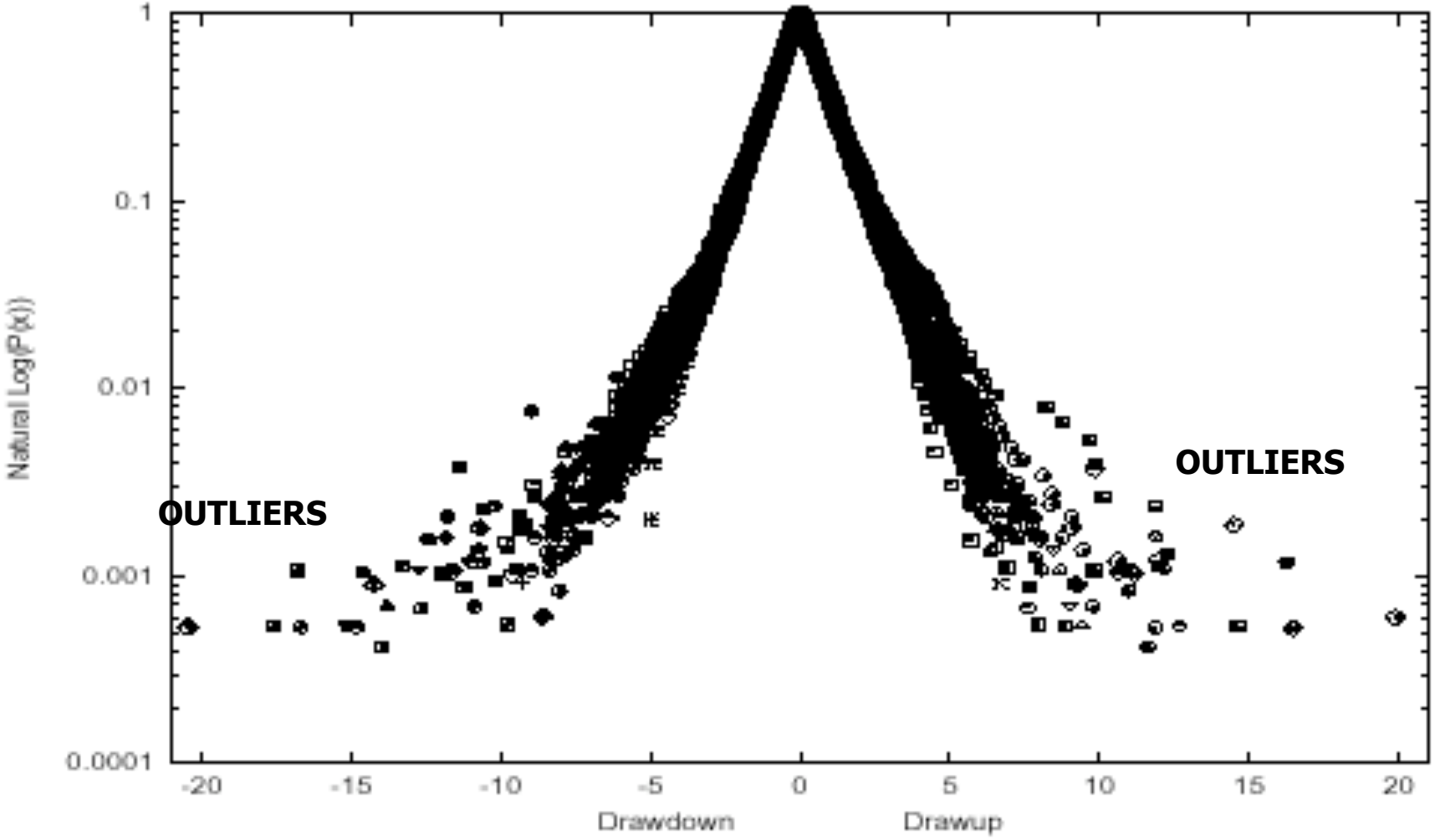
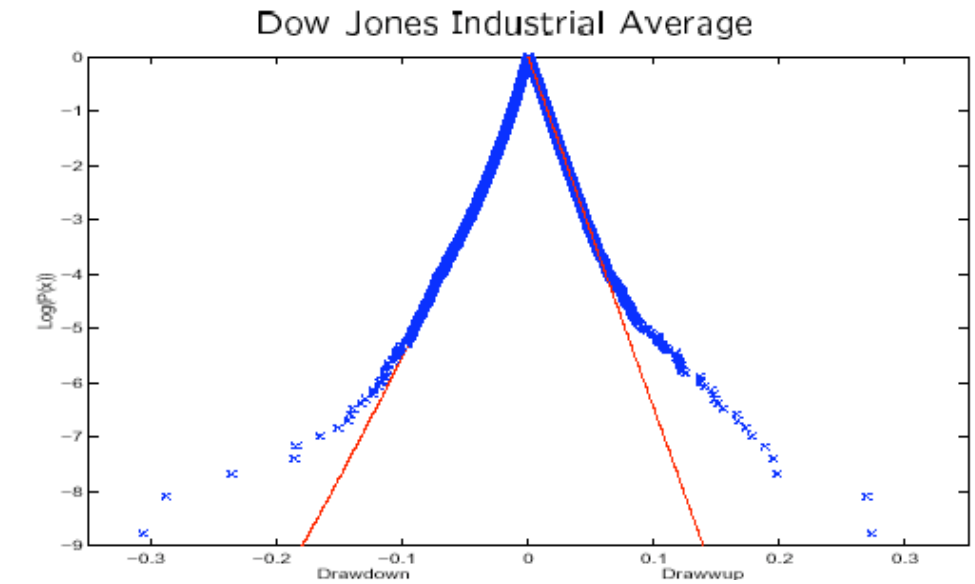


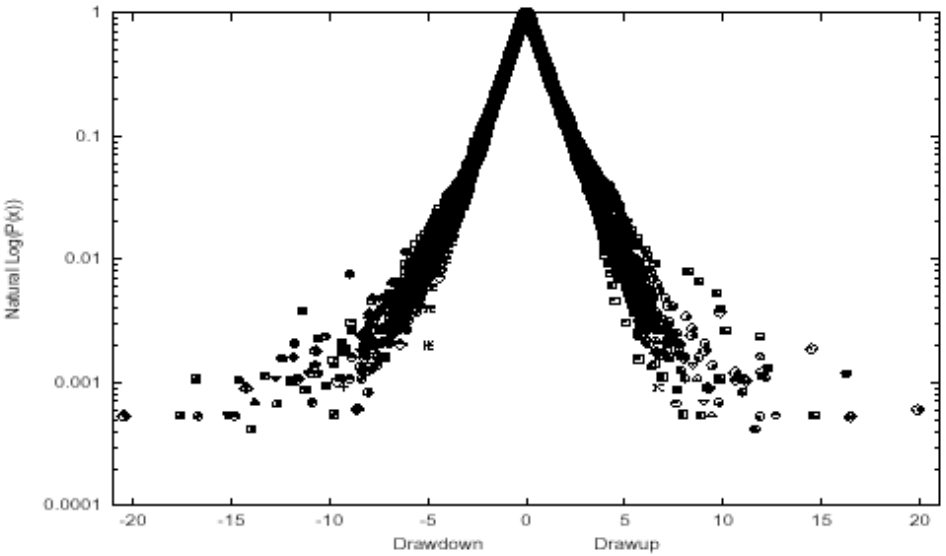
Table 1. NASDAQ composite index. The total number of drawdowns is 1495. The first column is the cut-off u such that the MLE of the two competing hypotheses (standard (SE) and modified (MSE) stretched exponentials) is performed over the interval $[0, u]$ of the absolute value of the drawdowns. The second column gives the fraction 'quantile' of the drawdowns belonging to $[0, u]$. The third column gives the exponents z found for the SE (first value) and MSE (second value) distributions. The fourth and fifth columns give the logarithm of the likelihoods (12) and (13) for the SE and MSE, respectively. The sixth column gives the variable T defined in (14). The last column 'proba' gives the corresponding probability of exceeding T by chance. For $u > 18\%$, we find that T saturates to 13.6 and 'proba' to 0.02%.

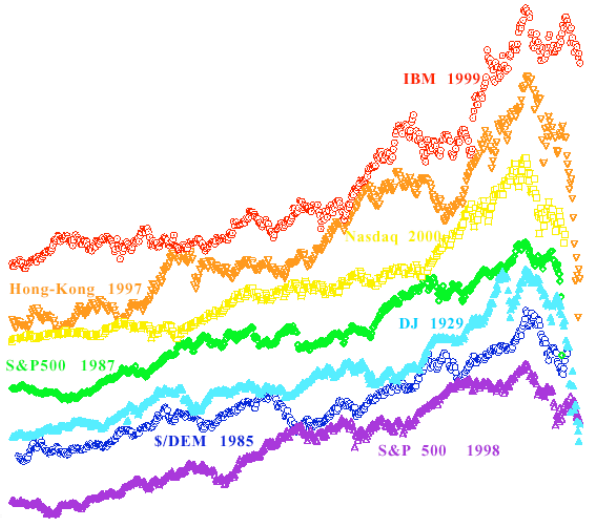
Cut-off u	Quantile	z	$\ln(L_0)$	$\ln(L_1)$	T	Proba
3%	87%	0.916, 0.940	4890.36	4891.16	1.6	20.5%
6%	97%	0.875, 0.915	4944.36	4947.06	5.4	2.0%
9%	99.0%	0.869, 0.918	4900.75	4903.66	5.8	1.6%
12%	99.7%	0.851, 0.904	4872.47	4877.46	10.0	0.16%
15%	99.7%	0.843, 0.898	4854.97	4860.77	11.6	0.07%
18%	99.9%	0.836, 0.890	4845.16	4851.94	13.6	0.02%

D. Sornette and A. Johansen
Significance of log-periodic precursors to financial crashes,
Quantitative Finance 1 (4), 452-471 (2001)

A. Johansen and D. Sornette,
Endogenous versus Exogenous Crashes in Financial Markets,
in press in ``Contemporary Issues in International Finance''
(Nova Science Publishers, 2004)
(http://arXiv.org/abs/cond-mat/0210509)







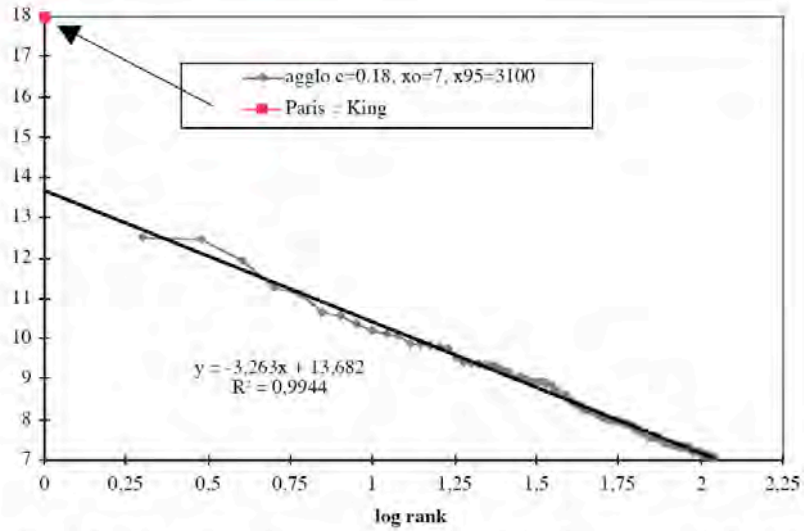


Fig. 7. French agglomerations: stretched exponential and "King effect".

Beyond power laws: five examples of "kings"

Paris as the king in the Zipf distribution of French city sizes.

Outliers and kings in the distribution of financial drawdowns.

Extreme king events in the pdf of turbulent velocity fluctuations.

Material failure and rupture processes.

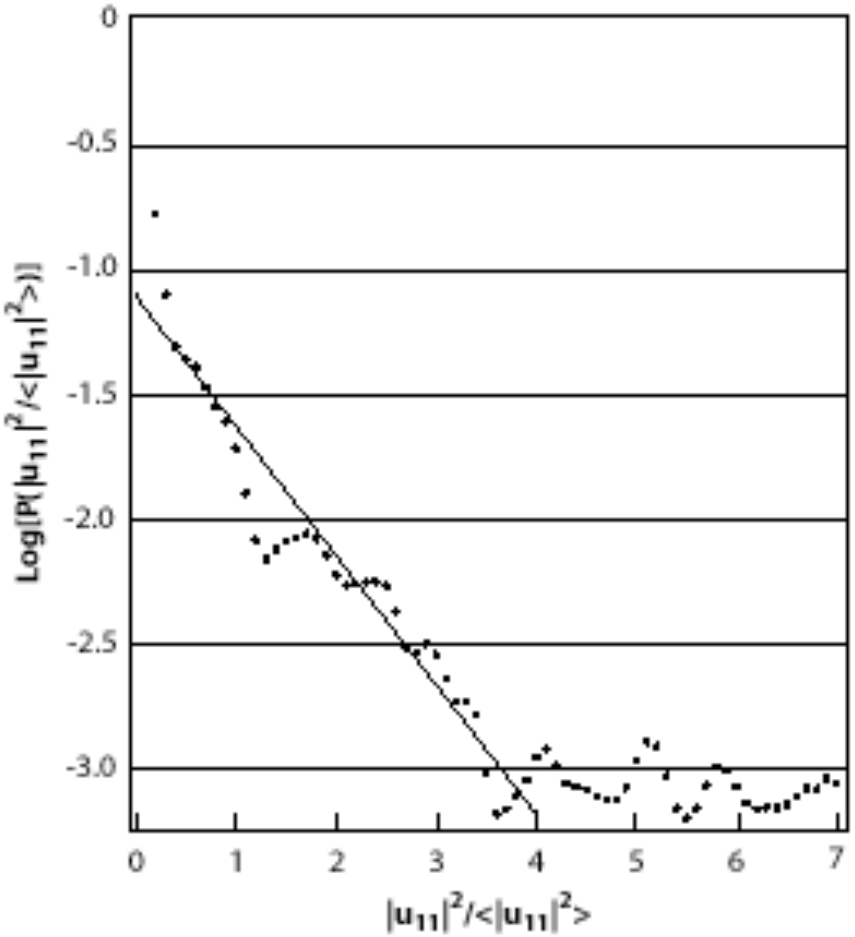
Epileptic seizures

Gutenberg-Richter law and characteristic earthquakes.

Mathematical Geophysics Conference

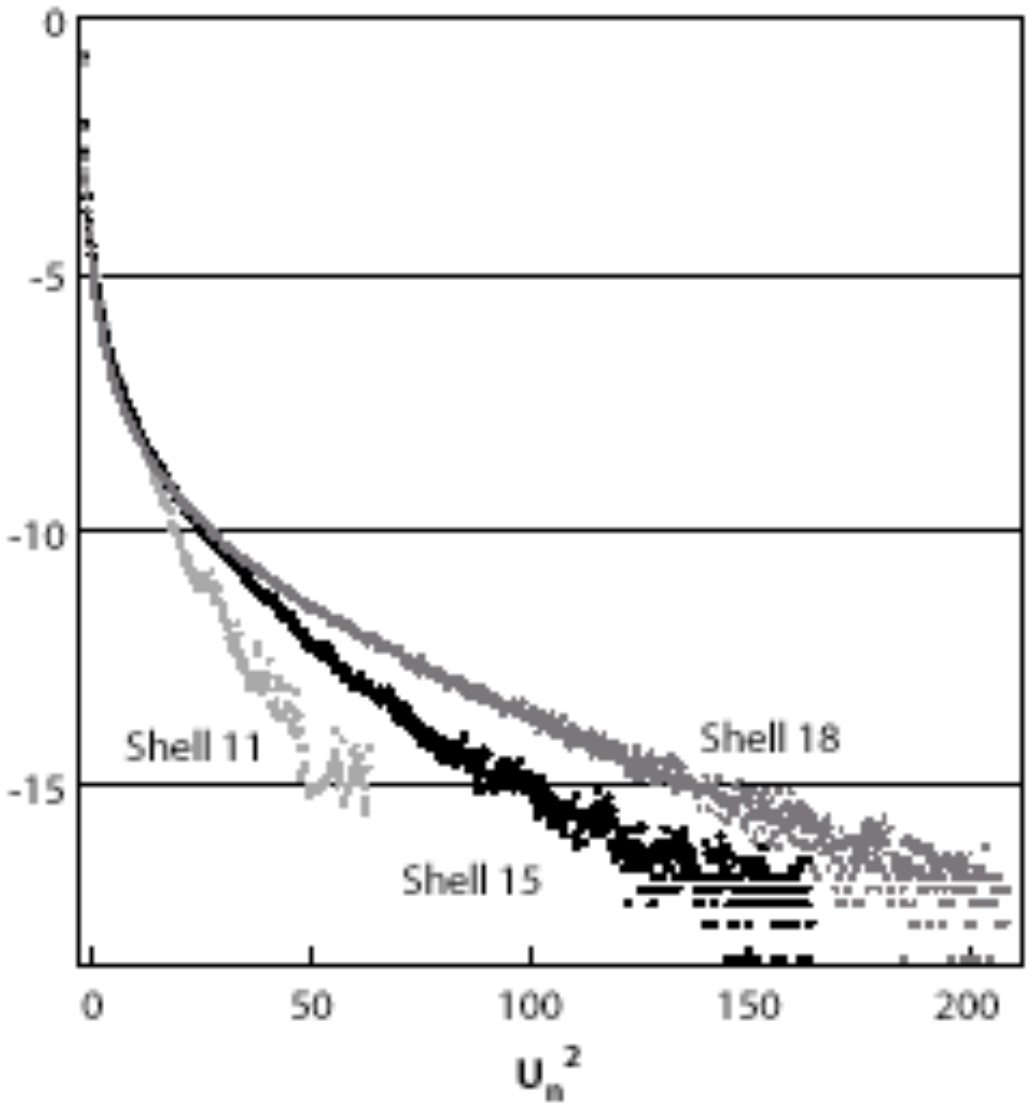
Villefranche-sur-Mer, 18-23 June 2000

Extreme Earth Events



L'vov, V.S., Pomyalov, A. and Procaccia, I. (2001) Outliers, Extreme Events and Multiscaling, Physical Review E 6305 (5), 6118, U158-U166.

FIG. 3.2. Apparent probability distribution function of the square of the fluid velocity, normalized to its time average, in the eleventh shell of the toy model of hydrodynamic turbulence discussed in the text. The vertical axis is in logarithmic scale such that the straight line, which helps the eye, qualifies as an apparent exponential distribution. Note the appearance of extremely sparse and large bursts of velocities at the extreme right above the extrapolation of the straight line. Reproduced from [252].



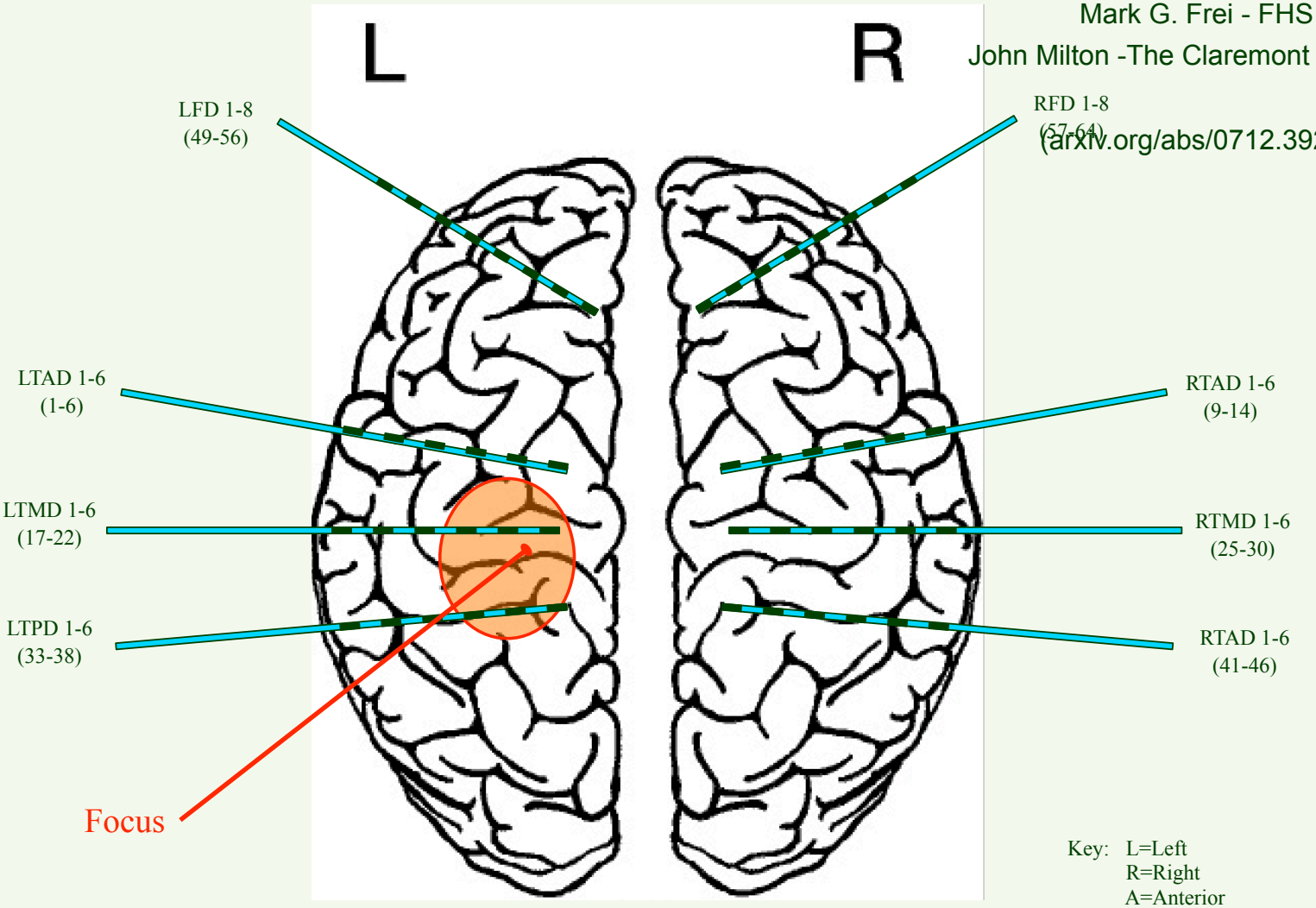
Pdf of the square of the Velocity as in the previous figure but for a much longer time series, so that the tail of the distributions for large Fluctuations is much better constrained. The hypothesis that there are no outliers is tested here by collapsing the distributions for the three shown layers. While this is a success for small fluctuations, the tails of the distributions for large events are very different, indicating that extreme fluctuations belong to a different class of their own and hence are outliers.

L'vov, V.S., Pomyalov, A. and Procaccia, I. (2001) Outliers, Extreme Events and Multiscaling, Physical Review E 6305 (5), 6118, U158-U166.

Epileptic Seizures – Quakes of the Brain?

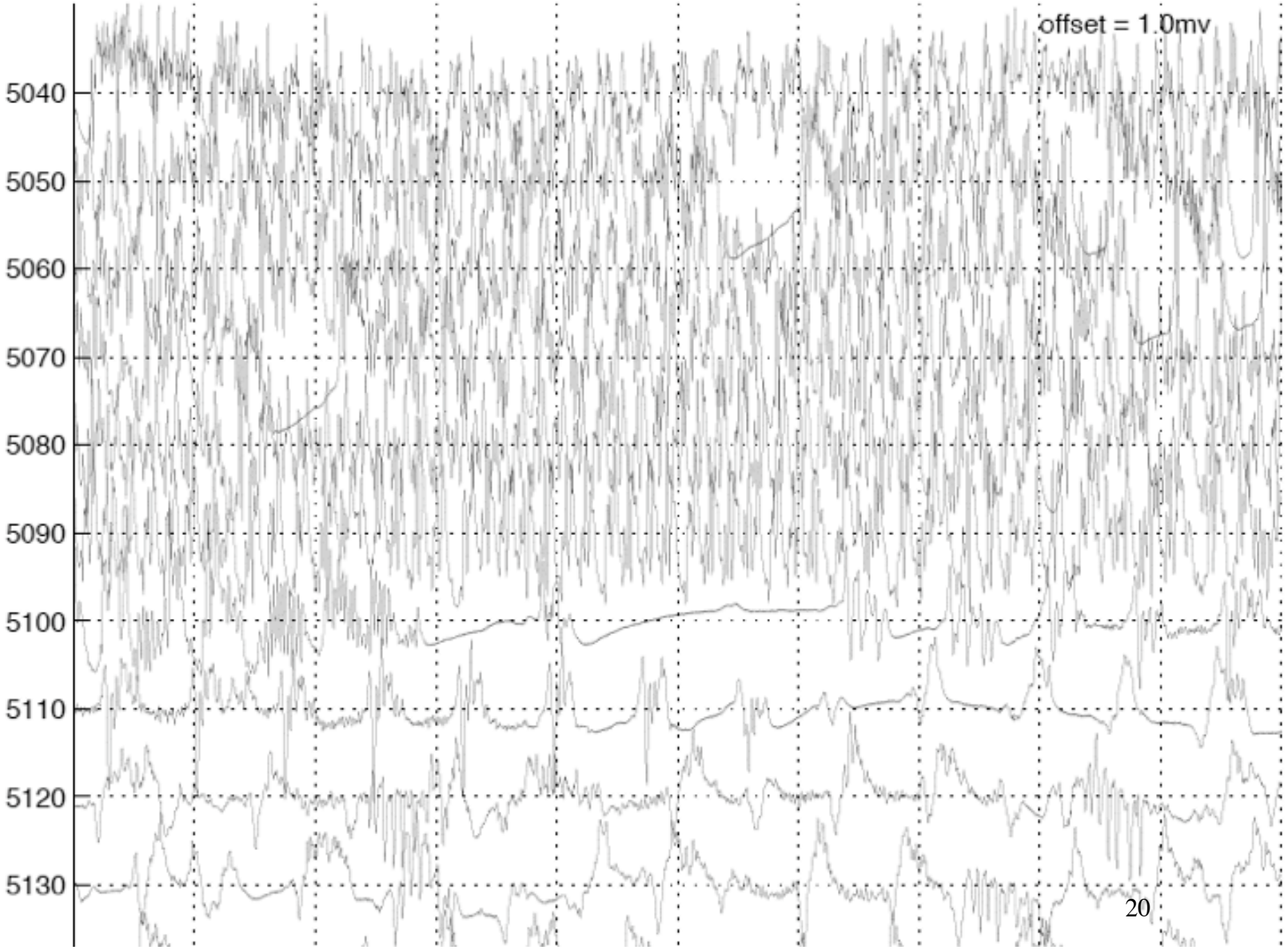
with Ivan Osorio – KUMC & FHS
Mark G. Frei - FHS
John Milton -The Claremont Colleges

(arxiv.org/abs/0712.3929)

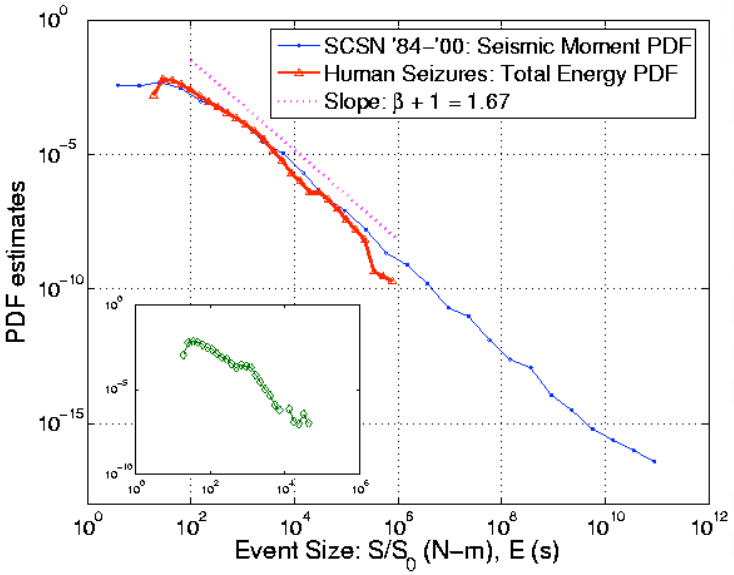


Key: L=Left
R=Right
A=Anterior
M=Mesial
P=Posterior
D=Depth
T=Temporal
F=Frontal

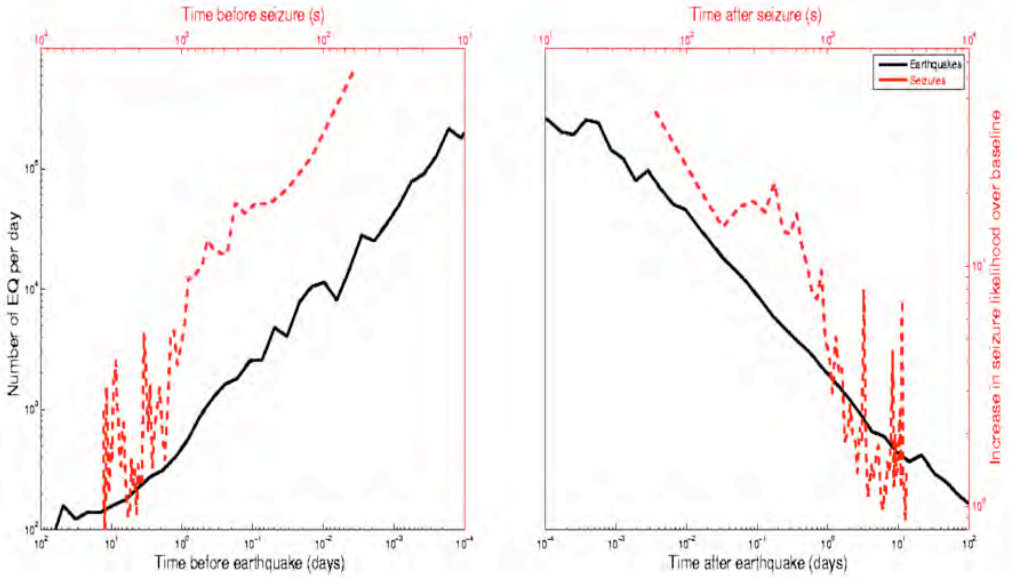
Bursts and Seizures



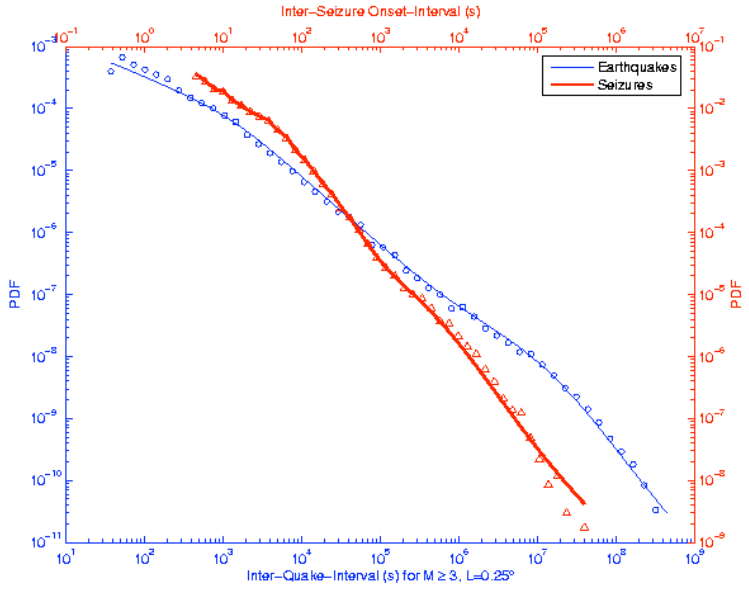
Gutenberg-Richter distribution of sizes



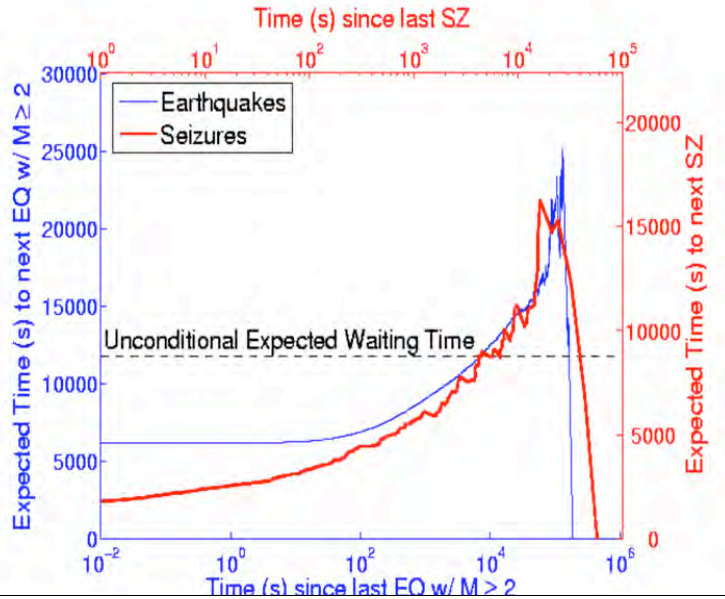
Omori law: Direct and Inverse



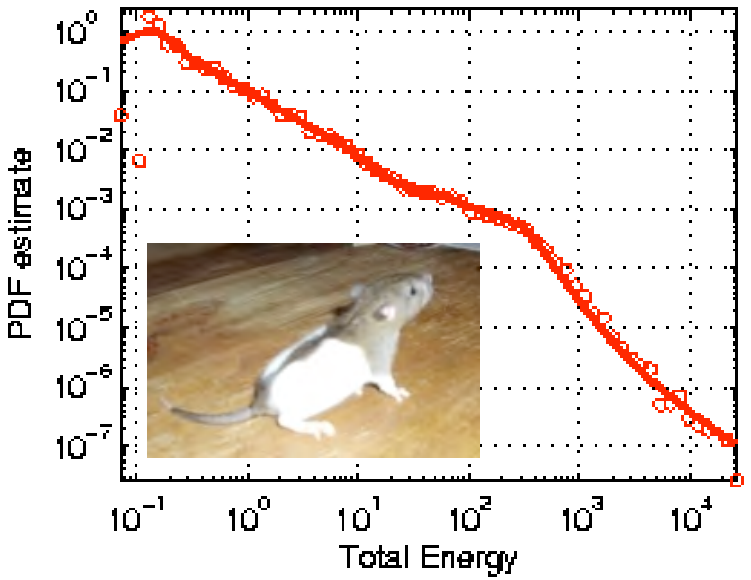
pdf of inter-event waiting times

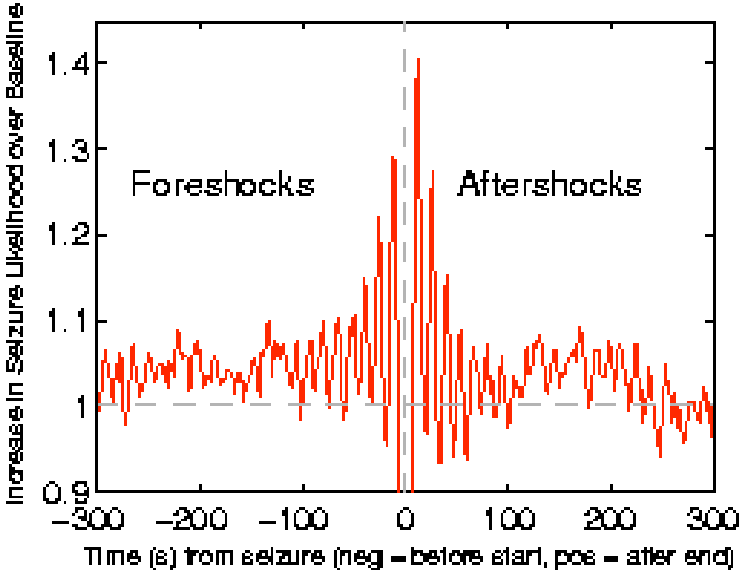


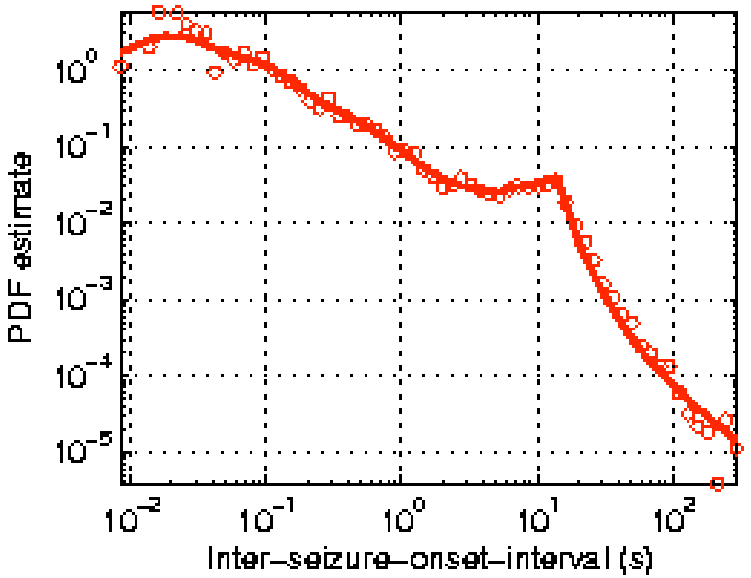
The longer it has been since the last event, the longer it will be since the next one!

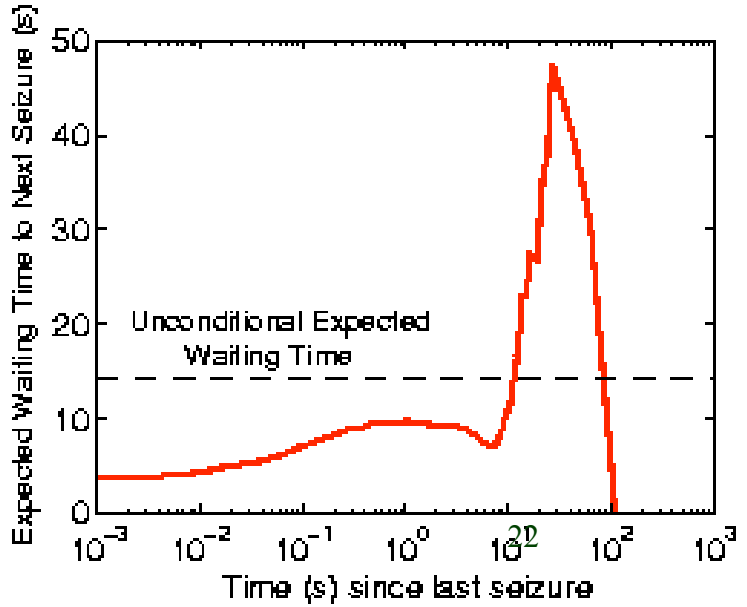


19 rats treated intravenously (2) with the convulsant 3-mercapto-proprionic acid (3-MPA)



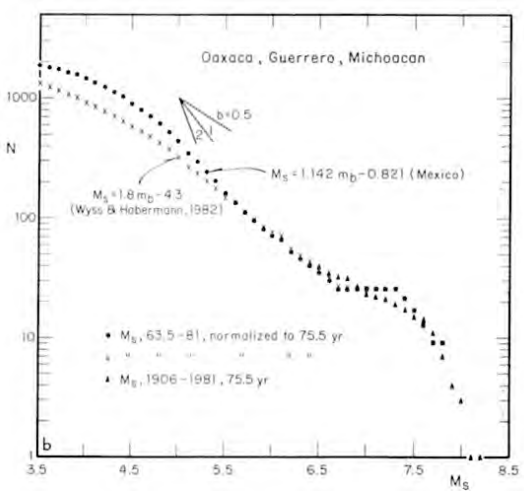




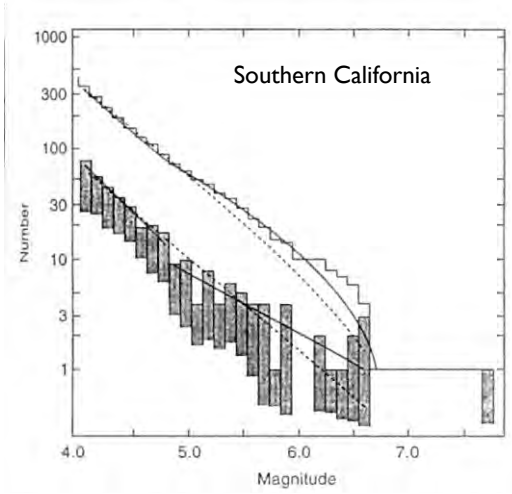


Complex magnitude distributions

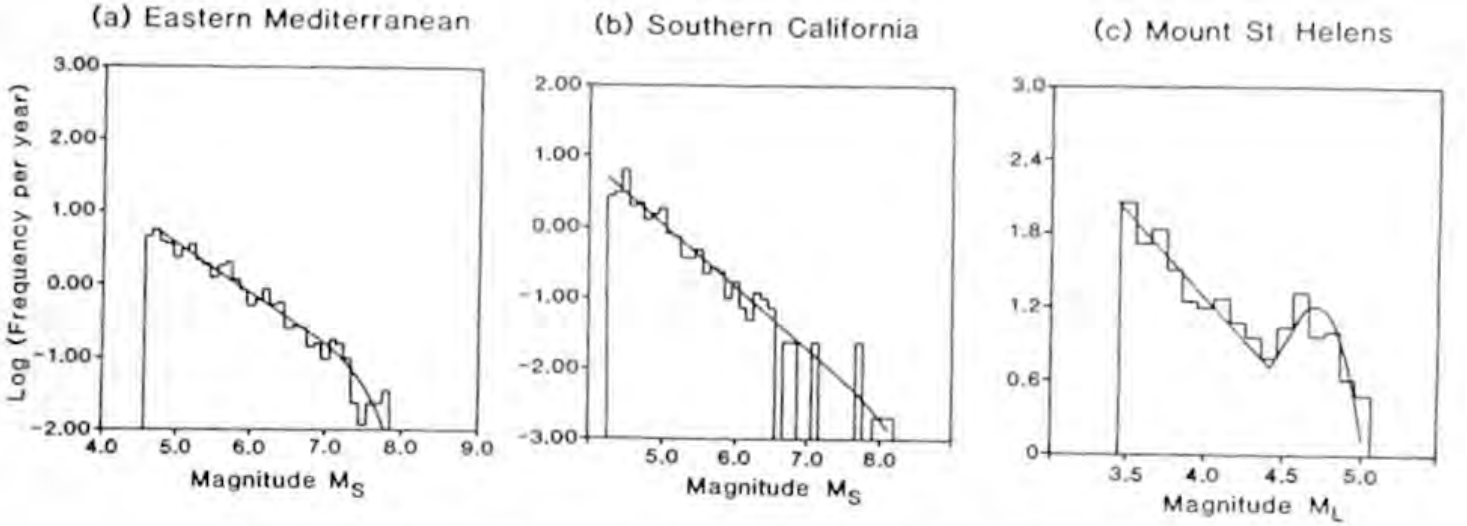
Characteristic earthquakes?



Singh, et. al., 1983, BSSA 73, 1779-1796



Knopoff, 2000, PNAS 97, 11880-11884



Main, 1995, BSSA 85, 1299-1308

Landau-Ginzburg Theory of Self-Organized Criticality

Dynamics of an order parameter (OP) and of the corresponding *control* parameter (CP): within the sandpile picture, $\frac{\partial h}{\partial x}$ is the slope of the sandpile, h being the local height, and S is the state variable distinguishing between static grains ($S = 0$) and rolling grains ($S \neq 0$).

L. Gil and D. Sornette "Landau-Ginzburg theory of self-organized criticality", Phys. Rev.Lett. 76, 3991-3994 (1996)

Normal form of sub-critical bifurcation

$$\frac{\partial S}{\partial t} = \chi \left\{ \mu S + 2\beta S^3 - S^5 \right\} \qquad (1)$$

where

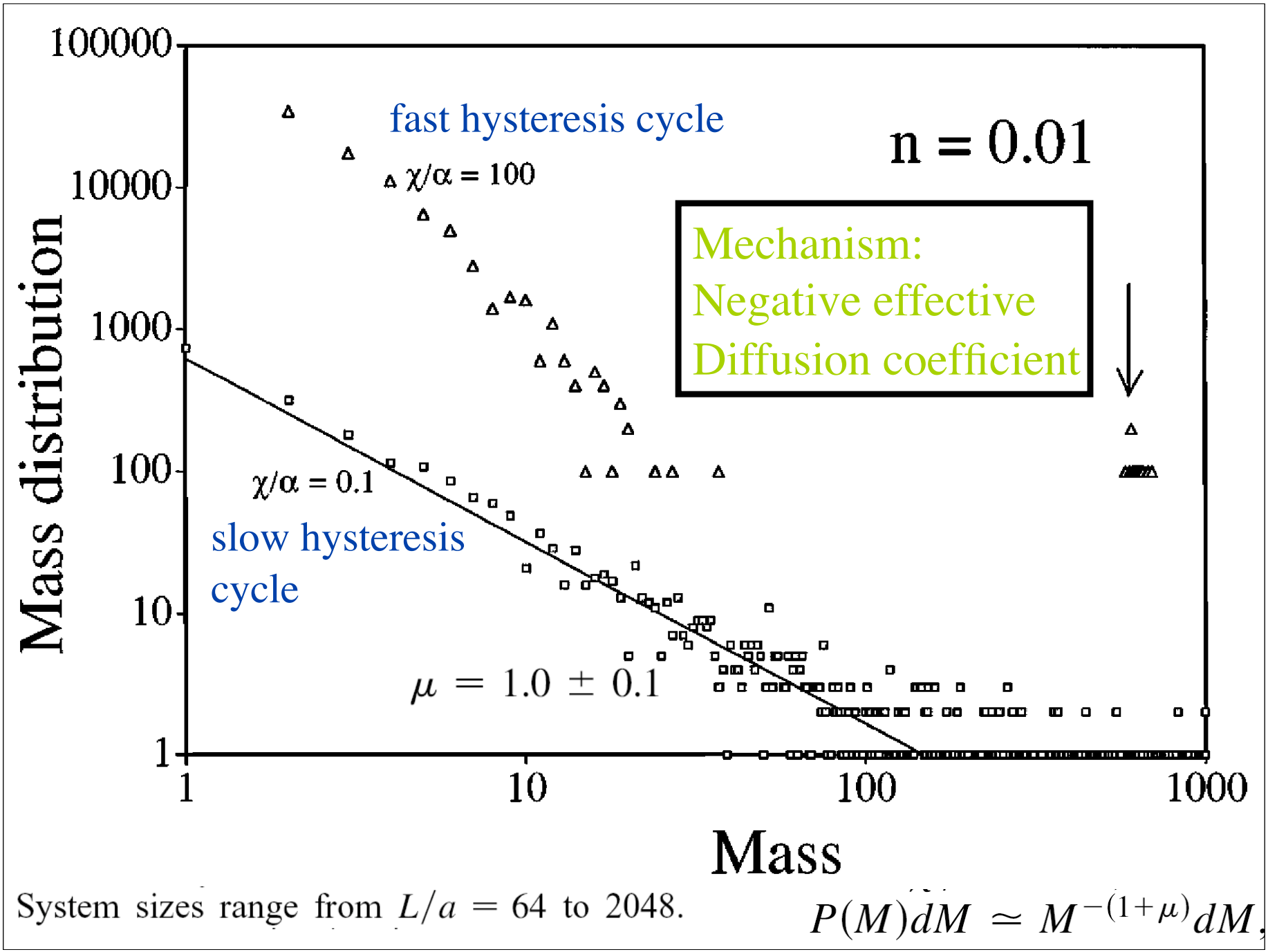
$$\mu = \left[\left(\frac{\partial h}{\partial x} \right)^2 - \left(\frac{\partial h}{\partial x}|_c \right)^2 \right] \qquad (2)$$

and $\beta > 0$ (subcritical condition).

$$F\left(S, \frac{\partial h}{\partial x} \right) = -\alpha \frac{\partial h}{\partial x} S^2, \qquad \alpha > 0$$

Diffusion equation

$$\frac{\partial h}{\partial t} = -\frac{\partial F\left(S, \frac{\partial h}{\partial x}\right)}{\partial x} + \Phi \qquad (3)$$



System sizes range from $L/a = 64$ to 2048.

$$P(M)dM \simeq M^{-(1+\mu)}dM,$$

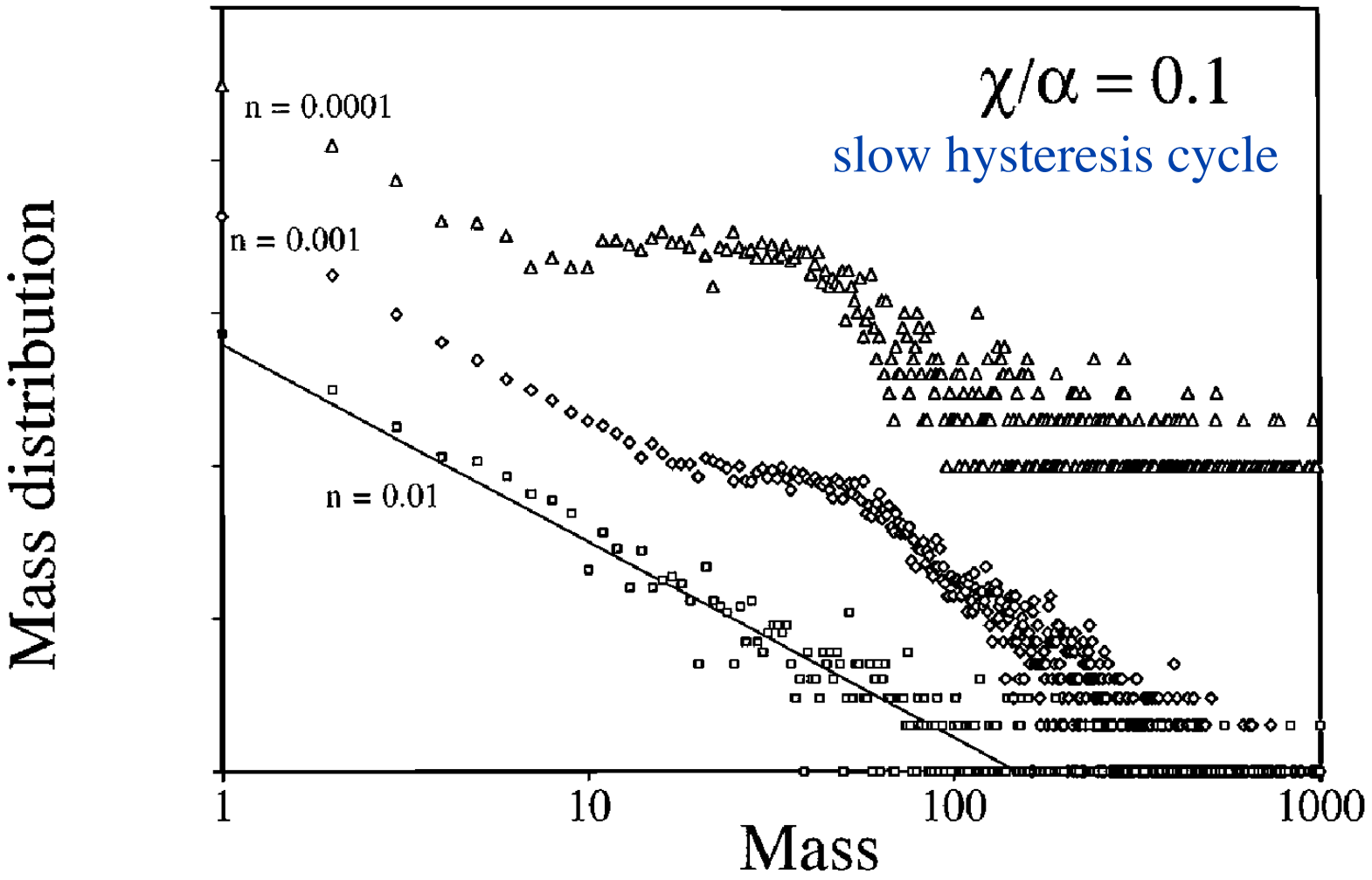


FIG. 2. Distributions $P(M)$ of avalanche sizes for the same $\chi/\alpha = 0.1$ but decreasing values, from bottom to top, of the noise. The curves have been moved with respect to each other for better clarity.

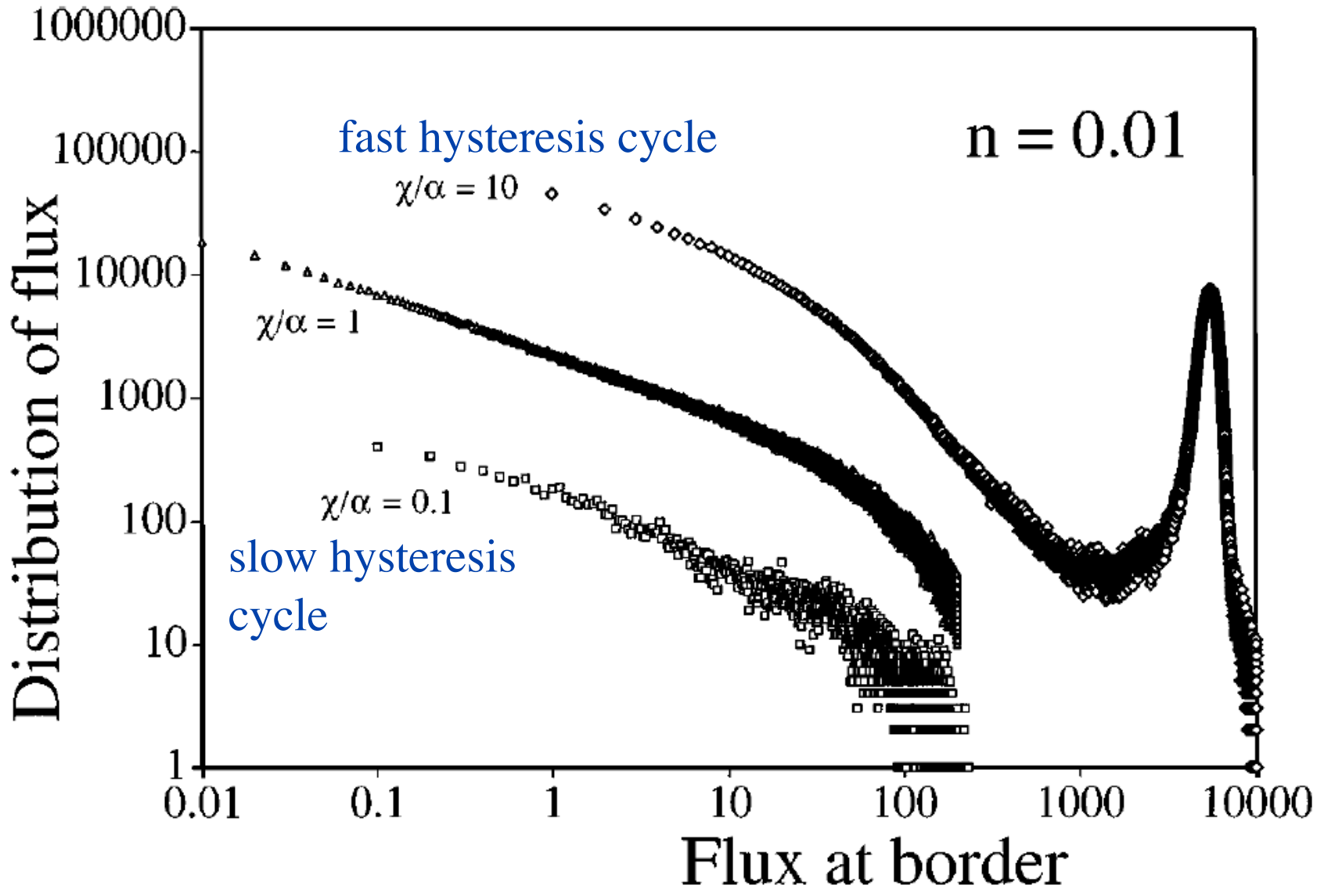
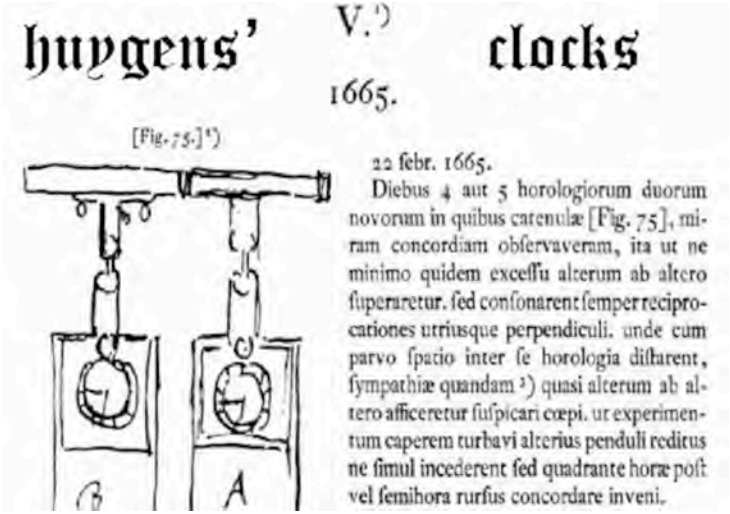


FIG. 3. Distribution $P(J)$ of flux amplitudes at the right border, in the same conditions as for Fig. 1.

SYNCHRONISATION AND COLLECTIVE EFFECTS IN EXTENDED STOCHASTIC SYSTEMS



huygens' V.[1] clocks

1665.

[Fig. 75.][4]

22 febr. 1665.

Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaverim, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3] quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

Fireflies

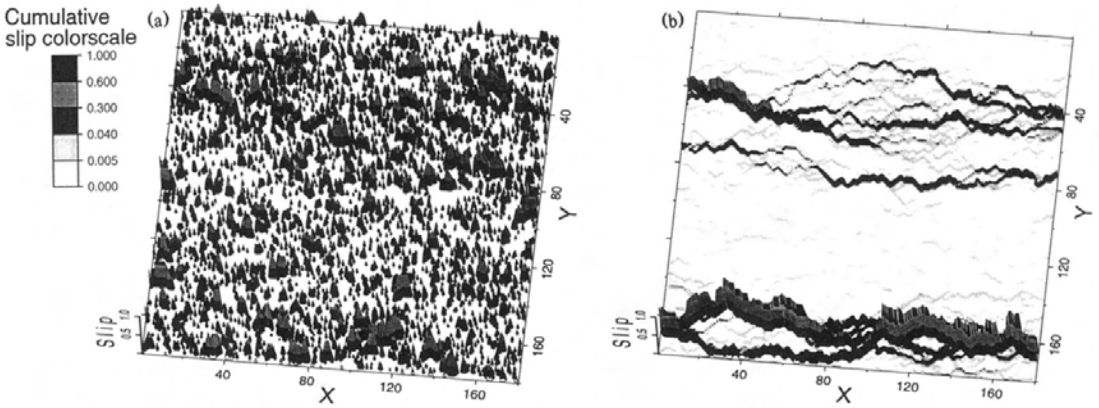
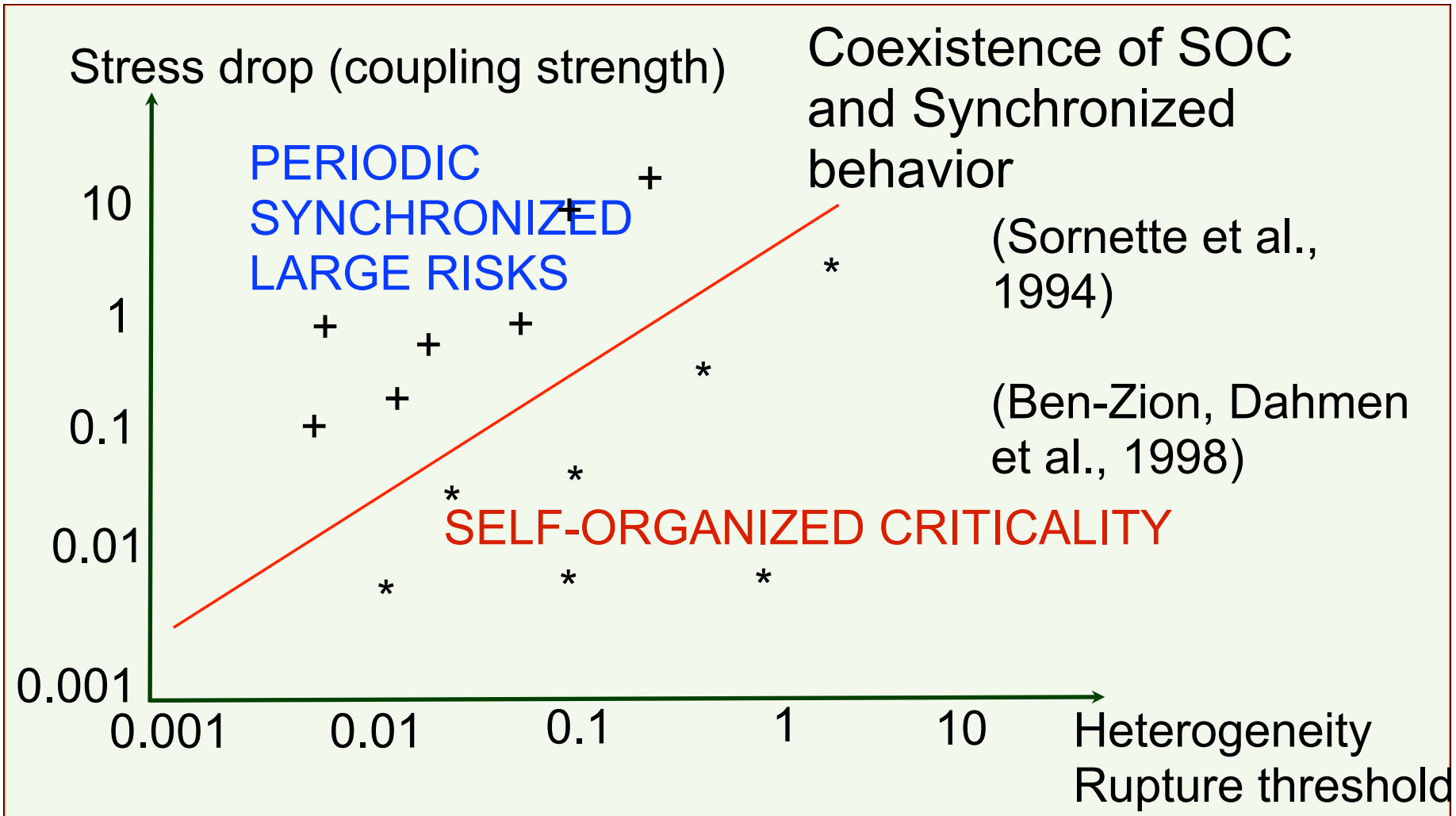


FIG. 1. Evolution of the cumulative earthquake slip, represented along the vertical axis in the white to black color code shown above the picture, at two different times: (a) early time and (b) long time, in a system of size $L=90$ by $L=90$, where $\Delta\sigma=1.9$ and $\beta=0.1$.

Miltenberger et al. (1993)



"Phase diagram" for the model in the space (heterogeneity, stress drop). Crosses (+) correspond to systems which exhibit a periodic time evolution. Stars * corresponds to systems that are self-organized critical, with a Gutenberg-Richter earthquake size distribution and fault localization whose geometry is well-described by the geometry of random directed polymers.

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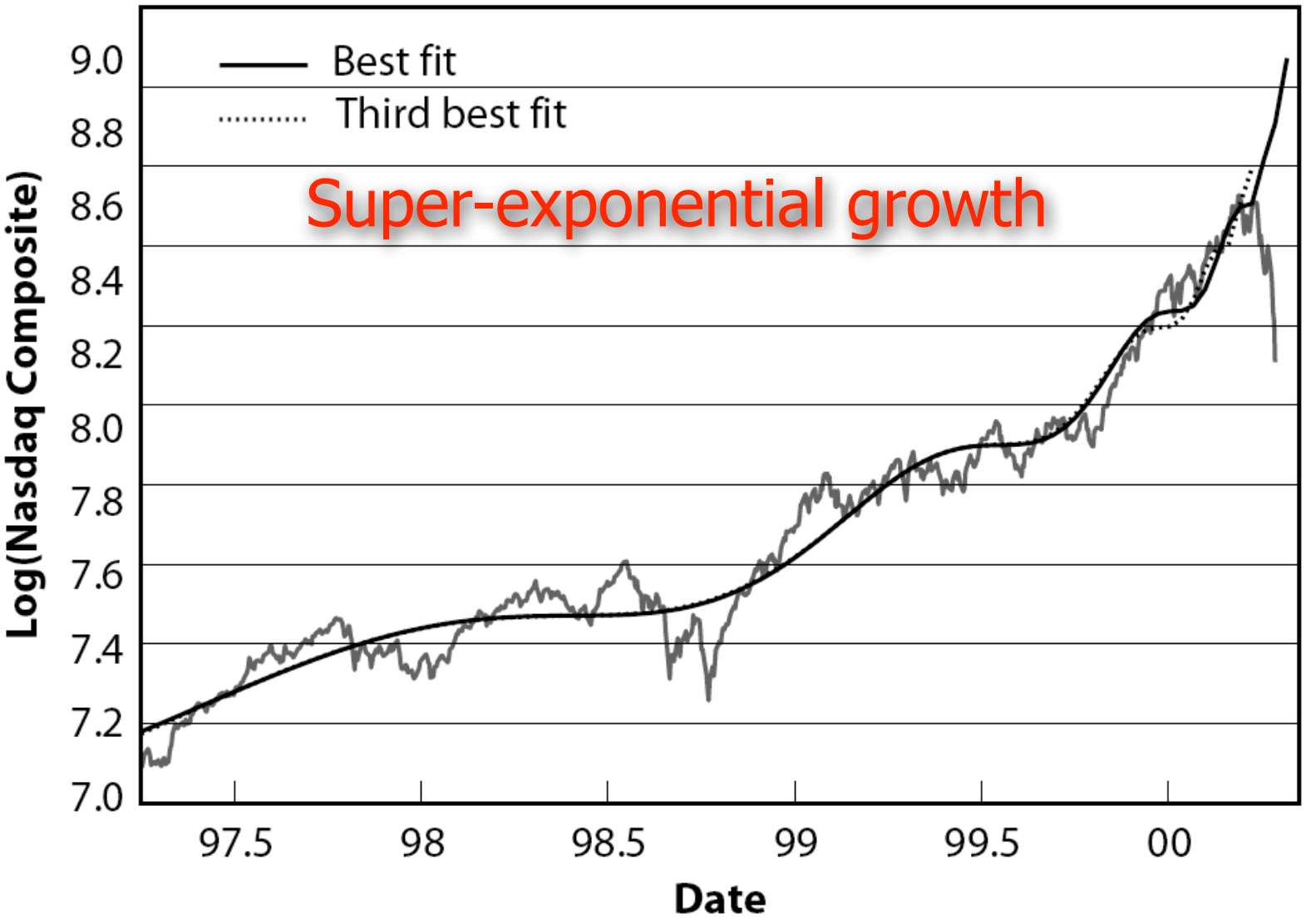
- Commodities and Oil bubbles (2006-2008) Consequences (deep loss of trust, systemic instability)
- Solution? Financial Ratio Index (FRI)

THE NASDAQ CRASH OF APRIL 2000

"New Economy": ICT



Foreign capital inflow

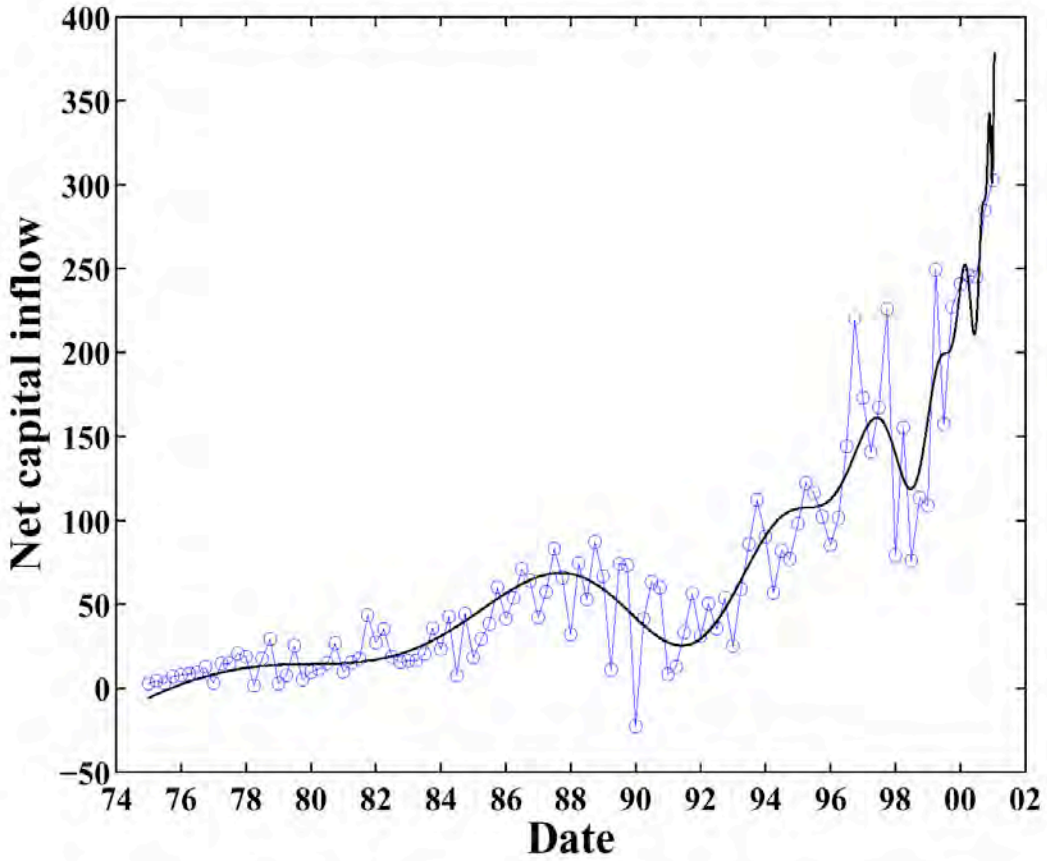


Fig. 2. Fit of the time evolution of the foreign net capital inflow $I(t)$ in the USA from 1975 till the first quarter of 2001 when it reached its maximum, by a second-order Weierstrass-type function given by expression (1). The predicted critical time is $t_c = 2001/03/12$, the power-law exponent is $m = 0.01$, and the angular log-frequency is $\omega = 4.9$. The fitted linear parameters are $A = 7355$, $B = -6719$, $C_1 = 21.5$ and $C_2 = 16.2$. The r.m.s. of the residuals of the fit is 22.810.

EXPECTATIONS of strong future growth

- better business models (small required capital, reduced delay in payments…)
- the network effect (positive returns and positive feedbacks)
- first-to-scale advantages
- real options (value of fast adaptation to grasp new opportunities)

Probably true… but problem of timing…

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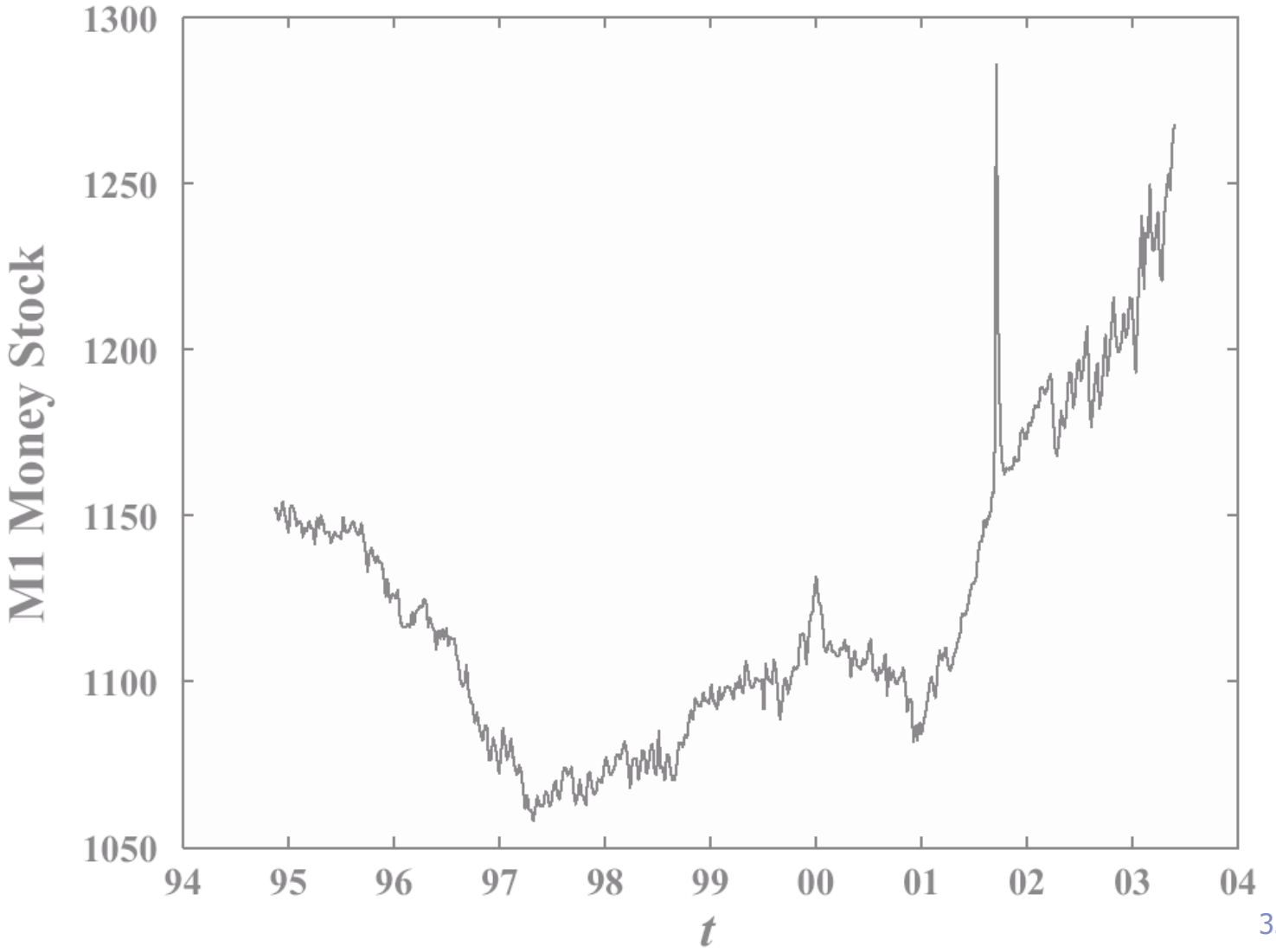
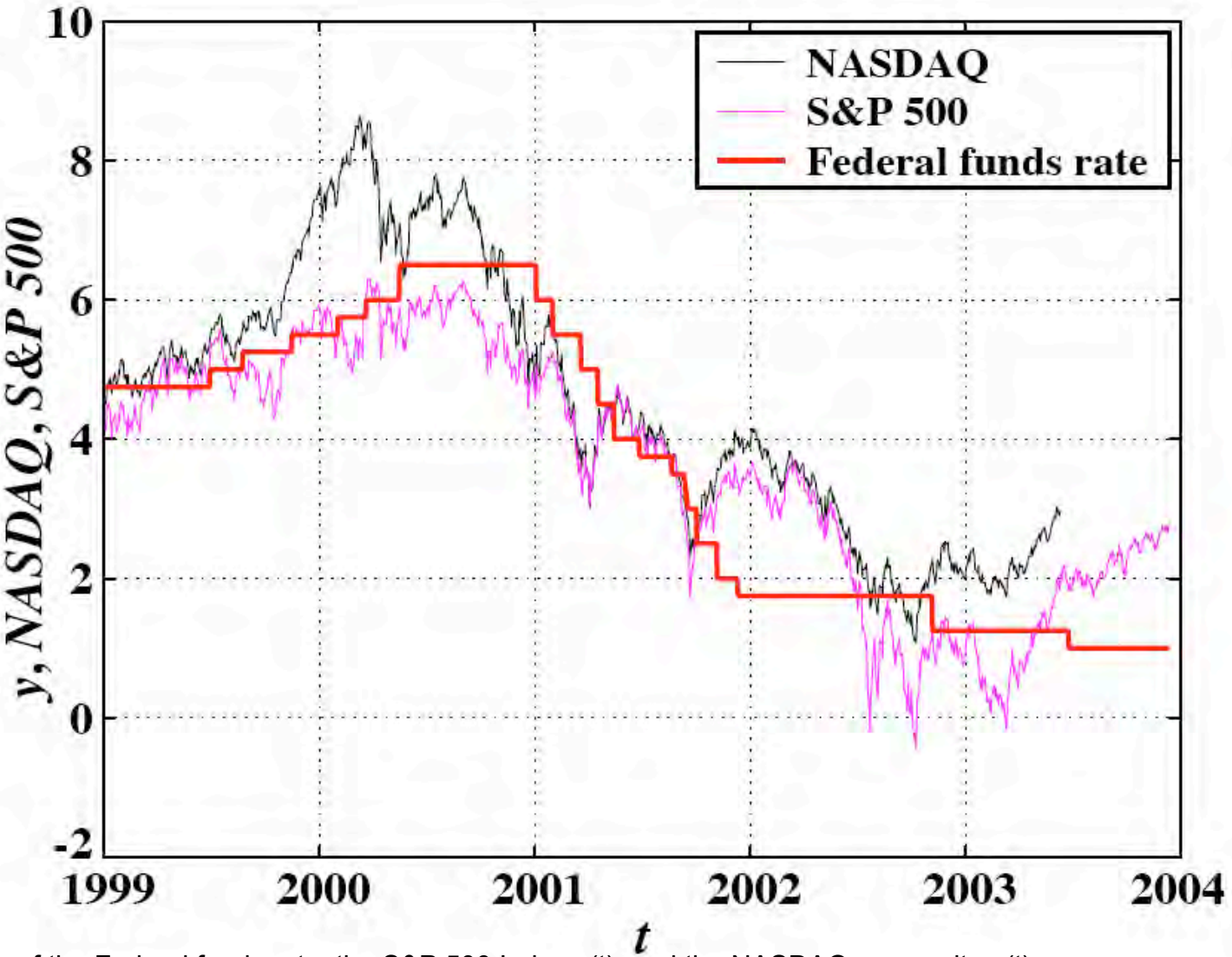
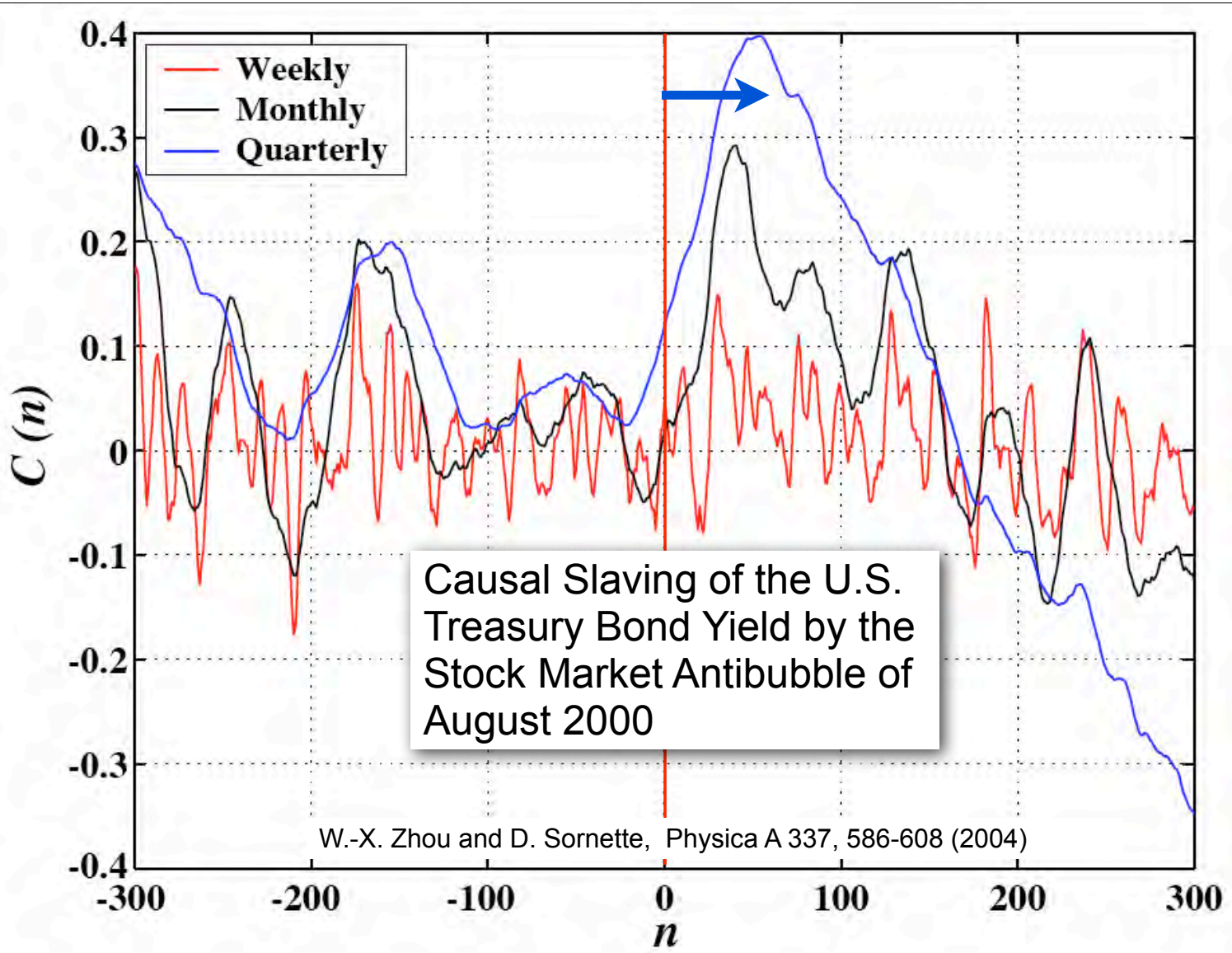
Growth of Money supply (M1)



"SLAVING OF THE FED TO THE STOCK MARKET



Comparison of the Federal funds rate, the S&P 500 Index x(t), and the NASDAQ composite z(t), from 1999 to mid-2003. To allow a illustrative visual comparison, the indices have been translated and scaled as follows: $x \to 5x - 34$ and $z \to 10z - 67$.



Cross-correlation coefficient C(n) between the increments of the logarithm of the S&P 500 Index and the increments of the Federal funds rate as a function of time lag n in days. The three curves corresponds to three different time steps used to calculate the increments: weekly, monthly and quarterly. A positive lag n corresponds to having the Federal funds rate posterior to the stock market.

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Real-estate bubbles

Sources: Shiller; BIS.

Real-estate in the UK

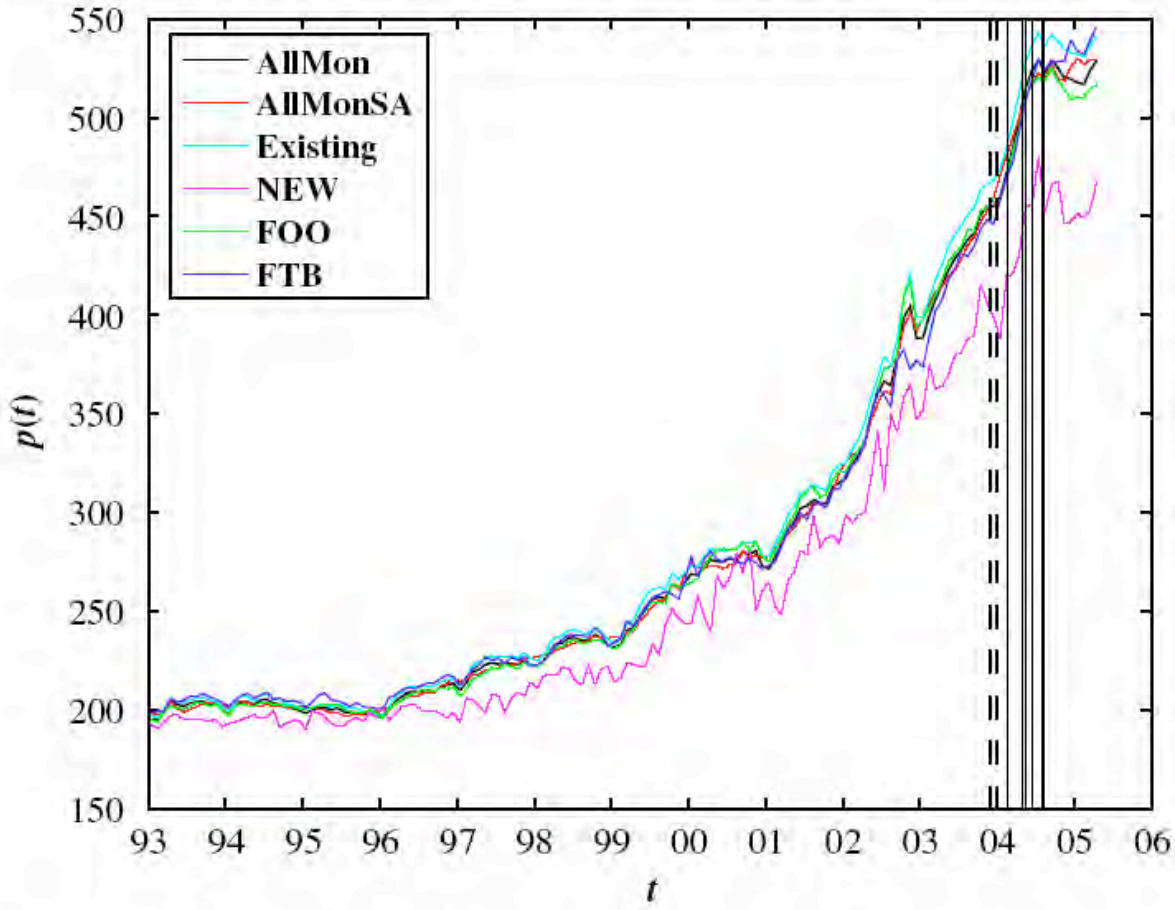


Fig. 1. (Color online) Plot of the UK Halifax house price indices from 1993 to April 2005 (the latest available quote at the time of writing). The two groups of vertical lines correspond to the two predicted turning points reported in Tables 2 and 3 of [1]: end of 2003 and mid-2004. The former (resp. later) was based on the use of formula (2) (resp. (3)). These predictions were performed in February 2003.

W.-X. Zhou, D. Sornette, 2000–2003 real estate bubble in the UK but not in the USA, Physica A 329 (2003) 249–263.

Real-estate in the USA

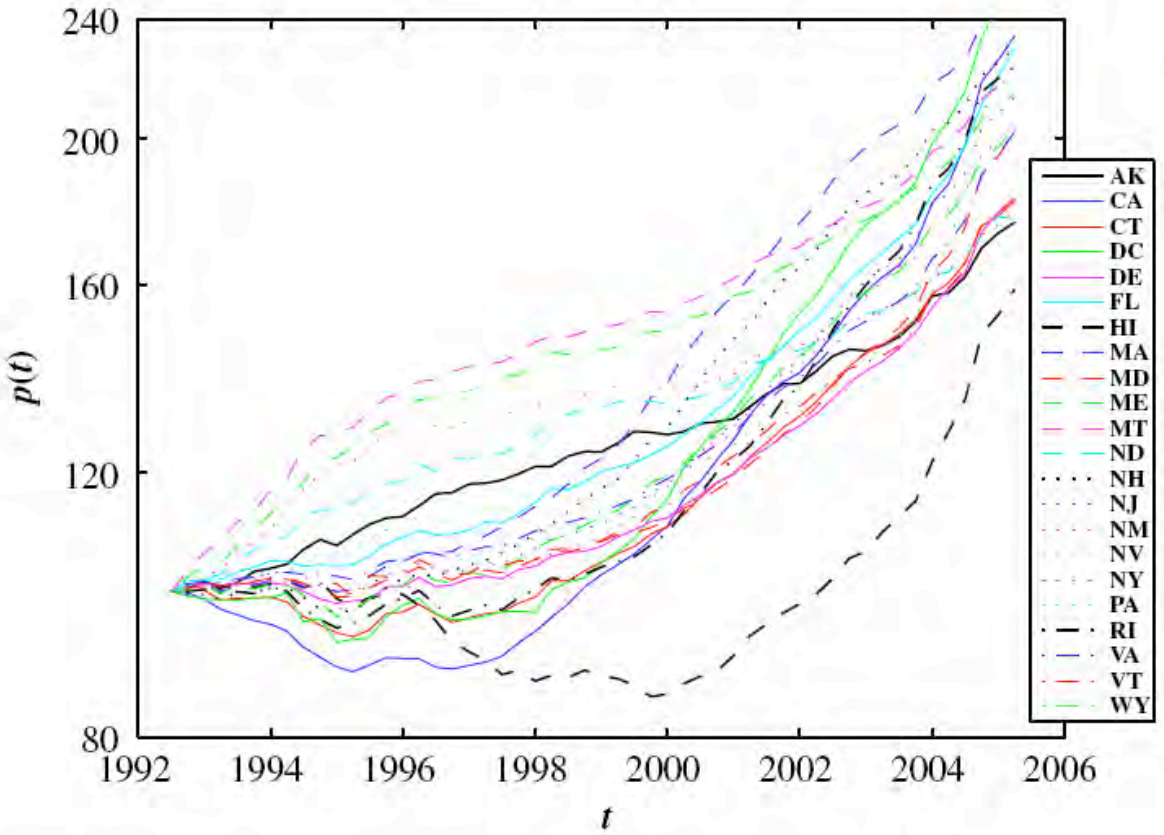
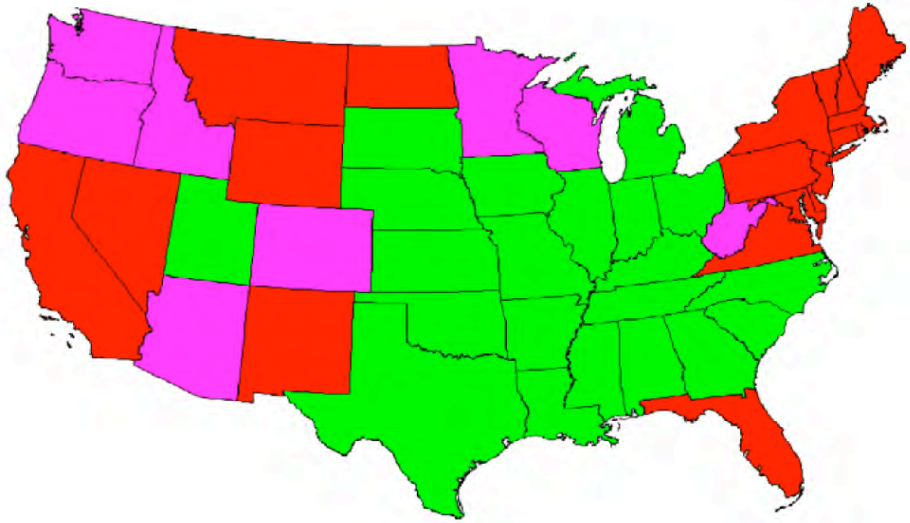


Fig. 5. (Color online) Quarterly average HPI in the 21 states and in the District of Columbia (DC) exhibiting a clear upward faster-than-exponential growth. For better representation, we have normalized the house price indices for the second quarter of 1992 to 100 in all 22 cases. The corresponding states are given in the legend.

W.-X. Zhou, D. Sornette / Physica A 361 (2006) 297–308

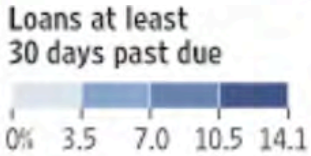
Our study in 2005 identifies the bubble states



Fall 2007

Hammered

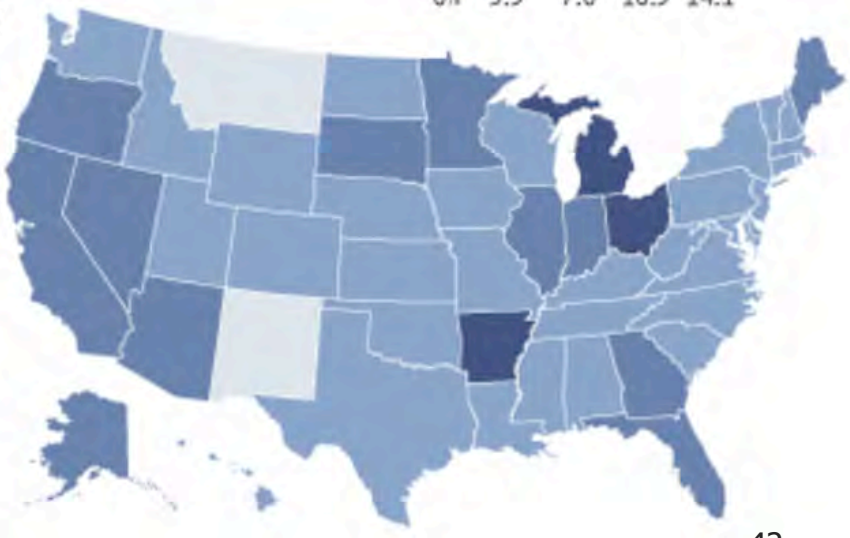
Delinquency rates for construction loans for single-family homes.

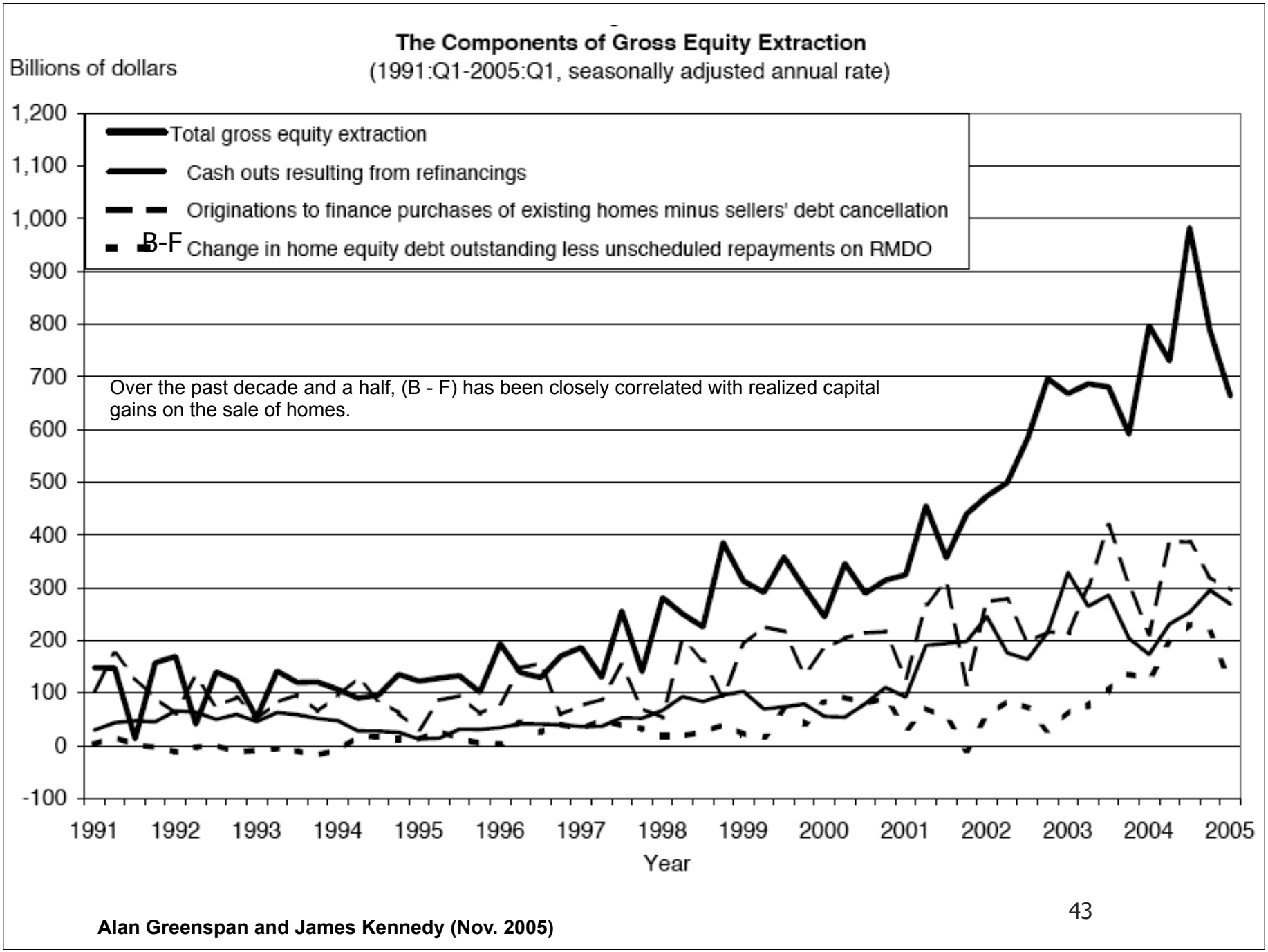


Fourth quarter 2007

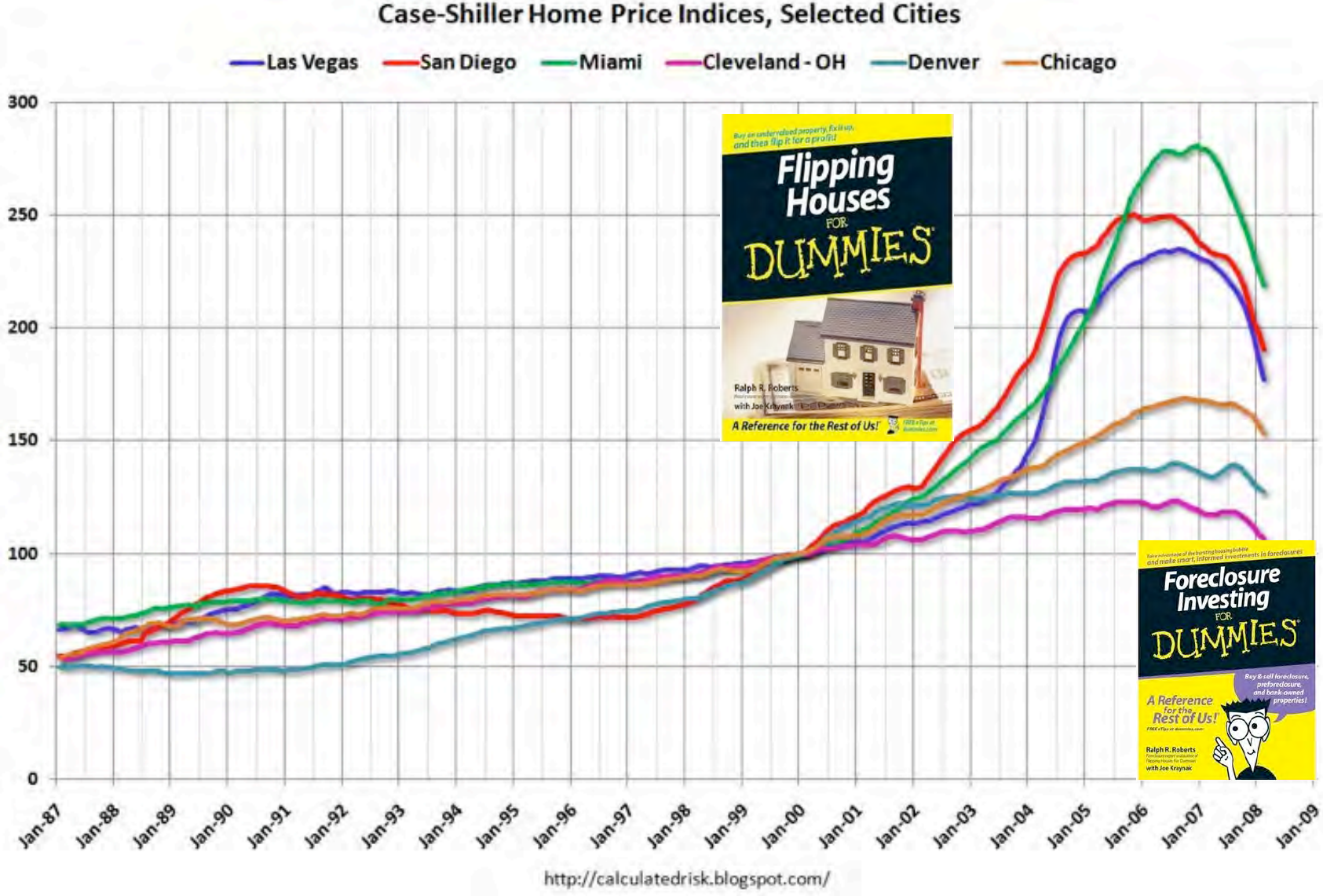
Highest rates

Michigan	14.0%
Ohio	13.7
Arkansas	11.1
Arizona	10.3
Minnesota	10.0
Florida	9.9
Georgia	9.6
South Dakota	9.2





Alan Greenspan and James Kennedy (Nov. 2005)



This graph shows the year-over-year price changes for the Case-Shiller composite 10 and 20 indices (through February), and the Case-Shiller and OFHEO National price indices (through Q4 2007).

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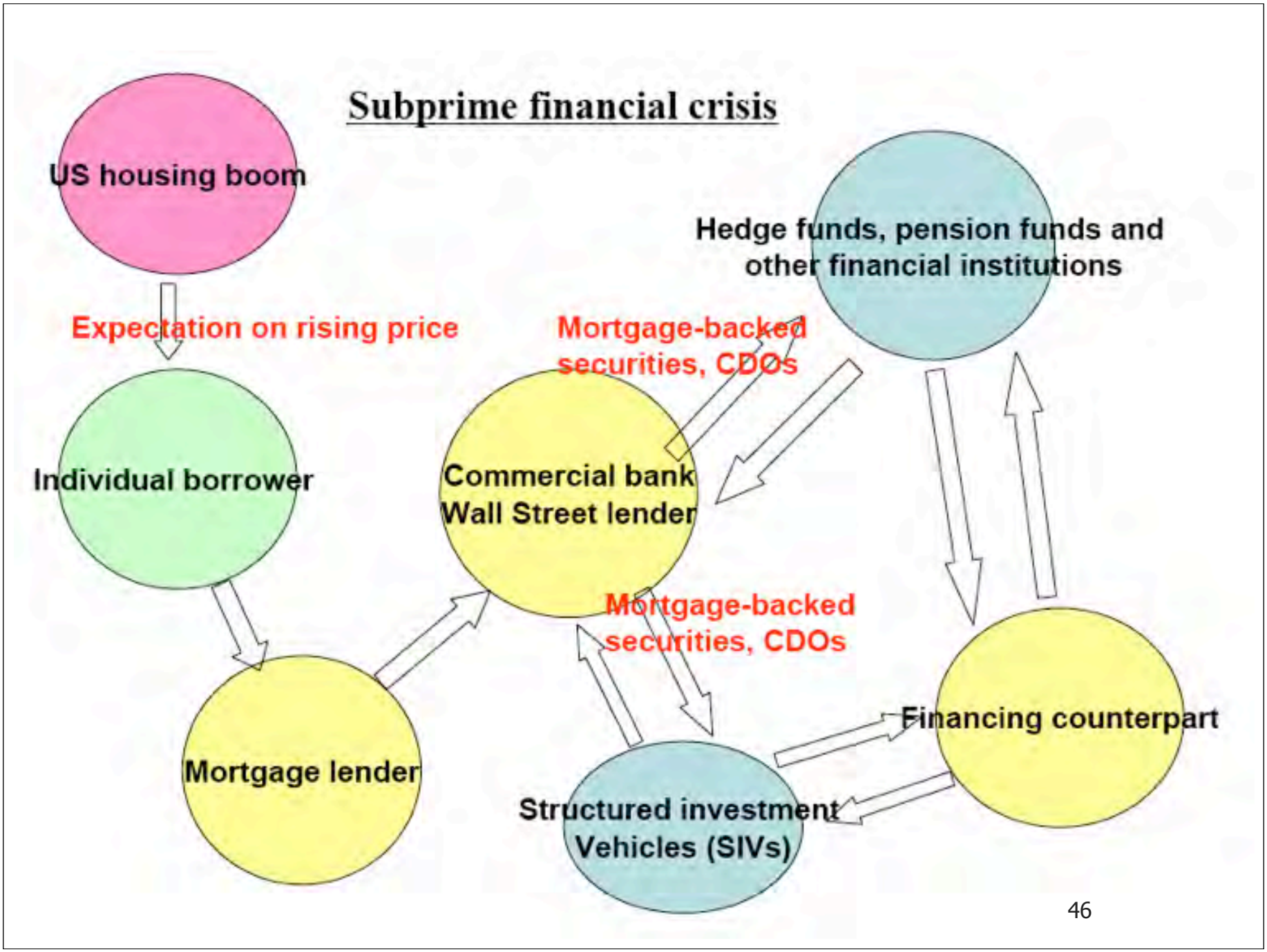
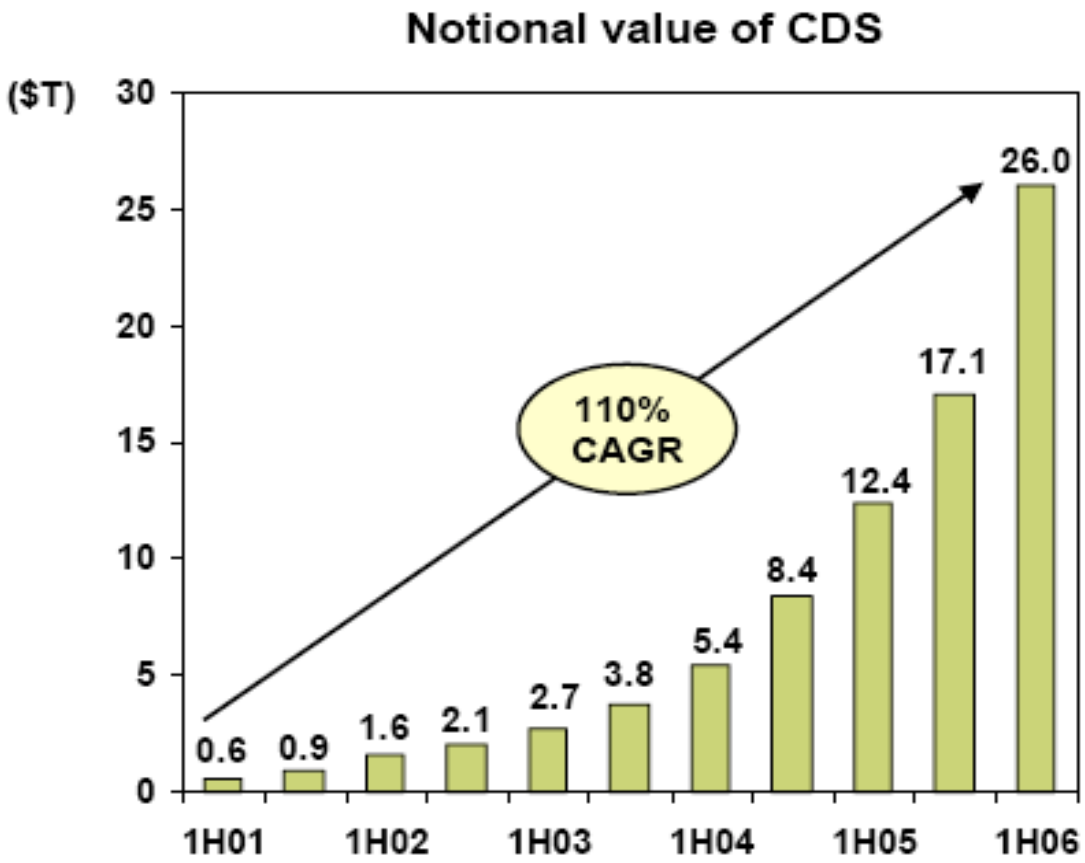
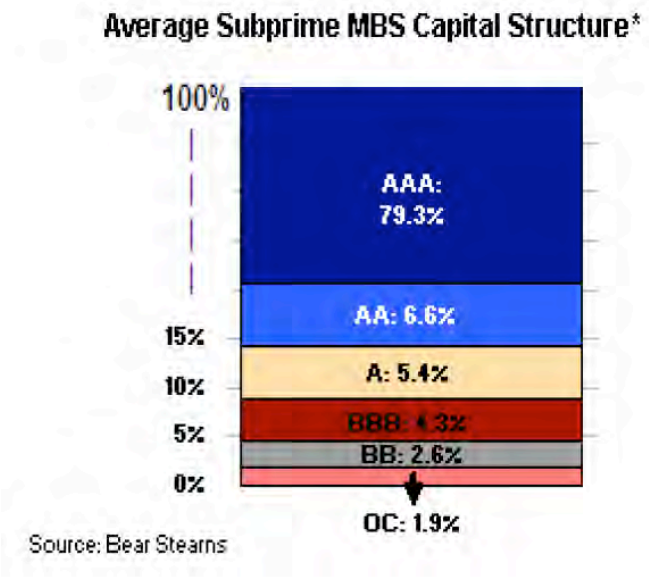
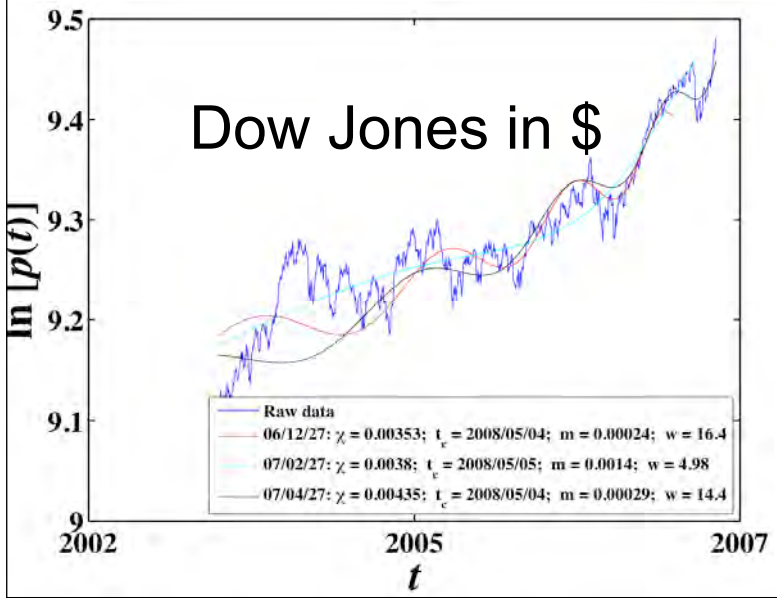

Subprime financial crisis
US housing boom
Hedge funds, pension funds and other financial institutions
Expectation on rising price
Mortgage-backed securities, CDOs
Individual borrower
Commercial bank Wall Street lender
Mortgage-backed securities, CDOs
Financing counterpart
Mortgage lender
Structured investment Vehicles (SIVs)

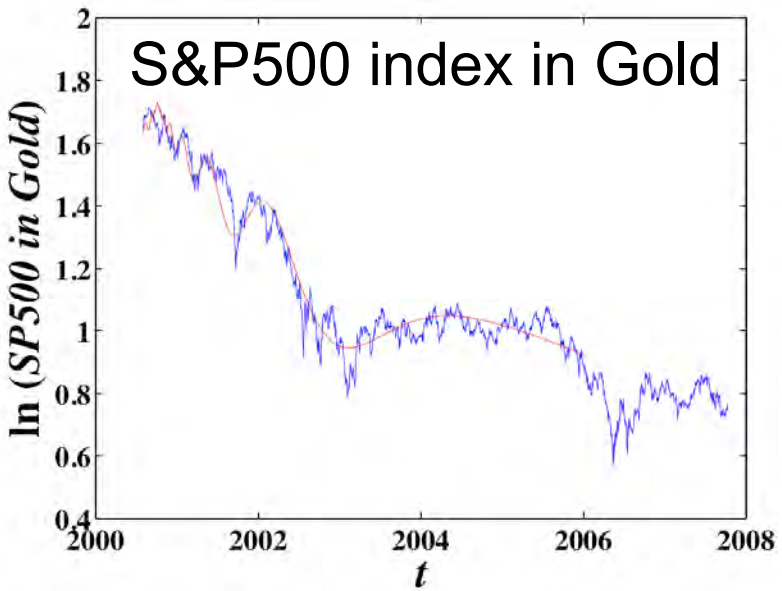
Securitization of non-financial assets (commodities, real-estate, credit)






S&P500 index in $
1500
1250
1000
source: Bloomberg
2004 May Sep 2005 May Sep 2006 May Sep 2007 May Sep 2008 May


Dow Jones in $
ln [p(t)]
9.5
9.4
9.3
9.2
9.1
9
2002
2005
2007
t
Raw data
06/12/27: χ = 0.00353; t_c = 2008/05/04; m = 0.00024; w = 16.4
07/02/27: χ = 0.0038; t_c = 2008/05/05; m = 0.0014; w = 4.98
07/04/27: χ = 0.00435; t_c = 2008/05/04; m = 0.00029; w = 14.4


S&P500 index in Gold
ln (SP500 in Gold)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
2000
2002
2004
2006
2008
t

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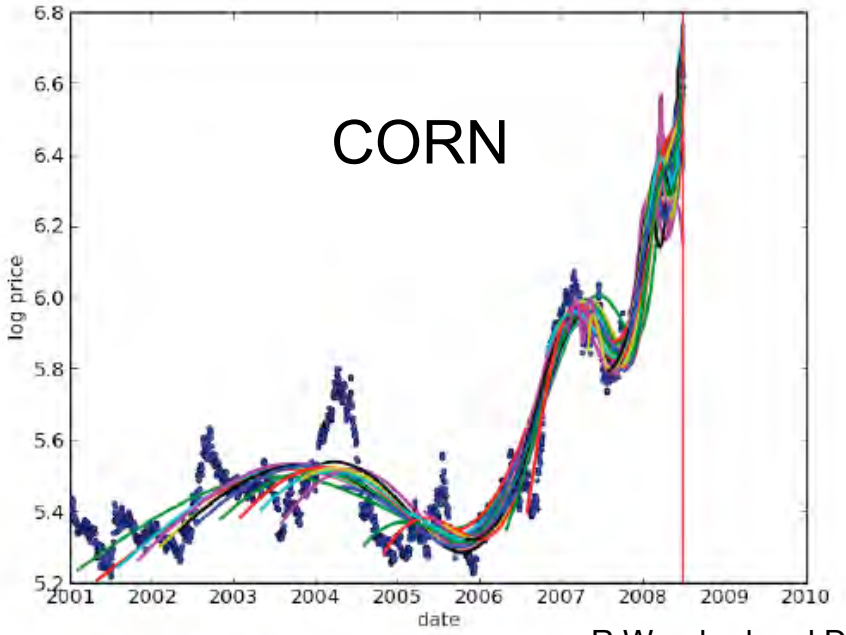
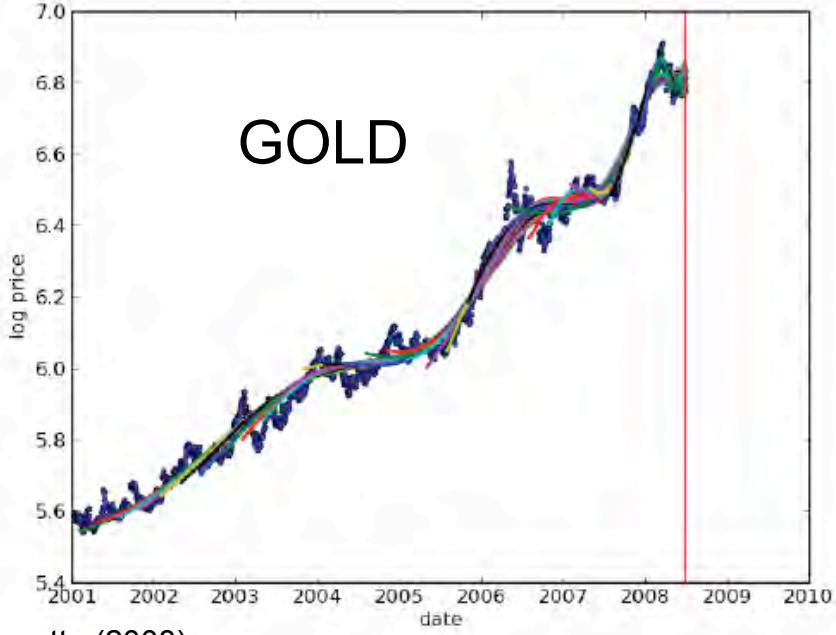
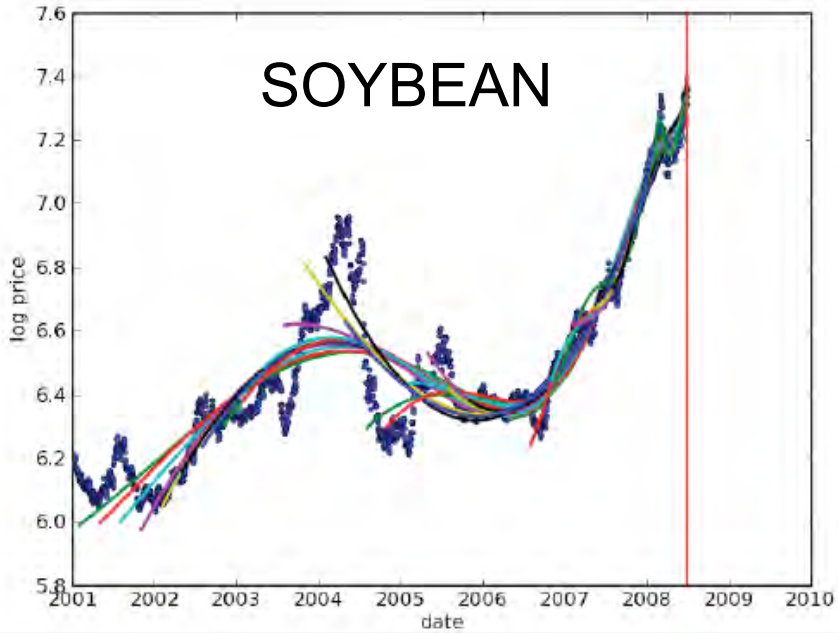
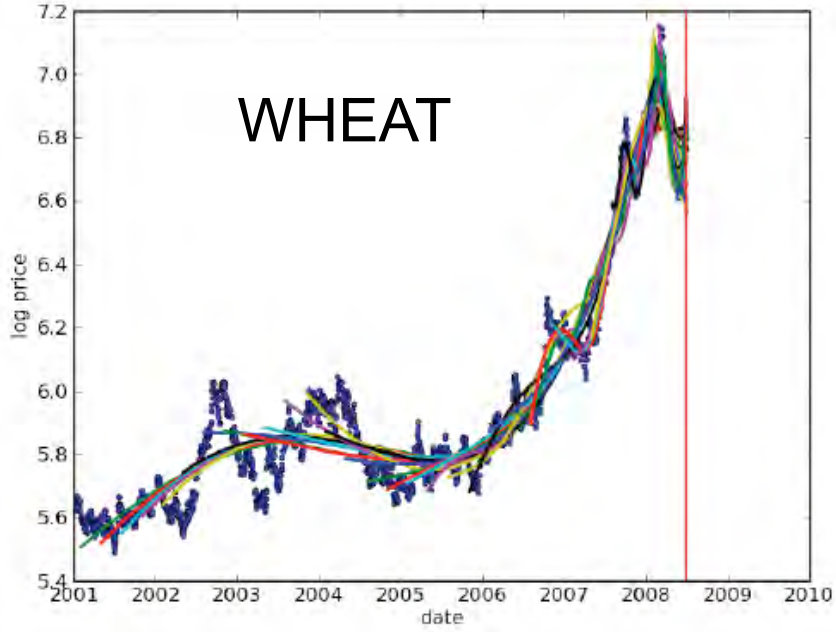
source: R. Woodard

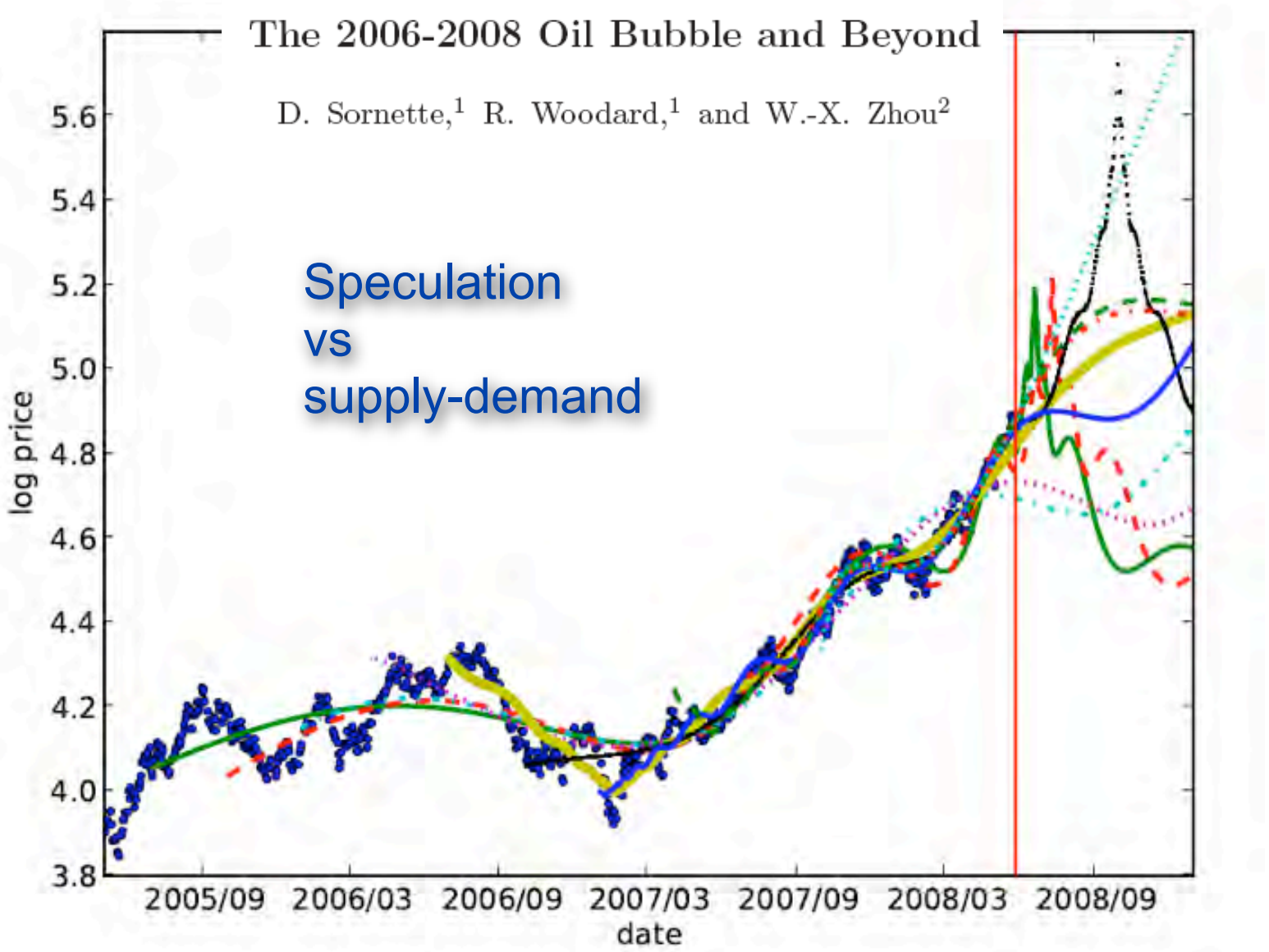




R.Woodard and D.Sornette (2008)







Typical result of the calibration of the simple LPPL model to the oil price in US$ in shrinking windows with starting dates t_{start} moving up towards the common last date t_{last} = May 27, 2008.

In summary

- Each excess was partially “solved” by the subsequent excess... leading to a succession of
  - -unsustainable wealth growth
  - -instabilities

- The present crisis+recession is the consolidation after this series of unsustainable excesses.

- One could conclude that the extraordinary severity of this crisis is not going to be solved by the same implicit or explicit “bubble thinking”.

"The problems that we have created cannot be solved at the level of thinking that created them." Albert Einstein

Recession-Plagued Nation Demands New Bubble To Invest In

The Onion, JULY 14, 2008 | ISSUE 44•29

A prominent finance expert asks Congress to help Americans rebuild their ficticious dreams.

"Every American family deserves a false sense of security," said Chris Reppto, a risk analyst for Citigroup in New York. "Once we have a bubble to provide a fragile foundation, we can begin building pyramid scheme on top of pyramid scheme, and before we know it, the financial situation will return to normal."

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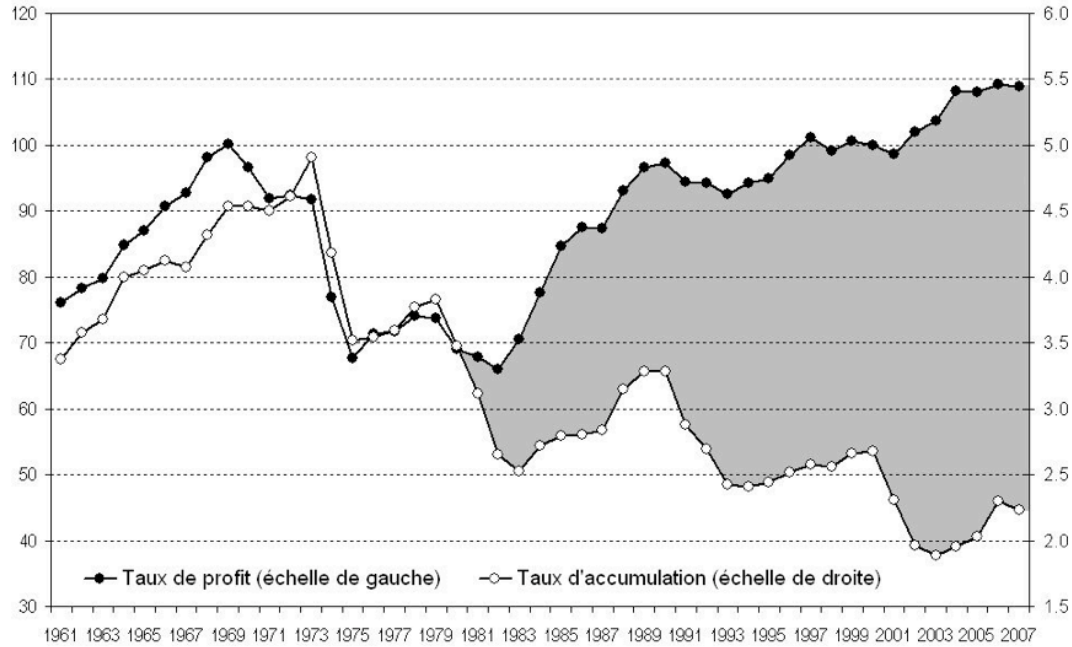
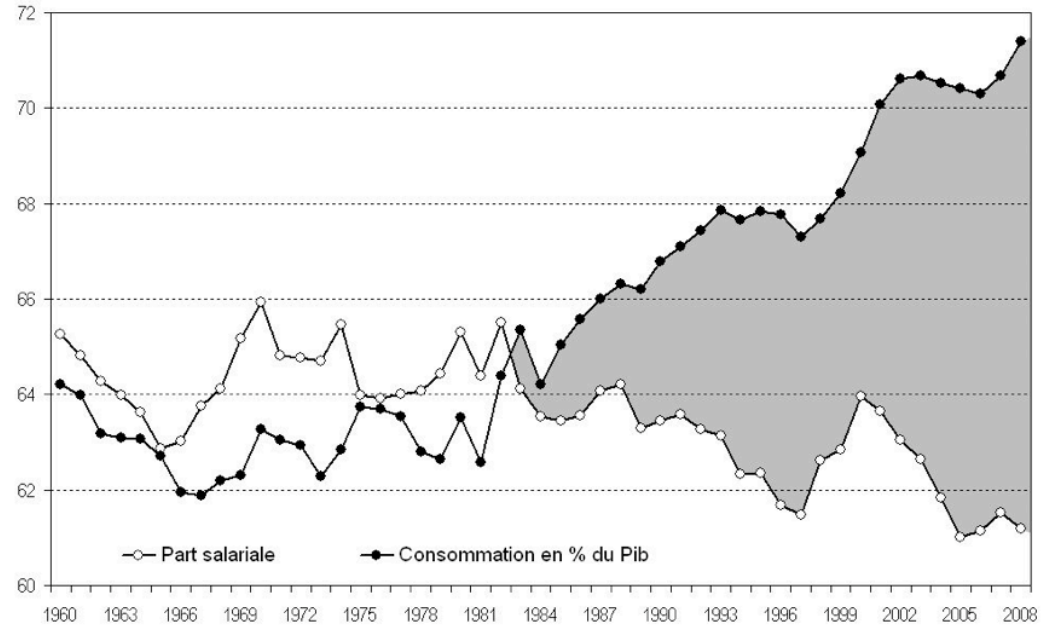


Rate of profit and rate of accumulation: The United States + European Union + Japan

* Rate of accumulation = rate of growth rate of the net volume of capital * Rate of profit = profit/ capital (base: 100 in 2000)

Sources and data of the graphs: http://hussonet.free.fr/toxicap.xls

Thee gap widens between the share of wages and the share of consumption (gray zones), so as to compensate for the difference between profit and accumulation. FINANCE allows increasing debt and virtual wealth growh... which can only be transitory (even if very long).



United States Share of wages and of private consumption in Gross Domestic Product (GDP)

Source of data and graphics: http://hussonet.free.fr/toxicap.xls

- An economy which grows at 2 or 3 per cent cannot provide a universal profit of 15 per cent, as some managers of equities claim and many investors dream of.

- As long as the incomes drawn from financial assets are re-invested, the fortunes increase independently of any material link with the real sphere and the variation can potentially become infinite.

- Financial assets represent the right to a share of the surplus value that is produced. As long as this right is not exercised, it remains virtual. But as soon as anyone exercises it, they discover that it is subject to the law of value, which means, quite simply, that you cannot distribute more real wealth than is produced.

We are witnessing a fundamental reassessment of the value of virtually every asset everywhere in the world.

- Intelligence of the crowd: general loss of trust can be restored by removing uncertainty through frank clarification
- Fight moral hazard (ex: clawback permission...)
- Regulations (illusion of control and the law of unintended consequences)
- Development of culture of integrity and ethical behavior (informed by behavioral psychology)
- “Robust Investment” approach (W. Buffet)
- The overgrowth of the “financial economy” versus the “real economy”
- Financial Ratio Index (FRI) (total fixed assets + working capital, excess supply of money...)