Generic multifractality in exponentials of long memory processes

- Multifractality in the time domain: conditional response functions
- Financial volatility
- Robust multifractality in exponential of long-memory processes
- Multifractal Omori law for the relaxation of earthquake aftershocks

Didier Sornette
Chair of Entrepreneurial Risks, ETH-Zurich
(Swiss Federal Institute of Technology, Zurich)
Department of Management, Technology and Economics
http://www.er.ethz.ch/

with G. Ouillon (Nice) and A. Saichev (Nizhny-Novgorov)

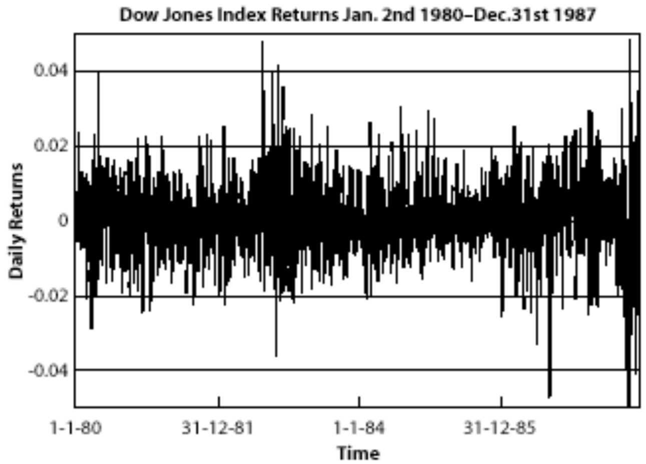
$$\delta_\tau X(t) = \int_{t-\tau}^{t} \mu(t')dt', \quad \text{with } \mu(t) = \kappa e^{\omega(t)},$$

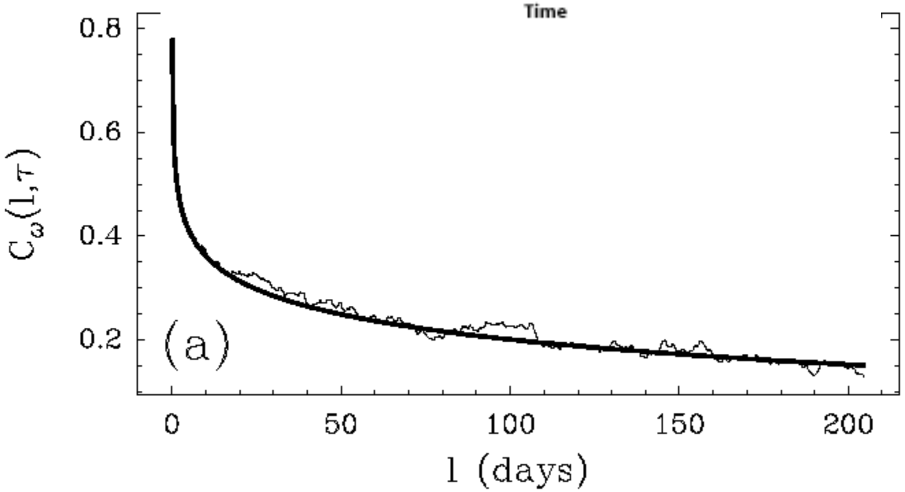
$$\omega(t) = \int_{-\infty}^{t} dW(t')h(t-t'),$$

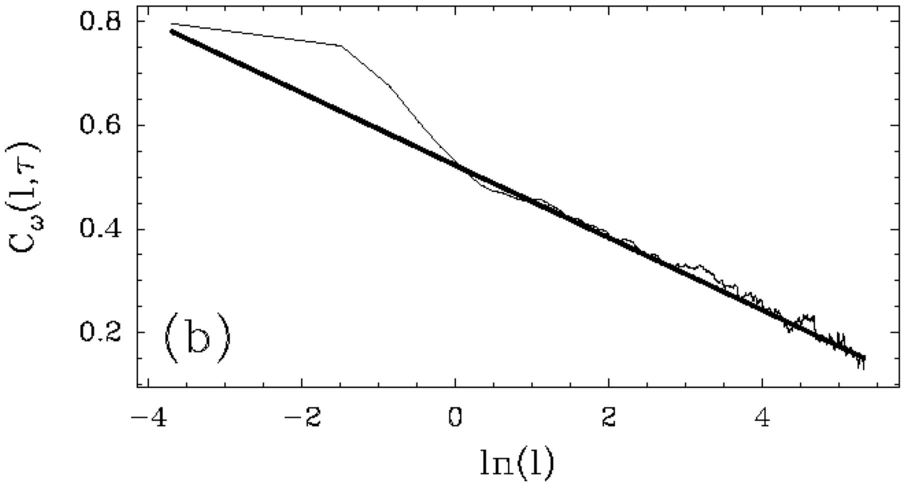
$$h(t) = \frac{h_0}{(1+x)^{\varphi+1/2}} H(t), \quad x = t/\ell,$$

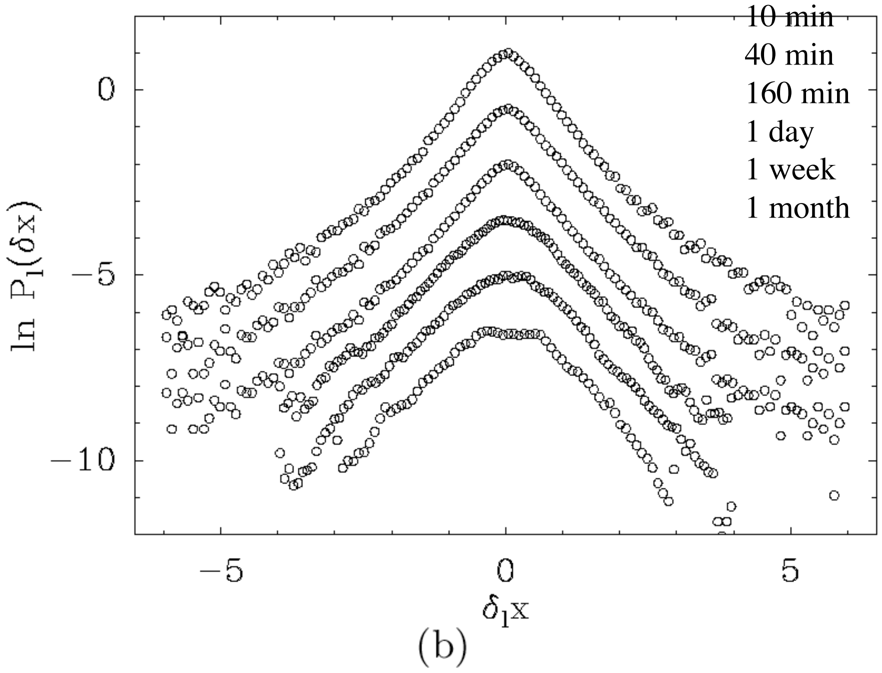
A. Saichev and D. Sornette, Generic Multifractality in Exponentials of Long Memory Processes, Physical Review E 74, 011111 (2006)

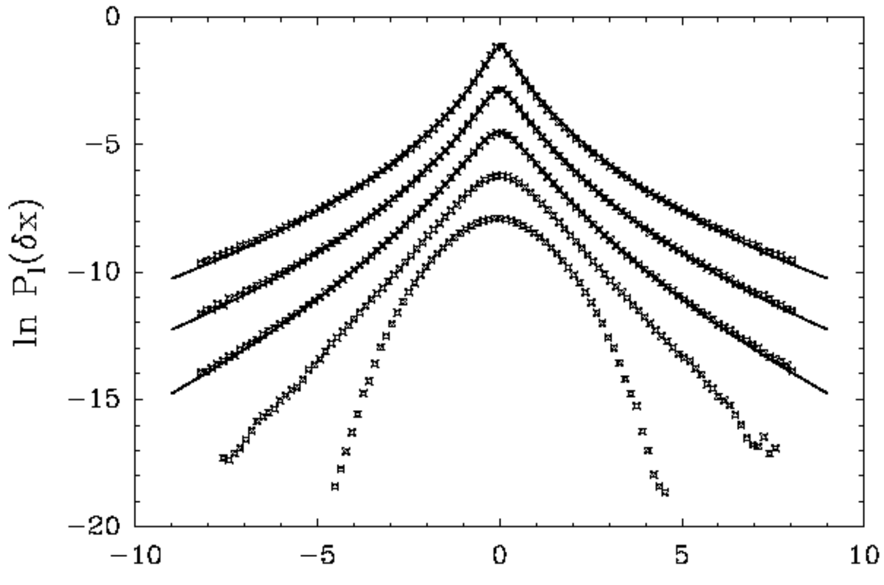
Stylized facts in financial markets





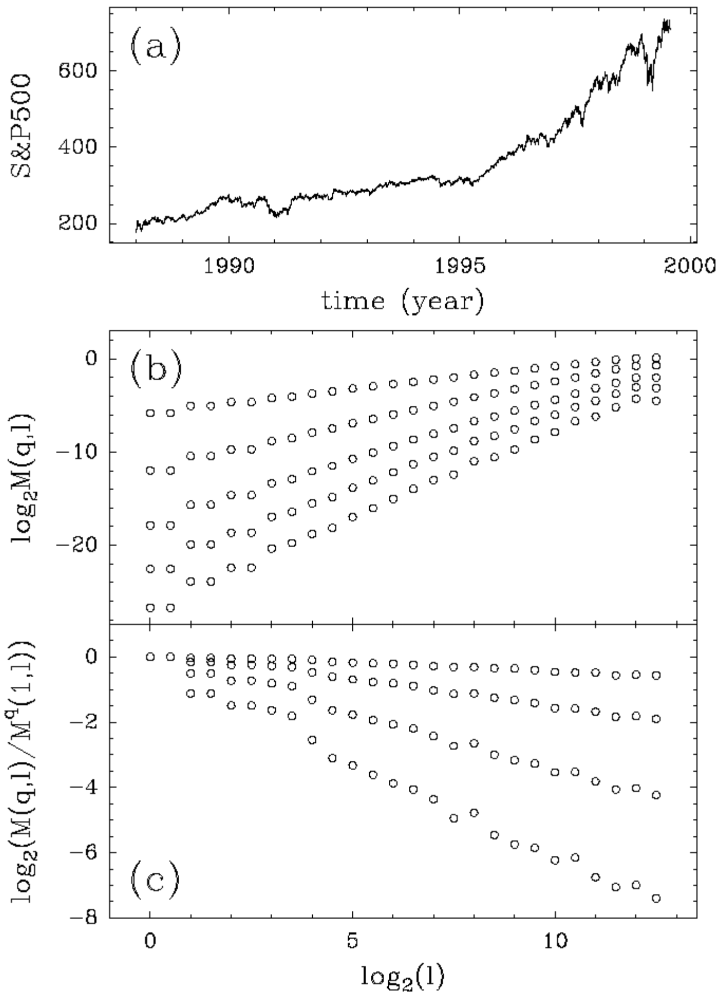






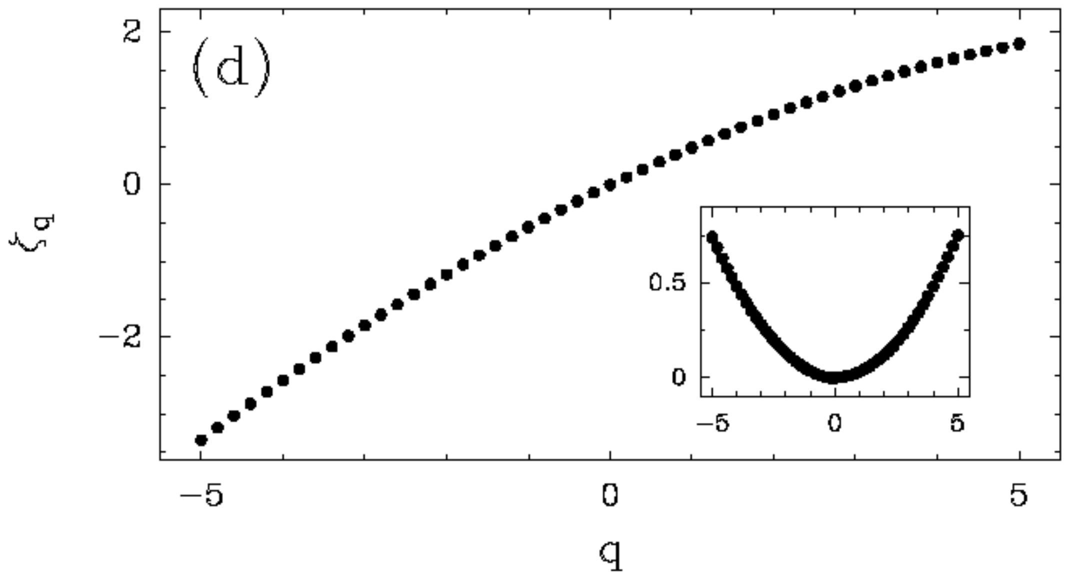
Multifractal random walk

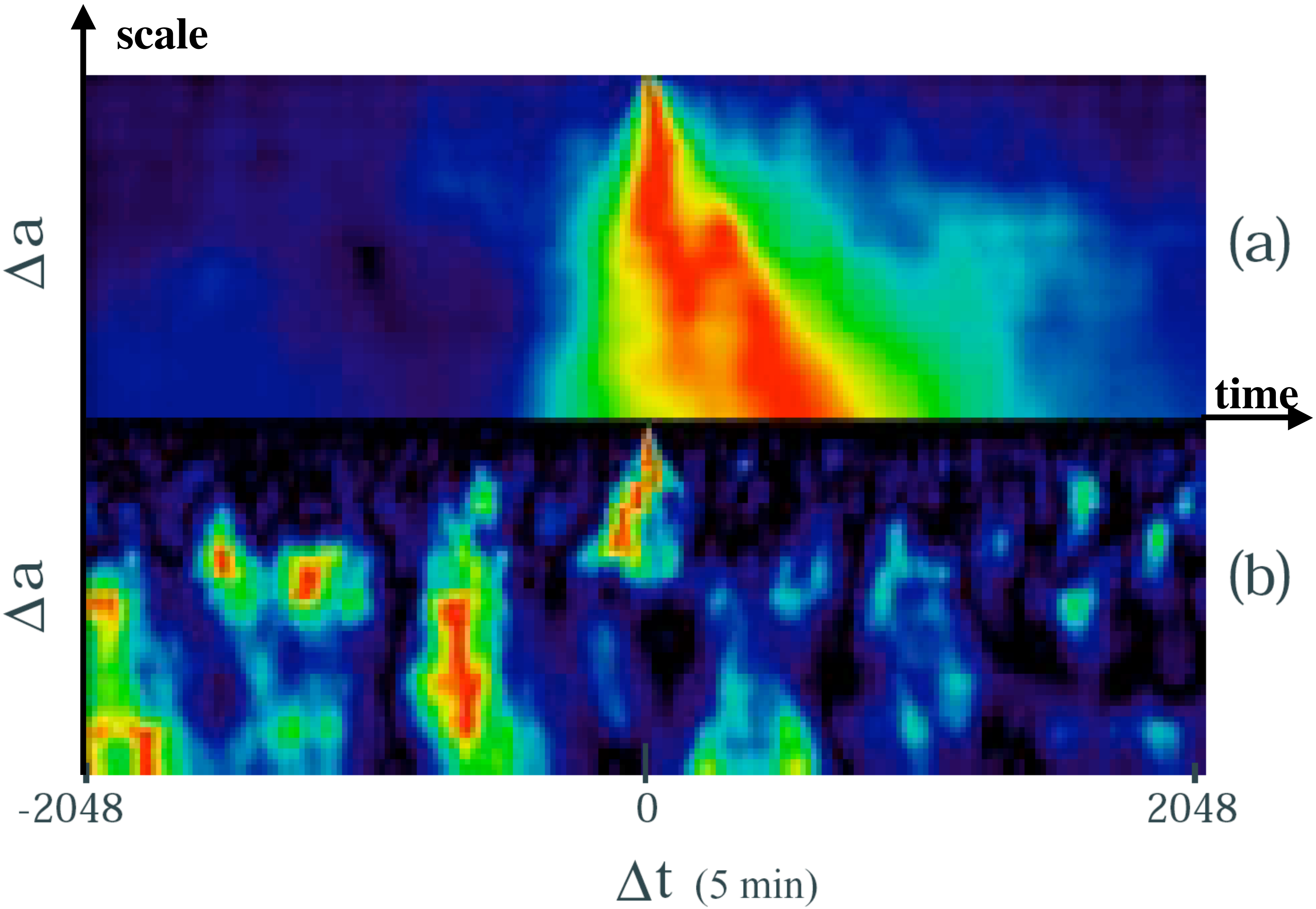
Multifractality: $\langle [\delta_\tau X(t)]^q \rangle = a(q)\,\tau^{\zeta(q)}, \quad \text{for } \tau < T.$



$$\zeta(q) = \left(1 + \frac{\lambda^2}{2}\right) q - \frac{\lambda^2}{2} q^2$$

$\lambda^2 = -\zeta''(0)$ is the so-called *intermittency coefficient*.



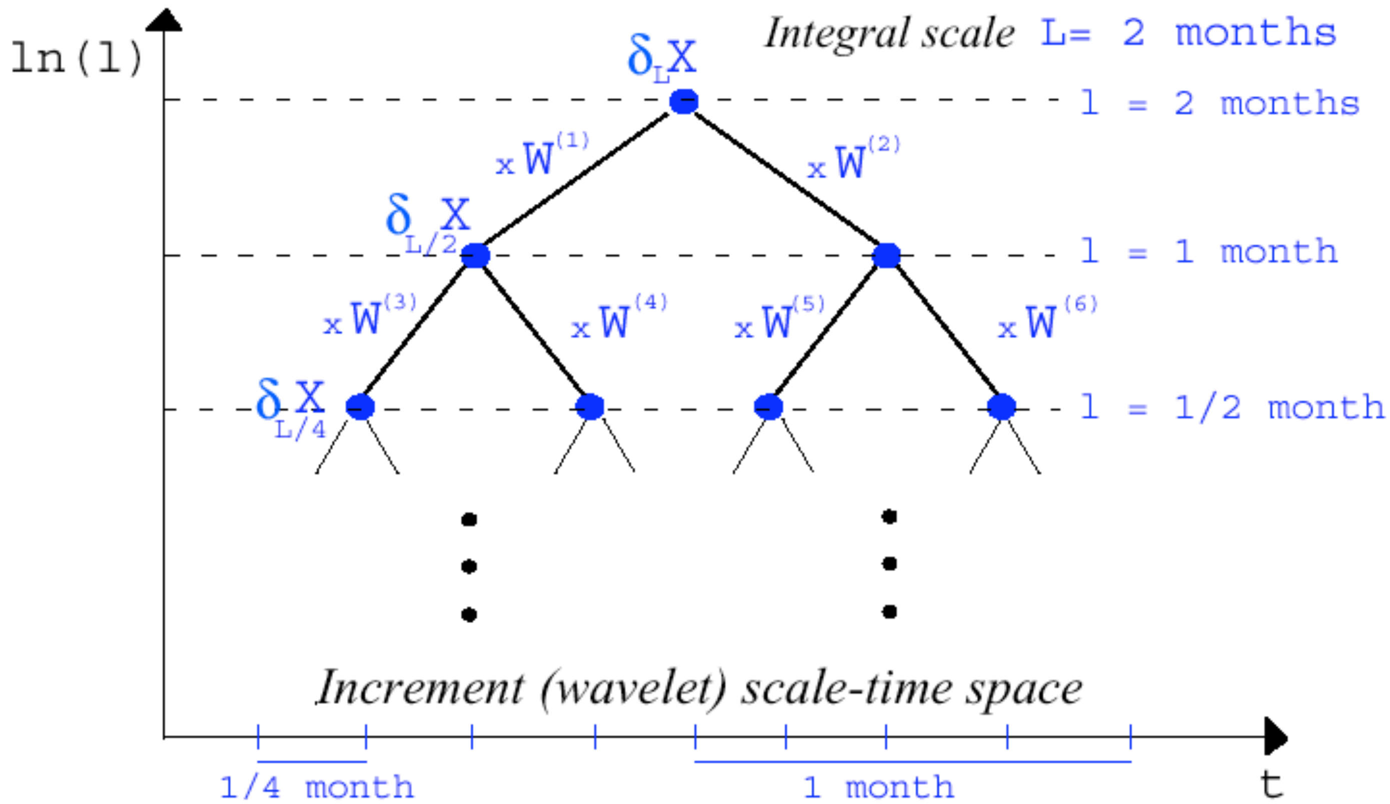


A. Arneodo, J.-F. Muzy and D. Sornette, Direct causal cascade in the stock market, European Physical Journal B 2, 277-282 (1998)

The multiplicative cascade paradigm

$$\delta_{\lambda l} X(\lambda t) = \lambda^H \delta_l X(t) = W_\lambda \delta_l X(t)$$

- $\mathcal{W}$-cascades (wavelet cascade)



The Multifractal Random Walk (MRW) model

$$r_{\Delta t}(t) = \epsilon(t) \cdot \sigma_{\Delta t}(t) = \epsilon(t) \cdot e^{\omega_{\Delta t}(t)}$$

$$\mu_{\Delta t} = \frac{1}{2}\ln(\sigma^2 \Delta t) - C_{\Delta t}(0)$$

$$C_{\Delta t}(\tau) = \mathrm{Cov}[\omega_{\Delta t}(t), \omega_{\Delta t}(t+\tau)] = \lambda^2 \ln\left(\frac{T}{|\tau| + e^{-3/2}\Delta t}\right)$$

$$\omega_{\Delta t}(t) = \mu_{\Delta t} + \int_{-\infty}^{t} d\tau\ \eta(\tau)\ K_{\Delta t}(t-\tau)$$

$\omega_{\Delta t}(t)$ is Gaussian with mean $\mu_{\Delta t}$ and variance $V_{\Delta t} = \int_0^\infty d\tau\ K^2_{\Delta t}(\tau) = \lambda^2 \ln\left(\frac{T e^{3/2}}{\Delta t}\right)$

$$C_{\Delta t}(\tau) = \int_0^\infty dt\ K_{\Delta t}(t) K_{\Delta t}(t + |\tau|)$$

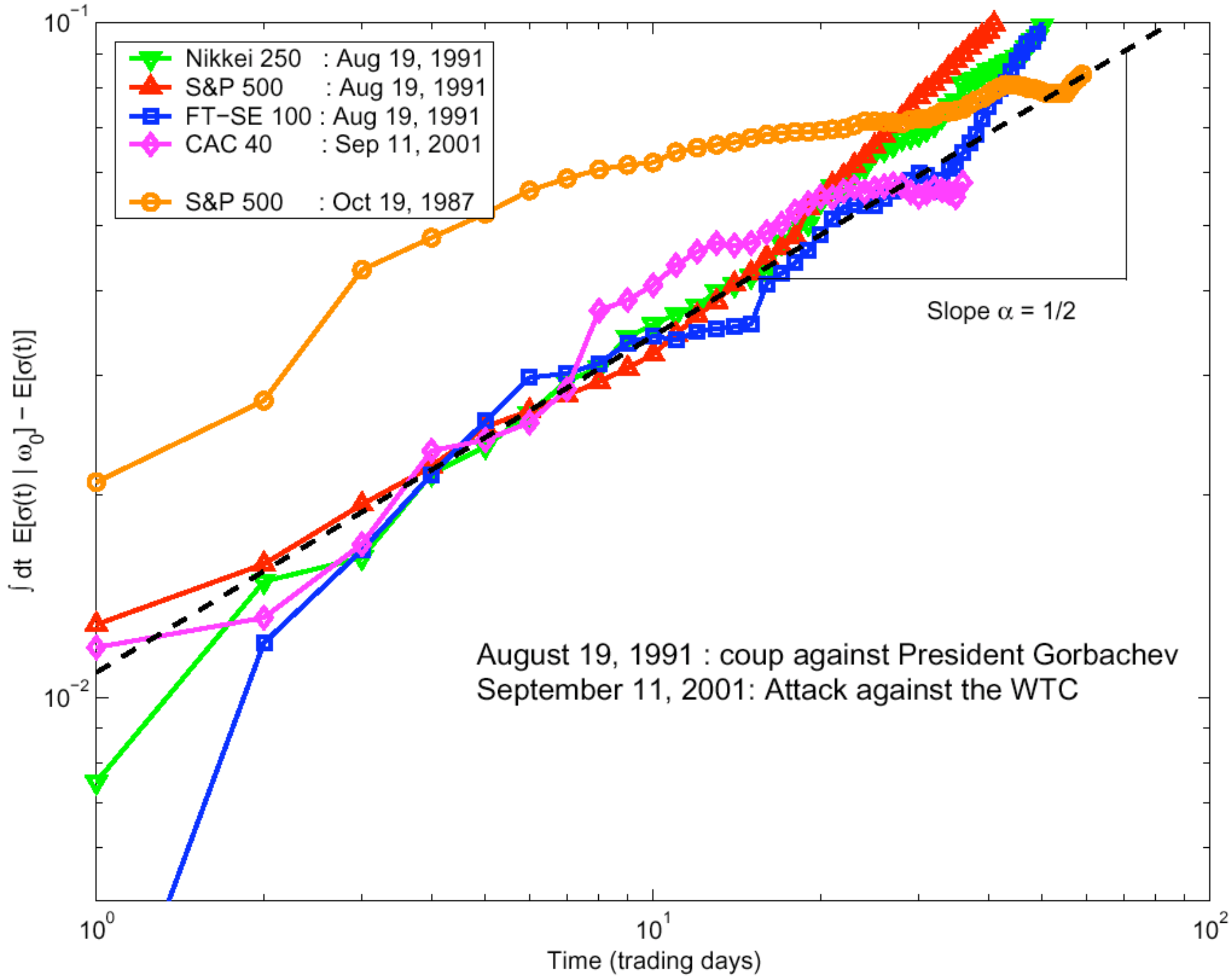
$$\hat{K}_{\Delta t}(f)^2 = \hat{C}_{\Delta t}(f) = 2\lambda^2\ f^{-1}\left[\int_0^{Tf} \frac{\sin(t)}{t} dt + O\left(f\Delta t \ln(f\Delta t)\right)\right]$$

$$K_{\Delta t}(\tau) \sim K_0 \sqrt{\frac{\lambda^2 T}{\tau}} \quad \text{for}\ \ \Delta t << \tau << T\ .$$

$\Rightarrow \quad \varphi = 0$

Linear response to an external shock

$$E_{exo}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



D. Sornette, Y. Malevergne and J.F. Muzy
Volatility fingerprints of large shocks: Endogeneous versus exogeneous,
Risk Magazine
(http://arXiv.org/abs/cond-mat/0204626)

"Conditional response" to an endogeneous shock

$$\begin{aligned} \mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] &= \overline{\sigma^2(t)} \exp\left[2(\omega_0 - \mu) \cdot \frac{C(t)}{C(0)} - 2\frac{C^2(t)}{C(0)}\right] \\ &= \overline{\sigma^2(t)} \left(\frac{T}{t}\right)^{\alpha(s)+\beta(t)} \end{aligned}$$

Interplay between
-long memory
-exponential

where

$$\alpha(s) = \frac{2s}{\ln(\frac{Te^{3/2}}{\Delta t})},$$

$$\beta(t) = 2\lambda^2 \frac{\ln(t/\Delta t)}{\ln(Te^{3/2}/\Delta t)}$$

Within the range $\Delta t < t << \Delta t e^{\frac{|s|}{\lambda^2}}$, $\beta(t) << \alpha(s)$

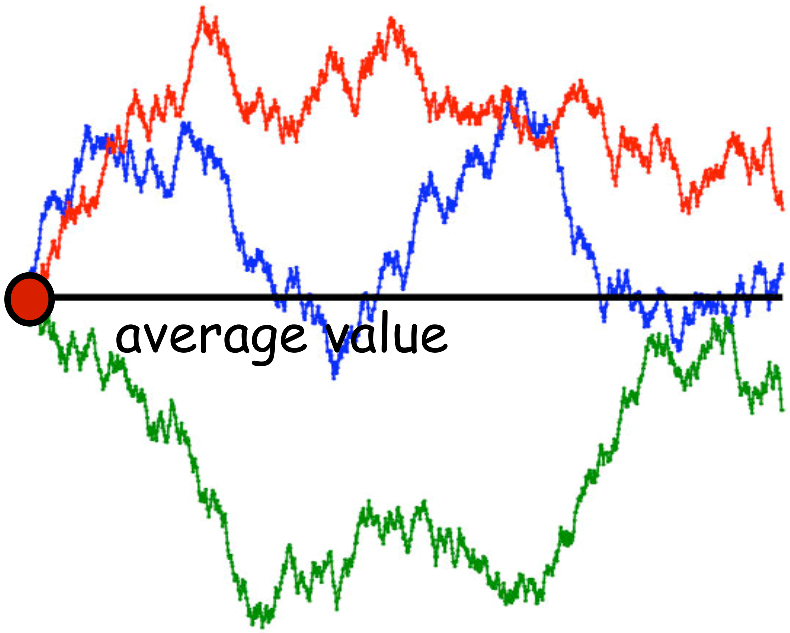
$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

D. Sornette, Y. Malevergne and J.F. Muzy, Volatility fingerprints of large shocks: Endogeneous versus exogeneous, Risk 16 (2), 67-71 (2003) (http://arXiv.org/abs/cond-mat/0204626)

Conditional expectations

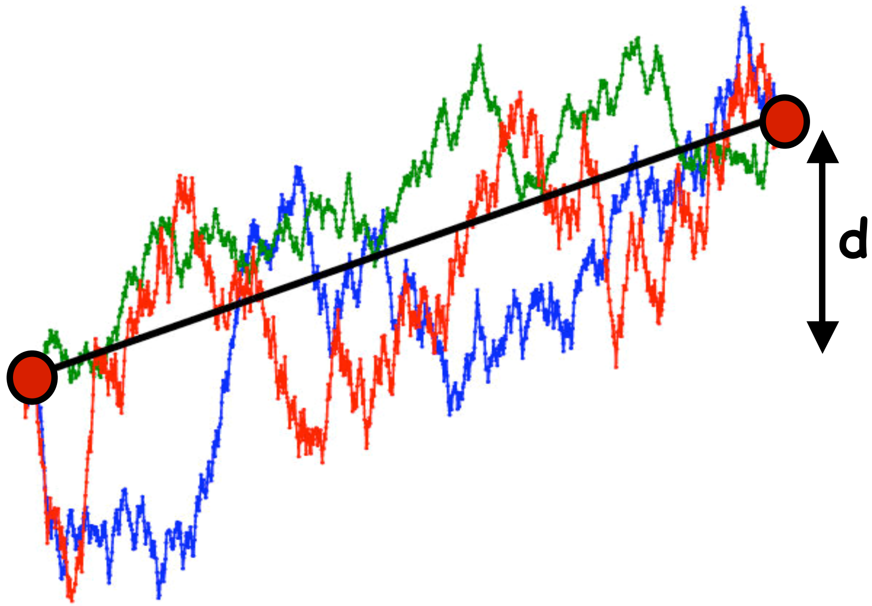
Analogy Brownian motions / seismicity rate in the ETAS model

without conditioning:
stationary process, average=0



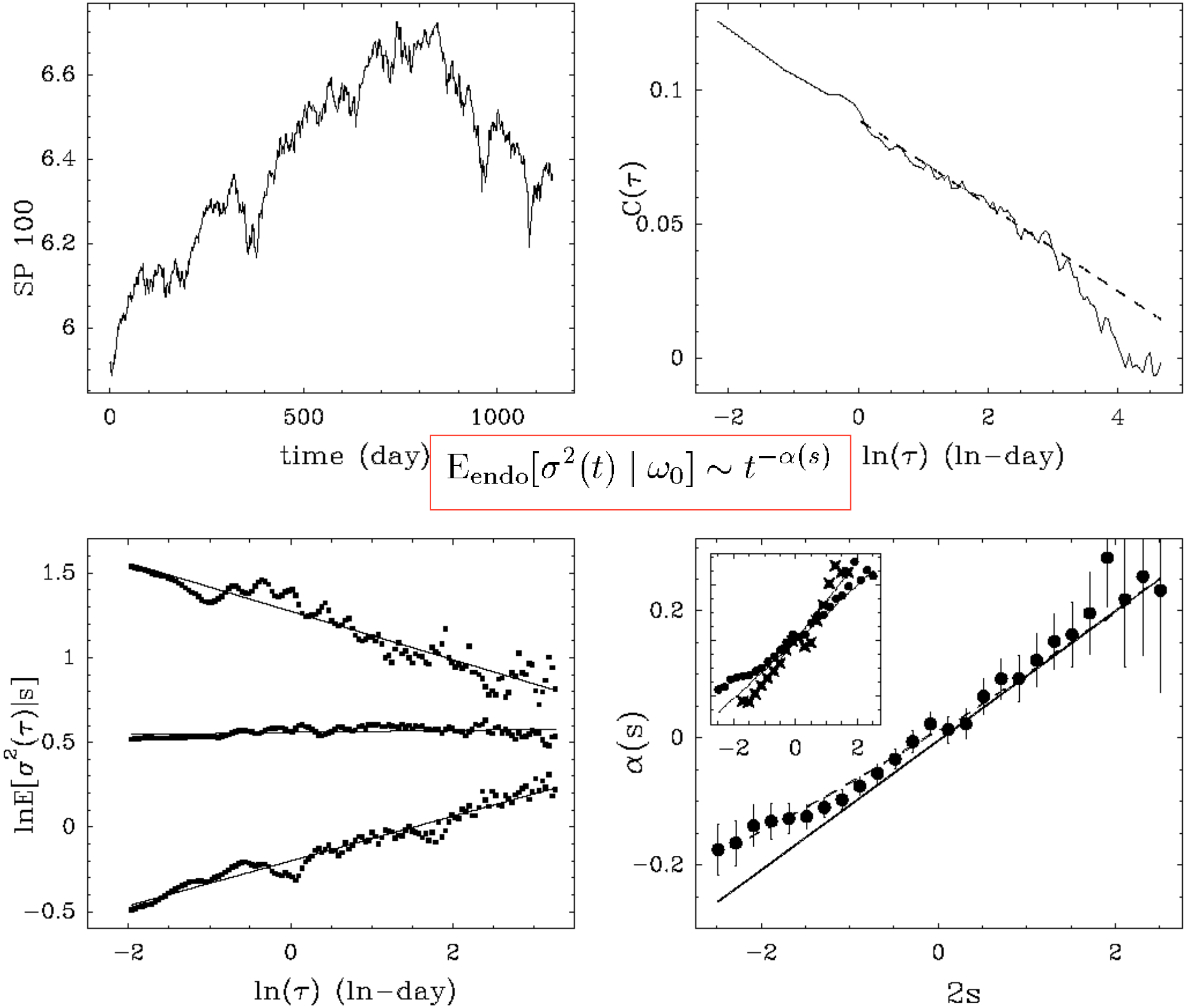
stationary rate

conditioning to a large value **W(t_c)=d** :
non-stationary process, average # 0



Conditional rate

Real Data and Multifractal Random Walk model



$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

Importance of Positive Feedbacks and Over-confidence in a Self-Fulfilling Ising Model of Financial Markets

$$s_i(t) = \text{sign}\left[\sum_{j \in \mathcal{N}} K_{ij}(t)\mathrm{E}[s_j](t) + \sigma_i(t)G(t) + \epsilon_i(t)\right]$$

Imitation

News

Private information

$$K_{ij}(t) = b_{ij} + \alpha_i K_{ij}(t-1) + \beta r(t-1)G(t-1)$$

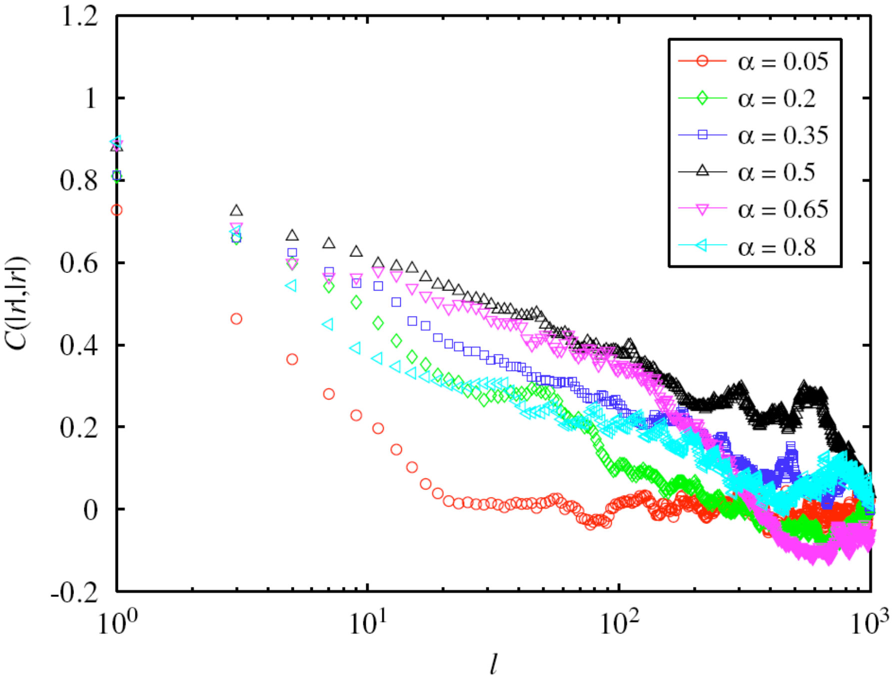
(generalizes Carlos Pedro Gonçalves, who generalized Johansen-Ledoit-Sornette)

β: propensity to be influenced by the felling of others

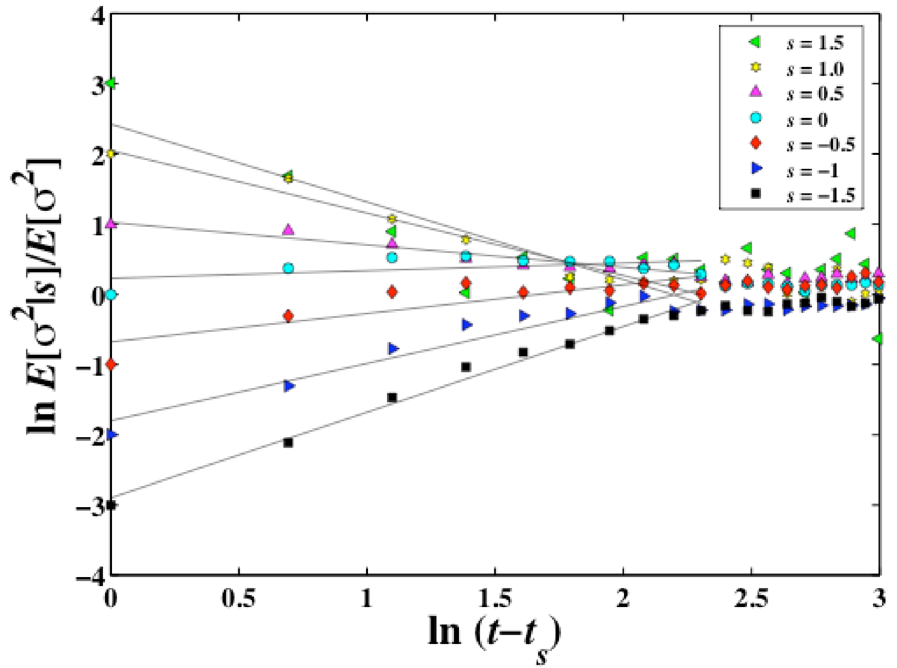
1. **β<0: rational agents**

- **β>0: over-confident agents**

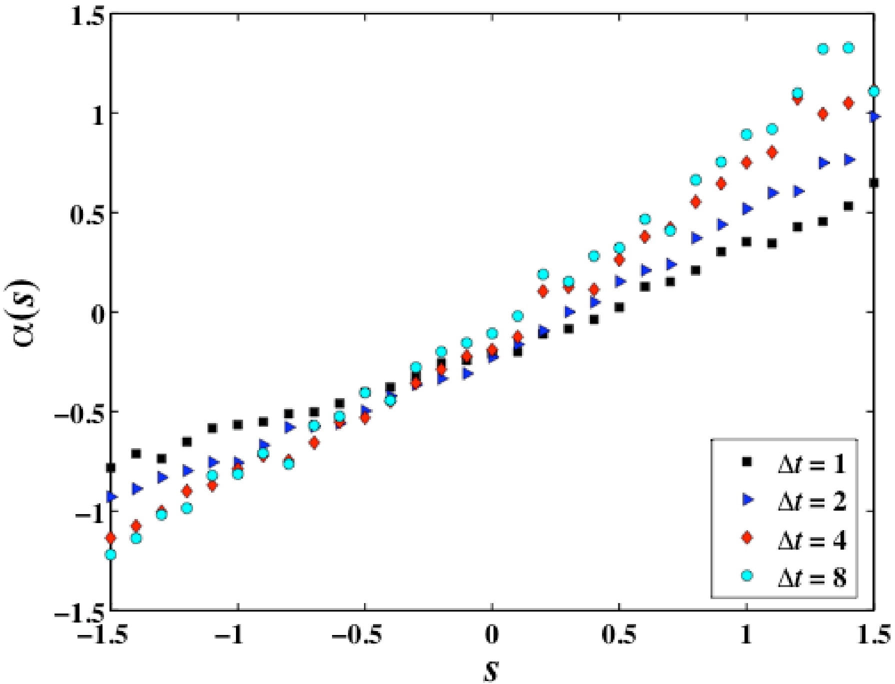
Didier Sornette and Wei-Xing Zhou, Physica A (2006)



C(|r|,|r|)
l
α = 0.05
α = 0.2
α = 0.35
α = 0.5
α = 0.65
α = 0.8



ln E[σ²|s]/E[σ²]
ln (t−t_s)
s = 1.5
s = 1.0
s = 0.5
s = 0
s = −0.5
s = −1
s = −1.5



α(s)
s
Δt = 1
Δt = 2
Δt = 4
Δt = 8

- Multifractality in the time domain: conditional response functions

- Financial volatility

- Robust multifractality in exponential of long-memory processes

- Multifractal Omori law for the relaxation of earthquake aftershocks

What is the origin of the robust multifractality?

$$\delta_\tau X(t) = \int_{t-\tau}^{t} \mu(t')dt', \quad \text{with } \mu(t) = \kappa e^{\omega(t)},$$

$$\omega(t) = \int_{-\infty}^{t} dW(t')h(t-t'),$$

$$h(t) = \frac{h_0}{(1+x)^{\varphi+1/2}} H(t), \quad x = t/\ell,$$

Case $\varphi > 0$

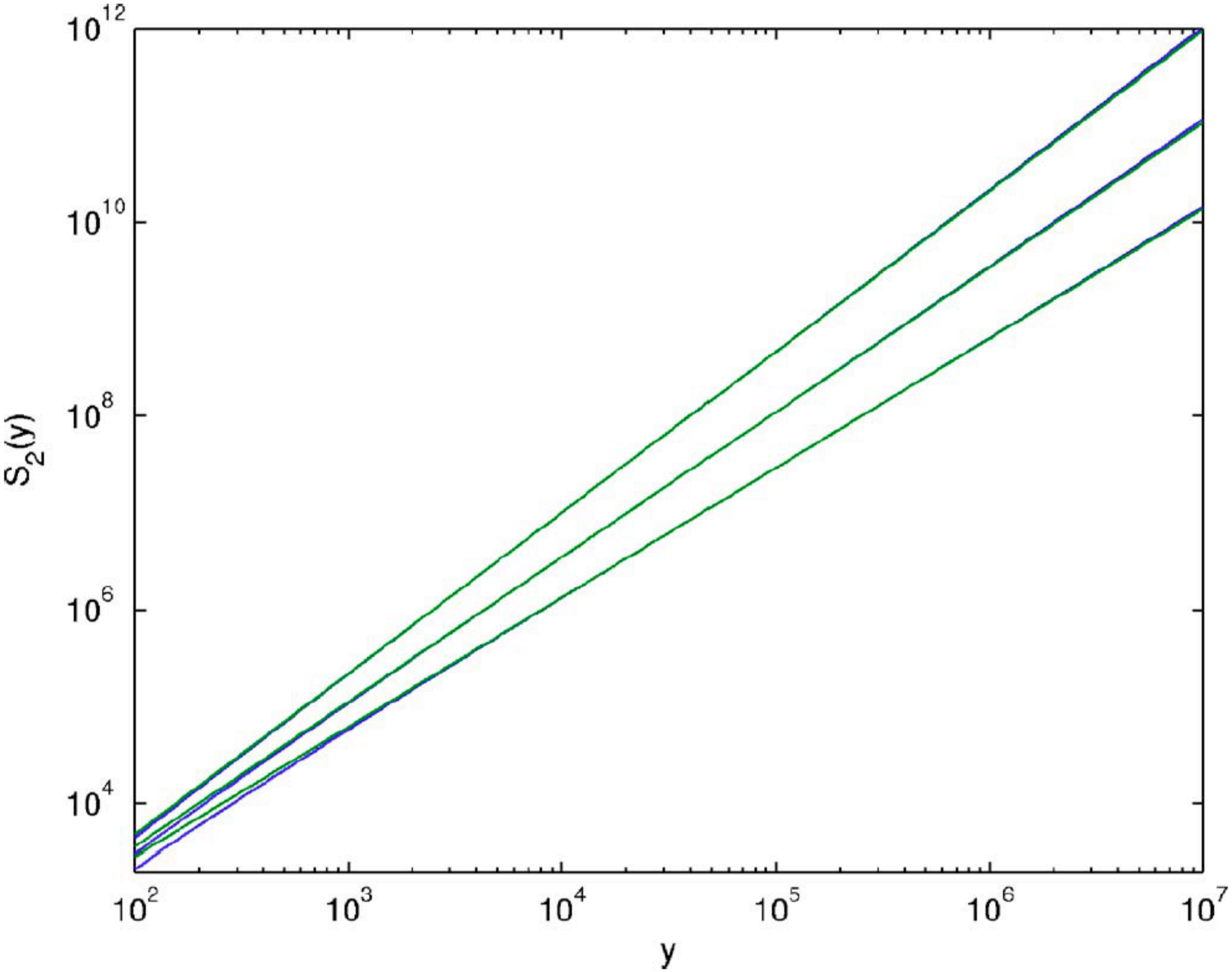
$$h(t) = \frac{h_0}{(1+x)^{\varphi+1/2}} H(t), \quad x = t/\ell,$$

$$S_2(y) = \frac{1}{\ell^2 \langle \mu^2 \rangle} \langle [\delta_\tau X(t)]^2 \rangle = \int_0^y dx_1 \int_0^y dx_2 G(x_2 - x_1)$$

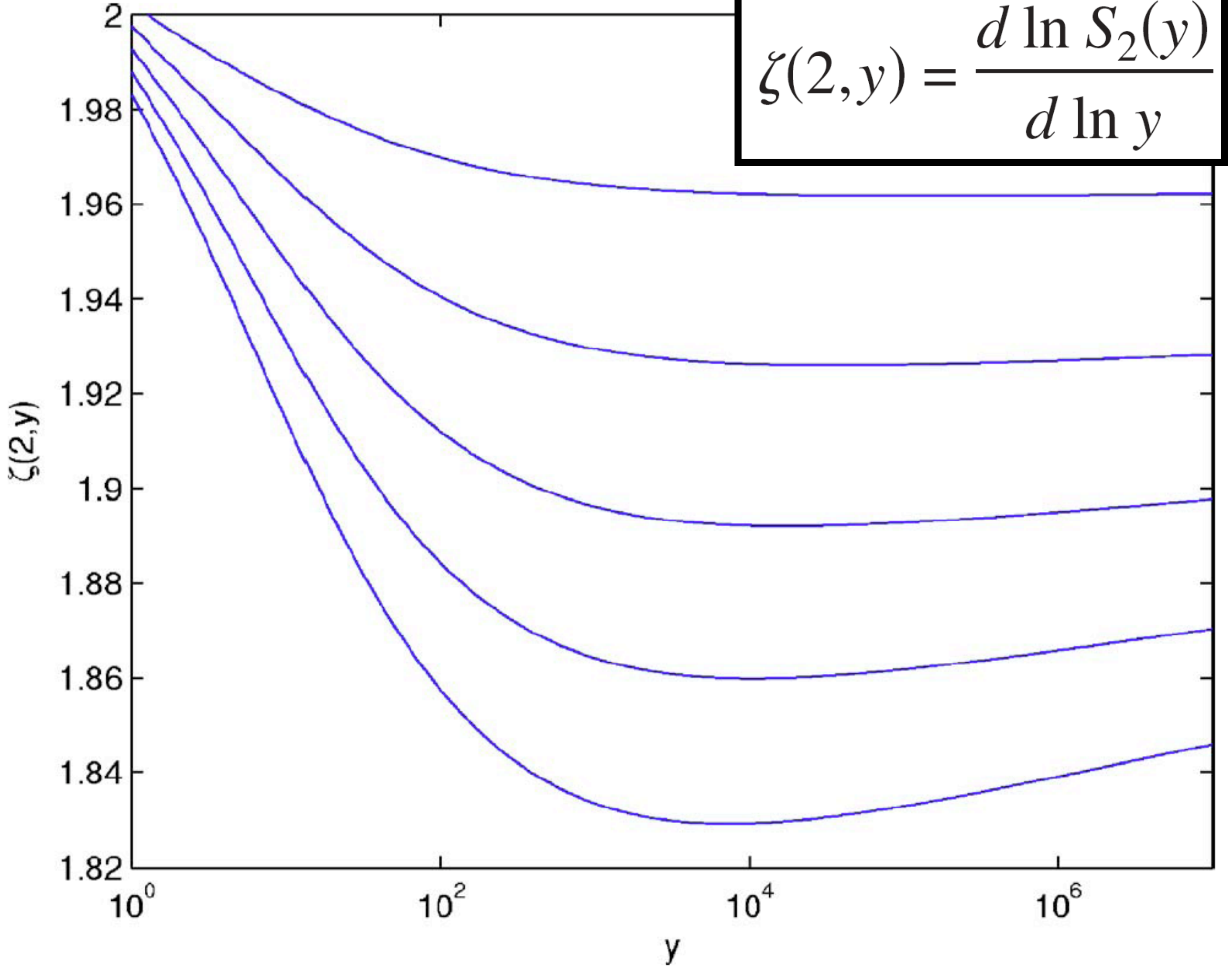
$$\langle \mu^q \rangle = \kappa^q e^{\sigma^2 q^2/2}, \quad G(y) = e^{-\sigma^2 d(y)}, \quad d(y) = 1 - C(y),$$

$$C\left(\frac{\tau}{\ell}\right) = \frac{1}{\sigma^2} \int_0^\infty h(t) h(t+\tau) dt. \qquad \sigma^2 = \int_0^\infty h^2(t) dt = h_0^2 \frac{\ell}{2\varphi}$$

$$S_2(y) = A_2 y^{\zeta(2)} \qquad ?$$

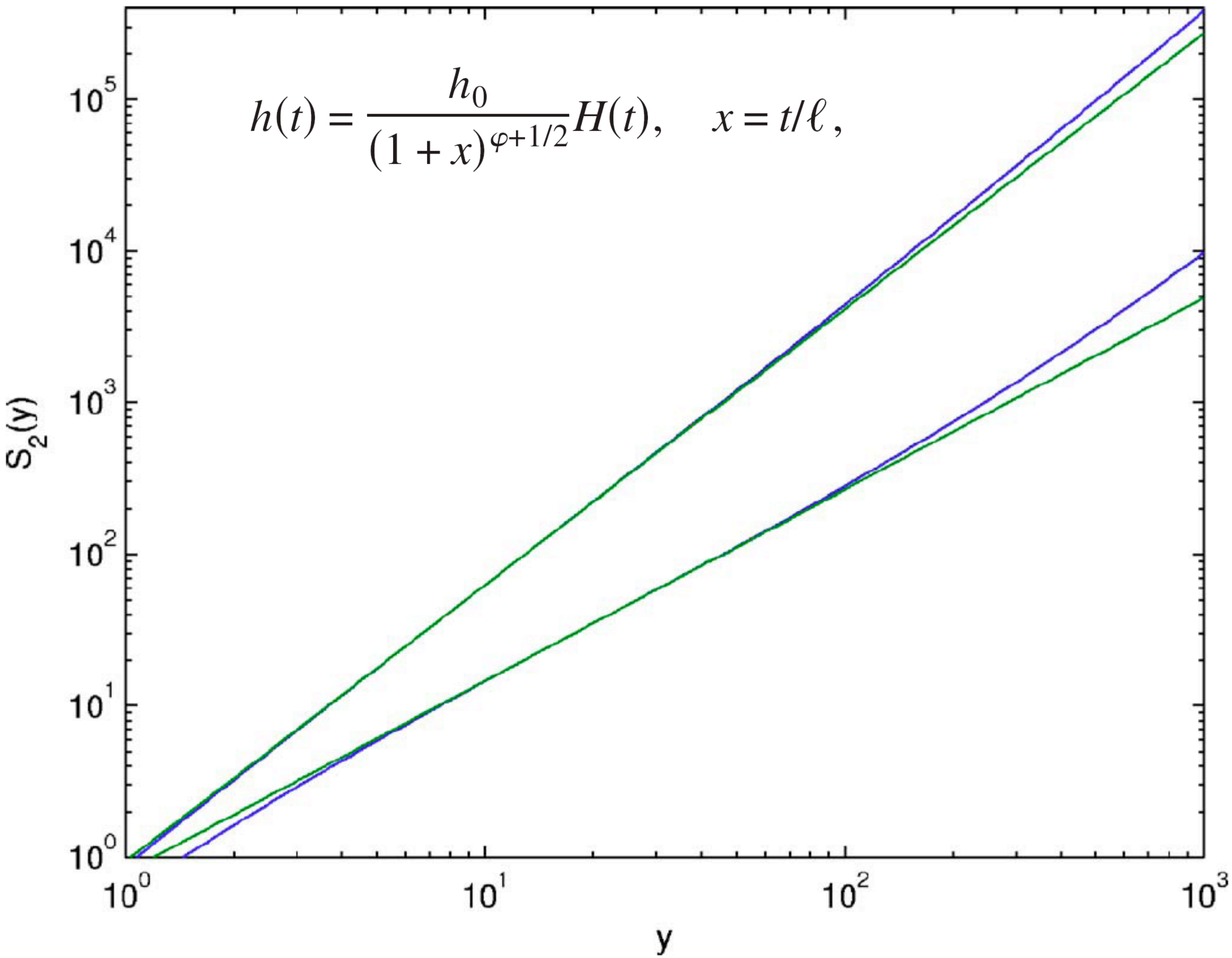


Log-log plot of the second-order moment and its power approximation for φ =0.01 and σ^2=20, 30, 40 (top to bottom) The corresponding exponents are equal to $\zeta(2)$ =1.66; 1.49; 1.34.

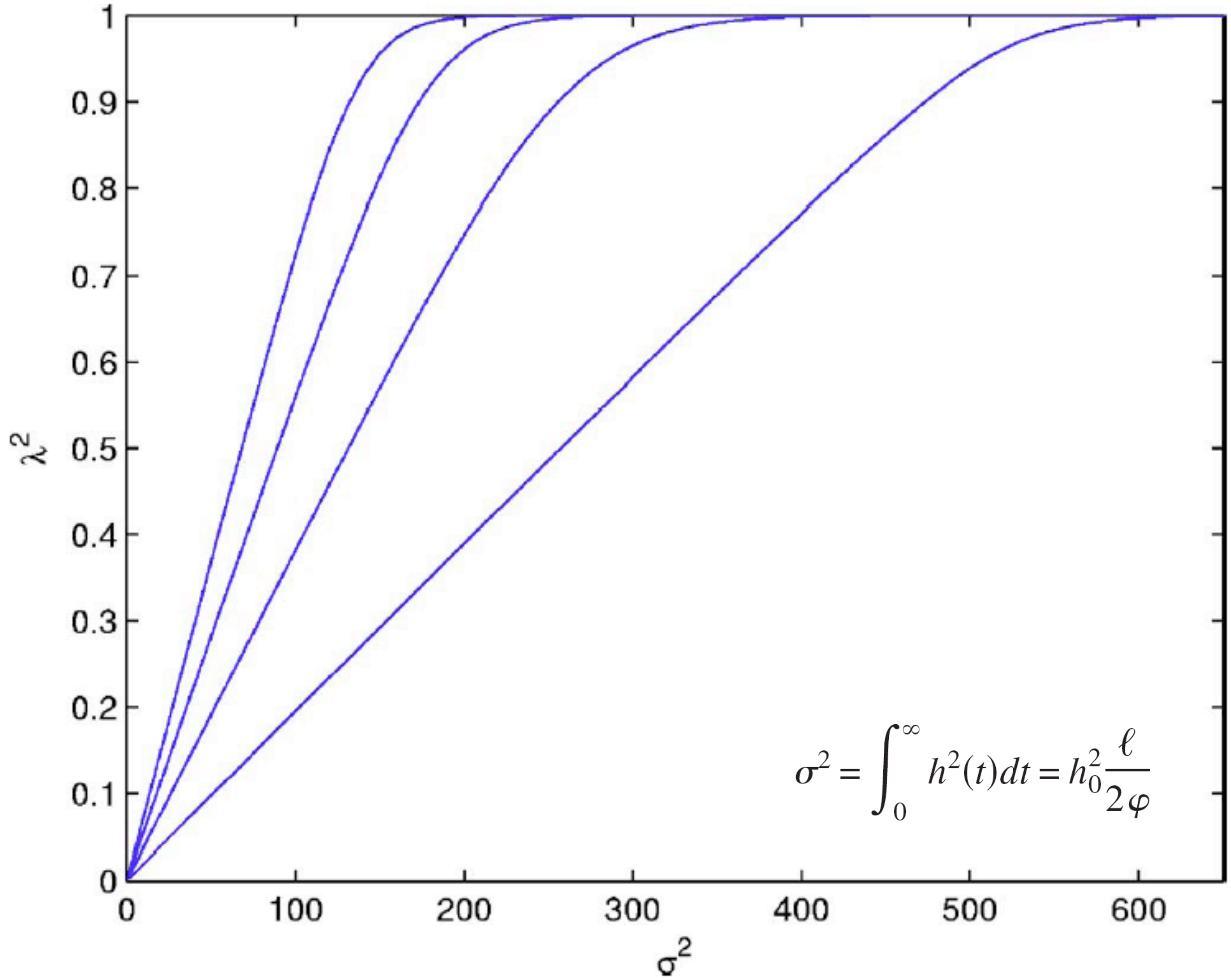


$$\zeta(2,y) = \frac{d \ln S_2(y)}{d \ln y}$$

Local exponent $\zeta(2,y)$ as a function of $y=\tau/\ell$ for $\sigma^2=10$ and $\varphi=0.002;0.004;0.006;0.008;0.01$ (top to bottom).



Log-log plot of the second-order moment and its power approximation for $\varphi = 0.5$ and $\sigma^2 = 1$ and 5 (top to bottom) The corresponding exponents are equal to $\zeta(2) = 1.82$ and 1.26.



Dependance of the intermittency coefficient $\lambda^2 = 2 - \zeta(2)$ as a function of σ^2 for different values of $\varphi = 0.01 - 0.04$ (bottom to top).

HIGHER-ORDER MOMENTS, UNIVERSAL SCALING FUNCTION, AND MULTIFRACTAL SPECTRA

$$\langle [\delta_\tau X(t)]^q \rangle$$

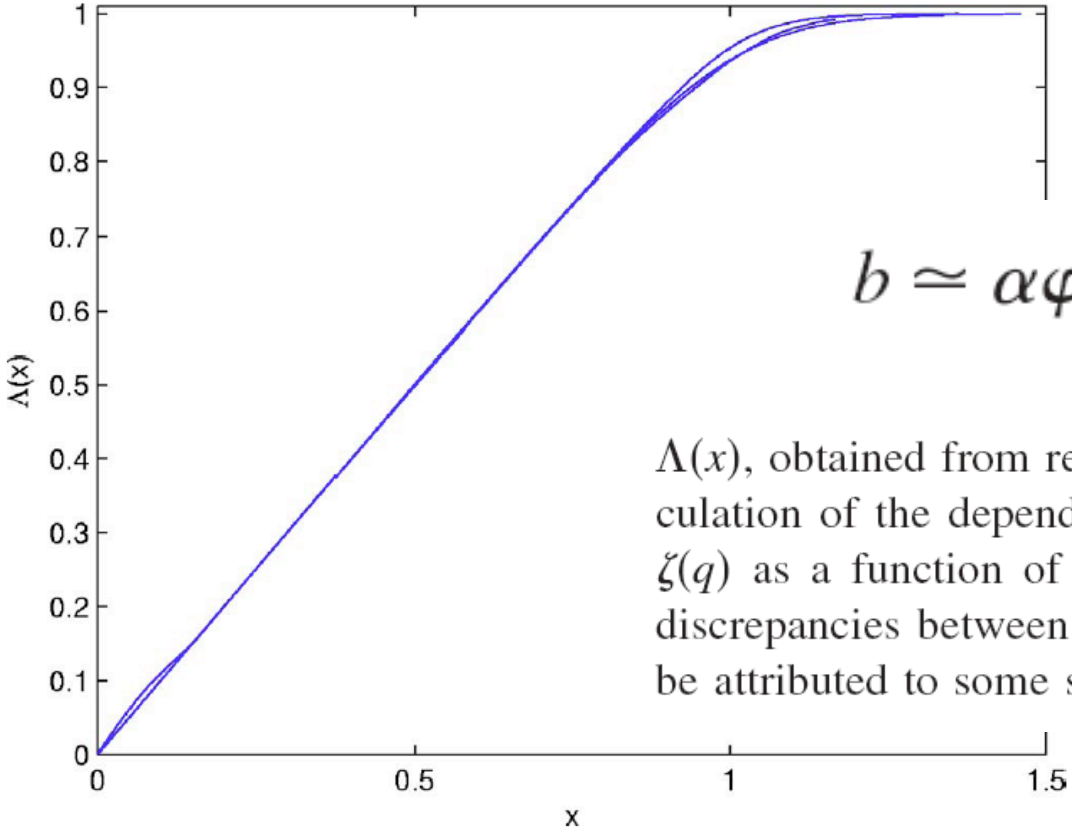
$$S_q(y) = q(q-1)\int_0^y (y-x)G_q(x)dx,$$

$$G_q(x) = G(x)\int_0^x du_1 \cdots \int_0^x du_{q-2} \prod_{\substack{i=1 \\ j=i+1}}^{q-2} G(x_i)G(u-x_i)G(x_i-x_j).$$

$$S_q(y) = A_q y^{\zeta(q)}, \quad A_q = S_q(y_m)y_m^{-\zeta(q)}$$

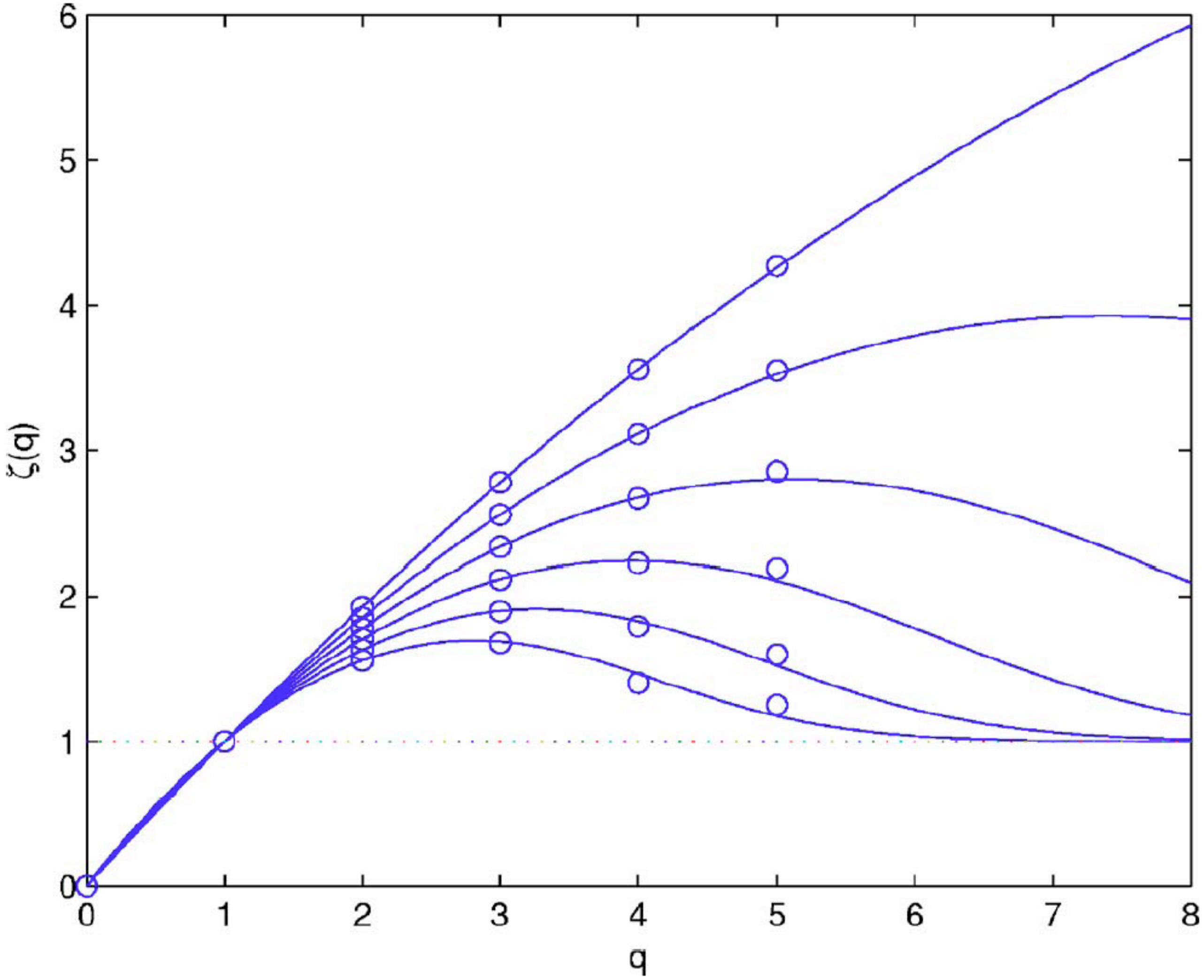
Define $$\lambda^2(q;\sigma^2,\varphi) = 2\frac{q-\zeta(q)}{q(q-1)} \quad (1)$$

$$\lambda^2(q;\sigma^2,\varphi) = \frac{2}{q}\Lambda(b\sigma^2 q). \quad (2)$$



$$b \simeq \alpha\varphi^{\beta}, \quad \alpha = 0.58, \quad \beta = 0.92.$$

Plot of the universal scaling function $\Lambda(x)$, obtained from relations (1) and (2) and the numerical calculation of the dependence of the effective multifractal exponents $\zeta(q)$ as a function of σ^2, for $\varphi=0.001$ and $q=2;3;4$. The slight discrepancies between the curves in the neighborhood of $x=1$ can be attributed to some systematic errors of numerical calculations.



Non-concave:
Holder inequality broken
(Intermediate asymptotics)

Universal multifractal spectra ζ(q) for φ=0.004 and σ^2=10, 20, 30, 40, 50, 60 (top to bottom).

Line: exponent obtained from $S_q(y) = q(q-1)\int_0^y (y-x)G_q(x)dx,$

o: $\zeta(q) = q + (1-q)\Lambda(b\sigma^2 q)$ where Λ(x) is obtained from previous scaling with q=2 and φ=0.001

What is the origin of the robust multifractality?

$$\delta_\tau X(t) = \int_{t-\tau}^{t} \mu(t')dt', \quad \text{with } \mu(t) = \kappa e^{\omega(t)},$$

$$\omega(t) = \int_{-\infty}^{t} dW(t')h(t-t'), \qquad h(t) = \frac{h_0}{(1+x)^{\varphi+1/2}}H(t), \quad x = t/\ell,$$

$$S_2(y) = \frac{1}{\ell^2\langle\mu^2\rangle}\langle[\delta_\tau X(t)]^2\rangle = \int_0^y dx_1 \int_0^y dx_2 G(x_2 - x_1)$$

$$\langle\mu^q\rangle = \kappa^q e^{\sigma^2 q^2/2}, \quad G(y) = e^{-\sigma^2 d(y)}, \quad d(y) = 1 - C(y),$$

$$C\left(\frac{\tau}{\ell}\right) = \frac{1}{\sigma^2}\int_0^\infty h(t)h(t+\tau)dt. \qquad \sigma^2 = \int_0^\infty h^2(t)dt = h_0^2\frac{\ell}{2\varphi}$$

Answer: *d(y)* is almost logarithmic

- Multifractality in the time domain: conditional response functions

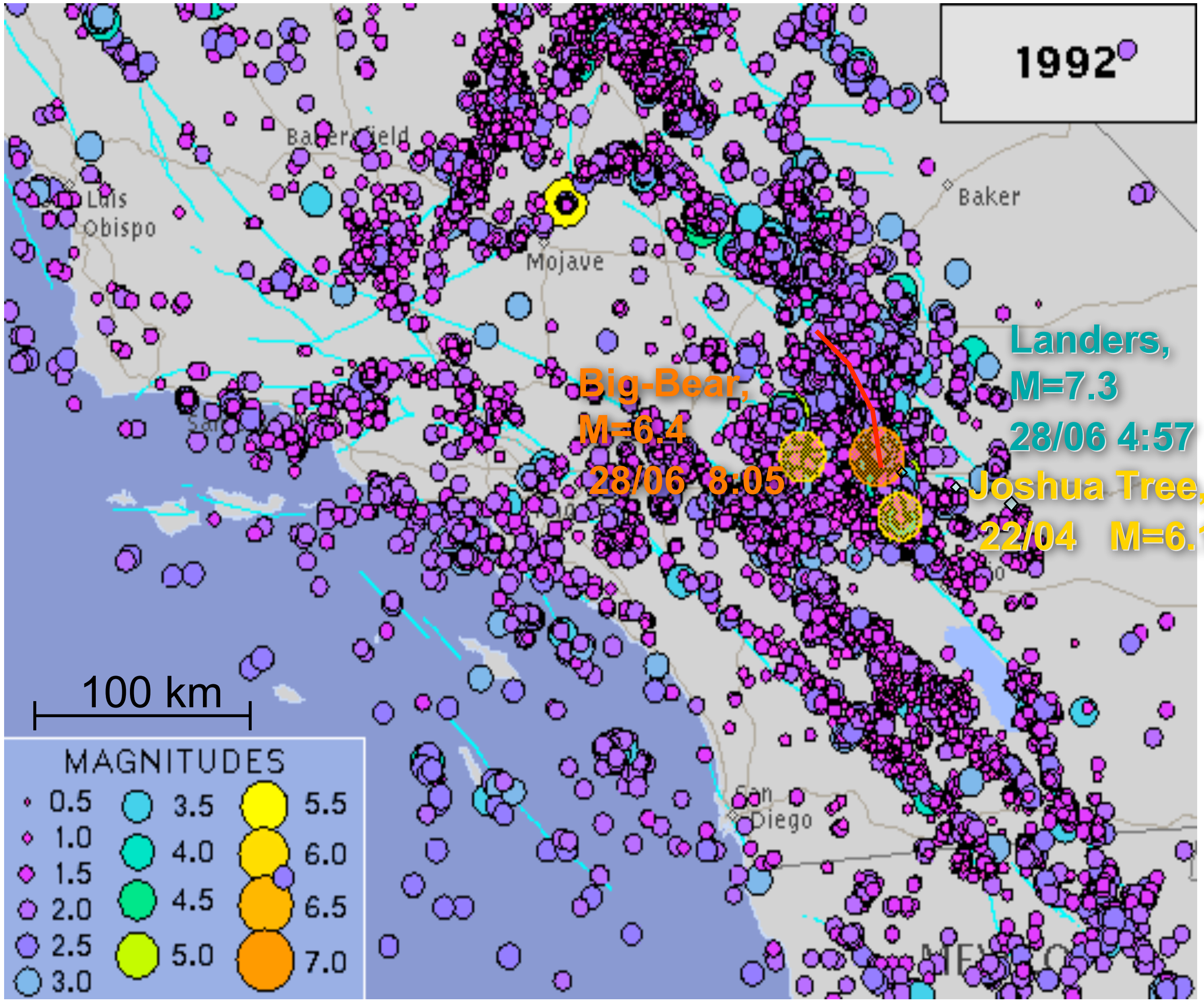
- Financial volatility

- Robust multifractality in exponential of long-memory processes

- Multifractal Omori law for the relaxation of earthquake aftershocks

Spatial and temporal organization of seismicity in Californie

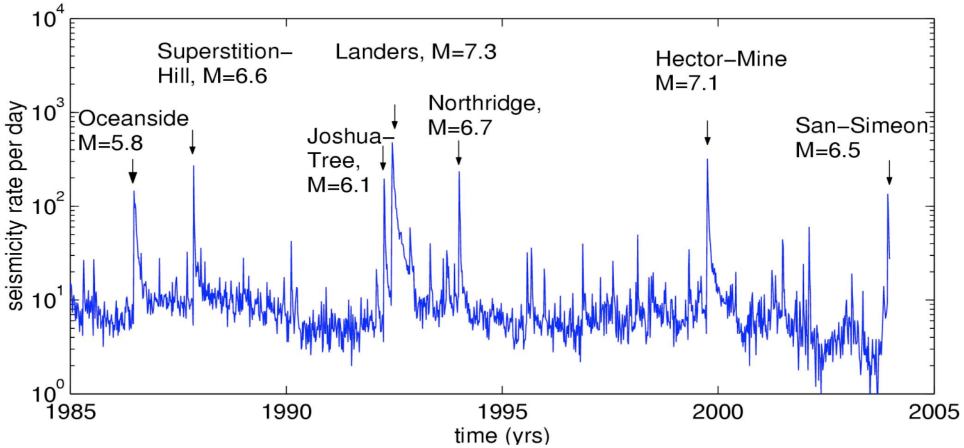
Landers
28 june 1992
M=7.3



Statistical laws of seismicity

- Gutenberg-Richter law: $\sim 1/E^{1+\beta}$ (with $\beta \approx 2/3$)
- Omori law $\sim 1/t^p$ (with $p \approx 1$ for large earthquakes)
- Productivity law $\sim E^a$ (with $a \approx 2/3$)
- PDF of fault lengths $\sim 1/L^2$
- Fractal/multifractal structure of fault networks $\zeta(q)$, $f(\alpha)$
- PDF of seismic stress sources $\sim 1/s^{2+\delta}$ (with $\delta \geq 0$)

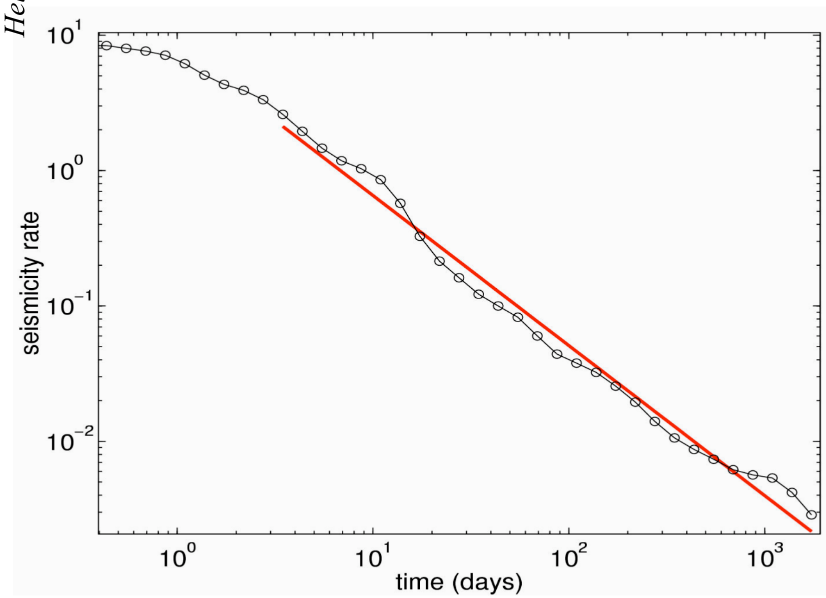
Aftershocks Time Series



Earthquake catalogs appear as a succession of bursts of activity – each event, whatever its magnitude, is followed by a decay of activity.

Events occurring during this relaxation phase are usually refered to as aftershocks or triggered seismicity.

Helmstetter, 2002



The time decay of triggered seismicity rate is measurable after large events, as their number of aftershocks is large enough.

This time decay is known as the Omori-Utsu law (1894):

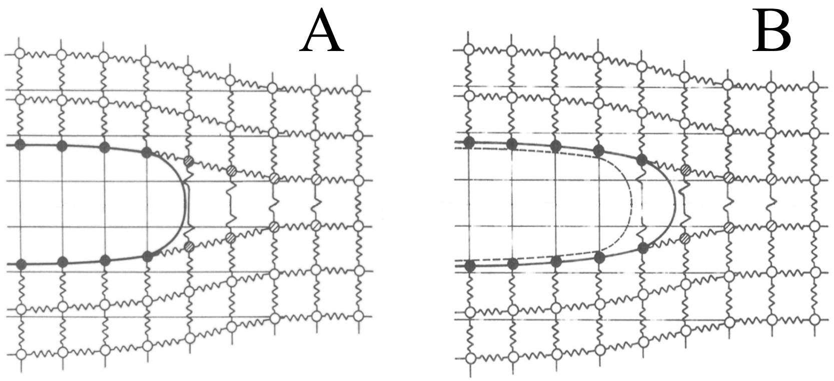
$$N(t)\, dt \sim t^{-p}\, dt$$

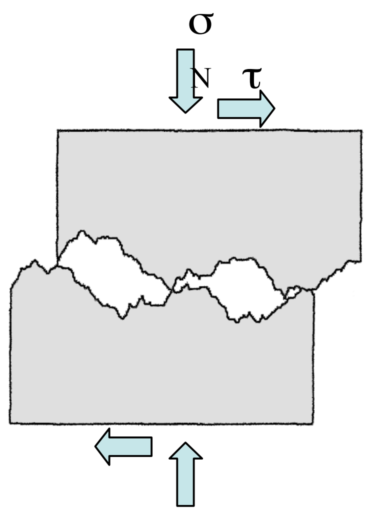
The exponent p is close to 1 for most sequences.

Each event thus defines a **mathematical singularity.**

Mechanics of Triggered Seismicity

Krausz and Krausz, 1987





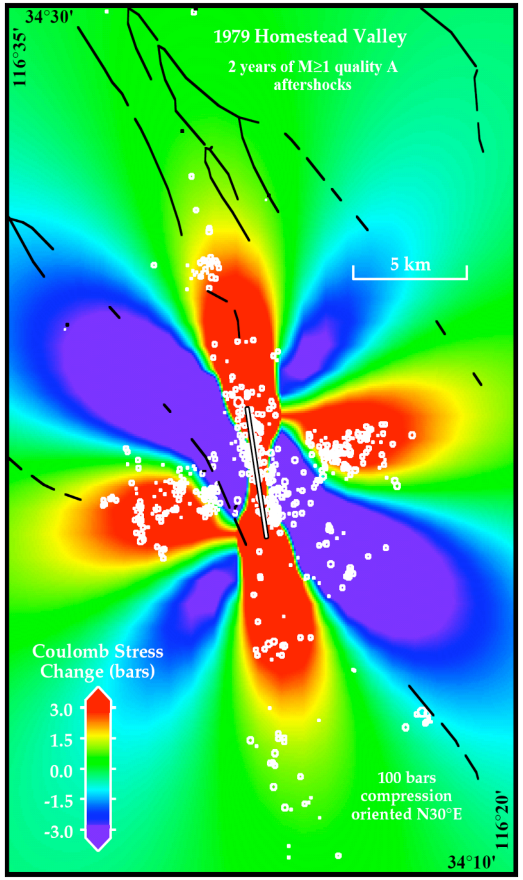
One class of models to explain triggered seismicity is **slow crack growth** *:* under the effect of applied stress and thermal agitation, cracks within rocks grow subcritically by breaking successive atomic bonds (represented by springs). After they reach a critical length, they propagate critically : this is the seismic event.

The second class is **state and rate-dependent friction**, which predicts a time shift between a stress perturbation and the possible slip instability. This process is also activated by stress and temperature.

First-order Models Predict a Universal Omori Law

Coupling those physical models with the linear elastic stress tensor perturbation induced by a given event (the so called *stress-transfer* patterns) predicts reasonable spatial locations of triggered events.

In the time domain, those models also predicts that the triggered seismicity sequence will taper with time as a power-law (the Omori law) with an exponent *p~1*.



Stein (1983)

Two very important remarks:

(1) The exponent *p* is then predicted to be **independent of the size** of the initial, triggering event (mainshock).

(2) These models **neglect interactions among triggered events** and take into account only the mechanical stress perturbation of the first event in the sequence.

Mathematics of Multifractal Time Series and the Omori law

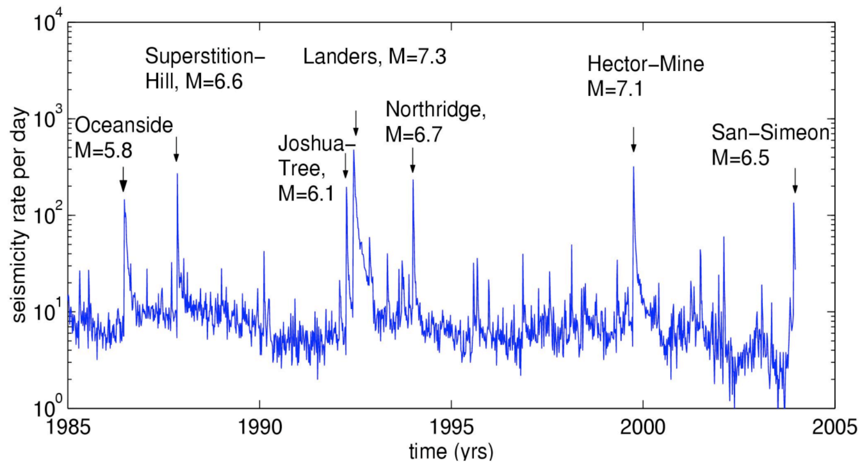
In self-similar time series, each event defines a singularity, which means that the seismic activity it triggers decays with time as :

$$N(t) \sim t^{-p}$$

where the exponent ***p*** **defines the *strength* of the singularity**. Singularities show up as **bursts of activity**.

In self-similar signals, the exponent *p* generally varies with the power of the burst: **the larger the burst, the larger the singularity *p***. This is known as multifractality, a phenomenology often observed for earthquakes time series when looking at correlation functions.

Independently, it is observed that **the larger the magnitude** of the initial event, the larger the average number of events it triggers (**the larger the burst**).



For a self-similar signal such as earthquake occurrence rate, we thus have only two possibilities :

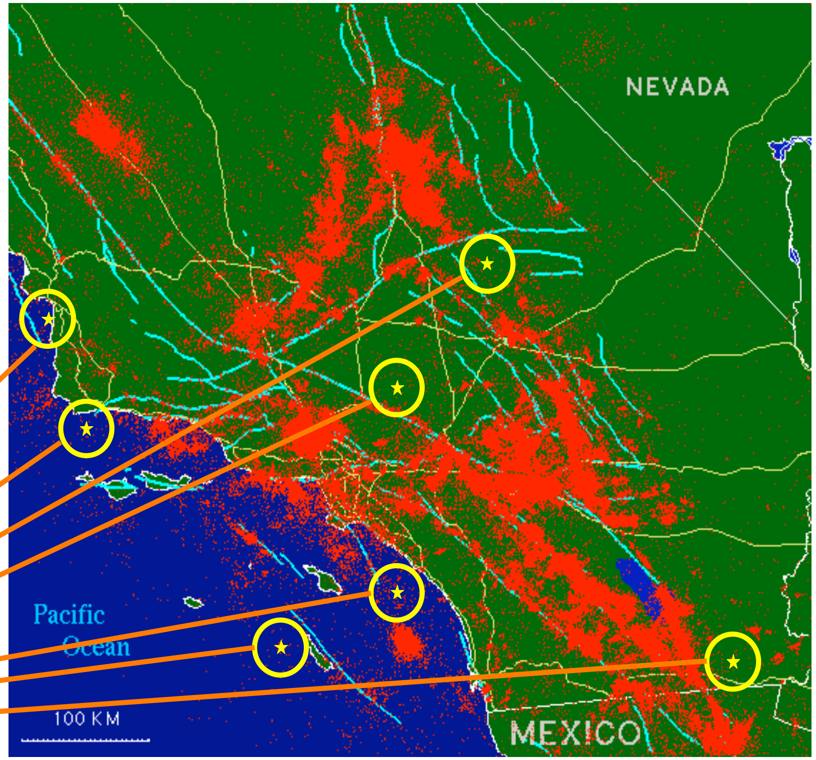
- **Monofractal case** : decay is the same for all bursts so that ***p*** **is constant with M**.

- **Multifractal case** : strong bursts decay faster and thus ***p*** **must increase with magnitude M** of the initial event.

Stacked Triggered Seismicity Sequences

Large events trigger enough aftershocks to allow us to compute a *p* exponent – but this is not the case for low-magnitude main events.

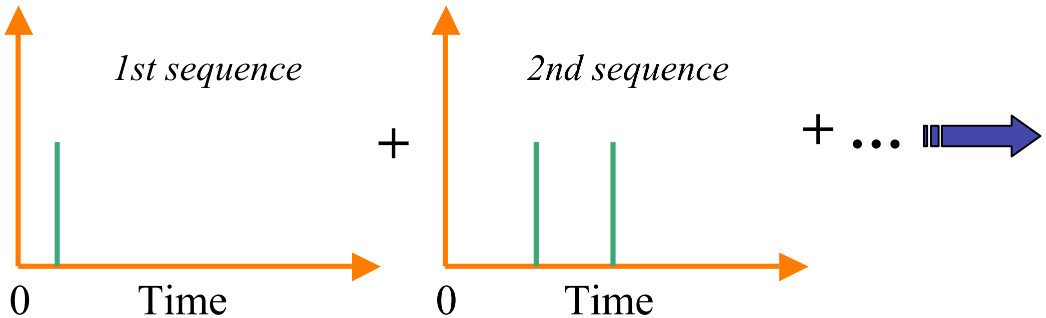
So we prefer to follow a **stacking strategy to improve the signal to noise ratio** :

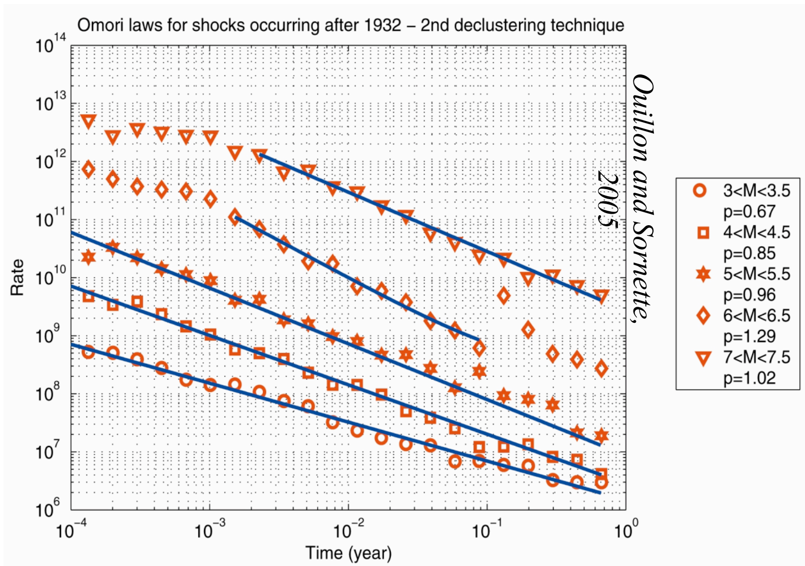


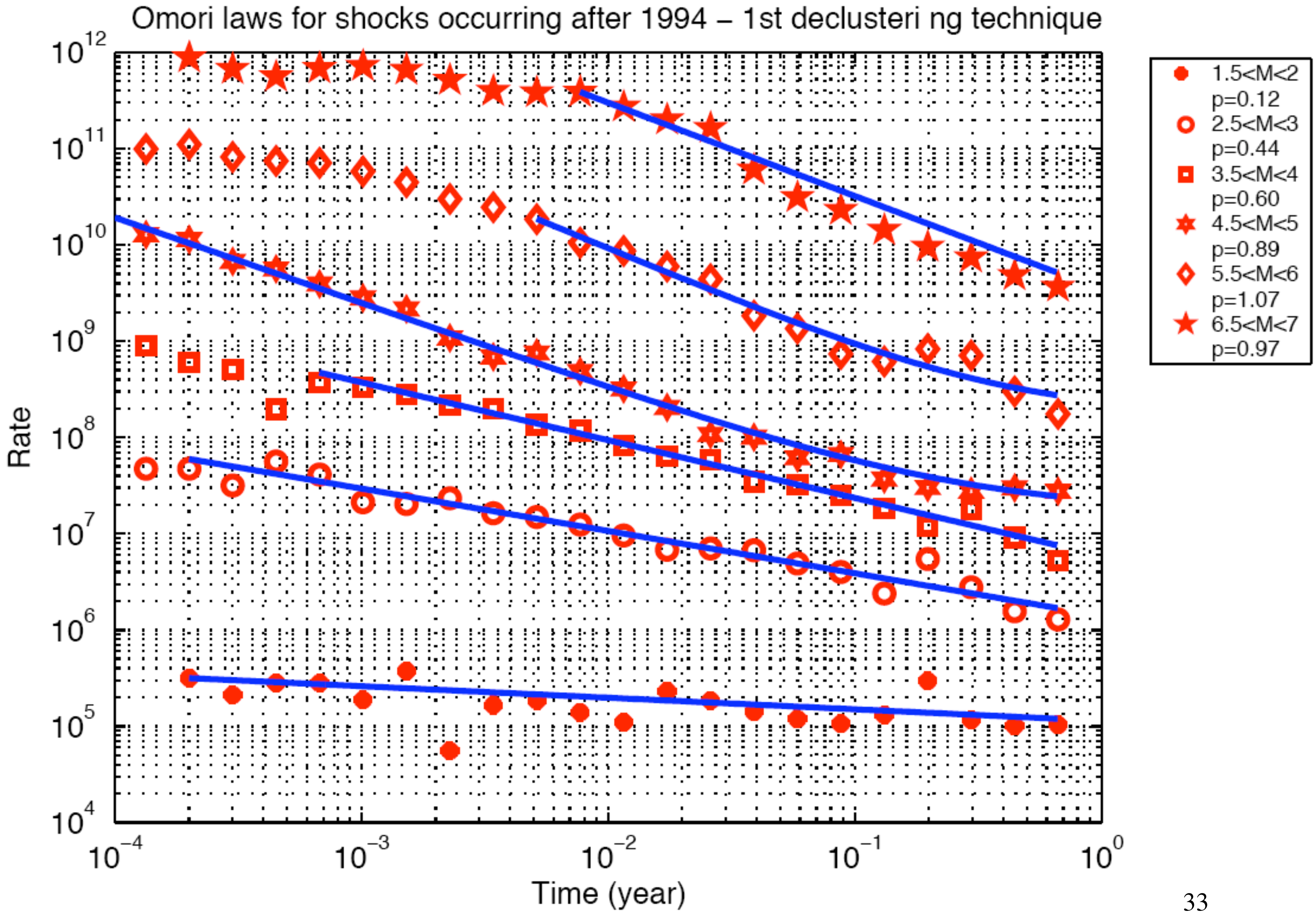
- look for isolated mainshocks according to magnitude range
- select and stack aftershocks sequences
- fit the stacks with :

$$\mathbf{N(t) = A\, t^{-p} + B}$$

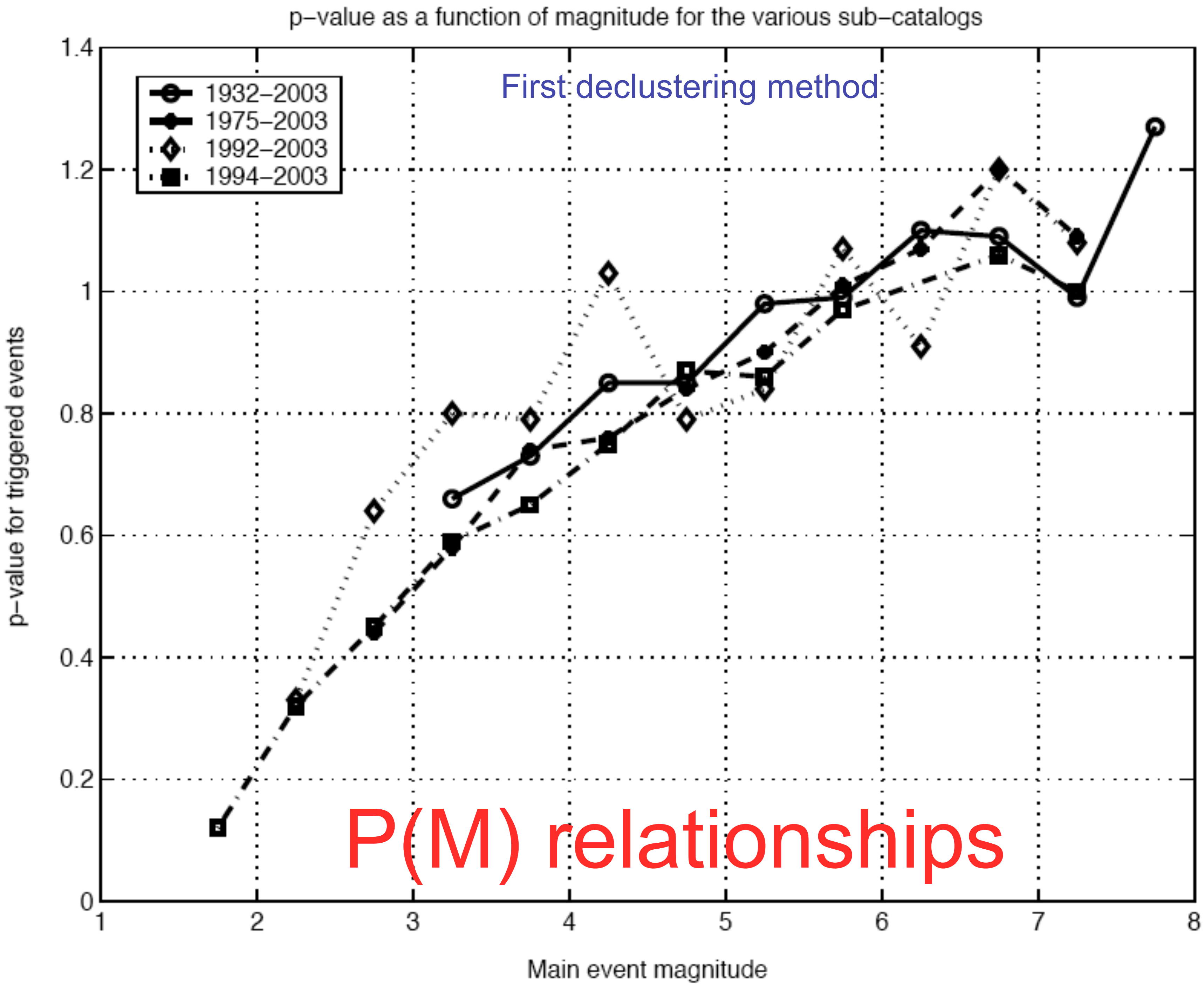
where B accounts for a constant background noise



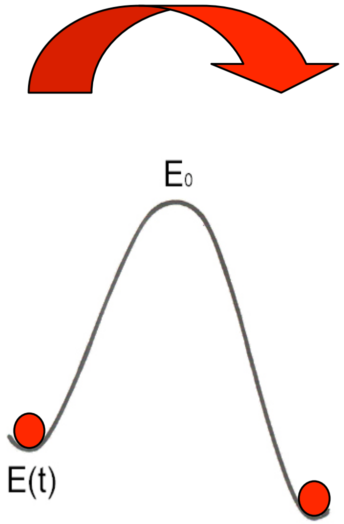




D. Sornette and G. Ouillon, Multifractal Scaling of Thermally-Activated Rupture Processes, Phys. Rev. Lett. 94, 038501 (2005)


p–value as a function of magnitude for the various sub–catalogs
First declustering method
1932–2003
1975–2003
1992–2003
1994–2003
p–value for triggered events
Main event magnitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1
2
3
4
5
6
7
8
P(M) relationships

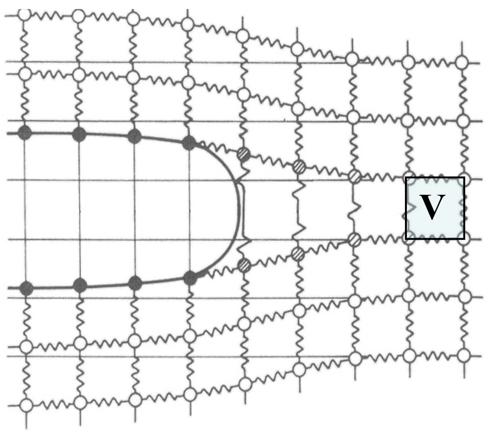
The Physics of Stress-Aided Thermal Activation of Rupture



In order to reach a state of lower energy, some microscopic physical systems must overcome an energy barrier.

The rate at which this is done depends **exponentially on the height of the barrier**, as well as on **the inverse of temperature**.

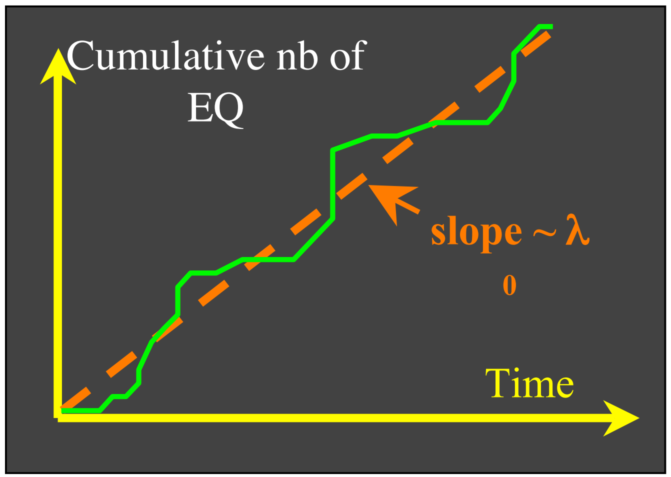
$$\lambda(t) = \lambda_0 \exp\left(-\frac{E_0 - E(t)}{kT}\right)$$



For example, the rate at which a bond breaks at a crack tip depends on the driving stress applied on that bond, its strength and temperature.

We simply assume that the relationship between the seismicity rate and the applied stress follows the same kind of law.

$$\lambda(t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(t)}{kT} V\right)$$



λ_0 ~ mean seismicity rate - $\lambda(t)$: seismicity rate - σ_0 : strength

$\sigma(t)$: applied stress - V : activation volume - T : temperature

k : Boltzmann’s constant

Experiments by Zhurkov   Int. J. Fract. Mech. **1**, 311 (1965)

$$\tau = \tau_o \exp\left(\frac{U}{k_B T}\right)$$

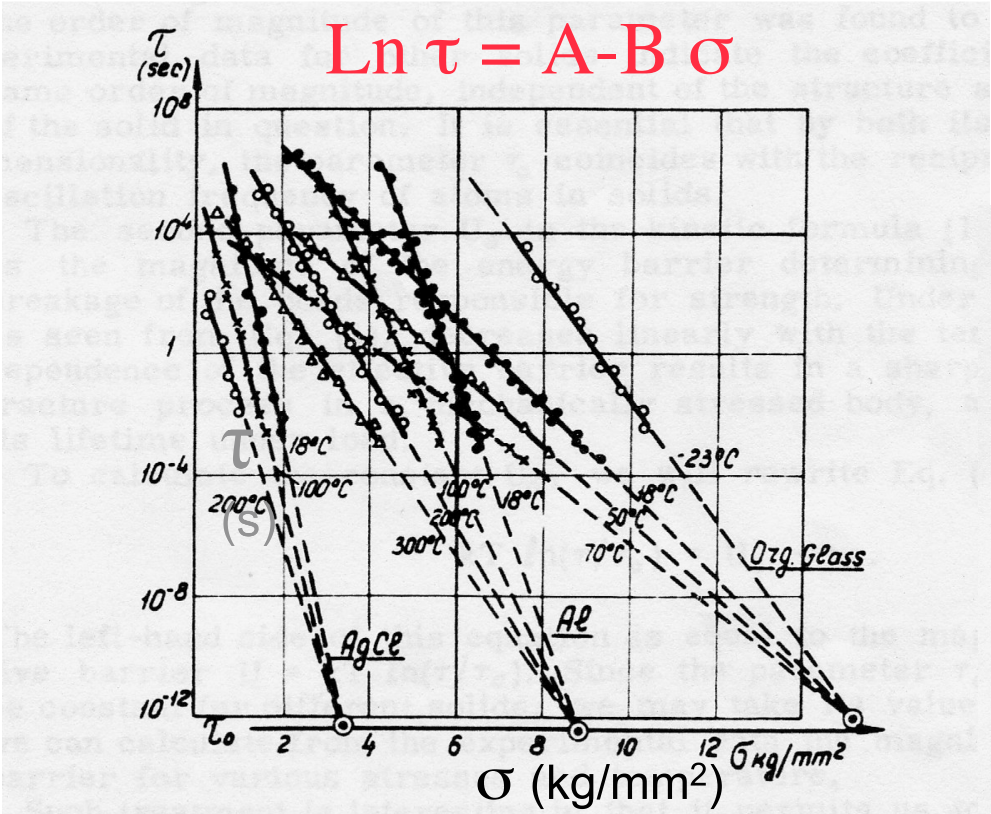


Fig. 5. Time and temperature dependence of the lifetime of solids on stress.
1. Silver chloride (Reference 4)
2. Aluminum (Reference 5)
3. Plexiglas (Reference 6)

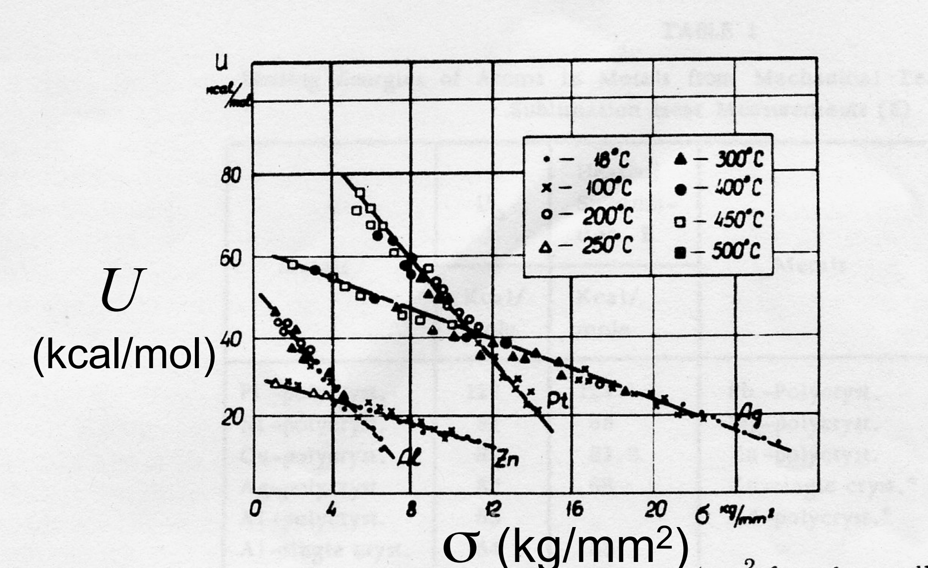


Fig. 6. Effective barrier U kkal/mol vs. tensile stress σ kg/mm² for polycrystall

Empirical energy barrier

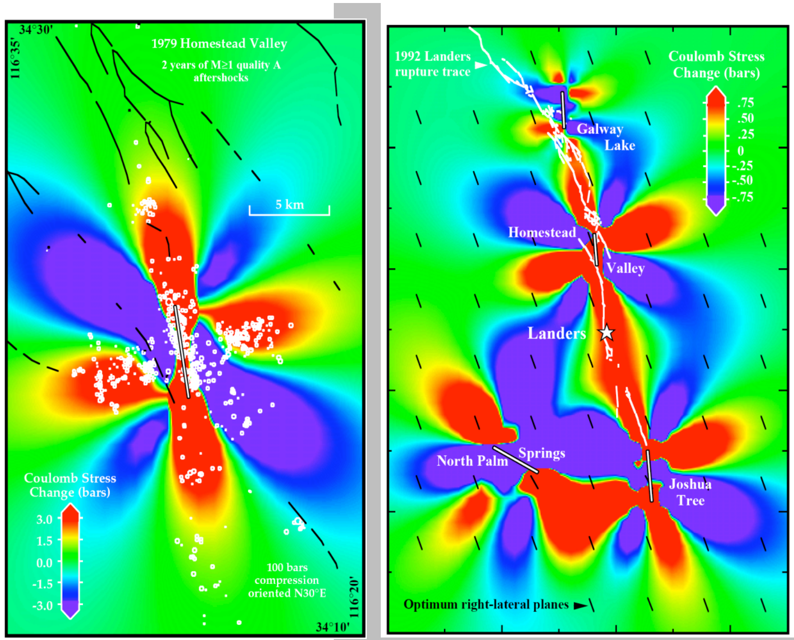
$$U = U_o - \alpha\sigma$$

où U_o : énergie de sublimation

A possible mechanism : thermal activated process

Taking account of history and boundary conditions

$$\lambda(\vec{r},t) = \lambda_0 \exp\left(-\frac{\sigma_0 - \sigma(\vec{r},t)}{kT}V\right)$$



$$\sigma(\vec{r},t) = \sigma(\vec{r})_{far\ field} + \int_{-\infty}^{t} \int_{space} dN\left[d\vec{\rho} \times d\tau\right] \Delta\sigma(\vec{\rho},\tau)\, G(\vec{r}-\vec{\rho},t-\tau)$$

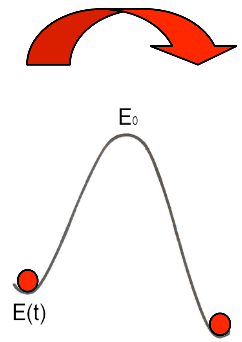
local stress — tectonic loading — Time and space density of past shocks — Stress fluctuations — Green function

Thermally activated multifractal rupture process

G. Ouillon and D. Sornette, Magnitude-Dependent Omori Law: Theory and Empirical Study, J. Geophys. Res., 110, B04306, doi:10.1029/2004JB003311 (2005); Multifractal Scaling of Thermally-Activated Rupture Processes, Phys. Rev. Lett. 94, 038501 (2005)

Intensity (average conditional seismicity rate)

At position r and time t



$$\lambda(\vec{r},t) \sim \exp\left[-\beta E(\vec{r},t)\right]$$

$$E(\vec{r},t) = E_0(\vec{r}) - V\Sigma(\vec{r},t)$$ (Zhurkov, 1965)

(due to stress corrosion, damage, state-and-velocity dependent friction and mechano-chemical effects

$$\Sigma(\vec{r},t) = \Sigma_{\text{far field}}(\vec{r},t) + \int_{-\infty}^{t} \int dN[d\vec{r}\,' \times d\tau]\Delta\sigma(\vec{r}\,',\tau) g(\vec{r} - \vec{r}\,', t-\tau)$$

$$g(\vec{r},t) = f(\vec{r}) \times h(t)$$

And sum over all spatial positions of sources gives

$$\lambda_i(t) = \lambda_{\text{tec}}(t) \exp\left[\beta \sum_j \int_{-\infty}^{t} d\tau\ \Delta\sigma_j(\tau)\ g_{ij}(t-\tau)\right]$$

Generalization of stress release models [Vere-Jones et al.]

Our Physical Picture of Seismicity

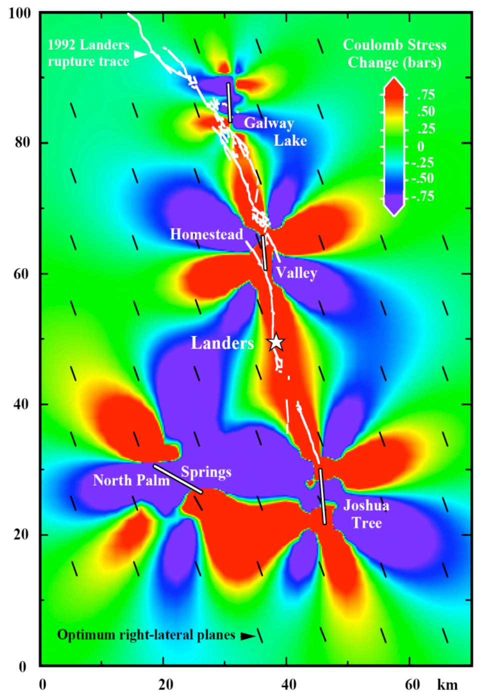
- The **rupture** of each event **is thermally activated**, driven by stress.
- Each shock induces instantaneously a **burst of aftershocks,** which amounts to 10^{qM} events.
- At each location, **stress fluctuations** due to previous events are distributed as:

$$P(\sigma)\,d\sigma \approx \frac{C}{(\sigma+\sigma_0)^{1+\mu}}\,d\sigma$$

- The **rheology is viscoplastic**, with a relaxation function featuring a very large relaxation time τ_M :

$$h(t) = \frac{h_0}{(t+t_1)^{1+\theta}} \exp\left(-\frac{t}{\tau_M}\right) \qquad \theta = -1/2+\varphi$$

- At any place *r* and any time *t*, **the seismicity rate** (on the left-hand side) **depends exponentially on the stress fluctuations** due to past earthquakes, mediated by the relaxation function:



This picture takes account of the fact that earthquakes generate stress fluctuations, which in turn modify the stress state in a feedback loop and cascading process, involving the whole history of the system.

$$\lambda(r,t) = \lambda_{tec} \exp\left[\frac{V}{kT}\sum_{passé}\sigma(t_i)\,h(t-t_i)\right]$$

Theoretical predictions using tail covariance (Ide-Sornette, 2001)

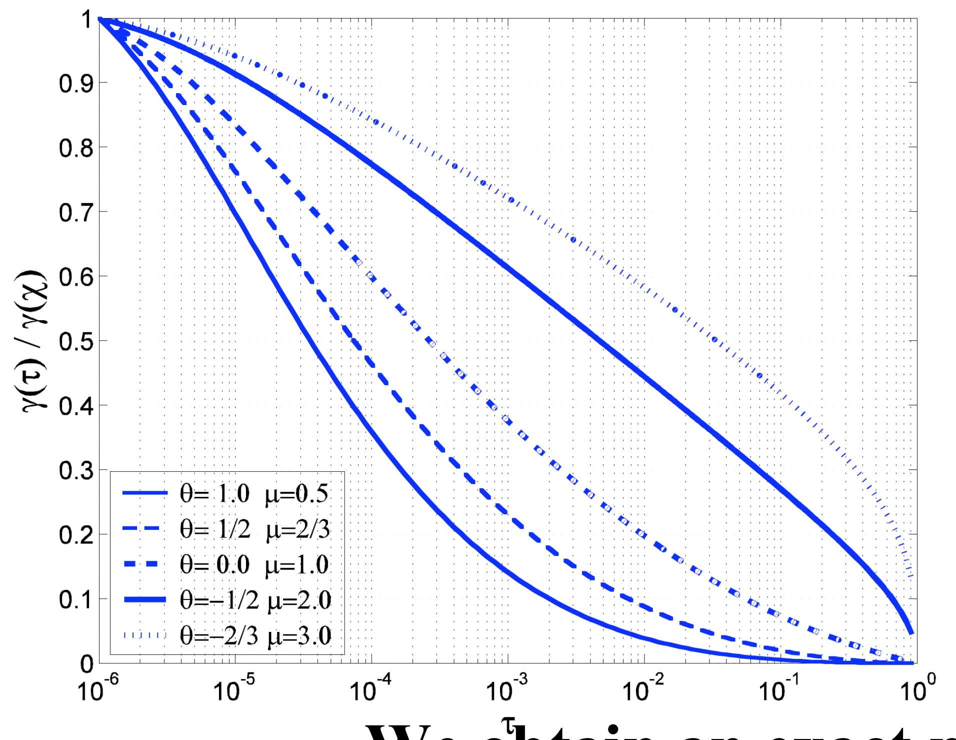
$$\Pr[\lambda(t) > \lambda_q | \lambda_M] = \Pr[e^{\beta\omega(t)} > \frac{\lambda_q}{\lambda_{\text{tec}}} | \omega_M] = \Pr[\omega(t) > (1/\beta)\ln\left(\frac{\lambda_q}{\lambda_{\text{tec}}}\right) | \omega_M]$$

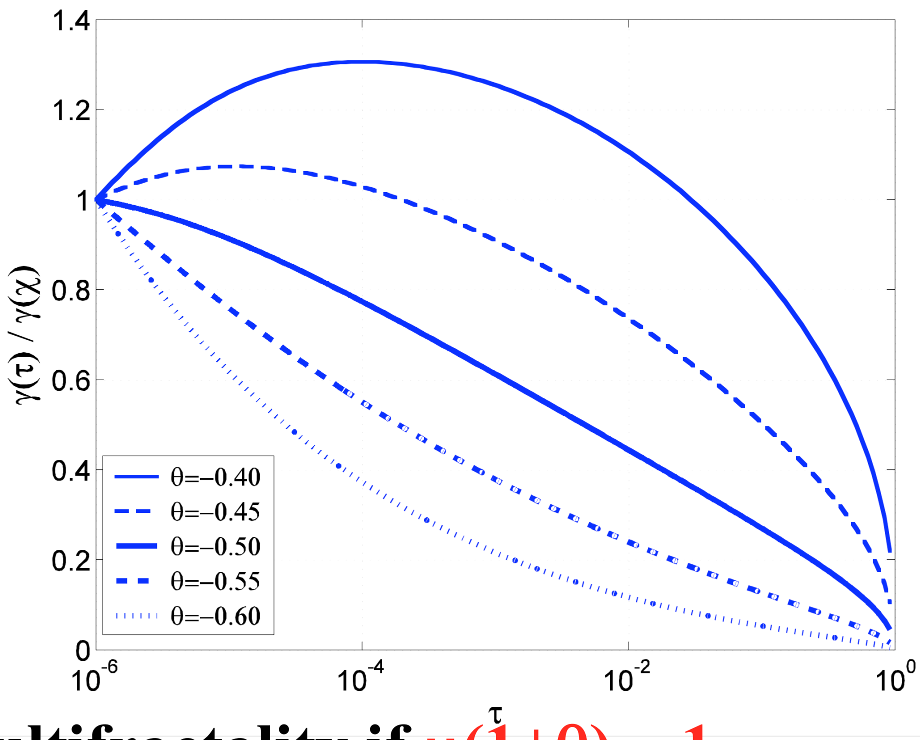
$$\lambda_q(t) = A_q \; \lambda_{\text{tec}} \; e^{\beta\gamma(t)\omega_M}$$

$$\gamma(t) = \frac{h_0^2}{\Delta t^{2/\mu}} \left(\frac{1}{t^{2m-1}} \int_{c/t}^{\frac{T+c}{t}-1} dy \frac{1}{(y+1)^m} \frac{1}{y^m} \right)^{\frac{2}{\mu}}$$

$$m = (1+\theta)\mu/2.$$

Since $\gamma(t) \sim \ln(t)$ and $\omega_m \sim M$, we obtain $p(M) = a\,M + b$





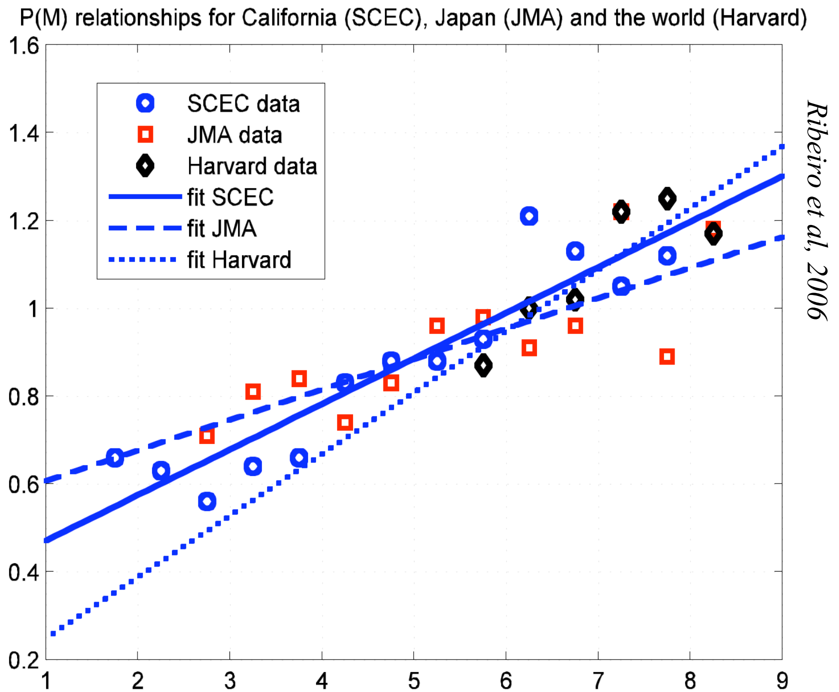
We obtain an exact multifractality if $\mu(1+\theta) \sim 1$

p(M) = aM + b

We processed three catalogs, that we pre-processed to check for their completude and its evolution with time.
We then computed stacked aftershocks time series, sorting them within intervals of 0.5 magnitude amplitudes.

We clearly observed a linear dependence of *p* with magnitude M.

Statistical tests have been performed using a bootstrap strategy, and we were able to show that all slopes were significantly different from 0, and that all linear relationships were significantly different from each other.



For Southern California (SCEC catalog):

$$p(M) = 0.10M + 0.37$$

For Japan (JMA catalog):

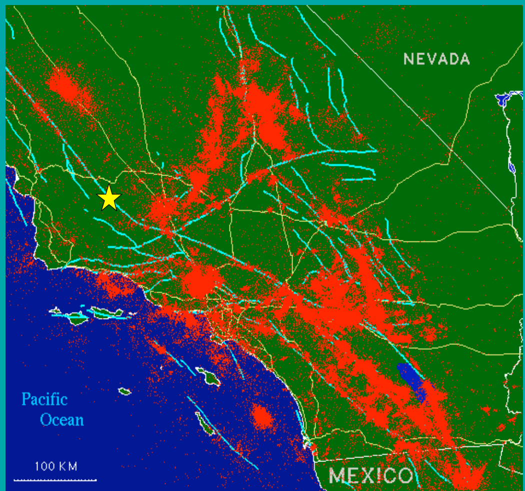
$$p(M) = 0.07M + 0.54$$

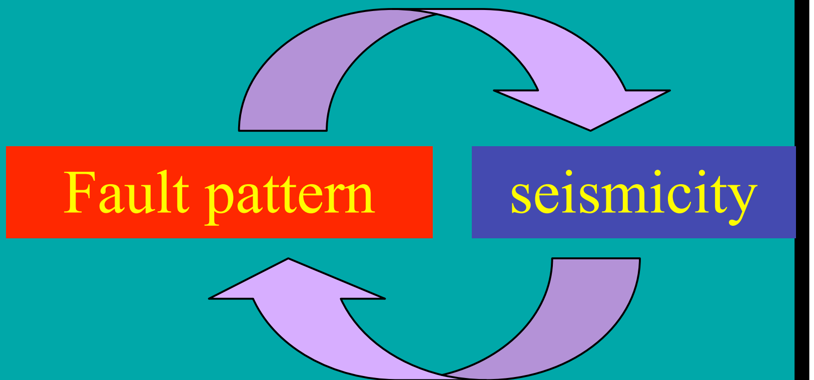
For the World (Harvard catalog):

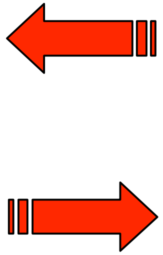
$$p(M) = 0.14M + 0.11$$

μ(1+θ) : evidence of self-organization ?

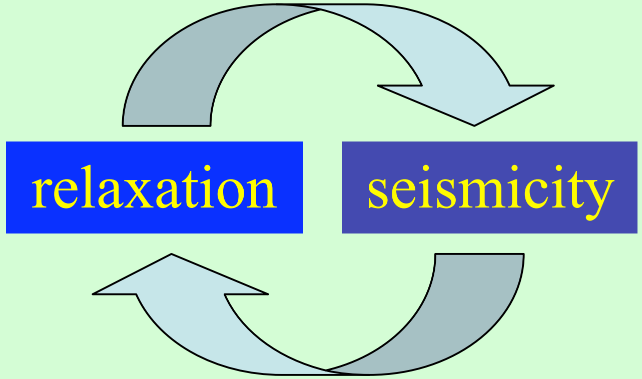
$$P(\sigma)\,d\sigma \approx \frac{C}{(\sigma+\sigma_0)^{1+\mu}}\,d\sigma$$







$$h(t) = \frac{h_0}{(t+t_1)^{1+\theta}} \exp\left(-\frac{t}{\tau_M}\right)$$



μ controls stress fluctuations, which mainly depend on the spatial structure of the fault network over which events occur – which make the fault pattern grow (left).

θ controls the stress relaxation in rocks. Stress determines the seismicity rate, but earthquakes are themselves part of the stress relaxation complex process (top).

All in all, the condition μ(1+θ) ~ 1 reflects the critical self-organization of brittle processes in the earth's crust.

Conclusion (earthquakes)

• The multifractal time distribution of earthquakes implies that **the exponent *p* of the Omori law increases with the magnitude M of the mainshock.**

• Empirical data observations on various catalogs suggest that *p* linearly increases with M: **p(M) = aM + b**

• We proposed a physical model where the **seismicity rate depends exponentially on stress and on inverse temperature**, and where **the rheology is viscoplastic** with a slow relaxation.

• A condition **linking the fault network geometry and the rheology of the tectonic system** emerges to explain such a multifractal phenomenology : $\mu(1+\theta) \sim 1$ – we speculate that it is a **fundamental equation of self-organized criticality.**

• For the first time, a physical microscopic model is proposed that is able to explain earthquakes time dynamics at all scales.

References:

G. Ouillon and D. Sornette: Magnitude-dependent Omori law: Theory and empirical study, J. Geophys. Res., VOL. 110, B04306, doi:10.1029/2004JB003311, 2005

E. Ribeiro, G. Ouillon and D. Sornette: New empirical results on the magnitude-dependent Omori law, submitted to Geophys. Res. Lett., 2006

A.S. Krausz and K. Krausz: Fracture kinetics of crack growth, Kluwer, 1987

Stein, R. S.: Earthquake conversations, Sci. Am., 288(1), 72–79, 2003

Summary

• Multifractality in the time domain: conditional response functions

•Financial volatility

•Robust multifractality in exponential of long-memory processes

•Multifractal Omori law for the relaxation of earthquake aftershocks