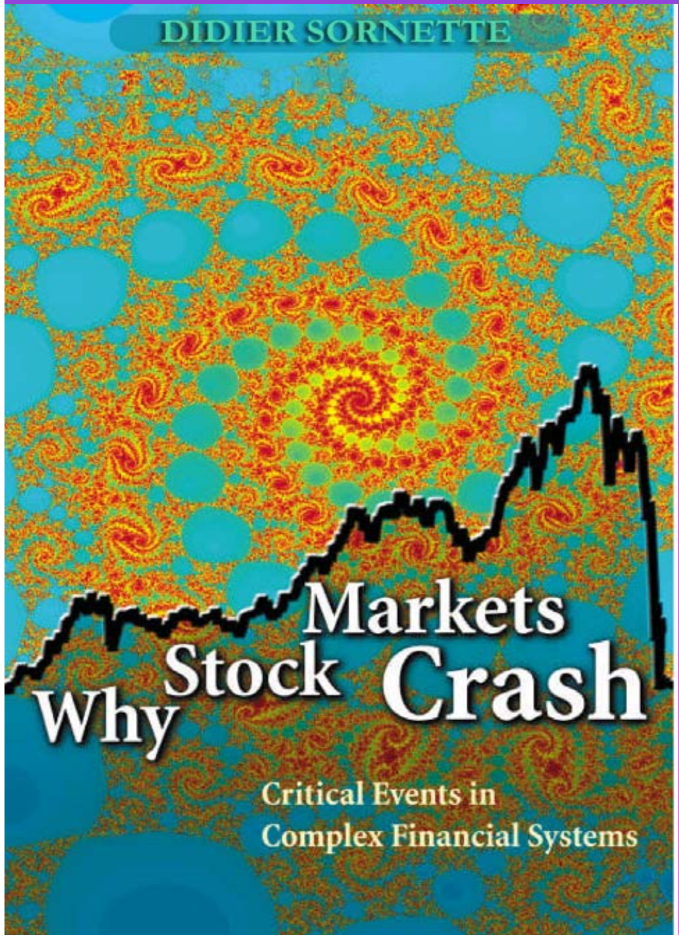
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Power laws and scaling in finance

Practical applications for risk control and management

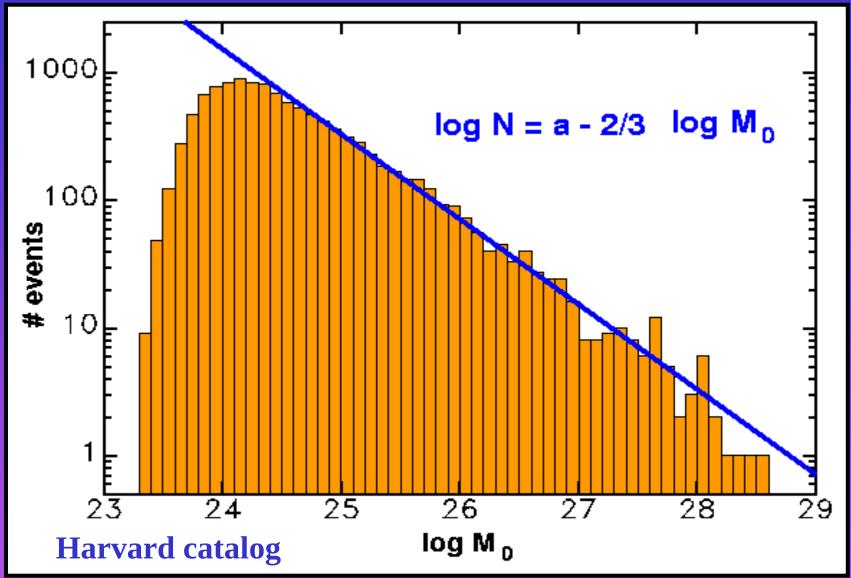


D. SORNETTE

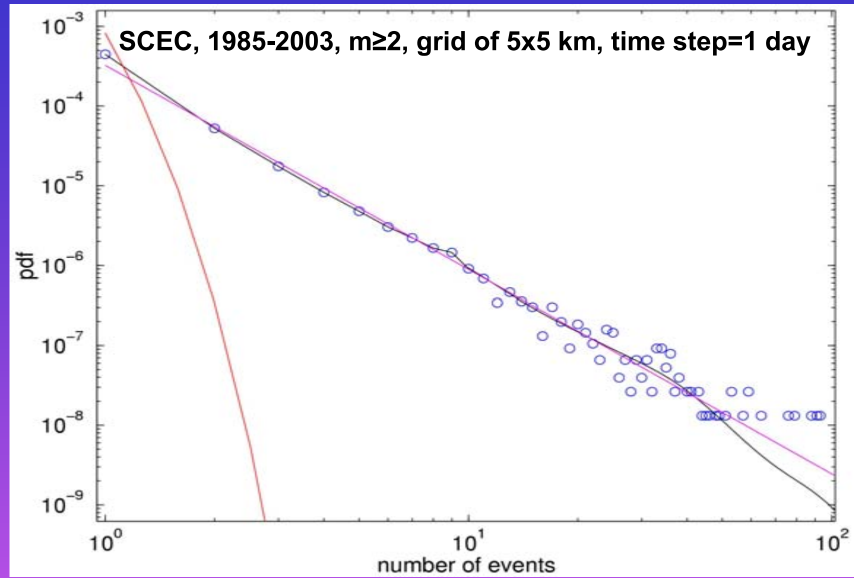
ETH-Zurich
Chair of Entrepreneurial Risks
Department of Management, Technology and Economics (D-MTEC)

http://www.mtec.ethz.ch/

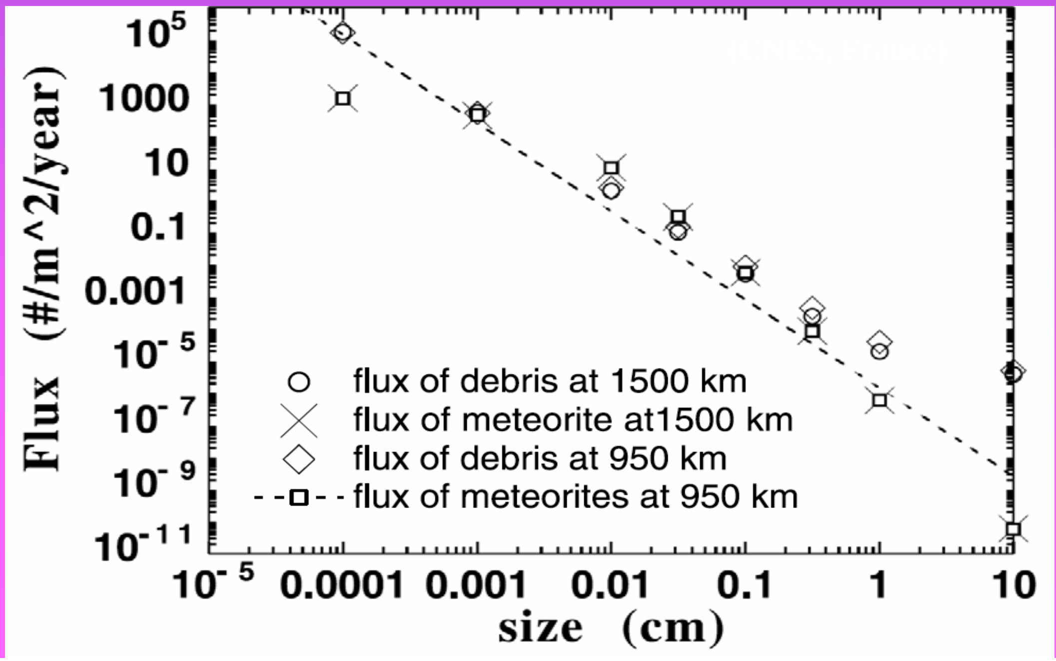
Heavy tails in pdf of earthquakes



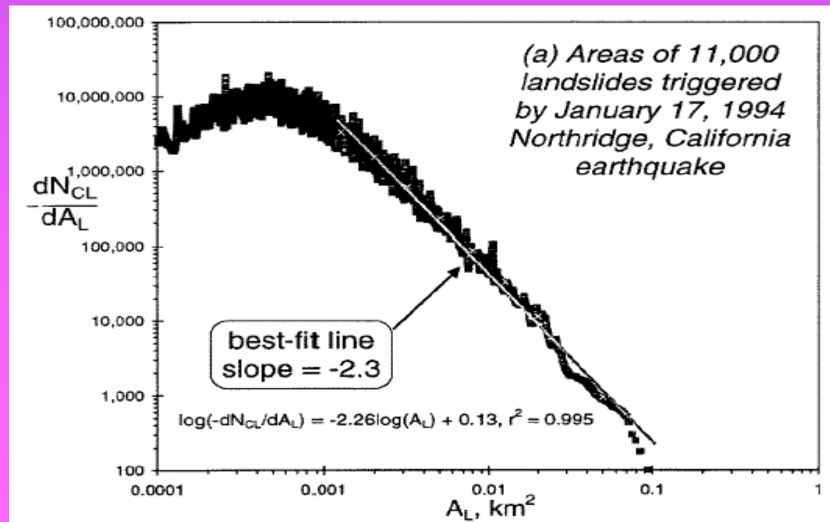
Heavy tails in pdf of seismic rates



Heavy tails in ruptures



Heavy tails in pdf of rock falls, Landslides, mountain collapses



Turcotte (1999)

Heavy tails in pdf of forest fires

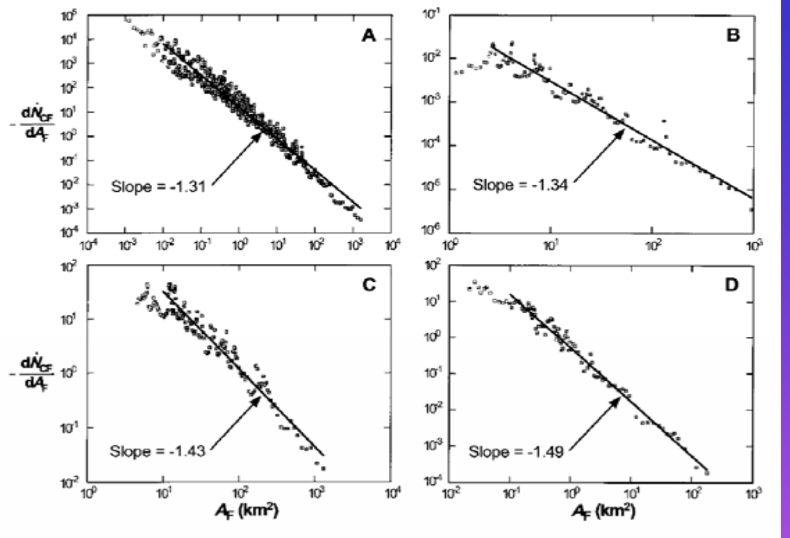
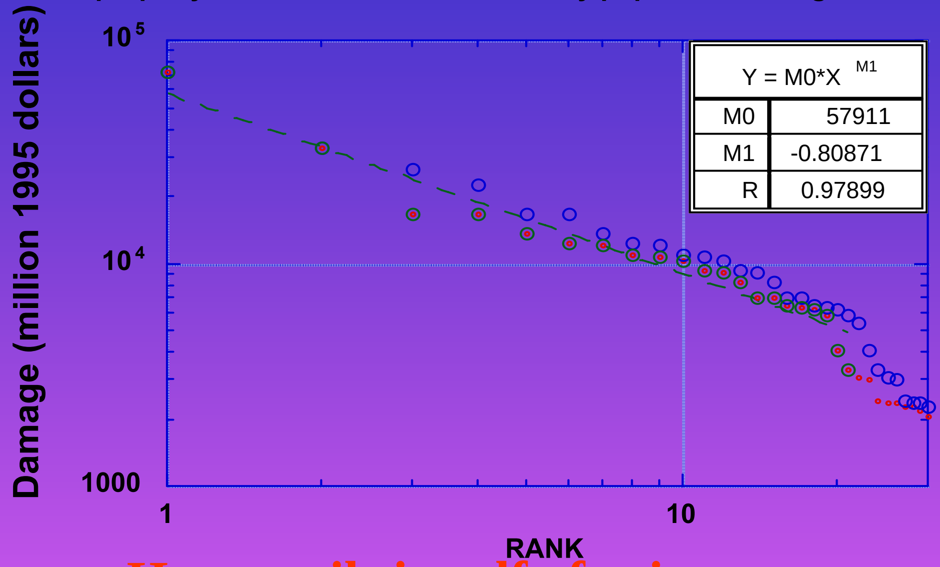


Fig. 2. Noncumulative frequency-area distributions for actual forest fires and wildfires in the United States and Australia: (**A**) 4284 fires on U.S. Fish and Wildlife Service lands (1986–1995) (*9*), (**B**) 120 fires in the western United States (1150–1960) (*10*), (**C**) 164 fires in Alaskan boreal forests (1990–1991) (*11*), and (**D**) 298 fires in the ACT (1926–1991) (*12*). For each data set, the noncumulative number of fires per year ($-d\dot{N}_{CF}/dA_F$) with area (A_F) is given as a function of A_F (*13*). In each case, a reasonably good correlation over many decades of A_F is obtained by using the power-law relation (Eq. 1) with α = 1.31 to 1.49; $-\alpha$ is the slope of the best-fit line in log-log space and is shown for each data set.

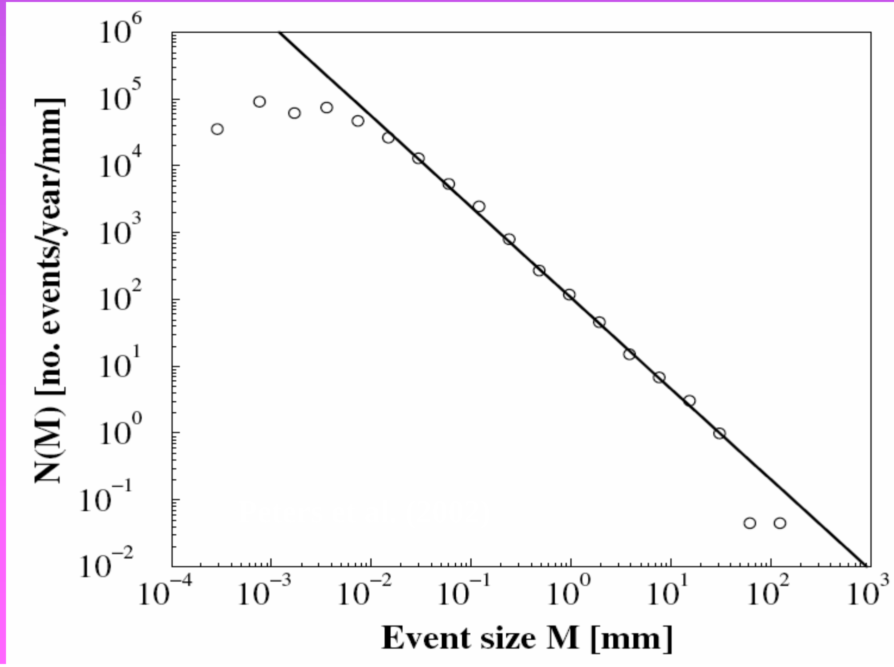
Malamud et al., Science 281 (1998)

Heavy tails in pdf of Hurricane losses

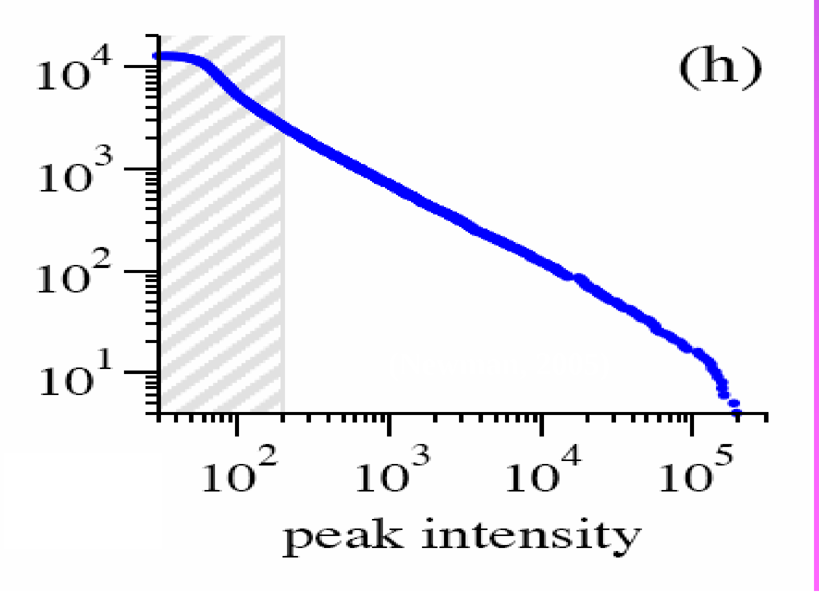
Damage values for top 30 damaging hurricanes normalized to 1995 dollars by inflation, personal property increases and coastal county population change

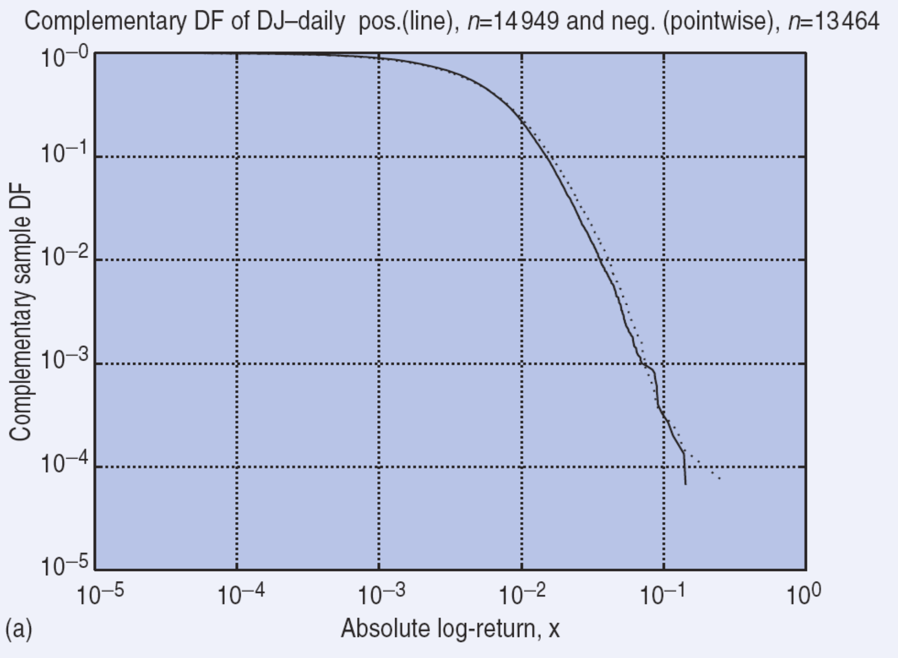


Heavy tails in pdf of rain events

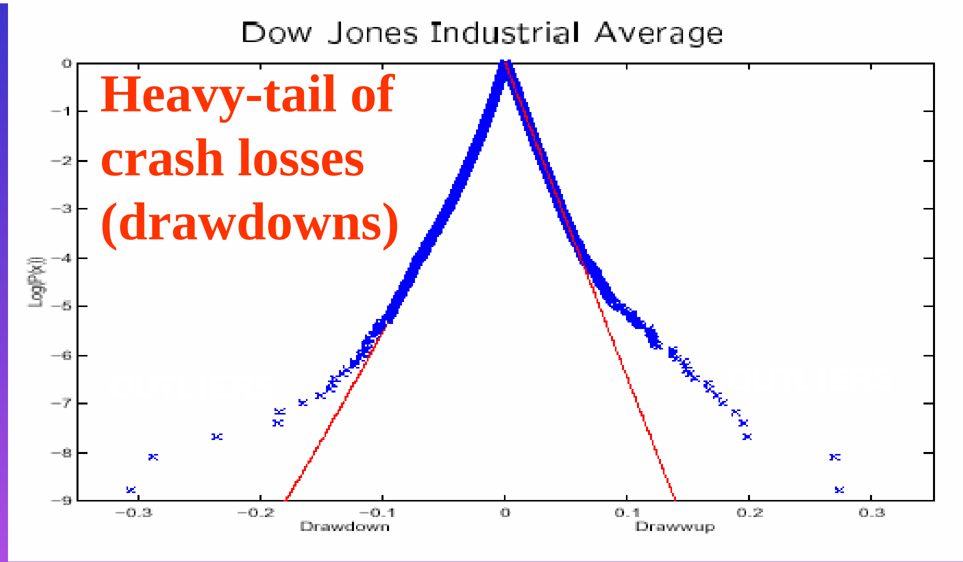


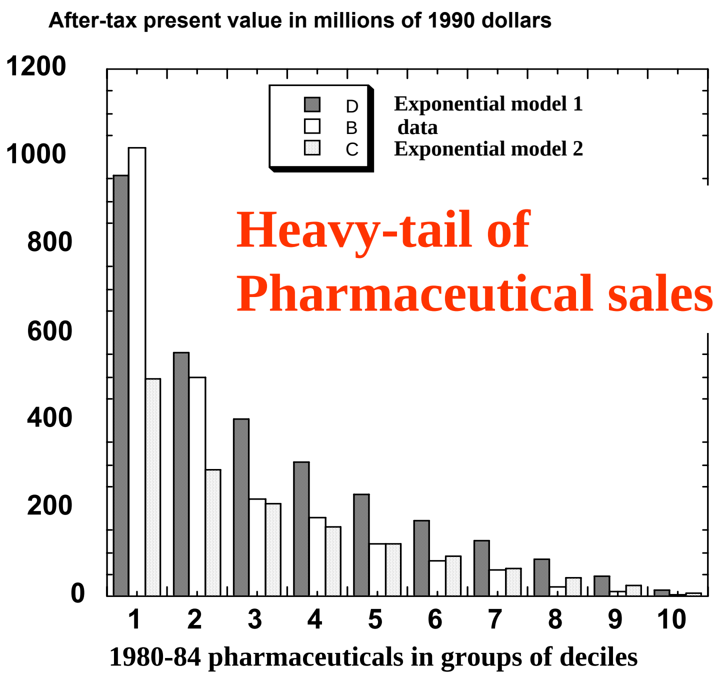
Heavy tails in pdf of Solar flares



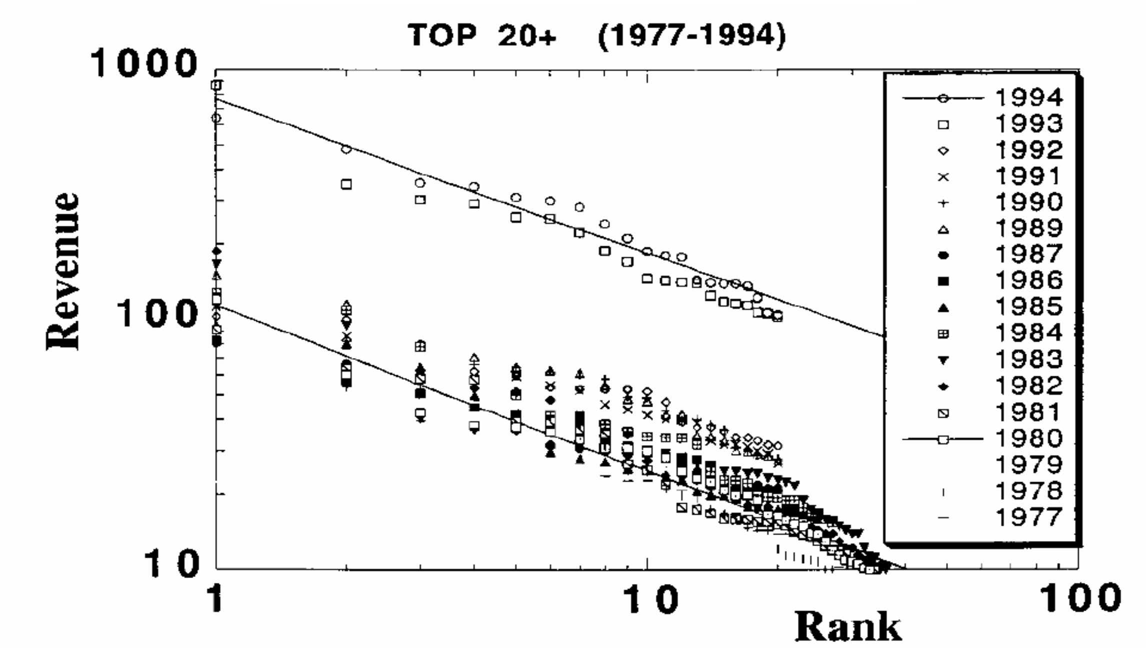


Heavy-tail of price changes

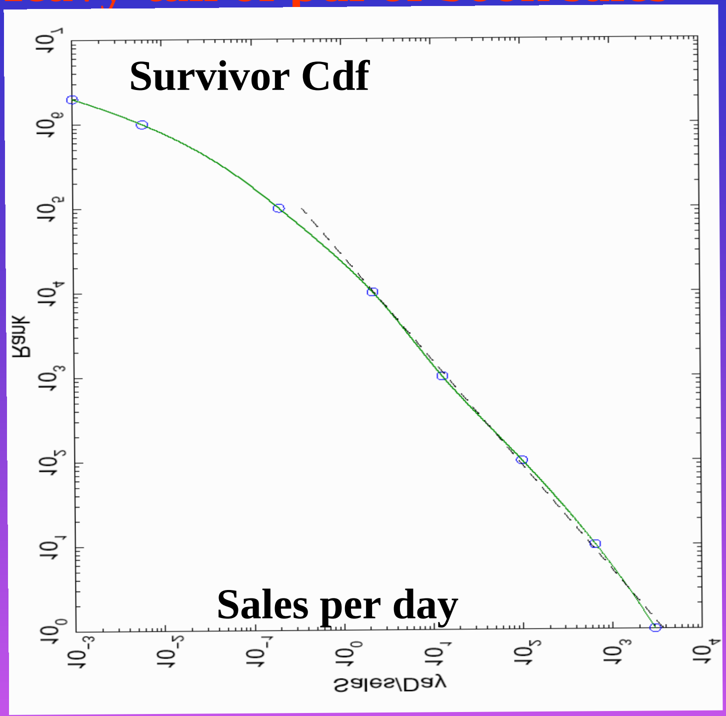




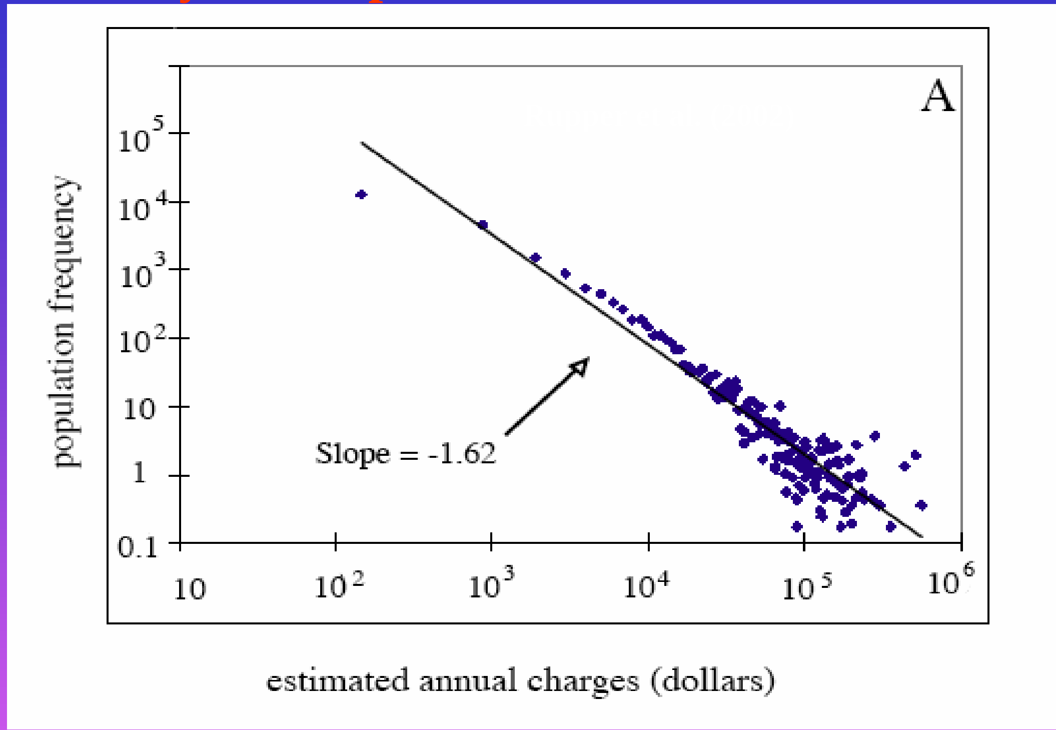
Heavy-tail of movie sales



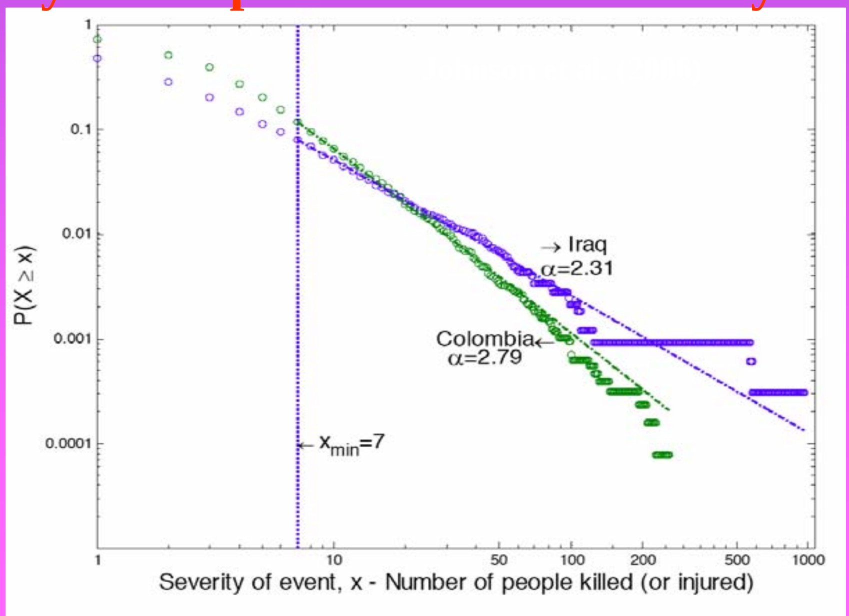
Heavy-tail of pdf of book sales

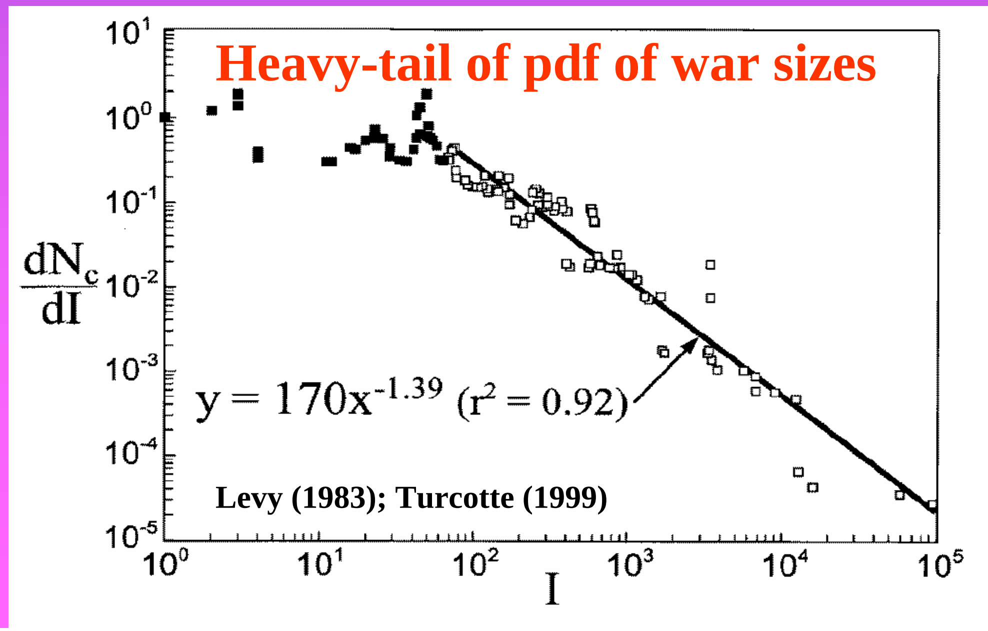


Heavy-tail of pdf of health care costs



Heavy-tail of pdf of terrorist intensity



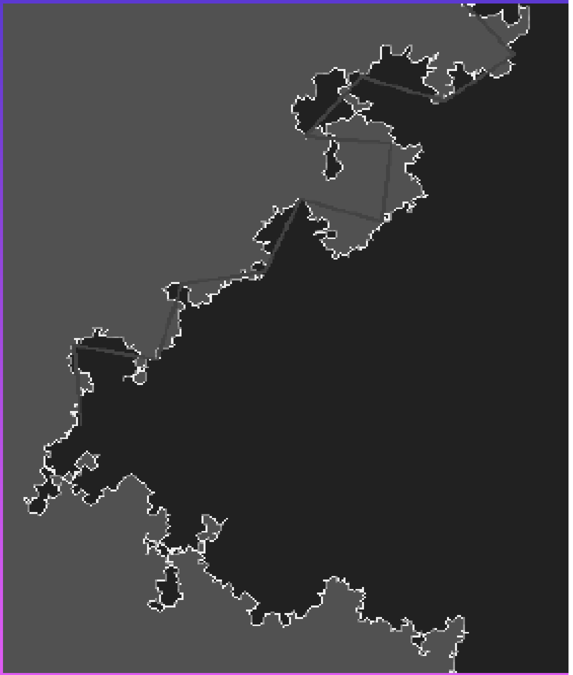


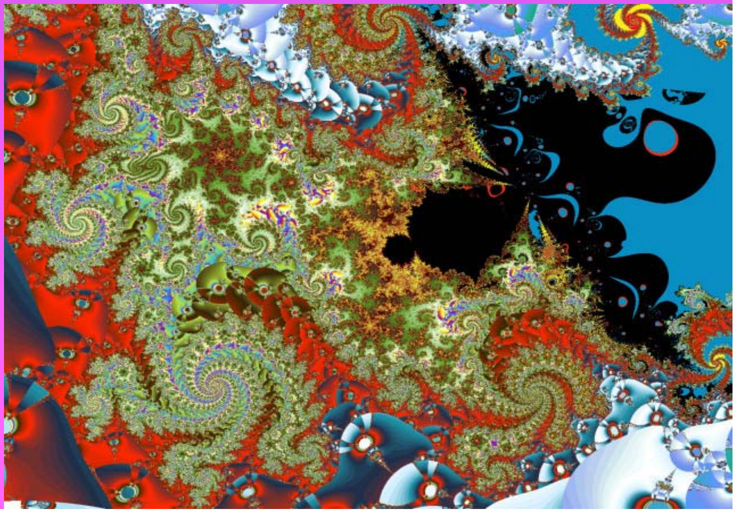
Power laws and large risks

- Power laws are ubiquitous
- They express scale invariance
- Probability of large excursion:

  -example of height vs wealth

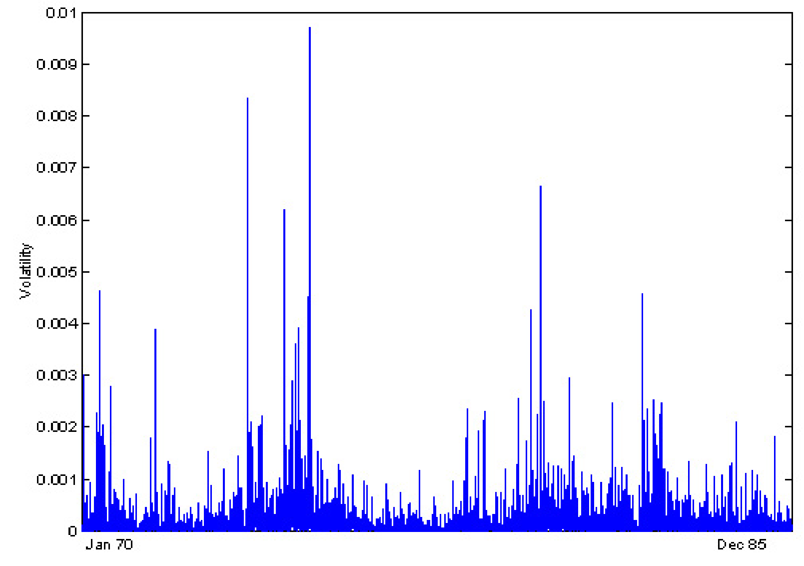
- Gaussian approach inappropriate: underestimation of the real risks
  - Markowitz mean-variance portfolio
  - Black-Scholes option pricing and hedging
  - Asset valuation (CAPM, APT, factor models)
  - Financial crashes

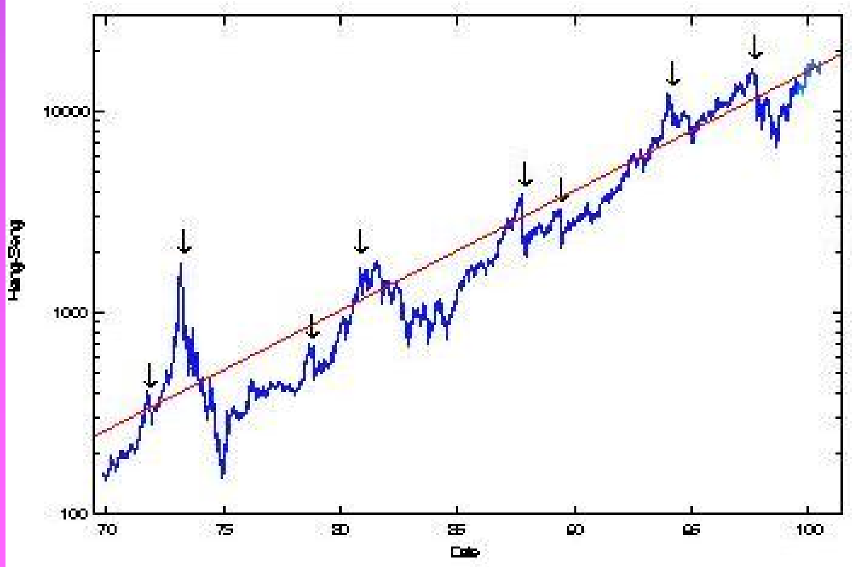




Stylized facts for financial data

- Distributions with heavy tails
- Clusters of volatility,
- Multifractality,
- Leverage effect,
- Super-exponential growth of speculative bubbles.





Empirical Results about the Distributions of Returns

- Models in terms of Regularly varying distributions:

$$\Pr\left[r_t \geq x\right] = \mathcal{L}(x) \cdot x^{-\mu} \qquad (\mu \approx 3-4)$$

Longin (1996) , Lux (1996-2000), Pagan (1996), Gopikrishnan et *al.* (1998)…

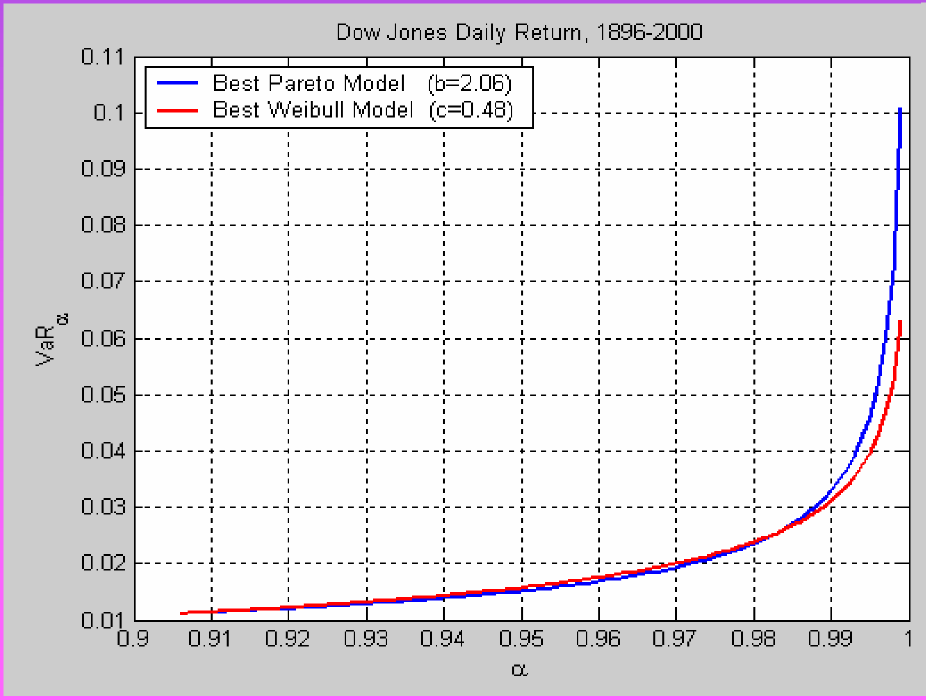
- Models in terms of Weibull-like distributions:

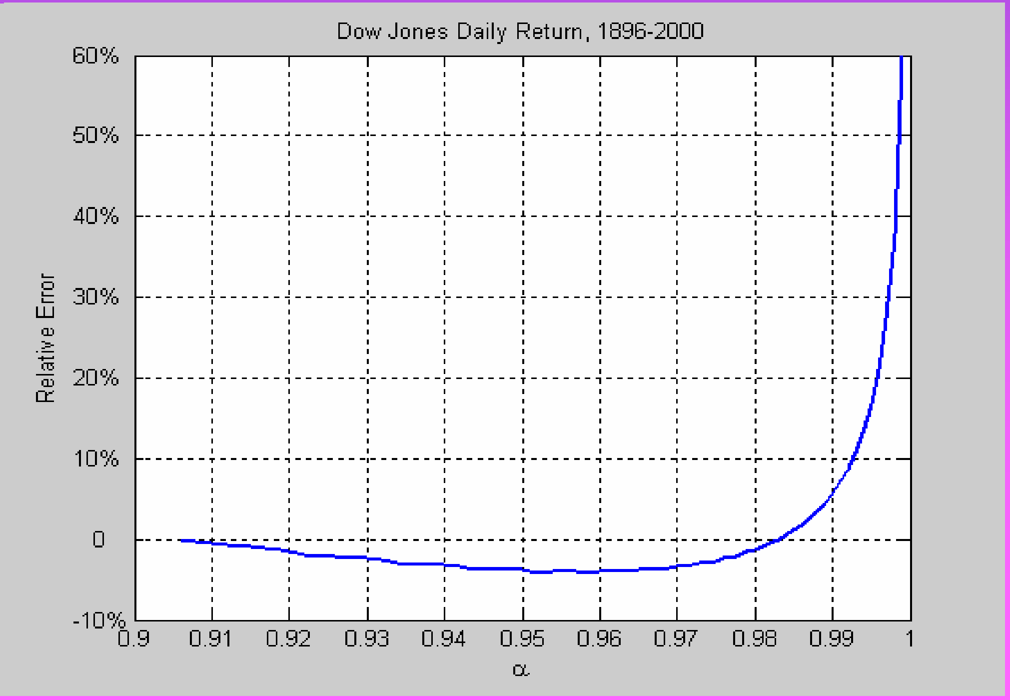
$$\Pr\left[r_t \geq x\right] = \exp\left[-\mathcal{L}(x) \cdot x^{c}\right] \qquad (c < 1)$$

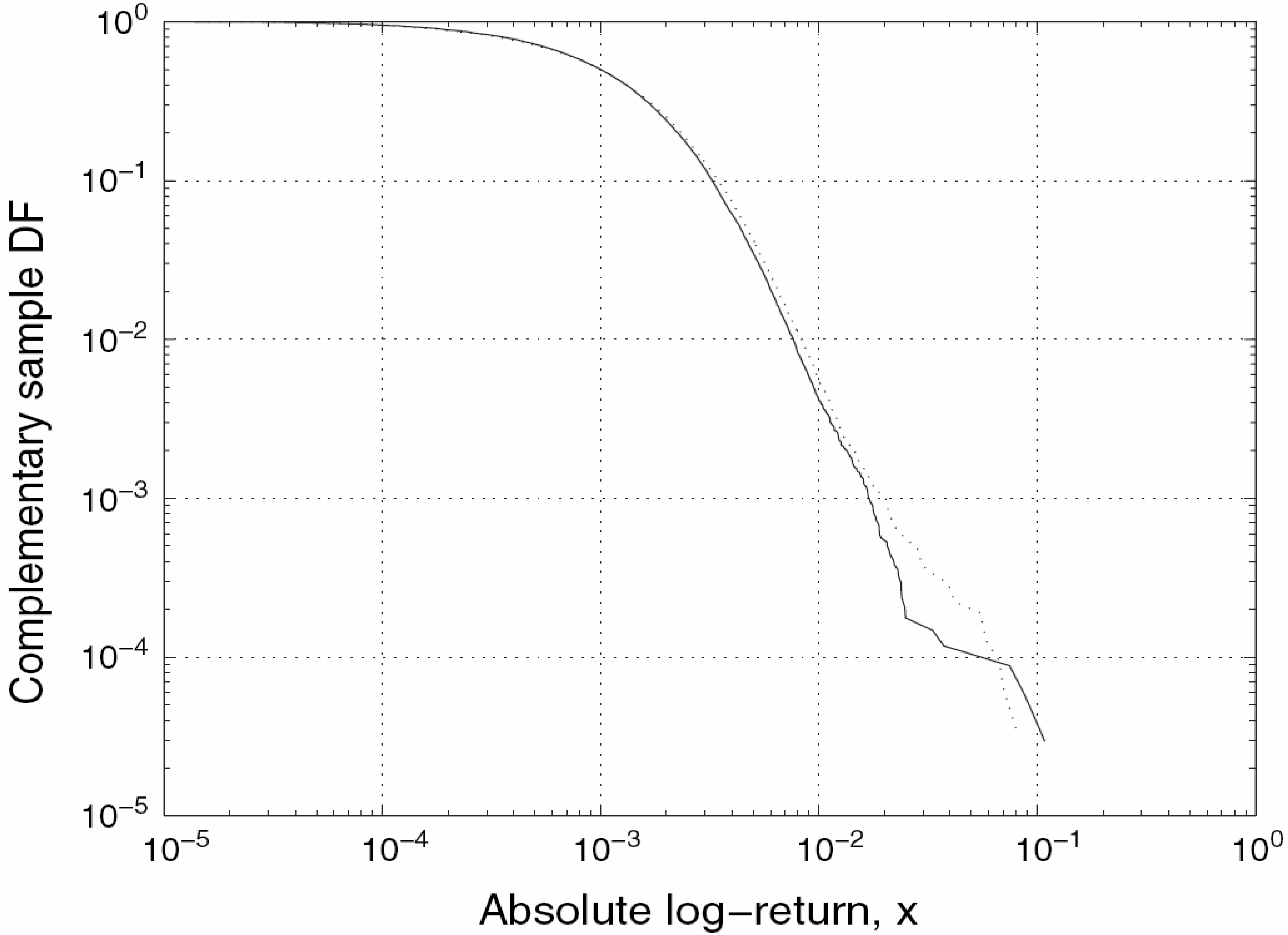
Mantegna and Stanley (1994), Ebernlein et *al.*(1998), Gouriéroux and Jasiak (1998), Laherrère and Sornette (1999)…

Implications of the two models

- Practical consequences :
  - Extreme risk assessment,
  - Multi-moment asset pricing methods.





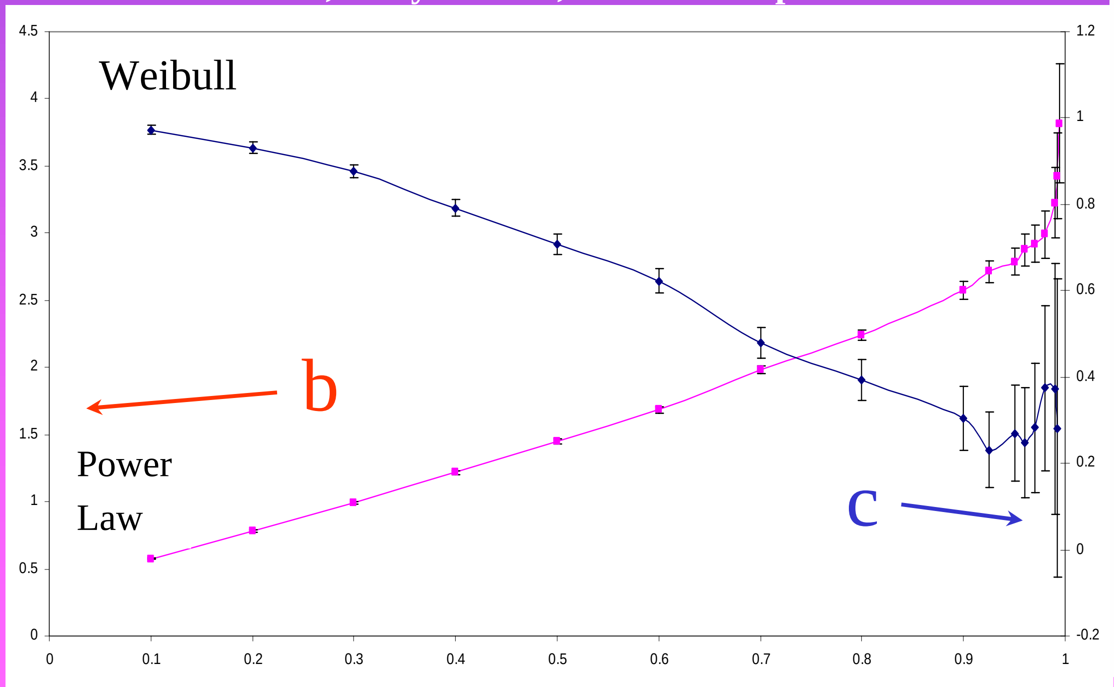


Complementary sample distribution function for the Standard & Poor's 500 30-minute returns over the two decades 1980–1999. The plain (resp. dotted) line depicts the complementary distribution for the positive (the absolute value of negative) returns.

Main Results

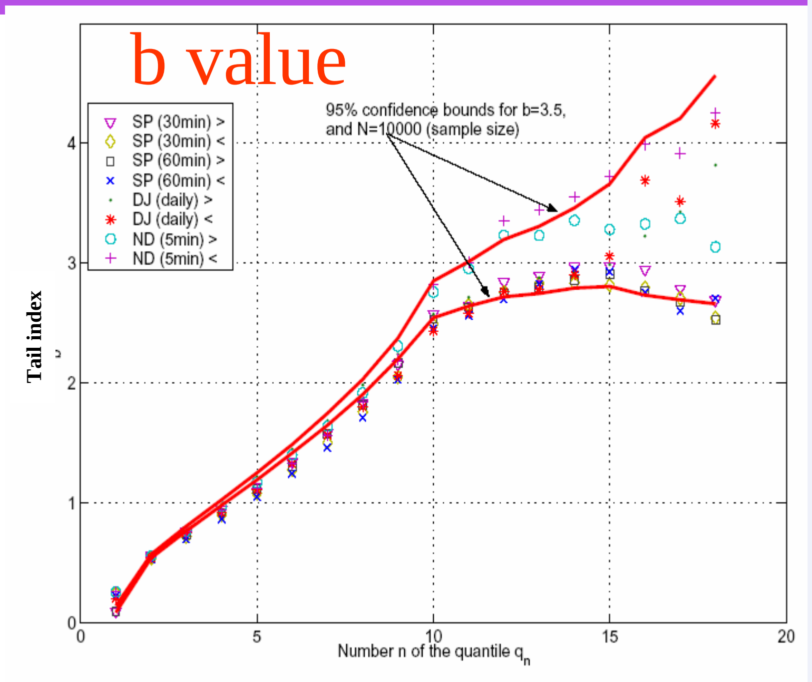
- For sufficiently high thresholds, both the Power laws and Weibull distributions comply with the data.
- For both models, the evolution of the parameters is not exhausted at the end of the range of available data.

Dow Jones, Daily returns, 1896-2000 – positive tail



Quantile U

Power law



$q_1 = 0$
$q_2 = 0.1$
$q_3 = 0.2$
$q_4 = 0.3$
$q_5 = 0.4$
$q_6 = 0.5$
$q_7 = 0.6$
$q_8 = 0.7$
$q_9 = 0.8$
$q_{10} = 0.9$
$q_{11} = 0.925$
$q_{12} = 0.95$
$q_{13} = 0.96$
$q_{14} = 0.97$
$q_{15} = 0.98$
$q_{16} = 0.99$
$q_{17} = 0.9925$
$q_{18} = 0.995$
$q_{19} = 0.999$
$q_{20} = 0.9995$
$q_{21} = 0.9999$

Value@Risk confidence level = 1%

Monthly Data		Realised losses <-VaR
>3 years	2528 Hedge funds (126100 mths)	**0.97%**
	5000 simulated portfolios of 100 funds	0.96%
	5000 simulated portfolios of 25 funds	0.97%
2 years	3067 Hedged Funds (156912 mths)	1.17%
1 year	3067 Hedged Funds (156912 mths)	1.53%

Track Value™

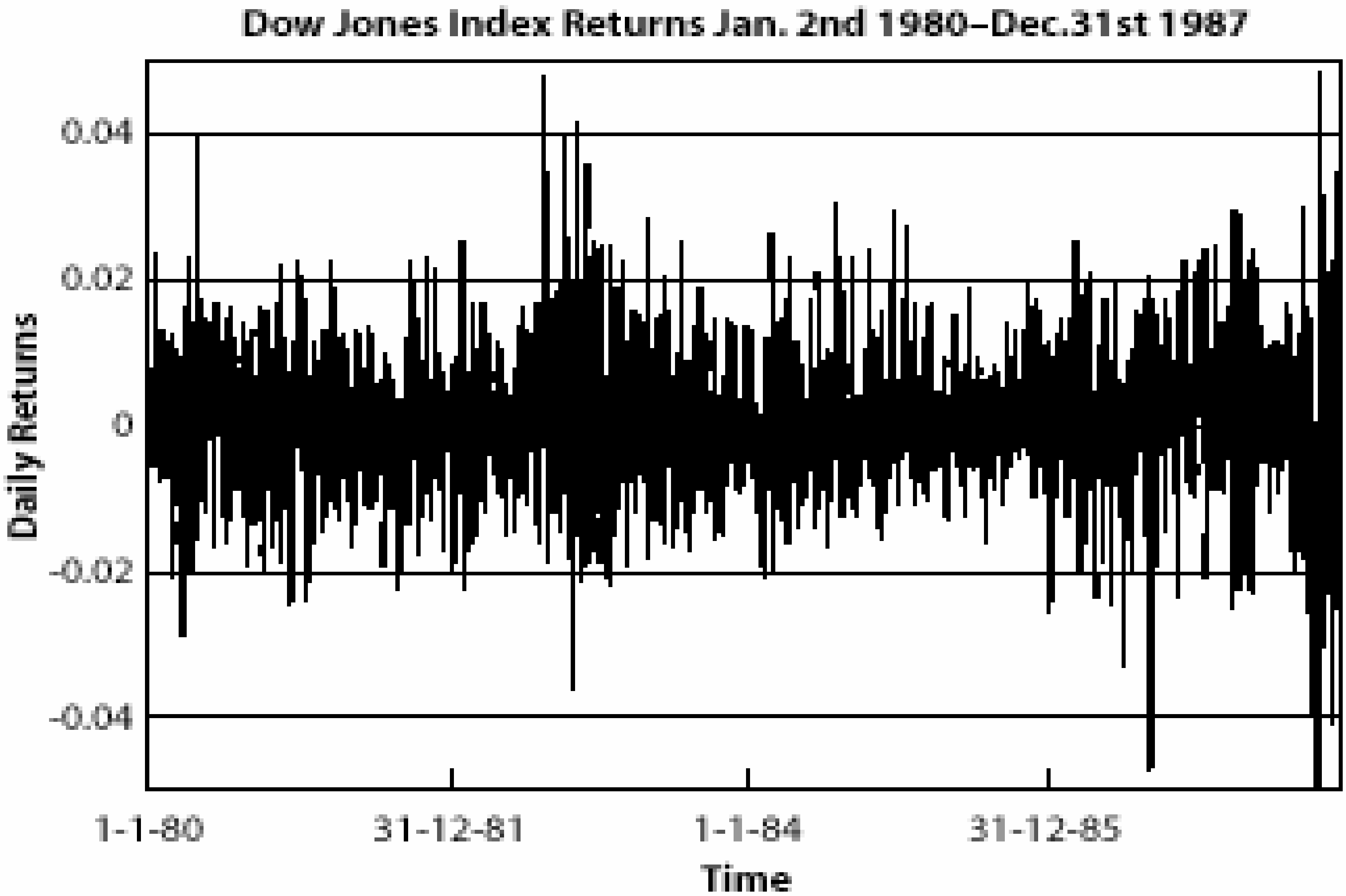
Software for funds of funds risk management

INSIGHT RESEARCH

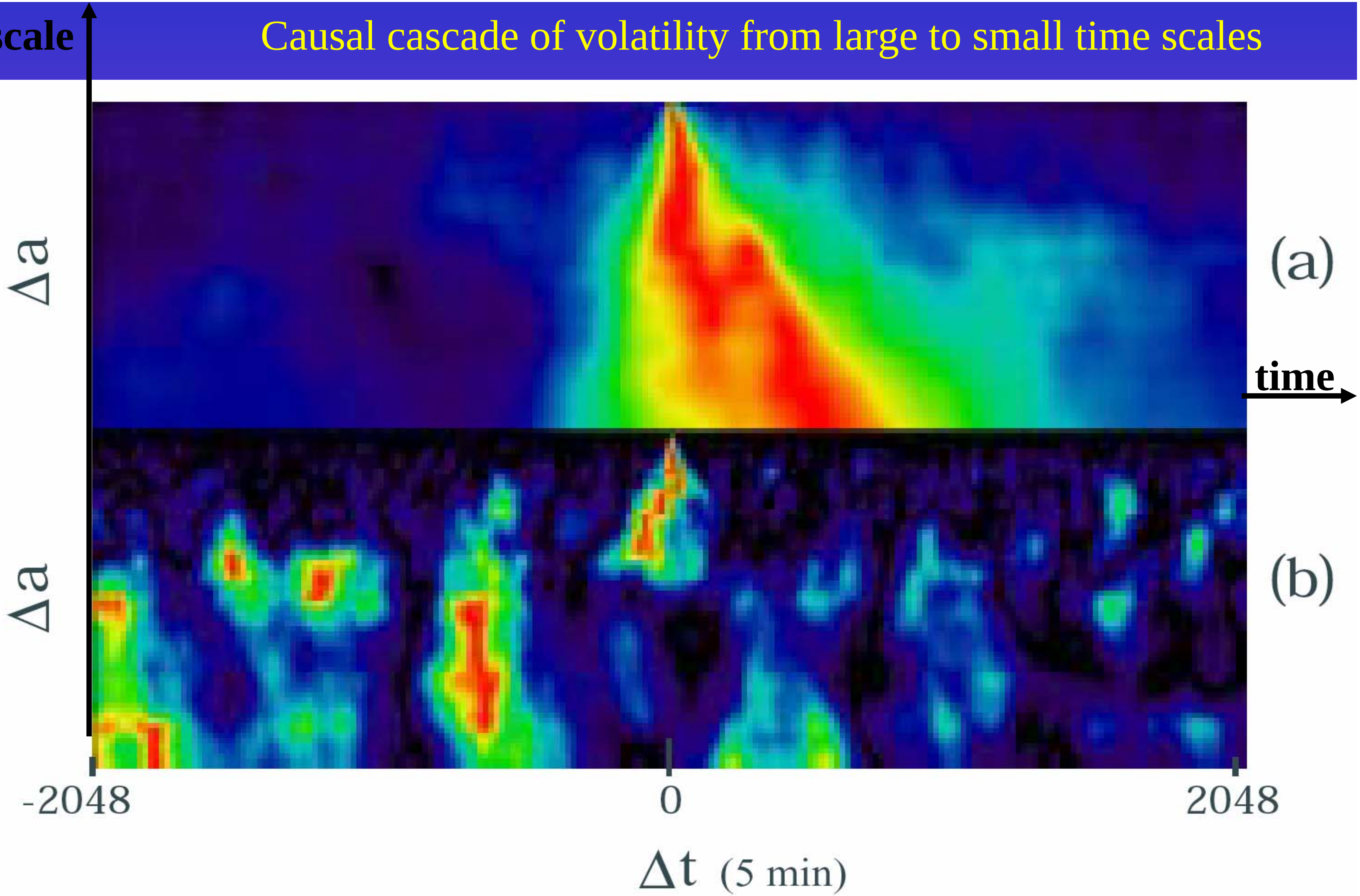
Value@Risk confidence level = 5%

Monthly Data		Realised losses <-VaR
>3 years	2528 Hedge funds (126100 mths)	**4.86%**
	5000 simulated portfolios of 100 funds	4.86%
	5000 simulated portfolios of 25 funds	4.87%
2 years	3067 Hedged Funds (156912 mths)	5.09%
1 year	3067 Hedged Funds (156912 mths)	5.79%

Forecast of Financial Volatility



Causal cascade of volatility from large to small time scales

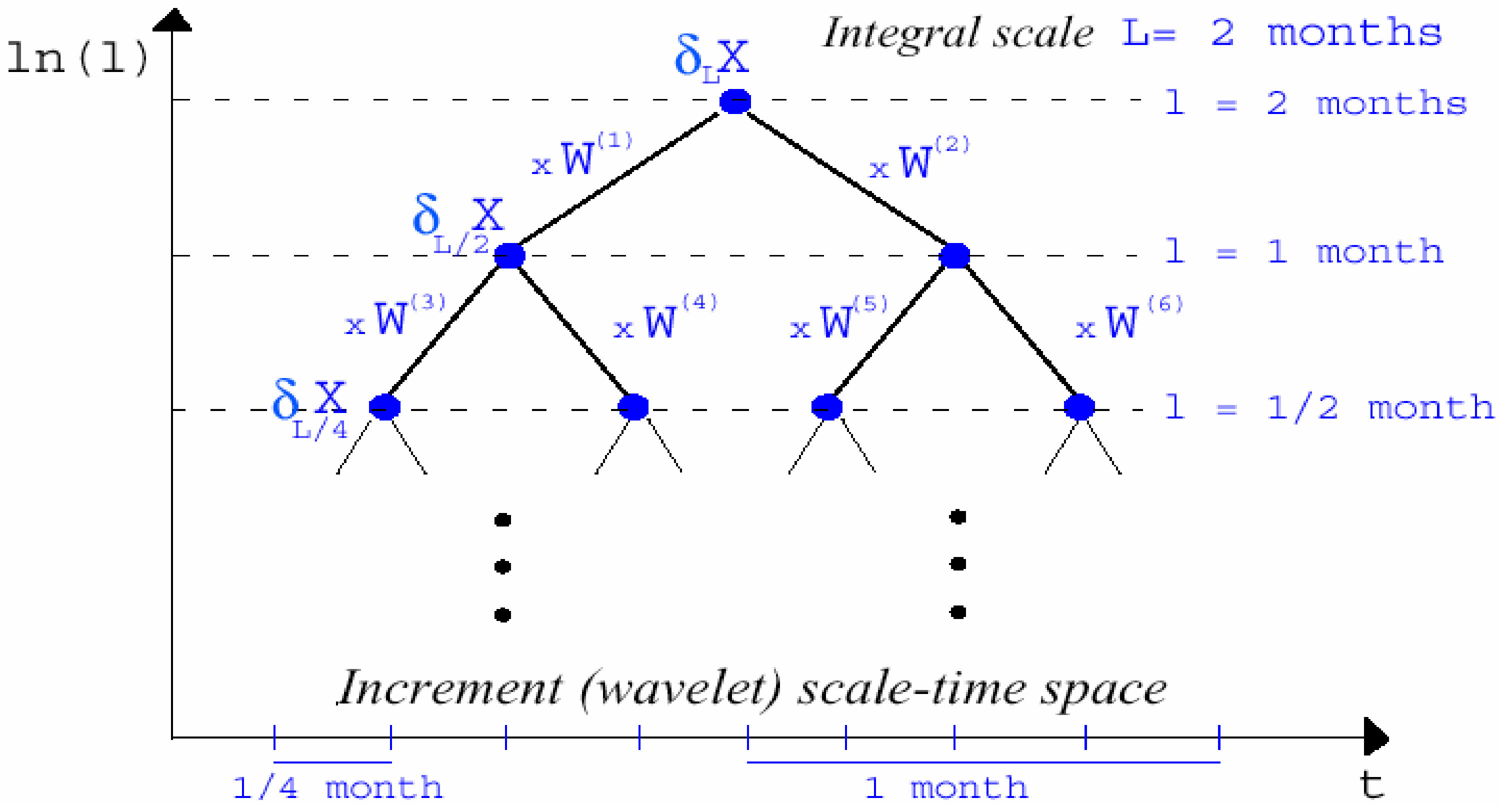


Arneodo, Muzy and Sornette (1998)

The multiplicative cascade paradigm

$$\delta_{\lambda l} X(\lambda t) = \lambda^H \delta_l X(t) = W_\lambda \delta_l X(t)$$

- $\mathcal{W}$-cascades (wavelet cascade)

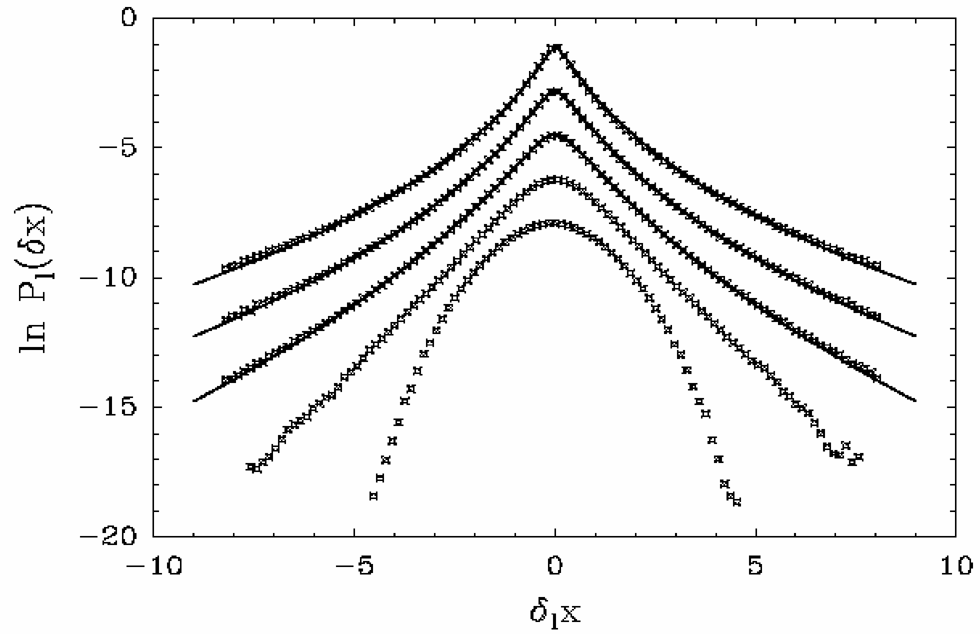


The Multifractal Random Walk Model

•Heavy tail is consequence of long-range time dependence

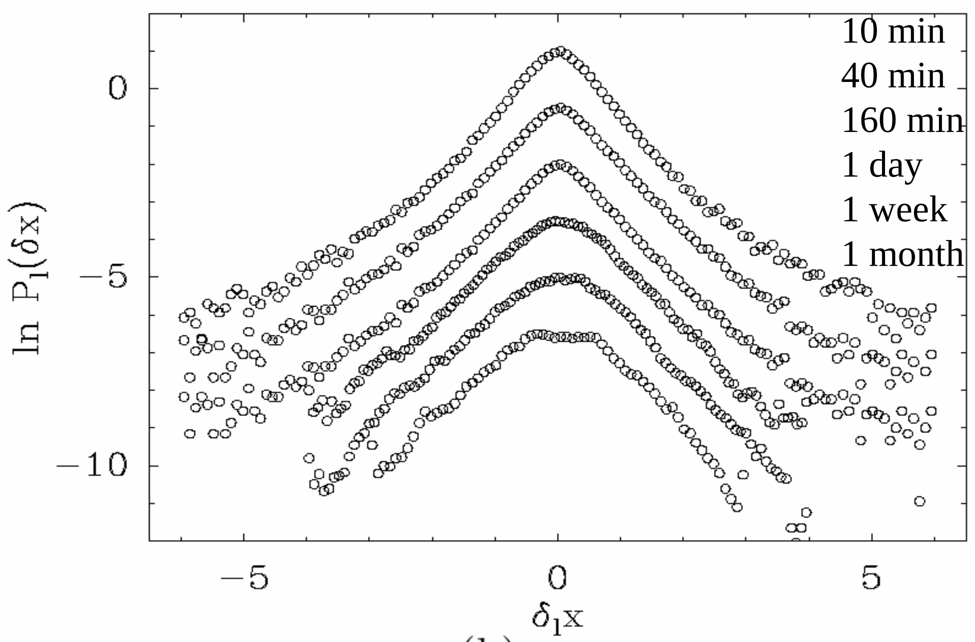
•Self-consistent coherent description of PDF and dependences at all scales simultaneously

Prediction



(a)

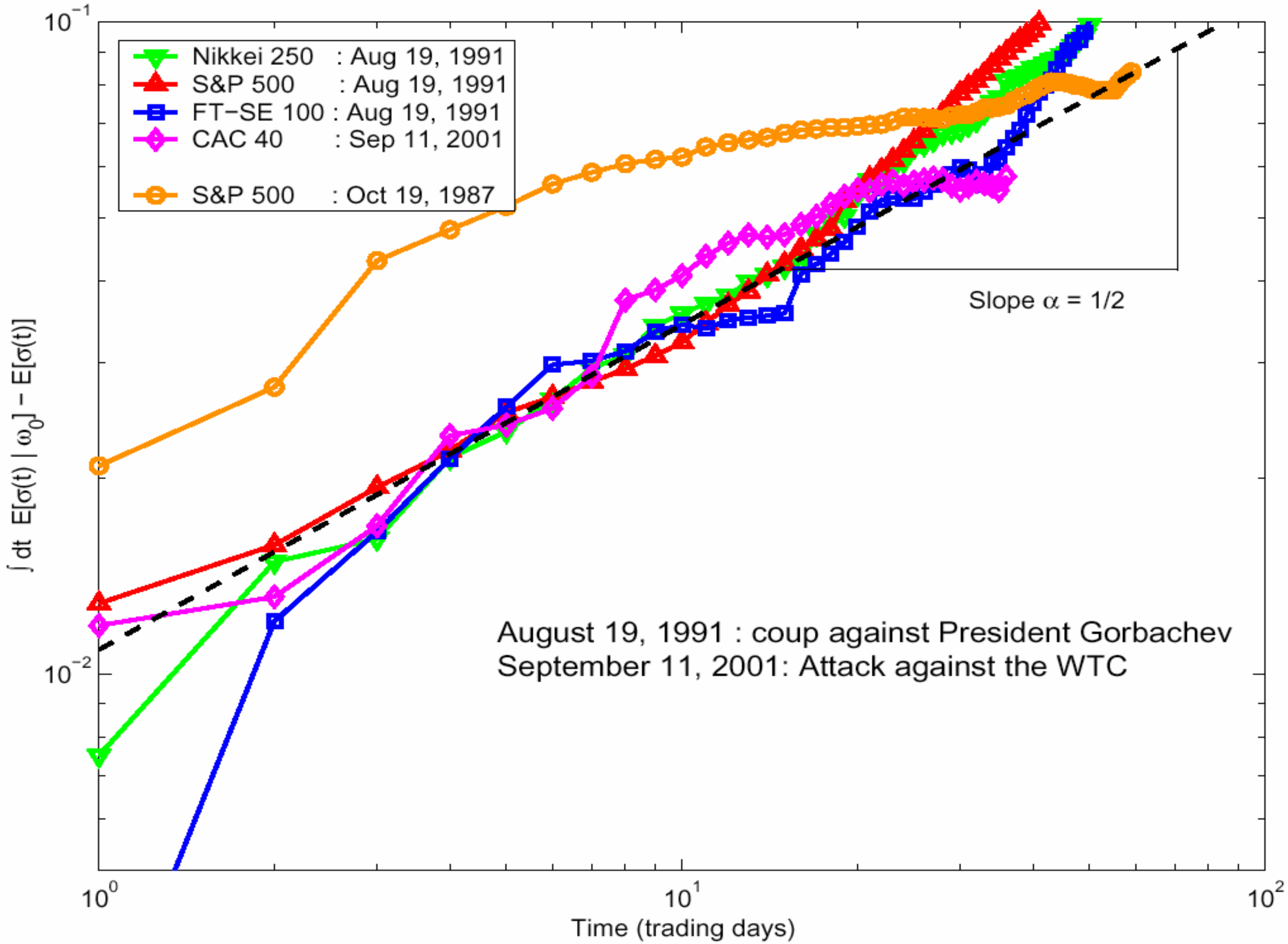
Data



(b)

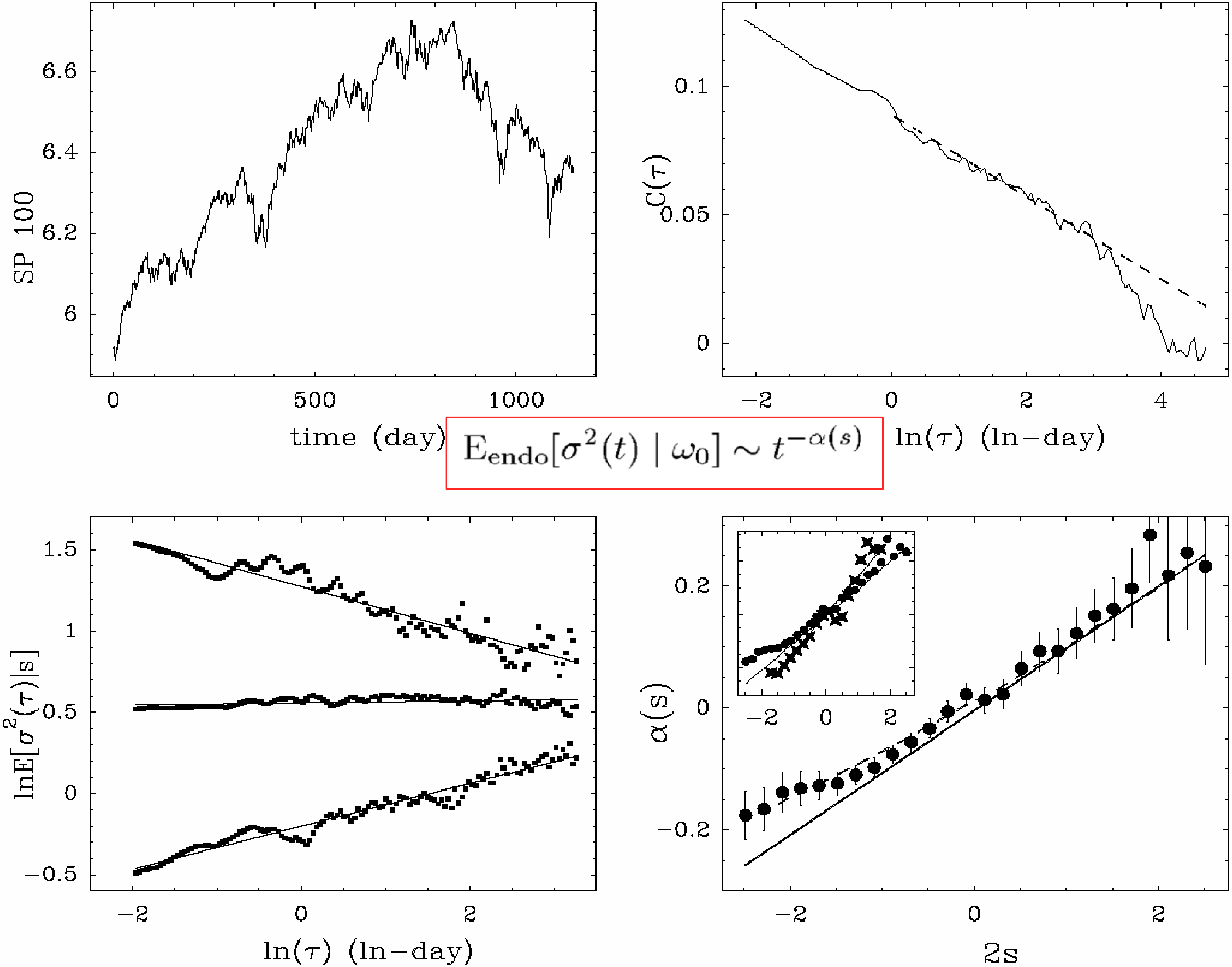
Linear response to an external shock

$$E_{exo}[\sigma^2(t) \mid \omega_0] - \overline{\sigma^2(t)} \propto e^{2K_0 t^{-1/2}} - 1 \approx \frac{2K_0}{\sqrt{t}}$$



D. Sornette, Y. Malevergne and J.F. Muzy
Volatility fingerprints of large shocks: Endogeneous versus exogeneous,
Risk Magazine (2003)
(http://arXiv.org/abs/cond-mat/0204626)

Real Data and Multifractal Random Walk model



$$\mathrm{E}_{\mathrm{endo}}[\sigma^2(t) \mid \omega_0] \sim t^{-\alpha(s)}$$

Forecasting historical and implied volatility with the MRW

$horizon = 1, 10, 20, 120$ days and $scale = 1, 10, 20, 120$ days,

1 day, 10 days, one month and six months future volatilities

Comparison with RiskMetrics and GARCH(1,1)

SP 500 index : 28/12/61 – 25/04/00

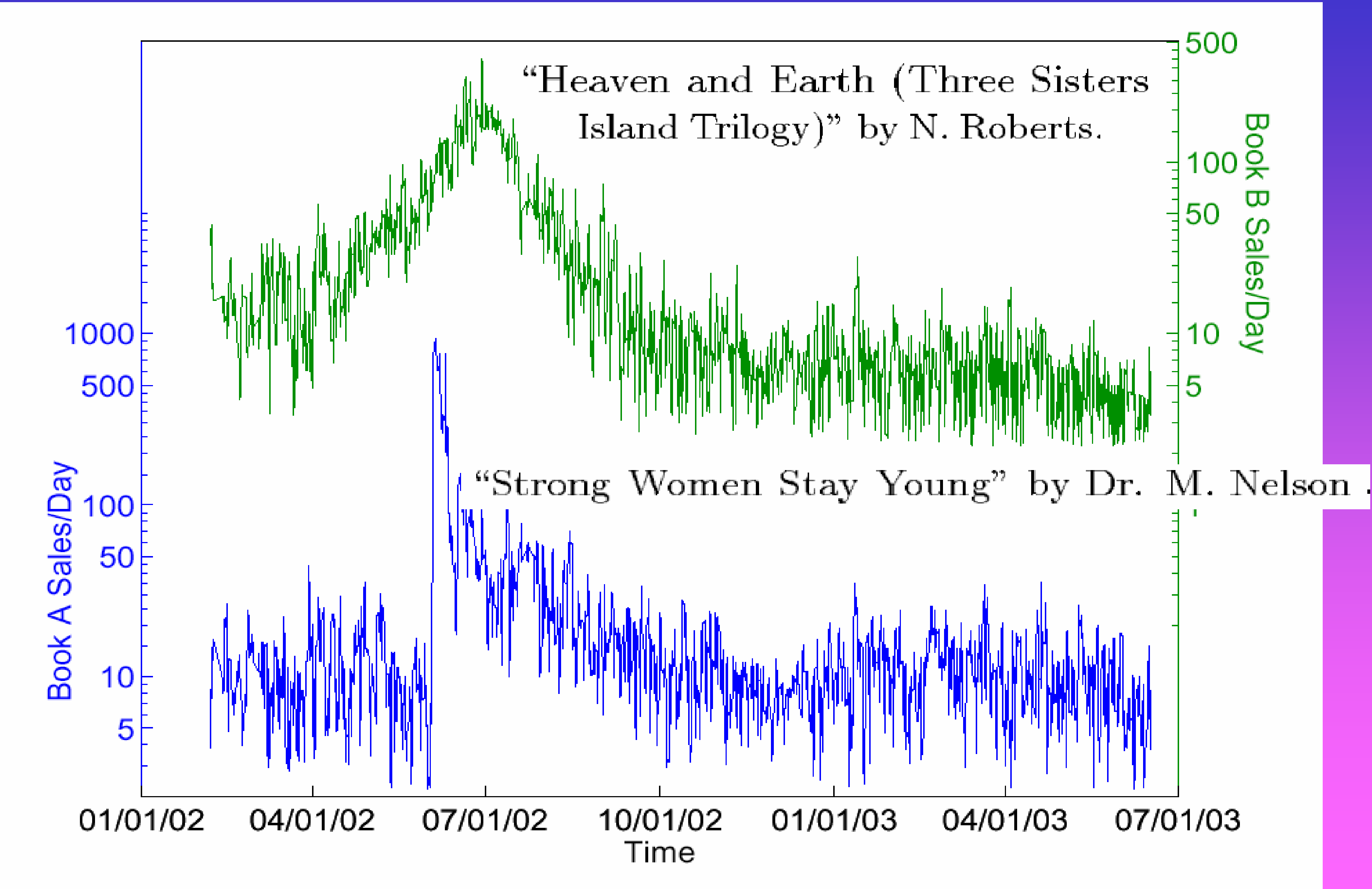
s = 1 day, h = ...	Historic	Garch(1,1)	MRWlin	MRWlog
RMSE 10 days	+37.41	-0.09	-1.67	**-1.97**
1 month	+37.65	-0.66	-1.92	**-2.06**
6 months	+37.17	**-2.57**	-2.42	-2.35
MAE 10 days	+20.38	-4.24	-10.60	**-22.83**
1 month	+20.92	-6.80	-12.82	**-23.96**
6 months	+18.35	-17.42	-19.09	**-26.25**

Implied volatility

R2 value/scale	Risk Metrics	Garch(1,1)	MRWlinc	MRWlog (intraday vol)
30 days	0.34	0.52	0.44	**0.61**

Table 1: Comparison of R2 values for different historical forecasts

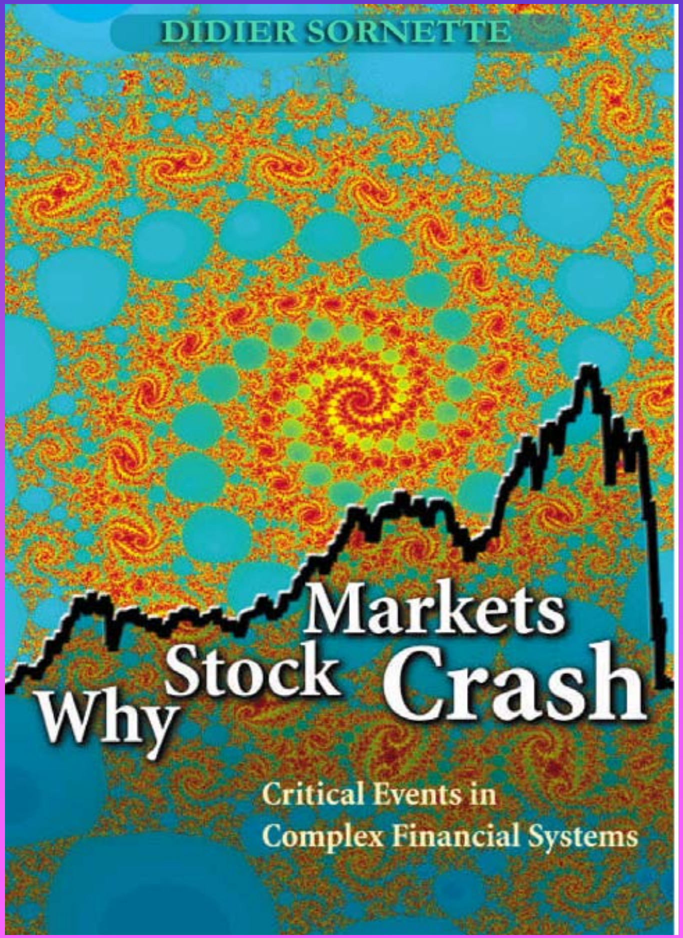
PREDICTING COMMERCIAL SALES



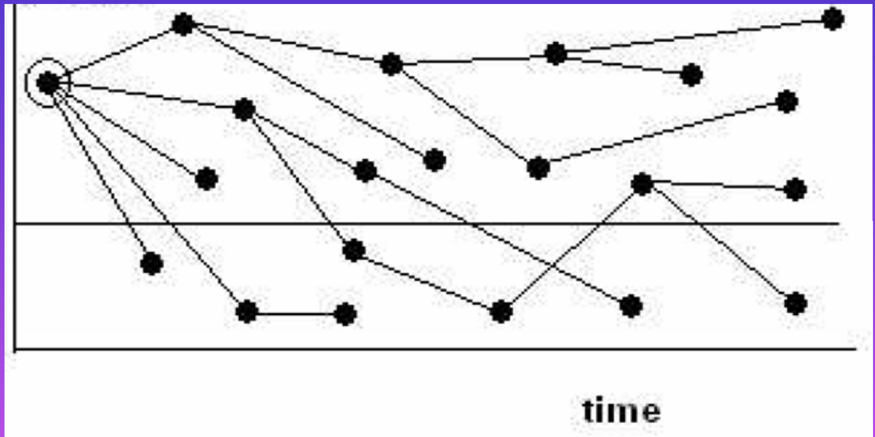
D. Sornette et al., Phys. Rev. Letts. 93 (22), 228701 (2004); F. Deschatres and D. Sornette, The Dynamics of Book Sales: Endogenous versus Exogenous Shocks in Complex Networks, Phys. Rev. E 72, 016112 (2005)

The Original "Crisis"

- On Friday January 17, 2003, WSMC jumped to rank 5 on Amazon.com's sales ranking (with Harry Potter as #1!!!)
- Two days before: release of an interview on MSNBC's MoneyCentral website

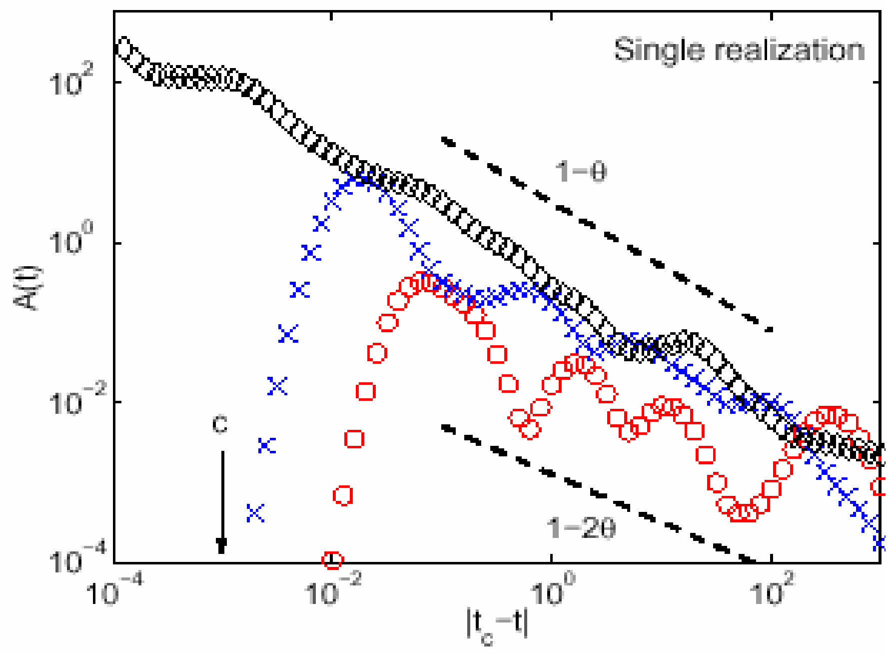


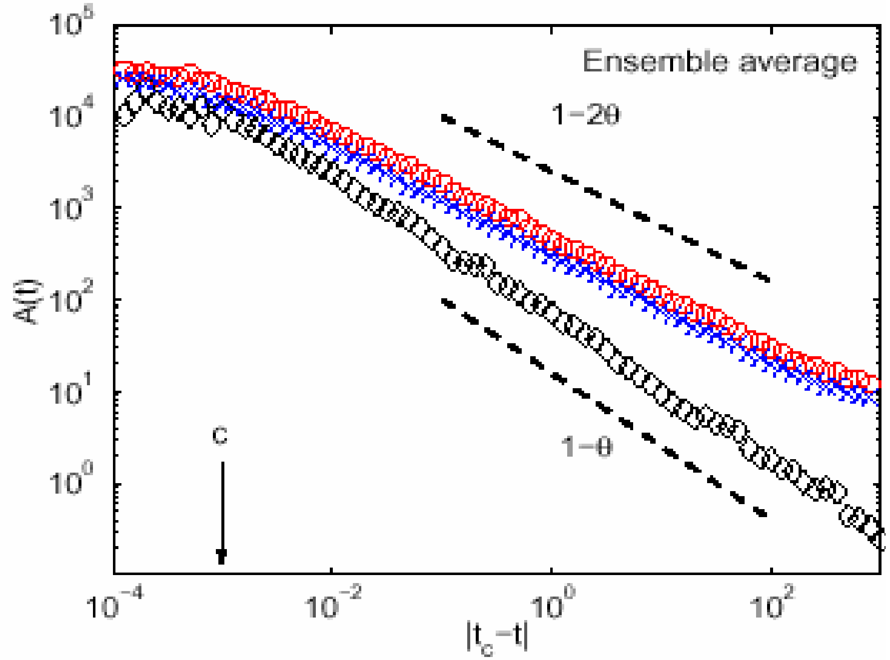
Epidemic branching process of word-of-mouth



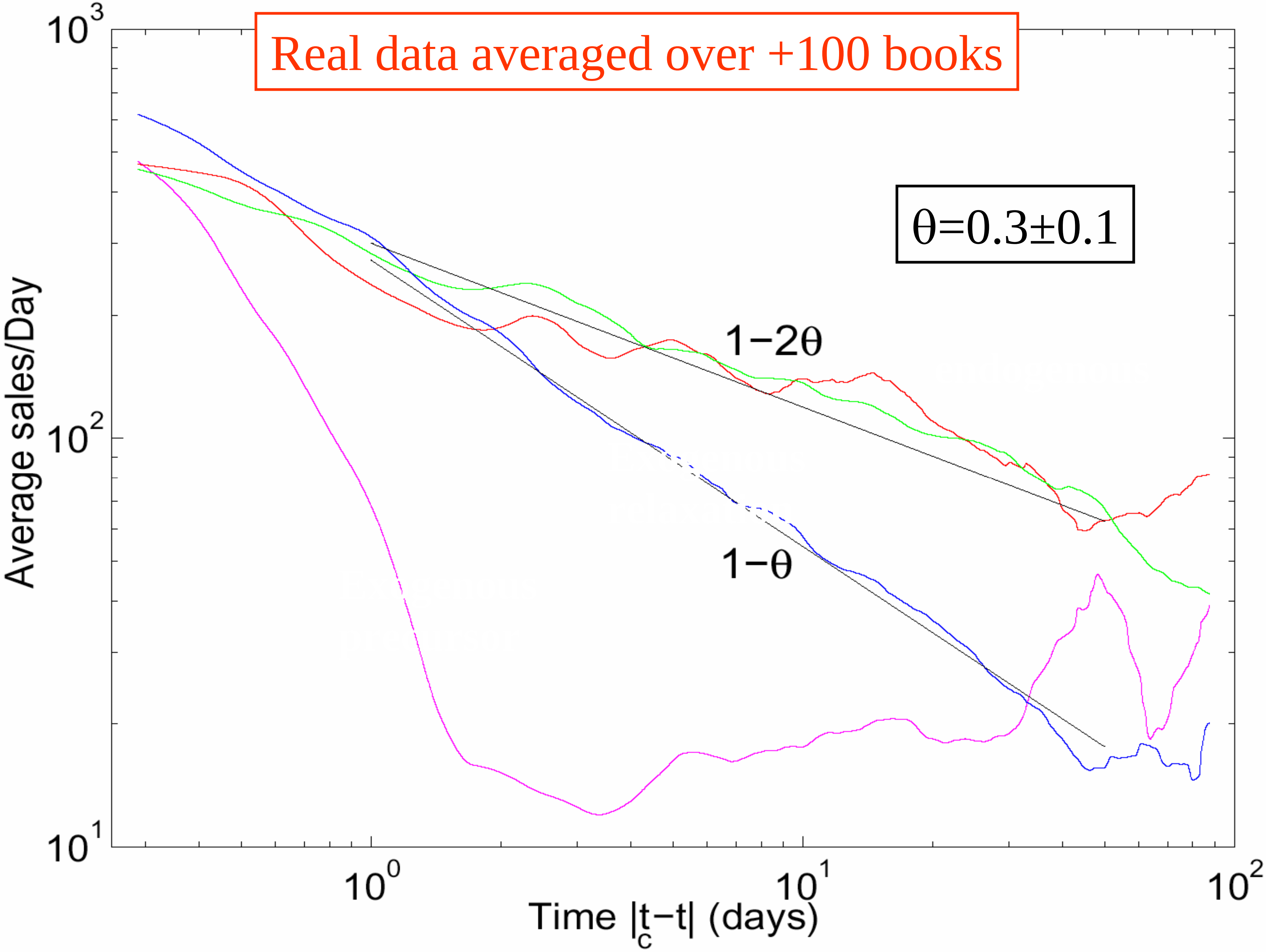
for memory kernels $K(t) \propto 1/t^{1-\theta}$ with $\theta > 0$

$$E_{\text{endo}}[A(t)|A(0)=A_0] \propto \frac{A_0}{t^{1-2\theta}} \gg E_{\text{exo}}[A(t)] \propto \frac{A_0}{t^{1-\theta}}$$



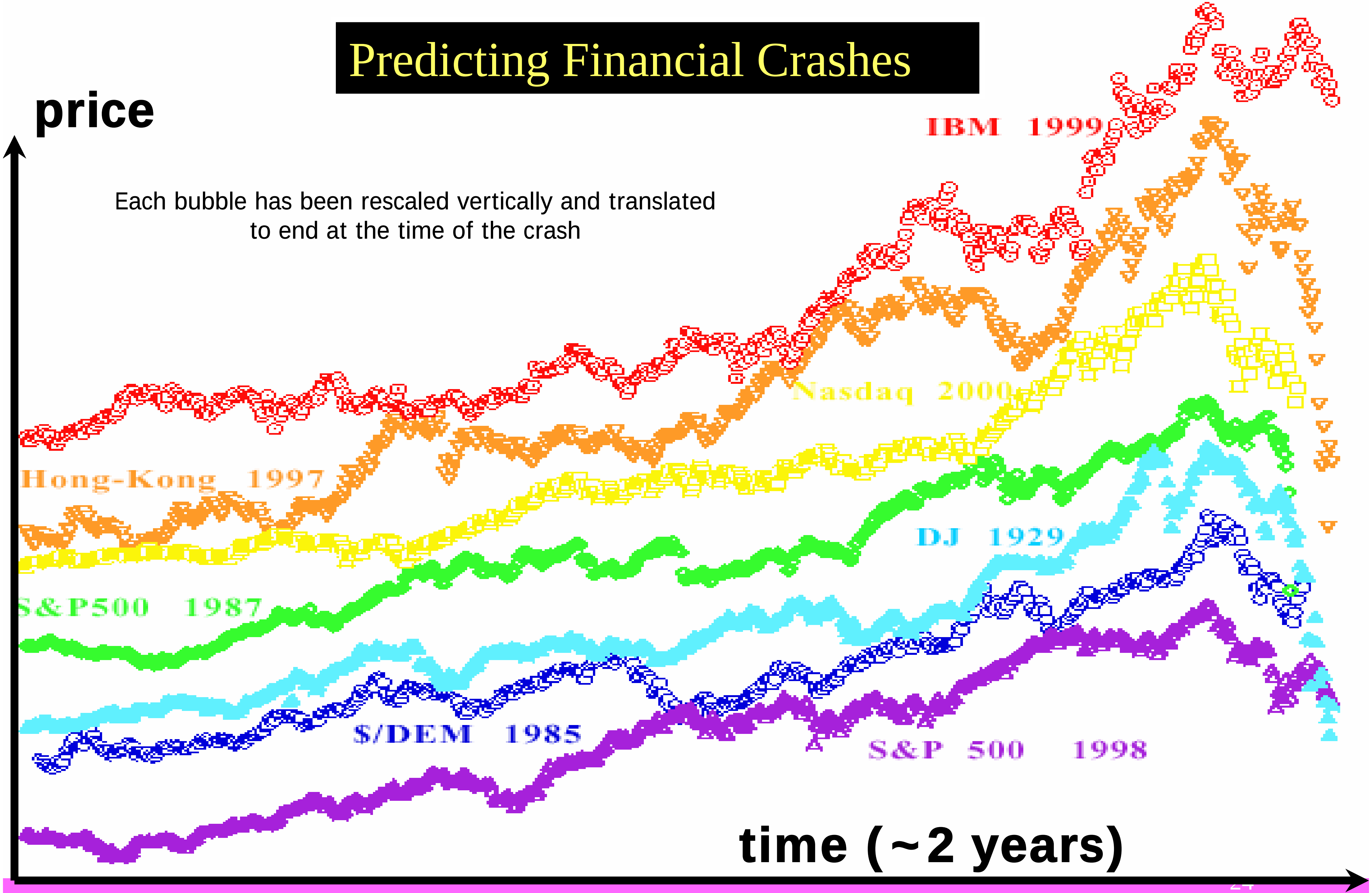


D. Sornette and A. Helmstetter
Endogeneous Versus Exogeneous Shocks in Systems with Memory, Physica A 318, 577 (2003)



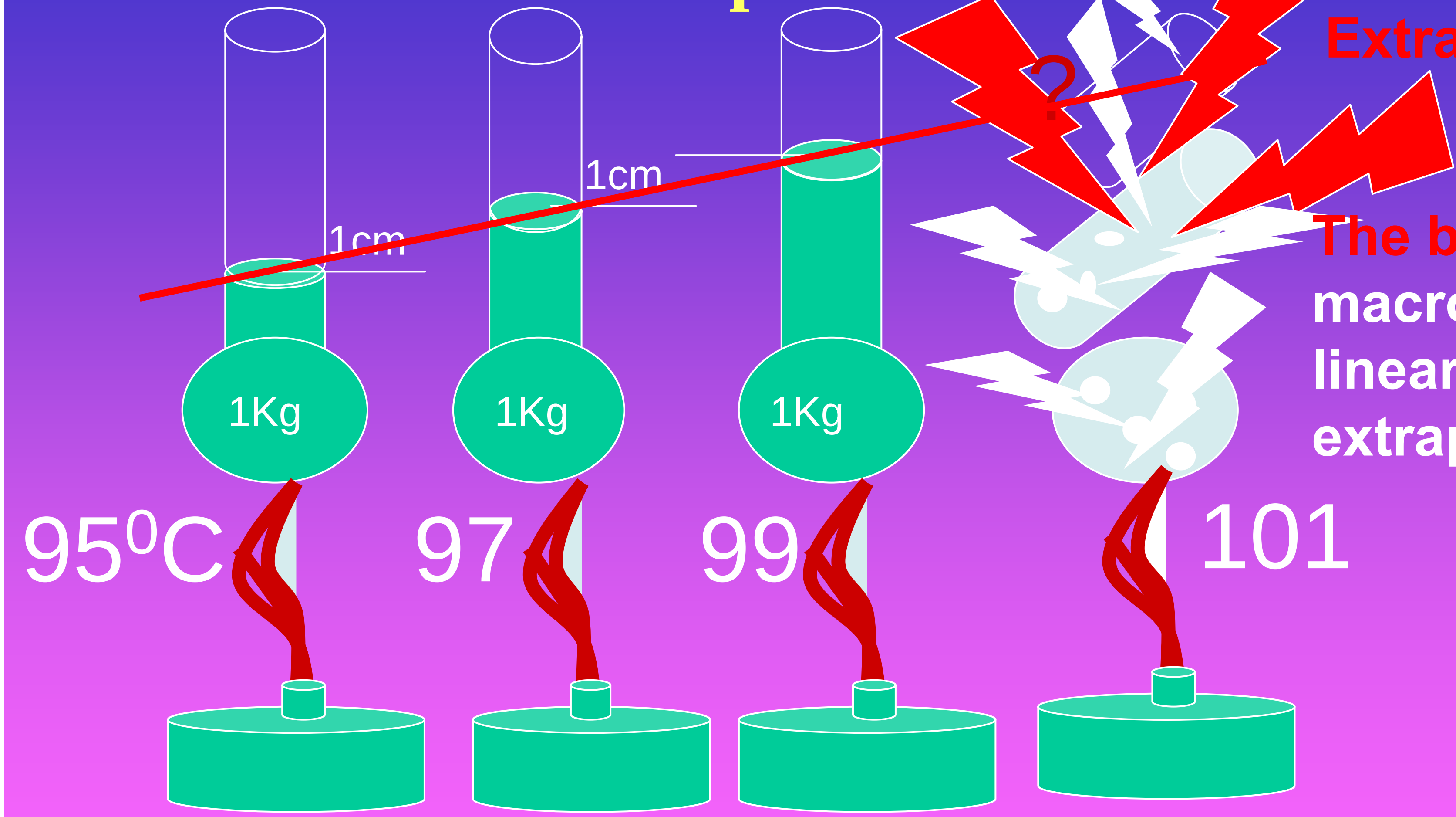
Real data averaged over +100 books
θ=0.3±0.1
1−2θ
1−θ
Average sales/Day
Time |t_c−t| (days)
10^3
10^2
10^1
10^0
10^1
10^2

Predicting Financial Crashes



Simplest Example of a “More is Different” Transition

Water level vs. temperature



Extrapolation?

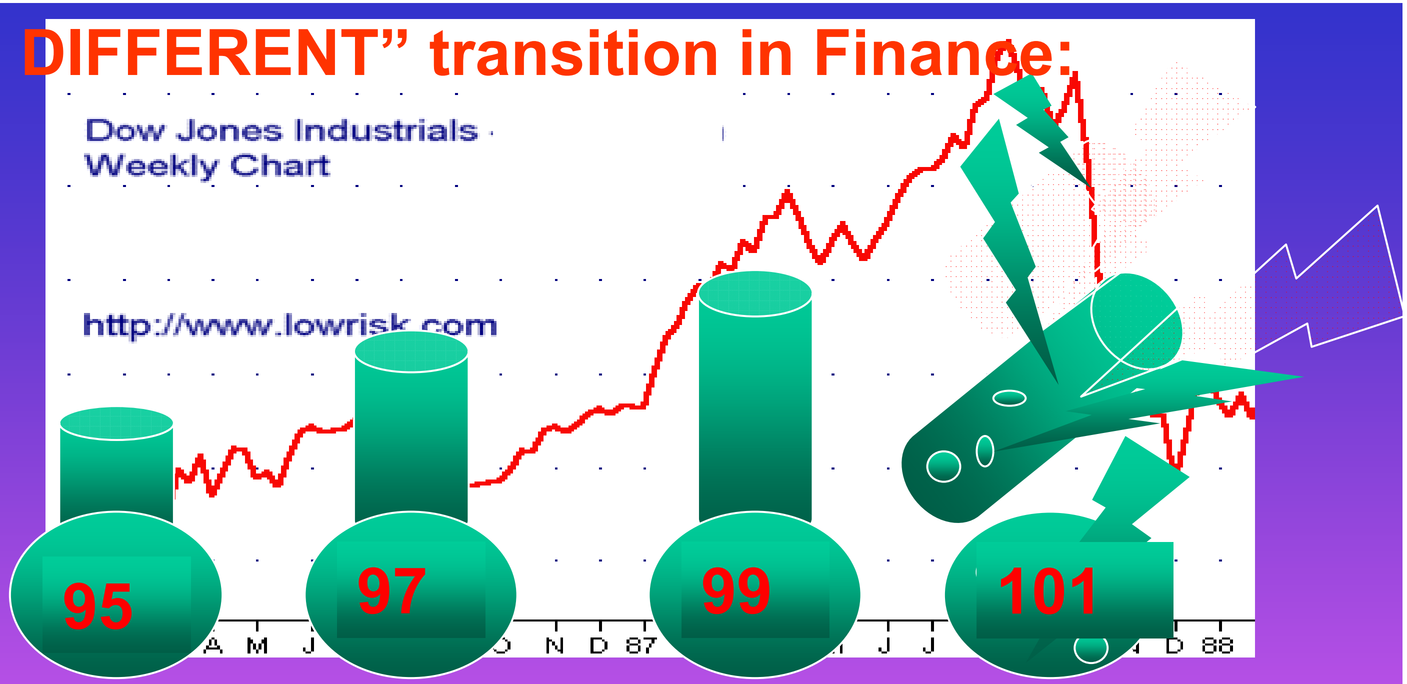
The breaking of macroscopic linear extrapolation

BOILING PHASE TRANSITION

More is different: a single molecule does not boil at $100C^0$

(S. Solomon)

Example of "MORE IS DIFFERENT" transition in Finance:

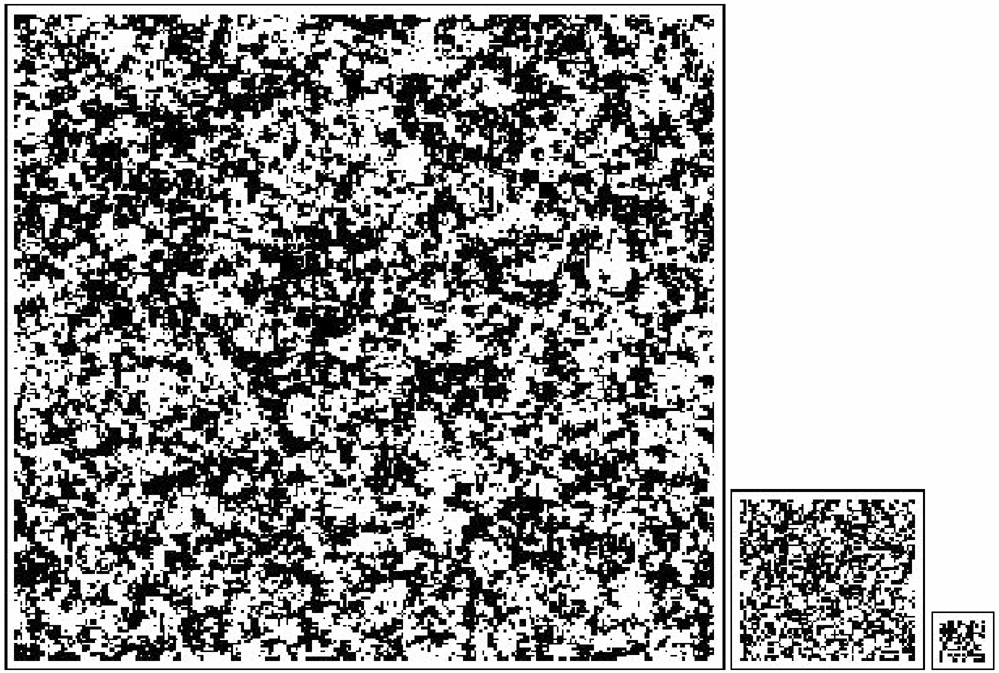


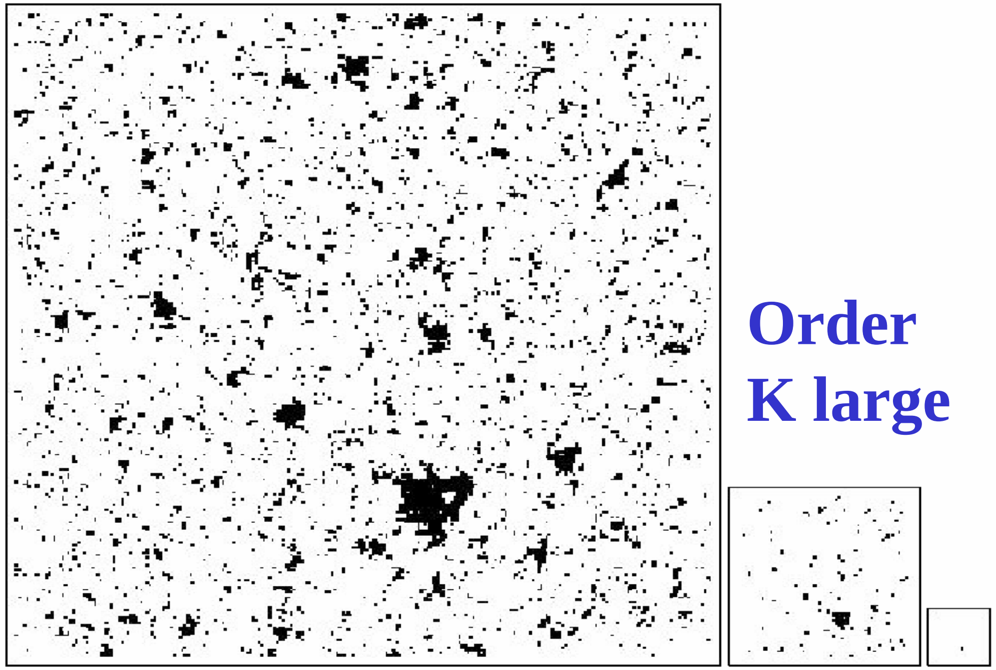
Instead of Water Level:

-economic index (Dow-Jones etc…)

Crash = result of collective behavior of individual traders

(S. Solomon)

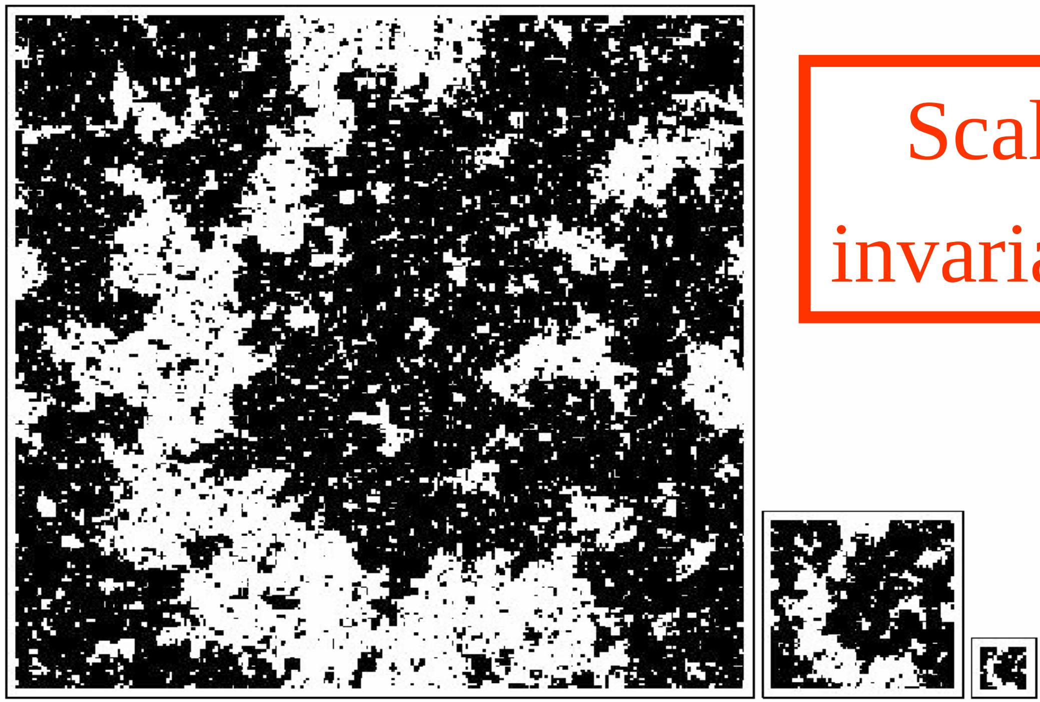




Order
K large

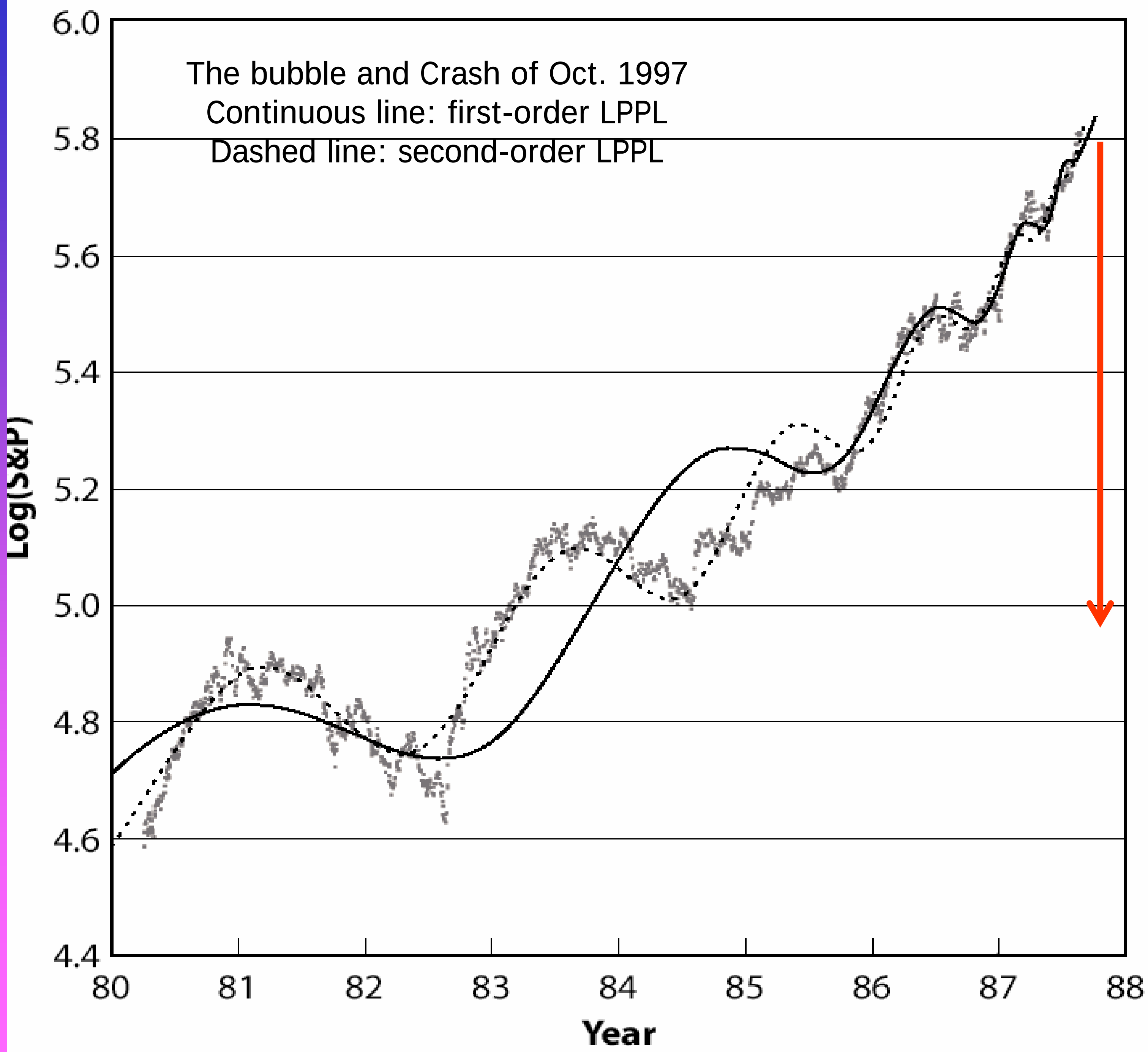
Disorder : K small

Renormalization group:
Organization of the description scale by scale



Scale invariance

Critical: K=critical value



The bubble and Crash of Oct. 1997
Continuous line: first-order LPPL
Dashed line: second-order LPPL
Log(S&P)
6.0
5.8
5.6
5.4
5.2
5.0
4.8
4.6
4.4
80
81
82
83
84
85
86
87
88
Year

Towards a methodology to identify crash risks

- Development of methods to diagnose bubbles
- Crashes are not predictable
- Only the end of bubbles can be forecasted
- 2/3 of bubbles end in a crash
- Multi-time-scales
- Probability of crashes; alarm index
  - Successful forward predictions: Oct. 1997; Aug. 1998, April 2000
  - False alarms: Oct. 1997

Summary

- Power laws are ubiquitous: large risks are common
- Robust reliable prediction of VaR with sparse data (hedge-funds)
- Forecasts of financial volatility (option market maker)
- Predicting commercial sales (books, CDs, movies…)
- Predicting financial instabilities

References

Y. MALEVERGNE, V. PISARENKO and D. SORNETTE (2005) “On the power of generalized extreme value (GEV) and generalized Pareto distribution (GPD) estimators for empirical distributions of log-returns.” *Applied Financial Economics* 16, 271-289 (2006)

Y. MALEVERGNE, V. PISARENKO and D. SORNETTE (2005) “Empirical distributions of stock returns: Exponential or power-like?” *Quantitative Finance* **5**, 379-401.

Y. MALEVERGNE and D. SORNETTE (2005) *Extreme Financial Risks*. Springer. Chapter 2.