Quantum Decision Theory with Prospect Interference and Entanglement

Didier Sornette (ETH Zurich)

(with V.I. Yukalov + PhD M. Favre and T. Kovalenko)

Professor of Entrepreneurial Risks at ETH Zurich

Professor of Finance at the Swiss Finance Institute

Director of the Financial Crisis Observatory

Founding member of the Risk Center at ETH Zurich (June 2011) (www.riskcenter.ethz.ch)

Professor of Geophysics associated with the Department of Earth Sciences (D-ERWD), ETH Zurich

Professor of Physics associated with the Department of Physics (D-PHYS), ETH Zurich

Financial Crisis Observatory

The Financial Crisis Observatory (FCO) is a scientific platform aimed at testing and quantifying rigorously, in a systematic way and on a large scale the hypothesis that financial markets exhibit a degree of inefficiency and a potential for predictability, especially during regimes when bubbles develop.

Current analysis and forecasts

www.er.ethz.ch

Quantum Decision Theory (QDT)

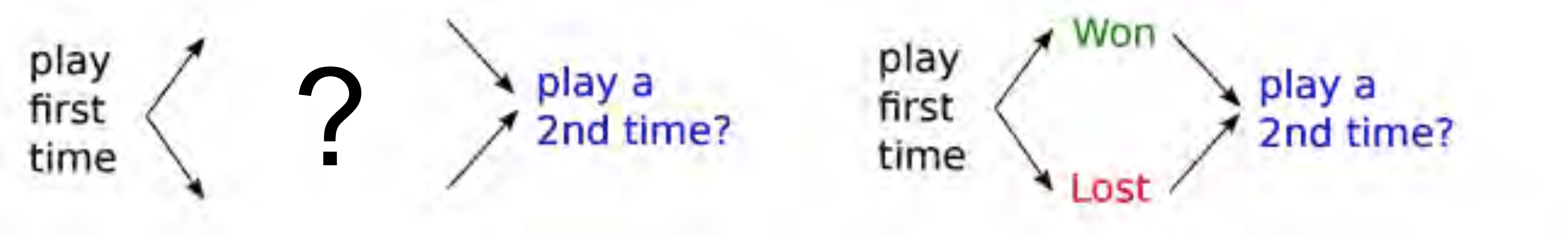
Vyacheslav I. Yukalov and Didier Sornette
+PhD Maroussia Favre and PhD Tatyana Kovalenko (ETH Zurich)

Stylized facts of real decision making:
.Many paradoxes of classical probability theory
.Decision-making problems are composite, consisting of several parts interconnected with each other and interfering
.Subjectivity, emotions, personality traits, biases, beliefs, framing, etc. can be seen as hidden variables
.Strong variability => need for probabilistic description of decision making

Ideas of QDT:
.based on quantum theory of measurement
.leads to non-additive probabilities, which contain interference terms
.Interference terms integrate hidden variables (variables we cannot access)

Classical paradoxes do not exist in QDT:
Compatibility violation (Allais); Independence violation; Ellberg paradox; Inversion paradox under uncertainty; Invariance violation; Certainty effects; Disjunction effect; Conjunction fallacy; Isolation effect; Combined paradoxes

Disjunction effect



Savage's 'sure-thing principle' (1954): if $P(A|X_j) > P(B|X_j)\ \forall j$, then $P(AX) > P(BX)$, where $X = \cup_j X_j$. Example of violation, from Tversky and Shafir (1992): 'to gamble or not to gamble'.

After first gamble,

- X_1 = learn about gain; X_2 = learn about loss ($X = X_1 \cup X_2$)
- A_1 = accept second gamble; A_2 = reject second gamble

Experiments by Tversky and Shafir (1992): $p(A_1|X_1) = 0.69$, $p(A_1|X_2) = 0.59$, $p(A_1X) = 0.36$.

Disjunction effect: QDT formulation (I)

- Elementary prospects: e_n (in disj. eff.: A_1X_1, A_1X_2, A_2X_1, A_2X_2)
- Elementary prospect states: $|e_n>$, $<e_n|e_m>=\delta_{nm}$
- Space of mind (Hilbert space): $\mathcal{H} = Span_n|e_n>$
- Prospect states: $|\pi_i> \in \mathcal{H}$
  ($|\pi_1>= a_{11}|A_1X_1> + a_{12}|A_1X_2>$,
  $|\pi_2>= a_{21}|A_2X_1> + a_{22}|A_2X_2>$)
- Strategic state: $|s> \in \mathcal{H}$, $|s>= \sum_n c_n|e_n>$, $<s|s>=1$
  ($|s>= c_{11}|A_1X_1> + c_{12}|A_1X_2> + c_{21}|A_2X_1> + c_{22}|A_2X_2>$)

Prospect probability

- Prospect operator: $\hat{P}(e_n) = |e_n><e_n|$
- Prospect probabilities: $p(\pi_i) =< s|\hat{P}(\pi_i)|s>$ (the prospect probability is defined as the expectation value of the prospect operator, with respect to the strategic state, and not with respect to an arbitrary basis.)

Disjunction effect: QDT formulation (II)

- Then we can write $p(\pi_i) = f(\pi_i) + q(\pi_i)$
- with $f(\pi_i) = \sum_n < s|\hat{P}(e_n)\hat{P}(\pi_i)\hat{P}(e_n)|s >$ (diagonal term)
- and $q(\pi_i) = \sum_{m \neq n} < s|\hat{P}(e_m)\hat{P}(\pi_i)\hat{P}(e_n)|s >$ (non-diagonal, interference effect)
- The utility factor $f(\pi_i)$ describes the weight of the prospect calculated classically, while $q(\pi_i)$ is called the attraction factor (contextual object describing subconscious feelings, emotions, and biases, playing the role of hidden variables)
- $p(\pi_i)$ represents the frequency with which the prospect π_i is chosen
- $0 \leq p(\pi_i) \leq 1$, $\sum_i p(\pi_i) = 1$
- $0 \leq f(\pi_i) \leq 1$, $\sum_i f(\pi_i) = 1$
- $-1 \leq q(\pi_i) \leq 1$, $\sum_i q(\pi_i) = 0$
- In disj. eff. : with $p(A_i X_j) \doteq |a_{ij} c_{ij}|^2$,
  $f(\pi_1) = p(A_1 X_1) + p(A_1 X_2)$, $f(\pi_2) = p(A_2 X_1) + p(A_2 X_2)$

Disjunction effect: QDT predictions (III)

- 'Quarter law': $|\bar{q}(\pi_i)| = 0.25$ (average over repetitions by a single decision maker or over several decision makers)

- π_1 is more attractive than π_2, i.e. $q(\pi_1) > q(\pi_2)$, if
  - π_1 offers a more certain gain or a less certain loss
  - π_1 offers activity under certainty or passivity under uncertainty

Reduction of QDT to classical decision making via decoherence:
$q(\pi_i) \to 0 \Rightarrow p(\pi_i) \to f(\pi_i)$

Choice of prospect with the largest probability

Probability: Frequency with which the prospect is chosen (by one or several decision makers)

Interference term (attraction factor): describes attractiveness of prospect (encapsulates feelings, etc.)

$$p(\pi_1) = f(\pi_1) + q(\pi_1)$$

Utility factor: weight of the prospect calculated classically

$q(\pi_1) > 0$: prospect is 'attractive'

$q(\pi_1) < 0$: prospect is 'unattractive'

Prediction of QDT:

QUARTER LAW: $|\bar{q}(\pi_1)| \approx 0.25$

Disjunction effect: comparison of QDT predictions with experiments (IV)

- QDT predicts $\bar{q}(\pi_2) = 0.25 = -\bar{q}(\pi_1)$ and $p(\pi_2) > p(\pi_1)$ (under uncertainty about first gamble, second gamble is rejected more often than accepted)
- $p(\pi_1) = p(A_1X_1) + p(A_1X_2) + q(\pi_1)$
- $p(\pi_2) = p(A_2X_1) + p(A_2X_2) + q(\pi_2)$
- $p(A_1X_1) = p_{exp}(A_1|X_1)p(X_1) = 0.69 \times 0.5 = 0.345$
- $p(A_1X_2) = p_{exp}(A_1|X_2)p(X_2) = 0.59 \times 0.5 = 0.295$
- $p_{exp}(\pi_1) = 0.36 = p(A_1X_1) + p(A_1X_2) + q(\pi_1) \Rightarrow$ $q(\pi_1) = -0.28$, $q(\pi_2) = 0.28$
- Also works with other instances of disjunction effect, conjunction fallacy, Allais paradox, etc. Suggests that the axioms of QDT correctly embody at a coarse-grained level the thought processes underlying decision making of humans.

Disjunction effect (violation of sure-thing principle): `To buy or not to buy'

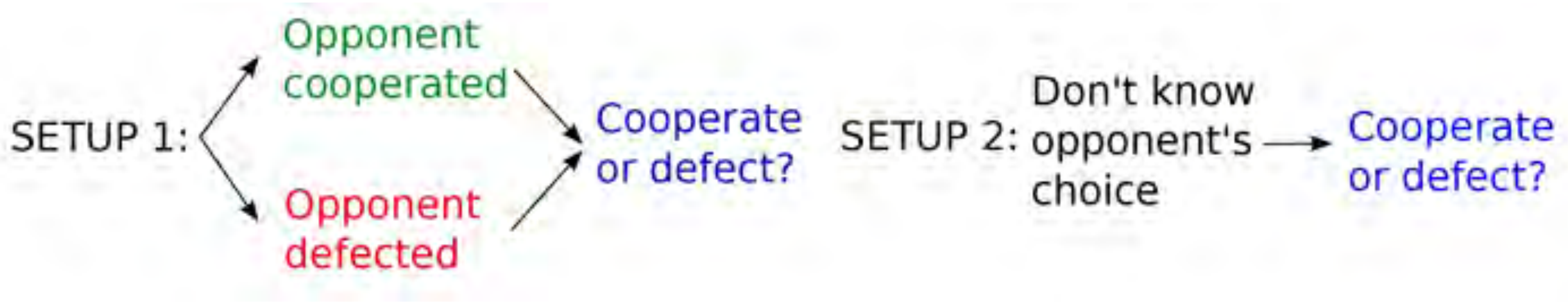
- X_1 = learn about success; X_2 = learn about failure, $X = X_1 \cup X_2$ = don't know exams results
- A_1 = buy vacation package; A_2 = don't buy vacation package
- Prospects in Setup 2: $\pi_1 = A_1 X$, $\pi_2 = A_2 X$

Experiments by Tversky and Shafir (1992):

$p_{exp}(A_1|X_1) = 0.54$, $p_{exp}(A_1|X_2) = 0.57$, $p_{exp}(A_1 X) = 0.32$.

- $p(A_1 X_1) = p_{exp}(A_1|X_1) p(X_1) = 0.54 \times 0.5 = 0.27$
- $p(A_1 X_2) = p_{exp}(A_1|X_2) p(X_2) = 0.57 \times 0.5 = 0.285$
- $p_{exp}(\pi_1) = 0.32 = p(A_1 X_1) + p(A_1 X_2) + q(\pi_1)$
  $\Rightarrow q(\pi_1) = -0.235$, $q(\pi_2) = 0.235$
- Quantitative agreement with quarter law

Disjunction effect (violation of sure-thing principle): `Prisoner's dilemma'



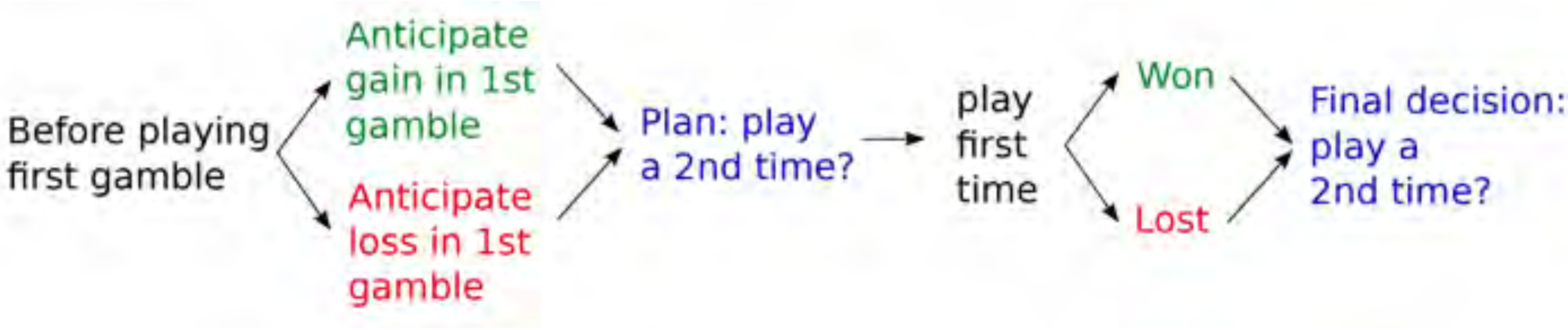
- X_1 = learn about cooperation; X_2 = learn about defection, $X = X_1 \cup X_2$ = don't know opponent's choice
- A_1 = cooperate; A_2 = defect
- Prospects in Setup 2: $\pi_1 = A_1 X$, $\pi_2 = A_2 X$

Experiments by Tversky and Shafir (1992):
$p_{exp}(A_1|X_1) = 0.16$, $p_{exp}(A_1|X_2) = 0.03$, $p_{exp}(A_1 X) = 0.37$.

- $p(A_1 X_1) = p_{exp}(A_1|X_1) p(X_1) = 0.16 \times 0.5 = 0.08$
- $p(A_1 X_2) = p_{exp}(A_1|X_2) p(X_2) = 0.03 \times 0.5 = 0.015$
- $p_{exp}(\pi_1) = 0.37 = p(A_1 X_1) + p(A_1 X_2) + q(\pi_1)$
  $\Rightarrow q(\pi_1) = 0.275$, $q(\pi_2) = -0.275$
- Quantitative agreement with quarter law

quantifies "strong cooperation" propensity of human beings

Disjunction effect and planning paradox



- First gamble: C_1 = anticipate a gain, C_2 = anticipate a loss, C_3 = experience a gain, C_4 = experience a loss
- Second gamble: A_1 = accept, A_2 = reject ; B_1 = win, B_2 = lose, $B = B_1 \cup B_2$
- Prospects: plans: $\pi_1 = A_1 B C_1$, $\pi_2 = A_1 B C_2$; final: $\pi_3 = A_1 B C_3$, $\pi_4 = A_1 B C_4$

QDT prediction:

- $q(\pi_1) > q(\pi_3)$ (gambler's fallacy: after winning, expectation to lose, more so in reality than in imagination), $q(\pi_2) < q(\pi_4)$ (after losing, expectation to win)
- $f(\pi_1) = f(\pi_3)$, $f(\pi_2) = f(\pi_4)$ (utility when anticipating = utility when experiencing)
- $\Rightarrow$ $p(\pi_1) > p(\pi_3)$ and $p(\pi_2) < p(\pi_4)$

Experiments by Barkan and Busemeyer (2003):
$p_{exp}(\pi_1) = 0.6 > p_{exp}(\pi_3) = 0.53$, $p_{exp}(\pi_2) = 0.63 < p_{exp}(\pi_4) = 0.69$

- $\Rightarrow$ Planning 'paradox' not a paradox in QDT

Conjunction fallacy

Example given by Tversky and Kahneman (1980): *Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Rate the probabilities of:*

1. *Linda is a bank teller*
2. *Linda is a bank teller and is active in the feminist movement.*

- A = object has primary feature (e.g. bank teller)
- Secondary feature (e.g. feminist): X_1 = decide that object has it; X_2 = decide that object does not have it, $X = X_1 \cup X_2$ = don't decide about it
- Subjects rate the probabilities of: $\pi_A = AX$, and AX_1

Experiments by Shafir and Smith (1990) (average over 14 cases similar to Linda's): $p_{exp}(AX) = 0.22$, $p_{exp}(AX_1) = 0.346$

- $p_{exp}(AX) = f(AX) + q(AX)$
- Incompatibility of primary and secondary features: $f(AX) = 0.5$
- $\Rightarrow q(AX) = -0.28$
- Quantitative agreement with quarter law

Conflicting traits

	Characteristics	$p(AX)$	$p(AX_1)$	$q(AX)$
A	Bank teller	0.241	0.401	−0.259
X_1	Feminist			
A	Bird watcher	0.173	0.274	−0.327
X_1	Truck driver			
A	Bicycle racer	0.160	0.226	−0.340
X_1	Nurse			
A	Drum player	0.266	0.367	−0.234
X_1	Professor			
A	Boxer	0.202	0.269	−0.298
X_1	Chef			
A	Volleyboller	0.194	0.282	−0.306
X_1	Engineer			
A	Librarian	0.152	0.377	−0.348
X_1	Aerobic trainer			
A	Hair dresser	0.188	0.252	−0.312
X_1	Writer			
A	Floriculturist	0.310	0.471	−0.190
X_1	State worker			
A	Bus driver	0.172	0.314	−0.328
X_1	Painter			
A	Knitter	0.315	0.580	−0.185
X_1	Correspondent			
A	Construction worker	0.131	0.249	−0.369
X_1	Labor-union president			
A	Flute player	0.180	0.339	−0.320
X_1	Car mechanic			
A	Student	0.392	0.439	−0.108
X_1	Fashion-monger			
	Average	0.220	0.346	−0.280

The average interference term is in good agreement with the interference-quarter law.

The empirical data are taken from Shafir et al. (1990)

Randomized Lottery task

Collaboration with A. Wittwer and H.R. Heinimann, Collegium Helveticum, ETH Zurich

Decision

π_1 = get CHF 50 with 60% chance (p) or nothing with 40% chance (1-p)

π_2 = get a guaranteed payoff CHF 25 (y)

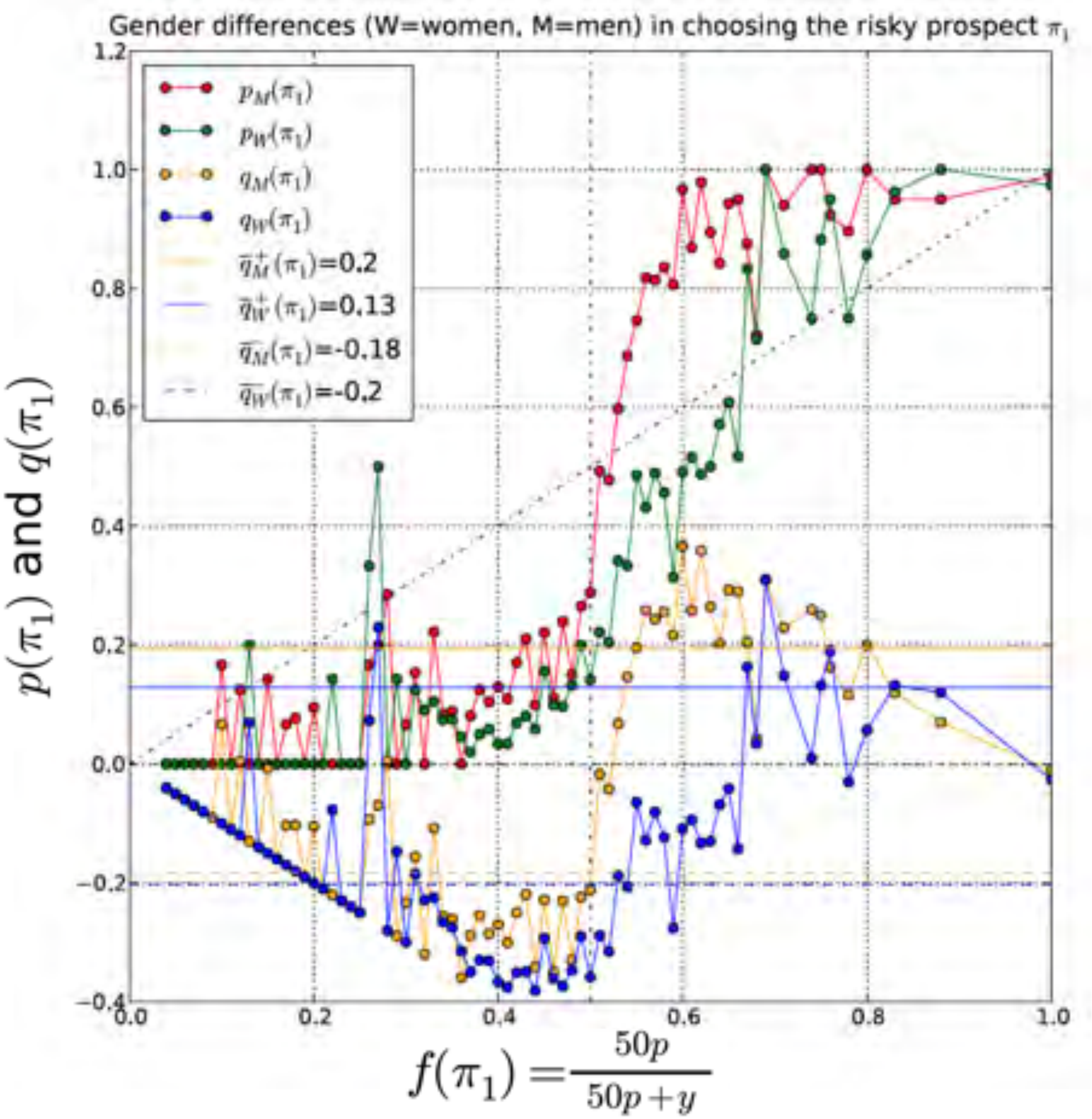
→ Lottery repeated n times with random p and y $(0<y<50)$
Final payoff: one of the n outcomes, picked randomly

- Prospects: π_1 = risky prospect, π_2 = certain prospect

Application in QDT:

- $f(\pi_1) = \frac{U(\pi_1)}{U(\pi_1)+U(\pi_2)} = \frac{50p}{50p+y}$
- $p(\pi_1) = p_{exp}(\pi_1)$ = fraction of people choosing π_1 for a given value of $f(\pi_1)$
- $q(\pi_1) = p_{exp}(\pi_1) - f(\pi_1)$
- Can we differentiate people or groups of people with regard to risk aversion, based on their attraction factor $q(\pi_1)$?
- Experiments with healthy subjects, Asperger patients, schizophrenics, nurses, traders, students in economics, etc.

Prospect π_1: risky prospect (CHF 50 with proba p, nothing with proba 1-p)
Prospect π_2: certain prospect (CHF y)



Data on 14 men, 13 women, 200 decisions per participant

Women have a smaller attraction factor towards the risky prospect than men, i.e. women are more risk averse than men

Influence of information
(learning by news or social interactions)

Agents A and B who interact (can be one agent + rest of the world)

Prospect probability: $p(\pi_j, \tau) \equiv \mathrm{Tr}_{AB}\hat{\rho}_{AB}(\tau)\hat{P}(\pi_j)$

prospect operator

$$\hat{\rho}_{AB}(\tau) = \hat{U}(\tau)\hat{\rho}_{AB}\hat{U}^{+}(\tau)$$

evolution operator

Normalization condition

$$\mathrm{Tr}_{AB}\hat{\rho}_{AB}(\tau) = 1 \quad \longrightarrow \quad \hat{U}^{+}(\tau)\hat{U}(\tau) = \hat{1}_{AB}$$

$$\frac{d\hat{U}(\tau)}{d\tau} + \hat{U}(\tau)\,\frac{d\hat{U}^{+}(\tau)}{d\tau}\,\hat{U}(\tau) = 0 \qquad i\,\frac{d\hat{U}(\tau)}{d\tau} = \hat{H}_{AB}\hat{U}(\tau) \qquad \hat{U}(\tau) = \exp\left(-i\hat{H}_{AB}\tau\right)$$

evolution generator

Influence of information
(learning by news or social interactions)

evolution generator: $\hat{H}_{AB} = \hat{H}_A + \hat{H}_B + \hat{H}_{int}$

$$p(\pi_j, \mu) = f(\pi_j) + q(\pi_j, \mu)$$

Attraction factor

$$q(\pi_j, \mu) = q(\pi_j) D(\mu)$$

$$D(\mu) = \exp\left(-\frac{\mu}{\mu_c}\right)$$

Decoherence

$$\lim_{\mu \to \infty} p(\pi_j, \mu) = f(\pi_j)$$

Information induced decoherence

Charness, G., Karni, E., Levin, D. (2010). On the conjunction fallacy in probability judgement: new experimental evidence regarding Linda. *Games and Economic Behavior*, 68, 551–556.

Decision making = self-organization

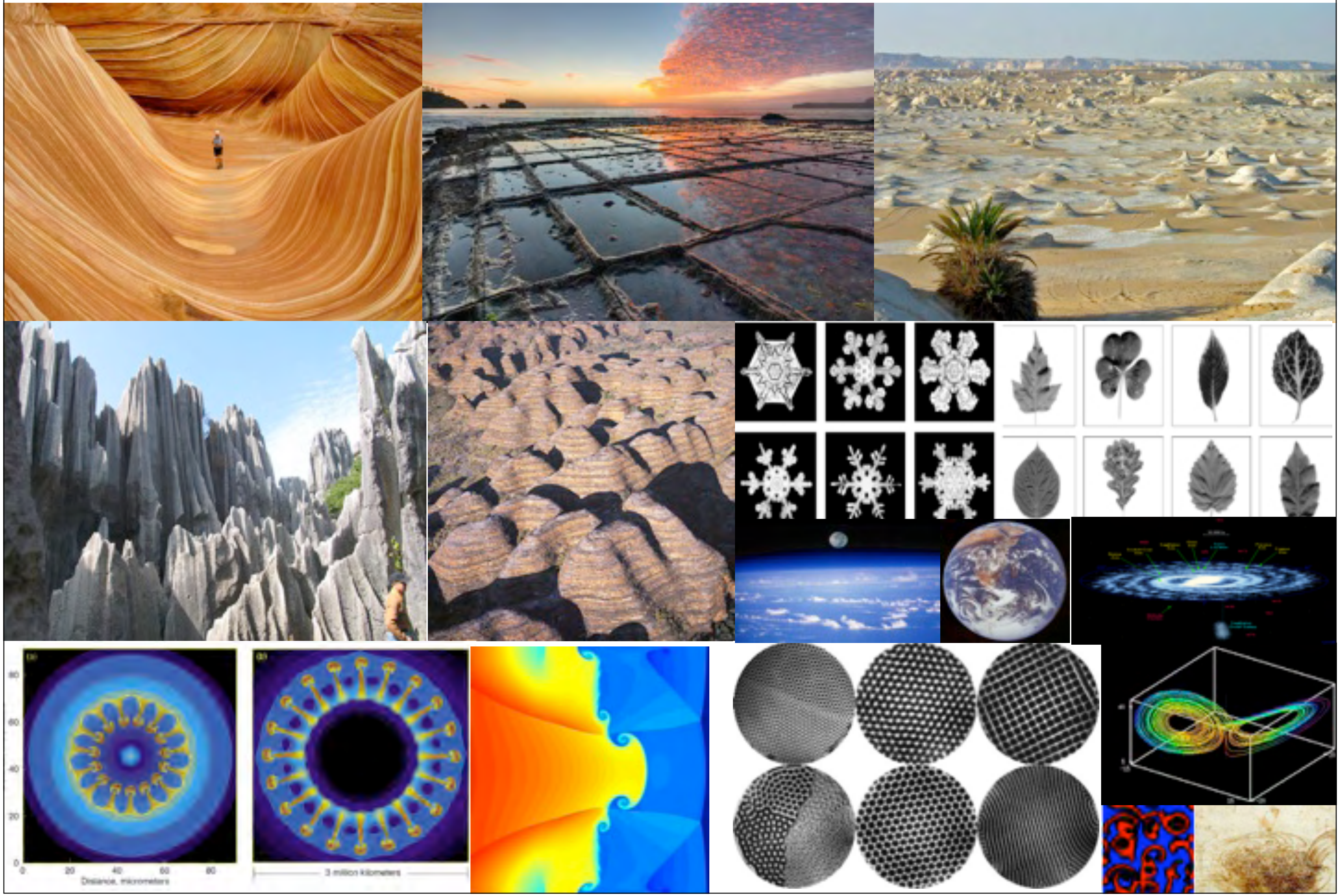
The watchmaker argument of Intelligent design

William Paley

. . . when we come to inspect the watch, we perceive. . . that its several parts are framed and put together for a purpose, e.g. that they are so formed and adjusted as to produce motion, and that motion so regulated as to point out the hour of the day; . . . *The inference we think is inevitable, that the watch must have had a maker.*

claim of ID: "what we observe and do must come from the will and *decision making* of a "super" watchmaker"

Self-organization



Decision making = self-organization

Self-organization is the process of evaluating the probabilities of system states in the search for the most stable state.

Decision making is the process of evaluating the probabilities of decision prospects in the search for the most preferable prospect.

Complex system	Decision maker
System states	Decision prospects
System fluctuations	Decision-maker deliberations
State probability	Prospect probability
System stability	Prospect preferability
Most stable state	Most preferable prospect
Self-organization	Decision making

Endogeneous decision making in self-organization and vice-versa

V.I. Yukalov and D. Sornette, Proc. Roy. Soc. A. (submitted 26 June 2012), Self-organization in nature and society as decision making.

Quantum Decision Theory (QDT)

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QDT:

- probabilistic formulation of decision making
- based on quantum theory of measurement
- leads to non-additive probabilities, which contain interference terms
- Interference terms integrate hidden variables (variables we cannot access)

Classical paradoxes do not exist in QDT:

Compatibility violation (Allais); Independence violation; Ellberg paradox; Inversion paradox under uncertainty; Invariance violation; Certainty effects; Disjunction effect; Conjunction fallacy; Isolation effect; Combined paradoxes

+ prediction that conjunction fallacy => disjunction effect

Publications by V.I. Yukalov, D. Sornette

1. Physics Letters A 372, 6867-6871 (2008), Quantum decision theory as quantum theory of measurement
2. Entropy 11, 1073-1120 (2009), Processing information in quantum decision theory.
3. European Physical Journal B 71, 533-548 (2009), Physics of risk and uncertainty in quantum decision making.
4. Laser Physics Letters 6, 833-839 (2009), Scheme of thinking quantum systems.
5. Physics of Atomic Nuclei 73, 559-562 (2010), Entanglement production in quantum decision making.
6. Advances in Complex Systems 13, 659-698 (2010), Mathematical structure of quantum decision theory.
7. Theory and Decision 70, 283-328 (2011), Decision theory with prospect interference and entanglement.
8. Game and Economic Behavior (submitted 22 Feb 2012), Quantum decision making by social agents.
9. Theory and Decision, submitted 30 March 2012 to FUR XV, Georgia State University, Atlanta, Georgia USA, Manipulating decision making of typical agents,,
10. Proc. Roy. Soc. A. (submitted 26 June 2012), Self-organization in nature and society as decision making.