

Practical implications for risk control and management

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•What tail risks? Power law vs Stretched exponentials

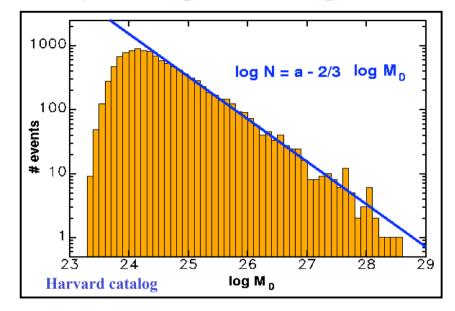
•Heavy-tail of PDF of firm sizes and new risk factors

•Power laws? No! Better measures of risks = "kings"

•Imitation, herding, conventions: bubbles and crashes

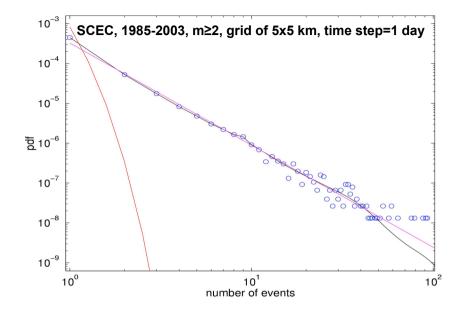
•Illusion of control

Heavy tails in pdf of earthquakes

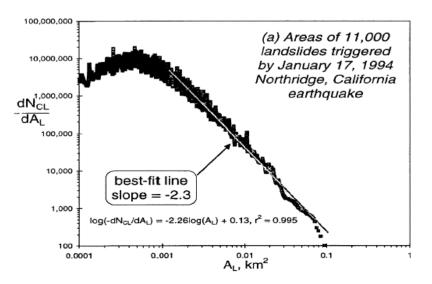


Heavy tails in ruptures 10⁵ (#/m^2/year) 1000 Д 10 8 0.1 0.001 10^{- 5} Flux flux of debris at 1500 km Ο **10^{- 7}** flux of meteorite at1500 km flux of debris at 950 km 10^{- 9} - - - - flux of meteorites at 950 km 10^{-11} 5 0.0001 0.001 0.01 0.1 10 10 1 size (cm)

Heavy tails in pdf of seismic rates



Heavy tails in pdf of rock falls, Landslides, mountain collapses



Heavy tails in pdf of forest fires

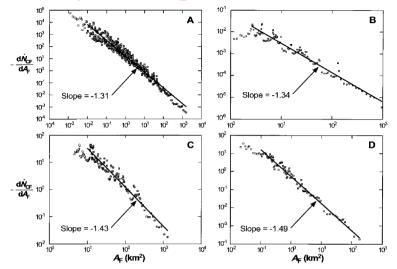
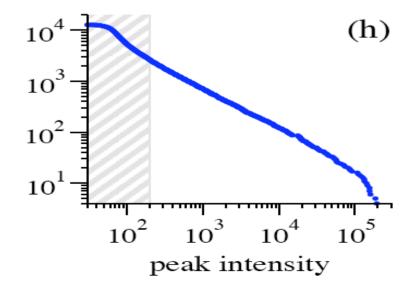
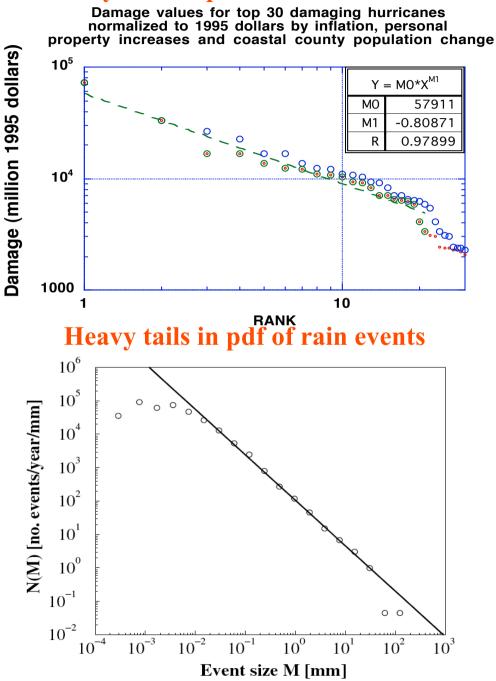


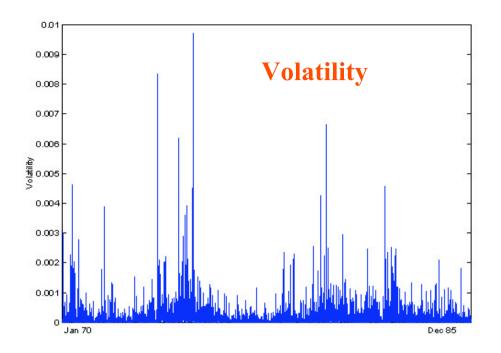
Fig. 2. Noncumulative frequency-area distributions for actual forest fires and wildfires in the United States and Australia: (A) 4284 fires on U.S. Fish and Wildlife Service lands (1986–1995) (9), (**B**) 120 fires in the western United States (1150–1960) (10), (C) 164 fires in Alaskan boreal forests (1990–1991) (11), and (**D**) 298 fires in the ACT (1926–1991) (12). For each data set, the noncumulative number of fires per year $(-d\dot{N}_{cc}/dA_{F})$ with area (A_{F}) is given as a function of A_{F} (13). In each case, a reasonably good correlation over many decades of A_{F} is obtained by using the power-law relation (Eq. 1) with $\alpha = 1.31$ to 1.49; $-\alpha$ is the slope of the best-fit line in log-log space and is shown for each data set.

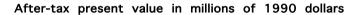
Heavy tails in pdf of Solar flares

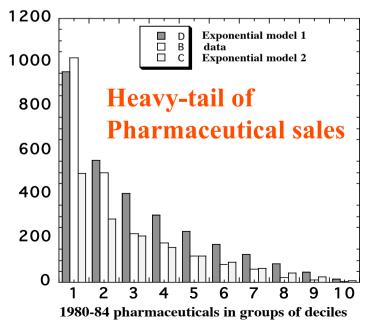


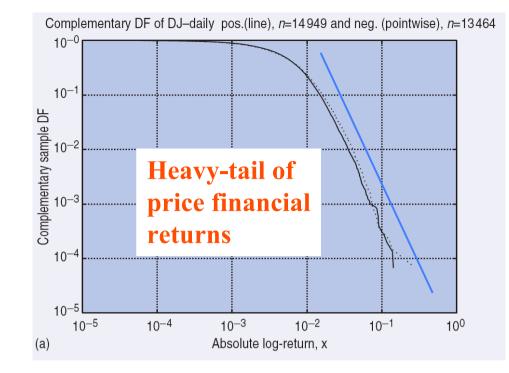
Heavy tails in pdf of Hurricane losses

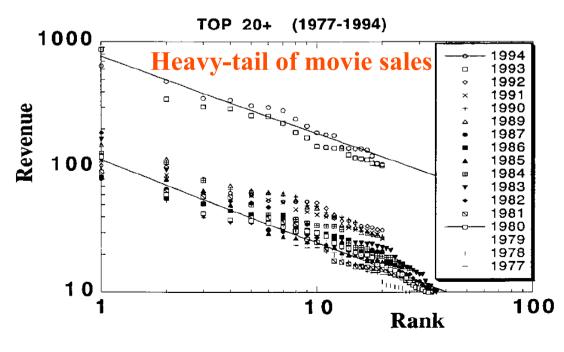




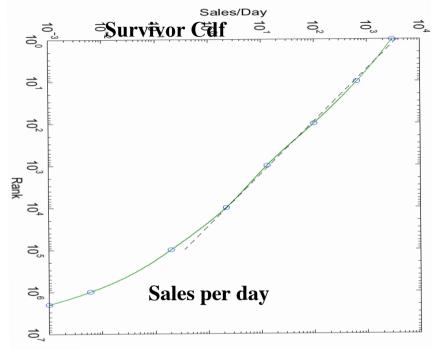




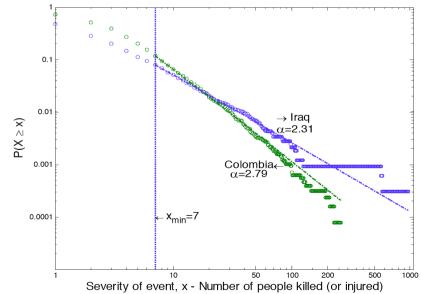




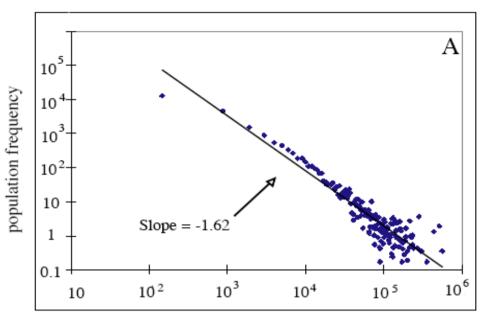
Heavy-tail of pdf of book sales



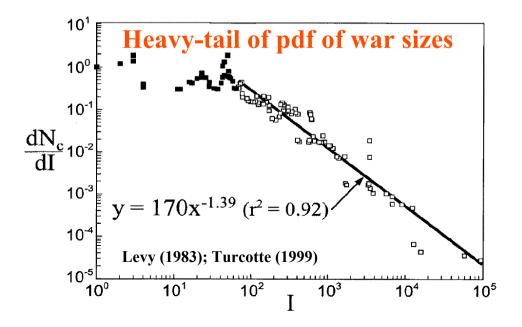
Heavy-tail of pdf of terrorist intensity



Heavy-tail of pdf of health care costs



estimated annual charges (dollars)



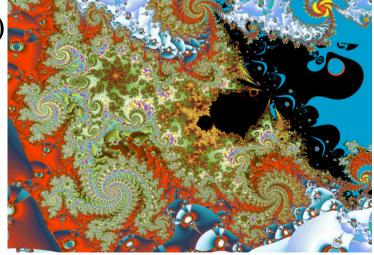
Power laws and large risks

- Power laws are ubiquitous
- They express scale invariance
- Large and extreme events -example of height vs wealth
- Gaussian approach inappropriate: underestimation of the real risks
 - Markowitz mean-variance portfolio
 - Black-Scholes option pricing and hedging
 - Asset valuation (CAPM, APT, factor models)
 - Financial crashes

TWO PROBLEMS

- ✓ What tail?
- ✓ What risks?





What model(s) for the Distributions of Returns?

• Models in terms of Regularly varying distributions:

$$\Pr[r_t \ge x] = \mathcal{L}(x) \cdot x^{-\mu} \qquad (\mu \approx 3 - 4)$$

Longin (1996), Lux (1996-2000), Pagan (1996), Gopikrishnan et al. (1998)...

• Models in terms of Weibull-like distributions:

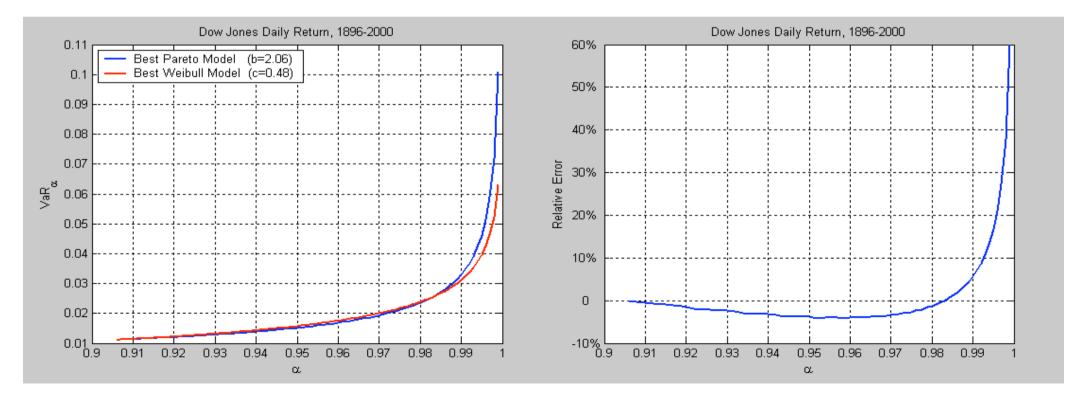
$$\Pr[r_t \ge x] = \exp[-\mathcal{L}(x) \cdot x^c] \quad (c < 1)$$

Mantegna and Stanley (1994), Ebernlein et *al.*(1998), Gouriéroux and Jasiak (1998), Laherrère and Sornette (1999)...

Implications of the two models

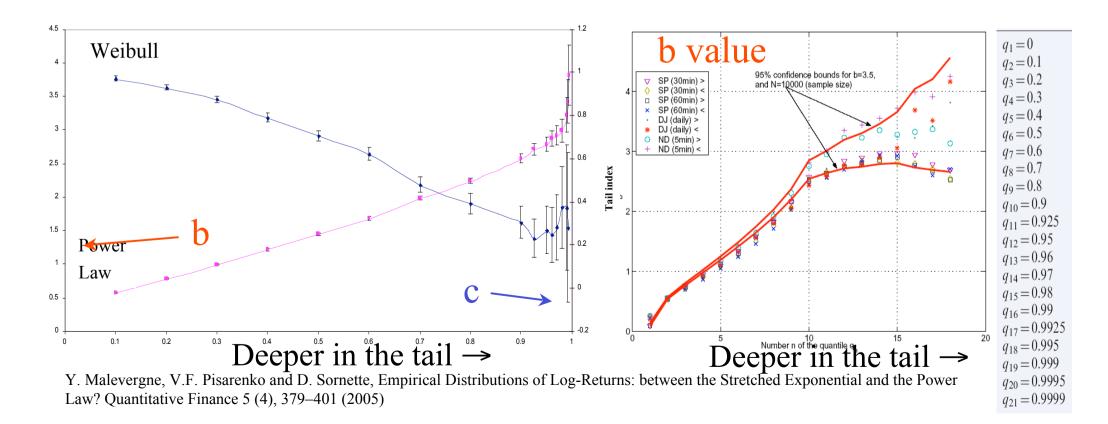
Practical consequences :

- •Extreme risk assessment,
- •Multi-moment asset pricing methods.



Main Results

- Power law model asymptotically embedded in SE model
- The SE model describes a much larger quantile domain
- For both models, the evolution of the parameters is not exhausted at the end of the range of available data.
- Different predictions for large risks (under- and over-estimation?)



Monthly Data		Realised losses <-VaR	
>3 year	rs 2528 Hedge fu	unds (126100 mths)	0.97%
_	5000 simulate	d portfolios of 100 funds	0.96%
		d portfolios of 25 funds	0.97%
2 years 3067 Hedged		Funds (156912 mths)	1.17%
1 year 3067 Hedged		Funds (156912 mths)	1.53%
	Track Value™]	
	TTACK VALUE	INSIGHT •	RESEARCH
	Software for funds of funds risk management		
Value	e@Risk confidence	e level = 5%	

Monthly Data		Realised losses <-VaR
>3 years	2528 Hedge funds (126100 mths)	4.86%
	5000 simulated portfolios of 100 funds	4.86%
	5000 simulated portfolios of 25 funds	4.87%
2 years	3067 Hedged Funds (156912 mths)	5.09%
1 year	3067 Hedged Funds (156912 mths)	5.79%

- •What tail risks? Power law vs Stretched exponentials
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Heavy distribution of firm's capitalizations, lack of diversification and the pricing anomalies

Y. Malevergne^{1,2} and D. Sornette¹ (2006)

 ¹ ETH Zurich – Department Management, Technology and Economics, Switzerland
 ² EM-Lyon Business School – Department Economics, Finance and Control, France e-mails: ymalevergne@ethz.ch and dsornette@ethz.ch

For arbitrary large economies, there may exist a new source of significant systematic risk, which has been totally neglected up to now but must be priced by the market. This is due to

(i) The "self-consistency" condition that the market portfolio (or factors) is constituted of the assets whose returns it is supposed to explain(ii) the distribution of the capitalization of firms is sufficiently heavy-tailed.

New risks in CAPM, APT and other factor models (<u>size effect</u> and <u>book-to-</u> <u>market effect</u>)

Hypotheses

- N risky assets and one risk-free asset,
- the excess returns on the risky assets over the risk-free rate write :

$$\vec{r}_t = \vec{\beta} \cdot r_m(t) + \vec{\varepsilon}(t),$$

- r_m and $\vec{\varepsilon}$ uncorrelated, and $\mathbf{E}[\vec{\varepsilon}] = 0$,
- complete market :

$$r_m(t) = \sum_{i=1}^N w_{m,i}(t) \cdot r_i(t) = \vec{w}_m(t)' \cdot \vec{r}(t).$$

The internal consistency condition

$$\vec{r}_t = \vec{\beta} \cdot r_m(t) + \vec{\varepsilon}(t)$$

$$\downarrow$$

$$r_m(t) \left[1 - \vec{w}_m(t)' \cdot \vec{\beta} \right] = \vec{w}_m(t)' \cdot \vec{\varepsilon}(t)$$

$$\downarrow$$

$$\vec{w}_m(t)' \cdot \vec{\beta} = 1$$
 and $\vec{w}_m(t)' \cdot \vec{\varepsilon} = 0$

There is a "self-consistency" factor f (Fama 1973)

• The condition $\vec{w}_m \cdot \vec{\varepsilon} = 0$ shows that the ε_i 's are *correlated*.

• Fama (1973), Sharpe (1992) : these correlations go to zero in the limit of an infinite number of assets.

Is it always true ???

Proposition

The asymptotic behavior of the excess return over the risk free interest rate of the equally-weighted portfolio is as follows :

1 provided that $E[S] < \infty$, or that S is regularly varying with tail index $\mu = 1$,

$$r_e = \beta_e \cdot r_m + o_p(1);$$

2 provided that S admits a regularly varying distribution with tail index $\mu \in (0, 1)$,

$$r_e = \beta_e \cdot r_m + \mathrm{E}[\gamma] \cdot \frac{\xi_N}{\zeta_N} + o_p(1) ,$$

where ξ_N and ζ_N are two sequences of dependent random variables with distributions given by two stable laws with the same tail index μ .

Concretely: well-diversified portfolios cannot be well-diversified

Variance of the equally weighted portfolio

It can be shown that

$$\operatorname{Var} \boldsymbol{r_e} \simeq \beta_e^2 \cdot \operatorname{Var} \boldsymbol{r_m} + \boldsymbol{K} \cdot \boldsymbol{H_N}$$

for some K > 0 that depends on the distributional properties of the η_i 's and γ_i 's.

Asymptotic behavior of H_N ???

Herfindahl index (participation ratio) of market portfolio $H_N = \sum_{i=1}^{N} \omega_i^2 \sim 1/N$ (well-diversified) to 1 (concentrated)

Proposition

The asymptotic behavior of H_N is as follows :

```
• provided that E[S^2] < \infty
```

 $H_N = O_p \left(1/N \right) \; ,$

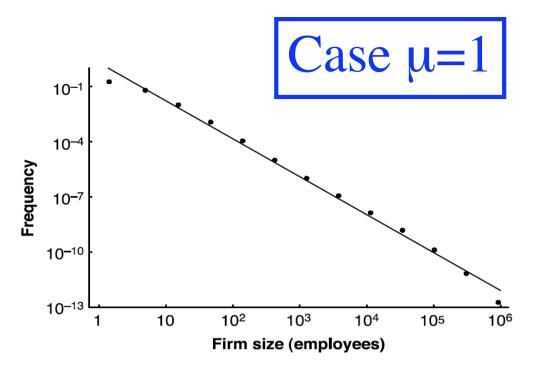
2 provided that $E[S] < \infty$ and admits a regularly varying distribution with tail index $\mu \in (1, 2)$,

$$H_N = O_p \left(1/N^{2(1-1/\mu)} \right) ,$$

3 provided that $E[S] = \infty$,

 $H_N \not\rightarrow 0.$

Concretely: H=0.04-0.05 \Rightarrow N_{eff} = 20-25 (and not 10000)



Zipf, Gabaix et al., Axtell, Simon et al. :

$$\Pr[S \ge s] \sim \frac{1}{s}$$
, for large *s*.

•
$$r_{e} = \beta_{e} \cdot r_{m} + O_{p}(1/\ln N),$$

- $\operatorname{Var} r_{\boldsymbol{\Theta}} = \beta_{\boldsymbol{\Theta}}^2 \cdot \operatorname{Var} r_m + O_p \left(1 / \ln^2 N \right),$
- $H_N = O_p \left(1 / \ln^2 N \right).$

US stock market (Amex + Nasdaq + Nyse) : $N \sim 7000 - 8000$ Non-diversified risk : $\sim 10 - 20\%$ of the total risk

Fig. 1. Histogram of U.S. firm sizes, by employees. Data are for 1997 from the U.S. Census Bureau, tabulated in bins having width increasing in powers of three (30). The solid line is the OLS regression line through the data, and it has a slope of 2.059 (SE = 0.054; adjusted R² = 0.992), meaning that α = 1.059; maximum likelihood and nonparametric methods yield similar results. The data are slightly concave to the origin in log-log coordinates, reflecting finite size cutoffs at the limits of very small and very large firms.

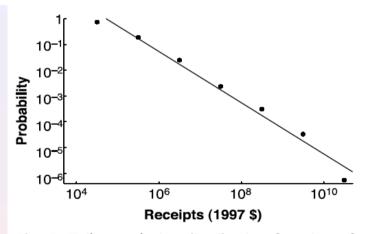


Fig. 2. Tail cumulative distribution function of U.S. firm sizes, by receipts in dollars. Data are for 1997 from the U.S. Census Bureau, tabulated in bins whose width increases in powers of 10. The solid line is the OLS regression line through the data and has slope of 0.994 (SE = 0.064; adjusted $R^2 = 0.976$).

Application to the APT (Arbitrage Pricing Theory)

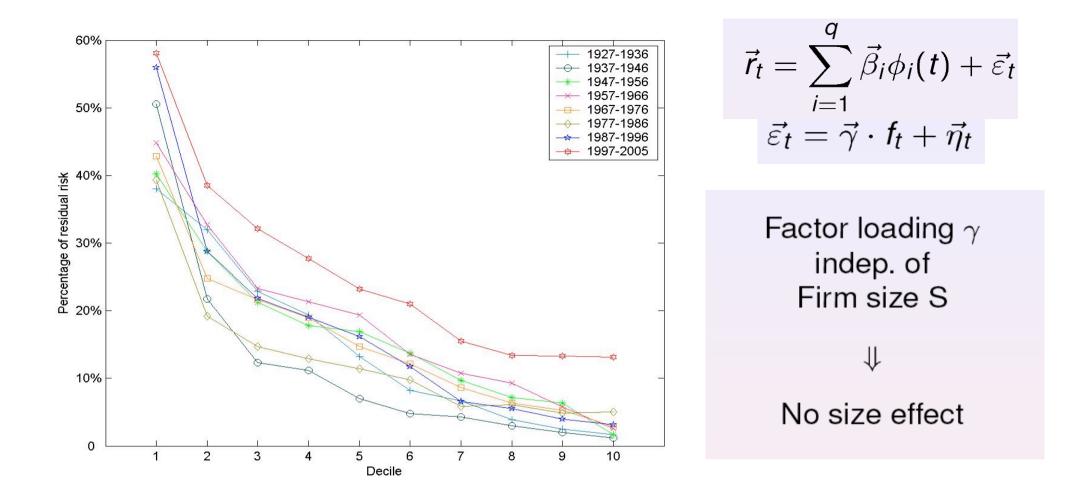
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APT applies : E[\vec{r}] = \vec{\beta} \cdot E[r_m]
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• when $E[S] = \infty$, the "self-consistent" factor f introduces an additional risk that must be remunerated by the market :

$$\mathbf{E}\left[\vec{r}\right] \simeq \vec{\beta} \cdot \mathbf{E}\left[r_{m}\right] + \left[\vec{\gamma} - \left(\vec{w}_{m}' \cdot \vec{\gamma}\right) \cdot \vec{\beta}\right] \cdot \mathbf{E}\left[r_{ICC}\right]$$

 $\vec{\varepsilon_t} = \vec{\gamma} \cdot f_t + \vec{\eta_t}$

Large book-to-market (value) firms have low beta's \Rightarrow larger returns (larger risks?)



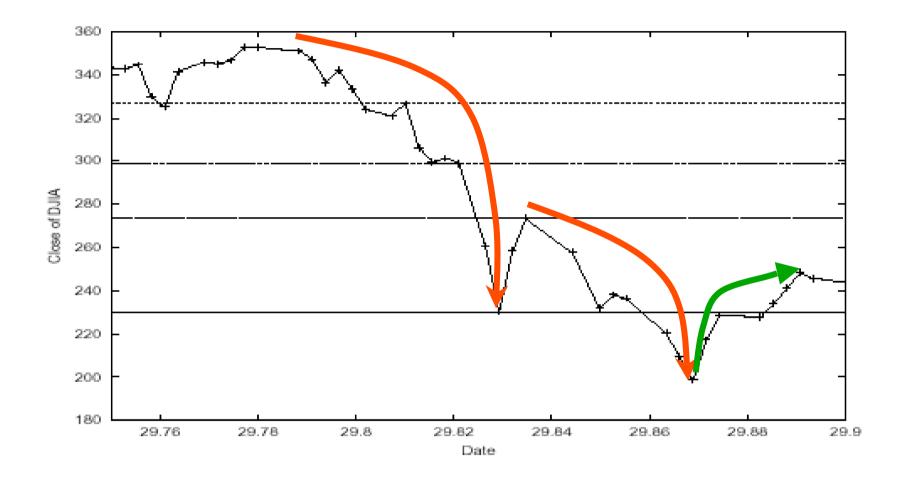
Assumption : Small caps are more sensitive to *f* than large caps

Size effect

 $\gamma(> 0)$ is Stochastically Decreasing in $S \implies 0$

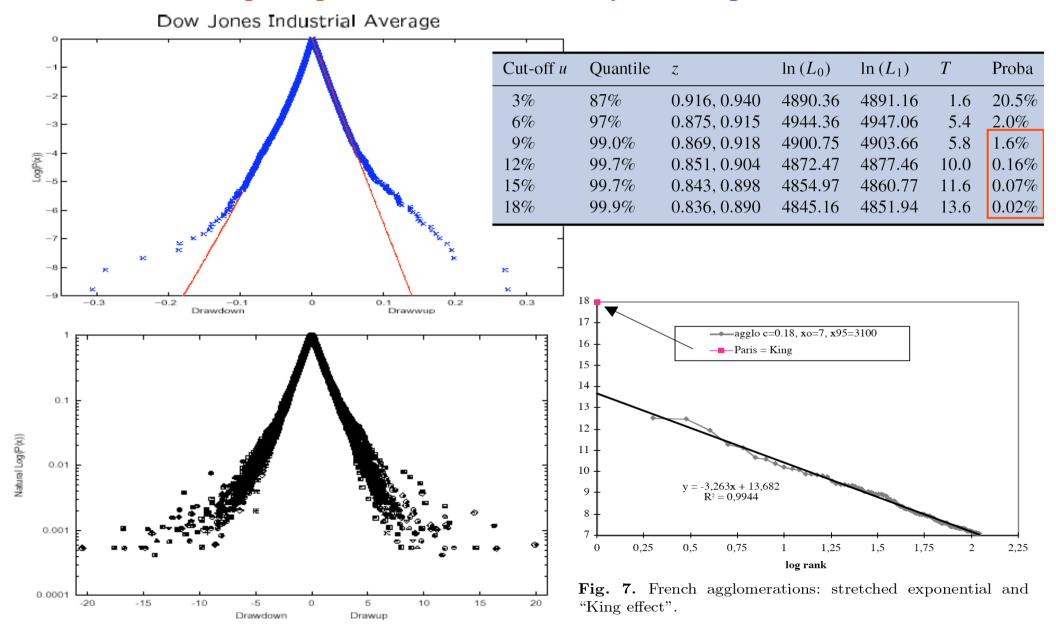
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Better risk measure: drawdowns



Outliers, Kings, "Black swans"

(require special mechanism and may be more predictable)



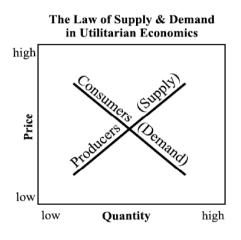
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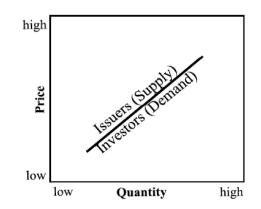
Feedbacks: negative but also POSITIVE

•Systemic risks: "In handling systemic issues, it will be necessary to address, on the one hand, risks to confidence in the financial system and contagion to otherwise sound institutions, and, on the other hand, the need to minimize the distortion of market signals and discipline." (Basle Committee on Banking Supervision)

Mechanisms for positive feedbacks in the stock markets

- Technical and rational mechanisms for positive feedbacks
 - 1. Option hedging
 - 2. Insurance portfolio strategies
 - 3. Trend following investment strategies
 - 4. Asymmetric information on hedging strategies
- Behavioral mechanisms for positive feedbacks
 - 1. It is rational to imitate
 - 2. It is the highest cognitive task to imitate
 - 3. We mostly learn by imitation
 - 4. The concept of "CONVENTION" (Orléan)





The problem of predictability

•Algorithmic complexity theory: most complex systems have been proved to be computationally irreducible, i.e. the only way to decide about their evolution is to actually let them evolve in time.

•The future time evolution of most complex systems appears inherently unpredictable... BUT...

... lesson from PHYSICS (RG)

Lesson from bottom-up hierarchical grouping

Computational Irreducibility and the Predictability of Complex Physical Systems

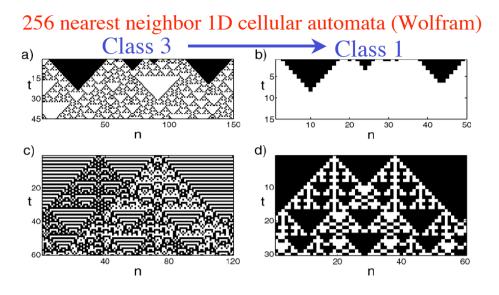


FIG. 1. Examples of coarse-graining transitions. (a) and (b) show coarse-graining rule 146 by rule 128. (a) shows results of running rule 146. The top line is the initial condition and time progress from top to bottom. (b) shows the results of running rule 128 with the coarse-grained initial condition from (a). (c) and (d) show coarse-graining rule 105 by rule 150. (c) shows rule 105 and (d) shows rule 150.

$$C(f_A^{T \cdot t}a(0)) = f_B^t C(a(0)).$$

Namely, running the original CA for Tt time steps and then coarse graining is equivalent to coarse graining the initial condition and then running the modified CA t time steps. The constant T is a time scale associated with the coarse graining.

240 coarse-grainable

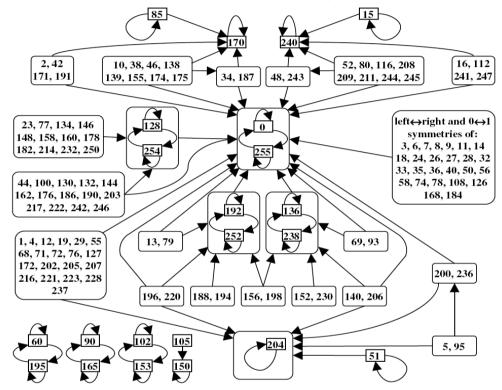
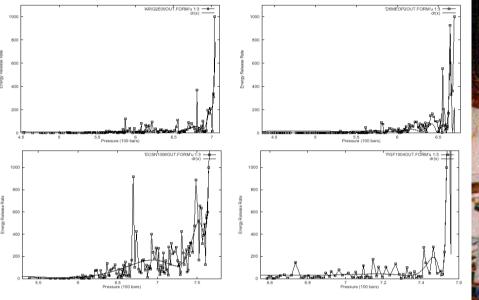


FIG. 2. Coarse-graining transitions within the family of 256 elementary CA. Only transitions with a cell block size N = 2, 3, and 4 are shown. An arrow indicates that the origin rules can be coarse grained by the target rules and may correspond to several choices of N and P.

Coarse-graining rule 110: CIR => C1 Navot Israeli and Nigel Goldenfeld PhysRevLett.92.074105

Strategy: look at the forest rather than at the tree



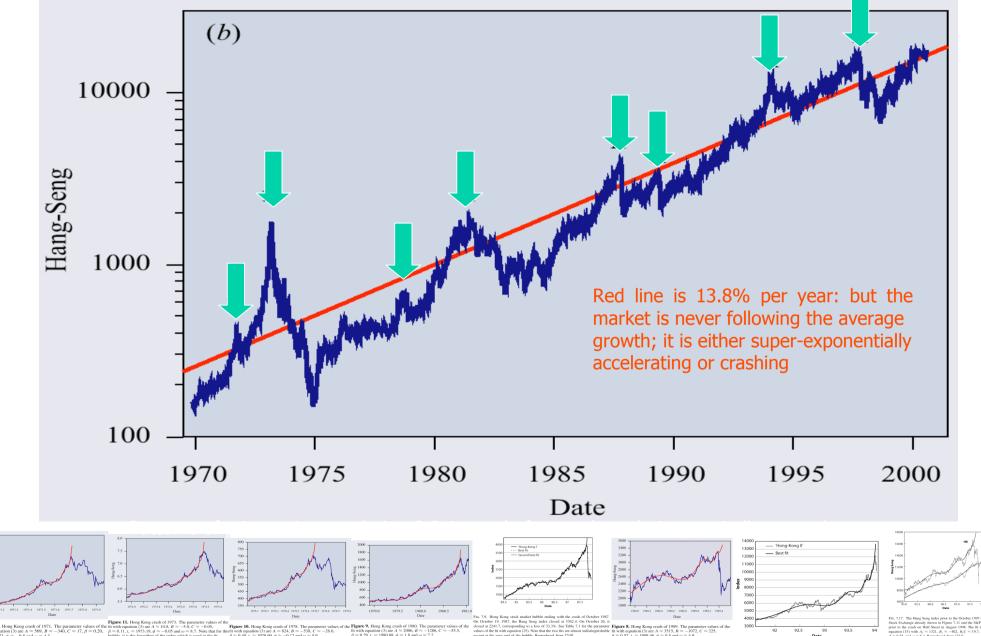
Our prediction system is now used in the industrial phase as the standard testing procedure.





J.-C. Anifrani, C. Le Floc'h, D. Sornette and B. Souillard "Universal Log-periodic correction to renormalization group scaling for rupture stress prediction from acoustic emissions", J.Phys.I France 5, n°6, 631-638 (1995)

Psychology of Investors and Entrepreneurs The "principle of Galilean invariance" in human psychology

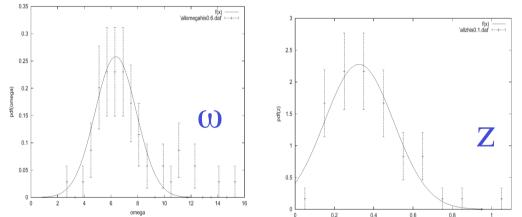


Endogenous vs exogenous crashes

1. Systematic qualification of outliers/kings in pdfs of drawdowns

2. Existence or absence of a "critical" behavior by LPPL patterns found systematically in the price trajectories preceding this

outliers $I(t) = A + B(t_c - t)^{\mathbf{Z}} + C(t_c - t)^{\mathbf{Z}} \cos(\omega \log(t_c - t) - \phi)$



Demonstration of universal values of z and ω across many different bubbles at different epochs and different markets

Results: In worldwide stock markets + currencies + bonds

•21 endogenous crashes

•10 exogenous crashes

A. Johansen and D. Sornette, Shocks, Crash and Bubbles in Financial Markets, in press in Brussels Economic Review on Non-linear Financial Analysis 149-2/Summer 2007 (http://arXiv.org/abs/cond-mat/0210509)

Main Messages

Investors, stock market regulators and macro-economic policy cannot ignore COLLECTIVE BEHAVIOR between AGENTS (with negative and positive feedbacks).

Imitation and herding behaviors lead to Positive and negative feedbacks AND vice-versa : the stock markets and the economy have never been more a CONFIDENCE "game".

Predictions and Preparation: complexity theory applied to such collective processes provides clues for precursors and suggests steps for precaution and preparation.

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The illusion of control

Information processing: normal people's high level of general intelligence makes them too smart for their own good.

✓After a full cycle of rise and fall after which stocks were valued just where they were at the start, all his clients lost money (Don Guyon, 1909)

✓Many academic works suggest that most managers underperform "buy-and-hold" strategy; persistence of winners is very rare, etc.

✓ Rats beat humans in simple games: People makes STORIES! Normal people have an "interpreter" in their left brain that takes all the random, contradictory details of whatever they are doing or remembering at the moment, and smoothes everything in one coherent story. If there are details that do not fit, they are edited out or revised! (T. Grandin and C. Johnson, Animals in translation (Scribner, New York, 2005)

The illusion of control: Minority game

(J. Satinover and D. Sornette, 2006)

Total action of agents
$$A^{\mu}(t) \equiv \sum_{i} a_{i}^{\mu}(t)$$

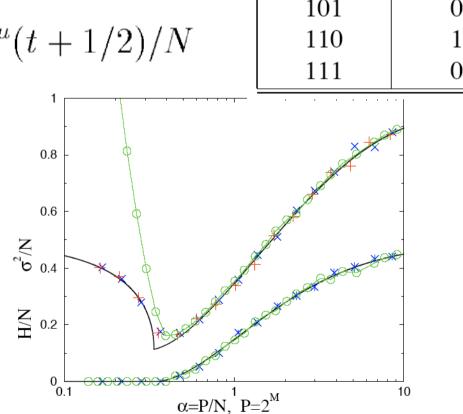
Parameters: m, s, τ , N

Price equation $\log (P(t+1)) = \log (P(t)) + A^{\mu}(t+1/2)/N$

MG payoff of strategy i :

$$g_i(t) = -a_i(t)A(t)$$

✓ Inductive reasoning✓ Minority mechanism

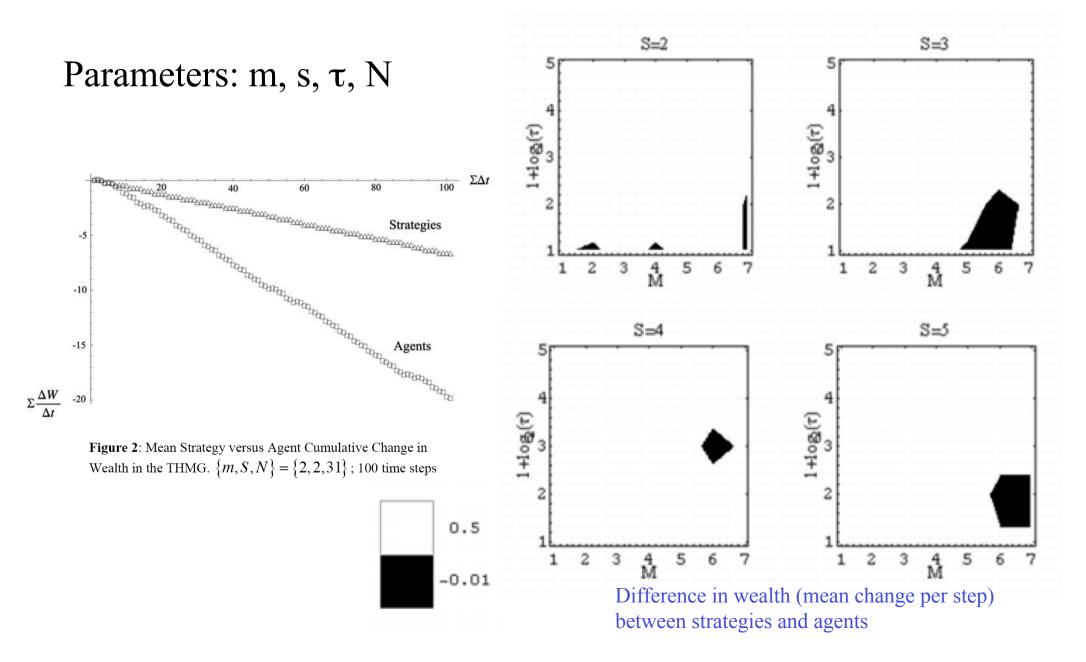


Example of strategy

signal (μ)	prediction
000	0
001	0
010	1
011	0
100	1
101	0
110	1
111	0

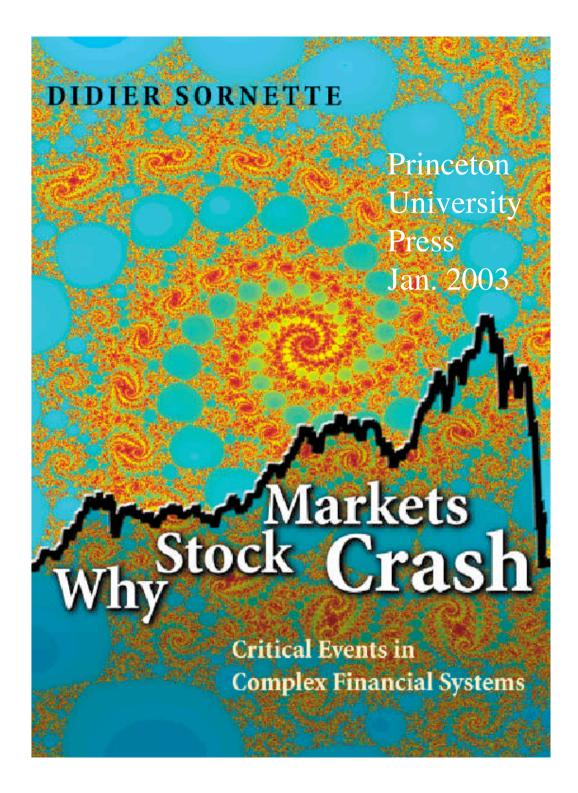
The illusion of control: Minority game example

(J. Satinover and D. Sornette, 2006)



My Research Agenda to Address Risks in Financial Management

- Added-value strategies / expected returns
 - 1. Asymmetric information between managers and investors
 - 2. Reverse engineering of hedge-funds and derivative strategies
 - 3. Combining portfolio and investment strategies
- Risk measure and control
 - 1. Scenario and crises analyses
 - 2. Robust statistical methods to address model error
- Bubbles, crashes and extreme risks of unsustainable regimes
 - 1. The "Crisis Observatory" and crash alarm index
 - 2. Robust multivariate scanning of world assets
 - 3. NL models with positive and negative feedbacks
- Macro and micro economic analyses
 - 1. Separating information from "noise" and false consensus
 - 2. Endogenous vs exogenous extreme risks



D. Sornette



Critical Phenomena in Natural Sciences

Chaos, Fractals, Selforganization and Disorder: Concepts and Tools

First edition 2000

Second enlarged edition 2004



Springer

Malevergne · Sornette

⑳

Extreme Financial Risks



Extreme Financial Risks

From Dependence to Risk Management

Nov 2005

