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**Thorsten Hens · Terje Lensberg ·
Klaus Reiner Schenk-Hoppé · Peter Wöhrmann**

Abstract As early as 1934 Graham and Dodd conjectured that excess returns from value investment originate from a tendency of stock prices to converge towards a fundamental value. This paper confirms their insights within the evolutionary finance model of Evstigneev et al. (Econ Theory 27:449–468, 2006). Our empirical results show the predictive power of the evolutionary benchmark valuation for the relative market capitalization and its dynamics in the sample of firms listed in the Dow Jones Industrial Average index in 1981–2009.

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1 Introduction

It is an empirically robust finding that value portfolios (i.e. portfolios with high value-to-price ratio) generate significantly higher returns than those with low value-to-price ratios. Practitioners often assign the latter class of assets colorful names such as growth, momentum or glamor portfolios.

This observation is a puzzle within the Capital Asset Pricing Model (CAPM) because the betas of value portfolios may even be smaller than those of growth portfolios. Starting with Basu (1977, 1983), the so-called value premium puzzle has been confirmed by empirical studies of different stock markets and time periods, e.g. Campbell (1999), Davis et al. (2000), Fama and French (1992) and Lettau and Wachter (2007). For instance, Lettau and Wachter (2007) analyzed monthly data from 1952 to 2002. They find an excess return of value over growth portfolios of about 4.01% p.a. when value is defined by the dividend-price-ratio. Even higher excess returns are found for other value criteria: For the earnings-price-ratio the excess return is 9.31% p.a., for the cash flow-price-ratio it is 8.04% p.a., and for the book to market-ratio it is 5.63% p.a. For all four value criteria the CAPM-betas of value portfolios are not higher and many times even smaller than those of growth portfolios, see (Lettau and Wachter 2007, Table III).

Our aim is to apply the evolutionary finance approach of Evstigneev et al. (2006, 2008) to the study of the value premium puzzle. We argue that, as originally claimed by Graham and Dodd (1934), excess returns from value investment stem from a tendency of asset prices to converge toward their fundamental value. Value investment works because financial markets may be temporarily displaced from the long-run fundamental equilibrium but find their way back. This convergence property gives rise to a predictability of asset returns based on fundamental criteria such as the dividend yield. In the evolutionary stock market model of Evstigneev et al. (2006, 2008), the convergence of asset prices stems from the wealth dynamics among heterogenous strategies interacting in the financial markets. Their main finding is that a particular investment strategy is selected by the market (as the long-term outcome of this wealth dynamics) which, in turn, provides an asset pricing benchmark. This benchmark predicts that the relative market value of a firm (relative to the total market value) is equal to the expected discounted *relative* dividend payments. The strategy of holding assets in proportions equal to their expected relative dividends is the unique evolutionary stable strategy (ensuring long-term survival and domination): Any mutant strategy entering the market will eventually lose its wealth to this incumbent investor. The effect on the asset price dynamics is that the market value of firms reverts to the benchmark in the long-term. The evolutionary model, in combination with the results obtained

in this framework, offers a novel approach to the study of the value premium puzzle.

By its very nature an evolutionary model has a long-term perspective. Thus our explanation of the value puzzle fits to recent explanations put forward by Lettau and Wachter (2007) and by Hansen et al. (2008) who show that the cash flows of value portfolios have a high duration and co-move with long-term risk like fluctuations in GDP-growth.

Two predictions derived from evolutionary finance will be tested empirically in this paper. The data sample consists of the Dow Jones Industrial Average (DJIA) index during the time period 1979–2009. The first hypothesis is concerned with pricing of firms in cross sections. We claim that our benchmark valuation has predictive power for the relative value of firms in any given year in the sample. The second hypothesis states that asset prices converge to this benchmark over time. If both predictions have strong empirical support, then the value premium puzzle ceases being a puzzle from an evolutionary perspective. Indeed, this is the case as we show in this paper.

The paper is organized as follows. Section 2 briefly reviews the underlying evolutionary model and the relevant results on evolutionary stable investment strategies. Section 3 shows how to justify the findings as an equilibrium in a standard asset pricing model (without out-of-equilibrium dynamics). Section 4 derives the implications for the value premium puzzle and presents the empirical results. Section 5 concludes.

2 Evolutionary stable markets

We consider a financial market with $I \geq 1$ investors and $K \geq 1$ long-lived assets (stocks). Asset k, $k = 1, ..., K$, pays a random dividend $D_{k,t} = D_k(s_t) \geq 0$ at time $t = 0, 1, ...$ Dividend payoffs are determined by the state of nature $s_t \in S$ (where S is a finite set). Each asset is assumed to be in unit supply. This is without loss of generality and facilitates the presentation: The asset prices in the model will correspond to the market values of firms. The dividend is not reinvested and plays the role of a perishable consumption good as in Lucas (1978).

Normalizing the price of the consumption good to one in all periods in time, an investor's wealth in terms of this numeraire is given by

$$w_{t+1}^i = \sum_{k=1}^{K} \left(D_{k,t+1} + P_{k,t+1} \right) \theta_{k,t}^i \tag{1}$$

where $(\theta_{1,t}^i, ..., \theta_{K,t}^i)$ denotes investor i's portfolio $(i = 1, ..., I)$ and $P_{k,t}$ is asset k's price in period t $(k = 1, ..., K)$. Investor i's portfolio holdings in asset k and the price of each asset k are determined by

$$\theta_{k,t}^i = \frac{\lambda_{k,t}^i w_t^i}{P_{k,t}} \quad \text{and} \quad P_{k,t} = \sum_{i=1}^{I} \lambda_{k,t}^i w_t^i \tag{2}$$

where $\lambda_{k,t}^i$ is investor i's budget share assigned to the purchase of asset k. Prices are determined by equating each asset's market value with the investment in that asset. Note that at these prices demand is equal to supply because $\theta_{k,t}^1 + \dots + \theta_{k,t}^I = 1$ (recall that supply is normalized to one). An investment strategy is formally defined as measurable functions $\lambda_t^i(s^t) = \left(\lambda_{0,t}^i(s^t), \lambda_{1,t}^i(s^t), \dots, \lambda_{K,t}^i(s^t)\right)$ with $\lambda_t^i(s^t) \geq 0$ and $\sum_{k=0}^K \lambda_t^i(s^t) = 1$, where $s^t = (s_t, s_{t-1}, \dots, s_0)$ is the history of states up to time t.

So far we made no assumptions other than those of Lucas (1978). But we have written his model in a way that is more suitable for our analysis. Our first assumption over and above the Lucas model is that all investors consume the same constant fraction $\lambda_0 > 0$ of their wealth in all periods in time, i.e., $\lambda_{0,t}^i(s^t) \equiv \lambda_0$. This assumption is hard to justify when investors' portfolio strategies λ are generated by intertemporal utility maximization since the consumption rate would then be endogenous and fluctuating with the asset returns. However, focussing on the long run survival, as we one does in an evolutionary model, this assumption is paramount since those investors with on average higher consumption rates would easily be driven out by those who consume less. Thus without this assumption nothing can be learned about the evolutionary fitness of portfolio strategies which is the question we address in this paper.

One has (using Eqs. 1 and 2 and the fact that $\lambda_0 = 1 - \sum_k \lambda_{k,t}^i$ for each i):

$$\sum_{k=1}^K D_{k,t} =: D_t = \lambda_0 \, W_t \qquad (3)$$

where

$$W_t = \sum_{i=1}^I w_t^i.$$

The dynamics of the investors' wealth shares $r_t^i = w_t^i / W_t$ can be derived from Eqs. 1, 2 and 3:

$$r_{t+1}^i = \sum_{k=1}^K \left(\lambda_0 \, d_{k,t+1} + \sum_{j=1}^I \lambda_{k,t+1}^j \, r_{t+1}^j \right) \frac{\lambda_{k,t}^i \, r_t^i}{\sum_{j=1}^I \lambda_{k,t}^j \, r_t^j} \qquad (4)$$

where $d_{k,t+1} = D_{k,t+1}/D_{t+1}$ denotes asset k's relative dividend payoff. It is assumed that at least one asset pays a dividend, $D_{t+1} > 0$. The last equation is linear in $r_{t+1} = (r_{t+1}^1, \dots, r_{t+1}^I)$. Its solution is given by

$$r_{t+1} = \lambda_0 \left(\mathrm{Id} - \left[\frac{\lambda_{k,t}^i r_t^i}{\sum_i \lambda_{k,t}^i r_t^i} \right]_i^k \Lambda_{t+1} \right)^{-1} \left[\sum_{k=1}^K d_{k,t+1} \frac{\lambda_{k,t}^i r_t^i}{\sum_i \lambda_{k,t}^i r_t^i} \right]_i \qquad (5)$$

where $\Lambda_{t+1}^T = \left(\lambda_{1,t+1}^T, \dots, \lambda_{K,t+1}^T\right) \in \mathbb{R}^{I \times K}$ denotes the matrix of budget shares in period $t+1$. The relative price of asset k is given by $p_{k,t} = \sum_{i=1}^I \lambda_{k,t}^i r_t^i$.

Equation 5 governs the evolution of wealth shares for given investment strategies. The first term captures the capital gains while the second term describes the change in relative wealth as resulting from the exogenous dividends. Our interpretation of the dynamics (Eq. 5) is that of a *market selection process* on a set of *investment strategies*.

The dynamics (Eq. 5) is well-defined (Evstigneev et al. 2006, Proposition 1) if there is an investor j with $r_0^j > 0$ and $\lambda_{k,t}^j > 0$ for all t, s^t, k.[1] Equation 5 generates a random dynamical system on the simplex $\Delta = \{r \in \mathbb{R}^I \mid r^i \geq 0, \sum_i r^i = 1\}$. For any initial distribution of wealth $w_0 \in \mathbb{R}_+^I$, Eq. 5 defines the path of wealth shares on the event tree with branches s^t. The initial distribution of market shares is given by $(r_0^i)_i = (w_0^i / W_0)_i$. The wealth of a strategy λ^i in any period in time can be derived from its wealth share and the aggregate wealth, defined by Eq. 3, as

$$w_{t+1}^i = \frac{D_{t+1}(s^{t+1})}{\lambda_0} r_{t+1}^i. \tag{6}$$

The further analysis will be restricted to the case of state-dependent strategies, i.e., $\lambda_t^i(s^t) = \lambda^i(s_t)$ and a stationary process s_t. Let us denote $\lambda_t^i := \lambda^i(s_t)$. Hence, summarizing our assumptions over and above the Lucas model, we assume a constant consumption rate that is common among all investors, adapted non-negative portfolio strategies, and stationary dividends. These assumptions reduce the Lucas model to a simple toy model that is nevertheless rich enough to explain important empirical observations, as we show in Section 4. In the context of this financial market model, Evstigneev et al. (2006) identified a unique investment strategy λ^* which is evolutionary stable. We present a heuristic derivation of this investment strategy here. Assume, for the sake of simplicity of presentation, strategies are slowly varying (nearly constant). Consider the market selection process close to the one-owns-all states. Suppose investment strategy λ^j owns almost all of the wealth, i.e. $r_t^j \approx 1$. Then it determines prices in the sense that $p_{k,t} \approx \lambda_{k,t}^j$ and $p_{k,t+1} \approx \lambda_{k,t+1}^j$. By assumption we also have $\lambda_{k,t+1}^j \approx \lambda_{k,t}^j$. Then Eq. 4 yields

$$r_{t+1}^i \approx \sum_{k=1}^K \left(\lambda_0 \, d_k(s_{t+1}) \frac{\lambda_{k,t}^i}{\lambda_{k,t}^j} + \lambda_{k,t+1}^i \right) r_t^i$$

$$= \left(1 - \lambda_0 + \lambda_0 \sum_{k=1}^K d_k(s_{t+1}) \frac{\lambda_{k,t}^i}{\lambda_{k,t}^j} \right) r_t^i.$$

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[1]Note that a strategies that could go short, i.e., allowing that $\lambda_{k,t}^j < 0$ for some k, cannot be evolutionary stable since there always exists a configuration of rival strategies that make it bankrupt, leading to extinction.

The exponential growth rate of strategy λ^i's market share at λ^j-prices can be inferred from this equation. It is given by

$$g_{\lambda^j}(\lambda^i) = \mathbb{E} \ln \left[1 - \lambda_0 + \lambda_0 \sum_{k=1}^{K} d_k(s) \frac{\lambda_k^i(s)}{\lambda_k^j(s)} \right] \tag{7}$$

where $\mathbb{E}$ denotes expected value with respect to the distribution on the set of states of nature S.

Evstigneev et al. (2006) establish the following result:

Theorem 1 (Evolutionary Stable Markets) *Suppose the states of the world s_t follow a stationary Markov process. Then the investment strategy λ^*, defined by*

$$\lambda_{k,t}^* = \lambda_0 \, \mathbb{E}_t \sum_{u=1}^{\infty} (1 - \lambda_0)^u \, d_k(s_{t+u}), \quad k = 1, ..., K \tag{8}$$

is the only investment strategy that is locally stable against any other investment strategy. More precisely, $g_{\lambda^*}(\lambda) < 0$ and $g_\lambda(\lambda^*) > 0$ for all $\lambda \neq \lambda^*$.

Here $\mathbb{E}_t = \mathbb{E}(\cdot \mid s_t)$ is the conditional expectation.

This result shows that there is always a tendency for the wealth process to converge to the λ^*-investor. Any other composition of the market can be invaded by some strategy growing faster than the incumbent strategy; only if λ^* governs the market then no further successful invasion is possible. The wealth dynamics implies that relative asset prices $p_{k,t}$ will converge to the process λ_t^*.

The above result assumes that investment strategies are distinct across investors. How can one analyze the case in which, for instance, more than one investor adopts the λ^* strategy? Fortunately, even the general case of investors pursuing the same portfolio strategy is straightforward: Since the relative wealth of two investors with the same portfolio strategy is fixed over time, it is equivalent to assume that investors with the same strategy set up a fund with claims equal to their initial share.

The stability result, Theorem 1, highlights the wisdom in the following comment of the fathers of value investment: Graham and Dodd (1934, page 36) state "We do not believe that short-run price movements–the day to day or the month-to-month variations–are a valid or profitable concern for the security analyst."

3 Equilibrium with a representative agent

The evolutionary stable investment strategy can also be obtained as an equilibrium within a standard representative agent model with a stochastic discount factor. This agent is a rational expected utility maximizer. Of course this approach lacks an 'out-of-equilibrium' dynamics. The finding rests on the observation that any arbitrage free process of asset prices can be represented by *some* stochastic discount factor of some utility function, Harrison and Kreps

(1979). Since there is no arbitrage in our model one should thus be able to represent asset prices as the outcome of a utility optimization problem. The following theorem assets that the asset prices derived from the investment strategy λ^* can also be generated in an economy with a representative investor who maximizes discounted expected utility and possesses a constant relative risk aversion of 1. This result connects our findings to asset pricing based on log-optimal investing. Following Kelly (1956) and Breiman (1961) an impressive literature including Algoet and Cover (1988), Cover (1984, 1991) has evolved in which log-optimal portfolios are characterized for various exogenous return processes. See the recent book edited by MacLean et al. (2011) for a full account of this literature. In particular Long (1990) has shown that the return of the log-optimal portfolio can be used to replace the risk free rate in discounted expected dividend pricing. Theorem 2 shows how the log-optimal pricing rule looks like when all asset returns are stripped down to the dividend processes. The unique pricing factor is then given by the relative dividends.

Theorem 2 (Stochastic Discount Factor) *Suppose a representative investor with constant relative risk aversion equal to* 1 *invests in* $k = 1, ..., K$ *stocks and uses the residual income for his consumption. Then, denoting his discount factor by β, the stock prices satisfy*

$$p_{k,t} = \frac{P_{k,t}}{\sum_n P_{n,t}} = \frac{1-\beta}{\beta} \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{D_{k,\tau}}{D_\tau} \right] \tag{9}$$

The stochastic discount factor associated with the standard moment condition with respect to the return of the market portfolio R_t,

$$1 = \mathbb{E} \left[\delta_{t,t+\tau} R_{t+\tau} \right] \tag{10}$$

is given by

$$\delta_{t,t+\tau} = \beta^\tau \frac{D_t}{D_{t+\tau}}, \quad t = 0, 1, ..., \quad \tau = 1, 2, ...$$

Proof It is well known that

$$P_{k,t} = u'(c_t^*)^{-1} \beta \mathbb{E} \left[u'(c_{t+1}^*)(D_{k,t+1} + P_{k,t+1}) \right] \tag{11}$$

for all t and k follows from the first order condition of an expected utility maximizer choosing among $k = 1, ..., K$ assets. Furthermore, in equilibrium we have $c_t^* = D_t$ for all t. Substituting $u'\left(c_t^*\right) = c_t^{*-1}$ (i.e. the agent's preferences are described through the instantaneous utility $u(c) = \ln(c)$) gives

$$P_{k,t} = D_t \beta \mathbb{E}_t \left[(D_{k,t+1} + P_{k,t+1})/D_{t+1} \right]$$

Forward iteration gives

$$P_{k,t} = D_t \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \frac{D_{k,\tau}}{D_\tau} \right] \tag{12}$$

Summing over all assets, we obtain

$$P_t = \sum_{k=1}^{K} P_{k,t} = D_t \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} = \frac{\beta D_t}{1-\beta} \tag{13}$$

Finally, combining Eqs. 12 and 13 gives Eq. 9.

Equation 10 is equivalent to

$$P_t = \beta^\tau \mathbb{E} \left[\frac{D_t}{D_{t+\tau}} (P_{t+\tau} + D_{t+\tau}) \right] \tag{14}$$

which is immediate from Eq. 13.   □

The result shows that stochastic discount factors can rationalize the asset prices obtained in our evolutionary finance model. The stochastic discount factors however are not defined outside the λ^*-equilibrium. Thus Theorem 1 can be seen as a justification for Theorem 2. Indeed the asset pricing model (Eq. 11) is frequently estimated at the equilibrium by using a first-order Taylor series approximation (log-linearization), see, e.g., Campbell and Cochrane (1999).[2] The evolutionary process allows for describing a stable dynamics converging to the pricing relation given in Theorem 2.

4 Implications for the value premium puzzle

In this section we derive two testable hypotheses from the theoretical results obtained for the evolutionary finance model. To test the model empirically we use constituents of the Dow Jones Industrial Average (DJIA).[3] The predictions follow from the stability properties of the wealth dynamics. They may help to explain why on long-term averages stock markets look quite rational while severe departures are possible in the short- and medium-term. The particular application of this line of thought is the value premium puzzle. First, relative market capitalizations and relative dividend payments should exhibit a strong relation in cross sections, i.e. the evolutionary asset pricing benchmark should be meaningful. Second, deviations from this benchmark should systematically decrease over time, i.e. the benchmark should be stable. From an evolutionary perspective, the value premium puzzle ceases being a puzzle if both hypotheses hold.

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[2] We are grateful to an anonymous referee for pointing this out.
[3] Data are taken from CRSP.

Hypothesis 1 (Market Capitalizations are Determined by λ^*) According to Theorem 1, the relative market valuations of firms should be given by the strategy λ^*. This investment strategy prescribes to divide wealth across assets proportional to the present expected value of their (relative) future dividend payoffs which is given by

$$p_{k,t} = \lambda^*_{k,t} = \lambda_0 \, \mathbb{E}_t \sum_{u=1}^{\infty} (1 - \lambda_0)^u \, d_k(s_{t+u}) \tag{15}$$

The relative market capitalization of a firm is simply calculated from the stock price and the number of shares issued for all firms in the sample. How to determine the relative fundamental value however is less straightforward and, obviously, leaves the econometrician with many options. Rather than attempting to formulate a specific model to calculate the future expected values, we take as a proxy for $\lambda^*_{k,t}$ the current relative dividend $d_{k,t}$ of all firms that were listed in DJIA for at least two subsequent years during the period under study. The joint hypothesis tested is that in the linear cross-sectional regression

$$p_{k,t} = a_0(t)d_{k,t} + a_1(t) + \varepsilon_t, \quad k = 1, ..., K \tag{16}$$

$a_0(t) > 0$ and $a_1(t) = 0$ for $t = 1979, ..., 2009$. ε_t is the error term. Notice that substituting $a_0(t) = \lambda_0 \, \mathbb{E}_t \sum_{u=1}^{\infty} (1 - \lambda_0)^u$ relates Eqs. 15 and 16. If this relation holds then, in each year, the relative market capitalization of a firm depends linearly on its current relative dividend payment.

Hypothesis 2 (Convergence of Market Capitalizations to λ^*) The convergence of asset prices to the Kelly prices λ^* is a consequence of the market dynamics. This dynamics implies that (small) deviations from the benchmark $\lambda^*_{k,t}$ should vanish over time. The empirical benchmark is provided by the valuation derived in Hypothesis 1, that is

$$\lambda^*_{k,t} = a_0(t)d_{k,t} + a_1(t),$$

where $a_0(t)$ and $a_1(t)$ denote the estimated values. Suppose there is one λ^*_t investor and a mutant investment strategy μ_t representing all the other investors in the market. Exponentially fast convergence of the Kelly investor's wealth share $r^*_t \to 1$ can be expressed as $[1 - r^*_{t+1}] = L_t[1 - r^*_t]$ with some random variable L_t, with $E \ln L_t < 0$. This parameter is determined by the exponential of the logarithmic growth rate $g_{\lambda^*}(\mu)$ as defined in Eq. 7. Since

$$p_{k,t} = \lambda^*_{k,t} r^*_t + \mu_{k,t}(1 - r^*_t)$$

one obtains (after some elementary calculations) the relation

$$\left[\lambda^*_{k,t+1} - p_{k,t+1}\right] = L_t \frac{\lambda^*_{k,t+1} - \mu_{k,t+1}}{\lambda^*_{k,t} - \mu_{k,t}} \left[\lambda^*_{k,t} - p_{k,t}\right] \tag{17}$$

Assuming that the relative position of μ_t to λ_t^* is approximately the same between two consecutive time periods, i.e., assuming $\lambda_{k,t}^* - \mu_{k,t} \approx \gamma_k$, which holds, e.g., with time constant strategies, we formalize our hypothesis as follows. Between any two consecutive years, t and $t+1$, $t = 1981, ..., 2008$, the linear regression

$$\left[\lambda_{k,t+1}^* - p_{k,t+1}\right] = a(t)\left[\lambda_{k,t}^* - p_{k,t}\right] + \epsilon_{k,t}, \quad k = 1, ..., K \qquad (18)$$

has a least-squares estimator $0 < a(t) < 1$.

The empirical results are summarized in Tables 1 and 2. Hypothesis 1 on the relevance of the Kelly rule as a pricing benchmark (for the relative valuation of firms) is supported by the data. In all but 3 years of the sample the coefficient $a_0(t)$ is significantly positive at the 5% level. In addition, the coefficients $a_1(t)$ are small, and not significantly different from zero in 2/3 of the sample. The adjusted R^2 values indicate that a considerable amount of the variation in the data is explained by the model, see Table 1.

Hypothesis 2 on the convergence of relative market capitalization towards the benchmark is tested in Table 2. Columns 2–4 contain coefficients, P-values

Table 1 Results on comparison of asset prices with the Kelly benchmark in cross sections (last trading day in a given year t)

Year t	$a_0(t)$	P-value	$a_1(t)$	P-value	R^2 adj.
1979	0.974	0.000	0.001	0.810	0.933
1980	0.947	0.000	0.002	0.608	0.964
1981	0.854	0.002	0.005	0.376	0.912
1982	0.864	0.143	0.005	0.682	0.717
1983	0.898	0.116	0.004	0.750	0.740
1984	0.964	0.042	0.001	0.888	0.802
1985	0.978	0.027	0.001	0.928	0.803
1986	0.866	0.000	0.005	0.194	0.890
1987	0.833	0.000	0.006	0.050	0.900
1988	0.855	0.000	0.005	0.196	0.920
1989	0.654	0.000	0.012	0.001	0.652
1990	0.833	0.000	0.006	0.043	0.844
1991	0.726	0.000	0.010	0.023	0.689
1992	0.619	0.001	0.013	0.002	0.616
1993	0.691	0.015	0.011	0.133	0.766
1994	0.679	0.005	0.011	0.075	0.739
1995	0.710	0.002	0.010	0.088	0.752
1996	0.750	0.002	0.009	0.164	0.713
1997	0.822	0.002	0.006	0.352	0.715
1998	0.799	0.002	0.007	0.340	0.631
1999	0.965	0.000	0.001	0.864	0.692
2000	0.643	0.001	0.012	0.078	0.547
2001	0.520	0.001	0.017	0.044	0.343
2002	0.515	0.000	0.017	0.027	0.365
2003	0.611	0.000	0.013	0.004	0.533
2004	0.283	0.709	0.024	0.132	0.350
2005	0.690	0.000	0.010	0.005	0.637
2006	0.705	0.000	0.010	0.003	0.691
2007	0.617	0.006	0.013	0.010	0.467
2008	0.731	0.033	0.009	0.261	0.412
2009	0.669	0.000	0.011	0.014	0.590

Coefficients, P-values and R^2 of the regression 16 testing the asset pricing Hypothesis 1. P-values are obtained from White-corrected covariance matrices to adjust for heteroscedastic errors

of two-sided t-tests and adjusted R^2's of the regression $v_{k,t+1} = a(t)v_{k,t} + \epsilon_{k,t}$, where $v_{k,t} = \lambda_{k,t}^* - p_{k,t}$, cf. Eq. 18. All coefficients are greater than zero, and all but 9 are less than 1. A coefficient $a(t) > 1$ implies divergence from the benchmark from the current to the next year. The nine 'no value' years in our sample are 1981, 1982, 1991, 1996, 1997, 1998, 2001, 2007 and 2008, and include the famous bubble years of the late nineties. However, a t-test across all years of the null hypothesis that $a(t) = 1$ is rejected at the 5% level (P-value 0.034), and the null hypothesis that $a(t) = 0$ is rejected at any level of significance (P-value 0.000).

It is well-known that regression models such as Eq. 18 may suffer from spurious mean reversion. We therefore perform a supplementary test of Hypothesis 2 on the convergence of asset prices to the λ^* benchmark. For each year t, we first regress $\log(p_{k,t})$ on $\log(d_{k,t})$ as in Eq. 16, and calculate the (negative) residuals $v_{k,t} = \hat{\lambda}_{k,t} - \log(p_{k,t})$, where $\hat{\lambda}_{k,t}$ is the the predicted log

Table 2 Results on the convergence of asset prices to the λ^* benchmark from year t to $t+1$

Year t to $t+1$	Regression results			Annual
	$a(t)$	P-value	R^2 adj.	return
1979–1980	0.545	0.105	0.532	0.033
1980–1981	1.261	0.000	0.767	0.074
1981–1982	1.899	0.000	0.868	0.010
1982–1983	0.965	0.000	0.966	0.177
1983–1984	0.868	0.000	0.942	0.084
1984–1985	0.969	0.000	0.936	0.110
1985–1986	0.489	0.002	0.642	0.097
1986–1987	0.818	0.000	0.759	0.069
1987–1988	0.769	0.000	0.826	0.040
1988–1989	0.866	0.129	0.211	−0.015
1989–1990	0.498	0.051	0.450	−0.095
1990–1991	1.159	0.000	0.671	0.146
1991–1992	0.951	0.000	0.820	0.046
1992–1993	0.521	0.035	0.445	0.088
1993–1994	0.959	0.000	0.829	−0.027
1994–1995	0.945	0.000	0.900	0.017
1995–1996	1.076	0.000	0.922	−0.019
1996–1997	1.028	0.000	0.912	−0.065
1997–1998	1.016	0.000	0.750	−0.138
1998–1999	0.901	0.000	0.754	−0.095
1999–2000	0.583	0.024	0.783	0.151
2000–2001	1.166	0.000	0.858	−0.068
2001–2002	0.866	0.000	0.863	0.108
2002–2003	0.731	0.000	0.783	0.059
2003–2004	0.391	0.323	0.074	0.067
2004–2005	0.304	0.054	0.144	−0.003
2005–2006	0.867	0.000	0.836	0.123
2006–2007	1.248	0.000	0.827	0.039
2007–2008	1.013	0.000	0.656	−0.030
2008–2009	0.230	0.095	0.117	0.035
Mean (μ)	0.863			0.034
Std. dev.	0.335			0.078
$\Pr(\mu = 0)$	0.000			0.025
$\Pr(\mu = 1)$	0.034			

Columns 2–4 contain coefficients, White-corrected P-values and adjusted R^2s of the regression 18. Column 5 contains annual returns of a self-financing strategy which invests in each asset k proportionally to the negative residuals from a regression of log relative values on log relative dividends at time t

value of firm *k* at time *t*.[4] We then construct a self-financing portfolio by going long or short in each asset *k* in proportion to the residuals $\nu_{k,t}$ over the period *t* to $t + 1$. If Hypothesis 2 holds, these $\hat{\lambda}$-portfolios should yield a positive return on average.

Column 5 of Table 2 contains the results. The mean return (including dividends) of the $\hat{\lambda}$ portfolios is 3.4% per year, and the null hypothesis of zero returns is comfortably rejected at the 5% level (P-value 0.025). Interestingly, the $\hat{\lambda}$ portfolios have negative returns during the dot-com bubble of the late nineties, but avoid large losses during the subsequent downturn in 2000–2002, as well as during the financial market crisis of 2008.

5 Conclusion

This paper offered an explanation of the value premium puzzle from an evolutionary perspective. The results derived in the evolutionary finance model by Evstigneev et al. (2006, 2008) (see also Hens and Schenk-Hoppé 2005) suggest a cross-sectional relation of relative market capitalization and relative dividends. The evolutionary model also provides a formal argument why deviations from this benchmark entail predictability of returns. The empirical evidence obtained in this paper supports the evolutionary explanation of excess returns from value investment.

While the cross-sectional relation could also be seen as resulting from an evaluation based on a representative agent with expected logarithmic utility, our explanation goes beyond the standard economic argument because we explicitly model the dynamics of asset prices for any market situation (i.e. 'off the representative agent equilibrium'). Our explanation is based on the fact that the value strategy of investing proportional to expected relative dividends has the highest growth rate against itself and is the only strategy with this property. Any other market valuation of assets is vulnerable to the invasion of strategies in the sense that the invaders grow against the incumbent strategies. This dynamics ensures the convergence of asset prices to this fundamental value. In the course of convergence the value strategy enjoys the highest growth of external wealth through dividend payments; an observation closely linked to Graham and Dodd (1934).

Our findings highlight that the simple logic of evolution and market selection can successfully compete with more sophisticated arguments that have been made recently to explain the value premium puzzle.

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[4]In those years where dividends are zero for one or more firms in the DJIA index, we translate each $d_{k,t}$ by +0.001 to obtain valued numbers from the log function.

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