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Biased Expectations in non-sustainable Financial and Economic Systems

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Abstract

The present thesis is concerned with biased expectations in unsustainable financial and economic systems. The term “biased expectations” implies that human beings are not cold calculating robots. Humans are subject to finite computing power, limited memory and emotions. Although the concept of perfectly rational “homo economicus” has been very useful in classical finance, the newer behavioral finance has helped to explain some remaining puzzles in the field. Starting with Tversky and Kahneman (1974), these two psychologists began to systematically examine how humans make judgments under uncertainty and identified a number of cognitive errors and emotional biases. This thesis will refer to the work by Tversky and Kahneman to explain some puzzles observed in financial and economic systems.

In the first segment, mutual funds are investigated. We start with a model assuming perfectly rational agents (“homo economicus”), but we will see that there is a mismatch between the value of the provided service by the fund manager and the service fee paid by the investor. This difference could be explained, for example, by over-optimism.

In the second segment, the dynamic of bubbles in a laboratory experiment is investigated. We find that bubbles can grow faster than exponentially and that agents seem to anchor their expectations on previous prices and extrapolate their expectations instead of considering the fundamental value. The resulting faster than exponential growth is non-sustainable and, therefore must burst in finite time.

In the last segment, we investigate non-linear processes and how they interact with each other: population and economic production per capita are the driving factors of atmospheric carbon dioxide content. Although each process by itself can grow only exponentially, the interplay can lead to faster than exponential growth and a finite time singularity. However, current efforts to bring emissions down to a sustainable level are anchored on past emissions which might no be sufficient, according the latest research.

In all three presented cases, the collective would be better off if the individuals would adopt a more rational view while avoiding biases to give up the their short term interests for the longer term good of all.

Zusammenfassung

Die vorliegende Arbeit beschäftigt sich mit “verzerrten Erwartungen” in nicht-nachhaltigen Finanz- und Wirtschaftssystemen. Der Bergriff “verzerrte Erwartungen” bringt zum Ausdruck, dass Menschen nicht emotionslose Roboter sind. Menschen haben ein beschränktes Erinnerungsvermögen, können Gleichungen oftmals nur näherungsweise lösen und haben Gefühle. Obwohl das Konzept eines “Homo economicus” in der Finanzliteratur äussert hilfreich ist, hat die neuere “Behavioural Finance” geholfen, mehrere bisher unverstandene Phänomene zu erklären. Tversky and Kahneman (1974), zwei Psychologen, haben erstmals die Mechanismen systematisch untersucht, wie Menschen Entscheidungen unter Unsicherheit treffen. Die Vorliegende Arbeit wird immer wieder Bezug auf die von Tversky und Kahneman gefundenen Erklärungen nehmen.

Im ersten Teil der vorliegenden Arbeit werden Aktienfonds untersucht. Das verwendete Modell setzt rationale Agenten voraus. Aber wie wir feststellen werden, besteht zwischen dem Wert der erbrachten Leistung des Manager des Aktienfonds, und des bezahlten Preises dafür, eine nicht zu vernachlässigenden Differenz. Diese Differenz könnte unter anderem durch übertriebener Optimismus erklärt werden.

Im zweiten Teil wird die Dynamik einer Blasenbildung in einem simulierten Aktienmarkt untersucht. Diese Blasen können schneller als exponentiell wachsen und die Händler scheinen ihre Erwartungen nicht am Fundamentalpreis, sondern aufgrund des letzten beobachten Wert zu bilden. Das resultierende “schneller-als-exponentielle” Wachstum ist nicht nachhaltig und die entsprechende Blase muss innert endlicher Zeit platzen.

Im dritten Teil untersuchen wir die Wechselwirkung zweier nicht-linearer Prozesse (Wirtschaftswachstum und Bevölkerung) auf Umweltverschmutzung. Obwohl jeder der verursachenden Prozesse für sich genommen nur linear wachsen kann, kann durch das Wechselspiel der beiden schneller als exponentielles Wachstum entstehen. Bei den Bemühungen, Karbon-Dioxidemissionen einzuschränken, sollte deswegen unbedingt beide Prozesse (Wirtschaftswachstum und Bevölkerung) gleichzeitig, sowie deren Wechselspiel berücksichtigt werden. Die derzeitigen Bemühungen die Verschmutzung einzugränzen Beschränken sich aber mehrheitlich darauf, wieder die Levels der 90er Jahre zu erreichen, was ungenügend ist.

In allen drei präsentierten Beispielen wäre es besser für die Gesellschaft als Ganzes, wenn die Individuen rationell Erwartungen hätten; dies würde dazu führen, dass weniger kurzfristige Ziele verfolgt würden und würde auch im langfristigen Interesse der Gesellschaft liegen.

1

Introduction

1.1 Organization of the Thesis

The thesis is organized as follows:

In the present chapter, we give a general introduction to expectations and cognitive errors. Each of the following three chapters is dedicated to a specific subject and starts with a brief introduction to the chapter's topic. We then present the respective original research paper. Finally, we end each chapter with some further perspective and concluding discussion.

Chapter 2 is dedicated to a model investigating the fee setting in mutual funds. We give a very short introduction to modern portfolio theory and efficient markets, before we present our research on mutual funds. Highlights and contribution of this paper are:

- We explain the relationship between a representative investor and a fund manager.
- Investor's perception plays a key role in the fund's fee-setting mechanism.
- US domestic equity mutual funds underperform the market expectation by about 1.5%.

We conclude this chapter with a short discussion on some of the limitations faced by classical finance and the economic vs. econophysics paradigm.

Chapter 3 is dedicated to a empirical test of an applied economic model of bubbles. We start by giving some background information on bubbles. We then present the research paper itself containing the following highlights:

- An interpretation of lab experiments is offered that exhibits financial bubbles.
- Our calibration reveals the existence of positive feedbacks.
- We find traders anchor expectations more on price than on returns in these bubbles.

We conclude this chapter by discussing possible extension such as the FTS-GARCH model and the LPPL model. A supplementary section is added to discuss an information theoretic approach to estimate parameters in non-stationary time series.

Chapter 4 is dedicated to a simple economic model linking atmospheric carbon dioxide content to population and GDP growth. The main results are as follows:

- A model coupling CO_2 emissions and macro-economic variables is developed.
- The model accounts for non-linear dynamic interaction between the variables.
- The growth rate of atmospheric CO_2 shows no sign of slowing down as we find strong evidence that it is in fact accelerating at least exponentially, with even evidence of super-exponential growth.

We conclude this chapter with a short assessment of the predictions by the Club of Rome and a general discussion on the limitations of forecasting.

Finally, chapter 5 concludes.

1.2 Expectations

Expectations play in all three presented papers a key role and are a central theme of the present thesis. Already Keynes (1937) noted that stock prices are mainly a "convention" of expectations: "A conventional valuation [...] is established as the outcome of the mass psychology of a large number of ignorant individuals." Expectations[1] have to be evoked as soon as an outcome is no longer certain (as opposed to a certain outcome with probability one). Knight (1964) distinguishes two kinds of uncertainty or risk: "There is a fundamental distinction between the reward for taking a known risk and that for assuming a risk whose value itself is not known." The latter is also referred to as Knightian uncertainty. Statistics allow to efficiently handle risk, when probability distributions are known or can be calibrated. It is interesting to note that even if the outcome is not certain, but the underlying probability distribution is known (for example an asset with normal distributed returns), there is very often a possibility to mitigate risk; it can be either hedged away or insured. In contrast, for Knightian uncertainty, the situation is becoming more complicated. It is virtually impossibly to risk manage such events as they would require to know the magnitude of the event as well as the probability. The concept of Knightian uncertainty has recently gained more attention with Taleb's "Black Swan", (Taleb, 2007), see also former US defense secretary Rumsfeld's somehow comical press conference where he refered to "known unknowns" and "unknown unknowns" (i.e. Knightian uncertainty)[2].

[1] from latin *expectatum* (look out for, await), see Webster's dictionary.

[2] *There are known knows; there are things we know we know.*
We also know there are known unknowns – that is to say we know, there are some things we do not know.
But there are also unknown unknowns – the ones we do not know we don't know.
available at `https://www.youtube.com/watch?v=GiPe1OiKQuk`

Financial markets are convenient to study expectations as prices reflect agents' aggregated expectations. Arrow (1964) described a perfect market where every kind of risk is replicable and tradeable. Such a market is referred as "complete". Up to a certain degree, some financial markets can be considered as complete as derivatives allow to be exposed only to specific risk factors. For example, fixed income derivatives allow to directly trade expectations in interest rate changes in a single transaction. Hence, in modern financial markets, it is no longer necessary to construct a complex fixed income portfolio to obtain a desired risk profile (see for example Hull, 2011). However, although there exists a plethora of derivatives, not all risks are tradeable on an exchange, in particular personal risks like for example a divorce.

Starting with Walras (1874) at the end of nineteenth century, economists began to model the economy as an aggregated sum of supply and demand of agents. The idea is that supply and demand are matched in order to clear the market. The resulting price, which balances the two, is the equilibrium price. This is called the "general equilibrium theory" (see Kurz (2007) for a short introduction and some generalization). Each of the agents in these models typically observe their environment and are equipped with an utility function, to make decisions which are most beneficial. It is important to note that it is generally assumed that the agents are perfectly rational, are only focused on their own personal interest and have the capability to perfectly compute their optimal decision. This idea is generally referred as "homo economicus". As discussed in Sargent (2008), a "homo economicus" is said to have "rational expectations". Kurz (1996) proposes a particularly interesting approach and develops an agent based model, which allows each agent to have its own expectations (which can be in "general wrong" in the sense that they are biased). These expectations, which agents form from their personal preferences and experiences are similar to a Bayesian priors, where agents try to assess parameters by conditioning them on what they have learned from the past (Hájek, 2012). The price itself is formed as an aggregate over the demand and supply of all agents in order to clear the market at the end of each period. Further, Kurz' model introduces an "endogenous uncertainty" component by amplifying exogenous fluctuations; Fitting his model to the US market, Kurz claims that "[...] more than 2/3 of the variability of stock returns is due to endogenous uncertainty" which would explain some of the excess volatility puzzle (Shiller, 2005).

1.3 Cognitive Errors and Emotional Biases

In everyday situations, people are not able to act as complete rational agents and as a result of this, they make suboptimal decisions; the "homo economicus" is an idealized construct of finance. Making decisions, humans suffer from a plenitude of cognitive errors and emotional biases. The former, cognitive errors, result from incomplete information whereas the latter, emotional biases, stem from spontaneous reaction (Kuhlman, 2012). Tversky and Kahneman (1974) were among the firsts to investigate how humans (as compared to a rational agent), in different environments and under uncertainty, behave. Kahneman won a Nobel prize in 2002 for this work (when Tversky had already died). Of the many cognitive errors and emotional biases described by Tversky and Kahneman and his colleagues[3], the over-optimism, anchoring and belief confirmation biases are most relevant for the present thesis.

- Over-optimism: "Perhaps the best documented of all psychological error is the tendency to be over-optimistic. People tend to exaggerate their own abilities (Montier, 2002)". "The main advantages of optimism may be found in increasing persistence and commitment during the phase of action toward a chosen goal, and in improving the ability to tolerate uncontrollable suffering. [...] Confidence, short of complacency, is surely an asset once the contest begins. The hope of victory increases effort, commitment, and persistence in the face of difficulty or threat of failure, and thereby raises the chances of success" (Kahneman, 2002). See also Kurz (2007) for a model with "rational over-confidence".
- Anchoring: "[...] people make estimates by starting from an initial value that is adjusted to yield the final answer" (Tversky and Kahneman, 1974). To consider something as cheap or expensive does not so much depend on the intrinsic value of a good, but more on the reference price; a good which is sold with a discount of 50% can be perceived as cheap, even if the discounted price is still well above the intrinsic value because the agent anchors its estimate to the original price.
- Belief confirmation: the subject may consider new information, but only if it matches his present expectations. He is mainly concerned to preserve the status quo and is not willing to adopt any new information. This can also lead to over-optimism, as the agent will only examine information which confirms his view (Rabin and Schrag, 1999).

Obviously, human agents who are participating in markets are not perfectly rational. It is impossible to "calculate" the optimal action for every problem, as the human brain is not a computer and agents have to make decision under limited information. Moreover, it has been shown in many studies that people are risk averse, i.e. people are willing to take upside risk, but only limited down side risk (see for example Tversky and Kahneman, 1974).

Behavioral finance tries to improve the classical models with perfectly rational agents by considering cognitive and emotional biases. See Camerer (2003) for a good general overview and Hens and Bachmann (2009) for an illustration of behavioral economics with many practical examples. We will refer to the biases discussed in this chapter in all of the remaining chapters.

[3] see for example `https://en.wikipedia.org/wiki/Cognitive_biases` for an extensive list.

2

Investors' Expectations, Management Fees and the Underperformance of Mutual Funds

It is hard to see how any rational man can ever invest.
—John Maynard Keynes

2.1 Introductory comments

Originally, investors were concerned with building a portfolio out of individual stocks in which each investment is judged individually (prudent man rule). In the early 1950s, Markowitz developed the modern portfolio theory where the portfolio is judged holistic. Portfolios are constructed as a combination of different assets, and diversification between stocks plays an important role to reduce the overall volatility (measured as standard deviation). Further, the portfolio with the highest return, given a specific level of risk, is called efficient portfolio (Markowitz, 1952).

The CAPM (capital asset pricing model) builds on top of Markowitz's idea: it assumes that all agents in a market invest only in the most efficient portfolio, which is the so called tangent portfolio. The model distinguishes specific risk from systematic risk: While specific risk is related to a specific asset and risk can be diversified away in the portfolio, systematic risk is inherent. The CAPM assumes the market is arbitrage free and the resulting prices form a price equilibrium at which agents will trade the respective assets.

Markovitz' approach as well as the CAPM are foremost based on purely theoretical consideration. If the CAPM holds in reality is subject to debate: while Kurz (2007) casts doubt, others (Levy and Roll, 2010; Ni et al., 2011) find that the CAPM can not be rejected, but crucially depends on the market proxy. The CAPM has been extended in several dimension which fix some of the observed anomalies: Fama and French (1993) use a three factor model to account for the large cap effect. The resulting classification in large-small and growth-value companies has been adopted by many practitioners. Daniel et al. (2001) offer an adoption of the CAPM model which takes behavioral biases in account. Finally, Malevergne and Sornette (2005) offer an overview over the various extensions of the CAPM model.

Fama proposed the "Efficient Market Hypothesis" in his seminal paper (Fama, 1970) to check if some investors are able to systematically outperform the market (which would be in contradiction to CAPM). Fama proposes three forms of efficiency of markets (known as efficient market hypothesis (EMH)):

- Weak form: The information set is just "past price (or return) history". Hence, historical patterns, etc. should not persist. The assumption of the weak form explains also why Brownian motion of prices of financial assets is a popular assumption.
- Semi-strong form: "the concern is the speed of price adjustment to other obviously publicly available information (e.g., announcements of stock splits, annual reports, new security issues, etc.)."
- Strong-form: To the previous sets, the information adds "monopolistic access to any information", i.e. it also includes insider information.

If the market actually follows all three forms of the EMH is subject to debate; whereas Fama and the Chicago school are strong advocates and belief that the EMH holds approximately in all three forms, others are more pessimistic (see for example Lo (2007) for a relatively recent review of the EMH). Moreover, behavioral scientists, as discussed in chapter 1, consider the assumption that agents act perfectly rational as flawed. To what degree departure of rationality leads also to deviations (and what deviations) from market efficiency is a hotly investigated issue in modern financial economics.

However, it is important to note that finance is a social science above all. As such, a hypothesis like the EMH does not get immediately rejected as soon as some weak evidence against it is found; a theory (even falsified) is only dropped when a new and better theory exists (McCauley, 2006). This should make clear why the Markovitz rule for portfolio allocation is still the working horse in finance.

The research paper on mutual funds presented in the next section follows the setup of Fama (1970) by assuming efficient markets. However, we will reveal that investors overpay mutual fund managers if they consider the Markovitz rule for their investments relative to the broad market.

2.2 Paper

On the following pages, we present the paper in full length. The paper is available as:

A. D. Huesler, Y. Malevergne, and D. Sornette. Investors' Expectations, Management Fees and the Underperformance of Mutual Funds. *Social Science Research Network Working Paper Series*, Feb. 2012

Investors' Expectations, Management Fees and the Underperformance of Mutual Funds

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Abstract

Why do investors buy underperforming mutual funds? To address this issue, we develop a one-period principal-agent model with a representative investor and a fund manager in an asymmetric information framework. This model shows that the investor's perception of the fund plays the key role in the fund's fee-setting mechanism. Using a simple relation between fees and funds' performance, empirical evidence suggests that most US domestic equity mutual funds have added high markups during the period from July 2003 to March 2007. For these fees to be justified, we show that the investor would have expected the fund manager to deliver an overall annual net excess-return of around 1.5% above the S&P 500 on a risk adjusted basis. In addition, our model offers a new classification of funds, based on their ability to provide benefits to investors' portfolios.

JEL-Classification: G23, G11, D82
Keywords: Mutual Fund Fee, Mutual Fund, Asymmetric Information, Principal-Agent Relationship, Markup

1 Introduction

The lack of performance of the mutual fund industry and the lack of investors' reaction to these poor performances is a widely reported phenomenon (see for instance Palmiter and Taha (2008), Berk and Green (2004), Nanigian et al. (2008), and Glode (2011)). However, despite its apparent underperformance, the total net assets managed by US mutual funds have increased from 7.0 trillion dollars in 2000 to 11.8 trillion dollars in 2010, according to the Investment Company Institute (2009). Why do investors buy these obviously underperforming investment vehicles? At the same time, how can one explain the long-standing puzzle of high markups in the mutual fund industry? These questions are even more crucial in recent years, given the emergence of new investment vehicles such as low-cost index funds (some index funds track S&P 500 indices with an annual expenses of 10 basis point and have no load fees) and exchange-traded funds (ETFs). With growing competition and increasing disclosure and transparency in the fund market, and given their relatively poor performance, one could expect that mutual funds would reduce their fees, but a report by the US Securities and Exchange Commission clearly shows that mutual fund total expense ratios (TER) have been overall on the rise since the late 1970s, see SEC (2001) for instance.

The present paper proposes a simple solution to these two related puzzles in terms of a principal-agent model in an asymmetric information framework. Our model provides a theoretical framework to account for the investors' limited abilities and to test some of its observable consequences. We find that the optimal fee level, determined by the fund managers in their own interest, depends only on the information and preferences of the investors. Because investors have only partial information on the fund managers' true abilities and limited knowledge of the financial markets, they may hold a biased view of the fund's true performance. Fund managers can then take advantage of the investors' biased view, attracting their investment and charging them an additional premium significantly above the competitive level of fund fees.

Our model assumes that the origin of the misperception of investors about the fund performance lies in their limited or misguided information (the two following perspectives about the rationality of investors can not be distinguished within our framework: either investors are rational but take their decisions based on limited information, or they have only bounded rationality) and in their inability to update their priors. Given these limitations, investors make rational decisions. The assumption of lack of learning has the advantage of reducing the problem to a simple one-period set-up and is justified by several survey evidence that show that most individual investors in mutual funds are unsophisticated and pay little attention to funds features that are not directly observable, so that their learning ability is actually questionable. Barber et al. (2005) and Choi et al. (2009) provide empirical evidence that investors are very sensitive to salient fees such as front-load fees. Surprisingly, investors appear to be unaware of the existence of mutual funds' expense ratios. Capon et al. (1996) and Alexander et al. (1998) demonstrated that investors are not familiar with many basic facts about mutual funds such as the level of fees they are paying to their funds. Further, investors' bias is probably influenced and reinforced by the marketing practices of mutual funds, which promote the sale of fund shares. Since 1980, after approval of the SEC Rule 12b-1, mutual funds have been allowed to charge marketing and distribution fees to their investors by adopting a 12b-1 plan, see Malhotra and McLeod (1997). Khorana and Servaes (2004) and Barber et al. (2005) have identified a positive impact of 12b-1 fees on funds' money flows. Murray (1991) explains various marketing strategies in promoting fund sales. Nonetheless, the debate is still open and another trend of literature suggests that investors are able to learn and adapt their decision making process to past information, which requires the introduction of a multi-period model with learning as in Berk and Green (2004) or Gil-Bazo and Ruiz-Verdú (2009) for instance.

In addition, our model suggests two alternative fee-setting mechanisms. First, when the fund provides diversification benefits from the perspective of an investor's global portfolio, investors have to pay higher fees to get access to this benefit. This scenario has been analyzed by Gil-Bazo and Ruiz-Verdú (2009). Second, when a fund does not provide diversification benefits, but actually adds additional returns to the investor's global portfolio, the fund manager will lower fund fees to attract more money inflow.

The existing literature attributes the investors' puzzling investment behavior either to the existence of redemption fees (Nanigian et al. (2008)), to the performance of funds in bad economic times (Glode (2011)), to competition among funds (Gil-Bazo and Ruiz-Verd (2008)), and to representativeness heuristic of investors influenced by recent returns independently of risk profiles (Harless and Peterson (1998)). Whether fund fees are excessive is a long-standing debate among academics. Several studies such as Coates and Hubbard (2007) and Grinblatt et al. (2008) argue that there is an adequate level of competition in the mutual fund industry and fees in the fund market are thus competitive. The result of our empirical study contradicts this view. More specifically, we show that after accounting for the returns on funds, diversification benefits and fees, most US domestic equity mutual funds, both actively and passively managed, have added markups. In addition, these mutual funds demonstrate competitive disadvantage to low-cost index funds or index ETFs. In this respect, our study provides additional evidence to previous works from School (1982), Freeman (2007), Freeman and Brown (2001), which report that mutual fund advisers charge significantly higher fees than free-market prices would suggest.

Our model is based on the observation that the relationship between a manager and a representative investor constitutes an example of the general principal-agent problem. Following the seminal work of Ross (1973) and Holmstrom (1979), numerous studies have applied the principal-agent model to various situations of economic exchange between two parties. In a nutshell, an investor "hires" a mutual fund manager and the fee structure of the mutual fund is the mechanism used to attempt to align their interest, under prevailing conditions of incomplete and asymmetric information between them.One can notice in passing that Cornell and Roll (2005) showed that the manager invests according her own objective function which can differ from the investor's utility function, even when she is compensated relative to a benchmark. In the language of the principal-agent problem, the manager sells her service, presented as information gathering ability and managerial efforts, to the investor in return for a compensation represented as the management fees (Golec (1992); Heinkel and Stoughton (1994); Starks (1987)). We assume that managers have full access to an investor's private information, whereas the investor has no access to a manager's private information.

We derive the demand function of the representative investor and the optimal level of management fees charged by the manager in an equilibrium. The information on the manager's skills is revealed to the investor by the return history of the managed fund. For a specified level of management fees, the demand function of the representative investor is determined by the composition of her optimal portfolio. This portfolio is defined as a mixture of the mutual fund investment and of other vehicles that she picks up herself. The fund manager uses her private knowledge of her own management skills and the full understanding of the investor's decision process to determine the optimal fee level, which maximizes her expected utility.

Our approach generalizes in three directions the work of Golec (1992), who also studied a one-period principal-agent model in which mutual fund managers trade their information-gathering abilities with investors: (I) we solve the principal-agent problem by fully considering the informational disadvantage of the investor; (II) we focus our attention on the fixed-fee compensation scheme as it is the most-used standard in the mutual fund industry; and (III) our treatment is not restricted to the mean-variance utility function, even if we eventually show that it is often sufficient to provide reasonable results. In contrast to other studies, such as Gruber (1996), Wermers (2000), Glode (2011) and French (2008), which focus on actively managed mutual funds, our model requires no assumption on whether the fund is actively or passively managed.

We obtain two main theoretical results. Firstly, the fee-setting mechanism including the fees at equilibrium is fully determined by the information available to the investor, while the manager's information is irrelevant. The investor's information includes her choice of the benchmark portfolio (The benchmark portfolio represents the investor's standard choice of investment vehicles. This can be ETFs or bank savings, depending on the investor's financial knowledge and risk preference.) and her anticipation of the fund's relative performance in terms of both diversification benefits and returns, when compared to her benchmark. Secondly, we provide a simple relation to analyze a fund's risk-adjusted performance, when fees come into play. In addition, our results do not require any restrictive assumptions on the form of the fund's return distribution and the investor's utility function as long as it is an increasing and concave function.

Then, we test whether the returns delivered by US equity mutual funds can justify their fees in recent years, given the perspective offered by our model. For this, we use a data set of 3,273 US domestic equity funds over the period from July 2003 to March 2007 from the CRSP Survivor-Bias-Free US Mutual Fund database. The results show that most funds have charged high markups to their investors. At the same time, we interpret the continued presence of high markups as an indication of investors' over-optimism about the funds' future performance when they make their investment decisions. The over-optimism of investors translates into decisions made on the basis of limited or misguided information, and leads to investments in underperforming mutual funds. This indicates the investor's incorrect selection of benchmark, possibly due to the lack of investment knowledge.

This paper is organized as follows. Section 2 describes the model, stressing its economic underpinning. In section 3, we present our main results, with the characterization of the equilibrium and its main properties. Section 4 presents the empirical framework and results. Section 5 summarizes our conclusions. An appendix with the proofs and additional table is available from the authors on request.

2 The Model

2.1 General Set-up

Let us consider a fund manager and a representative investor who play a one period game. Consistent with the literature on mutual fund fees (Freeman (2007); Herman (1963); Holmstrom (1979); Luo (2002b)), we focus on the situation in which the representative investor has no bargaining power. This setup implies a competitive supply of capital to mutual funds, as suggested by Berk and Green (2004). This assumption is realistic, given the large number of small investors and their weak market power in the mutual fund market. In addition, except for index funds or ETFs, the whole mutual funds market can be considered as a monopoly Chordia (1996). Further, Luo (2002a) points out that the vast majority of the management fee charge is due to the lack of competitiveness.

The investor is a utility-maximizer with incomplete information. The investor chooses the optimal amount of her money to invest in the managed fund, on the basis of its perceived average return and risk profile as well as the management fees charged by the fund manager. This latter is assumed to be a utility-maximizer too, privy of her own personal information.

We consider a one period game between the fund manager and the representative investor. This convenient simplification is not restrictive, because it actually reflects the reality that most mutual fund investors buy for the long-term and redeem their shares infrequently, about every four or five years (Investment Company Institute (2009)). This buy-and-hold strategy is encouraged by most funds, charging front- and back-loads or penalties for early redemption. Alves and Mendes (2007) showed that back-end load fees do have an impact on investor's redemption activity. On average, the Investment Company Institute (2001) suggested a redemption rate of 15% for domestic equity funds over the period from 1992 to 1999. Although our model is one-period, this does not imply that the fund is statically managed. We do not make any assumption about the underlying management process, which can include any general dynamic strategy. This makes our model quite versatile and relevant for both static and active mutual fund strategies.

The game unfolds as follows (see also Figure 1). At the beginning of the period, the fund manager decides on the level of management fees as a percentage of her asset under management. The representative investor observes the proposed fee structure and builds up her portfolio accordingly. She can purchase shares from the manager's fund, which involves a cost specified by the management fees. She can also buy shares from a "benchmark portfolio" which is accessible at zero management cost. This benchmark asset can be the risk-free asset or any exchange-traded fund (ETFs) that replicates a market index or a risk factor which is representative of the asset class used by the manager. Static index ETFs have no annual expenses. There is only a commission as low as 5-20 basis points when buying and selling the shares. Assuming 5 years holding period, this means 2-8 basis point per year. Poterba and Shoven (2002) compares the S&P 500 SPDR trust, the largest ETF to the Vanguard Index 500 and concludes that they both offer similar returns to investors.

The initial endowment of the investor is equal to one monetary unit. She invests ω in the managed fund and the rest, $1-\omega$, in the benchmark portfolio (a table summarizing the most important symbols can be found in Table 1). The benchmark portfolio can be sold short (unlike index-funds, short-selling ETFs is possible), but only long positions are allowed for the managed fund, so that $\omega \geq 0$. At the end of the period, the manager extracts her fees and then redeems the remaining capital to the investor.

Following the prevalent habit in the mutual fund industry, our set-up assumes a fixed-fee compensation scheme for the mutual fund, i.e., the fee is a fixed percentage per period of the net asset under management. This fee structure is to be contrasted with the incentive fee that usually includes a proportional base fee plus a percentage of the return above a certain benchmark, which is the prevalent compensation scheme in the hedge fund industry. For the mutual fund industry, Golec (1992) reported that, in 1985, only 27 out of 476 US equity funds used performance based compensation schemes. More recently, Elton et al. (2003) documented that, in 1999, only 108 out of 6716 US mutual funds specializing in bonds and stocks used incentive fees. This justifies our focus on the fixed-fee scheme.

We denote by f_e the expense ratio for the one-period and by f the corresponding management fees. Both are expressed as a percentage of the investor wealth under management in the fund. The compensation of the manager only includes the management fee f. Denoting by f_0 the remaining part of the expense ratio, we have

$$f_e := f_0 + f, \tag{1}$$

where f_0 gathers all the costs that do not contribute to the manager's compensation, such as annualized shareholder costs (when f_0 includes annualized shareholder costs, the expense ratio f_e is replaced by Total Shareholder Cost (TSC), denoted as f_{TSC}), distribution(12b-1) costs, legal costs and so on. The fund manager cannot benefit directly from f_0, but the investor must pay this cost.

Denoting by $\tilde{r}_m$ and $\tilde{r}_i$ the return on the mutual fund and on the benchmark portfolio respectively, the investor's terminal wealth $\tilde{W}_i$ reads

$$\tilde{W}_i = (1-\omega)(1+\tilde{r}_i) + \omega(1+\tilde{r}_m)(1-f_e), \tag{2}$$

while the manager's compensation $\tilde{W}_m$ is given by

$$\tilde{W}_m = \omega(1+\tilde{r}_m)f. \tag{3}$$

2.2 Description of the manager's and the investor's optimization problems

The game in our model is sequential. First, the fund manager announces the fee f she will charge. Then, the investor chooses the optimal amount ω of her initial wealth she wishes to invest in the mutual fund.

Definition 1. The *investor's demand function* is the mapping $\Omega : f_e \mapsto \omega = \Omega(f_e)$. It relates the expense ratio charged by the manager to the wealth invested by the investor in the mutual fund.

We assume that the investor is rational; her demand function $\Omega(f_e)$ is such that it maximizes the expected utility of her terminal wealth $\tilde{W}_i$, conditional on her information set $\mathcal{I}_i$,

$$\begin{aligned} \Omega(f_e) = \arg\max_{\omega} \mathbf{E}\left[U_i\left(\tilde{W}_i\right) \middle| \mathcal{I}_i \right], \\ s.t. \quad \omega \geq 0. \end{aligned} \tag{4}$$

A solution exists if and only if the managed fund is not *undesirable*.

Assumption 1. In the absence of any management fees ($f_e = f_0$), the managed fund is not undesirable if $\exists \omega > 0$, such that

$$\mathrm{E}\left[U_i\left((1-\omega)(1+\tilde{r}_i) + \omega(1+\tilde{r}_m)(1-f_0)\right) \middle| \mathcal{I}_i \right] \geq \mathrm{E}\left[U_i\left(1+\tilde{r}_i\right) \middle| \mathcal{I}_i \right]. \tag{5}$$

The demand function is strictly decreasing with respect to f_e. Since we do not allow short-selling of fund shares, it is convenient to define the *reservation* fee as the upper limit for f such that the demand remains always positive.

Definition 2. The reservation fee, i.e. the maximum level of management fees, denoted by $f_{\max}$, is

$$f_{\max} = \min\left\{1 - f_0, \inf\{f | \Omega(f_0 + f) > 0\}\right\}. \tag{6}$$

We immediately get the following result:

Proposition 1. *Given a non-undesirable managed fund, the reservation fee the manager can charge is*

$$f_{\max} = (1 - f_0) - \frac{\mathrm{E}\left[(1+\tilde{r}_i)\cdot U'\left(1+\tilde{r}_i\right)|\mathcal{I}_i\right]}{\mathrm{E}\left[(1+\tilde{r}_m)\cdot U'\left(1+\tilde{r}_i\right)|\mathcal{I}_i\right]}. \tag{7}$$

It is the largest fee that makes the managed fund non-undesirable.

Assumption 2. We assume that the manager knows the expression of the investor's demand function.

Such an assumption is rather strong and may seem both simplistic and unrealistic. On the contrary, as we will see latter, this assumption is reasonable from a practical point of view. Indeed, we shall prove that, irrespective of the specific shape of the investor's utility function, her optimal demand function always remains close to a linear (affine) function of the expense ratio.

Denoting by U_m the manager's utility function and by W_0 her initial personal wealth, her optimization problem reads

$$\begin{aligned} &\max_f \mathbf{E}\left[U_m\left(W_0 + \tilde{W}_m\right)\Big|\mathcal{I}_m\right], \\ &s.t. \left\{ \begin{array}{l} \omega = \Omega(f_0 + f) \\ f \in [0, f_{\max}], \end{array} \right. \end{aligned} \tag{8}$$

where $\tilde{W}_m$ in the compensation scheme (3). The fund manager thus chooses the optimal level of fees in response to her *expected* investor's demand, conditional on her own information set $\mathcal{I}_m$.

In our setting, the manager can only choose the percentage of the management fees f. She plays no role in the determination of f_0, which is exogenously set. The determination of the optimal value of f_0 is a subtle problem and is beyond the scope of this article. *A priori*, f_0 should be kept as small as possible in order to reduce the total fees and therefore attract the largest number of investors. But, among others, f_0 includes the advertisement costs, which may increase the demand for the fund as argued by Sirri and Tufano (1998). We make the assumption that the optimal levels for f_0 and f can be determined independently and that f_0 has already been fixed by the various running costs and the commercial strategy of the fund. We can then state the following important result, whose proof is given in supplementary material which is available on request.

Proposition 2. *Let f^* be the solution to the manager's optimization problem (8). If a solution f^* exists, it solves the optimization problem*

$$\begin{aligned} &\max_f f \cdot \Omega(f_0 + f), \\ &s.t. \quad f \in [0, f_{\max}]. \end{aligned} \tag{9}$$

The optimal management fee f^* depends neither on the manager's preferences U_m, nor on the manager's perceptions about the distribution of asset returns $(\tilde{r}_i, \tilde{r}_m|\mathcal{I}_m)$.

A priori, since the investors have no bargaining power, the management fees appear as a commitment from the manager, and therefore should depend on her own preferences. The fact that the optimal management fee does not depend on the manager's preferences is a result of the following assumptions: (i) Investors have no market power, they are price-takers and can only passively react to the fund manager's fee-setting strategy and (ii) the fund manager has a full knowledge of investor's preferences and therefore of her demand function.

It is worth noticing that this result is independent of (i) the distribution of both manager's and investor's portfolio, (ii) the form of the investor's utility function, as long as it is increasing and concave, (iii) the investor's rationality, as long as investors exhibit a decreasing demand function. In detail, as Capon et al. (1996) suggested, investors do not have to be utility-maximizers. They can exhibit some deviations from pure rationality in their decision process, leading to possibly nonlinear demand functions.

Proposition 2 suggests that all relevant information for the analysis of mutual fund fees is contained in the investor's information set on the fund and on the benchmark portfolio, which is a subset of all the public information available in the market. In the presence of search costs, the investor's limited information processing and gathering ability lead to some ignorance on otherwise accessible public information, a phenomenon referred to as bounded rationality by Simon (1982).

However, Proposition 2 is more interesting from an empirical point of view as it makes the fund manager's private information irrelevant to the determination of what should be the right fee level; it puts emphasis on the role of investor's limited ability and knowledge in explaining potential mispricings of the fund services. First, investors may receive biased information about the fund's historical performance. While financial information disclosed by mutual funds have to comply with SEC rules, there is still room for funds to make their performance appear better within these legal constraints. The Standards of Practice Handbook of the CFA Institute (2005) provides many examples of how funds may potentially beautify their performance. Second, unsophisticated investors may simply follow recommendations from their friends or from the funds themselves, instead of performing their own analysis. Third, it is difficult for most investors to assess correctly the fund's future performance due to lack of persistence in returns. Berk and Green (2004), Gruber (1996), Carhart (1997) and Busse et al. (2010) find that there is generally no persistence of funds' performance over the long term. Similarly, Fama and French (2009) and Barras et al. (2010) analyzed the persistence of abnormal returns (alpha) and show that a few fund managers have indeed the skills to produce abnormal returns that can not be explained by pure luck.

3 Characterization of the equilibrium, of the investor demand function and of the optimal management fee

In this section, we show the existence of an optimal allocation ω and an optimal management fee f and solve this problem in the equilibrium.

Definition 3. An equilibrium solution $(\omega^*, f^*) \in \mathbb{R}_+ \times [0, f_{\max}]$ is a solution to the optimization problems (8) and (4) with $\omega^* = \Omega(f_0 + f^*)$.

We first focus on the case where the investor chooses the risk-free rate as her benchmark. Then, we investigate the consequences of her choice of a risky portfolio as the benchmark.

3.1 Case where the benchmark portfolio is the risk-free asset

We first consider the case where the benchmark portfolio is the risk-free asset with return r_f. We assume that, conditional on the investor's information set, the returns of the managed fund are normally distributed

$$\tilde{r}_m|\mathcal{I}_i \sim \mathcal{N}\left(\bar{r}_m, \sigma_m^2\right), \tag{10}$$

with $\bar{r}_m > r_f$. In order to get a closed form expression, we restrict our attention to the case where the investor is equipped with a CARA utility function. Denoting by a the coefficient of absolute risk aversion of the investor, Assumption 1 is satisfied if and only if $(1 - f_0)(1 + \bar{r}_m) > (1 + r_f)$. This simply means that, in the absence of management fees, for the manager's fund to be non-undesirable, the expected return of the managed fund, net of operating costs, must be larger than the risk free rate. The demand function which solves the optimization problem (4) then reads

$$\Omega(f_e) = \frac{(1 + \bar{r}_m)(1 - f_e) - (1 + r_f)}{a \cdot \sigma_m^2 (1 - f_e)^2}. \tag{11}$$

It is a linearly increasing function of the after-fee excess return of the managed fund over the risk-free rate and a hyperbolically decreasing function of the risk of the managed fund. This leads to the following result:

Proposition 3. *An equilibrium solution exists if and only if assumption 1 holds. It is characterized by the demand*

$$\omega^* = \frac{(1 - f_0)^2 \cdot (1 + \bar{r}_m)^2 - (1 + r_f)^2}{4a \cdot \sigma_m^2 (1 - f_0)^2 \cdot (1 + r_f)}, \tag{12}$$

and by the optimal management fee charged by the fund manager

$$f^* = (1 - f_0) \cdot \frac{(1 + \bar{r}_m) \cdot (1 - f_0) - (1 + r_f)}{(1 + \bar{r}_m) \cdot (1 - f_0) + (1 + r_f)}, \tag{13}$$

provided that $f^* \in \left[0, \frac{\bar{r}_m - r_f}{1 + \bar{r}_m} - f_0\right]$.

Assumption 1 ensures that the interval $\left[0, \frac{\bar{r}_m - r_f}{1 + \bar{r}_m} - f_0\right]$ is non-empty, and therefore that an equilibrium solution exists. The proof of Proposition 3 is given in the supplementary material which is available from the authors on request.

While the optimal demand depends on the coefficient of risk aversion a and, therefore, may change from one investor to the other (characterized by different risk aversions), the optimal management fee level remains the same, irrespective of the risk aversion and, consequently, of the risk level σ_m of the managed fund.

Expression (13) can be approximated by

$$f^* \approx \frac{1}{2}\left(\frac{\bar{r}_m - r_f}{1+\bar{r}_m} - f_0\right) \equiv \frac{f_{max} - f_0}{2} \ , \tag{14}$$

where $f_{max} = \frac{\bar{r}_m - r_f}{1+\bar{r}_m}$ is the absolute maximum level of fees the manager can charge, as seen from expression (7) with (11). Formula (14) shows that the optimal management fee is close to one-half of this maximum value. We will show below that this result is quite general.

Further insight into this result can be obtained by remarking that, since $\bar{r}_m$ is usually much smaller than 1, the optimal management fee is approximately equal to one-half the excess return of the managed fund over the risk free rate minus all other fees. Thus, in equilibrium, the benefits of the fund management resulting in a non-zero excess return net of fees, the so-called α, should be equally shared between the investor and the manager. This theoretical result contrasts with the evidence presented by Fama and French (2009) and Barras et al. (2010) showing generally that managers do not share their excess returns, if any, and increase instead their own income by setting higher fees, leaving very few outperforming fund to the investor. However, in a more recent study, Cremers et al. (2011) distinguish between "closet indexing" and truly active mutual funds and find that "closet indexing" funds are related to higher fees and active funds are indeed able to earn excess returns.

An alternative interpretation is that, given the fee, the investor expects a rate of return on the managed fund equal to

$$\bar{r}_m \approx r_f + 2 \cdot f + f_0, \tag{15}$$

so that her expected gain, net of fees, is

$$(1+\bar{r}_m)(1-f_e^*) = (1 + \frac{f^*}{1-f_0})(1+r_f), \tag{16}$$

i.e., the risk-free rate plus the management fees. Thus, higher management fees must be justified by higher expected returns, both before and after fees. This shows that good managers can signal their performance by charging high management fees. However, this mechanism can also lead to adverse selection insofar as managers with bad performance will imitate the fee level of good managers in order to mislead and attract investors.

In fact, empirical evidence shows that funds with higher expense ratios deliver lower before-fee returns, as demonstrated by Elton et al. (1993), Gruber (1996) and Chevalier and Ellison (2002). Gil-Bazo and Ruiz-Verdú (2009) interpret this observation as a selection bias: underperforming funds target the pool of investors who are least sensitive to fund performance, while better performing funds charge lower fees to compete for and to attract performance sensitive investors. Christoffersen and Musto (2002) studied money market funds with a similar argument and showed that demand-curve variations explain fee variations. This argument can be rationalized within our model that managers exploit demand insensitive investors by charging them more fees. This will be discussed in more details in next subsection.

3.2 Case of a benchmark portfolio made of risky assets

We now assume that, conditional on the investor's information set $\mathcal{I}_i$, the joint distribution of returns of the benchmark portfolio and of the mutual fund is given by

$$\left(\begin{array}{c} \tilde{r}_i \\ \tilde{r}_m \end{array} \middle| \mathcal{I}_i \right) \sim \mathcal{N}\left(\left(\begin{array}{c} \bar{r}_i \\ \bar{r}_m \end{array} \right), \left(\begin{array}{cc} \sigma_i^2 & \rho\sigma_i\sigma_m \\ \rho\sigma_i\sigma_m & \sigma_m^2 \end{array} \right) \right). \tag{17}$$

We still restrict our attention to the case where the investor is equipped with a CARA utility function. We have also solved this problem for CRRA utility functions. In this case, the solution does not have a closed analytical form and requires numerical computations. The numerical results confirm the remarkably strong robustness of our analytical result derived for CARA utility functions. Calculations for CRRA utility functions can be found in the supplementary material which is available from the authors on request. We find that Assumption 1 is satisfied if and only if

$$(1 - f_0) \cdot (1 + \bar{r}_m - a\rho\sigma_i\sigma_m) > 1 + \bar{r}_i - a\sigma_i^2, \tag{18}$$

where a is the coefficient of absolute risk aversion of the investor. Hence, the manager's fund can be desirable for an investor with risk aversion a due to three effects: first, its expected return $\bar{r}_m$ is higher than the benchmark's $\bar{r}_i$, second, the fund's volatility σ_m is smaller then the benchmark's volatility σ_i and, third, the fund's diversification effect is high, i.e. ρ is small or even negative.

Proposition 4. *An equilibrium solution exists if and only if assumption 1 holds. It is then characterized by the demand function*

$$\Omega(f_e) = \frac{1}{a} \cdot \frac{(1 + \bar{r}_m - a\rho\sigma_i\sigma_m) \cdot (1 - f_e) - (1 + \bar{r}_i - a\sigma_i^2)}{\sigma_m^2(1 - f_e)^2 - 2\rho\sigma_i\sigma_m(1 - f_e) + \sigma_i^2} \tag{19}$$

and by the optimal management fee charged by the fund manager

$$f^* = \frac{R_m k_1}{k_2} - \frac{\sqrt{k_1 \cdot [R_m^2\sigma_i^2 - 2R_m R_i \rho\sigma_i\sigma_m + R_i^2\sigma_m^2]}}{k_2}, \tag{20}$$

where

$$\begin{aligned} R_m &= 1 + \bar{r}_m - a\rho\sigma_m\sigma_i, \\ R_i &= 1 + \bar{r}_i - a\sigma_i^2, \\ k_1 &= \sigma_i^2 - 2(1 - f_0)\rho\sigma_i\sigma_m + (1 - f_0)^2\sigma_m^2, \\ k_2 &= R_m(1 - f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2, \end{aligned} \tag{21}$$

provided that $f^* \in \left[0, \frac{R_m - R_i}{R_m} - f_0\right]$.

As stated previously, assumption 1 ensures that the interval $\left[0, \frac{R_m - R_i}{R_m} - f_0\right]$ is non-empty, and therefore that an equilibrium solution exists. The proof of Proposition 4 is given in the supplementary material which is available from the authors on request.

To provide more insight, we expand the cumbersome expression (20) to the first order with respect to $\hat{r}_i = R_i - 1$, $\hat{r}_m = R_m - 1$ and f_0. The optimal management fee can then be simplified into

$$f^* \approx \frac{\hat{r}_m - \hat{r}_i}{2} - \frac{f_0}{2} \approx \frac{f_{\max} - f_0}{2}. \tag{22}$$

This equation has the same structure as (14), except that r_f is now replaced by $\hat{r}_i$. There is also an adjustment for risk, as r_m and r_f are replaced by $\hat{r}_m$ and $\hat{r}_i$ respectively. Again, the optimal management fee is approximately half of the maximum level of fees the manager can charge to the investor. As proved in supplementary material, this rule is to a large extent independent of (i) the joint distribution of returns on the benchmark portfolio and the managed portfolio and (ii) the form of the investor's utility function, as long as it is increasing and concave. In fact, it holds as long as the investor's demand function is almost linear, which turns out to be a very good approximation for most practical situations.

Relation (22) links fees to the fund's performance conditional on the investor's information. It provides the amount of fees the investor is willing to pay for the fund's investment management service, in equilibrium. This fee is fully characterized by the investor's choice of the benchmark and her anticipation of the future performance of both the fund and the benchmark. More transparently, we have

$$f^* + f_e \approx (\bar{r}_m - \bar{r}_i) - a(\beta_m - 1)\sigma_i^2, \tag{23}$$

where

$$\beta_m = \frac{\mathrm{Cov}(\tilde{r}_m, \tilde{r}_i)}{\mathrm{Var}(\tilde{r}_i)} \ . \tag{24}$$

This general relation will be the cornerstone of our empirical analysis presented below.

Figure 2 plots the demand function (19) versus the total fee f_e for different values of the coefficient of absolute risk aversion a. The reservation fee $f_{\max}$ is the value corresponding to the intersection of the curves with the horizontal axis. As previously announced, the various demand functions are very close to straight lines. In addition, irrespective of the value of the coefficient of absolute risk aversion a, all the curves intersect at the point

$$f_p = \frac{\bar{r}_m - \bar{r}_i}{1 + \bar{r}_m} \ , \quad \Omega(f_p) := \Omega_p = \frac{\sigma_i^2 - \frac{1+\bar{r}_i}{1+\bar{r}_m}\rho\sigma_i\sigma_m}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}. \tag{25}$$

The approximate linear dependence of the demand function observed in Figure 2 can be rationalized analytically by a first order expansion of (19) around this fixed point, yielding

$$\Omega(f_e) \approx \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}\right] \cdot (f_e - f_p). \tag{26}$$

The two leftmost terms in the numerator of the fraction are generally much smaller than the rightmost one. Thus, the slope of the demand function is almost inversely proportional to the investor's absolute risk aversion. This linearized expression (26) of the demand function shows that the absolute value of the slope, i.e. the elasticity of the demand, decreases when the coefficient of absolute risk aversion increases. Investors tend to be less sensitive to a change in fees when they are more risk averse.

The management fee in equation (20) depends on the coefficient of risk aversion a only through the ratio

$$\frac{R_i}{R_m} = \frac{1+\bar{r}_i - a\sigma_i^2}{1+\bar{r}_m - a\rho\sigma_m\sigma_i} \tag{27}$$

Therefore, if the relation

$$\frac{1+\bar{r}_i}{\sigma_i} = \frac{1+\bar{r}_m}{\rho\sigma_m} \tag{28}$$

holds, the optimal management f^* fee is independent from a and it is given by

$$\begin{aligned} f^* &= \frac{\rho\sigma_i^2 - 2(1-f_0)\rho^2\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\rho}{\rho(1-f_0)\sigma_m^2 - 2\rho^2\sigma_i\sigma_m + \sigma_i\sigma_m} \\ &\quad - \frac{\sigma_i\sqrt{[(1-f_0)^2\sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2]\cdot[1-\rho^2]}}{\rho(1-f_0)\sigma_m^2 - 2\rho^2\sigma_i\sigma_m + \sigma_i\sigma_m} \end{aligned} \tag{29}$$

This result rationalizes in a general way the two distinct scenarios that we show in Figure 2. In one scenario we have

$$\frac{1+\bar{r}_i}{\sigma_i} \geq \frac{1+\bar{r}_m}{\rho\sigma_m}\ , \tag{30}$$

whereas the opposite inequality holds for the alternative scenario. The optimal management fee is either increasing or decreasing in the risk aversion coefficient a in these two scenarios. In the upper panel, the investor considers the fund to have diversification benefits in the context of her own portfolio strategy. This is the case when $\Omega_p > 0$. The fund's beta is then close to one (actually we have $\beta_m = \rho\sigma_m/\sigma_i < 1+\bar{r}_m/1+\bar{r}_i$, $\bar{r}_m$ is larger than $\bar{r}_i$ and the right term is usually slightly larger than 1). In the lower panel of Figure 2, the fund is a leveraged fund. In this case, we have $\Omega_p < 0$ and the fund's beta is strictly larger than one.

In the first case, the equilibrium solutions (f^*, ω^*) show that the manager actually exploits the diversification value perceived by the investor by charging higher fees when the investor coefficient of absolute risk aversion is larger. This rationalizes the interpretation of Gil-Bazo and Ruiz-Verdú (2009) according to which fees tend to increase when the elasticity of the demand decreases. In the case of a leveraged fund, a reversed relationship is revealed: risky funds attract risk-averse investors by charging smaller fees.

4 Empirical Analysis

In the light of the theoretical results presented in the previous section, we now analyze the management fees charged by fund managers between July 2003 and March 2007 using the CRSP Survivor Bias-Free US Mutual Fund Database.

4.1 Description of the Empirical Model

Our model shows that, in equilibrium, fees are determined solely by the investors' anticipations on the future performance of the mutual funds. Expression (23) quantifies how this anticipation is transformed into an equilibrium fee level that investors agree to pay.

The CRSP Mutual Fund Database only gives access to the *ex-post* performance of the funds through their historical returns. Our strategy is to infer the *ex-ante* expectations of the investors on the fund performance on the basis of the amount of fees they are willing to pay, by using relation (23). In this way, we test the following hypothesis:

(i) Does the performance achieved by funds justify the fees they charge, given a rational choice of the benchmark asset?

(ii) Do funds possess a competitive advantage in terms of fees, realized returns and diversification benefits, when compared to the benchmark asset?

(iii) Do investors correctly anticipate funds relative performances, given their benchmark asset?

As the industry practice suggests, a natural choice of the benchmark portfolio that investors should use is an index portfolio for the market in which the fund operates. In the following empirical test, we impose the S&P 500 total return index to be the investor's benchmark portfolio for US domestic equity mutual funds. This is in line with the fact that investors can buy Exchange Traded Funds (ETFs) or low-cost index funds to achieve index performance while paying a nearly-zero cost. One could however wonder whether the relevant benchmark to consider would not be a portfolio formed by a mixture of the S&P 500 and the risk-free asset. Indeed, a typical mutual fund investor may hold treasury bonds (or just put money in a savings account or a money market fund) and shares of an index fund or ETF that replicates some market index such as the S&P 500. This possibility is addressed in supplementary material (available on request), which shows that investors use an equity dominated benchmark: about 90% invested in the S&P 500 and only 10% in the risk-free asset so that the results do not significantly change with respect to the case considered in this section, namely the case where the benchmark portfolio is made of the S&P 500 alone.

For convenience, we define both the *after-fees excess return* and the *adjusted beta* for the fund j over a given period, as follows:

$$\text{after fees excess return}^j = (\bar{r}^j - \bar{r}_{index}) - (\bar{f}^j + \bar{f}^j_{TSC}) \ , \tag{31}$$

$$\text{adjusted beta}^j = (\beta^j - 1)(\sigma_{index})^2 \ , \tag{32}$$

where

$\bar{f}^j$ and $\bar{f}^j_{TSC}$	=	average management fee and average total shareholder cost,
$\bar{r}^j$ and β^j	=	fund's realized average return and beta,
$\bar{r}_{index}$ and σ_{index}	=	realized average return and volatility of the market index.

Similarly to Khorana et al. (2009), we define fund j's total shareholder cost (TSC) as a sum of both annual total expenses and annualized shareholder fees, given a five-year holding period in our analysis:

$$\bar{f}^j_{TSC} = \bar{f}^j_e(\text{average TER}) + \text{front-load}/5 + \text{back-end load at five years}/5. \tag{33}$$

Then, expression (23) leads to the regression model:

$$\text{after fees excess return}^j = a \cdot \text{adjusted beta}^j + b + \varepsilon^j , \tag{34}$$

where a stands for the investor's *relative* risk aversion. In the theoretical part, we have denoted by a the coefficient of *absolute* risk aversion while the total initial investor's wealth was set to one dollar. Our results generalize to an arbitrary initial endowment by replacing the absolute risk aversion by the relative risk aversion. b is an intercept that should equal zero if investor's ex-ante expectation matches exactly fund's ex-post performance and ε is a mean zero error term. The value of the bias b reflects the deviation of the ex-post performance of the funds from the investors' ex-ante expectations of the fund performance. A positive (respectively negative) value of the bias b can be interpreted as the fact that investors underestimate (respectively overestimate) the funds relative performance.

If our model was the whole story of what determines the strategic interactions between homogeneous investors and mutual fund managers, and what represents the risk-return performances of mutual funds and of the benchmark asset, then the regression model (34) should provide directly a unique estimation of investors' risk aversion for the whole mutual fund universe. However, this expectation is of course naive, given the heterogeneity of mutual funds and of investors. Notwithstanding the formidable problem of making sense of the heterogeneity in fund performance and in their fee structure, we can nevertheless identify robust and meaningful regularities. The key insight was to organize the universe of mutual funds into deciles of decreasing risk-adjusted return performance quantified by their Sharpe ratio. As we show in the following sections, remarkably good regressions with model (34) are found, which provide insightful economic interpretations. Particularly, we identify different groups of investors characterized by their specific risk aversion coefficient a. We are able to relate these groups to distinct fund characteristics, such as their leverage level and their relative performance. Our model also allows us to identify several subclasses of "abnormal" funds, which either provide good diversification and after-fees overperformance or give lower diversification benefits and sub-performance. This classification is performed on the basis of clustering analysis and the values of the regression intercept b.

4.2 Description of the Data Sample

We obtained our sample from the CRSP Survivorship-Bias-Free US Mutual Fund Database. This database contains as per June 2009 monthly data of more than 42,000 US mutual funds from January 1962 to June 2009.

In our analysis, we have focused on the time period from July 2003 to March 2007 which is characterized by strong growth. The rationale is the following: our model requires the investor to allocate her wealth between the fund manager and the benchmark portfolio. This assumption is only realistic if the investor is convinced that she will earn a positive return by combining the fund with the benchmark. Similar to the CAPM model, we use historic returns to proxy the future expectation of the investor (see Pettengill et al. (1995)). Hence, only periods with overall positive returns can be considered; if an investor really had expected negative returns in the market, she would have moved her wealth immediately to the risk free asset, which is not part of our model. Further, falling back to the risk-free asset as a benchmark in bull markets is not feasible; there are almost no mutual funds with significant negative beta to the market benchmark and consequently, none of these mutual fund can be desirable, i.e. fulfill

Assumption 1. Moreover, for the time before 2000, the CRSP data base is not complete with respect to availability of funds for retail investors. For all these reasons, we have restricted our analysis on the time period above.

We used the one month US treasury T-Bill (XIUSA04G7W) as a proxy for the risk-free rate, which is used to calculate the Sharpe ratio, and the S&P 500 TR (XIUSA04G92) from Morningstar as the benchmark.

Our initial sample is chosen to contain all mutual funds operating during the period from July 2003 to March 2007 having complete monthly returns history. Moreover, we require the funds to have no sale restrictions, to be open for investments and to be available to retail investors. Further, we consider only funds with the investment objectives from Table 2, i.e. funds which can be compared with the S&P 500 TR, and leave out funds with emerging markets, and fixed income strategies.

We do not include funds having negative fees (some funds in the CRSP database are listed with negative fees due to wavers or reimbursements). Because we take the perspective of a private investor for a one-period investment, we assume that the fees are locked-in at the beginning of the investment period when the current fees are observed by the investor, since the manager is not allowed to retrospectively change the terms of the agreement. Note that some funds are available in both (i) different share classes (for example A, B, C) that usually represent different fee contracts, and (ii) Service (service shares charge an additional small amount (less than 0.25%) to the investor to compensate a third party adviser) and Investor shares as well. As these different mutual fund shares and classes represent different choices that are open for investment, we let them coexist in the sample. For robustness, we have also considered removing all, but the main share class or weight each share class by its TNA and form value-weighted classes. However, this does not significantly influence any of the results.

Applying the above selection procedure, we are left with a sample to study, which contains 3,273 US domestic equity mutual funds. The total period covers 45 months. Table 3 characterizes our sample and presents descriptive statistics like weighted average total net assets (TNA), turnover ratio, Sharpe ratio and various fees such as the average TER, average management fees, average total shareholder cost (TSC), both front-load and back-end load fees.

4.3 Descriptive Statistics and Characteristics of Sharpe Ratio-Sorted Domestic Equity Funds

Figure 3 plots the after-fees excess returns for the whole sample of 3,273 US domestic equity mutual funds as a function of their adjusted betas. Since the benchmark portfolio corresponds to the origin of this plot, the set of positions with respect to the origin provide a natural classification of the performances of our fund universe. It is convenient to label funds with a beta smaller (respectively larger) than one as “diversification funds” (respectively “leveraged funds”). A beta β smaller (respectively larger) than one corresponds to an adjusted beta smaller (respectively larger) than zero. The top-left side of Figure 3 corresponds to the best funds which provide both diversification benefits and absolute benchmark-beating performances. Funds in the bottom-right side of the panels of Figure 3 are the worst funds, that are inferior to the index, even with leverage. Regardless of the investors’ risk aversion, funds in the top-left side provide superior relative performance and funds in the top-right side provide inferior relative performance, when compared with the benchmark portfolio. Funds in the top-right side of the panels provide index-beating performance, but at the price of higher risks (higher leverage). Funds in the bottom-left side of the panels provide diversification benefits, but fails to beat the benchmark's return after fees. The comparison of fund performance between funds from the top-right side and bottom-left side and the benchmark depends on how investors value risks against returns, i.e. on the risk aversion coefficient in our model.

Regressing this universe of funds on our model (34) gives a significant negative bias ($b = -0.5\%$), but the determination coefficient R^2 is very small (around 0.5%), so that our model has nearly no explanatory power when applied to the whole sample of mutual funds. However, this result is not so surprising in the presence of the huge heterogeneity of fund styles and investor preferences. Recall that our model assumes that (i) the investors are characterized by a unique risk-aversion coefficient a and (ii) the funds have a well-defined profile to sell to their clients, i.e. are desirable.

But, in reality, investors have heterogeneous risk aversions and, as a consequence, look for different risk vs. return trade-offs, and funds are very diverse in style and performance. In order to use our model, it is necessary to sort the universe of funds in what can be considered a priori to be homogeneous classes. With the data at our disposal, we cannot sort by classes of investors, but we hypothesize that there is a self-selection among investors, who cluster in those funds that best match their risk aversion profiles. Hartman and Smith (1990) have indeed suggested that funds investors can be best segmented by their risk tolerance. We consider the simplest and most generally used measure of risk-return profile, namely the Sharpe ratio. We thus sort and group funds by their Sharpe-ratios in 10 deciles, from the first decile corresponding to the best performing funds to the tenth decile of the worst funds, according to their Sharpe ratios. As we shall see later on, it turns out that all the investors expect more or less the same level of return (about 1.5% more than the S&P 500, see section 4.4), so that the Sharpe ratio is an indirect way to assess the level of risk aversion the investors are ready to bear and hence their risk appetite.

In Table 4, we sort the selected set of 3,273 domestic equity funds into 10 deciles according to their Sharpe ratios. We present several descriptive statistics. For each decile, we provide the mean value of the Sharpe ratio, net returns, fees, turnover, and beta.

Table 4 shows that the total expense ratio (TER), total shareholder costs (TSC) and management fees tend to increase when the Sharpe ratio decreases. Funds from the bottom decile charge on average 10 basis points more in management fee and 70 basis point more in TSC than top decile funds. These differences are economically significant. This is in line with existing empirical evidence that worse performing funds tend to charge higher management fees and total expenses. Worse-performing funds tend to also have a higher beta and a higher turnover ratio.

Another interesting observation provided by Table 4 is that funds from the last Sharpe ratio deciles charge higher back-end loads and lower front-loads, compared to better performing funds. This could be rationalized as a strategic behavior of mutual fund managers, exploiting deficient information gathering or inattention on the part of investors: poor-performing funds attract investor's initial investment by lowering front-load fees. This strategy makes sense, given the observation by Barber et al. (2005) that investors are more sensitive to salient fees such as front loads than to operating expenses. Then, by raising back-end load fees, bad-performing funds hinder investor's redemption activity. In support of this reasoning, Nanigian et al. (2008) indeed suggested that the lack of reaction of investors to bad-performing funds is caused by the existence of back-end loads.

Furthermore, Table 4 shows that both 12b-1 fee and the maximal level of 12b-1 fees tend to increase when the fund's Sharpe ratio decreases. This suggests that bad-performing funds tend to spend more on marketing and distribution than better-performing funds. This is in line with previous reports that worse-performing funds tend to charge higher fees than better-performing funds, see Elton et al. (1993), Gruber (1996) or Chevalier and Ellison (2002) for instance.

In each Sharpe-ratio deciles, funds have various investment styles. Table 2 lists all Lipper investment objectives used in this study. Empirical studies such as Wermers (2000), Brown and Goetzmann (1997), Chan et al. (2002), Brown and Harlow (2002), Barberis and Shleifer (2003) and Bogle (1998) have shown that funds with various styles exhibit different performances. These funds may target different groups of investors, with diverse degrees of risk aversion.

It might be fruitful to distinguish between different fund segments by an other criteria than Sharpe-ratio. A first idea would be to classify funds according to their investment styles, but the model did not perform and explanatory power was very low. This might show some limitations of our model. An other possible explanation could be that this approach is unreliable because funds often change their investment style and sometimes report misleading information; Brown and Goetzmann (1997) argued that reported investment styles are not satisfying in terms of reflecting a fund's true investment activities. Brown and Goetzmann (1997) and Donnelley (1992) have shown that some funds misclassify themselves. Below, we will use cluster analysis based on the formulation of our regression model, that reveals regularities in the best- and worst-performing funds.

4.4 Empirical Tests of the Model

As discussed in the previous section, we apply our regression model (34) separately on each of these ten deciles. If our model is correct, we should expect the following characteristics.

1. Good fits to the majority of the funds in each decile would define a market segment characterized by a given risk-return profile corresponding to a homogeneous class of investors with a common coefficient of risk aversion a. In other words, a well-defined risk aversion coefficient a would characterize that class of investors associated with that decile of mutual funds.

2. We would obtain a diagnostic of the efficiency of the fee structure in each fund decile via the bias b and its statistical significance,

3. An identification of over-performing and under-performing funds (with respect to our model) would become apparent for instance from the existence of a second cluster of funds in a given decile which is markedly different from the main regression. Our model could thus offer a new methodology for identifying desirable and non-desirable funds.

To test these hypotheses, we apply a mixture of regression of two regressions to each sub-sample fund decile. Brown and Goetzmann (1997) already developed a cluster classification scheme based on fund returns, that identified seven clusters. Our cluster analysis is however completely different because it uses the fund data organized according to the regression model (34). In other words, different clusters, if any, correspond in our approach to different pairs of after fees excess return and adjusted beta. This can be interpreted as corresponding to different investors risk aversion a and different abnormal excess returns b.

Figure 4 presents an example (plots of the remaining deciles are available on request) of the results of the mixture of regressions with our model (34) applied to the ten different Sharpe ratio fund deciles. Details about the funds' after fee excess returns as a function of their adjusted beta for each Sharpe ratio decile can be found in Table 5. We also provide further information about the mean value of the Sharpe ratio, net returns, fees, turnover, beta, and investors ex-ante expected returns. As explained above, we use the mixture of regressions method to test for the possible existence of abnormal funds.

For the deciles one to three, the normal funds is characterized by investors with a high risk aversion coefficient $a \approx 27$ and a significant positive bias b, indicating the fund managers of these funds are truly skilled and provide returns in excess to the benchmark on a risk and fee adjusted basis. Further, the mixture regression identifies a second cluster of funds; these abnormal funds are characterized by smaller risk aversion a and have a substantial positive bias b. According to our model, these funds in the abnormal cluster identify the best fund relative to a benchmark on a risk and fee adjusted basis and are the most favorable for the investor.

For the deciles four to ten, the results are qualitatively different. First, the intercept b for the normal funds is no longer positive, indicating the fund managers are no longer able to deliver excess returns over the benchmark. Second, the added performance for the abnormal funds is rather small and evidence for a second cluster is weaker. However, for the ninth and tenth decile, the abnormal cluster have very strong negative bias b; these funds correspond to a class of particularly under-performing funds, the least desirable funds of the whole sample.

More generally, b is significantly smaller than zero for seven of ten deciles for normal funds and for three of the 10 deciles for abnormal funds. Within our regression model, this can be interpreted as evidence that about 70% of US domestic equity funds have added markups over the period from July 2003 to March 2007. The rationalization is the following: In a market with perfect competition, if a fund underperforms the benchmark in terms of returns, diversification benefits and fees, for several years, it either has to lower down its fees to match its relative after-fees performance to the benchmark asset, or to exit the market. The continuing existence of these seriously underperforming funds is an indication for possible markups in the fund industry. Along with statistics on fees across all deciles in Table 5, our model provides a natural characterization for the well-known observation that worse-performing funds tend to charge higher fees than better-performing funds as reported in studies by Elton et al. (1993), Gruber (1996) and Chevalier and Ellison (2002).

For both the normal and the abnormal funds, we find that the coefficient a of risk aversion tend to be larger for the top decile (better-performing) funds than for the bottom deciles (worst-performing) funds. Interpreted within our model, this indicates investors' self-selection according to the inverse risk measure provided by the Sharpe ratio. This behavior is fully rational: the more risk-averse investors choose the less risky funds, i.e. those with the highest Sharpe ratio. In contrast, the less risk-averse investors choose the most risky funds, i.e., those with the smallest Sharpe ratio. In addition, the underperforming funds are mostly leveraged funds with an average beta around 1.30. Fund in the top deciles typically have betas slightly above 1.00, the abnormal funds have smaller betas. This is remarkable as our analysis focuses on a raising market and one could expect the deciles with higher betas would also deliver higher returns. Anyhow, this is in accordance with the second scenario obtained from our theoretical model: when funds are leveraged, managers tend to charge either higher fees to less risk averse investors or tend to offer lower fees to attract the more risk averse investors.

We now invert the logic of the model (34) and use the return bias b to estimate the investors ex-ante AFER, defined by

$$\text{ex-ante AFER} = \text{ex-post AFER} - b. \tag{35}$$

We find that the investors expected ex-ante AFER is remarkably well-behaved when the S&P 500 TR index is taken as the investor's benchmark; ANOVA tests for the mean show that we cannot reject the hypothesis that the mean value of investors' expected return of funds from the 4th, 5th, 6th, 7th, 8th and 9th deciles are equal, at the 5% significance level. We come to the same conclusion for funds from the 2nd, 3rd and 10th deciles. Similar results also hold for the Kruskal-Wallis test (see for instance Siegel and Castellan (1988) for more details about this tests). This results suggest that investor's ex-ante AFER are comparable for all ten Sharpe ratio deciles.

This indicates that, irrespective of their risk aversion, investors have homogeneous anticipations, as viewed from the perspective of our model. In this respect, sorting funds by Sharpe ratio is the same as sorting funds by risk level and hence by risk aversion, which legitimates the approach chosen in section 4.3. Overall, the investors expect a typical mutual fund to deliver an excess-return after fees of approximately 1.5% above the benchmark.

Investors' optimism bias is revealed by (i) the negative values of b and (ii) investors' high ex-ante expected returns. As negative b's are found for all normal fund deciles except the top three, this suggests that most investors overestimate the performance of funds in terms of fee-adjusted excess returns and of diversification potential. A possible origin of this optimism bias may be ascribed to the lack of financial literacy, as reported in Capon et al. (1996) and Alexander et al. (1998). Funds' marketing efforts, as reflected by their 12b-1 fees, could also play an important role. Table 5 shows that both normal and abnormal funds of the last two deciles tend to spend more on marketing and distribution than funds from the first two deciles.

The last column of Table 5 presents the expected return biases b_* of various deciles sorted by their Sharpe ratios, calculated according to the assumption that the investors choose a risk-free asset as their benchmark.

$$b_*^j = \bar{r}^j - \bar{r}^j_{risk-free} - \bar{f}^j_{TSC} - \bar{f}^j \ . \tag{36}$$

We observe that b_* is significantly positive for almost all fund funds. This suggests that the investors' optimism bias may come from the choice of such a suboptimal benchmark, possibly due to lack of sufficient financial literacy.

Finally, Table 6 presents the breakdown of the population of funds, classified according to the Sharpe ratio deciles, their leverage and excess returns, both for normal and abnormal funds in each decile. Unsurprisingly, diversification funds with positive after-fee excess return (AFER) are mostly concentrated in the top decile. At the other extreme, leveraged funds with negative after-fee excess return (AFER) are mostly concentrated in the bottom decile. The two other categories are more uniformly populated. One can also observe that most normal funds are leveraged funds with a negative after-fee excess return. In the last three deciles, approximately 80% of the funds belong to this category.

5 Conclusion

In order to understand why investors are buying underperforming investment vehicles, we have proposed a one-period principal-agent model based on a sequential game played by a representative investor and a fund manager in an asymmetric information framework. Our first main result is that only investor preference and information set determines the fee level of mutual funds. The manager's true ability is irrelevant here. Second, we have derived an analytical formula and provided an empirical framework that can help investors to gauge their funds and their portfolios. Third, our model has identified two alternative fee-setting scenarios depending on the fund's possible diversification benefits. Leveraged funds tend to exploit demand-insensitive investors by charging them higher fees, while funds providing diversification benefits lower fees to attract more risk averse investors and charge higher fees to the less averse investors. A salient point of our model is that investor are making rational decisions, but these are based on limited, misguided or incorrect information as a result of their possible misperception about the fund returns and the overall market. This misperception is identified in the later empirical results as investors' over-optimism about funds' future returns, which suggests possible mismatch between information perceived by the investors and the reality.

The excellent performance of our regression model on funds sorted in ten deciles according to their Sharpe ratios support the hypothesis that there is a self-selection among investors, who cluster in those funds that best match their risk aversion profiles. Then, just two variables, after fees excess return and adjusted beta, provides an accurate explanation of the impact of the risk aversion on the fund's performance relative to the market index. As a bonus, our mixture of regression method provides a novel methodology for identifying over-performing and under-performing funds, i.e., "skill" as well as "lemons."

Our empirical study of the U.S domestic equity fund market over the period from July 2003 to March 2007 has identified positive markups for around 70% of the funds in our sample. This basically means that these funds underperform low-cost index funds or ETFs, after taking the returns, the diversification benefits and the fees into account. However, investors keep investing in these underperforming funds. Within the information asymmetry framework of our model, we have shown that this puzzling investment behavior can be interpreted as an optimism bias towards funds' future performances. We have been able to estimate that, on an *ex-ante* basis, investors expect the fund managers to deliver an overall annual excess-return of around 1.5% over the S&P 500 TR, net of fees, irrespective of the investment style and of the risk level of the funds.

Investor's optimistic expectations of the fund market leads to the high markups in today's fund market. The correlation between investors' overconfidence and the high management fees and distribution (12b-1) fees found in our analysis suggest that the later play a role in promoting the former. Another element for investors' optimism bias is their lack of financial knowledge. More specifically, we demonstrated that this optimism bias can be rationalized by assuming that investors choose a risk-free asset as their benchmark. Our empirical analysis suggests that both factors may explain investor's overconfidence.

Our one-period model provides a static view of investors' behavior whose main advantage is its simplicity and versatility. We did not need any assumption on the fund management strategy. We focused on the crucial effect of information asymmetry on the pricing of mutual funds in order to disentangle it from learning effects. Our results raise the intriguing question of why investors have been continuously overoptimistic over time, apparently failing to learn the lessons of past under-performance of their investments. To address this question, a dynamic framework that includes learning would be needed. A priori, both asymmetric information and lack of learning may contribute to higher pricing of funds. This paper demonstrated the role of the former ingredient. The study of the impact of learning and of its lack thereof for this context is worthy of future research.

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f denotes the annual management fee which rewards the manager.

f_e denotes the total of all fees in percent. It includes the management fees, 12b-1 fees, brokerage fees caused by the manager for implementing his strategy and all other charges (except loads) annualized.

f_0 gathers all cost that do not contribute to the manager's compensation, i.e. $f_0 = f_e - f$.

ω denotes the initial wealth from the investor invested in the manager's fund.

r_i denotes the annualized return of the benchmark portfolio in which the investor can be invested at no additional cost.

r_m denotes the annualized *gross* return on the manager's portfolio. Before the returns are delivered to the investor, the fees (i.e. f_e) are subtracted.

f_{TSC} denotes the annualized total shareholder cost, i.e. f_e plus annualized front- and back-load which is included with 1/5th annual breakout.

r_{net} denotes the annualized return that the manager delivers to the investor, i.e. the return the manager actually realizes net of all fees (including loads).

Table 1: Summary of the most important symbols.

Code	Lipper Classification Name
EIE	Equity Income Funds
FS	Financial Services Funds
H	Health/Biotechnology Funds
LCC	Large-Cap Core Funds
LCG	Large-Cap Growth Funds
LCV	Large-Cap Value Funds
MCC	Mid-Cap Core Funds
MCG	Mid-Cap Growth Funds
MCV	Mid-Cap Value Funds
MLC	Multi-Cap Core Funds
MLG	Multi-Cap Growth Funds
MLV	Multi-Cap Value Funds
SPS	S&P 500 Index Objective Funds
TK	Science & Technology Funds
SCC	Small-Cap Core Funds
SCG	Small-Cap Growth Funds
SCV	Small-Cap Value Funds
TL	Telecommunication Funds
UT	Utility Funds

Table 2: Lipper Investment Objectives.

	Mean	S.D.	Median	min	max
Total Net Assets ($1M)	508.26	2478.04	55.20	0.10	65899.50
Expense Ratio (%/year)	1.72	1.01	1.70	0.00	42.26
Mgm Fee (%/year)	0.72	0.31	0.74	0.00	5.17
12b-1 Fee (%/year)	0.66	0.36	0.75	0.00	1.09
Front-load (%)	1.51	2.37	0.00	0.00	8.50
Back-end-load (%)	1.12	1.47	0.50	0.00	6.00
TSC (%/year)	2.24	1.12	2.33	0.00	43.86
Turnover	1.39	3.35	0.69	0.00	45.50
Net Return(%/year)	13.44	4.27	13.29	-20.94	35.47
Std. Dev. (%/year)	10.57	3.54	9.73	2.71	39.49
Beta	1.19	0.34	1.13	-0.10	3.65
Sharpe Ratio	1.07	0.45	1.09	-0.87	2.96

Table 3: Summary Statistics of US Domestic Equity Mutual Funds. This table summarizes the main features of the 3,273 US Domestic Equity Mutual Funds extracted from the CRSP Survivorship-Bias-Free US Mutual Fund Database over the period from July 2003 to March 2007. TSC refers to total shareholder costs. 12b-1 fee is the annual fee that funds have charged for marketing and distribution.

Decile	#	Sharpe Ratio	Net Return (%/year)	Std. Dev. (%/year)	Expense Ratio (%/year)	Mgm Fee (%/year)	12b-1 Fee (%/year)	Front Load (%)	Back Load (%)	TSC (%/year)	Beta	Turn-over	TNA (million $)	Ex-ante A (%/
best	327	1.87	17.90	8.35	1.53	0.70	0.58	1.82	0.99	2.10	0.91	0.78	59.00	
2nd	327	1.53	15.87	8.84	1.61	0.71	0.65	1.56	1.12	2.15	1.04	1.02	62.40	
3rd	328	1.37	14.81	9.07	1.45	0.66	0.60	1.44	0.98	1.93	1.08	0.80	62.90	
4th	327	1.25	14.47	9.69	1.51	0.66	0.61	1.72	0.96	2.04	1.15	1.00	78.30	
5th	328	1.15	13.78	9.99	1.68	0.69	0.70	1.40	1.29	2.21	1.17	1.24	57.40	
6th	327	1.03	13.33	10.69	1.68	0.73	0.66	1.42	1.08	2.18	1.21	1.82	55.75	
7th	327	0.88	11.91	10.85	1.72	0.75	0.65	1.46	1.03	2.21	1.21	1.86	48.80	
8th	328	0.74	10.92	11.62	1.84	0.74	0.70	1.57	1.23	2.40	1.28	1.72	47.70	
9th	327	0.59	9.50	12.14	1.83	0.75	0.69	1.48	1.27	2.38	1.31	1.34	41.90	
worst	327	0.32	6.65	14.44	2.30	0.83	0.71	1.26	1.21	2.79	1.49	2.31	34.90	
S&P 500		1.34	12.61	7.29										

Table 4: Summary statistics for Sharpe Ratio-Sorted US Domestic Equity Funds. This table summarizes the main features of a Sharpe Ratio-sorted deciles of the 3,273 US Domestic Equity Mutual Funds extracted from the CRSP Survivorship-Bias-Free US M Fund Database over the period from July 2003 to March 2007. TSC refers to total shareholder costs. 12b-1 fee is the annual fee funds have charged for marketing and distribution. TNA (weighted average total net assets) is given as the median of the correspon decile and all other fees and ratios are the mean of the corresponding deciles. AFER is the after-fee excess return.

Sharpe Ratio	a		b (%/year)		R^2	Net Return (%/year)	Std. Dev. (%/year)	Exp. Ratio (%/year)	Mgmt. Fee (%/year)	12b-1 Fee (%/year)	Front Load (%/year)	Back Load (%)	TSC (%/year)	Beta	Turn-over	Ex-an
Normal Funds																
1.80	28.54	***	3.89	***	0.80	16.87	8.16	1.49	0.72	0.57	1.93	0.99	2.07	0.98	0.69	
1.53	29.23	***	1.65	***	0.88	15.75	8.78	1.62	0.70	0.66	1.61	1.15	2.17	1.05	0.76	
1.37	26.90	***	0.26	***	0.86	14.69	8.99	1.44	0.65	0.60	1.49	0.98	1.93	1.08	0.70	
1.25	25.57	***	-1.04	***	0.93	14.21	9.54	1.49	0.64	0.60	1.85	1.02	2.07	1.15	0.88	
1.15	24.44	***	-1.87	***	0.91	13.63	9.89	1.68	0.68	0.71	1.46	1.34	2.24	1.17	1.14	
1.02	21.05	***	-2.70	***	0.93	13.03	10.45	1.64	0.72	0.66	1.54	1.12	2.17	1.21	1.48	
0.88	16.44	***	-3.43	***	0.89	11.81	10.75	1.71	0.76	0.64	1.48	1.03	2.21	1.22	1.67	
0.73	15.43	***	-5.29	***	0.95	10.56	11.43	1.84	0.76	0.68	1.93	1.35	2.50	1.30	1.32	
0.61	8.99	***	-4.80	***	0.79	10.21	12.91	1.90	0.78	0.72	1.03	1.18	2.34	1.34	1.56	
0.35	6.04	***	-7.75	***	0.52	7.08	13.72	2.06	0.81	0.72	1.27	1.26	2.57	1.44	1.93	
Abnormal Funds																
2.16	15.00	***	11.17	***	0.36	21.77	9.07	1.70	0.64	0.63	1.41	1.00	2.18	0.67	1.16	
1.55	10.65	*	5.74	***	0.23	17.82	9.81	1.47	0.75	0.42	0.85	0.54	1.75	0.77	5.46	
1.39	11.50	**	5.56	***	0.54	18.37	11.29	1.78	0.75	0.88	0.00	0.91	1.96	0.91	3.69	
1.27	20.05	***	1.18	***	0.93	16.22	10.74	1.61	0.74	0.67	0.78	0.56	1.88	1.16	1.82	
1.16	12.84	***	0.97	*	0.52	15.26	11.00	1.70	0.80	0.63	0.78	0.78	2.01	1.13	2.21	
1.03	16.18	***	-0.05		0.82	15.10	12.13	1.95	0.78	0.65	0.73	0.79	2.25	1.21	3.87	
0.89	22.04	***	0.15		0.94	15.67	14.73	1.95	0.58	0.85	0.81	1.00	2.32	1.20	9.50	
0.76	11.22	***	-3.19	***	0.85	11.59	11.99	1.84	0.71	0.74	0.88	1.00	2.22	1.24	2.47	
0.57	12.40	***	-6.73	***	0.95	8.34	10.90	1.73	0.72	0.66	2.20	1.42	2.46	1.26	0.99	
-0.08	10.45	**	-19.14	***	0.31	0.95	23.98	5.47	1.03	0.49	1.10	0.43	5.77	2.16	7.13	
1.34					1.00	12.61	7.29									

regression analysis of US domestic equity funds sorted by Sharpe Ratios. This table presents the results of the OLS regression of the afte
l a denote the regression coefficient to the risky-asset. $b*$ denotes the regression coefficient to the risk-free asset. The “normal” and “abn

Decile	# (=100%)	Leveraged Funds (in %) AFER > 0	AFER < 0	Diversification Funds (in %) AFER > 0	AFER < 0
		Normal Funds			
best	258	23.3	0.0	70.5	6.2
2nd	309	50.2	0.3	25.9	23.6
3rd	317	60.6	6.9	5.4	27.1
4th	286	51.7	28.3	0.0	19.9
5th	297	47.8	37.0	0.0	15.2
6th	280	46.1	37.5	0.0	16.4
7th	319	31.0	45.8	0.0	23.2
8th	215	10.2	79.1	0.0	10.7
9th	202	6.9	72.8	0.0	20.3
worst	304	2.0	84.5	0.0	13.5
		Abnormal Funds			
best	69	10.1	0.0	88.4	1.4
2nd	18	22.2	0.0	61.1	16.7
3rd	11	45.5	0.0	54.5	0.0
4th	41	65.9	0.0	9.8	24.4
5th	31	64.5	0.0	6.5	29.0
6th	47	63.8	2.1	2.1	31.9
7th	8	87.5	0.0	0.0	12.5
8th	113	28.3	45.1	0.0	26.5
9th	125	2.4	93.6	0.0	4.0
worst	23	4.3	95.7	0.0	0.0

Table 6: Breakout of US domestic equity funds sorted by Sharpe Ratios, according to different properties. For each decile of the distribution of Shape ratios, this Table reports the number of mutual funds in each of four following mutual fund categories: (i) leveraged funds with positive after-fee excess return (AFER), (ii) leveraged funds with negative AFER, (iii) diversification funds with positive AFER and (iv) diversification funds with negative AFER. We define a fund as leveraged (or diversifying) if $\beta_m - \frac{1+r_m}{1+r_i}$ is greater (or smaller, respectively) than zero. We perform count statistic for normal funds and abnormal funds separately.

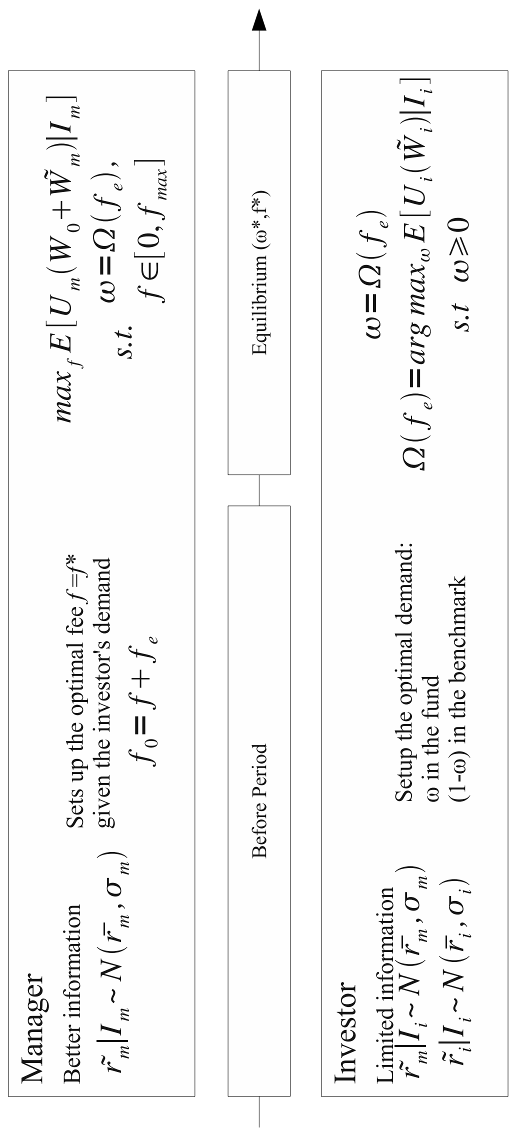


Figure 1: Summary of the model.

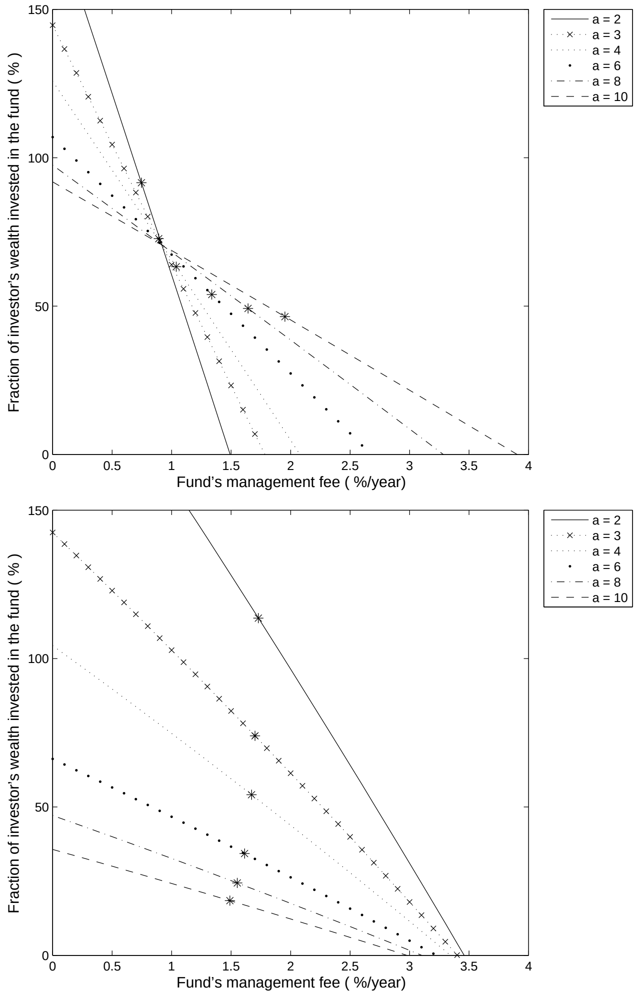


Figure 2: Investor's demand function for different values of the risk aversion coefficient a. The investor's invested amount is plotted against the fund's management fee. The equilibrium solutions $(f_p, \omega(f_p))$ for each value a has been marked with an asterisk. For diversification funds with a beta smaller than 1 (upper panel), the equilibrium weight is decreasing and the equilibrium fee is increasing when investors becomes more risk averse. For leveraged funds with a beta larger than 1 (bottom panel), both the equilibrium weight and the equilibrium fee are decreasing. The parameters for top figure are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.09$, $\sigma_i = 0.09$ and $\sigma_m = 0.08$; the parameters for bottom figure are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.12$, $\sigma_i = 0.08$, and $\sigma_m = 0.13$. Both have $\rho = 0.7$ and $f_0 = 0$.

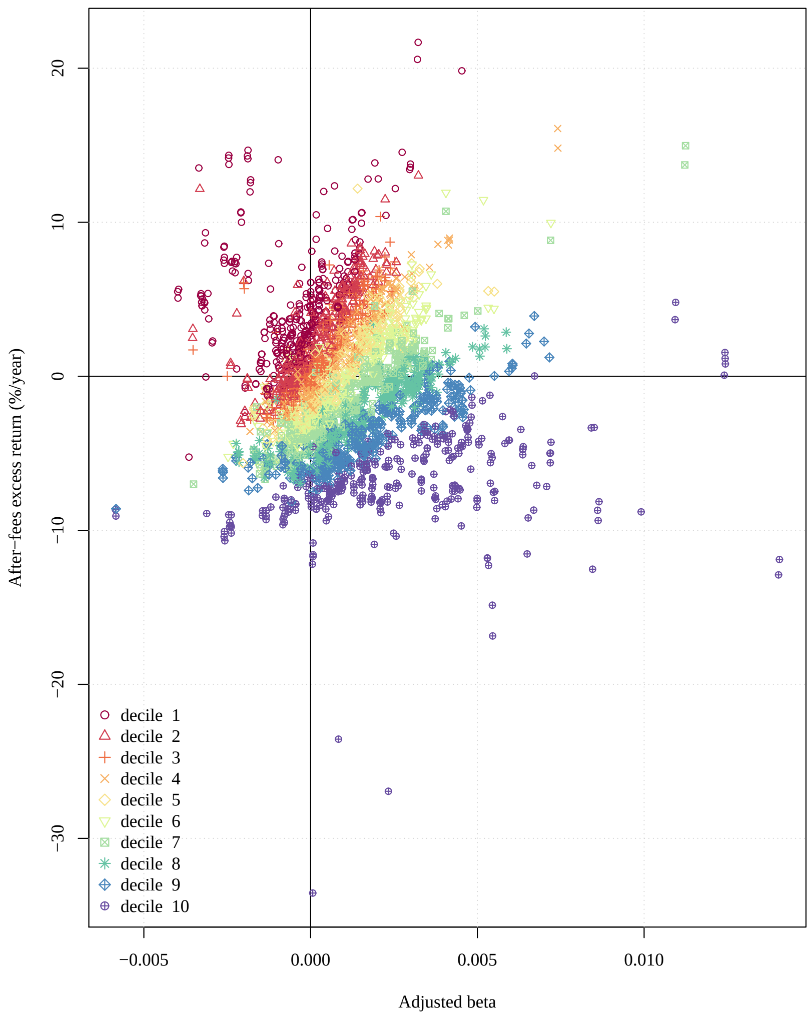
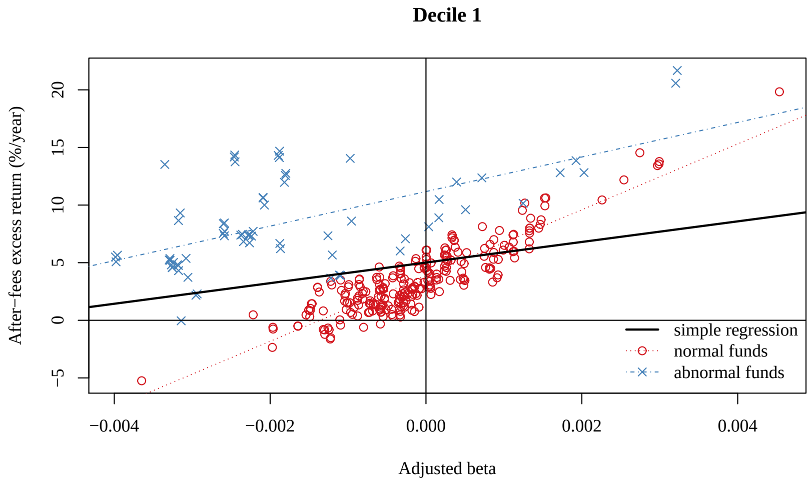


Figure 3: Plot of the after-fees excess returns versus their adjusted betas. For simplicity, six funds which are falling outside the margins of the plot, are not shown.



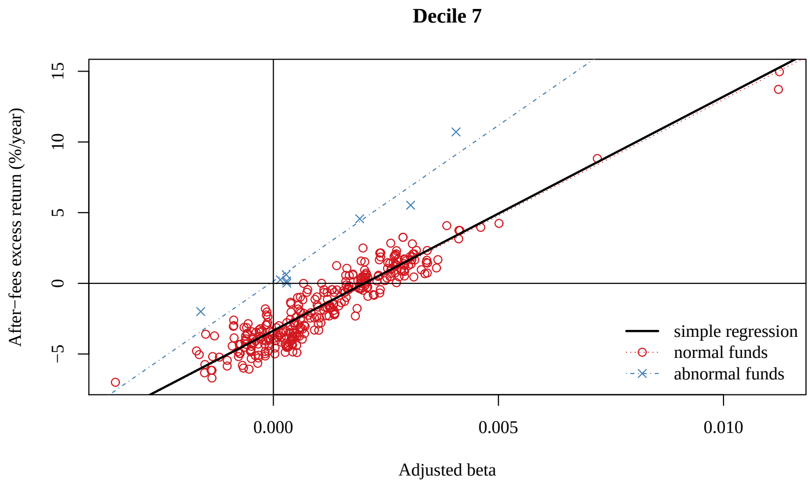


Figure 4: Cluster Analysis of the top and 7th deciles for the US Domestic Equity Mutual funds from July 2003 to March 2007. The details of the regression can be found in table 5.

Appendix

1 Proof of Proposition 2

The necessary condition for the existence of solutions to the problem

$$\max_{f|\omega(f)} \mathbf{E}[U_m(\tilde{W}_m)|\mathcal{I}_m] \tag{1}$$

is

$$\frac{\partial \mathbf{E}\left[U_m(W_0 + \Omega(f)(1+\tilde{r}_m)f)|\mathcal{I}_m\right]}{\partial f} = 0 \ , \tag{2}$$

namely

$$\mathbf{E}\left[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)(\Omega(f) + f\frac{\partial \Omega(f)}{\partial f})\bigg| \mathcal{I}_m\right] = 0. \tag{3}$$

The demand function $\Omega(f)$ is a deterministic function of f, therefore,

$$\mathbf{E}[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)|\mathcal{I}_m](\Omega(f) + f\frac{\partial \Omega(f)}{\partial f}) = 0 \ . \tag{4}$$

We have in reality $\tilde{r}_m > -1$ and because utility functions are increasing with wealth, namely $U_m^{'}(x) > 0$, we get

$$\mathbf{E}[U_m^{'}(W_0 + \Omega(f)(1+\tilde{r}_m)f)(1+\tilde{r}_m)|\mathcal{I}_m] > 0 \tag{5}$$

Therefore, the solution to (1) must be the solution to (9) in the main text. The concavity of U_m ensures the sufficiency of the first order condition. We stress that we did not need to specify the form of the utility function U_m, nor that of the demand function $\Omega(f)$. Q.E.D.

2 Proof of Proposition 3

Using equation (11) in the main text, the investor's demand function reads

$$\Omega(f_e) = \frac{(1+\bar{r}_m)(1-f_e) - (1+r_f)}{a \cdot \sigma_m^2 (1-f_e)^2} \tag{6}$$

with $f_e = f + f_0$. Then the solution to the manager's optimization problem is given by proposition 2. The first order condition yields

$$f = (1-f_0) \cdot \frac{(1+\bar{r}_m)\cdot(1-f_0) - (1+r_f)}{(1+\bar{r}_m)\cdot(1-f_0) + (1+r_f)}, \tag{7}$$

while the second order condition

$$(1-f_0)(1+r_f) \geq 0 \tag{8}$$

always holds. Q.E.D.

3 Proof of Proposition 4

The investor's demand function is solution to the problem

$$\max_{\omega|f_e} \mathrm{E}\left[-e^{-a((1-\omega)(1+\tilde{r}_i)+\omega(1+\tilde{r}_m)(1-f_e))}|\mathcal{I}_i\right] \tag{9}$$

with $\omega \geq 0$ and $(\tilde{r}_i, \tilde{r}_m)$ distributed according to (17). The expectation can be readily calculated

$$\mathrm{E}\left[-e^{-a((1-\omega)(1+\tilde{r}_i)+\omega(1+\tilde{r}_m)(1-f_e))}|\mathcal{I}_i\right] = -e^{\phi_\omega(-a)}, \tag{10}$$

where $\phi_\omega(-a)$ is the cumulant generating function of a Gaussian random variable at point $-a$, so that

$$\begin{aligned} \phi_\omega(-a) &= \frac{a^2}{2}\left[(1-\omega)^2\sigma_i^2 + 2\omega(1-\omega)(1-f_e)\rho\sigma_i\sigma_m + \omega^2(1-f_e)^2\sigma_m^2\right] \\ &- a\left[(1-\omega)(1+\bar{r}_i) + \omega(1-f_e)(1+\bar{r}_m)\right] \ . \end{aligned} \tag{11}$$

Maximizing the expectation in (9) is equivalent to minimize ϕ and therefore the first order condition yields

$$\Omega(f_e) = \frac{1}{a} \cdot \frac{(1+\bar{r}_m - a\rho\sigma_i\sigma_m)\cdot(1-f_e) - (1+\bar{r}_i - a\sigma_i^2)}{\sigma_m^2(1-f_e)^2 - 2\rho\sigma_i\sigma_m(1-f_e) + \sigma_i^2} \tag{12}$$

while the second order condition

$$a^2\left({\sigma_i}^2 - 2\left(1-f_e\right)\rho\sigma_i\sigma_m + \left(1-f_e\right)^2{\sigma_m}^2\right) \geq 0 \tag{13}$$

always holds.

Since f_e is defined within the range that satisfies $\omega \geq 0$, it is easy to check that

$$f_{max} = \frac{\bar{r}_m - \bar{r}_i + a\sigma_i^2 - a\rho\sigma_i\sigma_m}{1+\bar{r}_m - a\rho\sigma_i\sigma_m} - f_0. \tag{14}$$

As for the manager's optimization problem, proposition 2 shows that the optimal fee is solution to the first order condition

$$\Omega(f_0+f) + f\cdot\partial_f\Omega(f_0+f) = 0 \ , \tag{15}$$

namely, with the notations of proposition 4

$$\begin{aligned} &\left[R_m(1-f_0)\sigma_m - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2\right]f^2 \\ -\ &2R_m\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]f \\ +\ &\left[R_m(1-f_0) - R_i\right]\cdot\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right] = 0 \end{aligned} \tag{16}$$

whose solutions are

$$\begin{aligned} f_\pm &= \frac{R_m\left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right]}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2} + \\ &\pm \frac{\sqrt{\left[(1-f_0)^2\sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2\right]\cdot\left[R_m^2\sigma_i^2 - 2R_mR_i\rho\sigma_i\sigma_m + R_i^2\sigma_m^2\right]}}{R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2} \ . \end{aligned} \tag{17}$$

Since

$$\partial_f^2 \left(f \cdot \Omega(f_0 + f)\right)_{f=f_\pm} = \pm \sqrt{\begin{array}{l}\left[(1-f_0)^2\sigma_m^2 - 2(1-f_0)\rho\sigma_i\sigma_m + \sigma_i^2\right] \\ \times \left[R_m^2\sigma_i^2 - 2R_mR_i\rho\sigma_i\sigma_m + R_i^2\sigma_m^2\right]\end{array}}\ , \tag{18}$$

the second order condition leads us to choose f_-.

However, this solution is admissible if and only if $f_- \geq 0$, which requires

$$R_m^2 \left[\sigma_i^2 - 2(1-f_0)\rho\sigma_i\sigma_m + (1-f_0)^2\sigma_m^2\right] \gtreqless R_m^2\sigma_i^2 - 2R_mR_i\rho\sigma_i\sigma_m + R_i^2\sigma_m^2 \tag{19}$$

and

$$R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2 \gtreqless 0\ . \tag{20}$$

Factorizing (19), we get

$$[(1-f_0)R_m - R_i] \cdot \left[R_m(1-f_0)\sigma_m^2 - 2R_m\rho\sigma_i\sigma_m + R_i\sigma_m^2\right] \gtreqless 0, \tag{21}$$

so that, according to (19) and (20)

$$f_- \geq 0 \qquad \Longleftrightarrow \qquad (1-f_0)R_m \geq R_i \tag{22}$$

which holds by the assumption made in proposition 4. Q.E.D.

4 Generalization

We now consider the general case where the investor's utility function can be any increasing and concave function. We do not make any assumption on the joint distribution of returns on the benchmark portfolio and on the managed portfolio. We just assume that the funds are not too risky and that the investors are not too risk averse to justify a second order expansion of the investor's utility function. Then, as proved in 5 down-below, the following result holds

Lemma 1. *Up to second order terms in an expansion in powers of the returns and fees, the investor's demand function reads*

$$\Omega = \frac{\left[\sigma_i^2 - (1-f_e)\rho\sigma_i\sigma_m\right] + a(\Omega)^{-1} \cdot \left[(1+\bar r_i) + (1-f_e)(1+\bar r_m)\right]}{\sigma_i^2 - 2(1-f_e)\rho\sigma_i\sigma_m + (1-f_e)^2\sigma_m^2}, \tag{23}$$

where $a(\Omega)$ denotes the investor's absolute risk aversion at the point $(1-\Omega)(1+\bar r_i) + \Omega(1-f_e)(1+\bar r_m)$.

As previously, the demand is independent of the risk aversion for $f_e = f_p = \frac{\bar r_m - \bar r_i}{1+\bar r_m}$. Thus, up to the first order in $f_e - f_p$, we get

$$\Omega(f_e) \approx \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar r_i}{1+\bar r_m} \cdot {\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a^{-1}\left(1+\bar r_m\right)}{\sigma_m^2\left(\frac{1+\bar r_i}{1+\bar r_m}\right)^2 - 2\rho\sigma_i\sigma_m \cdot \frac{1+\bar r_i}{1+\bar r_m} + \sigma_i^2}\right] \cdot (f_e - f_p). \tag{24}$$

As checked later in 6, for the case of CRRA utility functions, the linear approximation of the demand function is quite good. This justifies assumption 2 according to which the fund manager knows the investors' demand function.

Without loss of precision, (24) can be simplified by replacing $a(\Omega)$ with $a(\Omega_p)$ in (23). Then, performing the same calculation as in 3, we can generalize the result of proposition 4. More importantly, we can state

Proposition 5. *Within the limits of the hypothesis of this section, irrespective of the investor's utility function and of the distributions of returns on the benchmark portfolio and on the managed portfolio, the optimal management fee the manager should charge is approximately half the fee that starts to make the managed fund undesirable to the investor:*

$$f^* \approx \frac{\hat{r}_m - \hat{r}_i}{2} - \frac{f_0}{2}, \tag{25}$$

where

$$\hat{r}_m = \bar{r}_m - a\left(\Omega_p\right)\rho\sigma_i\sigma_m \qquad \text{and} \qquad \hat{r}_i = \bar{r}_i - a\left(\Omega_p\right)\sigma_i^2 \tag{26}$$

are the risk-adjusted expected returns on the managed and on the benchmark portfolio.

As a corollary to this proposition, we can generalize the results derived when the benchmark portfolio is the risk-free asset.

Corollary 1. After adjustment for the level of risk, the expected return on the managed fund, net of all fees, is equal to the expected return on the benchmark plus the management fee

$$\hat{r}_m - f_e = \hat{r}_i + f^*. \tag{27}$$

5 Proof of lemma 1

Let us denote by U the investor's utility function. It is assumed increasing and concave. The demand function is solution to

$$\max_{\omega} \mathrm{E}\left[U\left((1-\omega)(1+\tilde{r}_i) + \omega(1-f_e)(1+\tilde{r}_m)\right)\right]. \tag{28}$$

We denote by $1+\bar{r}(\omega)$ the average gross rate of return and by $\sigma(\omega)^2$ the variance of the return on the investor's portfolio. Then expanding the utility function in the neighborhood of $\bar{r}$ up to the second order, the optimal demand solves

$$\partial_\omega \bar{r}(\omega) = -\frac{U''(\bar{r})}{U'(\bar{r})} \cdot \frac{\partial_\omega \sigma(\omega)^2}{2}, \tag{29}$$

where we recognize the coefficient of absolute risk aversion $-U''/U'$. The higher order term $\partial_\omega \bar{r}(\omega) U'''(\bar{r})\sigma(\omega)^2$ has been neglected.

Expressing $\bar{r}(\omega)$ and $\sigma(\omega)^2$ and substituting in the equation above, we get

$$\begin{aligned} &- (1+\bar{r}_i) + (1-f_e)(1+\bar{r}_m) \\ = -\frac{U''(\bar{r})}{U'(\bar{r})} &\left[\omega(1-f_e)^2\sigma_m^2 + (1-2\omega)(1-f_e)\rho\sigma_i\sigma_m - (1-\omega)\sigma_i^2\right]. \end{aligned} \tag{30}$$

To simplify notations, we define

$$a(\omega) := -\frac{U''(\bar{r}(\omega))}{U'(\bar{r}(\omega))}, \tag{31}$$

so that the optimal demand function reads

$$\Omega(f_e) = \frac{\left[\sigma_i^2 - (1-f_e)\rho\sigma_i\sigma_m\right] - a(\Omega)^{-1}\cdot\left[(1+\bar{r}_i) - (1-f_e)(1+\bar{r}_m)\right]}{\sigma_i^2 - 2(1-f_e)\rho\sigma_i\sigma_m + (1-f_e)^2\sigma_m^2}. \tag{32}$$

This equation remains implicit since the level of the demand appears in the right-hand side to set the level of absolute risk aversion. But, if it does not vary too fast, it is reasonable to make the approximation that it is locally constant.

Expanding this relation around $f_e = f_p$, we first evaluate

$$\Omega(f_p) = \Omega_p = \frac{\sigma_i^2 - \frac{1+\bar{r}_i}{1+\bar{r}_m}\rho\sigma_i\sigma_m}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m\cdot\frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}, \tag{33}$$

and

$$\Omega'(f_p) = \frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\cdot{\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a(\Omega_p)^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m\cdot\frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}, \tag{34}$$

so that

$$\Omega(f_e) \simeq \Omega_p + \left[\frac{\rho\sigma_i\sigma_m + \left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\cdot{\sigma_m}^2 - \rho\sigma_i\sigma_m\right)\Omega_p - a(\Omega_p)^{-1}\left(1+\bar{r}_m\right)}{\sigma_m^2\left(\frac{1+\bar{r}_i}{1+\bar{r}_m}\right)^2 - 2\rho\sigma_i\sigma_m\cdot\frac{1+\bar{r}_i}{1+\bar{r}_m} + \sigma_i^2}\right]\cdot(f_e - f_p). \tag{35}$$

6 Robustness Check: CRRA utility function with Log-Prices

The dependence of the results of our model is tested for a larger class of utility functions:

$$U_i(x) = \frac{x^{1-a}}{1-a} \tag{36}$$

$$U_i(x) = \frac{x^{1-b}}{1-b} \tag{37}$$

with $a, b > 0$ and $a, b \neq 1$, a, b represent respectively the constant relative risk aversion level of the investors and managers.

To avoid negative prices, we use log-returns. The wealth of the investor at period 1 is

$$\tilde{W}_i = (1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e) \ . \tag{38}$$

The wealth of the manager at period 1 is

$$\tilde{W}_m = \omega e^{\tilde{r}_m} f \tag{39}$$

The log-returns $\tilde{r}_i$ and $\tilde{r}_m$ are Gaussian distributed as

$$\tilde{r}_i | i \sim N(\bar{r}_i, \sigma_i^2) \tag{40}$$

$$\tilde{r}_m | i \sim N(\bar{r}_{m,i}, \sigma_{m,i}^2) \tag{41}$$

Their correlation perceived by the investor is denoted ρ.

The investor's optimization problem is formulated as:

$$\max_{\omega | f_e} \mathbf{E}[\frac{((1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e))^{1-a}}{1-a} | i] \tag{42}$$

with $\omega > 0$. The manager' optimization problem is formulated as

$$\max_{f | \omega^*} \mathbf{E}[\frac{(\omega e^{\tilde{r}_m} f)^{1-b}}{1-b} | m] \tag{43}$$

with $0 < f + f_0 < 1$ given

$$\tilde{r}_m | m \sim N(\bar{r}_{m,m}, \sigma_{m,m}^2) \tag{44}$$

In this setup, a closed form solution is difficult to find and we resort to numerical methods.

For the investor's optimization problem, we have

$$\mathbf{E}[((1-\omega)e^{\tilde{r}_i} + \omega e^{\tilde{r}_m}(1-f_e))^{1-a} | i] = \frac{1}{2\pi\sigma_i\sigma_{m,i}\sqrt{1-\rho^2}(1-a)}\mathbf{I} \tag{45}$$

with

$$\begin{aligned} \mathbf{I} \ = & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ((1-\omega)e^{r_i} + \omega e^{r_m}(1-f_e))^{1-a} \\ & \times e^{-\frac{1}{2(1-\rho^2)}(\frac{(r_i-\bar{r}_i)^2}{\sigma_i^2} + \frac{(r_{m,i}-\bar{r}_{m,i})^2}{\sigma_{m,i}^2} - \frac{2\rho(r_i-\bar{r}_i)(r_{m,i}-\bar{r}_{m,i})}{\sigma_i\sigma_{m,i}})} \mathbf{d}r_i \mathbf{d}r_{m,i} \end{aligned} \tag{46}$$

We numerically calculated this double integral and solved the optimization problem.

For the managers' optimization problem, we have (notice $\omega^*(f)$ is the same as $\omega^*(f_e)$ here)

$$\mathbf{E}[\frac{(\omega e^{\tilde{r}_m} f)^{1-b}}{1-b} | m] = \frac{1}{\sqrt{2\pi}\sigma_{m,m}(1-b)}(\omega^*(f)f)^{1-b} I(b, r_{m,m}, \sigma_{m,m}) \tag{47}$$

with

$$I(b, r_{m,m}, \sigma_{m,m}) = \int_{-\infty}^{+\infty} e^{r_m(1-b)} e^{-\frac{1}{2}(\frac{(r_m - \bar{r}_{m,m})^2}{\sigma^2_{m,m}})} \mathbf{d}r_m \tag{48}$$

Notice that only $(\omega^*(f)f)^{1-b}$ depends on f, therefore, the manager's optimization problem (43) is equivalent to

$$\max_{f|\omega^*} (\omega^*(f)f)^{1-b} \tag{49}$$

The numerical results confirm the remarkably strong robustness of our analytical result derived for CARA utility functions. Figures are available from the authors upon request.

7 Shape of the demand function Ω

Figure 1 depicts the changes of the optimal management fee (upper panel) and of the corresponding investor's demand (lower panel) as a function of the investor's anticipated correlation between the mutual fund and the benchmark portfolio, for different degrees of the investor's absolute risk aversion a. The upper panel predicts that the equilibrium fee decreases when the correlation between the two portfolios increases. This reflects the fact that investors put more value on those funds which provide a greater diversification potential with respect to their benchmark portfolio. In addition, this figure confirms that more risk averse investors accept higher fees when the fund provides diversification benefits.

In the upper panel of Figures 1, 2 and 3, all curves intersect at one single point. At this point, the optimal management fee is the same for investors with different value of the risk aversion coefficient a.

The lower panel shows the dependence of the demand as a function of the correlation, for different types of investors. For the chosen parameters, we observe that the less risk averse investors put a larger fraction of their wealth in the managed portfolio when the correlation increases, while the more risk averse investors invest less in this portfolio for the same correlation. This reflects the fact that more risk averse investors are more eager to seek diversification.

Figure 2 plots the equilibrium fee (upper panel) and the investors' demand (lower panel) as a function of the investor's expected future volatility (standard deviation) of the benchmark portfolio. The correlation coefficient is set to $\rho = 0.7$. We recall that the expected future volatility of the mutual fund is $\sigma_m = 0.08$.

The upper panel shows that, overall, investors accept higher fees when the expected volatility of the benchmark portfolio increases. This result is not surprising in so far as, everything else taken equal, the larger the benchmark volatility, the more attractive the managed portfolio. In addition, as previously, the more risk-averse investors are the more sensitive to an increase of the benchmark volatility and are thus more agreeable to paying higher fees. The lower panel confirms that the more risk-averse investors buy more mutual fund shares than the less risk-averse investors even if the number of shares they buy decreases, overall, when the benchmark volatility increases.

Figure 3 shows the equilibrium fee (upper panel) and the investor's demand (lower panel) as a function of the investor's expected return on the mutual fund. According to expression (22) in the main text, the equilibrium fee increases with the expected return on the mutual fund. The upper panel shows that the change is almost insignificant with respect to the different levels of risk aversion. In contrast, the lower panel shows that the level of risk aversion affects the demand significantly. More risk-averse investors are more sensitive.

8 Investment style versus Sharpe Ratio

Table 1 provides information on how the different investment styles are distributed over the ten Sharpe ratio deciles. The abbreviation codes used in the table are reported in Table 2 in the main text. The most frequent categories (about 27% of the total sample) in our data sample are large-cap (L) funds, closely followed by mid-cap (M) funds (23%) and small-cap (S) funds (17%). Most funds are classified after market capitalization and typically fall into the normal fund category, which is about 80% of the total sample. Further, value (V) funds tend to lie in the top deciles, whereas more growth (G) funds lie in the bottom deciles, as growth funds are more sensitive to market changes and experience higher volatility. The remaining funds (about one third of the sample) are not classified according to the market capitalization; most of these remaining fund invest into specific industry sectors with the exception of equity income (EIE) funds and S&P 500 index objective (SPS) funds. Unsurprisingly, all the S&P 500 index objective funds (SPS) funds have a very similar characteristic and fall in the middle deciles (fourth and fifth decile to be specific) of the normal funds. Equity Income (EIE) funds fall with very few exceptions into the normal cluster as they invest in dividend paying stocks, which can as well be found in large-cap portfolios. Utility (UT) funds performed particular well during the investment period and mostly fall into the top deciles of the abnormal funds. Financial services (FS) funds were similar successful. Finally, the performance of Health Care and Biotech (H) funds is more heterogeneous and these fund typically fall in the bottom deciles of the normal funds, with a few exceptions though. We can confirm that investment styles have a large impact on performance, which is in line with many other studies, for instance by Chan et al. (2002), Brown and Harlow (2002) and Bogle (1998).

9 Adding the risk-free asset to the equity benchmark portfolio

Consider the investor's benchmark as a mix of the risk-free asset (bond index) with return r_f and of a stock index, whose expected return is $\bar{r}_i$, risk σ_i and correlation ρ with the managed fund i. Let x denote the weight of the stock index in the benchmark portfolio. We consider the case where x is set exogenously. In this case, the results in section 3.2 of the main text hold with the correspondence

$$\begin{aligned} \bar{r}_i &\mapsto r_f + x\left(\bar{r}_i - r_f\right), && (50)\\ \sigma_i &\mapsto x\sigma_i, && (51)\end{aligned}$$

so that equation (17) in the main text becomes

$$f^* + f_e \simeq \left[\bar{r}_m - r_f - x\cdot(\bar{r}_i - r_f)\right] - a\left(\rho\frac{\sigma_m}{x\sigma_i} - 1\right)x^2\sigma_i^2, \qquad (52)$$

and x has to be considered as an (additional) independent variable in the regression model derived from this relation.

9.1 Regression framework

From eq. (52) we have

$$\bar{r}_m - r_f - f^* - f_e = x\cdot(\bar{r}_i - r_f) + a\cdot x\cdot\rho_m\sigma_m\sigma_i - a\cdot x^2\cdot\sigma_i^2, \qquad (53)$$

where m is the varying index from one managed fund to another. We can then perform the linear regression

$$\bar{r}_m - r_f - f^* - f_e = \alpha + \beta\cdot\rho_m\sigma_m + \epsilon_m, \qquad (54)$$

where α and β are the parameters to be estimated and ϵ_m is a zero mean error term. The parameters α and β have the following definitions

$$\begin{aligned} \alpha &= x\cdot(\bar{r}_i - r_f) - a\cdot x^2\cdot\sigma_i^2, && (55)\\ \beta &= a\cdot x\cdot\sigma_i, && (56)\end{aligned}$$

from which we get

$$\begin{aligned} a &= \frac{\beta}{\alpha}\cdot\frac{\bar{r}_i - r_f}{\sigma_i} - \frac{\beta^2}{\alpha}, && (57)\\ x &= \frac{\alpha}{(\bar{r}_i - r_f) - \beta\sigma_i}. && (58)\end{aligned}$$

With this approach, there is no intercept which can be interpreted in terms of investor's optimism. However, we can still compare the weight to the equity weight of the benchmark x, which depends on a.

9.2 Maximum Likelihood solution

The problem with the equation

$$(1+\bar{r}_m)(1 - f_e^m - f^m) - (1 + r_f) = \alpha + \beta\rho_m\sigma_m(1 - f_e^m - f^m) + \epsilon_m \qquad (59)$$

is that the condition

$$f^m(1 + \bar{r}_m - \beta\rho_m\sigma_m) \geq -\epsilon_m \tag{60}$$

should be considered in the regression, leading to a non-trivial maximum likelihood estimation procedure.

As we were not able to find a closed form solution for a maximum likelihood estimator for the problem (59) simultaneously respecting condition (60), we are estimating α and β from equation (55) and (56), respectively, by an iterative robust maximum likelihood methodology introduced by Dupuis and Morgenthaler (2002). This estimator weights observations with respect to the model. This is important as it allows us to down-weight observation which do no fulfill condition (60) and subsequently do not need to fulfill equation (59). Moreover, this estimator has some good properties, such as unbiasedness and consistency. The result of the methodology can be found in Figure 8. We obtain $\alpha = 0.074$ and slope $\beta = 0.225$ corresponding to $x = 0.904$ and $a = 4.3$, respectively. Note that this results should be interpreted with care because weighting observation by the model is a very strong assumption. However, applying this methodology, the results are in line with our previous results. The weight x is relatively close to 1, indicating that investors seem to use an equity dominated benchmark. The risk aversion $a \approx 4$ is also in line with the results presented previously where the sample was divided into ten Sharpe ratio deciles.

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10 Further Tables and Figures

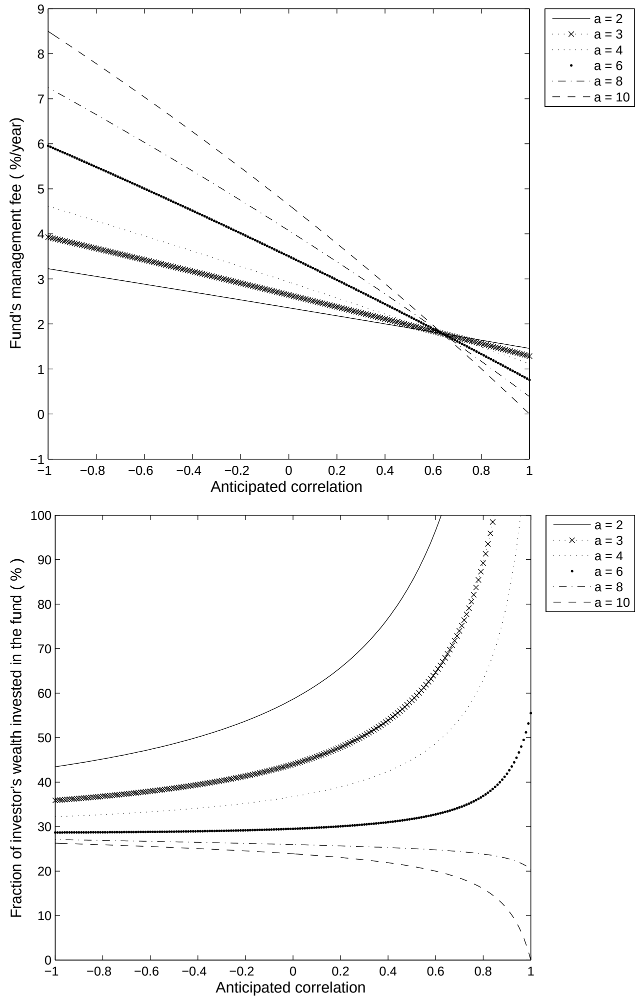


Figure 1: Fund's management fee and investor's invested amount in the mutual fund as a function of the anticipated correlation between the fund's portfolio and investor's benchmark portfolio, with all the other parameters being kept fixed. When the two portfolios are perfectly correlated, it should not be surprising to find that, depending on their risk aversion and taste for mean return, investors may invest fully in the benchmark portfolio or the mutual fund, or partly in the mutual fund and partly in the benchmark portfolio. The parameters are: $\bar{r}_i = 0.08$, $\bar{r}_m = 0.12$, $\sigma_m = 0.13$, $\sigma_i = 0.08$ and $f_0 = 0$.

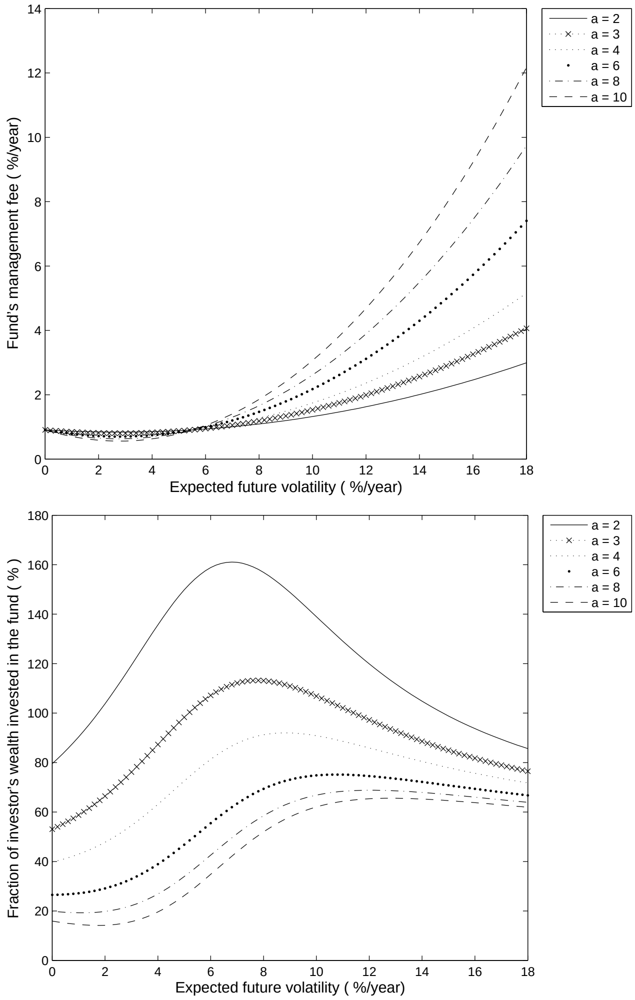


Figure 2: Fund's management fee and investor's invested amount in the mutual fund as a function of the expected future volatility (standard deviation) of the investor's benchmark portfolio, with all the other parameters being kept fixed. The parameters are: $\bar{r}_i = 0.08$,$\bar{r}_m = 0.10$, $\sigma_m = 0.08$, $\rho = 0.7$ and $f_0 = 0$

Decile	EIE	FS	H	LCC	LCG	LCV	MCC	MCG	MCV	MLC	MLG	MLV	SCC	SCG	SCV	SPS	TK	TL	UT	Tot…
Normal Funds																				
best	39	9	0	4	1	35	19	1	41	10	1	79	10	0	6	0	0	2	1	2…
2nd	34	2	0	19	0	51	21	9	29	30	2	59	22	0	27	0	0	1	3	3…
3rd	20	6	0	38	0	37	40	7	14	55	8	36	19	1	28	7	0	1	0	3…
4th	6	4	0	49	2	17	24	10	4	35	17	19	33	5	17	36	0	8	0	2…
5h	4	10	0	62	6	24	17	24	7	24	5	11	42	14	22	20	4	1	0	2…
6th	2	6	0	72	8	15	16	26	11	19	19	16	35	23	11	0	1	0	0	2…
7th	3	11	7	61	33	2	9	47	4	37	28	10	22	41	2	0	2	0	0	3…
8th	0	0	0	42	49	0	2	40	5	12	18	3	6	26	1	0	10	1	0	2…
9th	0	5	23	17	25	1	2	24	1	11	18	5	7	24	2	0	36	1	0	2…
worst	4	0	32	25	85	0	3	9	2	6	35	0	4	17	1	0	81	0	0	3…
Total	112	53	62	389	209	182	153	197	118	239	151	238	200	151	117	63	134	15	4	27…
Abnormal Funds																				
best	0	0	2	0	0	0	6	0	7	1	0	4	4	0	1	0	0	0	44	[illegible]
2nd	0	1	3	0	0	0	0	1	0	0	0	0	3	1	1	0	0	0	8	[illegible]
3rd	0	0	2	0	0	0	1	0	0	0	0	1	4	0	1	0	1	0	1	[illegible]
4th	0	3	0	2	0	1	7	5	2	2	3	3	5	2	6	0	0	0	0	[illegible]
5h	3	5	3	6	0	1	1	4	1	1	2	1	0	2	1	0	0	0	0	[illegible]
6th	0	8	5	4	1	1	2	4	0	1	2	1	4	6	3	0	4	1	0	[illegible]
7th	0	0	1	0	0	0	0	1	0	0	0	3	0	1	0	0	0	2	0	
8th	0	1	18	5	11	1	1	22	0	3	8	3	3	25	0	0	9	3	0	1…
9th	0	0	0	20	51	0	0	17	0	12	10	0	0	6	0	0	9	0	0	1…
worst	0	0	1	2	1	0	0	0	0	2	1	0	1	4	1	0	10	0	0	[illegible]
Total	3	18	35	39	64	4	18	54	10	22	26	16	24	47	14	0	33	6	53	4…

Table 1: Cluster Analysis and Investment Styles Statistics. For each decile of the distribution of Shape ratio, this table re… the results of the cluster analysis and the number of mutual funds in each of the Lipper investment objectives. We have a… of 3,273 US Domestic Equity Mutual Funds over the time period from July 2003 to March 2007. All deciles are classifie… two fund segments. All the abbreviations are reported in Table 2.

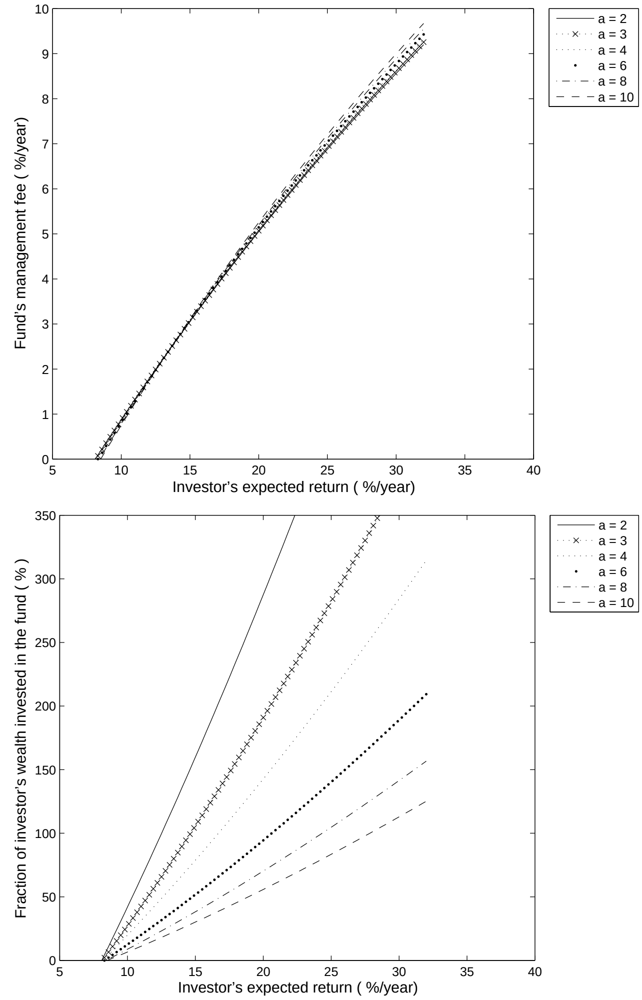
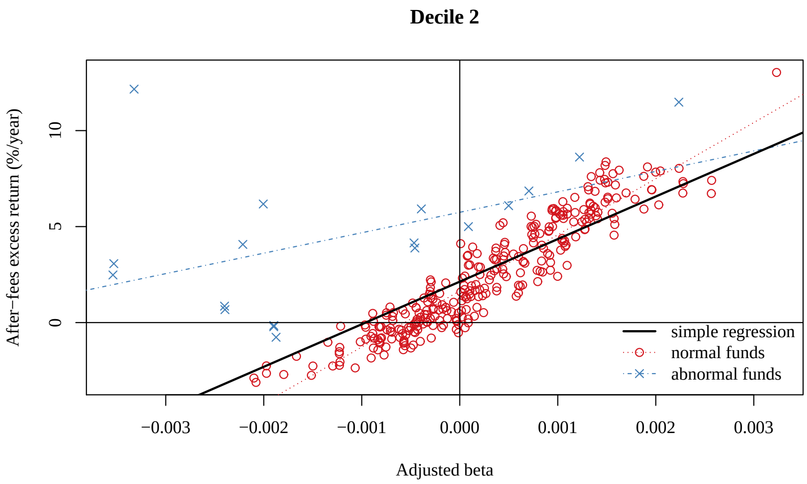


Figure 3: Fund's management fee and investor's invested amount in the mutual fund as a function of the expected return on the mutual fund, with all other parameters being kept constant. The parameters are: $\bar{r}_i = 0.08$, $\sigma_i = 0.10$, $\sigma_m = 0.15$, $\rho = 0.7$ and $f_0 = 0$.



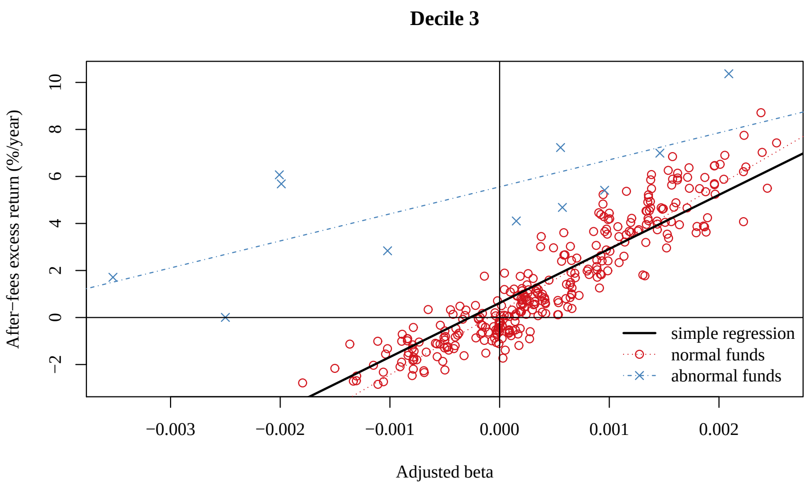
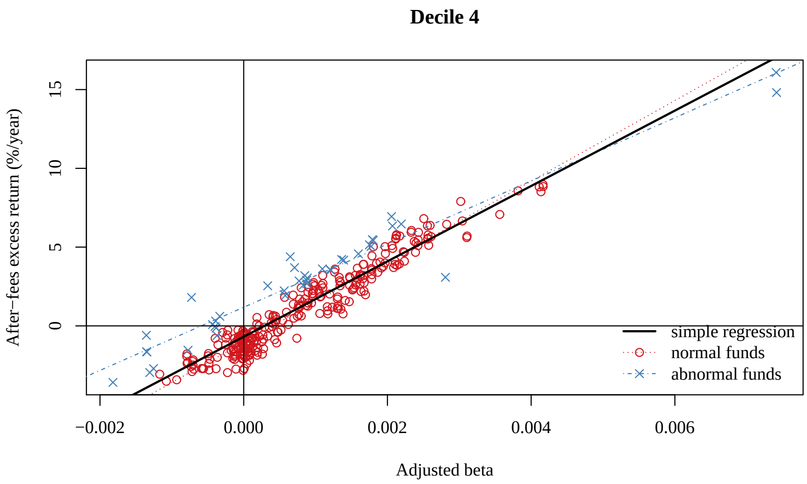


Figure 4: Cluster Analysis of deciles 2 and 3 for the US Domestic Equity Mutual funds from July 2003 to March 2007. The details of the regression can be found in table 5.



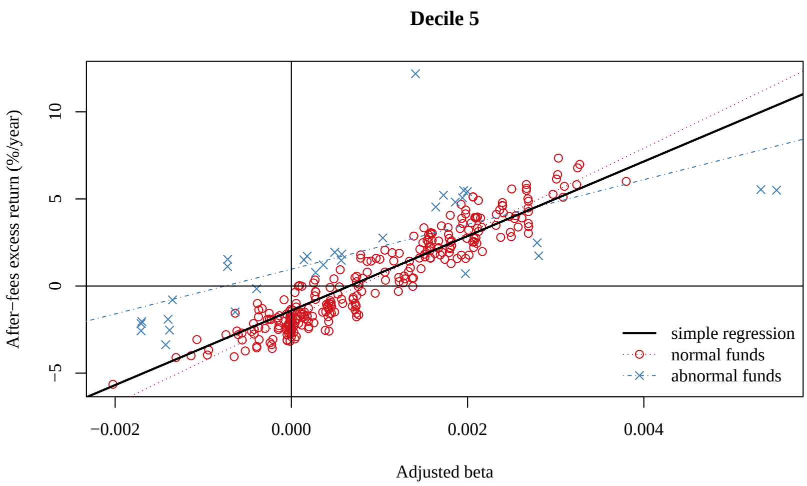


Figure 5: Cluster Analysis of the deciles 4 and 5 for the US Domestic Equity Mutual funds from July 2003 to March 2007. The details of the regression can be found in table 5.

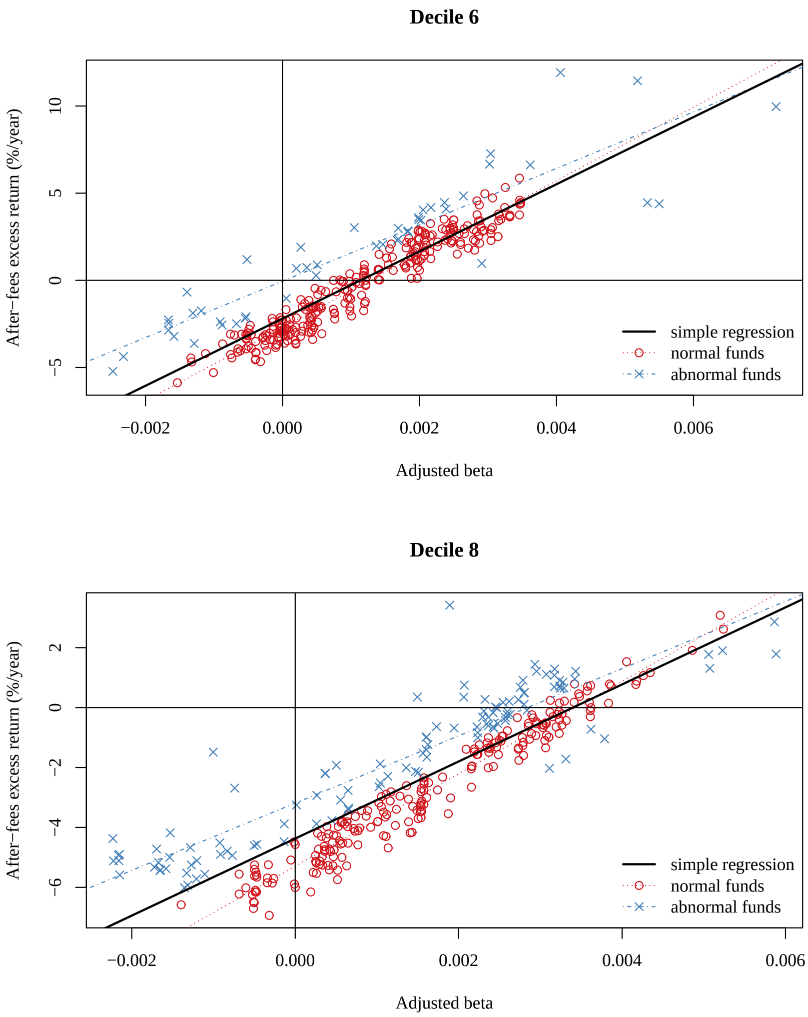


Figure 6: Cluster Analysis of the deciles 6 and 8 for the US Domestic Equity Mutual funds from July 2003 to March 2007. The details of the regression can be found in table 5.

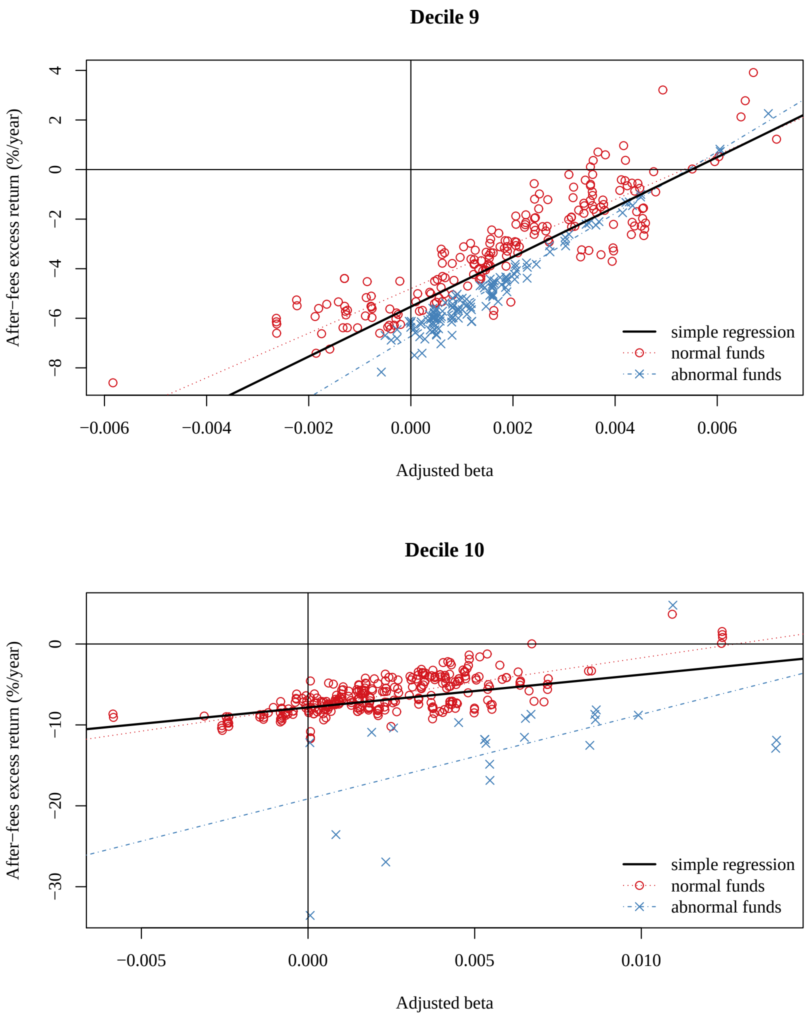


Figure 7: Cluster Analysis of the two bottom deciles for the US Domestic Equity Mutual funds from July 2003 to March 2007. The details of the regression can be found in table 5.

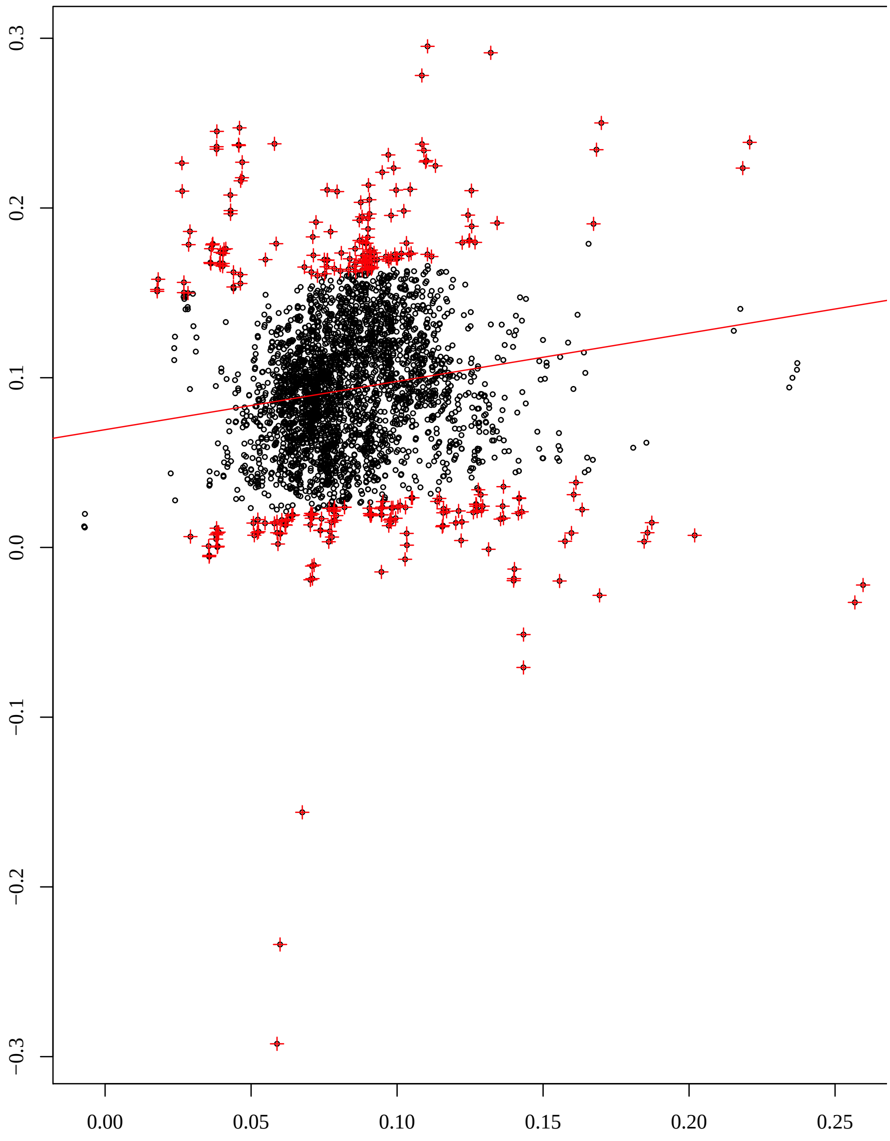


Figure 8: Fitting equation (59) using the methodology of Dupuis and Morgenthaler (2002). The red line presents the parameter estimates with intercept $\alpha = 0.0743$ and slope $\beta = 0.2252$ corresponding to $x = 0.9038$ and $a = 4.3176$, respectively. The funds with red crosses are assigned a weighted below 100%.

2.3 Discussion & Perspective

To be useful, the presented as well as the CAPM model have the problem that both have to be calibrated on future expectation. Of course, the problem is that these expectations can not be directly observed (latent variable). It is only possible to proxy them by past returns, i.e. basically assuming that the history repeats itself (see Pettengill et al. (1995) for a technical discussion). This is a common assumption in finance, nevertheless this assumption seems not very natural. Moreover, an intrinsic problem is that the fair value itsself is unobservable.

Starting around 1995, scientists from natural science and in particular physics increasingly stimulated the field of finance. They bring in new methods to the field of finance; for instance the analysis of massive data sets (developed originally for particle physics) or the use of statistical mechanics. "A market is a system of extremely many dynamically coupled variables. Theoretically, it is not obvious that such a system would have a stationary solution. For example, the system could behave periodic, quasi-periodic, chaotic, or turbulent [...]. In all these cases, there would be no convergence to a stationary solution" (Helbing, 2012).

Special attention should be drawn that econophysics is less concerned with giving an axiomatic explanation which is later verified with data. Rather, econophysics tries to describe and quantify phenomena observed in the data itself. For this reasons, it was partly ignored by economists, partly heavily criticized. Gallegati et al. (2006) for example point out, that econophysicists apply their model too often blindly, neither understanding the actual data nor giving an explanatory insight of the underlying interactions.

In his reply, McCauley (2006) argues that economists put to much emphasis on nice assumptions to keep model analytical tractable, for example assuming normal distributed noise, although it is well known from empirical observation that this is unrealistic. McCauley further criticizes that economist postulate models, even if they know that the assumptions are violated in reality, just because of the lack of any alternatives. In addition, even if classical finance can successfully model and explain historical relations, too often these relations can not be exploited for policy making; as soon as a policy maker tries to invoke a policy based on historical observations, they are no longer usable as they become expected or even manipulated. The former is known as Lucas critique (see Lucas, 1976), whereas the latter is known as Goodhart's law (Goodhart, 1975). Moreover, even Markowitz seminal paper was criticized to follow new paths and for not contributing to economics (Markowitz, 1990), however he was awarded a Nobel price in 1990.

Last, but not least, the current "financial crisis" is also a crisis of finance and economics. "We no longer know what things are worth. This is an advance on the previous situation, where we wrongly supposed they were worth whatever markets – or actuaries – told us they were. But for both financial folk and policy makers, it threatens paralysis." (Jackson, 2012). Financial theory did not only fail to predict the crisis, it seems also to miss solutions to solve it. Hence, the paper presented in the next chapter will use a more applied approach.

3

Super-exponential Bubbles in Lab Experiments: Evidence for Anchoring over-optimistic Expectations on Price

I can calculate the movement of the stars, but not the madness of men.
—Isaac Newton

3.1 Introductory comments

In this chapter, we use a more applied approach to describe prices during a bubble period. This reduces the hassle of financial modeling, and defining the expectations of our agents. Nevertheless, we will see later in the discussion and literature section of the paper that although the model has first and foremost only descriptive character, it can be motivated by well known models. We start by giving a short introduction to financial bubbles.

Bubbles have been with humans since a long time. Gisler and Sornette (2010 (available at http://ssrn.com/abstract=1590816) even argue that bubbles are the fundamental driving force of human development. Famous historical bubbles have been the tulip mania (1637) and the south sea bubble (1720), both induced by over-optimism of a new paradigm (see for example Garber, 2001). It is interesting to note that to some degree history indeed repeats itself. In every bubble, people tend to expect that "this time is different" (Reinhart and Rogoff, 2009), but after the initial euphoria, conventional valuation can not be completely overturned. Nobel laureate Shiller wrote an informative book about "Irrational exuberance"

(Shiller, 2005) discussing the psychological factors of bubbles.

Initially, all these bubbles are fueled by expectations in exogenous factors (new discoveries of better and more efficient technologies, etc.). However, as time progresses (and uncertainty in future prospect reduces), investors continue to jump on the bandwagon of rising prices. Prices continue to grow, but caused by endogenous factors; agents are not willing to adopt and preserve their expectations to the changed situation. The paper presented in the next section describes the dynamics relating to this second phase.

Many excellent reviews about bubbles have been written – including Brunnermeier (2008) and Brunnermeier and Oehmke (2012) which have a more classic stand point, and Kaizoji and Sornette (2010 (long version at http://arXiv.org/abs/0812.2449) a review with a broader perspective. Finally, the paper presented in the next section also contains an overview of the relevant literature of experimental bubbles which we do not want to replicate here.

For the last part of this section we focus on the dynamics of returns. In the most simple setting, returns are often modeled to be normal distributed (which is consistent with the weak form of the efficient market hypothesis). However, more complicated distributions are possible.

An interesting empirical pattern is the so called cobweb cycle or "hog cycle", first described by Harlow (1960) in which prices alternatively over- and undershoot the equilibrium price. Depending on the shape of the supply and demand curve, it is possible that the prices converges to an equilibrium, oscillates around the equilibrium, diverges or behave chaotic. The mispricing is generally the higher, the greater the confusion is about the pricing mechanism (Kirchler et al., 2012). Figure 1 shows an example where the cobweb will converge over time (the equilibrium price is where supply meets demand). Hommes (2006) presents agents based model where dynamics are chaotic.

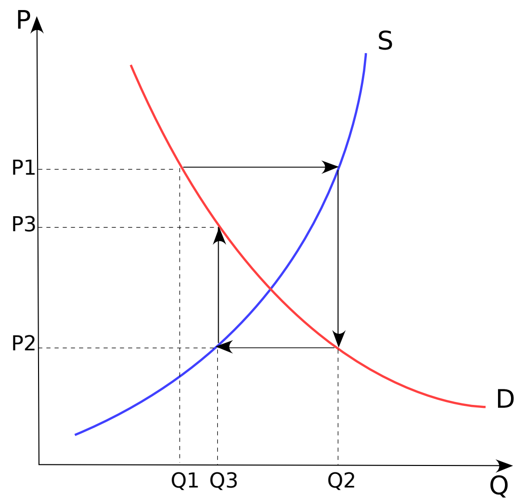


Fig. 1: Reproduced from `https://en.wikipedia.org/wiki/Cobweb_model`. Supply price (S) and demand price (d) are plotted as a function of quantity (Q) and price (P).

For the research paper presented in this chapter, Cars Hommes (which also co-authored the paper) kindly provided us with data from a laboratory market (please refer to the paper presented in the next section for more details). We find an interesting signature in the phase when a bubble is building up: Plotting returns r_t versus the previous period return r_{t-1} reveals the following pattern; although the fair price (which would be an equilibrium price) is already exceeded, returns r_t stay not only positive, they continue to grow, see Figure 2.

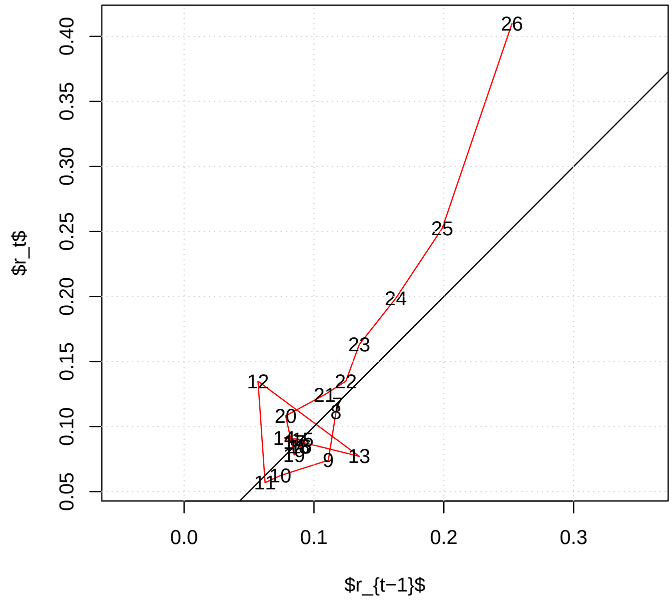


Fig. 2: Returns r_t versus previous period returns r_{t-1}. Reproduced from Hüsler et al. (2012). Note that the growing growth rate implies faster than exponential price acceleration.

Classical finance struggles to find an explanation for such a pattern. Prices in models should not deviate systematically from the equilibrium price. Phenomenological approaches have it easier: in physics and psychology such patterns are well known and can be explained by positive feedback or herding.

3.2 Paper

On the following pages, we present the paper in full length. The paper is available as:

A. Hüsler, D. Sornette, and C. H. Hommes. Super-exponential bubbles in lab experiments: evidence for anchoring over-optimistic expectations on price. May 2012

Super-exponential bubbles in lab experiments: evidence for anchoring over-optimistic expectations on price

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Abstract

We analyze a controlled price formation experiment in the laboratory that shows evidence for bubbles. We calibrate two models that demonstrate with high statistical significance that these laboratory bubbles have a tendency to grow faster than exponential due to positive feedback. We show that the positive feedback operates by traders continuously upgrading their over-optimistic expectations of future returns based on past prices rather than on realized returns.

Keywords: rational expectations; financial bubbles; speculation; anchoring; laboratory experiments; behavioral model; super-exponential growth; positive feedback; behavioral expectations

JEL: C92; D84; G12

Highlights:

- We offer an interpretation of lab experiments that exhibit financial bubbles.
- We show that bubbles in controlled experiments can grow faster than exponential.
- We find traders anchor expectations more on price than on returns in these bubbles.

1 Introduction

Bubbles, defined as significant persistent deviations from fundamental value, express one of the most paradoxical behaviors of real financial markets. Here, we analyze the dynamics of bubbles in a laboratory market (Hommes et al. (2008)) and focus on the regimes of strong deviations from the known fundamental values, which we refer to as the bubble regimes. Because this data is from a controlled environment, we can exclude exogenous influences such as news or private information. We show that a model with exponential growth, corresponding to a constant rate of returns, cannot account for the observed transient explosive price increases. Models that incorporate positive feedback leading to faster-than-exponential growth are found to better describe the data.

Research on financial bubbles has a rich literature (see e.g. Kaizoji and Sornette (2010) for a recent review) aiming at explaining the origin of bubbles, their persistence and other properties. The theoretical literature has classified different type of bubbles. For instance, Blanchard (1979) and Blanchard and Watson (1982) introduced rational expectation (RE) bubbles, i.e., bubbles that appear in the presence of rational investors who are willing to earn the large returns offered during the duration of the bubble as a remuneration for the risk that the bubble ends in a crash. Tirole (1982) argued that heterogeneous beliefs among traders is necessary for bubbles to develop. de Long et al. (1990) demonstrated that introducing noise traders in a universe of rational speculators can amplify the size and duration of bubbles. Brock and Hommes (1998) showed that endogenous switching between heterogeneous expectations rules, driven by their recent relative performance, generates bubble and crash dynamics of asset prices. Abreu and Brunnermeier (2003) explained the persistence of bubbles by the heterogeneous diagnostics of rational agents concerning the start time of the bubble, which leads to a lack of synchronization of their shorting of the underlying asset, and therefore prevents them from stopping the bubble to blossom. Lux and Sornette (2002) showed that the multiplicative stochastic process proposed by Blanchard and Watson (1982), together with the no-arbitrage condition, predicts a tail exponent of the distribution of returns smaller than 1, which is incompatible with empirical observations. Johansen et al. (1999) and Johansen et al. (2000) thus extended the Blanchard-Watson (1982) model of RE bubbles by proposing models for the crash hazard rate that exhibit critical bifurcation points reflecting the imitation and herding behavior of the noise traders. Gallegati et al. (2011) presented a model of bubbles and crashes, where crashes occur after a period of financial distress. Hommes (2006) reviewed behavioral models of bubbles with fundamentalists trading against chartists.

Jarrow et al. (2007), Jarrow et al. (2010) and Jarrow et al. (2011) developed local martingale models of bubbles within the arbitrage-free martingale pricing technology that underlies option pricing theory, based on the assumption that bubbles come together with (or are defined by) a volatility growing faster than linearly with the underlying price. But Andersen and Sornette (2004), among others, have shown that some (and perhaps most) bubbles are not associated with an increase in volatility. In particular,Bates (1991) documented that the famous worldwide October 1987 crash occurred at a minimum of the implied volatility, at least in the US. Gürkaynak (2008) surveyed econometric tests of asset price bubbles and showed that the econometric detection of asset price bubbles cannot be achieved with a satisfactory degree of certainty: for each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble.

The present paper represents the first detailed quantitative calibration of simple models with positive feedback that unambiguously demonstrates the existence of positive feedback mechanisms and super-exponential bubbles in the price formation process. It thus provides support within controlled laboratory set-ups for the empirical evidence presented by Sornette et al. on historical financial bubbles (see Jiang et al. (2010) and Kaizoji and Sornette (2010)[1] and references therein for an overview).

2 Material and methods

In the experiment of Hommes et al. (2008), participants ("traders") were asked to forecast the price of a single asset in every turn. The price of the asset evolves with the equation,

$$p_t = \frac{1}{1+r}\left[\frac{1}{H}\sum_{h=1}^{H} p^h_{t+1} + D\right], \tag{1}$$

where the market price p_t at time t is given as an average of the $H = 6$ traders discounted price expectations; $r = 5\%$ is the interest rate, p^h_{t+1} is the estimate of trader h for the price for period $t+1$ based on information up to time $t-1$ and $D = 3.00$ is the dividend. Hence, today's price p_t is simply the average of the current value of the traders' expectations for tomorrow p^h_{t+1}. Note that the traders have to make a two period forecast; for their forecast p^h_{t+1}, only the prices up to time $t-1$ are available.

Traders are given the parameters above (but not the price forming Equation 1 itself) and are rewarded according to their prediction accuracy[2]. The fundamental/equilibrium price p^f (which traders could calculate) is 60[3]. In our analysis, we focus on the realized price p_t and not on the traders' individual estimates p^h_t.

Notwithstanding the existence of a clearly defined market price formula, this experiment is remarkable in reporting realized prices that are quite loosely tied to the fundamental value, because traders are rewarded more by correctly foreseeing the other traders' forecasts than by correctly calculating the fundamental price p_f. Moreover, traders are allowed to estimate the asset value in a large price range between 0 and 1000 (where the upper bound is more than 16× the fundamental value p_f).

[1] An extended version is available at http://arxiv.org/abs/0812.2449
[2] The reward is proportional to the quadratic scoring rule $\max\{(1300 - 1300/49(p_t - p^h_t))^2, 0\}$
[3] $p^f = D/r = 3.00/5\% = 60$

3 Theory/calculation

3.1 Rational Bubble

Hommes et al. (2008) discussed the rational bubble

$$p_t = (1+\hat{r})^t a_1 + b_1 \ , \tag{2}$$

where a_1 is a positive constant. This process fulfills the "self-confirming" nature of rational expectations if the assumed interest rate $\hat{r}$ equals the interest rate r from Equation 1 and b_1 equals the fundamental value of $p^f = 60$[4]. In fact, Hommes et al. (2008) found that traders do use an interest rate $\hat{r}$ significantly larger than r in four of the six groups and hence their expectations are no longer rational (see section 4). Furthermore, the growth rate $\hat{r}$ is not constant, but is increasing as we will see later.

Todd and Gigerenzer (2000) argued that "decision-making agents in the real world must arrive at their inferences using realistic amounts of time, information, and computational resources. [..] The most important aspects of an agent's environment are often created by the other agents it interacts with." Moreover, Tversky and Kahneman (1974) presented three heuristics that are employed in making judgments under uncertainty. For our purposes, the heuristic that is relevant to interpret the groups' behaviors is the "adjustment from an *anchor*, which is usually employed in numerical prediction when a relevant value is available. These heuristics are highly economical and usually effective, but they lead to systematic and predictable errors." (Emphasis is ours).

In the rest of this section, we are presenting two models in which traders anchor their forecasts on (1) price or (2) return. Both models have in common that they can generate price growth that is significantly faster than exponential (as observed in the data) and generalize the rational bubble of Equation 2.

3.2 Anchoring on Price

Generalizing the constant growth generated by Equation 2, we specify a model which allows faster or slower than exponential growth. The growth rate $\log(\bar{p}_t/\bar{p}_{t-1})$ can be explained by the excess price $\bar{p}_{t-1}$ (which is the difference between the observed price p_t and the fundamental price p^f) plus a constant:

$$\log\left(\frac{\bar{p}_t}{\bar{p}_{t-1}}\right) = a_2 + b_2 \bar{p}_{t-1}. \tag{3}$$

$a_2 > 0$ and $b_2 > 0$ would imply faster than exponential growth i.e. the growth rate grows itself. For $b_2 = 0$, we recover the exponential growth (equivalent to the rational bubble Equation 2 with $r = \hat{r}$). We will see below that b_2 is typically significantly larger than zero, indicating faster than exponential growth and positive feedback on the price.

One justification for the functional form (Equation 3) is that anchoring on price is commonly used in technical trading. One of many patterns used are support and resistance levels which is nothing else but anchoring on price. Although in violation with the efficient markets hypothesis, Lo et al. (2000) studied technical trading rules and found "practical value" for such technical rules.

3.3 Anchoring on Return

Alternatively, we check if the growth rate can be explained by the excess log-return $\log(\bar{p}_t/\bar{p}_{t-1})$ following the following process

$$\log\left(\frac{\bar{p}_{t+1}}{\bar{p}_t}\right) = a_3 + b_3 \log\left(\frac{\bar{p}_t}{\bar{p}_{t-1}}\right). \tag{4}$$

The conditions that $a_3 > 0$ and $b_3 > 0$ implies again faster than exponential growth of the excess price $\bar{p}_t$ and positive feedback from past returns. This model can be interpreted as a second order iteration or adaptive form of the exponential growth.

[4] For a rational bubble, we have $\mathbb{E}_t[p_{t+1} + D] = c(1+r)^{t+1} + p^f(1+r) = (1+r)p_t$.

4 Results

In this section, we estimate the parameters of the two processes and check for the statistical significance of b_2 and b_3 that express a positive feedback of price (Equation 3) or of return (Equation 4) onto future returns. In particular, we are interested in the lower 95% confidence interval for the null hypothesis that b_2 and b_3 are zero, to check for significant deviations that can confirm or not that price growth is indeed significantly faster than exponential (which is the situation corresponding to b_2 and b_3 greater than zero). As the two models can be run over a multitude of different start and end points, we present the results in graphical form instead of tables to provide better insight.

Hommes et al. (2008) identified bubbles in five out of the six groups. Group one shows a somehow erratic price trajectory and no bubbles. Groups five and six show some tendency towards bubbles, but the time horizon is too short for our analysis to get significant results. Moreover, Hommes et al. (2008) found that the bubble in group five is compatible with the hypothesis of a rational bubble (Equation 2). Hence, we focus on group number two, three and four.

[Table 1 about here.]

4.1 Group 2

The bubble period identified by Hommes et al. (2008) runs from 7 – 26. Figure 1 shows that the price becomes larger than the fundamental value p^f at $t = 7$. Checking the returns vs. past returns in Figure 2, we see that the bubble initially grows approximately exponentially ($r_t \approx r_{t-1}$) as confirmed by the positions of the points along the diagonal. Later, at around $t = 14$, the returns become monotonous increasing (i.e. prices become faster than exponential growth) and are plotted above the diagonal. This is also confirmed by Figure 3 where, for low starting and ending values of the analyzing time window, the parameters estimated for Equation 3 are not distinguishable from exponential growth since the parameter b_2 is not significantly different from zero. However, towards the middle and the end of the bubble, the growth rate accelerates (b_2 becomes significantly larger then zero) before the bubble finally bursts. The parameter a_2 is positive over the whole analysis window (lower left panel) and almost always significantly larger than zero (lower right panel). The upper panels shows that b_2 (for low start and ending values) is not significant different from zero, but, later in the bubble, b_2 becomes positive (top left panel) and even significant positive (upper right panel).

Checking for the existence of feedback from past returns in Figure 4, we find that Equation 4 describes less accurately the experimental results; although the parameters a_3 and b_3 are both positive (left panels), the time windows where the parameters are both significantly positive (right panels) is relatively small (only for starting values $t = 7$ and $t = 8$).

Hence, in summary, the bubble in group 2 does not only grow significantly faster then exponential in the end phase, but traders seem to anchor their expectations more on price rather than on return.

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

4.2 Group 3

Group 3 (over the time horizon from 7 – 29) is the longest bubble among the six groups. From Figure 5 (which is plotted on log scale), the bubble seems to grow initially only exponentially (visible as a straight line in the plot), which is also confirmed by Figure 6, which shows that the growth rate is initially constant. At around $t = 20$, growth accelerates. This observation is also confirmed by the analysis of Equation 3, where a_2 is significant for almost all analysis windows. But, the positive feedback of the price on the growth rate embodied by b_2 becomes only significant in the later phase of the bubble. Analyzing this group for the possible existence of anchoring on return (Equation 4) in Figure 8, we find that the results are less clear cut: although a_3 and b_3 are positive, a_3 is not significantly different form zero for starting values after $t = 10$. Hence, we conclude that Equation 4 does not appropriately describe the price and traders tend to anchor their expectations on price rather than on return.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

4.3 Group 4

As can be seen from Figure 9, the bubble formed over the time window 7 – 29 is briefly disrupted by the intervention of trader number 6[5]. This can also be seen in Figure 6 where we plot the returns. Between $t = 7$ and $t = 13$, we have more or less a cobweb and then, starting with $t = 14$, the growth rate increases and a bubble is formed. For anchoring on price, we see in Figure 11 very strong evidence for faster then exponential growth; a_2 and b_2 are both significantly positive. Again, for very early and small analysis windows, only a_2 is positive, indicating exponential growth in the initial phase of the bubble. The analysis for Equation 4 in Figure 12 is less clear, but the signal for jointly positive a_3 and b_3 is relatively small (only for two smallest starting values), indicating that traders prefer to anchor their predictions on price and not on return.

[Figure 9 about here.]

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

5 Discussion

It is remarkable that we find many time windows where we can clearly reject the hypothesis of exponential growth and find evidence for faster than exponential growth. This is even more remarkable when taking into account that the data suffers some limitations which make detection of faster than exponential growth more difficult.

Price ceiling: Although the price is allowed to fluctuate over a relatively large range, it is capped at a maximum value of 1000. Because traders know and can anticipate this, we would expect traders to level off their price expectation much before reaching this upper bound. This turns out not to be the case.

Stable equilibrium price: The pricing formula Equation 1 assumes a fundamental value of 60 and thus biases the price towards this value. Even if all traders give an estimate of 1000, the realized market price from Equation 1 would be $(1000 + 3)/1.05 \approx 955$, i.e. the price is artificially deflated by almost 45 monetary units.

Mis-trades: There seems to be a few instances where trades' estimates are off by an order magnitude (i.e. some traders seem to fail to place the decimal point at the correct digit at some times).

Short data horizon: Although the experiments run over a time horizon of 50 time-steps; the bubbles appear in much shorter time, leaving relatively few points to estimate tight confidence intervals.

Heemeijer et al. (2009) ran a comparable experiment with a slightly different price forming mechanisms and focusing on the traders' individual price forecasts. Further, agents' predictions had to lie in a relatively narrow range (0 – 100) allowing relatively small deviations from the fundamental price compared to the data that we have analyzed here. In contrast to Heemeijer et al. (2009) who analyzed the data along the dimensions of trend following, fundamentalism and obstinacy, we focus on non-linear feedback

[5] The prediction of trader number six at time point $t = 10$ seems to be off by an order of magnitude as he has misplaced the decimal.

of realized price and return on the price growth rate. Anufriev and Hommes (2012) have fitted a heuristics switching model to a positive feedback asset pricing experiment in the presence of a fundamental robot trader, whose trading drives the price back towards its fundamental value. As a consequence, long lasting bubbles do not arise in that setting, but rather asset prices oscillate around the fundamental and individuals switch between different simple forecasting heuristics such as adaptive expectations and trend following rules.

Tirole (1982) noted that "[..] speculation relies on inconsistent plans and is ruled out by rational expectations." However, in the experiments of Hommes et al. (2008) that we analyze here, traders are rewarded, not on the basis of how well they predict the fundamental value of the assets they buy but rather, on the accuracy of their prediction of the realized price itself, similarly to real financial markets. Traders also do not need to invest their wealth into an asset, they do not worry about price fluctuations or care about supply & demand, which lead them to "ride the bubble" (see Abreu and Brunnermeier (2003), de Long et al. (1990) and De Long et al. (1990)). They rather give a forecast as in a Keynesian beauty contest Keynes (1936), where traders need to synchronize their beliefs. Such self-confirming predictions can easily lead to herding, in particular in situations where the fundamental value is not directly observable or when strong disagreement on the fundamentals between the traders occurs, such as in the dot-com bubbles, see Shiller (2005) for instance.

6 Conclusions

There have been many reports of super-exponential behavior in financial markets in a literature inspired by the dynamics of positive feedback leading to finite-time singularities in natural and physical systems (see for instance Johansen and Sornette (2001) and Sornette (2004) and references therein). However, the challenge has been and is still to confirm with more and more statistical evidence that the very noisy financial returns do contain a significant positive feedback component during some bubbles regimes. In the present paper, by analyzing a controlled experiments in the laboratory, we have the luxury of working with a low noise data set. With this advantage, we have presented the first detailed quantitative calibration of simple models with positive feedback that unambiguously demonstrate the existence of positive feedback mechanisms in the price formation process of controlled experimental financial markets.

Appendix

Faster than exponential growth means that there is a positive feedback loop, or as Andreassen and Kraus (1990) noted that "[..] subjects were more likely [..] to buy as prices rose [..]". The table down-below illustrates the difference between constant growth and positive feedback. Note that the prices in the two bubbles can be indistinguishable in the early phase of the bubble.

[Table 2 about here.]

[Figure 13 about here.]

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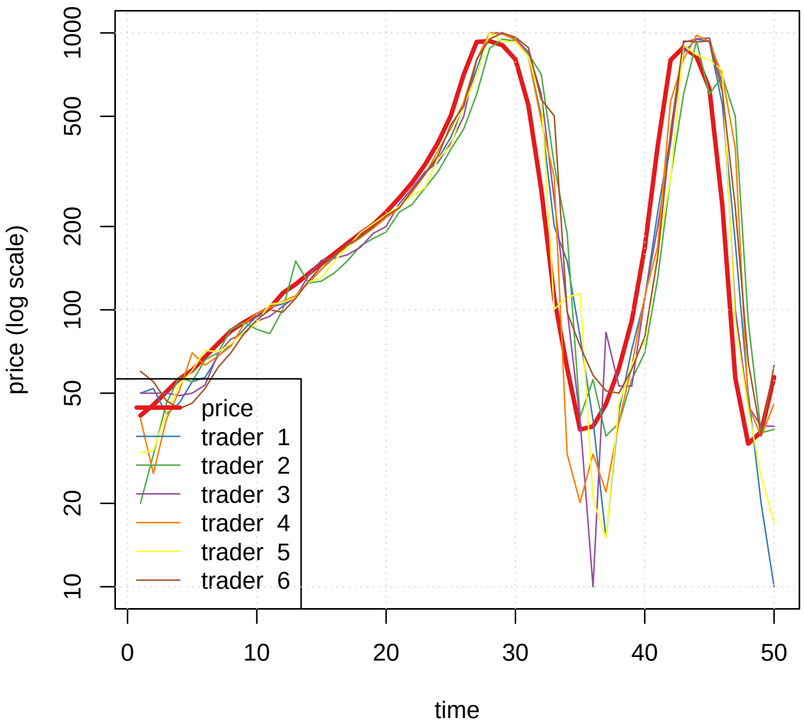

Figure 1: Price and traders' estimate over time for group 2. Note that traders' estimates p^h are for time $t+1$ and are used to form the price p_t at time t, i.e. $p_t = 1/H(\sum_h p^h_{t+1} + D)/(1+r)$.

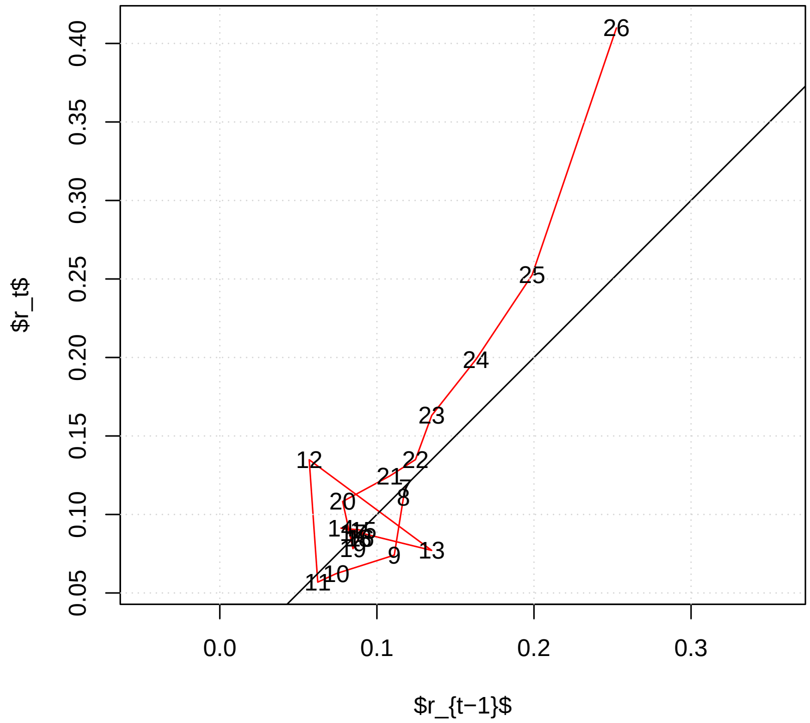

Figure 2: Next period returns r_{t+1} versus current returns r_t for group 2. Points on the diagonal correspond to constant growth ($r_{t+1} = r_t$), points above the diagonal ($r_{t+1} > r_t$) correspond to accelerating growth. Note that returns are defined as discrete returns, i.e. $r_{t+1} := (p_{t+1}/p_t) - 1$.

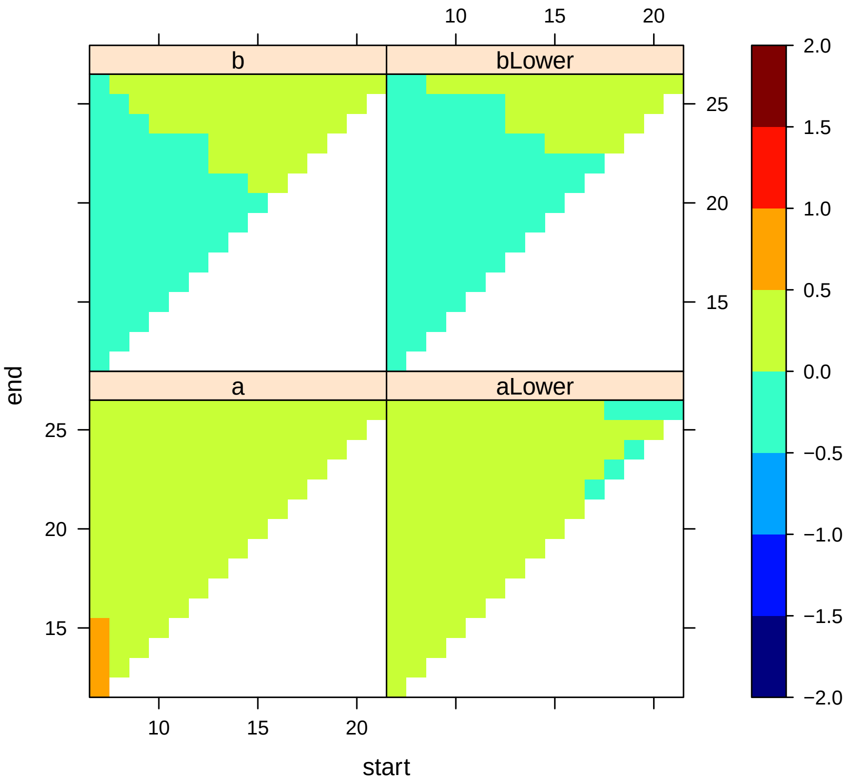

Figure 3: Parameter estimate of Equation 3 over the time interval [start, end] for group 2. The x-axis corresponds to the start point and the y-axis to the end point of the analyzed time window. The bar on the right gives the values of the parameters in color code, according to the indicated scale. aLower and bLower correspond to the lower 95% confidence level of a_2 and b_2 respectively of Equation 3. Note that b_2 is around 0 for small starting and end values implying exponential growth in the initial phase of the bubble. We observe a rather large domain in the parameter range describing the start time and end time of the window of calibration for which the parameter b_2 is positive at the 95% confidence level.

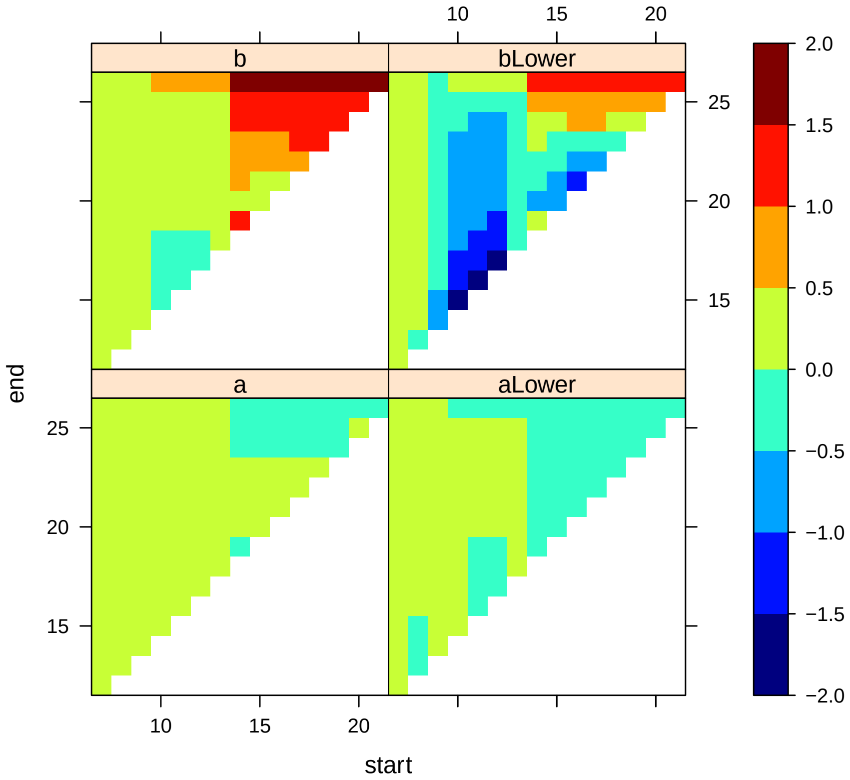

Figure 4: Parameter estimate of Equation 4 over the time interval [time, start] for group 2. The x-axis corresponds to the start point and the y-axis to the end point of the analyzed time window. aLower and bLower correspond to the lower 95% confidence level for a_3 and b_3 respectively of Equation 4. Note that the domain where a_3 and b_3 are both significantly larger than zero is restricted to the earliest two starting points.

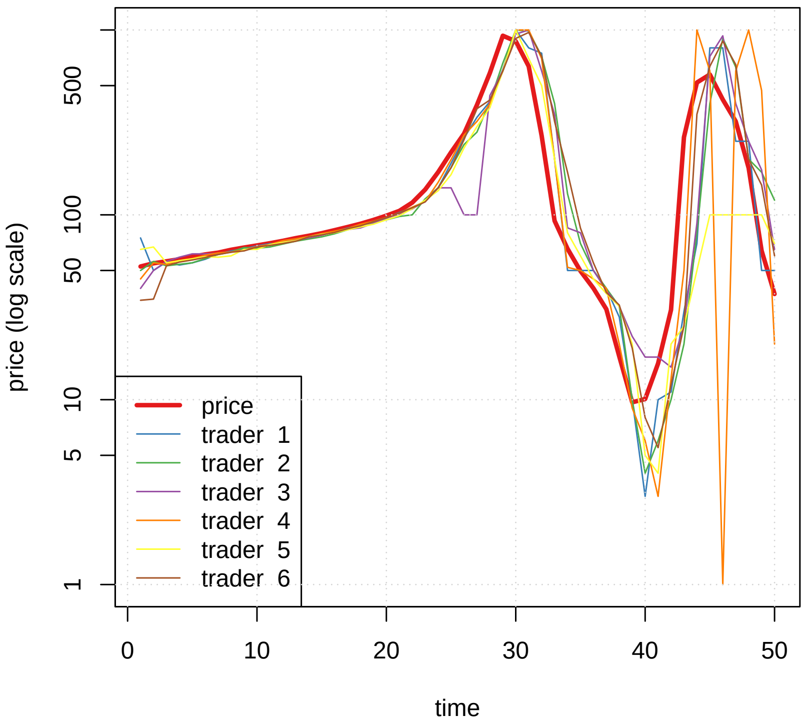

Figure 5: Price and traders' estimate over time for group 3. Same representation as Figure 1.

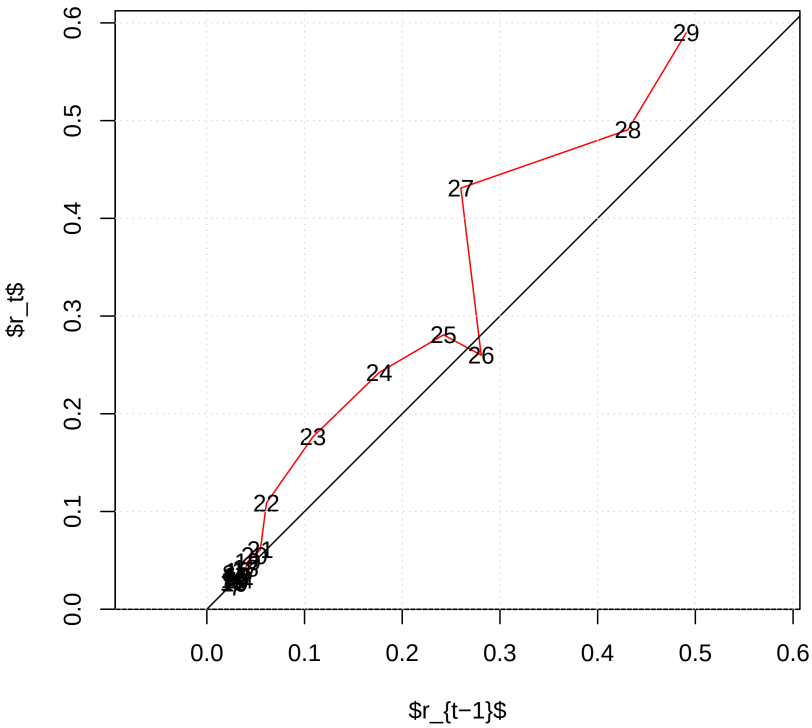

Figure 6: Next period returns r_{t+1} versus current returns r_t for group 3. Same representation as Figure 2.

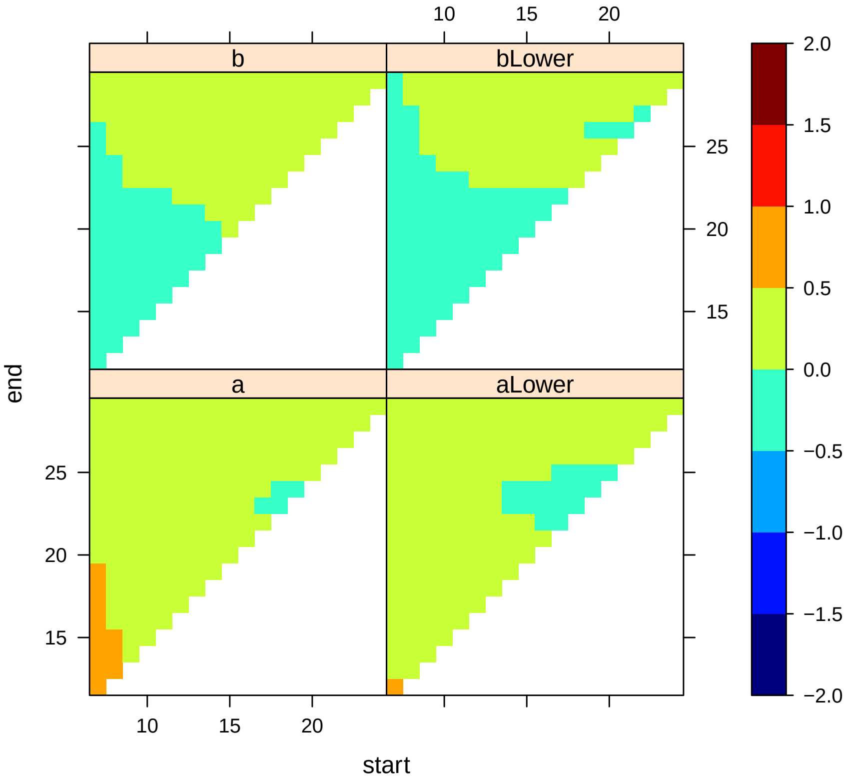

Figure 7: Parameter estimate of Equation 3 over the time interval [start, end] for group 3. Same representation as Figure 3.

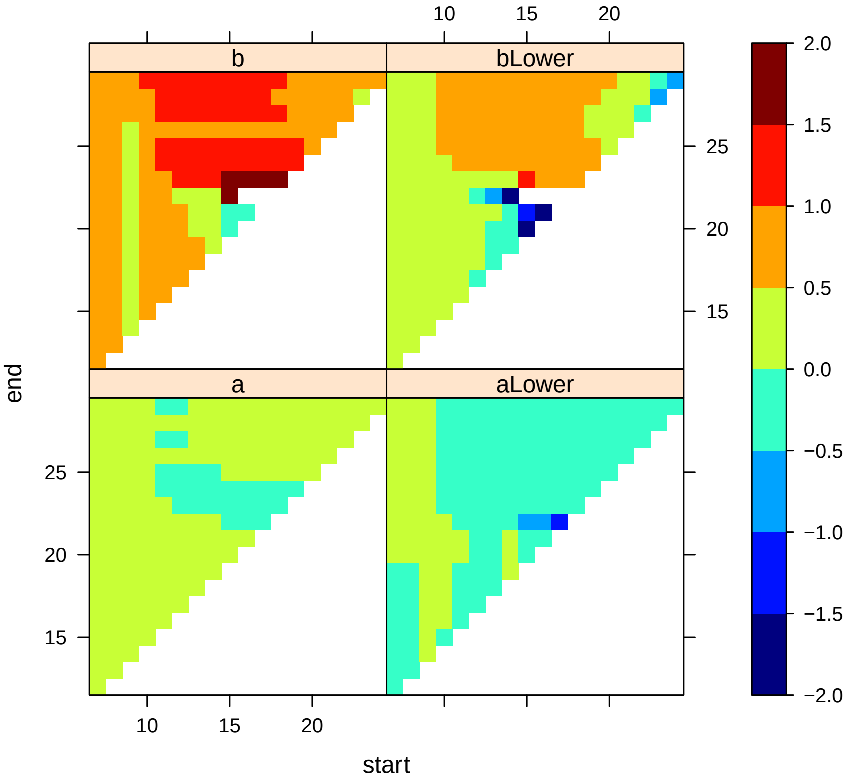

Figure 8: Parameter estimate of Equation 4 over the time interval [time, start] for group 3. Same representation as Figure 4.

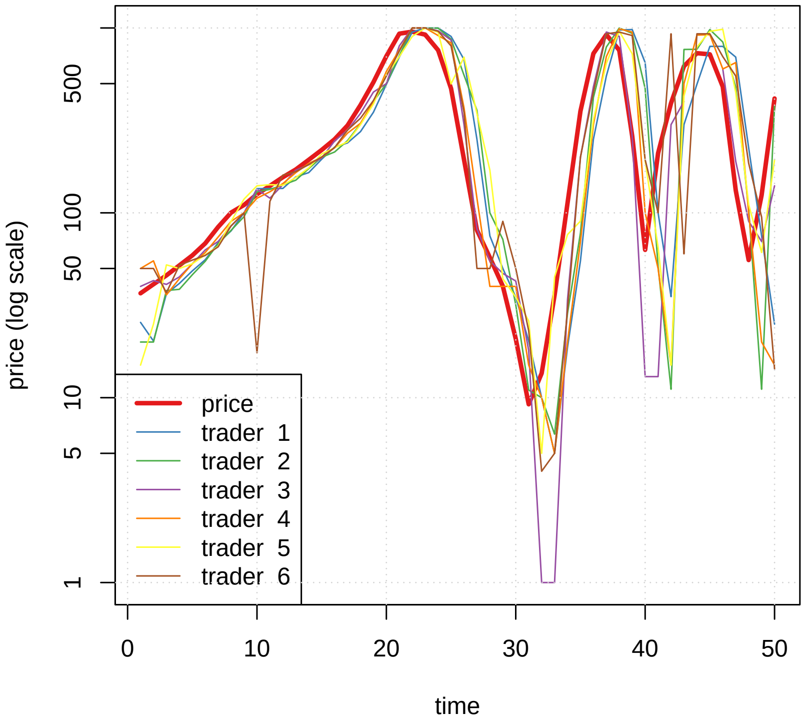

Figure 9: Price and traders' estimate over time for group 4. Same representation as Figure 1.

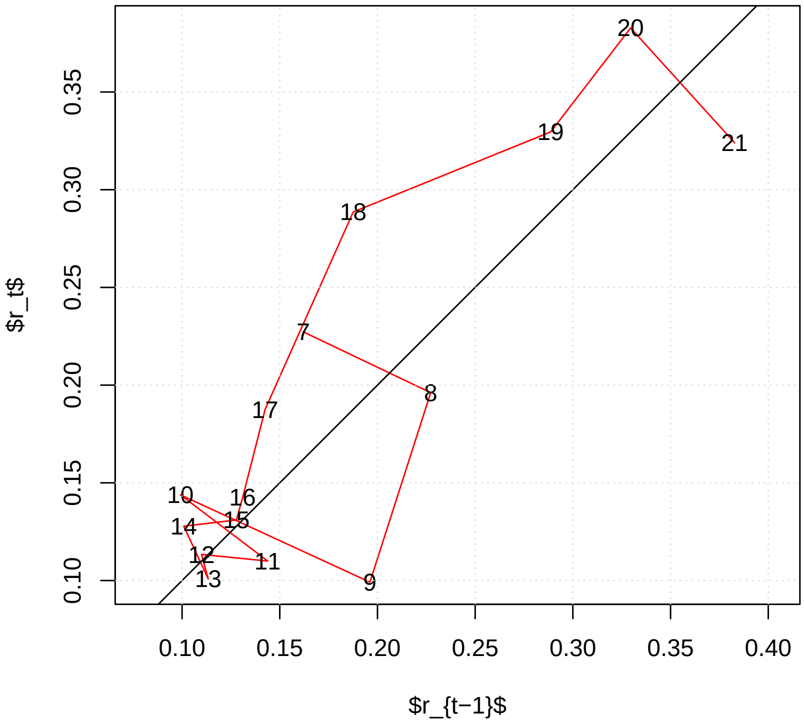

Figure 10: Next period returns r_{t+1} versus current returns r_t for group 4. Same representation as Figure 2.

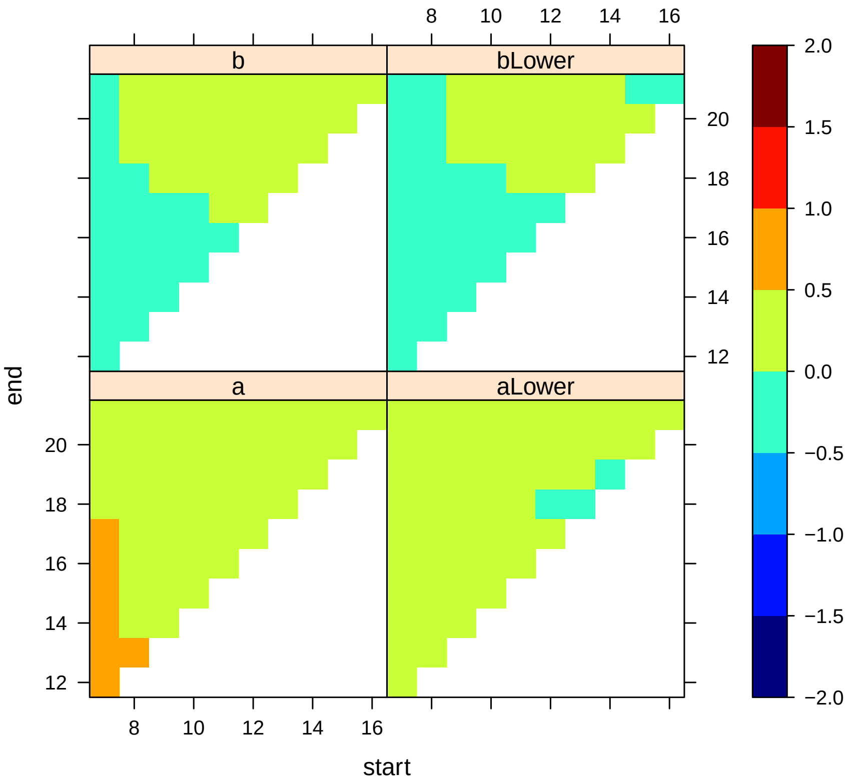

Figure 11: Parameter estimate of Equation 3 over the time interval [start, end] for group 4. Same representation as Figure 3.

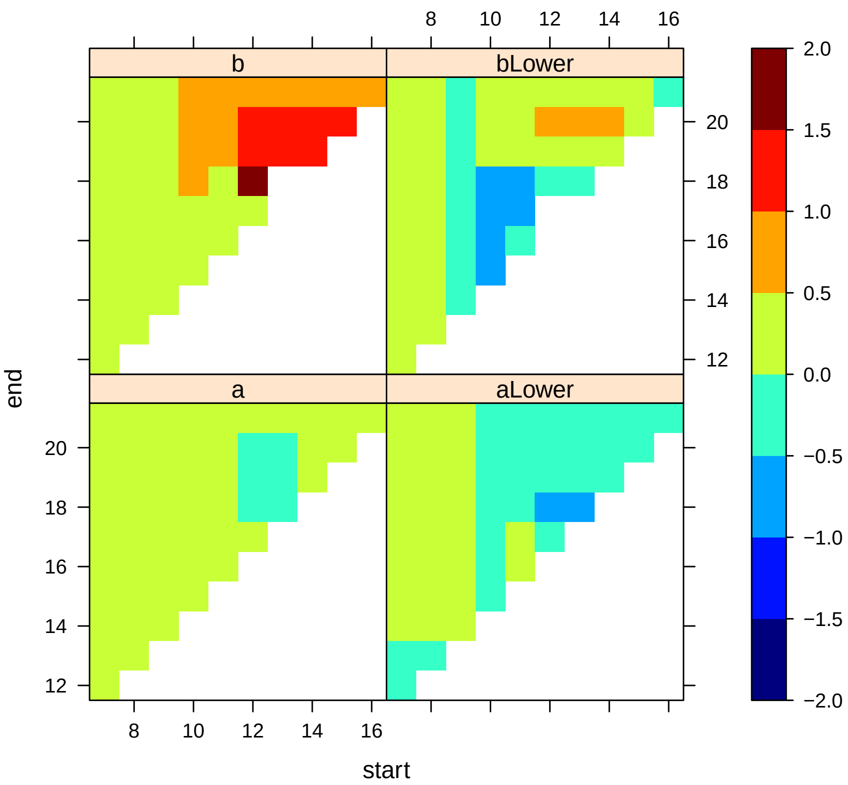

Figure 12: Parameter estimate of Equation 4 over the time interval [time, start] for group 4. Same representation as Figure 4.

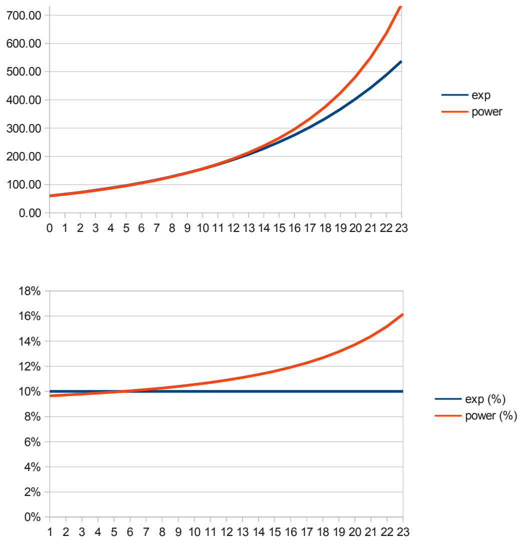

Figure 13: Graphical representation of Table 2. Top panel: prices. Bottom panel: returns.

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Group	Time window	Description	Classification
1	NA	erratic price trajectory	—
2	7 – 26	speculative bubble	anchoring on price
3	7 – 29	speculative bubble	anchoring on price
4	7 – 21	speculative bubble	anchoring on price
5	29 – 37	rational bubble	—
6	23 – 29	speculative bubble	(too short for analysis)

Table 1: Overview of bubbles reproduced from Hommes et al. (2008) with our own classification.

	$\log(\bar{p}_t/\bar{p}_{t-1}) = a_1$		$\log(\bar{p}_t/\bar{p}_{t-1}) = a_2 + b_2\bar{p}_{t-1}$	
t	$\bar{p}_t$	%	$\bar{p}_t$	%
0	60.00	–	60.00	–
1	66.00	10%	65.79	10%
2	72.60	10%	72.19	10%
3	79.86	10%	79.26	10%
4	87.85	10%	87.08	10%
5	96.63	10%	95.74	10%
6	106.29	10%	105.36	10%
7	116.92	10%	116.06	10%
8	128.62	10%	127.99	10%
9	141.48	10%	141.30	10%
10	155.62	10%	156.21	11%
11	171.19	10%	172.95	11%
12	188.31	10%	191.80	11%
13	207.14	10%	213.11	11%
14	227.85	10%	237.30	11%
15	250.63	10%	264.87	12%
16	275.70	10%	296.45	12%
17	303.27	10%	332.86	12%
18	333.60	10%	375.09	13%
19	366.95	10%	424.48	13%
20	403.65	10%	482.74	14%
21	444.01	10%	552.22	14%
22	488.42	10%	636.09	15%
23	537.26	10%	738.87	16%

Table 2: Table illustrating the difference between exponential growth ($a_1 = \log(1.1) \approx 0.095$, second column) and positive feedback by price on future returns ($a_2 = \log(1.09) \approx 0.086$, $b_2 = 0.0001$, fourth column). We let the bubbles start at $\bar{p}_t = 60 = 120 - 60 = p_t - p^f$. With the parameter above, the excess price $\bar{p}_t$ grows initially at around 10% at each time step. In the early phase, the prices grow approximately exponentially (the exponential growth is actually slightly faster). At time step $t = 10$, the bubble with positive feedback of the price on future returns overtakes the exponential growth benchmark and the growth rate start to accelerate.

3.3 Discussion & Perspective

As pointed out in the paper, this is one of the first studies, confirming super-exponential price growth in laboratory experiments. The presented laboratory experiments give us the luxury to rule out exogenous effects like news, moreover, the noise component in the signal is very small. The simplest model producing super exponential growth is a simple power-law:

$$p(t) = A + B(t_c - t)^{\beta}. \tag{3.1}$$

This model will be discussed in more detail in the next chapter. In the remainder of this chapter we briefly discuss two classes of models which generate super-exponential growth.

One of the remarkable models exploiting faster than exponential growth is Corsi and Sornette (2011) which extends the GARCH family with a finite time singularity. The model is such that as soon as the price component grows faster than exponential, an unsustainable state is identified and a crash alarm is issued, see Figure 3 for an illustration.

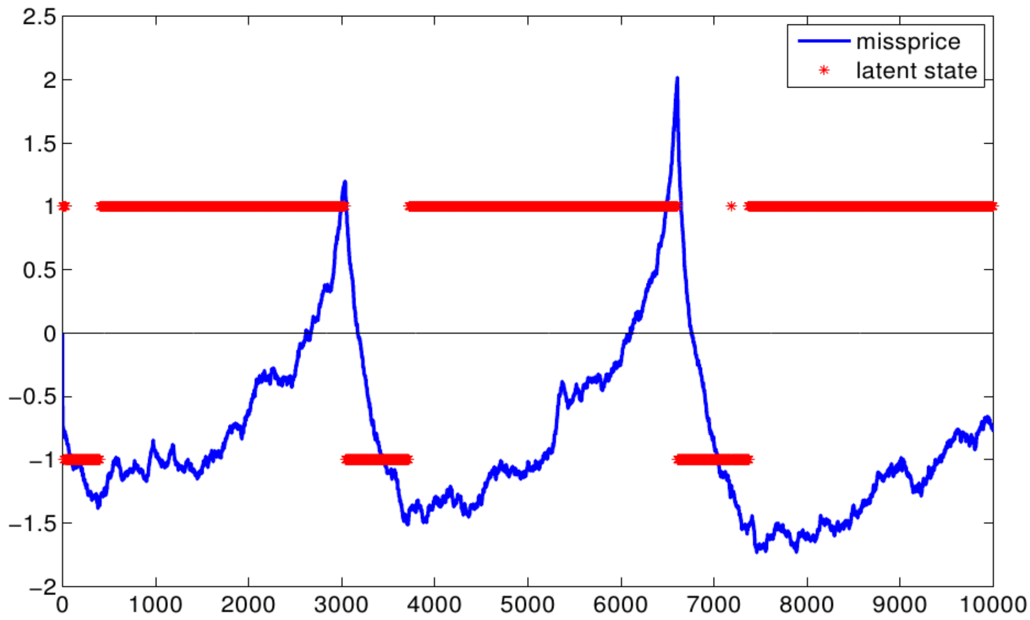

Fig. 3: FTS-GARCH model. As soon as prices are identified to grow significantly faster than exponential (latent state = 1), the FTS-GARCH model identifies and unsustainable state. Reprinted from Corsi and Sornette (2011).

Another class are the LPPL (log-periodic power law) models which consists of power-law decorated with log-periodic oscillation:

$$p(t) = A + B(t_c - t)^{\beta} + C(t_c - t)^{\beta} \cos(\omega \ln(t_c - t) - \phi) \ . \tag{3.2}$$

The application and quality of these models are extensively studied by Sornette et. al, see for example Sornette (2002) and in particular Sornette (2006) for a discussion of the technical underling concepts like fractals, phase transition, renormalization, etc. As pointed out in a case study (Sornette and Johansen, 2001), the model describes very accurately the hongkongese Hang-Seng Index (Figure 4). More recent experimental studies include the "Financial Bubble experiment" Sornette and Woodard (2010); Sornette et al. (2010). Finally, Sornette et al. (2011) discusses some of the caveats of the LPPL models.

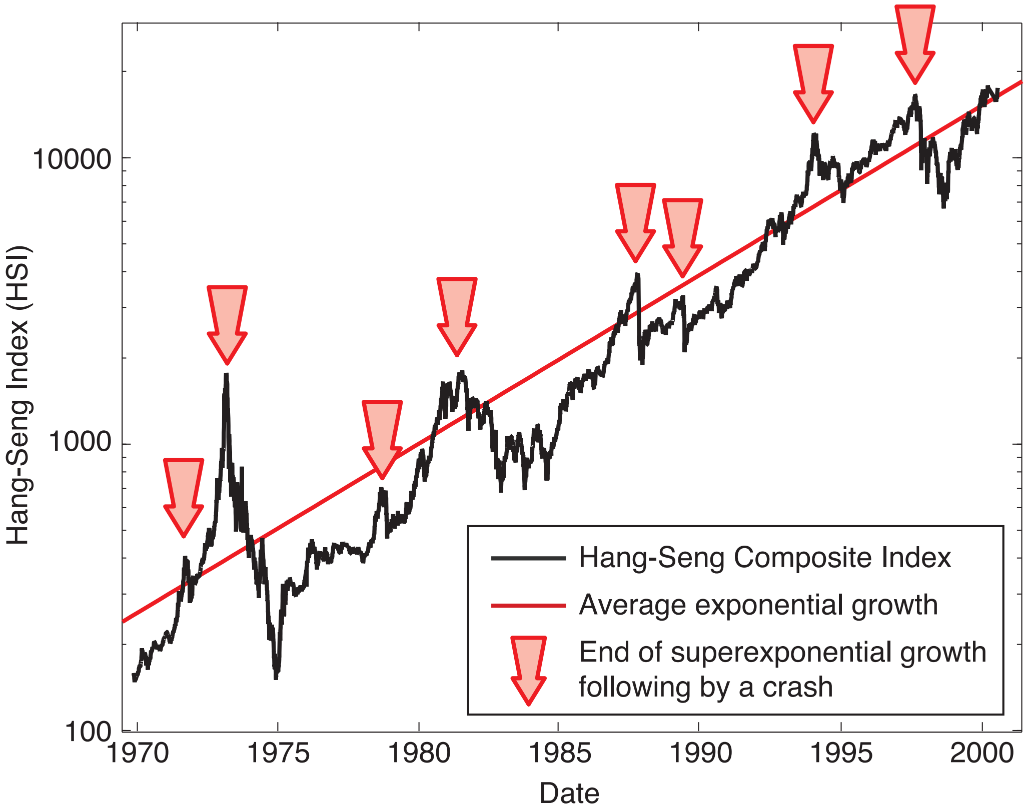

Fig. 4: Reprinted from Sornette and Johansen (2001). Note that the straight line would correspond to exponential growth. The shown time period illustrates that the Hang-Seng has a tendency to grow faster than exponential for some time (bubble phase) and then crashes.

3.4 Supplement: An Information Theory Approach

Heretofore, we have ignored one difficulty; in contrast to the standard financial literature, equation 3 in the previous presented paper, as well as in Sornette et. al's papers published on the calibration of financial bubbles, are calibrated on prices instead of returns. To understand why this difference is important, we have to understand that if returns are for example normally distributed (a standard assumption in finance), then the corresponding price trajectory follows a geometric Brownian motion. However, if the price follows a Brownian motion, the mean and standard deviation of the price diverge (they explode for progressing time). Hence, financial models are almost always calibrated on returns which are assumed to be stationary.

A F_t distributed time series X_t is defined as *stationary* if for an arbitrary τ, $X_{t+\tau} \sim F_t$, which means that the distribution of X_t is the same as the distribution of $X_{t+\tau}$, for any τ (for a comprehensive discussion, see Hamilton, 1993). Yule (1926) was among the first to point out "nonsense-correlations" between non-stationary time-series. The Nobel Prize laureates Engle and Granger (1987) illustrate the problem with the case of "spurious regression".

In this section, we will borrow some methods from information theory, with which we hoped to overcome some limitations in the estimation of parameters in non-stationary processes. Shannon (1948) laid the foundation of information theory just after World War II with his seminal paper: "A Mathematical Theory of Communication". At first sight, *(Shannon) entropy*,

$$H(X) := -\sum_{x \in X} p(x) \log p(x), \tag{3.3}$$

seems nothing else than a weighted average of the logarithm probability distribution. Further, *mutual information* is defined as

$$I(X,Y) = H(X) + H(Y) - H(X,Y), \tag{3.4}$$

where the mutual entropy $H(X,Y)$ is defined as

$$H(X,Y) := -\sum_{x \in X, y \in Y} p(x,y) \log p(x,y), \tag{3.5}$$

see Pierce (1980) and Cover and Thomas (2006) for more details.

Duncan (1970) presents the following theorem, which we tried to exploit in the parameter estimation for non-stationary time-series: Let Y and Z be two stochastic processes as follows:

$$dY_t = Z_t dt + dB_t, \tag{3.6}$$

where B_t denotes the Brownian motion, $t \in [0,1]$ and

$$\int \int Z_t^2 dP dt < \infty. \tag{3.7}$$

Then, the mutual information between the two processes can be calculated with the following equation:

$$I(Y,Z) = \frac{1}{2} E \int_0^1 (Z_u - \hat{Z}_u)^2 du \tag{3.8}$$

where $\hat{Z}_t := E\left[Z_t | Y_u, 0 \leq u \leq t\right]$.

Parameters are very often fitted by minimizing the least square (LQ) error. The right-hand-side of Equation 3.8 can be interpreted as sum of square errors. Hence, under the conditions of the theorem (Equation 3.6 and Equation 3.7) minimizing the sum of square errors is equivalent to minimizing the mutual information (Equation 3.8). The question arises whether mutual information can be used as an alternative to LQ to estimate parameters even in the non-stationary case. Can we induce a more general principle and estimate parameters by mutual information even if the conditions above are not necessarily fulfilled?

As an example, we have examined the time series

$$X_t := a + bX_{t-1} + \epsilon_t, \tag{3.9}$$

with $X_0 := 0$ and ϵ_t i.i.d. Gaussian noise. Our goal is to estimate the parameters a and b.

The "classical" least square (LQ) estimator for the parameters a and b has a closed form solution,

$$\hat{b} = cov(X_{t+1}, X_t)/var(X_t), \tag{3.10}$$

$$\hat{a} = \sum_{t=2,\dots,T-1} (X_{t+1} - X_t\hat{b})/T. \tag{3.11}$$

Note that the process X_t is only stationary for a certain range of parameters, for example when $|b| < 1$, we have an auto-regressive process. If the parameters do not lie in this range, the time series is no longer stationary and many problems in the estimation of the parameters arise; for example the variance (Equation 3.10) that we use to calculate $\hat{b}$ can not be defined in the limit of large times.

Can we estimate the parameters more reliably by minimizing the mutual information as in Equation 3.8? We set

$$Z_t := (\hat{a} + \hat{b}X_{t-1}) - X_t =: -\hat{\epsilon}_t, \tag{3.12}$$

$$Y_t := X_t. \tag{3.13}$$

Our study has shown that minimizing the mutual information between Equation 3.12 and Equation 3.13, i.e.

$$\{\hat{a}, \hat{b}\} = argmin_{\tilde{a},\tilde{b}} I(X_t, \hat{\epsilon}(\tilde{a}, \tilde{b})) \tag{3.14}$$

yields similar results to minimizing the sum of square errors to estimate a and b as long as the time series is stationary (i.e. when $|b| < 1$ for a long enough time series). It is slightly less efficient than the least square estimator (Equation 3.11 and Equation 3.11) because a good estimate of the mutual information requires more data points. Note also that Equation 3.7 in Duncan's theorem is fulfilled with $Z_t = -\hat{\epsilon}_t$ (Equation 3.12). However, the specification of Z_t and Y_t Duncan's theorem is not fulfilled (Equation 3.12 and Equation 3.13 can not be brought in form Equation 3.6). However, for two normal distributed variables V and W with correlation r, the mutual information can be rewritten as

$$I(V, W) = -\frac{1}{2}\log(1 - r^2) \tag{3.15}$$

(Kraskov et al., 2004). Hence, minimizing the mutual information in our example (Equation 3.14) is equivalent to minimizing the correlation between X and $\hat{\epsilon}$

which is almost equivalent to the LQ estimator [1]. However, for non-stationary time-series (i.e. for $|b| \geq 1$ in Equation 3.9) our estimator fails like (or performs even worse) than LQ estimator. Hence, a "mutual information" estimator works in principle when the time series is stationary, but fails in the more interesting case when the time series is non-stationary; it is not a valuable alternative to the least square estimator.

In our study, we have used the Kozachenko-Leonenko method (Kozachenko and Leonenko, 1987) to estimate mutual information, which uses the kNN (k nearest neighbors) approach to estimate the density of the empirical distribution function. By using the kNN methodology, we can be assured that the estimator is consistent (Devroye and Wagner, 1977) and unbiased (Kraskov et al., 2004; Luenberger and Woehrmann, 2007). In particular, we have used $k = \sqrt{T}$ (where T is the sample size) neighbors as suggested in Luenberger and Woehrmann (2007) for fastest possible convergence. We have used the implementation by Sales and Romualdi (2011), which is not only fast, but also considers the concerns by Kraskov et al..

Although our attempt has not worked, information theory has its place in applied finance; Darbellay and Wuertz (2000) demonstrate for example that the volatility is not helpful to predict returns in FX time series employing information theory. Maasoumi and Racine (2002) identify time pockets where systematic trading rules can be applied using entropy.

[1] For the LQ estimator, $cor(\hat{\epsilon}_t, \hat{X}_t) = 0$ holds because $\hat{\epsilon}_t \perp \hat{X}_t$.

4

Evidence for super-exponentially Accelerating Atmospheric Carbon Dioxide Growth

Prediction is very difficult, especially if it's about the future.
—Niels Bohr

4.1 Introductory comments

In the previous chapter, we have discussed some of the advantage and disadvantage of econophysics and how they can describe complex systems. In the paper presented in this chapter, we will use this methodology to describe the interaction between carbon dioxide content in the atmosphere and population. We find that carbon dioxide content in the atmosphere is growing faster than exponential, and are modeling this process with expressions of the form $(t_c - t)^\theta$ (i.e. a process with a singularity). The use of an applied approach seems natural as "phenomena such as catastrophes or phase transition ('system shifts') cannot be well understood within a linear modeling framework" (Helbing, 2012).

The "carrying capacity" of Earth and how many humans our planet can sustain is a long debated issue. Starting with Malthus (1798), scientists were most concerned with population growth and availability of resources. Malthus extrapolated that population will continue to increases exponentially, while resources, for example food production, did not. Verhulst (1838) refined the concept of Malthus; in his model, the logistic function, he extends the Malthusian model with a term accounting for finite carrying capacity avoiding unbounded population growth.

Verhulst's model can very accurately described the growth of, for example, bacteria. However, it is too simplistic to describe growth of human population; the actual carrying capacity of Earth is unknown and is thought to be limited rather by the absorptions of emissions than an absolute number of inhabitants. Further, the convergence does not need to be monotonously, and population might first overshoot and then undershoots an "idealized" carrying capacity.

Recently, less focus it put on population or carrying capacity itself, but on the interplay between population and resources, or pollution and absorbing capacity of Earth. A Nature paper "Approaching a state shift in Earth's biosphere" (Barnosky et al., 2012) has caught much attention and shows a myriad of illustrative examples on how to model the ecological system as complex systems, including population growth, ecosystems and climate change. Depending on the system, one can use phase transitions, singularities, bifurcations, etc; a comprehensive list can be found in Scheffer et al. (2009).

However, although Barnosky et al. (2012) call to forecast state shifts in order to prevent them, our ambition is not to make a faithful model about future developments. In the paper presented in the next section, we rather want to illustrate the interaction of non-linear processes, namely carbon dioxide and population.

4.2 Paper

On the following pages, we present the paper in full length. The paper is available as:

A. D. Hüsler and D. Sornette. Evidence for super-exponentially accelerating atmospheric carbon dioxide growth. July 2012

Evidence for super-exponentially accelerating atmospheric carbon dioxide growth

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July 18, 2012

Abstract

We analyze the growth rates of atmospheric carbon dioxide and human population by comparing the relative merits of two benchmark models, the exponential law and the finite-time-singular (FTS) power law. The later results from positive feedbacks, either direct or mediated by other dynamical variables, as shown in our presentation of a simple endogenous macroeconomic dynamical growth model. Our empirical calibrations confirm that the growth rate of human population has decelerated from super-exponential through 1960 to "just" exponential since, but with no sign of more deceleration. As for atmospheric carbon dioxide content, we find that it is at least exponentially increasing and most likely is characterized by an accelerating growth rate as of 2011, consistent with an unsustainable FTS power law regime announcing a drastic change of regime. The coexistence of quasi-exponential growth of human population with super-exponential growth of carbon dioxide content in the atmosphere is a diagnostic that, until now, improvements in carbon efficiency per unit of production worldwide has been dramatically insufficient.

1 Introduction

Today humanity uses the equivalent of 1.5 planets to provide the resources we use and absorb our waste. This means it now takes the Earth one year and six months to regenerate what we use in a year.[1] — Is humanity inevitably doomed?

During the 1960s, leaders were most concerned about human population growth (see for instance von Foerster et al. [1960]) and about depletion of energy resources (see for example the first report by the Club of Rome (Meadows [1972]) and its recent reassessment by Hall and Day [2009]). The

[1] http://www.footprintnetwork.org/en/index.php/GFN/page/world_footprint/

growth rate[2] of human population peaked in the late 1960s and although population is still growing, it is no longer the prime concern of policy leaders. This may be ill-advised as we show below that population growth is no longer decelerating anymore, but instead is on an exponential (proportional) growth trajectory.

More recently, scientists and politicians became aware of global warming (see Weart [2008] for a historic overview) due to or augmented by anthropogenic effects. We focus here on the undisputed fact that, due to the massive use of fossil energies the world economy emits, among many other products, large amounts of carbon dioxide go into the atmosphere. Part of this carbon dioxide is later absorbed by the oceans and plants. The fraction of carbon dioxide found in the atmosphere is currently around 50% of the total anthropogenic emissions, with a slight upward trend (Raupach et al. [2008]). Once in the atmosphere, this CO_2 is thought to play a pivotal role in global warming. In a recent Nature issue, Rockstrom et al. [2009] identify climate change due to CO_2 emissions as one of the most pressing problems that mankind needs to address.

Waggoner and Ausubel [2002] discusses the IPAT identity, which partitions factors that are believed to drive carbon dioxide emissions. They contribute carbon dioxide emissions to three factors

$$I = P \cdot A' \cdot T, \tag{1}$$

where I (impact) denotes the carbon dioxide emissions, P is human population, A' represents the affluence (measured as gross world product per capita) and T is technology.

The IPAT identity is useful to help thinking about the contributions of different variables and has been extensively used and discussed in the literature (see for instance Chertow [2000]) and will be used as a starting point. However, because one deals fundamentally with a complex dynamical system driven by entangled feedback loops with delays, the IPAT identity falls short, in our opinion, of providing the framework to understand the inter-relationships among the dynamical variables. It is especially important to develop a dynamical framework with delays, when studying the time-evolution of global variables such as atmospheric carbon dioxide content and human population.

Motivated by a dynamical view of the human-Earth system, we present here a framework borrowing from the theory of endogenous macroeconomic

[2]The growth rate r of the human population (or of any other variable) is defined by expression (2). Thus, a constant growth rate corresponds to a population growing exponentially, with a doubling time given by $(\log 2)/r$. As the present growth rate is $r(2011) \approx 1.0\%$ per year, this gives a present doubling time of 69 years. If nothing changes, the present 7 billion people (passed in the last quarter of 2011) will be more than 14 billion in 2080. This is in contradiction with projections of OECD for instance and other international organizations, which optimistically expect human population to stabilize around 9 to 10 billion individuals.

growth (Kremer [1993], Romer [2000]), whose feedback loops are shown to generate robust regimes of super-exponential growth. Mathematically, these regimes can be described by simple equations, whose solutions exhibit finite-time singular (FTS) power law behaviors. The interest in such solutions is that they point to changes of regime (see also Johansen and Sornette [2001], Sammis and Sornette [2002], Gluzman and Sornette [2002]).

Accelerating atmospheric carbon dioxide growth due to industrial activity has been previously reported by Canadell et al. [2007]. In contrast to Canadell et al. [2007], the present paper focuses on the interplay between and positive feedbacks on carbon-dioxide growth due to population and technology in a global macroeconomic model. Garrett [2011] develops global circulation model based on thermal conservation equations. It is interesting to note that, in such a model, energy consumption is super-exponential as well.

The article is organized as follows. We present a simple mathematical framework to model growth, first for a single variable like population in the presence of positive feedback, and then with several coupled variables, such as population, capital and technology. Two benchmark models, the exponential law and the FTS power law, are obtained as limiting cases of the theoretical framework. Then we describe the results of the calibration of these two models to some of the most extensive data sources on human population and atmospheric CO_2 content in the last two centuries up to present. The final section concludes.

2 Model

The benchmark for population growth is the Malthus model, which postulates that population growth is proportional to the population itself, capturing the simple idea that the number of children is proportional to the number of parents:

$$\frac{dp}{dt} = r \cdot p(t) \ . \tag{2}$$

The solution of equation (2) is the exponential function

$$p(t) = a' \exp(r \cdot t) + c' \ , \tag{3}$$

where c' is usually set to zero for population analysis.

Historically, equation (2) has been improved by Verhulst (see Verhulst [1845] and Verhulst [1847]) into the logistic equation to account for the competition for scarce resources between individuals. This competition can be embodied into the quadratic term $-r[p(t)]^2/K$, where K is the carrying capacity. This negative feedback of the population on the growth rate $r \to r(1-p(t))/K$ leads to a cross over from the exponential growth for $p(t) \ll K$ to a saturation of the population at long times, which asymptotically converges to K. Verhulst thought that Malthus was wrong (and therefore over-pessimistic when comparing human growth with food resources) not to take into account the negative feedbacks embodied in the quadratic term $-r[p(t)]^2/K$, that would lead naturally to an equilibrium.

But, the human population at the time of Verhulst and until around 1960 followed neither his specification, nor the Malthusian exponential growth. As reviewed in Johansen and Sornette [2001], Korotayev [2005] and Akaev et al. [2012] (and references therein), the human population grew faster than exponential, with the growth rate r itself growing.

The simplest generalization of equation (2) that accounts for this observation assumes that the growth rate r becomes $r \cdot [p(t)/p(0)]^\delta$, where $\delta > 0$ and p_0 is some reference population. The positivity of δ captures the positive feedback of population on the growth rate: the larger the population, the larger the growth rate. Equation (2) then transforms into

$$\frac{dp}{dt} = R \cdot p(t)^{1+\delta} , \tag{4}$$

where $R = r/p_0^\delta$. The solution of equation (4) reads

$$p(t) = \frac{p_0}{(1-(t/t_c))^{1/\delta}} \quad \text{if } \delta > 0 \ , \quad \text{with } t_c = \frac{1}{R}\,\frac{1}{\delta}\,\frac{1}{p_0^\delta} \ , \tag{5}$$

As can be seen, the critical time t_c at which the solution diverges is determined from the parameters of equation (4) and the initial population p_0 at time $t = 0$. For $\delta \to 0$, we recover the exponential solution (3), since

$$p(t) = p_0 \ \exp\left[-\frac{1}{\delta}\ln(1-(t/t_c))\right] \approx p_0 \ \exp\left[\frac{t}{\delta t_c}\right] \to p_0 \ e^{Rt} \ , \ \text{for } \delta \to 0 \ . \tag{6}$$

The standard exponential growth can thus be seen as the limit of a finite-time-singularity (FTS) power law with positive feedback exponent δ tending to zero. The singular solution (5) was first discussed by von Foerster et al. [1960] (see Umpleby [1990] for assessments of the relative merits of the "natural science" versus the "demographic" approach, Kremer [1993] for an economic underpinning that we explore later, and Johansen and Sornette [2001], Korotayev et al. [2006] for extensive generalizations). In ecology, the positive correlation between population density and the per capita population growth rate at the origin of the FTS behavior (5) is known as the Allee effect (Stephens et al. [1999]). More generally, Allee discovered the existence of an often present positive relationship between some component of individual fitness and either numbers or density of conspecifics. The Allee effect is usually used to refer to the self-reinforcing feedbacks that promote accelerated extinction of species and that can be modeled by finite-time crossing of zero (see Yukalov et al. [2009, 2012b] for detailed mathematical formulations). Finally, Goriely [2000] provides a rigorous mathematical framework with a generalized version of equation (4), where the right hand side is replaced by an arbitrary polynomial of $p(t)$.

The use of the mathematics of FTS to describe and diagnose changes of regime is not new. For instance, we refer to Johansen and Sornette [2001], Sornette [2003] for population dynamics and financial markets, Sammis and Sornette [2002] for applications to engineering failures and earthquakes, Sornette [2002, 2006] for a large variety of systems, Dakos et al. [2008] for climate systems, and Scheffer et al. [2009], Biggs et al. [2009], Drake and Griffen [2010] for environmental systems. These authors applied the concept of dynamical phase transitions and FTS behavior to different systems exhibiting a bifurcation, crisis, catastrophe or tipping point by showing how specific signatures can be used for advance warnings.

One can generalize (4) to take into account positive feedbacks of the growth rate $d \ln p/dt$ on its rate of change $d^2 \ln p/dt^2$ (see Ide and Sornette [2002]), to arrive at solutions that exhibit FTS not in the variable $p(t)$, but in its derivative dp/dt. We will thus use the slightly more general expression encompassing these cases:

$$p_{\text{power}}(t) = a(t_c - t)^{-1/\delta} + c \ . \tag{7}$$

A FTS in dp/dt and not in $p(t)$ corresponds to $-\infty < \delta < -1$ such that $0 < -1/\delta < 1$, together with $a < 0$ for an increase up to the value $p_{\text{power}}(t_c) = c$. Here, the meaning of the exponent δ is different from its use in equation (4).

We shall use the *exponential model* (3) and the *power law model* (7), as our two competing hypotheses. The essential difference between the *exponential model* and the power law model is that the former is defined for all times, while the later is valid only up to a finite time, the critical time t_c beyond which the solution ceases to exist. The singular behavior at t_c is not meant to predict a genuine divergence but only, as already stressed, that the system is exhibiting a transition to a qualitatively new regime.

Heated discussions among demographers greeted the publication of von Foerster et al. [1960] concerning the singular solution (5): the demographers criticized the use of mathematical models such as (4) as perhaps the clearest illustration of how bad use of mathematics may yield senseless results; actually, what the demographers missed was that the FTS should not be taken at face value, but as the signature of a transition to a new regime. Singularities do not exist in natural and social systems, but the singularities of our mathematical models, which are approximate representations of reality, are usually very good diagnostics of the changes of regime that occur in these systems [Sornette, 2002]. The perhaps clearest examples are the phase transitions between different states of matter (solid-liquid-gas-plasma, magnetized to non-magnetized, and so on) that statistical physics describes so well with its classification involving the nature of the singularity exhibited by the free energy of the system [Goldenfeld, 1992].

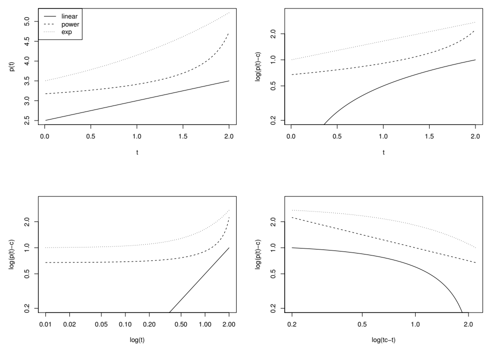

Figure 1: Illustration of the qualitatively different behaviors of the *exponential model* (3), the *power law model* (5) and a *linear model*, in different standard plot representations. For each of the four plots, the linear function $0.5t + 3.25$ is compared with the exponential function $e^{0.5t} + 2.5$ and with the power law $(2.2 - t)^{-0.5} + 2.5$. (a) is linear-linear, (b) is linear-log, (c) is log-log and (d) is log-log referenced to the singularity. The constant c is set to 2.5. The relative vertical positions of the three curves are arbitrarily chosen (from the above values) for the sake of a clear visualization (see Appendix for further discussion).

As t approaches t_c from below, two regimes can be observed for the power law model:

$\delta < 0$: $(t_c - t)^{-1/\delta}$ goes to zero for $t \to t_c$ and $p_{\text{power}}(t) \to c$.

$\delta > 0$: $(t_c - t)^{-1/\delta}$ goes to infinity for $t \to t_c$ and $p_{\text{power}}(t) \to \text{sign}(a) \cdot \infty$.

Figure 1 illustrates the qualitatively different behaviors allowing one to distinguish between the linear growth model ($dp(t)/dt \sim t$), the *exponential model* (3) and the *power law model* (7), in different standard plot representations.

Up to now, we have postulated the form (4) to capture the possible existence of a positive feedback of population on the population growth rate. Such a simplified ansatz leaves two issues unresolved. First, the positive feedback of population on growth rate may not be direct, but mediated by other variables via indirect mechanisms. Second, the consequences on the dynamics of carbon dioxide emissions are not clear. We thus address these two issues using an economic framework developed by Kremer [1993], following the approach of Johansen and Sornette [2001]. The following derivation is not intended to represent a faithful economic growth model that we would like to promote, but is offered to illustrate the importance of indirect mechanisms in growth processes. In particular, we would like to stress the fact that faster-than-exponential growth is a robust outcome of multi-dimensional loop processes. Even when each feedback process individually leads to an exponential or even a subdued sub-exponential growth, the overall dynamics can be super-exponential.

In economics, population $p(t)$ translates into labor force $L(t)$, which is assumed to be proportional to population. In addition to population represented by the labor force, we consider the effect of technology level $A(t)$ and of the amount $K(t)$ of available capital. In the presence of labor and capital, with a given technology level, the economy is going to produce an output $Y(t)$, for instance proxied by GDP. In the macroeconomics of endogenous growth (Romer [2000]), it is common to use the Cobb-Douglas equation (originally developed in Cobb and Douglas [1928] and extensively discussed in Romer [2000]) to relate the total output to labor, capital and technology as follows[3]:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \ , \text{with} \ \ 0 < \alpha < 1 \ . \tag{8}$$

Furthermore, we use the assumption by Solow that a constant fraction s of the economy goes to savings, i.e. capital grows according to

$$\frac{dK}{dt} = sY(t) \ . \tag{9}$$

Following Kremer [1993], we assume that, as already mentioned, labor is proportional to capital

$$K(t) \sim L(t) \ . \tag{10}$$

We further assume that technology change is depending on capital, labor and current level of technology according to

$$\frac{dA}{dt} = dK(t)^{\eta} \times L(t)^{\gamma} \times A(t)^{\theta} \ , \tag{11}$$

where the exponents η, γ and θ are all positive, expressing a positive feedback effect of each of the variables on the growth of technology. Putting together all these ingredients, we can rewrite the Kremer (10) and Solow (9) equations as a system of two coupled ordinary differential equations:

$$\frac{dA}{dt} = eL(t)^{\eta+\gamma} \times A(t)^{\theta}, \tag{12}$$

$$\frac{dL}{dt} = fL(t) \times A(t)^{1-\alpha} \ . \ , \tag{13}$$

[3] A' in the IPAT equations stands for gross world product per capita, whereas in the Cobb-Douglas equation A stands for technology. Further, the IPAT equation uses T instead of A to denote technology. Similar, the macro-economists refer to L as labor, whereas P in the IPAT equality stands for population. We will not distinguish between labor L and population P and use the terms interchangeably.

where e and f are two positive constants. Equation (13) basically states that labor (and thus population) is growing exponentially, holding technology constant. In other words, the growth rate of population is controlled by a nonlinear function of technology. Here, this nonlinear function is a power law with exponent $0 < 1 - \alpha < 1$, which embodies the benefits that technology brings in decreasing death rates, for instance via improvement in health care, or in feeding more mouths. Invoking these mechanisms is standard in demographic research.

We look for solutions exhibiting a FTS of the form

$$A(t) = A_0(t_c - t)^{-1/\mu} \ , \tag{14}$$

$$L(t) = L_0(t_c - t)^{-1/\kappa} \ . \tag{15}$$

Note that the critical time t_c of the singularity, if it exists, is necessarily the same for both variables, as seen from inspection of the two coupled equations (12,13). Inserting this ansatz in equations (12,13), we obtain a system of linear equations for the unknown inverse exponents $1/\mu$ and $1/\kappa$, whose solutions read

$$\mu = 1 - \alpha \ , \tag{16}$$

$$\kappa = \frac{\eta + \gamma}{2 - \theta - \alpha}(1 - \alpha) \ . \tag{17}$$

The condition for the solutions (14,15) to hold is that μ and κ be strictly positive. This implies $0 < \alpha < 1$ and $\alpha < 2 - \theta$. If $\theta \leq 1$, then the conditions are always satisfied in the regime where the Cobb-Douglas equation holds. The case $\theta \leq 1$ is particularly interesting because it corresponds to a sub-exponential growth of technology, for a fixed labor force. In other words, for a fixed population level, equation (12) gives a long-time growth of the form $A(t) \sim t^{\frac{1}{1-\theta}}$, which is sub-exponential (slower than exponential) for $\theta < 1$ and exactly exponential for $\theta = 1$. It is the coupling between a sub-exponential growth of $A(t)$ and an exponential growth of population $L(t)$ mediated by nonlinear feedback loops that create the super-exponential finite-time singularity. This behavior underlines the possible traps of single variable analysis.

These results can be translated into a prediction of carbon dioxide emission via the following simple assumption. Assuming that carbon dioxide emissions are proportional to production divided by some power of technology ξ, we have

$$\frac{dCO_2}{dt} = a\frac{Y(t)}{A(t)^{\xi}} = h(t_c - t)^{-1/\varphi}, \tag{18}$$

where $\varphi = (1/\kappa - \xi/\mu + 1)^{-1}$ (see appendix for details of the derivation) and CO_2 stands for the total carbon dioxide content in the atmosphere. The constant a absorbs the dimensional relations between the different variables.

The introduction of a non-zero exponent ξ accounts for the common observation that more developed countries tend to have a lower footprint and smaller carbon emissions per unit of output, due to the progressive adoption of more efficient technologies and the increasing importance of a clean environment in the utility functions of consumers.

Let us thus stress the main result of this exercise. We have $\frac{dA}{dt} \sim A(t)^\theta$ at fixed labor with $\theta < 1$ and $\frac{dL}{dt} \sim L(t)$ at fixed technology. Thus, there is no way to get a faster-than-exponential growth in any of these two variables alone. However, when coupling them via the feedback of labor on technology and that of technology on labor, the FTS power law solutions (14,15) emerge. Hence, a finite-time singularity can be created from the interplay of several growing variables resulting in a non-trivial behavior: the interplay between different quantities may produce an "explosion" in the population even though the individual dynamics do not!

Of course, infinities do not exist on a finite Earth! These singularities should not be interpreted as the prediction of real "blow-ups". They can be however faithful description of the transient dynamics up to a neighborhood of the predicted critical time t_c. Around t_c, new mechanisms kick in and produce a *change of regime*.

To illustrate the above point, let us go through a detailed scenario where the individual processes stay finite in finite time, but the combination via feedback can lead to finite time singularities. Consider the following parameters

$\alpha = \frac{1}{4}$**:** as in the seminal paper by Cobb and Douglas [1928].

$\theta = 1$**:** Linear feedback from technology A on itself. Holding all other factors constant, technology will grow exponentially (see equation (11)).

$\eta + \gamma = 1$**:** The simplest possible, non-trivial, assumption.

With these numbers, we obtain the two exponents $\mu = 3/4$ and $\kappa = 1$ for the equations (14) and (15), respectively, and the value $1/\varphi = 5/3$ for the rate of carbon dioxide emission given by equation (18), assuming carbon dioxide emission per capita technology is as efficient as general technology A, i.e. $\alpha = \xi = 1/4$. Although, we have only assumed exponential growth of all individual factors, carbon dioxide emission is predicted in this example to grow faster than exponential, leading to a mathematical FTS which is the signature of a non-sustainable regime towards a new behavior (see Figure 2). We refer to Yukalov et al. [2009, 2012b,a] for detailed classifications of possible regimes.

Even less stringent conditions for a FTS to occur are needed when the description of the dynamics of the system in terms of two coupled equations (12,13) is augmented to take into account the dynamics of additional coupled variables, leading to systems of three or four coupled equations. Such

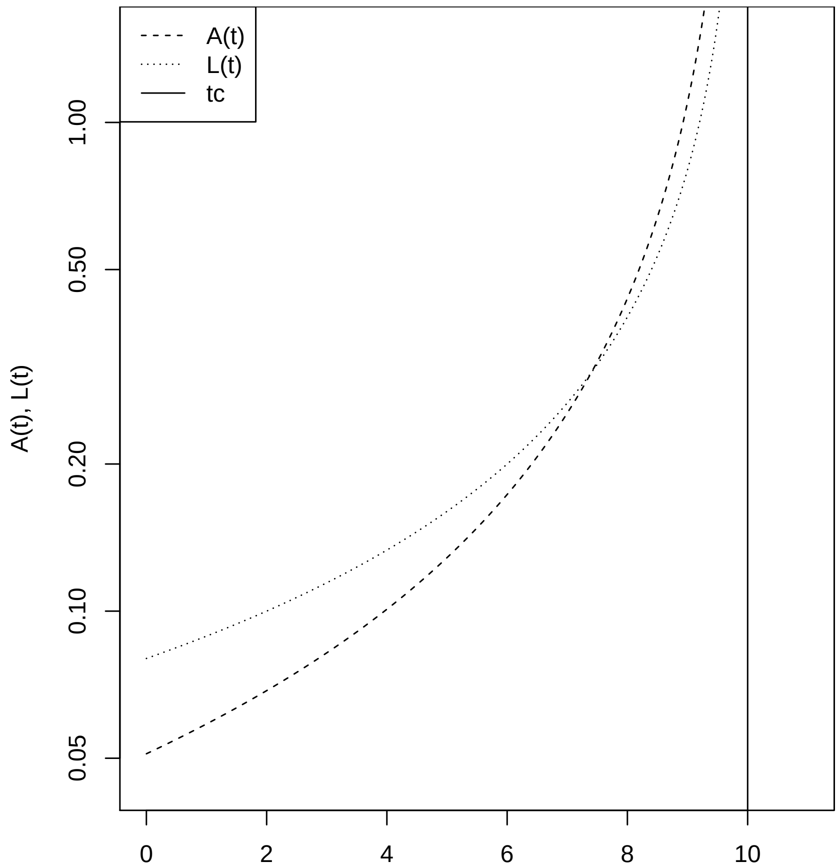

Figure 2: Numerical solution of equations (12) and (13) with $\alpha = \frac{1}{4}$, $\eta + \gamma = 1$, $\theta = 1$ and $t_c = 10$. The initial conditions are $A(0) = 1.1$ and $L(0) = 0.8$. We assume without loss of generality $e = f = 1$, as these coefficients can be absorbed in the units of A and L respectively. $L(t)$ and $A(t)$ grow super-exponentially towards a singularity occurring at the same time as a result of their coupling. $A(t)$ and $L(t)$ are plotted on a semi-log plot as function of time. The upward curvatures and approaches to the singular vertical asymptote exemplify the super-exponential growth.

additional positive feedback loops include nonlinear lagged dependencies of capital on labor (thus extending Kremer's simplifying assumption (10)).

3 Results

Figure 3 shows that the growth rate of the world population was a strongly increasing function of time till the late 1950s. A sharp decrease of the growth rate occurred, then followed by resumed acceleration until its peak in 1964, from which a slow decrease can be observed.

The first regime until about 1960 is incompatible with the exponential model, which corresponds to a constant growth rate. Figure 4 shows that, over the time period 1850 to 1965, the exponential model is greatly inferior to the FTS power law model. Using model (4), we estimate that the growth exponent δ is approximately equal to 2, that is, even larger than the value 1 estimated in von Foerster et al. [1960]: clearly, population growth over this time period was faster than exponential and the FTS power law model accounts parsimoniously for the data.

Figure 5 shows that, over the time period from 1970 to 2010, the exponential model (3) and the FTS power law model (7) are indistinguishable.

Figure 6 plots the carbon dioxide content in the atmosphere since 1000 CE. The dramatic acceleration due to anthropogenic forcing since the 1800s is clearly observed.

We calibrate the *exponential model* (3) and the *power law model* (7) separately to two time periods: (i) from 1850 to 1954 (Figure 7), for which the data originates from ice drill cores and (ii) from 1959 to 2011 (Figure 8), for which the data originates from air samples. The quality of the fits by the two models, as quantified by the sum of squared errors between theory and data, is practically equivalent. Therefore, we cannot reject the hypothesis that the exponential model is sufficient to explain the data for each time window *separately*.

However, the growth rate r calibrated with the exponential model (see equation (3) has more than doubled from the first period 1850 – 1954 ($r = 0.0066$) to the second period 1959 – 2011 ($r = 0.016$). While being not fully warranted given the heterogeneity of the data sources, we have fitted the two models to the whole period from 1850 to 2009. We find that the FTS power law is the clear winner (see Figure 9) which, together with the more than doubling of the growth rate r from the first to the second time intervals, suggests the existence indeed of a faster-than-exponential growth of the atmospheric content of carbon dioxide.

We now attempt to be more precise on the nature and evolution of the faster-than-exponential growth by estimating the exponent δ of equations (7) applied to the time series of carbon dioxide atmospheric content. We use the monthly data from the Mauna Loa site, as it is considered to be one of

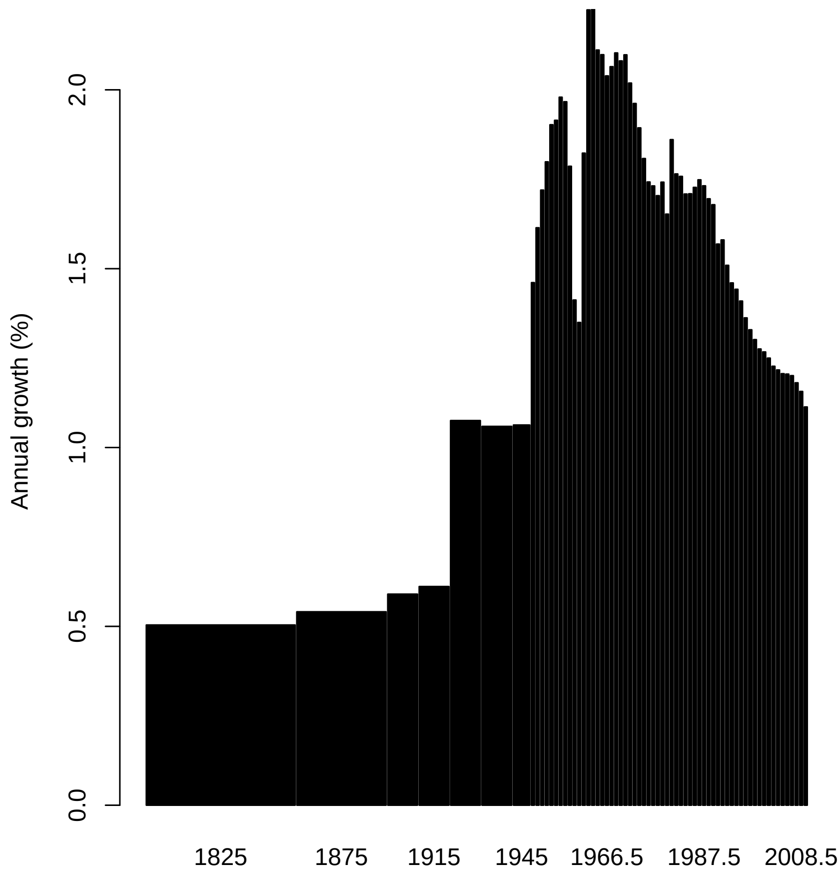

Figure 3: Annualized world population growth rate from year 1800 – 2010.

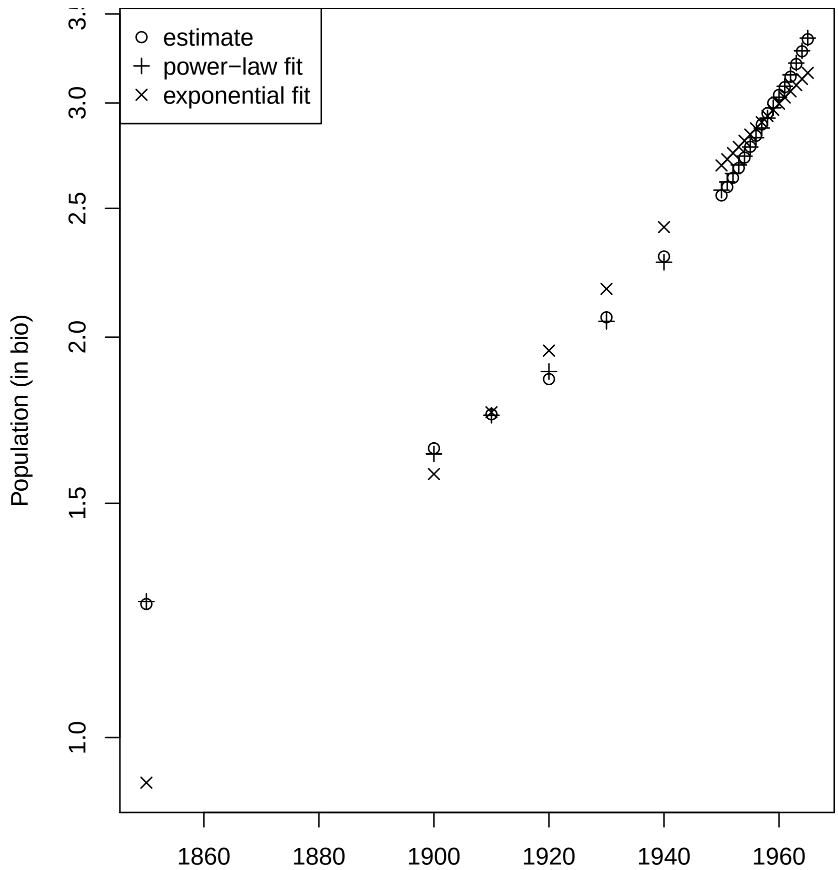

Figure 4: Population data represented by the empty circles (where "estimate" refers to the empirical estimation of the population) fitted over the time window from 1850 – 1965 by the FTS power-law (7) and the exponential model (3) with c' set to zero. The fitted parameters are $\delta = 2$ and $t_c = 1988$ for the power-law and $r = 0.011$ for the exponential fit.

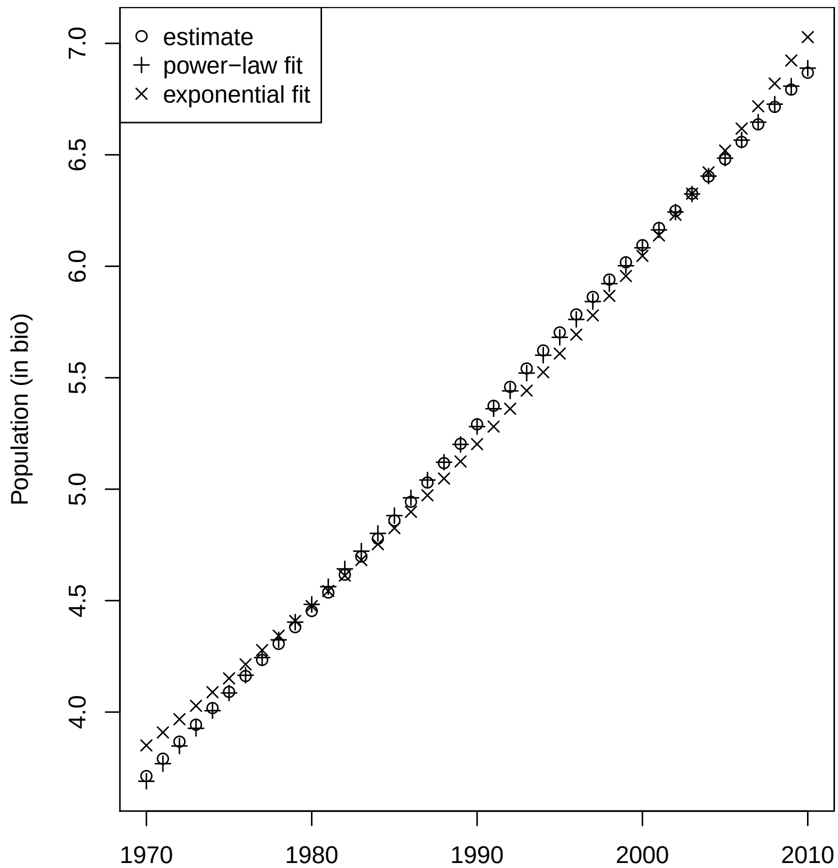

Figure 5: Population data fitted over the time window from 1970 – 2010 by the FTS power-law (7) and the exponential model (3) with c' set to zero. The fitted parameters are $\delta = 4.6$ and $t_c = 4312$ for the power-law and $r = 0.015$ for the exponential fit.

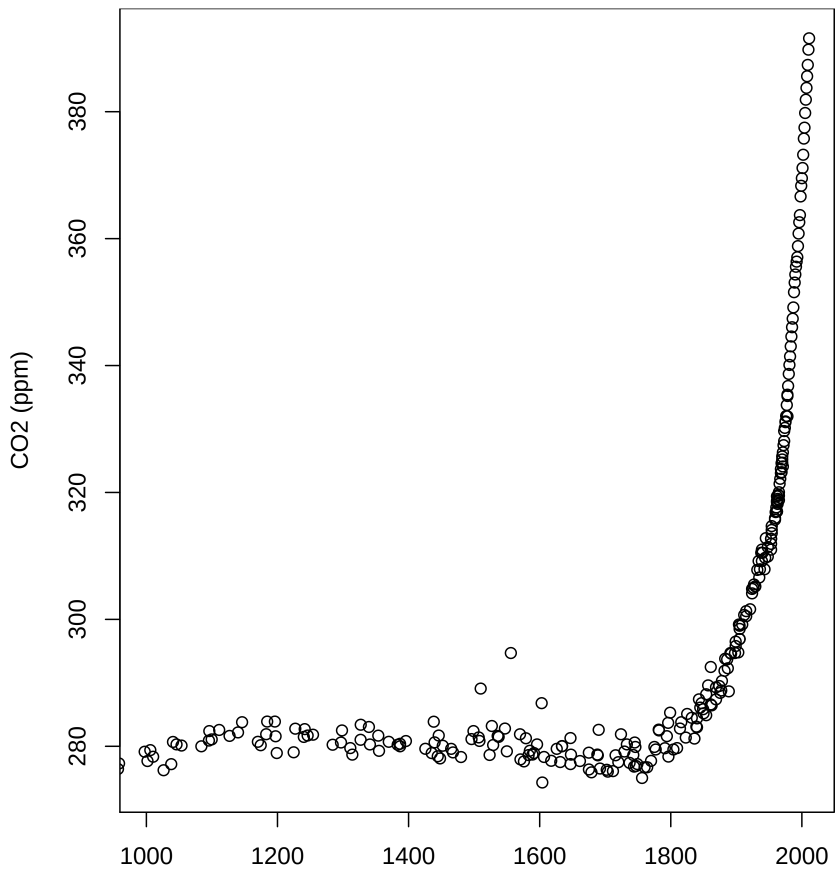

Figure 6: Atmospheric carbon dioxide since 1000 CE to present. The data shown combines ice core and air measurements from different sources. See data section for more details.

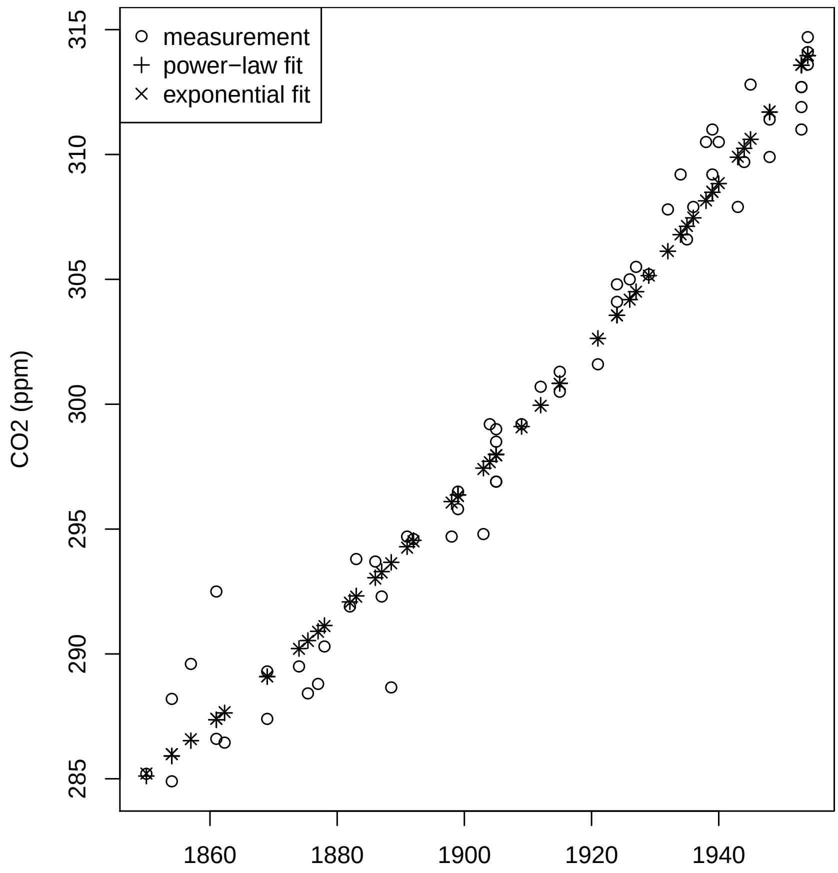

Figure 7: Carbon dioxide data fitted over the time window from 1850 – 1954 by the FTS power-law (7) and the exponential model (3). The fitted parameters are $\delta = 0.65$ and $t_c = 2304$ for the power-law and $r = 0.0066$ for the exponential fit.

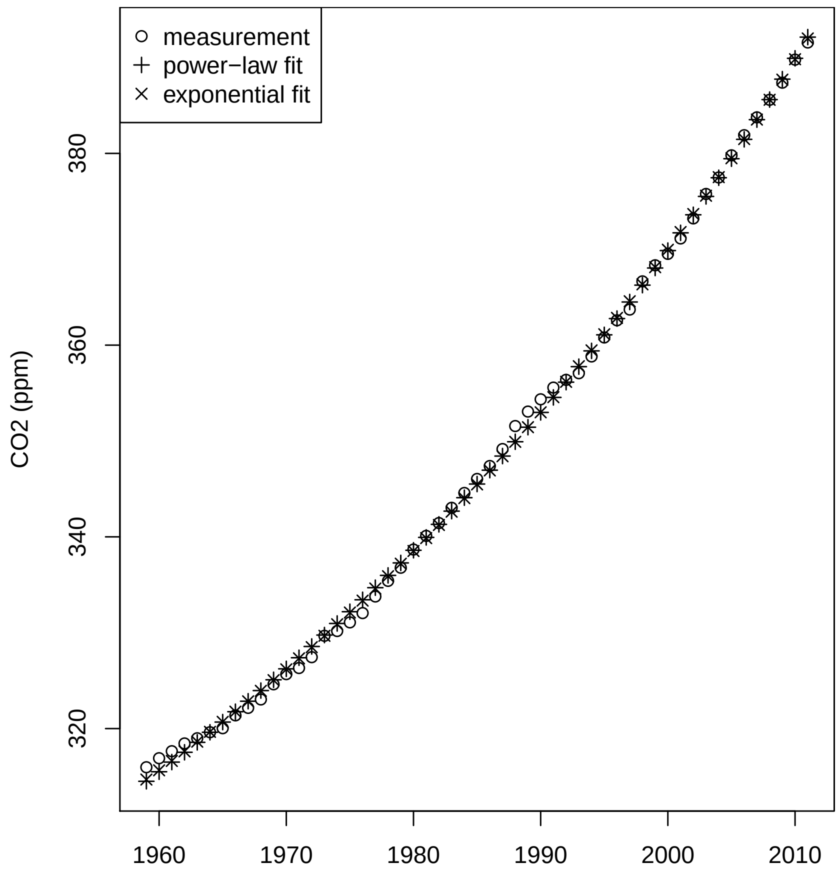

Figure 8: Carbon dioxide data fitted over the time window from 1959 – 2011 by the FTS power-law (7) and the exponential model (3). The fitted parameters are $\delta = 0.71$ and $t_c = 2141$ for the power-law and $r = 0.016$ for the exponential fit.

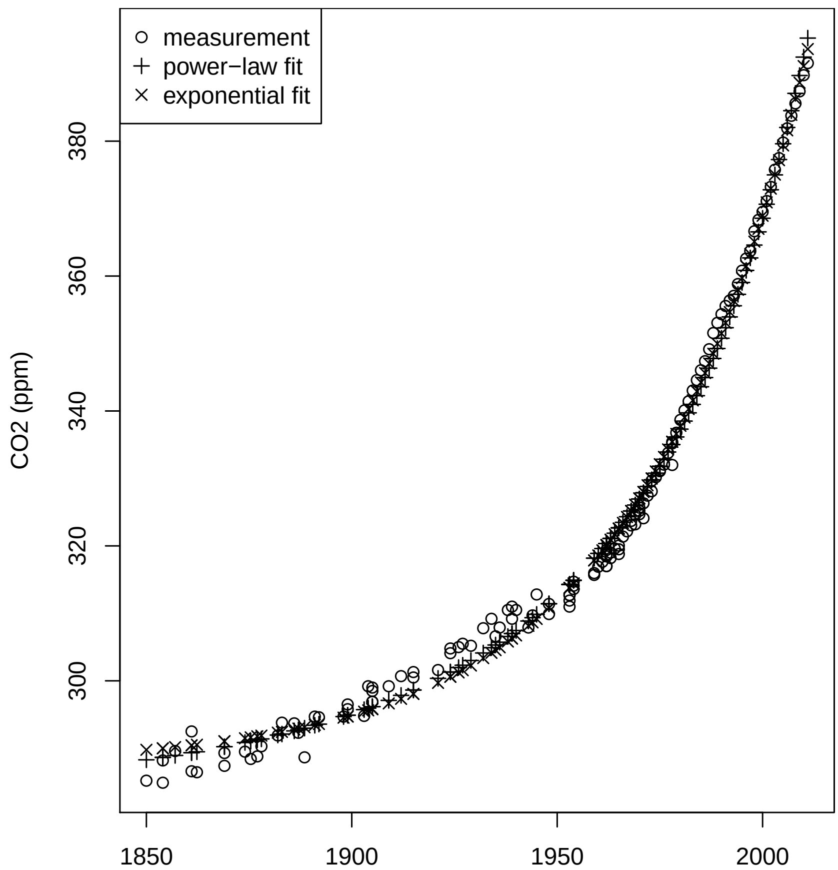

Figure 9: Carbon dioxide data fitted over the time window from 1850 – 2011 by the FTS power-law (7) and the exponential model (3). The fitted parameters are $\delta = 0.25$ and $t_c = 2167$ for the power-law and $r = 0.024$ for the exponential fit. The ratio of squared errors between the power-law and the exponential-fit is 0.92.

the most reliable and longest time-series. Before calibrating equation (7) to various time intervals $[t_1, t_2]$, we smooth the data by using a Gaussian kernel with a width of 10 years. Then, we estimate δ, with t_1 being scanned from 1958 to 2006 and t_2 being scanned from 1960 to 2010 as shown in Figure 10.

Two main results are obtained. First, the exponent δ is found almost always larger than or equal to 1, implying a growth significantly faster than exponential (which is recovered for $\delta \to 0$). Second, one can observe a systematic trend. For time intervals starting earlier (i.e., for t_1 in the late 1950s and in the 1960s), the exponent δ tends to be closer to 1, while for larger t_1, δ is significantly larger than 1. This leads to the conclusion that the carbon dioxide content in the Earth atmosphere is growing at least exponentially and probably faster-than-exponentially, with no sign of abating. The latest time intervals are characterized by the largest exponents δ's, significantly above the lower bound 0 that would correspond to an exponential growth. We thus conclude that the content of carbon dioxide in the atmosphere is accelerating super-exponentially.

4 Discussion

The previous empirical evidence suggests that the human population on Earth is growing now just exponentially, while there is suggestive evidence that the content of carbon dioxide in the atmosphere is accelerating super-exponentially. How are these two different behaviors compatible with the solutions (14,15) for $A(t)$ and $L(t)$ of equations (12,13)?

We consider two possible explanations. The first one would argue that until the 1960s both population and atmospheric carbon dioxide content were super-exponentially accelerating in accordance with expressions (14,15). Then, the slowing down from super-exponential to just exponential growth of the human population could be interpreted as a finite-size effect that is starting to be felt for this variable only, as physical limits are more stringent for the human carrying capacity and the response of human birth and death rates to policies than they are for carbon dioxide emissions.

The second explanation is that the two different behaviors of $A(t)$ and $L(t)$ may be resolved within the mathematical structure developed in equations (14) and (15). Indeed, let us assume that the growth of the human population is following solution (15), but with a small value of the exponent κ. For all practical purpose, a FTS power law with a small exponent is indistinguishable from an exponential growth over a finite time interval. This interpretation is reasonable in so far that human population growth has been unambiguously super-exponential until the 1960s, and it is only recently that this growth has somewhat abated.

Let us now turn to the dynamics of CO_2 content. The conditions for a super-exponential growth of the content of carbon dioxide in the atmosphere

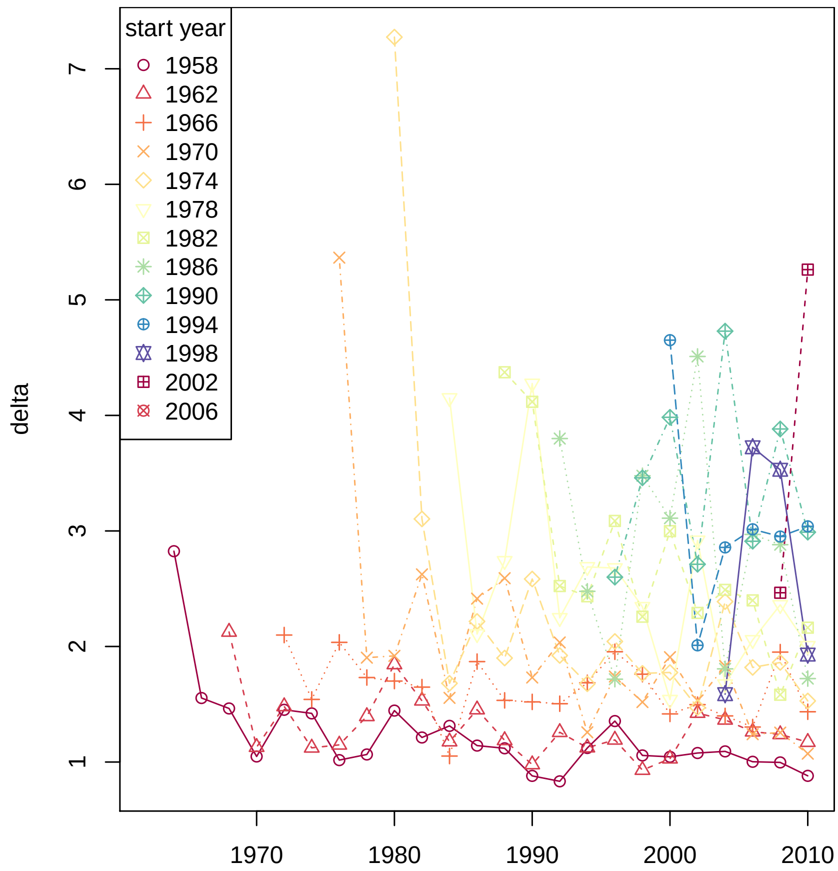

Figure 10: Estimates of the exponent δ of equation (7) on the monthly Mauna Loa carbon dioxide data obtained from air measurements in different intervals $[t_1, t_2]$. Each line corresponds to a specific start time t_1, as shown in the legend. The ending point t_2 is the variable on the abscissa.

are compounded by many complex processes involving, in addition to the emissions, the sequestrations of CO_2 by, and dynamics of, the ocean and biosphere. As a rough rule of thumb, we assume that the anthropogenic carbon dioxide in the atmosphere at time t is simply proportional to (but likely less than) the cumulative release of anthropogenic CO_2 until time t. In other words, CO_2 content is estimated as a finite fraction of the solution of equation (18). Under these assumptions, in order for CO_2 content to exhibit a FTS power law behavior, it is necessary and sufficient that the exponent $1/\varphi$ in (18) be larger than 1. Indeed, by integration, $CO_2(t)$ remains of the same form $(t_c - t)^{-1/\delta}$, with $1/\delta = (1/\varphi) - 1 > 0$, where δ is defined as in equation (7). This condition translates into the condition $\xi < \mu/\kappa$. As we have assumed that κ is small, corresponding to the closeness of the population dynamics to an exponential growth, this condition does not provide a strong constraint for ξ: CO_2 content can exhibit an (accelerated) FTS dynamics even if ξ is large, corresponding to a more efficient economy. If $1-\alpha$ is close to 0, corresponding to output mainly controlled by availability of capital, then ξ should be small. Small values of ξ correspond to the situation in which, taken globally over the whole Earth, the technological advances have not yet significantly abated carbon emission per unit of output. This statement may appear shocking and counter-factual for developed countries. But, at the scale of the whole planet, one can observe that improvement in carbon emissions in the developed countries are counteracted by the increases of carbon emissions in some major developing countries (Pielke et al. [2008]), such as China, India and Brazil, which use carbon emission inefficient technologies (for instance heavily based on coal burning).

In summary, we find a very robust FTS behavior for CO_2 over a broad and realistic range of parameters, which makes it difficult to constrain the impact of the advance of technology on production efficiency.

5 Conclusion

We have analyzed the growth of atmospheric carbon dioxide and of what constitutes arguably its most important underlying driving variable, namely human population. Our empirical calibrations suggest that human population has decelerated from its previous super-exponential growth until 1960 to “just” an exponential growth. As for atmospheric CO_2 content, we find that it is at least exponentially increasing and more probably exhibiting an accelerating growth rate, consistent with a FTS (finite-time singular) power law regime.

We have proposed a simple framework to think about these dynamics, based on endogenous economic growth theory. We showed that the positive feedback loops between several variables, such as population, technology and capital can give rise to the observed FTS behavior, notwithstanding the fact that the dynamics of each variable would be stable or at most exponential, conditional on the stationarity of the other variables. It is the joint growth of the coupled variables that may give rise to the enormous acceleration characterized by the FTS behavior both in the equation and in the carbon dioxide content in the atmosphere.

Overall, the evidence presented here does not augur well for the future.

- The human population is still growing at an unsustainable rate and there is no sign the population will stabilize anytime soon. Many argue that economic developments and education of women will lead to a decreased growth rate and an eventual stabilization of human population. This is not yet observed in the population dynamics, when integrated worldwide. Let us hope that the stabilization of the human population will occur endogenously by self-regulation, rather than by more stringent finite carrying capacity constraints that can be expected to lead to severe strains on a significant fraction of the population.
- Notwithstanding a lot of discussions, international meetings and prevalence in the media, atmospheric CO_2 content growth continues unabated with a clear faster-than-exponential behavior. On the face of this evidence using data until 2011, stabilizing atmospheric carbon dioxide emissions at levels reached in 1990 for instance seems very ambitious, if not utterly unrealistic. We are not pessimistic. We think that only evidence-based decision making can lead to progress. The present evidence gives some measure of the enormous challenges to control our carbon dioxide emissions to acceptable levels.

The International Energy Agency (IEA) has released its flagship publication of the year on the World energy outlook (see Organisation for Economic Co-operation and Development [2011]). The IEA reports that carbon dioxide emissions jumped by 5.3% last year to the record 30.4 gigatons, due mainly to increasing demand for coal in particular by China and India. The IEA raised its forecast for primary energy demand by a third between 2010 and 2035. The IEA report writes that, if there is no stringent new action by 2017, the energy related infrastructure in place would generate all the CO_2 emissions allowed up to 2035 for the World to meet its target of a maximum temperature increase of 2C. These conclusions are in line with the evidence presented here.

6 Data

Population data was obtained from the website of the United Nations (`http://www.un.org/esa/population/publications/sixbillion/sixbilpart1.`

pdf) and the website of U.S. Census Bureau (http://www.census.gov/population/international/data/idb/worldpoptotal.php).

Carbon dioxide data was collected from different sources: from the Carbon Dioxide Information Analysis Center (CDIAC) (http://cdiac.esd.ornl.gov/ftp/trends/co2/siple2.013, http://cdiac.ornl.gov/ftp/trends/co2/lawdome.combined.dat), the National Oceanic and Atmospheric Administration (NOAA) (ftp://ftp.cmdl.noaa.gov/ccg/co2/trends/co2_annmean_mlo.txt, ftp://ftp.ncdc.noaa.gov/pub/data/paleo/icecore/antarctica/maud/edml-co2-2005.txt, ftp://ftp.cmdl.noaa.gov/ccg/co2/trends/co2_mm_mlo.txt) and from Barnola et al. [1995][4].

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[4]leaving out "Bern" measurements

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A Discussion of exponential growth / FTS power-law

Depending on the scale of the abscissa and the ordinate, exponential growth and FTS power-law growth can look very different (see also Figure 1):

(a) the linear-linear plot shows the dual property of the FTS power law function, which is to both grow initially slower than the two other models, and then to catch up explosively.

(b) In this linear-log plot, by construction, the exponential function is a straight-line, thus a linear dependence in this representation qualifies an exponential growth. The linear model is concave (slower growth) and the power law FTS model is convex (faster growth).

(c) The log-log plot would qualify a power law t^β as a straight line whose slope is the exponent β. Hence the linear function is also linear in this representation with slope 1. Both the exponential and FTS power law model exhibit an upward convex shape. It is important not to confuse a power law and a FTS power law: the former is proportional to a power of t and thus exists for all times, while the later is proportional to a power of $t_c - t$ and is only defined for $t < t_c$.

(d) In this log-log plot in the variable $t_c - t$, by construction, the FTS power law is qualified by a straight line behavior, with a slope equal to the exponent $-1/\delta$. Both linear and exponential models are associated with concave curves, characterizing a slower growth in the vicinity of t_c. Note that time t increases from right to left.

B Exact Solution of the ODE system

This appendix provides the exact derivation of the system of equations (12) and (13), thus justifying the ansatz (14) and (15) used.

First, we combine equations (12) and (13) into a single equation:

$$\frac{dA}{dt} L(t)^{-\eta-\gamma} A(t)^{-\theta} - \frac{dL}{dt} L(t)^{-1} A(t)^{-1+\alpha} = 0 \ . \tag{19}$$

Without loss of generality, we can set $e = f = 1$ by defining appropriately the units of A and L. Separating the variables and integrating lead to

$$\frac{1}{2-\alpha-\theta} A(t)^{2-\alpha-\theta} - \frac{1}{\eta+\gamma} L(t)^{\eta+\gamma} = c' \ . \tag{20}$$

Looking for the large time asymptotic regime for which $L(T)$ and $A(t)$ (which are assumed to be monotonously increasing) become much larger that the constant c', we can solve for $A(t)$ and $L(t)$ as follows.

- Hence,

$$L(t) = \left[\frac{1}{2-\alpha-\theta}A(t)^{2-\alpha-\theta}(\eta+\gamma)\right]^{1/(\eta+\gamma)} \tag{21}$$
$$= c_2 A(t)^{\frac{2-\alpha-\theta}{\eta+\gamma}} \; . \tag{22}$$

Plugging this into equation (12) leads to

$$\frac{dA}{dt} = c_2 A(t)^{2-\alpha} \; . \tag{23}$$

By separating variables and subsequently integrating, we get:

$$A(t)^{\alpha-2}dA = c_2 dt \; , \tag{24}$$
$$\frac{1}{\alpha-1}A(t)^{\alpha-1} = c_2 t + c_2' \; , \tag{25}$$
$$A(t) = \left[(1-\alpha)c_2(\frac{-c_2'}{c_2} - t)\right]^{-1/(1-\alpha)} \tag{26}$$
$$\Leftrightarrow A(t) = A_0(t_c - t)^{-1/\mu} \; , \tag{27}$$

with $\mu = 1-\alpha$.

- Similarly, we find the solution for $L(t)$:

$$A(t) = \left[\frac{1}{\eta+\gamma}L(t)^{\eta+\gamma}\right]^{1/(2-\alpha-\theta)} \tag{28}$$
$$= c_3 L(t)^{\frac{\eta+\gamma}{2-\alpha-\theta}} \; . \tag{29}$$

Plugging this into equation (13) leads to

$$\frac{dL}{dt} = c_3' L(t)^{\frac{(\eta+\gamma)(1-\alpha)}{2-\alpha-\theta}+1} \tag{30}$$
$$=: c_3' L(t)^{\kappa+1} \quad \text{where } \kappa := \frac{(\eta+\gamma)(1-\alpha)}{2-\alpha-\theta} \; . \tag{31}$$

As before, we separate variables and integrate

$$L(t)^{-\kappa-1}dL = c_3' dt \; , \tag{32}$$
$$\frac{1}{-\kappa}L(t)^{-\kappa} = c_3' t + c_3'' \; , \tag{33}$$
$$L(t) = \left[\kappa c_3'(\frac{-c_3''}{c_3'} - t)\right]^{-1/\kappa} \tag{34}$$
$$\Leftrightarrow L(t) = L_0(t_c - t)^{-1/\kappa} \; , \tag{35}$$

with $\kappa = \frac{\eta+\gamma}{2-\alpha-\theta}(1-\alpha)$.

Of course, the solution for $L(t)$ could be directly obtained using (22) and (27), and reciprocally.

For a general mathematical rigorous theory of ordinary differential equations exhibiting finite-time singular behaviors, see Goriely [2000].

C Calculation of the exponent φ

Let us give some intermediate steps towards the solution of equation (18).

$$\frac{Y(t)}{A(t)^\xi} =^{(8)} \frac{K(t)^\alpha (A(t)L(t))^{1-\alpha}}{A(t)^\xi} \tag{36}$$

$$=^{(10)} L(t)A(t)^{1-\alpha-\xi} \tag{37}$$

$$=^{(14,15)} L_0(t_c-t)^{-1/\kappa}\left[A_0(t_c-t)^{-1/\mu}\right]^{1-\alpha-\xi} \tag{38}$$

$$= L_0A_0(t_c-t)^{-1/\kappa-(1-\alpha-\xi)/\mu} \tag{39}$$

$$=^! C_0(t_c-t)^{-1/\varphi} \ . \tag{40}$$

Hence,

$$\varphi = \frac{1}{1/\kappa - \xi/\mu + 1} \ , \tag{41}$$

using $\mu = 1-\alpha$ given by (16).

4.3 Discussion & Perspective

The "Club of Rome" was founded 1968 as a "structured responses to growing world-wide complexities and uncertainties". In their "World 3" model, Meadows et al. (1972) simulate the planet as complex system with various positive and negative feedback loops, for example like in Figure 5. Of course this simulation is so complex that it is no longer analytically traceable and has to be solved with the help of a computer.

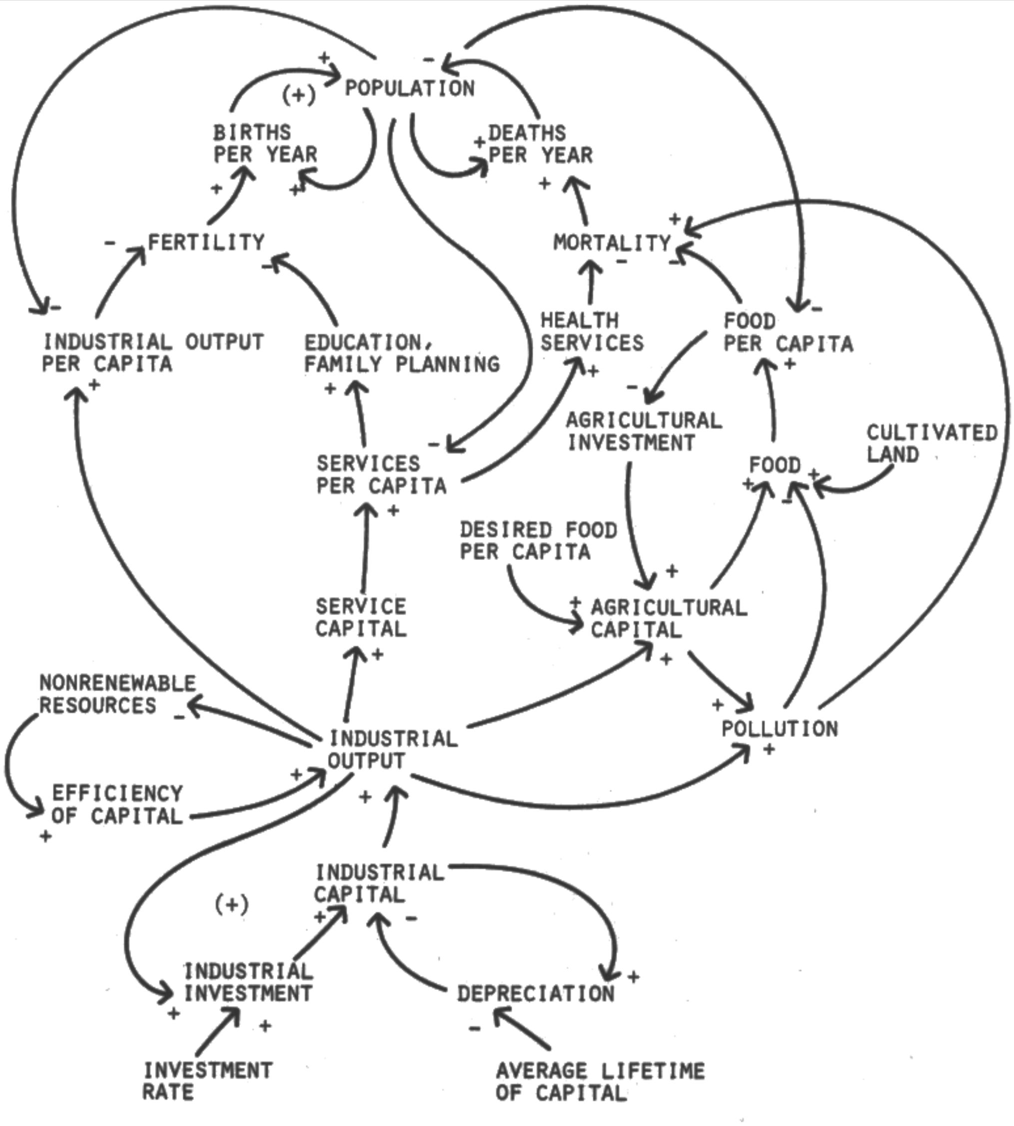

Figure 1-3 Causal-loop diagram of several important feedback loops in World3

Fig. 5: Reproduced from Meadows et al. (1972) showing the different feedback loops used in the original world 3 model.

The report was published as "Limits to Growth" by Meadows et al. (1972) and was an wake-up call for many politicians. Interestingly, Turner (2008) assessed the model 30 years later and finds that the "standard run" describes reality quite accurately. Further, Turner (2008) discusses some interesting prejudice about what Meadows et al. (1972) actually did not predict (for instance "peak oil"). Although the club has lost a lot of publicity and visibility in the last years, it continues to refine and update its forecasts (Randers, 2012).

It is interesting to note that efforts to reduce pollution in general and carbon dioxide emission in particular have weakened (The Economist, 2012). Politicians seem to be too over optimistic on the impact of new technologies (Pielke et al., 2008). Moreover, even stringent criteria like anchoring emissions on the 1990 level (at least for industrial nations), a 2°C global warming limit or 350ppm atmospheric CO_2limit (which is already exceeded as atmospheric CO_2 is currently at over 390ppm[1]) are not sufficient; even if these limits are maintained, they "[...] increase the risk of irreversible climate change, such as the loss of major ice sheets, accelerated sea-level rise and abrupt shifts in forest and agricultural systems" (Rockstrom et al., 2009). In addition, the current financial crisis shifts priorities away from environmental issues.

In contrast to Meadows et al. (1972) and our research paper presented in the previous section, Sanyal (2011) argues that world population growth (one of the most important drivers of pollution) has slowed and population will peak as early as 2050 and fertility falls below the reproduction level as early as 2030. However, the fertility rates are no longer as important, because population growth has be driven by increased life expectancy in the developed countries. This trend is now also visible in emerging countries; Brazil life expectancy has increased almost 50% over the last sixty years and now is at 74, for example (Sanyal, 2011). Moreover, Sanyal argues for example that China, the most populous country, has implemented a very effective "one-child policy". However, China's one child-policy is no longer strictly enforced; minorities are already granted generous exemptions, in addition some provinces push for a suspension (The Economist, 2011). Further, the latest official projections by UN experts expect 2.17 children per woman in the base line scenario. The high growth scenario assumes even exponential growth for the foreseeable future. And even for a very optimistic low growth scenario, "[...] population growth until 2050 is inevitable (United Nations, 2011)". In summary, the assumption that population grows exponentially seem realistic to us, at least for the next 20 years.

One inherent problem with forecasts is that sometimes not even the present value is well known and is clouded by uncertainty. One example is the prediction of economic growth measured as GDP growth. It is very common that predictions have to be revised and even the base for the current prediction is only known with uncertainty, see Figure 6 for an illustration.

[1] http://www.esrl.noaa.gov/gmd/ccgg/trends/global.html

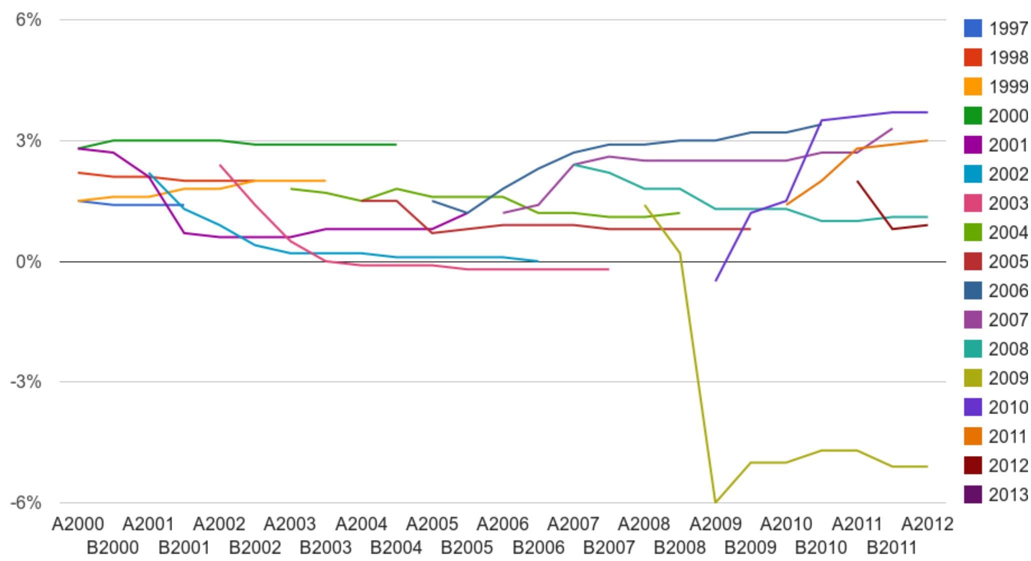

Fig. 6: GDP growth estimates can be subject to heavy revision. The chart above shows Germany's GDP estimates (one color corresponds to a specific year) vs. the issue time of the respective estimates. I.e., the beige colored line in the lower right of the chart shows the evolution of the GDP growth estimate for the year 2009. In spring and autumn of 2008 (2008A and 2008B, respectively), the growth was predicted to be positive. However, in the year 2009, the estimate was revised down to -5% to -6%. Ex-post, the GDP growth for 2009 has been several times restated (even 3 years later in 2012), but is now believed to be around -5.1%. Own work with data obtained from the "Center for Economic Studies" (`www.cesifo-group.de`).

5

Conclusions

In this thesis, we presented three research papers. In the first, we started wtih assumptions about the behavior of individual agents and how they compose their portfolios. Based on these assumptions, we formulated a mathematical framework and derived structural properties. Finally, by comparing the derived properties to actual data, we concluded that investors are over-optimistic regarding the performance of mutual funds or they overpay mutual fund mangers. In the second paper, we analyzed data from a laboratory stock market where a virtual asset is traded. We quantified the observed increase in price, interpreted as a bubble and found faster-than-exponential growth due to traders anchoring their expectations on past prices. In the final paper, we presented two competing models of atmospheric carbon dioxide growth, motivated by a simple economic model. We found that the atmospheric carbon dioxide content seems to grows than exponential. Further, we developed a scenario to project what could happen if the past trend continues.

In all three papers, the results depend crucially on expectations: the expectations of the representative agent, the expectations of the future price of the laboratory traders and the expectation that the present trend persists.

The concept of rationality is different in the three papers. In the first paper, the model assumes perfectly rational agents, however, we find in the data used to calibrate the model that the investors deviate from this ideal. In the second paper, agents, even if they make a rational decision, are forced to adapt to the non-rational agents as they drive the price upwards. Finally, in the third paper, we extrapolated the current trend of carbon dioxide and human population on Earth into the future, inferring a collective behavior using macro-level equations.

The problem is that each individual in the three discussed themes acts rationally for himself (even he is over-optimistic), but for the community as a whole, it would be more beneficial if they would behave also on a global scale rationally to allocate the resources more efficiently. This failure to coordinate is a well-known problem associated with the aggregation of preferences and calls for the introduction of regulations and policies (or specific incentives and structural designs) that may help coordinate the choices of the decision makers. In the case of mutual funds or more generally for investments, investors seem to inefficiently allocate their capital because they are over-optimistic and correspondingly simply overpay their managers. Regulations need to be adapted so that investors can make more rational decisions in their own interest, either by obtaining better information and transparency, or through better education. For bubbles, we find that traders try to adapt to the increasing growth rate and hence push the bubble to grow even faster. From the community's standpoint, it would be more desirable to mitigate bubbles, as very often the government has to step in at the end of bubbles and assume financial responsibilities. Hence, policies and incentives should be changed to encourage more fundamental investing and less momentum trading. Finally in the last paper, most nations are unable or unwilling to follow the recommendations by the IPCC (UN Intergovernmental Panel on Climate Change) to reduce carbon dioxide emissions to a sustainable level. A consensus between nations with very optimistic views (which believe Earth can absorb even higher levels of carbon dioxide and see no indication for climate change), and nations with pessimistic views (which are heavily exposed to potential climate change and/or make respective costly investments in technology to cut their emissions) has to be found.

In the end, it is a question of trading the short term interest of the individuals against the long-term well-being of all.

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