ETH
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Market Selection in an evolutionary Market with Dividends generated by a percolation Model

Master Thesis

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May 28, 2010

Abstract

The purpose of this study was to investigate the impact of a possible price dependency in dividends on market selection in an evolutionary stock market model. Dividends were hitherto assumed to be exogenously determined, but the existing literature on dividend policy leads one to suspect that dividends are more complex and may be influenced by the share price. Another aim was to study the resulting price processes. The study was conducted using computer simulations. Dividends were modeled through a directed percolation model, which was adjusted to take account of different possible price influences on dividends.

The results indicated that a possible price dependency in dividends does not affect the market selection process, but generating dividends by a percolation model leads to an interesting distribution of prices. It was discovered that the resulting market capitalization of firms were distributed according to a power law.

Acknowledgement

First and foremost, I want to thank my supervising professors Prof. Dr. Thorsten Hens and Prof. Dr. Didier Sornette. This thesis would not have been possible without all their encouragement, guidance and support. Prof. Dr. Hens provided me valuable insights into evolutionary finance and his counseling kept me from losing the right track over the six months of writing. Prof. Dr. Sornette supported me exploring the world of percolation and the fruitful discussions with him led me to many new interesting paths. My supervisors introduced me to the secrets of pursuing research and modeling. There are still many depths left to be discovered, but I certainly got a very good basis from which to start.

Special thanks to Roger Wiget for lending me a great hand with the layout of the thesis document. His help protected me from many LaTeX pitfalls and gave the document a stunning look. Special thanks also to my mother Virginia for being very unhappy about my English and thereby significantly improving my vocabulary. The grammar mistakes almost took flight in panic.

I am very grateful to my sister Fabienne and to my boyfriend Fabian for proofreading my thesis and almost willingly discussing with me new ideas.

Contents

List of Figures

List of Tables

1

Introduction

Evolutionary finance is a relatively new approach to model and study financial markets by applying Darwinian ideas. In a financial context, the concept of natural selection directly translates into dynamics of the wealth distribution among different investment styles. While in biology, different organisms are fighting for resources, in finance investment strategies rival for market capital. The main goals are to gain a better understanding of how different investment styles perform in varying environments and their effect on asset prices. For a fuller introduction into this emerging subfield of financial economics, see Hens and Schenk-Hoppé (2005).

Based on evolutionary theory, Evstigneev et al. (2008a) developed an evolutionary stock market model. They observe that one investment strategy, the so-called Kelly rule, is particularly successful in the longer term under certain conditions. The existence of such an outperforming strategy would be of particular interest for investors with a long time horizon like pension funds and insurance companies. Furthermore, a dominant strategy would have a strong impact on the valuation of assets according to the specification of their model. Since stock prices result from market clearing, a dominant strategy would almost entirely determine prevailing prices. The Kelly rule is an investment strategy that invests into assets according to the expected discounted relative dividends they pay. Thus in a market controlled by the Kelly rule, the relative prices of the assets would reflect their fundamental value in terms of their relative payoff.

In this evolutionary stock market model, dividends play an important role. Therefore, the assumptions made on dividends may have a strong impact on any model results. Up to now, dividends were assumed to develop independently from any other market variable in the model, but dividends may be more complex. For example, they may be influenced by the share price. A higher market value could for example increase a company's financial strength and promote new investments. This in turn could lead to higher earnings and allow for higher future dividend payments. Another reason for a price influence on dividends could be that managers may use stock performance as an indicator for shareholders' dividend preferences. The more investors prefer a particular dividend level, the higher valued will be the

stocks paying dividends in that range. Thus, managers may use stock prices to interpret investors' dividend preferences, when they set their dividend policy.

The goal of my thesis is to analyze the impact of a possible price-dependency in dividends on the model outcomes. The analysis will be conducted through computer simulations. As long as dividends do not depend on other variables, we can run the simulations on real dividend data. This has already been done with companies of the SMI and DJIA (see Hens et al. (2002) and Hens and Schenk-Hoppé (2004)). However in case of endogenously determined dividends, this approach is no longer possible. Therefore, we need an appropriate model that is able to capture possible influences from prices.

For this purpose, we will introduce a model based on directed percolation. Later on we will see that directed percolation models are able to capture some distributional properties of dividends and allow specifying different rules on how they evolve over time, which enables us to include possible price influences.

Section 2 examines the evolutionary stock market model. Section 3 shows some empirical evidence on dividends, which we will try to capture by our model. Section 4 summarizes existing theories and models on dividends. Section 5 introduces the basis of our dividend model. Section 6 explains the investment strategies considered in the simulations. Section 7, 8 and 9 discuss the simulation results. Section 10 concludes.

2

The Role of Dividends in the Evolutionary Stock Market Model

In this section, we will further introduce the subject of our analysis: the evolutionary stock market model developed by Evstigneev et al. (2008a). Their model allows including a wide range of investment strategies and investigating their relative performance to each other. Their main finding in this regard is that a strategy investing proportional to the discounted expected relative dividends of assets, the so-called Kelly rule, performs exceptionally well under certain conditions. We will further explain how the success of a strategy is defined in this setting below. Moreover, we will pay special attention to the assumptions made on the model and on dividends in particular and examine their importance for our work.

2.1 The Evolutionary Stock Market Model

Let us start with a concise definition of the general setting. The model considered here is placed in discrete time $t = \ldots, -2, -1, 0, 1, \ldots$ with $t = 0$ denoting the initial time period. There have been also attempts to restate the model in continuous time as in Evstigneev et al. (2008a), but the discrete time case has been better explored so far. There are a number of external random factors influencing the stock market, which are modeled by an exogenous stochastic process $s_1, s_2, \ldots$. Thus at any point in time t, all these influences are summarized by the current "state of the world" s_t. The space of possible states S is assumed to be finite. Market variables may not only depend on the current state, but also on past events. In order to ease notation, we define $s^t = (\ldots, s_{t-1}, s_t)$ as the history of events up to time t.

The stock market model consists of two main entities: assets and investors. Assets, which in our context are stocks, are characterized by the dividends they pay and their price, whereas investors are described in terms of investment strategies and wealth. The interaction of assets and investors then determine the market dynamics. A more detailed definition of all market variables follows below.

Assets There are $K \geq 1$ assets present in the market. The number of assets K is constant over time. Furthermore, we assume the supply of each asset k to be constant over time and normalize it to one.

At each time step $t = 1, 2, \ldots$, every asset k pays total dividends of $D_{t,k}(s^t) \geq 0$. Thus, dividend decisions may depend on the current and past states of the world. Dividend payments are in cash. Cash can either be consumed or reinvested, which means that assets are the only store of value. Additionally, we assume that at least one asset pays dividends in each period:

$$\sum_{k=1}^{K} D_{t,k}(s^t) > 0 \text{ at each time step } t \geq 0. \tag{2.1}$$

In contrast to dividend payments, asset prices $p_{t,k}$ are endogenously determined by market clearing. Since we have assumed unit supply, the price corresponds to the market capitalization of a firm. Throughout the thesis, we will use both terms interchangeably. The price mechanism will be further explained, when we have introduced all components of the model.

Investors There are $I \geq 1$ investment strategies present in the market. Every investment strategy is specified by a vector of time- and history-dependent portfolio weights $\lambda_t^i(s^t) = (\lambda_{t,1}^i(s^t), , \lambda_{t,K}^i(s^t))$. The portfolio weights $\lambda_{t,k}^i \in [0, 1]$ denote the budget share that an investor pursuing strategy i invests into asset k at time t, hence

$$\sum_{k=1}^{K} \lambda_{t,k}^i(s^t) = 1. \tag{2.2}$$

The total wealth at time t of all investors pursuing strategy i is denoted by w_t^i. We consider only strategies with positive initial wealth $w_0^i > 0$. Investors reinvest a fraction $0 < \rho < 1$ of their wealth into assets in the following period and spend the rest $(1-\rho)w_t^i$ on consumption. We assume the saving rate ρ to be the same for all investors and to be constant over time. This allows us to compare the performance of different investment styles without distortions from the choice of consumption plans.

Now that we have introduced all components of the model, let us get back to asset prices. As mentioned above, they are determined by market clearing. The price of asset k at time t is

$$p_{t,k} = \langle \lambda_{t,k}, \rho w_t \rangle = \sum_{i=1}^{I} \lambda_{t,k}^i \rho w_t^i. \tag{2.3}$$

The performance of investment strategies is governed by their interaction in the market. As mentioned before, all investors of type i purchase assets with a fraction of their wealth ρw_t^i at every time step t. In the following period, they will then benefit from dividend payments and capital gains according to their investment. If

they invest $\lambda^i_{t,k}(s^t)\rho w^i_t$ of their total wealth into asset k at time t, they currently own a number of shares issued by asset k

$$\frac{\lambda^i_{t,k}(s^t)\rho w^i_t}{p_{t,k}} = \frac{\lambda^i_{t,k}(s^t)w^i_t}{\langle \lambda_{t,k}(s^t), w_t\rangle}. \tag{2.4}$$

In the following period, they will then receive dividends of

$$D_{t+1,k}\frac{\lambda^i_{t,k}(s^t)w^i_t}{\langle \lambda_{t,k}(s^t), w_t\rangle}. \tag{2.5}$$

Accordingly, their share in asset k will be worth

$$p_{t+1,k}\frac{\lambda^i_{t,k}(s^t)w^i_t}{\langle \lambda_{t,k}(s^t), w_t\rangle}. \tag{2.6}$$

Thus, the wealth dynamics are governed by the following equation

$$w^i_{t+1} = \sum_{k=1}^{K}\left(D_{t+1,k}\left(s^{t+1}\right) + p_{t+1,k}\right)\frac{\lambda^i_{t,k}(s^t)w^i_t}{\langle \lambda_{t,k}(s^t), w_t\rangle} \tag{2.7}$$

for all investment strategies i in each time period t. The wealth dynamics can be restated in relative terms

$$r^i_{t+1} = \sum_{k=1}^{K}\left((1-\rho)\, d_{t+1,k}\left(s^{t+1}\right) + \rho\langle \lambda_{t+1,k}, r_{t+1}\rangle\right)\frac{\lambda^i_{t,k}r^i_t}{\langle \lambda_{t,k}, r_t\rangle} \tag{2.8}$$

by defining relative wealth shares as

$$r^i_t = \frac{w^i_t}{\sum_{n=1}^{I} w^n_t}, \tag{2.9}$$

relative dividends as

$$d_{t,k}(s^t) = \frac{D_{t,k}(s^t)}{\sum_{n=1}^{K} D_{t,n}(s^t)} \tag{2.10}$$

and inserting the market clearing condition (2.3). In matrix form, (2.8) becomes

$$r_{t+1} = (1-\rho)\,\Theta_t d_{t+1}\left(s^{t+1}\right) + \rho\Theta_t\Lambda_{t+1}r_{t+1} \tag{2.11}$$

where $r_t = (r^1_t, \ldots, r^I_t)^T \in \mathbf{R}^I$, $\Theta_t \in \mathbf{R}^{I\times K}$, $d_t(s^t) = \left(d_{t,1}\left(s^t\right), \ldots, d_{t,K}\left(s^t\right)\right)^T \in \mathbf{R}^K$ and $\Lambda_t = (\lambda^{1^T}_t, \ldots, \lambda^{I^T}_t) \in \mathbf{R}^{K\times I}$.

Evstigneev et al. (2006) provide a proof that the evolution of wealth is well-defined in all periods of time as long as at least one investment strategy is completely diversified. In their paper they use a slightly different notation by treating cash used for consumption as an asset 0. But we can translate the proofs to our case by noting $\rho = \lambda_0$ and that their $\lambda^i_{t,k}$ correspond to $\rho\lambda^i_{t,k}$ in our notation.

2.2 Survival and Stability

The main object is to analyze the performance of different investment styles in the model described above. We can measure the success of an investment strategy by its wealth gains relative to other strategies. Strategies that are more successful will accrue more wealth over time and drive out less successful ones. The question is which strategies will perform particularly well. Generally answering this question seems an almost impossible task, since the performance of a single investment strategy depends on what strategies the other market participants pursue. Nevertheless, it can be proved that one particular strategy, the so-called Kelly rule, will dominate the market in the longer term no matter which strategies it is competing with if some further restrictions are imposed on the model.

The Kelly rule is defined as

$$\lambda^{\star}_{t,k} = \frac{1-\rho}{\rho} E\left[\sum_{l=1}^{\infty} \rho^l d_{t+l,k}\left(s^{t+l}\right) \,\middle|\, s^t\right]. \tag{2.12}$$

Evstigneev et al. (2009) provide an overview over existing results on the dynamics of the model and in particular on the performance of the Kelly rule. We will summarize them as follows:

Survival Let us start with the definition of survival. An investment strategy $\lambda^i = (\lambda^i_t(s^t))$ is said to *survive* with probability one if $\inf_{t\geq 0} r^i_t > 0$ almost surely. Accordingly, we define an investment strategy to *become extinct* with probability one if $\lim_{t\to\infty} r^i_t = 0$ almost surely. These two definitions do not involve the choice of competing strategies. Thus, survival strategies in this sense are competitive in any pool of strategies.

Amir et al. (2009) prove that the Kelly rule is a survival strategy. The only additional assumption they need to show that the wealth of Kelly investors is not continually shrinking is

$$\lambda^{\star}_{t,k} = \frac{1-\rho}{\rho} E\left[\sum_{l=1}^{\infty} \rho^l d_{t+l,k}\left(s^{t+l}\right) \,\middle|\, s^t\right] > 0 \quad \text{a.s.} \tag{2.13}$$

for all k and t. Moreover, they show that the Kelly rule is asymptotically unique among all survival strategies which are either constant or depending only on the history of states s^t. Thus, the Kelly rule is the only survivor as long as we restrict competing strategies to be independent of market history. As soon as we allow strategies to depend on prices or past investments, there may be strategies co-existing or even better performing than the Kelly rule.

Stability The notion of stability is stronger than the one of survival. Stability means that a strategy will not only survive in the longer term, but also drive competing strategies out of the market. We distinguish between local and global stability.

To be more precise, an investment strategy λ^i is called *globally evolutionary stable* if it survives with probability one while all other strategies become extinct with probability one, of course provided that λ^i started with positive initial wealth. This means that a globally evolutionary stable investment strategy would be the ultimate winner rule. No matter which strategies it is competing with, it would eventually drive all others out of the market.

Indeed, the Kelly rule has been proved to be globally evolutionary stable if we impose further restrictions on the investment strategies and dividend payments considered. Assumed that the state of the world follows an i.i.d. process, there are no redundant assets and dividends are stationary, the Kelly rule then is globally evolutionary stable in the pool of constant and completely mixed strategies (cf. Evstigneev et al. (2008b). In addition, the Kelly rule reduces to the following form

$$\lambda_k^\star = E\left[d_k(s)\right]. \tag{2.14}$$

Local stability is a weaker notion of stability. It is defined analogously to global stability additionally requiring that the initial market share of the competing strategies is small enough. Thus if a locally evolutionary stable investment strategy has enough market power, it will drive any competitor out of the market.

The Kelly rule has been proved to be locally evolutionary stable in a more general setting than required for global stability. Assumed that the state of the world follows a Markov process and dividends are stationary, the Kelly rule then is locally evolutionary stable (cf. Evstigneev et al. (2006)).

2.3 Why Model Dividends?

In this section, we will discuss the assumptions made on dividends. The underlying assumptions are crucial for the validity of a model. Hence, it is important to get an understanding on how robust conclusions from the model are if any of the assumptions is relaxed in a meaningful way. The following explanations will show that the relaxation of some particular assumptions require modeling dividends.

The model assumes that dividend payments are nonnegative and that at each time step at least one asset pays a strictly positive amount. This seems a reasonable assumption. Considering a large number of assets, there will be most probably at least one asset paying out a strictly positive amount at each time step.

Furthermore, the model assumes that the number of assets as well as the supply of every asset is constant over time. These assumptions are far more restrictive. The issue of constant supply over time has already been tackled by adding a state- and asset-dependent supply to the model (see for example Amir et al. (2009)).

The number of assets is certainly not constant over time as well. Firms are entering and leaving the market continually. Schweri (2006) analyzes the extent and reasons of these changes and the resulting impact on the evolutionary stock market model. Particularly striking is the fact that companies on average remain listed on

the stock exchange for a rather short time. When analyzing companies listed at NYSE, AMEX and NASDAQ, Schweri (2006) observes that companies on average remained in the sample for less than ten years. Consequently, time series of single companies are too short to draw any conclusions for company evaluation. A way to include this effect into the above mentioned model can be found in Schweri (2006). According to his work, investors may circumvent this problem by considering cross-sectional information rather than analyzing the time series of a single company. Nevertheless, we assume the number of assets to be constant for the following dividend model, since we want to put a focus on another possibly strong assumption. A varying number of assets may still be incorporated to the model later on, but this will not be done in the following.

Besides the above-mentioned assumptions, the existing proofs on survival and stability of investment strategies require even more restrictions on the dividend process. As long as there are no less restrictive proofs, we can test the validity of the results on survival and stability in a more general framework only through computer simulations. Investment strategies then may be prone to estimation errors and do not properly reflect potential theoretical results, taking us one step closer to reality.

The advantage of computer simulations is that we can input real data on past dividends to the model. Thus, we do not rely further on possibly unrealistic restrictions. Results on simulations with companies from the SMI and DJIA can be found in Hens et al. (2002) and Hens and Schenk-Hoppé (2004). In the chosen set of strategies and assets, the Kelly rule still outperformed all other strategies in the longer term. This was even true when the investors following the Kelly rule were endowed with a smaller initial wealth relative to other strategies. These results require careful interpretation. As they were based on the model assuming a constant number of assets, only dividend streams that were long enough could be used for simulation. These survival effects may bias simulation results.

Another still unsettled issue is the possible dependency of dividends on other market variables. The currently available proofs and simulations assume that dividend payments are exogenously determined. The obtained results may no longer hold if dividends are influenced by the market history. We could imagine various ways in which past price developments may affect dividend payments. Managers may consider share price developments when deciding on dividend payouts. The dividend policy they set should of course attract investors. In order to identify investors' preferences, they may consider prices as an indicator on how investors valuate different levels of dividend payments. The more investors prefer a particular dividend level, the more will demand for stocks paying dividends in that range drive up stock prices.

Price influences on dividends may also occur in a more indirect way. If new investments would require equity, higher valued firms would be more likely to undertake them. These investments in turn may lead to higher future earnings. Thus, higher dividends could be distributed to shareholders. Unfortunately, the dimension

of price effects on dividends has been discussed very little in the literature. Therefore, we will check the robustness of the results obtained from the evolutionary stock market model under varying price influence on dividend payments.

If we assume the dividend process to be price-dependent, the currently available proofs for stability are no longer applicable. Moreover, a different approach for computer simulations is required. Since dividends then are endogenously determined, we can no longer input real data to the model. Thus, we need a dividend model that allows adjustment to potential share price influences.

3

Empirical Evidence on Dividends

A dividend model should be able to capture the most essential features that characterize dividend payments. Identifying some common features between companies has proved to be difficult and led to a variety of different dividend theories in the past. We will discuss these further in the next section. The problem is that the benefit of paying dividends is contentious and there are various dividend policies adopted by companies. Thus, some of the empirical facts given below will focus more to the differences between companies. The data on total dividends as used in the following stems from Schweri (2006), who calculated them from the CRSP database. It includes 23956 North American firms listed at NYSE, AMEX and NASDAQ between 1973 and 2008.

3.1 Aggregate Dividends

Nominal aggregate dividends have almost steadily increased over the last thirty years. Figure 3.1 shows dividends aggregated over sectors as in Schweri (2006). We observe a steep increase in total dividends as they rose at an average yearly growth rate of more than 8% since 1973. However, the global financial and economic crisis has left its mark in 2008. After more than 30 years of continuous increases or minor declines, aggregate dividends decreased by almost 3.4% in 2008.

Examining real aggregate dividends, we get a very different picture. Dividends are converted into 1973 dollars using the consumer price index. Aggregate dividends are no longer steadily increasing and show pronounced declines in some of the years. Dividends increased at an average yearly growth rate of around 3.2%. The difference between nominal and real growth is very large because of exceptionally high inflation rates in the early seventies. Overall, we can say that dividend growth moved pretty much in line with GDP growth until recent years. Figure 3.2 compares aggregate dividend growth with GDP growth. The dividend growth behavior seems to have changed markedly over the recent years and to overshoot GDP growth more than ever before. This abnormal behavior of dividends preceding the most recent crisis would be interesting to investigate further, but is beyond the scope of my thesis.

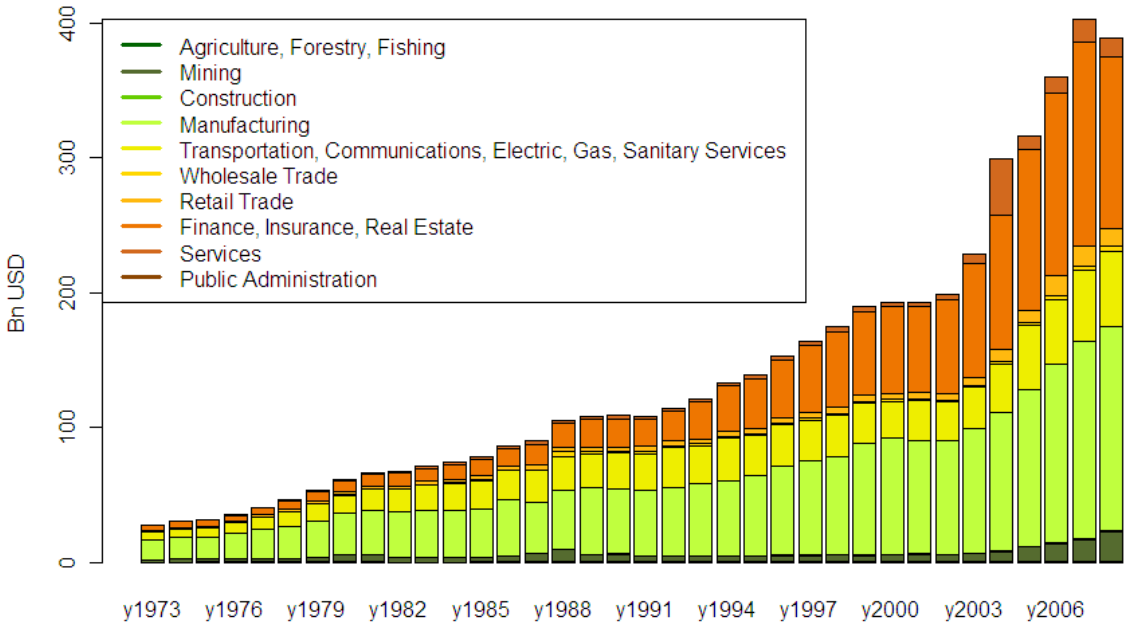

Figure 3.1: Nominal dividends aggregated over sectors

Obviously seen from figure 3.1, there are huge differences between sectors. The financial and the manufacturing sector in particular increased dividends more rapidly than any other sector. Thus, these two sectors accounted for around 71.7% of aggregate dividends paid in 2008. From the above analysis, we see that industries differ significantly in the total dividend amount they pay and in how total dividends develop over time. As we will see below, these differences become even more apparent when we break our analysis down to a company level.

3.2 Concentration

The total amount of dividends paid varies massively between different firms. According to Schweri (2006), there is a small fraction of firms accounting for the majority of aggregate dividends. On the other hand, there are more than fifty percent of the firms paying no dividends at all. This large concentration of dividend payments has even significantly increased over the past thirty years.

DeAngelo et al. (2004) further investigated this issue for nonfinancial and nonutility firms. Their empirical findings indicate that an increased concentration in earnings may have led to a similar development in total dividends. They found that both, earnings and total dividends are highly concentrated and that this concentration jointly increased between the years 1978 and 2000. Furthermore, the top earners in their sample account for the majority of aggregate dividends.

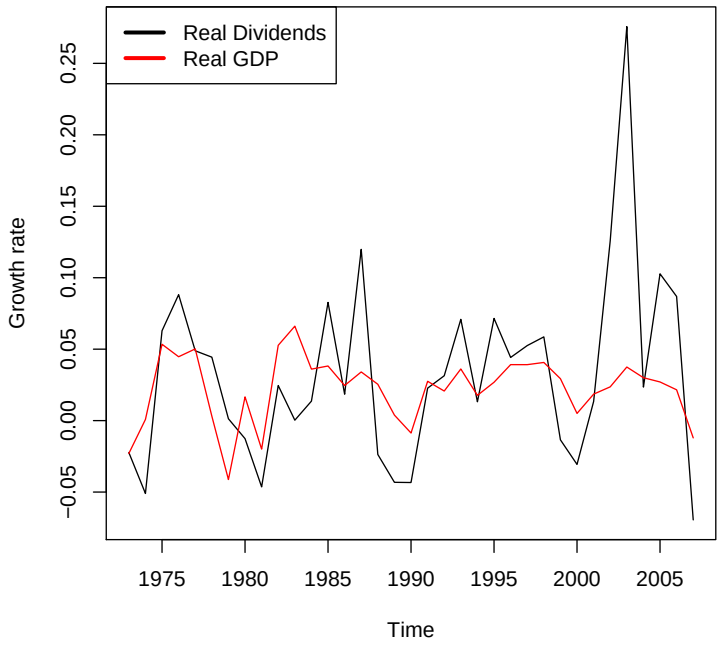

Figure 3.2: Real dividends growth compared to real GDP growth

In order to get a better understanding of the structure of total dividends, let us have a closer look at the distribution. Plotting the histogram of log dividends as in figure 3.3, total dividends seem log-normally distributed at first glance. Having a look at the QQ-plots as displayed in figure 3.4 and in appendix A, we see that log dividends are close to normal in the center part, but seem to differ from normal in the tails for most of the years.

Since firms in the tail of the distribution apparently account for a large majority of aggregate dividends, we are more interested in the tail of the distribution. As we will see below, the tails are closer to a Pareto distribution rather than lognormal. This means that the tail is fatter than in the case of a lognormal distribution and that particularly large dividend payments occur far more often than a lognormal distribution would suggest.

The Pareto and the lognormal distribution are often difficult to distinguish. Even though both distributions exhibit a clearly distinct behavior in the tails, they can look very similar over a large range of data. For discriminating between the Pareto and the lognormal distribution in the tails, we apply the uniformly most powerful unbiased (UMPU) test of the Pareto against the lognormal as suggested by Malevergne et al. (2009). The UMPU test is a statistical test that tests the null hypothesis of a power law against lognormality using a different number of data points in the

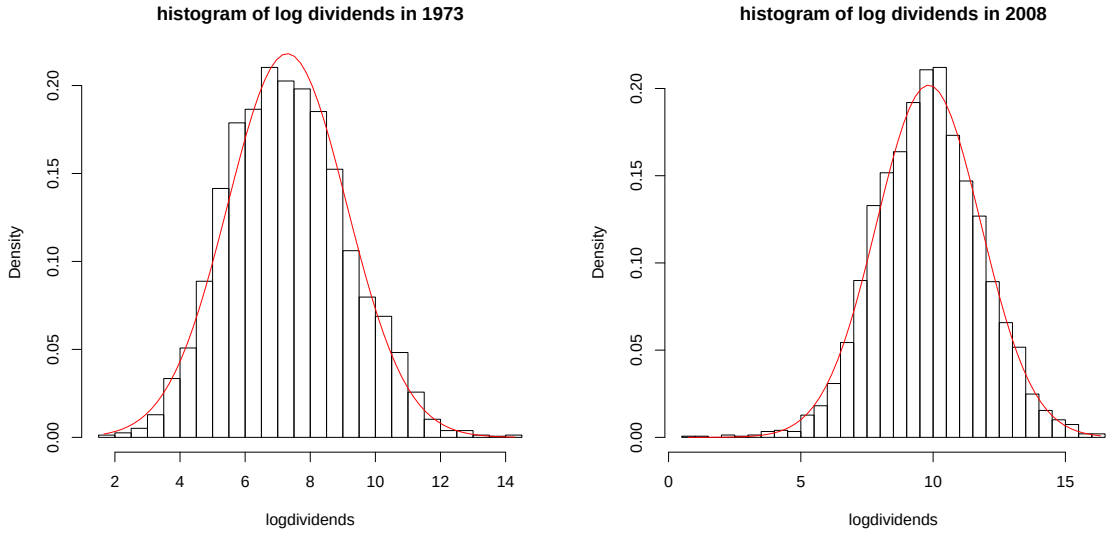

Figure 3.3: Histogram of log dividends in 1973 and 2008

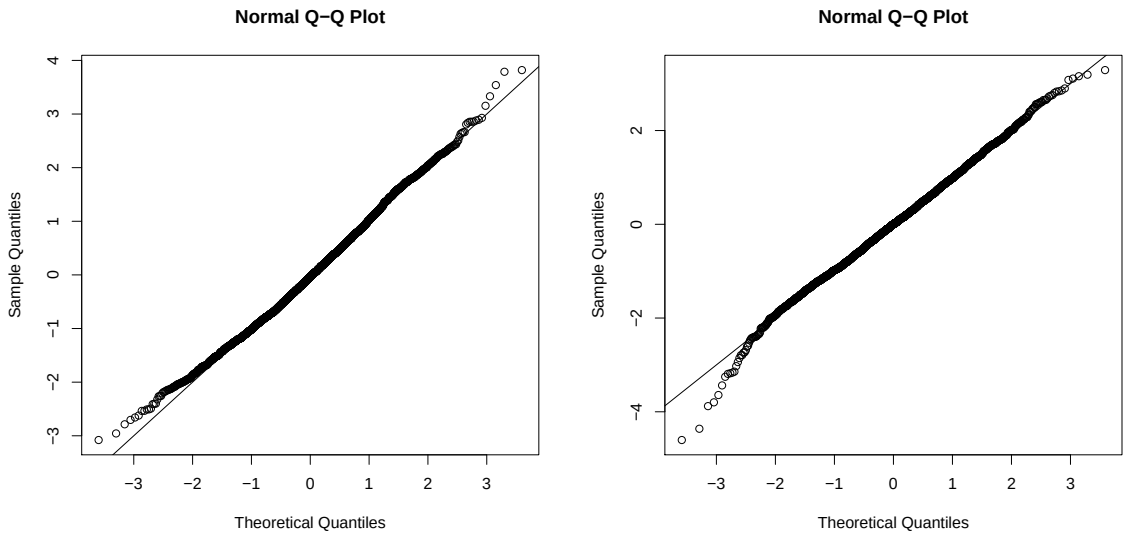

Figure 3.4: Normal QQ-plots of log dividends in 1973 and 2008

tail of the distribution. Thus, dividends are assumed to be distributed according to a power law above some threshold level under the null hypothesis. This hypothesis is tested against the alternative that dividends above that threshold level follow a lognormal distribution. Testing for different tail lengths allows to better explore the tail of the distribution. The two plots in figure 3.5 depict the p-values corresponding to different tail lengths in the years 1973 and 2008. The abscissae show how many of the largest dividend payments were included for testing at a time and the ordinates

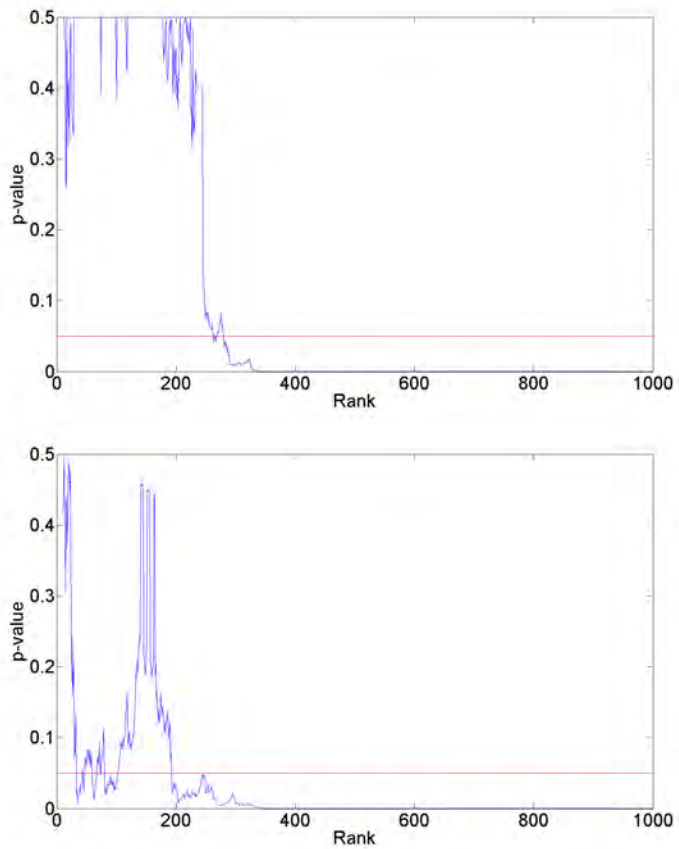

Figure 3.5: UMPU test applied to dividends in 1973 and 2008

show the respective p-values. The results for other years between 1973 and 2008 can be found in appendix B.

According to these graphs, we cannot reject the null hypothesis of a Pareto distribution in the tails. Moving towards the center part of the distribution however, we clearly reject the null hypothesis in favor of a lognormal distribution. Summing up, large occurrences seem to carry much more weight in aggregate dividends than a lognormal distribution would suggest. When modeling dividends, we therefore try to account for the heavy tail.

Assuming that the tails follow a Pareto distribution, we can now estimate how slow the tail decays. The complementary cumulative distribution function of dividends then is inversely proportional to a power of its size. Thus

$$1 - F(D) \sim D^{-\alpha}, \tag{3.1}$$

where α indicates the fatness of the tail. A smaller α means that the tail is fatter. In order to determine the size of the α, we employ the Hill estimator. Since the Hill estimator depends on where we cut-off the tail, we apply the Hill estimator to

different numbers of observations in the tail and plot the estimates in a Hill plot (see Drees et al. (2000)). Figure 3.6 shows the Hill plots of the years 1973 and 2008. The plots of other years can be found in appendix C.

We see that there is a plateau in the α-estimates of the approximately largest 200 dividends in 1973 and of the approximately largest 50 dividends in 2008. In all the years, we observe an α in a range of around $1.2 - 1.6$.

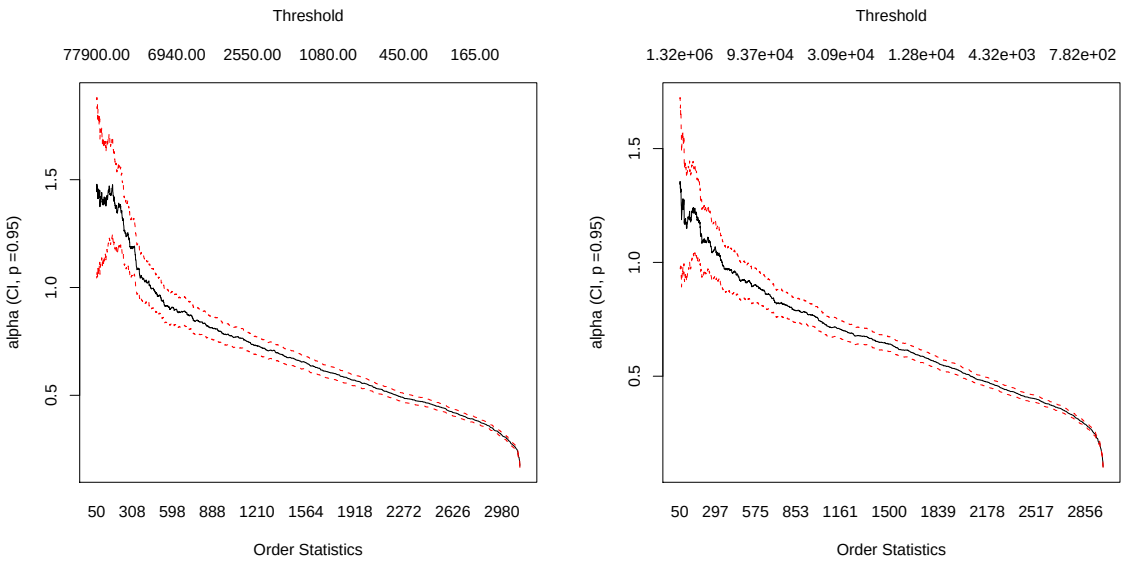

Figure 3.6: Hill plots of dividends in 1973 and 2008

3.3 Variability

The dividend patterns of different firms vary widely. Schweri (2003) analyzes in his work if relative dividends are stationary. He observes that no matter whether firms are grouped according to their market capitalization, return or dividend yield, at least one group of firms exhibits a significantly different average on yearly dividend growth. Thus, the mean of relative dividends seems to vary over time and relative dividends seem to be non-stationary. Individual firm growth patterns differ even more. Whereas for example Exxon Mobil, Wal-Mart and others almost continuously increased their dividends, there are many firms, particularly technology firms like for example Microsoft that never raised dividends for a prolonged time. Between these two extremes of continual growth and no growth at all, we find all kinds of different dividend growth paths. Thus, finding measures characterizing intertemporal correlations in general would be rather difficult. We would not only have to adjust for different levels in order to compare different dividend streams, but also to account for differing time trends.

However, there are other commonalities regarding variability. According to the management surveys conducted by Brav et al., managers are reluctant to cut dividends and follow rather conservative dividend policies. It seems to be a common held belief by managers that dividend cuts are likely to entail negative consequences. Besides, consistency with the past dividend policy of the firm seems to be an important factor for future dividend decisions. In our sample, around 72.7% of all dividend changes are positive. Thus, the data reflects indeed a reluctance to cut dividends. In more than half of the years that companies were in the sample however, dividends were left unchanged.

According to Schweri (2006), another characteristic of dividends are large jumps in their time series. To get an idea about how large these jumps are, we look at the median of log dividends and at the quantile differences. We ignore zero values in dividend payments to avoid problems with taking the logarithm. Considering nominal dividends, the median of the jumps is somewhere between 6% and 8% in most years. In years when inflation was high, the jumps tended to be much higher. In case of real dividends, the median of the jumps is only around 0% to 5% in most years and being negative in some years. The interquartile range of the jumps are somewhere between 18% and 24%. Thus, the dispersion of jumps is rather large.

4

Existing Dividend Theories

In order to find a starting point for our model, let us scan existing dividend theories and models for possible price influences on dividends. The literature on dividends is highly diverse and for most theories, there is only very limited empirical support. Black (1976) states appositely: "The harder we look at the dividend picture, the more it seems like a puzzle, with pieces that just don't fit together." Even though many researchers tried to solve the dividend puzzle, there are still many issues left open. Below we will briefly examine some main branches in dividend research and try to detect different reasons for a possible price influence on dividends. In order to structure the large variety of approaches, we will take the classification used by Frankfurter and Wood (2002). In their work, they classify dividend models according to the agents' rationality into full information models, models of information asymmetries and behavioral models.

4.1 Full Information Models

The work of Miller and Modigliani (1961) was a milestone in the development of full information models. Their main conclusion is that in an ideal economy with perfect capital markets and rational investors the dividend policy of a firm does not affect its market value. This is derived from the basic assumption that the rate of return has to be equal for every asset in the market over any period in time. The rationale behind this assumption is that investors would preferably sell low-return shares and buy high-return shares instead if the rates of returns differed between assets. This would drive down the prices of low-return shares and drive up the prices of high-return shares so that the return of both shares would equalize.

The assumption of equal rate of returns then implies that the dividend policy of a firm does not affect its market value. The intuition behind is that the value of the firm reflects total dividends paid over some period in time plus the value of the current number of shares at the end of that period. When firms finance their investments, they have to either lower dividends and rely on higher retained earnings or issue new shares, which would decrease the future value of the current

shares. Thus, the current value of the firm would be lower either because dividends are lowered or because of the lower future value of current shares. No matter how a firm decides to finance investments, the effect on the market value will be the same. The dividend policy does not affect the market value.

In their concluding remarks, Miller and Modigliani (1961) point out that the next step to their work would be the relaxation of the assumption of perfect capital markets. In particular, their suggestion of including tax effects has been analyzed thoroughly by further research. For a list of papers discussing taxation models see Frankfurter and Wood (2002). Differences in taxes may alter investors' preferences between capital gains and dividend payments. In countries with a much higher tax on dividends than on capital gains, investors would probably prefer stocks paying no dividends to stocks paying dividends. On the contrary Miller and Scholes (1978) state, that investors could circumvent dividend tax in several ways depending on the prevalent tax law. Thus, the impact of taxation on dividends remains questionable and the empirical evidence on tax effects is mixed (see for example Baker et al. (2002)).

4.2 Models of Information Asymmetries

This class of models focuses on information asymmetries between shareholders and managers of a corporation. Three particularly salient theories with regard to information asymmetries are the signaling theory, agency cost theory and the free cash flow hypothesis.

The signaling argument is based on the work of Bhattacharya (1979) and Miller and Rock (1985). Signaling models examine the role of dividend payments as a means for managers to communicate private information to investors. Most ways of reporting profitability are subject to moral hazard. If a manager possesses information that would positively affect the market value of the corporation, he strives to reduce this information asymmetry. Dividends can serve as a credible signal, since they come at a cost and the costs differ between the more and the less profitable companies.

In the model of Bhattacharya (1979) the underlying assumption is that managers operate in the interest of shareholders. Thus, they try to maximize the after-tax objective function of shareholders. The model assumes that shareholders benefit from a rise in liquidation value when dividends are raised. The signaling is costly, since dividend payments are taxed at a higher rate than capital gains. There are additional costs if dividends exceed the generated cash flow and have to be financed externally, which is more costly due to transaction costs. Thus sending wrong signals is more expensive.

Miller and Rock (1985) also assume that managers seek to maximize shareholder value, but in contrast to Bhattacharya (1979), their model does not take into account tax considerations. Moreover, the objective function depends on whether

investors are planning to hold or sell their shares. Dividends again are assumed to indicate earnings. By paying higher dividends, managers could temporarily increase the firm's value until true earnings are revealed. Shareholders selling their shares during that time then profit from the rise in price, while unsold shares adjust to the true value afterwards. Thus, the objective of maximizing shareholder value depends on whether investors sell or hold their assets. This issue is resolved by introducing a social welfare function attaching weights to the interests of sellers and holders proportional to the value of their shares. Miller and Rock (1985) show that there exists some dividend policy leading to an informational consistent equilibrium. However, levels of investment would then be lower than in the full-information case and the equilibrium would be less efficient.

Agency cost explanations focus on another kind of information asymmetry. Managers are no longer assumed to maximize shareholder value, but act on behalf of their own interests. Dividends are a way to align ownership and control interests. Easterbrook (1984) suggests two agency-cost explanations for the existence of dividend payments. One cause of agency costs is the monitoring of managers. If a shareholder decides on monitoring, he incurs the full monitoring cost, but his profits are proportional to the share of his holdings. Consequently monitoring through shareholders occurs too little. Another source of agency cost comes from the managers' risk aversion. Managers often are compensated with options and a large part of their wealth is tied to the company. In contrast to shareholders, they have fewer possibilities to diversify their risk. Thus, they favor projects that bear less risk, but imply lower returns than preferred by shareholders. Dividends help to overcome both problems. Dividend payments reduce free cash flow and force managers to raise funds more frequently in capital markets, where the firm is subject to the scrutiny of investment bankers and other financial intermediaries. Managers have an incentive to reduce agency-costs enabling them to have a more favorable access to capital markets. In case of higher agency-costs, new investors may require compensation in form of lower prices for any monitoring costs they incur.

According to signaling models, price considerations play an important role in managers' dividend decisions. Managers may take the past performance of prices as an indicator for their shareholders' preferences, which they can use to estimate the optimal dividend level.

4.3 Behavioral Models

Behavioral models analyze behavioral patterns in human decision-making. There are two approaches with respect to financial markets. One focuses directly on managers decisions. Another one examines investors' behavior. This in turn influences managers' decisions, assuming that manager seek to operate in investors' interest.

Lintner (1956) offers an approach that directly addresses managers' behavior. Through managerial surveys, he identifies the main drivers of dividend decisions.

An important finding is that most firms decide on the rate of dividend payment as a percentage of earnings rather than on the dividend level itself. Adjustments in the dividend payout ratio include longer-term considerations. Furthermore, dividend adjustments toward the target payout take place timely phased. Consequently, dividend payments are less volatile than net earnings.

The target payout ratio varies widely between firms. Dividend policy depends on the companies' experience and often evolves out of past decisions. However, Lintner (1956) mentions some important factors that enter more or less rationally dividend considerations. Probably the most important are the growth prospects of the industry and of the particular firm.

This model seems to have relatively strong empirical support. In his work, Lintner (1956) could explain a large part of the sample data with his model. According to Allen and Michaely (1995) later studies confirmed the good performance of the model. Moreover, Brav et al. conducted another managerial survey fifty years later checking the findings of Lintner (1956). Their findings are similar except that the importance of earnings for dividend decisions seems to have weakened. After all, the underlying concepts of the model seem to be still relevant today.

An approach focusing on the investor's side is offered by Shefrin and Statman (1984). They incorporate theories of individual choice behavior into standard financial theory. This leads to a number of explanations why investors demand cash dividends, among them are increased self-control, a different assessment of gains and losses in case of dividend payments and regret aversion. Under these aspects, capital gains and dividend payments are no longer substitutes, even if taxes and transaction costs are not taken into account.

An application of the theory of self-control is for example retirement planning. In order to increase their self-discipline to save enough money for future use when they get older, people impose rules on themselves like for example no investment capital is used for consumption. Therefore, the lower the dividend yield and the higher the capital gains the higher is the corresponding saving rate for retirement, but the lower is what they have got left for consumption.

Prospect theory provides another reason for why capital gains are not a perfect substitute for dividends. If there is a rise in price, people tend to segregate between capital gains and dividend yields and value them separately. However, in case of losses, they tend to value the capital loss and dividend yields jointly. Thus, dividends are perceived as an additional benefit when prices rise, but as a partly offset of losses when prices fall.

Another reason for dividend demand stems from the theory of regret aversion. The underlying assumption is that missing an opportunity feels worse if it is due to a deliberate action than not taking any action at all. If you finance consumption by the dividends you received, you would probably feel less regret in case of a following price increase than if you would have sold the same amount of your stocks in order to finance consumption.

These behavioral theories do not directly address the issue of a possible price

influence on dividends. Nevertheless, they do not necessarily exclude price effects. Let us reconsider the model of Lintner (1956). A higher market value may enable firms to access capital markets at terms that are more favorable. The firm's financial strength may influence managers in setting the dividend payout ratio or promote more investments, which may have an impact on future earnings.

5

The Dividend Percolation Model

Modeling dividend payments, we use a directed percolation model. Percolation models find many applications in several fields and disciplines such as chemistry, physics, geography and other sciences. They are a powerful tool to capture the structure of heterogeneous media. An example would be the spread of oil or gas in a porous rock as described by Stauffer and Aharony (1995). In our context, we will use a percolation model to capture the huge differences in the level of total dividends paid by different firms.

Directed percolation models are a special kind of percolation models. They introduce additionally the notion of direction. In our model, we will use direction to model time. As we will see, directed percolation models are able to capture many features that characterize dividends and they are flexible to include potential influences from other variables like for example prices. Before we start building a dividend model, we will further explain percolation in the following section.

5.1 Percolation Theory

Stauffer and Aharony (1995) provide an extensive introduction into percolation theory. In this section, we will focus on the part of their book on how percolation models are constructed. Let us start with a square lattice. This square lattice is assumed to be large so that we can neglect the influence from the boundaries. Each site on that lattice is either occupied or vacant. Two sites with a common edge are considered to be neighbors. Having defined neighborhood, we can now identify groups of neighboring sites. Such maximal connected sets of neighboring sites are called clusters.

Percolation theory deals with the number and properties of clusters. These properties depend on what topology we choose and how we populate the model with occupied and vacant sites. The topology does not necessarily have to be a square lattice. There are plenty of other structures and in other than two dimensions used in percolation models. In our model however, we will stick to the square lattice.

Likewise, there are many different ways to populate the model. We can choose rules like for example each site is occupied with a probability p and vacant with a probability $1 - p$. According to this rule, the state of each site would evolve independently of any other site. On the other hand, we could also choose rules that include dependencies between sites or other variables. The choice of a set of rules gets even more interesting in the case of directed percolation.

Sornette (2004) provides a brief introduction into directed percolation. He points out that there is a large variety of applications for this kind of model, since the notion of direction can be interpreted in many different ways. The interpretation we will focus on is time. In our model, we assume discrete time. At each time step, there is a new configuration of occupied and vacant sites on the lattice. We interpret the configurations on the lattice as the states of the world like we have introduced it in chapter 2.

The development of these states over time is specified by some transition rules. Whether a site will be occupied at the next time step will depend on whether it is already occupied at the current time step. Furthermore, the transition rules may depend on other factors from the evolutionary stock market model like for example prices. If transition rules depend on past realizations of either the sites themselves or other variables, we additionally have to care about the initialization of the model. The appropriate choice of the initial conditions entails some difficulties, since it may have long-lasting after-effects.

5.2 The Dividend Model

The economic interpretation of our model is inspired by Lintner (1956). In his work, he observes that the main determinants of a change in dividend payments are earnings and the companies' target payout ratio. Dividends are only partially adjusted to changes in earnings. As mentioned in section 3, managers mostly follow a rather conservative dividend policy. The partial adjustment reflects this cautiousness with respect to changes in dividends. Thus, dividend payments probably correspond to more persistent factors of success than earnings. Lintner (1956) lists some factors that may enter managerial decisions more or less consciously. Among the more important factors in his view are the soundness and growth opportunities of companies and of the respective industries, investment opportunities and shareholders' dividend preferences.

This is the starting point for our dividend model. We will now model the state process as the development of firms' businesses. Additionally, we introduce some similarity between firms, which should capture the notion of industries. Based on that, we then define a dividend function that reflects managers' dividend decisions considering the current state of the world.

The basis of our model is a large, but finite square lattice, which we denote as a $d \times d$ matrix $S_{i,j}$. Each column of this matrix represents a firm. Neighboring

firms are assumed to operate in the same or closely related businesses. Let us give an example. If we interpret one column as apple, the column next to it could be interpreted as microsoft. The rows of the matrix represent the potential business lines of a firm. The middle rows correspond to potential core business lines, while rows further away from the center denote less important business lines. Let us come back to the example of apple. Sites in rows towards the middle would then correspond to business lines that are related to the production and sales of core products like macs, ipods and iphones, whereas sites in rows further away from the middle would correspond to business lines related to products like accessories. This means that firms with a site occupied in the same row do not necessarily operate the same kind of business line, but the business lines in the same row are equally important for their respective core businesses.

All sites of the lattice can be either occupied or vacant. Thus, $S_{i,j} = 1$ if the site is occupied and $S_{i,j} = 0$ if vacant. An occupied site means that the corresponding business line of that firm exists. This means that the more occupied sites are in a column, the bigger the corresponding firm. The state space then is represented by all possible configurations of occupied and vacant sites on that lattice $\{0, 1\}^{d \times d}$.

At each time step, firm j pays dividends if the corresponding site in the middle row $S_{\lceil d/2 \rceil, j}$ is occupied. The level of dividend payment then is calculated as the size of the cluster containing that site times the size of the one-dimensional cluster in column j containing that site. By considering the two-dimensional cluster, we allow connected firms, mostly operating in similar businesses, to have an impact on the dividend payments of firm j. If we interpret firms that are close to each other on the lattice as an industry, this cluster captures similarities in dividend policies within industries.

The one-dimensional cluster in the column gives the company an additional weight and positions it within an industry. By considering only the middle row to determine whether a company pays dividends or not, we allow firms to grow without paying any dividends as long as they do not cross the middle row. Thus, firms can already be large when they start paying dividends and may immediately raise dividends to a substantial level.

In later sections, we will consider different kinds of transition rules. The rule we start with prescribes that the future state of site $S_{i,j}$ only depends on its current state:

- If a site is vacant, the site becomes occupied with some probability p.
- If a site is occupied, the site becomes vacant with probability q.

Once we have specified some initial conditions, the state process will evolve according to these rules.

In our simulations, we will choose the initial conditions so that we get a more or less stable proportion of occupied and vacant sites. In the initialization phase, a site is occupied with probability $\frac{p}{p+q}$ and vacant with probability $\frac{q}{p+q}$. This means

that a fraction $\frac{p}{p+q}$ of all sites is initially occupied and a fraction $\frac{q}{p+q}$ of all sites is initially vacant. A fraction $(1-q)$ of all occupied sites will still be occupied at the next time step and a fraction p of all vacant sites will become occupied at the next time step. Therefore a fraction

$$(1-q)\frac{p}{p+q} + p\frac{q}{p+q} = \frac{p}{p+q} \tag{5.1}$$

of all sites will be occupied at the next time step and a fraction

$$1 - \frac{p}{p+q} = \frac{q}{p+q} \tag{5.2}$$

of all sites will be vacant. By induction, we get the same proportion of occupied and vacant sites for all time steps.

5.3 Direct Consequences from the Model

In the previous section, we have introduced the model and provided a possible economic interpretation. The model qualitatively captures some features of dividends like similarities in dividend policies within industries or sudden initiations of payments. From the construction of the model, we can also draw some further conclusions on the concentration and variability of dividend payments.

The dividend payments in our model depend on the cluster sizes. Percolation theory tells us more on how these cluster sizes are distributed on our square lattice. The distribution depends on how we choose the probabilities p and q. The discussion of the cluster size distribution can be reduced to one parameter. According to our findings at the end of section 5.2, the distribution of the clusters on the lattice would be the same if we would choose sites to be active with a probability

$$p' = \frac{p}{p+q} \tag{5.3}$$

and inactive with a probability $1-p'$. The larger p compared to q, the larger is p' and the larger the clusters get. If p' is large enough, we start observing spanning clusters in the realizations of configurations on the lattice. The threshold value p', from which we start observing spanning clusters, is called the percolation threshold. We know from percolation theory that the distribution of cluster sizes behaves like a power law if we are close to the percolation threshold (cf. Herrmann and Singer (2008)).

Since dividend payments only relate to the clusters in one particular row, we get a slightly different distribution. Cluster sizes crossing the middle row still follow a power law, but with a different exponent. The probability that clusters of a particular size cross the middle row not only depends on the frequency of these clusters on the lattice, but also on the cluster size. Larger clusters are more likely to have at least one site on the middle row.

In order to get the dividend payments, we additionally multiply the cluster sizes in two dimensions by the one-dimensional ones in the corresponding column. The one-dimensional clusters are also highly concentrated. Thus, multiplying the one-dimensional and the two-dimensional clusters yields an even higher concentration.

The size of the jumps in dividend payments are also influenced by the choice of the probabilities p and q. If both probabilities are very small, we expect dividends to remain almost unchanged from one time step to the other. Probably also the difference between the two probabilities matters. Depending on the size of a cluster and its shape, the difference between the two probabilities determines whether the cluster is more likely to grow, to remain unchanged or to shrink. Since there are various shapes of clusters, the overall effect from different proportions between the two probabilities is difficult to tell and would need further analysis.

5.4 Implementation

The implementation of the stock market model that we use stems from Tupak (2009) and is written in C++. The program allows the user to extend conveniently the model by further plug-ins. In our case, we will derive a new dividend class from an existing dividend base class as provided by the program. The implementation of this new dividend module is given below.

The computationally costliest part of the model is to identify the clusters and determine their sizes. The most efficient algorithm for this purpose stems from Hoshen and Kopelman (1976). In their work, they introduce what they call a "cluster multiple labeling technique". While walking through the lattice once, the algorithm assigns each occupied site a number, which identifies it with a cluster. The cluster sizes are then stored in a separate array under the corresponding identification number.

Below we explain the algorithm of Hoshen and Kopelman (1976) in more detail. For implementation, we applied the pseudo-code provided by Herrmann and Singer (2008). At each time step, we have a lattice with occupied and vacant sites $S_{i,j}$ from which we want to determine the cluster sizes. Occupied sites are labeled with 1 and vacant sites with 0. For storing the cluster sizes, we need an additional array C_k. For every cluster identification number k, C_k corresponds to the size of cluster k.

The algorithm searches line-by-line for occupied sites and checks whether the site belongs to a new cluster or to an existing one. Let $S_{i,j}$ be the first occupied site. $S_{i,j}$ belongs to the first cluster that we have found and is thus assigned the identification number $k = 2$. We skip the identification numbers 0 and 1, since we have already used them to label vacant and occupied sites. Since cluster $k = 2$ consists of one site so far, we set $C_{k=2} \leftarrow 1$.

For every other occupied site $S_{i,j}$, we have to check, whether it is connected to a cluster we have already found. If it is not, it probably belongs to a new cluster:

- If the site on the left and the one on top is vacant, the site $S_{i,j}$ probably

belongs to a new cluster. Therefore we increase the identification variable by one $k \leftarrow k + 1$ and assign the new identification number $S_{i,j} \leftarrow k$. The array that counts cluster sizes is updated accordingly $C_k \leftarrow 1$.

- If either the left or the top site is occupied, the site $S_{i,j}$ certainly belongs to the same cluster. Let us assume that the already occupied neighboring site carries the identification number k_0. Since $S_{i,j}$ belongs to the same cluster, we assign the same identification number $S_{i,j} \leftarrow k_0$. Then cluster k_0 consists of one more site, thus $C_{k_0} \leftarrow C_{k_0} + 1$.

- If both the left and the top site are occupied, the site $S_{i,j}$ connects both clusters that we have previously found. Let us assume that the neighboring sites carry the identification numbers k_1 and k_2 respectively. We rename the cluster resulting from the two previous ones by the left identification number. Therefore site $S_{i,j}$ gets the identification number $S_{i,j} \leftarrow k_1$. The new cluster k_1 now consists of the two former clusters k_1 and k_2 plus the additional site $S_{i,j}$, thus $C_{k_1} \leftarrow C_{k_1} + C_{k_2} + 1$. Instead of renaming all sites of cluster k_2, we set $C_{k_2} \leftarrow -k_1$ to indicate that these sites now belong to cluster k_1. From now on, if we encounter a cluster k with a negative C_k, we know that it is part of another cluster $-C_k$. Thus, the corresponding cluster size is C_{-C_k} or if C_{-C_k} is negative, $C_{-C_{-C_k}}$ and so on.

Once we have scanned the entire lattice, all sites are identified with one particular cluster. Since our square lattice is finite, we have to care about the boundaries. In order not to discriminate the firms or business lines at the boundaries, we apply periodic boundary conditions. Hence, we no longer operate on a planar lattice, but on a torus. Since we will choose the number of firms and business lines to be large, the remaining distortions from the boundaries will be negligible. The one-dimensional cluster sizes in the columns can be found more easily. If the middle row of a column is occupied, we can simply count the number of occupied connected sites on top and below the middle row.

For determining the dividend payments, we first have to check for each firm the identification number in the middle row. Dividends are paid if the site in the middle row contains a number k greater than 0. This number k gives information about the identity of the two-dimensional cluster and the corresponding cluster size. If C_k contains a positive number, C_k is the cluster size. In case C_k is negative, we have to set $k' = -C_k$ until we find a $C_{k'}$ that contains a positive number. $C_{k'}$ then is the corresponding cluster size. Multiplying this cluster size by the size of the one-dimensional cluster in the column, we obtain the total dividends paid.

6

Investment Strategies under Consideration

The outcome of the simulations will strongly depend on which strategies we choose to compete. Particularly interesting will certainly be the Kelly rule and if it will still perform best in a changed setting, where dividends are no longer exogenously determined. As we have to confine ourselves to some set of strategies, we choose a number of popular strategies that differ regarding to the data they base their decisions on. In our choice of competing strategies, we include strategies that either depend on prices, dividends, total return or completely ignore the data. A more detailed description of the investment strategies follows below.

Kelly rule An investor following the Kelly rule invests his wealth proportionally to the expected discounted relative dividends of assets. In mathematical terms the Kelly rule reads $\lambda_t^* = (\lambda_{1,t}^*, \ldots, \lambda_{K,t}^*)$ with

$$\lambda_{k,t}^* = E\left[\sum_{l=1}^{\infty} \rho^{l-1} (1-\rho)\, d_k(s_{t+l}) \,\middle|\, s_t\right]. \tag{6.1}$$

Allowing dividends to be endogenously determined, the survival and stability proofs listed in section 2 are no longer applicable and the Kelly rule might perform worse than other strategies.

In our simulation studies, we cannot test the Kelly rule in its original form. The problem is the conditional expectation value. A Kelly investor would have to form expectations on infinitely many future relative dividend payments. An approximation for this conditional expectation value would have to be found. However, we will restrict ourselves to the form of the Kelly rule as it appears if the states follow an i.i.d. process. Thus, we use

$$\lambda_k^\star = E\left[d_k(s)\right]. \tag{6.2}$$

Under this assumption, we can now estimate the expectation value by averaging over past realizations of relative dividends. In previous simulations, the Kelly rule

in this form still performed particularly well even when states were not i.i.d.

Mean-variance optimization Since modern portfolio theory is a milestone in the development of portfolio theory, we are particularly interested in the performance of mean-variance optimizing strategies. In our simulations, we will include one type of mean-variance optimizers, investors investing their wealth according to the tangent portfolio. In the following, we will denote this strategy by λ^{Tan}.

In this case, we again have to take care about expectation values, namely the mean vector and the covariance matrix of the returns. We apply the same technique as used for the implementation of the Kelly rule. In our simulations, mean-variance optimizers will form their expectations by averaging over past realizations of prices and dividends.

Naïve diversification The third strategy we will include in our simulations is the naïve diversification. Naïvely diversifying investors invest their wealth equally in all assets, thus $\lambda^{ND} = (1/K, \ldots, 1/K)$. We will use naïve diversification as a benchmark as suggested by DeMiguel et al., partly due to its simplicity and popularity. Even though it completely ignores the data, it performed astonishingly well in their evaluation. Compared to other more sophisticated strategies, the $1/K$-rule is not prone to estimation errors. This may be the reason why it performed better than many theoretically superior investment strategies in practice.

Cyclical and anti-cyclical investment In addition to the strategies described above, we analyze the performance of momentum strategies. We include two types of investors betting on prices, trend followers and contrarians. In our context, they are defined as

$$\lambda^{Tr}_{t+1,k} \sim \left(\frac{p_{t,k}}{p_{t-1,k}} - 1\right)_+, \quad \text{and} \quad \lambda^{Co}_{t+1,k} \sim \left(1 - \frac{p_{t,k}}{p_{t-1,k}}\right)_+. \tag{6.3}$$

Additionally the strategy is set to $(1/K, \ldots, 1/K)$ if all portfolio weights according to the above definition would be equal to zero. This ensures that these investors will stay in the market even though there may be periods with no attractive investment opportunities.

The simulation program already includes all of the above-mentioned strategies. Estimates of expectation values are calculated according to an exponential moving average, for which the program allows choosing different smoothing factors. A problem with simulations is that some strategies rely on the realizations of past data. In theory, we can just assume the existence of a history of states at the initial time step. In simulations however, we have to provide strategies an explicit history of states. The simulation program circumvents this problem by allowing freezing market participants' wealth shares in an initial phase and using relative dividends as an approximation for relative prices. During this initial phase, prices and in the

endogenous case also dividends certainly follow a very different market mechanism, but this is the best solution currently available to cope with this problem.

With this selection of investment styles, we have hopefully chosen strategies that are particularly interesting with respect to the resulting wealth and price dynamics. Since the strategies differ in what input variables they use, they will probably behave very differently in the simulations.

One drawback is that all strategies in our simulations rely on past realizations of dividends and prices. None of them is able to anticipate higher future dividend payments if current payments are low. This means that we exclude the possibility to invest into growth firms that may not pay dividends for a prolonged time, but may substantially increase their payments in the future.

The way we set up our model gives the state process a new interpretation. This allows strategies to extract valuable information directly from the realizations of the state process. Investment strategies no longer have to rely only on the past realizations of dividends and prices. We could for example define strategies that invest into assets according to the occupied sites in the respective column, but which do not pay any dividends yet. Assuming that we impose additional rules on the lattice that imply a longer memory of states, these firms would be more likely to pay higher dividends in the future. However, we will not consider these kinds of strategies in our simulations. This would be beyond the scope of my thesis.

7

Simulations with Exogenous Dividends

In this section, we will conduct simulations with exogenously determined dividends as previously discussed. This means that the state process and the dividend function develop independently from any influences of other market variables. The states of the world in our model will follow a Markov process by construction. Once we have chosen the site activation and deactivation probabilities p and q, a configuration of occupied and vacant sites on the lattice only depends on the previous configuration. Furthermore, states are stationary. Since p and q have been chosen in order to get a more or less stable number of occupied and vacant sites on the lattice, the dividend distribution will remain the same over time.

7.1 The Choice of Parameters

In order to determine some probabilities p and q that yield more realistic dividends, we analyze the generated dividends for different levels of p and q. We note that the following results are only based on one simulation with each pair of values p and q. Thus, the results may strongly depend on the realizations and may not be representative. However, this approach suffices to get a feeling about how dividends depend on p and q. All simulations were conducted with 1000 assets.

Since we want to capture the tail of the dividend distribution, which follows a power law, we will try to find values for p and q so that the system is near percolation threshold. Figure 7.1 stems from the work of Fu et al. (2006) and shows the critical line of values p and q, where spanning clusters start to appear on the lattice. Their simulations were conducted on a 800×800 lattice. Apart from the lattice size, their specification of the lattice is the same as in our case. Simulations on our model would probably yield similar results. The area above the curve means that the system is below percolation threshold, whereas the area below means that it is above percolation threshold. In section 5.3 we have seen that the configurations on the lattice at each point in time could be generated by static percolation with activation

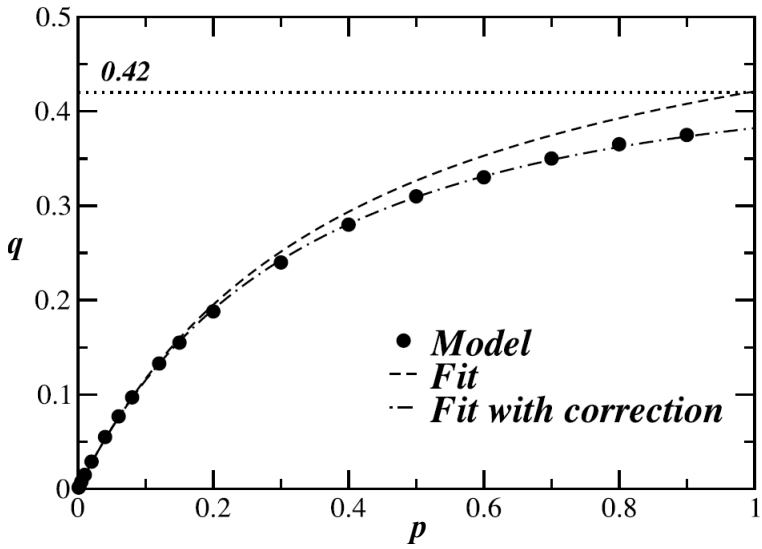

Figure 7.1: Critical values for p and q

probability $p' = \frac{p}{p+q}$. For static site percolation on a square lattice with activation probability p', we know that the percolation threshold is somewhere around 0.6 (cf. Herrmann and Singer (2008)). Thus, we expect p and q to be rather close to each other at percolation threshold. From figure 7.1 we see that for small probabilities, p and q indeed are almost the same at percolation threshold. In our simulations, we will therefore choose values for p and q that are close to each other.

From the Hill plots in appendix D.2, we see that the tail of the distribution seems heavier if q is larger than p and thinner if q is smaller than p. Choosing q large enough with respect to p will probably yield some distribution that behaves similarly to the distribution of the real data in the tails.

The effect on the dividend growth rate seems less clear. The respective tables are shown in appendix D.1. The median in all cases is zero or close to zero. This is what we expect, since p and q are chosen close to each other in our simulations. The cluster size growth related to p and the reduction in size related to q almost cancel each other for many clusters.

The interquartile distances draw a mixed picture. We cannot immediately identify a relation between the interquartile distances and the proportion between p and q. However, it seems that the variability increases for larger values p and q.

7.2 Simulation Results

For the following simulations we set $p = 0.05$ and $q = 0.07$, which yields a reasonable dividend concentration and variability. The strategies competing in the market are those described in section 6. We assume that all investors have a savings rate $\rho = 0.9$.

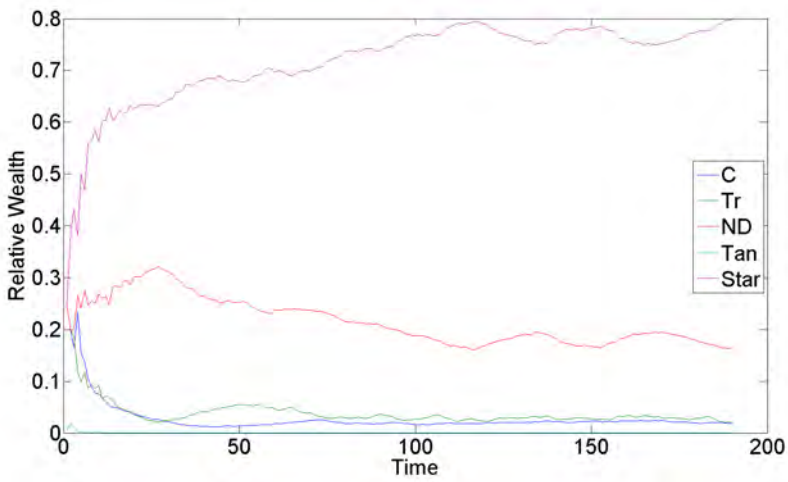

Figure 7.2: Relative wealth when $p = 0.05$ and $q = 0.07$

The simulations were conducted with 1000 assets over 200 time steps.

Figure 7.2 shows a typical evolution of market shares for one sample path of the dividend process. The Kelly rule rapidly acquired market share in an initial phase and kept dominating the market in the following time steps. Its market share more than tripled in the first twenty time steps. While most strategies lost market share at a very fast pace, the naïve diversification rule was able to keep its initial market share for a prolonged time.

Figure 7.3 shows relative dividends. Recall that relative dividends are defined as the share of dividends paid by a company in total dividends. They are characterized

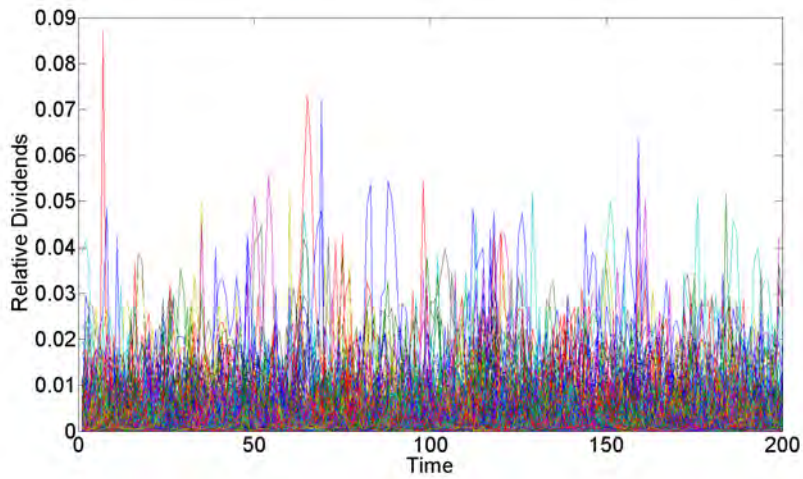

Figure 7.3: Relative dividends when $p = 0.05$ and $q = 0.07$

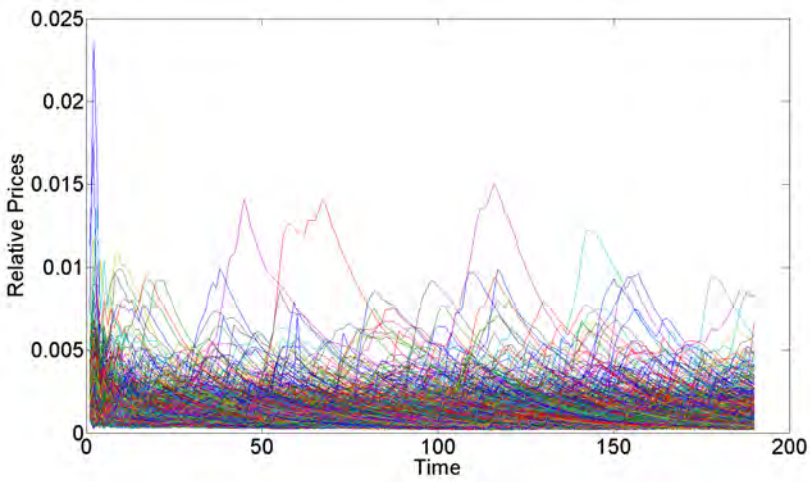

Figure 7.4: Relative prices when $p = 0.05$ and $q = 0.07$

by sharp spikes in our simulations. Even though dividend time series exhibit many different patterns, in reality most of them seem to be more persistent. Since we have chosen $q = 0.07$, an occupied site in the middle row of an asset will become vacant again in a relatively short amount of time. The time an asset keeps paying dividends will therefore be rather short in most cases. By choosing smaller values for p and q in our simulations, the relative dividend streams became smoother. However, there seemed to be no difference with regard to the market selection process.

Figure 7.4 shows relative prices. Recall that relative prices are defined as the price share of a company in aggregate price. We see that relative prices increased much faster than they declined. Relative price moves were probably initiated by dividend changes. Since strategies considering dividends have to base their dividend estimates on past data, they will allocate more of their wealth to assets when dividend payments increase. This in turn will drive up the price of these assets and attract investors basing their decision on prices. Thus, dividend developments seem to lead price changes.

More interesting than the time evolution of prices seems their structure. Prices seem to be highly concentrated according to figure 7.4. Let us recall at this point that prices mean the same as market capitalization in our model, since we have assumed unit supply of each asset. According to the literature on firm size distribution, firm sizes obey Zipf's law (for references see Malevergne et al. (2008)). This means that firm sizes are distributed according to a power law with an exponent close to one. Figure 7.5 shows the UMPU test and figure 7.6 the Hill plot of prices at the last time step of the simulation. The firm sizes generated by the percolation model seem to follow a power law, but with a far larger exponent than one.

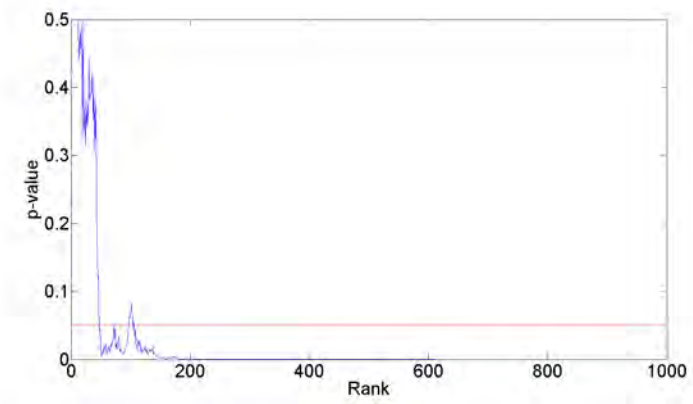

Figure 7.5: UMPU test of prices when $p = 0.05$ and $q = 0.07$

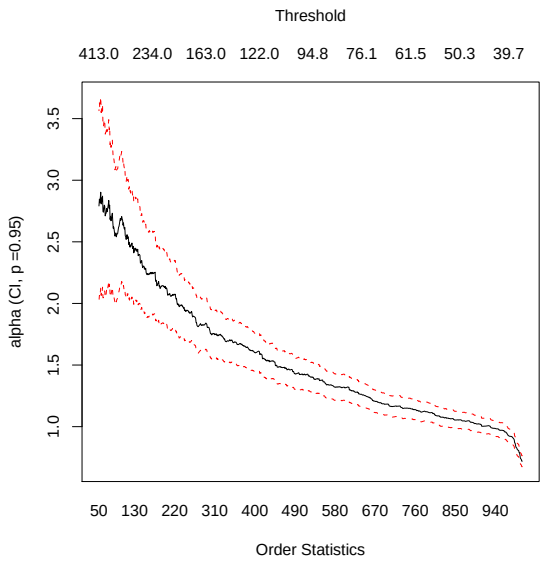

Figure 7.6: Hill plot of prices when $p = 0.05$ and $q = 0.07$

8

Simulations with Endogenous States

Strategies basing their asset allocations on dividends may have an advantage over those based on prices in the case of exogenous dividends, since dividends have an influence on prices, but prices have no influence on dividends. Therefore, strategies like the Kelly rule may implicitly anticipate prices, while strategies based on price developments do not consider any information that influences dividends. In the following simulations, we will decrease this asymmetry between prices and dividends. Dividends will no longer be exogenously determined, but depend on prices. In this section, we allow for a price influence on the state process.

8.1 The Dividend Model Adjusted for Prices

In order to account for price influences, we have to slightly modify our dividend model. In the following simulations, the activation probability p will no longer be constant, but depend on the price of an asset instead. This implies that p varies in time and between the columns of the lattice. We will denote the new activation probability of asset k at time t as $a_{t,k}$. $a_{t,k}$ is defined as

$$a_{t+1,k} = \max\left\{a \tanh\left(K \frac{p_{t,k}}{\sum_{k=1}^{K} p_{t,k}}\right), b\right\}, \tag{8.1}$$

where a and b are constants, K is the number of assets and $p_{t,k}$ is the price of asset k at time t. In the definition (8.1), relative prices are multiplied with the number of assets so that the probabilities do not get too small when the number of assets is increased. If the prices of all assets are the same, the relative price of each asset is $1/K$. Thus, independent of the number of assets we then get $a_{t+1,k} = a \tanh(1)$, assuming that b is smaller than this number. The choice of a partly controls the levels of the activation probabilities.

The hyperbolic tangent function ensures that the relative prices multiplied by the number of assets lie on the interval $[0, 1]$ so that they can be used as probabilities.

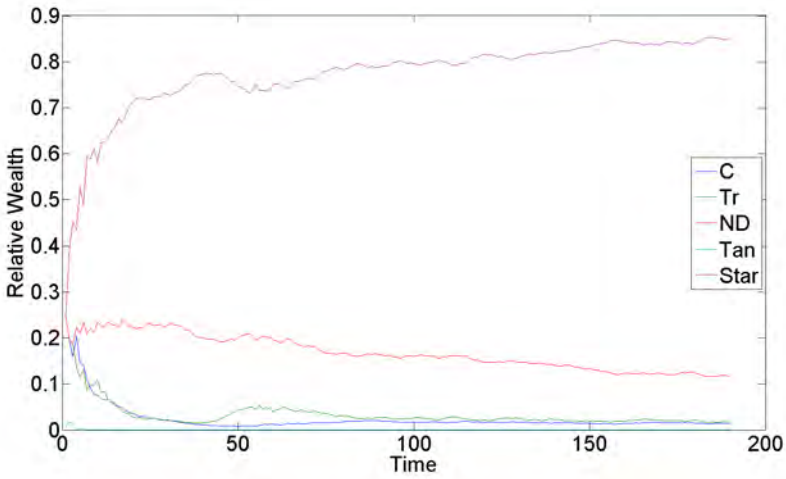

Figure 8.1: Relative wealth when $a = 0.05$, $q = 0.06$ and $b = 0.04$

The higher the price, the larger will be the activation probability at the next time step. A larger activation probability means in our interpretation of the model that companies are more likely to build up new business lines. A possible interpretation of this interrelation with prices could be that higher prices increase the company's financial strength and may increase investment activities.

The effects of different choices for a, q and b on dividend payments now additionally depend on the choice of strategies. For our simulations, we use the same strategies and consumption rate as before so that we can compare the results. From the Hill plots in appendix E.2 we see that the tail of the dividend distribution seems to be heavier than in the exogenously determined case. The exponent of the power law tends to be clearly below two, which is closer to what we observe from the real data.

The plotted aggregate dividends in appendix E.2 show the effect of the lower constraint b. If b is too small, aggregate dividends shrink over time. Furthermore, a higher value for b seems to increase the variability of dividends.

Due to the parameter b, there is some lower limit on activation probabilities. This condition ensures that companies still have some chance to establish a business line even though their price may be very low.

8.2 Simulation Results

In the following simulations with 1000 assets over 200 time steps, we chose $a = 0.05$, $q = 0.06$ and $b = 0.04$. The activation probabilities could therefore differ by 0.01 at maximum for different assets. Figure 8.1 shows the evolution of market shares for one sample path of the dividend process. The performance of all strategies is similar

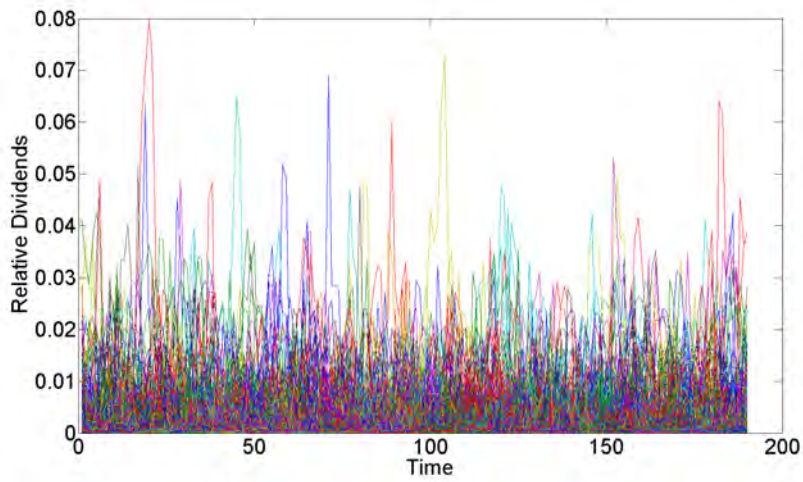

Figure 8.2: Relative dividends when $a = 0.05$, $q = 0.06$ and $b = 0.04$

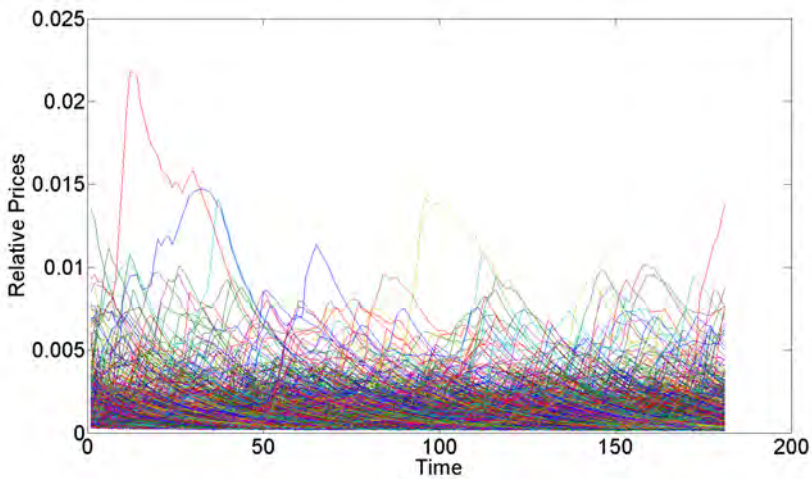

Figure 8.3: Relative prices when $a = 0.05$, $q = 0.06$ and $b = 0.04$

to what we observe from our simulations with exogenously determined dividends. The Kelly rule again performed exceptionally well and the relative wealth of the naïve diversification rule still declined only slowly.

The relative dividends in figure 8.2 and relative prices in figure 8.3 look almost unchanged compared to those in the simulations with exogenous dividends. Particularly prices still tended to increase faster than they decreased. Since the effect of prices on dividends is very limited according to our definition of the activation probabilities, we can observe almost no changes in all variables compared to the previous simulations. However, if we choose the price effect on dividends to be larger like in

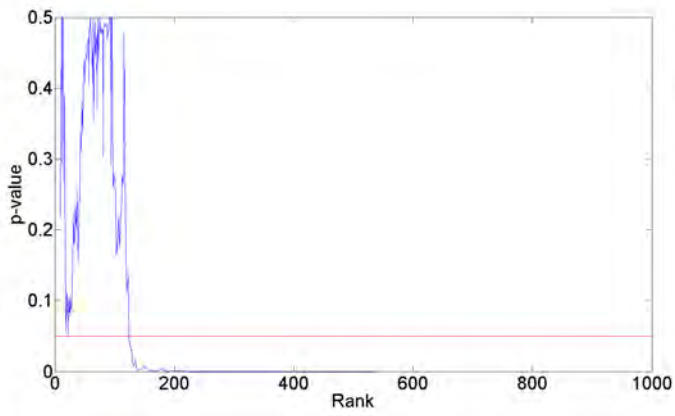

Figure 8.4: UMPU test of prices when $a = 0.05$, $q = 0.06$ and $b = 0.04$

appendix E.2, the exponent of the dividend distribution becomes smaller and we are even able to obtain an exponent close to one.

The structure of prices also remained more or less the same. Figure 8.4 shows the UMPU test and figure 8.5 the Hill plot of prices at the last time step of the simulation. Firm sizes are still distributed according to a power law, but with an exponent far larger than one.

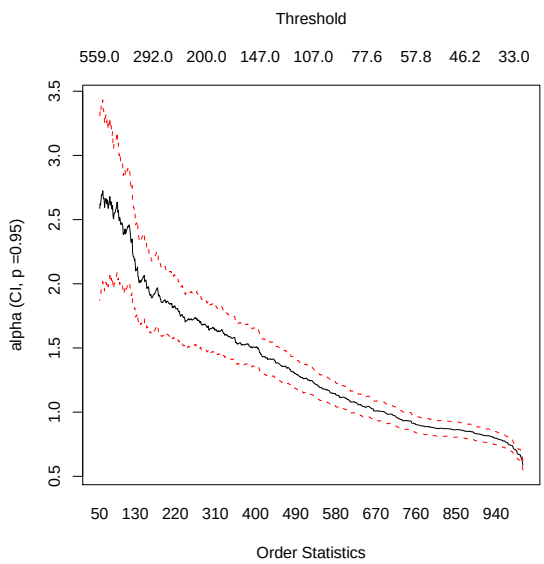

Figure 8.5: Hill plot of prices when $a = 0.05$, $q = 0.06$ and $b = 0.04$

9

Simulations with an Endogenous Dividend Function

In this section, we start another attempt to endogenize dividends. Instead of a price-dependent state process, we now let the dividend function depend on prices. This way, the price effect on dividends will probably be stronger by construction. Previously prices only had a very limited influence on dividends. An asset's price affected the activation probability in that asset's column, but the largest component of the dividend payment, which is the two-dimensional cluster, also depended on the activation probabilities of other columns. However in case of a price-dependent dividend function, an asset's dividend payments are influenced only by that asset's prices.

9.1 The Dividend Model Adjusted for Prices

The dividend payment $D^{endo}_{t,k}$ of asset k at time t is defined as

$$D^{endo}_{t+1,k} = D^{exo}_{t+1,k} p^{g}_{t,k}, \tag{9.1}$$

where $D^{exo}_{t,k}$ is the dividend payment from the exogenous case, $p_{t,k}$ denotes the price and g is some non-negative constant. In case g is equal to zero, we are in the exogenous case as described in section 5. If we choose g to be positive, higher prices will imply higher dividend payments. In our interpretation of the model, this means that managers would decide to pay higher dividends for some given state of the world if prices are high. A possible explanation of prices influencing managers' decisions could be that higher prices increase the company's financial strength, which enables the company to pay higher dividends. In this interpretation of the model, managers are assumed to believe that shareholders prefer higher dividends. Thus, they seek to influence the share price positively by paying out higher dividends.

In our simulations we will choose g to be smaller than one. Thus, we assume a diminishing marginal price effect on dividends. The Hill plots in appendix F.2 show

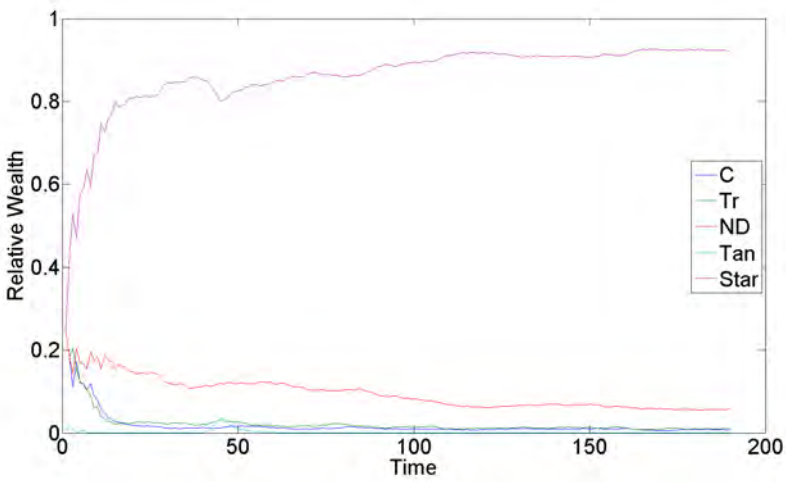

Figure 9.1: Relative wealth when $p = 0.03$, $q = 0.04$ and $g = 0.03$

how varying values of g affect the tail of the dividend distribution. It seems that the tail of the distribution tends to be heavier for larger values g. For $g = 0.8$ the exponent of the dividend distribution seems to hover somewhere around one. The reaction of aggregate dividends on different values of g are unclear from the plots in appendix F.2. At least, we cannot identify any pattern in the graphs as g varies. The effect on the jumps is also unclear if we only consider the values in appendix F.1.

9.2 Simulation Results

In the following simulations with 1000 assets over 200 time steps, we chose $p = 0.03$, $q = 0.04$ and $g = 0.3$. The problem again is how to start the simulations. Since some strategies require past data on dividends and prices, we first simulate ten time steps with exogenous dividends in order to generate some market history.

Even in the simulations with an endogenously determined dividend function, we observe almost the same evolution of relative wealth for all strategies as well as similar relative dividends and relative prices. Allowing a price influence on dividends does not seem to have an impact on market selection, at least not in our set of competing strategies and with the values p, q and g that we have chosen. Simulating with smaller values p and q and larger g, this is no longer true. Even though the Kelly rule still outperformed all other strategies in the long term, the tangent portfolio rule often managed to substantially increase market share in the first ten to twenty time steps. If p and q are small, relative prices and relative dividends again become smoother. In contrast to the exogenous case, relative prices no longer remain flat, but show some bulks.

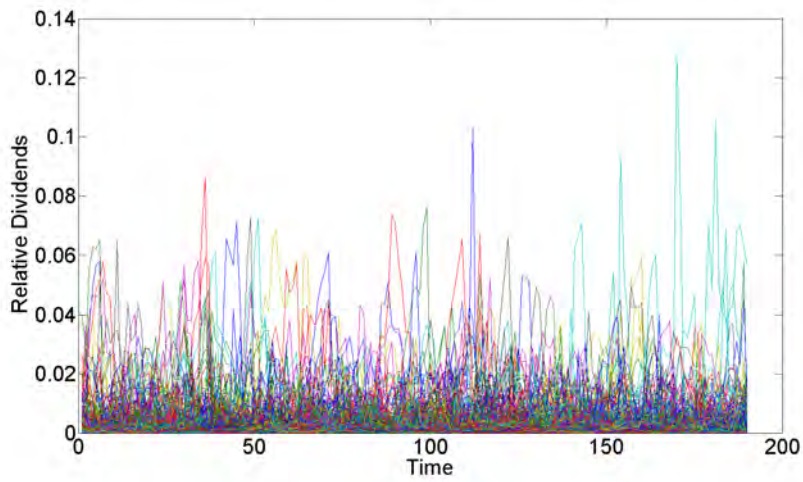

Figure 9.2: Relative dividends when $p = 0.03$, $q = 0.04$ and $g = 0.03$

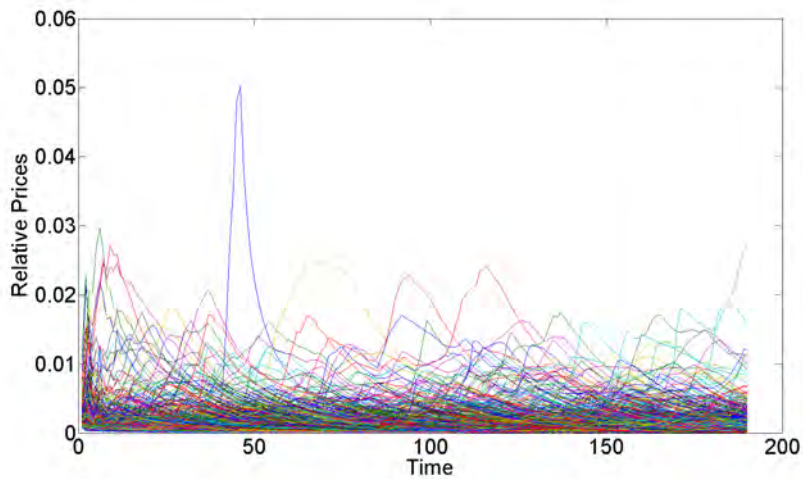

Figure 9.3: Relative prices when $p = 0.03$, $q = 0.04$ and $g = 0.03$

The structure of prices remained more or less the same as in the case of exogenous dividends and dividends generated with a price-dependent state process. Figure 9.4 shows the UMPU test and figure 9.5 the Hill plot of prices at the last time step of the simulation. Firm sizes were still distributed according to a power law, but with an exponent far larger than one.

Let us turn back to the wealth distribution. Even though the performance of all strategies generally seems to be the same as in the exogenous case, the often good performance of the tangent portfolio during the first few time steps raises questions. The tangent portfolio never performed well in the long term or in the

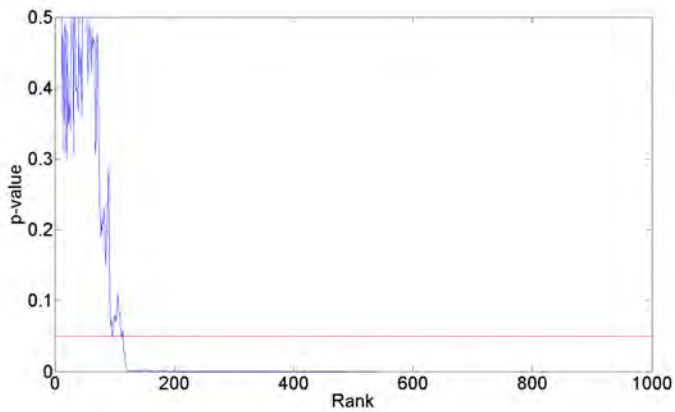

Figure 9.4: UMPU test of prices when $p = 0.03$, $q = 0.04$ and $g = 0.03$

exogenous case, but it quite often performed particularly well right after we have changed from exogenous to endogenous dividends. In my opinion, the persistence of dividends and prices may be an important issue in this regard. The way we have built our percolation dividend model, prices often have a longer memory than dividends, depending on the specifications of the dividend model and the strategies we choose. Dividends only depend on the current state, which in turn only depends on the previous state in the exogenous case. Dividends therefore have a very short memory in the exogenous case. Prices depend on strategies, which often depend on states that are further in the past. Strategies based on dividends may faster react on effects initiated by the random states than strategies considering prices. Dividends have a longer memory in the endogenous case, since they inherit some of the persistence of prices. This may partially explain why the tangent portfolio quite often perfomed very well in an initial phase. Considering prices apart from dividends may have been an advantage as long as the best performing stocks kept paying dividends. As soon as stocks stopped paying dividends, there was probably a significant wealth decrease driving the tangent portfolio out of the market.

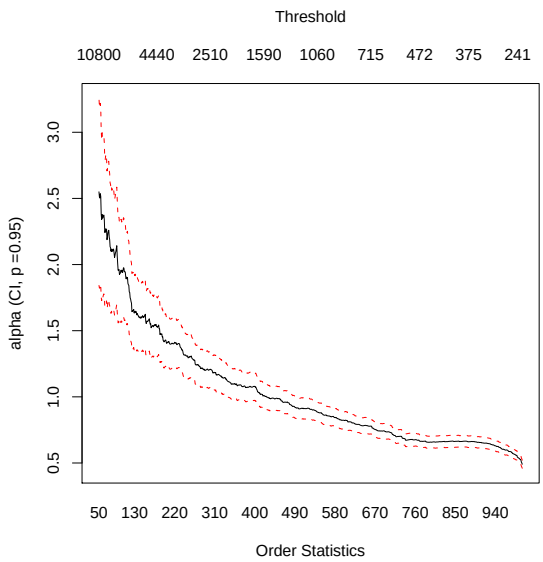

Figure 9.5: Hill plot of prices when $p = 0.03$, $q = 0.04$ and $g = 0.03$

10

Conclusion

This work has studied the effects of price-dependent dividends in an evolutionary stock market model of Evstigneev et al. (2008a) by using a percolation model. First, we examined facts and theories on dividends in order to identify possible price influences and to specify a model that accounts for the most essential features of dividends. Directed percolation models apparently are able to capture many dividend characteristics and allow for possible price influences. In a second step, we analyzed the consequences for relative wealth and relative prices when generating dividends by a percolation model with and without a price dependency.

Considering existing dividend theories, we cannot completely preclude price influences on dividends. However, various price effects on dividends seem to have no long-term effects on market selection according to our simulations. We conclude that previous findings on the performance of the Kelly rule probably are robust with respect to possible price influences on dividends. The resulting effect on prices was rather limited as well. Prices generally seemed to be more a reaction on past dividends than reflecting future expected dividends.

Generating dividends by a percolation model implied interesting price distributions. The tail of the distribution followed a power law in all our simulations. Since we have assumed unit supply of each asset, prices in our context are equal to market capitalization. The evolutionary stock market model may provide further insight on the mechanisms leading to a power law distribution of firm sizes.

Our percolation dividend model generates the states explicitly and gives the state of the world a more concrete interpretation. This allows us to implement strategies that directly depend on states. According to the interpretation of our model, such a strategy would represent a simplified form of fundamental analysis. The impact of fundamentalists on the evolutionary stock market model may be interesting for further simulation research. Prices may no longer be an image of past dividends. A strongly related issue is persistence. The results of our study indicate that the persistence of dividends may have been of importance in our simulations. Many strategies rely on expectations, which have to be determined. The predictability of market variables may have a large impact on the performance of different strategies.

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Appendix A

Histogram and QQ-plots of Log Dividends

The following figures show the histograms and normal QQ-plots of log dividends for the years 1973, 1980, 1987, 1994, 2001 and 2008. The red line displays the density function of the normal distribution with mean and variance as calculated from log dividends. The center part of the log dividend distribution is close to normal in all of the years, but the tails exhibit large deviations from the normal distribution. Thus, dividends seem to follow a lognormal distribution in the center part, but not in the tails.

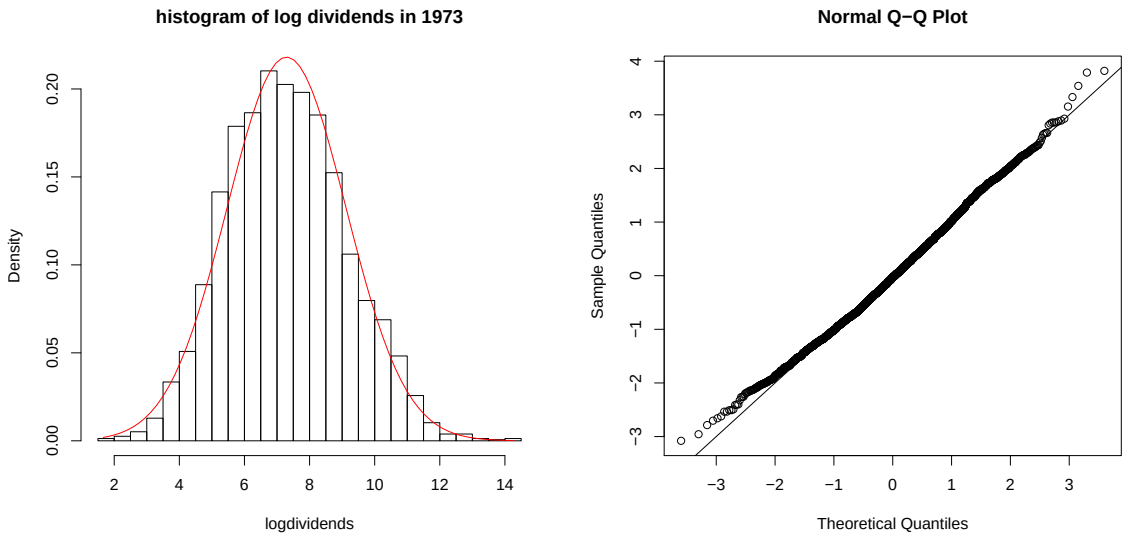

Figure A.1: Histogram and QQ-plot of log dividends in 1973

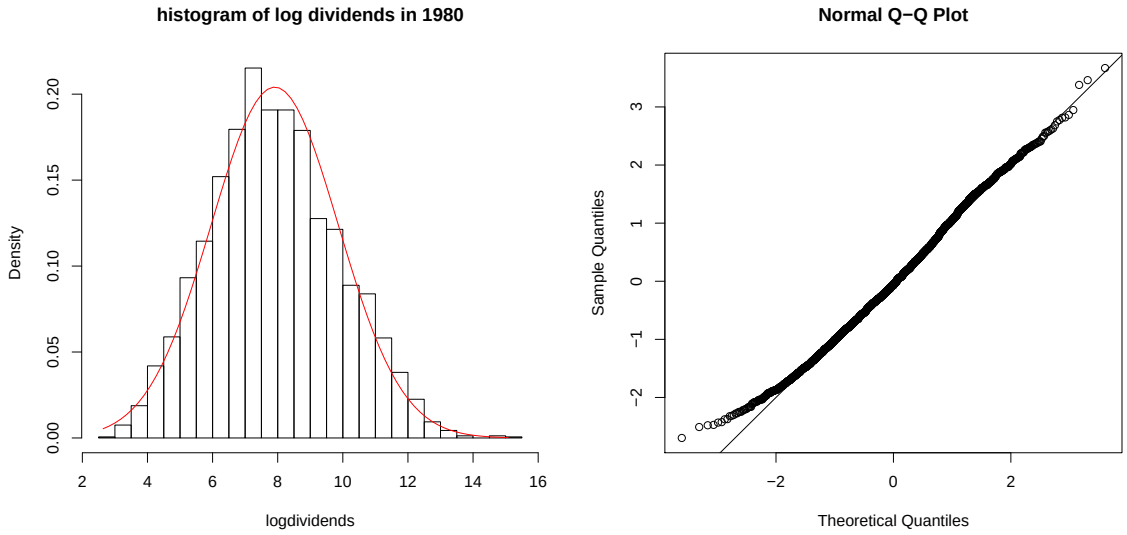

Figure A.2: Histogram and QQ-plot of log dividends in 1980

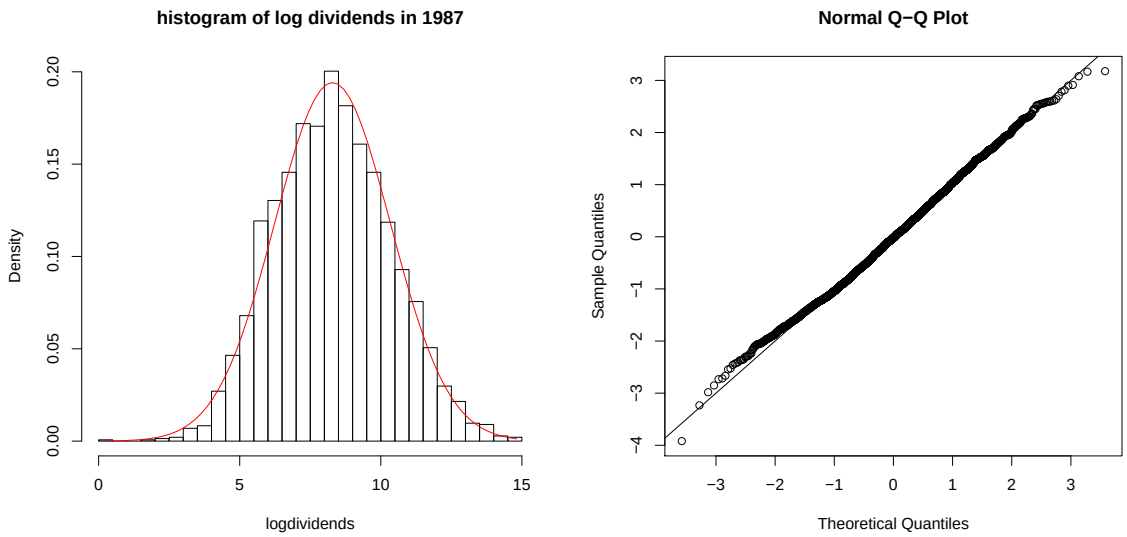

Figure A.3: Histogram and QQ-plot of log dividends in 1987

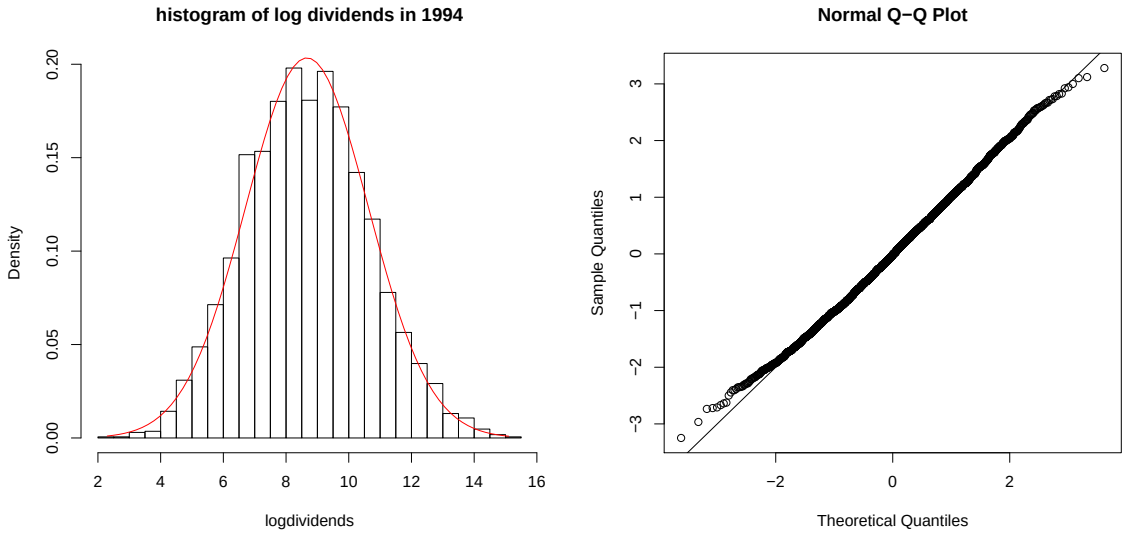

Figure A.4: Histogram and QQ-plot of log dividends in 1994

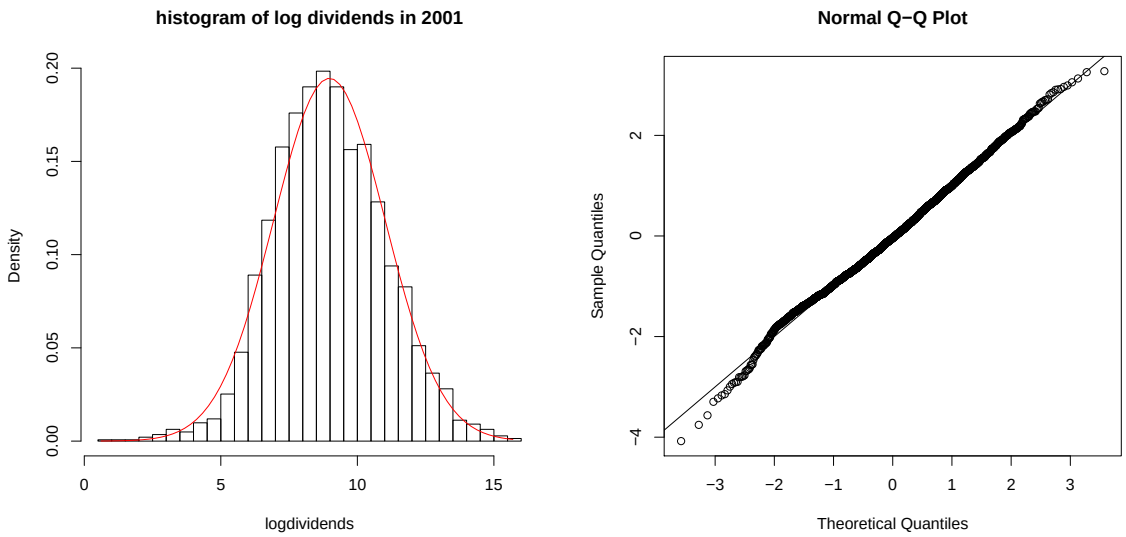

Figure A.5: Histogram and QQ-plot of log dividends in 2001

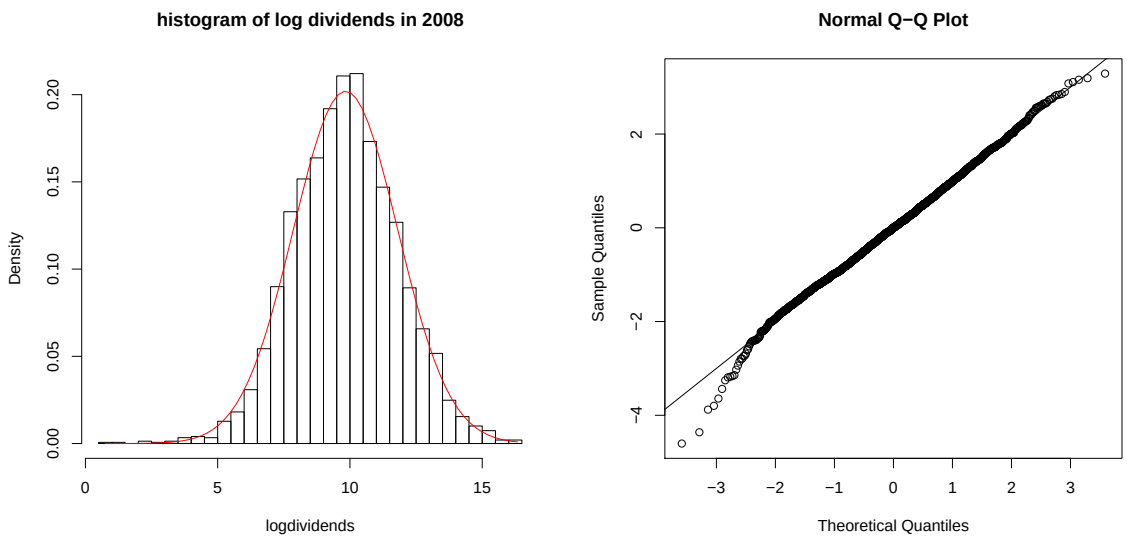

Figure A.6: Histogram and QQ-plot of log dividends in 2008

Appendix B

UMPU Tests of Dividends

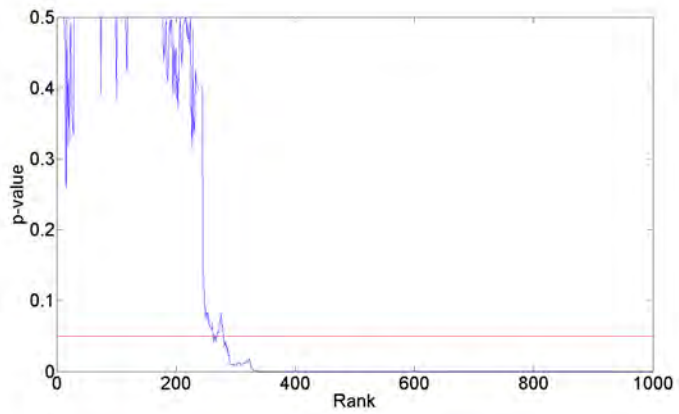

Figure B.1: UMPU test applied to dividends in 1973

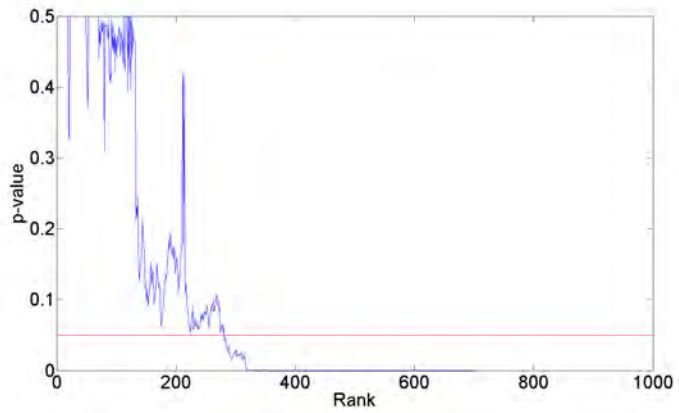

Figure B.2: UMPU test applied to dividends in 1980

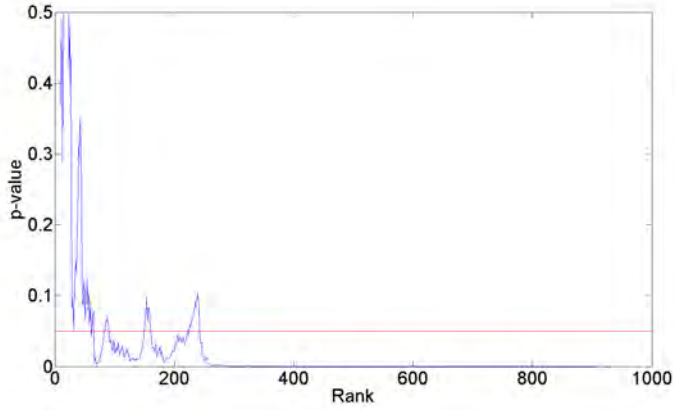

Figure B.3: UMPU test applied to dividends in 1987

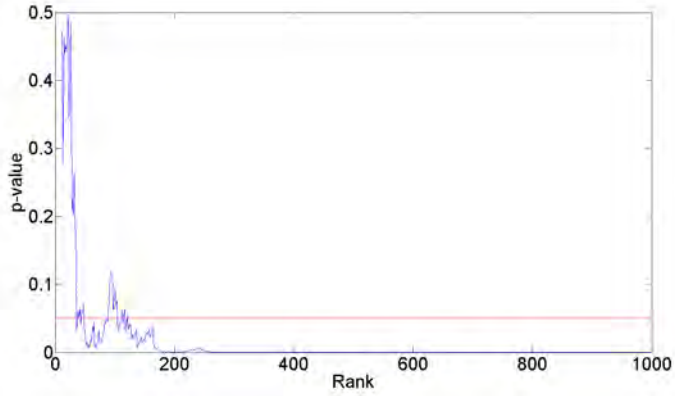

Figure B.4: UMPU test applied to dividends in 1994

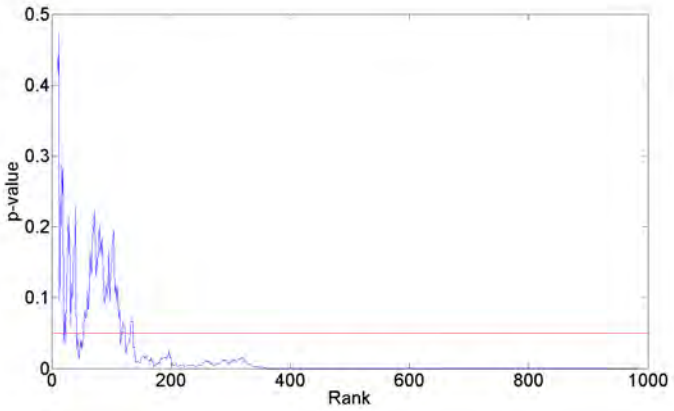

Figure B.5: UMPU test applied to dividends in 2001

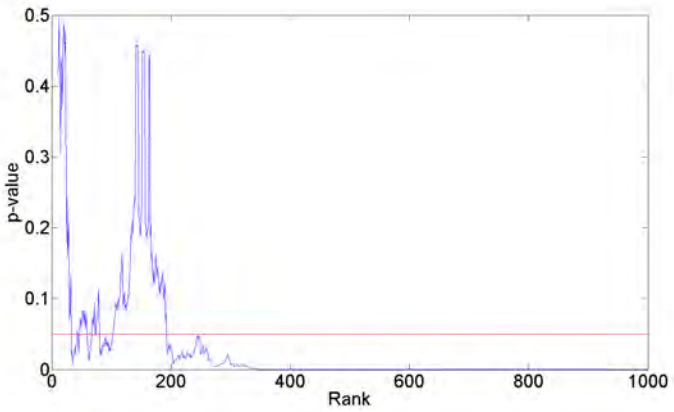

Figure B.6: UMPU test applied to dividends in 2008

Appendix C

Hill Plots of Dividends

The following figures show the Hill plots of dividends for the years 1973, 1980, 1987, 1994, 2001 and 2008. On top of the figure, we indicate the upper tail cut-off dividend values. The corresponding order statistics are given on the bottom of the figure. The red lines display the 95% confidence intervals of the exponent α. α ranges from about $1.2 - 1.6$ in the tails. The confidence invervals become quite large in the tails due to the small sample size for estimation.

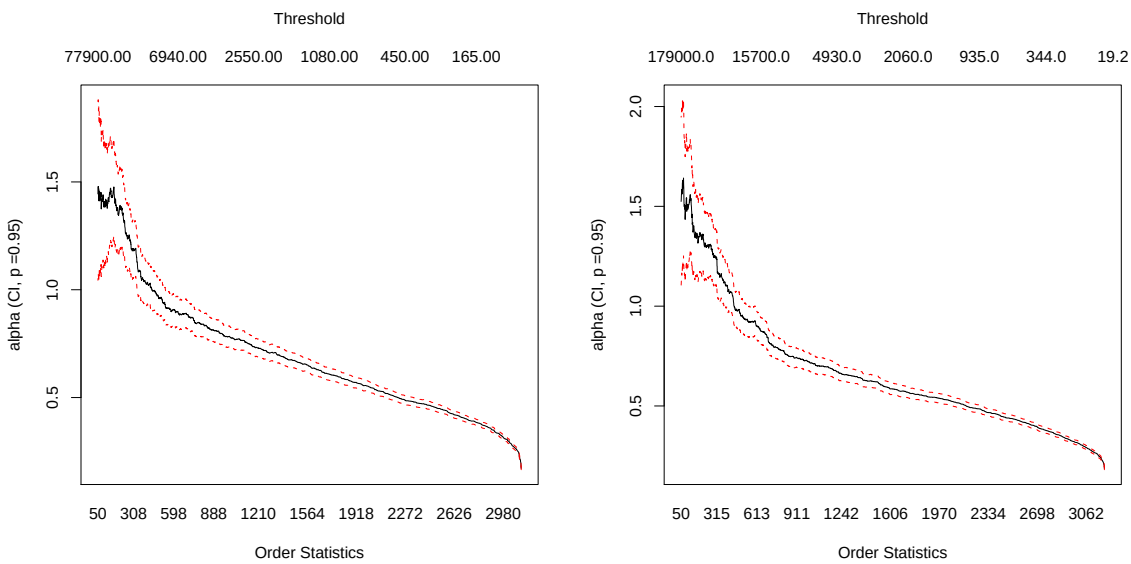

Figure C.1: Hill plots of dividends in 1973 and 1980

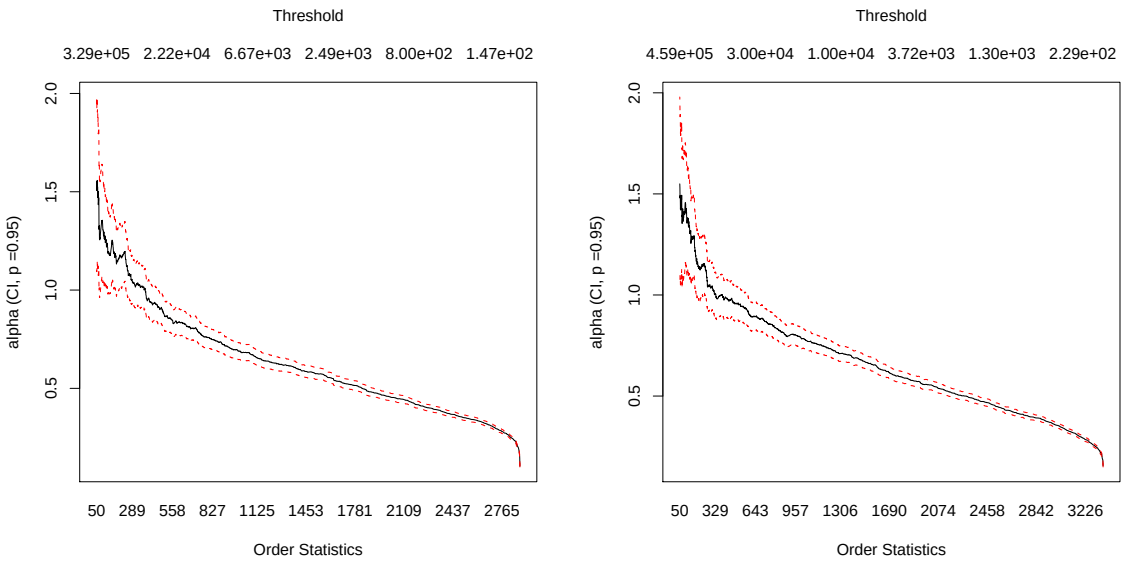

Figure C.2: Hill plots of dividends in 1987 and 1994

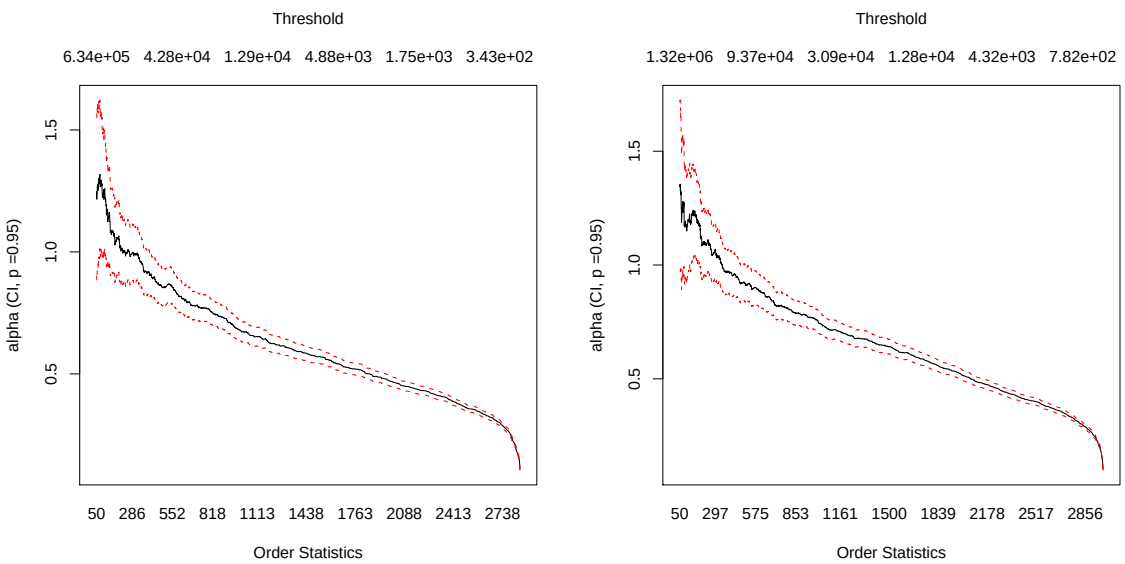

Figure C.3: Hill plots of dividends in 2001 and 2008

Appendix D

Simulation Results on Exogenous Dividends

All simulations were conducted with 1000 assets over 100 time steps. Independent of the choice of p and q, the Kelly rule clearly outperformed all other strategies. Relative dividends and relative prices were smoother for smaller values p and q and exhibited sharper spikes for larger values p and q.

D.1 Variability

Table D.1: Average median and interquartile distances of dividends over time

p	0.02	0.02	0.02	0.03	0.03	0.03	0.05	0.05	0.05	0.05
q	0.01	0.02	0.03	0.02	0.03	0.04	0.04	0.05	0.06	0.07
median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
intquartile distance	0.00	0.10	0.01	0.19	0.24	0.10	1.11	0.61	0.42	0.34

D.2 Concentration

The following figures show the Hill plots of dividends from simulations with different probabilities p and q at the 100th time step. The red lines display the 95% confidence intervals of the exponent α. The α-estimates in the tail seem to become smaller for larger values of q relative to p.

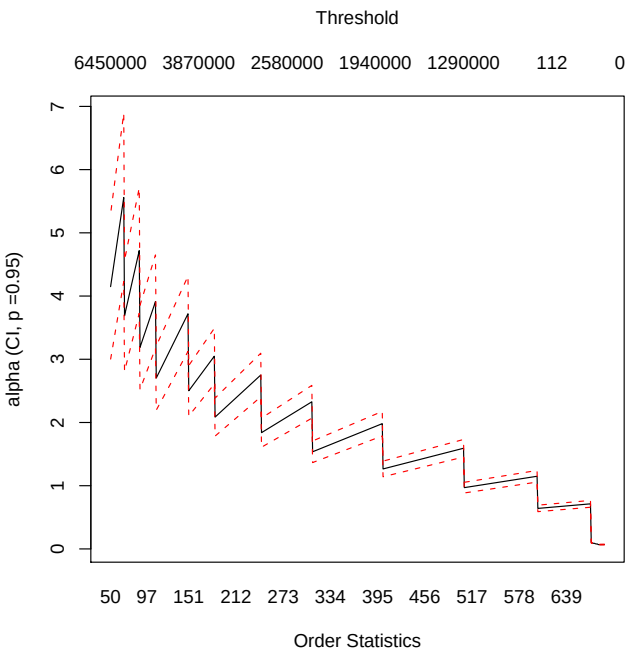

Figure D.1: Hill plot when $p = 0.02$ and $q = 0.01$

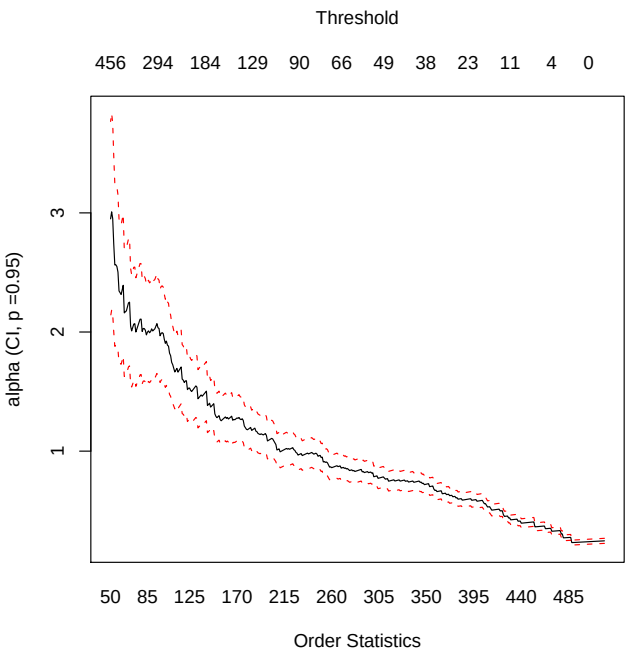

Figure D.2: Hill plot when $p = 0.02$ and $q = 0.02$

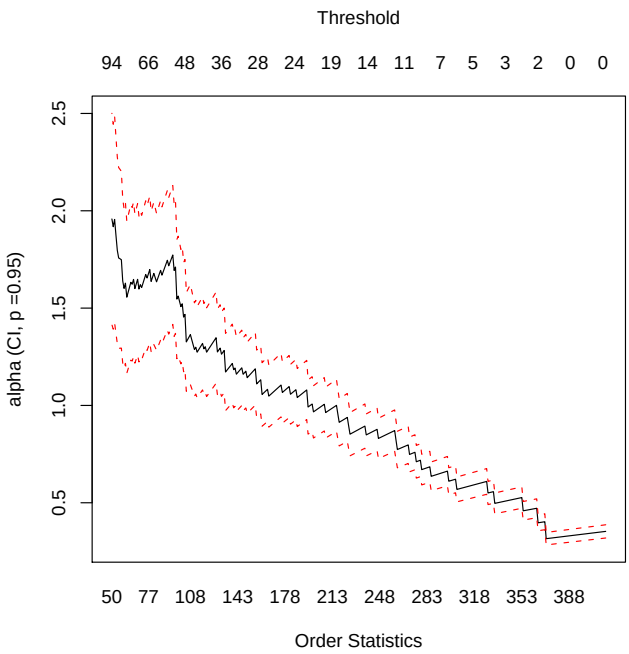

Figure D.3: Hill plot when $p = 0.02$ and $q = 0.03$

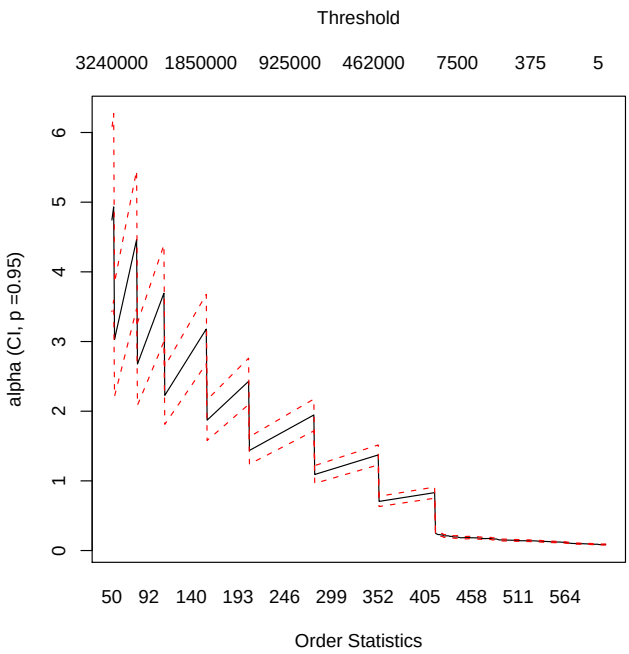

Figure D.4: Hill plot when $p = 0.03$ and $q = 0.02$

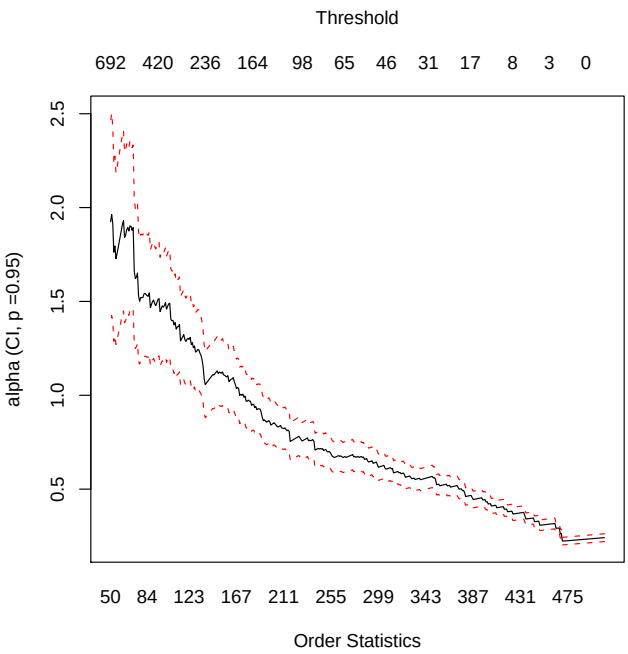

Figure D.5: Hill plot when $p = 0.03$ and $q = 0.03$

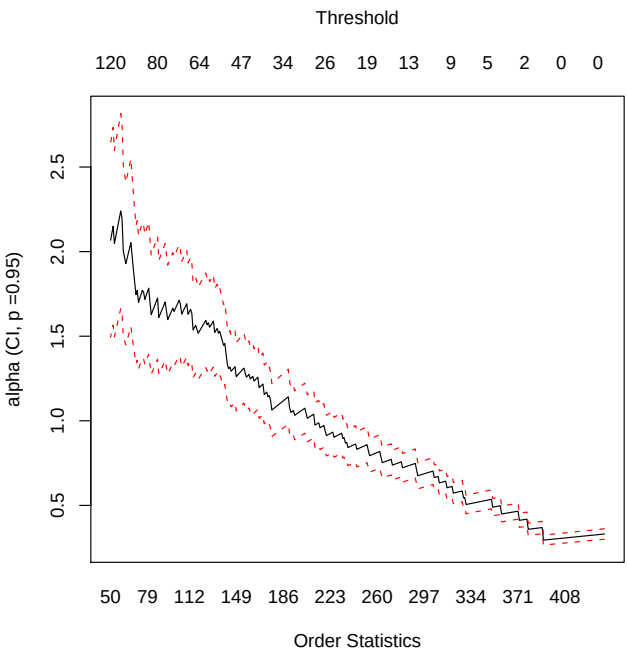

Figure D.6: Hill plot when $p = 0.03$ and $q = 0.04$

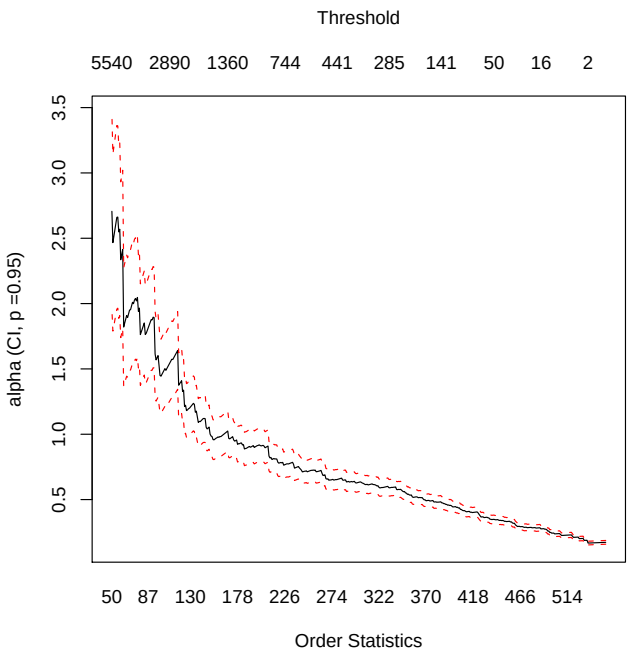

Figure D.7: Hill plot when $p = 0.05$ and $q = 0.04$

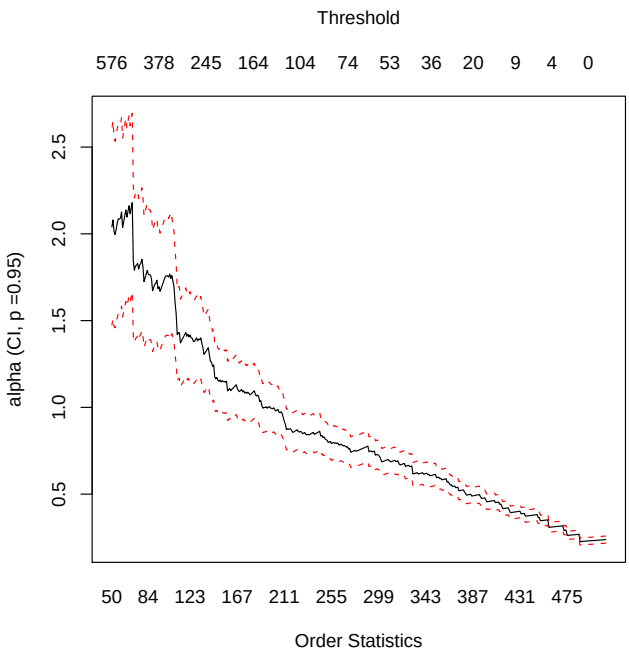

Figure D.8: Hill plot when $p = 0.05$ and $q = 0.05$

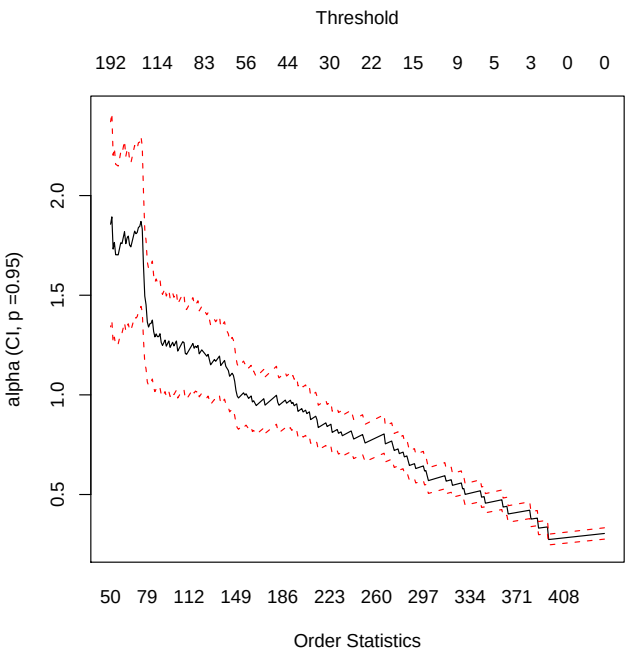

Figure D.9: Hill plot when $p = 0.05$ and $q = 0.06$

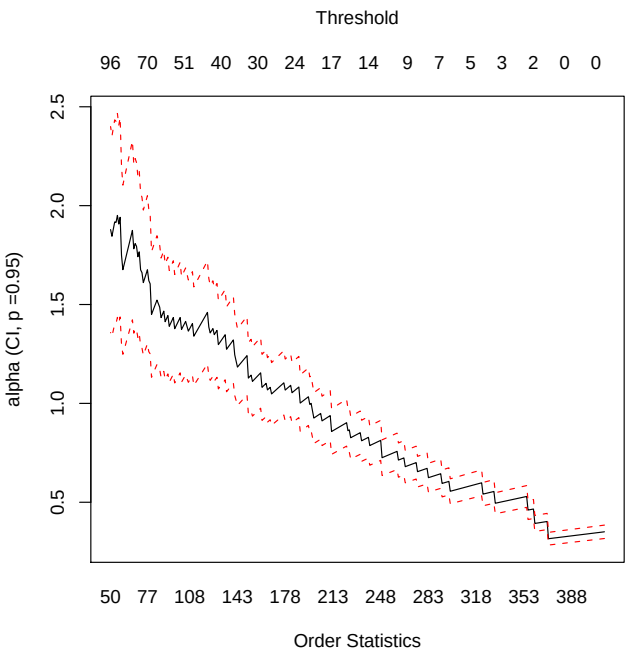

Figure D.10: Hill plot when $p = 0.05$ and $q = 0.07$

Appendix E

Simulation Results on Dividends with Endogenous States

All simulations were conducted with 1000 assets over 100 time steps. Independent of the choice of a and q and b, the Kelly rule clearly outperformed all other strategies. Relative dividends and relative prices were smoother for smaller values a and q and exhibited sharper spikes for larger values a and q. Varying b changed aggregate dividend growth.

E.1 Variability

Table E.1: Average median and interquartile distances of dividends after $t = 40$

a	0.03	0.03	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.05
q	0.02	0.03	0.04	0.04	0.05	0.06	0.06	0.06	0.06	0.06
b	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.02	0.03	0.04
median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
intquartile distance	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.08	0.22

E.2 Concentration

The following figures show the Hill plots of dividends from simulations with different parameters a, q and b at different time steps. The red lines display the 95% confidence intervals of the exponent α. The figures on top display the Hill plots at the 60th and 80th time step. The figures at the bottom left show the Hill plots at the 100th time step. The figures at the bottom right display how aggregate dividends developed over time. The initial 10 time steps, when dividends were exogenously determined, are not included in the plot.

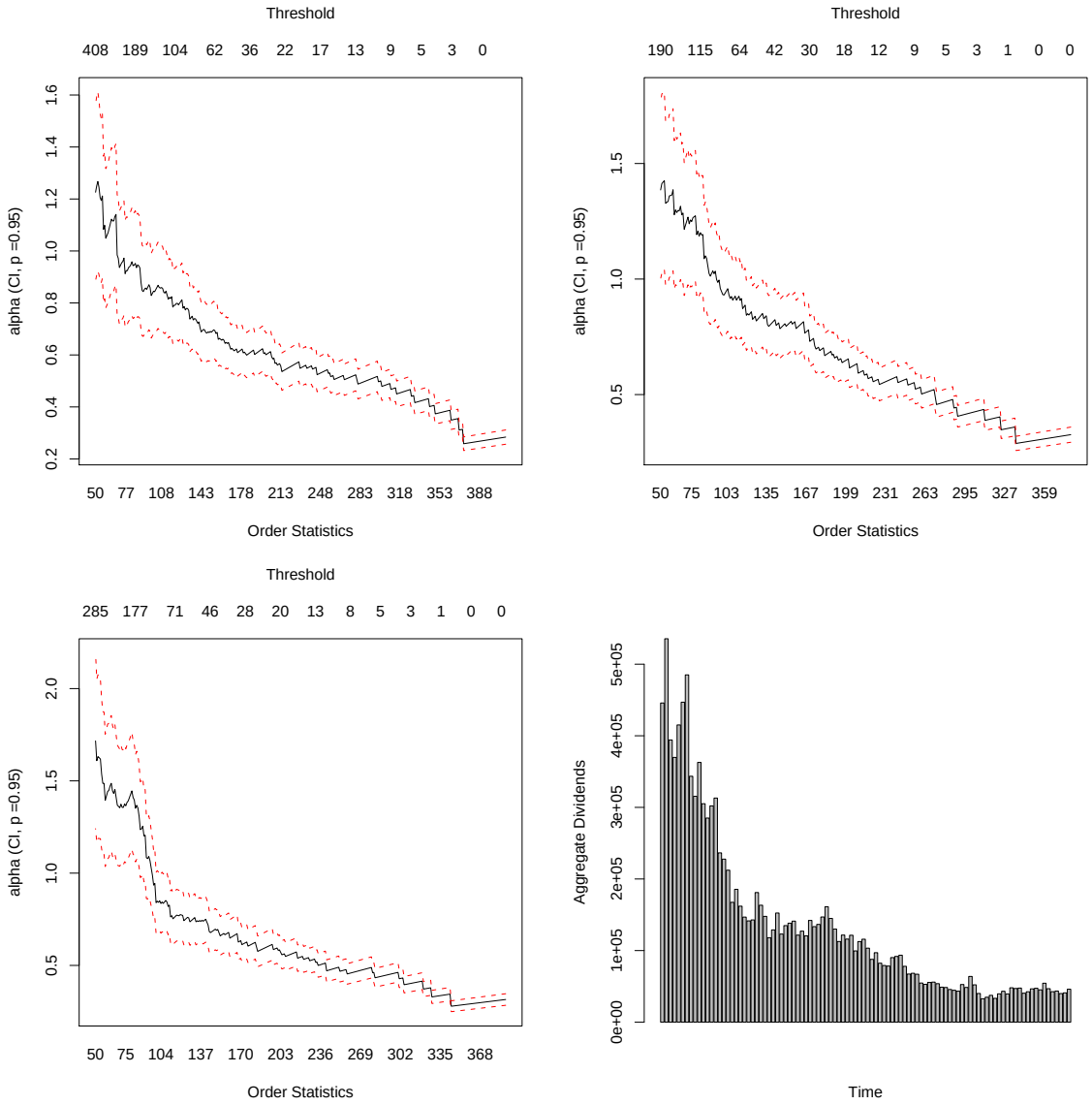

Figure E.1: Hill plot from simulations with $a = 0.03$, $q = 0.02$ and $b = 0.01$ after 60, 80 and 100 time steps

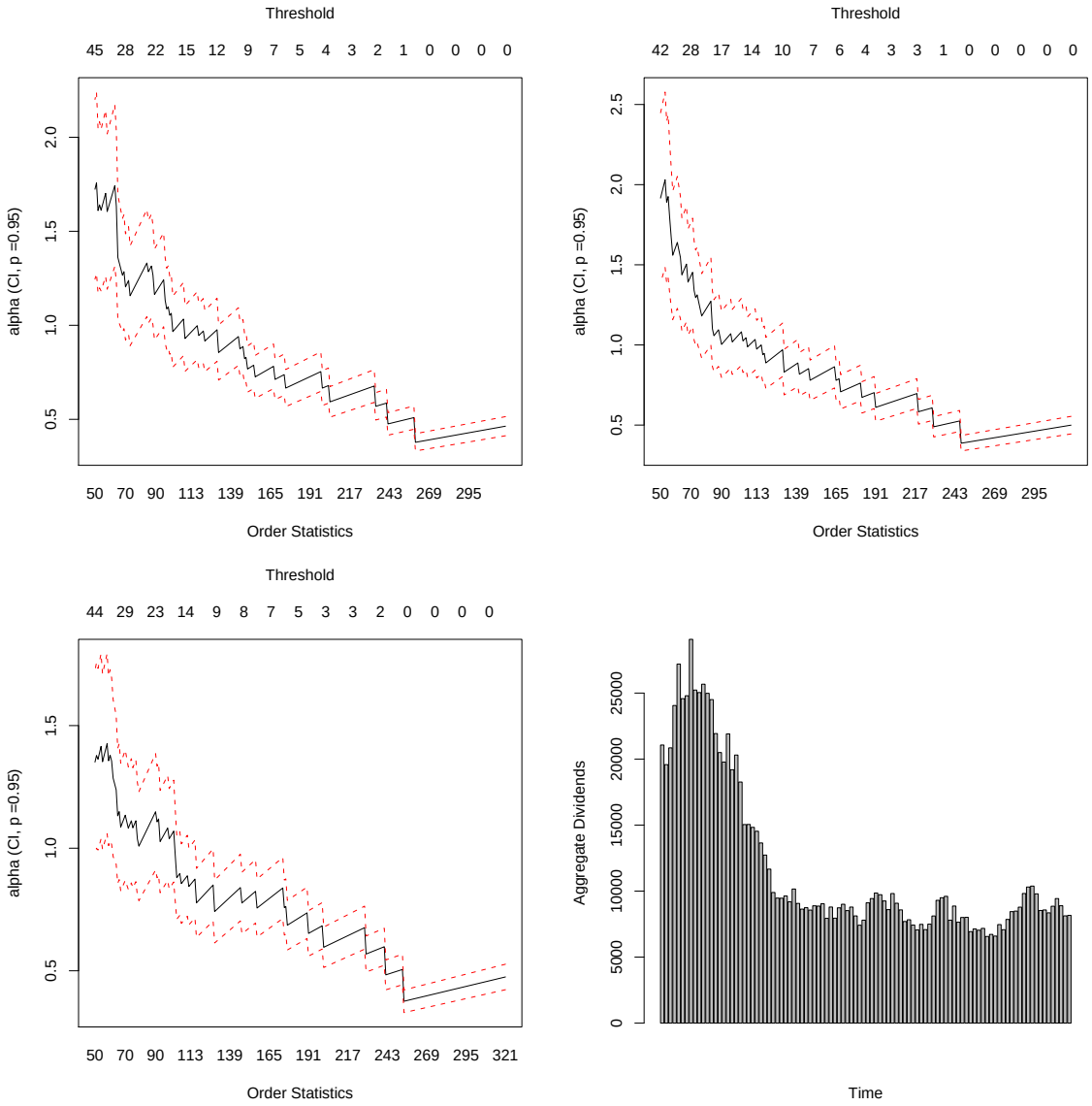

Figure E.2: Hill plot from simulations with $a = 0.03$, $q = 0.03$ and $b = 0.01$ after 60, 80 and 100 time steps

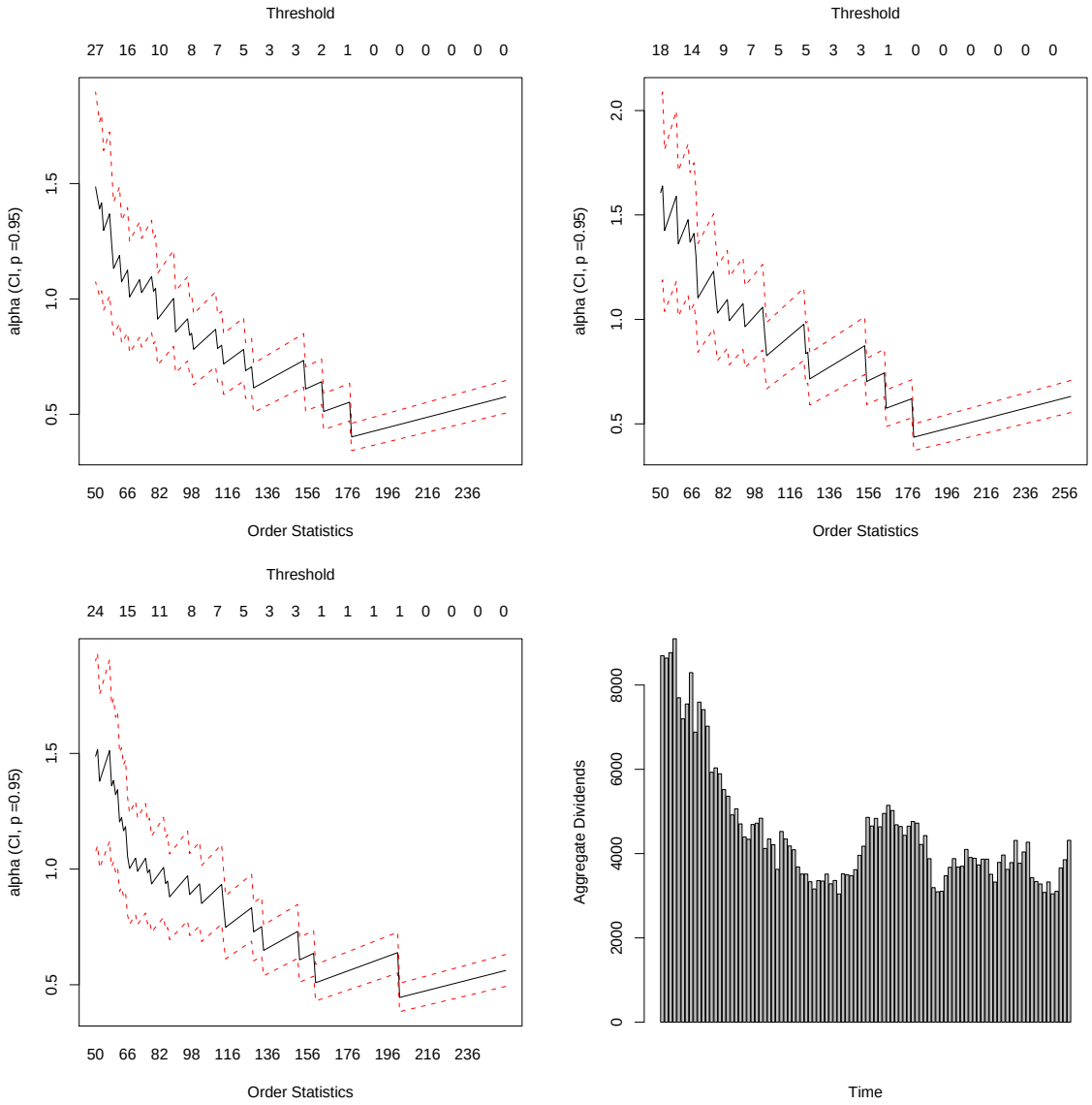

Figure E.3: Hill plot from simulations with $a = 0.03$, $q = 0.04$ and $b = 0.01$ after 60, 80 and 100 time steps

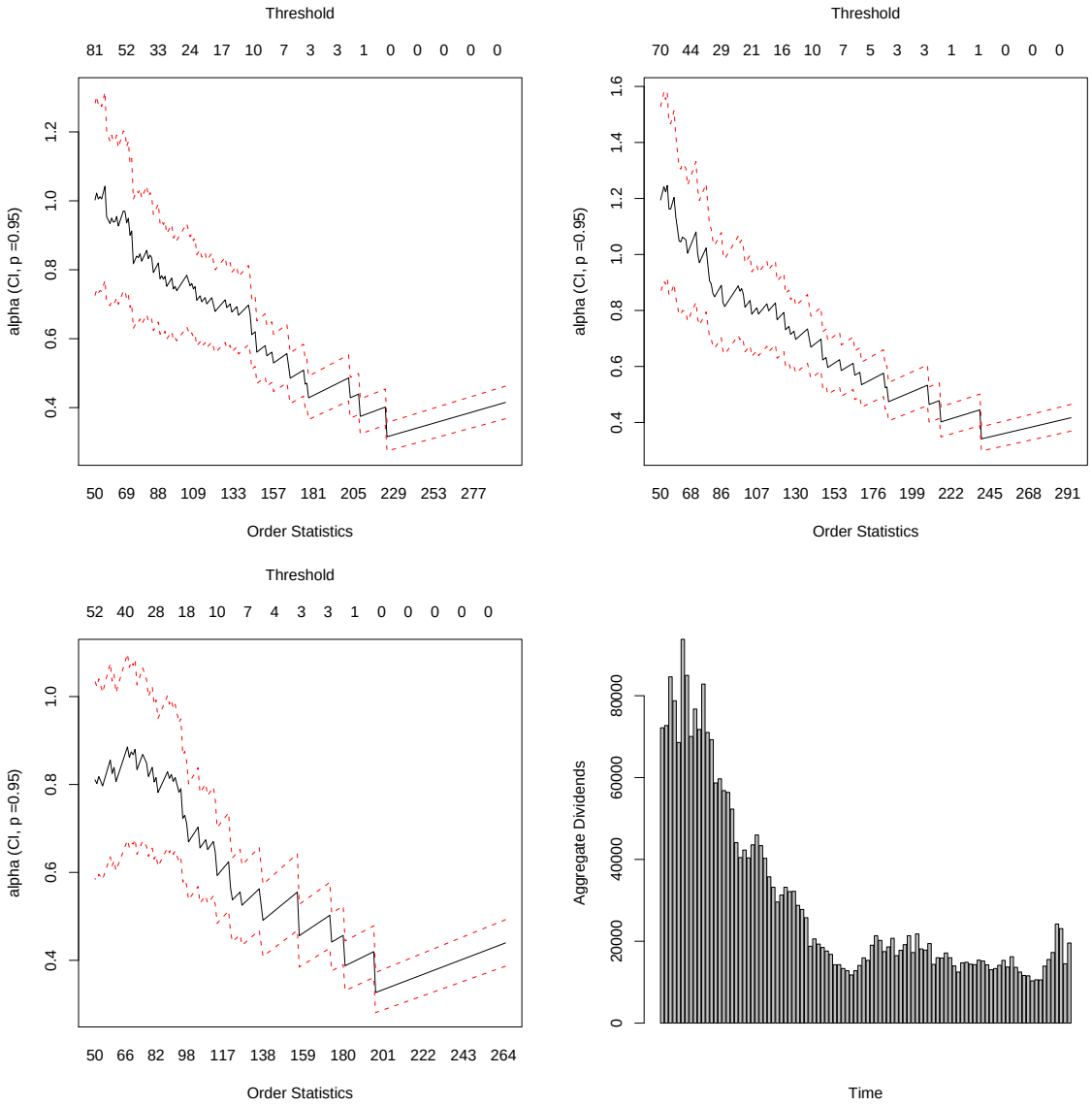

Figure E.4: Hill plot from simulations with $a = 0.05$, $q = 0.04$ and $b = 0.01$ after 60, 80 and 100 time steps

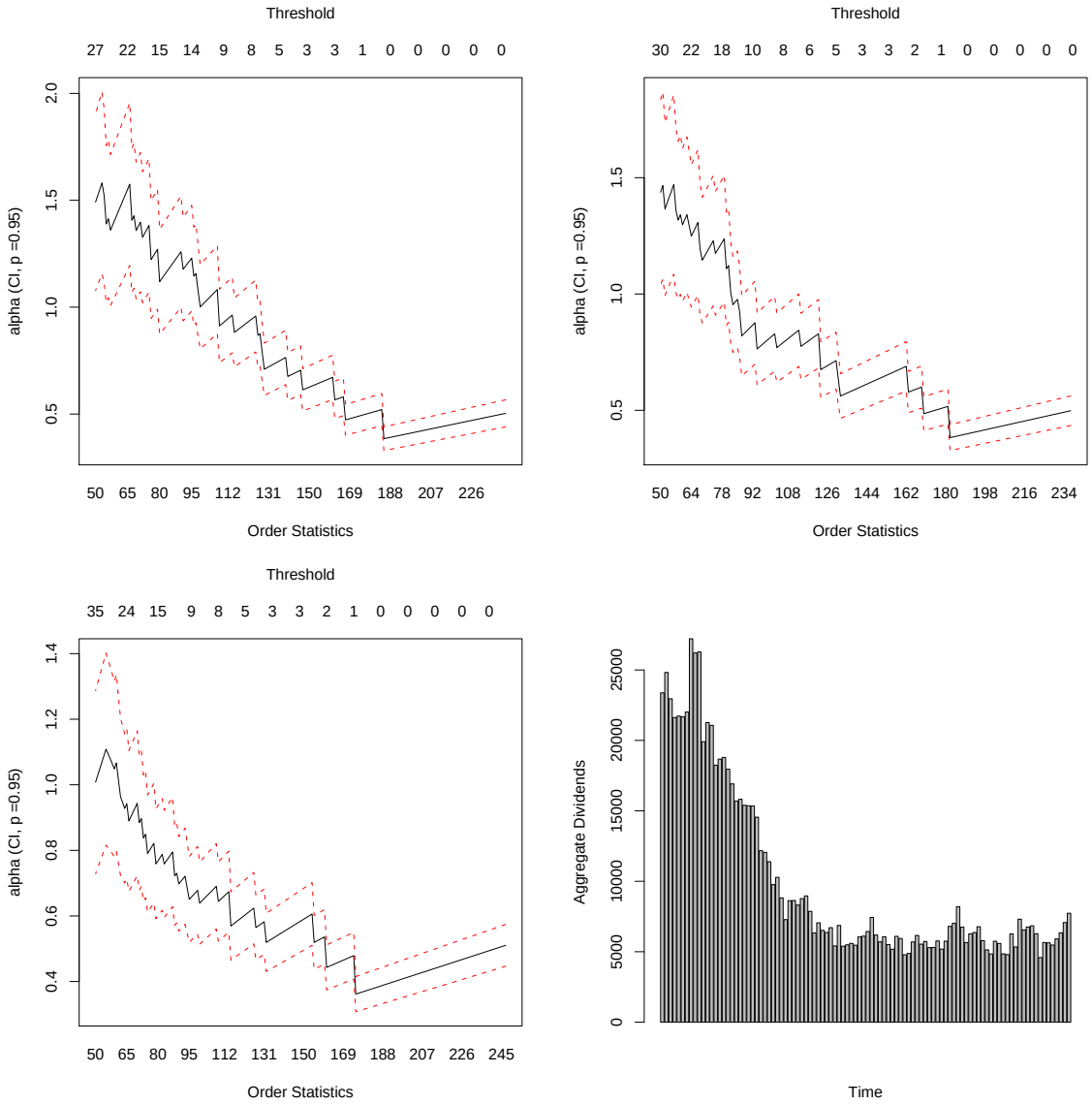

Figure E.5: Hill plot from simulations with $a = 0.05$, $q = 0.05$ and $b = 0.01$ after 60, 80 and 100 time steps

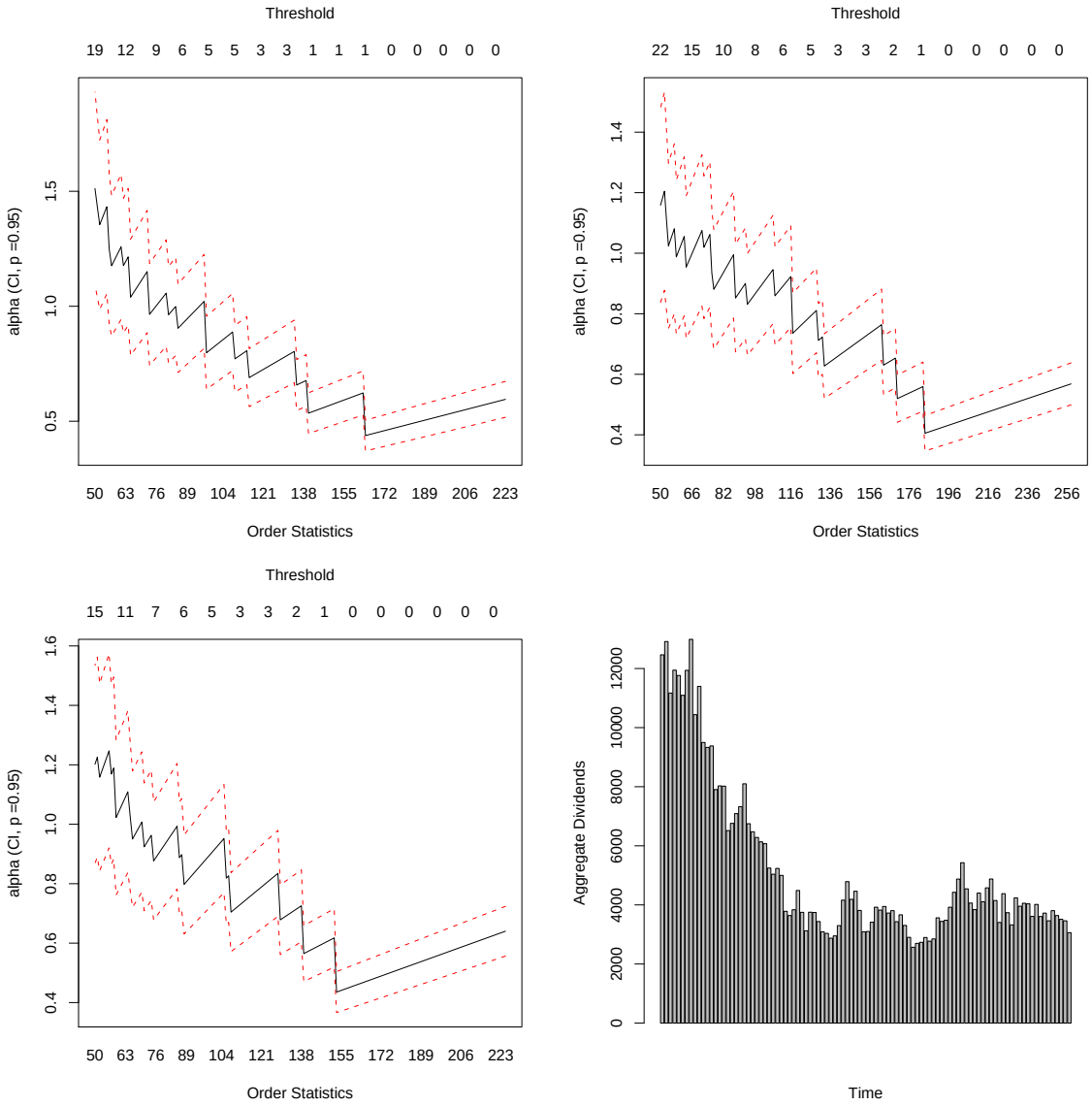

Figure E.6: Hill plot from simulations with $a = 0.05$, $q = 0.06$ and $b = 0.01$ after 60, 80 and 100 time steps

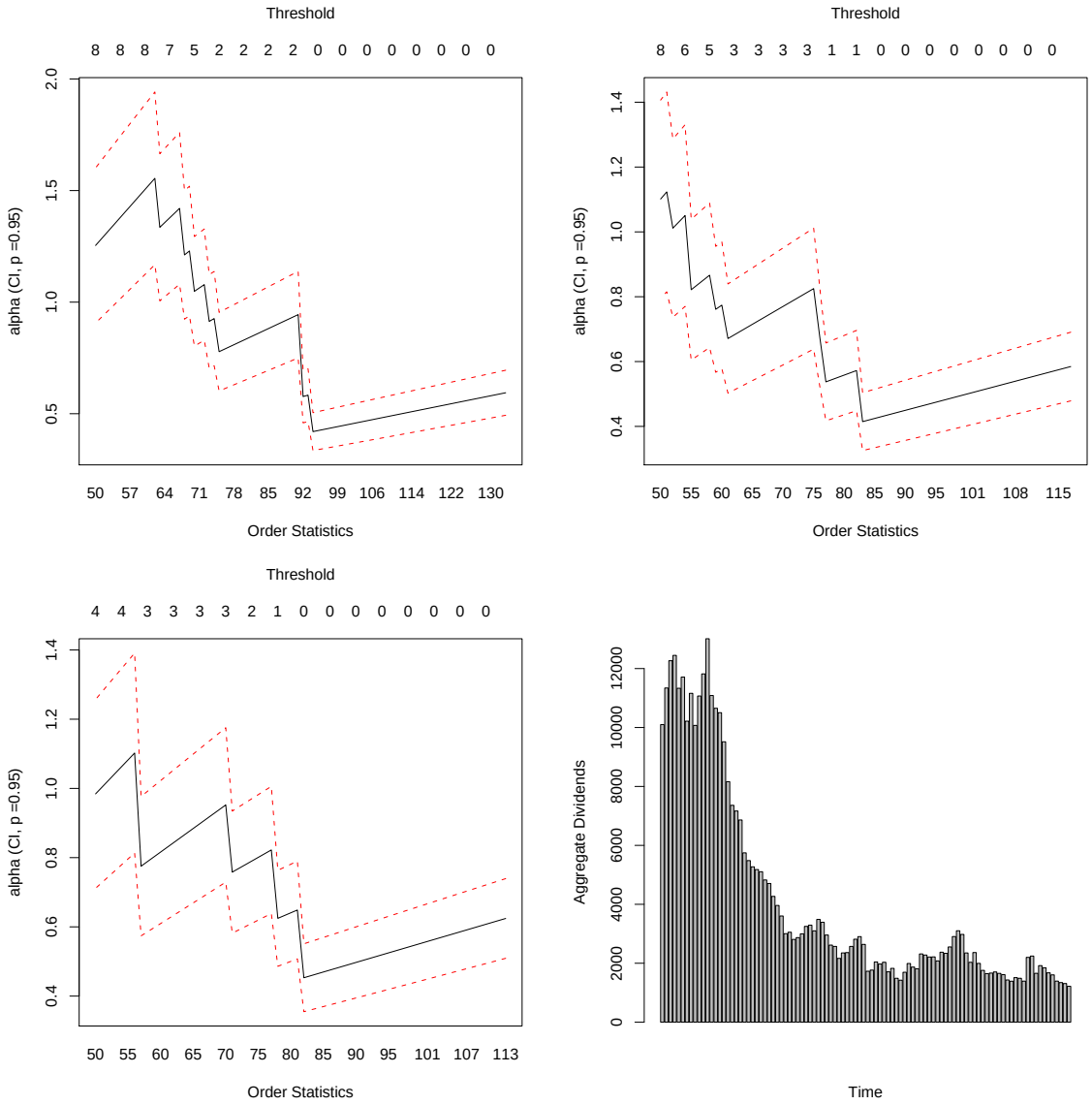

Figure E.7: Hill plot from simulations with $a = 0.05$, $q = 0.06$ and $b = 0.00$ after 60, 80 and 100 time steps

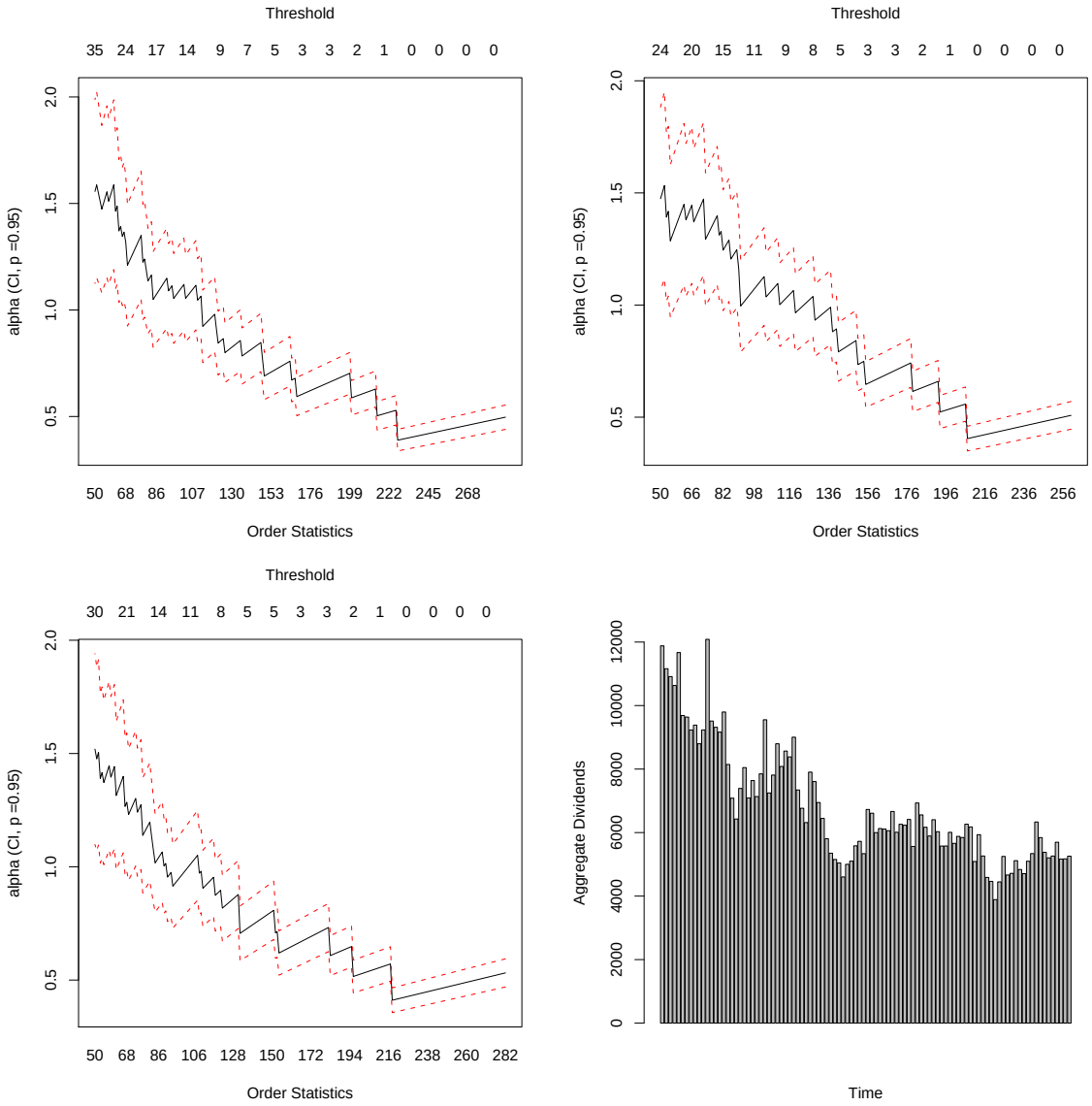

Figure E.8: Hill plot from simulations with $a = 0.05$, $q = 0.06$ and $b = 0.02$ after 60, 80 and 100 time steps

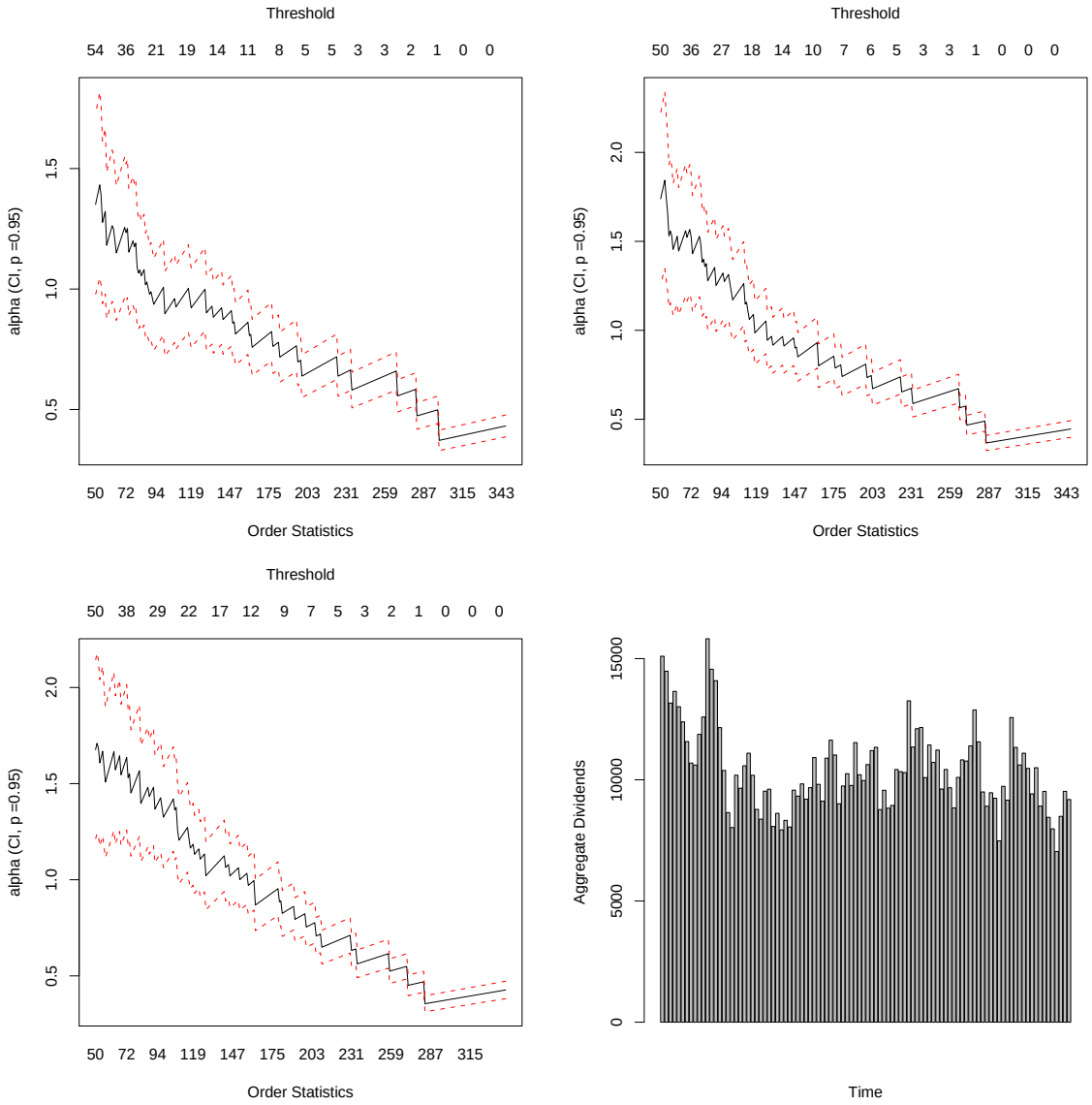

Figure E.9: Hill plot from simulations with $a = 0.05$, $q = 0.06$ and $b = 0.03$ after 60, 80 and 100 time steps

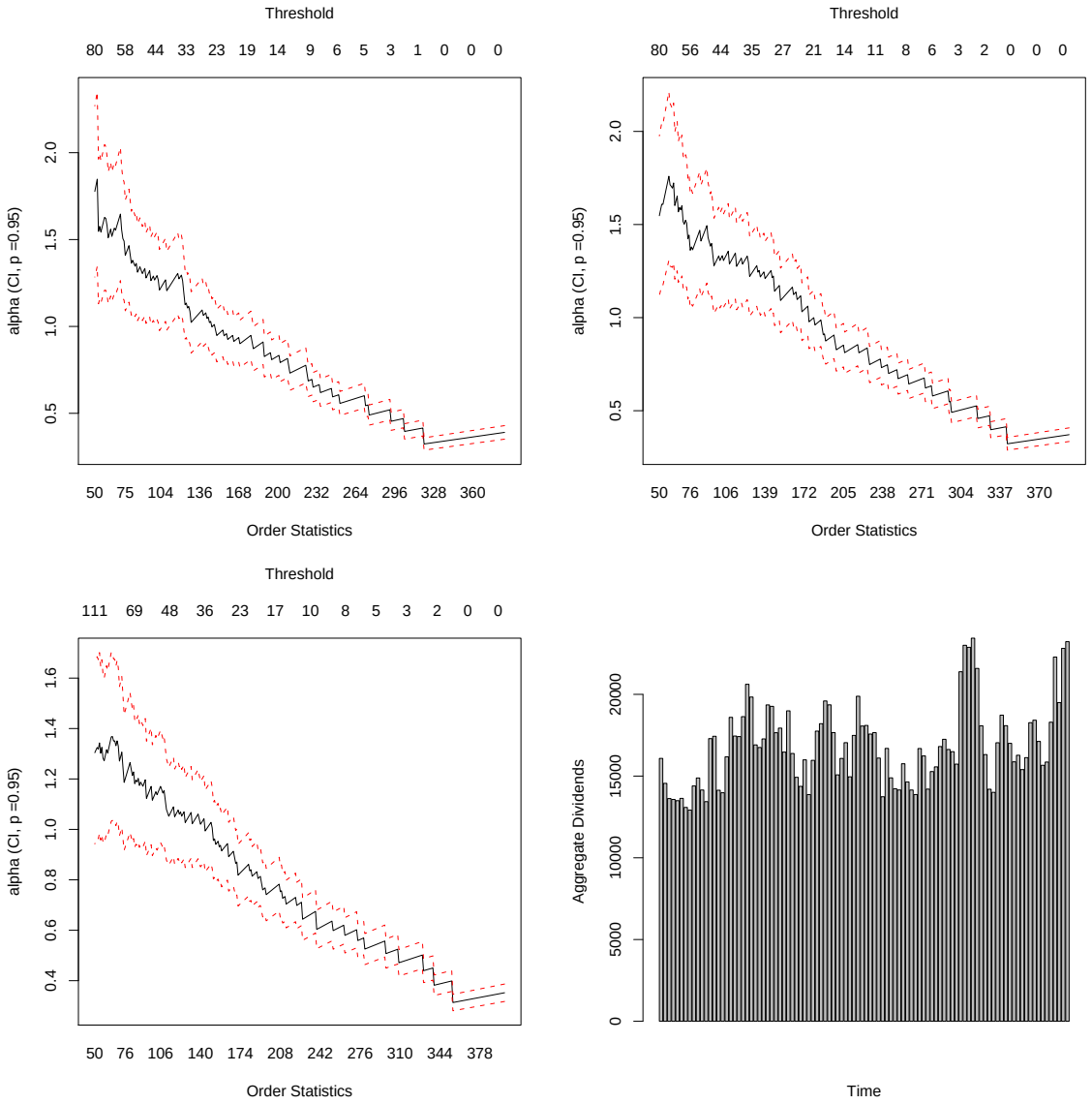

Figure E.10: Hill plot from simulations with $a = 0.05$, $q = 0.06$ and $b = 0.04$ after 60, 80 and 100 time steps

Appendix F

Simulation Results on Endogenous Dividends

All simulations were conducted with 1000 assets over 100 time steps. Independent of the choice of p and q and g, the Kelly rule clearly outperformed all other strategies in the long term. Relative dividends and relative prices were smoother for smaller values p and q and exhibited sharper spikes for larger values p and q. The effect of varying g is not that clear.

F.1 Variability

Table F.1: Average median and interquartile distances of dividends after $t = 40$

p	0.03	0.03	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.03
q	0.02	0.03	0.04	0.04	0.05	0.06	0.06	0.06	0.06	0.04
g	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.5	0.8	0.3
median	0.00	-0.01	0.00	-0.02	-0.01	0.00	-0.01	-0.01	-0.02	0.00
intquartile distance	0.02	0.25	0.15	1.15	0.60	0.48	0.42	0.48	0.52	0.17

F.2 Concentration

The following figures show the Hill plots of dividends from simulations with different parameters p, q and g at different time steps. The red lines display the 95% confidence intervals of the exponent α. The figures on top display the Hill plots at the 60th and 80th time step. The figures at the bottom left show the Hill plots at the 100th time step. The figures at the bottom right displays how aggregate dividends developed over time. The initial 10 time steps, when dividends were exogenously determined, are not included in the plot.

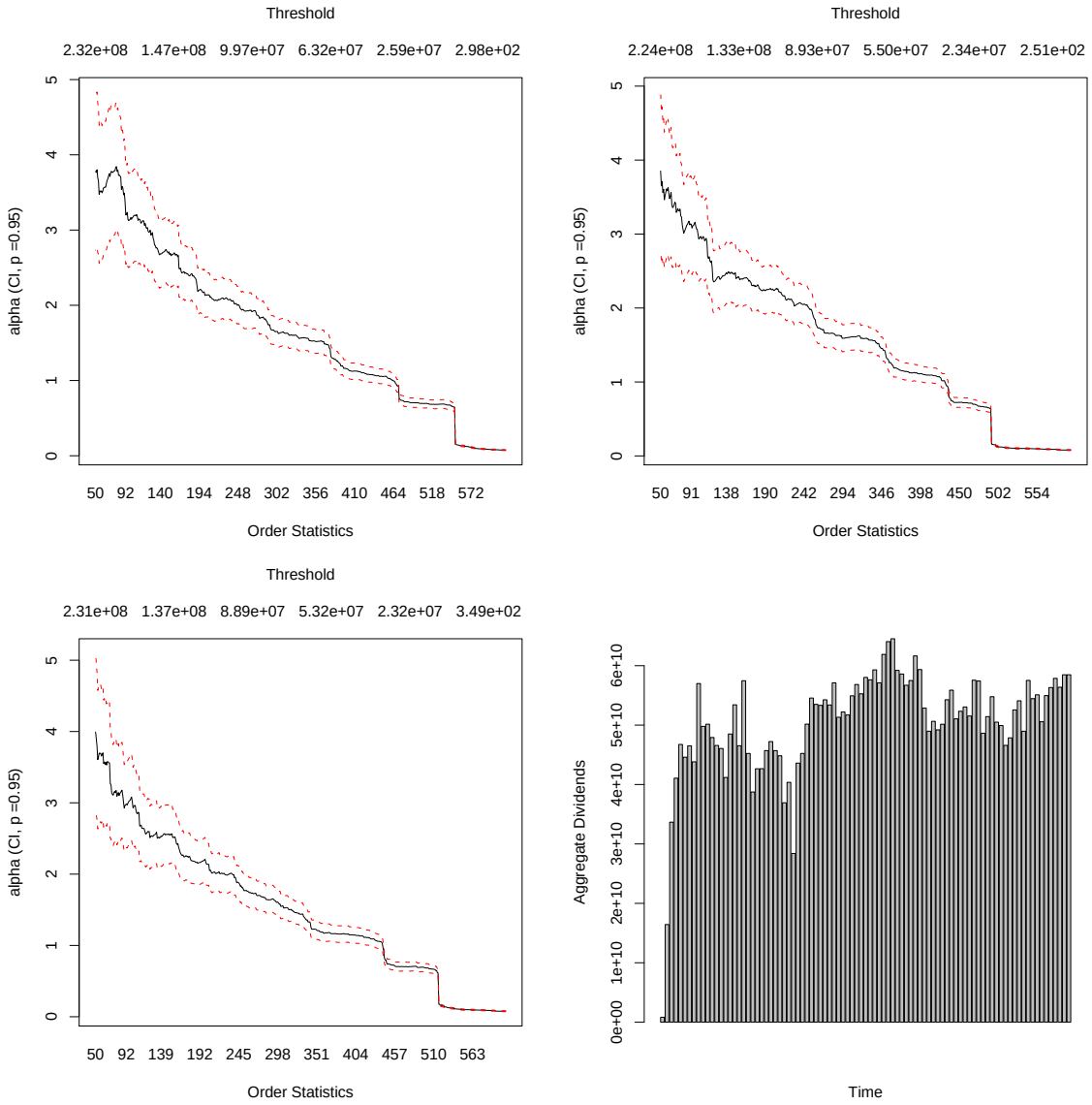

Figure F.1: Hill plot from simulations with $p = 0.03$, $q = 0.02$ and $g = 0.2$ after 60, 80 and 100 time steps

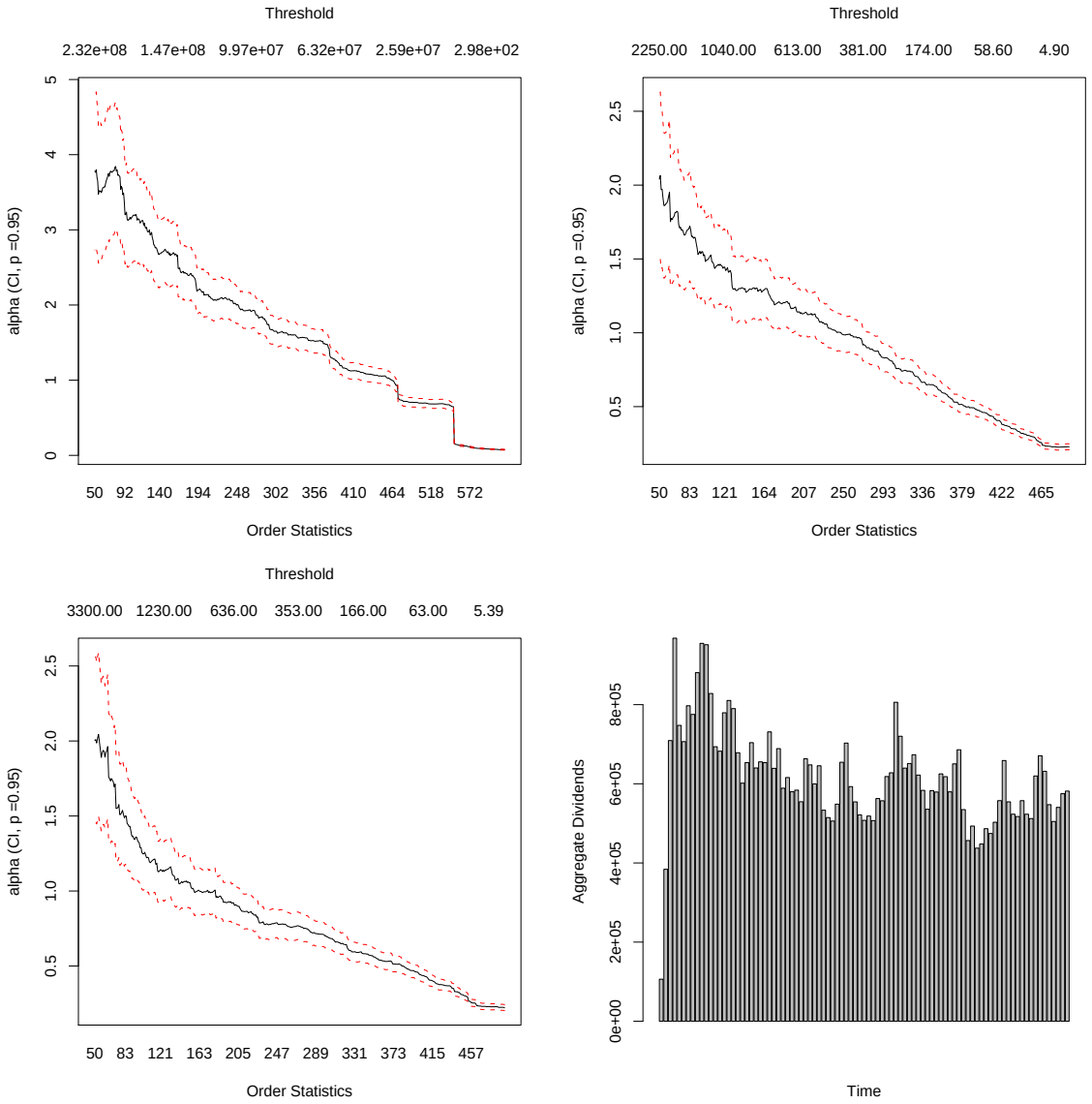

Figure F.2: Hill plot from simulations with $p = 0.03$, $q = 0.03$ and $g = 0.2$ after 60, 80 and 100 time steps

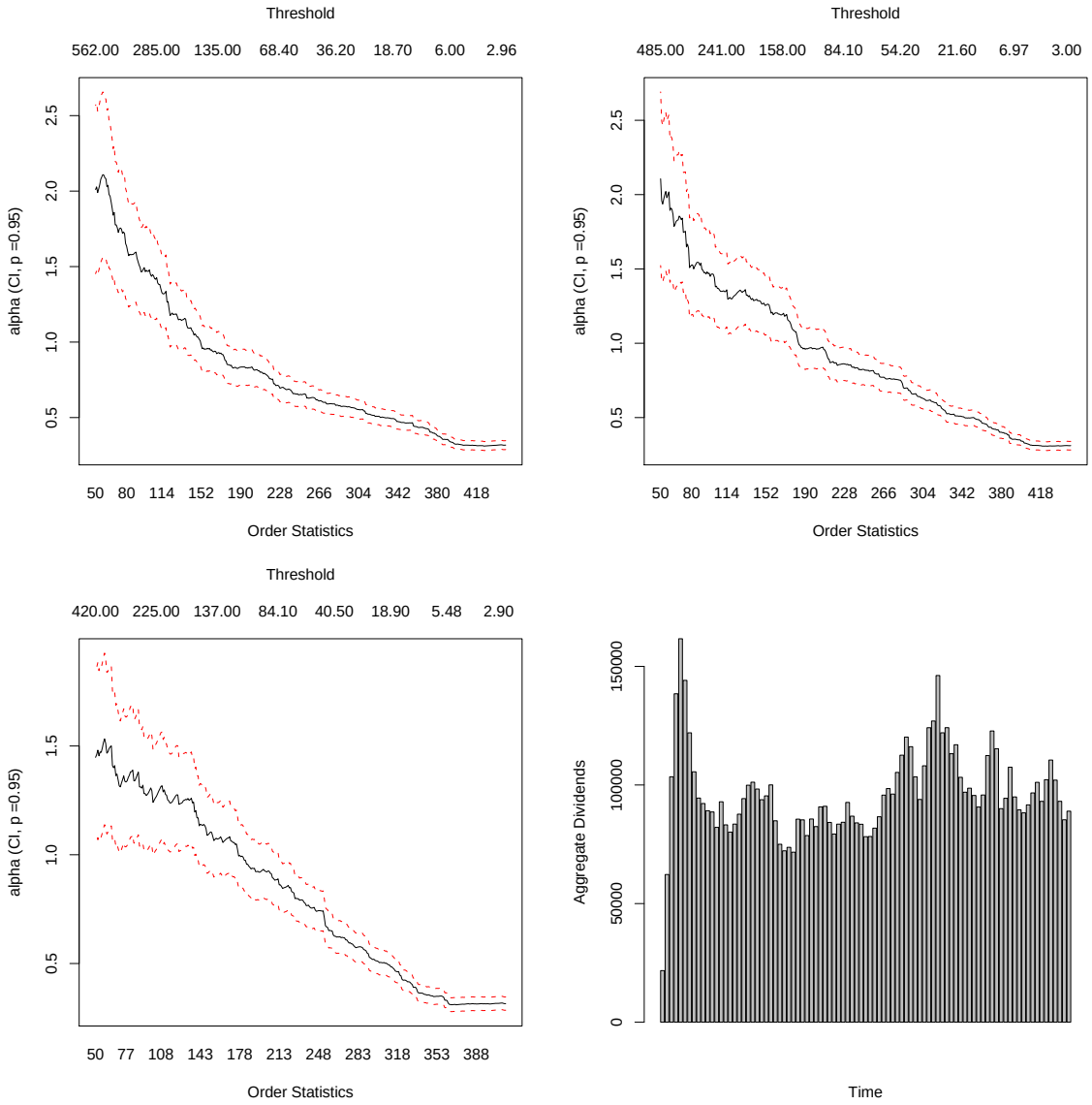

Figure F.3: Hill plot from simulations with $p = 0.03$, $q = 0.04$ and $g = 0.2$ after 60, 80 and 100 time steps

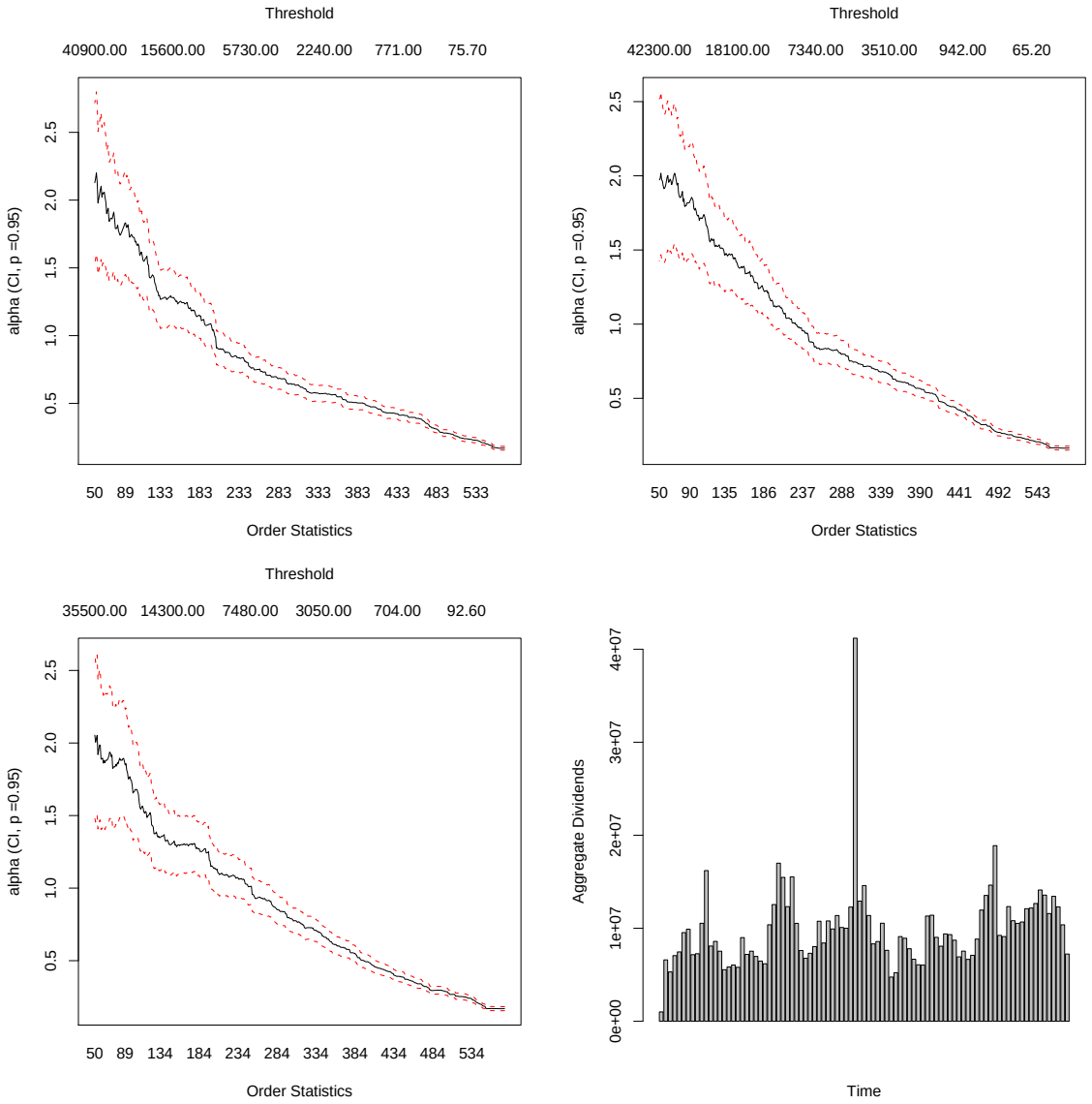

Figure F.4: Hill plot from simulations with $p = 0.05$, $q = 0.04$ and $g = 0.2$ after 60, 80 and 100 time steps

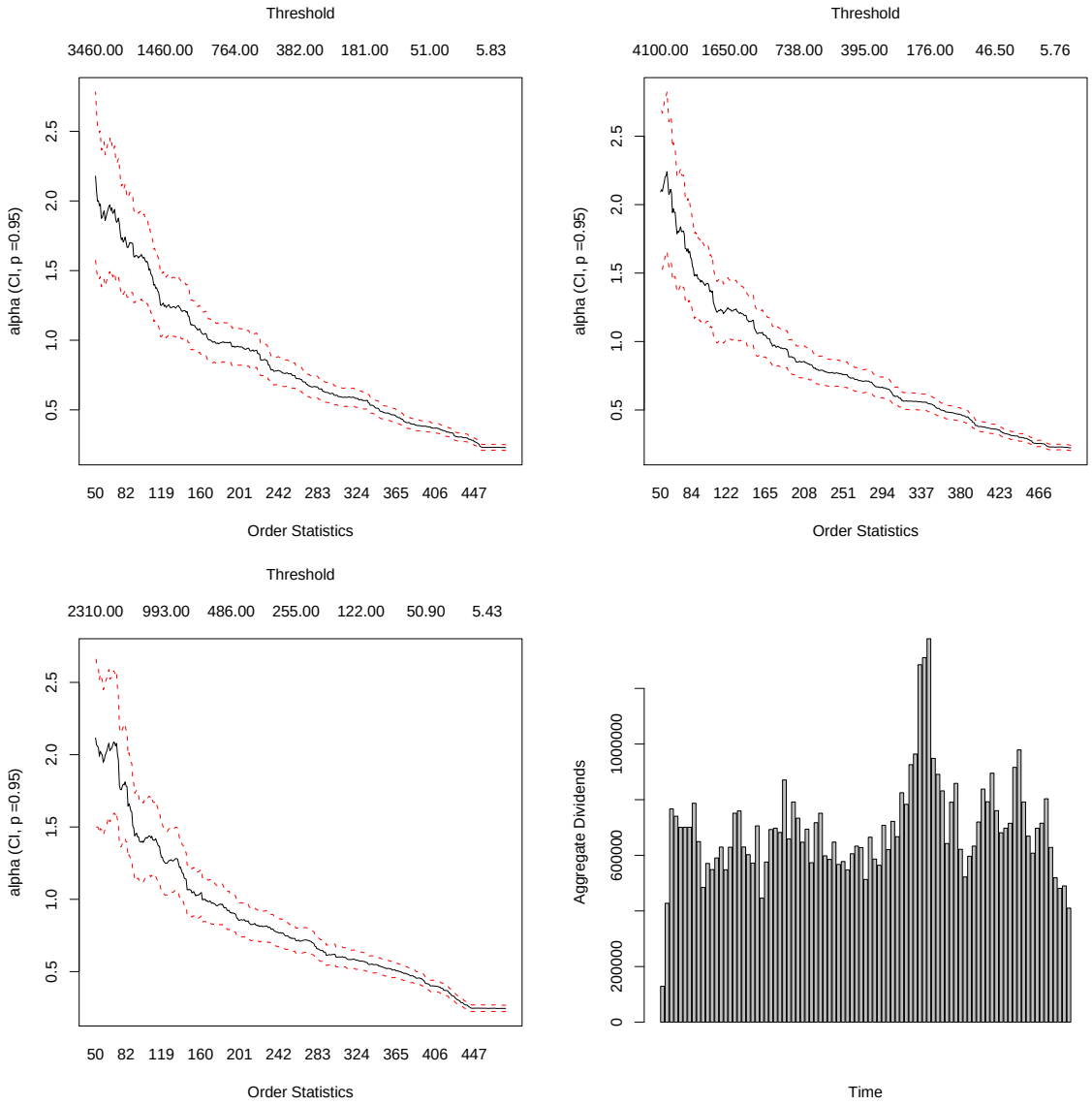

Figure F.5: Hill plot from simulations with $p = 0.05$, $q = 0.05$ and $g = 0.2$ after 60, 80 and 100 time steps

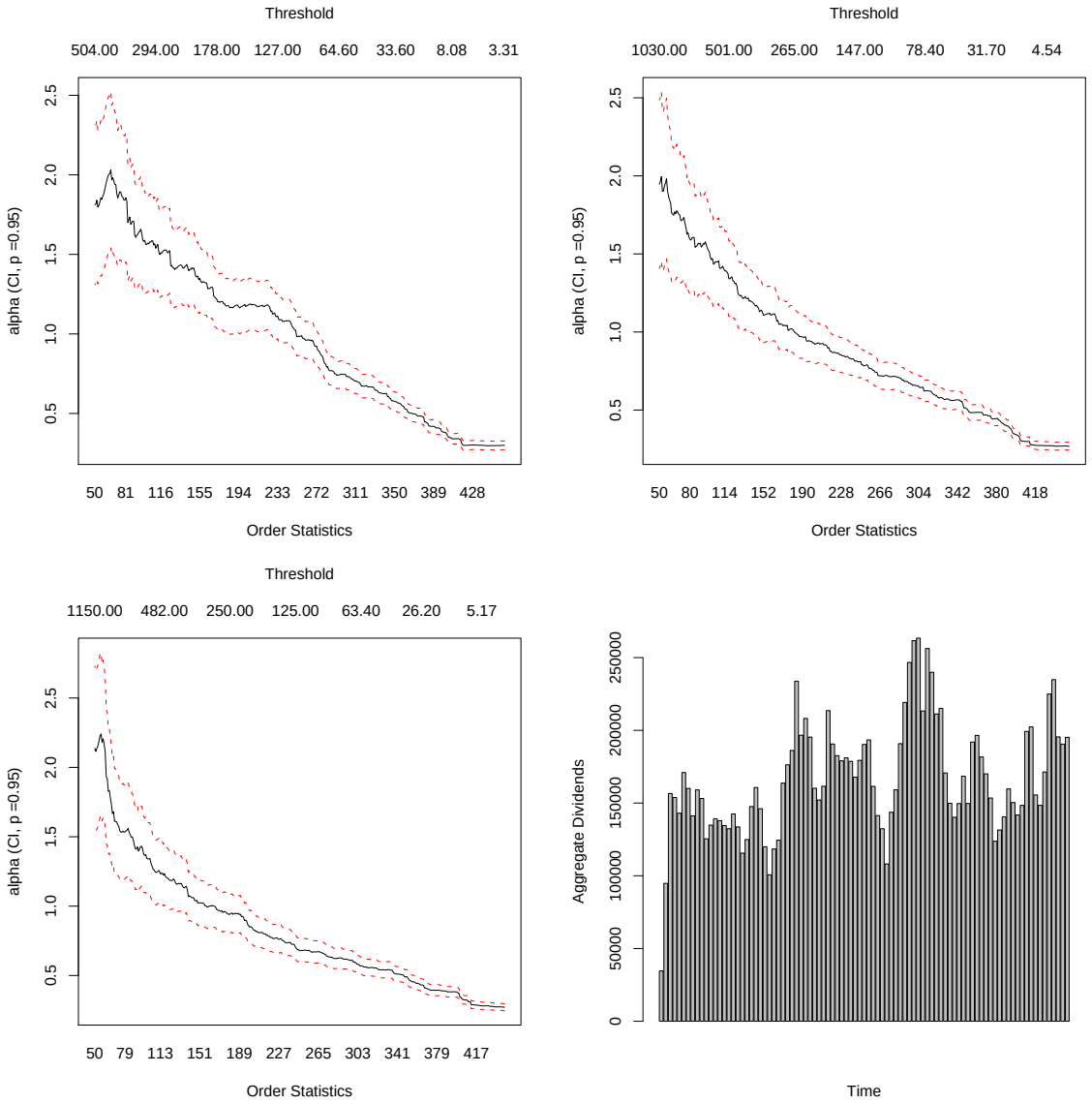

Figure F.6: Hill plot from simulations with $p = 0.05$, $q = 0.06$ and $g = 0.2$ after 60, 80 and 100 time steps

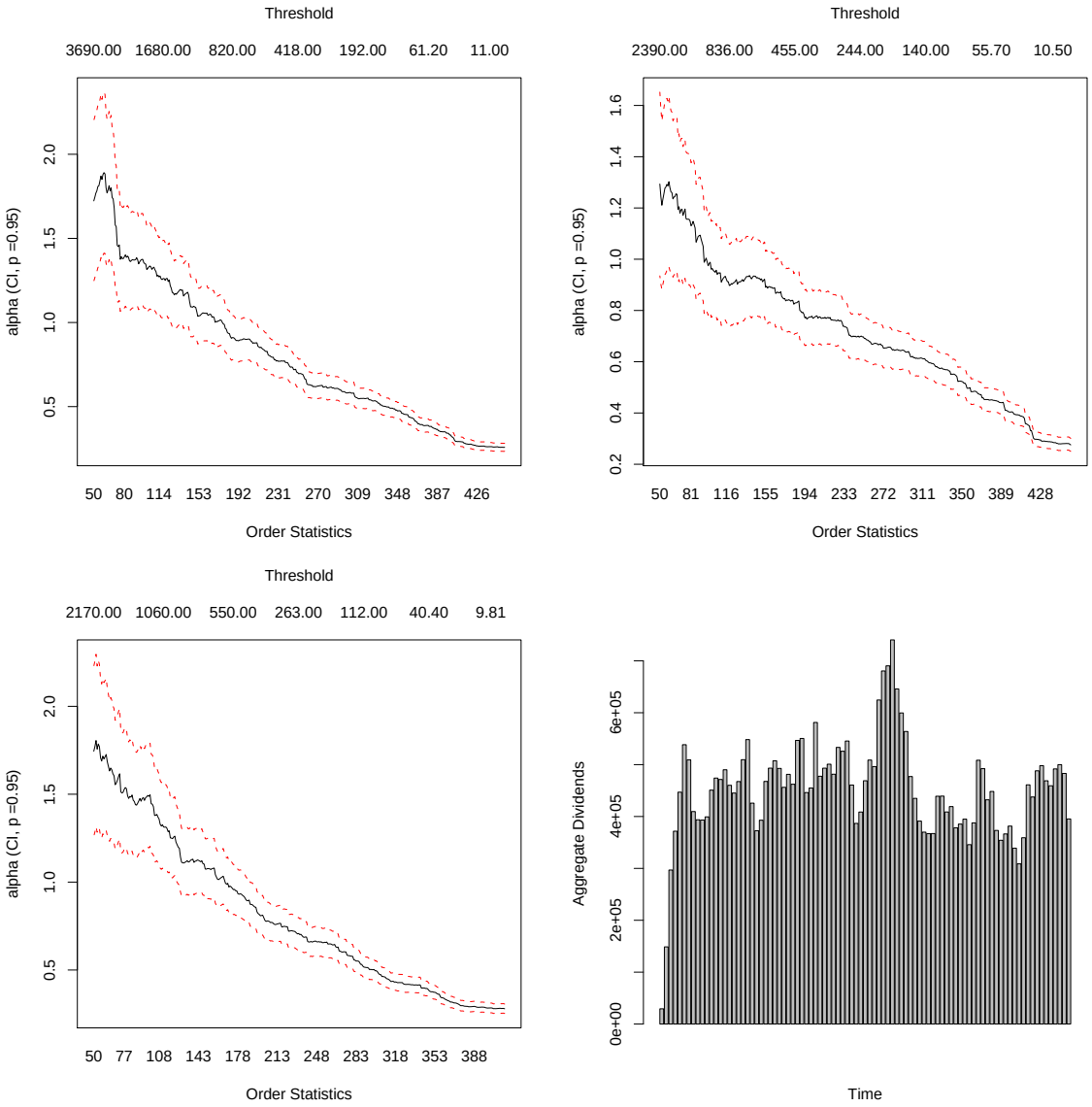

Figure F.7: Hill plot from simulations with $p = 0.05$, $q = 0.06$ and $g = 0.3$ after 60, 80 and 100 time steps

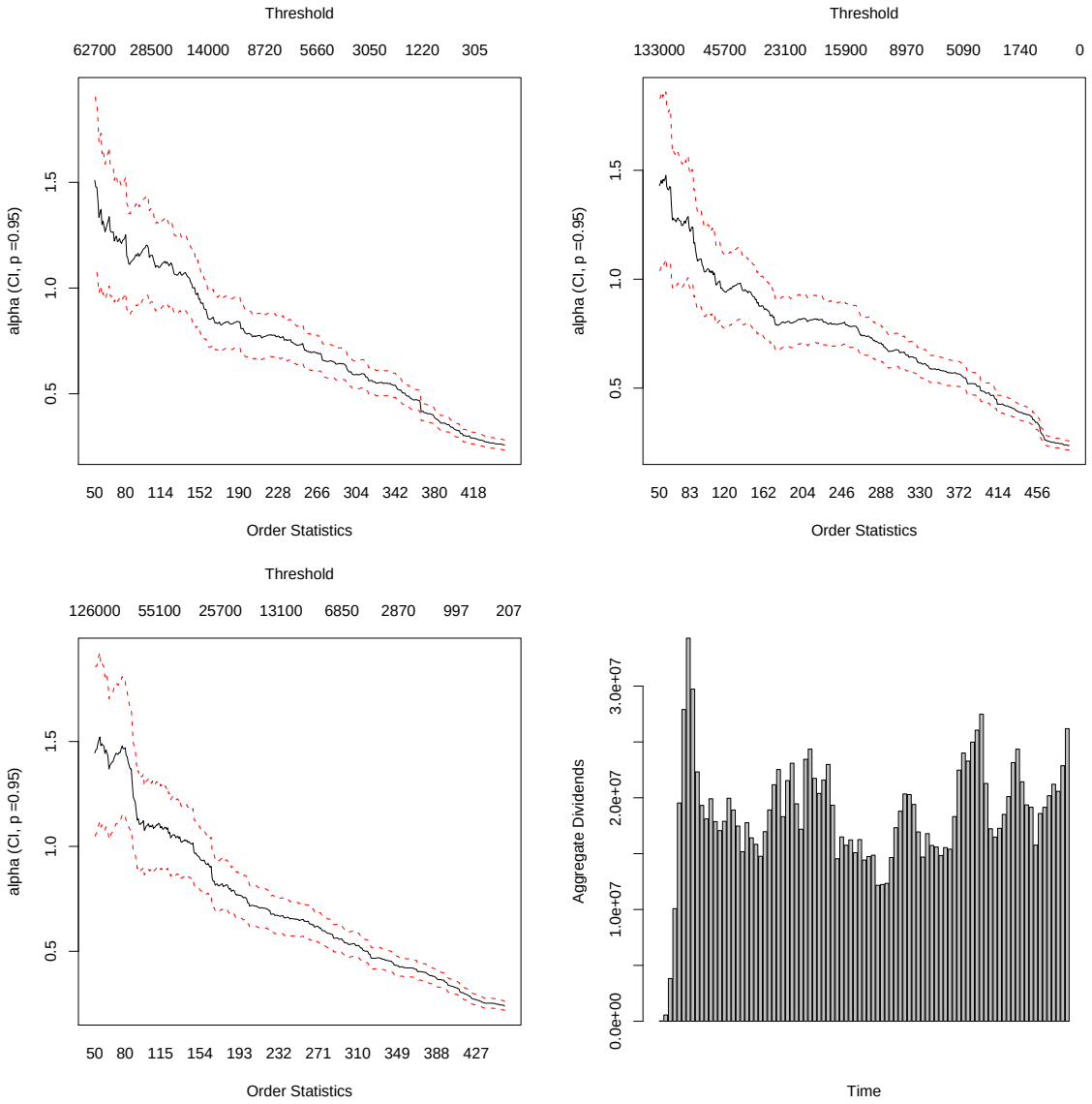

Figure F.8: Hill plot from simulations with $p = 0.05$, $q = 0.06$ and $g = 0.5$ after 60, 80 and 100 time steps

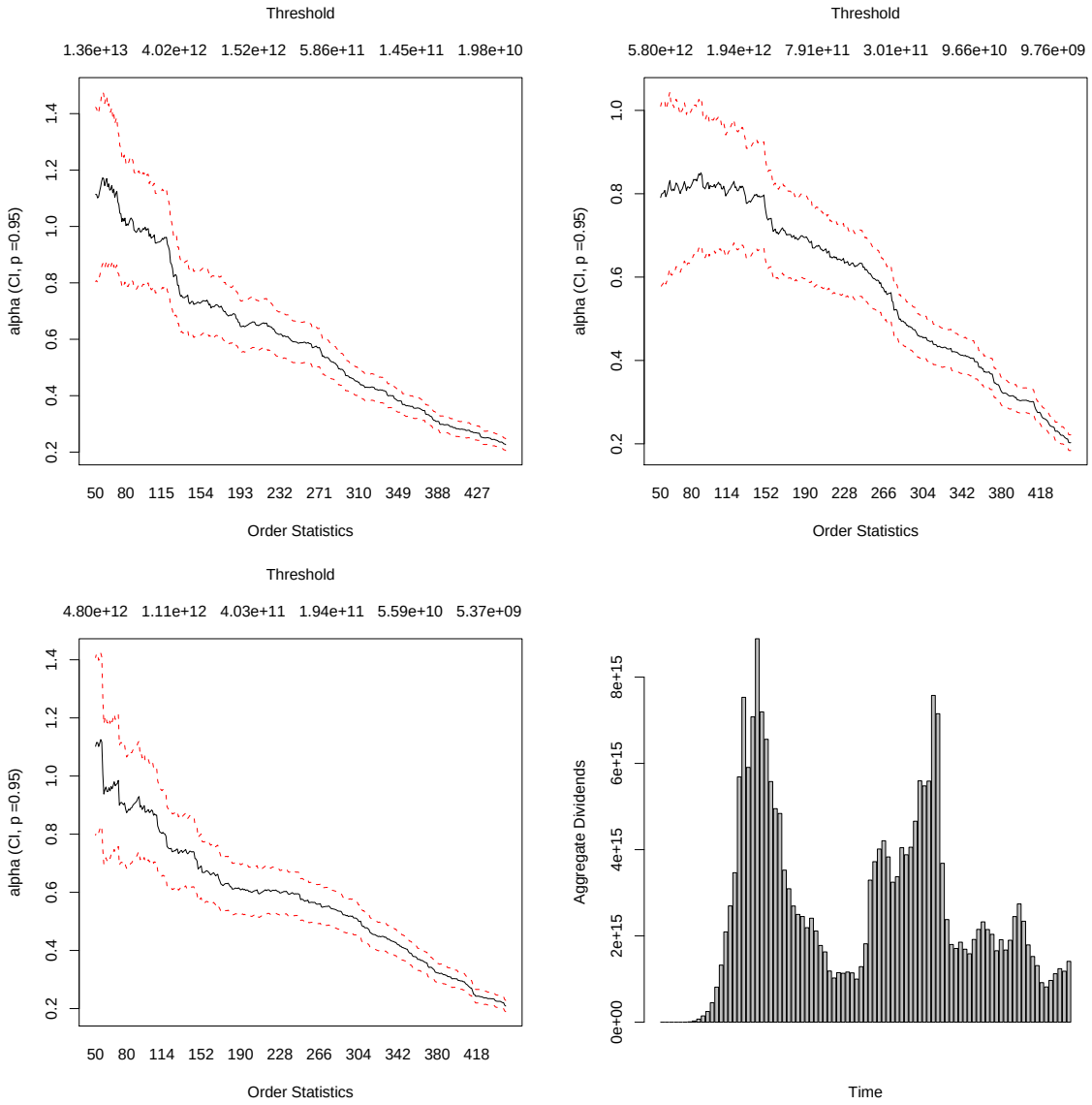

Figure F.9: Hill plot from simulations with $p = 0.05$, $q = 0.06$ and $g = 0.8$ after 60, 80 and 100 time steps

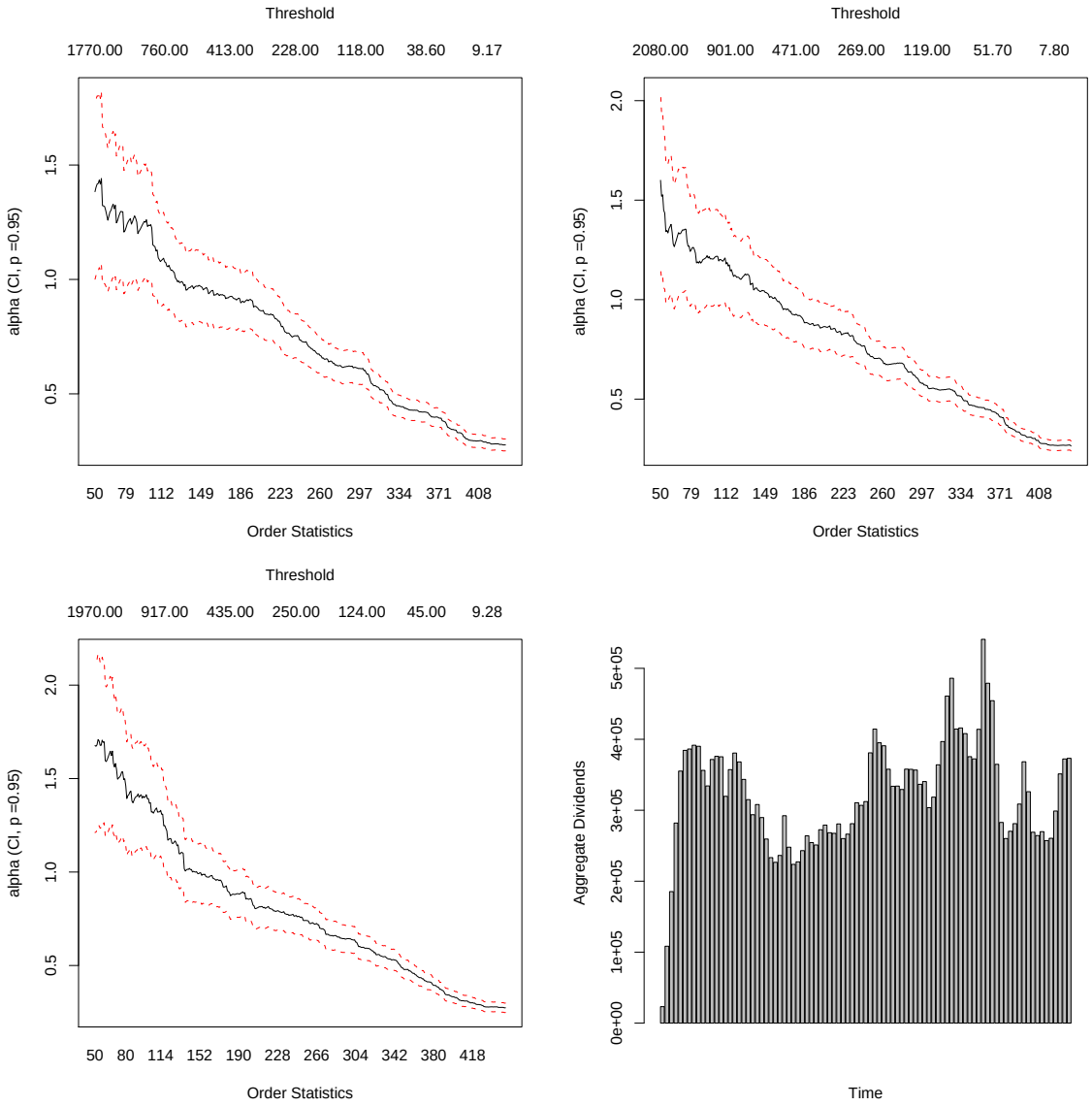

Figure F.10: Hill plot from simulations with $p = 0.03$, $q = 0.04$ and $g = 0.3$ after 60, 80 and 100 time steps