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Financial Market Anomalies: Acceleration Effect and Gamma Factor (Γ)

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Executive Summary

Over the last years, there has been increasing criticism of fundamental asset pricing labels, such as the Capital Asset Pricing Model (CAPM) or its extended version Fama-French Three-Factor Model as well as of theories such as the Efficient Market Hypothesis (Avramov & Chordia, 2006). Behavioral economists and other scientists have argued that due to a process of simplification – based on a number of strong assumptions such as the predictability and rationality of investor behavior - those asset pricing models are unable to explain numerous financial market anomalies which seem to stem from the behavioral irrationality of the investor (Fama, 1998). There is evidence that overconfidence, overreaction as well as other behavioral biases and risk-based sources are the cause of one of the most important asset pricing anomalies: "momentum" (Blitz, Hanauer, & Vidojevic, 2017) (Daniel, Hirshleifer, & Subrahmanyam, 1998).

According to Jegadeesh & Titman (1993): "if stock prices either overreact or underreact to information, then profitable trading strategies that select stocks based on their past returns exist". Indeed, the two most popular trading investment styles are the "contrarian" by De Bondt & Thaler (1985) and "momentum" documented for the first time by Jegadeesh & Titman (1993), which aim to achieve benefits from the "mean-reversion effect" (for the former) and the "short-term return persistence" anomalies (for the latter).

Momentum was first defined and documented in 1993 in a paper by Jegadeesh & Titman (1993); it was described by Fama & French (1993) as "the premier unexplained anomaly". Momentum consists in the persistence of a linear trend in the log-price process and there is empirical evidence across countries and time as well as asset classes (Fama & French, 1993). Nowadays, momentum-based strategies, i.e. strategies based on the Δ (delta) factor are widely implemented by asset managers.

Recently, Ardila, Forrò, & Sornette (2015) reported the evidence of an important novel effect complementing momentum: "acceleration", which is defined as the change in momentum and is quantified by "the first difference of successive returns", i.e. by the gamma (Γ) parameter. Ardila, Forrò, & Sornette (2015) defined acceleration as "transient (non-sustainable)" phenomenon related "to positive feedbacks influencing the price formation, which is prevalent during "special market regimes". Furthermore, it emerged that acceleration is related with procyclical mechanisms such as psychological and behavioral aspects. The study revealed that, on average, Γ-allocations have a positive performance and according to different parametrizations outperform momentum-based strategies in about two out of three cases.

This research is an extension of the previous paper by Ardila, Forrò, & Sornette (2015) and it aims firstly to develop better proxies to detect the Δ and the Γ parameter; successively two investment strategies (the Long-Short and the Relative Strength Weighted Portfolio) are optimized according to the Δ or the Γ factor to investigate portfolio performance. Three kind of detection methodologies are implemented to quantify the momentum and the acceleration effect: the simple, the trend-based and the wavelet transform (i.e. the Maximum Overlap Discrete Wavelet Transform, MODWT) approach. By applying the simple approach, the Δ and the Γ parameter are quantified as in the paper by Ardila, Forrò, & Sornette (2015), i.e. momentum is defined as the f-months cumulative return while acceleration is measured as the f-months difference in momentum. Moreover, since according to previous literature, "momentum" is defined as a short/medium-term persistence in log-returns, the trend-based detection aims to

improve the detection of the Δ factor using time series analysis tools as moving averages. Hence, a first trend-based approach implements an Exponential Moving Average to extrapolate the time series trend (i.e. momentum) whilst removing irregular fluctuations and noises. Moreover, a second trend-based detection estimates the trend as the difference between a short and a long Simple Moving Average. Thereafter, acceleration is computed as in the simple approach. According to Lera & Sornette (2017) and Shao & Ma (2003) the n-derivative of a signal is given by its convolution with a wavelet having n-vanishing moments. Since the momentum effect (Δ) might also be represented as the "velocity" of stock prices, it can be quantified by the first derivative of the log-price time series. Moreover, acceleration (Γ) might be defined (as in physics) as a change in velocity and it can be modelled by the second derivative. Hence, according to previous literature, in order to detect the delta factor, a wavelet transform using a Daubechies function with one vanishing moment (also named Haar wavelet function) is applied. Furthermore, the acceleration factor is captured by the detail coefficients of a MODWT performed with a Daubechies function with two vanishing moments (Db2). Moreover, the investigation is also executed using winsorized data.

Finally, a new hybrid portfolio optimization strategy which aims to consider both the momentum and the acceleration effect as factors for optimization has been developed, namely the Δ/Γ (Delta-Gamma) optimization. This strategy is an extension to the "traditional" time-series momentum strategy. More precisely, by applying the Δ/Γ optimization, a long and a short portfolio are constructed selecting stocks according to two conditions: the direction of momentum ("delta condition") and the direction of acceleration ("gamma condition"); moreover, equal-weights or relative Γ-weights are applied. More precisely, the long portfolio buys stocks with a positive momentum (i.e. a positive Δ) and having an upward accelerating price (i.e. a positive Γ), both factors are detected over the same formation period or at the same resolution level. Moreover, the short portfolio sells stocks having a negative momentum (i.e. a negative Δ) and a downward accelerating price (i.e. a negative Γ).

The investigation has been performed considering the U.S. equity market over two different periods in time: the distant past (1984-2002) and the recent past (2001-2016).

This study adds convincing evidence about the lower or even negative performance of momentum (as well as the acceleration) strategies, during the recent past (2001-2016), a period of time characterized by the dramatic impact of the global financial crisis (2007-2009) and therefore by a more volatile financial market regime. Previous literature on "momentum crashes" under unstable and stressed market states is sufficient to explain this outcome. Furthermore, on average, an improved portfolio performance is possible using Δ and Γ factors detected through the trend-based as well as the Maximum Overlap Discrete Wavelet Transform approach. Additionally, an important contribution is given by the newly developed hybrid Δ/Γ strategy. Indeed, there is significant evidence that implementing the hybrid portfolio optimization, i.e. a more "flexible" but more "selective" investment strategy which considers both the momentum and the acceleration as factors for optimization and which does not invest in a constant number of assets allows us to even gain good returns during stressed and more volatile market regimes.

Contents

List of Abbreviations

APT	Arbitrage Pricing Theory
ARP	Alternative Risk Premia
CAPM	Capital Asset Pricing Model
CWT	Continuous Wavelet Transform
DJIA	Dow Jones Industrial Average
DWT	Discrete Wavelet Transform
EMA	Exponential Moving Average
EMH	Efficient Market Hypothesis
EW	Equal Weights
GW	Gamma Weights (Γ-Weights)
IR	Information Ratio
LS	Long-Short
MODWT	Maximum Overlap Discrete Wavelet Transform
MPT	Modern Portfolio Theory
MW	Market Weights
RSWP	Relative Strength Weigthed Portfolio
SES	Simple Exponential Smoothing
SMA	Simple Moving Average
S&P500	Standard and Poor 500
SR	Sharpe Ratio
W	Detection using MODWT (daily)
WM	Detection using MODWT (monthly)
WC	Detection using calibrated MODWT (daily)
WMC	Detection using calibrated MODWT (monthly)
WT	Wavelet Transform
Δ	Delta (Momentum)
Γ	Gamma (Acceleration)
Δ/Γ	Delta-Gamma

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1 Insight on Financial Market Anomalies: Momentum and Acceleration Effect

1.1 Introduction

Recently, the efficiency of financial markets has been questioned by many economists (Avramov & Chordia, 2006). Under debate is the Efficient Market Hypothesis (EMH) - a cornerstone theory of finance- which posits that security market movements follow a random walk and the flow of information is efficiently accounted for in the stock value so that neither the past stock performance analysis (technical analysis) nor the consideration of financial news as "company earnings" or "asset values" (fundamental analysis) allows investor to predict stock prices and accordingly gain superior returns (Burton, 2003).
In fact, over the last few years, several financial publications have claimed evidence of the existence of financial market anomalies, i.e. particular "cross-sectional time series patterns" which are in contradiction with the EMH and supposedly can not be explained by fundamental asset pricing theories such as for example the Capital Asset Pricing Model (CAPM) or the Fama-French Three-Factor Model (Keim, 2006). Behavioural economists argue that due to over-simplification and strong assumptions, those fundamental asset pricing labels are unable to describe some financial market anomalies which seem to originate from investors' irrational behaviour as well as psychological effects (Fama, 1998).

"Momentum" is one of the most common anomalies, which consists in short-term return persistence and seems to originate from behavioural biases such as overconfidence and overreaction. At the beginning of the 21st Century economists started to believe that stock prices might be partially predictable on the basis of their past performance as well as fundamental metrics and that it was even feasible to earn risk-adjusted returns using technical as well fundamental indicators (Burton, 2003). Nowadays, technical as well as fundamental metrics are widely used among asset managers during portfolio optimization. The "momentum-based" strategy is one of the most widely implemented investment styles.

Nevertheless, Ardila, Forrò, & Sornette (2015) demonstrated that there is a significant "novel effect complementing momentum": "acceleration", defined as the change in momentum and quantified by "the first difference of successive returns", i.e. the gamma (Γ) parameter. The purpose of this research is to further investigate the acceleration effect and the Γ-parameter.

The first chapter aims to give a short introduction to fundamental asset pricing models as well as their inefficiency to explain particular financial market anomalies, which seem to originate from investor behavioural biases. The second section of this chapter briefly introduces the main features of some financial market anomalies and alternative risk premiums stemming from them. Specifically, there is an introduction and a literature review of the most important anomaly, namely "momentum", as well as the new factor, i.e. acceleration. Additionally, in the last section, the research question as well as the structure of the following chapters and the data set are defined.

1.2 Financial Asset Pricing Models

Risk, risk-based pricing in financial markets as well as project valuations are the main features of modern financial theory, where risk is seen as the principal component for value generation (Volkart, 2011). Assuming that returns are normal or log-normal distributed, the Modern Portfolio Theory (MPT) (Markowitz, 1952) reveals that risk-averse individuals select a portfolio of assets according to two stock price characteristics: return and risk (variance). In other words, investors have a mean-variance utility function and for an expected level of return they invest in the portfolio having the lower variance (Volkart, 2011). Moreover, Markowitz (1952) suggested that it is possible to achieve benefits in terms of risk reduction by focussing not only on the characteristics of a single asset but including more assets in the portfolio. Indeed, due to imperfectly correlated securities, a well-diversified portfolio lowers the idiosyncratic (asset-specific) risk to almost zero and its variance might be described only by the beta (systematic/market risk) of each single asset. Assuming that all investors have the same preferences, they would choose the same optimal portfolio and the asset prices are the result of this equilibrium.

The Capital Asset Pricing Model (CAPM) is a pioneer label of the Modern Portfolio Theory and a cornerstone of asset pricing. The Three-Factor Fama-French model as well as multi-factor models are extensions of the CAPM (Investopedia, 2018a). All these models assume that investors behave rationally and that their behaviour does not influence securities prices (Krause, 2001).

1.2.1 The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model is one of the most famous labels in finance, developed during the 60's by Treynor (1962), Sharpe (1964) and Lintner (1965). It aims to estimate the theoretical adequate rate of return of a risky asset. More precisely, it suggests that cross-sectional differences in average returns are determined only by the market risk which is measured through the parameter "beta" (β_i).

This parameter measures the sensitivity of the return of a risky asset in relation to the return of the market portfolio. The beta designs how the stock price co-varies with market price movements and it can theoretically take any value. Frequently, this number fluctuates between +0.5 and +1.5 (Volkart, 2011). This one period (static) model - commonly used in asset pricing - relies on several strong assumptions such as the predictability and rationality of investor behaviour, who are risk-adverse, have similar expectations regarding future outcomes and have mean-variance preferences. Additional statements are the existence of a risk-free asset so that one can borrow and lend at the risk-free rate and the completeness of financial markets, which have no friction: transaction costs and taxes are equal to zero (Investopedia, 2018a).

Under the CAPM the expected return of a risky security $E(r_i)$ is given by (Volkart, 2011):

$$E(r_i) = r_f + \beta_i \big(E(r_m) - r_f\big) \tag{1}$$

where r_f is the risk-free rate, β_i is the beta parameter and $E(r_m)$ is the expected return of the market portfolio.

1.2.2 The Arbitrage Pricing Theory

Multi-factor models are an alternative to CAPM and are based on the Arbitrage Pricing Theory (APT) developed by Ross in 1976. Both, the CAPM and the APT state that risk can be divided into unique and market risk. However according to the APT, the price of a security is determined by many sources of risk rather than one (Ross, 2017). In other words, in contrast to CAPM, where stock returns are explained only by one independent variable "beta", in the multi-factor models the sensitivity of the "beta" parameter is split into different risk-factors ($\beta_1, \beta_2, \beta_3, ...$) (Volkart, 2011). More precisely, the APT model states that the risk-premium originates from different external sources and not only market risk.

According to the APT model, the return of a security (r_i) is determined by several factors, as is shown by the following equation (Volkart, 2011):

$$r_i = \alpha_i + \beta_{i1} * F_1 + \beta_{i2} * F_2 + \beta_{i3} * F_3 + ... + \beta_{kn} * F_n + \varepsilon_i \tag{2}$$

where F_n is the return of a specific factor and ε_i is the regression residual.

Multi-factor models are more realistic and are based on fewer assumptions but they are harder to implement because - unlike the CAPM- there is no indication about which factors should be considered. In fact, stock returns are attributable to several factors, some systematic (macroeconomic) and other industry or company-specific and using factor analysis and statistical investigations it is possible to determine which are the most relevant current factors and which of them to select for pricing a singular security (Investopedia, 2018b) (Volkart, 2011). The criticism regarding pioneering models in Modern Portfolio Theory is strong and behavioural economists have demonstrated the inefficiency of those models to explain some financial anomalies, finding sources in investors' behavioural biases. As is explained in the next section, recently, there has been a trend in new labels to include anomalies and inefficiencies and obtain more realistic asset pricing estimations. An example is the Three-Factor model, which is a multi-factor model developed by Fama & French (1993) that aims to describe the differences in cross-sectional return according to firm size and book-to-market parameter. Moreover, some financial anomalies have been integrated in the multi-factor models as additional factors, such as for example momentum.

1.3 Introduction and Literature Review on Financial Market Anomalies

According to fundamental financial theories, participants in financial markets behave rationally and seek to maximize their wealth. Nevertheless, emotions and psychology can influence decisions and have an impact even on financial markets as well (Investopedia, 2018c). Over the last few years, several investigations in the field of behavioural finance – a relatively new discipline linking economy and finance with psychology – have aimed to explain, using assumptions of the neoclassical financial theory, the systematic divergence of asset prices and returns from predictions by fundamental labels (Volkart, 2011).

According to Volkart (2011), an anomaly in the point of view of financial managers is each factual observation which conflicts with market efficiency and the rationality of market participants and which leads to significant material or financial consequences to the company. In financial exchanges, anomalies are “cross-sectional and time series patterns in security returns that are not predicted by a central paradigm or theory” (Keim, 2006). Nowadays, there are a lot of recognized financial anomalies such as for example the January effect, the time instability of risk premium, the value and the size effect, the Equity Premium Puzzle or momentum (Volkart, 2011). In fact, for example, it appears that small-cap companies outperform large companies and “value” stocks (with high book-to-market value, B/M ratio) beat “growth” stock (with lower B/M ratio) (Blin, Lee, & Teiletche, 2017).

Due to the inefficiency of the CAPM in describing risk-contingent differences in returns, further research by Fama & French (1993) demonstrated that - with the inclusion of additional firm-specific factors such as company size and company value in this fundamental label - it is possible to obtain a better explanation of the power of return differences. As market capitalisation and book-to-market equity seems to have an impact on risk-premiums, the Three-Factor Fama-French model extends the CAPM with two related additional parameters: the Small-Minus-Big (SMB) and the High-minus-Low (HML). The size factor (SMB) plus the book-to-market factor (HML) combined with the market risk factor describe the expected risk premium of a risky asset in the following way (Fama & French, 1993):

$$E(r_i) - r_f = \beta_{i,m}\big(E(r_m) - r_f\big) + \beta_{i,SMB} * SMB + \beta_{i,HML} * HML \tag{3}$$

The expected risk premium ($E(r_i) - r_f$) is given not only by the sensitivity of the risky asset according to the expected market risk premium movements ($\beta_{i,m}$) but also by the sensitivity of the firm size ($\beta_{i,SMB}$) as well as its book-to-market ratio ($\beta_{i,HML}$) (Avramov & Chorida, 2006).

It is not clear if the outperformance tendency originated by SMB or HML factors might be a manifestation of market efficiency or inefficiency. Explanations sustaining the efficiency side define this abnormal excess return deriving from small-cap as a result of additional business risk and the higher cost of capital, which are characteristic of small companies. Theories supporting inefficiency claim that the origin of this outperformance might originate in company mispricing by market participants: the adjustment in the long-run of the value generates an excess return (Burton, 2003).

Moreover, even specific behavioural characteristics of market participants seems to influence share prices movements and be a driver of anomalies. For example, De Bondt & Thaler (1985) demonstrated that people “tend to overreact to unexpected and dramatic news events”. This leads to a mean-reversion in stock prices: previous losers tend to outperform previous winners. This is due to an over-reaction in the long-term which is the origin of the “mean reversion effect” (De Bondt & Thaler, 1985). Later, Jegadeesh & Titman (1993) demonstrated that - controversially - there is a short-term return persistence: “momentum”. Indeed, increasing share prices continue to increase and falling prices keep falling. Recently, several economists investigated the origin of momentum and the opportunity to achieve profits implementing momentum-based strategies. Behavioural aspects of investor such as herding, transient positive feedback, under and over-reactions seems to be potential sources of this anomaly (Jegadeesh & Titman, 1993).

Additionally, in their "Overconfidence Model", Daniel, Hirshleifer & Subrahmanyam (1998) explain how "overconfidence" bias – when the subjective confidence of one's own knowledge and ability is greater than the actual objective performance – as well as "self-attribution" bias - when successful outcomes are attributed to their own abilities but losses are seen as a "pech" - are further explanations of "momentum" and "reversal" in share prices. For their investigation, they used two sample of investors: informed and uninformed; the informed suffered from overconfidence and self-attribution bias. The biased self-confidence increased strongly after each successful trade; meanwhile negative outcomes were attributed to "pech" (Volkart, 2011) (Barberis & Thaler, 2003) (Hens & Bachmann, 2011).

According to Jegadeesh & Titman (1993): "if stock prices either overreact or underreact to information, then profitable trading strategies that select stocks based on their past returns exist".

Indeed, the two most popular trading investment styles are the "contrarian" by De Bondt & Thaler (1985) and "momentum" documented for the first time by Jegadeesh & Titman (1993), which aim to achieve benefits from the "mean-reversion effect" (for the former) and the "short-term return persistence" anomalies (for the latter). The contrarian strategy is seen as the strategy adopted by smart investors to beat the market and it consists "in buying past losers and selling past winners" (Jegadeesh & Titman, 1993). Contrarian investors believe that markets over-react and that herding behaviour leads to the mispricing of securities. According to Roncalli (2017) this investment style is also defined as value investing, where the intrinsic value (calculated fundamental value) is compared to the market value: undervalued assets are purchased and overpriced assets sold. Controversial, momentum-based strategies are associated to "naïve" investors who simply follow the trend of buying past winners while selling past losers – contradicting the "buy low and sell high" rule (Roncalli, 2017) (Blin, Lee & Teiletche, 2017).

1.3.1 Alternative Risk Premium

Those strategies are examples where investors seek to achieve "Alternative Risk Premia" (ARP) – in this case - implementing investment schemes based on assets' past returns. According to Naya & Tuschmid (2017), Alternative Risk Premia (ARP) are "systematic or rule-based strategies" and "value" or "size" also belong to the ARP family (Blin, Lee & Teiletche, 2017). The main idea is that investors should be rewarded for exposure to systematic (not diversifiable) risks. As opposed to traditional risk factors (such as equity or duration premium), ARP constitute compensation for "non-traditional" risks such as for example equity size, company value, interest rate and momentum. The ARP principle is based on the idea that the return of a risky security can be described by a multi-factor model where each factor describes a specific risk that should be remunerated by an adequate premium. The universe of Alternative Risk Premia is wide and selection might be difficult, also because of the low grade of homogeneity. Momentum, company size or book-to-market value are already established factors used by many asset managers; however, the gamma (Γ) parameter is a new factor which might give additional explanatory power to stock returns (Hamdan, Pavlowsky, Roncalli, & Zheng, 2016). In the following sections there is a deeper insight on the momentum effect as well as the new detected acceleration phenomena.

1.3.2 The Momentum Effect

The momentum risk premium is one of the most important ARP and it was first documented by Jegadeesh & Titman (1993). In their paper, they stated that "over an intermediate horizon of three to twelve months, past winners on average continue to outperform past losers" Roncalli (2017). According to Jegadeesh & Titman (1993), over the period between 1965 and 1989, trading strategies based on cross-sectional momentum ("relative strength" strategies[1]) achieved significant abnormal returns in the US market. However, others economists demonstrated evidence of momentum across asset classes and countries.

More precisely, in their study, Jegadeesh & Titman (1993) developed a J-month/K-month strategy which consists in ranking (ascending) stocks according to their J-months past returns and creating ten equally weighted decile portfolios; furthermore, there is a long position in the top decile (the "losers") and a short position in the bottom decile (the "winners"); both portfolios are hold for K-months. One of the most proficient investments was a six-month holding portfolio optimized by selecting stocks according to the past six-month returns (6-month/6-month strategy); the average compounded yearly excess return was around 12.01% (Jegadeesh & Titman, 1993).

The authors stated that performance by the relative strength portfolios seems not to originate from the systematic risk or a "lead-lag" effect resulting from a delayed reaction of stock prices to common factors but to a reaction to firm-specific news (Jegadeesh & Titman,1993). Moreover, long-term over-reaction and short-term under-reaction are too simplistic to uniquely describe the return patterns of relative strength momentum, i.e. the "initial positive and later negative portfolio performance". According to Jegadeesh & Titman (1993) it is instead the observed investor behaviour of buying past winners and selling past losers that might lead to a temporary price over-reaction which diverge from its long-run value. Alternatively, the authors suppose that it is possible "that the market under-reacts to information about the short-term prospects of firms" (such as earnings forecasts) but "overreacts to information about their long-term prospects" (vague or unclear information) and they reveal that there might be other explanation to their results.

As an alternative to the return patterns explanations of Jegadeesh & Titman (1993) regarding the cross-sectional momentum, there are other theories which attempt to explain the origin of the momentum profits. Recently, behavioural economists suggest that an over-reaction originating from the psychological feedback mechanism can be a source of momentum profitability (Burton, 2003).The positive feedback trading hypothesis (PFTH) is seen as an important explanation for momentum in financial markets and the idea is that "time traders" buy an asset simply due to its price increase; if a large number of investors behave in this way, there is buying pressure on the asset that drives the stock value even higher inducing more people to buy it (Liang, 2012). As is explained in the paper by DeLong, Shleifer, Summers,& Waldmann (1990) noise trading, i.e. irrational investors, who despite having no inside information, trade according to noise believing that there is an information value in it - might also lead to a divergence of prices from their fundamental value and investors might gain a premium

[1] Relative Strength investment strategies consist in comparing the performance of a security against a selected benchmark (a market index or a group of similar securities) in order to identify top-performing stocks in the universe of potential securities (Investopedia, 2018d).

as compensation for the self-created risk. Positive feedback is a sort of noise trading where "trend-chasers reinforce movement in stock prices even in absence of fundamental information", which leads to temporary returns followed by a sub sequential reversion effect in the long-term (Chan, Jegadeesh, & Lakonishok, 1996).

Contrary to behavioural models, according to Crombez (2001), even in efficient markets with rational investors the presence of information noise might be the origin of financial market anomalies. Other particular explanations regarding the momentum effect are for example the idea of compensation for systematic crash risk exposure (Ruenzi, & Weigert, 2018).

As was already mentioned above, momentum-based investment consists in buying securities that have performed well and selling those that have performed poorly. It is an established and very popular trading strategy widely used in investment industry (Roncalli, 2017). There follows a short insight on two additional investment styles based on the conventional momentum of Jegadeesh & Titmann (1993): the "time series momentum" and the "idiosyncratic momentum".

Beyond the cross-sectional momentum, which is based on the "relative" performance of stocks, i.e. selecting securities that outperformed their peers (going long only with the best-performing decile portfolio) or selling securities which underperformed (going short with the worst-performing decile portfolio), we can also find other momentum-based strategies such as the "time-series momentum" of Moskowitz et al. (2012), which is based on the "absolute" performance, i.e. only on an asset's own past return. However, if in the conventional strategy of cross-sectional momentum, the number of stocks in the winner or loser portfolio is constant, with the time-series momentum this number varies with the state of the market. According to Moskowitz et al. (2012), there is evidence of a "time series momentum effect" across futures contracts and several major asset classes, however the authors demonstrated that, consistent with the assumption of short-term under-reaction combined with long-term over-reaction, the time series momentum "partially reverses after one year" (Moskowitz et al., 2012). However, according to Bird, Gao & Yeung (2017), if on the one hand both styles of momentum strategies generate positive performances, on the other hand, due to the fact that "the information in the momentum signals is concentrated in the tails of the return distribution", the time-series momentum might be defined a better strategy.

Additionally, Blitz, Hanauer, & Vidojevic (2017) developed another strategy, which consists in selecting stocks according to their idiosyncratic (abnormal) past returns and not according to their total past performance as was done previously. This strategy is called " idiosyncratic momentum" and the authors demonstrated that it is "priced in the cross-section of stock returns" and that "it cannot be subsumed by any of the established asset pricing factors, including conventional momentum". Indeed, the idiosyncratic momentum should be considered an additional factor in asset pricing. This investment style seems to generate, on average, comparable performance with cross-sectional (conventional) momentum. Nevertheless, it boats of the half of the volatility as well as "lower non-linear crash risk exposure" and its performance is less influenced by market dynamics. Moreover, the authors documented the idiosyncratic momentum not only in international equity markets but even in Japan, where the effectiveness of conventional momentum was never confirmed. The underreaction hypothesis as a source of the premium of idiosyncratic momentum is supported by their empirical studies. Controlling for known forecasting variables for asset returns, conventional momentum "predicts high short to medium-term returns" but it becomes significantly negative within one year. Controversely, the idiosyncratic momentum forecasts high short and long-term returns and the authors demonstrated that it is feasible to use idiosyncratic momentum to differentiate between ”past total return winners that are prone to long term reversal and those that are not”. Since conventional and idiosyncratic momentum factors are not uniquely significantly and since there is ambiguity about the underlying mechanism linking both momentum phenomena, it is important to say that the authors designed both kinds of momentum styles as complementary and not as substitutes.

Moreover, Fama & French (2012) even included the conventional momentum factor in their previously developed Three- Factor model, which improved the explanatory power of the multi-factor model.

1.3.3 The Acceleration Effect

A recent research by Ardila, Forrò, & Sornette (2015) reported evidence that there is an innovative effect complementing momentum, ”acceleration”, which is defined as ”the first difference of sucessive returns” quantified by the parameter gamma (Γ). It was shown over the last 25 years, acceleration has been an important driver for momentum profits. Moreover, their research revealed that Γ-based strategies are profitable and considering different parametrizations beat momentum-based strategies in more than two-thirds of cases.

1.4 Research Purpose

This research is an extension of the previous paper by Ardila, Forrò, & Sornette (2015) and it aims to improve the detection of the Γ-factor in order to check if it is possible to obtain higher portfolio return using Γ-based portfolio optimization in the distant past (1984-2002) or in today's financial market environment (2001-2016). The second chapter aims to develop better proxies to detect the acceleration factor in daily log-returns. More precisely, three methods are implemented in order to quantify acceleration: a simple, a trend-based and a wavelet transform approach. The gamma parameter is computed for each security using different formation periods. Furthermore, the purpose of the third chapter is to set up portfolios based on the Γ-parameters in order to check their performance and compare it to momentum-based portfolios. Additionally, a hybrid portfolio - based on both, the momentum and the acceleration factor - is developed.

Moreover, this investigation might be a starting point for other analyses such as for example the idea to use “idiosyncratic” momentum as a basis to compute the Γ-factor or compute the acceleration factor not in an absolute quantification but relative, i.e. as a percentage of change in momentum.

1.5 Data

The detection of the acceleration effect has been studied using split and dividend-adjusted daily log-returns of the component of two important indices representing the U.S. market: the Dow Jones Industrial Average (DJIA) and the Standard and Poor 500 (S&P500). Firstly, there is momentum as well as acceleration detection using daily returns of 30 companies belonging to the Dow Jones Industrial Average (DJIA) from 5 January 1999 until 30 December 2016. The Dow Jones includes “blue chip” stocks, i.e. security with high capitalization and with a long history of paying out dividends in both good and bad financial time (Zacks, 2018). However, since it is the second-older U.S. index it has disadvantages because the companies included are not necessarily the most significant in the market. For this reason, it is less suitable to represent the whole U.S. market. On the other side, the Standard and Poor 500 included 500 U.S. companies with high capitalization (more than $5.3 billion) and high liquidity, therefore it is suitable to represent the United market. The detection is done using split and dividend-adjusted daily log-returns of 376 companies in the Standard and Poor 500 (S&P500) between 5 January 1999 and 16 May 2014 (Sharptrader, 2017).

The portfolio optimization is simulated using the daily detected parameters firstly for the Dow Jones components and after for the S&P500 companies. The first two years of data are removed due to estimation window constraints; for this reason the simulating portfolios start on 2 January 2001 until the availability of the detected data: 30 December 2016 for the DJIA and 16 May 2014 for the S&P500.

In order to have an additional comparison, detection as well as portfolio optimization has been implemented, also using a dividend-adjusted daily log-return of 21 companies included in the Dow Jones Industrial Average between 30 December 1981 and 31 December 2002 (i.e. between 1982 and 2002).

The Dow Jones Industrial Average (1999-2016) data set was downloaded from the Wharton Research Data Servies (WRDS) database while the Standard and Poor 500 stocks universe (2001-2014) as well as the DJIA (1982-2002) data set was acquired from Bloomberg Professional. This investigation is performed using MATLAB. Several key sections of the code are available in the attachment.

2 The Acceleration Effect and the Gamma Factor: Detection

The aim of this research is to employ different methods to detect the acceleration effect quantified by the gamma factor (Γ) in stock prices; it will be used later as a constraint in portfolio optimization or as an additional factor in asset pricing models. More precisely, the following three methods are employed for detection: a simple, a trend-based and a wavelet transform approach. Moreover, according to Welch (2017), "winsorized rates of returns predict their own future realization better than equivalents based on unwinsorized rates of returns"; therefore, some investigations are implemented twice, i.e. using unwinsorized as well as winsorized time series. Winsorizing methodology as well as each detection approach are explained in the following section. Furthermore, the MATLAB code for momentum as well as gamma detection and successive portfolio optimization is available from the author; each main part of this code is briefly transcribed and described in the Appendix.

2.1 Simple Detection

2.1.1 Approach

As per the paper by Ardila, Forrò & Sornette (2015) the first approach to detect the acceleration factor (Γ), consists simply of the first difference in returns ($r_{i,t}(f)$), i.e. the change in momentum between two points in time:

$$\Gamma_{i,t}(f) = \Delta_{i,t}(f) - \Delta_{i,t-f}(f) \tag{4}$$

where momentum is defined with the parameter delta (Δ) and is computed using the following equation:

$$\Delta_{i,t}(f) = r_{i,t}(f) = \ln\left(\frac{S_{i,t}}{S_{i,t-f}}\right) \tag{5}$$

and $S_{i,t}$ is the price of the stock i at the end of the day t. Moreover, the parameter f defines the formation period (expressed in days) used to estimate the momentum and the acceleration factor. The delta is the f-days cumulative log-return (momentum) of the stock i. The study is performed using daily or monthly data, i.e. the parameters t and f are expressed in days or months.

Since the original data set is in daily log-returns ($r_{i,k}$), the f-days cumulative return (momentum) or $\Delta_{i,t}(f)$ of a security i at the day t is given by the following equation:

$$\Delta_{i,t}(f) = r_{i,t}(f) = \sum_{t-f+1}^{t} r_{i,k} \tag{6}$$

2.1.2 Assumptions

According to Swingtradeystems (2018), a trading month consists on average of 21 days and a trading year or respectively a quarter (three months) are on average 252 resp. 63 days. Therefore, on the basis of daily returns, monthly returns have been calculated applying the Equation (6) with a time-step (f) of 21 days. Moreover, a period of 126 days defines approximately a time frame of six months. This assumption is applied to each analysis performed in this research.

As was done in the paper by Ardila, Forrò & Sornette (2015) and also according to previous literature as for example Jegadeesh & Titman (1993), the momentum as well as the acceleration factor are computed using different formation periods (f) of three, six and twelve-months (i.e. approximately 63, 126 and 252 days). Additionally, given the availability of daily data, a five-day period (i.e. a week) as well as a one-month formation period have been considered too.

Detection of the delta and gamma parameters is also executed by operating with winsorized data, which consists in removing outliers from the data set by setting a lower and an upper bound for return values and replacing each value outside these limits with the corresponding threshold value Welch (2017). According to Welch (2017) an absolute winsorization level of 10-15% is adequate to filter daily stock returns. However, for large stocks, a more conservative absolute level of 25-50% is recommended. Hence, considering the presence of large stocks in this research and taking into account the fact that acceleration depends directly on return values, a conservative level of 20% is chosen. Winsorization is an additional analysis which has only been applied to selected portfolios.

2.2 Trend-based Detection

2.2.1 Approaches and Assumptions

According to previous literature, "momentum" is defined as a short/medium-term persistence in log-returns. Therefore, a second approach to detect acceleration (i.e. a change in momentum) in stock returns is to firstly extrapolate the time series trend (i.e. momentum) whilst removing irregular fluctuations and noises.

The trend is estimated on the basis of standardized stock prices $S_{i,t}$ which are calculated by inverting the Equation (5) and inserting the earliest daily log-return ($r_{i,t=1}$) and a standardized starting price ($S_{i,t=0}$) equal to 100 as is shown in the following equation:

$$S_{i,1} = S_{i,0} * \exp(r_{i,1}) = 100 * \exp(r_{i,1}) \tag{7}$$

Successive stock prices are computed analogously; $S_{i,t}$ is calculated by applying the above equation and inserting the previously calculated $S_{i,t-1}$:

$$S_{i,t} = S_{i,t-1} * \exp(r_{i,t}) \tag{8}$$

The trend-based detection is performed through moving averages (MA), which are a common technique implemented by asset managers in order to assess momentum. Two types of MA are executed in order to better estimate the trend (momentum) and compute the delta and gamma parameters.

The first procedure for trend-based detection is Simple Exponential Smoothing (SES), also named Exponential Moving Average (EMA). It aims to estimate a "cleaned" trend of stock prices which is used as an input variable in the Equation (5) to compute the delta parameter. The gamma factor is calculated on the basis of the computed delta factor by applying the Equation (4). The momentum as well as the acceleration parameters are detected using a formation period of five days (one week) as well as one, three, six and twelve months.

Subsequently, considering the fact that "Simple Moving Average Crossovers" are widely adopted as a signal to buy or sell a security, a second trend-based approach which defines momentum (i.e. the delta) as the difference between long-term (slower) and a short-term (faster) asymmetric simple moving average (SMA) is employed (Bruder, Dao, Richard & Roncalli, 2011).

The next section gives a short overview on the Simple Moving Average (SMA) and the Simple Exponential Smoothing (SES) as well as the Moving Average Crossovers approach.

Simple Moving Average

The Simple Moving Average (SMA) is a kind of linear filter and it simply computes the average price of an asset over a selected number of periods (lag) in order to remove irregular price fluctuations and estimate the local mean (Investopedia, 2018e) (Chatfield, 2004).

Using a symmetric simple moving average approach, the time series trend ($x_{i,t}$) is calculated with the following equation (Chatfield, 2004):

$$SMA_q(S_{i,t}) = x_{i,t}(q) = \sum_{r=-q}^{+q} a_r * S_{i,t+r} \tag{9}$$

where $a_r = \frac{1}{2q+1}$ is the set of equal weights with $\sum a_r = 1$ and q is the number of previous and following days to be included in the "average" window.

Moreover, an alternative kind of simple moving average, which is defined as "asymmetric SMA", is used as technical trading rules. It consists of a simple moving average over the last K trading periods (here it is given in days) and it is shown by the following formula (Chatfield, 2004):

$$SMA_K(S_{i,t}) = x_{i,t}(K) = \frac{1}{K} \sum_{r=-K+1}^{0} S_{i,t+r} \tag{10}$$

Usually, common moving average lengths (lag) which can be applied to any plot time-frame (minute, day, weeks, etc.) are 20, 50, 100 as well as 200. For example, a 20-day lag "it follows the price more closely" and it might be used as an analytical tool for short-term trading. Nevertheless, the estimated trend lags the original time series, i.e. potential reversals or changes in trend are "only seen with a delay (lag)" (Learndatasci, 2018).

Using the Simple Moving Average Crossovers approach, two asymmetric simple moving averages of 50-days and 200-days on stock price series ($S_{i,t}$) are performed in order to compute the delta factor (Δ). Afterwards, the Γ-parameters are computed using the Equation (4). The actual procedure is clearly explained in the corresponding paragraph (Bruder, Dao, Richard & Roncalli, 2011).

Simple Exponential Smoothing

Since current and recent prices are more relevant than distant data points, exponential smoothing is a weighted moving average where weights are higher for recent prices and their rate of decrease is exponential (Investopedia, 2018f). Therefore, using a SES the "lag" problem of the estimated series is greatly reduced (Learndatasci, 2018).

According to Chatfield (2004), when using the SES procedure, the estimated time series trend ($x_{i,t}$) is:

$$SES_K(S_{i,t}) = x_{i,t}(K) = \sum_{j=0}^{\infty} \alpha * (1-\alpha)^j S_{i,t-j} \tag{11}$$

where α is a constant and $0 < \alpha < 1$. The weights (α) are calculated with the following formula:

$$\alpha = \frac{2}{K+1} \tag{12}$$

where K is the "lag" parameter (i.e. the window size). In order to extrapolate the trend (momentum) from the daily (unwinsorized and winsorized) price series, this study employed a set of exponential moving averages with different lag parameters of 20, 50, 100. Afterwards, on the basis of the filtered series (the trend), the delta as well as the gamma parameters are detected applying the Equations (5) resp. (4) and using a formation period of one, three, six and twelve months.

Given the exponential weighting which is decreasing at the rate j, this approach seems to be more effective in determining the trend than a SMA [2] and it is a useful method when we are dealing with non-seasonal data. Moreover, the SES is more responsive and more adequate to describe changes in trends (here defined as acceleration) (Chatfield, 2004).

Simple Moving Averages Crossovers

As was explained previously, a shorter simple moving average is faster and follows the price more closely, indicating more reversal signals (as well as more false signals) than a longer SMA. Moreover, the larger the lag parameter, the greater the delay in the estimated trend (see Figure A1 in the Appendix). The asymmetric simple moving average is a common technical indicator in order to determine trend reversal (i.e. regime shifts in momentum), i.e. the crossover of a short (as for example 10-days) and a long (20-days) moving average on a chart is an indication of changes in the trend (see Figure A2 in the Appendix) (Bruder, Dao, Richard & Roncalli, 2011).

An example is the "death cross" indicator, which results when a short-term (SMA_{50}) moving average crosses a long-term moving average (SMA_{200}) from above, signaling a downward trend (bearish signal). The opposite indicator (bullish signal) is the "golden cross" and is given by the crossover of the two SMAs but on the opposite sense, i.e. the short SMA_{50} crosses the long SMA_{200} from below. In the former case the trend shifts down after the crossover and in the latter situation the trend reverses positively (Investopedia, 2018f) (Investopedia, 2018g).

Therefore, the second trend-based approach to detect the gamma parameter is to firstly calculate the momentum as the difference between a 50-day simple moving average and a 200-day simple moving average (Bruder, Dao, Richard, & Roncalli, 2011):

$$\Delta_{i,t} = SMA_{50-200}(S_{i,t}) = SMA_{50}(S_{i,t}) - SMA_{200}(S_{i,t}) \tag{13}$$

A negative $SMA_{50-200}(S_{i,t})$ or delta (momentum) indicates that the long-term moving average is located above the short-term and is assumed to be a signal of a downtrend. A positive $SMA_{50-200}(S_{i,t})$ is supposed to be an indicator of a bullish market. Afterwards, it is possible to compute the Γ-factor using the calculated $\Delta_{i,t}$ and $\Delta_{i,t-f}$ as input variables in the Equation (4).

The Moving Average (MA) as well as other statistical methods rely on the assumptions of normally distributed and stationary stock price series. However, asset prices are mostly non-stationary and non-linear and through the application of time series decomposition techniques it might be possible

[2] However, the SMA has the advantage that it allows to find easier resistance and support areas, i.e. stock prices limits (below or above) which stock prices series have difficult to cross and which once they are touched the prices reverse (Investopedia, 2018h)

to detect the delta and the gamma parameters more accurately and to test whether it is possible to increase the portfolio performance or to improve prediction power in asset pricing models (Jothimani, Shankar, & Yadav, 2015). More precisely, many statistical models such as the Autoregressive (AR) or the Autoregressive Moving Average (ARMA) might be insufficient tools to analyze non-stationary time series such as stock prices. Furthermore, successive procedures such as the Generalized Autoregressive Conditional Heteroskedastic (GARCH) aim to model the change in volatility but are not able to capture irregular financial market phenomena (Jothimani, Shankar, & Yadav, 2015). A recent approach is the Maximum Overlap Discrete Wavelet Transform (MODWT) which might be useful to gain more accuracy during the time series decomposition and analysis and in this investigation it might increase the precision of the detected delta and Γ-parameter. The MODWT method as well as the underlying assumptions regarding its use for this investigation are explained in the next section.

2.3 Wavelet Transform Detection (MODWT)

2.3.1 Wavelet Transform Overview and Approach

Wavelet transform (WT) is a useful mathematical tool which makes it possible to decompose a time series into time-frequency (or time-scale) components and it helps to investigate localized variations of the signal that is incorporated in the time series (Math is in the air, 2017). Moreover, according to (Shao & Ma, 2003) by using the appropriate wavelet function, the wavelet transform approach enables us to directly compute the n-derivative of the input function. In this section there is a short overview on wavelet transform and how this method is implemented in order to detect the momentum as well as the acceleration factor in stock prices.

The main idea behind the WT is that a signal can be modelled using scaling and translation of an "oscillating wave", i.e. a real function the shape of which is similar to a wave in a limited interval of time (Math is in the air, 2017).

In general, the mother wavelet $\psi(t) \in L^2(\mathbb{R})$ is a function which must satisfy two main conditions: admissibility and regularity. The admissibility requirement states that the function can be used to restructure a signal without losing information or in a mathematical way that the integral of the function must be equal to zero (Ortega & Khashanah, 2014):

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \tag{14}$$

Under the admissibility condition, a signal can be completely reassembled through a set of coefficients of the wavelet transform, which capture information at different frequency (scale) and time resolutions. Moreover, under the regularity conditions (Ortega & Khashanah, 2014), the wavelet has "finite energy". In other words, the wavelet function activity is limited to the interval $[-T, T]$ and outside this range is equal to zero. It guarantees that the energy is the same for all scales. Mathematically, the regularity condition is given by the following equation:

$$\int_{-\infty}^{+\infty} |\psi(t)|^2 dt < \infty \tag{15}$$

that, if normalized results in

$$\int_{-\infty}^{+\infty} \psi^2(t) dt = 1 \tag{16}$$

There are many wavelet functions available which differ in shape and size. An example are Daubechies, Morlet, Symlet or the Biorthogonal wavelets (Alexander & Poularikas, 1998)(Misiti M., Misiti, Y., Oppenheim & Poggi, 2015). Another important property of a wavelet function is the number of vanishing moments. A wavelet $\psi(t) \in L^2(\mathbb{R})$ possess n-vanishing moments if it satisfies the following equation (Shao & Ma, 2003):

$$\int_{-\infty}^{+\infty} t^k \psi(t) dt = 0 \quad for\ 0 \leq k \leq n \tag{17}$$

or alternatively, it is orthogonal to a polynomial of $n-1$ degrees, i.e. every polynomial with a degree of $n-1$ or less can be suppressed (it integrates to zero) by convolution with the mother wavelet and the polynomial features are caught solely by the father wavelet (Ramsey, 2002). This is very useful for data compression because if the signal is mostly smooth the wavelet transform output will be sparse, which means that many wavelet coefficients are equal to zero. The non-zero coefficients represent discontinuities or non-smooth parts (Nason, 2008).

The Continuous Wavelet Transform (CWT)

Wavelet transform is used in different fields such as for example in biology and physics and it finds a fundamental application in image compression but also in denoising of non-stationary signals (Verdoliva, 2014). There are two types of WT: Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT) and they differ according to how (i.e. which kind of discretization) the wavelets are scaled or shifted and the underlying algorithm.

Briefly, the continuous wavelet transform coefficients ($\Psi_x^\psi(\tau, s)$) result from a convolution between a signal $x(t)$ and a mother wavelet (ψ^*)as is shown by the following simplified equation (Polikar, 1996):

$$CWT_x^\psi(\tau, s) = \Psi_x^\psi(\tau, s) = \frac{1}{\sqrt{s}} \int x(t) \psi^* \left(\frac{t-\tau}{s} \right) dt \tag{18}$$

As was already mentioned, two key concepts of WT are scaling and shifting and they are executed by applying the scaling (s) and the shifting (k) factors as is shown in the following equation:

$$scaling: \quad \psi\left(\frac{t}{s}\right) \quad with\ s > 0 \tag{19}$$

$$shifting: \quad \phi(t-k) \quad with\ k > 0 \tag{20}$$

The scaling factor is a positive value, which stretches or shrinks the function ψ in time. This factor is inversely proportional to the equivalent wavelet frequency, hence an $s = 2$ reduces its frequency by a half (or by an octave). A large scale factor ($s > 1$), stretches the wavelet so that it takes low-frequency. On the other hand, a small scale factor ($0 < s < 1$) shrinks the wavelet and gives rise to a function with higher frequency. The low-frequency wavelet is good to model slow variations in the signal (trend) while the contracted high-frequency wavelet is better to catch the localized changes (or noises). Moreover, using the shifting factor (k) , it is possible to advance the wavelet function in time along the signal and align it; this makes it possible to capture the features of the signal (MathWorks,2018a).

The Discrete Wavelet Approach (DWT)

The Discrete Wavelet Transform (DWT) is a suitable tool for signal denoising and makes it possible to decompose the time series into time and frequency domains. DWT is an extension of the Fourier Transform, which is a procedure for data analysis, representing data as the sum of different sine waves with infinite lifespan. Unfortunately, the Fourier approach only considers time and frequency separately, i.e. it only decomposes the signal in frequency spaces and is unable to detect time-variant characteristics so that anomalies and local or transient effects in the signal are not precisely localized and detected[3] (MathWorks,2018a). Since the acceleration effect is assumed to be an anomaly arising from transient positive feedbacks which are prevalent only during special market regimes (i.e. from localized events in the time series), wavelet transform might be a better proxy than the Fourier analysis to quantify more precisely the associated Γ-factor. As wavelets are limited functions in frequency and time intervals, signal decomposition into wavelet domain rather than into frequency domain through the Fourier Transform, gives a better resolution in the corresponding domain (George Dallas, 2014).

The DWT is based on "dyadic scaling and shifting" (MathWorks, 2018a) where the scaling factor is in the form of 2^j (with j being the level parameter, an integer such as for example, $j = 1, 2, 3, 4, \ldots$) and the translation is given by the parameter k in the following way: $2^j k$ (with $k = 1, 2, 3, 4, \ldots$). The underlying process of the DWT is multiscale or multiresolution analysis, more precisely the Mallat

[3] The Fourier transform is affected by the Heisenberg's Uncertainty Principle, which states that "you can know where a particle is or how fast it is going but not both". For this reason, in order to acquire more certainty about the position, you must have more uncertainty about its speed. This uncertain is named "resolution". This happens in the Fourier Transform: you can know with certainty the frequency or the time of the signal but not both contemporaneously. Therefore, Fourier Transform has "a lack of resolution between time and frequency domain", i.e. is not able to catch with certainty the instantaneous frequency, which is an important feature in signal processing (George Dallas, 2014)

pyramidal algorithm, where detail ($d_{j,k}$) and coarse (scale or approximation) coefficients ($a_{j,k}$) are computed for each scale recursively. Using a simplified description, the multiscale procedure enables us to decompose a discrete series of real number $x = (x_1, x_2, \ldots, x_n)$ of length equal to $n = 2^J$ (with J being an integer larger than zero) in two sequences using two kind of operation: addition and subtraction between non-overlapping pairs. The two coefficients for the first (finer) scale ($d_{1,k}$ and $a_{1,k}$) are calculated in the following way (Nason, 2008):

$$d_{1,k} = x_{2k} - x_{2k-1} \tag{21}$$

$$a_{1,k} = x_{2k} + x_{2k-1} \tag{22}$$

where $k = 1, 2, \ldots, 1/n$.

"Coarse "detail and smoother scale coefficients for the following scale can be calculated using the coefficient series ($a_{1,k}$) as input variables and by applying the same procedure. We can also see these computations differently: the former as a differentiation and the latter as a (simple moving) average smoothing operation (without the division by 2); however, both are applied to non-consecutive pairs. Moreover, without some modification these formulas are not energy preserving, i.e. the modulus[4] of is not equal to the sum of the modulus of the coefficient. Therefore, the DWT approach is based on the multiresolution analysis but filtering is performed through appropriate wavelet functions which satisfy the admissibility/energy requirement and which enable us to decompose and reconstruct the signal properly. The Haar wavelet, developed in 1909, is the first and the simplest wavelet function and it is derived from a modification of the Equations (21) and (22) in the following way (Nason, 2008):

$$d_{j,k} = \alpha(x_{2k} - x_{2k-1}) \tag{23}$$

$$a_{j,k} = \alpha(x_{2k} + x_{2k-1}) \tag{24}$$

with $\alpha = 1/\sqrt{2}$.

More precisely, the DWT approach works with the Mallat (pyramidal) algorithm (see Figure 3 in the Appendix) and the multi-scale decomposition is performed by two fundamental wavelets (filters): the mother wavelet (ψ) and the father wavelet (ϕ). The multiscale information is extracted through two "basis" wavelets which are determined by the following operations (Jothimani, Shankar, & Yadav, 2015):

[4] The modulus of a vector x is given by: $||x||^2 = \sum_{i=1}^{n} x_i^2$ (Nason, 2008).

$$\psi_{j,k} = 2^{-\frac{j}{2}}\psi\left(\frac{t-2^{j}k}{2^{j}}\right) \tag{25}$$

$$\phi_{J,k} = 2^{-\frac{J}{2}}\phi\left(\frac{t-2^{J}k}{2^{J}}\right) \tag{26}$$

with $j = 1, 2, \ldots, J$ and $k \in \mathbb{Z}$.

The fundamental mother wavelet performs as a high-pass filter which is used to represent high-frequency components or abnormal deviations from the trend and it generates a "child" basic wavelet $\psi_{j,k}$ that is used in order to calculate the detail coefficients ($d_{j,k}$). The father wavelet, which integrates to one, generates another corresponding basis wavelet $\phi_{J,k}$ implemented to compute the "scaling" (approximation) coefficients ($a_{j,k}$) ; it acts as a low-pass filter representing the long scale smooth features of the signal.

From a simplified mathematical point of view, the discrete wavelet transform is a convolution between a signal ($x(t)$) and the impulse response function of the corresponding filter. The output is a sequence of coefficients representing the projection of the function onto the wavelet basis. The DWT decomposition coefficients $a_{j,k}$ and $d_{j,k}$ are computed in the following way (Jothimani, Shankar, &Yadav, 2015):

$$a_{J,k} = \int x(t)\phi_{J,k} \tag{27}$$

$$d_{j,k} = \int x(t)\psi_{j,k} \tag{28}$$

with $j = 1, ...J$.

The approximation coefficient captures the trend of the signal while the detail coefficients are able to catch localized and faster variations (noises). The signal can be represented through the sum of the detail (d_j) coefficients of each scale and the approximation coefficient of the finer scale (a_J) which are computed using the wavelet transformation process (Ramsey, 2002):

$$x(t) = \sum_{k\in\mathbb{Z}} a_{J,k}\phi_{J,k}(t) + \sum_{j=1}^{J}\sum_{k\in\mathbb{Z}} d_{j,k}\psi_{j,k}(t) \tag{29}$$

or

$$x(t) = A_J + D_J + D_{J-1} + ... + D_1 \tag{30}$$

where

$$A_J = \sum_k a_{J,k}\phi_{J,k}(t) \tag{31}$$

$$D_j = \sum_k d_{j,k}\psi_{j,k}(t) \tag{32}$$

The DWT enables us to decompose the signal in order to zoom in and to obtain a different "amount of detail information", i.e. resolution (Polikar, 1996). The signal decomposition and reconstruction are done by sub- and upsampling operations, which make it possible to change the scale. Subsampling by a factor "n" means reducing the samples n times ($1/n$) (Polikar, 1996) (Haddadi, Abdelmounim, El Hanine, & Belaguid, 2014). In the above case, the decomposition is obtained with a dyadic scale and each subsampling decreases the sample by a factor of $n = 2$ (see Figure A3 in the Appendix). The upsampling enables us to reconstruct the signal (Haddadi, Abdelmounim, El Hanine, & Belaguid, 2014). Since it is based on the Mallat (pyramidal) algorithm, the DWT approach is different to a discretized CWT, where the computation is still challenging and it results in redundant information. It consists in passing the time domain of a signal through different high-frequency and low-frequency filters (Polikar, 1996).

The Maximum Overlap Discrete Wavelet Transform (MODWT)

The Maximum Overlap Discrete Wavelet Transform (MODWT) is an extension of the DWT, which makes it possible to gain resolution in the signal and is useful when the data set length has not a dyadic lenght ; however it loses the property of orthogonality (Ramsey, 2002). It is also named "non-decimate DWT" or "time-invariant DWT"; it is able to handle each sample size of without dyadic length requirements and is not affected by the problem of "circular shift[5] ". Indeed, the DWT is highly dependent on the origin of the analyzed signal and a small shift in the starting point changes the output, so that an alignment of the signal with time is difficult; on the other hand, the MODWT is translation invariant (Dghais & Ismail, 2013) (Ortega & Khashanah, 2014). Moreover, the MODWT considers the difference at each scale and not only on a dyadic scale (i.e. non-overlapping differences), so that each variability of the signal and the whole information can be captured by the coefficient more appropriately. Additionally, it uses a different algorithm during the scale transform, which does not down-sample and insert zero between coefficients. Hence, the scale number as well as wavelet coefficients at each transform is the same as the number of sample observations (Gallegati, 2008).

It is possible to obtain the MODWT coefficients by rescaling the DWT coefficient in the following way (Gallegati, 2008):

[5] The circular shift problem comes about when "a small shift in origin affects the outputs generated" (Jothimani, Shankar & Yadav, 2015).

$$\tilde{a}_{J,k} = \frac{a_{J,k}}{2^{j/2}} \tag{33}$$

$$\tilde{d}_{j,k} = \frac{d_{j,k}}{2^{j/2}} \tag{34}$$

Wavelets are an adequate tool for signals having low-frequency features in long durations and a few high-frequency features in short durations (Polikar, 1996). Due to the time-invariant property as well as the fact that the MODWT do not requires a dyadic length of the signal and gives as output as many coefficients as the input series, i.e. the output is easier to interpret for time signal analysis, therefore this method has been applied in order to detect the momentum as well as the acceleration factor. Moreover, it is more appropriate than the DWT because it considers each difference in the data at any scale. For appropriate portfolio-optimizations (in order to have full information to create portfolios in the next chapter) it is very important that there be no down-sampling in the number of observations. Given the large number of observations, the loss of orthogonality should not have a significant impact.

2.3.2 MODWT detection: Assumptions and Procedure

According to Lera & Sornette (2017) and Shao & Ma (2003) the n-derivative of a signal is given by its convolution with a wavelet having n-vanishing moments. Therefore, a wavelet transform performed by a mother wavelet function with n-vanishing moments ($\psi^n(t)$) results in a n-differentiation of the signal being analyzed. As per the last method, in this section, the momentum as well as the acceleration factor are detected using a non-calibrated as well as a calibrated version of the Maximum Overlap Discrete Wavelet Transform.

Since the momentum effect (Δ) might also be represented as the "velocity" of stock prices, it can be quantified by the first derivative of the log-price time series. Moreover, acceleration (Γ) might be defined (as in physics) as a change in velocity and it can be modelled by the second derivative. Hence, according to previous literature, in order to detect the delta factor, we performed a wavelet transform using a Daubechies function with one vanishing moment (also named Haar wavelet function). Furthermore, the acceleration factor is captured by the detail coefficients of a MODWT performed with a Daubechies function with two vanishing moments (Db2).

The wavelet transform is applied using the natural logarithm of a daily stock price series ($S_{i,t}$) as input variable:

$$x(i,t) = x_{i,t} = ln(S_{i,t}) \tag{35}$$

Before performing this approach on stock prices series, the MODWT is applied on simulated signals to calibrate its precision in measuring the first and second derivative. On this purpose the MODWT is firstly applied on the Gaussian, the Sigmoid and the Gaussian Pulse signals which have been simulated with MATLAB and they are represented in the Figures A4-A6.

The first derivative is computed applying the MODWT approach having as mother wavelet the Haar (or Daubechies 1) function. Moreover, the second derivative is performed similarly but using as mother wavelet a Daubechies function with two vanishing moments (Db2). Figure A7 and A8 in the Appendix shows the computation of the first and the second derivative of the Gaussian Signal at different scales; the MODWT estimation is compared to the "classical" first order differentiation performed by MATLAB. Figures A9-A12 in the Appendix represent the same approach applied to the Sigmoid and the Gaussian Pulse signals. It is clear that for all signals the first and second derivative might be approximate quite well. However, the second derivative results using the "negative" of the MODWT output performed with a Daubechies function with two vanishing moments.

Therefore, using the MODWT procedure, we can define the Δ and the Γ parameters for a specific security i in the following way:

$$\textit{Momentum Factor}: \quad \Delta_{i,t}(j) = d_{j,t}(i) \tag{36}$$

$$\textit{Acceleration Factor}: \quad \Gamma_{i,t}(j) = -\hat{d}_{j,t}(i) \tag{37}$$

where $d_{j,t}(i)$ is the detail coefficient calculated using the Haar mother wavelet with one-vanishing moment and $\hat{d}_{j,t}(i)$ is the detail coefficient calculated using the Db2 mother wavelet with two vanishing moments, both measured at the level j and with input variable being the signal $x_{i,t}$.

Moreover, for the second derivative there is a small lag, which is almost the same for each signal and growth proportionally to the level (j). For this reason, at each level (j), the lag has been recorded from the pure signals (see in Table A1 in the Appendix) and the wavelet transform on stock price series has been calibrated as follow:

$$d^c_{j,t}(i) = d_{j,t}(i + calibration(j, 1)) \tag{38}$$

$$\hat{d}^c_{j,t}(i) = \hat{d}_{j,t}(i + calibration(j, 2)) \tag{39}$$

where the parameter "$calibration(j, derivative)$" is the calibration for the scale j for the first and the second derivative, respectively.

According to Lera & Sornette (2017) and Ardila & Sornette (2016), detail coefficients of different scales (2^j) are able to capture different time-scale components. Considering the daily frequency of the data, the detail coefficients of the level $j = 1$ detects the delta and gamma factors within a scale of 2 days. Furthermore, because the MODWT is based on the dyadic scaling (2^j), the level $j = 2$ is able to estimate the factors considering a time difference between a scale of 2 (2^1) and 4 days (2^2) days and a $j = 3$ between 4 and 8 days; the conversion of the level (j) in the consequent time-scale which corresponds to a daily or monthly periodicity (i.e. the equivalent to the formation period (f)) is summarized in Table A2 in the Appendix.

The Δ and the Γ parameters have been quantified with different parametrization of the level j. Moreover, in order to detect further time-scale components, a short analysis of monthly prices has been performed; the conversion of the level (j) in the monthly time-scale is also available in the Table A2.

In conclusion, using the third detection approach, i.e. the wavelet transform method, the delta and the gamma parameters are computed using a non-calibrated as well as a calibrated version of the Maximum Overlap Discrete Wavelet Transform.

3 The Gamma Factor in Portfolio Optimization

3.1 Portfolio Selection and Construction

This chapter aims to set up portfolios based on the previously detected momentum (Δ) and acceleration (Γ) factors and backtest their performance. As was done in the paper by Ardila, Forrò & Sornette (2015) two type of portfolio optimization strategies are implemented: the "Long-Short" (LS) and the "Relative Strength Weighted Portfolio" (RSWP). Both strategies are based on cross-sectional (relative strength) stock selection. Additionally, a hybrid portfolio optimization, i.e. a Δ/Γ (Delta-Gamma) strategy has been developed.

According to Bird, Gao & Yeung (2017) the portfolio optimization consists of two main tasks: stock selection and portfolio construction.

Stock selection is the first and most important procedure to be executed during portfolio optimization. It consists in identifying stocks in which to invest - long or short -in. It is very important that the trend or acceleration is identified on time and not too late, hence the formation period f plays an important role. Therefore, this study is performed using numerous formation periods of five days (one week) as well as one, three, six and twelve months. Another important aspect during the stock selection process is the "cut-off" rule, through which securities are included in the winner (or long) portfolio and in the loser (short) portfolio. The cut-off might depend on a benchmark (as the market past return, i.e. market momentum or acceleration) as is implemented in the "Relative Strength Weighted Portfolio" optimization. Alternatively, as was done in the paper by Jegadeesh & Titman (1993), the cut-off rule consists in ranking the stock according to its past returns (i.e. momentum or acceleration): the winners and the losers are identifying through a top respectively the bottom percentile of the distribution. The second selection process takes place in the "Long-Short" portfolio optimization.

The portfolio construction involves three decisions: the holding period (h), portfolio rebalancing and weights determination (Bird, Gao, & Yeung, 2017). The holding period should be in harmony with "oscillation" and should "approximate the upward and downward cycle" of typical stocks. In this investigation, portfolio optimization is based on holding periods of 1, 3, 6 and 12 months. As was done in the investigation by Ardila, Forrò & Sornette (2015), there is no rebalancing. This means that portfolios follow the buy and hold strategy. The simulation consists in setting up a portfolio every day and selecting stock according to the delta or gamma parameter computed on the previous day using a specific formation period and, afterwards, holding the portfolio until the last day of the holding period.

Furthermore, equal weights (EW) and market weights (MW) are the most common methods to determine the share of the portfolio, which is assigned to a specific security. Equal weights are applied in the "Long-Short" portfolio optimization strategy while market weights are explained in the following section and are used in the "Relative Strength Weighted Portfolio" strategy (Bird, Gao & Yeung, 2017). In this research, a long-short market-neutral weighting rule is used. This mean that the amounts in the long and short position are equal: long position securities are bought through the sale of the securities in the short position, i.e. there is self-financing and it results a zero-cost strategy (BarclayHedge, 2018) (Investopedia, 2018i).

The last important parameter is the investment delay "s" , which specifies the "delay time" of the trading execution after the portfolio construction, i.e. after the identification of a security as a winner or loser. In fact, the bid-ask spread might play a role in the pricing of an asset because the stocks that have performed better (poorly) might be overpriced (under-priced), i.e. the bid-ask spread might be very high (low). However, the price should move back to "the midpoint of the bid-ask spread" and postponing the trading might increase performance (Bird, Gao, & Yeung, 2017). In this investigation a delay of one month ($s = 1$) and six months ($s = 6$) are also considered as additional parameters.

Since the original data set includes daily data, as it is stated in section 2.1.2 Assumptions, a month is approximated to 21 days.

3.1.1 Relative Strength Portfolios

According to "relative strength" strategies which have been briefly explained in the first chapter, one way to set up a portfolio is to rank the available stocks according to their delta or gamma and select a top (bottom) percentile creating a long (short) equal-weighted (EW) portfolio. Another way is to create a portfolio which invest long or short in the universe of securities and which weights are quantified according to the relative performance to the market (MW).

Long-Short Portfolio

In the Long-Short portfolio optimization strategy, considering the universe of N securities, at each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameter computed on the previous day over a specific formation period of f days (or months). More precisely, as was done in the paper by Ardila, Forrò & Sornette (2015) and similarly to Jegadeesh & Titman (1993) as well as many other investigations, the ranked stocks are divided into successive sub-groups representing Q-percentile equal weighted portfolios. This strategy consists in going long for h months in the top-ranked Q-percentile stocks (usually decile or quintile) and short for h months in the bottom-ranked Q-percentile stocks. The trading might be delayed by s months after the ranking day.

Portfolios based on the momentum factor can be defined in the following way:

- $\Delta^{L}_{s,f}$: defines the long Δ-based portfolio which includes stocks from the top Δ-ranked Q-percentile
- $\Delta^{S}_{s,f}$: defines the short Δ-based portfolio which includes stocks from the bottom Δ-ranked Q-percentile

Portfolios based on the gamma factor can be defined in the following way:

- $\Gamma^{L}_{s,f}$: defines the long Γ-based portfolio which includes stocks from the top Γ-ranked Q-percentile
- $\Gamma^{S}_{s,f}$: defines the short Γ-based portfolio which includes stocks from the bottom Γ-ranked Q-percentile

More precisely, the performance of the long and the short portfolio at the end of the holding period are calculated by summing the discretized h-months cumulative log-return of the stocks included in the respective Q-percentile portfolio. Moreover, the performance of the long and the short equal weighted sub-portfolios are described by the following notations: $\pi^{L}_{t+h-1}(f,h,s)$ and $\pi^{S}_{t+h-1}(f,h,s)$ or directly by the variable $\Gamma^{L}_{s,f(LS)}(h)$ and $\Gamma^{S}_{s,f(LS)}(h)$ for the Γ-based portfolio optimization and $\Delta^{L}_{s,f(LS)}(h)$ and $\Delta^{S}_{s,f(LS)}(h)$ for the momentum-based strategy.

Considering equal weighting, the total portfolio return ($\pi_{t+h-1}(f,h,s)$) is computed in general in the following way:

$$\pi_{t+h-1}(f,h,s) = \pi^{L}_{t+h-1}(f,h,s) - \pi^{S}_{t+h-1}(f,h,s) \tag{40}$$

and is also described with the notation $\Delta_{s,f(LS)}(h)$ or $\Gamma_{s,f(LS)}(h)$. Moreover, the total sum of weights of the long (+1) and the short (-1) positions is zero (i.e. a market-neutral weighting rule is applied).

Relative Strength Weighted Portfolio

Relative Strength Weighed Portfolio (RSWP) optimization differs from the first strategy in two respects: the first is that all market stocks are considered for the investment (and not only the best and worst percentile) and the second that the weights of each security are based on its relative delta or gamma compared to the market delta or gamma.

Similarly to the investigation carried out by Ardila, Forrò & Sornette (2015), the Relative Strength Weighted Portfolio constructed thorough the acceleration factor consists firstly in determining weights. More precisely, at each day t, the weight of a specific security i is determined by comparing its gamma factor computed the previous day $t-1$ (minus a delay of s months, which represents the delay in the investment) over a formation period of f days or months ($\Gamma_{i,t-1}(f)$) with the acceleration factor of the equal weighted index (the market) ($\Gamma_{m,t-1}(f)$):

$$w^{\Gamma}_{i,t}(f,s) = \frac{1}{N}\Big(\Gamma_{i,t-1-s}(f) - \Gamma_{m,t-1-s}(f)\Big) \tag{41}$$

where

$$\sum_{i=1}^{N} w^{\Gamma}_{i,t}(f,s) = 0 \tag{42}$$

Moreover, in order to get market-neutral weights, there is a standardization so that weights of the long positions (when the gamma of a specific security is larger than the gamma of the market, i.e. weights are positive) add up to one (+1) and the weights of the short position add up to minus one (-1).

The total Γ-portfolio performance is measured with the following equation:

$$\pi^{\Gamma}_{t+h-1}(f,h,s) = \sum_{i=1}^{N} w^{\Gamma}_{i,t}(f,s) * r_{i,t+h-1}(h) \tag{43}$$

where $r_{i,t+h-1}(h)$ is the discretized h-months cumulative return of the security i at time $t+h-1$.

The computation of portfolio optimization on the basis of the momentum factor (Δ) is the same, hence the Γ-factors are replaced with the Δ-parameters. The profit is computed with the following equation:

$$\pi^{\Delta}_{t+h-1}(f,h,s) = \sum_{i=1}^{N} w^{\Delta}_{i,t}(f,s) * r_{i,t+h-1}(h) \tag{44}$$

3.1.2 Hybrid Portfolio: the Δ/Γ-Portfolio Optimization

The third portfolio strategy aims to consider both the momentum and the acceleration effect as factors for optimization. The developed strategy is called "Delta-Gamma" portfolio optimization strategy (Δ/Γ) and is an extension to the "elementary" time-series momentum strategy which invests long in stocks with a momentum factor greater than zero and short in those with a delta less than zero.

The Δ/Γ optimization consists in selecting securities to include in the long and in the short portfolio according to two conditions: the direction of the momentum ("delta condition") and the direction of the acceleration ("gamma condition"). Considering the magnitude of the previously detected delta and gamma factor, each stock is assigned to the long resp. the short portfolio by applying the following delta and gamma conditions.

The first condition (the delta condition) for the security selection states that:

- At each time t a long portfolio is constructed including securities which have at time $t-1-s$ a momentum (delta) factor ($\Delta_{i,t-1-s}(f)$) greater than zero computed according to the parametrization f (i.e. the portfolio includes securities that have shown a positive past trend which is assumed to be persistent).

- At each time t a short portfolio is constructed including securities which have at time $t-1-s$ a momentum (delta) factor ($\Delta_{i,t-1-s}(f)$) less than zero computed according to the parametrization f (i.e. the portfolio includes securities that have shown a negative past trend which is assumed to be persistent).

Moreover, the second condition (the gamma condition) considers the acceleration factor of securities selected after the delta condition and it states:

- In order to still remain included, securities selected in the long portfolio must show at time $t-1-s$ a positive change in momentum (computed over the last f-days), hence the acceleration factor ($\Gamma_{i,t-1-s}(f)$) should be positive. This mean that the price of the stock included is rising at an increasing rate (i.e. upward accelerating price) over the last f-days.
- In order to still remain included, securities selected in the short portfolio must show at time $t-1-s$ a negative change in momentum (computed over the last f-days), hence the acceleration factor ($\Gamma_{i,t-1-s}(f)$) should be negative. This mean that the price of the stock included is falling at an increasing rate (i.e. downward accelerating price) over the last f-days.

Mathematically, departing from the previously detected delta factor matrix $[\Delta(f)]$ which indicates the momentum magnitude of each stock i at each day t of the whole universe of stocks (data set), two signals matrices (one for the long portfolio and one for the short portfolio selection) are originated. Therefore, applying the delta-condition gives rise to a Long Delta Signal Matrix $\left(\left[\Delta^{Long}\right]\right)$ and a Short Delta Signal Matrix $\left(\left[\Delta^{Short}\right]\right)$.

Long Delta Signal Matrix $\left(\left[\Delta^{Long}(f)\right]\right)$ (first condition):

$$\Delta_{i,t}(f) > 0 \longrightarrow \text{ the signal is one} \longrightarrow \left[\Delta_{i,t}^{Long}(f)\right] = 1 \tag{45}$$

$$\Delta_{i,t}(f) < 0 \longrightarrow \text{ the signal is zero} \longrightarrow \left[\Delta_{i,t}^{Long}(f)\right] = 0 \tag{46}$$

Short Delta Signal Matrix $\left(\left[\Delta^{Short}(f)\right]\right)$ (first condition):

$$\Delta_{i,t}(f) < 0 \longrightarrow \text{ the signal is one} \longrightarrow \left[\Delta_{i,t}^{Short}(f)\right] = 1 \tag{47}$$

$$\Delta_{i,t}(f) > 0 \longrightarrow \text{ the signal is zero} \longrightarrow \left[\Delta_{i,t}^{Short}(f)\right] = 0 \tag{48}$$

For the long portfolio, the signal is equal to one when the delta ($\Delta_{i,t}(f)$) is positive and otherwise zero; viceversa for the short portfolio. Bot, the long and short signal matrices encompass only values of zero or one.

Thereafter, both signal matrices are multiplied with the previously detected acceleration factor matrix $\left[\Gamma(f)\right]$. Following this procedure, we are able to assess the value of the acceleration[6] of each security

[6] The securities that are not included after the application of the first condition have a signal parameter of zero in the respective (Long or Short) Delta Signal matrix and for this reason the multiplication with the acceleration force do not change their inclusion.

that we have already included; furthermore, it is possible to compare it across the universe of securities. Hence, there is a Long Gamma Signal Matrix $\left(\left[\Gamma^{Long}(f)\right]\right)$ and a Short Gamma Signal Matrix $\left(\left[\Gamma^{Short}(f)\right]\right)$:

$$\left[\Gamma^{Long}(f)\right] = \left[\Delta^{Long}(f)\right] * \Gamma(f) \tag{49}$$

$$\left[\Gamma^{Short}(f)\right] = \left[\Delta^{Short}(f)\right] * \Gamma(f) \tag{50}$$

Successively, the second condition can be applied which discards the stock with a negative (positive) acceleration factor from the long (short) portfolio:

$$\left[\Gamma^{Long}(f)\right]\left(\left[\Gamma^{Long}(f)\right] < 0\right) = 0 \tag{51}$$

$$\left[\Gamma^{Short}(f)\right]\left(\left[\Gamma^{Short}(f)\right] > 0\right) = 0 \tag{52}$$

After this procedure we get two signal matrices which are able to indicate which stocks should be selected at each time t in order to construct the long and the short portfolio. Moreover, at this point we can decide to invest applying equal weights or according to the acceleration magnitude ("relative weights"). As was explained in the Relative Strength Weighted Portfolio, weights are standardized so that they satisfy the market neutral condition, i.e. weight for the long (short) portfolio add up to one (minus one). It is essential to be aware that when using the Δ/Γ optimization, the number of securities included in the portfolios is not constant. In extreme situations, at some days t, this procedure gives a non-investment strategy as the optimal strategy (the signal is zero for each stock).

As in long-short portfolio optimization, profits from the long and the short portfolio at the end of the holding period are calculated by adding the discretized h-months cumulative log-return of the stocks which are included in the respective portfolios and putting "equal" weights or "relative" standardized weights.

3.2 Performance Marks and Additional Assumptions

3.2.1 Portfolio Performance

For each optimization strategy, Δ and Γ-based portfolios performances are evaluated through the following parameters: the annualized average return ($\mu\left[\pi(f,h)\right]$) of all portfolios set up at each day (or month) using that specific strategy, the annualized volatility ($\sigma\left[\pi(f,h)\right]$) of the annualized average return and the Sharpe Ratio (SR). Additionally, a t-student test has been performed, in order to check the statistical significance of the results obtained.

Given the average discrete portfolio return of an h-days investment ($\mu_h [\pi(f,h)]$) , it is possible to compute the average annualized return by applying the following formula:

$$\mu [\pi(f,h)] = (1 + \mu_h [\pi(f,h)])^{252/h} - 1 \tag{53}$$

where h is the number of days of the holding-period.

Furthermore, annualized volatility is calculated as following:

$$\sigma [\pi(f,h)] = \sigma_h [\pi(f,h)] * \sqrt{252/h} \tag{54}$$

The Sharpe Ratio is a measure of the risk-adjusted performance and it is computed by the following equation (Investopedia, 2018j):

$$SR = \frac{\mu [\pi(f,h)] - r_f}{\sigma [\pi(f,h)]} \tag{55}$$

where r_f is assumed to be equal to zero. The Sharpe Ratio is a good measure in order to compare different investments because it takes into account not only the average return of a strategy (in excess to the risk free rate) but also the amount of risk (volatility) involved. Therefore, this ratio gives an indication about the average return for each unity of risk. The higher the SR is, the more return the strategy gives per unit of risk (DBF, 2018) (InvestingAnswers, 2018). Hence, investors aim to achieve a larger Sharpe Ratio; while a SR larger than one is considered acceptable, a ratio of two is a sign of a very good investment and three suggests the performance excellence of the investment strategy (Investopedia, 2018k).

Another interesting metric used to evaluate the performance of a specific portfolio optimization strategy is the Information Ratio (IR). More precisely, the information ratio determines the ability to generate excess returns in relation to a benchmark, i.e. it is a measure of the risk-adjusted excess return. In this study, the benchmark is the market return, i.e. the cumulative return of the equal-weighted index including all the securities in a stocks universe (data set) over the portfolio holding period (see Equation 60). Hence, the excess return is the excess performance above the return of the market index. The Information Ratio is computed by dividing the excess return by its standard deviation as is show by the following formula:

$$IR = \frac{R_p - R_m}{\sigma_{p-m}} \tag{56}$$

where R_p is the return of the portfolio, R_m is the return of the equal-weighted index over the portfolio holding period and σ_{p-m} is the volatility (i.e. the standard deviation) of the excess return (i.e. $R_p - R_m$). A higher IR is desired since its magnitude gives an indication about the strategy consistency.

Moreover, the one sample student's t-test performed in this research aims to determine if the average annualized return ($\mu\left[\pi(f,h)\right]$) from portfolio optimization is statistically significantly different from zero. Therefore, assuming normal distributed portfolio returns, a two-sides t-test is performed and relies on the null hypothesis (H_0) that the average annualized portfolio return is equal to zero against the alternative hypothesis (H_1) of an annualized average return different from zero (Stock & Watson, 2012). The t-value is calculated as follow:

$$t\text{−value}= \frac{\mu\left[\pi(f,h)\right]}{\sigma\left[\pi(f,h)\right]\sqrt{T}} \tag{57}$$

where T represent the number of observations, i.e. the number of portfolio returns computed. Critical values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). More precisely, a t-value above the critical value as for example above 1.96 indicates that the null hypothesis (i.e. return equal to zero) can be rejected with a confidence of 95% (Stock & Watson, 2012).

3.2.2 Additional Assumptions

Since the delta and gamma factors have been computed using continuous (log-)return (in the simple, as well as the exponential smoothing and wavelet detection[7]), and since "relative" (or market) weights are directly determined by the magnitude of the detected factor, there is an additional analysis which, instead continuous detected factors, implements "discrete" delta ($\Delta^d_{i,t}(f)$) and gamma ($\Gamma^d_{i,t}(f)$ parameters. This short analysis is applied to some RSWP as well as Δ/Γ portfolio strategies.

The original factors are converted in the following way:

$$\Delta^d_{i,t}(f) = exp\left(\Delta_{i,t}(f)\right) - 1 \tag{58}$$

$$\Gamma^d_{i,t}(f) = exp\left(\Gamma_{i,t}(f)\right) - 1 \tag{59}$$

Lastly, in order to obtain a comparison with the market investment, the return of the equal-weighted

[7] Since the crossover detection approach quantifies the momentum directly from price unities, it is not given the continuous form.

market index ($r_{m,t}(h)$) has been calculated as follows:

$$\pi^{f}_{t+h-1}(h,s) = r_{m,t}(h) = \frac{1}{N}\sum_{i=1}^{N} r_{i,t+h-1}(h) \tag{60}$$

where N represents the number of assets in the market.

4 Portfolio Optimization Results

4.1 Results: The Momentum and Acceleration Effect in the 21th Century

4.1.1 Dow Jones Industrial Average (1984-2002)

The first analysis aims to ascertain that strategies based on the momentum factor (such as for example the Long-Short strategy or the Relative Strength Weighted Portfolio) which have been implemented in several previous papers are still also generating positive returns in today's financial market environment.

To this end, as was carried out in the paper by Jegadeesh & Titman (1993), a first investigation has the purpose of determining the performance of Long-Short (LS) strategies based on momentum by examining the universe of stocks (data set) which includes the Dow Jones Industrial Average components over the period of time starting on 2 January 2001 and ending on 30 December 2016 (recent past) and comparing it with the "distant past" performance (i.e. between 1984 and 2002). Moreover, a set of Γ-based strategies that were previously studied in the paper by Ardila, Forrò & Sornette (2015) are newly tested using the Dow Jones Industrial Average data set over the above mentioned investigation window and compared to strategies based on momentum.

In this section, momentum as well as the acceleration factors, are quantified through the simple approach. Furthermore, the Relative Strength Weighted Portfolio (RSWP) optimization strategy is also applied to determine the persistence of these financial market anomalies in stock prices.

Long-Short Portfolio Optimization

Table 1 (in the following page) shows the average annualized return (μ), the annualized volatility (σ) both expressed as a percentage as well as the average annualized Sharpe Ratio (SR) and the significance test (t-test) for 24 Δ_{LS} and 24 Γ_{LS} portfolios having a holding period of one and six months. Moreover, both factors have been quantified using the simple approach; the long portfolio includes stocks of the top Δ or Γ-ranked (in ascending order) quintile while the short portfolio includes securities of the bottom quintile. Equal weights are applied to each stock in the corresponding portfolio; weights of the long (short) portfolio add up to +1 (-1), hence market-neutral weights are implemented (it is a zero-cost strategy). A comprehensive table also including holding periods of three and twelve months (i.e. including the performance of 48 Δ_{LS} and Γ_{LS} strategies) is available in the attachment (Table A3 in the Appendix)[8].

[8] The portfolio optimization strategy is written in the text in this form: $FACTOR_{strategy}$ (detection mode), for example the Δ_{LS} (WMC) notation indicates the momentum-based Long-Short portfolio optimization performed using a Δ-factor detected with the calibrated MODWT approach on a monthly basis or the Γ^{D}_{RSWP} (simple) is the acceleration-based Relative Strength Weighted Portfolio optimization performed using factors detected with the simple approach and converted in the discrete form. The analysis has been performed through MATLAB.

Annualized performance of Δ_{LS} and Γ_{LS} (simple)
DJIA (2001-2016)

		Δ_{LS}				Γ_{LS}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-4.35	18.31	-0.24	-4.43	-1.42	17.05	-0.08	-1.53
	3	-3.87	19.29	-0.2	-3.73	0.23	16.38	0.01	0.25
	6	-5.44	19.91	-0.27	-5.12	-6.73	19.14	-0.35	-6.63
	12	-2.43	21.5	-0.11	-2.09	-1.59	18.57	-0.09	-1.58
1	1	-1.1	17.45	-0.06	-1.15	-1.15	16.38	-0.07	-1.29
	3	-3.37	19.03	-0.18	-3.28	0.23	16.75	0.01	0.25
	6	-3.41	19.29	-0.18	-3.27	-3.53	18.51	-0.19	-3.53
	12	-2.52	21.08	-0.12	-2.2	-1.43	18.27	-0.08	-1.43
6	1	-1.26	16.46	-0.08	-1.39	-1.3	16.17	-0.08	-1.46
	3	2.8	17.72	0.16	2.8	-4.2	15.96	-0.26	-4.83
	6	4.39	18.4	0.24	4.21	1.97	16.27	0.12	2.16
	12	1.66	19.38	0.09	1.53	-1.22	16.28	-0.07	-1.35
					$h = 6$				
0	1	-1.5	16.51	-0.09	-4.03	-0.58	16.38	-0.04	-1.58
	3	-1.75	17.31	-0.1	-4.48	-1.76	15.31	-0.12	-5.1
	6	0.13	19.67	0.01	0.3	-2.52	18.86	-0.13	-5.93
	12	-0.47	21.67	-0.02	-0.97	-1.21	18.44	-0.07	-2.9
1	1	-0.85	15.84	-0.05	-2.36	-0.6	15.61	-0.04	-1.71
	3	-0.76	18.39	-0.04	-1.82	-2.62	15.82	-0.17	-7.33
	6	1.69	20.51	0.08	3.61	-1.27	18.3	-0.07	-3.06
	12	0.32	21.75	0.01	0.65	-1.08	18.19	-0.06	-2.63
6	1	1.77	14.87	0.12	5.14	-0.8	14.84	-0.05	-2.33
	3	2.07	17.39	0.12	5.14	1	15.69	0.06	2.77
	6	1.2	17.41	0.07	2.97	0.53	17.31	0.03	1.33
	12	1.76	18.64	0.09	4.09	-1.98	15.59	-0.13	-5.55

Table 1: The figure shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ (simple) portfolios set up considering daily split and dividend-adjusted log-returns of securities included in the Dow Jones Industrial Average (2001-2016). At each day t stocks are ranked in ascending order according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked quintile while the short portfolio sells stocks of the bottom-ranked quintile. Equal (market-neutral) weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$-factors are detected through the simple approach using different formation periods (f) expressed in months. The parameter s represents the delay in the investment. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical values are: 1.64 (90%), 1.96 (95%) and 2.58 (99%).

It is clear (see Table 1) that in the recent past, both portfolios ($\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$) are performing negatively in about half of the scenarios. The Δ_{LS} (simple) strategy leads to statistically significant positive returns in 7 out of 24 scenarios. Moreover, it achieves better results when the investment is delayed by six months ($s = 6$) and it reaches the maximal average annualized return of 4.39 % for the $\Delta_{6,6(LS)}$ (simple) portfolio held for one month. As is reported in Table A3 in the Appendix, the Δ_{LS} (simple) strategy with a twelve-month holding period ($h = 12$) always generated positive performance and, in general, a formation period (f) of six or twelve months produced better results than a formation period of one month for $\Delta_{s,f(LS)}$ (simple) portfolios.

$\Gamma_{s,f(LS)}$ (simple) portfolios performed negatively in 19 out of 24 scenarios; 9 of them are statistically significant. The maximal performance (average annualized return) is 1.97% and it results in the $\Gamma_{6,6(LS)}$ (simple) portfolio held for one month. Contrary to the Δ_{LS} (simple) strategy, a holding period of one year leads to negative results for Γ_{LS} (simple) strategy independently of the formation period or the delay in the investment (see Table A3 in the Appendix). Moreover, the Γ_{LS} (simple) portfolio optimization only beats the Δ_{LS} (simple) strategy in situations of negative returns, i.e. in 5 scenarios out of 48 where the rate of return of the Γ_{LS} (simple) strategy is statistically slightly less negative than the return of the Δ_{LS} (simple).

To gain a deep understanding of the negative performance of the above Δ_{LS} and Γ_{LS} (simple) portfolio optimizations, Tables A4 and A5 in the Appendix indicate the performance of the long and the short equal-weighted sub-portfolios separately; the former is constructed buying stocks of the top quintile while the latter invests short in securities of the bottom quintile. It is surprising to see that ”short” sub-portfolios, which according to the theory of momentum should have a lower (or negative) performance (because they should track stocks with a negative trend), show on average a positive annualized return which is greater than the rate of return of long sub-portfolios or, in some cases, short sub-portfolio returns even outperform the rate of return in the long sub-portfolios.

Furthermore, as is shown in Table 2, the LS (simple) strategy computed using a formation period of one week does not seem to improve the performance of portfolios based on momentum or acceleration.

Annualized performance of Δ_{LS} and Γ_{LS} (simple) DJIA (2001-2016)
($f = 5$ days)

		Δ_{LS}				Γ_{LS}			
h	s	μ	σ	SR	t-test	μ	σ	SR	t-test
					f = 5 days				
1	0	-3.81	17.47	-0.22	-4.06	-1.85	16.96	-0.11	-2.01
	3	-1.07	16.45	-0.07	-1.19	-0.68	16.58	-0.04	-0.75
	6	-0.95	16.65	-0.06	-1.04	-0.49	15.97	-0.03	-0.56
3	0	-1.99	16.48	-0.12	-3.83	-1.01	16.28	-0.06	-1.97
	3	-0.82	16.16	-0.05	-1.59	-0.12	16.4	-0.01	-0.23
	6	-0.56	16.07	-0.04	-1.09	0.1	15.64	0.01	0.19
6	0	-1.13	16.43	-0.07	-3.04	-0.44	16.49	-0.03	-1.17
	3	-0.44	16.07	-0.03	-1.22	-0.15	16.61	-0.01	-0.39
	6	0.63	15.17	0.04	1.79	0.27	15.29	0.02	0.78
12	0	-0.27	16.56	-0.02	-1.02	-0.03	16.43	0	-0.13
	3	-0.25	16.07	-0.02	-0.94	-0.09	16.18	-0.01	-0.35
	6	0.11	15.68	0.01	0.44	0.13	15.69	0.01	0.5

Table 2: The table shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ (simple) portfolios set up considering the DJIA stocks universe (2001-2016) and using a formation period (f) of one week (i.e. five days). Portfolios have a h-month holding period and the investment might be delayed of s months. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 (99%).

To have a comparison with a previous time period (distant past), the LS (simple) strategy is applied to the components of the Dow Jones Industrial Average considering the period of time starting on 23 December 1983 and ending on 31 December 2002 (i.e. 1984-2002). Results for a holding period (h) of one or six months are available in Table 3 while a complete overview on additional holding periods is illustrated in Table A6 in the Appendix.

Annualized performance of Δ_{LS} and Γ_{LS} (simple)
DJIA (1984-2002)

		Δ_{LS}				Γ_{LS}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
$h = 1$									
	1	-13.7	24.91	-0.55	-11.73	-9.17	25.45	-0.36	-7.51
0	3	15.75	28.37	0.56	10.35	-5.48	26.22	-0.21	-4.28
	6	22.21	29.54	0.75	13.67	-10.22	26.64	-0.38	-8.04
	12	32.95	30.93	1.07	18.6	10.18	27.42	0.37	7.09
	1	-3.42	25.97	-0.13	-2.66	-21.51	28.28	-0.76	-16.89
1	3	17.86	29.09	0.61	11.32	-5.15	25.82	-0.2	-4.07
	6	26.55	29.26	0.91	16.19	-5.52	26.08	-0.21	-4.32
	12	37.89	30.8	1.23	21.05	14.38	27.34	0.53	9.84
	1	-1.71	25.5	-0.07	-1.33	-2.37	25.39	-0.09	-1.86
6	3	24.32	28.19	0.86	15.34	-2.74	25.45	-0.11	-2.15
	6	38.63	28.59	1.35	22.81	19.2	26.36	0.73	13.22
	12	28.12	29.75	0.95	16.58	11.69	26.91	0.43	8.13
$h = 6$									
	1	7.13	33.35	0.21	10.16	-1.73	33.36	-0.05	-2.51
0	3	27.91	40.88	0.68	30.98	-3.88	34.03	-0.11	-5.57
	6	37.05	39.57	0.94	41.71	-0.46	32.46	-0.01	-0.68
	12	45.31	41.36	1.1	48.03	17.34	36.88	0.47	21.83
	1	9.86	33.83	0.29	13.72	-0.57	34.57	-0.02	-0.8
1	3	29.64	39.62	0.75	33.75	-3.18	32.88	-0.1	-4.7
	6	40.01	38	1.05	46.54	5.04	32.14	0.16	7.47
	12	44.56	41.08	1.09	47.52	17.79	36.85	0.48	22.33
	1	13.6	32.64	0.42	19.24	-0.6	33.86	-0.02	-0.85
6	3	34.38	39.15	0.88	38.79	5.35	32.28	0.17	7.81
	6	38.68	38.56	1	43.94	16.42	34.01	0.48	22.16
	12	31.93	39.86	0.8	35.56	16.51	37.26	0.44	20.32

Table 3: The table shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ (simple) portfolios set up using daily split and dividend-adjusted log-returns of securities included in the DJIA considering the distant past (1984-2002). Portfolios have a h-month holding period and the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 (99%).

According to Table 3, in the distant past (1984-2002), the performance of the Δ_{LS} (simple) strategy was significantly positive in 22 out of 24 scenarios and the average positive annualized rate of return ranged from around 7% to around 45%; this confirms the presence of the momentum effect over the last two decades of the 20th century in the U.S. securities market. The acceleration factor generated positive returns in 10 out of 24 cases and it reached a maximal average rate of return of about 16% (annual). Furthermore, considering a holding period of one year ($h = 12$), the performance of Δ_{LS} and Γ_{LS} (simple) portfolios based on formation windows of three, six and twelve months is always positive (see Table A6 in the Appendix).

In general, slightly greater portfolio returns are generated using a formation period f of six or twelve

months for both the Δ_{LS} and the Γ_{LS} (simple) portfolio optimizations; while an f of one month always generates negative returns for the Γ_{LS} (simple) portfolio and it leads to a lower performance for corresponding Δ_{LS} portfolio. The best performance in terms of Sharpe Ratio derives from the strategies $\Delta_{6,6(LS)}$ (simple) and $\Gamma_{6,6(LS)}$ (simple) using a holding period of one month ($h = 1$) which reconfirms what was discovered in the paper by Ardila, Forrò & Sornette (2015). The longer the holding period, the higher the portfolio performance for both factors; however there is also an increment in the volatility of the portfolio return and for the momentum strategy, a holding period of one year is no more profitable if we consider the risk-adjusted annualized return (SR).
Moreover, excluding a formation period of one month $f = 1$, there are some similarities to what was discovered in the study by Ardila, Forrò & Sornette (2015). Both Δ_{LS} and Γ_{LS} (simple) portfolios are performing significantly well and the Γ_{LS} (simple) strategy shows an increase in the rate of return for longer holding periods. Moreover, it is less likely to find an acceleration effect in the short-term (for f equal to one or three months) and the best portfolios are the same as in the previous investigation (i.e. $\Delta_{6,6(LS)}$ and $\Gamma_{6,6(LS)}$). Nevertheless, in this study there is only a significant demonstration that the Γ_{LS} (simple) strategy might be more profitable than the Δ_{LS} (simple) optimization; a possible explanation is given in the discussion part.

Relative Strength Weighted Portfolio

Tables A7 and A8 in the Appendix report the averaged annualized performance of Δ_{RSWP} and Γ_{RSWP} (i.e. Relative Strength Weighted Portfolio) strategies optimized using the Dow Jones Industrial Average (DJIA) stocks universe over the period of time between 2001 and 2016 (recent past); factors are detected through the simple approach. Table A7 in the Appendix is based on Δ and Γ factors computed directly from log-returns, while Table A8 aims to test if the conversion of these factors in discrete form (Δ^D and Γ^D) might have an impact on the portfolio outcome. In the RSWP strategy, the weight of each stock is determined by the magnitude of its Δ or Γ-parameter compared to the Δ or Γ of the market (i.e. to the factor quantified from an equal weighted index of all stocks in the universe examined (data set)). Moreover, weights of the short and the long positions have been standardized in order to get market-neutral weights, i.e. weights of stocks in the long (short) portfolio add up to +1 (-1) so that total weights add up to zero (i.e. it is a zero-cost strategy).

Comparing Table A7 and A8 in the Appendix, it appears that the RSWP (simple) strategy based on discrete factors (Δ^D and Γ^D) always beats the RSWP strategy computed directly through continuously detected factors (Δ and Γ). For this reason only results of Δ^D_{RSWP} and Γ^D_{RSWP} (simple) strategies are described and the discrete version of the RSWP strategy is applied to further analysis.

Table A8 indicates that a a holding period (h) of one month leads to negative portfolio returns in more than half of the parametrizations for both Δ^D_{RSWP} and Γ^D_{RSWP} (simple) portfolios. As was already established using the Long-Short strategy, the best performance is generated by the RSWP (simple) portfolios $\Delta^D_{6,6(RSWP)}$ and $\Gamma^D_{6,6(RSWP)}$ held for one month. However, the annualized Sharpe Ratios generated by the RSWP (simple) strategy ($SR = 0.35$ for the $\Delta^D_{6,6(RSWP)}$ and $SR = 0.37$ for the $\Gamma^D_{6,6(RSWP)}$) are greater than the corresponding Sharpe Ratios generated by the LS (simple) strategy (0.24 and 0.12).
The return of the Γ^D_{RSWP} (simple) portfolio optimization is significantly negative in half of the parametrizations while positive statistically significant returns are always generated by the strategy $\Gamma^D_{6,6(RSWP)}$

(simple) for each holding period (h). The Δ^D_{RSWP} (simple) strategy generated a positive significant average annualized return in over half the scenarios. A negative performance arises mainly when factors are detected over a short formation period (f) or when there is no delay in the investment execution. However, a one-year holding period leads to statistically significant positive returns for all kind of parametrizations for the momentum strategy. Considering the whole number of parametrization (see Table A8), Γ^D_{RSWP} (simple) strategies over-perform Δ^D_{RSWP} (simple) in 6 scenarios out of 48 (all statistically significant) where 5 of them are in circumstances of negative return and one scenario regards the best-performing portfolio: $\Gamma^D_{6,6(RSWP)}$.

Table A9 in the Appendix reconfirms that a portfolio optimization based on factors detected with a formation period (f) of one week is not associated to significantly (greater) positive annualized returns.

The RSWP strategy has also been backtested in the distant past, i.e. in the time period between 1984 and 2002, using the data set including the DJIA components; Table A10 shows the Δ^D_{RSWP} and Γ^D_{RSWP} (simple) performances. The annualized return of Δ^D_{RSWP} as well as Γ^D_{RSWP} (simple) strategies are very large compared to the return generated in the recent past (between 2001 and 2014) by the same strategy; the same also appeared using the Long-Short strategy.
Considering holding periods of one and six months, the Δ^D-factor generated an average statistically significant positive return (annualized) in 21 scenarios out of 24. As has been documented in previous analyses in this study, a formation period (f) of one month always generates a worse performance for the Δ^D_{RSWP} (simple) portfolio. Γ^D_{RSWP} (simple) performances are always positive and greater if one uses a formation period (f) of one year. The best returns are generated by postponing the investment by six months ($s = 6$). However, the Γ^D_{RSWP} is more profitable than Δ^D_{RSWP} (simple) strategy only in two scenarios. The best performance in terms of Share Ratio appears in the following (simple) portfolios: $\Delta^D_{6,6(RSWP)}$ and $\Gamma^D_{6,6(RSWP)}$, both with a one month holding period ($h = 1$). Another aspect to be considered is that the volatility increases strongly with the holding period.

To sum up, in general, the average annualized return generated by the RSWP (simple) strategy (i.e. the investment which considers the whole universe of stocks) is greater than the return with the LS (simple) strategy. Moreover, applying both portfolio optimization strategies, there was no acceleration effect in the short-term (i.e. using a formation period of one or three months) while a formation period (f) of 6 or 12 months generated better returns for both factors. Additionally, in the recent past (2001-2016), a delay in the investment of six months ($s = 6$) leads (in general) to improvements for both, momentum and the acceleration-based strategies.

4.1.2 Standard and Poor 500 (2001-2014)

As demonstrated in the previous section, it appears that nowadays (in the recent past) it has not been possible to obtain such great returns - through portfolio optimizations based on momentum and the acceleration factor - as was possible at the end of the last century (when momentum was first discovered). Therefore, this section aims to conduct a similar analysis using split and dividend-adjusted log-returns of the components of the Standard and Poor 500 considering the period between the year 2001 and the year 2014 (recent past).

Long-Short Portfolio Optimization

Table A11 in the Appendix reports the performance of Δ_{LS} and Γ_{LS} (simple) portfolios. More precisely, the strategy invests long in the top-ranked decile (according to the Δ or the Γ parameter) and short in the bottom-ranked decile.

Results are fairly similar as in the analysis of the Dow Jones Industrial Average data set. However, here it is more clearly visible that there is a negative performance for both, Δ_{LS} and Γ_{LS} (simple) strategies. Indeed, the momentum leads to a statistically significant positive performance only in 2 cases out of 48, while the Γ_{LS}(simple) strategy generated a positive average annualized return in 3 cases out of 48. It is interesting to see that despite the overall negative performance, the previously documented best-performing LS (simple) portfolios $\Delta_{6,6(LS)}$ and $\Gamma_{6,6(LS)}$ both held for one month are performing well and they exhibit an average annualized return of 2.68% and a SR of 0.12 (for the Δ_{LS}) and a rate of return of 3.58% and a SR of 0.18 (for the Γ_{LS} strategy).
Furthermore, considering a holding period of one month Γ_{LS} beats the Δ_{LS} (simple) strategy in 8 out of 12 scenarios while a holding period of one year leads to negative returns in all scenarios for both strategies. In general, Γ_{LS} (simple) optimizations are more profitable (or less negative) than Δ_{LS} in 18 out of 48 scenarios; all of them are statistically significant.

Relative Strength Weighted Portfolio

Furthermore, as was already documented by Ardila, Forrò & Sornette (2015) regarding the Γ-factor: an investment which considers the whole universe of stocks and not only securities having extreme returns is more profitable. This can also be reconfirmed by Table A12 in the Appendix, which illustrates the portfolio performance of different RSWP (simple) strategies optimized by the Δ^D_{RSWP} and Γ^D_{RSWP} performed by applying different parametrizations. This study is executed by investing in the components of the Standard and Poor 500 in the recent past (2001-2014).

The Δ^D_{RSWP} strategy generated a statistically significant positive performance in 33 out of 48 scenarios; 17 of them are statistically significant. Moreover, 25 Γ^D_{RSWP} (simple) portfolio optimizations out of 48 generated a positive performance and 20 of them are statistically significant. As in previous analyses, the best performance is generated by $\Delta^D_{6,6(RSWP)}$ and $\Gamma^D_{6,6(RSWP)}$ (simple) portfolios held for one month: the average annualized rate of return amounts to 1.33% (SR= 0.20) for the Δ^D_{RSWP} strategy and 15.31% (SR= 0.27) for the Γ^D_{RSWP} strategy. Moreover, portfolio optimizations based on the acceleration effect beat momentum strategies in 26 out of 48 cases; 23 of them are statistically significant.

Considering longer investments (h) of six or twelve months, what was already discovered in the paper by Ardila, Forrò & Sornette (2015) is more evident: the acceleration leads to better portfolio performance compared to the momentum factor in more than half of the parametrizations.

Short Conclusion

To sum up, Table 4 in the next pages provides an overview of the average annualized return (μ), the annualized volatility (σ) both expressed as a percentage as well as the annualized Sharpe Ratio (SR) and the significance test (t-test) for the best performing LS (simple) and RSWP (simple) portfolios optimized for the momentum or the acceleration factor, i.e. the $\Delta_{6,6}$ and $\Gamma_{6,6}$ (simple) portfolios with a holding period (h) of one month. The table aims not only to compare the divergences in the average annualized rate of return between Δ and Γ portfolios but also between different optimization strategies (i.e. the LS and the RSWP strategy) as well as over two different periods in time: the distant past (1984-2002) and the recent past (2001-2014/2016)

Annualized performance of different $\Delta_{6,6}$ and $\Gamma_{6,6}$
LS (simple) and RSWP (simple) portfolio optimizations

	$\Delta_{6,6}$				$\Gamma_{6,6}$			
Strategy	μ	σ	SR	t-test	μ	σ	SR	t-test
$h = 1$								
LS - DJIA (2001-2016)	4.39	18.4	0.24	4.21	1.97	16.27	0.12	2.16
LS - DJIA (1984-2002)	38.63	28.59	1.35	22.81	19.2	26.36	0.73	13.22
RSWP - DJIA (2001-2016)	5.9	16.92	0.35	6.11	6.01	16.24	0.37	6.47
RSWP - DJIA (1984-2002)	49.97	35.12	1.42	23.11	19.33	25.14	0.77	13.95
LS - S&P500 (2001-2014)	2.68	21.77	0.12	1.98	3.58	19.9	0.18	2.88
RSWP - S&P500 (2001-2014)	3.43	17.03	0.2	3.23	15.31	56.66	0.27	4.12

Table 4: Comparison of the performance of different $\Delta_{6,6}$ and $\Gamma_{6,6}$ LS (simple) and RSWP (simple) portfolio optimizations using the S&P500 as well as the DJIA stocks universe for the distant and the recent past. The analysis is performed with daily split and dividend-adjusted log-returns and portfolios are constructed on a daily basis using a formation period of six months; the investment is delayed by six months after the portfolio construction and the portfolio is held for one month. Weights of the LS as well as the RSWP strategy have been standardized to get market-neutral weights (i.e. it is a zero cost strategy). A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 (99%).

Table 4 reconfirms what was discovered in the paper by Ardila, Forrò & Sornette (2015), i.e. that the Γ-factor might generate better performances than Δ-based (simple) portfolios. Moreover, it is possible to see that strategies based on momentum as well acceleration lead to a lower performance in today's market environment compared to the two last decades of the 20th century in the U.S. market. Furthermore, as it was already documented by Ardila, Forrò & Sornette (2015) and as has already been reconfirmed, an investment which considers the whole universe of stocks (i.e. the RSWP strategy) and not only securities having extreme returns (i..e the LS strategy) is more profitable.

4.1.3 Trend-based Portfolio Optimization

This section describes the outcome of a short analysis that aims to determine if trend-based detection approaches, i.e. the Exponential Moving Average (EMA) and the Simple Moving Average Crossovers (crossovers) implemented in the quantification of the momentum as well as the acceleration factor, might have an influence on the performance of Δ and Γ Long-Short or RSWP portfolios.

The investigation is executed for the DJIA stocks universe (data set) considering the distant past (between 1984 and 2002), i.e. a period of time when the momentum effect appeared to be more evident.The EMA detection approach has been tested using Long-Short optimizations while for the crossover detection approach both strategies (i.e. RSWP and LS) are implemented.

Dow Jones Industrial Average (1984-2002)

Table A13 illustrates the performance of Long-Short portfolios based on factors detected through the EMA detection for a holding period of one or six months. The long portfolio buys stocks in the top-ranked quintile (according to the Δ or Γ-factor) while the short portfolio sells stocks in the bottom quintile. Moreover, three different moving averages windows (K) of 20, 50 and 100 days have been applied to compute the factors.

Table A13 in the Appendix indicates that the trend-based detection (EMA) leads to better results for the Δ_{LS} (EMA) strategy in 42 out of 44 scenarios compared Δ_{LS} (simple) portfolios (for the comparison see Table A6 in the Appendix). Moreover, cleaning stock prices with an Exponential Moving Average computed over a window of 100 days ($K = 100$) generated better performances than using shorter windows. However, the implementation of the EMA approach has no benefits in terms of volatility, which is quite similar as in the portfolio Δ_{LS} (simple) portfolio. Furthermore, the use of an Exponential Moving Average also improved the performance of Γ_{LS} portfolios. Indeed, in 30 out of 44 cases, the Γ_{LS} (EMA) strategy generated statistically significant annualized returns which outperform the same strategy based on Γ-factors detected with the simple approach. Similarly, as was reported in Table A6 in the Appendix, there is no significant evidence that strategies based on the acceleration are more profitable than momentum. The best performing portfolios are the same as previously, i.e. the $\Delta_{6,6(LS)}$ and the $\Gamma_{6,6(LS)}$, both with factors detect using the EMA approach with a moving average window (K) of 20 days.

Another approach implemented for the detection of the delta and gamma factor is the "crossovers" approach, which computes the momentum (Δ) as the difference between a short (50 days) and long (200 days) asymmetric Simple Moving Average applied to stock prices. Thereafter, the acceleration (Γ) is computed as the f-months difference in momentum.

Table A14 in the Appendix illustrates the annualized performance of Long-Short as well as the RSWP optimizations constructed on the basis of Δ and Γ-factors detected with the crossover methodology. We can observe that using the crossover detection approach, there is no improvement in terms of performance for the Δ_{LS} (crossovers) strategy, since using the simple approach with a formation period (f) of six or twelve months (more frequently twelve) the Δ_{LS} (simple) strategy generated a greater risk-adjusted performance (see Table A6). A similar conclusion is arrived at by using the RSWP optimization, i.e. a portfolio constructed on a Δ-factor detected using the simple approach over a formation period (f) of twelve months always showed higher Share Ratios (see Table A10), except for the scenario designed for a one-month holding period ($h = 1$) and no delay in the investment ($s = 0$) where the crossover detection outperformed the simple approach for all formation periods (f).

It is interesting to observe, that the performance of Long-Short and Relative Strength Weighted Portfolio strategies based on the Γ-factor is strongly significantly positive (in the distant past) if this factor is detected through the crossover approach, i.e. it is quantified on the basis of a Δ computed as the difference between a short and a long SMA. As is illustrated in Table A14 in the Appendix, a longer formation period (f) improves the performance of acceleration-based portfolio for both optimization the, LS (crossover) and the RSWP (crossover) strategies. Moreover, the RSWP strategy always generated better returns than the LS strategy; this is an additional confirmation of what was discovered in the previous paper and already reconfirmed in the previous section with the simple approach. Additionally, compared to the simple approach (see Table A6 and A10 in the Appendix), LS, as well as RSWP strategies based on a Γ-factor detected with the crossover approach, always outperforms the corresponding strategy where factors are quantified through the simple approach. Furthermore, a Γ-factor computed using the crossovers approach over a formation period (f) of six or twelve months generated a higher return compared to optimization performed with the Δ-factor in 6 (5) set of parametrizations (h, s) out of 6 for the RSWP (LS) strategy.

To sum up, time series tools (such as moving averages) which allow us to clean irregular fluctuations from stock prices and to estimate the trend might lead to an improvement in terms of portfolio performance for Δ and Γ-based optimizations - not only in the past but also in today's financial market environment. In particular, the asymmetric simple moving average crossover allows us to increase the performance of Γ strategies.

4.2 Wavelet Transform and Portfolio Optimization

This section describes the performance of portfolios optimized on the basis of Δ and Γ-factors detected through the Maximum Overlap Discrete Wavelet Transform (MODWT) approach. The investigation is executed using daily and monthly data and it aims to test if the wavelet approach might lead to better results in terms of portfolio performance. In the following analyses, both factors are detected through the MODWT approach, i.e. momentum is defined as the first derivative of the logarithm of the stock price series and it is quantified by the magnitude of detail coefficients (d_j) of a MODWT performed by a Haar (Db1) mother wavelet while the second derivative (acceleration) is detected by the negative of detail coefficients ($(-1) * \tilde{d}_j$) of a MODWT performed by a Daubechies function with two vanishing moments (Db2).

The first part of this section determines the impact of implementing wavelet-based optimization strategies on the portfolio performance in the distant past (1984-2002); the investigation is executed using the Dow Jones Industrial Average (DJIA) stocks universe (data set). Thereafter, there is a short investigation to understand the influence of the level of resolution (j) on the rate of return of wavelet-based portfolios. Moreover, an additional analysis aims to implement a calibrated version of the MODWT approach to increase the accuracy of the quantification of both derivatives (i.e. of the parameters Δ and Γ) and check if there is also an improvement in portfolio performance. Finally, the wavelet-based portfolio optimization is briefly tested in today's financial environment, i.e. implementing a Relative Strength Weighted Portfolio strategy using the Standard and Poor 500 (S&P500) universe of stocks considering the recent past (2001-2014).

4.2.1 Dow Jones Industrial Average (1984-2002)

The wavelet-based portfolio optimization is first tested for the universe of stocks consisting in securities included in the DJIA; the investigation is executed on a daily (W) and a monthly (WM) basis (i.e. using daily or monthly stock prices (logarithm)). Since the start of 1984 until February 2002 a portfolio is set up each day (or month) according to Δ or Γ-factors detected using the MODWT approach and considering different levels of resolution (j), a delay in the investment (s) of one or six months as well as different portfolio holding periods (h). First, portfolios are constructed according to the Long-Short strategy. Thereafter, the Relative Strength Weighted Portfolio optimization is applied.

Table A15 in the Appendix indicates the annualized performance of Δ_{LS} (W) and Γ_{LS} (W) portfolios where the investment is computed on a daily basis and factors are extracted through the MODWT from the logarithm of the daily stock price series. According to previous papers, the momentum as well as the acceleration effect seem to be more evident over longer formation periods, such as for example between three and twelve months; the implementation of analysis which considers monthly data might improve the detection of factors and therefore the portfolio optimization. Indeed, a MODWT approach performed using monthly stock prices allows to detect Δ and Γ-factors over longer time-scales (at is explained in Table A2). Table A16 thus illustrates the results for the performance of Δ_{LS} (WM) and Γ_{LS} (WM) strategies, i.e. optimization that implements factor detected through the MODWT approach on a monthly basis.

As expected, Table A15 in the Appendix confirms that Δ_{LS} (W) strategy (performed with daily data) achieved on average a very low rate of return which is not even comparable to returns from standard momentum investing (i.e. returns from the Δ_{LS} (simple) strategy, see in Table A6 in the Appendix). However, as reported in Table A16 in the Appendix, a wavelet-based detection performed on a monthly time-scale (i.e. computing the derivatives from monthly stock price series) leads to a similar or even better performance than the ”simple” momentum investing strategy.

The following table (Table 5 in the following page) includes a comparison between the ”traditional” (simple) and the wavelet-based Δ_{LS} (WM) portfolios having a holding period of one month ($h = 1$). The long portfolio buys stocks from the top Δ-ranked quintile while the short portfolio sells stocks from the bottom Δ-ranked quintile. Equal (market-neutral) weights are applied for both, the long and the short portfolio. The total (long plus short) annualized portfolio performance in percentage terms (i.e. μ, σ) as well as the Sharpe Ratio (SR) are represented here below.

As is illustrated in Table 5, a Δ-factor detected through the wavelet approach at the resolution level four ($j = 4$) always improved the average annualized return for scenarios with no or a one month delay in time in the investment execution ($s = 0$ and $s = 1$) of a portfolio held for one month. However, for a delay in the investment of one month ($s = 1$) even factors detected at the resolution levels $j = 2$ and $j = 3$ generated an increased annualized performance. Moreover, Table A16 reports that for a holding period (h) of three months the wavelet-based strategy is more profitable only if the investment is not delayed ($s = 0$). For a longer holding period as ($h = 6$) there is just a slight improvement obtained through wavelet-based momentum detection when the investment is postponed by one month but for a holding period of one year, a Δ computed at a resolution level of $j = 3$ always improved performance in comparison to the standard momentum strategy (see Table A6 in the Appendix for the comparison). Momentum-based Long-Short portfolios constructed using a Δ-factor detected with the wavelet-based approach are statistically and significantly positive in 35 out of 36 scenarios (see Table A16 in the Appendix).

Annualized performance of $\Delta_{s,j(LS)}$ (WM) and $\Delta_{s,f(LS)}$ (simple)
DJIA (1984-2002)

			Δ_{LS} (simple)				Δ_{LS} (wavelet (WM))		
s	$f(j)$	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1 (1)	-13.7	24.91	-0.55	-11.73	-15.51	23.74	-0.65	-3.07
	3 (2)	15.75	28.37	0.56	10.35	13.63	29.45	0.46	1.9
	6 (3)	22.21	29.54	0.75	13.67	23.5	31.13	0. 76	2.98
	12 (4)	32.95	30.93	1.07	18.6	37.08	30.9	1.20	4.51
1	1 (1)	-3.42	25.97	-0.13	-2.66	-8.03	29.25	-0.27	-1.24
	3 (2)	17.86	29.09	0.61	11.32	25.99	30.99	0.84	3.27
	6 (3)	26.55	29.26	0.91	16.19	30.01	30.09	1	3.84
	12 (4)	37.89	30.8	1.23	21.05	41.19	30.8	1.34	4.94
6	1 (1)	-1.71	25.5	- 0.07	-1.33	-4.95	24.39	-0.20	-0.89
	3 (2)	24.32	28.19	0.86	15.34	20.84	27.76	0.75	2.96
	6 (3)	38.63	28.59	1.35	22.81	35.26	30.16	1.17	4.36
	12 (4)	28.12	29.75	0.95	16.58	27.56	29.85	0.92	3.54

Table 5: Comparison between the performance of ”traditional” $\Delta_{s,f(LS)}$ (simple) and wavelet-based $\Delta_{s,j(LS)}$ (WM) portfolio optimizations performed on the DJIA (1984-2002) for a one-month holding period. The first portfolio examined is constructed optimizing for Δ-factors detected with the simple approach while the second is a wavelet-based portfolio set up on a monthly basis (WM). Moreover, factors are detected for different formation periods f (expressed in months) or different resolution levels (j), which according to the time-scale conversion are approximately comparable. The actual investment might be delayed of s months after the portfolio construction. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

A similar result is also documented for Γ_{LS} (W) portfolios optimized through the MODWT approach: daily data do not lead to a general increment in annualized risk-adjusted profitability in comparison to the simple approach (see Table A15 in the Appendix and Table A10 in the Appendix for comparison). However, a Γ-factor detected on a monthly basis (see Table A16 in the Appendix) might positively influence the performance compared to the simple approach (see Table A6 in the Appendix) mostly for portfolios with short holding periods.

Therefore, Table 6 illustrates a comparison between the ”traditional” (simple) and the wavelet-based Γ_{LS} (WM) strategy considering a holding period of one month.

Annualized performance of $\Gamma_{s,j(LS)}$ (WM) and $\Gamma_{s,f(LS)}$ (simple)
DJIA (1984-2002)

			Γ_{LS} (simple)				Γ_{LS} (wavelet (WM))		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1 (1)	-9.17	25.45	-0.36	-7.51	-30.63	29.59	-1.04	-5.31
0	3 (2)	-5.48	26.22	-0.21	-4.28	-16.06	27.57	- 0.58	-2.75
	6(3)	-10.22	26.64	-0.38	-8.04	-5.7	22.95	-0.25	-1.11
	12 (4)	10.18	27.42	0.37	7.09	12.12	26.64	0.45	1.88
	1 (1)	-21.51	28.28	-0.76	-16.89	32.5	29.2	1.11	4.24
1	3 (2)	-5.15	25.82	-0.20	-4.07	12.45	28.92	0.43	1.77
	6 (3)	-5.52	26.08	-0.21	-4.32	17.94	28.07	0.64	2.57
	12 (4)	14.38	27.34	0.53	9.84	15.51	26.83	0.58	2.35
	1 (1)	-2.37	25.39	-0.09	-1.86	-31.14	30.08	-1.04	-5.25
6	3 (2)	-2.74	25.45	-0.11	-2.15	-10.28	27.17	-0.38	-1.71
	6 (3)	19.2	26.36	0.73	13.22	6.58	25.84	0.25	1.06
	12 (4)	11.69	26.91	0.43	8.13	11.07	27.7	0.40	1.64

Table 6: Comparison between the performance of "traditional" $\Gamma_{s,f(LS)}$ (simple) and wavelet-based $\Gamma_{s,j(LS)}$ (WM) portfolio optimizations performed on the Dow Jones Industrial Average (1984-2002) for a one-month holding period. The first portfolio examined is constructed optimizing for Γ-factors detected with the simple approach while the second is a wavelet-based portfolio set up on a monthly basis (WM). Moreover, factors are detected for different formation periods f (expressed in months) or different resolution levels (j), which according to the time-scale conversion are approximately comparable. The actual investment might be delayed of s months after the portfolio construction. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Indeed, Table 6 shows that the return of the Γ_{LS} (WM) portfolio with a holding period of one month outperforms "traditional" Γ_{LS} portfolio optimization, when the investment has no or a one month delay in time; the latter allows us to obtain a statistically significant risk-adjusted return (SR) of 1.1. Moreover, Table A16 shows that for a longer holding period of three months the wavelet-based strategy only outperforms the classical one for $s = 0$ and $s = 1$ while for a six-month and one-year holding period it is not more profitable.

Table A17 and A18 in the Appendix shows the performance of Relative Strength Weighted Portfolios; the former is an analysis executed using daily data while the latter implements monthly stock prices. Results in Table A17 (daily data) are quite similar to using the Long-Short strategy based on the wavelet approach: there is not a significant scenario where the wavelet-based (daily) RSWP (W) strategy generated better performance than the simple approach (see Table A10 in the Appendix for a comparison). However, it is interesting to see that at a resolution level of $j = 4$, the annualized performance is quite similar to the return generated by the "traditional" (simple) portfolio constructed using a formation period (f) of one month. This is a logical consequence, since the MODWT performed through daily data is able to detect the momentum as well as the acceleration factor from a time-scale of two-days (i.e. a change in price (or return) within two days) at a resolution level of $j = 1$ until a time-scale of 16 days at a resolution level of $j = 4$ (i.e. first and second order price changes within 8 and 16 days), i.e. it detects factors over very short formation periods.

The very low or even negative performance of the MODWT approach computed with daily prices indicates that both the momentum and the acceleration effect do not exist at a very short scale. For this reason, a formation period of two weeks or one months is too short to detect both factors adequately, this was also confirmed in Table 2 and Table A9 in the Appendix, where the formation period was one week.

Table A18 illustrates wavelet-based RSWP performances obtained using monthly data. The outcome for Δ_{RSWP} (WM) portfolio optimization is much similar to what is documented according to the classical momentum (see Table A10 in the Appendix), however the wavelet-based approach slightly outperforms the simple approach only in three set of parametrization (h, s) out of nine. According to Table A18 in the Appendix, the Γ_{RSWP} (WM) strategy never outperforms the corresponding "traditional" Γ_{RSWP} (simple) strategy, except for one scenario, which was already document using the Long-Short strategy: the RSWP portfolio optimized for a Γ-factor detected at the resolution level $j = 1$ and having one month holding period after a delay in the investment of one month, i.e. $\Gamma_{1,1(RSWP)}$ (WM) with $h = 1$.

This short investigation revealed that the wavelet-based portfolio performance is higher when the factor detection is performed on a monthly basis (i.e. from monthly stock prices). Moreover, the MODWT approach only leads to significant improvements for investments with a short holding period ($h = 1$) for both factors. Otherwise, results are very similar to the "traditional" momentum and acceleration investing (i.e. the simple approach).

4.2.2 The Influence of the Resolution Level (j) in Portfolio Optimization

In the previous section, we observed that the resolution level (j) plays the same role as the formation period (f) in "traditional" investments optimized with factor detected applying the simple approach. Indeed, using daily data, even a resolution level of $j = 4$ (i.e. which considers first and second order price changes over a time-scale between 8 and 16 days) was not adequate to capture the momentum as well as the acceleration factor. However, optimizations performed with monthly data generated similar (or in some cases) even better results as in optimizations based on the simple detection approach. This analysis is performed implementing Relative Strength Weighted portfolios.

Therefore, here there is a short investigation which aims to emphasize the role of the parameter j (i.e. the resolution level) and to demonstrate that even using daily data it is possible to achieve reasonable portfolio results by increasing the resolution level (see Table A19 in the Appendix). For example, a resolution level of $j = 8$, i.e. a time-scale between 128 days (or approximately six months) and 256 days (one year), generated a quite similar return as in the simple approach or as in wavelet-based optimizations performed on a monthly basis. Indeed, Table A19 in the Appendix illustrates that the performance of both factors increases with the resolution level (j) on a daily basis. For a holding period of one month, despite the fact that portfolio performance generated by a level $j = 8$ is quite significant and very similar to the simple approach, the (daily) wavelet-based detection does not lead to higher results (compared to Table A10 in the Appendix) for both Δ^D and Γ^D portfolios. However, if we consider a holding period of six months, a Δ^D-factor detected on a daily time-scale at a resolution level $j = 8$ improved the performance in comparison to the "traditional" simple portfolio when there is no or a one month delay in time ($s = 0$ and $s = 1$). Moreover, for the same holding period, the Γ^D-factor detected at the resolution level ($j = 8$) increases the performance of the portfolio delayed by six months ($s = 6$), which achieves an annualized risk-adjusted profitability (SR) of 0.52 outperforming the simple approach.

Furthermore, this section also reconfirms that momentum (but also acceleration) is a financial anomaly which can be documented through cumulative past returns of securities over a window (or a formation period f) of three-twelve months (or by applying resolution levels of j equal to 2, 3 and 4 to monthly data). Indeed, as can be seen in Table A20 in the Appendix (monthly data), further resolution levels generated a lower or clearly negative performance for the momentum factor, i.e. a j equal to 5 or 6 (i.e. $j = 6$ defines a time-scale between 2 and a half years and 5 and a half years) generated lower performances while a portfolio computed using a delta factor detected at the resolution level $j = 7$ always performs negatively. The Γ^D-factor is more sensitive and it performs negatively in each scenario where j is equal to 6 or 7.

However, it is interesting to observe (Table A20 in the Appendix) that if we construct Γ^D portfolios, using a Γ^D-factor detected through the MODWT (monthly) at a resolution level j equal to 5 (i.e. a time-scale between 16 and 32 months), performance is always much better compared to what we obtain with a formation period of one year ($f = 12$) in the simple approach (see Table A10 in the Appendix) and also compared to a level of $j = 4$, except for the scenario of a one month holding period and a one month delay in the investment where the fourth resolution level generated an higher return (see Table A18 in Appendix). This might be an indication that the acceleration effect is more evident in the medium-term, i.e. between one and 2 and a half years. For this reason, computing the Γ^D-factor using larger formation periods or through a MODWT approach at higher resolutions levels, seems to give rise to an increment in portfolio performance.

4.2.3 Calibration of the Wavelet Approach

The application of the MODWT approach on pure signals (see Section 2.3.2) to compute the first and the second derivative revealed that this approach should be calibrated because both derivatives are approximately similar as using the traditional differentiation approach but lagged in time. As is visible in Figure A7-A12 in the Appendix, the lag is similar for each of the three pure signals; therefore, it has been recorded from pure signals to calibrate factors used in the portfolio optimization.

Table A21 in the Appendix illustrates the performance of the calibrated wavelet-based (WMC) RSWP strategy, i.e. executed by optimizing for calibrated Δ and Γ-factors. As previously, the data set investigated is the DJIA for the distant past, i.e. between 1984 and 2002 (period of time when the momentum effect was largely documented). The investigation is performed with monthly data.

If we look at the performance of RSWP (WMC) strategy (Table A22) for a holding period (h) of one month or six months and we compare it with the non-calibrated version (see Table A18 in the Appendix), we can observe that calibrated factors might improve portfolio performance. More precisely, strategies based on a calibrated momentum (Δ_{RSWP} (WMC)) achieved a greater annualized performance compared to non-calibrated Δ_{RSWP} (WM) portfolio optimizations when there is no or a one month delay in the investment, while it is never profitable for a delay of six months. A similar conclusion is shown by comparing the Δ_{RSWP} (WMC) strategy to the "traditional" Δ_{RSWP} (simple) strategy (see Table A10 in the Appendix), i.e. a calibrated MODWT might increase the annualized return and decrease the volatility; this leads to an increment in risk-adjusted profitability for the momentum strategy.

$\Gamma_{RSWP(WMC)}$ strategies always generated better performances than classical (simple) or non-calibrated optimizations for a holding period of six months. While for a holding period of one month they are never more profitable than "traditional" or non-calibrated investments.

To conclude, the following table (Table 7) aims to compare the annualized Share Ratio of Δ_{RSWP} and Γ_{RSWP} strategies optimized for simple or wavelet-based detected factors. The first portfolio is the "classical" RSWP (simple) strategy where factors are computed on a daily basis using different formation periods. Moreover, a second portfolio is constructed on a monthly basis by optimizing for wavelet-based detected factors (WM). The last portfolio is constructed using factors quantified with the calibrated wavelet approach form monthly stock price series (WMC).

Comparison of annualized Sharpe Ratios (SR): simple and wavelet-based detected factors
DJIA (1984-2002)

			Δ			Γ	
s	*j (f)*	SR simple	SR wavelet (WM)	SR wavelet (WMC)	SR simple	SR wavelet (WM)	SR wavelet (WMC)
				$h = 1$			
	1 (1)	-0.67	-0.79	1.61	-0.41	-0.94	-0.91
0	2 (3)	0.64	0.41	1.83	-0.25	-0.67	-0.32
	3 (6)	0.9	0.81	1.64	-0.35	-0.24	0.21
	4 (12)	1.29	1.2	1.66	0.53	0.43	0.3
	1 (1)	-0.06	-0.3	-0.17	-0.78	1.17	-0.21
1	2 (3)	0.88	1.1	1.89	-0.11	0.57	1.07
	3 (6)	1.03	0.87	1.36	-0.21	0.55	1.01
	4 (12)	1.34	1.32	1.73	0.55	0.53	0.51
	1 (1)	-0.02	-0.11	0.39	0.07	-0.98	-0.98
6	2 (3)	1	0.65	0.83	-0.2	-0.51	-0.32
	3 (6)	1.42	1.28	0.85	0.77	0.26	0.13
	4 (12)	1.14	1.01	0.97	0.58	0.41	0.38
				$h = 6$			
	1 (1)	0.37	0.32	0.52	-0.01	0.02	0.06
0	2 (3)	0.76	0.69	0.85	-0.07	-0.09	0.32
	3 (6)	0.93	0.92	0.96	0.01	0.43	0.55
	4 (12)	1.04	1.05	1.22	0.52	0.47	0.49
	1 (1)	0.42	0.39	0.39	0.03	0.04	0.07
1	2 (3)	0.8	0.73	0.73	-0.06	-0.07	0.34
	3 (6)	1	1	0.86	0.19	0.51	0.53
	4 (12)	1.02	1.02	1.14	0.52	0.48	0.53
	1 (1)	0.51	0.47	0.39	0.03	0.07	0.07
6	2 (3)	0.94	0.85	0.73	0.2	0.13	0
	3 (6)	0.98	0.98	0.92	0.46	0.11	0.15
	4 (12)	0.83	0.78	0.77	0.51	0.38	0.51

Table 7: The figure shows the average annualized Sharpe Ratio of three different Δ_{RSWP} and three different Γ_{RSWP} strategies considering holding periods (h) of one and six months. The investigation is executed through daily or monthly split and dividend-adjusted log-returns of securities included in the DJIA (1984-2002). The first portfolio examined is constructed optimizing for discretized factors detected with the simple approach (Δ^D and Γ^D). Moreover, there are two wavelet-based portfolios set up on a monthly basis. The first portfolio ("wavelet (WM)") is constructed according to Δ and Γ-factors computed through the MODWT approach while the second (WMC) implements a calibrated version of those parameters. Factors are detected for different formation periods f (expressed in months) or different resolution levels (j), which according to the time-scale conversion are approximately comparable. The actual investment might be delayed of s months after the portfolio construction. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Here, it is clearly visible that the calibrated wavelet-based Δ_{RSWP} (WMC) strategy outperforms the "normal" wavelet-based (WM) as well as the traditional (simple) strategy when there is no or a one month delay in time for a holding period of one month and it allows to generate very high risk-adjusted returns. Furthermore, the calibrated Γ_{RSWP} (WMC) portfolio optimization is profitable for the scenario of a one month holding period and a delay in the investment of one month or for six months holding period and no or a delay in the investment of one month. Otherwise, it generated a similar or a lower annualized performance compared to the simple approach.

Nevertheless, as was documented previously and reported in Table A20 in the Appendix, the RSWP strategy optimized for a non-calibrated Γ-factor detected through a Maximum Overlap Discrete Transform at the resolution level $j = 5$ generated a Sharpe Ratio of 0.6 ($s = 0$), 0.63 ($s = 1$) and 0.68 ($s = 6$) for portfolios having a holding period of one month and 0.54 ($s = 0$), 0.55 ($s = 1$) and 0.51 ($s = 6$) for a holding period of six months. Hence, an acceleration factor detected through the non-calibrated MODWT approach at the level $j = 5$ slightly improved the risk-adjusted returns compared to "traditional" Γ_{RSWP} (simple) strategies in 5 set of parametrizations (h, s) out of 6 or it generated a similar or more frequently a higher performance than wavelet-based calibrated portfolio optimization (see Table A21), except for the scenario of a one month holding period and a one month delay in time where the Sharpe Ratio of the calibrated Γ_{RSWP} (WMC) portfolio is 1.07 for a resolution level $j = 2$.

4.2.4 Wavelet-based Portfolio optimization - Today

This short section aims to determine if the calibrated version of the Maximum Overlap Discrete Wavelet Transform might also improve the performance of Δ and Γ portfolio in today's financial market environment. For this reason, the calibrated wavelet-based RSWP strategy is back-tested using the S&P500 stocks universe, considering the recent past (2001-2014).

As is shown in Table A22 in the Appendix and as was already documented in the distant past, the calibrated wavelet-based Δ_{RSWP} (WMC) outperforms the "traditional" Δ_{RSWP} (simple) strategy when there is no or a one month delay in the investment for holding periods of one and six months (i.e. it improved the results in 4 out of 6 scenarios); moreover in these scenarios a Δ-factor detected at the resolution level (j) equal to two or three is always more profitable.

The conclusion regarding Γ_{RSWP} (WMC) portfolio optimization is the same; contrary to the Δ strategy, the Γ_{RSWP} (WMC) strategy also generated a higher risk-adjusted return than the "traditional" Γ_{RSWP} (simple) optimization in the scenario of six month delay in the investment for a holding period of one month (i.e it leads to better results in 5 out of 6 scenarios). The Γ_{RSWP} (WMC) strategy beats the calibrated Δ_{RSWP} (WMC) optimization in 4 out of 6 scenarios. The best risk-adjusted return for the Δ_{RSWP} (WMC) strategy is about 0.68 and it can be seen in the portfolio held for one month where the investment is not delayed and at a resolution level (j) equal to two. The best SR of calibrated Γ portfolios is about 0.50 and is generated by a portfolio held for one month after a delay of six months in the investment execution and the portfolio is optimized for factors detected at the resolution level $j = 4$. Both Sharpe Ratios are statistically significant. The calibrated Γ_{RSWP} (WMC) strategy generated a positive annualized return in 16 out of 24 scenarios, 6 of them are statistically significant while the Δ_{RSWP} (WMC) optimization in 16 out of 24 scenarios, 4 statistically significant.

4.3 Hybrid Δ/Γ Portfolio Optimization Strategy

As was demonstrated in previous analyses performed in this study, in today's financial environment (i.e. in the recent past) portfolios based on the momentum or the acceleration factor return lower performances (sometimes even negative) compared to the distant past (i.e. the last two decades of the 20th century). Therefore, it seems not to be possible to gain superior returns by investing according to the momentum or to the newly discovered effect which complements momentum, i.e. acceleration. A possible explanation is that nowadays both factors might be not adequate to track the performance (or the trend) in stock prices, because the persistence in returns is shorter and the trend (the momentum) seems to reverse more quickly. A deeper explanation is given in the discussion part in the next chapter.

The Δ/Γ strategy which has been newly developed in this study considers both factors simultaneously and it aims to achieve benefits by investing in upward or downward trending stocks before the trend reverses. More precisely, this strategy consists in buying stocks (go long) not only when the cumulative past return (the momentum) is positive but only if those stocks show increasingly positive past returns (i.e. upward accelerating stock price). Moreover, the short portfolio aims to tracks stock which performed badly in the past and whose price is falling at an increasing rate (i.e. downward accelerating stock price).

The Δ/Γ portfolio optimization strategy is first implemented using a Δ and a Γ-factor detected with the simple approach. Moreover, additional Δ/Γ portfolios are optimized through non-calibrated as well as calibrated factors detected with the Maximum Overlap Discrete Wavelet Transform approach on a daily or on a monthly basis.

This strategy is tested in two different time periods: between 1984 and 2002 (distant past) and between 2001 and 2014 (recent past).

Moreover, a short study aims to investigate potential sources of the success (in terms of portfolio performance) of the newly developed Δ/Γ optimization, i.e. it aims to enhance differences between the hybrid and the Long-Short strategy in the number of stocks over time included in the long and short sub-portfolios. Finally there is a short comparison of the Information Ratio (IR) resulting from the Δ/Γ strategy as well as from the best LS and RSWP simple and wavelet-based strategies.

4.3.1 Hybrid Δ/Γ Strategy: Simple Approach

Δ/Γ Strategy in the Distant Past (1984-2002)

Table 8 shows the performance of Δ/Γ (simple) portfolios having a holding period of one and six months where factors are detected with the simple approach using different formation periods (f). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative

$\Gamma(f)$ at the time t. Moreover, the investment might be delayed by one or six months after the portfolio construction. Two kind of weighting rule are applied: the first portfolio is based on equal weights (EW) whilst the second applies relative "Gamma" weights (GW or Γ-weights), which are computed according to the relative magnitude of the acceleration factor across all stocks in the corresponding (long or short) portfolio. Moreover, weights of the long and the short portfolio have been standardized in order to obtain market-neutral weights. A complete table including also holding periods of three and twelve months is available in the attachment (Table 23 in the Appendix).

Annualized performance of Δ/Γ (simple)
DJIA (1984-2002)

		Δ/Γ (EW)				Δ/Γ (GW)			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
$h=1$									
	1	33.93	24.35	1.39	24.24	27.91	28.13	0.99	17.65
0	3	36.91	25.13	1.47	25.28	29.73	28.57	1.04	18.38
	6	32.56	26.85	1.21	21.2	31.63	30.63	1.03	18.11
	12	37.68	29.03	1.3	22.28	35.44	34.22	1.04	17.92
	1	43.91	24	1.83	30.66	53.77	27.78	1.94	31.4
1	3	47.81	22.81	2.1	34.67	52.89	26.98	1.96	31.9
	6	60.54	24.32	2.49	39.53	69.24	28.04	2.47	38.2
	12	77.71	25.16	3.09	46.62	88.03	31.01	2.84	41.63
	1	29.32	24.24	1.21	21.11	32.45	28.24	1.15	19.83
6	3	27.85	23.33	1.19	20.95	29.01	26.98	1.08	18.79
	6	30.86	25.11	1.23	21.33	40.23	29.48	1.36	22.9
	12	29.44	25.99	1.13	19.77	39.48	32.45	1.22	20.48
$h=6$									
	1	50.48	31.91	1.58	68.71	63.67	43.5	1.46	62.09
0	3	47.15	29.49	1.6	69.87	51.18	33.75	1.52	65.76
	6	51.59	31.51	1.64	70.97	58.27	37.46	1.56	66.61
	12	57.48	31.64	1.82	77.9	65.86	40.82	1.61	68.19
	1	49.08	33.39	1.47	63.84	64.13	45.9	1.4	59.1
1	3	44.85	29.61	1.52	66.32	50.22	34.87	1.44	62.43
	6	50.7	32.61	1.56	67.34	59.18	38.94	1.52	64.82
	12	55.23	31.62	1.75	75.02	65.96	41.46	1.59	67.08
	1	4.98	31.13	0.16	7.54	15.92	40.58	0.39	18.02
6	3	6.11	31.94	0.19	8.99	11.92	39.82	0.3	13.87
	6	10.19	34.2	0.3	13.87	17.97	39.45	0.46	20.83
	12	10.47	33.51	0.31	14.53	20.96	41.35	0.51	23.02

Table 8: Performance of Δ/Γ (simple) portfolio optimizations performed using components of the DJIA (1984-2002) for a holding period of one and six months. This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

It is clear (see Table 8) that this strategy worked very well in the distant past, indeed the average annualized return is positive in 24 out of 24 scenarios and all results are statistically significant. If we take into consideration additional holding periods of three and twelve months (see Table 23 in the Appendix), the average annualized return is positive in 44 (46) out of 48 scenarios for the equal-weighted (GW) strategy. Negative performances are generated by a holding period (h) of three months and a delay in the investment (s) of six months. Moreover, due to a decrease in the return and an increase in volatility, a holding period of twelve months generated the lower risk-adjusted performance for both the EW and the GW strategies. In general, the equal-weighted Δ/Γ (simple) portfolio has a lower volatility and achieves a maximal average annualized return of 77.71% and an annualized risk-adjusted return (SR) of 3.09 using a formation period (f) of 12 months, holding the portfolio for one month with a delay in the investment (s) of one month while the same portfolio constructed using Γ-weights gave rise to a greater annualized return of 88.03% but due to an increase in volatility the Share Ratio (SR) is 2.84. EW portfolios outperform GW portfolios in 17 out of 24 scenarios but never when the holding period is six months ($h = 6$) and the investment is delayed by six months ($s = 6$).

On average, considering shorter holding periods of one or three months, a delay in the investment (s) of one or six months is profitable but with an increase in the holding period (as for example $h = 6$), a delay decreases risk-adjusted profitability. This means that Δ/Γ (simple) portfolios are able to capture an increase (decrease) in the stock price that in the long-term seems to disappear. For this reason, holding the portfolio for too long or delaying the investment might adversely influence performance.

Compared to previous Δ_{LS} and Γ_{LS} (simple) strategies (see Table A6 in the Appendix), the equal-weighted (EW) Δ/Γ (simple) strategy leads to profitable returns and it outperforms the Δ_{LS} (simple) portfolio optimization in 32 out of 48 scenarios, all of them statistically significant. The Δ_{LS} (simple) strategy beats the (EW) Δ/Γ (simple) strategy when the holding period is long and there is an additional delay in the investment. Moreover, the best $\Delta_{6,6(LS)}$ (simple) portfolio held for one month generated a greater averaged annualized return compared $(\Delta/\Gamma)_{6,6}$ (simple) with a one month holding period. Similar results are shown by the comparison of Γ-weighted (GW) Δ/Γ (simple) portfolios with the Δ_{LS}(simple) strategy. However, using Γ-weights it is possible to achieve a higher Sharpe of 1.36 for the $(\Delta/\Gamma)_{6,6}$ portfolio. Furthermore, Δ/Γ (simple) returns are always larger than returns of the Γ_{LS} (simple) strategy, except for the scenario where the newly developed strategy generated negative returns (i.e. when $h = 3$ and $s = 6$).

Furthermore, if we compare the performance of the new hybrid strategy to Δ^D_{RSWP} (simple) or Γ^D_{RSWP} (simple) optimizations (see Table A10 in the Appendix), we can observe a similar pattern: both (EW and GW) Δ/Γ (simple) strategies significantly outperform the Δ^D_{RSWP} (simple) strategy in 34 out of 48 scenarios. An underperformance of the Δ/Γ (simple) strategy is generated by longer holding periods or a delay in the investment. Moreover, the GW $(\Delta/\Gamma)_{6,6}$ (simple) portfolio underperform the best Δ^D_{RSWP} (simple) strategy, i.e. the $\Delta^D_{6,6(RSWP)}$ (simple) portfolio. Conclusions regarding the comparison between the hybrid strategy and the Γ_{RSWP} (simple) strategy are the same as above.

Δ/Γ Strategy in the Recent Past (2001-2016)

Table 9 in the next page illustrates the average annualized performance of Δ/Γ (simple) equal-weighted (EW) and Γ-weighted (GW) portfolios with a one and a three month holding period built considering the Dow Jones Industrial Average stocks universe in the recent past (2001-2016). Table A24 in the Appendix gives a complete overview of an additional holding period of six months.

Annualized performance of Δ/Γ (simple)
DJIA (2001-2016)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-37.3	29.81	-1.25	-28.06	-44.89	35.78	-1.25	-29.67
	3	-34.1	27.83	-1.23	-26.9	-42.22	32.8	-1.29	-29.86
	6	-41.2	29.46	-1.4	-32.21	-47.46	34.93	-1.36	-32.77
	12	-50.32	31.88	-1.58	-38.93	-53.53	37.97	-1.41	-35.71
1	1	114.2	24.71	4.62	57.98	124.3	28.26	4.4	53.87
	3	117.2	23.68	4.95	61.63	122.3	28.13	4.35	53.5
	6	138.4	25.6	5.41	64.13	151.1	29.86	5.06	58.4
	12	135.5	24.35	5.56	66.42	158.4	29.26	5.41	61.49
6	1	0.98	20.99	0.05	0.84	8.89	24.16	0.37	6.36
	3	-1.25	20.43	-0.06	-1.1	3.59	23.64	0.15	2.69
	6	-1.25	19.29	-0.07	-1.17	5.98	21.81	0.27	4.8
	12	3.37	18.41	0.18	3.24	5.31	21.33	0.25	4.37
					$h = 3$				
0	1	60.55	21.17	2.86	74.7	57.54	25.15	2.29	60.22
	3	62.76	20.68	3.04	78.83	58.61	24.64	2.38	62.46
	6	65.55	25.6	2.56	66.04	65.16	31.5	2.07	53.4
	12	58.61	20.86	2.81	73.76	61.5	26.33	2.34	60.88
1	1	24.29	20.48	1.19	34.25	24.43	23.71	1.03	29.74
	3	24.74	19.34	1.28	36.88	23.97	23.25	1.03	29.8
	6	29.42	23.63	1.25	35.38	31.65	29.01	1.09	30.8
	12	25.44	19.32	1.32	37.9	28.98	24.6	1.18	33.53
6	1	56.39	23.33	2.42	62.82	66.58	26.07	2.55	64.64
	3	57.03	23.82	2.39	62.1	67.3	26.79	2.51	63.5
	6	55.21	19.33	2.86	74.46	68.24	22.04	3.1	78.08
	12	59.8	19.75	3.03	77.97	69.08	22.95	3.01	75.74

Table 9: Performance of Δ/Γ (simple) portfolio optimizations performed using components of the DJIA (2001-2016) for a holding period of one and six months. This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

It interesting to see that this strategy also leads to positive returns in today's financial market environment (recent past) and it generated significantly positive averaged annualized returns in 18 out of 24 scenarios for the equal-weighted Δ/Γ (simple) strategy and in 20 out of 24 if Γ-weights (GW) are applied. Negative returns are generated mostly by a holding period of one month and no delay in the investment. However, delaying the investment by one month ($s = 1$) and holding the portfolio for one month ($h = 1$) makes it possible to achieve very high Share Ratios which vary from about 4.62 to about 5.56 (with f=12) for the EW Δ/Γ (simple) strategy and from about 4.34 to about 5.41 for the GW Δ/Γ (simple) strategy. Moreover, a holding period of three months also generated very good performances and it always outperforms portfolios with holding periods of six months (see Table A24 in the Appendix). The EW strategy outperforms the GW Δ/Γ (simple) strategy in 23 out of 36 scenarios.

If we compare the results of the Δ/Γ (simple) strategy to Δ_{LS} or Γ_{LS} (simple) portfolio optimizations (see Table A3 in the Appendix) as well as to the Δ_{RSWP} or Γ_{RSWP} (simple) strategies (see Table A8 in the Appendix), we can observe that the hybrid strategy is always much more profitable than both previous optimizations, except for the scenario where the hybrid strategy is performing negatively. More precisely EW and GW Δ/Γ (simple) portfolios outperform Δ_{LS} or Γ_{LS} (simple) or Δ_{RSWP} or Γ_{RSWP} (simple) strategies in 30 (EW) or 32 (GW) out of 36 scenarios. Moreover, the hybrid strategy also performed better than the long and short sub-portfolio of the LS (simple) strategy observed separately (see Table A4 and A5 in the Appendix).

Table A25 in the Appendix shows the performance of Δ/Γ (simple) portfolios constructed using the data set which includes the components of the Standard and Poor 500 in the recent past (2001-2014). We can observe that applying this strategy to a larger market, there is a decrease in the annualized portfolio volatility as well as in the number of the negative returns (which are generated only by the scenario $h = 1$ and $s = 0$). Nevertheless, there is also a reduction in the magnitude of positive returns but despite that the strategy remains very profitable and allows us to achieve average annualized returns between 54% and 75% (i.e. SR of about 2.04 and 2.75) investing for one month ($h = 1$) and delaying the investment by one month ($s = 1$), using "relative" Γ-weights. Moreover, considering a larger market the GW strategy seems to be more profitable than the EW Δ/Γ (simple) strategy: indeed it outperforms the equal-weighted investment in 28 out of 36 scenarios. As before, holding the stocks too long ($h = 6$) has a negative impact on performance.

As was shown previously, the Δ/Γ (simple) strategy outperforms previous Δ_{LS} and Γ_{LS} (simple) and Δ_{RSWP} and Γ_{RSWP} (simple) strategies in each scenario, but not in the scenario where the hybrid strategy generated negative returns (see Table A11 and A12 in the Appendix for a comparison).

To sum up, the Δ/Γ (simple) strategy seems to be significantly profitable in today's market environment. A holding period of three months seems to be adequate and always leads to high positive annualized returns. However, investing with one month delay ($s = 1$) and holding the portfolio for one month ($h = 1$) allows us to achieve the maximum performances, i.e. very high Sharpe Ratio which varies from a minimum of 2.76 (investing using the S&P500 stocks universe) to a maximum of 5.41 (for the DJIA stocks universe) using a formation period (f) of one year. Moreover, we can see that holding the stock for too long (six or twelve months) is not profitable in the Δ/Γ (simple) strategy. This strategy also generated higher returns in the distant past, but the best performance arose in the recent past.

4.3.2 Hybrid Δ/Γ Strategy: Wavelet Approach

This section implements a series of EW and GW Δ/Γ strategies which are optimized for factors detected through the Maximum Overlap Discrete Wavelet approach. For this analysis, only the recent past is considered, i.e. investment are back-tested in the period of time between 2001 and 2016.

First, the investment strategy is tested for the DJIA stocks universe and considering Δ and Γ-factors detected through the wavelet approach on a daily (W) or on a monthly (WM) basis (i.e. applying the wavelet transform on daily or monthly stock price series). Moreover, calibrated daily (WC) and monthly (WMC) factors are also implemented. Afterwards, the hybrid strategy is also optimized by considering the S&P500 stocks universe for non-calibrated and calibrated factors.

Dow Jones Industrial Average (2001-2016)

First, the investment strategy is tested for the DJIA stock universe.

Tables A26-A29 in the Appendix show the average annualized performance of equal-weighted (EW) and Γ-weighted (GW) Δ/Γ portfolios constructed using the Dow Jones Industrial Average stock universe and considering the period of time between 2001 and 2016 (recent past). Tables A26 and A27 in the Appendix are optimized for factors detected from daily stock price series while Tables A28 and A29 implement factors detected on a monthly basis (from monthly price series). Furthermore, Tables A26 and A28 use a non-calibrated Δ and a Γ-factor while Tables A27 and A29 are optimized for calibrated factors.

Irrespective of the calibration or the time-scale of detection (daily or monthly), EW and GW Δ/Γ portfolios display negative returns for a holding period (h) of one month and no or six months delay in the investment. Moreover, very low returns are generated for a "long" holding period of six months ($h = 6$) in comparison to "shorter" holding periods of one or three months. As was shown previously, by implementing the Δ/Γ (simple) strategy a holding period of three months always leads to positive averaged annualized returns, which are relatively high for no or a delay of six months in the investment; however, the best risk-adjusted performance was investing by using the Δ/Γ (simple) strategy for a period of one month ($h = 1$) but delaying the investment by one month ($s = 1$) after portfolio construction. Furthermore, equal-weighted hybrid strategies outperform the relative-weighted (Γ-weighted) strategies in more than half the scenarios; however when factors are detected on a monthly basis, the GW portfolios might lead to higher SR compared to EW portfolios.

Moreover, if we consider only the "best" scenarios, i.e. a one month holding period with one month delay in the investment ($h = 1$ and $s = 1$) and three months holding periods with no or six months delay ($h = 3$ with $s = 0$ or $s = 6$), there is not a clear pattern about the influence of the resolution level j to the portfolio performance. In general higher resolution levels (j) such as for example $j = 4$ and $j = 5$ seem to improve the risk-adjusted return; however, sometimes the highest Sharpe Ratio is obtained using a resolution level of $j = 3$.

Furthermore, comparing Table A26 with Table A28 in the Appendix, we can observe that on average detecting factors on a monthly time-scale (using monthly stock price series) improves the performance of the portfolios compared to a daily time-scale and an additional calibration of the detected factors (see Table 29 in the Appendix) might further raise portfolio performance. Indeed, considering a holding period of three months, Δ/Γ (WMC) portfolios optimized for calibrated factors detected on a monthly time-scale generated higher risk-adjusted returns compared to non-calibrated Δ/Γ (WM) portfolio in 9 out of 15 scenarios; if we considers only the "best" scenario, i.e. a one month holding period and a one month delay in the investment execution, the calibrated version generated an higher risk-adjusted return in comparison to the non-calibrated strategy in 4 out of 5 cases.

More precisely, if we look only at the previously defined "best" parametrizations ($h = 1, s = 1$), a Δ/Γ (W) strategy, i.e. a portfolio optimized with not-calibrated factors detected on a daily basis, achieved the maximal annualized Sharpe Ratio of about 4.47 for a resolution level $j = 3$ using equal weights while the same portfolio optimized for calibrated factors detected at the same resolution level improves the Sharpe Ratio to 4.4. Furthermore, in the best scenario, the risk-adjusted profitability generated might further rise if we construct a Δ/Γ (WM) portfolio (optimized for non-calibrated factors detected on a monthly time-scale), i.e. the annualized Sharpe Ratio reached the value of 5.26 for the Γ-weighted portfolio (GW) when factors are detected at the resolution level $j = 5$. Finally, implementing calibrated factors detected on a monthly basis at the level $j = 5$, the (GW) Δ/Γ (WMC) portfolio generated the overall higher risk-adjusted return of 5.76 in the best scenario.

Comparing Δ/Γ (simple) portfolios (see Table A24 in the Appendix) to wavelet-based Δ/Γ portfolios (see Table A27-A29 in the Appendix), it appears that factors detected using a wavelet transform performed on a daily time-scale do not improve the portfolio performance compared to simple detected factors. However, using monthly time series as input variables for the wavelet transform, it is possible to reach portfolio performances that are more similar to the performance of the Δ/Γ (simple) portfolio and if the factors are calibrated there is a significant improvement in the portfolio return compared to the simple approach (see Table A29 in Appendix). Indeed, the calibrated (GW) Δ/Γ (WMC) portfolio reached a Sharpe Ratio of about 5.76 in the "best" scenario, which is higher compared to the risk-adjusted return of the (EW) Δ/Γ (simple) portfolio (SR=5.56); moreover, the wavelet-based strategy optimized for calibrated factors detected at resolution levels of $j = 3$ and $j = 4$ increased the risk-adjusted profitability not only in the best scenario but also for other parametrizations compared to the simple approach, except for a holding period of three months with a delay in the investment of six months, where the strategy based on the simple approach outperforms the calibrated wavelet-based optimization, i.e. the Δ/Γ (WMC) portfolios.

Standard and Poor 500 (2001-2014)

Tables A30 and A31 report the performance of equal-weighted (EW) and Γ-weighted (GW) Δ/Γ strategies optimized using the data set including stocks from Standard and Poor in the period of time between 2001 and 2014. Portfolios have a holding period of one or three months and factors are detected through the MODWT approach. Moreover, Table A30 shows the results of portfolios optimized for non-calibrated factors while Table A31 shows the performance of wavelet-based calibrated Δ/Γ (WMC) strategy.

Performances are very similar as previously, i.e. as for Δ/Γ strategies constructed using the DJIA stocks universe. More precisely, the average annualized return is fairly negative if the investment is only held for one month and is not delayed in time. Moreover, lower or even negative annualized returns also appear if the investment is delayed by six months and the holding period is very short ($h = 1$). However, as before, the best performance is given by a one-month holding period and a delay in the investment of one month: the portfolio implementing non-calibrated factors reached the maximal Sharpe Ratio of about 2.72 (in the GW portfolio) while the calibrated portfolio (GW) generated an annualized return of about 2.88. Considering a three-month holding period, factors detected at the resolution level $j = 4$ generated improved performance compared to other resolution levels; moreover, on average calibrated portfolios show a higher annualized risk-adjusted performance. Finally, comparing wavelet-based Δ/Γ strategies to Δ_{LS} and Γ_{LS} (simple) or Δ_{RSWP} or Γ_{RSWP} (simple) strategies (see Table A11 and A12), we can observe that the hybrid strategy generated strong improvements in terms of annualized performance in the S&P500 stocks universe, except for the scenario when it generated a negative return ($h = 1$ and $s = 0$).

To sum up, the calibrated wavelet transform approach performed on a monthly basis allows us, on average, to also increase performance for the Δ/Γ portfolio optimization strategy

4.3.3 Δ/Γ Strategy: Number of Assets in the Portfolio

It has been demonstrated through back-tests, that the hybrid Δ/Γ strategy was able to generate very high risk-adjusted returns in the distant past and it performs even better in today's financial market environment. This newly developed strategy does not invest in a constant number of assets as happens in the Long-Short strategy (where the long and the short portfolio always contains the same number of securities according to a selected percentile) or as happens in the RSWP strategy, which invests in all stocks of a market. This strategy has the purpose of picking up ″booming″ or ″falling″ securities which are generating increasingly positive or negative returns as quickly as possible in order to benefit from the short-term upward or downward price acceleration. Therefore, this section briefly investigates the composition of the best simple and the best wavelet-based Δ/Γ portfolio by determining the number of assets over the time in the long and in the short sub-portfolios. This analysis is performed using the DJIA stocks universe for the recent past, i.e. between 2001 and 2016. The best portfolio generated by the Δ/Γ (simple) strategy as well as by the wavelet-based approach is the same and it is defined with the following parametrization: a one year holding period ($h = 1$), one month delay in the investment execution after the portfolio construction ($s = 1$) and the simple approach implemented a formation period (f) of one year while the wavelet approach detected the factor on a monthly basis at the resolution level ($j = 3$).

The first chart (see Figure 1 in the next page) indicates the number of assets over the time in the long Δ/Γ (simple) sub-portfolio. More precisely, at each day t, a long portfolio is built according to the hybrid strategy, i.e. the long portfolio buys stocks with a positive momentum (i.e. a positive Δ) plus an upward accelerating price (i.e. a positive Γ); both factors are detected with a simple approach over a formation period (f) of one year. It is clear that there is a cyclical ″trend″ in the number of stocks held in the long portfolio.

The second chart (Figure 2 in the next page) illustrates the number of assets over the time in the short $(\Delta/\Gamma)_{1,12}$ sub-portfolio. As previously, at each day t a short sub-portfolio is built according to the hybrid strategy, i.e. the short portfolio sells stocks having a negative momentum (i.e. a negative Δ) and a downward accelerating price (i.e. a negative Γ), both factors are detected with a simple approach over a formation period (f) of one year. As for the long sub-portfolio, the number of assets included in the short sub-portfolio seems also to have a cyclical behaviour. Furthermore, there is a symmetric relation between the long and the short cyclical pattern.

Moreover, Figure A13 in the Appendix gives a joint overview of the two previous charts, i.e. it represents the number of assets over the time in the long (blue line) and in the short (orange line) sub-portfolios.

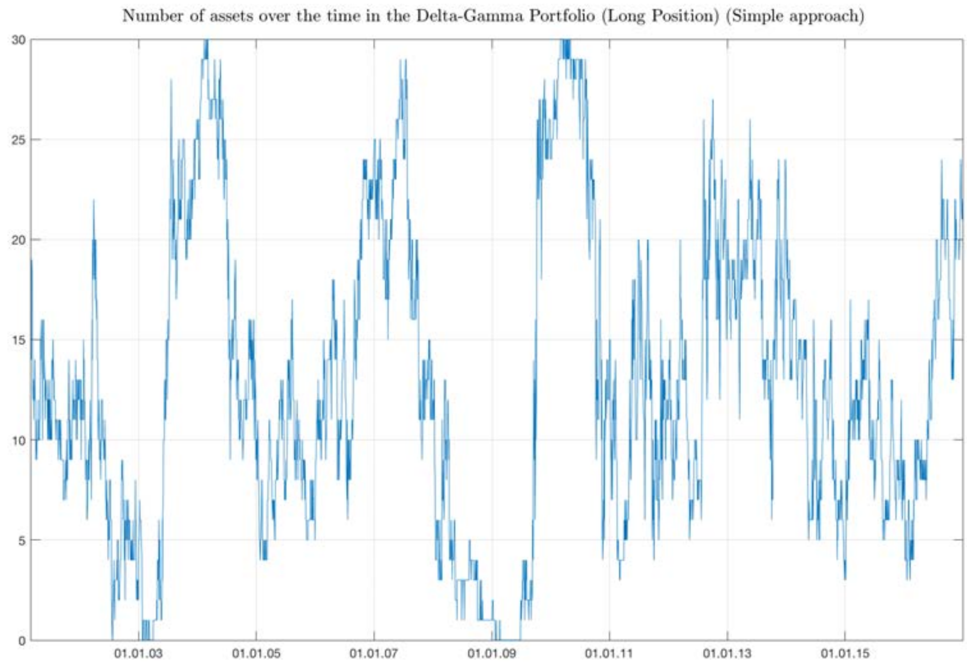

Figure 1: The chart indicates the number of assets in the $(\Delta/\Gamma)_{1,12}$ long sub-portfolio for optimizations performed between 2001 and 2016 and using the universe of securities (data set) including the component of the Dow Jones Industrial Average. More precisely, at each day a long sub-portfolio is built according to the hybrid strategy, i.e. the long portfolio buys stocks with a positive momentum (i.e. a positive Δ) and with an upward accelerating price (i.e. a positive Γ), both quantified over the last $f = 12$ months. Factors are detected with the simple approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time of $s = 1$ months. This analysis is performed using MATLAB.

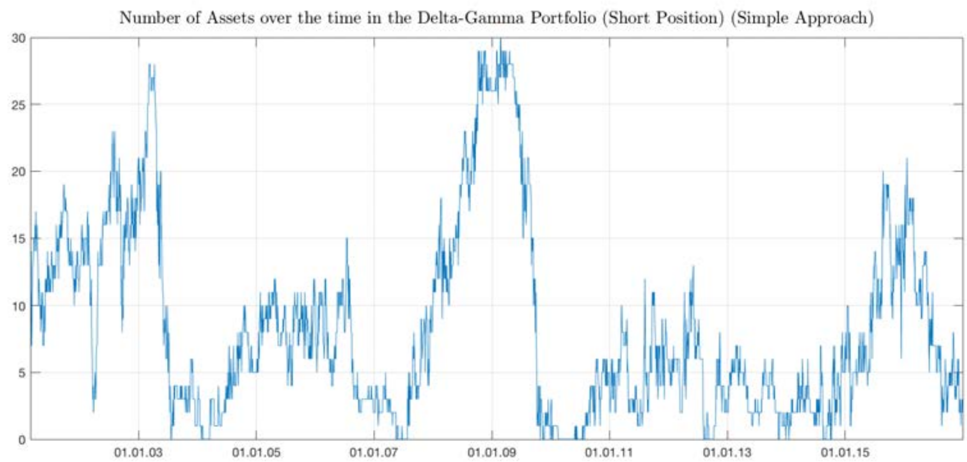

Figure 2: The chart indicates the number of assets in the $(\Delta/\Gamma)_{1,12}$ short sub-portfolio for optimizations performed between 2001 and 2016 using the universe of securities (data set) including the component of the Dow Jones Industrial Average. More precisely, at each day a short sub-portfolio is built according to the hybrid strategy, i.e. the short portfolio sells stocks having a negative momentum (i.e. a positive Δ) and a downward accelerating price (i.e. a positive Γ), both quantified over the last $f = 12$ months. Factors are detected with the simple approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time by $s = 1$ months. This analysis is performed using MATLAB.

In general, we can notice that there are four different "patterns" in the number of assets in the long or the short sub-portfolio: the first pattern starts in 2001 until the start of 2003, the second is between 2003 and 2007, the third between the end of 2007 and the middle of 2010 and the last between the second quarter of 2010 and 2016.

More precisely, considering a market of 30 securities (i.e. the Dow Jones Industrial Average), between 2001 and 2002, there is a downward trend in the number of securities included in the long portfolio, i.e. in 2001 this number fluctuated between 10 and 15 but in 2002 there was first a significant increase to 20 and then a fall to zero. The short portfolio behaves in the opposite direction: in general, there was an upward trend which reached 28 securities for the short portfolio in 2003.

Moreover, in 2003 the number of securities in the long portfolio increased again to 30 and then fluctuated around 10-12 securities until the end of 2006 when this number rose again to around 29. Conversely, for the short portfolio after the growth of 2003, there was a strong decrease in the number of securities and then this number fluctuated between 5-10 securities before dropping to zero at the end of 2006. The strong, long and weak short holding behaviour continued until the first and the second quarter of 2007.

After the second quarter of 2007, a new pattern started which was more volatile: there was first a significant decrease in the number of assets held in the long portfolio and a significant increase in the number of securities in the short portfolio. We can notice that from 2008 until the second quarter of 2009, the Δ/Γ strategy was investing predominating short (i.e. during the financial crisis). Thereafter, in the second half of 2009, there was a strong decrease in the number of assets in the short portfolio to give space to an investment focussed on the long side. The transition lasted until around the second quarter of 2010. Thereafter, from the second half 2010 and 2016, there was an assessment of the number of assets held in both sub-portfolio, i.e. the number of stocks in the long sub-portfolio fluctuated between about 5 and 20 (with some exceptions to 25 stocks) while the short sub-portfolio counted about 5 and 10 securities. The only particularity is a short increment (decrease) int he number of stocks in the short (long) portfolio at the end of 2014.

Furthermore, Figure A14-A16 in the Appendix illustrates the same analysis performed with the best wavelet-based Δ/Γ portfolio which is optimized for Δ and Γ factors detected from monthly stock price series using the MODWT approach. Figure A14 illustrates the number of assets in the long Δ/Γ sub-portfolio while Figure A15 in in the short Δ/Γ sub-portfolio. Finally, Figure A16 gives a joint overview of the number of securities in each sub-portfolio, where the number of stocks in the long are sketched by the blue line and the amounts of securities in the short sub-portfolio by the orange line. The conclusion are very similar as above, since portfolios have been constructed on a monthly frequency (i.e. each month), the line is smoother and represents lower variation. The patterns explained above are also quite clear by investing using the wavelet-based approach. Here one can clearly see the predominantly short investment during the financial crisis. Moreover, we can observe that on average the number of stocks in the long portfolio fluctuates between 5 and 20 while the average number of securities in the short sub-portfolio oscillated between 0 and 10 with some exception, i.e. some peaks above 15 or even three peaks above 25.

To give a general overview of the number of assets in both portfolios, the long sub-portfolios have an average number of assets equal to 13 while the maximal securities held in a long sub-portfolio is

30 and the minimum is zero. Furthermore, the short portfolio has an average number of assets (between 2001 and 2016) equal to 8.3 and the maximal number of securities held in the short portfolio is 29 while the minimum is zero. This means that there are circumstances where this strategy invests only long or short in but on average the number of securities held in the long (short) portfolio is 13 and 8.3. To sum up, unlike the Long-Short strategy which invests in a constant number of securities or unlike the RSWP strategy which invests in the whole market, the Δ/Γ strategy does not invest in a constant number of securities. Moreover, this short study documented a cyclical pattern in the number of stocks included in the long or short sub-portfolio.

4.3.4 Δ/Γ Information Ratio: Comparison with other Strategies

In order to have a further comparison between the Δ/Γ strategy and LS or RSWP optimizations, below there is a short analysis comparing the Information Ratio of the best performing portfolios. Each month, a portfolio having a holding period of one month is constructed according to a selected strategy and using a specific stocks universe (DJIA or S&P500), considering the distant and the recent past. Moreover, the monthly performance of the equal-weighted market index (including all the securities in the implemented data set) is computed to calculated the excess return of the selected strategy. The section 3.2.1 Performance Marks gives an explanation about the Information Ratio.

Table 10 shows the IR for the best performing $\Delta_{s,f}$ (simple) and $\Gamma_{s,f}$ (simple) portfolios as well as the $\Delta_{s,j}$ (WM), $\Gamma_{s,j}$ (WM) portfolios optimized using the LS and the RSWP strategies and considering the S&P500 as well as the DJIA stocks universe for two different period of time: the distant past (1984-2002) and the recent past (2001-2014/2016). Moreover, Table 11 shows the IR for the Δ/Γ strategy computed using factors detected with the traditional (simple) and the calibrated wavelet approach (WMC).

Comparison of the Information Ratio (IR)
of the best performing LS and RSWP strategies

Strategy	IR $\Delta_{6,6}(simple)$	$\Gamma_{6,6}(simple)$	$\Delta_{6,4}(WM)$	$\Gamma_{6,4}(WM)$
		$h = 1$		
LS - DJIA (1984-2002)	0.04	-0.15	-0.05	-0.18
LS - DJIA (2001-2016)	-0.03	-0.03	-0.09	-0.08
LS - S&P500 (2001-2014)	-0.08	-0.08	-0.11	-0.12
RSWP - DJIA (1984-2002)	0.09	-0.2	0.01	-0.17
RSWP - DJIA (2001-2016)	-0.02	-0.02	-0.08	-0.08
RSWP - S&P500 (2001-2014)	-0.09	0.02	-0.13	-0.13

Table 10: Comparison of the Information Ratio (IR) of the best performing $\Delta_{s,f}$ (simple) and $\Gamma_{s,f}$ (simple) as well as $\Delta_{s,j}$ (WM) and $\Gamma_{s,j}$ (WM) LS and RSWP strategies using the S&P500 as well as the DJIA stocks universe and considering the distant (1984-2002) and the recent past (2001-2014/2106).

As demonstrated in Table 11 (in the following page), in today's financial environment (2001-2014/2016) the Δ_{LS} or Γ_{LS} as well as Δ_{RSWP} or Γ_{RSWP} strategy optimized for factors detected with the simple or the wavelet approach (WM) do not generate a positive Information Ratio; this means that the portfolio performance is lower than the benchmark return (i.e. the market return). However, using the Δ/Γ strategy it is possible to achieve adequate risk-adjusted excess returns in today's financial environment.

Comparison of the Information Ratio (IR) of the best performing Δ/Γ strategies

Strategy	IR $(\Delta/\Gamma)_{1,12}(simple)$	$(\Delta/\Gamma)_{1,3}(WMC)$
	$h = 1$	
DJIA (1984-2002)	0.49	0.45
DJIA (2001-2016)	1.13	0.90
S&P500 (2001-2014)	0.49	0.45

Table 11: Comparison of the Information Ratio (IR) of the best performing $(\Delta/\Gamma)_{s,f}$ (simple) and $(\Delta/\Gamma)_{s,j}$ (WMC) portfolios using the S&P500 as well as the DJIA stocks universe considering the distant (1984-2002) and the recent past (2001-2014/2106).

4.4 Winsorization and Portfolio Performance

In this section, a few selected portfolio optimizations are performed using winsorized split and dividend-adjusted log-returns. More precisely the Δ/Γ as well as the RSWP strategy is implemented using the DJIA or S&P500 stocks universe (winsorized), in order to test if the winsorization approach (which is explained in the section 2.1.2 Assumptions) might improve the portfolio return.

Table A32 in the Appendix reports the performance of $(\Delta^D_{RSWP})^{wins}$ and $(\Gamma^D_{RSWP})^{wins}$ (simple) strategies performed using winsorized returns considering the S&P500 stocks universe (2001-2014). If we compare it with the performance of non-winsorized Δ^D_{RSWP} and Γ^D_{RSWP} (simple) portfolio optimizations (see Table A12 in the Appendix), we can observe a slight improvement mostly in scenarios with negative returns. The winsorized $(\Delta^D_{RSWP})^{wins}$ (simple) strategy generated improved results in 16 out of 24 scenarios whilst the winsorized $(\Gamma^D_{RSWP})^{wins}$ (simple) strategy improved the performance in 12 out of 24 scenarios. Δ^D_{RSWP} (simple) strategy outperformed the non-winsorized corresponding strategy mostly for a holding period of six months but it always performs better for a holding period of one month and a delay in the investment of six months. Furthermore, the winsorized $(\Gamma^D_{RSWP})^{wins}$ (simple) optimization always generated greater results when the factor is detected over a formation window of $f = 12$ months for a holding period of six months.

Moreover, Table A33 reports the performance of winsorized $(\Delta^D_{RSWP})^{wins}$ (WMC) and $(\Gamma^D_{RSWP})^{wins}$ (WMC) portfolio optimizations performed using the S&P500 stocks universe. The winsorization approach lead to a better performance compared to non-winsorized optimizations in 18 out of 24 scenarios for momentum-based portfolio but never in the scenario of a month holding period and no delay in the investment (see Table A22 in the Appendix for a comparison). Moreover, the winsorized $(\Gamma^D_{RSWP})^{wins}$ (WMC) strategy increased the performance compared to non-winsorized Γ^D_{RSWP} (WMC) in 14 out of 24 and it outperforms the winsorized $(\Delta^D_{RSWP})^{wins}$ (WMC) strategy in 12 out of 24. Furthermore, we can observe that using the winsorization methodology, the number of negative returns in the momentum-based strategy is drastically reduced, for example the non-winsorized Δ^D_{RSWP} (WMC) investment generated a negative average annualized return in 8 scenarios out of 24 while the corresponding winsorized strategy only in 3 scenarios. However, applying the winsorization to the acceleration ($(\Gamma^D_{RSWP})^{wins}$ (WMC)) strategy it decreased the number of scenarios showing negative returns only by one.

Table A34 and A35 in the Appendix reports the annual performance of the winsorized $(\Delta/\Gamma)^{wins}$ (simple) portfolio optimization performed using the DJIA stocks universe; the former considers the recent past (2001-2016) while the latter the distant past (1984-2002).

Comparing Table A34 with Table A24 we can observe, that the winsorization approach generated on average the same performance as the corresponding non-winsorized optimization. There is only one scenario where the use of winsorized log-returns as an input variable might increase the portfolio performance and it is the scenario characterized by a month holding period and a month delay in the investment for a Δ and a Γ-factor detected using formation periods (f) of six and twelve months. Moreover, this is also confirmed by comparing Table A35 to Table A23 in the Appendix, i.e. the performance of the winsorized $(\Delta/\Gamma)^{wins}$ (simple) strategy is similar to the previous corresponding non-winsorized strategy; however, we can find 5 scenarios out of 24 where winsorization generated a slightly improved performance.

To conclude, applying the winsorization approach, it is possible to decrease the number of scenarios having negative performance but it seems not to be possible to achieve a significant increase in the profitability of both the RSWP and the Δ/Γ portfolio optimization.

5 Discussion

This research investigated two important financial market anomalies which seemed to stem from the behavioural irrationality of investors: the momentum and the acceleration effect (Fama, 1998). Concluding my investigation, I will aim to give a critical explanation and interpretation of the Δ and Γ portfolio optimization results which have been performed considering the U.S. equity stocks market.

The main evidence documented in this study is that in today's financial environment (i.e. in the recent past) portfolios based on the momentum (Δ) or the acceleration factor (Γ) are returning lower (sometimes even negative) performances compared to the distant past (i.e. the last two decades of the 20th century). However, a new hybrid strategy has been developed, i.e. the Δ/Γ strategy, which generated very good performances also in the today's financial market regime.

To give a general overview regarding portfolio optimization results: by applying different Δ or Γ-based investment strategies and considering a large number of parametrizations, it is interesting to see that this study reconfirms several facts already documented in the paper by Ardila, Forrò & Sornette (2015) as well as in previous articles about momentum strategies (Jegadeesh & Titman, 1993, 2001). In particular, there is evidence that investment strategies according to the Δ or Γ parameter in the whole universe of stocks and not only in securities having extreme returns are more profitable. Moreover, Γ-allocations seem to perform better in the long-term (i.e. for longer investment holding periods) while there is no acceleration in the short-term, i.e. it is less or even not profitable to detect the Γ-factor over a short formation period (of one or three months). Unlike the paper by Ardila, Forrò & Sornette (2015), there is little evidence that an acceleration-based portfolio outperforms momentum; a possible explanation is given later in this section. Furthermore, similarly to previous researches, a strategy which invests with a delay of six months after portfolio construction performed with factors detected using the "traditional" simple approach over a formation period of six months generates the best performance in term of risk-adjusted return, if we hold the portfolio for one month. This is the most proficient strategy already documented by Jegadeesh & Titman (1993), i.e. the 6/6 momentum strategy in this study denominated as $\Delta_{6,6}$-allocation. However, if we compare the momentum as well as the acceleration performance of the last two decades to the performance of the two last decades of the 20th century (1980-2000), there is a strong reduction in the portfolio return which is not even fair comparable with the distant past profitability.

Additionally, this study developed two tools to improve the detection of the Δ and the Γ-factors: the trend-based and the wavelet-approach. As was stated in the previous section, the trend-based (EMA) detection increased the performance of both the momentum and the acceleration portfolios in most scenarios while computing the momentum using the Simple Moving Average Crossovers approach allows to increase the performance of Γ-allocations. Furthermore, the MODWT approach performed on a monthly basis (WM) generated an improvement in the performance for both factors only for short-term investments with one-month and with no or one-month delay in the investment execution while the calibrated version of the MODWT approach seems to further increase the risk-adjusted return. However, since the calibration shifts the stock's price series backwards (i.e. it corrects the lag), its implementation in the portfolio optimization requires firstly a forecast and an estimation of future stock prices; therefore, this approach is not directly applicable. Moreover, by analysing the behaviour of the portfolio performance for strategies optimized with the Maximum Overlap Discrete Wavelet Transform tool at a different resolution levels (i.e. time-scale), it is clearly visible that momentum is more evident over a period of three-twelve months while portfolios based on the acceleration factor achieve a higher return when the Γ is measured over a formation period between 16 and 32 months. Furthermore, applying the winsorization approach, it is possible to decrease the number of scenarios having negative performance but it seems not to be possible to achieve a significant increase in the profitability

Nevertheless, even using better tools to quantify the momentum or the acceleration factor, the discrepancy between the distant past and recent past performance remains significant. The fact that the momentum strategy is not profitable by the start 21st century was also documented for example by the following paper: Jegadeesh and Titman (2011), Essay UK (2018), Hwang & Rubesam (2008) and Abourachid, Kubo& Orbach (2017). The first part of this section indicates four possible interpretations for the lower or even negative performance of momentum and acceleration allocations.

The first suggested explanation regarding the lower recent performance of Δ and Γ portfolios (the first two decades of the 21st century) assumes that there are potential biases affecting the endogenous structure of this investigation, i.e. the sample selection bias and survivorship bias (Investopedia, 2018m). The sample selection bias happens when non-random data is selected for a study. Moreover, the survivorship bias belongs to the sample selection bias group and it might only be common during back-testing tasks if securities with data available for the whole investigation period are selected (Investopedia, 2018n). Indeed, this analysis is performed considering companies which were included in the DJIA or S& P500 index for the whole length of the analysis; therefore, companies which failed during the investigation period as well as new included stocks have not been considered. This might have an impact on both the momentum and the acceleration allocations performance, since for example potential profits deriving from shorting stocks exhibiting a drop in the price which led to a default are missed. Moreover, the lower evidence of the Γ-profitability over Δ might also stem from selection bias: the acceleration has been previously documented as a ″transient (non-sustainable)″ phenomenon related “to positive feedbacks influencing the price formation” and since securities involved in particular events might be discarded from the analysis, a portion of this profit might be lost. However, since the data set for the distant past is also affected by selection bias, but there is evidence of momentum in this time period, this bias might not be the only cause of the temporal divergence in the performance, in particular for momentum strategies.

A second interpretation of the discrepancy in the performance of Δ or Γ strategies between the distant and the recent past might be found in the data mining bias. While data mining indicates an action to find and extract patterns from a large volume of historical data to build predictive financial models or to develop winning investment strategies, the data mining bias consist in remaining erroneously stuck in such data mining practice (Investopedia, 2018m). Indeed intergenerational data mining, i.e. the on-going use of information already revealed in prior financial papers might be ineffective, since it might happen that when a phenomenon is widely recognized from market participants, the implementation of trading strategies geared to anticipating and taking advantages of this effect has an impact on the stock price which will be adjusted for the anomaly, i.e. the effect is priced into the stock value (Investopedia, 2018m). Thus, the momentum effect has been largely investigated and momentum-based strategies are widely adopted. Moreover, since acceleration is an effect which complements momentum, the data mining bias might negatively affect also Γ-profits.

However, as is stated in the paper by Hwang & Rubesam (2008): ″Considering that momentum has been very popular in academic works as well as in practice since the seminal paper by Jegadeesh & Titman (1993), we wonder why it took such a long time for investors to erode the profit opportunity from simple momentum strategies. If the market is efficient, its participants are expected to act quickly in exploiting arbitrage opportunities if momentum is not related to priced risk". The authors illustrated a possible explanation for the delayed erosion of momentum profits in the late 1990s: not all market participants are able to easily perform momentum strategies since the short (i.e. the loser) portfolio might mainly include small cap companies which are illiquid. However, the study by Hwang & Rubesam (2008) also suggests a complementary explanation which might also be adequate to interpret the results obtained in this research. More precisely, investigating the U.S. equity market between 1927 and 2006 with a multi structural breaks model, they documented that "momentum profits are driven from different sectors in different periods" as for example by the energy sector during the period of time (1977-1982) or the financial sector (1982-1994). Thus, according to Hwang & Rubesam (2008), momentum only generated significantly positive profits during certain time periods and this strategy performed poorly since the last structural break of the year 2000. Therefore, an explanation for delayed disappearance (considering that momentum was firstly reported in 1993) might be found in the unexpected hi-tech and telecom stocks bubble in the late 1990s. More precisely, according to Hwang & Rubesam (2008), at the start of the 1990s the momentum premium increased due to the inclusion of booming high-tech and telecom stocks in the long (winners) portfolio while the "old-economy" was sold, i.e. the momentum was driven by winners. Afterwards, after the "burst of the bubble", momentum profit was still high for a few years since the "previous inflated" high-tech and telecom prices dropped and they were selected in the looser portfolio (Hwang & Rubesam (2008)). Moreover, since the momentum strategy also consists in shorting stocks, as was stated previously, according to the authors in the last decade of the 20th century due to the complexity of this operation investors were not induced to invest according to this strategy; however, due to the profits deriving from momentum investing during the telecom bubble, the strategy became more popular and the number of hedge funds increased. Due to the entrance into the market of these specialized financial institutions which were easily able to undertake active trading strategies (such as momentum) at lower costs and with fewer constraints, momentum (as well as acceleration) profits might be cancelled by successive price adjustments in the market following the widespread implementation of this strategy.

Additionally, a fourth and final interpretation relies on the assumption that momentum and acceleration anomalies also continue to persist in today's financial environment but, as in the paper by Daniel et al. (2012), their performance is affected by "turbulent" (i.e. more volatile) financial market regimes. In particular, the paper by Daniel et al. (2012) revealed that the performance of the momentum strategy is "highly left skewed and significantly leptokurtic [9]", i.e. it is characterized by "infrequent but larger loss". According to Daniel et al. (2012), this distribution might be drawn by a "mixture of distributions", more precisely, by two hidden states: a "calm" and a "turbulent" market state. Previous studies by Abourachid, Kubo,& Orbach (2017) or by Maheshwari & Dhankar (2017) documented a profound low profitability of the momentum strategy during the global financial crisis period (2007-2009), generally defined as "momentum crash". Moreover, according to Maheshwari & Dhankar (2017) market volatility is "almost twice during financial crisis" compared to "calm" market regimes, i.e. the pre- and post-crisis period. Therefore, the low return of Δ and the Γ strategies is consistent with those previous studies as well as with the theory of momentum crashes in "turbulent" market regimes. In particular, according to Daniel et al. (2012), the poor performance of momentum during turbulent and

[9] A leptokurtic distribution is characterized by an excess positive kurtosis (kurtosis >3) and by fatter tails (Statistic How To, 2018).

more volatile markets might be attributed to ”strong short-term reversal effects instead of trend continuation”. Fortunately, as is stated in the paper by Daniel et al. (2012), turbulent market regimes are predictable and, therefore, the negative performance of momentum strategies can be avoided. Furthermore, as per the papers by Daniel & Moskowitz (2016), there is an additional interesting explanation regarding the low performance of momentum strategies outside the normal (”calm”) market environment; indeed, the author stated that: ”in panic states, following multi-year market drawdown and in periods of high market volatility, the price of past losers embodies a high premium”. This mean that in the final phase of ”turbulent” market regimes, i.e. during the rebound phase, the looser portfolio ”experience strong gains” and due to the shorting of those assets a momentum crash is generated, i.e. the momentum profit is reversed. This is also confirmed in this study by the fact that the performance of the short sub-portfolio computed using the Long-Short ”traditional” strategy in the previous section was at least greater than the long portfolio performance considering the recent past; this outcome supports the contrarian investment strategy which, as was explained in the first chapter, consists in buying past losers and selling past winners. Moreover, since the acceleration effect is computed directly by the momentum factor, the bad performance of acceleration-based (Γ) strategies in the recent past might also be related to the state of the market, i.e. by the fact that the recent past includes the severe impact of the global financial crisis.

Therefore, in my opinion, relying purely on past trends (i.e. on the direction of the trend) without deeply observing the pattern of the trend as well as shifting regimes, might lead to setting up a portfolio whose profit reverses within the investment period, mainly during ”turbulent” market regimes. Therefore, in my view, it might still be possible to achieve profits by implementing strategies based on the technical analysis also during a stressed and a more volatile financial environment, but we should be more aware of the stocks which should be selected in the long or short portfolio.

Hence, from my perspective, investing according to the momentum or the acceleration factor separately, without considering their relationship, might lead to unsuccessful investment as well as missed potential profitable investments. Therefore, observing both signals at the same time might help to elucidate patterns which might not otherwise be detected. An example is, if selecting a security to include in the short portfolio only using the parameter Γ, i.e. we are going to include it if the Γ is negative (or strongly negative), however if this security still has a positive Δ we might incur a loss in the short term: the security is characterized by positive returns which decrease over time and shorting a portfolio with positive return is not profitable (i.e. this asset has a decreasing upward price over the formation window). The same happens if we buy a portfolio with a positive acceleration but whose momentum is still negative: a security having decreasing negative returns (or a decreasing downward price).

As was explained previously, the Δ/Γ is an extension to the “elementary” time-series momentum strategy which invests long in stocks with a momentum factor greater than zero and short in those with a delta less than zero. The developed hybrid strategy aims to consider both factors simultaneously and it consists in selecting securities to include in the long and in the short portfolio according to two conditions: the direction of the momentum (“delta condition”) and the direction of the acceleration (“gamma condition”). For each security, a signal is generated according to its Δ and its Γ factor and it indicates which kind of trading activity is appropriate: buy, sell or not invest. Furthermore, according to the magnitude of the acceleration factor one can choose to assign ”relative” Γ-weights to securities in the corresponding long or short portfolio.

In my opinion, the success of the Δ/Γ strategy is due to two important aspects; firstly the hybrid strategy gives a clear signal about which kind of stocks to select, i.e. securities with a lower probability to definitely reverse their trend in the short-term and which have a potential growth (or drop in case of shorting securities) prospect; secondly a large part of the success is attributed to the fact that the hybrid strategy allows the number of stocks in each portfolio to vary over time. Therefore it might be a better strategy than to just invest in a constant number of stocks having extreme returns or in the whole market; this allows us to keep track of the general market movements and does not include securities which display an uncertain past performance (as for example positive momentum and negative acceleration). As was shown in the previous chapter (section 4.3.3 Δ/Γ Strategy: Number of Assets in the Portfolio), there is a cyclical pattern in the number of stocks held in the long or in the short Δ/Γ sub-portfolio over time. It is clear that under a distressed market regime (as for example the worldwide financial crisis 2007-2009) this strategy predominantly invests short. Moreover, it is interesting to see that a good positive performance is possible by investing for three months, while longer holding periods lead to a decrease in performance levels. This might indicate that if a stock price has a positive increasing past (or negative decreasing) past performance, this will continue for a period of around three months and thereafter the trend comes to an end, i.e. holding the portfolio too long is not profitable.

However, another important aspect that should be highlighted is the divergence of performance between the distant and the recent past if we apply the Δ/Γ for a one-month holding period. Indeed, if in the distant past a Δ/Γ portfolio held for one month had a positive performance for each kind of further parametrization (i.e. for each formation period and each delay in the investment), conversely in the recent past the strategy generated strong negative returns if the investment was not delayed in time but it generates the overall highest performance by delaying the investment by one month (as was described in the previous section). My point of view is that waiting for one month before implementing the strategy might be more profitable because it allows us to avoid ”transient” short-term reversal and to gain an extra profit deriving from readjustment from this short-term drop. However, this is only a hypothesis and further analysis should be performed to investigate this divergence.

Therefore, since Δ/Γ strategy is more ”flexible” as it do not invest in a constant number of stocks and is more ”selective” because it only invests in a portion of stocks previously selected by the momentum strategy, it allow us to avoid momentum crashes originate by volatile and ”turbulent” market regimes.

To sum up, the momentum as well as the acceleration profitability might depend on the ”state” of the market (Cooper, Gutierrez,& Hameed, 2004). More precisely, as is documented by previous studies such as the paper by Daniel et al. (2012), momentum crash is possible in more volatile and ”turbulent” financial regimes. Therefore, this might be an explanation for the relatively low performance of Δ and Γ strategies in the recent past, which is characterized by the dramatic impact of the global financial crisis (2007-2009). However, this study adds evidence, that selecting stocks according to both the momentum and the acceleration factors allows us to even generate a high return in a stressed financial environment. Indeed, the Δ/Γ strategy performed well not only in the distant past but also in the recent past. However, to test the practical implementation of the delta-gamma strategy, further simulations which consider transaction costs as well as additional market frictions should be executed.

Moroever, this analysis might be a starting point for further investigations, such as for example testing the relation between the number of stocks held in each sub-portfolio to macroeconomics variables as well as other financial indicators. Furthermore, another interesting analysis might be to measure acceleration not in an absolute quantification but in a relative quantification: as the percentage change in momentum or to implement the idiosyncratic momentum to detect the Γ-factor.

6 Conclusions

Summarizing, this research revealed the profitability of momentum and acceleration-based portfolio optimization strategies considering the recent (2001-2016) and the distant past (1983-2002) and using different detection proxies to quantify the momentum (Δ) as well as the acceleration (Γ) factor: the "traditional" (simple), the trend-based and the wavelet transform (i.e. the Maximum Overlap Discrete Wavelet Transform) approach; moreover, the winsorization methodology is also applied. Additionally, a new hybrid strategy has been developed, i.e. the Δ/Γ (Delta-Gamma) allocation and it aims to consider both the Δ and Γ to optimize portfolios.

There are two main evidences documented in this study, i.e. the lower profitability of Δ and Γ-based strategies in today's financial environment (i.e. in the recent past, 2001-2016) compared to the distant past (i.e. the two last decades of the 20th century) and the good performance of the Δ/Γ allocation, which generated a good return in the distant past and performs even better in today's financial regime.

On average, the implementation of additional proxies to detect the momentum and the acceleration effect revealed that the trend-based as well as the wavelet transform approach, in particular the (calibrated) MODWT performed on a monthly basis (i.e. using monthly stock prices as input variable), allows us to improve the performance of Δ, Γ as well as Δ/Γ allocations; however the return of Δ and Γ strategies remains very low in today's financial environment. Moreover, the calibrated version of the MODWT approach requires a forecast of future stock prices in order to be implemented, i.e. it is not directly applicable.

This study adds convincing evidence about the negative performance of momentum (as well as the acceleration) strategies during "turbulent" and more volatile market regimes, i.e. in the analysis performed on the recent past (2001-2016), a period of time characterized by the dramatic impact of the global financial crisis (2007-2009). Moreover, there is significant evidence that implementing the hybrid Δ/Γ portfolio optimization, i.e. a more "flexible" but more "selective" investment strategy which considers both momentum and acceleration as factors for the optimization and does not invest in a constant number of assets allows us to even gain a good return during stressed and more volatile market regimes.

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Appendix

All figures and tables were created using MATLAB by the author, unless otherwise specified. The MATLAB code as well as the data sets are available from the author.

The portfolio optimization strategy is written in this form: $FACTOR_{strategy}$ (detection mode), for example the Δ_{LS} (WMC) notation indicates the momentum-based Long-Short portfolio optimization performed using a Δ-factor detected with the calibrated MODWT approach on a monthly basis.

Figures

Figure A1: The Simple Moving Average (SMA) is a good tool to remove the noise from stock prices. Nevertheless, there is a lag between the original price series and its SMA signal. A shorter SMA (blue) ($K = 20$) is faster and follows the price more closely, indicating more reversal signals (as well as more false signals) than a longer SMA (red) ($K = 100$). Moreover, the larger the lag parameter, the greater the delay in the estimated trend. Source: Learndatasci (2018).

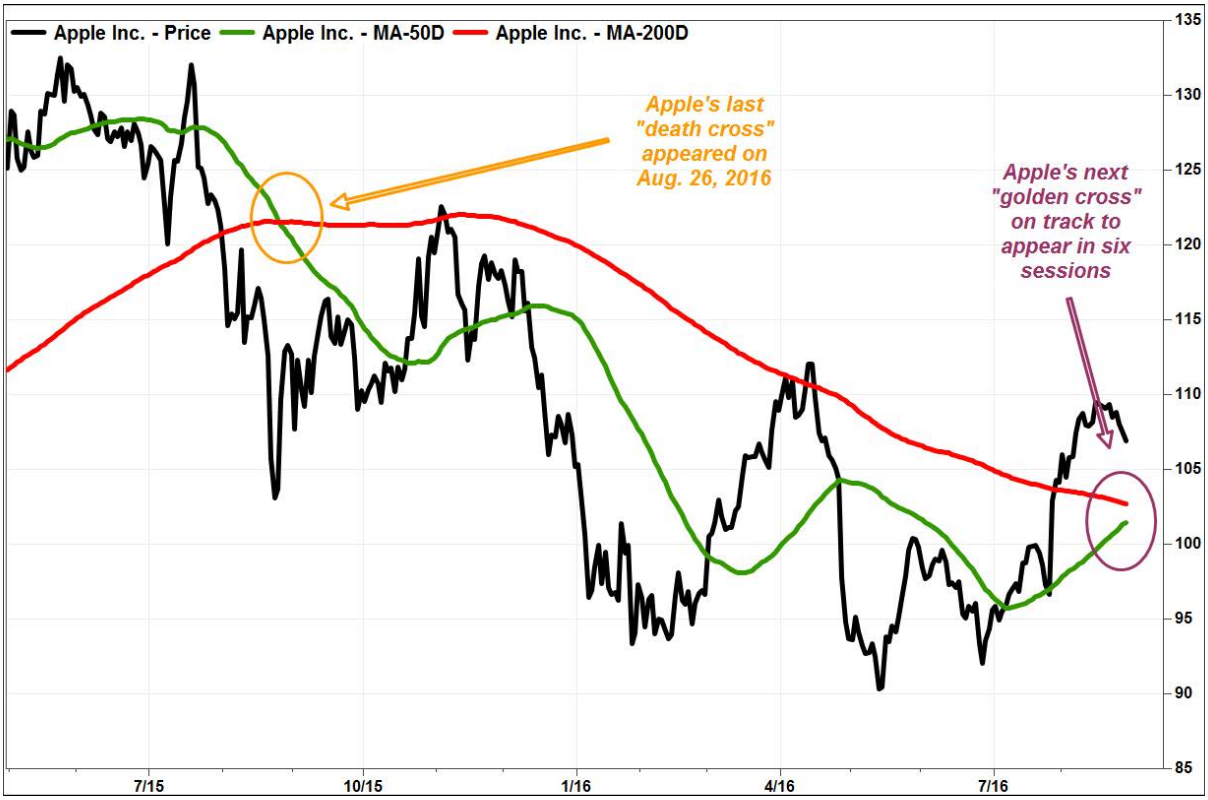

Figure A2: The asymmetric simple moving average is a common technical indicator to determine trend reversals (i.e. a reversal in the momentum). The crossover of a short (as for example 50-days) and a long (200-days) moving average on a chart is an indication of a change in trend. This figure shows an example of ”death cross” for the share price of Apple on the August 26 2016, hence when the short (50-days) moving average (green) of the Apple stock price crossed below the longer (200-days) moving average (red). The Death Cross indicates a negative reversal in the future trend; indeed, the Apple stock price (black) fell. Moreover, it is clear how this tool might anticipate future changes in trends (i.e. the chart shows a potential upcoming ”golden cross” which indicates a positive reversion in the trend). Source: ETFDailyNews (2016).

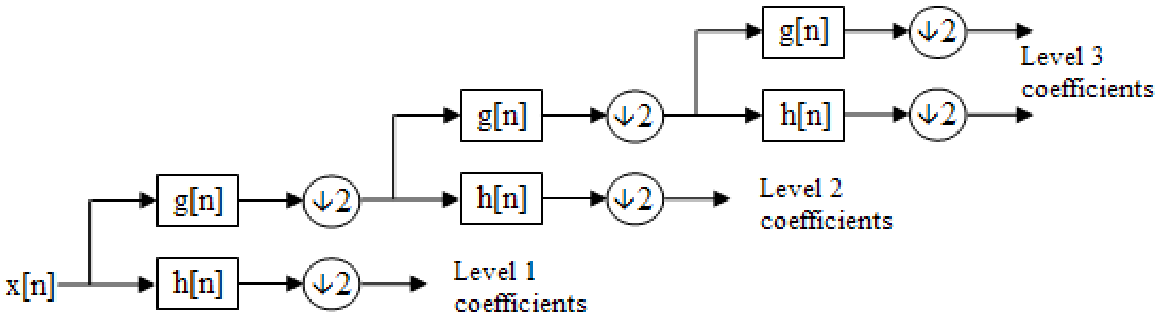

Figure A3: The Discrete Wavelet Transform works with the Mallat (pyramidal) algorithm. The multi-scale decomposition is performed by two fundamental wavelets (filters): the mother wavelet (i.e. the high-frequency filter here represented by the function ($h\,[j]$)) and the father wavelet (i.e. the low frequency filter, here $g\,[j]$). The signal is simultaneously filtrated by a low-pass and a high-pass function; since there is a relation (”quadrature mirror”) between the two filters, the sample is automatically divided in two parts: a high-frequency series which is extracted with the high-pass function (i.e. the detail coefficient) and a low-frequency part (i.e. approximation coefficients). Both series have half of the length of the original signal, i.e. at each level there is a subsampling by a factor of two. This procedure is repeated at each level j and it allows us to extract further high-frequency detail and approximation coefficients. The signal must have a dyadic length of 2^J. Soruce: Wikipedia (2018).

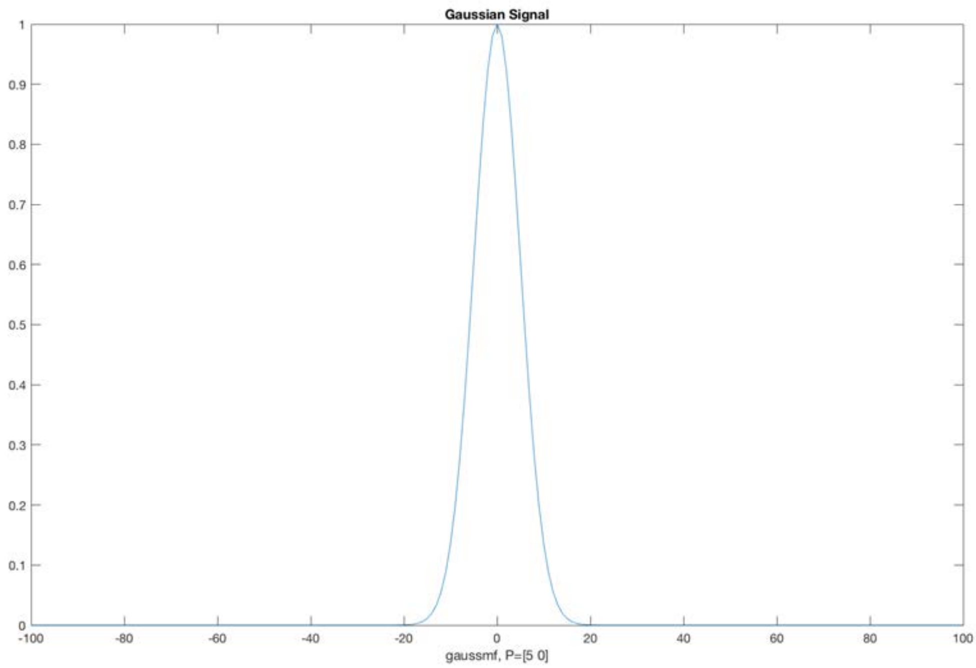

Figure A4: Gaussian Signal simulated using MATLAB. The symmetric Gaussian Signal is determined by two parameters: the volatility (σ) and its center (μ) and it is given by the following function: $f(x;\mu,\sigma) = exp(\frac{-(x-\mu)^2}{2\sigma^2})$. The above Gaussian Signal has been simulated with a $\mu = 0$ and a $\sigma = 5$. Source: MATLAB(2017b).

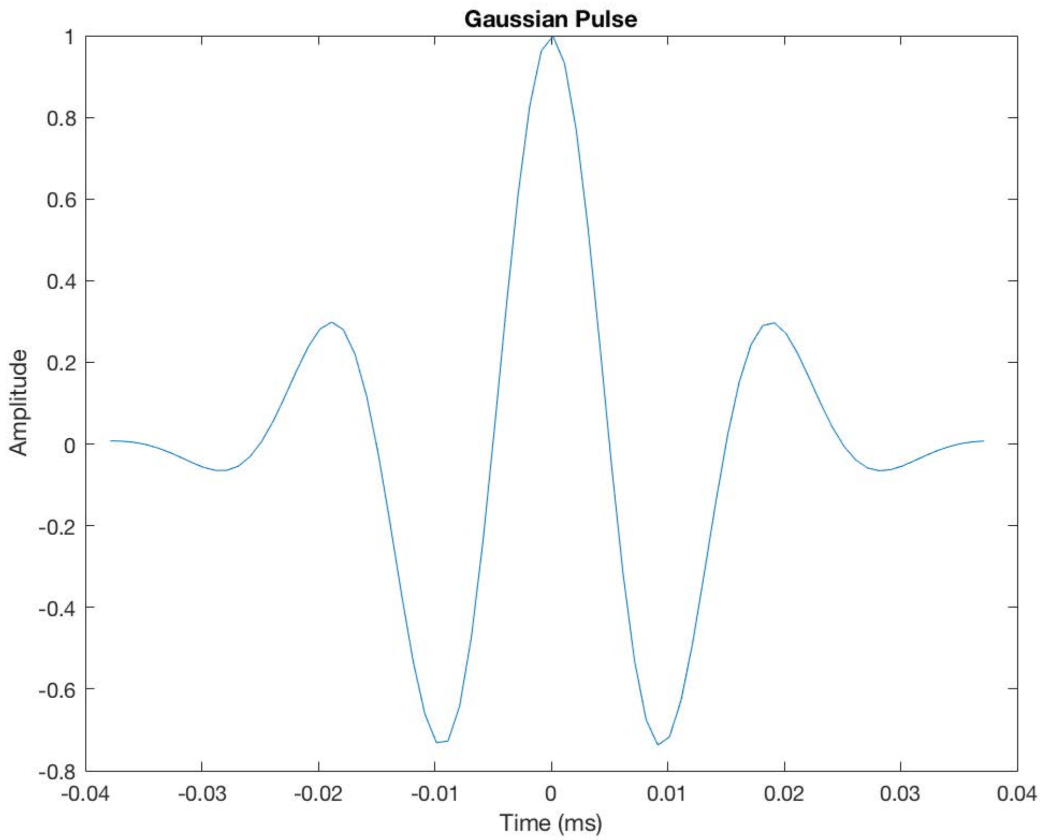

Figure A5: Gaussian Pulse Signal simulated using MATLAB. The Gaussian Pulse signal is defined by three parameters: the time array (t), the center frequency (herz) (fc) and the fractional bandwidth ($bw > 0$). This signal has been simulated using the following values: $fc = 50'000$ and a $bw = 0.6$. It gave rise to a 50kHz Gaussian RF pulse with a bandwidth equal to 60%. The signal is sampled at a rate of 1MHz and the pulse is truncated where the envelope drops 40dB below the peak. Source: MATLAB(2017b).

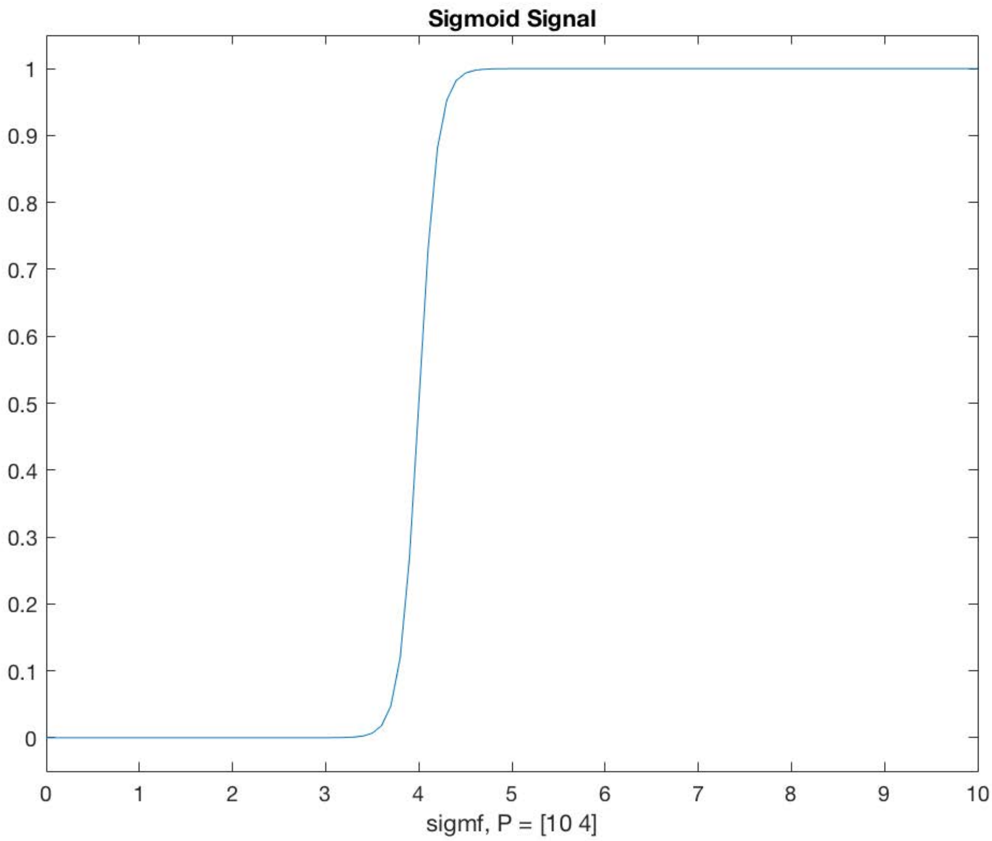

Figure A6: Sigmoid Signal simulated using MATLAB. The Sigmoid Signal is an S-shaped curve generated by the following function: $f(x; [a\, c]) = \frac{1}{1+exp(-a(x-c))}$. The parameter "$c$" indicates the "center" whilst the parameter "a" determines how much the function is open to the right and to the left. The above Signal has been simulated with the following parameters: $c = 4$ and $a = 10$. Source: MATLAB(2017b).

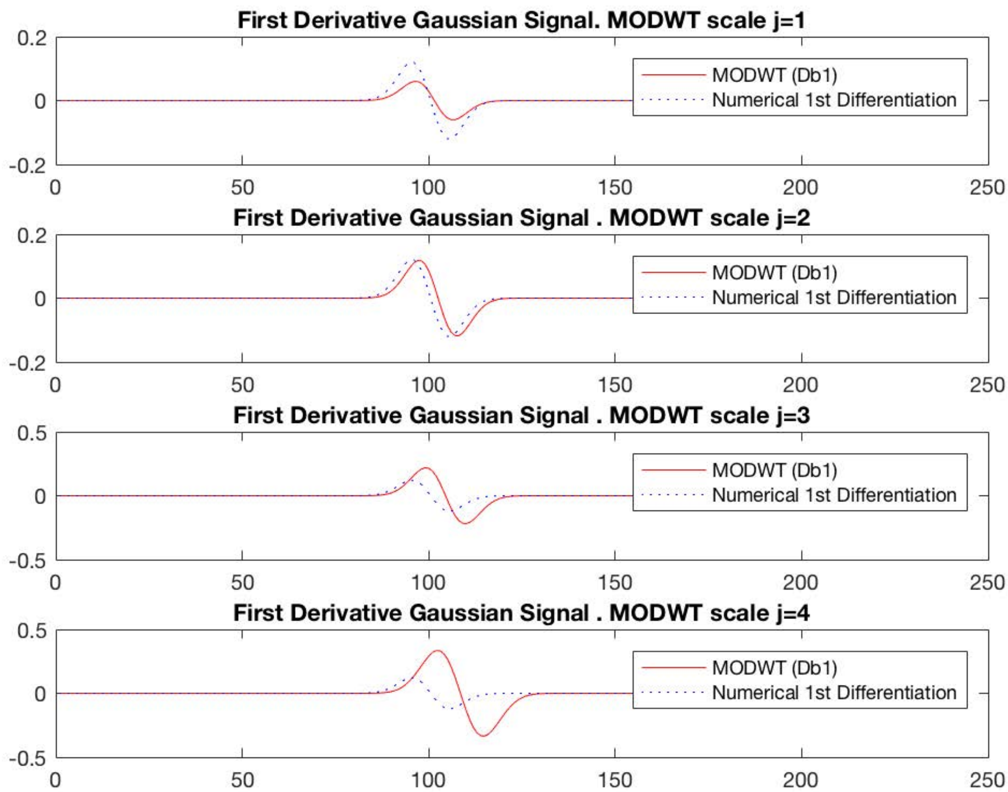

Figure A7: The figure shows the first derivative of the Gaussian Signal computed with two different approaches. The dotted line shows the first derivative computed through the common numerical approach while the red line illustrates the output, i.e. detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through the Haar (also named Daubechies function with one vanishing moment, Db1) mother wavelet. It is clear that the Wavelet Transform is a good tool to compute the first derivative which is approximated quite well compared to the numerical method, except for a small lag in time. This analysis as well as the figure are performed using MATLAB.

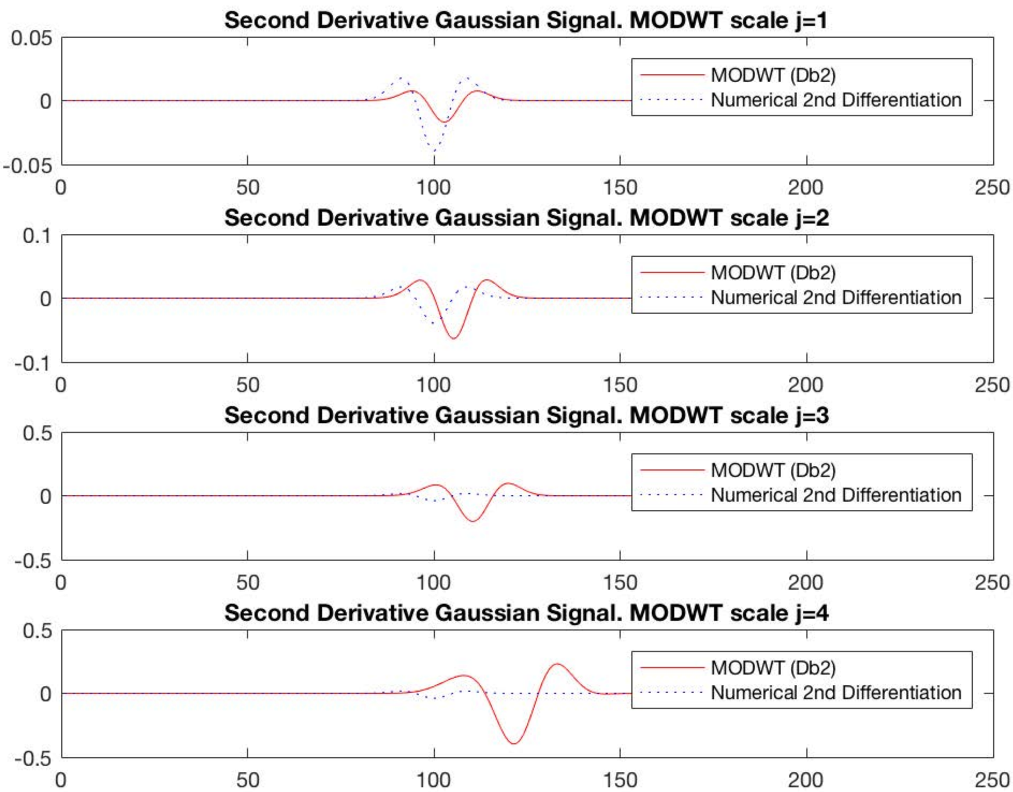

Figure A8: The figure illustrates the second derivative of the Gaussian Signal computed with two different approaches. The dotted line shows the second derivative computed through the common numerical approach while the red line illustrates the output, i.e. the negative of detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through a Daubechies function with two vanishing moments (Db2) mother wavelet. It is clear that the Wavelet Transform is a good tool to compute tool also for the calculation of the second derivative which is approximated quite well compared to the numerical method, except for a small time lag, which is slightly larger as in the computation of the first derivative. This analysis as well as the figure are performed using MATLAB.

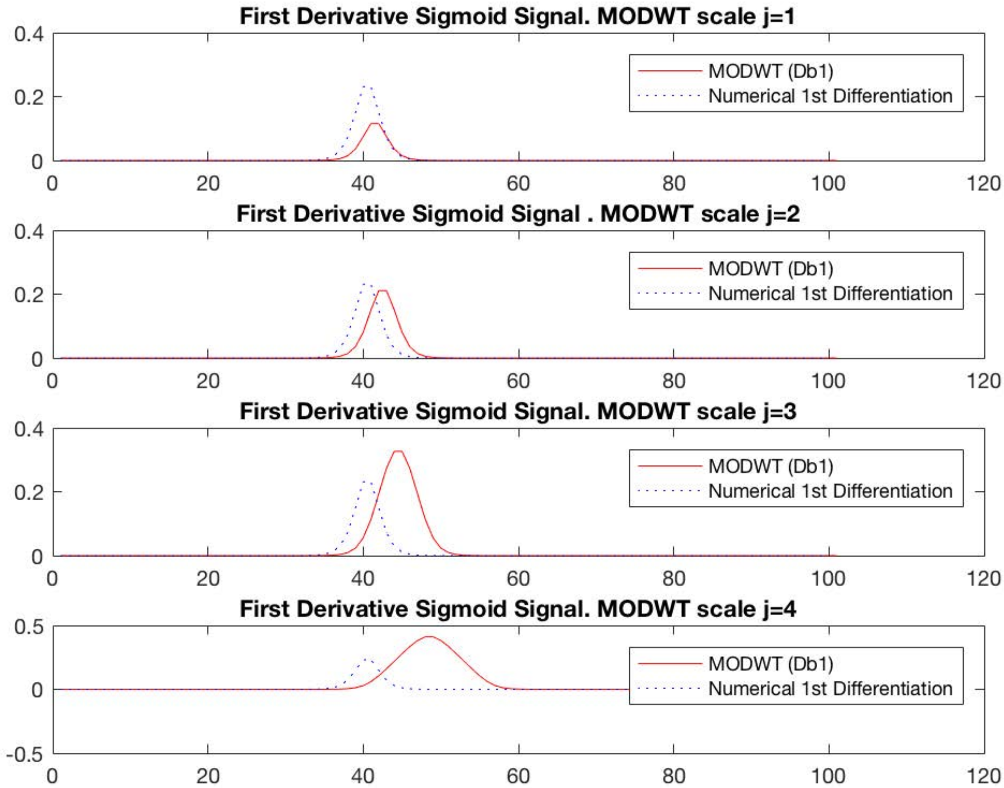

Figure A9: The figure shows the first derivative of the Sigmoid Signal computed with two different approaches. The dotted line shows the first derivative computed through the common numerical approach while the red line illustrates the output, i.e. detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through the Haar (als named Daubechies function with one vanishing moment, Db1) mother wavelet. The first 5 (16) detail coefficients at the resolution levels j=1,2 (j=3,4) have been normalized to zero since their magnitude is an abnormal result deriving from the ”overlapping” effect of the MODWT approach. It is clear that the Wavelet Transform is a good tool to compute the first derivative which is approximated quite well compared to the numerical method, except for a small time lag. This analysis as well as the figure are performed using MATLAB.

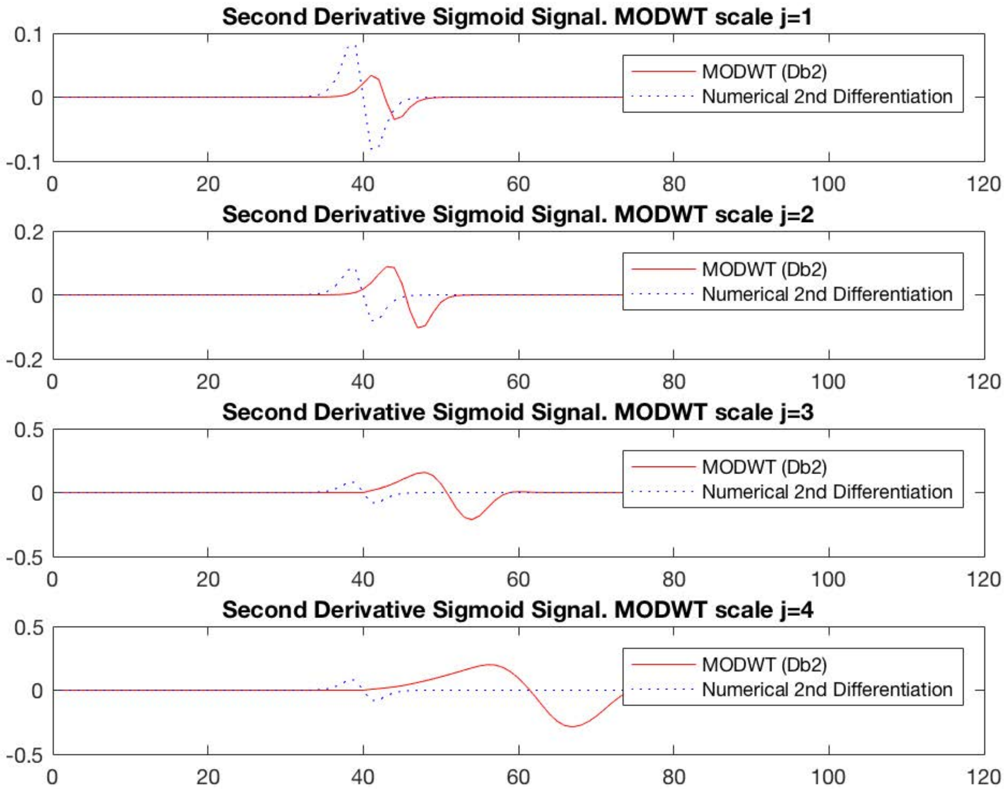

Figure A10: The figure illustrates the second derivative of the Sigmoid Signal computed with two different approaches. The dotted line shows the second derivative computed through the common numerical approach while the red line illustrates the output, i.e. the negative of detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through a Daubechies function with two vanishing moments (Db2) mother wavelet. The first 20 (40) detail coefficient at the resolution levels j=1,2 (j=3,4) have been normalized to zero since their magnitude is an abnormal result deriving from the "overlapping" effect of the MODWT approach. It is clear that the Wavelet Transform is a good tool also for the calculation of the second derivative which is approximated quite well compared to the numerical method, except for a small time lag, which is slightly larger as in the computation of the first derivative. This analysis as well as the figure are performed using MATLAB.

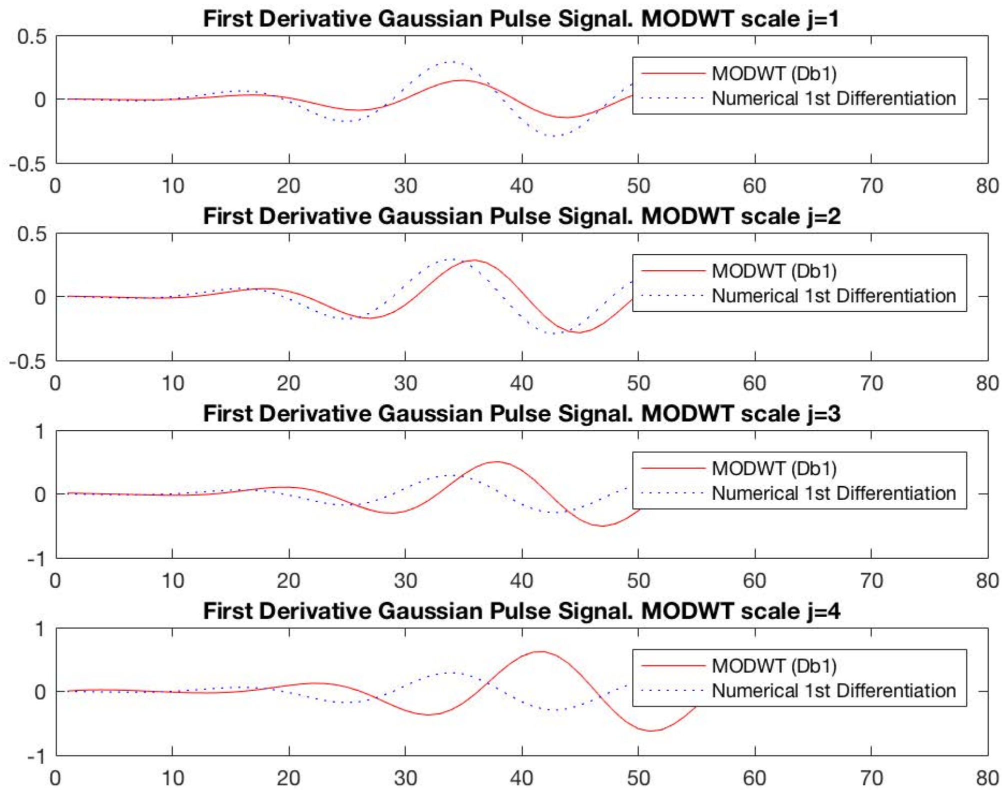

Figure A11: The figure shows the first derivative of the Gaussian Pulse Signal computed with two different approaches. The dotted line shows the first derivative computed through the common numerical approach while the red line illustrates the output, i.e. detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through the Haar (also named Daubechies function with one vanishing moment, Db1) mother wavelet. It is clear that the Wavelet Transform is a good tool to compute the first derivative which is approximated quite well compared to the numerical method, except for a small time lag. This analysis as well as the figure are performed using MATLAB.

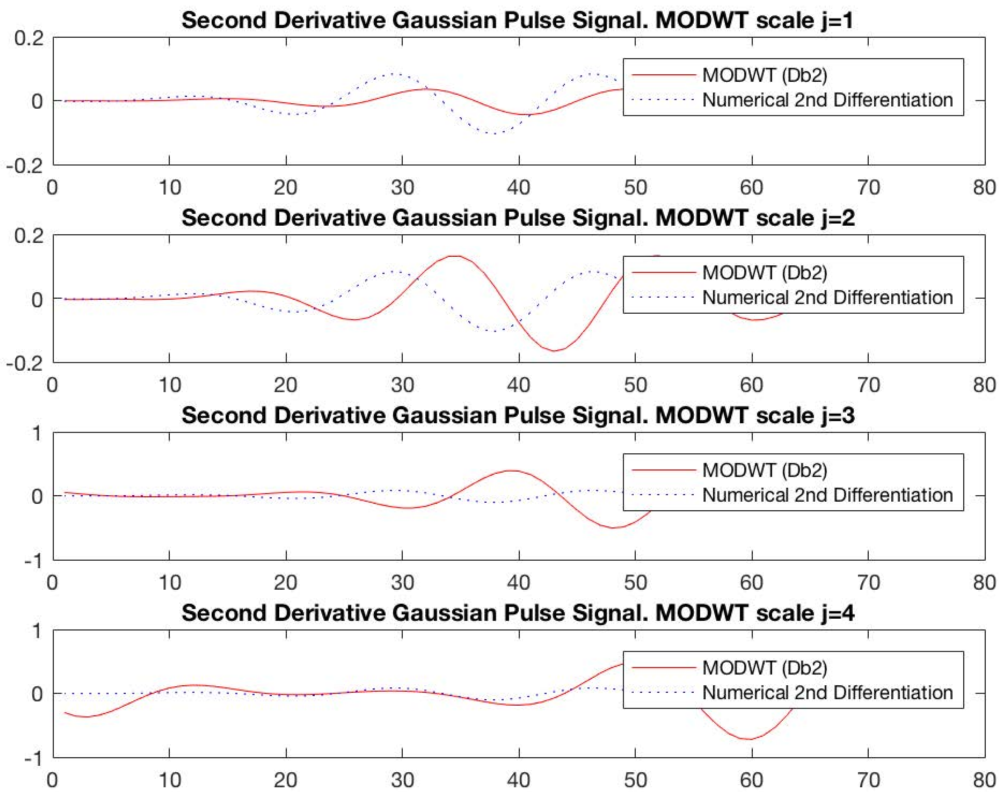

Figure A12: The figure illustrates the second derivative of the Gaussian Signal computed with two different approaches. The dotted line shows the second derivative computed through the common numerical approach while the red line illustrates the output, i.e. the negative of detail coefficients at different resolution levels (j), of the Maximum Overlap Discrete Wavelet approach performed through a Daubechies function with two vanishing moments (Db2) mother wavelet. It is clear that the Wavelet Transform is a good tool also for the calculation of the second derivative which is approximated quite well compared to the numerical method, except for a small time lag, which is slightly larger as in the computation of the first derivative. This analysis as well as the figure are performed using MATLAB.

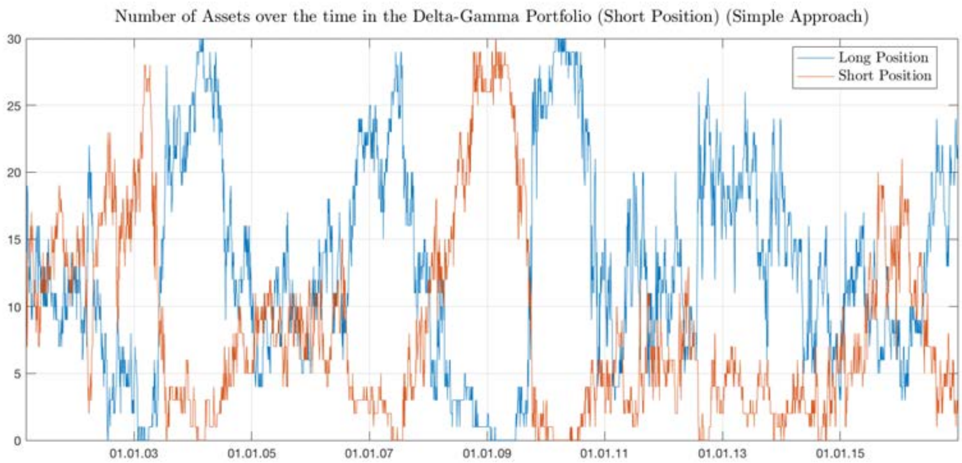

Figure A13: The chart indicates the number of total assets in the $(\Delta/\Gamma)_{1,12}$ (simple) portfolio for optimizations performed between 2001 and 2016 using the universe of securities (data set) including the component of the Dow Jones Industrial Average. The blue line shows the total assets held in the long sub-portfolio while the orange line the number of securities held in the short sub-portfolio. More precisely, at each day a long and a short sub-portfolio is built according to the hybrid strategy, i.e. the long sub-portfolio buys stocks with a positive momentum (i.e. a positive Δ) and with an upward accelerating price (i.e. a positive Γ) and the short sub-portfolio sells stocks having a negative momentum (i.e. a positive Δ) and a downward accelerating price (i.e. a positive Γ), both quantified over the last f=12 months. Factors are detected with the simple approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time by $s = 1$ months. This analysis is performed using MATLAB.

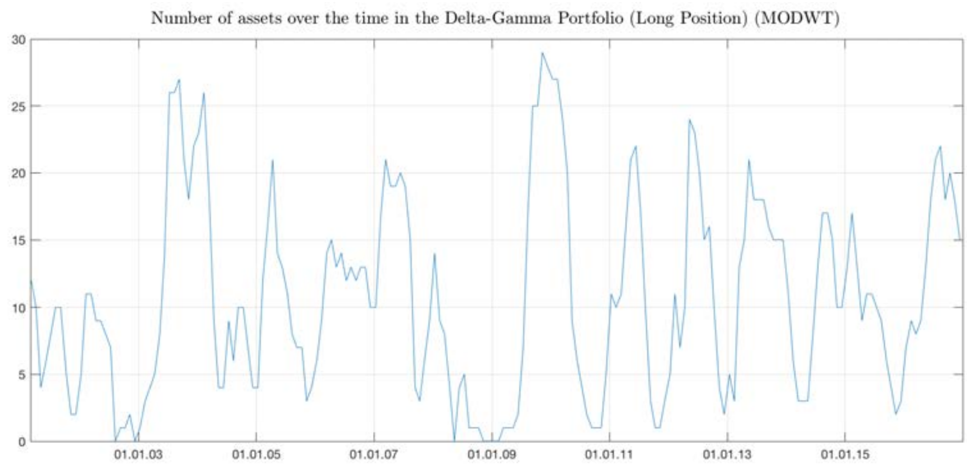

Figure A14: The chart indicates the number of assets in the $(\Delta/\Gamma)_{1,3}$ (WM) long sub-portfolio for optimizations performed on a monthly basis between 2001 and 2016 using the universe of securities (data set) including the component of the Dow Jones Industrial Average. More precisely, at each month a long sub-portfolio is built according to the hybrid strategy, i.e. the long portfolio buys stocks with a positive momentum (i.e. a positive Δ) and with an upward accelerating price (i.e. a positive Γ), both quantified at the resolution level $j = 3$. Factors are detected with the MODWT approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time by $s = 1$ months. This analysis is performed using MATLAB.

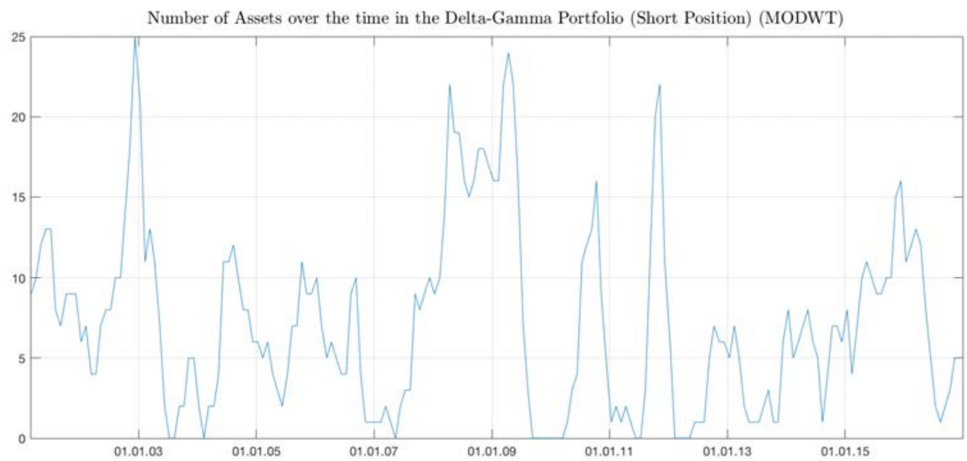

Figure A15: The chart indicates the number of assets in the $(\Delta/\Gamma)_{1,3}$ (WM) short sub-portfolio for optimizations performed on a monthly basis between 2001 and 2016 using the universe of securities (data set) including the component of the Dow Jones Industrial Average. More precisely, at month day a Short portfolio is built according to the hybrid strategy, i.e. the short portfolio sells stocks having a negative momentum (i.e. a positive Δ) and a downward accelerating price (i.e. a positive Γ), both quantified at the resolution level $j = 3$. Factors are detected with the MODWT approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time by $s = 1$ months. This analysis is performed using MATLAB.

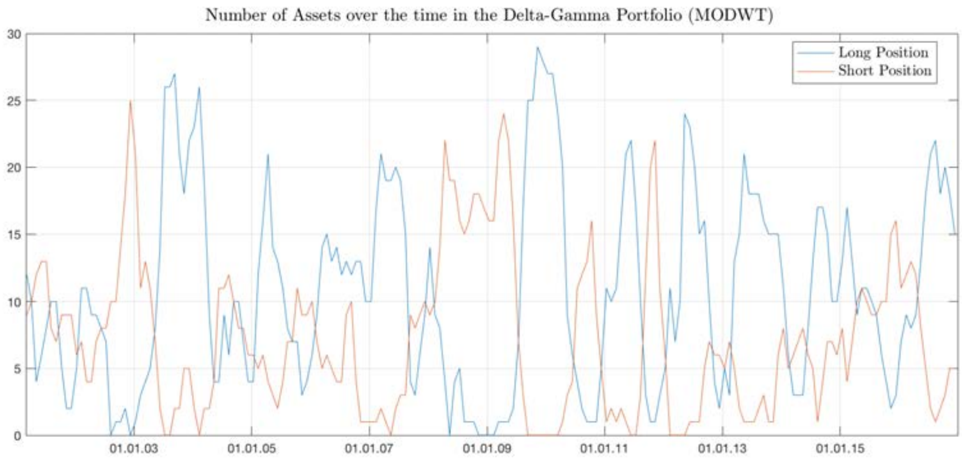

Figure A16: The chart indicates the number of total assets in the $(\Delta/\Gamma)_{1,3}$ (WM) portfolio for optimizations performed on a monthly basis between 2001 and 2016 using the universe of securities (data set) including the component of the Dow Jones Industrial Average. The blue line shows the total assets held in the long portfolio while the orange line the number of securities held in the short portfolio. More precisely, at each day a long and a short sub-portfolio is built according to the hybrid strategy, i.e. the long portfolio buys stocks with a positive momentum (i.e. a positive Δ) and with an upward accelerating price (i.e. a positive Γ) and the short portfolio sells stocks having a negative momentum (i.e. a positive Δ) and a downward accelerating price (i.e. a positive Γ), both quantified at the resolution level $j = 3$. Factors are detected with the MODWT approach. The portfolio has a holding period of one month ($h = 1$); moreover, the investment is delayed in time by $s = 1$ months. This analysis is performed using MATLAB.

Tables

Detail coefficients calibration (MODWT)

Level (j)	Calibration First Derivative (time units) (i)	Calibration Second Derivative (time units) (i)
1	1	4
2	2	5
3	3	10
4	6	20
5	8	25

Table A1: According to simulated pure signal, there is a lag in the first and second derivative computed with the MODWT approach. The first derivative is computed applying a MODWT with a Haar (also named Daubechies 1) mother wavelet function (Db1) whilst the second derivative is calculated through a Daubechies function with two vanishing moments (Db2). Since the lag is almost the same for each signal and growth proportional to the level (j), it might be measured in order to calibrate both derivatives in stock prices during the wavelet detection. This table shows the magnitude of the lag according to the first and the second derivative and according to the resolution level j. Since the MODWT is applied to daily or monthly stock prices, the lag is given in time units (i), i.e. days or months. The analysis is performed using MATLAB.

Conversion of the MODWT resolution level (j) to
scale and time-scale format

Level (j)	Scale (2^j)	Time-Scale in days (months) (f)
1	2	within 1 and 2 days (months)
2	4	within 2 and 4 days (months)
3	8	within 4 and 8 days (months)
4	16	within 8 and 16 days (months)
5	32	within 16 and 32 days (months)
6	64	within 32 and 64 days (months)
7	128	within 64 and 128 days (months)
8	256	within 128 and 256 days (months)

Table A2: The table shows the conversion of different levels of resolution (j) resulting from a Wavelet Transform decomposition to the respective time-scale (2^j) (i.e. the equivalent to the formation period). For example, a $\Delta_{i,t}(j)$ detected at a level j= 4 quantifies changes in stock log-prices which results in a time frame between 8 and 16 days (using daily stock prices as input variable) or between 8 and 16 months (performing the analysis with monthly stock prices).

Annualized performance of Δ_{LS} and Γ_{LS} (simple)
DJIA (2001-2016)

		Δ_{LS}				Γ_{LS}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h=1$				
0	1	-4.35	18.31	-0.24	-4.43	-1.42	17.05	-0.08	-1.53
	3	-3.87	19.29	-0.2	-3.73	0.23	16.38	0.01	0.25
	6	-5.44	19.91	-0.27	-5.12	-6.73	19.14	-0.35	-6.63
	12	-2.43	21.5	-0.11	-2.09	-1.59	18.57	-0.09	-1.58
1	1	-1.1	17.45	-0.06	-1.15	-1.15	16.38	-0.07	-1.29
	3	-3.37	19.03	-0.18	-3.28	0.23	16.75	0.01	0.25
	6	-3.41	19.29	-0.18	-3.27	-3.53	18.51	-0.19	-3.53
	12	-2.52	21.08	-0.12	-2.2	-1.43	18.27	-0.08	-1.43
6	1	-1.26	16.46	-0.08	-1.39	-1.3	16.17	-0.08	-1.46
	3	2.8	17.72	0.16	2.8	-4.2	15.96	-0.26	-4.83
	6	4.39	18.4	0.24	4.21	1.97	16.27	0.12	2.16
	12	1.66	19.38	0.09	1.53	-1.22	16.28	-0.07	-1.35
					$h=3$				
0	1	-2.5	16.94	-0.15	-4.68	-0.87	16.05	-0.05	-1.71
	3	-3.31	18.5	-0.18	-5.71	-1.26	16.2	-0.08	-2.45
	6	-3.28	18.13	-0.18	-5.76	-4.34	18.75	-0.23	-7.41
	12	-2.21	20.19	-0.11	-3.47	-0.81	17.41	-0.05	-1.47
1	1	-2.03	16.44	-0.12	-3.91	-1.11	15.57	-0.07	-2.25
	3	-2.7	18.52	-0.15	-4.63	-1.93	17.14	-0.11	-3.56
	6	-1.7	17.75	-0.1	-3.02	-3.05	17.44	-0.17	-5.55
	12	-1.94	20.15	-0.1	-3.05	-0.47	17.71	-0.03	-0.83
6	1	-0.4	15.62	-0.03	-0.8	-1.19	15.25	-0.08	-2.42
	3	2.08	18.12	0.11	3.53	-1.08	15.57	-0.07	-2.15
	6	1.55	18.39	0.08	2.59	1.21	16.97	0.07	2.2
	12	0.87	19.26	0.05	1.39	-2.87	16.52	-0.17	-5.43
					$h=6$				
0	1	-1.5	16.51	-0.09	-4.03	-0.58	16.38	-0.04	-1.58
	3	-1.75	17.31	-0.1	-4.48	-1.76	15.31	-0.12	-5.1
	6	0.13	19.67	0.01	0.3	-2.52	18.86	-0.13	-5.93
	12	-0.47	21.67	-0.02	-0.97	-1.21	18.44	-0.07	-2.9
1	1	-0.85	15.84	-0.05	-2.36	-0.6	15.61	-0.04	-1.71
	3	-0.76	18.39	-0.04	-1.82	-2.62	15.82	-0.17	-7.33
	6	1.69	20.51	0.08	3.61	-1.27	18.3	-0.07	-3.06
	12	0.32	21.75	0.01	0.65	-1.08	18.19	-0.06	-2.63
6	1	1.77	14.87	0.12	5.14	-0.8	14.84	-0.05	-2.33
	3	2.07	17.39	0.12	5.14	1	15.69	0.06	2.77
	6	1.2	17.41	0.07	2.97	0.53	17.31	0.03	1.33
	12	1.76	18.64	0.09	4.09	-1.98	15.59	-0.13	-5.55
					$h=12$				
0	1	0.27	17.18	0.02	0.95	-0.47	16.16	-0.03	-1.8
	3	0.39	18.94	0.02	1.25	-0.13	16.21	-0.01	-0.51
	6	0.91	21.31	0.04	2.63	-1.52	19.46	-0.08	-4.78
	12	1.24	22.12	0.06	3.43	-1.32	18.39	-0.07	-4.42
1	1	0.6	16.45	0.04	2.24	-0.52	15.65	-0.03	-2.05
	3	0.46	19.37	0.02	1.47	-0.49	16.63	-0.03	-1.79
	6	1.58	21.19	0.07	4.56	-0.92	18.89	-0.05	-2.99
	12	1.5	22.04	0.07	4.17	-1.32	18.34	-0.07	-4.39
6	1	1.36	15.45	0.09	5.3	-0.3	15.53	-0.02	-1.17
	3	1.12	17.84	0.06	3.79	-0.03	16.26	0	-0.11
	6	1.69	18.34	0.09	5.56	-0.19	17.81	-0.01	-0.63
	12	1.52	18.29	0.08	5	-1.69	17.48	-0.1	-5.83

Table A3: The figure shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ portfolios set up considering daily dividend-adjusted log-returns of securities included in the Dow Jones Industrial Average (2001-2016). At each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked quintile while the short portfolio sells stocks of the bottom-ranked quintile. Equal weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$ factors are detected through the simple approach using different formation periods (f) expressed in months. Moreover, the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of $\Delta_{LS(Long)}$ and $\Gamma_{LS(Long)}$ (simple)
DJIA (2001-2016)

		$\Delta_{LS(Long)}$				$\Gamma_{LS(Long)}$			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	9.63	18.68	0.52	9.02	10.93	20	0.55	9.51
0	3	9.55	17.52	0.54	9.54	11.81	19.37	0.61	10.58
	6	8.86	17.14	0.52	9.08	7.92	19.44	0.41	7.19
	12	13.09	16.68	0.78	13.53	12.79	17.26	0.74	12.8
	1	10.98	18.43	0.6	10.34	12.08	19.89	0.61	10.49
1	3	8.9	17.73	0.5	8.79	10.45	19.51	0.54	9.33
	6	10.89	17.24	0.63	10.97	9.17	19.25	0.48	8.34
	12	12.39	16.61	0.75	12.88	12.76	17.6	0.73	12.5
	1	12.4	19.81	0.63	10.66	13.37	20.02	0.67	11.33
6	3	13.37	18.86	0.71	12.02	9.03	19.66	0.46	7.93
	6	15.21	17.5	0.87	14.63	15.75	20.22	0.78	13.09
	12	13.71	17	0.81	13.66	11.9	18.05	0.66	11.25
					$h = 3$				
	1	10.2	17.14	0.59	18.05	11.21	17.94	0.63	18.89
0	3	9.55	17.19	0.56	16.89	10.26	18	0.57	17.29
	6	10.37	17.66	0.59	17.8	8.85	20.05	0.44	13.46
	12	12.19	15.65	0.78	23.48	12.03	16.15	0.75	22.46
	1	10.6	17.93	0.59	17.86	11.46	17.91	0.64	19.28
1	3	10.35	18.34	0.56	17.07	10.04	19.01	0.53	16
	6	11.72	17.73	0.66	19.89	9.61	20.21	0.48	14.42
	12	11.75	15.66	0.75	22.59	11.93	16.43	0.73	21.83
	1	12.21	18.21	0.67	19.87	11.79	18.79	0.63	18.63
6	3	13.24	16.1	0.82	24.29	10.4	17.58	0.59	17.65
	6	13.41	15.19	0.88	26.07	14.05	18.43	0.76	22.45
	12	13.39	15.37	0.87	25.73	11.21	17.54	0.64	19.01
					$h = 6$				
	1	10.66	18.9	0.56	24.29	11.46	18.99	0.6	25.91
0	3	11.17	19.48	0.57	24.64	10.14	20.13	0.5	21.71
	6	12.53	18.44	0.68	29.12	10.22	20.03	0.51	21.98
	12	12.22	16.36	0.75	32.04	11.77	17.13	0.69	29.5
	1	11.19	18.93	0.59	25.33	11.86	18.55	0.64	27.36
1	3	11.82	19.8	0.6	25.56	9.72	20.79	0.47	20.12
	6	13.51	17.62	0.77	32.71	11.29	18.85	0.6	25.65
	12	12.35	16.32	0.76	32.34	11.74	17.26	0.68	29.12
	1	13.38	17.79	0.75	31.64	12.05	18.9	0.64	26.91
6	3	12.71	15.78	0.81	33.94	12.39	17.28	0.72	30.23
	6	12.4	14.97	0.83	34.91	13.48	20.02	0.67	28.33
	12	13.74	14.64	0.94	39.44	10.98	18.01	0.61	25.79
					$h = 12$				
	1	11.83	19.38	0.61	37.5	11.56	19.73	0.59	36
0	3	11.9	19.12	0.62	38.23	11.14	20.47	0.54	33.43
	6	12.43	17.28	0.72	44.17	11.24	18.98	0.59	36.37
	12	12.9	15.69	0.82	50.5	11.46	18.07	0.63	38.97
	1	12.18	19.07	0.64	39.1	11.63	19.44	0.6	36.66
1	3	12.1	18.64	0.65	39.77	11.17	19.94	0.56	34.31
	6	12.85	16.8	0.76	46.83	11.72	18.88	0.62	38.02
	12	12.9	15.46	0.83	51.1	11.39	17.9	0.64	38.96
	1	13.02	18.58	0.7	42.34	12.02	19.4	0.62	37.43
6	3	12.58	17.29	0.73	43.94	11.99	18.7	0.64	38.73
	6	13.32	16.91	0.79	47.56	13.07	20.81	0.63	37.94
	12	12.96	15.42	0.84	50.73	10.95	17.51	0.63	37.75

Table A4: The figure shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ long sub-portfolios set up considering daily dividend-adjusted log-returns of securities included in the Dow Jones Industrial Average (2001-2016). At each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked quintile. Equal weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$ factors are detected through the simple approach using different formation periods (f) expressed in months. Moreover, the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of $\Delta_{LS(Short)}$ and $\Gamma_{LS(Short)}$ (simple)
DJIA (2001-2016)

			$\Delta_{LS(Short)}$				$\Gamma_{LS(Short)}$		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	14.56	23.14	0.63	10.79	12.51	21.4	0.58	10.11
0	3	13.91	25.04	0.56	9.55	11.56	22.17	0.52	9.05
	6	15.05	25.46	0.59	10.12	15.61	23.51	0.66	11.34
	12	15.87	26.06	0.61	10.39	14.59	24.69	0.59	10.13
	1	12.2	23.19	0.53	9.09	13.37	21.72	0.62	10.58
1	3	12.66	24.62	0.51	8.86	10.21	22.08	0.46	8.05
	6	14.76	24.31	0.61	10.38	13.12	22.17	0.59	10.19
	12	15.26	25.79	0.59	10.09	14.38	24.25	0.59	10.15
	1	13.82	20.49	0.67	11.42	14.85	20.52	0.72	12.2
6	3	10.31	22.42	0.46	7.9	13.76	20.46	0.67	11.38
	6	10.4	23.32	0.45	7.66	13.54	20.65	0.66	11.12
	12	11.87	23.67	0.5	8.56	13.27	22.12	0.6	10.17
					$h = 3$				
	1	12.93	22.15	0.58	17.55	12.16	20.87	0.58	17.55
0	3	13.19	22.51	0.59	17.59	11.63	20.91	0.56	16.79
	6	13.99	22.47	0.62	18.65	13.63	20.16	0.68	20.28
	12	14.64	24.5	0.6	17.85	12.92	22.97	0.56	16.9
	1	12.83	21.21	0.6	18.13	12.68	20.6	0.62	18.46
1	3	13.32	21.7	0.61	18.38	12.15	20.31	0.6	17.97
	6	13.58	22.23	0.61	18.28	12.95	19.03	0.68	20.39
	12	13.89	24.54	0.57	16.92	12.44	22.97	0.54	16.25
	1	12.65	20.43	0.62	18.32	13.09	19.56	0.67	19.78
6	3	10.99	22.41	0.49	14.6	11.57	19.76	0.59	17.39
	6	11.73	22.87	0.51	15.23	12.73	20.91	0.61	18.01
	12	12.45	22.66	0.55	16.27	14.38	21.57	0.67	19.62
					$h = 6$				
	1	12.25	21.44	0.57	24.51	12.07	20.82	0.58	24.87
0	3	13.03	22.49	0.58	24.79	12.01	19.15	0.63	26.89
	6	12.39	23.39	0.53	22.71	12.89	21.29	0.61	25.93
	12	12.72	24.86	0.51	21.92	13.05	23.88	0.55	23.4
	1	12.08	20.99	0.58	24.62	12.5	21.01	0.59	25.43
1	3	12.63	22.74	0.56	23.73	12.5	19.23	0.65	27.77
	6	11.73	23.93	0.49	20.98	12.63	21.42	0.59	25.2
	12	12.01	24.75	0.49	20.76	12.89	23.76	0.54	23.17
	1	11.51	21.14	0.54	23.02	12.9	20.46	0.63	26.55
6	3	10.53	22.84	0.46	19.53	11.33	20.94	0.54	22.88
	6	11.14	22.44	0.5	20.99	12.92	21.5	0.6	25.31
	12	11.87	23.12	0.51	21.68	13.08	22.28	0.59	24.72
					$h = 12$				
	1	11.56	21.77	0.53	32.64	12.04	20.62	0.58	35.86
0	3	11.51	23.13	0.5	30.57	11.27	19.24	0.59	35.98
	6	11.52	24.11	0.48	29.35	12.76	22.96	0.56	34.12
	12	11.67	24.78	0.47	28.92	12.79	24.29	0.53	32.33
	1	11.57	21.81	0.53	32.5	12.16	20.85	0.58	35.72
1	3	11.64	23.35	0.5	30.53	11.66	19.98	0.58	35.73
	6	11.27	24.2	0.47	28.52	12.64	23.17	0.55	33.42
	12	11.4	24.56	0.46	28.42	12.7	24.38	0.52	31.91
	1	11.67	20.91	0.56	33.69	12.32	20.11	0.61	37.01
6	3	11.46	22.37	0.51	30.95	12.02	22.02	0.55	32.98
	6	11.63	22.09	0.53	31.79	13.26	21.43	0.62	37.36
	12	11.44	22.67	0.5	30.48	12.63	23.01	0.55	33.16

Table A5: The figure shows the annualized performance of different $\Delta_{s,f}$ and $\Gamma_{s,f}$ short sub-portfolios set up considering daily dividend-adjusted log-returns of securities included in the Dow Jones Industrial Average (2001-2016). At each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The short portfolio is constructed selling stocks of the bottom-ranked quintile. Equal weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$ factors are detected through the simple approach using different formation periods (f) expressed in months. Moreover, the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).The analysis has been performed through MATLAB

Annualized performance of Δ_{LS} and Γ_{LS} (simple)
DJIA (1984-2002)

		Δ_{LS}				Γ_{LS}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-13.7	24.91	-0.55	-11.73	-9.17	25.45	-0.36	-7.51
0	3	15.75	28.37	0.56	10.35	-5.48	26.22	-0.21	-4.28
	6	22.21	29.54	0.75	13.67	-10.22	26.64	-0.38	-8.04
	12	32.95	30.93	1.07	18.6	10.18	27.42	0.37	7.09
	1	-3.42	25.97	-0.13	-2.66	-21.51	28.28	-0.76	-16.89
1	3	17.86	29.09	0.61	11.32	-5.15	25.82	-0.2	-4.07
	6	26.55	29.26	0.91	16.19	-5.52	26.08	-0.21	-4.32
	12	37.89	30.8	1.23	21.05	14.38	27.34	0.53	9.84
	1	-1.71	25.5	-0.07	-1.33	-2.37	25.39	-0.09	-1.86
6	3	24.32	28.19	0.86	15.34	-2.74	25.45	-0.11	-2.15
	6	38.63	28.59	1.35	22.81	19.2	26.36	0.73	13.22
	12	28.12	29.75	0.95	16.58	11.69	26.91	0.43	8.13
					$h = 3$				
	1	2.95	25.41	0.12	3.95	-3.1	25.93	-0.12	-4.16
0	3	19.11	29.42	0.65	20.91	-5.63	25.48	-0.22	-7.76
	6	27.44	30.07	0.91	28.61	-6.44	27.12	-0.24	-8.38
	12	38.39	30.95	1.24	37.65	12.81	27.8	0.46	15.15
	1	6.63	26.43	0.25	8.41	0.75	27.12	0.03	0.95
1	3	21	30.24	0.69	22.17	-4.29	26.17	-0.16	-5.72
	6	30.77	28.78	1.07	33.11	-1.73	26.24	-0.07	-2.28
	12	40.53	30.63	1.32	39.81	14.16	27.52	0.51	16.8
	1	10.23	25.9	0.39	12.92	-1	26.64	-0.04	-1.28
6	3	27.06	29.32	0.92	28.58	0.06	24.99	0	0.08
	6	36.96	29.8	1.24	37.29	17.51	25.74	0.68	21.72
	12	28.01	30.11	0.93	28.72	13.66	27	0.51	16.36
					$h = 6$				
	1	7.13	33.35	0.21	10.16	-1.73	33.36	-0.05	-2.51
0	3	27.91	40.88	0.68	30.98	-3.88	34.03	-0.11	-5.57
	6	37.05	39.57	0.94	41.71	-0.46	32.46	-0.01	-0.68
	12	45.31	41.36	1.1	48.03	17.34	36.88	0.47	21.83
	1	9.86	33.83	0.29	13.72	-0.57	34.57	-0.02	-0.8
1	3	29.64	39.62	0.75	33.75	-3.18	32.88	-0.1	-4.7
	6	40.01	38	1.05	46.54	5.04	32.14	0.16	7.47
	12	44.56	41.08	1.09	47.52	17.79	36.85	0.48	22.33
	1	13.6	32.64	0.42	19.24	-0.6	33.86	-0.02	-0.85
6	3	34.38	39.15	0.88	38.79	5.35	32.28	0.17	7.81
	6	38.68	38.56	1	43.94	16.42	34.01	0.48	22.16
	12	31.93	39.86	0.8	35.56	16.51	37.26	0.44	20.32
					$h = 12$				
	1	14.58	63.48	0.23	15.49	-2.31	68.61	-0.03	-2.27
0	3	46.41	78.7	0.59	39.77	1.04	61.86	0.02	1.13
	6	54.9	77.87	0.71	47.55	11.83	65.5	0.18	12.18
	12	56.44	81.72	0.69	46.58	28.11	75.28	0.37	25.18
	1	16.93	64.38	0.26	17.7	-0.24	70.5	0	-0.23
1	3	46.12	77.78	0.59	39.9	2.37	60.71	0.04	2.63
	6	53.87	76.77	0.7	47.22	13.12	66.38	0.2	13.3
	12	54.96	81.64	0.67	45.3	28.55	76.18	0.37	25.22
	1	15.12	63.82	0.24	15.76	-0.16	67.41	0	-0.16
6	3	39.4	76.25	0.52	34.36	4.82	60.69	0.08	5.28
	6	43.77	77.3	0.57	37.65	15.17	65.57	0.23	15.39
	12	42.45	77.04	0.55	36.64	26.91	71.95	0.37	24.87

Table A6: The figure shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ portfolios set up considering daily dividend-adjusted log-returns of securities included in the Dow Jones Industrial Average (1983-2002). At each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked quintile while the short portfolio sells stocks of the bottom-ranked quintile. Equal weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$ factors are detected through the simple approach using different formation periods (f) expressed in months. Moreover, the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB

Annualized performance of Δ_{RSWP} and Γ_{RSWP} (simple)
DJIA (2001-2016)

			Δ_{RSWP}				Γ_{RSWP}		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-3.34	19.46	-0.17	-3.19	-1.54	15.94	-0.1	-1.78
0	3	-3.14	21.2	-0.15	-2.75	-0.47	16.38	-0.03	-0.52
	6	-4.35	21.12	-0.21	-3.84	-7.62	19.77	-0.39	-7.3
	12	0.14	22.8	0.01	0.11	-2.51	17.84	-0.14	-2.61
	1	-2.16	18.02	-0.12	-2.2	-1.58	15.54	-0.1	-1.87
1	3	-4.17	19.91	-0.21	-3.89	-0.71	17.38	-0.04	-0.75
	6	-3.14	19.75	-0.16	-2.94	-5.83	17.87	-0.33	-6.11
	12	-0.54	21.88	-0.02	-0.45	-1.89	16.78	-0.11	-2.07
	1	-1.42	15.97	-0.09	-1.61	-1.75	15.04	-0.12	-2.11
6	3	0.35	16.96	0.02	0.37	-5.22	14.86	-0.35	-6.47
	6	5.53	17.26	0.32	5.62	4.92	15.79	0.31	5.49
	12	2.59	19.71	0.13	2.33	-0.58	15.61	-0.04	-0.67
					$h = 3$				
	1	-1.89	17.19	-0.11	-3.49	-0.48	14.58	-0.03	-1.05
0	3	-2.89	18.96	-0.15	-4.85	-1.36	15.4	-0.09	-2.8
	6	-2.2	18.66	-0.12	-3.74	-5.07	18.82	-0.27	-8.65
	12	0.58	20.98	0.03	0.87	-1.31	16.38	-0.08	-2.52
	1	-2.14	16.5	-0.13	-4.1	-0.94	14.74	-0.06	-2.01
1	3	-2.53	18.53	-0.14	-4.33	-1.99	16.79	-0.12	-3.74
	6	-1.35	17.88	-0.08	-2.38	-3.79	16.55	-0.23	-7.3
	12	0.42	20.99	0.02	0.63	-0.82	16.34	-0.05	-1.58
	1	-0.44	14.97	-0.03	-0.91	-1.03	13.81	-0.07	-2.32
6	3	1.71	17.24	0.1	3.05	-1.52	14.56	-0.1	-3.26
	6	3.7	17.57	0.21	6.44	3.07	15.49	0.2	6.07
	12	2.29	19.6	0.12	3.59	-1.79	15.54	-0.12	-3.59
					$h = 6$				
	1	-1.18	16.83	-0.07	-3.11	-0.46	14.56	-0.03	-1.4
0	3	-1.26	18.39	-0.07	-3.03	-1.88	14.33	-0.13	-5.83
	6	0.32	20.83	0.02	0.68	-3.21	19.02	-0.17	-7.5
	12	1.83	22.1	0.08	3.63	-0.86	17.55	-0.05	-2.16
	1	-0.91	15.95	-0.06	-2.52	-0.69	14.7	-0.05	-2.08
1	3	-0.84	18.91	-0.04	-1.96	-3.01	15.83	-0.19	-8.42
	6	1.91	20.93	0.09	4	-1.27	17.12	-0.07	-3.28
	12	2.15	22.14	0.1	4.26	-0.54	16.83	-0.03	-1.41
	1	2.13	13.94	0.15	6.61	-0.49	13.22	-0.04	-1.62
6	3	3.24	16.73	0.19	8.34	1.65	14.13	0.12	5.04
	6	3.23	16.48	0.2	8.44	1.97	16.31	0.12	5.22
	12	3.19	19.08	0.17	7.19	-1.47	14.91	-0.1	-4.31
					$h = 12$				
	1	0.69	16.52	0.04	2.55	-0.3	14.51	-0.02	-1.28
0	3	1.41	18.58	0.08	4.66	0.11	14.55	0.01	0.46
	6	2.16	20.91	0.1	6.34	-1.14	18	-0.06	-3.88
	12	3.21	21.28	0.15	9.26	-0.99	16.59	-0.06	-3.68
	1	0.89	15.69	0.06	3.46	-0.31	14.3	-0.02	-1.31
1	3	1.38	18.37	0.08	4.6	-0.28	14.94	-0.02	-1.15
	6	2.57	20.66	0.12	7.63	-0.53	17.09	-0.03	-1.91
	12	3.32	20.99	0.16	9.69	-0.85	16.32	-0.05	-3.2
	1	1.53	14.66	0.1	6.28	-0.12	14.83	-0.01	-0.47
6	3	1.99	16.92	0.12	7.11	-0.09	15.3	-0.01	-0.35
	6	3.15	16.69	0.19	11.38	0.68	16.46	0.04	2.49
	12	3.41	17.78	0.19	11.57	-1.15	15.66	-0.07	-4.43

Table A7: The figure shows the annualized performance of different $\Delta_{s,f(RSWP)}$ and $\Gamma_{s,f(RSWP)}$ portfolios set up considering securities included in the DJIA (2001-2016). The $\Gamma_{s,f}$ portfolio is constructed as follows: at each day t, the weight of a specific security i is determined by the magnitude of its Γ factor computed the previous day $t-1-s$ (where s represents the delay in the investment in months) over a formation of f months ($\Gamma_{i,t-1}(f)$) relatively to the Γ of the equal weighted index (the market) ($\Gamma_{m,t-1}(f)$). Hence, $w^{\Gamma}_{i,t}(f,s) = \frac{1}{N}(\Gamma_{i,t-1-s}(f) - \Gamma_{m,t-1-s}(f))$ where $\sum_{i=1}^{N} w^{\Gamma}_{i,t}(f,s) = 0$. In order to get market-neutral weights, there is a standardization, i.e. weights of the long positions add up to one (+1) and the weights of the short position add up to minus one (-1). The Δ portfolio is computed similarly. Each portfolio is held for h months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ^{D}_{RSWP} and Γ^{D}_{RSWP} (simple)
DJIA (2001-2016)

		Δ^{D}_{RSWP}				Γ^{D}_{RSWP}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
$h = 1$									
0	1	-3.2	18.62	-0.17	-3.18	-1.65	15.91	-0.1	-1.91
	3	-2.94	19.74	-0.15	-2.75	-0.42	15.87	-0.03	-0.48
	6	-3.77	19.55	-0.19	-3.58	-7.37	18.71	-0.39	-7.45
	12	1.15	21.17	0.05	0.99	-1.35	16.98	-0.08	-1.46
1	1	-2.09	17.51	-0.12	-2.19	-1.4	15.37	-0.09	-1.67
	3	-4.04	18.84	-0.21	-3.98	-0.76	16.86	-0.04	-0.82
	6	-2.68	18.85	-0.14	-2.62	-6.28	17.92	-0.35	-6.58
	12	0.19	20.68	0.01	0.16	-1.03	16.35	-0.06	-1.15
6	1	-1.25	15.77	-0.08	-1.44	-1.64	14.94	-0.11	-1.99
	3	0.75	16.58	0.05	0.81	-5.24	14.8	-0.35	-6.53
	6	5.9	16.92	0.35	6.11	6.01	16.24	0.37	6.47
	12	2.89	19	0.15	2.7	-0.62	15.28	-0.04	-0.73
$h = 3$									
0	1	-1.82	16.69	-0.11	-3.46	-0.47	14.57	-0.03	-1.02
	3	-2.77	18.31	-0.15	-4.82	-1.3	15.2	-0.09	-2.7
	6	-1.84	17.94	-0.1	-3.25	-5.27	18.4	-0.29	-9.2
	12	1.23	20.31	0.06	1.9	-0.63	15.71	-0.04	-1.26
1	1	-2.1	16.25	-0.13	-4.09	-0.83	14.58	-0.06	-1.78
	3	-2.42	18.02	-0.13	-4.25	-1.74	16.6	-0.1	-3.3
	6	-0.96	17.53	-0.05	-1.73	-4.05	16.82	-0.24	-7.68
	12	0.97	20.43	0.05	1.48	-0.36	15.85	-0.02	-0.71
6	1	-0.35	14.92	-0.02	-0.73	-0.99	13.89	-0.07	-2.22
	3	2.06	17.15	0.12	3.68	-1.38	14.61	-0.09	-2.93
	6	3.91	17.53	0.22	6.81	3.57	15.7	0.23	6.95
	12	2.65	19.45	0.14	4.18	-1.7	15.3	-0.11	-3.46
$h = 6$									
0	1	-1.1	16.26	-0.07	-3	-0.38	14.57	-0.03	-1.15
	3	-1.14	17.57	-0.06	-2.86	-1.81	13.95	-0.13	-5.74
	6	0.73	20.04	0.04	1.6	-3.05	18.04	-0.17	-7.52
	12	2.28	21.75	0.11	4.61	-0.37	16.88	-0.02	-0.98
1	1	-0.82	15.63	-0.05	-2.32	-0.56	14.48	-0.04	-1.7
	3	-0.67	18.24	-0.04	-1.61	-2.9	15.19	-0.19	-8.47
	6	2.29	20.69	0.11	4.85	-1.05	17.01	-0.06	-2.73
	12	2.53	21.96	0.12	5.04	-0.21	16.53	-0.01	-0.55
6	1	2.18	14	0.16	6.72	-0.43	13.24	-0.03	-1.43
	3	3.37	17.05	0.2	8.52	1.81	14.34	0.13	5.45
	6	3.35	16.87	0.2	8.56	2.42	16.66	0.14	6.26
	12	3.52	19.45	0.18	7.79	-1.54	14.91	-0.1	-4.51
$h = 12$									
0	1	0.74	16.56	0.04	2.74	-0.23	14.46	-0.02	-0.99
	3	1.51	19	0.08	4.88	0.19	14.6	0.01	0.81
	6	2.43	21.58	0.11	6.91	-0.9	18.15	-0.05	-3.04
	12	3.57	22.21	0.16	9.87	-0.73	16.61	-0.04	-2.72
1	1	0.94	15.81	0.06	3.64	-0.2	14.26	-0.01	-0.87
	3	1.49	18.73	0.08	4.88	-0.16	14.92	-0.01	-0.64
	6	2.82	21.31	0.13	8.11	-0.27	17.41	-0.02	-0.95
	12	3.65	21.74	0.17	10.29	-0.71	16.44	-0.04	-2.66
6	1	1.59	14.74	0.11	6.52	-0.01	14.9	0	-0.06
	3	2.13	17.15	0.12	7.48	0.05	15.4	0	0.19
	6	3.35	16.98	0.2	11.92	1.05	16.58	0.06	3.83
	12	3.75	18.15	0.21	12.48	-1.22	16.16	-0.08	-4.56

Table A8: The figure shows the annualized performance of different $\Delta_{s,f}{}^{D}_{(RSWP)}$ and $\Gamma_{s,f}{}^{D}_{(RSWP)}$ portfolios set up considering securities included in the DJIA (2001-2016). The $\Gamma^{D}_{s,f}$ portfolio is constructed as follows: at each day t, the weight of a specific security i is determined by the magnitude of its (discretized) Γ^{D} factor computed the previous day $t-1-s$ (where s represents the delay in the investment in months) over a formation of f months ($\Gamma^{D}_{i,t-1}(f)$) with the (discretized) acceleration factor of the equal weighted index (the market) ($\Gamma^{D}_{m,t-1}(f)$). Hence, $w^{\Gamma^{D}}_{i,t}(f,s) = \frac{1}{N}(\Gamma^{D}_{i,t-1-s}(f) - \Gamma^{D}_{m,t-1-s}(f))$ where $\sum_{i=1}^{N} w^{\Gamma^{D}}_{i,t}(f,s) = 0$. In order to get market-neutral weights, there is a standardization. The Δ portfolio is computed similarly. The portfolio is held for h months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ^{D}_{RSWP} and Γ^{D}_{RSWP} (simple) DJIA (2001-2016)
($f = 5$ days)

			Δ^{D}_{RSWP}				Γ^{D}_{RSWP}		
h	s	μ	σ	SR	t-test	μ	σ	SR	t-test
					f = 5 days				
1	0	-3.64	18.89	-0.19	-3.58	-1.95	18.36	-0.11	-1.96
	1	-0.46	16.48	-0.03	-0.51	-0.71	16.55	-0.04	-0.79
	6	-0.53	15.78	-0.03	-0.61	-0.75	15.37	-0.05	-0.88
3	0	-1.73	17.34	-0.1	-3.15	-0.86	16.93	-0.05	-1.6
	1	-0.26	16.17	-0.02	-0.51	0.13	16.14	0.01	0.25
	6	-0.17	15.46	-0.01	-0.34	-0.07	14.73	0	-0.14
6	0	-0.81	17.21	-0.05	-2.07	-0.33	17.11	-0.02	-0.84
	1	-0.07	15.76	0	-0.21	-0.04	16.35	0	-0.1
	6	1.5	14.83	0.1	4.37	0.18	14.19	0.01	0.54
12	0	0.38	16.48	0.02	1.43	-0.04	16.02	0	-0.17
	1	0.49	16.12	0.03	1.85	-0.02	15.74	0	-0.07
	6	0.69	15.33	0.05	2.73	0.02	14.85	0	0.07

Table A9: The figure shows the annualized performance of different $\Delta_{s,f}{}^{D}_{(RSWP)}$ and $\Gamma_{s,f}{}^{D}_{(RSWP)}$ portfolios set up considering daily dividend-adjusted log-returns of securities included in the DJIA (2001-2016) and using a formation period (f) of 5 days (i.e. one week). The portfolio is held for h months and the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ^{D}_{RSWP} and Γ^{D}_{RSWP} (simple)
DJIA (1984-2002)

		Δ^{D}_{RSWP}				Γ^{D}_{RSWP}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-14.93	22.21	-0.67	-14.44	-9.67	23.45	-0.41	-8.62
0	3	19.4	30.39	0.64	11.73	-6.08	23.94	-0.25	-5.21
	6	32.13	35.9	0.9	15.67	-8.45	24.29	-0.35	-7.23
	12	53.21	41.22	1.29	21.03	14.21	26.79	0.53	9.95
	1	-1.45	23.83	-0.06	-1.22	-24.47	31.43	-0.78	-17.57
1	3	27.99	31.65	0.88	15.69	-2.69	23.97	-0.11	-2.26
	6	37.9	36.65	1.03	17.69	-4.91	23.3	-0.21	-4.3
	12	56.51	42.14	1.34	21.57	14.91	27.31	0.55	10.19
	1	-0.58	26.74	-0.02	-0.43	1.74	25.97	0.07	1.31
6	3	31.46	31.4	1	17.35	-4.67	23.83	-0.2	-3.95
	6	49.97	35.12	1.42	23.11	19.33	25.14	0.77	13.95
	12	46.31	40.68	1.14	18.72	17.12	29.54	0.58	10.61
					$h = 3$				
	1	8.99	28.44	0.32	10.53	-2.69	28.62	-0.09	-3.27
0	3	26.28	31.62	0.83	26.15	-3.78	23.78	-0.16	-5.55
	6	37.26	36.09	1.03	31.43	-5.28	23.9	-0.22	-7.76
	12	56.17	40.94	1.37	39.65	14.24	26.83	0.53	17.37
	1	14.12	29.41	0.48	15.68	1.5	30.44	0.05	1.68
1	3	29.59	32.9	0.9	27.95	-1.66	23.92	-0.07	-2.4
	6	40.57	35.64	1.14	34.25	-1.15	23.35	-0.05	-1.7
	12	55.86	41.13	1.36	39.18	14.76	28.21	0.52	17.05
	1	16.09	29.38	0.55	17.57	0.68	29.34	0.02	0.79
6	3	34.61	31.61	1.1	33.14	-0.99	23.72	-0.04	-1.42
	6	47.21	35.14	1.34	39.23	17.07	24.65	0.69	22.14
	12	45.08	41.23	1.09	32.11	16.66	28.43	0.59	18.76
					$h = 6$				
	1	15.64	42.76	0.37	17.04	-0.6	42.63	-0.01	-0.68
0	3	37.09	49.08	0.76	33.66	-2.23	31.1	-0.07	-3.48
	6	51.14	55.24	0.93	40.15	0.32	28.02	0.01	0.56
	12	67.38	64.79	1.04	43.84	19.75	37.92	0.52	24.05
	1	19.08	45.41	0.42	19.38	1.51	46.08	0.03	1.58
1	3	39.59	49.2	0.8	35.59	-1.68	30.1	-0.06	-2.7
	6	54.42	54.63	1	42.86	5.49	28.67	0.19	9.11
	12	66.54	65.51	1.02	42.78	20.95	40.52	0.52	23.75
	1	22.07	42.98	0.51	23.27	1.53	44.3	0.03	1.64
6	3	43.73	46.49	0.94	40.81	6.41	31.84	0.2	9.45
	6	51.83	52.64	0.98	42.07	15.16	32.9	0.46	21.21
	12	52.91	63.89	0.83	35.32	21.51	42.56	0.51	22.93
					$h = 12$				
	1	22.07	42.98	0.51	23.27	1.53	44.3	0.03	1.64
0	3	43.73	46.49	0.94	40.81	6.41	31.84	0.2	9.45
	6	51.83	52.64	0.98	42.07	15.16	32.9	0.46	21.21
	12	52.91	63.89	0.83	35.32	21.51	42.56	0.51	22.93
	1	32.5	104.9	0.31	20.85	3.16	109.9	0.03	1.93
1	3	62.24	109	0.57	38.43	4.41	61.01	0.07	4.86
	6	77.47	126.4	0.61	41.23	12.76	66.37	0.19	12.94
	12	89.86	153.6	0.58	39.36	35.55	92.29	0.39	25.92
	1	27.98	95.96	0.29	19.39	2.44	98.83	0.02	1.64
6	3	55.72	106	0.53	34.95	6.75	67.34	0.1	6.66
	6	67.52	120.1	0.56	37.4	16.05	70.37	0.23	15.17
	12	73.32	141.2	0.52	34.53	37.02	102	0.36	24.14

Table A10: The figure shows the annualized performance of different $\Delta_{s,f}{}^{D}_{(RSWP)}$ and $\Gamma_{s,f}{}^{D}_{(RSWP)}$ portfolios set up considering daily securities included in the DJIA (1984-2002). The $\Gamma^{D}_{s,f}$ portfolio is constructed as follows: at each day t, the weight of a specific security i is determined by the magnitude of its (discretized) Γ^{D} factor computed the previous day $t-1-s$ (where s represents the delay in the investment in months) over a formation of f months($\Gamma^{D}_{i,t-1}(f)$) with the (discretized) acceleration factor of the equal weighted index (the market) ($\Gamma^{D}_{m,t-1}(f)$). Hence, $w^{\Gamma^D}_{i,t}(f,s) = \frac{1}{N}(\Gamma^{D}_{i,t-1-s}(f) - \Gamma^{D}_{m,t-1-s}(f))$ where $\sum_{i=1}^{N} w^{\Gamma^D}_{i,t}(f,s) = 0$. In order to get market-neutral weights, there is a standardization. The Δ portfolio is computed similarly. Each portfolio is held for h months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{LS} and Γ_{LS} (simple)
S&P500 (2001-2014)

		Δ_{LS}				Γ_{LS}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-8.07	21.71	-0.37	-6.42	-2.44	18.37	-0.13	-2.23
0	3	-7.75	24.62	-0.31	-5.43	-7.06	17.51	-0.4	-6.93
	6	-7.22	28.64	-0.25	-4.33	-10.85	23.54	-0.46	-8.07
	12	-4.35	30.76	-0.14	-2.4	-3.3	26.56	-0.12	-2.1
	1	-7.03	21.55	-0.33	-5.58	-6.75	18.64	-0.36	-6.2
1	3	-3.16	23.89	-0.13	-2.23	-1.11	16.54	-0.07	-1.12
	6	-7.2	28.59	-0.25	-4.32	-5.27	21.26	-0.25	-4.21
	12	-5.81	30.73	-0.19	-3.22	-3.57	25.34	-0.14	-2.38
	1	-0.69	17.31	-0.04	-0.66	-3.15	16.33	-0.19	-3.19
6	3	-1.02	20.26	-0.05	-0.83	-4.01	19.14	-0.21	-3.48
	6	2.68	21.77	0.12	1.98	3.58	19.9	0.18	2.88
	12	-2.88	25.44	-0.11	-1.87	-1.87	20.83	-0.09	-1.48
					$h = 3$				
	1	-4.73	21.78	-0.22	-6.32	-3.88	17.24	-0.23	-6.54
0	3	-3.27	23.71	-0.14	-4	-3.72	17.12	-0.22	-6.3
	6	-4.74	28.95	-0.16	-4.77	-5.04	23.44	-0.21	-6.27
	12	-3.91	32.21	-0.12	-3.53	-1.43	26.82	-0.05	-1.53
	1	-1.05	21.18	-0.05	-1.42	-2.7	19.4	-0.14	-4.01
1	3	-0.57	23.63	-0.02	-0.69	-1.84	16.87	-0.11	-3.13
	6	-3.11	26.45	-0.12	-3.39	-2.72	21.75	-0.12	-3.6
	12	-3.59	31.1	-0.12	-3.34	0.02	24.7	0	0.02
	1	-0.43	15.69	-0.03	-0.77	-1.58	15.8	-0.1	-2.82
6	3	0.43	22.08	0.02	0.55	-1.77	18.62	-0.09	-2.68
	6	-0.34	22.68	-0.02	-0.42	1.14	19.22	0.06	1.66
	12	-0.02	0.26	0	-2.53	-0.03	0.2	0	-3.73
					$h = 6$				
	1	-1.96	22.83	-0.09	-3.45	-2.19	18.44	-0.12	-4.79
0	3	-1.7	26.49	-0.06	-2.58	-3.2	18.65	-0.17	-6.94
	6	-2.86	33.38	-0.09	-3.45	-3.64	28.04	-0.13	-5.25
	12	-4.13	35.21	-0.12	-4.75	-2.35	28.45	-0.08	-3.33
	1	-0.96	23.27	-0.04	-1.66	-2.63	20.33	-0.13	-5.2
1	3	-0.93	27.13	-0.03	-1.38	-3.11	19.34	-0.16	-6.48
	6	-1.14	31.51	-0.04	-1.45	-1.27	26.29	-0.05	-1.93
	12	-4.15	33.64	-0.12	-4.98	-2.24	26.44	-0.08	-3.41
	1	2.02	17.61	0.11	4.47	-0.93	17.58	-0.05	-2.09
6	3	0.93	22.2	0.04	1.63	2.19	19.08	0.11	4.48
	6	-1.42	23.17	-0.06	-2.42	-0.81	18.42	-0.04	-1.74
	12	-0.6	25.11	-0.02	-0.94	-2.39	19.64	-0.12	-4.81
					$h = 12$				
	1	-0.18	27.65	-0.01	-0.36	-1.18	23.25	-0.05	-2.82
0	3	-0.6	33.63	-0.02	-0.99	-0.57	24.01	-0.02	-1.32
	6	-2.7	40.85	-0.07	-3.67	-3.21	34.9	-0.09	-5.1
	12	-3.18	40.18	-0.08	-4.4	-2.79	33.57	-0.08	-4.62
	1	-0.34	29.02	-0.01	-0.64	-2.02	25.39	-0.08	-4.4
1	3	-0.45	34.69	-0.01	-0.72	-0.04	23.68	0	-0.08
	6	-2.48	39.77	-0.06	-3.45	-2.59	32.19	-0.08	-4.45
	12	-3.03	39.22	-0.08	-4.28	-2.71	32.15	-0.08	-4.67
	1	0.07	20.89	0	0.18	-1.33	20.67	-0.06	-3.49
6	3	-0.8	25.76	-0.03	-1.69	-1.52	21.77	-0.07	-3.8
	6	-1.6	25.35	-0.06	-3.44	-0.81	21.01	-0.04	-2.11
	12	-1.35	29.31	-0.05	-2.51	-3.03	23.42	-0.13	-7.04

Table A11: The figure shows the annualized performance of different $\Delta_{s,f(LS)}$ and $\Gamma_{s,f(LS)}$ portfolios set up considering daily split and dividend-adjusted log-returns of securities included in the Standard and Poor 500 (2001-2014). At each day t stocks are ranked ascending according to their delta ($\Delta_{i,t-1-s}(f)$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked decile while the short portfolio sells stocks of the bottom-ranked decile. Equal weights are applied and the portfolio is held for h months. $\Delta(f)$ and $\Gamma(f)$ factors are detected through the simple approach using different formation periods (f) expressed in months. Moreover, the investment might be delayed of s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR) . Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ^D_{RSWP} and Γ^D_{RSWP} (simple)
S&P500 (2001-2014)

		Δ^D_{RSWP}				Γ^D_{RSWP}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
				$h = 1$					
0	1	-6.56	17.54	-0.37	-6.41	0.4	16.73	0.02	0.39
	3	-5.23	19.08	-0.27	-4.66	-3.6	16.1	-0.22	-3.78
	6	-1.7	21.76	-0.08	-1.31	-4.84	19.7	-0.25	-4.18
	12	0.61	23.51	0.03	0.43	6.01	53.88	0.11	1.8
1	1	-7.27	17.1	-0.43	-7.29	-5.62	15.8	-0.36	-6.05
	3	-2.07	18.46	-0.11	-1.87	-0.42	15.55	-0.03	-0.45
	6	-0.93	20.85	-0.04	-0.75	-2.99	18.46	-0.16	-2.72
	12	-1.6	22.45	-0.07	-1.19	0.5	35.23	0.01	0.24
6	1	0.33	13.01	0.03	0.41	-0.87	12.79	-0.07	-1.11
	3	-0.69	15.02	-0.05	-0.75	-2.36	16.7	-0.14	-2.32
	6	3.43	17.03	0.2	3.23	15.31	56.66	0.27	4.12
	12	-0.33	18.46	-0.02	-0.29	-4.59	19.23	-0.24	-3.98
				$h = 3$					
0	1	-4.13	17.34	-0.24	-6.92	-1.71	15.34	-0.11	-3.21
	3	-1.61	18.47	-0.09	-2.5	-0.81	14.49	-0.06	-1.6
	6	0.28	20.48	0.01	0.39	-2.12	19.16	-0.11	-3.19
	12	0.17	22.11	0.01	0.22	2.75	40.35	0.07	1.93
1	1	-0.88	17.22	-0.05	-1.46	-1.42	16.43	-0.09	-2.47
	3	0.81	18.93	0.04	1.22	1.54	15.16	0.1	2.89
	6	0.97	19.34	0.05	1.43	-0.91	18.74	-0.05	-1.4
	12	0.46	21.58	0.02	0.61	3.85	45.28	0.08	2.39
6	1	0.49	12.33	0.04	1.12	1.1	13.3	0.08	2.32
	3	0.72	16.71	0.04	1.21	-0.71	15.91	-0.04	-1.25
	6	1.96	16.54	0.12	3.29	10.01	43.69	0.23	6.2
	12	1.38	17.46	0.08	2.2	-5.3	21.29	-0.25	-7.12
				$h = 6$					
0	1	-1.24	18.63	-0.07	-2.67	-0.08	17.6	0	-0.17
	3	0.13	20.33	0.01	0.26	0.29	17.32	0.02	0.67
	6	0.64	23.22	0.03	1.1	0.07	29.32	0	0.1
	12	0.16	24.25	0.01	0.27	-1.03	43.6	-0.02	-0.95
1	1	-0.24	19.2	-0.01	-0.5	-0.56	17.86	-0.03	-1.24
	3	0.52	20.72	0.03	1	0.12	16.95	0.01	0.28
	6	1.36	22.57	0.06	2.4	3.17	36.22	0.09	3.47
	12	-0.12	23.56	-0.01	-0.21	-2.54	38.29	-0.07	-2.66
6	1	2.04	14.54	0.14	5.49	0.89	14.96	0.06	2.32
	3	1.73	16.68	0.1	4.06	2.37	18.55	0.13	5
	6	1.44	17.42	0.08	3.23	5.84	39.19	0.15	5.77
	12	3.13	17.81	0.18	6.85	-5.92	24.49	-0.24	-9.63
				$h = 12$					
0	1	0.33	22.78	0.01	0.82	0.82	22.32	0.04	2.04
	3	0.92	25.74	0.04	2	1.84	24.7	0.07	4.13
	6	1.07	28.77	0.04	2.07	2.36	39.03	0.06	3.36
	12	1.73	28.52	0.06	3.37	-3.48	49.04	-0.07	-3.95
1	1	0.41	24.53	0.02	0.93	0.03	23.17	0	0.06
	3	1.14	26.29	0.04	2.4	3.03	33.93	0.09	4.94
	6	1.07	28.14	0.04	2.1	2.5	39.6	0.06	3.5
	12	2.23	28.62	0.08	4.31	-4.14	43.26	-0.1	-5.29
6	1	0.67	17.82	0.04	2.06	0.83	23.65	0.03	1.9
	3	0.42	18.91	0.02	1.21	0.63	21.99	0.03	1.57
	6	1.7	18.78	0.09	4.92	2.07	46.23	0.04	2.44
	12	6.82	26.36	0.26	14.07	-1.61	30.21	-0.05	-2.89

Table A12: The figure shows the annualized performance of different $\Delta_{s,f}{}^D_{(RSWP)}$ and $\Gamma_{s,f}{}^D_{(RSWP)}$ portfolios set up considering securities included in the S&P500 (2001-2016). The $\Gamma^D_{s,f}$ portfolio is constructed as follows: at each day t, the weight of a specific security i is determined by the magnitude of its (discretized) Γ^D factor computed the previous day $t-1-s$ (where s represents the delay in the investment in months) over a formation of f months($\Gamma^D_{i,t-1}(f)$) with the (discretized) acceleration factor of the equal weighted index (the market) ($\Gamma^D_{m,t-1}(f)$). Hence, $w^{\Gamma^D}_{i,t}(f,s) = \frac{1}{N}(\Gamma^D_{i,t-1-s}(f) - \Gamma^D_{m,t-1-s}(f))$ where $\sum_{i=1}^{N} w^{\Gamma^D}_{i,t}(f,s) = 0$. In order to get market-neutral weights, there is a standardization. The Δ portfolio is computed similarly. Each portfolio is held for h months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{LS} and Γ_{LS} (EMA) DJIA (1984-2002)

				Δ_{LS}				Γ_{LS}		
s	K	f	μ	σ	SR	t-test	μ	σ	SR	t-test
						$h = 1$				
0	20	1	-9.82	25.23	-0.39	-8.14	-20.86	25.28	-0.83	-18.29
		3	20.13	29.28	0.69	12.6	-5.18	26.72	-0.19	-3.96
		6	26.44	29.91	0.88	15.81	-8.27	26.2	-0.32	-6.56
	50	1	1.15	26.76	0.04	0.85	-24.71	26.27	-0.94	-21.31
		3	21.28	29.44	0.72	13.18	-8.61	25.84	-0.33	-6.93
		6	29.73	29.96	0.99	17.53	-6.09	26.13	-0.23	-4.79
	100	1	15.04	28.43	0.53	9.9	-26	27	-0.96	-21.98
		3	23.54	30.03	0.78	14.17	-10.09	27.02	-0.37	-7.82
		6	35.48	29.8	1.19	20.6	-2.12	25.55	-0.08	-1.68
1	20	1	15.85	27.53	0.58	10.71	-13.91	28.09	-0.5	-10.55
		3	17.93	29.66	0.6	11.15	-5.93	26.16	-0.23	-4.64
		6	29.37	29.7	0.99	17.46	-3.88	25.84	-0.15	-3.04
	50	1	26.53	29.3	0.91	16.15	-4.62	27.05	-0.17	-3.47
		3	24.22	29.92	0.81	14.56	-8.87	27.09	-0.33	-6.8
		6	33.58	29.54	1.14	19.76	-2.59	25.05	-0.1	-2.08
	100	1	27.43	29.79	0.92	16.37	-1.24	27.16	-0.05	-0.91
		3	29.19	29.48	0.99	17.49	-6.68	26.84	-0.25	-5.11
		6	36.37	29.25	1.24	21.39	-0.5	24.88	-0.02	-0.4
6	20	1	1.25	26.46	0.05	0.92	-15.01	26.88	-0.56	-11.83
		3	26.64	28.68	0.93	16.38	-3.46	25.32	-0.14	-2.73
		6	41.22	29.27	1.41	23.56	23.22	26.57	0.87	15.61
	50	1	11.74	27.13	0.43	8.1	-20.65	27.75	-0.74	-16.26
		3	31.07	29.51	1.05	18.26	-2.3	25.37	-0.09	-1.81
		6	38.58	29.8	1.3	21.86	22.79	26.49	0.86	15.39
	100	1	21.88	28.01	0.78	14.03	-22.29	28.3	-0.79	-17.36
		3	33.32	29.49	1.13	19.44	2.48	26.49	0.09	1.83
		6	36.97	29.93	1.24	20.97	20.83	27.91	0.75	13.45
						$h = 6$				
0	20	1	16.08	36.21	0.44	20.66	-3.56	33.7	-0.11	-5.15
		3	31.51	41.31	0.76	34.36	-2.83	33.78	-0.08	-4.09
		6	40.13	38.24	1.05	46.47	1.76	31.33	0.06	2.7
	50	1	25.93	39.38	0.66	30	-4.36	34.01	-0.13	-6.26
		3	35.03	40.35	0.87	38.83	-4.82	33.5	-0.14	-7.04
		6	43.79	37.89	1.16	50.81	6.32	30.47	0.21	9.87
	100	1	34.67	39.66	0.87	39.12	-5.82	34.15	-0.17	-8.37
		3	39.38	39.09	1.01	44.67	-3.76	35.06	-0.11	-5.23
		6	45.46	38.74	1.17	51.44	11.29	30.72	0.37	17.29
1	20	1	18.69	36.91	0.51	23.38	-2.32	34.24	-0.07	-3.29
		3	32.76	40.28	0.81	36.46	-2.46	32.97	-0.07	-3.62
		6	42.65	37.22	1.15	50.38	7.34	31.11	0.24	11.18
	50	1	28.27	38.8	0.73	32.96	-3.16	34.28	-0.09	-4.48
		3	37.13	39.64	0.94	41.63	-3.51	32.73	-0.11	-5.22
		6	45.3	37.47	1.21	52.88	11.65	30.76	0.38	17.76
	100	1	36.18	38.49	0.94	41.85	-4.55	34.79	-0.13	-6.38
		3	41.21	38.2	1.08	47.56	-1.24	33.1	-0.04	-1.81
		6	45.54	38.9	1.17	51.19	15.74	31.52	0.5	23.21
6	20	1	22.36	34.4	0.65	29.44	-1.14	33.87	-0.03	-1.61
		3	36.45	39.96	0.91	40.14	5.6	32.18	0.17	8.19
		6	38.79	38.82	1	43.76	16.63	34.08	0.49	22.38
	50	1	30.39	37.15	0.82	36.43	-0.6	33.77	-0.02	-0.85
		3	36.97	39.72	0.93	40.9	7.31	32.59	0.22	10.51
		6	36.82	38.38	0.96	42.18	15.11	32.72	0.46	21.25
	100	1	35.58	38.71	0.92	40.51	0.04	33.74	0	0.05
		3	36.4	39.05	0.93	41.01	10.32	34.15	0.3	14.06
		6	34.38	38.55	0.89	39.39	14.19	33.12	0.43	19.76

Table A13: The figure shows the average annualized performance of Δ_{LS} and Γ_{LS} (EMA) strategies set up using the DJIA (2001-2016) data set. Factors are detected applying the Exponential Moving Average (EMA) approach, i.e. the trend is estimated applying a K-days moving average on daily stock prices. The Δ is quantified as the cumulative return over the previous f months while the Γ is computed as the simple f-months difference in momentum. Each portfolio is held for h months. Factors are detected using different asymmetric moving average windows (K) expressed in days. The investment might be delayed by s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ and Γ RSWP and Long-Short (crossovers) strategies
DJIA (1984-2002)

		RSWP Δ^D and Γ^D					Long-Short Δ and Γ			
s	f	μ	σ	SR	t-test		μ	σ	SR	t-test
						$h = 1$				
0	Momentum	66.32	47.78	1.39	21.71	Momentum	30.48	28.83	1.06	18.62
	Acceleration					Acceleration				
	1	12.66	44.12	0.29	5.42	1	-1.43	24.99	-0.06	-1.15
	3	25.52	44.11	0.58	10.38	3	7.83	25.23	0.31	5.98
	6	41.99	45.31	0.93	15.67	6	13.9	26.25	0.53	9.95
	12	60.12	49.91	1.21	19.2	12	30.52	26.22	1.16	20.5
1	Momentum	61.17	46.26	1.32	20.96	Momentum	32.14	28.63	1.12	19.61
	Acceleration					Acceleration				
	1	42.88	46.36	0.93	15.56	1	14.35	25.82	0.56	10.4
	3	35.65	45.84	0.78	13.42	3	15.14	26.66	0.57	10.6
	6	51.23	46.09	1.11	18.18	6	20.76	26.16	0.79	14.47
	12	66.01	48.08	1.37	21.44	12	31.8	25.91	1.23	21.46
6	Momentum	46.98	44.16	1.06	17.46	Momentum	33.51	28.31	1.18	20.35
	Acceleration					Acceleration				
	1	21.99	42.97	0.51	9.18	1	10.91	24	0.45	8.54
	3	36.32	41.77	0.87	14.8	3	18.84	24.62	0.77	13.91
	6	45.88	42.74	1.07	17.67	6	30.12	25.32	1.19	20.7
	12	54.33	46.08	1.18	18.88	12	26.66	25.94	1.03	18.11
						$h = 6$				
0	Momentum	67.26	66.57	1.01	42.6	Momentum	38.63	37.32	1.03	45.97
	Acceleration					Acceleration				
	1	30.74	65.87	0.47	21.05	1	12.72	37.3	0.34	15.99
	3	45.31	65.02	0.7	30.56	3	21.23	37.33	0.55	26.18
	6	58.95	62.7	0.94	40.21	6	29.45	36.16	0.81	36.84
	12	72.93	66.49	1.1	45.81	12	36.92	35.15	1.05	46.81
1	Momentum	63.71	65.31	0.98	41.29	Momentum	39.03	36.88	1.05	46.86
	Acceleration					Acceleration				
	1	33.36	66.25	0.50	22.55	1	15.58	35.49	0.44	20.41
	3	48.79	63.64	0.77	33.32	3	23.95	35.97	0.67	30.4
	6	61.04	61.64	0.99	42.11	6	32.76	34.99	0.94	41.98
	12	72.11	64.73	1.11	46.49	12	36.47	34.33	1.06	47.28
6	Momentum	51.12	60.45	0.85	36.18	Momentum	31.61	32.22	0.98	43.58
	Acceleration					Acceleration				
	1	29.66	60.37	0.49	21.91	1	20.26	32.19	0.63	28.63
	3	39.68	58.4	0.68	29.71	3	25.01	32.26	0.76	34.91
	6	47.3	55.6	0.85	36.66	6	27.53	29.21	0.94	42.22
	12	50.54	59.83	0.84	36.18	12	28.56	29.16	0.98	43.78

Table A14: The figure shows the annualized performance of different Δ_{LS} and Γ_{LS} (crossovers) as well as Δ_{RSWP} and Γ_{RSWP} (crossovers) portfolios set up considering securities included in the DJIA (2001-2016) where the momentum (Δ) is detected using the asymmetric Simple Moving Average (SMA) Crossovers approach on daily stock prices. More precisely, the Δ factor is quantified as the difference of a short (50 days) and a long (200 days) SMA while the Γ factor is computed as the simple difference in momentum considering different formation periods (f) expressed in months. In the Long-Short Portfolio, at each day t stocks are ranked in ascending order according to their delta ($\Delta_{i,t-1-s}$) or gamma ($\Gamma_{i,t-1-s}(f)$) parameters. The long portfolio is constructed buying stocks of the top-ranked quintile while the short portfolio sells stocks of the bottom-ranked quintile. Equal weights are applied and the portfolio is held for h months. In the RSWP, the $\Gamma^D_{s,f}$ portfolio is constructed as follows: at each day t, the weight of a specific security i is determined by the magnitude of its (discretized) gamma factor computed the previous day $t-1-s$ (where s represents the delay in the investment, here expressed in months) over a formation of f months($\Gamma^D_{i,t-1}(f)$) with the (discretized) acceleration factor of the equal weighted index (the market) ($\Gamma^D_{m,t-1}(f)$). Hence, $w^{\Gamma^D}_{i,t}(f,s) = \frac{1}{N}(\Gamma^D_{i,t-1-s}(f) - \Gamma^D_{m,t-1-s}(f))$ where $\sum_{i=1}^{N} w^{\Gamma^D}_{i,t}(f,s) = 0$. In order to get market-neutral weights, there is a standardization. The Δ portfolio is computed similarly. Moreover, the investment might be delayed by s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB

Annualized performance of Δ_{LS} and Γ_{LS} (W)
DJIA (1984-2002)

		Δ_{LS}				Γ_{LS}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-7.02	25.11	-0.28	-5.77	-1.23	24.41	-0.05	-1.01
0	2	-10.29	25.3	-0.41	-8.53	-2.72	24.38	-0.11	-2.26
	3	-11.03	24.9	-0.44	-9.32	-0.24	24.15	-0.01	-0.2
	4	-10.97	24.46	-0.45	-9.44	-3.49	25.12	-0.14	-2.82
	1	-2.26	24.04	-0.09	-1.9	-0.89	24.16	-0.04	-0.74
1	2	-3.69	24.64	-0.15	-3.03	-1.04	24.39	-0.04	-0.85
	3	-5.25	25.1	-0.21	-4.27	-2.28	25.25	-0.09	-1.81
	4	-5.92	26.66	-0.22	-4.55	-11.21	26.68	-0.42	-8.83
	1	-1.28	24.29	-0.05	-1.04	2.63	24.53	0.11	2.09
6	2	1.4	25.02	0.06	1.09	0.2	24.58	0.01	0.16
	3	0.77	25.43	0.03	0.59	2.58	24.44	0.11	2.05
	4	-3.36	25.36	-0.13	-2.65	1.14	25.64	0.04	0.87
					$h = 3$				
	1	-3.24	24.39	-0.13	-4.62	-0.6	24.31	-0.02	-0.85
0	2	-3.93	24.67	-0.16	-5.57	-0.88	24.65	-0.04	-1.23
	3	-2.66	25.16	-0.11	-3.68	-0.42	24.72	-0.02	-0.58
	4	0.61	25.18	0.02	0.83	0.53	25.64	0.02	0.71
	1	-0.42	24.26	-0.02	-0.59	0.19	24.56	0.01	0.26
1	2	0.07	24.91	0	0.1	0.57	24.65	0.02	0.79
	3	1.56	25.28	0.06	2.1	0.35	25.86	0.01	0.46
	4	3.27	26.02	0.13	4.26	3.01	26.38	0.11	3.88
	1	0.01	24.45	0	0.02	1.1	24.39	0.05	1.53
6	2	1.64	25.12	0.07	2.21	0.42	24.54	0.02	0.58
	3	3.47	25.79	0.13	4.52	1.45	26.49	0.05	1.84
	4	5.47	25.72	0.21	7.08	1.18	26.72	0.04	1.49
					$h = 6$				
	1	-1.69	31.23	-0.05	-2.62	0.14	31.45	0	0.21
0	2	-1.71	32.33	-0.05	-2.57	0.36	32.17	0.01	0.54
	3	-0.12	32.38	0	-0.18	0.2	32.98	0.01	0.3
	4	3.37	32.77	0.1	4.93	2.2	32.69	0.07	3.24
	1	-0.55	31.28	-0.02	-0.85	0.92	32.03	0.03	1.38
1	2	0.54	32.53	0.02	0.8	1	32.27	0.03	1.49
	3	2.27	32.93	0.07	3.3	0.86	33.91	0.03	1.21
	4	4.99	32.64	0.15	7.28	2.85	33.3	0.09	4.1
	1	0.52	30.55	0.02	0.81	1.35	31.12	0.04	2.06
6	2	2.39	32.2	0.07	3.52	1.11	31.47	0.04	1.67
	3	4.91	33.08	0.15	7	1.6	34.11	0.05	2.23
	4	7.62	32.11	0.24	11.11	2.3	34.05	0.07	3.2
					$h = 12$				
	1	-1.13	58.76	-0.02	-1.29	1.16	61.28	0.02	1.28
0	2	0.03	63.13	0	0.03	1.05	61.73	0.02	1.15
	3	2.81	62.83	0.04	3.02	1.53	65.43	0.02	1.57
	4	7.4	62.07	0.12	8.04	3.67	67.78	0.05	3.65
	1	-0.18	58.22	0	-0.21	1.61	61.67	0.03	1.76
1	2	1.85	62.42	0.03	1.99	1.95	61.84	0.03	2.13
	3	4.86	63.58	0.08	5.15	2.16	66.72	0.03	2.18
	4	9.81	63.44	0.15	10.4	4.66	70.04	0.07	4.47
	1	-0.21	54.72	0	-0.25	1.49	59.15	0.03	1.68
6	2	1.88	59.63	0.03	2.1	1.97	59.96	0.03	2.18
	3	4.37	62.26	0.07	4.66	2.49	65.13	0.04	2.54
	4	8.39	63.22	0.13	8.83	3.62	69.1	0.05	3.48

Table A15: Performance of Δ_{LS} and Γ_{LS} (W) optimizations performed using the DJIA (1984-2002) data set. Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to daily stock prices at resolution levels ($j = 1, 2, 3, 4$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). The portfolio is held for h months. Moreover, the investment might be delayed by s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{LS} and Γ_{LS} (WM)
DJIA (1984-2002)

		Δ_{LS}				Γ_{LS}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-15.51	23.74	-0.65	-3.07	-30.63	29.59	-1.04	-5.31
	2	13.63	29.45	0.46	1.9	-16.06	27.57	-0.58	-2.75
	3	23.5	31.13	0.76	2.98	-5.7	22.95	-0.25	-1.11
	4	37.08	30.9	1.2	4.51	12.12	26.64	0.45	1.88
1	1	-8.03	29.25	-0.27	-1.24	32.5	29.2	1.11	4.24
	2	25.99	30.99	0.84	3.27	12.45	28.92	0.43	1.77
	3	30.01	30.09	1	3.84	17.94	28.07	0.64	2.57
	4	41.19	30.8	1.34	4.94	15.51	26.83	0.58	2.35
6	1	-4.95	24.39	-0.2	-0.89	-31.14	30.08	-1.04	-5.25
	2	20.84	27.76	0.75	2.96	-10.28	27.17	-0.38	-1.71
	3	35.26	30.16	1.17	4.36	6.58	25.84	0.25	1.06
	4	27.56	29.85	0.92	3.54	11.07	27.7	0.4	1.64
					$h = 3$				
0	1	2.52	25.67	0.1	0.73	2.51	26.68	0.09	0.7
	2	21.51	30.15	0.71	4.98	-2.57	27.26	-0.09	-0.72
	3	30.45	29.61	1.03	6.98	8.67	26.05	0.33	2.42
	4	39.78	30.8	1.29	8.53	12.9	26.64	0.48	3.48
1	1	7.76	26.75	0.29	2.12	4.08	27.72	0.15	1.09
	2	22.31	30.29	0.74	5.11	-3.22	28.93	-0.11	-0.85
	3	31.94	28.67	1.11	7.51	11.46	26.1	0.44	3.16
	4	40.47	31.39	1.29	8.48	13.92	26.84	0.52	3.7
6	1	9.58	26.71	0.36	2.57	2.9	26.3	0.11	0.81
	2	26.3	28.71	0.92	6.21	6.91	28.42	0.24	1.76
	3	35.08	30.03	1.17	7.71	8.33	26.75	0.31	2.24
	4	27.01	29	0.93	6.3	11.84	26.43	0.45	3.18
					$h = 6$				
0	1	6.98	35.13	0.2	2.06	1.93	34.62	0.06	0.59
	2	28.32	43	0.66	6.52	-2.53	35.74	-0.07	-0.75
	3	38.02	40.16	0.95	9.19	12.76	32.88	0.39	3.98
	4	44.24	40.99	1.08	10.35	16.96	37.18	0.46	4.63
1	1	9.22	33.92	0.27	2.8	3.33	36.01	0.09	0.97
	2	29.42	41.05	0.72	7.07	-1.96	36.68	-0.05	-0.57
	3	39.95	38.85	1.03	9.93	15.49	32.62	0.47	4.82
	4	42.75	41.22	1.04	9.96	16.91	37.09	0.46	4.62
6	1	12.36	32.88	0.38	3.8	7.26	34.57	0.21	2.15
	2	32.29	39.11	0.83	8	3.48	34.96	0.1	1.03
	3	38.08	37.51	1.02	9.73	2.18	31.51	0.07	0.72
	4	28.36	39.44	0.72	7.02	12.59	33.44	0.38	3.81
					$h = 12$				
0	1	13.69	65.81	0.21	3.06	7.67	72.43	0.11	1.56
	2	45.33	80.3	0.56	8.32	1.26	70.18	0.02	0.27
	3	56.04	78.96	0.71	10.46	13.7	61.34	0.22	3.29
	4	54.67	82.09	0.67	9.81	26.47	73.43	0.36	5.31
1	1	15.01	65.06	0.23	3.39	9.81	72.37	0.14	1.99
	2	44.89	77.21	0.58	8.55	2.45	72.25	0.03	0.5
	3	54.88	77.08	0.71	10.46	14.87	60.04	0.25	3.64
	4	52.76	82.06	0.64	9.45	25.54	72.12	0.35	5.21
6	1	12.88	62.74	0.21	2.98	8.73	71.53	0.12	1.77
	2	36.88	73.19	0.5	7.32	2.27	72.92	0.03	0.45
	3	46.03	75.93	0.61	8.81	13.14	57.89	0.23	3.3
	4	40.6	77.65	0.52	7.6	19.7	67.37	0.29	4.25

Table A16: Performance of Δ_{LS} and Γ_{LS} (WM) optimizations performed using the DJIA (1984-2002) data set. Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). The portfolio is held for h months. Moreover, the investment might be delayed by s months. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{RSWP} and Γ_{RSWP} (W)
DJIA (1984-2002)

		Δ_{RSWP}				Γ_{RSWP}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					h = 1				
	1	-6.63	23.6	-0.28	-5.79	-1.33	23.45	-0.06	-1.14
0	2	-10.32	23.59	-0.44	-9.18	-2.79	22.4	-0.12	-2.52
	3	-10.26	22.55	-0.45	-9.54	0.19	21.92	0.01	0.18
	4	-10.44	22.75	-0.46	-9.63	-4.08	23.36	-0.17	-3.55
	1	-2.33	21.89	-0.11	-2.14	-0.56	21.99	-0.03	-0.51
1	2	-3.57	22.29	-0.16	-3.25	-0.7	22.3	-0.03	-0.63
	3	-4.59	22.8	-0.2	-4.09	-2.08	23.03	-0.09	-1.81
	4	-3.96	24.38	-0.16	-3.3	-17.29	33.07	-0.52	-11.34
	1	1.67	24.91	0.07	1.31	1.75	27.67	0.06	1.24
6	2	2.69	27.28	0.1	1.92	1.43	25.6	0.06	1.1
	3	1.96	27.77	0.07	1.38	2.97	23.6	0.13	2.45
	4	-1.53	26.07	-0.06	-1.16	0.88	23.65	0.04	0.73
					h = 3				
	1	-2.51	23.13	-0.11	-3.76	-0.56	23.95	-0.02	-0.81
0	2	-2.36	24.13	-0.1	-3.4	-0.59	25.15	-0.02	-0.81
	3	0.58	25.06	0.02	0.79	-0.53	26.01	-0.02	-0.7
	4	5.38	26.88	0.2	6.76	-0.96	28.45	-0.03	-1.17
	1	0.67	24.02	0.03	0.96	0.39	25.36	0.02	0.53
1	2	2.42	25.65	0.09	3.2	0.91	25.68	0.04	1.21
	3	5.23	26.38	0.2	6.68	0.18	27.14	0.01	0.23
	4	8.87	27.71	0.32	10.64	0.97	29.76	0.03	1.11
	1	1.52	24.95	0.06	2.05	0.58	26.02	0.02	0.75
6	2	3.49	26.69	0.13	4.38	0.35	25.77	0.01	0.47
	3	6.24	27.25	0.23	7.6	0.55	27.32	0.02	0.69
	4	9.49	27.01	0.35	11.53	0.31	28.78	0.01	0.37
					h = 6				
	1	-0.57	33.35	-0.02	-0.83	0.01	34.83	0	0.01
0	2	0.46	34.06	0.01	0.65	0.02	36.76	0	0.03
	3	4.1	35.92	0.11	5.47	-0.22	39.43	-0.01	-0.27
	4	9.59	38.97	0.25	11.62	0.3	43.94	0.01	0.33
	1	1.19	34.74	0.03	1.64	0.71	38.72	0.02	0.88
1	2	3.48	37.95	0.09	4.39	1.09	39.24	0.03	1.34
	3	6.92	39.67	0.17	8.28	0.58	40.85	0.01	0.68
	4	11.63	40.92	0.28	13.33	1.04	45.11	0.02	1.1
	1	2.12	34.6	0.06	2.91	0.78	37.17	0.02	1.01
6	2	4.89	37.01	0.13	6.23	0.46	38.11	0.01	0.58
	3	8.7	38.32	0.23	10.59	1.22	41.09	0.03	1.41
	4	13.84	39.58	0.35	16.13	0.81	42.86	0.02	0.9
					h = 12				
	1	0.94	72.48	0.01	0.87	0.68	78.69	0.01	0.59
0	2	3.64	74.94	0.05	3.27	0.34	81.49	0	0.28
	3	9.29	78.44	0.12	7.99	0.59	91.61	0.01	0.43
	4	17.57	88.03	0.2	13.46	0.75	105.3	0.01	0.48
	1	2.53	75.2	0.03	2.26	1.27	85.63	0.01	1
1	2	6.55	82.01	0.08	5.37	1.81	87.32	0.02	1.4
	3	12.36	88.53	0.14	9.39	1.76	93.92	0.02	1.26
	4	20.74	93.27	0.22	14.96	1.73	107.5	0.02	1.08
	1	2.28	74.1	0.03	2.04	1.07	85.85	0.01	0.82
6	2	5.83	79.78	0.07	4.86	1.5	83.76	0.02	1.19
	3	10.41	82.75	0.13	8.37	1.73	89.16	0.02	1.29
	4	17.15	84.96	0.2	13.43	1.03	98.33	0.01	0.69

Table A17: Performance of Δ_{RSWP} and Γ_{RSWP} (W) optimizations performed using the DJIA (1984-2002) data set. Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to daily stock prices at resolution levels ($j = 1,2,3,4$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). The portfolio is held for h months. Moreover, the investment might be delayed by s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{RSWP} and Γ_{RSWP} (WM)
DJIA (1984-2002)

		Δ_{RSWP}				Γ_{RSWP}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h=1$				
	1	-16.24	20.47	-0.79	-3.75	-36.42	38.8	-0.94	-4.99
0	2	11.92	28.94	0.41	1.7	-18.82	28.13	-0.67	-3.2
	3	29.23	36.25	0.81	3.12	-5.03	21.31	-0.24	-1.05
	4	46.18	38.39	1.2	4.38	11.98	27.63	0.43	1.79
	1	-8	26.35	-0.3	-1.37	38.53	33.02	1.17	4.35
1	2	45.22	41.21	1.1	4	15.83	27.87	0.57	2.31
	3	31.28	36.09	0.87	3.32	13.82	25.07	0.55	2.26
	4	51.04	38.76	1.32	4.71	14.66	27.65	0.53	2.16
	1	-2.74	23.85	-0.11	-0.5	-37.38	38.29	-0.98	-5.16
6	2	17.9	27.52	0.65	2.59	-13.71	26.85	-0.51	-2.35
	3	41.87	32.8	1.28	4.65	6.03	23.54	0.26	1.07
	4	35.89	35.68	1.01	3.74	11.74	28.44	0.41	1.69
					$h=3$				
	1	8.14	28.25	0.29	2.1	0.44	29.74	0.01	0.11
0	2	24.53	30.68	0.8	5.53	-1.46	24.39	-0.06	-0.45
	3	34.37	34.66	0.99	6.65	7.06	22.38	0.32	2.31
	4	47.78	35.32	1.35	8.73	13.1	26.67	0.49	3.52
	1	12.42	29.16	0.43	3.06	1.74	29.6	0.06	0.44
1	2	25.98	32.41	0.8	5.5	-1.26	26.14	-0.05	-0.36
	3	36.96	33.53	1.1	7.32	11.94	23.01	0.52	3.73
	4	47.67	34.92	1.37	8.79	13.72	26.63	0.52	3.68
	1	14.46	29.23	0.49	3.49	-1.11	29.72	-0.04	-0.28
6	2	30.18	30.07	1	6.72	7.63	25.39	0.3	2.17
	3	42.47	31.52	1.35	8.71	6.26	24.59	0.25	1.85
	4	34.49	34.76	0.99	6.56	10.9	25.96	0.42	2.99
					$h=6$				
	1	13.88	43.12	0.32	3.29	1.05	45.74	0.02	0.24
0	2	33.42	48.37	0.69	6.77	-3.18	33.53	-0.09	-1.01
	3	47.15	51.29	0.92	8.77	12.51	29.07	0.43	4.41
	4	55.83	53.22	1.05	9.85	18.75	39.51	0.47	4.8
	1	16.93	43.5	0.39	3.94	1.66	46.44	0.04	0.37
1	2	34.79	47.87	0.73	7.09	-2.56	34.88	-0.07	-0.78
	3	49.56	49.4	1	9.51	14.6	28.89	0.51	5.14
	4	53.92	52.77	1.02	9.61	18.76	39.2	0.48	4.83
	1	19.42	41.07	0.47	4.71	3	44.58	0.07	0.69
6	2	37.93	44.73	0.85	8.12	4.31	32.65	0.13	1.36
	3	45.15	45.92	0.98	9.29	3.15	27.95	0.11	1.16
	4	40.01	51.6	0.78	7.4	13.96	36.49	0.38	3.86
					$h=12$				
	1	26.03	98.95	0.26	3.88	2.78	110.8	0.03	0.37
0	2	55.21	103.6	0.53	7.85	1.7	69.4	0.02	0.36
	3	70.77	113.4	0.62	9.2	12.95	53.77	0.24	3.55
	4	74.22	121.3	0.61	9.01	31.15	88.36	0.35	5.19
	1	27.99	101.2	0.28	4.07	4.7	107.8	0.04	0.64
1	2	55.52	104.1	0.53	7.84	3.31	74.49	0.04	0.65
	3	69.73	111.1	0.63	9.22	14.15	54.09	0.26	3.85
	4	71.79	119.2	0.6	8.85	30.39	87.34	0.35	5.11
	1	24.16	91.37	0.26	3.84	2.35	101	0.02	0.34
6	2	48.11	97.83	0.49	7.14	2.82	70.36	0.04	0.58
	3	59.72	100.8	0.59	8.6	12.73	53.17	0.24	3.48
	4	56.54	108.9	0.52	7.54	22.6	82.72	0.27	3.97

Table A18: Performance of Δ_{RSWP} and Γ_{RSWP} (WM) optimizations performed using the DJIA (1984-2002) data set. Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). The portfolio is held for h months. Moreover, the investment might be delayed by s months. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{RSWP} and Γ_{RSWP} (W) DJIA (1984-2002) considering further resolution levels (j)

		Δ_{RSWP}				Γ_{RSWP}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
$h = 1$									
	5	-13.51	22.4	-0.60	-12.85	-29.03	33.74	-0.86	-20
0	6	9.98	26.3	0.38	7.25	-11.3	26.51	-0.43	-8.98
	7	31.27	33.74	0.93	16.28	-6.38	22.2	-0.29	-5.91
	8	44.37	36.8	1.21	20.22	9.47	24.69	0.38	7.34
	5	10.83	25.75	0.42	7.99	29.66	30.74	0.97	17.01
1	6	45.11	37.53	1.20	20.07	-17.14	28.48	-0.60	-13.05
	7	31.76	34.29	0.93	16.2	-5.44	21.92	-0.25	-5.07
	8	47.24	36.07	1.31	21.71	6.45	24.41	0.26	5.11
	5	-4.08	24.61	-0.17	-3.33	-28.7	31.96	-0.90	-20.56
6	6	13.51	26.28	0.51	9.55	-10.44	25.27	-0.41	-8.55
	7	41.24	32.07	1.29	21.51	10.04	22.72	0.44	8.33
	8	41.02	34.05	1.20	20.16	11.91	25.04	0.48	8.89
$h = 6$									
	5	17.87	42.95	0.42	19.29	0.61	44.52	0.01	0.66
0	6	31.36	46.96	0.67	30.09	-1.84	37.39	-0.05	-2.39
	7	44.99	50.47	0.89	39.11	4.64	24.65	0.19	8.99
	8	58.27	52.3	1.11	47.71	10.2	30.35	0.34	15.86
	5	20.11	43.63	0.46	21.22	1.22	43.17	0.03	1.36
1	6	32.42	47.26	0.69	30.77	-1.21	37.21	-0.03	-1.57
	7	46.94	48.84	0.96	41.91	7.78	27.39	0.28	13.45
	8	57.51	51.96	1.11	47.35	10.65	30.67	0.35	16.33
	5	23.19	40.22	0.58	26.06	1.91	41.72	0.5	2.17
6	6	35.44	44.23	0.80	35.33	2.74	36.43	0.08	3.56
	7	44.3	45.99	0.96	41.74	5.91	29.66	0.20	9.36
	8	41.81	52.52	0.80	34.66	17.39	33.68	0.52	23.63

Table A19: Performance of Δ_{RSWP} and Γ_{RSWP} (W) optimizations performed using the DJIA (1984-2002) data set. MODWT approach (on a daily basis) using additional resolution levels ($j = 5, 6, 7, 8$). Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to daily stock prices. The portfolio is held for h months. Moreover, the investment might be delayed by s months. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of Δ_{RSWP} and Γ_{RSWP} (WM) DJIA (1984-2002) considering further resolution levels (j)

		Δ_{RSWP}				Γ_{RSWP}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
h = 1									
0	5	34.83	35.99	0.97	5.25	20.43	34.07	0.6	2.4
	6	13.28	33.63	0.4	1.63	-9.21	38.9	-0.24	-1.08
	7	-25.36	39.82	-0.64	-3.16	-35.7	38.31	-0.93	-4.93
1	5	36.77	36.16	1.02	3.82	21.52	34.34	0.63	2.49
	6	12.05	34.29	0.35	1.45	-9.68	38.71	-0.25	-1.14
	7	-26	39.88	-0.65	-3.24	-36.25	38.33	-0.95	-5.01
6	5	26.51	32.84	0.81	3.11	24.08	35.25	0.68	2.66
	6	4.09	37.87	0.11	0.46	-13.76	38.27	-0.36	-1.65
	7	-29.09	39.98	-0.73	-3.65	-39.11	38.32	-1.02	-5.46
h = 6									
0	5	41.91	53.52	0.78	7.55	30.08	55.44	0.54	5.35
	6	11.77	66.19	0.18	1.83	-16.5	69.02	-0.24	-2.64
	7	-36.49	68.45	-0.53	-6.27	-49.75	59.97	-0.83	-10.25
1	5	39.59	52.9	0.75	7.23	30.79	56.15	0.55	5.39
	6	9.9	67.48	0.15	1.51	-17.58	69.04	-0.25	-2.81
	7	-37.28	68.47	-0.54	-6.4	-50.48	59.76	-0.84	-10.45
6	5	26.64	51.59	0.52	5.06	30.12	59.2	0.51	4.95
	6	1.56	70.19	0.02	0.23	-22.19	69.66	-0.32	-3.53
	7	-40.67	68.04	-0.60	-7.04	-54.19	58.85	-0.92	-11.44

Table A20: Performance of Δ_{RSWP} and Γ_{RSWP} (WM) optimizations performed using the DJIA (1984-2002) data set. MODWT approach (on a monthly basis) using additional resolution levels ($j = 5, 6, 7$). Both factors are detected using the Maximum Overlap Discrete Wavelet Transform applied to monthly stock prices. A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The portfolio is held for h months. Moreover, the investment might be delayed by s months. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of calibrated Δ_{RSWP} and Γ_{RSWP} (WMC)
DJIA (1984-2002)

			Δ_{RSWP}				Γ_{RSWP}		
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					h = 1				
	1	41.9	26.05	1.61	5.94	-30.47	33.32	-0.91	-4.68
0	2	61.2	33.41	1.83	6.36	-10.02	30.85	-0.32	-1.49
	3	57.42	35.13	1.64	5.74	5.79	27.34	0.21	0.9
	4	55.9	33.71	1.66	5.85	8.74	29.12	0.3	1.26
	5	43.94	35.41	1.24	4.55	11.5	25.61	0.45	1.86
	1	-4.78	27.95	-0.17	-0.76	-9.51	44.57	-0.21	-0.97
1	2	75.09	39.65	1.89	6.29	35.41	33.16	1.07	4.03
	3	46.29	34.11	1.36	4.93	31.57	31.37	1.01	3.85
	4	57.51	33.19	1.73	6.07	13.57	26.67	0.51	2.09
	5	41.59	35.71	1.17	4.3	10.58	25.4	0.42	1.73
	1	10.85	27.72	0.39	1.61	-33.67	34.41	-0.98	-5.05
6	2	31.62	38.13	0.83	3.14	-8.52	26.72	-0.32	-1.43
	3	29.11	34.37	0.85	3.23	4.3	32.64	0.13	0.56
	4	34.21	35.45	0.97	3.61	10.47	27.87	0.38	1.54
	5	27.94	32.71	0.85	3.27	9.21	26.66	0.35	1.43
					h = 6				
	1	22.37	42.7	0.52	5.25	3.13	48.51	0.06	0.68
0	2	43.59	51.28	0.85	8.17	11.26	35.06	0.32	3.3
	3	50.67	52.58	0.96	9.14	24.74	45.33	0.55	5.44
	4	60.46	49.45	1.22	11.39	17.99	37.09	0.49	4.91
	5	47.21	54.97	0.86	8.19	13.53	34.1	0.4	4.06
	1	16.35	42.28	0.39	3.92	3.44	47.78	0.07	0.75
1	2	37.74	51.64	0.73	7.08	11.42	33.85	0.34	3.46
	3	44.72	51.9	0.86	8.24	23.89	44.91	0.53	5.31
	4	56.53	49.48	1.14	10.69	18.61	35.36	0.53	5.31
	5	43.67	54.09	0.81	7.74	13.01	33.59	0.39	3.96
	1	15.25	38.88	0.39	3.94	2.96	42.45	0.07	0.72
6	2	33.22	45.26	0.73	7.1	0.1	31.44	0	0.03
	3	41.83	45.56	0.92	8.73	5.94	40.83	0.15	1.49
	4	42.4	55.4	0.77	7.27	17.87	34.91	0.51	5.11
	5	27.52	50.09	0.55	5.38	7.84	33.59	0.23	2.38

Table A21: Performance of calibrated Δ_{RSWP} and Γ_{RSWP} (WMC) optimizations performed using the DJIA (1984-2002) data set. Both factors are detected using the calibrated Maximum Overlap Discrete Wavelet Transform applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by calibrated detail coefficients ($d^c_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the calibrated detail coefficients ((-1)$\hat{d}^c_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The portfolio is held for h months. Moreover, the investment might be delayed by s months. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of calibrated Δ_{RSWP} and Γ_{RSWP} (WMC)
S&P500 (2001-2014)

		Δ_{RSWP}				Γ_{RSWP}			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	12.13	18.3	0.66	2.28	-2.59	18.33	-0.14	-0.52
	2	13.83	20.25	0.68	2.33	7.01	32.77	0.21	0.75
	3	10.06	22.99	0.44	1.52	1.49	19.89	0.08	0.27
	4	5.61	26.34	0.21	0.75	-2.78	31.42	-0.09	-0.32
1	1	-3.62	22.84	-0.16	-0.58	-16.31	21.38	-0.76	-2.99
	2	0.14	24.66	0.01	0.02	8.51	17.2	0.49	1.72
	3	6.67	21.45	0.31	1.09	5.79	18.63	0.31	1.1
	4	3.4	27.17	0.13	0.45	0.67	27.39	0.02	0.09
6	1	-5.38	15.94	-0.34	-1.23	5.84	15.32	0.38	1.32
	2	-3.98	18.13	-0.22	-0.8	0.52	16.19	0.03	0.11
	3	-2.22	22.18	-0.1	-0.36	-4.99	15.99	-0.31	-1.14
	4	-3.37	23.54	-0.14	-0.52	8.97	17.84	0.5	1.72
					$h = 6$				
0	1	3	23.18	0.13	1.12	-3.9	26.2	-0.15	-1.32
	2	5.57	22.71	0.25	2.12	4.06	31.44	0.13	1.12
	3	5.22	21.39	0.24	2.11	7.03	16.49	0.43	3.67
	4	2.83	28.1	0.1	0.87	5.78	21.53	0.27	2.32
1	1	-1.37	32.46	-0.04	-0.37	-2.82	27.12	-0.1	-0.91
	2	1.23	31.12	0.04	0.34	1.56	19.44	0.08	0.7
	3	3.27	22.51	0.15	1.26	5.62	16.22	0.35	2.98
	4	0.75	29.93	0.03	0.22	8.11	19.86	0.41	3.49
6	1	1.89	20.77	0.09	0.78	-0.36	19.63	-0.02	-0.16
	2	0.2	17.31	0.01	0.1	0.11	16.52	0.01	0.06
	3	-1.6	28.27	-0.06	-0.49	-5.29	16.3	-0.32	-2.82
	4	-1.57	27.69	-0.06	-0.49	0.77	17.9	0.04	0.37

Table A22: Performance of calibrated Δ_{RSWP} and Γ_{RSWP} (WMC) optimizations performed using the S&P500 (2001-2014) data set. Both factors are detected using the calibrated Maximum Overlap Discrete Wavelet Transform applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4$). The Δ is quantified by calibrated detail coefficients ($d^c_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the calibrated detail coefficients ((-1)$\hat{d}^c_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). A one-month return is assumed to correspond to the cumulative return over the previous 21 days. The portfolio is held for h months. Moreover, the investment might be delayed by s months. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). The analysis has been performed through MATLAB.

Annualized performance of (EW) and (GW) Δ/Γ (simple)
DJIA (1984-2002)

		Δ/Γ (EW)				Δ/Γ (GW)			
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	33.93	24.35	1.39	24.24	27.91	28.13	0.99	17.65
0	3	36.91	25.13	1.47	25.28	29.73	28.57	1.04	18.38
	6	32.56	26.85	1.21	21.2	31.63	30.63	1.03	18.11
	12	37.68	29.03	1.3	22.28	35.44	34.22	1.04	17.92
	1	43.91	24	1.83	30.66	53.77	27.78	1.94	31.4
1	3	47.81	22.81	2.1	34.67	52.89	26.98	1.96	31.9
	6	60.54	24.32	2.49	39.53	69.24	28.04	2.47	38.2
	12	77.71	25.16	3.09	46.62	88.03	31.01	2.84	41.63
	1	29.32	24.24	1.21	21.11	32.45	28.24	1.15	19.83
6	3	27.85	23.33	1.19	20.95	29.01	26.98	1.08	18.79
	6	30.86	25.11	1.23	21.33	40.23	29.48	1.36	22.9
	12	29.44	25.99	1.13	19.77	39.48	32.45	1.22	20.48
					$h = 3$				
	1	49.83	25.45	1.96	57.53	59.81	32.49	1.84	52.69
0	3	45.49	25.66	1.77	52.71	47.17	29	1.63	48.15
	6	47.24	25.61	1.85	54.6	51.08	29.1	1.76	51.4
	12	59.64	27.81	2.15	61.41	65.63	33.21	1.98	55.74
	1	41.01	27.78	1.48	44.37	56.05	35.61	1.57	45.39
1	3	36.12	25.11	1.44	43.84	40.72	29.64	1.37	41.32
	6	42.24	26.54	1.59	47.66	47.42	31.41	1.51	44.56
	12	52.97	28.42	1.86	54.19	62.16	35.5	1.75	49.7
	1	-9.73	25.4	-0.38	-13.51	-0.74	30.91	-0.02	-0.82
6	3	-9.4	25.98	-0.36	-12.75	-6.95	30.4	-0.23	-7.97
	6	-4.03	28.33	-0.14	-4.91	3.65	31.89	0.11	3.83
	12	-3.84	28.98	-0.13	-4.56	5.07	33.19	0.15	5.09
					$h = 6$				
	1	50.48	31.91	1.58	68.71	63.67	43.5	1.46	62.09
0	3	47.15	29.49	1.6	69.87	51.18	33.75	1.52	65.76
	6	51.59	31.51	1.64	70.97	58.27	37.46	1.56	66.61
	12	57.48	31.64	1.82	77.9	65.86	40.82	1.61	68.19
	1	49.08	33.39	1.47	63.84	64.13	45.9	1.4	59.1
1	3	44.85	29.61	1.52	66.32	50.22	34.87	1.44	62.43
	6	50.7	32.61	1.56	67.34	59.18	38.94	1.52	64.82
	12	55.23	31.62	1.75	75.02	65.96	41.46	1.59	67.08
	1	4.98	31.13	0.16	7.54	15.92	40.58	0.39	18.02
6	3	6.11	31.94	0.19	8.99	11.92	39.82	0.3	13.87
	6	10.19	34.2	0.3	13.87	17.97	39.45	0.46	20.83
	12	10.47	33.51	0.31	14.53	20.96	41.35	0.51	23.02
					$h = 12$				
	1	33.34	55.14	0.6	40.78	49.19	85.24	0.58	38.92
0	3	31.43	45.91	0.68	46.17	37.46	57.9	0.65	43.64
	6	36.41	50.34	0.72	48.78	44.88	61.54	0.73	49.19
	12	39.91	47.22	0.85	57.01	53.51	69.58	0.77	51.86
	1	26.89	53.53	0.5	33.8	44.63	87.25	0.51	34.42
1	3	24.53	46.16	0.53	35.75	31.91	59.91	0.53	35.83
	6	29.81	50.1	0.6	40.04	39.25	63.92	0.61	41.32
	12	33.17	46.93	0.71	47.57	48.12	70.12	0.69	46.17
	1	5.65	51.22	0.11	7.34	22.57	78.26	0.29	19.18
6	3	6.94	52.8	0.13	8.75	16.75	73.32	0.23	15.19
	6	14.38	58.3	0.25	16.4	26.36	75.95	0.35	23.09
	12	18.5	57.79	0.32	21.29	36.84	83.58	0.44	29.31

Table A23: Performance of Δ/Γ (simple) portfolio optimizations performed using the DJIA (1984-2002) stocks universe. This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of (EW) and (GW) Δ/Γ (simple)
DJIA (2001-2016)

			Δ/Γ (EW)					Δ/Γ (GW)		
s	j	μ	σ	SR	t-test		μ	σ	SR	t-test
						$h=1$				
	1	-37.3	29.81	-1.25	-28.06		-44.89	35.78	-1.25	-29.67
0	3	-34.1	27.83	-1.23	-26.9		-42.22	32.8	-1.29	-29.86
	6	-41.2	29.46	-1.4	-32.21		-47.46	34.93	-1.36	-32.77
	12	-50.32	31.88	-1.58	-38.93		-53.53	37.97	-1.41	-35.71
	1	114.2	24.71	4.62	57.98		124.3	28.26	4.4	53.87
1	3	117.2	23.68	4.95	61.63		122.3	28.13	4.35	53.5
	6	138.4	25.6	5.41	64.13		151.1	29.86	5.06	58.4
	12	135.5	24.35	5.56	66.42		158.4	29.26	5.41	61.49
	1	0.98	20.99	0.05	0.84		8.89	24.16	0.37	6.36
6	3	-1.25	20.43	-0.06	-1.1		3.59	23.64	0.15	2.69
	6	-1.25	19.29	-0.07	-1.17		5.98	21.81	0.27	4.8
	12	3.37	18.41	0.18	3.24		5.31	21.33	0.25	4.37
						$h=3$				
	1	60.55	21.17	2.86	74.7		57.54	25.15	2.29	60.22
0	3	62.76	20.68	3.04	78.83		58.61	24.64	2.38	62.46
	6	65.55	25.6	2.56	66.04		65.16	31.5	2.07	53.4
	12	58.61	20.86	2.81	73.76		61.5	26.33	2.34	60.88
	1	24.29	20.48	1.19	34.25		24.43	23.71	1.03	29.74
1	3	24.74	19.34	1.28	36.88		23.97	23.25	1.03	29.8
	6	29.42	23.63	1.25	35.38		31.65	29.01	1.09	30.8
	12	25.44	19.32	1.32	37.9		28.98	24.6	1.18	33.53
	1	56.39	23.33	2.42	62.82		66.58	26.07	2.55	64.64
6	3	57.03	23.82	2.39	62.1		67.3	26.79	2.51	63.5
	6	55.21	19.33	2.86	74.46		68.24	22.04	3.1	78.08
	12	59.8	19.75	3.03	77.97		69.08	22.95	3.01	75.74
						$h=6$				
	1	9.02	21.9	0.41	17.79		7.5	25.88	0.29	12.57
0	3	9.67	22.73	0.43	18.35		7.28	27.33	0.27	11.55
	6	9.71	25	0.39	16.75		9.54	31.17	0.31	13.21
	12	5.45	20.16	0.27	11.79		6.94	25.16	0.28	11.98
	1	18.15	18.39	0.99	41.65		20.27	20.66	0.98	41.2
1	3	17.58	20.45	0.86	36.33		18.35	23.94	0.77	32.34
	6	19.52	20.87	0.94	39.35		22.77	26.46	0.86	35.95
	12	17.93	18.05	0.99	41.94		21.27	21.93	0.97	40.65
	1	10.91	18.81	0.58	24.54		13.48	21.95	0.61	25.83
6	3	10.56	16.89	0.63	26.49		13.26	19.6	0.68	28.48
	6	8.88	15.2	0.58	24.83		11.97	17.33	0.69	29.15
	12	7.49	15.06	0.5	21.2		8.66	17.53	0.49	21.01

Table A24: Performance of Δ/Γ (simple) portfolio optimizations performed using the DJIA (2001-2016) stocks universe. This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of (EW) and (GW) Δ/Γ (simple)
S&P500 (2001-2014)

s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
		Δ/Γ (EW)				Δ/Γ (GW)			
				$h = 1$					
	1	-43.68	18.89	-2.31	-49.32	-47.96	23.35	-2.05	-45.23
0	3	-43.63	17.95	-2.43	-51.84	-49.16	21.62	-2.27	-50.56
	6	-43.96	17.21	-2.55	-54.59	-48.55	20.74	-2.34	-51.8
	12	-45.91	17.32	-2.65	-57.48	-50.24	23.54	-2.13	-47.87
	1	32.81	20.44	1.61	23.27	54	26.37	2.05	27.62
1	3	33.89	19.46	1.74	25.15	54.49	25.58	2.13	28.69
	6	39.17	18.4	2.13	30.18	61.15	23.71	2.58	34
	12	49.04	19.34	2.54	34.76	74.83	27.12	2.76	34.93
	1	4.49	20.11	0.22	3.57	7.18	23.43	0.31	4.84
6	3	1.79	19.09	0.09	1.51	3.35	22.72	0.15	2.37
	6	6.63	19.49	0.34	5.38	10.91	23.14	0.47	7.33
	12	4.66	20.12	0.23	3.69	13.66	25.78	0.53	8.14
				$h = 3$					
	1	26.03	18.85	1.38	36.14	33.01	23.28	1.42	36.32
0	3	25.08	18.87	1.33	34.89	29.93	24.22	1.24	31.95
	6	28.34	17.21	1.65	42.77	35.43	21.22	1.67	42.47
	12	32.67	18.95	1.72	44.21	40.41	25.79	1.57	39.28
	1	11.75	19.02	0.62	16.88	17.06	24.02	0.71	19.07
1	3	9.59	18.82	0.51	14.04	11.89	24.44	0.49	13.28
	6	12.97	17.96	0.72	19.66	17.55	23.05	0.76	20.41
	12	14.74	18.49	0.8	21.57	20.31	23.7	0.86	22.77
	1	35.04	19.82	1.77	44.14	43.46	23.31	1.87	45.45
6	3	34.88	19.13	1.82	45.56	42.27	23.68	1.79	43.65
	6	37.45	18.34	2.04	50.63	44.29	22.1	2	48.73
	12	39.96	18.87	2.12	52.13	52.45	25.1	2.09	49.68
				$h = 6$					
	1	2.83	20.54	0.14	5.47	5.94	25.83	0.23	9.08
0	3	2.39	18.82	0.13	5.05	3.24	23.9	0.14	5.39
	6	3.92	18.44	0.21	8.44	6.41	23.09	0.28	10.94
	12	4.79	19.93	0.24	9.51	8.05	24.83	0.32	12.74
	1	12.41	20.5	0.61	23.46	17.31	25.68	0.67	25.84
1	3	11.24	19.16	0.59	22.81	13.25	25.05	0.53	20.45
	6	14.24	19.34	0.74	28.43	18.94	24.55	0.77	29.46
	12	15.14	20.44	0.74	28.52	21.51	25.83	0.83	31.62
	1	10.86	21.18	0.51	19.62	14.78	25.51	0.58	21.96
6	3	10.81	19.26	0.56	21.46	13.79	23.65	0.58	22.15
	6	10.35	19.28	0.54	20.56	11.63	21.79	0.53	20.38
	12	12.37	21.49	0.58	21.95	17.74	31.66	0.56	21.1

Table A25: Performance of Δ/Γ (simple) portfolio optimizations performed using the S&P500 (2001-2014) stocks universe. This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of (EW) and (GW) Δ/Γ (W)
DJIA (2001-2016)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	j	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-35.7	32.37	-1.1	-24.47	-41.66	38.48	-1.08	-25.01
0	2	-36.46	31.09	-1.17	-26.15	-42.38	36.72	-1.15	-26.81
	3	-36.36	30.38	-1.2	-26.67	-43.58	36.16	-1.21	-28.24
	4	-36.3	31.62	-1.15	-25.57	-42.51	37.95	-1.12	-26.04
	5	-36.07	30.96	-1.17	-25.91	-43.38	35.71	-1.22	-28.42
	1	113	27.07	4.18	52.54	122.1	31.03	3.94	48.44
1	2	112	25.96	4.32	54.44	120.7	30.12	4.01	49.5
	3	113.4	25.37	4.47	56.2	125.7	30.97	4.06	49.55
	4	111.5	26.26	4.24	53.6	121.5	29.46	4.13	50.86
	5	113.2	25.51	4.44	55.8	123.4	29.09	4.24	52.06
	1	-0.45	21.9	-0.02	-0.37	3.91	25.16	0.16	2.75
6	2	-1.21	22.35	-0.05	-0.98	2.93	25.6	0.11	2.03
	3	-0.55	22.33	-0.02	-0.45	4.72	26.09	0.18	3.19
	4	2.38	21.58	0.11	1.96	8.21	25.36	0.32	5.61
	5	-0.64	22.18	-0.03	-0.52	3.46	25.92	0.13	2.36
					$h = 3$				
	1	58.13	24.2	2.4	63.15	55.9	29.06	1.92	50.86
0	2	58.02	23.28	2.49	65.53	56.02	27.52	2.04	53.8
	3	57.94	22.15	2.62	68.79	55.78	26.98	2.07	54.69
	4	58.34	22.54	2.59	68.02	56.54	27.09	2.09	55.09
	5	59.3	22.17	2.68	70.1	56.6	26.07	2.17	57.3
	1	24.36	22.04	1.11	31.91	24.82	26	0.95	27.51
1	2	23.49	21.45	1.1	31.71	23.95	24.87	0.96	27.84
	3	24.13	21.19	1.14	32.9	25.46	25.99	0.98	28.17
	4	23.18	20.91	1.11	32.12	24.41	24.5	1	28.76
	5	24.08	20.7	1.16	33.62	24.52	23.82	1.03	29.71
	1	54.84	24.3	2.26	58.88	62.77	27.45	2.29	58.45
6	2	53.54	24.02	2.23	58.37	60.85	27.47	2.22	56.9
	3	54.05	24.34	2.22	58.06	62.53	28.1	2.23	56.92
	4	56.78	24.07	2.36	61.24	64.28	27.37	2.35	59.79
	5	54.99	23.94	2.3	59.91	63.54	27.04	2.35	59.94
					$h = 6$				
	1	9.18	23.94	0.38	16.56	8.21	28.84	0.28	12.31
0	2	8.73	23.21	0.38	16.25	7.86	27.17	0.29	12.52
	3	9.33	22.87	0.41	17.61	8.42	27.03	0.31	13.47
	4	8.41	22.46	0.37	16.2	7.76	26.79	0.29	12.54
	5	9.02	22.83	0.4	17.07	7.6	26.41	0.29	12.47
	1	17.7	20.28	0.87	36.86	19.37	23.59	0.82	34.57
1	2	17.31	19.47	0.89	37.59	18.9	22.12	0.85	35.99
	3	18.17	19.82	0.92	38.68	20.45	23.26	0.88	36.9
	4	17.71	18.37	0.96	40.71	19.88	20.73	0.96	40.31
	5	17.78	19.42	0.92	38.66	19.19	21.77	0.88	37.1
	1	9.85	19.39	0.51	21.55	11.41	22.25	0.51	21.67
6	2	9.28	19.3	0.48	20.42	10.66	21.85	0.49	20.65
	3	10.13	19.81	0.51	21.67	12.1	23.18	0.52	22.03
	4	11.17	19.68	0.57	24.01	12.96	22.84	0.57	23.9
	5	10.65	17.85	0.6	25.26	12.72	20.18	0.63	26.55

Table A26: Performance of Δ/Γ (W) portfolio optimizations performed using the DJIA (2001-2016) stocks universe. Both factors are detected using the MODWT applied to daily stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of calibrated (EW) and (GW) Δ/Γ (WC)
DJIA (2001-2016)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-41.9	37.29	-1.12	-26.01	-48.07	43.47	-1.11	-26.8
0	2	-42.01	36.55	-1.15	-26.62	-48	43.09	-1.11	-26.98
	3	-35.87	33.22	-1.08	-23.99	-40.55	39.34	-1.03	-23.63
	4	-35.96	34.31	-1.05	-23.3	-40.42	40	-1.01	-23.15
	5	-41.11	34.15	-1.2	-27.7	-51.65	42.1	-1.23	-30.6
	1	116.6	26.97	4.33	53.94	126.7	31.05	4.08	49.69
1	2	116.2	26.14	4.45	55.52	125	30.29	4.13	50.46
	3	114.8	25.57	4.49	56.23	127.2	31.23	4.07	49.54
	4	114	26.03	4.38	54.95	124.5	29.28	4.25	52.05
	5	113.3	25.36	4.47	56.18	124.7	29.22	4.27	52.22
	1	5.43	23.88	0.23	3.99	11.03	27.67	0.4	6.83
6	2	5.2	24.12	0.22	3.79	11.34	27.77	0.41	6.98
	3	1.53	23.63	0.06	1.15	4.94	27.68	0.18	3.14
	4	3.5	22.47	0.16	2.76	8.11	26.26	0.31	5.36
	5	6.68	23.57	0.28	4.95	16.33	27.84	0.59	9.83
					$h = 3$				
	1	54.93	28.6	1.92	50.92	52.3	33.76	1.55	41.35
0	2	55.57	28.04	1.98	52.43	52.99	33.1	1.6	42.65
	3	59.2	24.74	2.39	62.72	59.43	30.26	1.96	51.45
	4	59.96	24.87	2.41	63.06	59.96	29.15	2.06	53.81
	5	56.65	25.02	2.26	59.75	51.37	31.66	1.62	43.42
	1	23.44	23.66	0.99	28.69	23.68	27.73	0.85	24.71
1	2	23.06	23.11	1	28.92	23.09	26.68	0.87	25.09
	3	24.46	21.88	1.12	32.27	26.31	26.95	0.98	28.02
	4	24.2	21.17	1.14	33.01	26.14	24.79	1.05	30.27
	5	23.79	21.63	1.1	31.81	23.91	25.65	0.93	26.96
	1	60.37	25.52	2.37	60.82	69.47	28.74	2.42	60.77
6	2	59.71	25.34	2.36	60.7	68.71	29.03	2.37	59.61
	3	56.14	24.54	2.29	59.49	63.82	28.52	2.24	57.05
	4	58.17	24.27	2.4	61.98	65.35	27.62	2.37	60.09
	5	60.38	24.96	2.42	62.22	72.54	28.89	2.51	62.64
					$h = 6$				
	1	7.45	27.86	0.27	11.59	6.2	32.96	0.19	8.18
0	2	7.43	27.62	0.27	11.67	6.22	32.42	0.19	8.34
	3	9.67	24.89	0.39	16.75	9.67	29.54	0.33	14.12
	4	9.56	24.83	0.39	16.62	9.75	29.31	0.33	14.34
	5	7.99	25.33	0.32	13.65	5.44	31.65	0.17	7.49
	1	19.05	21.5	0.89	37.31	20.92	25.05	0.84	35.03
1	2	19.01	20.98	0.91	38.16	20.77	23.82	0.87	36.58
	3	18.78	20.58	0.91	38.47	20.75	24.26	0.86	35.88
	4	18.97	19.33	0.98	41.34	21.17	21.96	0.96	40.41
	5	19.78	21.11	0.94	39.4	22.36	24.36	0.92	38.37
	1	11.02	20.57	0.54	22.66	12.71	23.41	0.54	22.87
6	2	10.98	21.02	0.52	22.09	12.76	23.91	0.53	22.48
	3	10.71	20.22	0.53	22.41	12.36	23.71	0.52	21.99
	4	11.33	19.75	0.57	24.25	13.02	22.7	0.57	24.16
	5	11.38	18.59	0.61	25.86	13.67	21.65	0.63	26.54

Table A27: Performance of Δ/Γ (WC) portfolio optimizations performed using the DJIA (2001-2016) stocks universe. Both factors are detected using the calibrated MODWT applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by calibrated detail coefficients ($d^c_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the calibrated detail coefficients ((-1)$\hat{d}^c_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of (EW) and (GW) Δ/Γ (WM)
DJIA (2001-2016)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h=1$				
	1	-37.66	29.93	-1.26	-6.18	-45.9	36.52	-1.26	-6.54
0	2	-33.2	31.51	-1.05	-5.02	-42.78	38.64	-1.11	-5.63
	3	-39.36	35.58	-1.11	-5.49	-48.11	40.2	-1.2	-6.34
	4	-41.26	32.7	-1.26	-6.35	-46.35	37.92	-1.22	-6.38
	5	-55.89	40.33	-1.39	-7.83	-61.26	41.16	-1.49	-8.84
	1	118.5	27.54	4.3	11.67	119.2	30.13	3.96	10.71
1	2	122.7	26.24	4.68	12.56	134.9	31.43	4.29	11.21
	3	138.5	27.75	4.99	12.93	164.8	31.87	5.17	12.66
	4	103.3	26.61	3.88	10.93	131.5	31.15	4.22	11.11
	5	167.4	33.66	4.97	12.12	180.4	34.26	5.26	12.5
	1	-4.12	23.18	-0.18	-0.71	2.53	28.01	0.09	0.35
6	2	0.41	19.99	0.02	0.08	8.37	22.64	0.37	1.4
	3	0.99	20.92	0.05	0.18	4.8	23.13	0.21	0.8
	4	8.19	19.18	0.43	1.62	6.34	20.9	0.3	1.16
	5	-3.26	28.17	-0.12	-0.46	0.59	28.48	0.02	0.08
					$h=3$				
	1	58.9	21.92	2.69	15.4	55.35	25.61	2.16	12.5
0	2	60.27	23.03	2.62	14.94	56.95	28.57	1.99	11.48
	3	65.35	26.78	2.44	13.76	67.05	30.12	2.23	12.5
	4	54.35	21.23	2.56	14.84	60.02	24.91	2.41	13.77
	5	53.13	33.18	1.6	9.32	49.45	35.01	1.41	8.3
	1	24.94	20.31	1.23	7.73	24.88	22.83	1.09	6.86
1	2	29.02	21.31	1.36	8.46	31.01	26.03	1.19	7.36
	3	28.68	22.41	1.28	7.96	33.43	25	1.34	8.2
	4	20.47	21.23	0.96	6.16	25.84	24.49	1.06	6.62
	5	30.18	28.66	1.05	6.52	30.7	30.44	1.01	6.24
	1	50.03	23.82	2.1	12.12	58.73	27.96	2.1	11.85
6	2	52.33	23.57	2.22	12.73	63.04	26.72	2.36	13.16
	3	56.96	21.04	2.71	15.34	67.83	23.86	2.84	15.67
	4	57.99	22.85	2.54	14.35	65.42	25.37	2.58	14.3
	5	58.87	26.42	2.23	12.57	64.95	26.5	2.45	13.61
					$h=6$				
	1	9.26	21.39	0.43	4.08	6.78	25.42	0.27	2.53
0	2	11.47	22.98	0.5	4.68	9.69	29.47	0.33	3.1
	3	9.5	22.67	0.42	3.95	9.81	25.61	0.38	3.61
	4	5.69	19.86	0.29	2.73	7.77	22.31	0.35	3.3
	5	7.49	31.86	0.24	2.23	6.33	34.35	0.18	1.75
	1	18.15	19.1	0.95	8.76	19.46	20.93	0.93	8.54
1	2	19.95	20.29	0.98	9.03	22.44	25.78	0.87	7.95
	3	18.92	17.24	1.1	10.1	22.54	19.76	1.14	10.41
	4	15.67	18.57	0.84	7.82	18.53	21.6	0.86	7.9
	5	21.84	24.14	0.9	8.27	23.39	25.2	0.93	8.46
	1	9.52	17.47	0.55	5.05	11.13	21.31	0.52	4.83
6	2	11.42	16.12	0.71	6.54	14.03	18.95	0.74	6.79
	3	7.32	15.03	0.49	4.54	9.41	19.37	0.49	4.51
	4	10.41	17.46	0.6	5.52	13.48	21.44	0.63	5.78
	5	7.13	22.59	0.32	2.94	9.06	23.43	0.39	3.59

Table A28: Performance of Δ/Γ (WM) portfolio optimizations performed using the DJIA (2001-2016) stocks universe. Both factors are detected using the MODWT applied to daily stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of calibrated (EW) and (GW) Δ/Γ (WMC)
DJIA (2001-2016)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h=1$				
0	1	-40.09	34.35	-1.17	-5.83	-51.63	41.49	-1.24	-6.78
	2	-31.85	30.31	-1.05	-4.97	-41.04	37.69	-1.09	-5.47
	3	-31.44	31.7	-0.99	-4.68	-38.95	36.26	-1.07	-5.32
	4	-38.82	31.66	-1.23	-6.07	-40.78	35.62	-1.15	-5.74
	5	-54.98	40.43	-1.36	-7.62	-58	42.5	-1.37	-7.86
1	1	111	26.33	4.22	11.64	110.9	28.78	3.85	10.65
	2	133.1	28.03	4.75	12.45	152.6	31.76	4.81	12.07
	3	147.2	28.45	5.17	13.15	175.2	32.39	5.41	12.98
	4	111.2	26.81	4.15	11.45	143.1	31.37	4.56	11.69
	5	169.8	32.72	5.19	12.58	189.2	32.81	5.77	13.47
6	1	1.77	24.53	0.07	0.28	10.51	29.09	0.36	1.36
	2	1.11	21.05	0.05	0.21	6.74	24.17	0.28	1.06
	3	-3.23	18.86	-0.17	-0.68	-0.74	21.45	-0.03	-0.14
	4	8.49	18.89	0.45	1.7	3.15	19.83	0.16	0.61
	5	-2.83	27.82	-0.1	-0.41	-0.39	28.22	-0.01	-0.05
					$h=3$				
0	1	57.79	25.93	2.23	12.8	51.92	31.32	1.66	9.68
	2	65.06	24.34	2.67	15.08	65.89	31.51	2.09	11.77
	3	75.44	24.96	3.02	16.62	81.29	28.84	2.82	15.29
	4	60.12	22.32	2.69	15.39	70.53	25.39	2.78	15.46
	5	55.22	32.19	1.72	9.92	56.57	35.24	1.61	9.25
1	1	24.12	21.5	1.12	7.08	24.55	25.39	0.97	6.09
	2	33	22.64	1.46	8.95	37.95	28.49	1.33	8.06
	3	35.9	23.11	1.55	9.46	43.6	26.79	1.63	9.69
	4	25.92	22.4	1.16	7.26	34.14	25.68	1.33	8.14
	5	31.45	27.79	1.13	6.98	35.44	30.55	1.16	7.07
6	1	55.66	25.44	2.19	12.44	65.76	28.99	2.27	12.57
	2	52.47	24.78	2.12	12.14	61.86	28.07	2.2	12.33
	3	53.26	20.56	2.59	14.82	63.43	24.84	2.55	14.23
	4	59.27	22.54	2.63	14.81	64.46	23.89	2.7	15
	5	60.51	27.37	2.21	12.41	65.88	27.21	2.42	13.41
					$h=6$				
0	1	8.24	25.51	0.32	3.05	6.37	32.69	0.19	1.85
	2	14.78	24.82	0.6	5.55	15.58	33.88	0.46	4.27
	3	16.11	23.08	0.7	6.48	19.69	29.31	0.67	6.19
	4	10.2	22.25	0.46	4.31	15.12	25.42	0.59	5.53
	5	8.81	31.33	0.28	2.66	11.01	36.39	0.3	2.84
1	1	18.99	19.78	0.96	8.83	22.26	24.58	0.91	8.27
	2	23.14	21.97	1.05	9.6	27.71	29.87	0.93	8.38
	3	22.36	20.62	1.09	9.91	28.39	27.22	1.04	9.41
	4	19.55	21.06	0.93	8.53	23.43	24.35	0.96	8.77
	5	22.89	24.23	0.94	8.62	26.54	27.68	0.96	8.68
6	1	11.08	18.19	0.61	5.63	12.67	21.42	0.59	5.45
	2	12.3	18.29	0.67	6.19	13.96	21.26	0.66	6.03
	3	7.19	14.95	0.48	4.48	8.42	17.58	0.48	4.45
	4	12.56	17.1	0.73	6.76	14.94	18.82	0.79	7.27
	5	9.19	24.57	0.37	3.47	12.14	26.58	0.46	4.21

Table A29: Performance of Δ/Γ (WMC) portfolio optimizations performed using the DJIA (2001-2016) stocks universe. Both factors are detected using the calibrated MODWT applied to monthly stock prices at resolution levels ($j = 1,2,3,4,5$). The Δ is quantified by calibrated detail coefficients ($d^c_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the calibrated detail coefficients ((-1)$\hat{d}^c_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ), annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of (EW) and (GW) Δ/Γ (WM) S&P500 (2001-2014)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-43.11	22.66	-1.9	-8.83	-49.64	25.15	-1.97	-9.62
	2	-42.51	22.29	-1.91	-8.81	-47.98	23.68	-2.03	-9.75
	3	-41.73	18.31	-2.28	-10.46	-45.97	22.07	-2.08	-9.87
	4	-46.1	18.68	-2.47	-11.7	-49.27	23.45	-2.1	-10.21
	5	-44.12	21.08	-2.09	-9.78	-52.09	24.59	-2.12	-10.53
1	1	36.67	21.38	1.72	5.35	60.34	27.9	2.16	6.24
	2	34.84	19.92	1.75	5.5	57.65	23.69	2.43	7.08
	3	38.4	19.79	1.94	6.02	60.51	24.11	2.51	7.24
	4	45.84	19.71	2.33	7.03	69.95	28.14	2.49	6.97
	5	56.64	24.72	2.29	6.69	82.99	30.55	2.72	7.34
6	1	5.73	21.94	0.26	0.91	9.21	25.62	0.36	1.23
	2	2.9	19.81	0.15	0.51	6.63	23.63	0.28	0.97
	3	1.34	21.2	0.06	0.22	-0.77	23.78	-0.03	-0.12
	4	5.85	21.6	0.27	0.94	7.67	24.56	0.31	1.07
	5	-1.66	23.04	-0.07	-0.26	6.37	26.08	0.24	0.85
					$h = 3$				
0	1	26.63	23.26	1.15	6.53	33.51	28.18	1.19	6.64
	2	26.21	19.78	1.33	7.57	31.75	23.8	1.33	7.49
	3	29.01	18.28	1.59	8.98	35.9	22.86	1.57	8.71
	4	31.18	19	1.64	9.23	38.53	25.28	1.52	8.39
	5	34.51	23.31	1.48	8.24	36.8	28.4	1.3	7.17
1	1	11.76	20.07	0.59	3.5	16.4	24.6	0.67	3.92
	2	10.87	18.85	0.58	3.45	15.07	22.84	0.66	3.89
	3	12.11	19.09	0.63	3.78	13.56	23.75	0.57	3.39
	4	17.19	18.49	0.93	5.45	20.96	23.14	0.91	5.24
	5	14.44	23.67	0.61	3.61	14.68	30.02	0.49	2.89
6	1	35.63	22.52	1.58	8.61	41.16	25.59	1.61	8.61
	2	35.1	18.65	1.88	10.26	44.15	23.18	1.9	10.11
	3	34.19	20.33	1.68	9.19	38.67	25.76	1.5	8.1
	4	41.7	19.67	2.12	11.34	51.04	24.44	2.09	10.88
	5	35.09	23.1	1.52	8.28	40.11	28.54	1.41	7.55

Table A30: Performance of Δ/Γ (WM) portfolio optimizations performed using the S&P500 (2001-2014) stocks universe. Both factors are detected using the MODWT applied to daily stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by detail coefficients ($d_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the detail coefficients ($(-1)\hat{d}_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). This strategy consists in buying a long portfolio and selling a short portfolio. The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of calibrated (EW) and (GW) Δ/Γ (WMC)
S&P500 (2001-2014)

			Δ/Γ (EW)				Δ/Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	-42.99	22.73	-1.89	-8.76	-49.57	25.33	-1.96	-9.53
0	2	-41.94	22.64	-1.85	-8.52	-47.04	25.52	-1.84	-8.8
	3	-41.04	18.7	-2.2	-10.03	-45.12	22.85	-1.97	-9.29
	4	-45.7	18.61	-2.46	-11.61	-49.56	24	-2.07	-10.06
	5	-44.96	20.69	-2.17	-10.21	-54.13	23.69	-2.29	-11.56
	1	35.63	21.46	1.66	5.2	56.81	28.31	2.01	5.86
1	2	34.7	20.21	1.72	5.4	57.62	24.72	2.33	6.79
	3	39.61	20.02	1.98	6.12	63.73	24.45	2.61	7.45
	4	47.12	19.85	2.37	7.15	72.55	28.9	2.51	6.99
	5	56.38	24.26	2.32	6.79	83.11	28.85	2.88	7.78
	1	6.54	21.93	0.3	1.03	10.65	25.62	0.42	1.41
6	2	2.9	19.84	0.15	0.51	6.54	23.99	0.27	0.94
	3	1.54	21.24	0.07	0.26	-0.26	23.77	-0.01	-0.04
	4	6.53	21.84	0.3	1.03	9.17	25.54	0.36	1.23
	5	1.77	22.69	0.08	0.28	15.42	25.14	0.61	2.04
					$h = 3$				
	1	26.29	23.31	1.13	6.44	32.17	28.44	1.13	6.34
0	2	26.34	20.07	1.31	7.49	32.1	24.81	1.29	7.26
	3	30.17	18.64	1.62	9.13	38.08	23.51	1.62	8.93
	4	31.73	19.27	1.65	9.25	38.79	26.26	1.48	8.12
	5	32.73	22.48	1.46	8.15	31.76	27.17	1.17	6.56
	1	11.43	20.08	0.57	3.4	15.39	24.92	0.62	3.64
1	2	10.9	19.11	0.57	3.41	15.18	23.55	0.64	3.8
	3	13.13	19.38	0.68	4.03	15.59	24.54	0.64	3.74
	4	17.78	18.71	0.95	5.56	22.31	24.03	0.93	5.35
	5	14.13	22.87	0.62	3.66	14.84	27.52	0.54	3.19
	1	35.82	22.37	1.6	8.71	41.62	25.68	1.62	8.67
6	2	35.22	18.66	1.89	10.28	44.42	23.13	1.92	10.19
	3	34.57	20.45	1.69	9.23	39.55	26.07	1.52	8.16
	4	42.32	19.64	2.16	11.5	51.69	24.33	2.12	11.05
	5	37.14	22.76	1.63	8.84	44.53	27.11	1.64	8.71

Table A31: Performance of Δ/Γ (WMC) portfolio optimizations performed using the S&P500 (2001-2014) stocks universe. Both factors are detected using the calibrated MODWT applied to monthly stock prices at resolution levels ($j = 1, 2, 3, 4, 5$). The Δ is quantified by calibrated detail coefficients ($d^c_{j,t}(i)$) calculated using the Haar (also named Daubechies) mother wavelet with one-vanishing moment (Db1). The acceleration factor (Γ) is captured by the calibrated detail coefficients ((-1)$\hat{d}^c_{j,t}(i)$) applying a MODWT performed with a Daubechies function with two vanishing moments (Db2). The long portfolio includes stocks having a positive momentum ($\Delta(f)$) at the time t computed over the previous f months; moreover, the selected stocks must have a positive acceleration ($\Gamma(f)$) at time t quantified using the same formation period. The short portfolio sells stocks having a negative $\Delta(f)$ and additionally a negative $\Gamma(f)$ at the time t. Two kind of weighting rule are applied: equal weights (EW) or relative "Gamma" weights (GW). Moreover, the investment might be delayed by one or six months after the portfolio construction. A one-month period is assumed to correspond to 21 days. The performance is given as an average annualized return (μ) and annualized volatility (σ) both expressed as a percentage as well as annualized Sharpe Ratio (SR). A t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%).

Annualized performance of winsorized Δ^D_{RSWP} and Γ^D_{RSWP} (simple)
S&P500 (2001-2014)

		Δ^D_{RSWP}				Γ^D_{RSWP}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-6.5	16.62	-0.39	-6.7	0.09	14.89	0.01	0.09
	3	-5.13	18.03	-0.28	-4.84	-4.13	13	-0.32	-5.38
	6	-2.8	20.04	-0.14	-2.35	-4.77	15.82	-0.3	-5.12
	12	-0.81	20.97	-0.04	-0.65	1	20.58	0.05	0.8
1	1	-6.45	16.58	-0.39	-6.64	-4.86	14.84	-0.33	-5.55
	3	-1.4	17.6	-0.08	-1.33	0.48	13	0.04	0.62
	6	-0.89	19.44	-0.05	-0.76	-0.58	16.7	-0.03	-0.58
	12	-1.39	20.79	-0.07	-1.11	-0.64	16.88	-0.04	-0.63
6	1	0.51	12.64	0.04	0.65	-1.07	12.04	-0.09	-1.45
	3	0.52	14.12	0.04	0.59	-0.8	12.63	-0.06	-1.04
	6	3.11	15.1	0.21	3.31	3.72	16.94	0.22	3.52
	12	1.33	17.36	0.08	1.24	2.47	14.58	0.17	2.73
					$h = 6$				
0	1	-1.22	18.55	-0.07	-2.65	-0.2	17.03	-0.01	-0.47
	3	0.24	20.01	0.01	0.48	-0.28	14.57	-0.02	-0.76
	6	1.46	22.6	0.06	2.59	1.1	22.22	0.05	1.97
	12	0.58	23.53	0.02	0.99	0.79	18.06	0.04	1.74
1	1	-0.14	18.96	-0.01	-0.3	-0.69	16.99	-0.04	-1.63
	3	0.93	19.84	0.05	1.86	0.02	14.82	0	0.06
	6	2.35	21.53	0.11	4.33	2.45	21.4	0.11	4.55
	12	0.75	22.81	0.03	1.31	0.91	16.36	0.06	2.22
6	1	1.59	13.03	0.12	4.77	0.73	13.82	0.05	2.08
	3	1.82	16.14	0.11	4.4	2.52	15.15	0.17	6.49
	6	1.28	17.03	0.08	2.95	-0.38	12.57	-0.03	-1.2
	12	3.6	17.47	0.21	8.01	1.57	13.88	0.11	4.42

Table A32: Performance of winsorized Δ^D_{RSWP} and Γ^D_{RSWP} (simple) portfolio optimizations performed using the S&P500 (2001-2014) stocks universe for a holding period of one and six months Winsorization consists in removing outliers from the data set by setting a lower and an upper bound for return values and replacing each value outside these limits with the corresponding threshold value (Welch, 2017). Winsorization level: 20%. The performance is given as average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). A one-month period is assumed to correspond to 21 days. The analysis has been performed through MATLAB.

Annualized performance of winsorized Δ^D_{RSWP} and Γ^D_{RSWP} (WMC)
S&P500 (2001-2014)

		Δ_{RSWP}				Γ_{RSWP}			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	7.99	15.05	0.53	1.86	-1.12	13.85	-0.08	-0.3
	3	7.82	15.65	0.5	1.75	-3	13.51	-0.22	-0.82
	6	4.61	17.61	0.26	0.93	5.48	18.09	0.3	1.07
	12	4.79	21.3	0.22	0.8	5.89	15.47	0.38	1.35
1	1	0.66	17.68	0.04	0.13	-9.14	14.28	-0.64	-2.42
	3	2.63	18.84	0.14	0.5	2.03	12.77	0.16	0.57
	6	5.04	16.62	0.3	1.07	6.72	13.52	0.5	1.74
	12	3.71	21.36	0.17	0.62	5.56	13.41	0.41	1.46
6	1	-4.54	15	-0.3	-1.1	3.67	11.96	0.31	1.07
	3	-1.6	14.06	-0.11	-0.41	-0.55	12.07	-0.05	-0.16
	6	1.87	17.13	0.11	0.39	-1.97	11.56	-0.17	-0.61
	12	-1.06	20.51	-0.05	-0.18	7.57	14.7	0.51	1.77
					$h = 6$				
0	1	3.34	17.08	0.2	1.69	-1.49	18.12	-0.08	-0.72
	3	4.13	15.69	0.26	2.28	0.46	12.82	0.04	0.32
	6	3.97	16.41	0.24	2.09	5.69	14.38	0.4	3.41
	12	3.99	21.42	0.19	1.61	8.75	14.65	0.6	5.11
1	1	0.46	21.46	0.02	0.19	-0.71	18.3	-0.04	-0.34
	3	1.53	20.31	0.08	0.65	0.7	12.19	0.06	0.5
	6	3.46	16.34	0.21	1.83	4.42	13.12	0.34	2.9
	12	2.55	22.61	0.11	0.98	9.43	15.49	0.61	5.19
6	1	2.04	17.92	0.11	0.97	-0.07	14.74	0	-0.04
	3	2.15	12.68	0.17	1.45	1.25	11.47	0.11	0.93
	6	1.94	19.74	0.1	0.84	-1.65	12.19	-0.14	-1.17
	12	-0.07	24.23	0	-0.02	0.93	15.91	0.06	0.5

Table A33: Performance of winsorized Δ_{RSWP} and Γ_{RSWP} (WMC) portfolio optimizations performed using the S&P500 (2001-2014) stocks universe for a holding period of one and six months. Winsorization consists in removing outliers from the data set by setting a lower and an upper bound for return values and replacing each value outside these limits with the corresponding threshold value (Welch, 2017). Winsorization level: 20%. The performance is given as average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). A one-month period is assumed to correspond to 21 days. The analysis has been performed through MATLAB.

Annualized performance of winsorized (EW) and (GW) Δ/Γ (simple)
DJIA (2001-2016)

		Δ/ Γ (EW)				Δ/ Γ (GW)			
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
0	1	-37.3	29.81	-1.25	-28.06	-44.91	35.76	-1.26	-29.71
	3	-34.18	27.86	-1.23	-26.95	-42.55	32.81	-1.3	-30.15
	6	-40.76	29.03	-1.4	-32.23	-46.77	34.31	-1.36	-32.71
	12	-50.38	32.01	-1.57	-38.85	-53.4	37.84	-1.41	-35.72
1	1	114.2	24.7	4.62	58.02	124.3	28.22	4.4	53.94
	3	117.2	23.64	4.96	61.77	122	28.09	4.34	53.48
	6	139.3	25.31	5.5	65.15	153.4	29.54	5.19	59.64
	12	136.5	24.3	5.62	66.93	161.3	29.44	5.48	61.89
6	1	0.97	20.98	0.05	0.83	8.91	24.16	0.37	6.37
	3	-1.33	20.4	-0.07	-1.18	3.62	23.56	0.15	2.72
	6	-1.21	19.11	-0.06	-1.15	5.41	21.63	0.25	4.39
	12	3.25	18.58	0.18	3.1	4.69	21.37	0.22	3.87
					$h = 3$				
0	1	60.54	21.17	2.86	74.7	57.51	25.12	2.29	60.26
	3	62.73	20.72	3.03	78.64	58.38	24.64	2.37	62.23
	6	66.14	25.52	2.59	66.76	66.46	31.48	2.11	54.34
	12	58.8	20.89	2.82	73.86	62.56	26.27	2.38	61.89
1	1	24.29	20.47	1.19	34.27	24.41	23.68	1.03	29.76
	3	24.72	19.41	1.27	36.73	23.95	23.33	1.03	29.68
	6	29.23	23.35	1.25	35.6	31.73	28.81	1.1	31.08
	12	25.55	19.27	1.33	38.14	29.53	24.68	1.2	34
6	1	56.4	23.32	2.42	62.84	66.61	26.1	2.55	64.6
	3	56.97	23.84	2.39	62	67.2	26.82	2.51	63.33
	6	55.3	19.34	2.86	74.49	68.1	21.97	3.1	78.17
	12	59.48	19.86	3	77.22	68.9	22.99	3	75.42

Table A34: Performance of winsorized Δ/Γ (simple) portfolio optimizations performed using the DJIA (2001-2016) stocks universe for a holding period of one and six months. Winsorization consists in removing outliers from the data set by setting a lower and an upper bound for return values and replacing each value outside these limits with the corresponding threshold value (Welch, 2017). Winsorization level: 20%. The performance is given as average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). A one-month period is assumed to correspond to 21 days. The analysis has been performed through MATLAB.

Annualized performance of winsorized (EW) and (GW) Δ/ Γ (simple)
DJIA (1984-2002)

			Δ/ Γ (EW)				Δ/ Γ (GW)		
s	f	μ	σ	SR	t-test	μ	σ	SR	t-test
					$h = 1$				
	1	33.63	24.37	1.38	24.04	28.12	28.04	1	17.82
0	3	36.9	24.91	1.48	25.5	29.87	28.1	1.06	18.77
	6	32.68	26.81	1.22	21.3	32.79	29.97	1.09	19.11
	12	38.65	29	1.33	22.8	33.94	32.7	1.04	18.05
	1	43.77	24.06	1.82	30.51	51.68	27.3	1.89	30.92
1	3	47.29	23.01	2.06	34.06	53.35	27.14	1.97	31.94
	6	57.15	23.38	2.44	39.23	64.54	26.57	2.43	38.11
	12	79.88	25.31	3.16	47.34	84.93	29.23	2.91	42.98
	1	29.21	24.23	1.21	21.05	31.66	27.68	1.14	19.79
6	3	28.81	23.99	1.2	21	29.52	27.29	1.08	18.87
	6	30.43	24.44	1.25	21.65	38.83	28.21	1.38	23.22
	12	31.02	25.97	1.2	20.72	37.67	30.51	1.24	20.91
					$h = 3$				
	1	49.24	25.08	1.96	57.78	56.3	30.11	1.87	54.01
0	3	44.98	24.04	1.87	55.71	47.09	27.57	1.71	50.56
	6	46.15	24.71	1.87	55.45	49.89	27.91	1.79	52.52
	12	61.92	27.45	2.26	64.22	63.82	30.35	2.1	59.58
	1	40.39	27.59	1.46	44.06	51.8	33.66	1.54	44.88
1	3	35.45	24.69	1.44	43.85	40.94	29.48	1.39	41.73
	6	40.52	24.88	1.63	49.02	44.96	28.82	1.56	46.36
	12	55.17	28.36	1.95	56.23	60.66	32.77	1.85	52.75
	1	-9.96	25.33	-0.39	-13.89	-3.13	29.36	-0.11	-3.66
6	3	-9.42	26.16	-0.36	-12.69	-7.18	30.22	-0.24	-8.29
	6	-5.11	28.01	-0.18	-6.32	1.33	30.81	0.04	1.45
	12	-2.23	29.14	-0.08	-2.62	3.69	32.41	0.11	3.81

Table A35: Performance of winsorized Δ/Γ (simple) portfolio optimizations performed using the DJIA (1984-2002) stocks universe for a holding period of one and six months. Winsorization consists in removing outliers from the data set by setting a lower and an upper bound for return values and replacing each value outside these limits with the corresponding threshold value (Welch, 2017). Winsorization level: 20%. The performance is given as average annualized return (μ), annualized volatility (σ) both expressed as a percentage and annualized Sharpe Ratio (SR). Moreover, a t-test is employed to check the statistical significance of the results. Critical t-values are: 1.64 (90%), 1.96 (95%) and 2.58 ((99%). One month period is assumed to correspond to 21 days. The analysis has been performed through MATLAB.

MATLAB Code

In the following section, there is a short introduction about the MATLAB code which has been developed to investigate the momentum and acceleration factors as well as the portfolio optimizations. The main piece of this code is transcribed and explained here below.

The first part of the code allows us to set up investigation variables, as well as to select the detection approach and the portfolio optimization strategy (the portfolio optimization strategies are explained later in this section):

```
% 1.1 SELECT INPUT VARIABLES

    dataset = 'DJIA';     % ('SP500', 'DJIA', 'DJIA_81')
    factor = 'acc'          %('acc' or 'mom')
    detection = 'simple'   % ('simple', 'trend-based', 'wavelet')
    trend= 'C' ;           % ('C' vs 'EMA')

    w = 1;                  % (non-winsorized -> 0, winsorized -> 1)
    discretefactor = 'yes';  % (convert delta and gamma in discrete form: 'yes' or 'no')

    pfo = 'RSWP';             % ('LS', 'RSWP', 'DeltaGamma', 'Market')
    LS = 'Total';           % ('Total', 'Long', 'Short')
    DGWeights = 'GW'       % ('EW','GW')

    DGNumberAssets= 'no'    % (to create a file with number of assets in the long
                                      %   or short sub-portfolio)
    calibration = 'no'     % (calibration of wavelet coefficient: 'yes' or 'not')
    Q = 5;                  % (ex. 10 = decile portfolio, 5 = quintile portfolio)

    f5D= 5; % 5 days (i.e. 1 week)
    f1M= 21;  % 1 month
    f3M= 63;   % 3 months
    f6M= 126;   % 6 months
    f1Y= 252;     % 1 year
```

More precisely, the first part of the code makes it possible to load the desired data set (Standard and Poor 500 or two data sets for the Dow Jones Industrial Average over two different time-periods, see the section 1.5 Data in Chapter 1). Successively, one can choose which 'factor' (delta or gamma) to compute and can select the approach for detection: simple, trend-based or wavelet (daily basis). The trend-based approach can be executed by applying the Exponential Moving Average ('EMA') or the Moving Average Crossover ('C'). The computation of the Wavelet approach on a monthly basis required the corresponding "monthly" MATLAB code (2).

In order to obtain a winsorization of the data set (i.e. log-returns) the parameter 'w' should be set to 1. Additionally, the simple and the trend-based (EMA) detection quantify the delta and gamma factors directly from log-return, i.e. in "continuous" form, the factors can be converted in "discrete" form by inserting the order 'discretefactor' = 'yes' (see. Section 3.2.2 Additional Assumptions). The second part of the above code enables us to fix parameters concerning the portfolio optimization 'pfo' and it is explained later in this section. The f-parameter designs the formation period while the parameter K designs the exponential moving average window for trend-detection; neither parameters is visible here since this version of the code includes loops which execute computations automatically for different variables f, K (as well h and s and $levj$ for the portfolio optimization) and the output from each loop is automatically stored in a separate file, so that we achieve a comprehensive overview for different parametrizations.

Here below there is an overview of predetermined parameters as well as the winsorization approach and other useful parameters which are implemented in further calculations. The variables 'wname1' and 'wname2' delineate the two wavelet functions applied during the wavelet transform (wavelet detection approach). Moreover, in this section of the code, the h-days cumulative return as well as the return of the equal market index (equal-weighted average return of the securities in the data set) is computed.

```
1.2 FIXED PARAMETERS
                    nasset = size(ret, 2);
                    retlength = length(ret);
                    retTS = fints(date, ret);
                    dateobj = cdfepoch(date);
                    retW = ret;
                    wname1 = 'haar';
                    wname2 = 'db2';
                    lev = 9;
                    f5D = 5; %  5 days (one week)
                    f1M = 21;  % 1 month
                    f3M = 63;   % 3 months
                    f6M = 126;   %  6 months
                    f1Y = 252;     %  1 year

                    % winsorizing
                    trup = 0.20;
                    trdown = -0.20;
                    retW(retW > trup) = trup;
                    retW(retW < trdown) = trdown;
                    K1 = 50 ;       % for SMA crossovers detection
                    K2 = 200;       % for SMA crossovers detection


% 1.3 CUMULATIVE RETURN (H—DAYS HOLDING PERIOD)

                    for i = 1: retlength- h+ 1 ;
                        for j = 1: nasset;
                            retholding(i, j) = sum(ret(i:i+h-1, j),1);
                        end
                    end

% 1.4 WINSORIZED RETURNS SELECTION
                    if w == 0
                        retI = ret;
                    end

                    if w == 1
                        retI = retW;
                    end

% 1.5  MARKET RETURN (Equal Market Index)
                    returnmarket = sum( exp(retI)-1, 2 ).* 1/nasset;
                    returnmarket = log( 1+ returnmarket );
```

The subsequent part of the program makes it possible to compute the delta and the gamma factor for the universe of securities in the selected data set as well as for the equal market index according to the simple detection:

```
% 2 SIMPLE DETECTION

% 2.1 MOMENTUM — Simple Detection
```

```
                    if strcmp('simple', detection);
                        for i = 1: length(retI)- f+ 1 ;
                            for j = 1: nasset;
                                delta(i,j)= sum(retI(i:i+f-1, j));
                            end
                        end

% 2.2 ACCELERATION — SIMPLE DETECTION

                        if factor == 'acc';

                            for i = 1:length(delta)- f+ 1
                                for j = 1: nasset
                                    gamma(i, j)= delta(i+f-1, j)-delta(i, j);
                                end
                            end
                        end

% 2.3 DELTA AND GAMMA FOR MARKET INDEX — SIMPLE DETECTION
                        for i = 1:length(returnmarket)- f+ 1 ;
                            deltamarket(i) = sum(returnmarket(i:i+f-1));
                        end
                        deltamarket = deltamarket';

                        if factor == 'acc';
                            for i = 1: length(deltamarket)-f+1
                                gammamarket(i) = deltamarket(i+f-1)- deltamarket(i);
                            end

                            gammamarket = gammamarket';
                        end

% 2.4  CONVERT DELTA AND GAMMA IN "DISCRETE"
                        if strcmp('yes',discretefactor)

                            delta=exp(delta)-1;
                            deltamarket=exp(deltamarket)-1;

                            if strcmp('acc',factor)
                                gammamarket=exp(gammamarket)-1;
                                gamma=exp(gamma)-1;
                            end
                        end

                    end
```

The code for the trend-based detection (Exponential Moving Average as well as Simple Moving Average Crossovers) is available here below:

```
% 3. TREND—BASED DETECTION

                    if strcmp('trend-based', detection);

% 3.1 STOCK PRICES COMPUTATION

                        if w == 0
                            [St, tSt] = ret2tick(ret,100, 1, 1, 'Continuous');
                            St= St(2:end,:);
                        end
                        if w == 1
                            [St, tSt] = ret2tick(retW,100, 1, 1, 'Continuous');
                            St= St(2:end, :);
                        end

                        [Market, tMarket] = ret2tick(returnmarket,100, 1, 1,
                                                          'Continuous');
```

```
                        Market= Market(2:end, :);

% 3.2 MOMENTUM — SMA CROSSOVERS DETECTION

                        if strcmp('C', trend)

                            SMAshort= tsmovavg(St, 's', K1, 1);
                            SMAshort= SMAshort(K1:end, :);

                            SMAlong= tsmovavg(St,'s', K2, 1);
                            SMAlong= SMAlong(K2:end, :);

                            SMAshort= SMAshort(K2-K1+1:end, :);

                            delta= SMAshort-SMAlong;

%  3.3 MOMENTUM MARKET— SMA CROSSOVERS DETECTION

                            SMAshortMarket= tsmovavg(Market, 's', K1, 1);
                            SMAshortMarket= SMAshortMarket(K1:end, :);

                            SMAlongMarket= tsmovavg(Market,'s', K2, 1);
                            SMAlongMarket= SMAlongMarket(K2:end, :);

                            SMAshortMarket= SMAshortMarket(K2-K1+1:end, :);

                            deltamarket= SMAshortMarket-SMAlongMarket;


                        end

% 3.4 MOMENTUM — EMA
                        if strcmp('EMA',trend)

                            EMA= tsmovavg(St,'e', K, 1);
                            EMA= EMA(K:end, :);


                            for i=1:length(EMA)-f;
                                for j=1:nasset
                                    delta(i, j)= log(EMA(i+f, j)/EMA(i, j));
                                end
                            end

% 3.5 MOMENTUM MARKET — EMA
                            [Market, tMarket] = ret2tick(returnmarket,100, 1, 1,
                                                                   'Continuous');
                            EMAmarket= tsmovavg(Market,'e', K, 1);
                            EMAmarket= EMAmarket(K:end, :);

                            for i=1:length(EMAmarket)-f;
                                deltamarket(i)= (log(EMAmarket(i+f)/EMAmarket(i)));
                            end
                            deltamarket=deltamarket';
                        end
```

The delta (momentum) factor can be computed using the Exponential Smoothing ('EMA') or the Moving Average Crossovers ('C'). Firstly, in section 3.1 of the code, stock prices as well as the price of the equal weighted market index are computed. Afterwards through the "EMA" approach, the time series trend is estimated in order to compute the momentum using the Equation 5. Conversely, the Crossovers methodology computes the momentum as a difference between the two moving averages. The gamma factor is assessed in the same manner as in the section of the code 2.3. Moreover, since the output of the 'EMA' procedure is in continuous form, it might be discretized as in the section of the code 2.4.

The code for the wavelet detection in MATLAB works as follows:

```
% 4.1.1 STOCK PRICES ON THE BASIS OF MONTHLY RETURNS


                           [St, tSt] = ret2tick(retI, 100, 1, 1, 'Continuous');
                            St = St(2: end, :);
                           [Market, tSt] = ret2tick(returnmarket, 100, 1, 1,
                                                    'Continuous');
                           Market = Market(2: end);

                       St = log(St);
                       Market = log (Market);
                            for i = 1:nasset
                                A{i} = St(:, i);
                            end

 %4.2 MOMENTUM — MODWT (HAAR or DB1)

                            for i = 1 : nasset
                                wt1{i}= modwt(A{i}, lev, wname1);
                            end

                            for i= 1 : nasset
                                delta{i} = wt1{1,i}(levj, :);
                            end

                            delta = cell2mat(delta(:));
                            delta = delta';

 %4.3 MOMENTUM — MODWT (HAAR or DB1)— Market

                             wt1Market= modwt(Market, lev, wname1);
                             deltamarket = wt1Market(levj, :);
                             deltamarket = deltamarket';


%4.4 ACCELEARATION — MODWT (DB2)
if strcmp ('acc', factor)
                            for i= 1 : nasset
                                wt2{i} = -modwt(A{i},lev, wname2);
                            end

                            for i = 1 :nasset
                                gamma{i} = wt2{1,i}(levj,:);
                            end

                            gamma = cell2mat(gamma(:));
                            gamma = gamma';

%4.4 ACCELEARATION — MODWT (DB2)— MARKET
                            wt2Market= - modwt(Market, lev, wname2);
                            gammamarket = wt2Market(levj, :);
                            gammamarket = gammamarket';

end

 %4.4 CALIBRATION
 if strcmp ('yes', calibration)
                      if levj == 1
                                for i = 1: length(delta) -1
                                    delta(i) = delta(i +1);
                                end

                                for i = 1: length(deltamarket) -1
                                    deltamarket(i) = deltamarket(i +1);
                                end
                            end
```

```
                        if levj == 2
                                 for i = 1: length(delta) -2
                                     delta(i) = delta(i +2);
                                 end
                                  for i = 1: length(deltamarket) -2
                                     deltamarket(i) = deltamarket(i +2);
                                 end
                             end

                         if levj == 3
                                 for i = 1: length(delta) -3
                                    delta(i) = delta(i +3);
                                 end
                                 for i = 1: length(deltamarket) -3
                                    deltamarket(i) = deltamarket(i +3);
                                 end
                             end

                             if levj == 4
                                 for i = 1: length(delta) -6
                                     delta(i) = delta(i +6);
                                 end
                                   for i = 1: length(deltamarket) -6
                                     deltamarket(i) = deltamarket(i +6);
                                 end
                             end

                             if levj == 5
                                 for i = 1: length(delta) -8
                                     delta(i) = delta(i +8);
                                 end
                                   for i = 1: length(deltamarket) -8
                                     deltamarket(i) = deltamarket(i +8);
                                 end
                             end

 if strcmp ('acc', factor)

                             if levj == 1
                                 for i = 1: length(gamma) -4
                                     gamma(i) = gamma(i +4);
                                 end
                                 for i = 1:length(gammamarket) -4
                                     gammamarket(i) = gammamarket( i+4);
                                 end
                             end

                             if levj == 2
                                 for i= 1: length(gamma)-5
                                     gamma(i)=gamma(i +5);
                                 end
                                 for i= 1: length(gammamarket)-5
                                     gammamarket(i)=gammamarket(i +5);
                                 end
                             end
                             if levj == 3
                                 for i= 1 :length(gamma) -10
                                     gamma(i)=gamma(i +10);
                                 end
                                 for i= 1 :length(gammamarket)-10
                                     gammamarket(i)=gammamarket(i +10);
                                 end

                             end

                             if levj == 4

                                 for i= 1:length(gamma) -20
                                     gamma(i)=gamma(i +20);
                                 end
```

```
                                    for i= 1: length(gammamarket) -20
                                        gammamarket(i)=gammamarket(i +20);
                                    end
                                end
                                if levj == 5

                                    for i= 1: length(gamma) -25
                                        gamma(i)=gamma(i +25);
                                    end
                                    for i= 1: length(gammamarket) -25
                                        gammamarket(i)=gammamarket(i +25);
                                    end
                                end
end
end
end
```

As in trend-based detection, the first part of the above code computes the stock prices departing from non-winsorized or winsorized daily log-returns. Additionally, the wavelet transform can be applied to monthly returns which are calculated at the start of the 4. section of the code. Moreover, the MODWT approach is applied firstly using the Haar wavelet mother and it gives as output the MODWT signal 'wt1' which represents the momentum effect, the delta is extrapolated according to different time-scale (resolution) 'levels' ('levj') and it might be calibrated. The calibration is measured on the basis of pure signals (as is explained in the previous chapter). The procedure for the acceleration detection is the same, however the mother wavelet function is the Daubechies with two vanishing moments and the calibration magnitude is different.

As was already mentioned above, the first part of the code makes it possible to decide which kind of optimization strategy to apply ('pfo') and which factor to use in order to compute the portfolio return. Moreover, for the Long-Short strategy, it is possible to compute the return of the long or the short portfolio separately, indicating the desired ('Long' or 'Short') portfolio under 'LS' instead of 'Total'. For the Delta/Gamma strategy, it is possible to decide which kind of weighting rule to use (equal weights or "gamma-dependent" weights, i.e. 'EW' or 'GW'). Here below are transcribed and explained the main parts of the three portfolio optimization strategies.

The first strategy is Long-Short portfolio optimization and it consists in ranking the stock (descending) according to the momentum or acceleration parameter and then to investing long (short) in the top Q-ranked (bottom Q-ranked) percentile.

```
% 5.1 PORTFOLIO OPTIMIZATION — LS — LONG/SHORT

                    if  strcmp('LS',pfo)

                        % 2.1 RUN  stock rank

                        if factor== 'mom'
                            [srtcum, idxcum]  = sort(delta, 2, 'descend');
                        end

                        if factor== 'acc'
                            [srtcum, idxcum]  = sort(gamma, 2, 'descend');
                        end

                        % PORTFOLIO RETURNS

                        for i= 1:length(retholding)-h-s
                            retlong(i) = sum(exp(retholding(i+1+h-1+s,
                                        idxcum(i,round(1:nasset/Q))))-1, 2)
                                        .*1/round(nasset/Q);
```

```
                retshort(i) = (sum(exp(retholding(i+1+h-1+s,
                 idxcum(i,round(nasset-nasset/Q+1):nasset)))-1,2))
                                            .*1/round(nasset/Q);
            end

            retlong= retlong';
            retshort= retshort';
            PFreturnLS= retlong- retshort;

        end
```

The second optimization strategy the is the Relative Strength Weighted Portfolio. Weights of each security are determined comparing its delta or gamma factor with the corresponding market factor; moreover, positive and negative weights are divided into two different matrices in order to standardize them and to obtain market-neutral weights (the sum of weight in the long resp. the short position is +1 and -1).

```
% 5.2 PORTFOLIO OPTIMIZATION — Relative Strength Weighted Portoflio (RSWP)

        if strcmp('RSWP',pfo)

            retholdingD=exp(retholding)-1;

            if factor == 'mom'
                weights = (delta-deltamarket)./nasset;
            end

            if factor == 'acc'
                weights = (gamma-gammamarket)./nasset;

            end
            % computation market—neutral weights
            weightpositiveIndicator = weights;
            weightpositiveIndicator(weightpositiveIndicator > 0)= 1;
            weightpositiveIndicator(weightpositiveIndicator < 0)= 0;

            weightnegativeIndicator = weights;

            weightnegativeIndicator(weightnegativeIndicator > 0)= 0;
            weightnegativeIndicator(weightnegativeIndicator < 0)= 1;

            Longweights = weightpositiveIndicator.*weights ;
            Shortweights = weightnegativeIndicator.*weights ;

            Longweights = Longweights./sum(Longweights, 2) ;
            Shortweights = Shortweights./-sum(Shortweights,2) ;

            LongReturn= sum(retholdingD(h+1+s:end,:)
                                           .*Longweights(1:end-h-s,:), 2
                                           );
            ShortReturn= sum(retholdingD(h+1+s:end,:)
                                            .* Shortweights(1:end-h-s,:), 2
                                            );
            PFreturn= LongReturn+ShortReturn;

            PFreturn(isnan(PFreturn))=0;

        end
```

Furthermore, in order to better capture the paybacks of the acceleration factor and its synergy with the momentum effect a third hybrid investment strategy has been developed. It is called Delta-Gamma strategy and it aims to consider both the momentum and the acceleration effect. The theoretical and mathematical explanation is given in the section 3.1.2.

```
% 5.3 PORTFOLIO OPTIMIZATION — Hybrid Delta—Gamma (DG)

                    if strcmp('DeltaGamma',pfo)

                        %first condition long portoflio: if momentum > 0  —-> signal = 1
                        deltalong = delta;
                        deltalong(delta > 0) = 1;
                        deltalong(delta < 0) = 0;


                        %second condition long portfolio: if gamma < 0    —-> signal = 0
                        weightlong = deltalong .* gamma;
                        weightlong(weightlong<0) = 0;

                        %first condition short portfolio: if momentum < 0  —-> signal = 1
                        deltashort = delta;
                        deltashort(deltashort > 0) = 0;
                        deltashort(deltashort < 0) = 1;


                        %second condition short portfolio: if gamma > 0    —-> signal = 0
                        weightshort = deltashort .* gamma;
                        weightshort(weightshort > 0) = 0;

                        %Equal Weights (EW)

                        if strcmp('EW', DGWeights)

                            weightlong(weightlong > 0) = 1;
                            nassetlong = 1 ./ sum(weightlong, 2);
                            weightshort(weightshort < 0) = 1;
                            nassetshort = 1 ./ sum(weightshort, 2);

                            retholdingD = exp(retholding)-1;

                            retlong = retholdingD(h+s+1: end,:)
                                                    .* weightlong(1: end-h-s,:);
                            retlong = sum(retlong,2)
                                                      .* nassetlong(1: end-h-s);
                            retlong(isnan(retlong)) = 0;

                            retshort=retholdingD(h+s+1, :).* weightshort(1: end-h-s,:);
                            retshort=sum(retshort, 2).* nassetshort(1: end-h-s);
                            retshort(isnan(retshort)) = 0;

                            PFreturn = retlong-retshort;

                            %number of asset in each portfolio
                            if strcmp('yes',DGNumberAssets)
                                longassetinportfolio = mean(1./nassetlong);
                                maxlongasset = max(1 ./nassetlong);
                                minlongasset = min(1 ./nassetlong);
                                shortassetinportfolio = mean(1 ./ nassetshort)
                                maxshortasset = max(1 ./ nassetshort);
                                minshortasset = min(1 ./ nassetshort);
                            end

                        end

                        % "Relative" weights (Gamma Weights)

                        if strcmp('GW', DGWeights)

                            retholdingD = exp(retholding)- 1;

                            weightlong = weightlong ./ sum(weightlong, 2);
                            weightshort = weightshort ./ -sum(weightshort, 2);
                            weightlong(isnan(weightlong)) = 0;
                            weightshort(isnan(weightshort)) = 0;
```

```
                    retlong = sum(retholdingD(h+s+1: end, :)
                                        * weightlong(1: end-h-s, :), 2);
                    retlong(isnan(retlong)) = 0;

                    retshort = sum(retholdingD(h+s+1, :)
                                        .* weightshort(1: end-h-s, :), 2);
                    retshort(isnan(retshort)) = 0;

                    PFreturn = retlong + retshort;
                end
            end
```