# Diagnostics and Pricing Models of Employee Stock Options

**by**

Bernhard Andreaus

INFICON Ltd, Balzers, Liechtenstein

**Supervisors**

Prof. Didier Sornette

Chair of Enterpreneurial Risk, D-MTEC, ETH Zürich

Matthias Troendle

CFO, INFICON Holding AG, Bad Ragaz, Switzerland

**MAS–MTEC Thesis, ETH Zürich**

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# Summary

In this report, we analyze the employee stock option (ESO) program of the company INFICON Holding AG in terms of the optionees' exercise behavior, the expected lifetime of the options and the performance of the currently used Black–Scholes valuation. After a brief introduction into the theory of option pricing, we dwell on the employee stock option specifics and the regulations of US–GAAP (and IFRS2) on how to valuate them. The valuation regulations are highly restricting in terms of finding a 'fair' value that matches as accurately as possible the intrinsic value of the options once they are exercised. In order to better incorporate the particularities of ESOs into the valuation, we propose a well–established lattice model, the Hull–White model, as an alternative to the Black–Scholes formula. We analyze individual input parameters to the valuation models, such as the volatility, the expected lifetime, the optionees' exit rate, for both Hull–White and Black–Scholes models, and seek for more accurate formulations of those parameters in order to improve the accuracy of the option value estimate. We propose a highly simplifying method of how to include the non–hedgeability of ESOs. Finally we compare the Hull–White and the Black–Scholes models for their sensitivity to the fundamental input parameters such as volatility, expected lifetime, risk–free interest rate and the dividend yield.

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# 1 Motivation

The company INFICON Holding AG, provider of instruments for gas analysis, measurement and control, warrants a stock option plan for its employees and the board of directors as an incentive for the employees and the board members to contribute to the future success and prosperity of the company and to maintain continued employment. To ensure the goals of the program, the option plan includes a vesting delay as well as transferability restrictions and the possibility for expiration in case of termination of the employment/board membership. The strike price corresponds to the share price at the day of grant. When these options are exercised, new shares will be issued from a conditional share capital thus diluting the present equity.

In the income statement, the expenses for the prospective costs of the option plan amount to a significant number in the order of $800 - 1000$ kCHF per year , compared to a net income of 2.27 M$ in 2009. This value is obtained using the Black-Scholes (B–S) option-pricing model.

However, the B-S model was originally developed to estimate the fair value of exchange-traded equity options, which neither have the above-mentioned restrictions of the employee option plan, nor (as a purely financial instrument) want to consider fundamental strategic issues such as ensuring a long-term commitment to the company and the long-term success of the company. As such, it is e.g. questionable whether a B-S type model with its positive relationship between volatility and the option price can at all provide a measure for the valuation of an employee option plan.

Moreover, it is the feeling of the INFICON management that the B–S valuation of the options has resulted in a much too high value (thus much too high expenses in the income statement) compared to the benefit the employees have gained from exercising them. As such, the B–S model itself seems to be too limited as to adequately indicate the value of the options.

The aim of the thesis is therefore to (a) compare the historical expenses due to the B-S option valuation with the revenues due to the exercised options to judge whether today's feeling of the too high B–S valuation is justified and (b) find more adequate pricing models for the company's stock option plan, both in term of proper valuation and also in the sense of accounting for the above-mentioned strategic issues such a stock option plan addresses.

# 2 Fundamentals of Employee Stock Option Valuation

## 2.1 Option Pricing

With the publication of the Black–Scholes–Merton model [1, 2] in 1973, the valuation of many types of commonly traded stock options has seen a significant advancement despite the fact that the formalism used in the calculation of the option value has been developed already around the turn of the 20th century by Bachelier (1900) [3] and Bronzin (1908) [4]. Due to the ease of use, the so-called Black–Scholes formula for the valuation of derivatives has become ubiquitous.

In this section, I try to give a brief overview over the mechanics of option pricing in an arbitrage–free world which allows us to formulate the classic option pricing models such as the closed–form Black–Scholes–Merton model or the binomial lattice model of Cox–Ross–Rubinstein [5]. For this purpose a small detour into the maths of stochastic processes is inevitable. From this protoype models for over–the–counter and exchange–traded options, we will then move on to the employee stock option specifics. An excellent introduction into all option matters discussed here is given by Hull [6].

There are six basic factors that determine the price of a stock option, the stock price at grant, $S_0$, the strike price $K$, the time to expiration of the option, $T$, the volatility of the stock price $\sigma$, the risk–free interest rate $r$ and the dividend yield $q$. The stock price as a function of time $t$ will be denoted as $S(t) = S_t$.

### 2.1.1 The Stochastic Process of a Stock Price

#### Preliminaries

A variable that changes over time in an uncertain way is said to follow a *stochastic process* and obviously the stock price is such a process. Extensive data analysis has shown [7] that daily stock returns $S_t - S_{t-1}$, from one trading day at time $t-1$ to the next at time $t$, are not correlated, meaning that we cannot predict the stock price $S_{t+1}$ from the return at time $t$. In mathematics, such a stochastic process where only the last value of the variable is kept in memory and where

any longer past history of the variable is irrelevant is termed *Markov process*. The prototype process of such a Markov process is the *Brownian motion*, the motion of small particles subject to a large number of molecular shocks. This process, equivalent to the *random walk process*, is usually denoted as the *Wiener process*. It has the following properties:
The stochastic variable $z$ follows a Wiener process if

1. the change of $\Delta z$ during the time period $\Delta t$ is
$$\Delta z = \epsilon\sqrt{\Delta t} \tag{2.1}$$
where $\epsilon$ has a standardized normal distribution, i.e. $\epsilon \in \mathcal{N}(0,1)$, meaning that the expectation $E(z)$ of the distribution is zero, $E(z) = 0$ and that the square root of the variance $Var(z)$, which ist the standard deviation, equals 1, $\sqrt{Var} = 1$. (1)
2. the values of $\Delta z$ for any two different short intervals of time are independent.

Thus, for a process of $N$ steps of width $\Delta t$, the expectation value of $z$ will be $E(z) = 0$, the standard deviation will be $\sqrt{Var} = \sqrt{N\Delta t}$.
If we want to complicate matters in order to have a non-zero expectation value for a stochastic variable $x$, i.e. when we ask for a drift $a$ as $a = \mathrm{d}x/\mathrm{d}t$, the resulting *Generalized Wiener process* writes as
$$\mathrm{d}x = a\mathrm{d}t + b\mathrm{d}z \tag{2.2}$$
where $\mathrm{d}z$ follows a Wiener process and $a$ and $b$ are constants. For a small time interval $\Delta t$, we can therefore write
$$\Delta x = a\Delta t + b\epsilon\sqrt{\Delta t} \tag{2.3}$$
The average change in $x$ during the period $T$ is $E(x) = aT$, the standard deviation of $x$ is $\sqrt{Var(x)} = b\sqrt{T}$. Noting that the variance is the square of the standard deviation, we can regard $b^2$ as the variance rate, the variance per unit time. Thus, the stochastic variable $x$ of a generalized Wiener process is normally distributed according to $\mathcal{N}(a\Delta t, b^2\Delta t)$
We can once more generalize in that we make the constants $a$ and $b$ now functions of the process variable $x$ and the time $t$, ie.
$$\mathrm{d}x = a(x,t)\mathrm{d}t + b(x,t)\mathrm{d}z \tag{2.4}$$
This process is then called a *Itô process* in honour of K. Itô. It is also known as *geometric Brownian motion*. Itô discovered an important result for this process, termed *Itô's Lemma* which states:

Given a stochastic variable $x$ that follows an Itô process according to Eq. 2.4 and define a function $G$ of $x$ and $t$. Then $G$ follows the process
$$\mathrm{d}G = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t}\mathrm{d}t + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)\mathrm{d}t + \frac{\partial G}{\partial x}b\mathrm{d}z \tag{2.5}$$

(1) Note that the volatility $\sigma$, defined later on again, is the square root of a variance *rate*, and its greek letter $\sigma$ should not be confounded with the square root of the variance, the standard deviation, for which the same letter is typically used in literature.

Thus, $G$ also follows an Itô process with a drift rate $\alpha$ and variance rate $\beta$ given by

$$\alpha = \frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t}\mathrm{d}t + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2 \tag{2.6}$$

$$\beta = \left(\frac{\partial G}{\partial x}\right)^2 b^2 \tag{2.7}$$

Hence if we take a function $G(x)$ of an Itô process $x$, Itô's Lemma tells us how the process for this function $G(x)$ will look like: it's again an Itô process.

### The Stock Price Process

Let's now turn to how to describe the evolution of a stock price in mathematical terms. Generally speaking, an investor expects a certain return from his investment, if he invests into a stock he therefore asks for a certain return which is independent of the stock price. For a stock with price $S_t$ at time $t$, he therefore expects the stock to increase during the period $\Delta t$ to some $\mu S_t \Delta t$, where $\mu$ is the expected rate of return on the stock. Thus in the limit $\Delta t \to 0$ we can write

$$\mathrm{d}S = \mu S \mathrm{d}t \qquad \text{or} \qquad \frac{\mathrm{d}S}{S} = \mu \mathrm{d}t \tag{2.8}$$

As for the volatility of the stock price, a reasonable assumption is that the variability of the percentage return in a short period of time is independent of the stock price, implying that the standard deviation of the change in a short time interval is proportional to the stock price. This leads to the following stochastic process model for a stock price

$$\text{continuous time limit} \quad : \quad \mathrm{d}S = \mu S \mathrm{d}t + \sigma S \mathrm{d}z \tag{2.9}$$

$$\text{discrete time} \quad : \quad \Delta S = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \tag{2.10}$$

where $\epsilon \in \mathcal{N}(0,1)$. Dividing Eq. 2.10 by $\Delta S$ shows that $\frac{\Delta S}{S} \in \mathcal{N}\left(\mu\Delta t, \sigma^2 \Delta t\right)$.
The process given above leads to the exponential evolution of the expectation and the variance as

$$E(S_T) = S_0\, \mathrm{e}^{\mu T} \tag{2.11}$$

$$Var(S_T) = S_0^2\, \mathrm{e}^{2\mu T}\left(\mathrm{e}^{\sigma^s T} - 1\right) \tag{2.12}$$

Now, it can be shown that the above form of $E(S_T)$ and $Var(S_T)$ satisfy the prerequisites of a *lognormal distribution*, i.e. where the $\ln(S_T)$ follows a normal distribution. In fact, we can take advantage of Itô's Lemma by defining $G = \ln S_T$ to obtain from the process in Eq. 2.9:

$$G = \ln S \qquad \Longrightarrow \qquad \mathrm{d}G = \left(\mu - \frac{\sigma^2}{2}\right)\mathrm{d}t + \sigma \mathrm{d}z \tag{2.13}$$

So that $\ln S_T$ follows the normal distribution according to

$$\ln S_T - \ln S_0 \in \mathcal{N}\left[\left(\mu - \frac{\sigma^2}{2}\right), \sigma^2 T\right] \tag{2.14}$$

$$\ln S_T \in \mathcal{N}\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right), \sigma^2 T\right] \tag{2.15}$$

On the other hand, if we define the continuously compounded rate of return per annum[(2)] realized between times 0 and $T$ by $\bar{\mu}$, so that we can write the time dependence of the stock price as

$$S_T = S_0 \, e^{\bar{\mu} T} \tag{2.16}$$

we get $\bar{\mu} = \frac{1}{T} \ln S_T S_0$. But, in conjunction with Eq. 2.14 this is equivalent to saying that $\bar{\mu}$ follows the distribution

$$\bar{\mu} \in \mathcal{N}(\mu - \frac{\sigma^2}{2}, \frac{\sigma^2}{T}) \tag{2.17}$$

Conclusively, in all the calculations regarding option pricing, we use the *log returns* of the stock, i.e. $\ln \frac{S_t}{S_{t-1}}$ and not the stock returns.

### 2.1.2 Volatility

The volatility of the stock price is defined here in its strict sense as the square root of the variance *rate*. Volatility is a measure of the spread of the stock return, thus closely related to the return's standard deviation. Given the daily log return of the stock as

$$u_i = \ln \frac{S_i}{S_{i-1}} \qquad \text{for all } n+1 \text{ observations, with} \quad i = 0 \ldots n \tag{2.18}$$

the estimator $s$ of the standard deviation is given by

$$s = \sqrt{\frac{1}{n-1} \sum_i (u_i - \bar{u})^2} \qquad \text{where } \bar{u} = E(u) \tag{2.19}$$

Now, from Eq..2.14, we see that this estimate of the standard deviation equals $\sigma \Delta t$ where $\Delta t$ is the time span (one trading day). Thus we can calculate the volatility from

$$\sigma = \frac{s}{\Delta t} \tag{2.20}$$

depending on the period $\Delta t$. Note that the volatility is estimated from the number of trading days and not from real days. Thus, the volatility per annum which is used in the standard option pricing calculations is given by

$$\text{volatility per annum} = (\text{volatility per trading day}) \times \sqrt{\text{Number of trading days per year}} \tag{2.21}$$

Note that this definition of the volatility conforms to the definition of the continuously compounded rate of return in Eq. 2.17, where $\bar{\mu}$ is distributed with mean $\mu - \frac{\sigma^2}{2}$ and standard deviation $\frac{\sigma^2}{T}$. Hence, $\sigma$ is the volatility per annum, as a the standard deviation for the time period $T$ is $\frac{\sigma}{\sqrt{T}}$. At INFICON, the number of trading days per year was set to 257, except for the 2010 grant where it is set to 252.

(2) See Appendix A on continuously compounded rates

### 2.1.3 How to Price an Option

The fundamental idea of option pricing is to set up a *replicating portfolio*, an option equivalent, by combining common investments of stocks and borrowing. Such a portfolio is said to be a replicated one if it matches the behavior of the option exactly, e.g. it has the same net cost as the option.

An advantageous way to set up the replicating portfolio is to make it risk–neutral by having it contain the option and the underlying stock. The stock price process can be described according to Sect. 2.1.1. In that case, the option value can be found straightforwardly because the present value of the portfolio can be obtained from discounting with the risk-free interest rate.

Here, we exemplify the procedure where the portfolio consists of writing a covered call. The portfolio $\Pi$ consists of

- 1 short position of a call (i.e. we *write* a call) with the unknown value $f$.
- The fraction $\Delta = \partial f/\partial S$ of shares[3] of the underlying stock valued at $S$.

We will now valuate this portfolio in the continuous, closed form and as a discrete scheme. In the following, we derive the value of a call option. If we need to price a put option, we simply use the put–call parity to obtain the put value.

#### In Continuous Formulation: The Black–Merton–Scholes Equation

The value of the portfolio is given by

$$\Pi = \frac{\partial f}{\partial S} S - f = \Delta S - f \tag{2.22}$$

Note that we substract $f$ as we *write* the call.

We have already all ingredients to calculate the option value $f$. As $f$ is some function of $S$ and the time $t$, we can use the stock price process, Eq. 2.9 and Itô's Lemma, Eq. 2.5

$$\mathrm{d}S = \mu S \mathrm{d}t + \sigma S \mathrm{d}z \tag{2.23}$$

$$\mathrm{d}f = \left( \frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \mathrm{d}t + \frac{\partial f}{\partial S} \sigma S \mathrm{d}z \tag{2.24}$$

If we put Eqs. 2.23–2.24 into the change of the replicating portfolio $\mathrm{d}\Pi$ during the time interval $\mathrm{d}t$ given by[4] $\mathrm{d}\Pi = \partial f/\partial S \mathrm{d}S - \mathrm{d}f$, we will obtain after some rearranging

$$\mathrm{d}\Pi = \left( -\frac{\partial f}{\partial t} - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \mathrm{d}t \tag{2.25}$$

(3) Note that the hedging Delta $\Delta = \partial f/\partial S$ should not be confounded with the Delta of the notation for the discrete time step $\Delta t$.

(4) Note that the hedging Delta: $\Delta = \partial f/\partial S$ is fixed at time $t$.

The paramount property of Eq. 2.25 is that it does neither involve $\mathrm{d}z$ nor the rate of return $\mu$. Because of the advantageous variable change, Eq. 2.25 is independent of risk preferences, which are inherent in $\mu$, so that risk cannot affect the portfolio. Thus the portfolio must be riskless during time $\mathrm{d}t$. Therefore it must earn the same rate of return as other risk–free securities, hence

$$\mathrm{d}\Pi = r\Pi\mathrm{d}t \tag{2.26}$$

where $r$ is the risk–free interest rate. Combining Eqs. 2.22, 2.25 and 2.26 leads to the fundamental *Black–Merton–Scholes differential equation*

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{2.27}$$

This differential equation has many different solutions depending on the boundary conditions. The *Black–Scholes formula* is the solution of Eq. 2.27 for the following particular, but very important boundary conditions for an European call of strike price $K$ and maturity $T$

$$\text{B.C.}: \; f = max(S-K,0) \qquad \text{for} \quad t = T \tag{2.28}$$

**Black–Scholes Formula** The Black–Scholes formula for the price at time 0 of an European call option on a non–dividend paying stock is

$$f = S_0\Phi(d_1) - K\,\mathrm{e}^{-rT}\Phi(d_2) \qquad \text{with} \tag{2.29}$$

$$d_1 = \frac{\ln(S_0/K) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} \tag{2.30}$$

$$d_2 = \frac{\ln(S_0/K) + \left(r - \sigma^2/2\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{2.31}$$

where $\Phi(x)$ is the cumulative probability distribution function for a variable $x$ which has a standardized normal distribution. $S_0$ is the stock price at time $t = 0$.

### Discrete Formulation: Lattice Model

The discrete approach for option valuation was spearheaded by Cox, Ross and Rubinstein [5]. We consider again the same portfolio $\Pi$, that contains for each option of value $f$ a fraction $\Delta = \partial f/\partial S$ of the underlying stock $S$. Given the option price $S_0$ at $t = 0$, the stock price may move in discrete steps, either up through the factor $u$, $u > 1$ or down by the factor $d$, $0 < d < 1$, which will result in an adjusted option value $f_u$ and $f_d$, respectively. In order to make the portfolio riskless, we require its value to be equivalent independent of whether the stock price moves up or down, hence we require

$$\Pi_{\text{Stock up}} = \Pi_{\text{Stock down}} \qquad \Longleftrightarrow \qquad \Delta S_0 u - f_u = \Delta S_0 d - f_d \tag{2.32}$$

i.e.

$$\Delta = \frac{f_u - f_d}{S_0 u - S_o d} \tag{2.33}$$

Moreover, as we ask for a riskless portfolio, the cost of setting up the portfolio must equal the present value of the future price in $t = \Delta t$ discounted by the risk–free rate $r$:

$$\Delta S_0 - f = (\Delta S_0 u - f_u)\ \mathrm{e}^{-r\Delta t} = (\Delta S_0 d - f_d)\ \mathrm{e}^{-r\Delta t} \tag{2.34}$$

Rearranging the above equation with Eq. 2.33 allows to calculate the option price at $t = 0$, $f$ as:

$$f = \mathrm{e}^{-r\Delta t}\,[p f_u + (1-p) f_d] \qquad \text{with} \qquad p = \frac{\mathrm{e}^{r\Delta t} - d}{u - d} \tag{2.35}$$

From the above equation, we can interprete $p$ as the probability of an up–move of the stock, $(1-p)$ as a downward move, in a risk–neutral world. Finally, we want to connect the factors $u$ and $f$ for the stock movements with the volatility of the stock, which is at their origin. For this purpose, we match the expected return and the standard deviation of the portfolio with those of the tree model (i.e. we match the first and the second moments). In mathematical terms, this states as

$$\text{expected return} \qquad p S_0 u + (1-p) S_0 d = S_0\,\mathrm{e}^{r\Delta t} \tag{2.36}$$

$$\text{variance} \qquad p u^2 + (1-p) d^2 - (pu + (1-p)d)^2 = \sigma^2 \Delta t \tag{2.37}$$

In Eq. 2.37, the variance of the stock is given as $\sigma^2\Delta t$ as defined in Eq. 2.20, Section 2.1.2. The variance of our tree model, the left side of the equation, is obtained from the definition of the variance: $Var(X) = E(X^2) - [E(X)]^2$. Whereas Eq. 2.36 gives as result $p = p(u, d, r, \Delta t)$ as in Eq. 2.35, Eq. 2.37 allows to extract the values of $u$ and $d$ as a function of $\sigma$ — when the higher orders in $\Delta t$ are ignored, a solution is [5]:

$$u = \mathrm{e}^{\sigma\sqrt{\Delta t}} \qquad \text{and} \qquad d = \frac{1}{u} \tag{2.38}$$

We have shown so far the calculation of $f$ involving one single time step. The procedure is easily extended to an arbitrary number of time steps as we build a binomial tree, see Fig. 2.1 below. We are now in the position to calculate all values of $S_{i,j}$ and $f_{i,j}$ in the tree. In practice, the procedure for calculating the value for an option with maturity $T$ is the following

1. Calculate the values of $u$, $d$ and $p$ given $\sigma$, $r$, $\Delta t = T/N$ where $N$ is the number of steps of the tree. Time progresses in steps $\Delta t$ so that its discrete values are $t = i\Delta t$, with $i = 0, \ldots, N$.
2. Calculate all possible stock prices of the tree at maturity $t = T$, i.e. calculate $S_{N,j}$ with $j = 0, \ldots, N$. The values are $S_{N,j} = S_0 u^j d^{N-j}$.
3. Calculate all possible option values at maturity, $f_{N,j}$. The values as $f_{N,j} = max(S_{N,j} - K, 0)$
4. Work backwards in time utilizing $f_{i,j} = p f_{i+1,j+1} + (1-p) f_{i+1,j}$.
5. The option value at present is obtained at $f = f_{0,0}$.

The accuracy of a lattice model strongly depends on the way the lattice is set up. In particular, a sufficient number of nodes in decisive for an accurate estimate. Several modifications of the prototype binomial model of Cox et al. have been proposed (see e.g. in Hull [6], chapters 19 and 26). One modification is to use a trinomial lattice with three pathways emanating from each

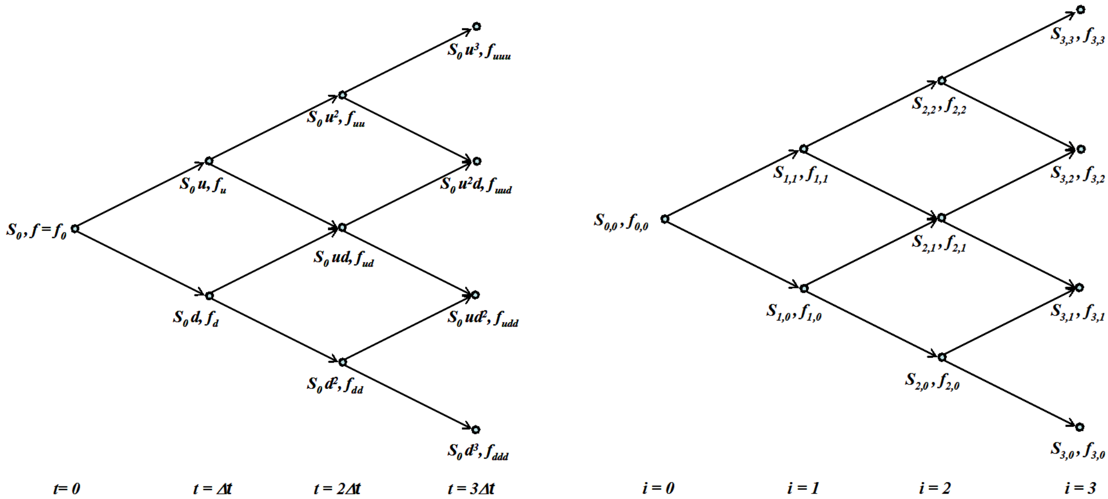


**Figure 2.1:** Left: Sketch of a three–step binomial tree showing the evolution of the stock price $S_t$ and the option price $f_t$, Right: Codification of $S_{i,j} and f_{i,j}$ as a function of time $t = i\Delta t$.

node, one up, one down and a center path. For the subsequent simulations using a lattice model, I have tried different approaches, binomial as well as trinomial lattices [6], partly optimized for speed [8, 9]. I found that the gain in accuracy obtained from the trinomial lattice was regularly offset by a decreased speed of the trinomial approach. Another modification is required if a exercise–trigger (kind of a barrier option) or time–dependent variables such as $r = r(t)$ or $\sigma = \sigma(t)$ are introduced. In the former case, it is advantageous that the trigger level is at certain nodes using the approach by Levitan [10], for the latter, in case of time–varying volatility, the time steps are modified in a way so that for each time step, the standard deviation remains the same (see [6], chap. 19). All variants used in the simulations are given as Matlab code in Appendix B.

Having seen the valuation of regular options, we now move on to the particularities that are present with employee stock options.

## 2.2 Particularities of Employee Stock Options

Employee stock options (ESOs) differ fundamentally from standard exchange-traded and over-the-counter options in several aspects and their particular properties are not reflected in standard option valuation tools such as the widely used simple Black–Scholes formula. The main differences of ESOs to exchange-traded options are:

**Non-transferability** Usually, an employee is not allowed to sell the ESO granted to him. This non-transferability or non-marketability represents the most intricate property from a valuation perspective. In fact, if the employee wants to realize cash or diversify his portfolio, his only possibility is to exercise the option. From a theoretical point of view however, such an American call option (on a non-dividend paying stock, to be exact), should never be exercised early, an option holder who wants to diversify should sell an option rather than exercise early. Several studies have shown that early exercise is indeed common practise [11, 12, 13, 14]. This early exercise possibility in turn reduces the value of the option and hence should be accounted for in the valuation model.

**Non-hedgeability** Non–hedgeability is closely related to the non–transferability of ESOs. The option valuation models presented in Sect. 2.1 are valid only in a complete, arbitrage–free market. This requires that the replicating portfolio can be hedged, i.e. that we can adapt the fraction $\Delta = \partial f / \partial S$ of our portfolio $\Pi$ to keep it risk–free. However, the optionee of an ESO does not have this possibility as he has no other choice than to accept the option grant or not, and cannot take another financial equivalent for it.

**Vesting period** Typically, ESOs have a vesting period during which the options cannot be exercised. This vesting period can be seen as a service period for which the employee is granted the intrinsic value of the option, which is the stock price at exercise minus the exercise price. According to IFRS 2, the vesting period is 'The period during which all the specified vesting conditions of a share–based payment arrangement are to be satisfied' [IFRS 2 Appendix A]. Furthermore, the vesting period should ensure the long-term benefits of ESO plans for the firm.

**Forteiture and cancellation of options upon termination of employment** When the employment ends, the option becomes nullified after a certain delay period to ensure the long term binding of the employee to the company. When the options become nullified during the vesting period, the options are said to *forfeit*, whereas they are *cancelled* when they have vested.

**Dilution of the present equity** When ESOs are exercised, the company issues new shares which results in the dilution of the present shareholders equity. In that sense, an employee stock option is a type of a warrant. Paragraph [718-10-55-49] of ASC Topic 718 states that if the market for a firm's share is reasonably efficient and the exercise of the options is anticipated by the market, the effect of the dilution from exercising ESOs will be reflected in the market price of the underlying share. Therefore, dilution effects should not be included in the calculation of the option value. An exception to this rule can occur when a large, unexpected number of option grant takes place or when this grant is generally not believed to be at the benefit of the company. Also Hull [15] states that the effect of dilution is already contained in the underlying share price and is therefore not to be included in the valuation of the ESOs.

Option pricing theory shows that it is never optimal to exercise an European or a non–dividend paying American call option before its expiration date, only for dividend–paying American call options it can be advantageous to exercise early. For ESOs however, it is a common finding that the options are exercised early. The optimal time of exercise of an ESO from the optionees point of view depends on several factors such as the current employment relationship (if employment terminates), the liquidity and portfolio diversification needs of the exercising employee, his/her risk aversion and of course his/her view of the prospects of the company he/she is working for.

Another point that promotes early exercising deserves attention, especially for firms that issue large ESO grants: Whereas it is in principle disadvantageous to exercise early, defectors of the wait–until–maturity strategy who exercise early will profit from the undiluted stock price, those who wait until maturity will see their profits slip due to dilution of the defectors' exercised options. Hence, a social dilemma forms, because in awareness of the above defector strategy, most optionees tend to exercise early, which in turn will be anticipated by the market so that the stock price is adjusted accordingly. In turn, the average profit from exercising early will be suboptimal, reducing the overall benefit of the optionees.

## 2.3 Regulations for ESO Valution according to US–GAAP

The accounting regulations concerning ESOs within US–GAAP are summarized in the FASB Accounting Standards Codification 718, denoted in short form as ASC 718 or Topic 718, in effect since 09.2009. Topic 718 is the successor of the regulations FASB 123(R). As for the valuation of INFICON's stock option program, an award classified as equity, the two regulations are basically equivalent. They are in fact very similar to the IFRS2 accounting standards as well.

### 2.3.1 Why ESOs should be Recognized

When with the creation of Topic 718's precessor, FASB123(R), it became mandatory to recognize share–based payments as expenses under US–GAAP, it created a considerable dispute between the FAS board and the community in general. Substantial reductions in the average income of the companies were predicted and triggered widespread disagreement against any recognition [12]. A study of Firicon on the effect of expense recognition of 110 German companies in 2006 found an average cost of share–based payments of 5.06% of the companies income [16].
An important argument of the opponents of the recognition of ESOs as expense is that the expenses for the company are limited to the transaction costs for the issuance of the stock upon exercise of an option, but that no real expenses in terms of a cash–out occur. In addition, as in the case of INFICON, when ESOs are granted as a gratification and a way to bind the employee to the company but are not contractually guaranteed, it can be argued that the employee does not have to render any service in order to obtain the options intrinsic value. In the words of the FAS Board [FASB123(R),B16]

> Some respondents to the Exposure Draft said that an entity does not receive an asset, and thus does not incur compensation cost, when it receives employee services in exchange for equity instruments. The Board disagrees; employers receive employee services in exchange for all forms of compensation paid. Those services, like services received from nonemployees, qualify as assets, if only momentarily because receipt of a service and its use occur simultaneously.

An *asset* in turn has, according to B17. FASB Concepts Statement No. 6, the following three essential characteristics:

> (a) it embodies a probable future benefit that involves a capacity, singly or in combination with other assets, to contribute directly or indirectly to future net cash inflows, (b) a particular entity can obtain the benefit and control others' access to it, and (c) the transaction or other event giving rise to the entity's right to or control of the benefit has already occurred.

The proponents of the expense recognition note the example where shares of stock were issued to acquire legal or consulting services, tangible assets, or an entire business in a business combination. To omit such assets and the related costs would give a misleading picture of the company's financial position and financial performance. The Board's opinion is summarized in FAS123(R), B21:

> Accounting for assets received (and the related expenses when the assets are consumed) has long been fundamental to the accounting for all freestanding equity instruments except one-fixed equity share options that had no intrinsic value at the grant date (...). This Statement remedies that exception.

Also, any pro forma disclosure seems not to be an acceptable substitute for recognition [FAS123(R), B23].

In principle, according to corporate finance, a fundamental goal of a corporation is to maximize the shareholder(5) value. In that sense, ESOs create more than just transaction costs to the firm, as the real cost at exercise of an ESO is carried by the stockholders who see their investment being diluted. A study by Aboody on the effects of ESOs on the company's stock price from 1996 [17] concludes that there is a *negative* correlation between the value of outstanding options and the firm's stock price for medium–sized and large firm's whose outstanding options exceed 5 % of the registered shares. This negative correlation becomes stronger when the outstanding options are exercisable and in–the–money. A slight positive correlation however exists when the options are still vesting. This study therefore corroborates the opinion that the firm's cost of issuing ESOs is not only transaction cost, but that it's value is negatively influenced by the dilution effect of ESOs for the current stockholders and that ESOs should therefore considered as real cost for the

(5) shareholders being an important part of the firm's stakeholders

company(6).

Next to fundamentally agreeing on recognizing ESO grants as expenses, another dispute dealed and still deals with the valuation method, in particular whether the expenses shall be based on an option valuation method that fixes the value at a specific 'measurement date', typically the grant date, or whether the intrinsic value, i.e. the stock price at exercise minus the strike price shall be taken as a base for the expenses. The latter is simple and straightforward and reflects the true value the optionnee obtains for his service. The significant problem of finding the fair value by extrapolation of present and past knowledge is therefore easily circumvented. The usual accounting response to major problems in measuring the effects of a transaction is to defer final measurement until the measurement difficulties are resolved [FAS123(R),B45]. Nevertheless, it was decided to retain the grant date as the measurement date, as it was argued that the value of that service should be measured and recognized based on the share price at the date the parties reach a mutual understanding (which is the grant date) and that the board wanted to achieve convergence with international accounting standards [FAS123(R),B46–48].
Despite the FAS board's decision, there are strong reasons why an intrinsic valuation of ESOs would be advantageous from a accounting point of view. Most importantly, the true value of the option grant could be written as expense at exercise. The ESOs would be treated similar to other derivative transactions of the company. If the ESO became more or less valuable from one accounting period to the other, there would be an additional charge to the income statement or previous expenses could be clawed back respectively. Advocates of the grant date option valuation might argue that the uncertain outcome of the option value creates unwanted volatility in the income statement of future years, but as Hull remarks [6](p.314f), the valuation of the options according to the intrinsic value will most likely even have a stabilizing effect on the income statement, as in years when the company does well, the stock price will likely be high which translates into higher ESO expenses to be written and vice versa.

### 2.3.2 Requirements from ASC Topic 718

According to ASC Topic 718, which is more or less equivalent to IFRS 2 in terms of share–base payment recognition, employee options should be recognized as expenses when the options are part of an employee share scheme, as these benefits form part of the remuneration of the employees for services provided to the company [718-10-25-2, IFSR 2.7](7). For awards classified as equity, which is the case for Inficon's stock option program, the amount to expense should correspond to the fair value of the option [718-10-30-2], defined as *The amount for which an asset could be bought, or incurred, sold or settled in a current transaction between willing parties, that is, other than in a forced or liquidation sale* [718-10-20]. The fair value should be fixed at the grant date [718-10-30-3]. In the case of INFICON's stock option program, the option grants are linked to a

(6)Note however that the dilution should not be considered in the valuation of the ESOs because the exercise of the options is anticipated by the market and therefore already reflected in the share price [15].

(7)In the following, all references to ASC Topic 718 regulation articles are given in square brackets such as these.

service condition in that the options vest only after a certain time [718-10-55-67]. In topic 718 (as in IFRS 2), the expenses shall be recognized after the requisite service by the employee has been received [718-10-25-2]. No compensation cost shall be recognized if the requisite service is not rendered [718-10-25-21]. Hence, when optionees terminate their employment and have to forfeit[(8)] their unvested option rights, the fair value of these unvested options should not be expensed. However, vesting conditions and the forfeiture rate of unvested options shall not influence the calculation of the fair value, the latter will be calculated as if the options had vested immediately [718-10-30-17]. This means that in order not to recognize forfeited options, their fair value will not be expensed, instead of lowering their fair value by the inclusion of a pre-vesting exit rate. Finally, previously recognized expenses shall not be reversed for options that have vested (i.e. the requisite service has been rendered) but expire unexercised [718-10-35-3].

The method to valuate the fair value is not fixed [718-10-55-10 to 20, in particular 718-10-55-17, IFRS 2.B4], in [718-10-55-10], topic 718 states that if equivalent options are on the market, that those market prices should be taken as fair value, *... observable market prices of identical or similar equity or liability instruments in active markets are the best evidence of fair value...*. In case such options are not available, as it is the case for INFICON, an established valuation model shall be used [718-10-55-11]. Closed–form or lattice models can be used as other methods such as Monte Carlo models. But the model must estimate the fair value at a single point in time, the grant date [718-10-55-15] and it is explicitely accepted that the valuation method will provide erronous results, especially given the long time period to consider [718-10-55-15]. Nevertheless, a re–valuation at a later stage is not allowed. Factors to consider in the valuation of the options are, at minimum, all of the following:

1. **the exercise price of the option** The exercise price is fixed at grant date, for INFICON, the exercise price corresponds to the market price at grant date.
2. **the current price of the underlying shares**
3. **the expected term of the option** Several factors influence the expected life of an ESO, denoted *expected term*, such as the length of the vesting period, historical data on exercise behavior within the firm, the price of the underlying share, the expected volatility, the risk aversion of the optionees and the optionees employment level within the organization. Section 2.2 summarizes all the effects that lead to early exercising. For closed–form models, the expected term is an input to the model, whereas it is an output for lattice models [718-10-55-30]. The expected term could be the result of an exercising trigger barrier, a stock–price to strike–price ratio at which the optionees on average exercise. Such a barrier can be modeled in a lattice approach.
4. **the expected volatility of the share price** The topic 718 does not specify a method for calculating the expected volatility. The expected volatility can be based on historical volatilities of the stock price when a comparable period to the option life is considered [718-10-55-36], or it can be based on implied volatilities or a mixture of both. When the volatility

[(8)] options that expire during the vesting period are said to *forfeit*.

is calculated from implied volatilities of traded options, it should be noted that the terms for such options are in general much shorter as those of ESOs. Weighting is also an accepted practise. When using a closed–form model, the expected term is at the basis of the volatility estimate, whereas it is the contractual term for lattice models [718-10-55-37].

5. **the dividends expected on the shares** Dividend yields or dividend amounts may be used in the calculation of the fair value. Additionally, a company's historical pattern of dividends shall be considered [718-10-55-42].

6. **the risk–free interest rate for the expected term of the option** The interest rate to use must be the risk–free interest rate, in particular the implied yield of the currently available zero–coupon government issues over the contractual term of the options (for lattice models) or the expected term (for closed–form models), denominated in the currency of the market in which the share primarily trades, in our case the interest rates of the Swiss government bond [718-10-55-28].

Note that dilution is not to be considered in the fair value determination as the dilution will be reflected in the share price [718-10-55-49]. Finally, the valuation model should be consistent and changes in the valuation should be well founded.

## 2.4 Regulative Limits for Adequate ESO Valuation

The main difficulty for adequately valuating ESOs lies in the combination of valuing the options at grant date together with the very long contractual terms of typically 7 to 10 years for ESOs, which is at least an order of magnitude more than typical exchange–traded options. US–GAAP disallows to use methods that expense the true intrinsic value, e.g. by initially setting up provisons for the options value and adjusting these provisions at each financial year ends until the ESOs are either exercised or cancelled.

### 2.4.1 ESO's as an Non-Replicable Portfolio

The different nature of an ESO in comparison to a regular call were described above, namely their non-transferability, the dilution of the present equity, the existence of a vesting period and their forfeiture upon termination of the employment. These differences are taken into account in ASC Topic 718 compatible models by implementing adaptions, more properly termed fudge–factors, such as an exercise barrier $M$, an exit rate $w$ or an expected life calculation. However, all of the above models are based on the risk–neutrality assumption and therefore discount at the risk–free interest rate. The risk–neutrality in turn is based on the assumption that the fair value of the option is obtained because a risk–neutral replicating portfolio can be set up (due to arbitrage–free market) that matches the option's behavior. In the case of an ESO, the argument

of market completeness (= replication is possible) is not given, as the following arguments show: The ESO is granted to the optionees in return for their service delivered (this is after all the fundamental argument of FASB to expense ESOs), however, the employee cannot choose the form of the payment for his or her service. Accordingly, the optionee is denied the freedom to choose an alternative portfolio as a payment, in particular a risk-free portfolio that replicates the option. Thus, instead of taking the risk–free interest rate in either the B–S model or any other lattice–based model, the interest rate $r$ (that is used in the option valuation models to denominate the rate of return for the non–stock asset part, ie the risk–free asset part of the replicating portfolio) should now reflect the systematic risk in dealing with the stock of the company and should thus include a risk premium, e.g. given by the market risk and the $\beta$ of the company in the capital asset pricing model view. This is because we cannot set up a risk–free asset. At the same time, we can ask whether the calculation of the present value of the strike price should also be done with the same $r$, or whether we would need another rate, e.g. a WACC type interest rate.
The FASB's response to this inability of the optionees to hedge his or her call option position was that (see [718-10-30-10 to 14 and 718-10-55-29]) the need for a higher rate of return $r$ was considered in the adjusted expected life of the options. As shown above, Topic 718, 718-10-55-28 requires $r$ to be the risk–free interest rate, in particular the implied yield over the contractual (lattice models) or expected life (B–S) term of the option currently available on zero–coupon government issues denominated in the currency of the market in which the share primarily trades. Whereas the FASB's propositon of incorporating the risk premium into the expected life term may empirically work for vested options (although it is rather dubious from a methodological point of view), the argument doesn't hold for options in the vesting period, which make a substantial portion of the expected life.

## 2.5 ASC Topic 718 Compatible Valuation Models

### 2.5.1 Modified Black–Scholes Formula

The B–S formula is most often used for the valuation of ESOs [18]. The only 'modification' usually consists to use the expected term as the maturity of the options instead of the contractual term, as put forward by ASC 718. Several complications to the simple B–S formula have been suggested within the Black–Merton–Scholes type model that includes e.g. early exercise [19, 20, 21, 22]. However, with those models, the simplicity of the B–S formula drops so that the main advantage of B–S over lattice models vanishes.
A number of issues make the B–S model a poor choice for employee options valuation. First of all, B–S is strictly only valid for European option that cannot be exercised until maturity. This is not the case for ESOs which are American options. Moreover, time dependent parameters and barrier– or utility–type models are not possible with the simple B–S formula and require partly sophisticated solutions for the Black–Merton–Scholes equation.

### 2.5.2 Lattice Models

Many models for ESO valuation found in literature are lattice models. Their attraction mainly lies in the fact that they allow to include time varying processes, such as time varying volatilities, and enable comparisons at every single node of the lattice. The latter is e.g. used for barrier–type or utility–type option valuations. On the one hand, utility maximing ESO exercise behavior, as e.g. described by [11, 13, 23] measure the cost of ESOs under the assumption that optionees behave optimally but are constrained from selling their ESOs or hedging their ESO risks. Carpenter [23], Hull and White [24, 15], Bajaj et al. [14], Ammann and Seiz [25] or Brisley et al. [26] develop simpler non-utility-based binomial models. Carpenter assumes an exogenous stopping rate at which exercise occurs if the ESO is in-the-money. Hull—White [24, 15] include the possibility of an employee exit rate, which is analogous to Carpenter's stopping rate, and an early exercise trigger, a multiple of the strike price at which voluntary exercise occurs with certainty. Brisley et al. [26] modify the Hull–White trinomial lattice model to allow for a declining exercise multiple as expiration nears. Bajaj et al. [14] use historical ESO exercise patterns to develop a grant date early exercise probability matrix as a function of the option's moneyness and its remaining life. Ammann and Seiz [27] compare several ESO valuation models. They find that except for the standard B–S model and the permitted Topic 718 simplification, the various models provide consistent pricing when they are all calibrated to the same expected term.

#### Hull–White Model

The Hull–White model [15] is a modified lattice model that extends the prototype Topic 718 model. The prototype Topic 718 incorporates the particularities of ESOs in comparison to a traded stock–option mentioned in Section 2.2 by reducing the life of the option to an average 'expected life' usually determined in hindsight from the average option lifetime. According to Hull et al., the true value of an option once it has vested is determined by a) the exercise strategy of the employee, and b) the possibility that the employee may be forced to exercise the option early or abandon it because he or she leaves the company. In particular, the optionees exercise policy, based on many factors such as his or her risk averseness or the liquidity needs can result in an intricate exercise pattern over time. The Hull–White model tries to explicitly incorporate the employee's early exercise policy by assuming that early exercise happens when the stock price is a certain multiple, $M$, of the exercise price. Thus, $M$ serves as a trigger for the exercise and the approach is in fact a model for a barrier option with the barrier:

$$\frac{S}{K} \geq M \tag{2.39}$$

Furthermore, Hull–White include a vesting and a post–vesting employee exit rate constant, $w_1$ and $w_2$ respectively, which defines the proportion of optionees exits per year during the vesting and during the post–vesting periods. Note that the model reduces to a simple American call option model for $M = 1$ and $w_1 = w_2 = 0$.

Given the stock price $S_{i,j}$ for all possible settings $j$ of the binomial tree at times $i$, the exercise price $K$, the trigger multiple of the exercise price, $M$, the vesting period $\nu$, $r$ the risk–free interest rate, and the employee exit rate $w_2$, the rules for calculating the option value $f_{i,j}$ are the following: At the end of the option's lifespan, $T$, we set

$$f_{N,j} = \max(S_{N,j} - K, 0) \tag{2.40}$$

Then, for the discrete time steps $0 \leq i \leq (N-1)$ of duration $\delta t = N/T$, and defining the probability of an up-step during the interval $\delta t$ as $p$:

$$\begin{aligned}
&\text{if} \quad i\delta t \geq \nu \quad \text{if} \quad S_{i,j} > KM \quad \Rightarrow f_{i,j} = S_{i,j} - K && (2.41)\\
&\qquad\qquad\quad\;\; \text{if} \quad S_{i,j} \leq KM \quad \Rightarrow f_{i,j} = (1 - w_2\delta t)\,\mathrm{e}^{-r\delta t}\,[pf_{i+1,j+1} + (1-p)f_{i+1,j}] \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + w_2\delta t \max\,(S_{i,j} - K) && (2.42)\\
&\text{if } i\delta t < \nu \qquad\qquad\qquad\qquad\;\; \Rightarrow f_{i,j} = \mathrm{e}^{-r\delta t}\,[pf_{i+1,j+1} + (1-p)f_{i+1,j}] && (2.43)
\end{aligned}$$

The option value at grant date, $f$, is then given by:

$$f = f_{0,0}(1 - w_1)^{\nu} \tag{2.44}$$

Hull and White also give a method to calculate an expected life $L$, which is set up similar to the option value $f_{i,j}$, i.e. they define $L_{i,j}$ as:
For i = N:

$$L_{N,j} = 0 \tag{2.45}$$

For $0 \leq i \leq (N-1)$;

$$\begin{aligned}
&\text{if} \quad i\delta t \geq \nu \quad \text{if} \quad S_{i,j} > MK \quad \Rightarrow L_{i,j} = 0 && (2.46)\\
&\qquad\qquad\quad\;\; \text{if} \quad S_{i,j} \leq KM \quad \Rightarrow L_{i,j} = (1 - w_2\delta t)\,[pf_{i+1,j+1} + (1-p)f_{i+1,j} + \delta t] && (2.47)\\
&\text{if} \quad i\delta t < \nu \qquad\qquad\qquad\quad\; \Rightarrow L_{i,j} = \mathrm{e}^{-r\delta t}\,[pf_{i+1,j+1} + (1-p)f_{i+1,j} + \delta t] && (2.48)
\end{aligned}$$

The expected life $L_0 = L_{0,0}$ is defined as the length of time that options reamin unexercised on average given that they vest.
Ammann and Seiz [25] argue that the annual employee exit rate should be written as a compounded rate, which then leads to the slight modification of Eq. 2.42 in that $(1 - w2\delta t)$ becomes $\mathrm{e}^{-w_2\delta t}$ and $w_2\delta t$ becomes $(1 - \mathrm{e}^{-w_2\delta t})$ (i.e. the Hull–White formulation is the linearized form of the Ammann–Seiz version of the exit rate). However, the difference is in practise negligible.
In order to comply with ASC Topic 718, the pre–vesting exit rate $\nu$ is to be set to zero. In Topic 718, forfeited options will not be recognized but have to be valued as vested ones in the valuation.

A dividend yield $q$ is easily included by redefining in the above algorithm $r$ as $r = \tilde{r} - q$ where $\tilde{r}$ is the risk–free interest rate.

**Connections to the Ammann–Seiz model** Ammann–Seiz present a subtle change to the Hull–White model in that the barrier $M$ is only triggered if

$$\max\left(S_{i,j} - MK\right) \geq \mathrm{e}^{-r\delta t}\left[p f_{i+1,j+1} + (1-p) f_{i+1,j}\right] \tag{2.49}$$

given that $\max\left(S_{i,j} - K\right) > 0$. Hence, the above condition replaces the condition of Equation 2.46. The calculation of the fair option value proceeds as in the Hull–White model. Again, for $M = 1$ they obtain the valuation for an American call. For $M < 1$, the exercise is accelerated, it is delayed in the case $M > 1$.
In their model, Ammann and Seiz primarily consider the expected life $L_{0,0}$ as an input to calculate the barrier variable $M$ using a self–consistent solution approach.

In a comparison of different models, Ammann and Seiz [27] show that the differences between the above approaches are of minor importance, as are the differences to so-called utility-maximization models. This is not really surprising. All of the models do not reflect the true intricate nature of the optionee's exercise behavior (modeled as a barrier option in the Hull–White approach, or as a power law in the utility-maximization model of Kulatilaka and Marcus [13]), and the model parameters have therefore a limited meaning and are easily adjustable to fit past option programs.

# 3 Analysis of INFICON's ESO program

## 3.1 Nature of INFICON's ESO plans

To ensure the goals of the program, the option plan includes a vesting delay as well as transferability restrictions and the possibility for expiration in case of termination of the employment/board membership. The strike price corresponds to the share price at the day of grant. When these options are exercised, new shares will be issued from a conditional share capital so that the present equity will be diluted.

Three option programs are currently used, the *Key Employee*, the *Management* and the *Director* plan. The first two are in fact identical, and differ only in the persons they are granted to: The *Management* program is restricted to the CEO, CFO and the heads of the business units. The *Director* program serves the members of the board of directors. The terms of the programs are given in Table 3.1. Thus, for the Management & Key Employee plan, the options vest after 1, 2,

**Plan Details**

| | Director plan | Key Employee & Management plan |
|---|---|---|
| vesting | immediately at grant | 25% each year from grant date |
| exercisable | one year from grant date | 25% each year from grant date |
| expiration time | 7 years | 7 years |

**Remaining Lifetime for Options upon Termination of Employment**

| Reasons | Director plan | Key Employee & Management plan |
|---|---|---|
| Resignation (voluntarily) | 12 months | 6 months |
| Resignation (with adverse charge) | 12 months | 6 months |
| Termination by company not for cause | 12 months | 6 months |
| Termination of removal for cause | 0 | 0 |
| Retirement | 12 months | 24 months |
| Disability | 12 months | 24 months |
| Death | 12 months | 12 months |

**Table 3.1:** Plan details of the Inficon employee stock option program consisting of the three plans Key Employees, Management and Director's.

3 and 4 years, whereas they immediately vest for the Director plan, although the options can be exercised only after one year.

A summary of the different grants for the 3 plans, their parameters for the calculation of the current B–S valuation, and the B–S value for each grant is given in Table 3.2. Note that for the year 2002, the Key Employee and the Management plan for that year were in fact separate plans, called KeyEmployee2002 and Management2002 respectively. But as their properties equal those of the later plans, their are for simplicity integrated into this summary.

Key Employee Plan

| Grant Date | Option Price | # granted | Expected Life | Volatility | Risk free rate | r 7 years | Dividend yield | $f_{B\text{-}S}$ |
|---|---|---|---|---|---|---|---|---|
| 15.01.2002 | 165.00 | 11440 | 5 | 63.43% | 3.07% | 3.17% | 0.00% | 92.10 |
| 17.02.2003 | 50.00 | 20300 | 5 | 63.30% | 1.96% | 1.90% | 0.00% | 27.21 |
| 30.09.2003 | 91.50 | 6411 | 5 | 64.06% | 2.23% | 2.23% | 0.00% | 50.55 |
| 17.03.2004 | 116.75 | 15500 | 5 | 61.07% | 2.54% | 2.01% | 0.00% | 62.64 |
| 16.02.2005 | 93.00 | 15800 | 4.75 | 46.82% | 1.56% | 1.85% | 3.30% | 29.03 |
| 10.02.2006 | 188.00 | 15300 | 4.75 | 41.29% | 1.88% | 2.02% | 3.33% | 52.14 |
| 04.05.2007 | 213.80 | 15500 | 4.75 | 36.87% | 2.68% | 2.76% | 3.22% | 55.66 |
| 02.05.2008 | 172.00 | 18600 | 5.52 | 40.39% | 2.83% | 3.02% | 5.14% | 42.06 |
| 19.05.2008 | 175.00 | 200 | 5.52 | 39.89% | 2.80% | 2.97% | 5.14% | 42.14 |
| 11.08.2008 | 156.80 | 200 | 5.52 | 37.35% | 2.61% | 2.79% | 5.14% | 34.68 |
| 27.10.2008 | 102.00 | 300 | 5.52 | 36.39% | 2.03% | 2.30% | 5.14% | 21.12 |
| 12.05.2009 | 102.00 | 20500 | 5.54 | 35.48% | 1.69% | 2.02% | 8.22% | 13.60 |
| 20.05.2010 | 127.00 | 21600 | 5.71 | 35.38% | 0.99% | 1.22% | 4.04% | 27.28 |
| **Average** | **130.46** | **161651** | **5.15** | **46.89%** | **2.07%** | **2.16%** | **3.14%** | **41.40** |

Management Plan

| Grant Date | Option Price | # granted | Expected Life | Volatility | Risk free rate | r 7 years | Dividend yield | $f_{B\text{-}S}$ |
|---|---|---|---|---|---|---|---|---|
| 15.01.2002 | 165.00 | 11200 | 5.00 | 63.43% | 3.07% | 3.17% | 0.00% | 92.10 |
| 17.02.2003 | 50.00 | 15700 | 5.00 | 63.30% | 1.96% | 1.90% | 0.00% | 27.21 |
| 17.03.2004 | 116.75 | 12000 | 5.00 | 61.07% | 2.54% | 2.01% | 0.00% | 62.64 |
| 01.01.2004 | 105.00 | 4000 | 5.00 | 63.08% | 2.41% | 2.41% | 0.00% | 57.55 |
| 16.02.2005 | 93.00 | 14000 | 4.75 | 46.82% | 1.56% | 1.85% | 3.30% | 29.03 |
| 25.04.2005 | 105.40 | 2000 | 4.75 | 46.38% | 1.43% | 1.76% | 3.30% | 32.41 |
| 10.02.2006 | 188.00 | 15500 | 4.75 | 41.29% | 1.88% | 2.02% | 3.33% | 52.14 |
| 04.05.2007 | 213.80 | 6250 | 4.75 | 36.87% | 2.68% | 2.76% | 3.22% | 55.66 |
| 02.05.2008 | 172.00 | 5000 | 4.45 | 31.99% | 2.66% | 3.02% | 5.14% | 30.75 |
| 01.09.2008 | 149.10 | 1250 | 4.45 | 31.01% | 2.41% | 2.72% | 5.14% | 25.24 |
| 12.05.2009 | 102.00 | 6250 | 4.65 | 34.58% | 1.45% | 2.02% | 8.22% | 13.10 |
| 20.05.2010 | 127.00 | 6250 | 5.11 | 35.07% | 0.89% | 1.22% | 4.04% | 26.25 |
| **Average** | **127.71** | **99400** | **4.86** | **49.72%** | **2.08%** | **2.19%** | **2.35%** | **45.26** |

Director Plan

| Grant Date | Option Price | # granted | Expected Life | Volatility | Risk free rate | r 7 years | Dividend yield | $f_{B\text{-}S}$ |
|---|---|---|---|---|---|---|---|---|
| 15.05.2002 | 170.00 | 1728 | 5.00 | 63.43% | 3.07% | 3.17% | 0.00% | 94.90 |
| 15.11.2002 | 69.00 | 3173 | 5.00 | 63.43% | 3.07% | 3.17% | 0.00% | 38.52 |
| 15.05.2003 | 75.00 | 3817 | 5.00 | 66.37% | 1.88% | 1.88% | 0.00% | 42.25 |
| 15.11.2003 | 97.50 | 1709 | 5.00 | 63.57% | 2.51% | 2.51% | 0.00% | 53.87 |
| 17.11.2003 | 97.50 | 328 | 5.00 | 63.57% | 2.51% | 2.51% | 0.00% | 51.91 |
| 15.05.2004 | 107.00 | 1930 | 5.00 | 61.07% | 2.54% | 2.54% | 0.00% | 57.41 |
| 15.11.2004 | 70.75 | 2999 | 5.00 | 59.19% | 2.11% | 2.11% | 0.00% | 36.68 |
| 15.05.2005 | 110.00 | 1927 | 4.00 | 47.55% | 1.26% | 1.71% | 3.30% | 32.76 |
| 15.11.2005 | 181.90 | 1165 | 4.00 | 44.86% | 1.88% | 2.09% | 3.30% | 52.27 |
| 15.05.2006 | 161.00 | 1973 | 4.00 | 45.60% | 2.38% | 2.72% | 3.33% | 47.81 |
| 04.05.2007 | 213.80 | 3150 | 4.00 | 31.33% | 2.64% | 2.76% | 3.22% | 44.60 |
| 02.05.2008 | 172.00 | 3150 | 4.51 | 32.37% | 2.67% | 3.02% | 5.14% | 31.28 |
| 12.05.2009 | 102.00 | 3150 | 4.69 | 34.46% | 1.47% | 2.02% | 8.22% | 13.03 |
| 20.05.2010 | 127.00 | 3150 | 4.84 | 35.70% | 0.84% | 1.22% | 4.04% | 26.44 |
| **Average** | **122.65** | **33349** | **4.66** | **49.71%** | **2.19%** | **2.33%** | **2.40%** | **41.17** |

**Table 3.2:** Overview over the grant parameters of the different grants for all three plans, from 2002 to 2010.

By Dec. 31, 2009, a total of 2'150'056 shares at a nominal value of 5 CHF were registered [28]. The number of 31'000 options granted in May 2010 represents therefore a conditional fraction of 1.44 % of the registered shares. The total number of outstanding options at the above date is 147'061 or a 6.84 % of the registered shares. Of these outstanding options, 82'154 were exercisable

(3.82 % of registered shares), with an weighted average exercise price of 148.26 CHF. Hence, on average, they were out–of–the–money as the stock price on Dec. 30, 2009 was 117.50 CHF. Relating to the discussion in Sect. 2.3.1, the negative influence of the considerable number of outstanding and exercisable options on the stock price is likely diminished as the options are on average out–of–the–money.

## 3.2 Evolution of INFICON's Stock Price

We observe that the rather volatile stock price is related to the economic cycles of the semiconductor industry given that an important share of INFICON's revenues stem from this sector. For the years 2005–2009, approx. 30 % of the revenues originated from the semiconductor industry segment (between 27 % and 34 %). The latter industry saw a major downturn in 2001, followed by a steady rise, with only small downturns in the 3rd quarters of '06 and '07 until the latest crisis set in again in the last quarter of 2008, see the following Fig. 3.1. Except for the rise in 2002, the IFCN stock price reflects the Semi industry's economic cycles quite well.

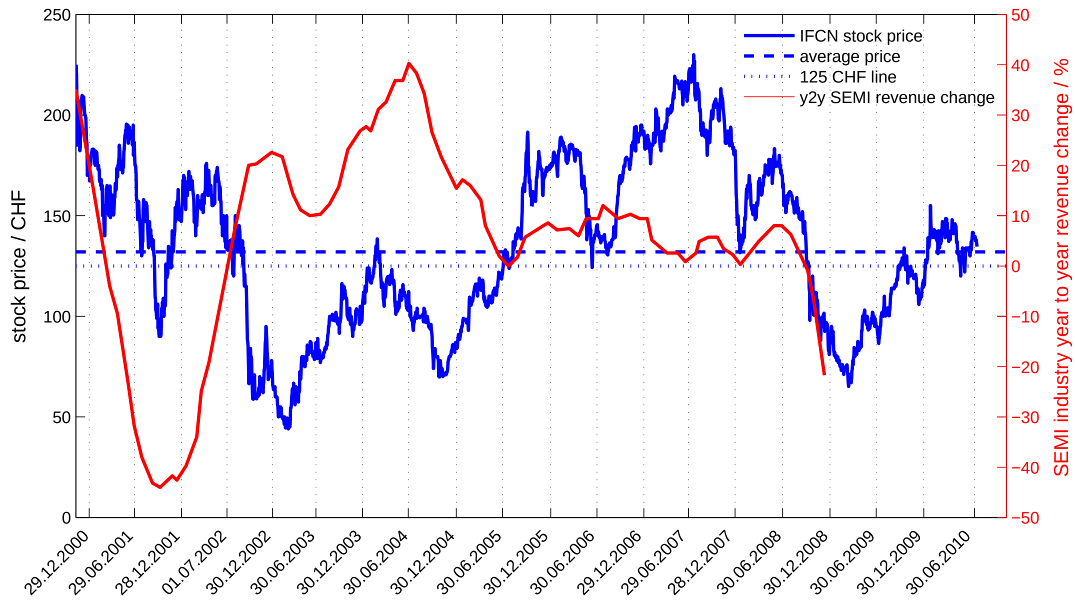


**Figure 3.1:** Evolution of the IFCN stock price over the years, the average stock price, the 125 CHF line as well as the year–to–year revenue change in % of the SEMI industry (data from [29]).

## 3.3 Exercise Behavior

The exercise behavior of INFICON employee stock options seems to follow typical ESO trends. In the following, the relevant parameters will be presented as a function of the option's age: a) The fraction of options exercised from the total number issued from a specific grant during a specific time interval, which is a quarter year. b) The moneyness, which is defined as the ratio of the stock price at exercise to the strike price, $S/K$, also plotted vs the option's age in intervals of quarter years. c) The intrinsic value gathered as a function of maturity.
Starting with the Key Employee program between 2002 and 2009, we can see from the top graph in Figure 3.2 that after every vesting period of 1, 2, 3 and 4 years there is a spike in the number of exercises. Here, in all three graphs, the exercise dates on the x–axis are gathered in time spans of 3 month in the graph. Thus, many optionees exercise as soon as they can. This seems to be a typical outcome of the non-marketability issue with employee stock options [14]. A certain fraction of people exercise once the options vest independent whether the conditions are particularly appealing for exercising, because the intrinsic value and the moneyness, defined as the quotient of stock price to the exercise price, $S/K$, were not higher at early exercise dates than at later stages. Therefore, there is strong indication that people exercise for liquidity and diversification reasons. On the other hand, when we compare the exercise dates to the stock price at that time, we observe that the optionees indeed exercise at moments when the stock price is high, compare the first and the second graph in Fig. 3.2. Hence, a significant fraction of optionees seem to follow attentatively the trend of the stock price and exercise when they deem the situation most favorable. Thus, overall, the exercise behavior is a balance of a need for liquidity and diversification vs. optimal profit strategy.

The average 'Moneyness' for the Key Employee option program was $S/K = 2.34$, i.e. the stock price was 2.34 times the strike price. This value could now be used in a binomial model (such as the Hull–White model) to calculate the option price for future grants. The average intrinsic value of an option was 86.25 CHF.

The results for the management program are quite similar. One could say that they 'know better' and keep the options late. But management does not as can be seen in Figure 3.3. We see again the exercise spikes that follow the vesting periods. But again, as for the Key Employee program, exercising occurs mostly during times of relatively high stock prices. Overall, the management's moneyness is not better than the key employees, but in fact slightly lower: $S/K = 2.14$ on average. The average intrinsic value of 71.27 CHF was slightly lower as well in comparison to the Key Employee program.
Finally, I have analyzed the exercise behavior of the Director program. In contrast to the other two programs, the options of the Director program become all exercisable after only one year. Again, many options are exercised early, many not at very high moneyness (especially at the time $t = 1.5 - 2$ years). However, at $t = 4$ years, the spike in the exercises is accompanied with a high moneyness - and reflect good timing, this is also reflected in Fig 3.4. Note that as very few options have been exercised for grants after 2005 and two grants were issued per year, we have restricted

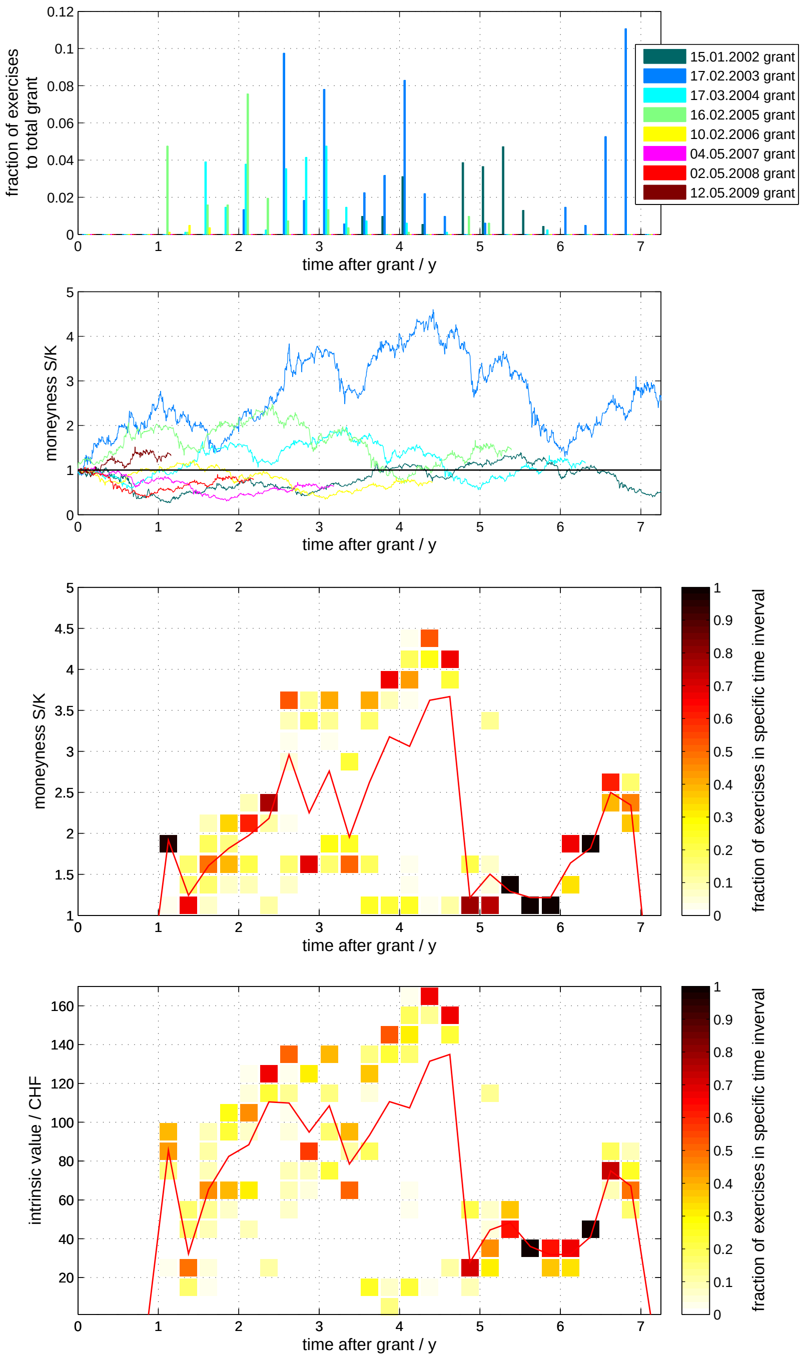


**Figure 3.2:** Key Employee plan: The top graph shows the fraction of exercises compared to the total options issued per grant as a function of maturity. The 2$^{\text{nd}}$ graph shows the 'moneyness' of the stock price, the 3$^{\text{rd}}$ graphs shows the 'moneyness' of the exercises as a function of the option's age The bottom graph shows the intrinsic value gathered at exercise as a function of the option's age. The coloured dots of the center and bottom graph show how many of the exercised option for a certain date (i.e. a certain quarter of the figure above) fall into a certain moneyness (gathered into bins of 0.25) or intrinsic value (bins of 10 CHF). The total for each column (i.e. each exercise quarter) sums up to 100 %. So a black coloured dot means that practically 100 % of the exercises at a certain date (quarter) had the same moneyness or intrinsic value.

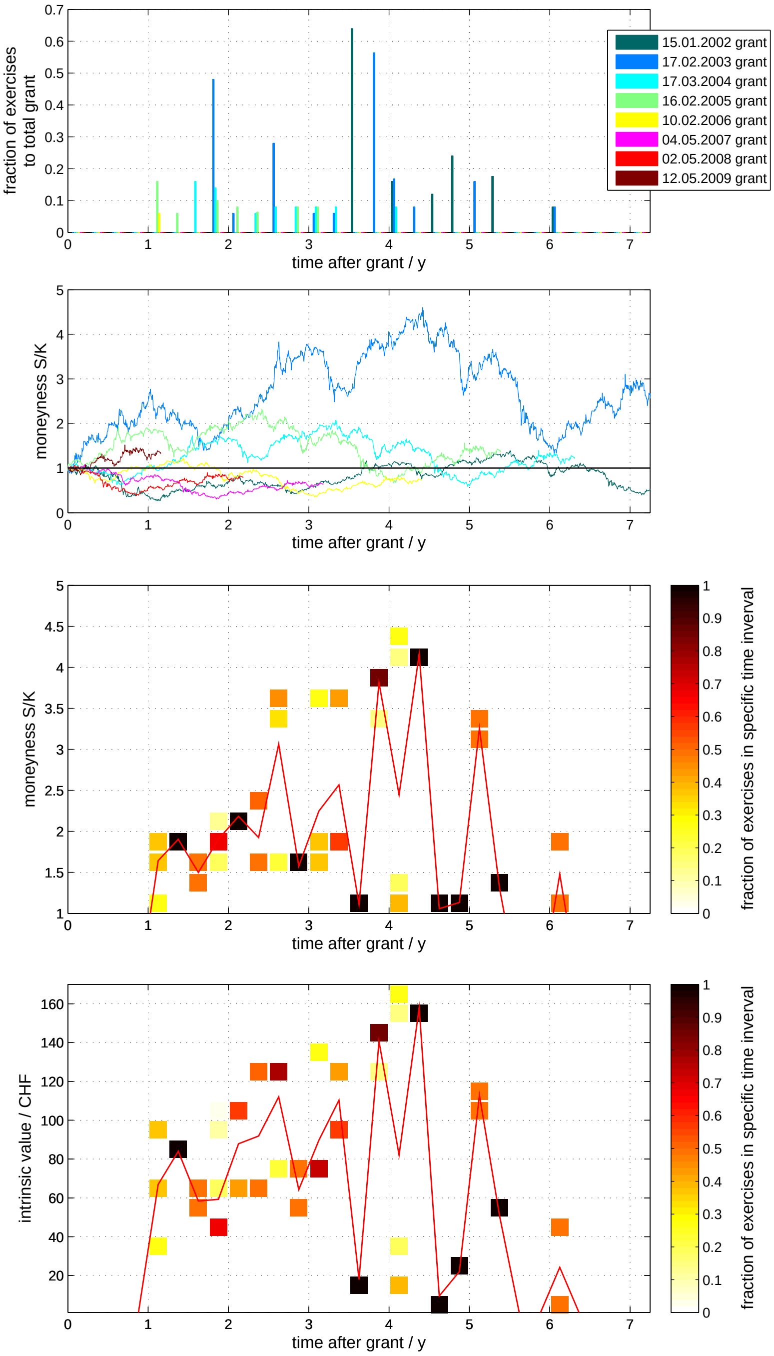


**Figure 3.3:** Management plan: Exercise behavior over the lifetime of the option (top), the moneyness as a function of the ($2^{nd}$ graph) stock and the lifetime ($3^{rd}$ graph), and the intrinsic value at exercise as a function of the lifetime (bottom) for the grants 2002 – 2009.

the analysis in the first two graphs of Fig. 3.4 to the years 2002 to 2005 (The lower two graphs span all the grants from 2002 to 2009). Interestingly, the overall moneyness, for the years 2002 – 2009, at $S/K = 2.05$ is the lowest of all plans, whereas the average intrinsic value of 87.53 CHF exceeds that of the Management plan.

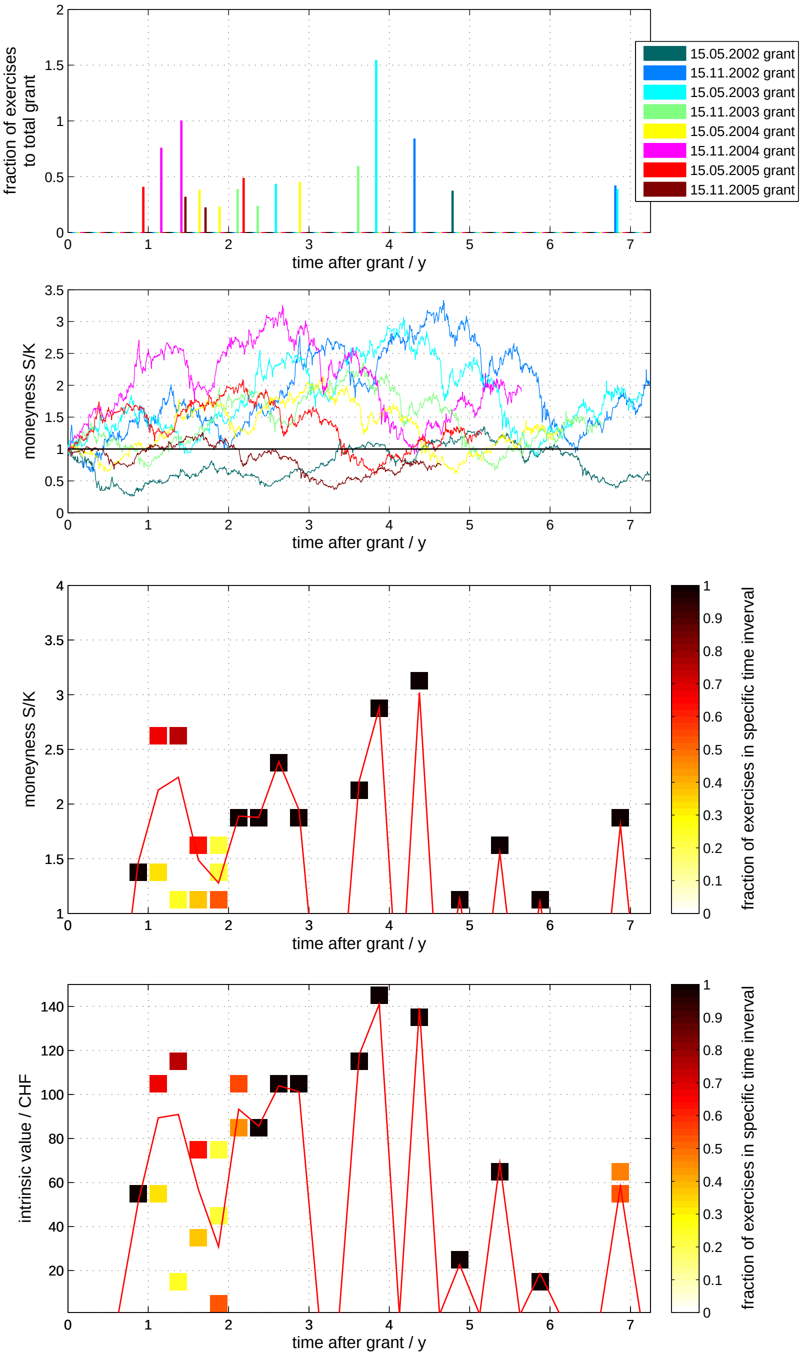


**Figure 3.4:** Director plan: Exercise behavior over the lifetime of the option (top), the moneyness as a function of the stock, $2^{nd}$ graph, and the lifetime ($3^{rd}$), and the intrinsic value at exercise as a function of the lifetime (bottom) for the grants 2002 – 2009.

## 3.4 Expected Option Life

The software used at Inficon, *Express Options* [30], provides an automated report generating the expected life, $L_0$, of the different option plans. It sums up the lifetimes of the exercised, expired, those cancelled after they have vested as well as the forfeited (cancellations during the vesting period) options and adds to those the contractual term (7 years) for each outstanding option.

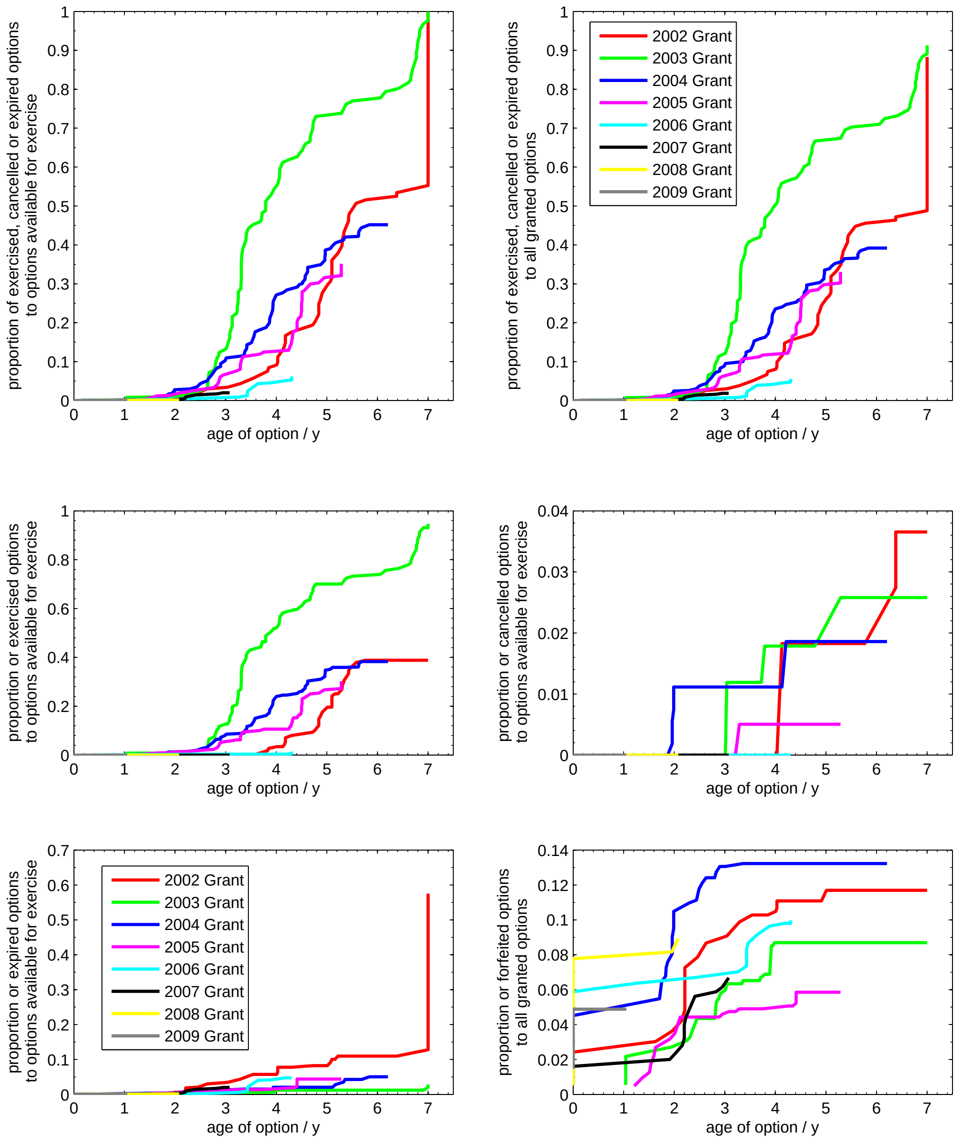


**Figure 3.5:** Top row: Evolution of the proportion of inactivated options by exercising, expiring, cancelling and forfeiting to the number of vested options (left) and to all options (right) for the Key Employee plan. The lower two rows show the details for the proportions of all four inactivating actions, namely exercising, expiring, cancelling and forfeiting.

However, this method overestimates the expected life because the outstanding options should be given an expected term instead of the full contractual term. Thus, for the very recent 2010 grant for instance, all Key Employee and Management options will have the full 7 years of lifetime added to the total grants issued. The overestimation becomes even more severe when the number of options issued increases.

A proper estimation of the expected life of an option can therefore only be done for a grant that has expired its contractual term. Starting with the 2002 option program, we have only two grant years, namely 2002 and 2003, which have expired as of today. For the more recent grants, we can compare their evolution of the exercised, expired, cancelled and forfeited proportion of the options to those of the fully expired grants in order to dare a forecast. In the following we analyze the evolution of the grants for all plans. For the expired grants of 2002 and 2003, the expected life/expected term, $L_0$, is given in Table 3.3.

**Table 3.3:** Expected life in years for different option plans

| Grant | Key Employees | Management | Directors |
|---|---|---|---|
| 2002 | 5.77 | 4.49 | 4.56 |
| 2003 | 4.32 | 3.30 | 2.99 |
| average | 5.05 | 3.89 | 3.77 |

We note that the expected lifes $L_0$ extracted using this method is substantially smaller than the values found by the *Express Option* software, which give values ranging from 4.84 to 5.71 years for the recent Key Employee grants, see Fig. 3.2. We have to keep in mind that smaller expected terms will lead to reduced option prices, a reason why many companies try to reduce their expected lifes in order to spare on option expenses [18].
With a historical data base for the expected life calculation of only two years, it is highly recommended to include $L_0$–forecasts using the more recent grants to obtain meaningful expected life values.

As for all grants, the strike prices correspond to the stock prices at the day of grant, the expected life is closely related to the evolution of the stock price onwards from the day of grant. The overall evolution of the stock price is depicted in Fig. 3.1. Given the strong dependence on the overall economic cycle, the spread in $L_0$ from year to year is expectedly large. This gives a second reason to include a trend for the more recent grants, as e.g. shown in Fig. 3.5 for the Key Employee plan. Obviously, from Fig. 3.5, we can easily relate the low expected life of the 2003 grant to the large and early exercising portion in comparison to the 2002 grant. A low strike price and the stock price peak in 2007 followed by the substantial crisis in 2008 make $L_0$ of the 2003 grant comparably small, especially as the time span of the economic upswing was long enough so that all options could vest and were available for exercise. For the more recent grants in 2004 and 2005 we see a behavior close to that of the 2002 grant but with a higher exercising portion. Possibly, sensing the coming 2008 crisis, the optionees tried to cash in on the vested options. As the 2006 and 2007 grants from the 'real estate bubble years' are still deep out–of–the–money, we might expect higher $L_0$ values for the future, possibly in the $L_0 = 5 - 5.5$ years region. In turn, the low strike prices

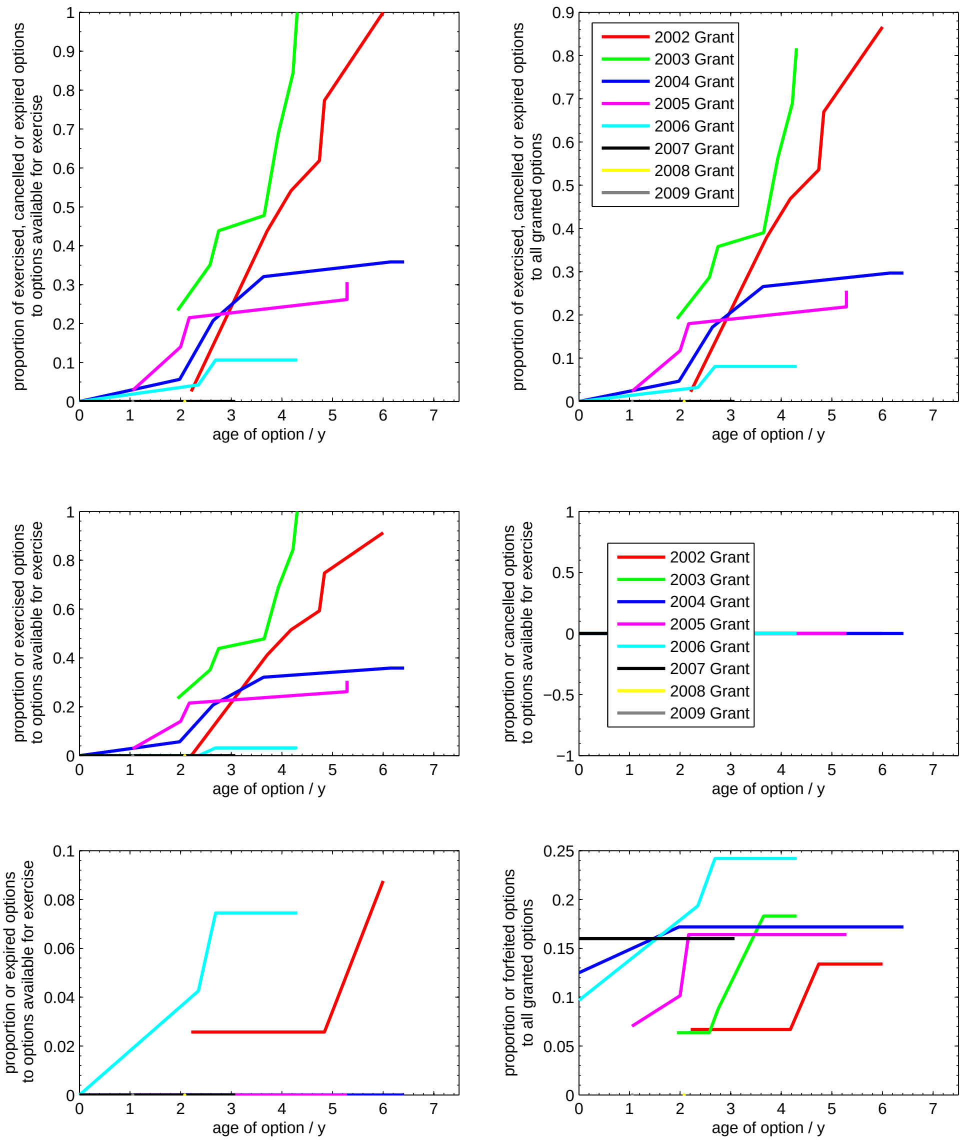


**Figure 3.6:** Top row: Evolution of the proportion of inactivated options by exercising, expiring, cancelling and forfeiting to the number of vested options (left) and to all options (right) for the Management plan. The lower two rows show the details for the proportions of all four inactivating actions, namely exercising, expiring, cancelling and forfeiting.

of the 2008 and 2009 grants due to the recent severe crisis will likely result again in significantly earlier exercising, thus reducing $L_0$. In summary, the expected term is a rather volatile number, especially for a company that is significantly exposed to the economic cycle.

For the Management plan, Fig. 3.6 indicates that we have to expect an increase in the $L_0$ value in the coming years towards the values for the Key Employee plan, i.e. $L_0 \approx 5$ years, as the respective proportions become very similar to the inactivating procedures of the Key Employee plan.

Finally, as depicted in Fig. 3.7, for the more recent evolution of the Director's grants, we may expect lower $L_0$ values than for the other two plans, namely in a region of $4 - 4.5$ years.

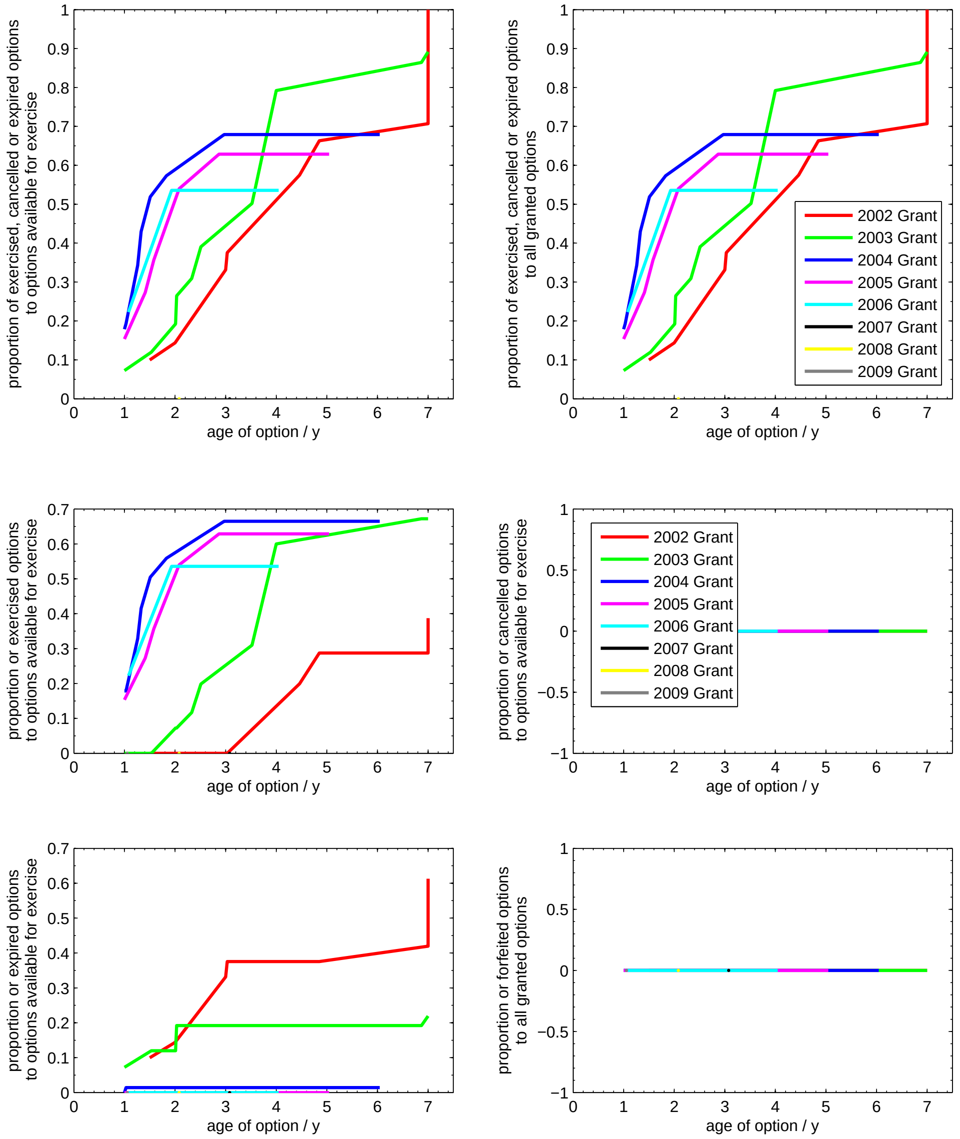


**Figure 3.7:** Top row: Evolution of the proportion of inactivated options by exercising, expiring, cancelling and forfeiting to the number of vested options (left) and to all options (right) for the Director's plan. The lower two rows show the details for the proportions of all four inactivating actions, namely exercising, expiring, cancelling and forfeiting.

## 3.5 Performance of the current Black–Scholes Valuation

This work was to a significant extend triggered by the perception that the expenses for the ESOs at INFICON for the years 2008 and 2009 were largely exceeding the total intrinsic value of the exercised options for those years. Thus, in a first analysis, I tried to compile the relationship between recognized expenses and the intrinsic value from excercises over a longer period of time.

| year | $f_{\text{B-S}}$ kCHF | $f_{\text{I}}$ kCHF | $f_{\text{B-S}} - f_{\text{I}}$ kCHF | Revenue kCHF | EBIT % | EPS | $r$ % | $\sigma$ | $q$ % | $L_0$ y |
|---|---|---|---|---|---|---|---|---|---|---|
| 2009 | 889 | 264 | 625 | 181696 | 2.66 | 1.06 | 1.62 | 35.18 | 8.22 | 5.3 |
| 2008 | 1111 | 259 | 852 | 256489 | 12.78 | 11.34 | 2.76 | 37.57 | 5.14 | 5.2 |
| 2007 | 1083 | 2045 | -962 | 236557 | 14.23 | 10.73 | 2.68 | 36.87 | 3.22 | 4.8 |
| 2006 | 786 | 1466 | -680 | 211694 | 13.42 | 9.41 | 1.88 | 41.29 | 3.33 | 4.8 |
| 2005 | 0 | 907 | -907 | 191300 | 10.75 | 6.68 | 1.55 | 46.8 | 3.3 | 4.7 |

**Table 3.4:** Employee option related data from INFICON's annual reports showing the expenses for its ESOs, $f_{\text{B-S}}$, calculated using the Black–Scholes formula, the true instrinsic value of exercised options for that year, $f_{\text{I}}$, the difference $f_{\text{B-S}} - f_{\text{I}}$, the company revenue, EBIT, EPS, the risk-free interest rate $r$, the yearly volatility $\sigma$, the dividend yield $q$, and the expected life, $L_0$.

Expenses were published in the annual report only starting 2006. As we can see from Table 3.4, the recognized expenses for the years 06–09 were in fact lower than the intrinsic value, $f_I$ captured by exercising the options, by 165 kCHF. Thus, the company has even spared expenses in comparison to the true value given to its employees.

Whereas the mismatch over these 4 years between the expenses and the intrinsic value is surprisingly low (the 165 kCHF represent a mere 4% of the expenses). From Table 3.4, it is not obvious where the mismatch should stem from, there is no clear trend regarding the difference $f_{\text{B-S}} - f_{\text{I}}$ that would relate to other critical parameters such as the EBIT, the earnings per share or the dividend yield. The following Figure 3.8 shows that the quotient of the Black–Scholes value, $f_{\text{B-S}}$ and the intrinsic value at exercise date, $f_{\text{I}}$, reported as a function of the strike price are in a strong disagreement. The graph shows that when the the strike price is below approx. 125 CHF, the intrinsic value significantly bigger than the calculated fair value $f_{\text{B-S}}$, except for the spike at 91.5 CHF (the latter is really an exception, a special grant given on 30.9.2003 to one person only, who probably had to exercise very quickly at a non-ideal moment). For strike prices $K > 125$ CHF, the Black-Scholes model overestimates the option values. The under- and overestimates are quite significant[(1)], and it is interesting to note that they could be even fitted with a straight line in the log-plot. Which means that there are easily mismatching factors of 2 - 5 found between the real intrinsic value and the calculated one. We conclude that whereas the total mismatch over the last 4 years is neglibibly small, the individual option fair values did largely differ from their

[(1)]Note that we plot the $\log_{10}$ of the quotient $f_{\text{B-S}} - f_{\text{I}}$.

intrinsic values at exercise. An explanation of this behavior might be extracted from analyzing

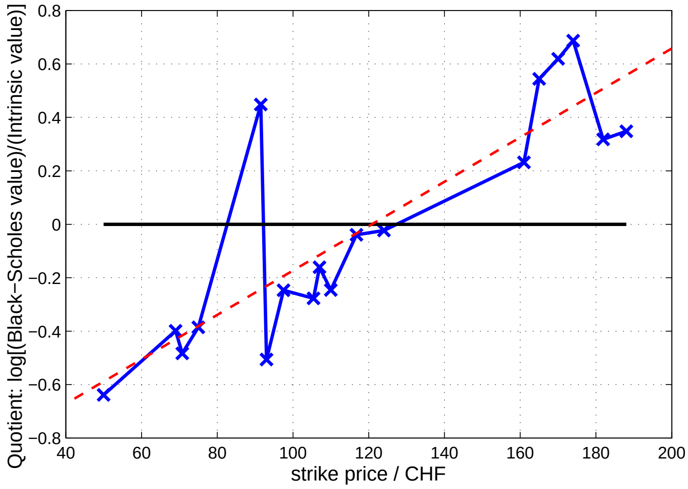


**Figure 3.8:** Logarithm, $\log_{10}$, of the quotient of the Black–Scholes option value to the intrinsic value at exercise, $\log f_{(\text{B-S}}/f_{\text{I}})$, as a function of the exercise price $K$.

the evolution of the stock price over the years as depicted in Fig. 3.1. It can be argued that the mismatch between the intrinsic value and the B–S option value should become minimal around the average stock price. Indeed, the average stock price of 132 CHF for the time frame between 01.01.2001 and 30.06.2010 is very close to the minimal mismatch at approx. 125 CHF in Fig. 3.8. The significant mismatches for strike prices far away from this average stock price may then be explained by the high volatility of the stock, due to the very pronounced economic cycles during the last decade.

# 4 Pathways for Future Valuations

The main driver for this work was the perception that the ESO expenses did neither reflect the aspect of long-term commitment such options should entail nor was it properly reproducing the conditions the company was in financially. In contrast, in rough times such as in 2008–2009, option values went up as the volatiliy of the stock inevitably rose — thus creating another financial burden for the firm.
In addition, given (a) the stringent restrictions of ASC Topic 718 and (b) the substantial mismatches between the expensed option value and the intrinsic value of most of the grants (although overall, the match is rather good), I see two different pathways for future ESO valuations:

1. Either we try to forecast as accurately as possible the value of the options in order to match as accurately as possible the expenses with the intrinsic value of exercised options
2. Or try to minimize the expenses in accordance with ASC Topic 718 regardless of any mismatch between intrinsic and expensed option value.

## 4.1 Accuracy–Improving Measures Compliant with Topic 718

A proper valuation of ESOs seems like a formidable task: Obviously, already the very long expiration times of many years does not lend it for forecasting, all the more the regulations require a valuation at grant date without the possiblilty to set provisions. And this is just one of the several disputable decisions the FASB has taken with respect to share–based payments. In fact, it is highly questionable whether ESOs can be valuated using standard option valuation theory at all.

### 4.1.1 Relevant Model Parameters

Given the long contractual term of ESOs together with their particular nature, a multitude of influencing parameters have been considered in literature in order to get a better hold of the ESO's value. Typical examples are the introduction of an exercise trigger $M$ (i.e. a barrier type option behavior) to mimic early exercise, utility functions for the same reason and optionees exit rates $w$ to handle forfeiture and option cancelling or expected lifetimes/terms. In addition, we'll have the usual parameters such as the interest rate $r$ and of course the volatility $\sigma$.

**Exercise Trigger Multiple** $M$ Ammann and Seiz [27] have shown that the differences in the option value are small for the different approaches to handle early exercises due to the inability to trade and to hedge ESOs, whether utility-maximizing functions (e.g. by Huddart or Kulatilaka et al. [11, 13]) or exercise triggers (Hull–White [15] and Ammann–Seiz [25]) are used. Even more complex models of time dependent triggering such as the model by Bajaj et al. [14], where the trigger depends on a historical early exercise probability matrix of maturity and moneyness have been suggested. Brisley & Anderson [26] suggest a lattice with a declining trigger barrier as expiration nears. It is however questionable whether such detailed models give better insights or estimates of the option pricing, mainly because of the uncertainty and the respectable spread of the model parameter values. As shown in Section 3.3, the moneyness, i.e. the stock–price to strike–price ratio, of the exercised options varies without any clear pattern along the time axis. Certain optionees tend to exercise immediately upon vesting, others wait until the end of the contractual term. From Figure 3.2 for example, it is not obvious that there would be a declining trigger barrier towards the end of the option's lifetime. Moreover, the exercise pattern is likely to change throughout the economic cycles, which are of great importance for a company such as INFICON which is closely linked to the highly cyclic semiconductor industry.

**Exit Rate** On the grounds of the economic cycles, it also remains to be discussed whether factors such as the optionees exit rate should be specifically incorporated into a option pricing model given that the fluctuation is strongly dependent on the overall economic situation.

**Expected Life or Expected Term** $L_0$ Given the reasoning above, instead of considering different effects separately using ill-defined parameters, we might get to similar results using a single factor that incorporates different effects. Such a factor can be the expected life, also called expected term, $L_0$. It can easily be estimated on historical grounds by averageing the lifetimes of all options, and there is no obvious reason why its uncertainty should be greater than that of the different individual parameters. In fact, in the following, we have tried to analyze the ESOs in terms of the expected option life, $L_0$ instead of taking a trigger $M$ and exit rates $w$ as parameters.

**Interest Rate** $r$ ASC Topic 718 strictly defines the interest rate and leaves no room for accuracy improving measures such as considering a risk–premium or implementing autoregressive models.

**Volatility** $\sigma$ In contrast to the interest rate, the decisive parameter $\sigma$ can be obtained by different means according to Topic 718, with the aim to obtain the best–possible fair value estimation. In the following, we will sketch a possible alternative to a simple averaging over the expected term or the contractual term of the options.

### 4.1.2 Improved Forecast for the Expected Volatility

ASC Topic 718 does not specify a method to calculate the expected volatility. The expected volatility can be calculated from historical returns or from implied volatilities. In lack of the latter, INFICON uses a system that calculates the expected volatility from historical volatilities. Topic 718 suggests a historical period commensurate with the expected life (closed-form valuations) or the contractual term (lattice models). However, weighting according to the general evolution of the economy is allowed. For lattice models, a range of expected volatilities can be incorporated into the lattice over the contractual term of the option [718-10-55-39].

#### GARCH Model for the Volatility

Time series of financial data, such as our time series on the log returns which form the basis of our estimate for the expected volatility, are a typical application for the so-called GARCH model, introduced by Bollerslev [31]. In literature, GARCH models have been used to estimate the expected volatilities of ESOs, starting with the work of Duan [32, 33] but also by Ritchken et al. [34] or more recently by Léon [35].
GARCH is an abbreviation for *generalized autoregressive conditional heteroskedasticity*. *Heteroskedasticity* means that the expected value of the error term is not constant in time, but will be larger for some points or range in time than for others. For our financial time series, this means that some time periods are riskier than others. In addition these riskier times with larger errors will not occur randomly in time, instead we will often find a so-called volatility clustering, periods of high volatility following calm periods. The word *autoregressive* indicates a feedback mechanism to incorporate past observations to estimate the present, *conditional* means that the error term has a dependence on the immediate past.
GARCH allows to analyze time series that not only exhibit volatility clustering but also show fat tails (kurtosis), i.e. a much higher proportion of the observed values lies far away from the mean than what is expected from e.g. the normal distribution (which has a very slim tail). This phenomenon is often observed in financial time series (due to crisis and speculative stock price increases).

**GARCH(1,1) Model** In the following, we define the daily return at day $n$ as $u_n = \ln(S_n/S_{n-1})$ and the daily variance rate at day $n$ as the squared volatility $\sigma_n^2$ in accordance with the volatility definition in Sect. 2.1.2. In the simplest GARCH model, GARCH(1,1) the variance rate is a linear combination of the daily variance rate of the preceding trading day, the squared return of the preceding trading day and a long–run variance rate $V_L$.

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{4.1}$$

Note that weights have to sum up to one, $\alpha + \beta + \gamma = 1$. When Eq. 4.1 is fitted to real data (e.g. using a maximum likelyhood method), it is not possible to discriminate the product $\gamma V_L$, hence, for fitting, we will replace $\gamma V_L = \kappa$.
The variance rate for any trading day $n + t$ in the future, which we denote by $V(t)$, is given by the expectation value of $\sigma^2_{n+t}$, obtained by at $t$–times successive application of Eq. 4.1:

$$V(t) = E(\sigma^2_{n+t}) = V_L + (\alpha+\beta)^t(\sigma^2_n - V_L) = V_L + \mathrm{e}^{-at}\,[V(0) - V_L] \quad \text{with} \quad a = \ln\frac{1}{\alpha+\beta} \tag{4.2}$$

where $V(0)$ is the daily variance rate at time $t = 0$, i.e. trading day $n$. The GARCH(1,1) process is exhibiting a so–called *mean reversion*, meaning that $V(t)$ goes to $V_L$ for $t \to \infty$. Hence, the daily volatility on the long run will be a constant, it's long–term value, within that process.
The total variance during a time span $T$ is given by

$$\int_0^T V(t)\mathrm{d}t = T\left(V_L + \frac{1-\mathrm{e}^{-aT}}{aT}\,[V(0) - V_L]\right) \tag{4.3}$$

Note that total variance for the time span $T$ in the above equation is nothing else than $\sigma^2(T)T$, where $\sigma(T)$ is the volatility for the time span $T$, i.e. $\int_T V(t)\mathrm{d}t = \sigma^2(T)T$.

Although financial return series typically exhibit little correlation, the squared returns often indicate significant correlation and persistence. This implies correlation in the variance process, and is an indication that the data is a candidate for GARCH modeling. The advantage of this simple GARCH(1,1) model is on the one hand its simplicity, as a stock price process with a drift as in Eq. 2.9 requires only 4 parameters to be modeled, $\alpha$, $\beta$ and $\kappa$ for the volatility and one for the return's drift (denoted $\mu$ here). The other strong point is its ability to capture volatility clustering and fat tails.
In Figure 4.1 shows the INFICON (IFCN) stock log returns. The log returns show typical volatility clustering. When we analyze this time series for autocorrelation, i.e. the correlation between values of the process at different times and thus the occurrence of repeating patterns, we observe no autocorrelation for the log return but significant autocorrelation for the squared log return. This is seen in the left graph of Fig 4.2 below) for the grants of 2005 to 2010. This indicates that we have a GARCH type dependence as noted in equation 4.1.
We quantitatively tested for correlations of the squared returns using hypothesis tests such as Engle's test for the presence of ARCH effects as well as the Ljung-Box-Pierce Q-Test by implementing MATLAB's *archtest* and *lbqtest* functions [36]. Both tests show significant evidence in support of GARCH effects, in particular the heteroscedasticity. Using MATLAB's *garchfit* routine, the model parameters of the GARCH(1,1) model were estimated as summarized in Table 4.1.

Thus, we are now in the position to forecast expected daily volatilities given by $V(t)$ in Eq. 4.2. The resulting time dependent daily (trading day) standard deviation, as depicted in Figure 4.2 for the Key Employee plan, can be implemented into a lattice model such as the Hull–White model.(1)

(1) consisting of a modified lattice calculation scheme which adapts each time step in order to keep the standard deviation for each time step constant.

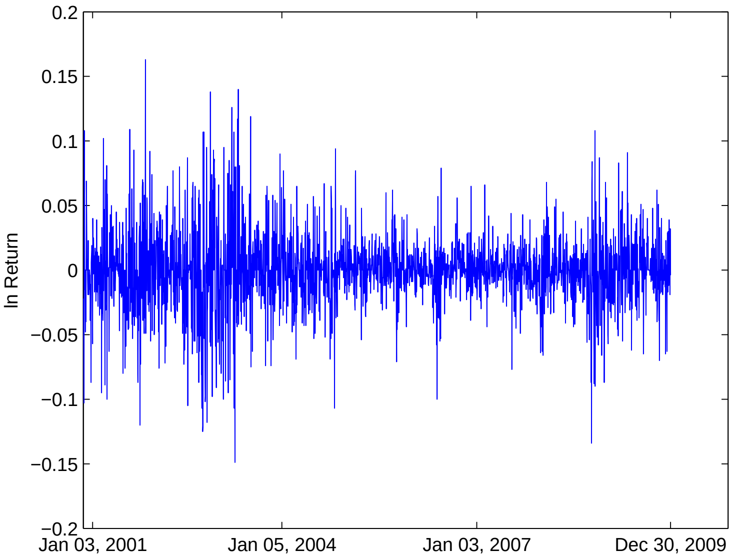


**Figure 4.1:** The daily log returns, $ln(S_t/S_{t-1})$ as a function of time for IFCN stocks.

**GARCH(1,1) parameters**

**Key Employee Plan**

| Grant | $\mu$ value | $\mu$ T-statistic | $\alpha$ value | $\alpha$ T-statistic | $\beta$ value | $\beta$ T-statistic | $\kappa$ value | $\kappa$ T-statistic |
|---|---|---|---|---|---|---|---|---|
| 10.02.2006 | 1.027E-03 | 1.94 | 1.052E-01 | 10.09 | 8.665E-01 | 75.64 | 1.716E-05 | 5.69 |
| 04.05.2007 | 1.016E-03 | 1.99 | 1.219E-01 | 9.68 | 8.479E-01 | 65.94 | 1.889E-05 | 5.90 |
| 02.05.2008 | 1.567E-03 | 3.05 | 1.461E-01 | 11.15 | 8.362E-01 | 69.57 | 1.687E-05 | 5.65 |
| 12.05.2009 | 1.948E-03 | 3.39 | 1.407E-01 | 9.61 | 8.484E-01 | 67.02 | 1.442E-05 | 4.82 |
| 20.05.2010 | 1.780E-03 | 2.44 | 8.027E-02 | 6.99 | 9.137E-01 | 82.12 | 9.337E-06 | 2.91 |

**Management Plan**

| Grant | $\mu$ value | $\mu$ T-statistic | $\alpha$ value | $\alpha$ T-statistic | $\beta$ value | $\beta$ T-statistic | $\kappa$ value | $\kappa$ T-statistic |
|---|---|---|---|---|---|---|---|---|
| 10.02.2006 | 1.780E-03 | 2.44 | 8.027E-02 | 6.99 | 9.137E-01 | 82.12 | 9.337E-06 | 2.91 |
| 04.05.2007 | 1.921E-03 | 3.34 | 1.406E-01 | 9.60 | 8.483E-01 | 67.04 | 1.443E-05 | 4.83 |
| 02.05.2008 | 1.502E-03 | 2.82 | 1.498E-01 | 9.03 | 7.982E-01 | 53.84 | 2.655E-05 | 6.74 |
| 12.05.2009 | 1.007E-03 | 1.84 | 1.315E-01 | 9.63 | 8.377E-01 | 63.99 | 1.848E-05 | 5.96 |
| 20.05.2010 | 8.100E-04 | 1.45 | 9.891E-02 | 9.38 | 8.771E-01 | 71.47 | 1.425E-05 | 4.80 |

**Director Plan**

| Grant | $\mu$ value | $\mu$ T-statistic | $\alpha$ value | $\alpha$ T-statistic | $\beta$ value | $\beta$ T-statistic | $\kappa$ value | $\kappa$ T-statistic |
|---|---|---|---|---|---|---|---|---|
| 15.05.2006 | 1.582E-03 | 2.33 | 1.054E-01 | 7.63 | 8.888E-01 | 71.87 | 9.665E-06 | 4.65 |
| 04.05.2007 | 6.579E-04 | 1.13 | 9.816E-02 | 9.29 | 8.783E-01 | 70.90 | 1.460E-05 | 6.07 |
| 02.05.2008 | 9.427E-04 | 1.73 | 1.336E-01 | 9.64 | 8.342E-01 | 62.93 | 1.920E-05 | 6.49 |
| 12.05.2009 | 1.415E-03 | 2.67 | 1.430E-01 | 8.94 | 8.086E-01 | 54.80 | 2.484E-05 | 6.11 |
| 20.05.2010 | 2.078E-03 | 3.55 | 1.610E-01 | 9.19 | 7.995E-01 | 54.55 | 2.617E-05 | 3.33 |

**Table 4.1:** Results of the GARCH(1,1) model estimates for grants between 2006 and 2010 for the different option plans.

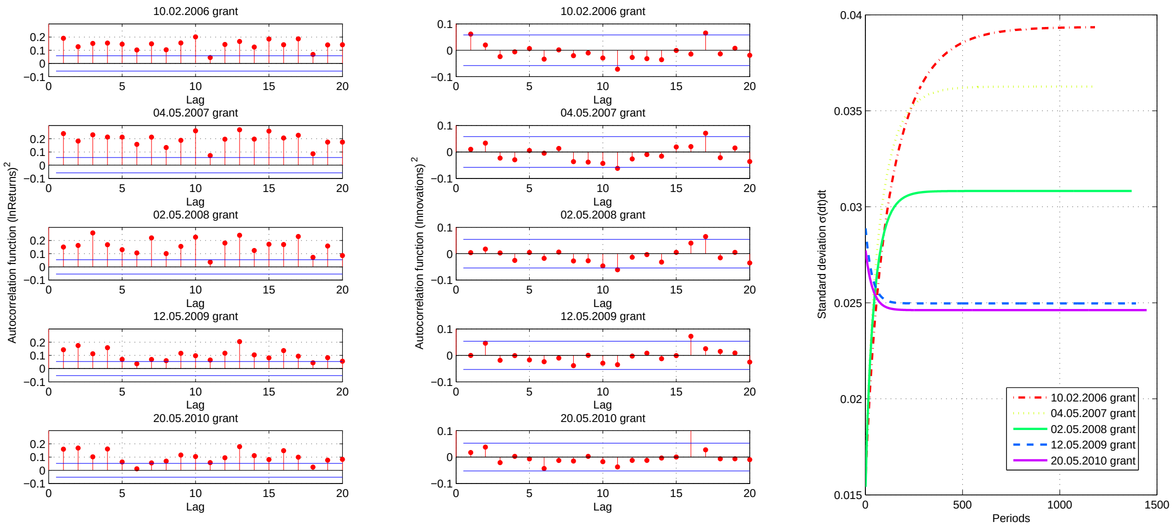


**Figure 4.2:** Left: The autocorrelation for the squared ln(Return), Mid: The autocorrelation function of the squared innovation (The innovations are equivalent the the error term $\epsilon$ in the Wiener process) after the garch modeling. The blue lines in the left and mid graphs indicate the 5 % confidence interval bounds. Right: the evolution of the daily (trading day) standard deviation. Data for the Key Employee plan.

**Key Employee Plan**

| Grant | Strike CHF | $\sigma$ | $r_7$ | $q$ | $L_0$ | $f_{B\text{-}S}$ CHF | $V(0)$ | $\sigma_L$ | $f_{GARCH11}$ CHF | $\Delta_{GARCH11}$ % |
|---|---|---|---|---|---|---|---|---|---|---|
| 10.02.2006 | 188.00 | 41.29% | 2.02% | 3.33% | 4.75 | 52.14 | 0.0158 | 60.68% | 59.88 | 14.8% |
| 04.05.2007 | 213.80 | 36.87% | 2.76% | 3.22% | 4.75 | 55.66 | 0.0152 | 55.89% | 69.59 | 25.0% |
| 02.05.2008 | 172.00 | 40.39% | 3.02% | 5.14% | 5.52 | 42.06 | 0.0196 | 47.51% | 55.76 | 32.6% |
| 12.05.2009 | 102.00 | 35.48% | 2.02% | 8.22% | 5.54 | 13.60 | 0.0293 | 38.49% | 20.82 | 53.1% |
| 20.05.2010 | 127.00 | 35.38% | 1.22% | 4.04% | 5.71 | 27.28 | 0.0233 | 37.94% | 34.90 | 27.9% |

**Management Plan**

| Grant | Strike CHF | $\sigma$ | $r_7$ | $q$ | $L_0$ | $f_{B\text{-}S}$ CHF | $V(0)$ | $\sigma_L$ | $f_{GARCH11}$ CHF | $\Delta_{GARCH11}$ % |
|---|---|---|---|---|---|---|---|---|---|---|
| 10.02.2006 | 188.00 | 41.29% | 2.02% | 3.33% | 4.75 | 52.14 | 0.015849367 | 60.68% | 59.79 | 14.7% |
| 04.05.2007 | 213.80 | 36.87% | 2.76% | 3.22% | 4.75 | 55.66 | 0.016218918 | 55.55% | 68.44 | 23.0% |
| 02.05.2008 | 172.00 | 31.99% | 3.02% | 5.14% | 4.45 | 30.75 | 0.015290588 | 34.84% | 30.23 | -1.7% |
| 12.05.2009 | 102.00 | 34.58% | 2.02% | 8.22% | 4.65 | 13.10 | 0.028713246 | 37.73% | 13.61 | 3.9% |
| 20.05.2010 | 127.00 | 35.07% | 1.22% | 4.04% | 5.11 | 26.25 | 0.027463567 | 37.59% | 30.14 | 14.8% |

**Director Plan**

| Grant | Strike CHF | $\sigma$ | $r_7$ | $q$ | $L_0$ | $f_{B\text{-}S}$ CHF | $V(0)$ | $\sigma_L$ | $f_{GARCH11}$ CHF | $\Delta_{GARCH11}$ % |
|---|---|---|---|---|---|---|---|---|---|---|
| 15.05.2006 | 161.00 | 45.60% | 2.72% | 3.33% | 4.00 | 47.81 | 0.0207 | 62.70% | 52.18 | 9.1% |
| 04.05.2007 | 213.80 | 31.33% | 2.76% | 3.22% | 4.00 | 44.60 | 0.0172 | 39.66% | 52.91 | 18.6% |
| 02.05.2008 | 172.00 | 32.37% | 3.02% | 5.14% | 4.51 | 31.28 | 0.0153 | 34.89% | 38.71 | 23.7% |
| 12.05.2009 | 102.00 | 34.46% | 2.02% | 8.22% | 4.69 | 13.03 | 0.0287 | 37.62% | 20.70 | 58.9% |
| 20.05.2010 | 127.00 | 35.70% | 1.22% | 4.04% | 4.84 | 26.44 | 0.0275 | 38.38% | 33.24 | 25.7% |

**Table 4.2:** Comparison of the average historical volatility and GARCH(1,1) approaches for the calculation of the option value for the grants between 2006 and 2010. In the table, $r_7$ corresponds to the risk-free interest rate for the contractual term of 7 years, $q$ is the dividend yield, $L_0$ the expected term, $V_0$ the initial daily volatility for the GARCH(1,1) forecast, $\sigma_L$ the long–term annual volatility of the GARCH approach. $\Delta_{GARCH11}$ gives the percentage deviation of the GARCH(1,1) valuation, $f_{GARCH11}$ from the B–S valuation, $f_{B-S}$.

From the middle graph in Fig. 4.2, we can observe that the result from the GARCH(1,1) approach is not yet ideal, because there remains still autocorrelations after the fit to GARCH(1,1). Improvement might be found in using a more sophisticated GARCH approach such as the leveraged EGARCH or GJR approaches [36].

The use of GARCH(1,1) to forecast the daily volatility results in generally higher option value, approx. 23 % for the grants between 2006 and 2010 for all plans. The option values with the GARCH(1,1) time dependent volatilities were calculated using the Hull–White model. As Table 4.2 shows, all calculated option values were higher than the values obtained using B–S and the average historical volatility over the expected term period, except for the 2008 grant for the Management plan. In this table, the long–term annual volatility $\sigma_L$ of the GARCH(1,1) approach, calculated as $\sigma_L = \sqrt{252 \cdot V_L}$, is constantly above the volatility values obtained from averageing the historical volatility over period of the expected life. The data indicates that GARCH modeling of the volatility has a stabilizing effect: For the grants just before the financial crisis (2008), the option value for a comparably high stock price is rather moderate whereas for the grant during the financial crisis (2009), when also the stock price was very low, there is a higher valuation premium due to GARCH modeling. It is likely that GARCH estimates of the volatility will reduce the mismatch between intrinsic value and the fair value set at grant date.

#### Fractioning the Volatility into Positive and Negative Volatilities

A very simple approach to differentiate between the effects of an upswing or a downswing in the stock prices is to split the volatility into a positive part, resulting from the volatility of positive daily returns, and a negative volatility part from the negative daily returns. Note that the average of the positive and the negative volatilities equals to the regular volatility. Defining $u_i$ as the daily log return (see Eq. 2.18) and $n_{ytd}$ as the number of trading days per year, these volatilities are given by

$$\sigma_{\pm} = \sqrt{n_{ytd} \frac{1}{n-1} \sum_i (u_i^{\pm} - \bar{u})^2} \tag{4.4}$$

where $u_i^{\pm}$ are fraction of the daily log returns which are positive (+), respectively negative (-). $\bar{u}$ is the expectation value of $u$.

The positive and negative volatilities $\sigma_+$ and $\sigma_-$ can then be used in a slightly modified lattice model. Here, the procedure is exemplified for a binomial lattice, where instead of the definition for the factors of an up– and downstep, $u$ and $d$, from Eq. 2.38, we use

$$u = \mathrm{e}^{\sigma_+ \sqrt{\Delta t}} \qquad \text{and} \qquad d = \mathrm{e}^{-\sigma_- \sqrt{\Delta t}} \tag{4.5}$$

This simplistic model was tested for the Hull–White model for the Key Employee plan between 2005 and 2010 and compared to the regular H–W valuation. Although for all grants within this time frame, I found $\sigma_+ < \sigma_-$ (i.e. the option price is likely to decrease), the differences between the two valuation methods were negligible, and accounted usually to less than 0.1 CHF. Hence, this approach was not followed up.

### 4.1.3 Estimate of the Expected Life

We had seen in Section 3.4 that the current calculation of the expected term as done with the software *Express Options* overestimates the expected life value.
However, the expected life has a considerable influence on the option value, as can be seen from Fig. 4.3. Here it is shown that for typical option values found for Inficon, the option value changes by approx. 5 % for the Black–Scholes model when $L_0$ is varied by only 9 months, and even roughly 20 % when the Hull–White model is used. This is ultimately an important reason why many companies try to use minimal expected terms in order to reduce their expenses [18]. In Sect. 3.4,

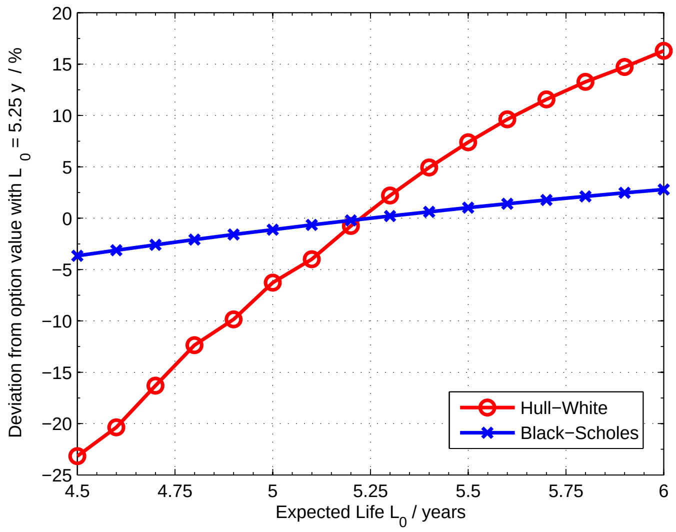


**Figure 4.3:** Percentage change of the option values $f_{B-S}$ and $f_{H-W}$ for the B–S and the H–W model as a function of the expected life. The base case is $L_0 = 5.25\,\text{y}$, $\sigma = 48\,\%$, the risk–free interest rates are $r = 2.1\,\%$ (for B–S), $r_7 = 2.2\,\%$ (for H–W), the dividend yield is $q = 4\,\%$, the contractual term is $T = 7\,\text{y}$. For this base case, the option values are $f_{B-S} = 47.26\,\text{CHF}$ and $f_{H-W} = 46.91\,\text{CHF}$ respectively. Starting from this base case, $L_0$ is then varied between $L_0 \in [4.5, 6]\,\text{y}$.

it was argued that there are up to now only two grants which have fully expired and from which the expected life can in principle be extracted. With this point of view, the values noted in Table 3.3 should be used. These values are significantly lower than the current values, which are for the 2010 grant given by 5.71 y for Key Employees, 5.11 y for Management and 4.84 y for the Director plan, see Table 3.2. The implications for the option values are significant, especially for the H–W model. However, it should be noted that basis for the expected life estimation is, with two years only, rather small, and we might need to add forecasts for the grants following the two earliest ones as we have sketched in Sect. 3.4.
As many companies are dealing with the same issue of an ill–defined expected life estimate, the SEC has issued in SAB 107, D2, Question 6, a guideline for how $L_0$ can be calculated in the absence of historical values [37]. They propose that:

$$L_0 = \frac{v + T}{2} \tag{4.6}$$

where $v$ is the vesting period and $T$ is the contractual term. In our case, according to Eq. 4.6, the expected term would be $L_0 = (2.5 + 7)/2 = 4.75\,\mathrm{y}$ for the Key Employee and the Management plan, and $L_0 = (1 + 7)/2 = 4\,\mathrm{y}$ for the Director plan respectively. This value is still below the current (wrong) estimates.
In Table 4.3, I summarize the effects of different expected life estimates on the 2010 grant option value: From the table, we observe that as shown in Fig. 4.3 that the H–W model depends much

| Key Employee Plan | | | Management Plan | | | Director Plan | | |
|---|---|---|---|---|---|---|---|---|
| $L_0$ y | $f_{B-S}$ | $F_{H-W}$ | $L_0$ | $f_{B-S}$ | $F_{H-W}$ | $L_0$ | $f_{B-S}$ | $F_{H-W}$ |
| 5.05 | 26.65 | 27.03 | 3.89 | 24.66 | 13.86 | 3.77 | 24.91 | 23.28 |
| 4.75 | 26.30 | 24.26 | 4.75 | 25.86 | 24.02 | 4.00 | 25.30 | 24.88 |
| 5.71 | 27.28 | 31.55 | 5.11 | 26.25 | 27.33 | 4.84 | 26.44 | 29.90 |

**Table 4.3:** Comparison of the option values according to B–S, $f_{B-S}$, and H–W, $f_{H-W}$ depending on the option's expected life, calculated for the 20.05.2010 grant. All option values in CHF. The first row gives the expected life according to Table 3.3, the second according to Eq. 4.6, the last one according to the current assumptions for this years' grant.

stronger on $L_0$ than the B–S model. Hence for the H–W model with an expected term given from the 2002 and 2003 grant results, i.e. Table 3.3, the option values are significantly lower than today's estimates with B–S, given in the third row. Note however that e.g. for the Director plan with $L_0 = 4.84\,\mathrm{y}$ that the H–W value is above the B–S value even though the expected term is quite low. This is mostly due to the significantly higher risk–free interest rate that has to be taken in the H–W approach (1.22 % for a 7 year bond) than for the B–S value (0.84 % for a 4.84 year bond).
Conclusively, the current assumptions of the expected life are too high, leading to a comparably high option value. If the H–W approach is taken, the changes due to an adaption of $L_0$ to more moderate values in the order of $4.5 - 5.2$ years, depending on the plan, will lead to significant reductions in the option value. For this purpose, it will be necessary to discuss what a good estimate of $L_0$ would be.

### 4.1.4 Optionee Exit Rates $w_1$ and $w_2$

One strong point of lattice models is that it is straightforward to include pre– and post–vesting optionee exit rates. Typically, the models differentiate between the pre–vesting exit rate $w_1$ during the vesting period, and the post–vesting exit rate $w_2$, both given as annual rates. Literature [24, 15, 17] suggests to obtain this data from independent sources about the employee exit rate, keeping in mind that different plans might have different exit schemes as different hierarchies tend to have different exit behavior. However, this approach has the drawback that we don't actually need the annual exit rates of employees but the exit rate per granted option. Thus, as the number of options granted to an employee can differ significantly (none, 100, 300, etc.), it will be difficult to obtain the appropriate fraction of options forfeited or cancelled. I therefore extracted the optionee exit rate from the fraction of forfeited options, as I could not obtain the data of the fraction of cancelled options. The difficulty with the latter is that there is a grace period of $6 - 24$ months after termination, see Table 3.1, so that many options that would expire as cancelled will be exercised whenever they are in–the–money and will not appear as cancelled.

**Key Employee Plan**

| Grant | # granted | # forfeited | Period / y | $w_1$ |
|---|---|---|---|---|
| 17.03.2004 | 15500 | 1475 | 3.0 | 3.2% |
| 16.02.2005 | 15800 | 575 | 3.0 | 1.2% |
| 10.02.2006 | 15300 | 1250 | 3.0 | 2.7% |
| 04.05.2007 | 15500 | 1150 | 2.5 | 3.0% |
| 02.05.2008 | 18600 | 1425 | 2.0 | 3.8% |
| 12.05.2009 | 20500 | 1000 | 1.5 | 3.3% |
| | | | $< w_1 >$ | 2.9% |

**Management Plan**

| Grant | # granted | # forfeited | Period / y | $w_1$ |
|---|---|---|---|---|
| 17.03.2004 | 12000 | 2750 | 3.0 | 7.6% |
| 16.02.2005 | 14000 | 1125 | 3.0 | 2.7% |
| 10.02.2006 | 15500 | 3000 | 3.0 | 6.5% |
| 04.05.2007 | 6250 | 1000 | 2.5 | 6.4% |
| 02.05.2008 | 5000 | 0 | 2.0 | 0.0% |
| 12.05.2009 | 6250 | 1000 | 1.5 | 10.7% |
| | | | $< w_1 >$ | 5.6% |

**Table 4.4:** Annual exit(forfeiture) rates $w_1$ for the Key Employee and Management plans, starting with 2004 grants. The forfeiture period (in years) column estimates the number of years for which all the forfeitures were obtained. For Inficon's graded vesting system, the aggregate vesting period is 2.5 years. To this 2.5 years, we add 6 months as the minimum grace period granted after termination (if not removal for cause). Hence, the period here is probably underestimated, as e.g. for retirement, there is a 2 year grace period. Thus, $w_1$ is most probably overestimated.

Data was available for grants starting in 2004. There was no forfeitured option for the Director plan during that time, so the estimate of the annual optionee exit rate is given only for the Management and the Key Employee plans. Note that the pre–vesting exit rate $w_1$ obtained in this way might not coincide with the post–vesting exit rate $w_2$, but typically, literature suggests to use identical values [24].

From Table 4.4, we obtain average exit rates for the Key Employee plan in the order of 3 %, with rather little spread, whereas, the spread is large for the Management plan, as is also the value for the forfeiture rate, $w_1 = 5.6\,\%$. These values are consistent with findings in literature [24]. Nevertheless, it remains unclear whether the forfeiture rate also reflects the post–vesting exit rate and whether this is a well–defined stable number over the economic cycles. Certaintly, a longer history will help here.

## 4.2 Accuracy–Improving Measures Non-Compliant with Topic 718

A fundamental assumption in the option pricing theory that leads to solutions such as the B–S formula or the Hull–White approach is that we can hedge our portfolio in order to keep it risk–neutral. The possiblity of hedging facilitates the pricing task enormously as it allows us to discount at a well–defined risk–free interest rate. However, as it was pointed out in Sect. 2.4.1, employee options cannot be hedged by the optionee, it's either take it or leave it. In the following we try to valuate the ESOs based on the risk–premium that should be required by the optionee for his risk. Topic 718 simply incorporates the non–hedgeability into a reduction of the option's maturity, i.e. Topic 718 states that the effect of non–hedgeability is already contained in the expected term of the option. Thus, as a shorter time to maturity reduces the option's price, Topic 718 states that the option value is reduced by the non–hedgeability.

When a risk–premium is included in the portfolio, this risk–premium the optionee should ask for as his expected rate of return, $\mu_r$, is often estimated in terms of the capital pricing model view:

$$\mu_i = r_f + \beta_i \left[ r_m - r_f \right] \tag{4.7}$$

where $r_f = r$ is the risk free interest rate, $r_m$ is the expected return for the market and $\beta_r$ is the sensitivity of the expected excess specific asset returns to the expected excess market return, e.g. for an asset $i$, $\beta_i = Cov(\mu_i, r_m)/Var(r_m)$. Note that this expected rate of return is equivalent to the cost of equity.

Presently, for INFICON, the market risk is $r_m = 0.07$ [38], the risk–free interest rate for a 7 year Swiss govt. bond is at $r_f = 0.0122$ and the $\beta$ values $\beta_{\mathrm{IFCN}} = 0.889$ [39]. This results in an expected return of

$$\mu_{\mathrm{IFCN}} = 0.0122 + 0.889 \cdot [0.07 - 0.0122] = 0.0636 \tag{4.8}$$

As INFICON does not have debt [28], the weighted average cost of capital (WACC), defined as

$$\mathrm{WACC} = \text{Cost of equity} + \text{Cost of debt} \tag{4.9}$$

equals the cost of equity, $\mu_{\mathrm{IFCN}}$.

Now, for the option valuation, I make the following very simplifying assumption: Consider a stock from which the stock holder and also the optionee expects a return $\mu$, and which is paying a dividend yield $q$. The total return from dividends and capital gains would be in principle $\mu$. Now, an optionee who cannot hedge his portfolio but calculates the option price in the risk–neutral world, expects a total return equal to $r_f$. This can be seen as if his capital gains provided a return of $r - \mu - q$. That is, the expected value from an initial stock price of $S_0$ after the time interval $\Delta t$ will be $S_0 \, \mathrm{e}^{(r-\mu-q)\Delta t}$.

Now given our expected return as in Eq. 4.7, we get a modified return in the capital gains as

$$\tilde{r} = r_f - \mu_i - q = -\beta_i (r_m - r_f) - q \tag{4.10}$$

This $\tilde{r}$ is then put into the valuation model instead of the regular risk–free interest rate minus the dividend yield, $r_f - q$. Note that this assumption has a stabilizing effect on the option valuations over the economic cycle compared to the regular B–S type valuation. In an economically declining time period, the market risk and the beta are likely to increase and thus reduce the option value. The option can now be valuated using $\tilde{r}$ and the full contractual term as the non–hedgeability is now included in the model through the interest rate. B–S or H–W can be used. The valuation can be refined for optionee exit rates and an exercise trigger to account for the non–marketability of the options.

For INFICON, with the above assumptions for $r_f$, $f_m$ and $\beta$ as well as an dividend yield of $q = 0.04$, we get

$$\tilde{r}_{\mathrm{IFCN}} = -0.889(0.07 - 0.0122) - 0.04 = -0.0914$$

we assume therefore that the risk–free interest rate is replaced by $\tilde{r}_{\mathrm{IFCN}}$. Additionally, we may include a post–vesting exit rate, e.g. 3 % (Key Employee plan), an exercise trigger $M$ (for the Key Employee plan $M \approx 2.3$) and also a GARCH–type volatility. This results in the option values of the 2010 grant given in Table 4.5. Both the current B–S as well as the H–W simulation with a

| | Current | with $\tilde{r}$ | | with $M$ | with $w_2$ | with $\sigma_{\mathrm{GARCH}}$ |
|---|---|---|---|---|---|---|
| Plan | $f_{B-S}$ | $f_{B-S}$ | $f_{H-W}$ | $f_{H-W}$ | $f_{H-W}$ | $f_{H-W}$ |
| Key Employee | 27.28 | 14.67 | 25.80 | 22.26 | 21.95 | 24.90 |
| Management | 26.25 | 14.12 | 25.55 | 22.52 | 22.02 | 25.29 |
| Director | 26.44 | 14.39 | 26.11 | 25.26 | 25.25 | 27.43 |

**Table 4.5:** Estimate of the 2010 grant value using a risk–premium. New features are added for each column to the left, the existing features are kept. $r_f$ and $\sigma$ (except for $\sigma_{\mathrm{GARCH}}$) as in Table 3.2. $w_2$ as in Section 4.1.4, $M$ as in Section 3.3.

risk–adjusted rate of return give similar results, with the H–W values being slightly lower. This is consistent with the hypothesis that the optionee will get less because he cannot hedge. Further refinements such as including a post–vesting exit rate do not change the option value by much. A slight increase is seen when the volatility is that of a GARCH(1,1) type estimate. Interestingly, the expected life resulting from the simulations were in the region of $L_0 \in (6, 6.7)$ years, approximately. Hence, these $L_0$ values are significantly higher than those given by the findings in Section 3.4, i.e. $L_0 \approx 5$ y. An adjustment in $M$ towards lower values in order to mimic the measured $L_0$'s would translate in lower option values, in the order of the $\tilde{r}$ risk–adjusted B–S values.

## 4.3 Expense–Reducing Measures Compliant with Topic 718

Given the difficulties in appropriately valuating the ESOs, we can as well try to simply minimize the expenses incurred by the ESO grants in disregard of any match between the intrinsic value and the calculated option value. However, it is important to note that the following procedure will not clarify the question of how to properly valuate options. But it gives an idea of the relationsship between different model parameters and their impact on the valuation models.
From literature, different authors [24, 12, 14] have pointed out that lattice models such as the Ammann–Seiz model or the Hull–White model often lead to lower option values. Hence, we have tried to compare the current Black–Scholes (B–S) model to the Hull–White (H–W) lattice model.

### 4.3.1 Black–Scholes vs Hull–White Valuation

As discussed before in Section 4.1.1, most of the ESO valuation parameters are ill-defined so that we might as well incorporate them into a single fudge–factor such as the expected lifetime, which then contains a whole range of different effects. The FASB suggests a similar approach throughout Topic 718. When this expected lifetime approach is taken, the Hull–White and the Ammann–Seiz model become analogous (the difference is anyway very small).
We thus follow this approach and solve the Hull–White model iteratively until the expected lifetime of the options matches the set lifetime. Given one single expected lifetime, we aggregate the graded vesting of the management and the key employee plans into a single vesting period equivalent of the average vesting, ie. 2.5 years instead of 25 % at 1, 2, 3 and 4 years. Simulations with different realistic option parameters have been conducted in the H–W approach to compare the single 2.5 y vesting period result with the outcome of 4 different grants with the 1, 2, 3 and 4 years vesting periods. The results showed full equivalence between the two approaches for all simulations so that for all H–W model runs with either the Key Employee or Management plan, the aggregate single vesting period approach was taken.
I have compared the valuations all grants of the three plans, Director, Management and Key Employee, according to B–S and Hull–White (H–W) valuation. I have used a binomial lattice model as well as a trinomial lattice model adapted to barrier options for the H–W calculations. The results depend critically on the number of time steps. Unfortunately, *none* of the publications I consulted that use lattice models ever specified the number of steps for their results. Here I have used typically 1000 steps. When the error in the result of the expected term was $> 1\,\%$, I increased the number of steps to up to 1e4. The trinomial model which is, according to Hull, advantageous as we put the barrier values, i.e. the value of $M$, onto lattice points does not seem to provide better results. As we can see from Table 4.6, the Hull–White model is approx. 9 % lower in option values than the B–S model. Given the very large number of options granted, this single digit savings translate into respectable numbers, on average, approx. 120 kCHF could have been saved per year (non–discounted) using the H–W approach.

**Comparison of Hull-White and Black-Scholes Valuation (average values)**

| Plan | | | Weighted average values | | | | Option value | |
|---|---|---|---|---|---|---|---|---|
| | # granted | Option Price | Expected Life | Volatility | Risk free rate | Dividend yield | Black-Scholes | Hull-White |
| **Director** | 32399 | 122.65 | 4.66 | 49.71% | 2.19% | 2.40% | 41.1728 | 38.7411 |
| **Management** | 99400 | 127.71 | 4.86 | 49.72% | 2.08% | 2.35% | 45.2629 | 39.2080 |
| **KeyEmployee** | 161651 | 130.46 | 5.15 | 46.89% | 2.07% | 3.14% | 41.3994 | 38.8599 |
| **Weighted Totals** | **293450** | **128.67** | **5.00** | **48.16%** | **2.09%** | **2.79%** | **42.6831** | **38.9647** |
| d(HW/BS) | **-8.71%** | | | | | | | |
| Total savings by using HW | **1091158.5** | | | | | | | |
| Savings/year | **121239.83** | | | | | | | |

**Table 4.6:** Comparison between the B–S and the H–W approaches in terms of option values for the different plans, summarized as the average of all grants weighted to their respective numbers of grants. Period 2002 – 2010. Values in CHF.

However, things are more delicate at a closer look. In fact, the B–S valuation can be advantageous costwise under the right circumstances. As an example, we show valuations of different grants for for the key employee plan: As we can observe from Table 4.7, the B–S model becomes advantageous

**Key Employee Plan**

| Grant Date | Option Price | BS Option value | # granted | Expected Life | Volatility | Risk free rate | r 7 years | Dividend yield | Hull-White |
|---|---|---|---|---|---|---|---|---|---|
| 15.01.2002 | 165 | 92.1049 | 11440 | 5 | 63.43% | 3.07% | 3.17% | 0.00% | 76.6722 |
| 17.02.2003 | 50 | 27.2089 | 20300 | 5 | 63.30% | 1.96% | 1.90% | 0.00% | 21.9177 |
| 30.09.2003 | 91.5 | 50.55 | 6411 | 5 | 64.06% | 2.23% | 2.23% | 0.00% | 41.6534 |
| 17.03.2004 | 116.75 | 62.6409 | 15500 | 5 | 61.07% | 2.54% | 2.01% | 0.00% | 52.4421 |
| 16.02.2005 | 93.00 | 29.0277 | 15800 | 4.75 | 46.82% | 1.56% | 1.85% | 3.30% | 25.2948 |
| 10.02.2006 | 188.00 | 52.1418 | 15300 | 4.75 | 41.29% | 1.88% | 2.02% | 3.33% | 47.2105 |
| 04.05.2007 | 213.80 | 55.6551 | 15500 | 4.75 | 36.87% | 2.68% | 2.76% | 3.22% | 52.0900 |
| 02.05.2008 | 172.00 | 42.0622 | 18600 | 5.52 | 40.39% | 2.83% | 3.02% | 5.14% | 47.2311 |
| 19.05.2008 | 175.00 | 42.1393 | 200 | 5.52 | 39.89% | 2.80% | 2.97% | 5.14% | 47.3951 |
| 11.08.2008 | 156.80 | 34.6802 | 200 | 5.52 | 37.35% | 2.61% | 2.79% | 5.14% | 39.3451 |
| 27.10.2008 | 102.00 | 21.1151 | 300 | 5.52 | 36.39% | 2.03% | 2.30% | 5.14% | 24.4356 |
| 12.05.2009 | 102.00 | 13.6015 | 20500 | 5.54 | 35.48% | 1.69% | 2.02% | 8.22% | 17.8661 |
| 20.05.2010 | 127.00 | 27.2825 | 21600 | 5.71 | 35.38% | 0.99% | 1.22% | 4.04% | 31.5269 |
| **Weighted Totals** | **130.46** | **41.3994** | **161651** | **5.15** | **46.89%** | **2.07%** | **2.16%** | **3.14%** | **38.8599** |

**Table 4.7:** Comparison between the B–S and the H–W approaches in terms of option values for the different plans, summarized as the average of all grants weighted to their respective numbers of grants. Values in CHF.

in 2008 through 2010. The rather small difference between B–S and H–W does not create a clear winner, and the above observations of lower B–S cost shows that we should analyze the regions for which each valuation method is to be preferred. This allows for a sensitivity analysis of the valuation parameters.

### Regions of Comparative Advantages

To analyze regions of cost advantage for both methods, we plotted all the model parameters, namely the dividend yield, the risk free interest rate, the volatility as well as the post–vesting exit rate as a function of the expected lifetime, which is the overall determining input parameter, and the option price ratio of the H–W model to the B–S model, defined by $f_{H-W}/f_{B-S} - 1$. The latter ratio is $> 0$ when the H–W value is higher than the B–S value and vice versa.

We use contour plots to depict the value $f_{H-W}/f_{B-S} - 1$ (multiply by 100 to get the deviation in percent) on a graph that has the expected lifetime on the x–axis and a chosen parameter ($r$, $\sigma$, the dividend yield $q$) on the y–axis. The parameter values of the other parameter than the chosen one are set to typical values (average over the period 2002 – 2009). The results are given in Fig. 4.4 below. First of all, in all of the graphs it is advantageous to use the H–W approach only when the post-vesting exit rate $w_2$ is not explicitely noted but rather included implicitely in the expected lifetime. Hence, for H–W to be cost–advantageous, we should set $w_2 = 0$. Furthermore, the cost deviation ratio critically depends on the expected lifetime for all parameters $r$, $\sigma$ and $q$. The change-over from an advantage of H–W to an advantage of B–S is generally around an expected life of $L_0 = 5 - 5.5$ years. This stems from the much larger sensitivity of the H–W approach on the expected term in comparison to the B–S model, see e.g. Fig 4.3. Thus, H–W has a cost–advantage only when the expected life clearly below 5.5 years[(2)]. For 2009 it was at 5.54 years, for 2010 even 5.71 y for the Key Employee plan. This is the main reason why B–S is currently the better solution for this plan.

In addition, low dividend yields favor H–W. At zero dividends, the H–W is advantageous even for very long expected lifes.

Changes in the risk–free interest rate do not affect the change-over of the advantageous valuation method in terms of the expected lifetime, the contours of $f_{H-W}/f_{B-S} - 1$ have almost zero slope. As for the volatility, Figure 4.4 shows that low volatilites require short expected lifetimes in order to use H–W.

Conclusively, at present, H–W has an edge over the B–S valuation method in terms of cost, an approximate 8–9% of the costs can be saved. However, the above discussion shows that the decision of the valuation model has to be taken with precaution, estimating the future values of the model parameters $r$, $\sigma$, $q$ and the all-encompassing expected lifetime. As we have seen in the preceding section, the expected life values of the software are overestimated, but they will likely increase from the values noted in Table 3.3.

Finally, it should be emphasized again that the Hull–White model should not be taken into considaration because of some cost-advantage, but because it allows for the inclusion of many ESO specifics and can therefore estimate the option value in a more realistic way than the simple Black–Scholes model.

[(2)]Note that the current valuation is according to B–S.

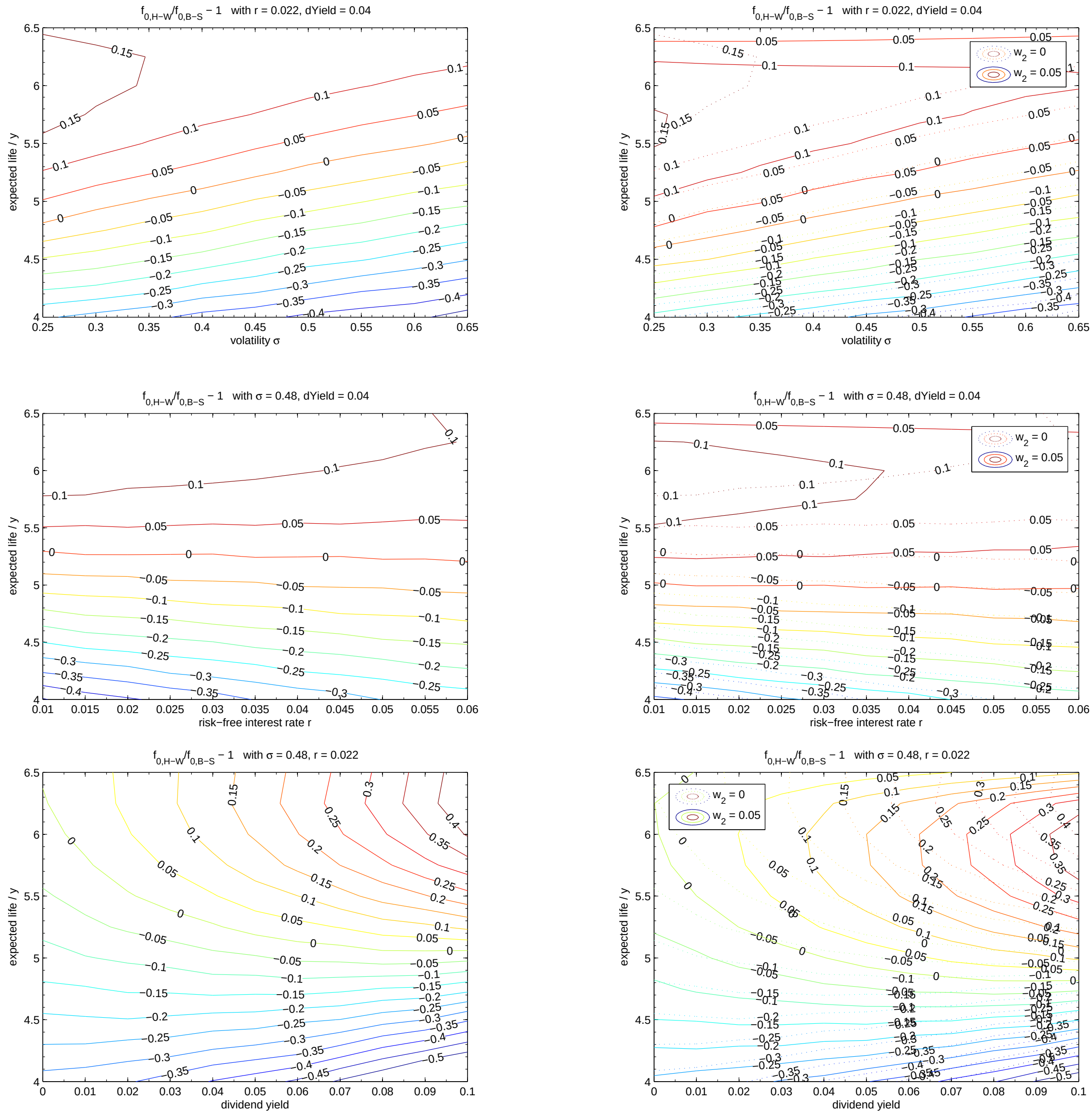


**Figure 4.4:** The left column of the graphs show the dependence of the cost deviation $f_{H-W}/f_{B-S} - 1$ on volatility $\sigma$ (top graph), the risk free interest rate $r$ (middle graph) as well as the dividend yield $q$ (bottom graph), and, for all graphs, the expected lifetime. For all graphs the post–vesting exit rate is set to $w_2 = 0$. On the right, the influence of the post-vesting exit rate $w_2 = 0.05$ (rate per year) is included for all three parameters $sigma$, $r$ and the dividend yield. Number of steps in the underlying binomial lattice model for the H–W model were 10000.

# 5 Conclusions

The initial analysis of INFICON's employee stock option (ESO) program showed that the optionees follow the typical behavior of ESO holders, namely that they exercise early and often non–ideally. This is the typical outcome when options cannot be sold, cannot be hedged and can become nullified upon termination of the employment and reflects the optionees' need for liquidity, their risk averseness and partly their exit from the company. The specifics of ESOs such as their non–marketability, their non–hedgeability, the vesting period and the possibility of cancellation and forfeiture, and last but not least the very long maturity period make them very difficult to valuate. Nevertheless, companies that follow US–GAAP (such as INFICON does) or IFRS2 are forced to recognize the fair option value as expenses. From a fair value estimate point of view (in the literal sense of the word) even worse are the stringent restriction on how to value the option: (a) their value must be determined at grant date, meaning that we have to predict the evolution of an option price over the next 7 years. (b) the discounting factor must be a clearly defined risk–free interest rate. This basically limits the valuation methods to the classical option pricing models and it is not possible to make provisons from year to year in order to expense the intrinsic value gained from exercising the option(1). It is therefore not surprising that many companies use the Black–Scholes formula to valuate their ESO grants. This prototype model does however not reflect the underlying goals of ESO grants, namely providing an incentive for a long–term success of the company and to ensure a low key–employee fluctuation rate. Nor do the alternative models that are compliant with ASC Topic 718, the underlying regulations for US–GAAP in share–based payment matters. For all of the valuation methods will have high option values when the volatility is increased, thus seemingly giving the optionees a better reward when they achieve to increase their firm's volatility, but usually high volatilities do not go together with long–term success. Surprisingly enough, our analysis of INFICON's past option grants show that whereas the B–S option value and the intrinsic value per grant differ significantly, that the overall expenses for the years $2006 - 2009$ match the B–S valuation quite well, the difference is a mere $4\,\%$. The analysis showed as well that the B–S approach is rather conservative, the intrinsic value over these 4 years was higher than the B–S valuation. Nevertheless, there are alternative models to B–S, which can mimic the ESO specifics better than the simple B–S approach and are compliant with US–GAAP or IFRS2. Typically these are lattice models which allow for time dependent parametrization of the option valuation. As such, vesting periods, employee exit rates and exercise trigger barrier can be included. Such models also allow for simple implementation of time dependent parameters

(1) It was shown by Hull that this could have a stabilizing effect, making the options more expensive in economically sound years, and reducing the expenses in economically hard years [6].

such as GARCH type volatilities.
In this report, we have taken the lattice model by Hull–White [15] as a prototype ESO valuation model that complies with Topic 718. It incorporates a vesting period, a post–vesting employee exit rate, a exercise trigger barrier for the stock price at which the options are exercised immediately. We have modified H–W's original paper with a time dependent volatility in order to implement an improved GARCH(1,1) forecast for the volatility. GARCH allows to analyze time series that exhibit volatility clustering and non–constant error terms for the stochastic stock process, i.e. time series where some times are riskier than others and where periods of high volatility follow calm periods.

Given the stringent restrictions of ASC Topic 718, I suggest two pathways for future ESO valuation at Inficon, either (a) try to valuate as accurately as possible even though this task is a formidable one; (b) valuate in a way that is consistent with Topic 718 but has the least expenses. If we blank out the fundamental goals in corporate finance of maximizing the stakeholder (and therefore also the shareholder) value, the costs for INFICON are pure transaction costs, as upon exercise of an option, new shares will be issued from conditional capital. Hence, all costs are carried by the stock holder in that his portfolio is being diluted.

For pathway (a), we found that the expected term, the average option lifetime, is overestimated by the current valuation program. The adequate expected terms for the three option plans will be up to 1 year shorter than specified today. This will result in lower option values, in particular if the Hull–White approach is taken.
Furthermore, we estimate the post–vesting employee exit rate (per annum) based on the number of forfeited options during the vesting period. We find that for the Key Employee plan, the exit rate amounts to 3 % whereas it is slightly higher for the Management plan with 5 %. These values are in line with findings from literature.
Then we analyzed the impact of a GARCH(1,1) type volatility. The current volatilites are estimated on historical grounds. Given the cyclic market the company INFICON is in, it is not surprising that the GARCH(1,1) long–term volatilities are in general higher than the values used today. This leads to higher option values. However, the use of GARCH type volatilities has a stabilizing effect in that the mismatch between the intrinsic value and the estimate fair value is likely to decrease.
Finally, we have tried to estimate as accurately as possible the option value using a ASC 718 *non-compliant* model that incorporates a risk–premium because of the non–hedgeability of ESOs. The calculations are based on the highly simplifying assumption that the return from the capital gains are strongly reduced by the non–hedgeability. This reduction amounts to the difference between the expected return by the optionee due to his risk and the risk–free rate. The risk is measured in the terms of the capital asset pricing model.

Concerning pathway (b), we show that the Hull–White lattice model overall has an edge in terms of expenses. However, the difference in the option price between H–W and B–S is an intricate relationship depending on the expected term, the volatility and the dividend yield. The main finding here is that the option pricing according to H–W has a much stronger dependency on the expected term than with the B–S formula. I give a plot of the regions of comparative option price advantage showing the dependence of the H–W vs B–S pricing in terms of the parameters volatility, expected term, dividend yield and risk–free interest rate. It should however be emphasized that the H–W model should not only be taken into considaration because of some cost-advantage, but because it allows for the inclusion of many ESO specifics and can therefore estimate the option value in a more realistic way than the simple B–S model

The conclusion is that with the Hull–White model, a more adequate option valuation is possible in comparison with the presently used Black–Scholes formula, as it allows for the methodologically correct inclusion of employee stock option specifics. And the model complies with ASC Topic 718. The decision–makers have however to keep in mind that Topic 718 requires consistency in the valuation method, so that year–to–year changes in the valuation method will hardly find acceptance. Moreover, once a company has switched to a more detailed lattice model, it will be difficult to justify a return to a simple Black–Scholes valuation.

Finally, all models for share–based awards classified as equity, such as INFICON's ESOs, will always produce mismatches between the intrinsic value and the 'fair' value estimated from the option pricing model, because the fair value has to be estimated at grant.[(2)]

[(2)] The only possibility to set up provisions for payment expenses is to grant awards classified as liabilities, but in that way, the simplicity of the ESO grant model for employee awards within the company, in terms of accounting and legal matters is given up. This topic is however beyond the scope of this work.

# Appendix

## A Continuous Compounding

If an amount $M_0$ is invested for $n$ years at an interest rate $R$ per annum, then the value of the investment after these years is

$$M_n = M_0(1+R)^n \tag{.1}$$

If the rate is compounded $p$ times per year, the value $M_t$ is

$$M_n = M_0\left(1+\frac{R}{p}\right)^{np} \tag{.2}$$

Now, continuous compounding means increasing the number of times the rate is compounded towards infinity, $p \to \infty$. In that case, using the definition of the exponential function $\mathrm{e}^x$ with $\mathrm{e}^x = \lim_{n\to\infty}(1+x/n)^n$, we obtain

$$\lim_{p\to\infty}\left(1+\frac{R}{p}\right)^{np} = \mathrm{e}^{Rn} \qquad \text{so that} \qquad \lim_{p\to\infty} M_0\left(1+\frac{R}{p}\right)^{np} = M_0\,\mathrm{e}^{Rn} \tag{.3}$$

To switch between continously compounded interest rate $R_c$ to an equivalent rate $R_p$ with $p$ times compounding per annum, the following relationsships hold

$$R_c = p\ln\left(1+\frac{R_p}{p}\right) \tag{.4}$$

$$R_p = p\left(\mathrm{e}^{R_c/p}-1\right) \tag{.5}$$

## B MATLAB Codes

### B.1 Hull–White Model

This section contains the MATLAB routine to calculate the option value of an American call with a dividend yield, a vesting period and a pre– and post–vesting exit rate and a trigger multiple $M$ of the strike price $K$. When the stock price reaches the trigger multiple, the option will be exercised. The model is based on a binomial lattice.
The routine allows for a set expected life $L_0$ using MATLAB's *fzero* routine. In that case, $L_0$ is an input to the routine and the trigger level $M$ is the outcome of the routine.

```
function [f0,MLout,varargout] = HullWhite_Exp(S0,K,v,T,ML,sigma,r,D,w1,w2,N,solType)
% Hull White model for ESOs, 2002, according to the solution scheme by Manfred Gilli and
% Enrico Schumanna, using exponentials for exit rate.
%
% INPUT
% =====
% S0    : the underlying share price at grant date
% K     : the strike price.
% ML    : the expected life EL, OR the strike price adjusting factor M according to
%         solType
% v     : vesting period in years
% T     : lifetime of option in years
% sigma : volatiliy of underlying stock per annum
% r     : risk free interest rate per annum
% D     : dividend yield
% w1    : employee exit rate in vesting period
% w2    : post-vesting employee exit rate
% N     : number of binomial time steps at [0,T/N,2*T/N,...,N*T/N=T].
% solType : is 'M' or 'L': If set to 'M' it sets M and calculates the expected life L. If
%           'L' is chosen, it sets the expected life to 'L' and tries to find the matching
%           M by an optimization routine (fzero to minimize the mismatch in the expected
%           life).
% OUTPUT
% ======
% f0    : the option value
% MLout : either M or L as output, M in case 'L' is chosen as solType, in case of solTyp =
%         M the return of MLout = expected life.
% varargout: The calculated expected life L0 when solType = 'L'.
%
% B. Andreaus, 11.05.2010
%
% parameter definition
ix0     = 1;
dt      = T/N;
u       = exp( sigma * sqrt(dt));
d       = 1 / u;
p       = ( exp((r-D) * dt) - d) / (u - d);
% initialise asset prices at maturity ( period N)
S   = zeros(1,N+1);
S(ix0+0) = S0 * d^N;
for j = 1:N
    S(ix0+j) = S(ix0+j - 1) * u / d;
end
% calculate f0 and L0 according to solType
switch lower(solType)
    case {'m'}
```

```
        [f0,MLout] = calcfL_HullWhite(ML,S,K,v,dt,r,p,d,w1,w2,N);
    case {'l'}
        % we must find M such that L(1,1) = EL by using fzero.
        % find M for set EL, which is ELM. Tolerances are set to 0.5 %
        zeroOpt         = optimset('TolFun',5e-3,'TolX',5e-3);
        Minit           = 1;
        const           = [ML,K,v,dt,r,p,d,w1,w2,N,S];
        [MLout, res]    = fzero(@calcResidL_HullWhite,Minit,zeroOpt,const);
        % found Mout is put into calcfL_HullWhite of type 'M' to get f and L
        [f0,L0]         = calcfL_HullWhite(MLout,S,K,v,dt,r,p,d,w1,w2,N);
        if nargout > 2
            varargout{1} = L0;
        end
    otherwise
        error('wrong lType case, must be lower(''M'') or lower(''L'')');
end

function [f0,L0] = calcfL_HullWhite(M,S,K,v,dt,r,p,d,w1,w2,N)
%
ix0     = 1;
rc      = exp(-r * dt);
w2c     = exp(-w2*dt);
iVested = ceil(v/dt);
% initialize L
L   = zeros(1,N+1);
% initialise option values at maturity ( period N)
f = max(S - K, 0);
% step back through the tree in the vested period
for i = N-1:-1:iVested
    for j = 0:i
        S(ix0+j) = S(ix0+j) / d;
        if S(ix0+j) > K*M
            f(ix0+j) = S(ix0+j) - K;
            L(ix0+j) = 0;
        else
            f(ix0+j) = w2c*rc*(p*f(ix0+j+1) + (1-p)*f(ix0+j)) + ...
                       (1-w2c)*max(S(ix0+j) - K,0);
            L(ix0+j) = w2c*(p*L(ix0+j+1) + (1-p)*L(ix0+j) + dt);
        end
    end
end
% step back through the tree in vesting period
for i = iVested-1:-1:0
    for j = 0:i
        f(ix0+j) = rc * (p * f(ix0+j + 1) + (1-p) * f(ix0+j));
        L(ix0+j) = p*L(ix0+j+1) + (1-p)*L(ix0+j) + dt;
```

```
    end
end
f0  = f(ix0+0)*(1-w1)^v;
L0  = L(ix0+0);
return


function ResidL = calcResidL_HullWhite(M,const)
% calculate the residual of the calculated EL given M and the set EL.
%
setL = const(1);
K       = const(2);
v       = const(3);
dt      = const(4);
r       = const(5);
p       = const(6);
d       = const(7);
w1      = const(8);
w2      = const(9);
N       = const(10);
S       = const(11:length(const));
%
[f0,L0] = calcfL_HullWhite(M,S,K,v,dt,r,p,d,w1,w2,N);
ResidL  = L0 - setL;
return
```

## B.2 Hull–White Model: Trinomial Lattice

The Hull–White Model is mapped onto a trinomial lattice. We use Levitan's approach [10] to get the trigger value $M$ onto the grid.

```
function [f0,L0] = calcfL_HullWhite_Trinomial(M,S0,K,v,T,sigma,r,D,w1,w2,N)
% Hull White model for ESOs, 2002, according to the trinomial solution scheme according to
% Hull p.608 used for Barrier options. We use the general layout of Gilli and Schumann.
% Use Levitan's approach to get the Barrier onto the grid.
%
% INPUT
% S0    : the underlying share price at grant date
% K     : the strike price.
% ML    : the expected life EL, OR the strike price adjusting factor M according to
%         solType
% v     : vesting period in years
% T     : lifetime of option in years
% sigma : volatiliy of underlying stock per annum
% r     : risk free interest rate per annum
```

```
% D      : dividend yield
% w1     : employee exit rate in vesting period
% w2     : post-vesting employee exit rate
% N      : number of binomial time steps at [0,T/N,2*T/N,...,N*T/N=T].
% OUTPUT
% f0     : the option value
% L0     : Expected life in years
%
% B. Andreaus, Inficon AG, 27.07.2010
%
ix0     = 1;
dt      = T/N;
rc      = exp(-r * dt);
w2c     = exp(-w2*dt);
iVested = ceil(v/dt);
%

sqrtdt  = sqrt(dt);
% define lambda, use n0 which is at least 1
n0      = max(1,floor(log(M)/sigma/sqrtdt));
lambda  = log(M)/(n0*sigma*sqrtdt);     %sqrt(3);       %
mu      = r - D - 0.5*sigma^2;
u       = exp(sigma*lambda*sqrtdt);
d       = 1/u;
pud1sum = 0.5*mu*sqrtdt/lambda/sigma;   % = sqrt(dt/(12*sigma^2))*mu;
pd      = -pud1sum + 0.5/lambda^2;
pm      = 1 - 1/lambda^2;
pu      = pud1sum + 0.5/lambda^2;
%
% initialise asset prices at maturity ( period N)
S        = zeros(1,2*N+1);
S(ix0+N) = S0;
for j = 0:(N-1)
    S(ix0+j)     = S0*d^(N-j);
    S(ix0+N+1+j) = S0*u^(j+1);
end
% initialize L
L = zeros(2*N+1,1);
% initialise option values at maturity ( period N)
f = max(S - K, 0);
%
% step back through the tree in the vested period
for i = N-1:-1:iVested
    tempf = zeros(2*N+1,1);
    tempL = zeros(2*N+1,1);
    for j = N + (-i:i)
```

```
        % S(ix0+j) = S(ix0+j) / d;
        if S(ix0+j) > K*M
            tempf(ix0+j) = S(ix0+j) - K;
        else
            tempf(ix0+j) = w2c*rc*(pu*f(ix0+j+1) + pm*f(ix0+j) + pd*f(ix0+j-1)) + ...
                           (1-w2c)*max(S(ix0+j) - K,0);
            tempL(ix0+j) = w2c*(pu*L(ix0+j+1) + pm*L(ix0+j) + pd*L(ix0+j-1) + dt);
        end
    end
    f   = tempf;
    L   = tempL;
end
% step back through the tree in vesting period
for i = iVested-1:-1:0
    tempf = zeros(2*N+1,1);
    tempL = zeros(2*N+1,1);
    for j =  N + (-i:i)
        tempf(ix0+j) = rc*(pu*f(ix0+j+1) + pm*f(ix0+j) + pd*f(ix0+j-1));
        tempL(ix0+j) = pu*L(ix0+j+1) + pm*L(ix0+j) + pd*L(ix0+j-1) + dt;
    end
    f   = tempf;
    L   = tempL;
end
f0  = f(ix0+N)*(1-w1)^v;
L0  = L(ix0+N);
return
```

## B.3 Hull–White Model with GARCH–type Volatility

The H–W model is modified in order to allow for time–dependent volatilities. The time steps are adapted in order to keep the standard deviation of the stock return over each time step constant, see Hull's book, chapter 19 [6]. A binomial lattice is used.

```
function [f0,L0] = calcfL_HullWhite_GARCHSigma(M,S0,K,v,T,dpy,sigmaC,r,D,w2,N)
% Hull-White model for ESOs, 2002, according to the solution scheme by Manfred Gilli and
% Enrico Schumanna, using exponentials for exit rate (Ammann and Seiz).
% INPUT
% S0    : the underlying share price at grant date
% K     : the strike price.
% ML    : the expected life EL, OR the strike price adjusting factor M according to
%         solType
% v     : vesting period in years
% T     : lifetime of option in years
% dpy   : trading days per annum
```

```
% sigmaC: structure of GARCH coefficients acccording to garchpred, containing
%           .GARCH  coefficient (beta)
%           .ARCH   coefficient (alpha)
%           .K      coefficient
%           .sigma0 standard deviation at time 0 for a unit time interval (day).
% r     : risk free interest rate per annum
% D     : dividend yield
% w1    : employee exit rate in vesting period
% w2    : post-vesting employee exit rate
% N     : number of binomial time steps at [0,T/N,2*T/N,...,N*T/N=T].
% OUTPUT
% f0    : option value
% L0    : expected life in years
%
% B. Andreaus, Inficon AG, 08.08.2010
%
% parameter definition
ix0     = 1;
T       = T*dpy;
a       = log(1/(sigmaC.ARCH + sigmaC.GARCH));
VL      = sigmaC.K/(1 - sigmaC.ARCH - sigmaC.GARCH);
V0      = sigmaC.sigma0^2;
V       = T*(VL + (1-exp(-a*T))*(V0-VL)/(a*T));
Ttot    = zeros(1,N+1);
for i = 1:N
    if ~isinf(a)
        Ttot(i+1) = fzero(@(t)t*VL + (1-exp(-a*t))*(V0-VL)/a - i*V/N, i*T/N);
    else
        Ttot(i+1) = fzero(@(t)t*VL - i*V/N, i*T/N);
    end
end
dt      = diff(Ttot/dpy);
u       = exp(sqrt(V/N));
d       = 1/u;
p       = ( exp((r-D) * dt) - d) / (u - d);
rc      = exp(-r*dt);
w2c     = exp(-w2*dt);
iVested = find(cumsum(dt)>=v,1,'first');
% initialise asset prices at maturity ( period N)
S   = zeros(1,N+1);
S(ix0+0) = S0 * d^N;
for j = 1:N
    S(ix0+j) = S(ix0+j - 1) * u / d;
end
% initialize L
L   = zeros(1,N+1);
```

```
% initialise option values at maturity ( period N)
f = max(S - K, 0);
% step back through the tree in the vested period
for i = N-1:-1:iVested
    for j = 0:i
        S(ix0+j) = S(ix0+j) / d;
        if S(ix0+j) > K*M
            f(ix0+j) = S(ix0+j) - K;
            L(ix0+j) = 0;
        else
            f(ix0+j) = w2c(i+1)*rc(i+1)*(p(i+1)*f(ix0+j+1) + (1-p(i+1))*f(ix0+j)) +...
                       (1-w2c(i+1))*max(S(ix0+j) - K,0);
            L(ix0+j) = w2c(i+1)*(p(i+1)*L(ix0+j+1) + (1-p(i+1))*L(ix0+j) + dt(i+1));
        end
    end
end
% step back through the tree in vesting period
for i = iVested-1:-1:0
    for j = 0:i
        f(ix0+j) = rc(i+1)*(p(i+1)*f(ix0+j + 1) + (1-p(i+1))*f(ix0+j));
        L(ix0+j) = p(i+1)*L(ix0+j+1) + (1-p(i+1))*L(ix0+j) + dt(i+1);
    end
end
f0  = f(ix0+0);
L0  = L(ix0+0);
return
```

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