Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Master Thesis (MAS MTEC)

Tail Dependence of Hedge Funds

David Trudel

Department of Management, Technology, and Economics (D-MTEC)
Chair of Entrepreneurial Risks (ER)
and
ISPartners AG

Supervisors:
Prof Dr Didier Sornette
Dr Rainer Rueppel

November 2008

Abstract

This thesis assesses the tail dependence of hedge funds applying various linear and nonlinear methods of tail dependence estimation. The nonlinear methods are all based on concepts from extreme value theory. The methods applied have been developed by Poon et al., Malevergne and Sornette, and Schmidt and Stadtmüller. The linear measures of tail dependence are the classical linear measures of correlation (Pearson's r, Spearman's ρ, and Kendall's τ).

The data that has been investigated stems from the S&P 500, the Tremont Hedge Fund Index, and from ISPartners, a hedge fund situated in Zurich and collaborator of this thesis.

The gained results are manifold, but some general conclusions can be drawn. For small data sets the linear measures of tail dependence performed better than expected and sometimes almost as good as the nonlinear measures (assuming that their results are more reliable than those obtained with linear measures of tail dependence). The method of Poon et al. performed poorly and was only applied to a small set of the investigated data. The performance of the various tail index estimators that were needed for the methods of Poon et al. and Malevergne and Sornette was quite differing. The Gabaix estimator seems to perform best for small data sets and has been applied to all relations investigated. In general the nonlinear measures of tail dependence seem to perform better than the linear ones, as the variance of their results is generally smaller than the ones of the results derived with linear measures of tail dependence. However, the methods of Malevergne and Sornette might produce spurious results due to an intrinsic asymmetry caused by including the β of an underlying factor model in their methods. Therefore, the method by Schmidt and Stadtmüller might be superior, although its behavior is less smooth than that of Malevergne and Sornette's methods.

The economic interpretation of the results proved to be difficult, as only few tendencies could be extracted from the gained results. It seems that managed futures (CTAs) have small tail dependencies related to market movements, as they often apply trading software that does not show herding behavior in extreme situations. This holds also for hedge funds that do not contain a lot of equity. On the other hand, we have hedge funds that are mainly equity based (e.g event driven funds that try to profit from mispricings in securities related to specific corporate or market events) and, hence, show a higher tail dependence with the market movements.

To clarify open questions future research is needed. The next steps could be the prolongation of short times series using benchmarks or to test the various methods with synthetic data sets that possess known properties.

Acknowledgments

First, I would like to thank Prof Didier Sornette from the ETH and Dr Rainer Rueppel from ISPartners that made this work possible. It was a very interesting time and I could learn a lot about field that was completely new to me. It was a very productive environment and I am sure that this work will be of benefit for my future career and hopefully also for the work of Prof Sornette's group and Dr Rueppel's company ISPartners AG.

A special thanks goes to Sebastian Schmuki, my predecessor. His master thesis was an excellent basis for my work and it would not have possible for me to write this thesis without his input.

I would also like to thank Andreas Hüsler, a former employee of ISPartners and now PhD student at Prof Sornette's chair for entrepreneurial risks. His help was crucial for the successful completion of this work and it was a very fruitful experience to work together with a professional statistician. Here, I would also like to say thank you to Dr Bastian Hertstein, the successor of Andreas Hüsler at ISPartners. He also gave valuable inputs to my work.

Another important contributor for this thesis was Prof Yannick Malevergne. He brought up the idea to compare the THFI with the S&P 500 and was author of the equations for the factor model where the dependent and the independent variable have been exchanged and I would like to thank him for this.

Finally, I would like thank the team at Kreuzplatz! You made this work a really great experience and I hope that we can stay in contact.

Contents

1. Introduction **1**

2. Methodology **3**
2.1. Extreme Value Theory (EVT) . . . 3
2.1.1. Copulas . . . 4
2.1.2. Tail Dependence . . . 5
2.2. Nonlinear Measures of Tail Dependence . . . 5
2.2.1. Tail Index Estimation . . . 6
2.2.2. Approach by Poon, Rockinger and Tawn . . . 8
2.2.3. Approaches by Malevergne and Sornette . . . 11
2.2.4. Approach by Schmidt and Stadtmüller . . . 15
2.3. Linear Measures of Tail Dependence . . . 17
2.3.1. Pearson's Product-Moment Correlation . . . 18
2.3.2. Spearman's Rank Correlation . . . 18
2.3.3. Kendall's Rank Correlation . . . 19
2.4. Uncertainty Analysis / Bootstrapping . . . 19
2.5. Comparison of Rankings . . . 19
2.6. Data . . . 20
2.6.1. S&P 500 . . . 20
2.6.2. Hedge Funds (HF) . . . 20
2.6.3. Tremont Hedge Fund Index (THFI) . . . 24
2.6.4. Fund of Hedge Funds (FoHF) . . . 24
2.6.5. Calculation of Returns . . . 24
2.7. Software . . . 26

3. Results and Discussion **27**
3.1. THFI vs. S&P 500 . . . 27
3.1.1. Moving Threshold Graphs . . . 27
3.2. THFI vs. THFI_{sub} . . . 36
3.2.1. Moving Threshold Graphs . . . 36
3.2.2. Detailed Results . . . 43
3.3. THFI vs. Hedge Funds of ISPartners (HF_{ISP}) . . . 53
3.3.1. Moving Threshold Graphs . . . 53
3.3.2. Detailed Results . . . 60
3.4. S&P 500 vs. HF_{ISP} . . . 69
3.4.1. Moving Threshold Graphs . . . 69
3.4.2. Detailed Results . . . 74
3.5. S&P 500 vs. THFI_{sub} . . . 83
3.5.1. Moving Threshold Graphs . . . 83
3.5.2. Detailed Results . . . 88

4. Conclusions **97**

A. Appendix **102**
A.1. THFI vs. S&P 500 103
A.2. S&P 500 vs. THFI 105
A.3. THFI vs $THFI_{sub}$ 110
A.4. THFI vs. HF_{ISP} 116
A.5. S&P 500 vs. HF_{ISP} 121
A.6. S&P 500 vs. $THFI_{sub}$ 126
A.7. Source Code 131
A.7.1. Help Scripts 131
A.7.2. Calculation of β 131
A.7.3. Tail Index Estimators 132
A.7.4. Tail Dependence Coefficients (TDCs) 133

List of Figures

3.1. Tail index estimates for THFI and S&P 500 29
3.2. $\overline{\chi}$ and χ for THFI vs. S&P 500 with Hill and Gabaix estimator, and non-parametric method by Poon et al. 30
3.3. λ for THFI vs. S&P 500 with non-parametric method by Malevergne and Sornette 31
3.4. λ for THFI vs. S&P 500 with parametric method by Malevergne and Sornette 32
3.5. λ for THFI vs. S&P 500 with non-parametric method by Schmidt and Stadtmüller 33
3.6. λ for THFI vs. S&P 500 with linear measures of correlation 34
3.7. λ for S&P 500 vs. THFI with adjusted parameters 35
3.8. Tail index estimates for THFI and $\mathrm{THFI_{sub}}$ 37
3.9. $\overline{\chi}$ and χ for THFI vs. $\mathrm{THFI_{sub}}$ with Hill and Gabaix estimator, and non-parametric method by Poon et al. 38
3.10. λ for THFI vs. $\mathrm{THFI_{sub}}$ with non-parametric method by Malevergne and Sornette 39
3.11. λ for THFI vs. $\mathrm{HF_{ISP}}$ with parametric method by Malevergne and Sornette 40
3.12. λ for THFI vs. $\mathrm{THFI_{sub}}$ with non-parametric method by Schmidt and Stadtmüller 41
3.13. λ for THFI vs. $\mathrm{THFI_{sub}}$ with linear measures of correlation 42
3.14. Tail index estimates for THFI and $\mathrm{HF_{ISP}}$ 54
3.15. $\overline{\chi}$ and χ for THFI vs. $\mathrm{HF_{ISP}}$ with Hill and Gabaix estimator, and non-parametric method by Poon et al. 55
3.16. λ for THFI vs. $\mathrm{HF_{ISP}}$ with non-parametric method by Malevergne and Sornette 56
3.17. λ for THFI vs. $\mathrm{HF_{ISP}}$ with parametric method by Malevergne and Sornette 57
3.18. λ for THFI vs. $\mathrm{HF_{ISP}}$ with non-parametric method by Schmidt and Stadtmüller 58
3.19. λ for THFI vs. $\mathrm{HF_{ISP}}$ with linear measures of correlation 59
3.20. λ for S&P 500 vs. $\mathrm{HF_{ISP}}$ with non-parametric method by Malevergne and Sornette 70
3.21. λ for S&P 500 vs. $\mathrm{HF_{ISP}}$ with parametric method by Malevergne and Sornette 71
3.22. λ for S&P 500 vs. $\mathrm{HF_{ISP}}$ with non-parametric method by Schmidt and Stadtmüller 72
3.23. TDC for S&P 500 vs. $\mathrm{HF_{ISP}}$ with linear measures of correlation 73
3.24. λ for S&P 500 vs. $\mathrm{THFI_{sub}}$ with non-parametric method by Malevergne and Sornette 84
3.25. λ for S&P 500 vs. $\mathrm{THFI_{sub}}$ with parametric method by Malevergne and Sornette 85
3.26. λ for S&P 500 vs. $\mathrm{THFI_{sub}}$ with non-parametric method by Schmidt and Stadtmüller 86

3.27. TDC for S&P 500 vs. $THFI_{sub}$ with linear measures of correlation . . . 87

A.1. $\overline{\chi}$ and χ for THFI vs. S&P 500 with Clauset and Huisman estimator . 103
A.2. Parameters to calculate λ according to methods from Malevergne and Sornette 104
A.3. $\overline{\chi}$ and χ for S&P 500 vs. THFI with Hill and Gabaix estimator, and non-parametric method by Poon et al. 105
A.4. λ for S&P 500 vs. THFI with non-parametric method by Malevergne and Sornette 106
A.5. λ for S&P 500 vs. THFI with parametric method by Malevergne and Sornette 107
A.6. λ for S&P 500 vs. THFI with non-parametric method by Schmidt and Stadtmüller 108
A.7. λ for S&P 500 vs. THFI with linear measures of correlation 109
A.8. $\overline{\chi}$ and χ for THFI vs. $THFI_{sub}$ with Clauset and Huisman estimator . . 110
A.9. $\overline{\chi}$ and χ for THFI vs. HF_{ISP} with Clauset and Huisman estimator . . . 116
A.10. $\overline{\chi}$ and χ for S&P 500 vs. HF_{ISP} with Hill and Gabaix estimator 121
A.11. $\overline{\chi}$ and χ for S&P 500 vs. $THFI_{sub}$ with Hill and Gabaix estimator . . . 126

List of Tables

2.1. Coding of THFI 24
2.2. Coding of FoHF 25

3.1. Bootstrapping of λ for THFI vs. $THFI_{sub}$ according to non-parametric approach by Malevergne and Sornette applying the Gabaix estimator . 46
3.2. Bootstrapping of λ for THFI vs. $THFI_{sub}$ according to parametric approach by Malevergne and Sornette applying the Gabaix estimator . . . 47
3.3. Bootstrapping of λ for THFI vs. $THFI_{sub}$ according to non-parametric approach by Schmidt and Stadtmüller 48
3.4. Bootstrapping of r for THFI vs. $THFI_{sub}$ according to Pearson's product-moment correlation 49
3.5. Ranking of TDCs for THFI vs. $THFI_{sub}$ 50
3.6. Correlation between TDCs for THFI vs. $THFI_{sub}$ for upper tail 51
3.7. Correlation between TDCs for THFI vs. $THFI_{sub}$ for lower tail 52
3.8. Bootstrapping of λ for THFI vs. HF_{ISP} according to non-parametric approach by Malevergne and Sornette applying the Gabaix estimator . 62
3.9. Bootstrapping of λ for THFI vs. HF_{ISP} according to parametric approach by Malevergne and Sornette applying the Gabaix estimator 63
3.10. Bootstrapping of λ for THFI vs. HF_{ISP} according to non-parametric approach by Schmidt and Stadtmüller 64
3.11. Bootstrapping of r for THFI vs. HF_{ISP} according to Pearson's product-moment correlation 65
3.12. Ranking of TDCs for THFI vs. HF_{ISP} 66
3.13. Correlation between TDCs for THFI vs. HF_{ISP} for upper tail 67
3.14. Correlation between TDCs for THFI vs. HF_{ISP} for lower tail 68
3.15. Bootstrapping of λ for S&P 500 vs. HF_{ISP} according to non-parametric approach by Malevergne and Sornette applying the Gabaix estimator . 76
3.16. Bootstrapping of λ for S&P 500 vs. HF_{ISP} according to parametric approach by Malevergne and Sornette applying the Gabaix estimator . 77
3.17. Bootstrapping of λ for S&P 500 vs. HF_{ISP} according to non-parametric approach by Schmidt and Stadtmüller 78
3.18. Bootstrapping of r for S&P 500 vs. HF_{ISP} according to Pearson's product-moment correlation 79
3.19. Ranking of TDCs for S&P 500 vs. HF_{ISP} 80
3.20. Correlation between TDCs for S&P 500 vs. HF_{ISP} for upper tail 81
3.21. Correlation between TDCs for S&P 500 vs. HF_{ISP} for lower tail 82
3.22. Bootstrapping of λ for S&P 500 vs. $THFI_{sub}$ according to non-parametric approach by Malevergne and Sornette applying the Gabaix estimator . 90
3.23. Bootstrapping of λ for S&P 500 vs. $THFI_{sub}$ according to parametric approach by Malevergne and Sornette applying the Gabaix estimator . 91
3.24. Bootstrapping of λ for S&P 500 vs. $THFI_{sub}$ according to non-parametric approach by Schmidt and Stadtmüller 92

3.25. Bootstrapping of r for S&P 500 vs. THFI_{sub} according to Pearson's product-moment correlation . . . 93
3.26. Ranking of TDCs for S&P 500 vs. THFI_{sub} . . . 94
3.27. Correlation between TDCs for S&P 500 vs. THFI_{sup} for upper tail . . . 95
3.28. Correlation between TDCs for S&P 500 vs. THFI_{sup} for lower tail . . . 96

A.1. Bootstrapping of λ for THFI vs. THFI_{sub} according to non-parametric approach by Malevergne and Sornette applying the Hill estimator . . . 111
A.2. Bootstrapping of λ for THFI vs. THFI_{sub} according to parametric approach by Malevergne and Sornette applying the Hill estimator . . . 112
A.3. Bootstrapping of ρ for THFI vs. THFI_{sub} according to Spearman's rank correlation . . . 113
A.4. Bootstrapping of τ for THFI vs. THFI_{sub} according to Kendall's rank correlation . . . 114
A.5. Bootstrapping of λ for THFI vs. HF_{ISP} according to non-parametric approach by Malevergne and Sornette applying the Hill estimator . . . 117
A.6. Bootstrapping of λ for THFI vs. HF_{ISP} according to parametric approach by Malevergne and Sornette applying the Hill estimator . . . 118
A.7. Bootstrapping of ρ for THFI vs. HF_{ISP} according to Spearman's rank correlation . . . 119
A.8. Bootstrapping of τ for THFI vs. HF_{ISP} according to Kendall's rank correlation . . . 120
A.9. Bootstrapping of λ for S&P 500 vs. HF_{ISP} according to non-parametric approach by Malevergne and Sornette applying the Hill estimator . . . 122
A.10. Bootstrapping of λ for S&P 500 vs. HF_{ISP} according to parametric approach by Malevergne and Sornette applying the Hill estimator . . . 123
A.11. Bootstrapping of ρ for S&P 500 vs. HF_{ISP} according to Spearman's rank correlation . . . 124
A.12. Bootstrapping of τ for S&P 500 vs. HF_{ISP} according to Kendall's rank correlation . . . 125
A.13. Bootstrapping of λ for S&P 500 vs. THFI_{sub} according to non-parametric approach by Malevergne and Sornette applying the Hill estimator . . . 127
A.14. Bootstrapping of λ for S&P 500 vs. THFI_{sub} according to parametric approach by Malevergne and Sornette applying the Hill estimator . . . 128
A.15. Bootstrapping of ρ for S&P 500 vs. THFI_{sub} according to Spearman's rank correlation . . . 129
A.16. Bootstrapping of r for S&P 500 vs. THFI_{sub} according to Kendall's rank correlation . . . 130

1. Introduction

Recent developments at the stock markets have again revealed the need for a better understanding of extreme risks related to financial assets and products in turbulent times. As the concepts traditionally applied are based on linear models (e.g. Pearson's product-moment correlation or Spearman's rank correlation) covering the whole return distribution, they are not able to correctly reflect the behavior of assets in extreme situations that generally does not show a linear behavior. Therefore, new concepts have been developed that focus especially on the tail of the return distributions and its nonlinear behavior. In 1959, Sibuya was the first to introduce the concept of tail-dependence and thus tackling the special behavior of assets in extreme situations [22]. To quantify this tail dependence a coefficient of tail dependence has been developed and defined as the probability that one asset undergoes a large loss (or gain) simultaneously with another asset.

Several concepts have been developed to estimate tail dependence coefficients (abbreviated here with TDC(s), λ, or χ) [14, 18, 19]. However, it is still an open question which method is most reliable for the analysis of data ones is interested in. Schmuki [20] showed in his master thesis that the approach developed by Poon et al. [18] does not produce useful results when applied to daily data of the S&P 500 index and some selected assets. Better results can be expected from the application of the non-parametric and the parametric approach developed by Malevergne and Sornette [14, 15] and the non-parametric method developed by Schmidt and Stadtmüller [19]. Although the absolute values of the calculated TDCs still show distinct variations among the different methods, the relative ranking of the calculated TDCs is similar for the approaches developed by Malevergne and Sornette, and Schmidt and Stadtmüller.

This work is mainly based on the master thesis of Sebastian Schmuki [20], but with a different focus. The focus of this thesis is set on the application of the methods to assess the TDC on financial data related to hedge funds. This is especially challenging, as hedge fund data is sparse, i.e. only monthly data is available and the number of data points has a direct influence on the quality of the TDC estimates. Therefore, special emphasis has been set on the proper estimation of parameters (tail indices) that are very sensitive to small data sets.

The code for the calculation of the TDCs has been generated on Matlab. A large part of the code used for this master thesis is based on the code developed by Schmuki [20] for his master thesis. However, new algorithms have been implemented to account for small samples and the structure of the code has been improved to make it easier for non-expert users to follow and apply the code. The interested reader can find the code in the appendix of this thesis.

The structure of this thesis is as follows: Following this introduction, the methodology part will explain the applied concepts and data sets used. The results will be presented and discussed in the third section and the conclusions will summarize the most important findings and give a brief outlook of open questions and possible ways to tackle them.

2. Methodology

Most parts of the methodology presented here are strongly influenced by the master thesis of Schmuki [20]. However, I rephrased and reordered several parts to make them easier to understand and added also some new parts with focus on the investigation of small data sets (especially two additional methods to estimate tail indices of power-law distributions with small datasets).

Firstly I will give some background information on Extreme Value Theory (EVT) and the concept of tail dependence. Secondly a section that describes the applied methods to estimate tail dependence coefficients based on EVT will be presented, followed by a section about the tail dependence coefficients based on linear measures of correlation. The next section gives some information on the bootstrapping procedure applied for uncertainty analysis, followed by a section about how I compared the different results among the various methods. Thereafter a section that describes the investigated data will follow and the last and very short section about the software used will close this methodology chapter.

2.1. Extreme Value Theory (EVT)

Extreme value theory is the branch of statistics that deals with extreme and, therefore, rare events. EVT has applications in various areas of science such as in geology to calculate the probability of large earthquakes or floodings. Traditional statistical methods are not able to correctly reflect the behavior of a system in extreme situations as they focus on the whole distribution and, hence, fail to account for extreme behavior of a system that is reflect in the tails of the distribution. If, for example, financial data is analyzed traditionally and the distribution of returns is assumed to be Gaussian, the tails are too thin and do not properly reflect the possible up- and downside risks.

Some basic properties of EVT [6] are presented below and should help the reader to better understand the following methodological sections. The explanation is given for upside risks (gains) but the same holds for downside risks (losses).

Let $x_1, \ldots, x_N$ be N independently and identically distributed (i.i.d.) realizations of the random variable X. F stands for the cumulative distribution function (CDF) with $F(x) = \Pr(X \leq x)$. $x_{1,N} \leq \cdots \leq x_{N,N}$ denotes the ordered statistics so that $x_{i,N}$ is the ith smallest variable among the ordered $x_{\cdot,N}$.

The main theorem of EVT (Fisher Tippett theorem) describes the asymptotic behavior when the sample maxima $x_{N,N}$ goes to infinity. There are just three possible extreme value distributions G of $x_{N,N}$ to correctly reflect the asymptotic behavior. These three distributions are the Gumbel law that is related to light-tailed distributions such as normal, log-normal or exponential, the Fréchet law to heavy-tailed distributions such as Pareto, Cauchy or Student distribution, and the Weibull law to finite support distributions such as the uniform distribution.

Due to the Jenkins representation the CDF of the extreme value distribution G can be written as a function of G_γ depending on the index γ. Where $\gamma \rightarrow 0$ corresponds to the Gumbel distribution, the case $\gamma > 0$ to the Fréchet distribution, and the case $\gamma < 0$ to the Weibull distribution.

As it is reasonable to assume that returns of financial assets show a heavy-tail distribution, we can model them with a power-law distribution. So, for any x that belongs to the tail of the distribution, we can assume that $1 - F(x) = Cx^{-1/\gamma}$ and the maximum is of Fréchet type with $\gamma > 0$.

As this thesis investigates the dependence structure among various random variables we need to apply multivariate EVT. Multivariate EVT does not only reflect the risk related to the marginal distribution of one asset, but also the risk that arises due to dependence structures between various assets or random variables, respectively.

In the following two subsections important concepts of multivariate EVT will be described.

2.1.1. Copulas

Copulas have been developed only relatively recently by Sklar in 1959 [23]. They describe the dependence between several random variables. If we assume a bivariate distribution, it is important to see that it contains two pieces of information: namely the two marginal distributions as well as information about the dependence of them. With the help of copulas is possible to clearly differentiate this two pieces of information and extract only the information about the dependence structure of the two marginal distributions. For the definitions below I will focus on the bivariate case only.

A bivariate distribution function F of two random variables X and Y with marginal distribution $F_x(\cdot)$ and $F_y(\cdot)$ can be expressed as:

$$\begin{aligned} F(x, y) &= \Pr(X \leq x, Y \leq y), \\ &= C(F_X(x), F_Y(y)), \end{aligned} \tag{2.1}$$

where $C(\cdot, \cdot)$ with domain $\mathcal{A} = [0, 1] \times [0, 1]$ is the copula of the two random variables X and Y. The copula C thus contains the complete information about the dependence structure except the information contained in its marginals. In other words, the copula C reflects the joint distribution of X and Y after transformation to variables U and V with Uniform[0,1] margins.

If we assume independence we have $F(x, y) = F_X(x)F_Y(y)$, so $C(u, v) = uv$ on $\mathcal{A}$

If we assume perfect dependence we have $Y = F_Y^{-1}\{F_X(X)\}$ with probability 1, so $F(x, y) = min\{F_X(x), F_Y(y)\}$ and $C(u, v) = min(u, v)$ on $\mathcal{A}$

Denoting the joint survivor function $\Pr(X > x, Y > y)$ by $\overline{F}(x, y)$ we may write:

$$\begin{aligned} \overline{F}(x, y) &= 1 - F_X(x) - F_Y(y) + F(x, y), \\ &= \overline{C}\{F_X(x), F_Y(y)\}, \end{aligned} \tag{2.2}$$

where $\overline{C}(\cdot,\cdot)$ with domain $\mathcal{A} = [0,1] \times [0,1]$ is the survival copula of the two random variables X and Y.

2.1.2. Tail Dependence

Copulas described in the previous section reflect the dependence structure of the whole bivariate distribution. However, we are primarily interested in the dependence of the tails of the distribution of two random variables. In addition, classical measures of dependence like Person's product-moment correlation are linear and focused on the variance around the mean. Therefore, a new measure of tail dependence between two random variables is needed. In 1996, Ledford and Tawn (Ledford1996) introduced a coefficient of tail dependence (TDC) between two assets X_i and X_j that maps extreme events that occur simultaneously. The TDC is defined as the probability that the asset X_i incurs a large loss (or gain) assuming that the asset X_j also undergoes a large loss (or gain) at the same probability level.

The coefficient of lower tail dependence λ_{ij}^- and the coefficient of upper tail dependence λ_{ij}^+ are defined by:

$$\lambda_{ij}^- = \lim_{u \to 0^+} \Pr\{X_i < F_i^{-1}(u) | X_j < F_j^{-1}(u)\}, \tag{2.3}$$

$$\lambda_{ij}^+ = \lim_{u \to 1^-} \Pr\{X_i > F_i^{-1}(u) | X_j > F_j^{-1}(u)\}, \tag{2.4}$$

where $F_i^{-1}(u)$ and $F_j^{-1}(u)$ represent the quantiles of assets X_i and X_j at the level u. So, if we assume X_i and X_j to represent the volatilities of two different stocks, λ^- gives the probability that both stocks exhibit a large loss simultaneously.

As shown in equations (2.5) for the lower and (2.6) for the upper tail dependence, the TDC can be expressed in terms of the copulas of X_i and X_j and is therefore independent of the marginals and symmetric in X_i and X_j:

$$\lambda_{ij}^- = \lim_{u \to 0^+} \frac{C(u,u)}{u}, \tag{2.5}$$

$$\begin{aligned} \lambda_{ij}^+ &= \lim_{u \to 1^-} \frac{\overline{C}(u,u)}{1-u}, \\ &= \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u}. \end{aligned} \tag{2.6}$$

2.2. Nonlinear Measures of Tail Dependence

This section provides information about the theoretical as well as the practical background for the methods that have been used in this thesis to estimate the tail dependence for the given data sets applying nonlinear measures of correlation based on EVT. There are three different approaches presented to calculate a TDC. The one developed by Poon, Rockinger and Tawn is presented in subsection 2.2.2 and based on the concept of tail copulas. Those developed by Malevergne and Sornette can be found in subsection 2.2.3 and are based on a factor model with a parametric and a non-parametric implementation. The approach developed by Schmidt and Stadtmüller is based on rank order statistics and can be found in subsection 2.2.4.

As the first two author teams both explicitly assume a power-law behavior of the investigated data, they need to estimate a tail-index for their calculations. Therefore, the following first subsection presents the methods that have been applied to estimate the tail-index throughout this work.

2.2.1. Tail Index Estimation

The tail index of a power-law distribution of the form $p(x) \propto \mathcal{L}(x)x^{-\alpha}$ where $\alpha > 1$ and $\mathcal{L}(x)$ is a slowly varying function can be estimated with various methods. In this work four methods have been applied such as the classical Hill estimator [9] or the Huisman estimator that includes corrections for small sample bias.

Hill Estimator

The tail index α may be estimated by the Hill estimator $\hat{\nu}$ as presented in equation (2.7), a maximum-likelihood estimator derived by Hill [9] in 1975. This estimator is asymptotically unbiased and easy to implement, but is biased and shows some serious instabilities for small samples. For a random variable Y the estimator is given by

$$\hat{\nu} = \left[\frac{1}{k} \sum\nolimits_{j=1}^{k} \log \frac{y_{j,N}}{y_{k,N}} \right]^{-1}, \tag{2.7}$$

where k is the threshold number that defines the tail of the distribution, and $y_{1,N} \geq y_{2,N} \geq \ldots \geq y_{N,N}$ is the ordered statistics of random variable Y.

Huisman Estimator

To correct the bias of the Hill estimator for small samples, Huisman et al. [11] developed a weighted Hill estimator. The difficult part while using the Hill estimator is to select a threshold number k such that the variance as well as the bias are minimized. This problem cannot be easily solved and the selection of k is normally based on expert judgment. However, the Huisman estimator presented here, does not base its estimate of the tail index on one threshold number k only, but exploits the information contained in a set of Hill estimators each based on a different number of tail observations.

The approach proposed by Huisman et al. [11] uses an important characteristic of the bias function. For values of k smaller than a threshold κ, the estimates for $\gamma = 1/\alpha$ increase almost linearly. This suggests that for small κ the bias term can be approximated with linear regression or ordinary least square (OLS) method, respectively. So, instead of calculating one Hill estimator for a given k, Huisman et al. propose to calculate $\gamma(k)$ for all k's below κ as shown in equation (2.8)

$$\gamma(k) = \beta_0 + \beta_1 k + \epsilon(k) \quad \forall\, k = 1, \ldots, \kappa. \tag{2.8}$$

Equation (2.8) can then be rewritten as:

$$\boldsymbol{\gamma} = \boldsymbol{Z}\boldsymbol{\beta}^{\mathrm{T}} + \boldsymbol{\epsilon}, \tag{2.9}$$

with $\boldsymbol{Z}$ as a $(\kappa \times 2)$ matrix with ones in the first column and the vector $\{1, 2, \ldots, \kappa\}'$ in the second.

To correct for the heteroscedasticity of the error term $\epsilon(k)$, a weighted least square (WLS) approach has been used. As the Hill estimator is inversely related to k, a weighting matrix $\boldsymbol{W}$ has been implemented that has the dimensions $(\kappa\times\kappa)$, $\left\{\sqrt{1}, \sqrt{2}, \ldots, \sqrt{\kappa}\right\}$ as diagonal elements and zeros elsewhere. Rearranging of equation (2.9) and solving for the Huisman estimator yields:

$$\hat{\boldsymbol{h}} = (\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{Z})^{-1}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\gamma}, \tag{2.10}$$

where $\boldsymbol{\gamma}$ equals one over the Hill estimator as defined in equation (2.7), i.e. $\gamma(k) = 1/\hat{v}(k)$. The tail index we are looking for equals the first element of the vector $\hat{\boldsymbol{h}}$.

It can be shown that the Huisman estimator is a weighted version of the traditional Hill estimators for $k = 1, 2, \ldots, \kappa$:

$$\hat{h}(\kappa) = \sum_{k=1}^{\kappa} w(k)\gamma(k), \tag{2.11}$$

with weights $w(k)$ depending on k.

Clauset Estimator

The Clauset estimator has been developed by Clauset et al. [1] and is based on the Hill estimator as shown in equation (2.7). The difference, however, is the use of the Kolmogorov-Smirnov or KS statistic to define the best threshold k that defines the upper (or lower) tail of the distribution. This procedure assumes that only the tail of the distribution follows a power-law behavior and using the KS statistics calculates the point in the empirical distribution where the change from the assumed power-law to another distribution will most likely happen.

Mathematically spoken, the KS statistic simply calculates the maximum distance between the cumulative distribution function (CDF) of the investigated data and the fitted model:

$$D = \max_{y \geq y_{(k,N)}} \left| Q(y) - P(y) \right|, \tag{2.12}$$

where $Q(y)$ stands for the CDF of the data with values of at least $y_{k,N}$ and $P(y)$ for the power-law model that shows the best fit for the data in the region $y \geq y_{k,N}$. The optimal threshold k is where D is minimized.

Gabaix Estimator

The Gabaix estimator [7] is an improved ordinary least square (OLS) estimator for the tail index α. It has been a popular method using OLS log-log rank-size regression to estimate the tail index of power-laws according to the following equation:

$$\log(t - \gamma) = a - b \log y_{t,N}, \tag{2.13}$$

or in other words

$$\log(\mathrm{Rank} - \gamma) = a - b \log(\mathrm{Size}), \tag{2.14}$$

where $y_{t,N}$ is an ordered statistics with $y_{1,N} \geq \ldots \geq y_{N,N}$, $\gamma = 0$, and $\hat{b}$ is the Gabaix estimator.

However, the estimates derived with the OLS method show an important bias especially for small samples. Gabaix and Ibragimov [7] could show that this bias is reduced the most when $\gamma = 1/2$.

2.2.2. Approach by Poon, Rockinger and Tawn

Basically the dependence structure among variables can be distinguished into four different groups: independence, perfect dependence, asymptotic independence, and asymptotic dependence. The approach by Poon et al. offers the possibility to cover all four cases. The theory below will give the basis to calculate measures of dependence that can distinguish between these four groups.

The basis for the concept of Poon et al. presented here has been laid in antecedent papers published by Heffernan [8], by Coles, Heffernan and Tawn [2], and by Ledford and Tawn [12]. Whenever needed, respective citations are given.

Measures of Dependence

Coles et al. [2] developed two measures of dependence to project the information contained in the copula C of two random variables into one parameter. χ and $\overline{\chi}$ were introduced as dependence measures and used to distinguish whether and to what extent two variables are asymptotically dependent or asymptotically independent, respectively.

To calculate χ, first the bivariate random variables X and Y need to be transformed to remove the marginal aspects. This is performed by transformation to unit Fréchet marginals S and T because the distributions of the returns are assumed to show fat-tail behavior:

$$S = -1/\log\left(F_X(X)\right) \quad T = -1/\log\left(F_Y(Y)\right), \tag{2.15}$$

where F_X and F_Y are the marginal distributions for X and Y. The transformed variables S and T posses the same dependence structure as X and Y, but are now on a common scale. This means that events of the form $S > s$ and $T > s$ for large s correspond to events that are equally extreme for both variables.

The measure for asymptotic dependence χ is then given by:

$$\begin{aligned} \chi &= \lim_{s \to \infty} \Pr(T > s \,|\, S > s), \\ &= \lim_{s \to \infty} \frac{\Pr(T > s, S > s)}{\Pr(S > s)}, \end{aligned} \tag{2.16}$$

or expressed through copula C:

$$= \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u}, \tag{2.17}$$

$$= \lim_{u \to 1^-} 2 - \frac{1 - C(u,u)}{1-u},$$

$$\sim \lim_{u \to 1^-} 2 - \frac{\log C(u,u)}{\log u}, \tag{2.18}$$

with $0 \leq \chi \leq 1$. To put the measure χ into perspective we can see that equation (2.17) is the same as equation (2.4) given for the upper tail dependence coefficient beforehand. So χ can be used to investigate whether S and T are asymptotically dependent, perfectly dependent, or asymptotically independent. If $\chi > 0$ then S and T are asymptotically dependent, if $\chi = 1$ they are perfectly dependent, and if $\chi = 0$, S and T are asymptotically independent.

Ledford and Tawn [12] and Coles et al. [2] have developed a second measure of dependence $\overline{\chi}$ that contains information about the degree of asymptotic independence of two variables. This information is not given by χ in the case of asymptotic independence where $\chi = 0$.

$$\overline{\chi} = \lim_{s \to \infty} \frac{2 \log\left(\Pr(S > s)\right)}{\log\left(\Pr(S > s, T > s)\right)} - 1, \tag{2.19}$$

$$= \lim_{u \to 1^-} \frac{2 \log(1-u)}{\log \overline{C}(u,u)} - 1, \tag{2.20}$$

where $-1 < \overline{\chi} \leq 1$, and $\overline{\chi}$ is a measure of the rate at which $\Pr(T > s \mid S > s)$ approaches zero. For perfect dependence $\overline{\chi} = 1$ and for independence $\overline{\chi} = 0$. Therefore values of $\overline{\chi} > 0$ indicate that S and T are positively associated, and $\overline{\chi} < 0$ indicate that S and T are negatively associated in the end of the tails. For the bivariate Gaussian dependence structure $\overline{\chi} = \rho$, where ρ denotes the Pearson product-moment correlation coefficient. Further examples can be found in Heffernan [8].

The process to investigate the dependence structure of two random variables with the method by Poon et al. [18] is two-tiered. First it is important to check whether $\overline{\chi} = 1$ holds. If $\overline{\chi} = 1$ cannot be rejected, the variables are asymptotically dependent with χ as respective measure of dependence. If $\overline{\chi} < 1$ then the variables are asymptotically independent with $\chi = 0$ and degree of dependence given by $\overline{\chi}$.

Non Parametric Estimation of χ and $\overline{\chi}$

The tail of a univariate heavy-tailed random variable Z above a high threshold number k is defined by the following equation:

$$\Pr(Z > z) = \mathcal{L}(z) \cdot z^{-\alpha}, \quad \forall\, z > z_{k,N}, \tag{2.21}$$

where $\mathcal{L}(z)$ is a slowly varying function of z, that is

$$\lim_{k \to \infty} \frac{\mathcal{L}(k \cdot z)}{\mathcal{L}(k)} = 1, \quad \forall\, z > 0. \tag{2.22}$$

If we assume $\mathcal{L}(z)$ to be constant above a threshold k, we can estimate $\mathcal{L}(z)$ by equation (2.23)

$$\hat{\mathcal{L}}(z) = \frac{k}{N} \cdot (z_{k,N})^{\hat{\alpha}}, \tag{2.23}$$

with tail index α estimated by any of the four methods presented in section 2.2.1 and $z_{1,N} \geq z_{2,N} \geq \cdots \geq z_{N,N}$ as the ordered statistics of the sample containing N i.i.d. realizations of the return vector $\boldsymbol{z}$. The threshold number k denotes the number of data points that are treated as tail of the distribution.

As we are investigating the dependence structure between two variables, i.e. return vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, we also define two slowly varying functions. To smooth the strong fluctuations in the tails of the distribution we take the mean values of the slowly varying function as defined by equation (2.23):

$$\bar{\hat{\mathcal{L}}}(x) = \frac{1}{k} \sum_{j=1}^{k} \frac{j}{N} \cdot (x_{j,N})^{\hat{\alpha}}, \tag{2.24}$$

$$\bar{\hat{\mathcal{L}}}(y) = \frac{1}{k} \sum_{j=1}^{k} \frac{j}{N} \cdot (y_{j,N})^{\hat{\alpha}}. \tag{2.25}$$

Ledford and Tawn [12, 13] characterized the joint tail behavior by constant η denoting the coefficient of tail dependence and the slowly varying function $\mathcal{L}(s)$. Under weak conditions the following relation holds:

$$\Pr(S > s, T > s) \sim \mathcal{L}(s) \cdot s^{-1/\eta}, \quad \text{as } s \to \infty, \tag{2.26}$$

with $0 < \eta \leq 1$. From this representations it follows that

$$\overline{\chi} = 2\eta - 1, \tag{2.27}$$

and

$$\chi = \begin{cases} c & \text{if } \overline{\chi} = 1 \text{ and } \mathcal{L}(s) \to c > 0, \quad \text{as } s \to \infty, \\ 0 & \text{if } \overline{\chi} = 1 \text{ and } \mathcal{L}(s) \to 0, \quad \text{as } s \to \infty, \\ 0 & \text{if } \overline{\chi} < 1, \end{cases} \tag{2.28}$$

where $\overline{\chi} = 1$ corresponds to $\eta = 1$ and yields $\chi = \lim_{s\to\infty} \mathcal{L}(s)$. Therefore η and $\lim_{s\to\infty} \mathcal{L}(s)$ are needed to estimate χ and $\overline{\chi}$.

Based on univariate extreme value techniques it follows that if $Z = \min(S, T)$ then:

$$\begin{aligned} \Pr(Z > z) &= \Pr\left(\min(S,T) > z\right), \\ &= \Pr(S > z, T > z), \\ &= \mathcal{L}(z) \cdot z^{-1/\eta}, \quad \forall\, z > z_{k,N}, \end{aligned} \tag{2.29}$$

for some high threshold number k. Whereas η can be estimated with the Hill or any other tail index estimator that is restricted to the interval $(0,1]$ and $\mathcal{L}(z)$ by applying equation (2.23).

For the actual implementation, however, return vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ are transformed according to equation (2.15) to S and T that can now be estimated by

$$\hat{S} = -\frac{1}{\log\left(\hat{F}_X(x)\right)}, \tag{2.30}$$

$$\hat{T} = -\frac{1}{\log\left(\hat{F}_Y(y)\right)}. \tag{2.31}$$

with $\hat{F}_X(x) = 1 - \hat{\overline{F}}_X(x)$ and $\hat{F}_Y(y) = 1 - \hat{\overline{F}}_Y(y)$.

The survival functions $\overline{F}_X(x)$ and $\overline{F}_Y(y)$ of the two heavy-tailed random variables X and Y can be estimated using the following two equations:

$$\hat{\overline{F}}_X(x) = \hat{\overline{\mathcal{L}}}(x) \cdot x^{-\hat{\alpha}}, \tag{2.32}$$

$$\hat{\overline{F}}_Y(y) = \hat{\overline{\mathcal{L}}}(y) \cdot y^{-\hat{\alpha}}, \tag{2.33}$$

with $\hat{\overline{\mathcal{L}}}(x)$ and $\hat{\overline{\mathcal{L}}}(y)$ being estimated by equation (2.24) and (2.25), respectively, and the tail index α with any of the methods presented in section 2.2.1.

Finally, assuming i.i.d. observations on Z and $Z = \min(S, T)$, we get the estimators for $\overline{\chi}$ and χ:

$$\hat{\overline{\chi}} = \frac{2}{k}\left(\sum_{j=1}^{k} \log\left(\frac{z_{j,N}}{z_{k,N}}\right)\right) - 1, \tag{2.34}$$

$$\text{var}\left(\hat{\overline{\chi}}\right) = \frac{\left(\hat{\overline{\chi}} + 1\right)^2}{k}, \tag{2.35}$$

and

$$\hat{\chi} = \frac{z_{k,N} \cdot k}{N}, \tag{2.36}$$

$$\text{var}\left(\hat{\chi}\right) = \frac{{z_{k,N}}^2 k(N-k)}{N^3}. \tag{2.37}$$

However, χ only needs to be calculated, if there is no significant evidence to reject $\overline{\chi} = 1$. If one likes to compare the TDCs from Poon et al. with other TDCs, it is recommended to use $\hat{\chi}$, because $\hat{\chi}$ is a measure for asymptotic tail dependence like the others that will be presented later in this work.

2.2.3. Approaches by Malevergne and Sornette

Malevergne and Sornette [14, 15, 16] developed two approaches to estimate TDCs based on a linear factor model between e.g. market and stock returns. We will first present the theory of the factor model that has been used and that is valid for both, the non-parametric as well as the parametric approach and then explain the non-parametric method in detail followed by the parametric approach.

The Factor Model

The additive factor model that has been applied by Malevergne and Sornette links fluctuations of asset returns with returns of the market in the following way:

$$\boldsymbol{x} = \beta \cdot \boldsymbol{y} + \boldsymbol{\epsilon}, \tag{2.38}$$

where $\boldsymbol{x}$ represents a vector of asset returns and $\boldsymbol{y}$ the vector of corresponding index returns. In other words, we only assume one factor in the model with $\boldsymbol{y}$ as the risk factor for the asset returns $\boldsymbol{x}$. β is the regression coefficient as generally used in capital asset pricing models (CAPM) [21] and determined by the least squares method. $\boldsymbol{\epsilon}$ denotes the vector of idiosyncratic noises assumed independent of $\boldsymbol{y}$. Although this model is a one factor model only, $\boldsymbol{\epsilon}$ may contain other factors as long as they remain independent of $\boldsymbol{y}$.

If we wish to exchange the dependent with the independent variable, i.e. the underlying asset with the risk or explaining factor, we have to adjust equation (2.38) of the factor model to

$$\boldsymbol{y} = \beta_{\text{ex}} \cdot \boldsymbol{x} + \boldsymbol{\epsilon}_{\text{ex}}, \tag{2.39}$$

with

$$\begin{aligned} \beta_{\text{ex}} &= \frac{\operatorname{cov}(\boldsymbol{x}, \boldsymbol{y})}{\operatorname{var}(\boldsymbol{x})}, \\ &= \frac{\beta \operatorname{var}(\boldsymbol{y})}{\left(\beta^2 \operatorname{var}(\boldsymbol{y}) + \operatorname{var}(\boldsymbol{\epsilon})\right)}, \end{aligned} \tag{2.40}$$

and

$$\boldsymbol{\epsilon}_{\text{ex}} = \frac{\boldsymbol{y} \operatorname{var}(\boldsymbol{y})}{\left(\beta^2 \operatorname{var}(\boldsymbol{y}) + \operatorname{var}(\boldsymbol{\epsilon})\right)} - \frac{\beta \operatorname{var}(\boldsymbol{y}) \boldsymbol{\epsilon}}{\left(\beta^2 \operatorname{var}(\boldsymbol{y}) + \operatorname{var}(\boldsymbol{\epsilon})\right)}. \tag{2.41}$$

By applying the modified parameters β_{ex} and $\boldsymbol{\epsilon}_{\text{ex}}$ to the methods described below, we should get the same TDC as with the parameters calculated for equation (2.38).

Schmuki [20] could show in his thesis that the β calculated for the factor model shows a strong bias that is amplified by the calculation of the TDC. Therefore, he tried to improve the performance of the approaches of Malevergne and Sornette with different ways of calculating the β of the underlying factor model. However, Schmuki found that these improvements are only valid for a very small part of the data range and they are therefore not considered for this thesis.

Non Parametric Approach

Malevergne and Sornette [15] provide a general non-parametric result for the upper tail TDC of a one factor model:

$$\lambda^+ = \int_{\max\{1, \frac{l}{\beta}\}}^{\infty} f(x) \mathrm{d}x, \tag{2.42}$$

with

$$l = \lim_{u \to 1^-} \frac{F_X^{-1}(u)}{F_Y^{-1}(u)}, \tag{2.43}$$

and

$$f(x) = \lim_{t \to \infty} \frac{t \cdot P_Y(t \cdot x)}{\overline{F}_Y(t)}, \tag{2.44}$$

where F_X and F_Y are the marginal distribution functions of $\boldsymbol{x}$ and $\boldsymbol{y}$. P_Y is the pdf of $\boldsymbol{y}$ and $\overline{F}_Y = 1 - F_Y$ is the complementary cumulative distribution function of $\boldsymbol{y}$. An expression similar to equation (2.42) holds for the coefficient of lower tail dependence. To ensure that the TDC is non-vanishing two conditions must be fulfilled: firstly the limit function $f(x)$ must be non-zero and secondly the constant l must remain finite.

Malevergne and Sornette have shown [15] that for rapidly varying factors such as Gaussian, exponential or gamma laws, i.e. all factors whose distributions decrease faster than the power law, the TDC is equal to zero. However, if the factors show a regularly varying behavior as defined in equation (2.45) for $\overline{F}_Y(y)$

$$\overline{F}_Y(y) = \mathcal{L}(y) \cdot y^{-\alpha}, \tag{2.45}$$

with $\mathcal{L}(y)$ as a slowly varying function with tail index α, then:

$$\lambda^+ = \frac{1}{\max\{1, \frac{l}{\beta}\}^{\alpha}}, \tag{2.46}$$

with l defined by equation (2.43). The TDC for the lower tail: λ^- can be obtained by replacing $u \to 1^-$ with $u \to 0^+$.

For implementation purposes l can be estimated by the following equation:

$$\hat{l} = \lim_{u \to 1^-, 0^+} \frac{F_X^{-1}(u)}{F_Y^{-1}(u)} \cong \frac{x_{k,N}}{y_{k,N}}, \quad \text{as} \quad k \to N, 0. \tag{2.47}$$

To smooth the strong fluctuations of l in the tails the mean value of $\hat{l}(j)$ with $j = 1, \ldots, k$ has been calculated and used for the estimation of the TDC according to equation (2.48)

$$\bar{\hat{l}}(k) = \frac{1}{k} \sum_{j=1}^{k} \frac{x_{j,N}}{y_{j,N}}. \tag{2.48}$$

The coefficient β has been estimated applying ordinary least squares regression to the factor model $\boldsymbol{x} = \beta \cdot \boldsymbol{y} + \boldsymbol{\epsilon}$ according to equation (2.49):

$$\hat{\beta} = (\boldsymbol{x}^{\mathrm{T}} \boldsymbol{x})^{-1} \boldsymbol{x}^{\mathrm{T}} \boldsymbol{y}. \tag{2.49}$$

We finally get an non-parametric estimator for the TDC given by equation (2.50)

$$\hat{\lambda}^{+,-} = \frac{1}{\max\left\{1, \frac{\bar{\hat{l}}}{\hat{\beta}}\right\}^{\hat{\alpha}}}, \tag{2.50}$$

with the above defined estimators for l and β, and any of the estimators defined in section 2.2.1 as tail index α.

For the factor model where $\boldsymbol{x}$ and $\boldsymbol{y}$ are exchanged the TDC is estimated by

$$\hat{\lambda}_{\text{ex}}^{+,-} = \frac{1}{\max\left\{1, \frac{\bar{\hat{l}}}{\hat{\beta}_{\text{ex}}}\right\}^{\hat{\alpha}}}, \tag{2.51}$$

with $\bar{\hat{l}}$ being estimated according to equation (2.48) but with $x_{j,N}$ in the denominator and $y_{j,N}$ in the numerator and $\hat{\beta}_{\text{ex}}$ as defined in equation (2.40).

Parametric Approach

The second approach developed by Malevergne and Sornette is parametric and based on the assumption that the distribution of the idiosyncratic noise ϵ is also regularly varying with tail index α. If we assume $\overline{F}_Y(y) \sim C_{\text{Y}} \cdot y^{-\alpha}$ and $\overline{F}_\epsilon(\epsilon) \sim C_\epsilon \cdot \epsilon^{-\alpha}$ for large y and ϵ, then the TDC is given by the ratio of the scale factors C_ϵ and C_{Y}:

$$\lambda^{+,-} = \frac{1}{1 + \beta^{-\alpha} \cdot \frac{C_\epsilon}{C_{\text{Y}}}}. \tag{2.52}$$

However, if the tail indices α_{Y} and α_ϵ of the distribution of the factors and the residue are different, then $\lambda = 1$ for $\alpha_{\text{Y}} < \alpha_\epsilon$ and $\lambda = 0$ for $\alpha_{\text{Y}} > \alpha_\epsilon$.

For implementation purposes we consider N sorted realizations of return vector $\boldsymbol{y}$ and residual vector $\boldsymbol{\epsilon}$ denoted by $y_{1,N} \geq y_{2,N} \geq \cdots \geq y_{N,N}$ and $\epsilon_{1,N} \geq \epsilon_{2,N} \geq \cdots \geq \epsilon_{N,N}$. We can now calculate $\boldsymbol{\epsilon}$ by using the relation $\boldsymbol{\epsilon} = \boldsymbol{x} - \beta \cdot \boldsymbol{y}$. The two scale factors C_y and C_ϵ can then be estimated as follows:

$$\hat{C}_{\text{Y}} = \frac{k}{N} \cdot (y_{k,N})^{\hat{\alpha}}, \quad \text{as} \quad k \to N, 0, \tag{2.53}$$

$$\hat{C}_\epsilon = \frac{k}{N} \cdot (\epsilon_{k,N})^{\hat{\alpha}}, \quad \text{as} \quad k \to N, 0, \tag{2.54}$$

with α estimated by any of the methods presented in section 2.2.1.

To smooth the behavior of the two estimators $\hat{C}_{\text{Y}}$ and $\hat{C}_\epsilon$ we use the mean value over the whole range of the tail as defined by the threshold number k:

$$\bar{\hat{C}}_{\text{Y}} = \frac{1}{k} \sum_{j=1}^{k} \frac{j}{N} \cdot (y_{j,N})^{\hat{\alpha}}, \tag{2.55}$$

$$\bar{\hat{C}}_\epsilon = \frac{1}{k} \sum_{j=1}^{k} \frac{j}{N} \cdot (\epsilon_{j,N})^{\hat{\alpha}}. \tag{2.56}$$

Using the estimator for β as defined in equation (2.49) and putting the other parameters in equation (2.52) we get the estimator for λ:

$$\hat{\lambda}^{+,-} = \frac{1}{1 + \hat{\beta}^{-\hat{\alpha}} \cdot \frac{\bar{\hat{C}}_\epsilon}{\bar{\hat{C}}_{\text{Y}}}} = \frac{1}{1 + \left(\frac{\epsilon_{k,N}}{\hat{\beta} \cdot y_{k,N}}\right)^{\hat{\alpha}}}, \quad \text{as} \quad k \to N, 0. \tag{2.57}$$

It is important to remark, that if $\beta < 0$ for any asset relation, relation (2.57) cannot be applied, since it assumes $\beta > 0$ and therefore $\hat{\lambda} = 0$.

For the factor model with exchanged $\boldsymbol{x}$ and $\boldsymbol{y}$ we need the estimator for β_{ex} as defined in equation (2.40) and for $\boldsymbol{\epsilon}_{\text{ex}}$ as defined in equation (2.41). $\hat{\bar{C}}_\epsilon$ is calculated by applying the values of $\boldsymbol{\epsilon}_{\text{ex}}$ and $\hat{\bar{C}}_{\text{X}}$ by applying the values of $\boldsymbol{x}$. Hence, we get the following estimation for the TDC:

$$\begin{aligned} \hat{\lambda}_{\text{ex}}^{+,-} &= \frac{1}{1 + \hat{\beta}_{\text{ex}}^{-\hat{\alpha}} \cdot \frac{\hat{\bar{C}}_{\epsilon_{\text{ex}}}}{\hat{\bar{C}}_{\text{X}}}} \\ &= \frac{1}{1 + \left(\frac{\epsilon_{\text{ex}(k,N)}}{\hat{\beta}_{\text{ex}} \cdot x_{k,N}}\right)^{\hat{\alpha}}}, \quad \text{as} \quad k \to N, 0. \end{aligned} \tag{2.58}$$

Tail Dependence between Two Assets of a Factor Model

If we want to know the TDC between two random variables (assets) X_1 and X_2 that are coupled via the same factor Y, we can write the model as:

$$\boldsymbol{x}_1 = \beta_1 \cdot \boldsymbol{y} + \boldsymbol{\epsilon}_1, \tag{2.59}$$

$$\boldsymbol{x}_2 = \beta_2 \cdot \boldsymbol{y} + \boldsymbol{\epsilon}_2, \tag{2.60}$$

with $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ being the vector of the idiosyncratic noises of the two assets. The coefficient for (upper) tail dependence $\lambda^+ = \lim_{u \to 1} \Pr\left\{X_1 > F_1^{-1}(u) | X_2 > F_2^{-1}(u)\right\}$ is defined by the equation

$$\lambda^+ = \int_{\max\left\{\frac{l_1}{\beta_1}, \frac{l_2}{\beta_2}\right\}}^{\infty} f(x) \mathrm{d}x, \tag{2.61}$$

which is close to equation (2.42) for an asset and its explaining factor. Equation (2.61) can now be rephrased for the two random variables X_1 and X_2 and we get:

$$\lambda(X_1, X_2) = \min\left\{\lambda(X_1, Y), \lambda(X_2, Y)\right\}. \tag{2.62}$$

This means that the tail dependence between two assets can not be stronger than the weakest tail dependence between an asset and its explaining (risk) factor.

2.2.4. Approach by Schmidt and Stadtmüller

This section describes the methods developed by Schmidt and Stadtmüller [19] to estimate a TDC between two random variables. Their concept is based on tail copulas from multivariate extreme value theory and rank order statistics to derive measures of tail dependence. They propose a set of non-parametric estimators for the upper and lower tail copula $\Lambda_{\text{U}}(x, y)$ and $\Lambda_{\text{L}}(x, y), (x, y)' \in \mathbb{R}_+^2$. To estimate the TDCs they apply a non-parametric method as no general finite-dimensional parametrization of tail copulas exists.

Let C_{m} denote the empirical copula defined by:

$$C_{\text{m}}(u, v) = F_{\text{m}}(G_{\text{m}}^{-1}(u), H_{\text{m}}^{-1}(v)), \quad (u, v)' \in [0, 1]^2, \tag{2.63}$$

with $F_{\mathrm{m}}, G_{\mathrm{m}}, H_{\mathrm{m}}$ representing the empirical distribution functions of $F, G,$ and H. Analogously we define the 'empirical survival copula' by:

$$\overline{C}_{\mathrm{m}}(u,v) = \overline{F}_{\mathrm{m}}(\overline{G}_{\mathrm{m}}^{-1}(u)\overline{H}_{\mathrm{m}}^{-1}(v)), \quad (u,v)' \in [0,1]^2, \tag{2.64}$$

with

$$\overline{F}_{\mathrm{m}}(x,y) = \frac{1}{N}\sum_{j=1}^{N} \boldsymbol{I}_{\{x_{j,N}>x,\, y_{j,N}>y\}}, \tag{2.65}$$

and $\overline{G}_{\mathrm{m}} = 1 - G_{\mathrm{m}}, \overline{H}_{\mathrm{m}} = 1 - H_{\mathrm{m}}$. Let $r_{\mathrm{X},j}$ and $r_{\mathrm{Y},j}$ denote the rank of $x_{j,N}$ and $y_{j,N}, j = 1,\ldots,N$ respectively. The estimators are then given by:

$$\hat{\Lambda}_{\mathrm{L}}(x,y) := \frac{N}{k} C_{\mathrm{m}}\left(\frac{kx}{N}, \frac{ky}{N}\right) \approx \frac{1}{k}\sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j}>kx \text{ and } r_{\mathrm{Y},j}>ky\}}, \tag{2.66}$$

$$\hat{\Lambda}_{\mathrm{U}}(x,y) := \frac{N}{k} \overline{C}_{\mathrm{m}}\left(\frac{kx}{N}, \frac{ky}{N}\right) \approx \frac{1}{k}\sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j}>N-kx \text{ and } r_{\mathrm{Y},j}>N-ky\}}, \tag{2.67}$$

with threshold number $k \in \{1,\ldots,N\}$. The estimators $\hat{\Lambda}_{\mathrm{U}}(x,y)$ and $\hat{\Lambda}_{\mathrm{L}}(x,y)$ are referred to as 'empirical tail copulas'. For the asymptotic result we further assume that $k = k(N) \to \infty$ and $k/N \to 0$ as $N \to \infty$.

A related estimator was introduced by Huang in [10] (see also [17], and [3]). The relation between the upper tail copula and the stable tail dependence l is given by

$$\Lambda_{\mathrm{U}}(x,y) = x + y - l(x,y), \tag{2.68}$$

with

$$l(x,y) = \lim_{t\to\infty} t\left(1 - C\left(1 - \frac{x}{t}, 1 - \frac{y}{t}\right)\right). \tag{2.69}$$

The corresponding estimator for $\Lambda_{\mathrm{L}}(x,y)$ on $\mathbb{R}_+^2$ and function l estimated with respect to EVT is then:

$$\begin{aligned}\hat{\Lambda}_{\mathrm{L}}^{\mathrm{EVT}}(x,y) &= x + y - \frac{N}{k}\left(C_{\mathrm{m}}\left(\frac{kx}{N}, \frac{ky}{N}\right)\right)\\ &\approx x + y - \frac{1}{k}\sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j}>kx \text{ or } r_{\mathrm{Y},j}>ky\}}, \quad (x,y) \in \mathbb{R}_+^2,\end{aligned} \tag{2.70}$$

and the corresponding estimator for $\Lambda_{\mathrm{U}}(x,y)$ on $\mathbb{R}_+^2$ is:

$$\begin{aligned}\hat{\Lambda}_{\mathrm{U}}^{\mathrm{EVT}}(x,y) &= x + y - \frac{N}{k}\left(\overline{C}_{\mathrm{m}}\left(\frac{kx}{N}, \frac{ky}{N}\right)\right)\\ &\approx x + y - \frac{1}{k}\sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j}>N-kx \text{ or } r_{\mathrm{Y},j}>N-ky\}}, \quad (x,y) \in \mathbb{R}_+^2,\end{aligned} \tag{2.71}$$

with $k = k(N) \to \infty$ and $k/N \to 0$ as $N \to \infty$. Based on the above estimates of the lower and upper tail copula we can define lower and upper TDCs according to

$$\hat{\lambda}_{\mathrm{L}} := \hat{\Lambda}_{\mathrm{L}}(1,1), \quad \text{and} \quad \hat{\lambda}_{\mathrm{L}}^{\mathrm{EVT}} := \hat{\Lambda}_{\mathrm{L}}^{\mathrm{EVT}}(1,1), \tag{2.72}$$

$$\hat{\lambda}_{\mathrm{U}} := \hat{\Lambda}_{\mathrm{U}}(1,1), \quad \text{and} \quad \hat{\lambda}_{\mathrm{U}}^{\mathrm{EVT}} := \hat{\Lambda}_{\mathrm{U}}^{\mathrm{EVT}}(1,1). \tag{2.73}$$

Hence, we get:

$$\hat{\lambda}_{\mathrm{L}} = \frac{1}{k} \sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j} > k \text{ and } r_{\mathrm{Y},j} > k\}}, \tag{2.74}$$

$$\hat{\lambda}_{\mathrm{U}} = \frac{1}{k} \sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j} > N-k \text{ and } r_{\mathrm{Y},j} > N-k\}}, \tag{2.75}$$

and

$$\hat{\lambda}_{\mathrm{L}}^{\mathrm{EVT}} = 2 - \frac{1}{k} \sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j} > k \text{ or } r_{\mathrm{Y},j} > k\}}, \tag{2.76}$$

$$\hat{\lambda}_{\mathrm{U}}^{\mathrm{EVT}} = 2 - \frac{1}{k} \sum_{j=1}^{N} \boldsymbol{I}_{\{r_{\mathrm{X},j} > N-k \text{ or } r_{\mathrm{Y},j} > N-k\}}, \tag{2.77}$$

with N denoting the size of the return sample i.e. number of monthly observations, k denoting the threshold number that defines the tail of the return distribution, and $r_{\mathrm{X},j}$ and $r_{\mathrm{Y},j}$ with $j = 1, \ldots, N$ the rank order statistics of index returns Y and asset returns X in ascending order. If two or more return values are equal (i.e. tied), then $r_{\mathrm{X},j}$ and $r_{\mathrm{Y},j}$ have been calculated by the following equation:

$$r_{\cdot,\mathrm{eq}} = \frac{\sum_{i=1}^{n} r_{\cdot,\mathrm{pre}} + i}{n}, \tag{2.78}$$

assuming n equal return values with $r_{\cdot,\mathrm{eq}}$ denoting the ranks of equal returns and $r_{\cdot,\mathrm{pre}}$ denoting the rank of the return(s) previous to the set of considered equal returns. In other words: the rank for a tied observation is equal to the average rank of all observations tied with it.

As the results based on the method of Schmidt and Stadtmüller are rather bumpy, a smoothing algorithm has been implemented to ease their interpretation, which is especially helpful for finding an optimal threshold number k. The best smoothing effect that still keeps the underlying results visible, could be achieved by applying the Savitzky-Golay filter, a generalized moving average with filter coefficients determined by an unweighted linear least-squares regression and a polynomial model of degree 2.

2.3. Linear Measures of Tail Dependence

An important question of this work is whether we need to use sophisticated models to calculate TDCs based on EVT or if the traditional way of investigating dependence structures between random variables by calculating linear measures of correlation is sufficient. To give the methodological background to answer this question later in this work, three measures of correlation are presented here shortly: Firstly Pearson's product-moment correlation, secondly the Spearman's rank correlation, and thirdly Kendall's rank correlation.

2.3.1. Pearson's Product-Moment Correlation

The correlation coefficient r between two random variables X and Y is defined by the following relation

$$r_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}, \tag{2.79}$$
$$= \frac{\operatorname{E}((X-\mu_X)(Y-\mu_Y))}{\sigma_X \sigma_Y}, \tag{2.80}$$
$$= \frac{\operatorname{E}(XY) - \operatorname{E}(X)\operatorname{E}(Y)}{\sqrt{\operatorname{E}(X^2) - \operatorname{E}^2(X)}\sqrt{\operatorname{E}(Y^2) - \operatorname{E}^2(Y)}}, \tag{2.81}$$

with σ_X and σ_Y being the standard deviations and μ_X and μ_Y the expected values of the two random variables X and Y.

As we are interested in the tails of the distribution we only calculate r for the k largest or smallest values of the distributions, i.e. we sort either X or Y and calculate the correlation between the smallest or largest values of X or Y and the corresponding values of the other random variable.

2.3.2. Spearman's Rank Correlation

Another measure of correlation that has been applied in this work is Spearman's rank correlation ρ. Spearman's non-parametric rank correlation is very close to Pearson's r but instead of the size, the ranks of the random variables are correlated with each other. The ranks are calculated using the same procedure as described in section 2.2.4. Hence, Spearman's ρ does not make any assumption about the frequency distribution of the investigated random variables and is therefore more robust to outliers. If the ranks are not tied, i.e. none of the values of the random variables are similar and equation (2.78) is not needed to calculate the ranks, ρ is defined by the following equation:

$$\rho_{X,Y} = 1 - \frac{6\sum d_i^2}{N(N^2-1)}, \tag{2.82}$$

where $d_i = r_{X,i} - r_{Y,i}$ represents the difference between the ranks of the values x_i and y_i with $i \in [1,\ldots,N]$ and N stands for the number of values in each data set (same for both sets).

However, if tied ranks exist, the following equation (a special case of equation (2.81)) has to be applied

$$\rho_{X,Y} = \frac{N(\sum r_{X,i} r_{Y,i}) - (\sum r_{X,i})(\sum r_{Y,i})}{\sqrt{N(\sum r_{X,i}^2) - (\sum r_{X,i})^2}\sqrt{N(\sum r_{Y,i}^2) - (\sum r_{Y,i})^2}}. \tag{2.83}$$

As we are interested in the tails of the distribution, the k smallest and largest values are investigated only (see section 2.3.1).

2.3.3. Kendall's Rank Correlation

Kendall's tau is a linear measure of correlation that measures the degree of correspondence between two rankings of random variables. The coefficient is defined as:

$$\tau_{X,Y} = \frac{n_c - n_d}{1/2N(N-1)}, \tag{2.84}$$

where n_c denotes the number of concordant pairs, n_d the number of discordant pairs, and N the size of the sample. Concordant pairs and discordant pairs are defined as follows: Let $x_{1,N} \geq x_{2,N} \geq \ldots \geq x_{N,N}$ be the ordered statics of random variable X and $y_1, y_2, \ldots, y_N$ the related values of the second random variable Y. A concordant pair is where:

$$\operatorname{sgn}(x_{j,N} - x_{i,N}) = \operatorname{sgn}(y_j - y_i), \quad \forall (j,i) \in [1,\ldots,N] \wedge j > i, \tag{2.85}$$

and discordant where

$$\operatorname{sgn}(x_{2,N} - x_{1,N}) = -\operatorname{sgn}(y_2 - y_1), \quad \forall (j,i) \in [1,\ldots,N] \wedge j > i. \tag{2.86}$$

As the tails of the distribution are of primary interest, only the k smallest and largest values, respectively, of the investigated random variables are used for calculation (see section 2.3.1).

2.4. Uncertainty Analysis / Bootstrapping

To check for uncertainty of the estimates a bootstrapping procedure has been implemented. Bootstrapping is a rather new and computer intensive method of statistical inference that belongs to the broader group of resampling methods [4]. The bootstrapping method assumes i.i.d. observations and resamples from the original sample by drawing randomly with replacement. This is repeated until the resample has the same number of observations as the original sample. Then the whole procedure is repeated to produce x-numbers of resamples. In our case we produced 1000 and if possible 5000 resamples to base our statistical inferences on. The number of resamples drawn is somehow an arbitrary decision and depends also on the available computer power, but we think that 5000 or 1000 resamples, respectively, should be enough for our purposes. To avoid statistical artifacts caused by very small samples, I resampled always from the whole distribution, i.e. not only the data points from the tail of the return distribution. For more details about this method please see the book written by Efron and Tibshirani [5].

2.5. Comparison of Rankings

To investigate whether the TDCs calculated with the various methods are different, I performed a Kruskal-Wallis. This test is non-parametric and does not need an assumption about the distribution of the data. This test compares the median of the various samples and gives a p-value that signifies, if all medians are from the same population (null hypothesis) or not (alternative hypothesis). The lower the p-value the larger the probability that at least one of the samples is not from the same population, i.e. that the null hypothesis can be rejected.

Additionally, I calculated the correlation coefficients between the results of the various methods with Spearman's rank correlation method. This was an important tool to help me check whether the results have the similar rankings or if they are different. I chose Spearman's method as it does not make any assumptions about the distribution of the investigated data.

2.6. Data

The topic of this work is the analysis of data that is related to hedge funds. As this work has been written in collaboration with ISPartners AG (ISP), a special focus is set on the investigation of data provided by ISPartners. ISPartners is a hedge fund with own fund of funds (FoF) and a fund of hedge funds (FoHF). This FoHF has been analyzed in comparison to the S&P 500 as well as the Tremont Hedge Fund Index (THFI), all three described in more detail below. Additionally, the relation between the S&P 500 and the THFI, the relation between the hedge funds of ISPartners (HF_{ISP}) that are part of the FoHF of ISPartners ($FoHF_{ISP}$), and finally the dependence structure between the THFI and its subindices ($THFI_{sub}$) have been investigated.

The data for the S&P 500 has been downloaded from the finance portal of Yahoo (`http://finance.yahoo.com/`), the data for the THFI from the respective homepage (`www.hedgeindex.com`) and the data set for the $FoHF_{ISP}$ has been provided by ISPartners AG. The analysis has been conducted for monthly data only and for the S&P500 opening courses have been used. The time span for investigation was from April 1994 up to September 2008 (174 observations) for comparisons between the THFI and the S&P 500 and from June 2004 up to September 2008 (53 observations) for comparisons that involved data from the FoHF.

Below will follow a overview over the investigated data, the various strategies that are applied for hedge funds and a short explanation on how we calculated the returns that we used for further analysis.

2.6.1. S&P 500

This is a stock market index containing 500 large-cap companies from the US traded on the New York Stock Exchange (NYSE) or the NASDAQ. The S&P 500 is maintained by the rating agency Standard and Poor's and published daily. The companies of the S&P 500 are selected to be representative for the various industries that are domiciled in the US.

2.6.2. Hedge Funds (HF)

A hedge fund is a private investment vehicle for a limited range of investors that has more freedom to invest than normal funds. If the investor wants to invest in a hedge fund he has to pay a management fee to the investment manager. Hedge funds have traditionally been seen as a measure to hedge away risks with various methods such short selling. However, in the mean time hedge funds have developed various strategies and some of them can be very risky. There are hedge funds that are focused e.g. on shares, commodities, works of art, etc. The following strategies (as defined by Credit Suisse and Tremont for the THFI) have been applied by hedge funds investigated here:

Convertible Arbitrage (COA)

Convertible Arbitrage funds aim to profit from the purchase of convertible securities and the subsequent shorting of the corresponding stock when there is a pricing error made in the conversion factor of the security. Managers typically build long positions of convertible and other equity hybrid securities and then hedge the equity component of the long securities positions by shorting the underlying stock or options. The number of shares sold short usually reflects a delta neutral or market neutral ratio. As a result, under normal market conditions, the arbitrageur generally expects the combined position to be insensitive to fluctuations in the price of the underlying stock [24].

Dedicated Short Bias (DSB)

Dedicated Short Bias funds take more short positions than long positions and earn returns by maintaining net short exposure in long and short equities. Detailed individual company research typically forms the core alpha generation driver of dedicated short bias managers, and a focus on companies with weak cash flow generation is common. To affect the short sale, the manager borrows the stock from a counter-party and sells it in the market. Short positions are sometimes implemented by selling forward. Risk management consists of offsetting long positions and stop-loss strategies [24].

Emerging Markets (EMM)

Emerging Markets funds invest in currencies, debt instruments, equities and other instruments of countries with 'emerging' or developing markets (typically measured by GDP per capita). Such countries are considered to be in a transitional phase between developing and developed status. Examples of emerging markets include China, India, Latin America, much of Southeast Asia, parts of Eastern Europe, and parts of Africa. There are a number of sub-sectors, including arbitrage, credit and event driven, fixed income bias, and equity bias [24].

Equity Market Neutral (EMN)

Equity Market Neutral funds take both long and short positions in stocks while minimizing exposure to the systematic risk of the market (i.e., a beta of zero is desired). Funds seek to exploit investment opportunities unique to a specific group of stocks, while maintaining a neutral exposure to broad groups of stocks defined for example by sector, industry, market capitalization, country, or region. There are a number of sub-sectors including statistical arbitrage, quantitative long/short, fundamental long/short and index arbitrage. Managers often apply leverage to enhance returns [24].

Event Driven (EVD)

Event Driven funds invest in various asset classes and seek to profit from potential mispricing of securities related to a specific corporate or market event. Such events can include: mergers, bankruptcies, financial or operational stress, restructurings, asset sales, recapitalizations, spin-offs, litigation, regulatory and legislative changes as well as other types of corporate events. Event Driven funds can invest in equities, fixed income instruments (investment grade, high yield, bank debt, convertible debt and distressed), options and various other derivatives. Many managers use a combination of strategies and adjust exposures based on the opportunity sets in each sub-sector [24].

Distressed (EDDI) Event Driven funds that focus on distressed situations invest across the capital structure of companies subject to financial or operational distress or bankruptcy proceedings. Such securities trade at substantial discounts to intrinsic value due to difficulties in assessing their proper value, lack of research coverage, or an inability of traditional investors to continue holding them. This strategy is generally long-biased in nature, but managers may take outright long, hedged or outright short positions. Distressed managers typically attempt to profit on the issuer's ability to improve its operation or the success of the bankruptcy process that ultimately leads to an exit strategy [24].

Multi-Strategy (EDMS) Multi-Strategy Event Driven managers typically invest in a combination of event driven equities and credit. Within the equity space, sub-strategies include risk arbitrage, holding company arbitrage, equity special situations, and value equities with a hard or soft catalyst. Within the credit-oriented portion, sub-strategies include long/short high yield credit (sub-investment grade corporate bonds), leveraged loans (bank debt, mezzanine, or self-originated loans), capital structure arbitrage (debt vs. debt or debt vs. equity), and distressed debt (workout situations or bankruptcies) including post-reorganization equity. Multi Strategy Event Driven managers have the flexibility to pursue event investing across different asset classes and take advantage of shifts in economic cycles [24].

Risk Arbitrage (EDRA) Risk Arbitrage Event Driven hedge funds attempt to capture the spreads in merger or acquisition transactions involving public companies after the terms of the transaction have been announced. The spread is the difference between the transaction bid and the trading price. Typically, the target stock trades at a discount to the bid in order to account for the risk of the transaction not closing successfully. In a cash deal, the manager will typically purchase the stock of the target and tender it for the offer price at closing. In a fixed exchange ratio stock merger, one would go long the target stock and short the acquirer's stock according to the merger ratio, in order to isolate the spread and hedge out market risk. The principal risk is deal risk, should the deal fail to close [24].

Fixed Income Arbitrage (FIA)

Fixed Income Arbitrage funds attempt to generate profits by exploiting inefficiencies and price anomalies between related fixed income securities. Funds limit volatility by hedging out exposure to the market and interest rate risk. Strategies include leveraging long and short positions in similar fixed income securities that are related either mathematically or economically. The sector includes credit yield curve relative value trading involving interest rate swaps, government securities and futures; volatility trading involving options; and mortgage-backed securities arbitrage (the mortgage-backed market is primarily US-based and over-the-counter) [24].

Global Macro (GLM)

Global Macro funds focus on identifying extreme price valuations and leverage is often applied on the anticipated price movements in equity, currency, interest rate and commodity markets. Managers typically employ a top-down global approach to concentrate on forecasting how political trends and global macroeconomic events affect the valuation of financial instruments. Profits are made by correctly anticipating price movements in global markets and having the flexibility to use a broad investment mandate, with the ability to hold positions in practically any market with any instrument. These approaches may be systematic trend following models, or discretionary [24].

Long/Short Equity (LSE)

Long/Short Equity funds invest on both long and short sides of equity markets, generally focusing on diversifying or hedging across particular sectors, regions or market capitalizations. Managers have the flexibility to shift from value to growth; small to medium to large capitalization stocks; and net long to net short. Managers can also trade equity futures and options as well as equity related securities and debt or build portfolios that are more concentrated than traditional long-only equity funds [24].

Managed Futures (MAF or CTA)

Managed Futures funds (often referred to as CTAs or Commodity Trading Advisors) focus on investing in listed bond, equity, commodity futures and currency markets, globally. Managers tend to employ systematic trading programs that largely rely upon historical price data and market trends. A significant amount of leverage is employed since the strategy involves the use of futures contracts. CTAs do not have a particular bias towards being net long or net short in any particular market [24].

Multi-Strategy (MST)

Multi-Strategy funds are characterized by their ability to allocate capital based on perceived opportunities among several hedge fund strategies. Through the diversification of capital, managers seek to deliver consistently positive returns regardless of the directional movement in equity, interest rate or currency markets. The added diversification

Table 2.1.: This table contains the names, abbreviations, and allocations of the subindices of the THFI ($THFI_{sub}$).

Name	Abbreviation	Allocation
Tremont Hedge Fund Index	THFI	99.9%
Tremont Hedge Fund Index Convertible Arbitrage	COA	1.8%
Tremont Hedge Fund Index Dedicated Short Bias	DSB	0.7%
Tremont Hedge Fund Index Emerging Markets	EMM	8.9%
Tremont Hedge Fund Index Equity Market Neutral	EMN	5.6%
Tremont Hedge Fund Index Event Driven	EVD	24.1%
Tremont Hedge Fund Index Event Driven Distressed	EDDI	
Tremont Hedge Fund Index Event Driven Multi-Strategy	EDMS	
Tremont Hedge Fund Index Event Driven Risk Arbitrage	EDRA	
Tremont Hedge Fund Index Fixed Income Arbitrage	FIA	4.2%
Tremont Hedge Fund Index Global Macro	GLM	14.4%
Tremont Hedge Fund Index Long/Short Equity	LSE	25.8%
Tremont Hedge Fund Index Managed Futures	MAF or CTA	4.0%
Tremont Hedge Fund Index Multi-Strategy	MST	10.4%

benefits reduce the risk profile and help to smooth returns, reduce volatility and decrease asset-class and single-strategy risks. Strategies adopted in a multi-strategy fund may include, but are not limited to, convertible bond arbitrage, equity long/short, statistical arbitrage and merger arbitrage [24].

2.6.3. Tremont Hedge Fund Index (THFI)

Credit Suisse/Tremont Hedge Fund Index is compiled by Credit Suisse [24]. It is an asset-weighted hedge fund index and includes only hedge funds. The Index consists only of hedge funds with a minimum of US$50 million under management, a 12-month track record, and audited financial statements. It is calculated and rebalanced on a monthly basis, and shown net of all performance fees and expenses. Table 2.1 displays the coding of the various subindices that are part of the THFI. Part of this work was also investigate whether we can identify tail dependence between the THFI and the subindices of the THFI ($THFI_{sub}$)

2.6.4. Fund of Hedge Funds (FoHF)

The investigated $FoHF_{ISP}$ consists of 27 different hedge funds. Their strategies and substrategies, as well all their allocation percentages are listed in table 2.2. Number 9 has been omitted, as the time series was too short for analysis.

2.6.5. Calculation of Returns

As we are interested in the relative volatility of the price development, only the returns have been investigated. They have been calculated according to equation (2.87)

$$\text{return}(t) = \log(\text{price}(t)) - \log(\text{price}(t-1)), \tag{2.87}$$

Table 2.2.: Strategies, substrategies, and allocation of hedge funds that are part of the investigated $FoHF_{ISP}$ with the respective number that has been used in the results and discussion section. Number 9 has been omitted, as the times series was too short.

Name	Strategy	Substrategy	Allocation
1	Long/Short	L/S USA	5.71%
2	Global Macro	Global Macro	5.52%
3	Global Macro	Global Macro	4.04%
4	Event Driven	Special Situations	4.19%
5	Global Macro	Global Macro	2.61%
6	CTA	CTA	5.00%
7	Relative Value	Multi-Strategy	3.82%
8	Relative Value	Multi-Strategy	6.66%
10	Long/Short	L/S Global	3.05%
11	Event Driven	Special Situations	3.17%
12	Global Macro	Global Macro	2.69%
13	Global Macro	Global Macro	3.49%
14	Event Driven	Distressed	6.81%
15	Long/Short	L/S Global	5.65%
16	Event Driven	Special Situations	2.98%
17	Long/Short	L/S USA	2.65%
18	Event Driven	Distressed	2.56%
19	Global Macro	Global Macro	1.67%
20	Long/Short	L/S Global	4.40%
21	Relative Value	Multi-Strategy	2.08%
22	Long/Short	L/S Global	4.82%
23	Event Driven	Distressed	3.26%
24	Event Driven	Special Situations	1.69%
25	CTA	CTA	1.30%
26	Long/Short	L/S Global	3.38%
27	CTA	CTA	4.10%

where log(·) denotes the natural logarithm.

If the price changes were already given in percentages, the following equation has been used to calculate the returns:

$$\text{return}(t) = \log(\text{price}_{\text{change}}(t) + 1), \tag{2.88}$$

with

$$\text{price}_{\text{change}}(t) = \frac{\text{price}(t) - \text{price}(t-1)}{\text{price}(t-1)}. \tag{2.89}$$

2.7. Software

All calculations were performed under Windows Vista using Matlab R2008a. A selection of the most important codes (m-files) can be found in appendix A.7.

3. Results and Discussion

This chapter contains the results that have been obtained by applying the above described methods. This chapter also contains the discussion of the results, i.e. the interpretation is not part of a separate chapter to avoid unneeded duplication of texts, figures, and tables, and to make this work as easy to read as possible.

3.1. THFI vs. S&P 500

We first present the results that have been found for the comparison between the data set for the Tremont Hedge Fund Index (THFI) and the S&P 500. In this constellation the THFI is an explaining factor for the S&P 500 in the factor model: S&P 500 = $\beta \cdot \mathrm{THFI} + \epsilon$. Hence, λ is a measure for the probability that the S&P 500 incurs a large loss or gain conditional on a large loss or gain of the THFI, respectively (see equations (2.3) and (2.4) for the definition of λ for lower and upper tail of the return distribution). The results for the relation THFI = $\beta \cdot$ S&P 500 + ϵ, i.e. the probability that the THFI incurs a large loss or gain conditional on a large loss or gain of the S&P 500, can be found in appendix A.2.

3.1.1. Moving Threshold Graphs

Figure 3.1 contains the results of the various tail index estimators applied in this work in relation to varying threshold values k. If we compare the different estimators, we can see that they perform quite differently: the Hill, the Gabaix, and the Clauset estimator seem to behave rather stable for comparably small values of k, whereas the Huisman estimator shows a relatively unstable regime even for large k's (especially in the lower tail). It is surprising that the Huisman estimator does not show a more stable behavior, as this estimator has been especially developed to adjust the unstable and biased behavior of the Hill estimator. I will therefore discard the results that are based on the Huisman estimator or move them to the appendix, if I think they could be of interest. The Clauset estimator seems to behave rather stable, however, for the upper tail we can see some sort of lock-in effect for the tail index calculated for the S&P 500 data. This can be explained by the min max procedure that is part of the method developed by Clauset et al. (see section 2.2.1). If the maximal distance calculated with the KS statistics is minimal where, e.g. $k/N \approx 0.12$, then the tail index calculated for that ratio is assumed to reflect most accurately the assumed power-law distribution. However, it might happen that the relatively high tail index for the upper tail drops to a lower value, if k would be increased further (as we can see in the lower tail for the S&P 500 data). Based on this findings, I conclude that the most reliable estimator is the Gabaix, followed by the Hill, and maybe the Clauset estimator, dependent on the situation. This is also the reason why in the following sections only the results based on the Gabaix estimator are presented in the 'detailed results' section (valid for sections 3.2.2, 3.3.2, 3.4.2, and 3.5.2).

If we look at the TDCs calculated by applying the tail index estimates presented above, we can see that the results based on the method of Poon et al. (Figure 3.2) are rather heterogeneous. The behavior of $\hat{\bar{\chi}}$, a measure for asymptotic independence, is rather unstable, independent of the tail index estimator or the size of the tail of the distribution. So, it is difficult to draw any conclusions based on these results. If we look at $\hat{\chi}$, a measure for asymptotic dependence and equivalent to what we have defined to be a TDC, the situation looks better and we have stable results for the calculations based on the Hill estimator. However, if we apply the Gabaix estimator the situation looks again rather unpleasant as $\hat{\chi}$ shows large fluctuations due to $\hat{\chi}$ being set to zero. This is the case, if we can assume that it is unlikely that $\hat{\bar{\chi}}$ is equal to one and we therefore experience asymptotic independence. The correct measure for this case is given by $\hat{\bar{\chi}}$ and not $\hat{\chi}$. However, $\hat{\bar{\chi}}$ should lie between the boundaries -1 and 1 and not exceed 1. This is obviously not the case and the reason for this behavior might be statistical artifacts caused by small samples and/or strange behavior of certain tail index estimators in company with the method developed by Poon et al. (For results based on the Clauset and the Hill estimator please see Figure A.1 in the appendix.)

Figure 3.3 shows the results based on the non-parametric approach by Malevergne and Sornette. We can see that the estimates in the upper tail seem to be slightly higher than in the lower tail. This would mean that the probability that the S&P 500 will experience large gains given that the THFI also experiences large gains is higher than the probability that the S&P 500 will experience large losses given that the THFI also experiences large losses. The same results are obtained when applying Malevergne and Sornette's parametric approach. This can be seen in Figure 3.4. However, if we look at the results obtained by Schmidt and Stadtmüller's method (see Figure 3.5), we cannot see such a difference as the TDC is almost similar for the upper and the lower tail, independent of the estimator (i.e. λ or λ^{EVT}) applied. The results based on linear measures of correlation as presented in Figure 3.6 are inconclusive as well. For Pearson's r the correlation seems to be larger for the lower tail than for the upper tail, whereas for Spearman's ρ and Kendall's τ we have slightly higher correlation in the upper tail compared to the lower tail.

To check whether the results found with Malevergne's and Sornette's methods are spurious, I exchanged the THFI data with the data of the S&P 500 and vice versa, and recalculated the TDC with the adjusted model as it has been presented in section 2.2.3. If the TDC is in fact higher for the upper tails, then the results for the non-parametric and the parametric method of Malevergne and Sornette, as presented in Figure 3.7, should be the same as presented in Figures 3.3 and 3.4 for the Gabaix estimator. As I cannot find such a consistency between the results, I am unable to conclude whether the tail dependence between the THFI and the S&P 500 is asymmetric in respect to the upper and lower tail or if we face a statistical artifact due to small samples or another yet unknown reason. Interestingly, however, the results for the TDCs calculated with the methods of Schmidt and Stadtmüller are exactly the same, independent of whether we treat the data of the THFI as $\boldsymbol{x}$ and those of the S&P 500 as $\boldsymbol{y}$ in the factor model (see equation (2.38)) or vice versa.

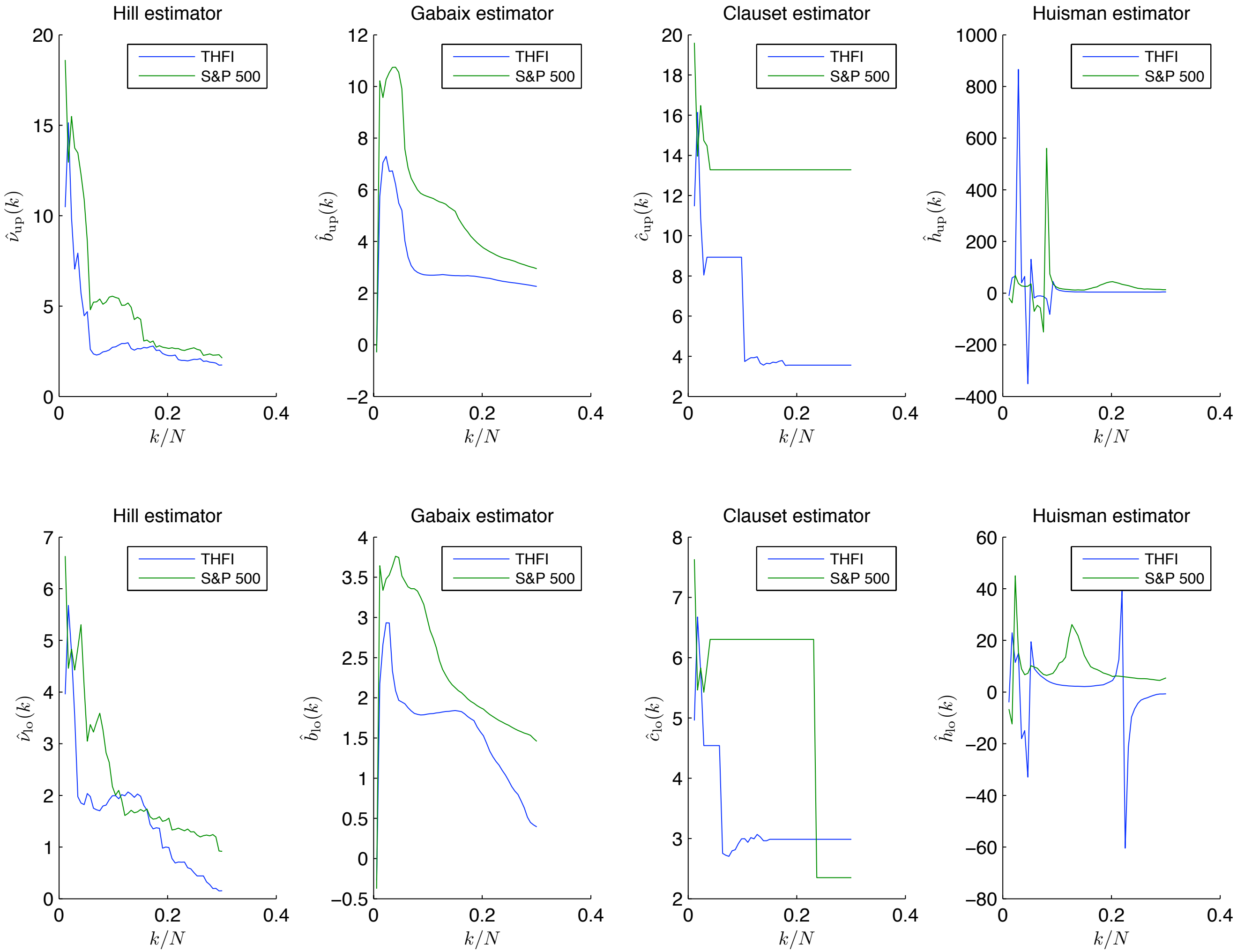
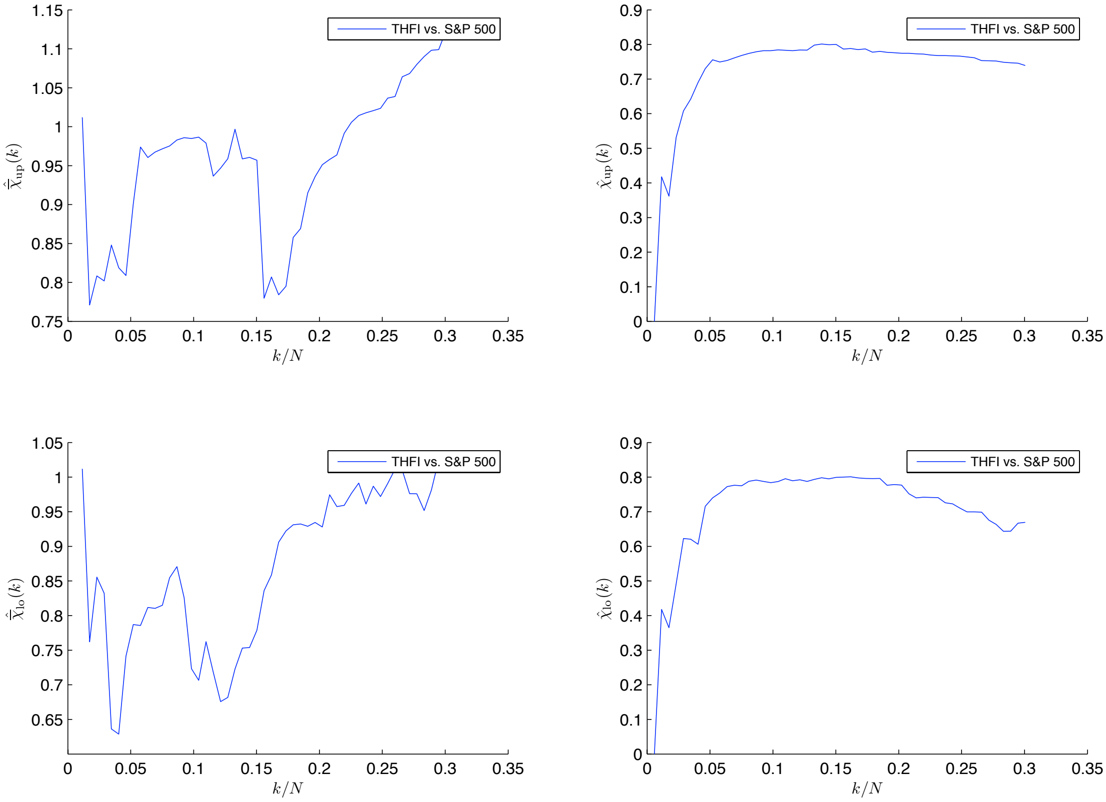
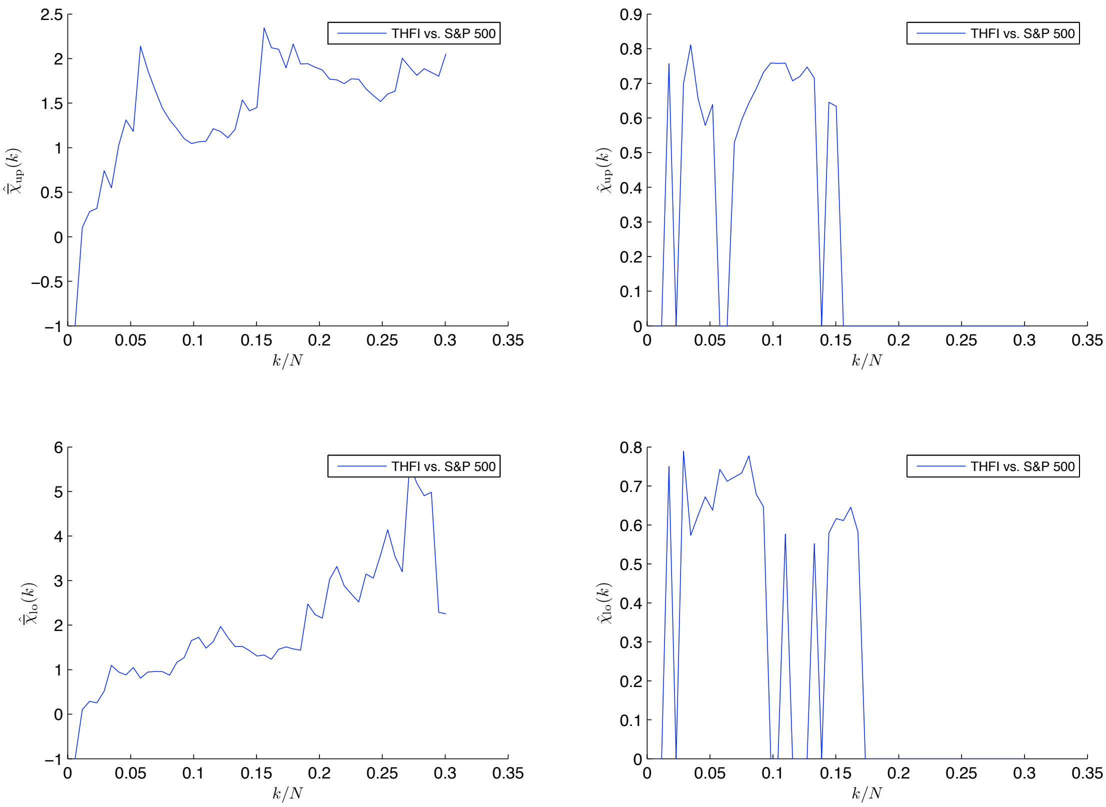


Figure 3.1.: The above figure shows the estimates of the four *tail index* estimator for the THFI and the S&P 500. From left to right we have the Hill, the Gabaix, the Clauset, and finally the Huisman estimator. The first row contains the tail index estimators for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) $\overline{\chi}$ and χ with Hill estimator



(b) $\overline{\chi}$ and χ with Gabaix estimator

Figure 3.2.: $\overline{\chi}$ and χ calculated for THFI vs. S&P 500 applying the Hill estimator in sub-figure (a) and the Gabaix estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

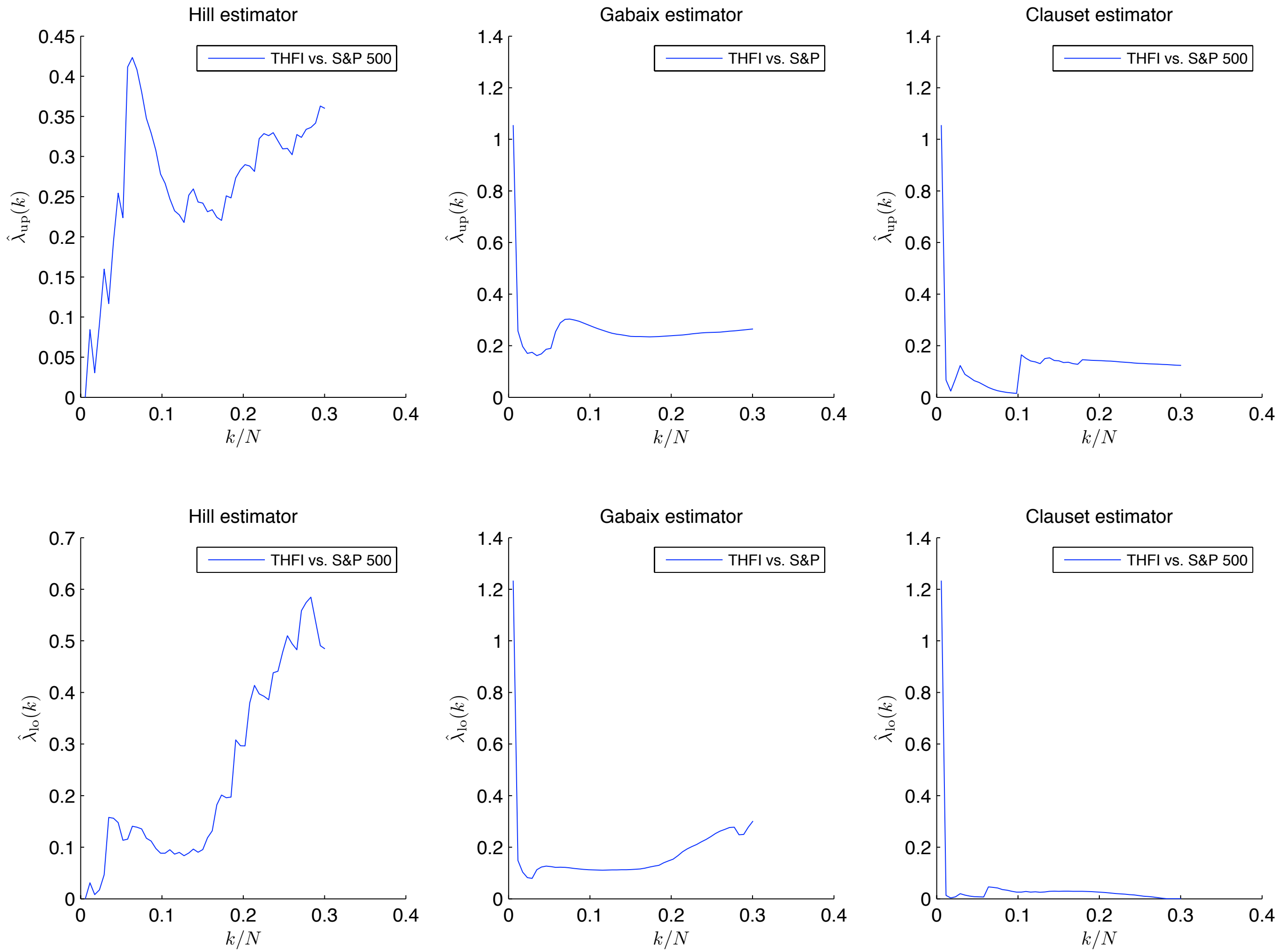


Figure 3.3.: λ for THFI vs. S&P 500 with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

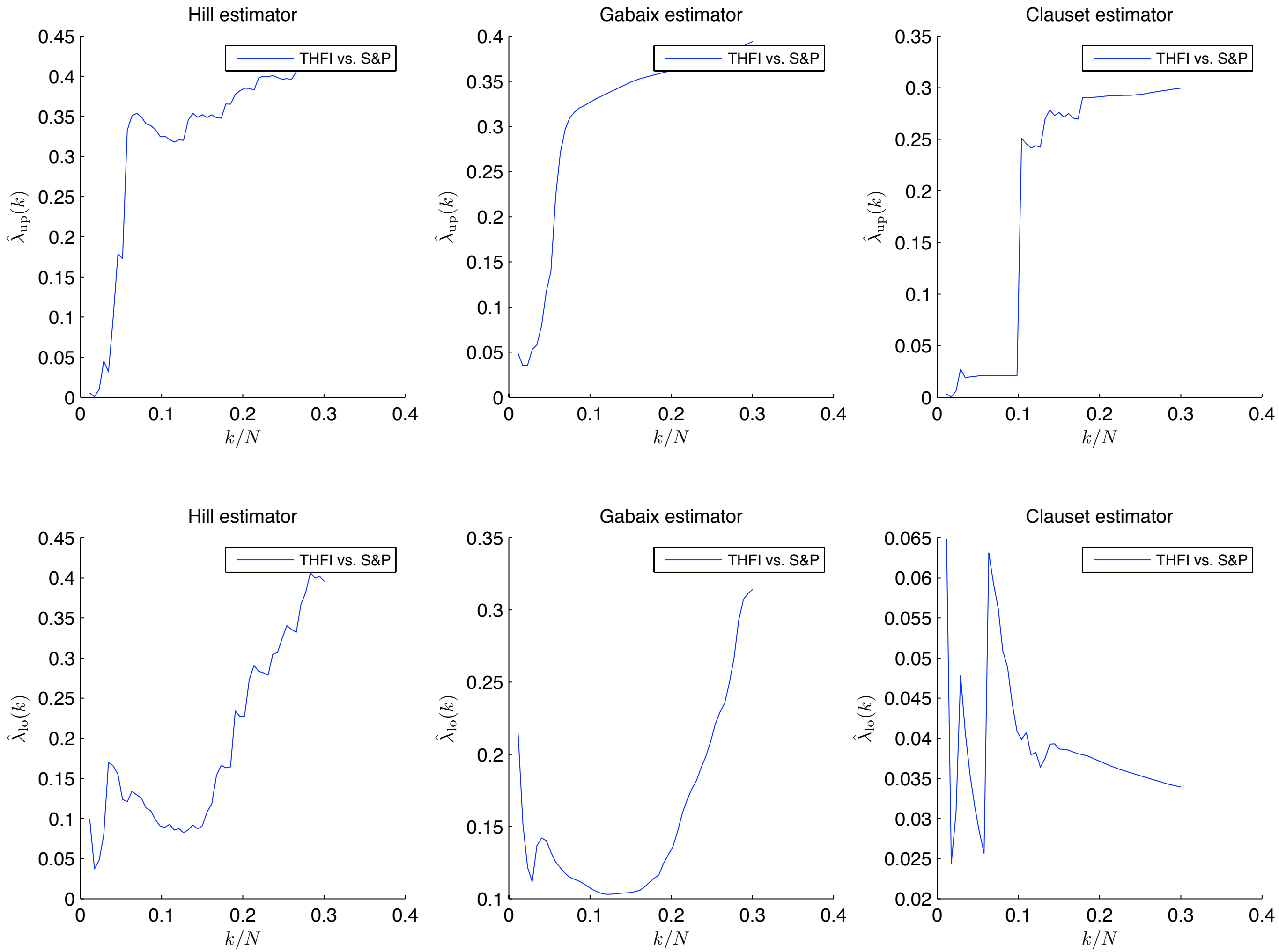


Figure 3.4.: λ for THFI vs. S&P 500 with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

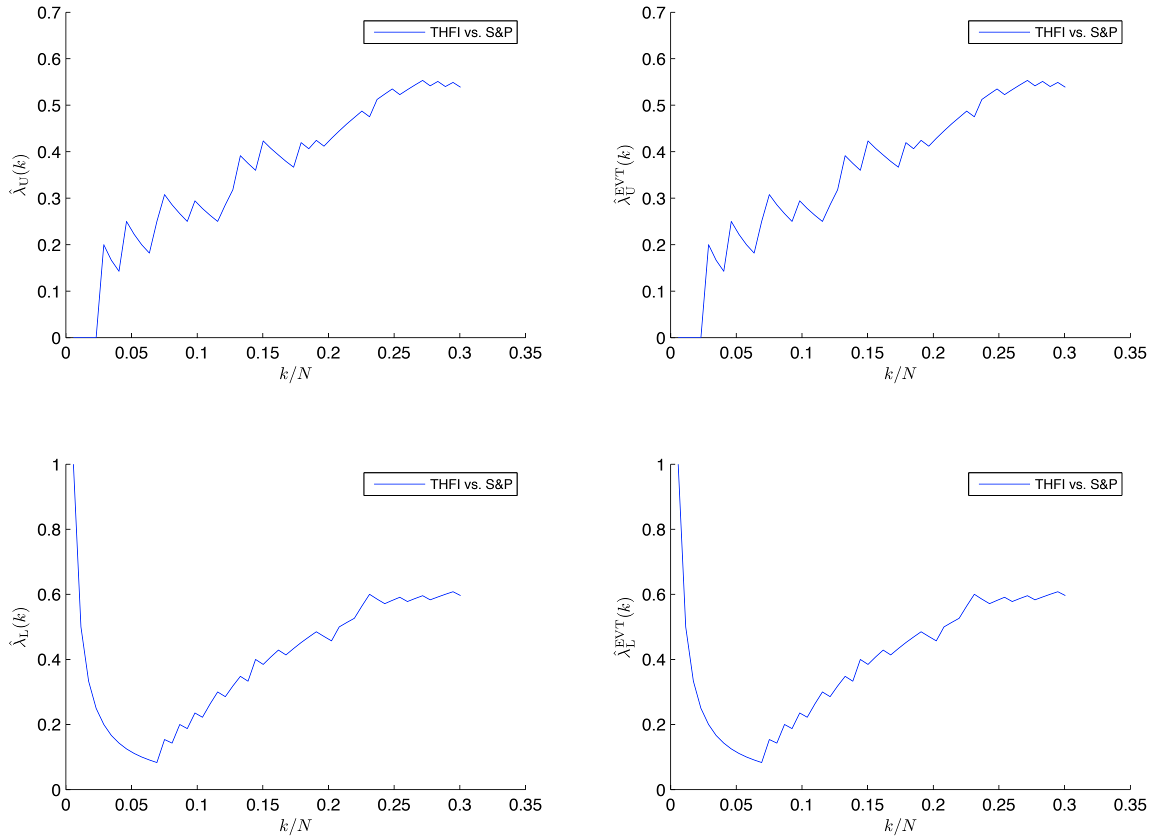


Figure 3.5.: λ for THFI vs. S&P 500 by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

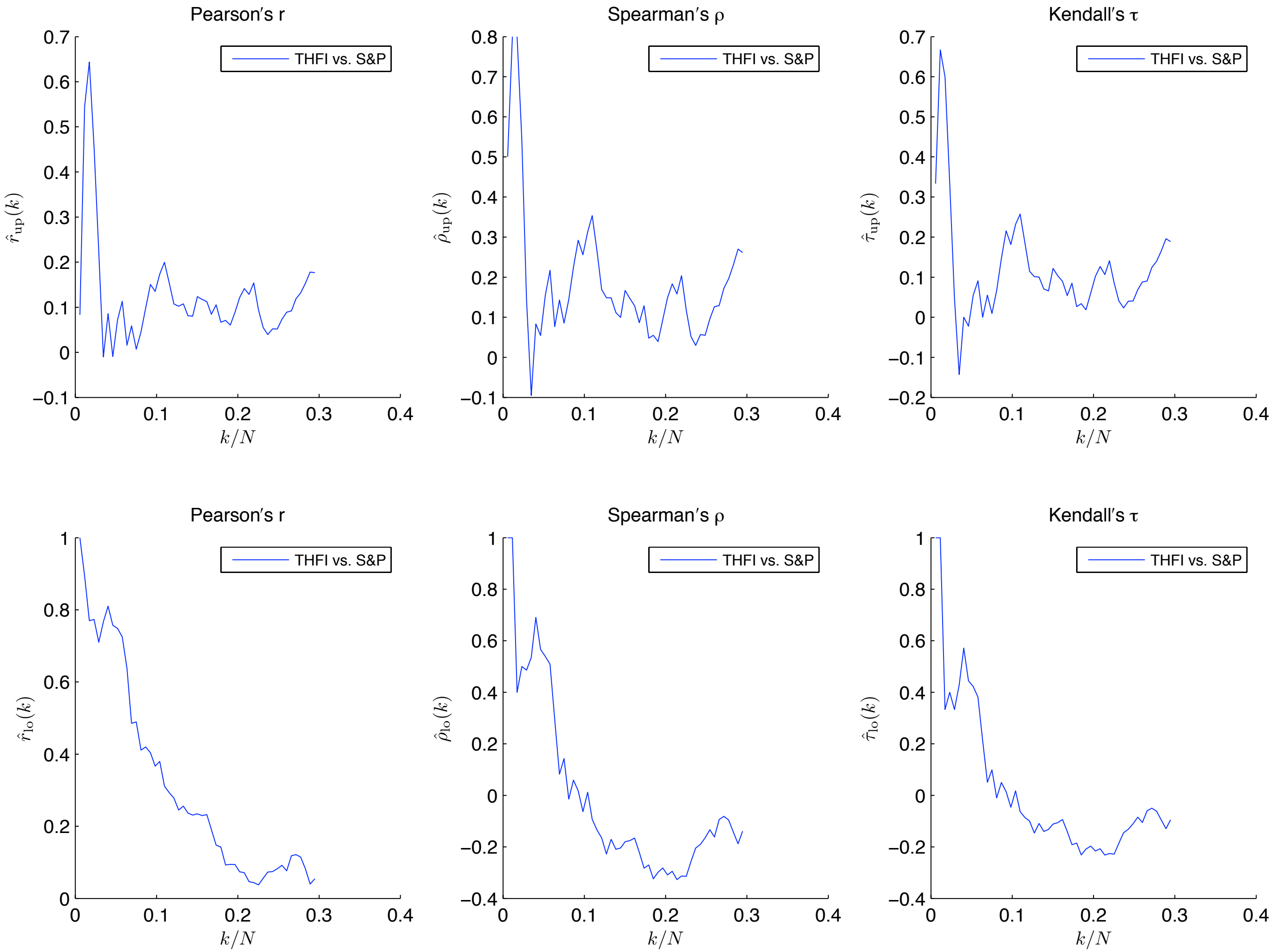


Figure 3.6.: λ for THFI vs. S&P 500 calculated with *linear measures of correlation.* From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

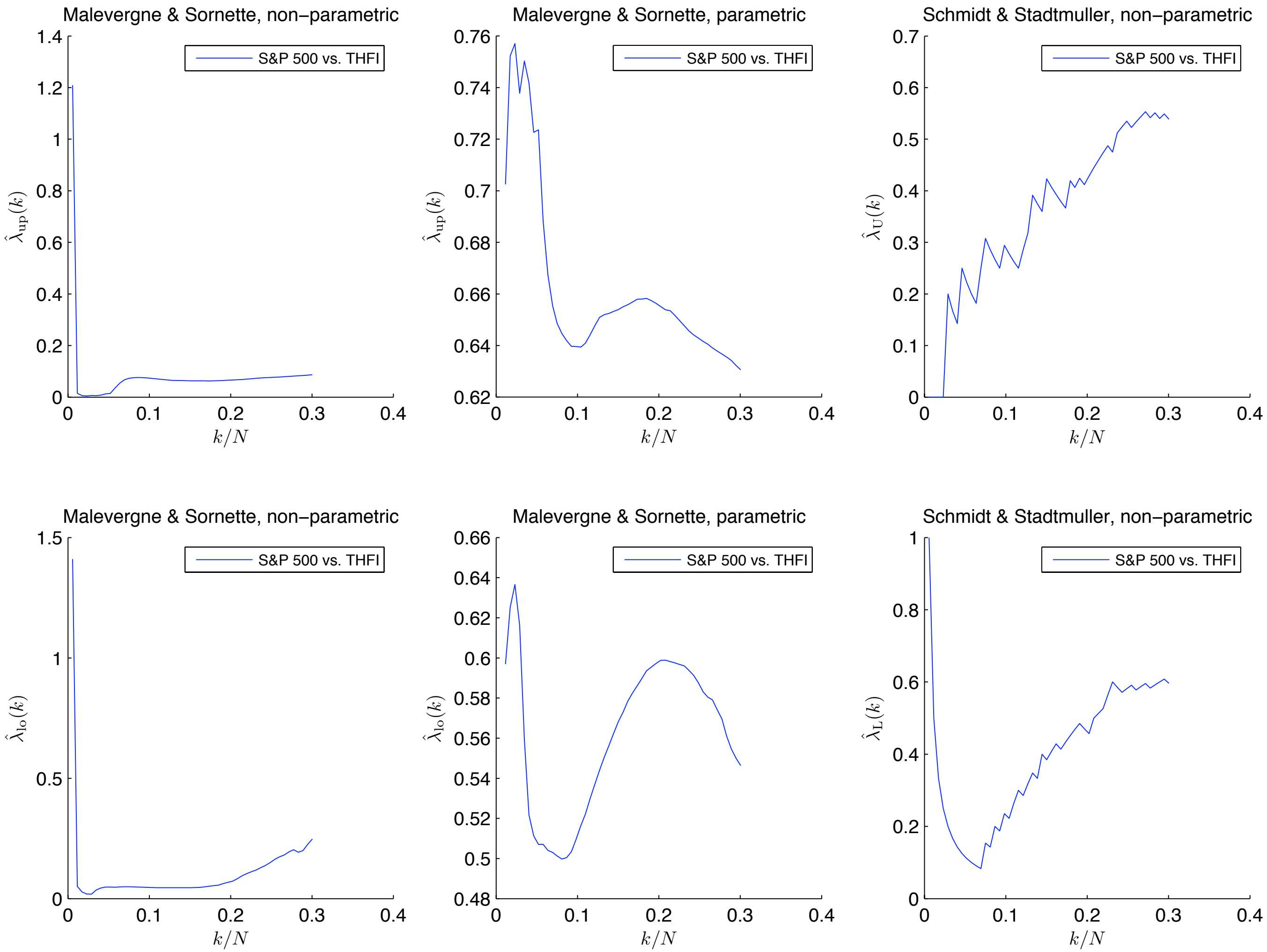


Figure 3.7.: λ for S&P 500 vs. THFI with *adjusted parameters* calculated with, from left to right, the non-parametric method of Malevergne and Sornette applying the Gabaix estimator, the parametric methods by Malevergne and Sornette also applying the Gabaix estimator, and the non-parametric method of Schmidt & Stadtmüller. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

3.2. THFI vs. THFI_{sub}

In this section I present the results for the tail dependence between the THFI and its subindices THFI_{sub}. The strategies of these subindices are all described in the methodology section, including the abbreviations used here. The return data used for this section are composed of 13 subindices that consist of 173 observations each. The underlying factor model can be written with THFI as the explaining factor for the various subindices: $\text{THFI}_{\text{sub},i} = \beta_i \cdot \text{THFI} + \epsilon_i$, with $i = 1, \ldots, 13$. Hence, the TDC calculated here, reflects the probability that given the THFI experiences a large loss or gain the various subindices also experience a large losses or gains (see equations (2.3) and (2.4) for definition of the TDC).

3.2.1. Moving Threshold Graphs

In Figure 3.8 we can see that the tail index estimates show a rather stable behavior. The results for Poon et al. (Figure 3.9) are again conflicting. On the one hand we see a more or less stable behavior of $\hat{\bar{\chi}}$ and $\hat{\chi}$ in sub-figure (a) for the upper tail and partly for the lower tail. On the other hand, the results for $\hat{\chi}$ are almost identical, so no additional information can be gained by looking at the detailed values for the respective TDCs. In subfigure (b) $\hat{\bar{\chi}}$ behaves unstable and $\hat{\chi}$ is in the same range for all subindices, independent of the value of k. Therefore the results based on the method by Poon et al. are not considered further for in depth analysis.

Figures 3.10 and 3.11 presenting the results for the non-parametric and the parametric method of Malevergne and Sornette show a partly stable behavior around 0.12 for the ratio k/N. Therefore it seems reasonable to define the optimal value for the ratio k/N to be 0.12. I assume this to be a reasonable trade off between a low bias (small k) and a low variance (large k) for the applied tail index estimators.

The results gained with the method of Schmidt and Stadtmüller (Figure 3.12) might also display a partly stable behavior around the value of 0.12 for k/N. However, it is hard to argue for or against a specific value for k based on Figure 3.12. It is also interesting to see that, as already seen in Figure 3.5, we cannot see any differences between the two estimators ($\hat{\lambda}$ and $\hat{\lambda}^{\text{EVT}}$) developed by Schmidt and Stadtmüller. That is, way I will only present detailed results for $\hat{\lambda}$.

The results for Pearson's r, as presented in Figure 3.13, show a more or less stable behavior for $k/N \geq 0.12$. The other linear measures are less stable, but a value of 0.12 for k/N still seems to be reasonable.

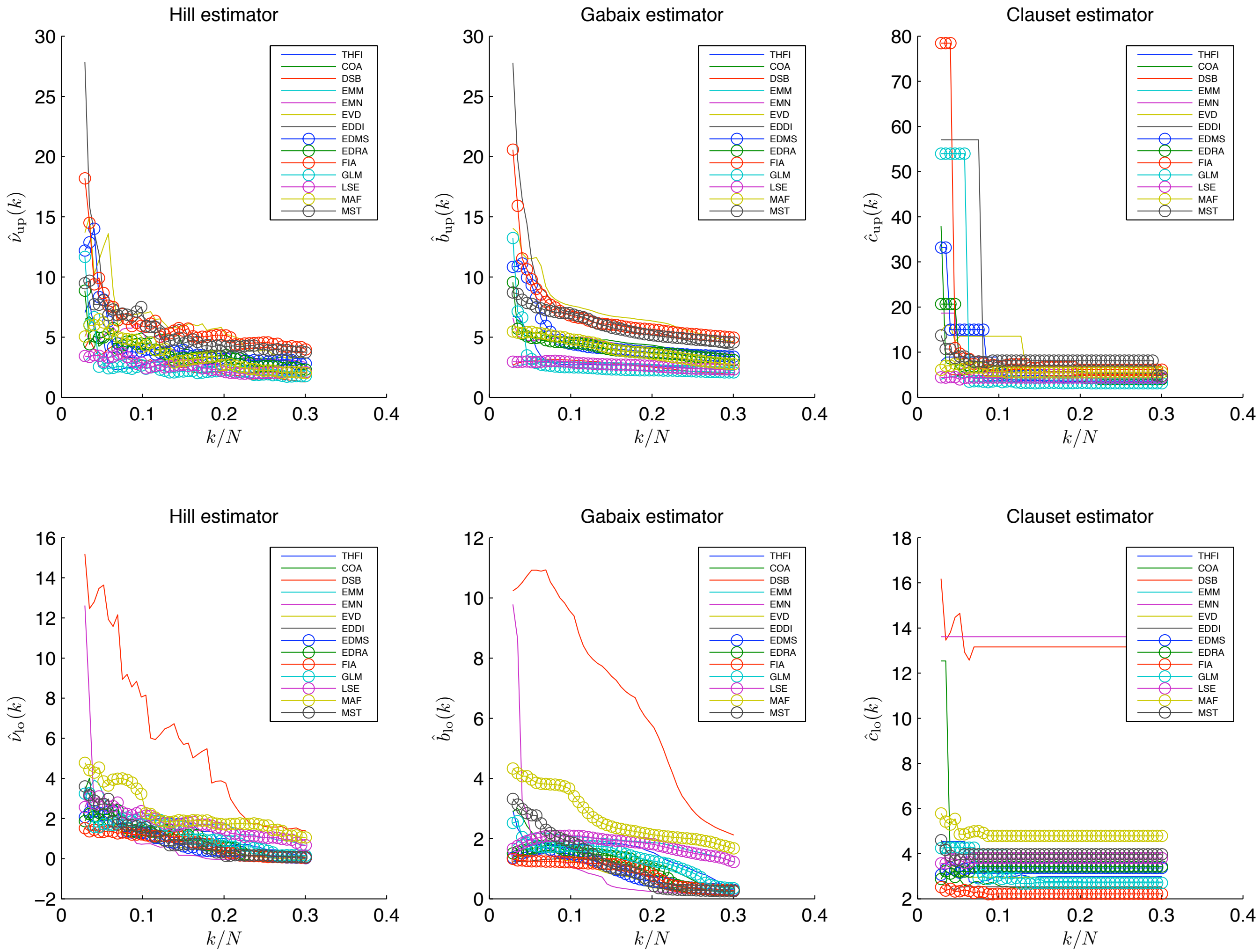
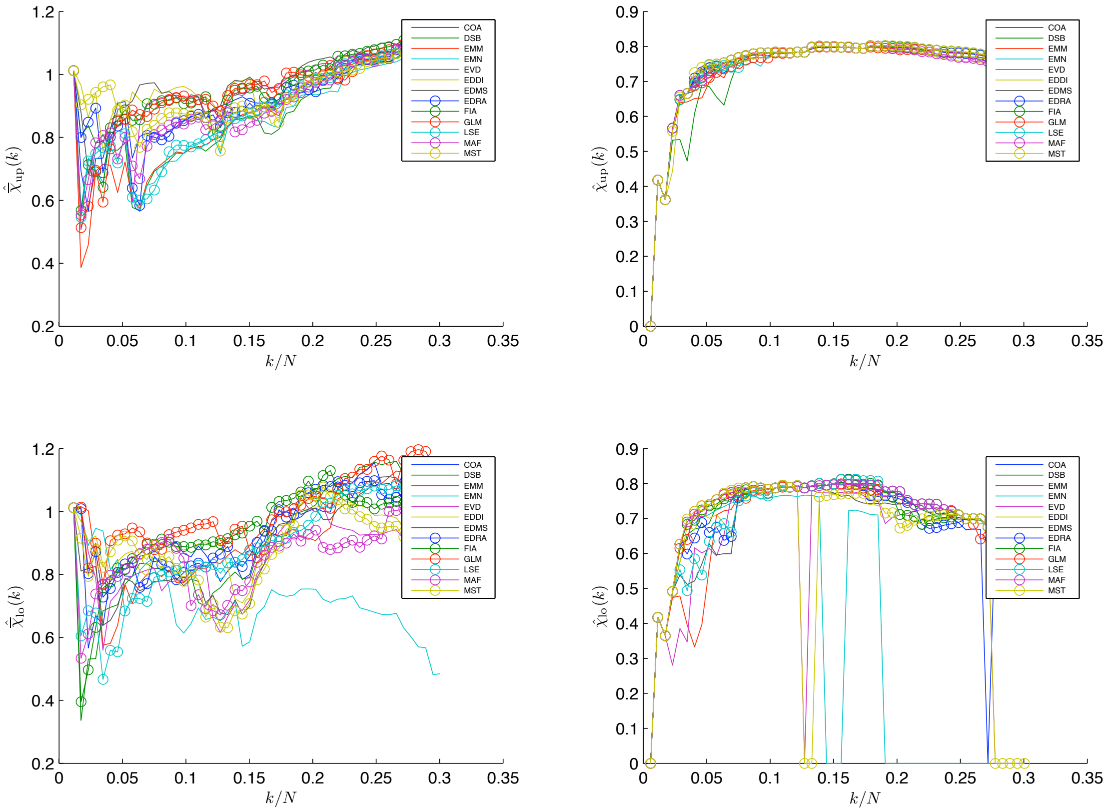
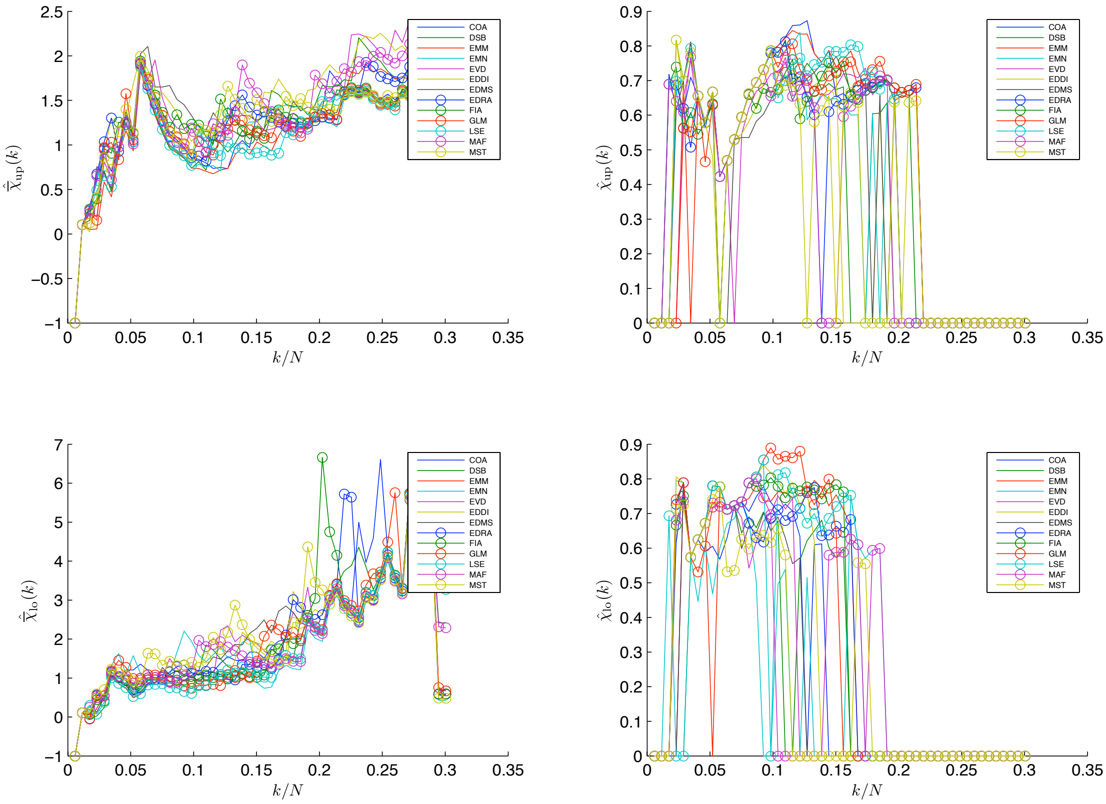


Figure 3.8.: The above figure shows the estimates for three *tail index* estimator for the THFI and its subindices $THFI_{sub}$. From left to right we have the Hill, the Gabaix, and the Clauset estimator. The first row contains the tail indices for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) $\overline{\chi}$ and χ with Hill estimator



(b) $\overline{\chi}$ and χ with Gabaix estimator

Figure 3.9.: $\overline{\chi}$ and χ calculated for THFI vs. THFI_{sub} applying the Hill estimator in sub-figure (a) and the Gabaix estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

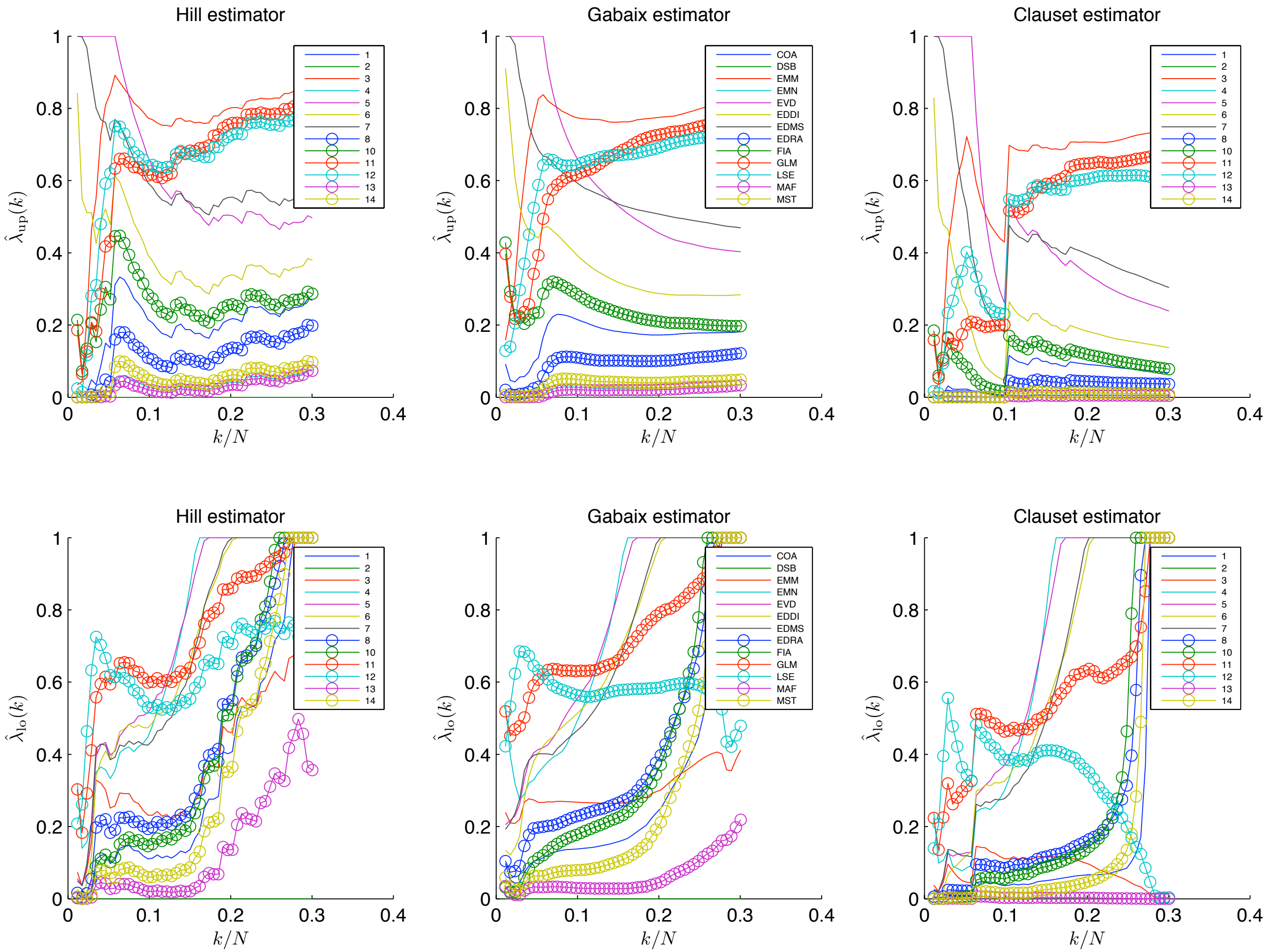


Figure 3.10.: λ for THFI vs. $THFI_{sub}$ with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

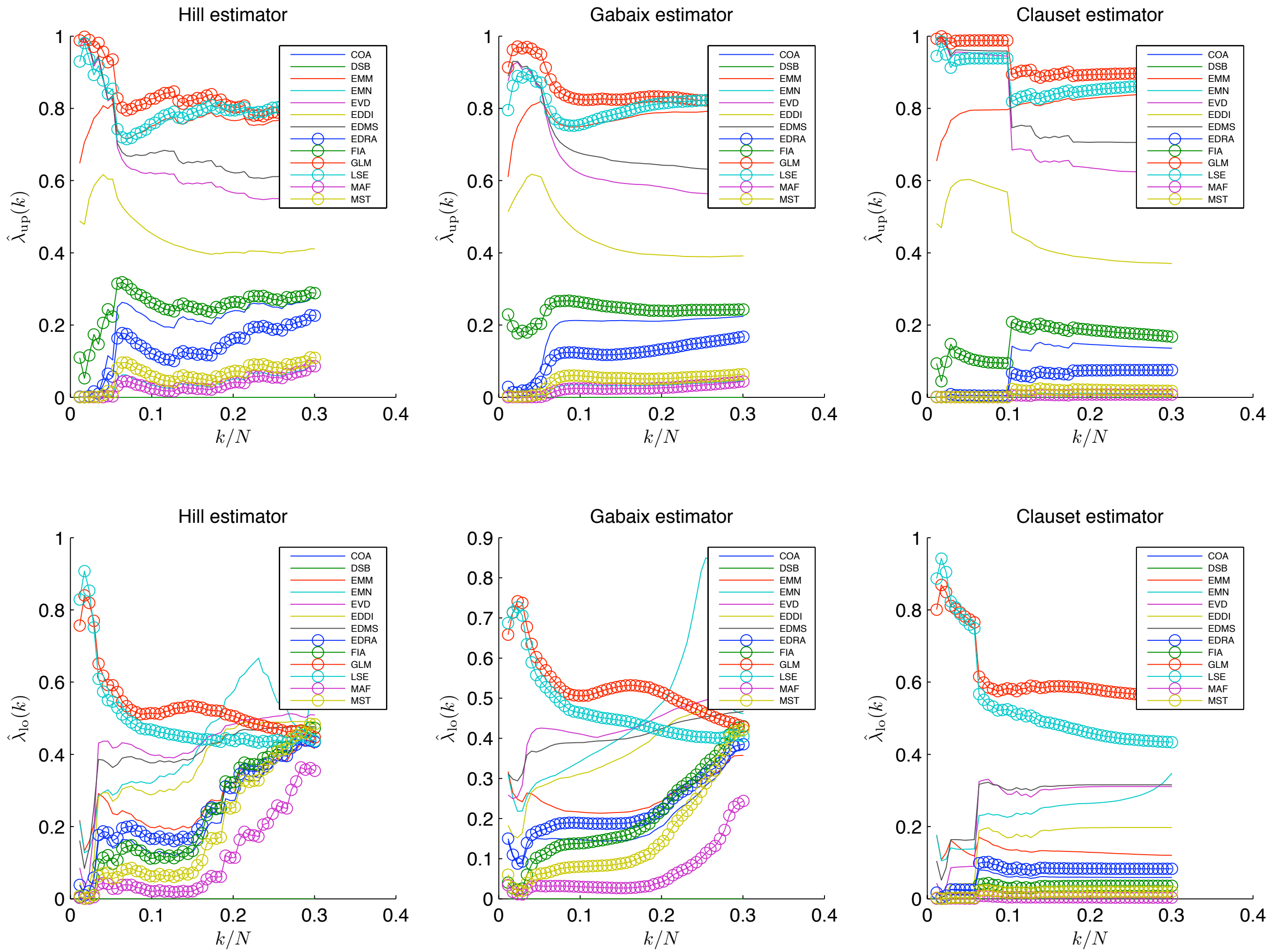
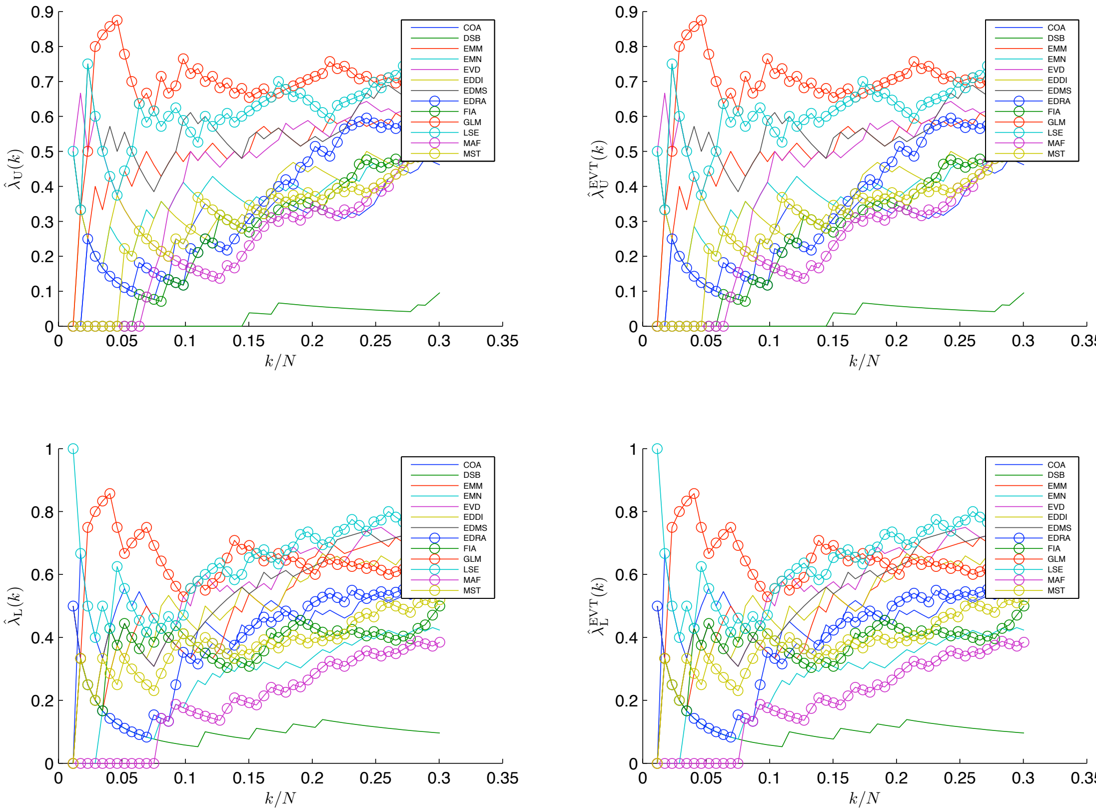
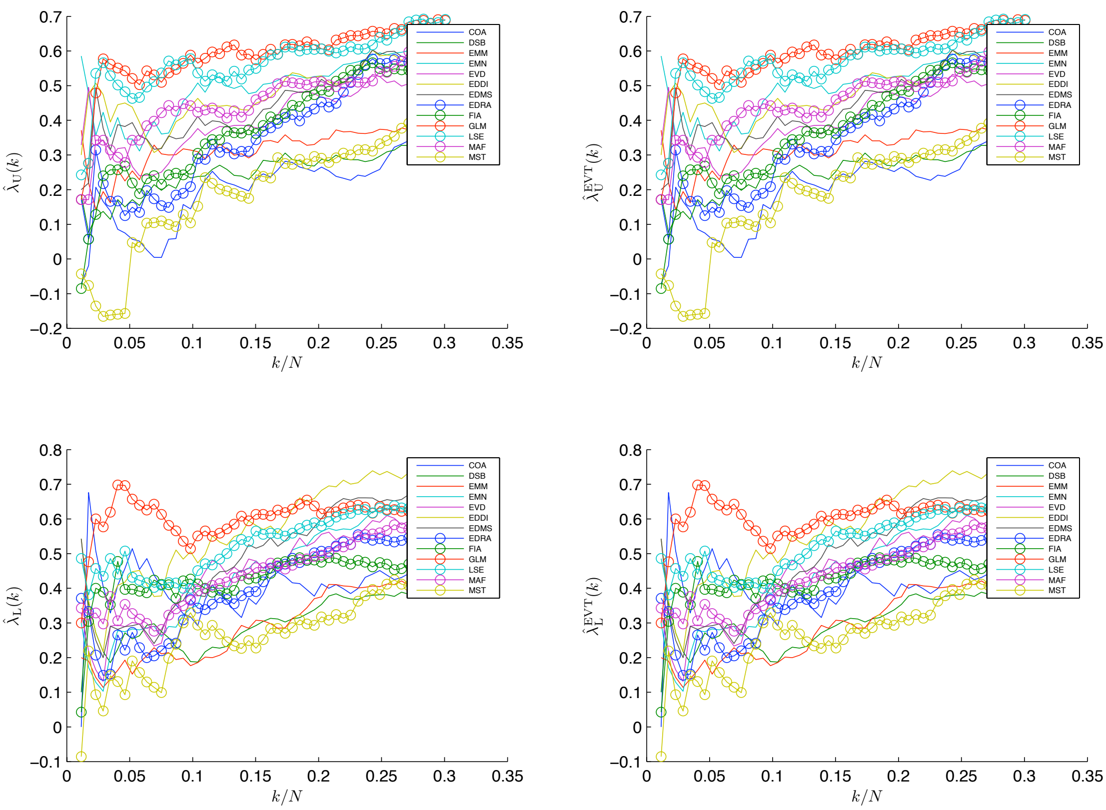


Figure 3.11.: λ for THFI vs. $\mathrm{THFI}_{\mathrm{sub}}$ with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) original results



(b) smoothed results

Figure 3.12.: λ for THFI vs. THFI$_{sub}$ by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

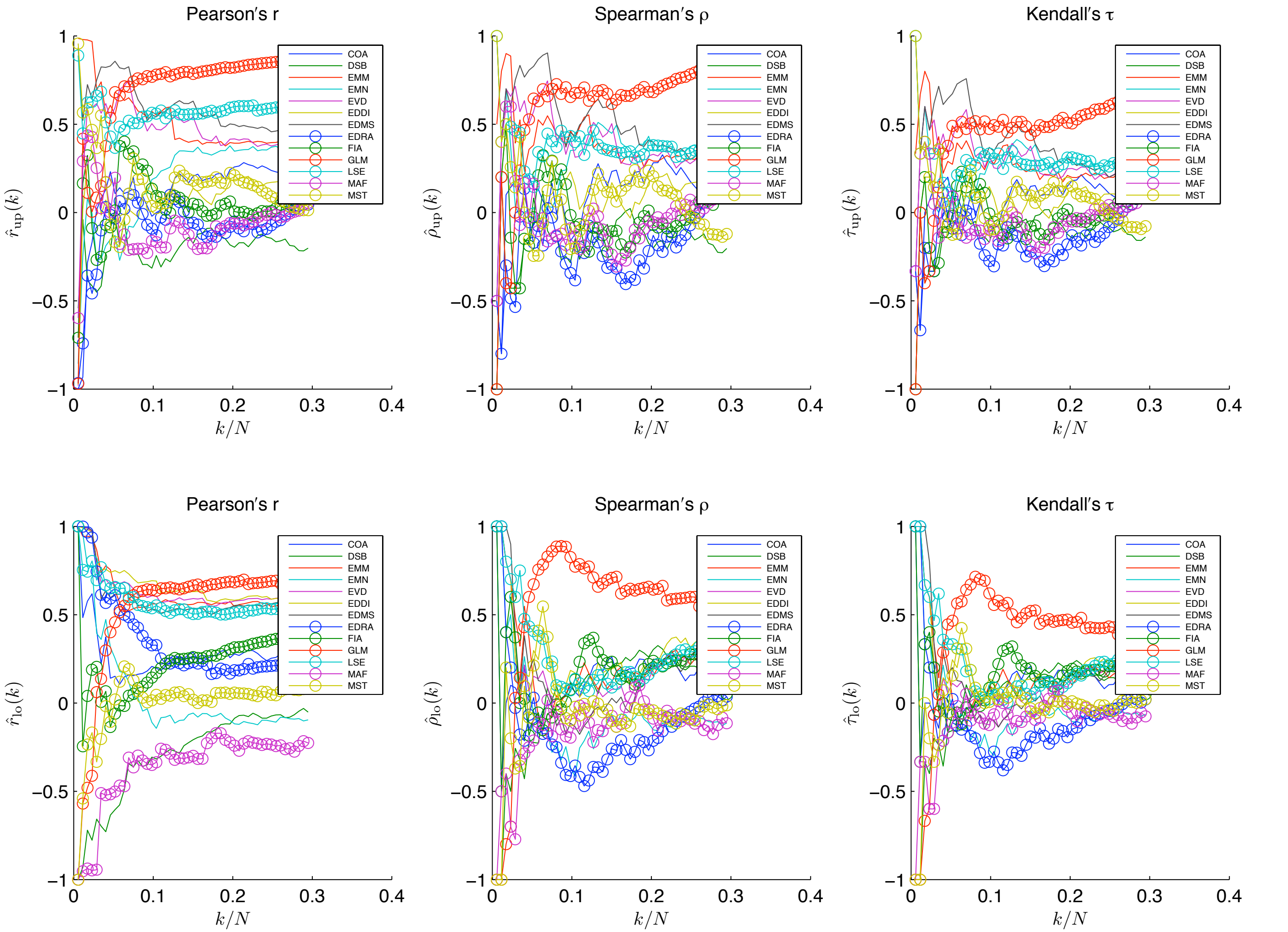


Figure 3.13.: TDC for THFI vs. THFI_{sub} based on linear measures of correlation. From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

3.2.2. Detailed Results

In this section the detailed results are presented and their uncertainty is tested with bootstrapping as described in section 2.4 of the methodology chapter. To make the tables easier to read, the three (if there were more similar TDCs, e.g. zeros, sometimes also more) highest and lowest values of the TDCs for the upper and lower tail are highlighted by setting them bold or italic, respectively. In the following paragraphs I will mainly focus my analysis on these highest (top group) and lowest (bottom group) values, as I assume that they are of most interest.

The results presented below are all calculated for the ratio $k/N = 0.12$ corresponding to a rounded value of 21 (data points) for k as $N = 173$. As the estimates of λ derived with the parametric and the non-parametric method of Malevergne and Sornette are very similar for the Hill and the Gabaix estimator, I placed the results that are based on the Hill estimator in appendix A.3. There are no detailed results for the Clauset and Huisman estimator, as I do not expect to gain additional insight from them. Due to the findings in the above section 3.2.1, I also do not present detailed results based on Poon et al.'s method.

When investigating the results gained with the non-parametric (Table 3.1) and the parametric (Table 3.2) approach of Malevergne and Sornette, we can see that, despite different absolute values for $\hat{\lambda}$, the top and bottom groups are almost similar (for overview see Table 3.5). There is one little difference in the lower tail: subindex EDMS is part of the top group for the non-parametric method, whereas the subindex GLM is part of top group in case of the parametric approach. It is interesting so see that the standard deviations are relatively small, considering that the sample is small too. When comparing the standard deviations among the various subindices, we can see a general tendency that larger TDCs have a larger standard deviation. However, if we look at the coefficient of variation (not shown in the tables), the situation changes, as the lower TDCs generally have higher coefficients of variation than the larger TDCs. Considering these two findings and the fact that for values with a mean close to zero, the coefficient of variation is very sensitive to small changes, we can conclude that the precision of the TDC seems to be in the same range for small and large TDCs.

In Table 3.3 the results for the TDC calculation based on the method of Schmidt and Stadmüller are presented. If we compare these results with the ones obtained with the methods of Malevergne and Sornette, we can see that the top group of the upper tail is different by one subindex and the bottom group by two. For the lower tail we have two differences in the top group and one difference in the bottom group. However, the overall rankings are again quite similar and we can conclude that the method of Schmidt and Stadtmüller as well as the approaches of Malevergne and Sornette produce comparable results.

Regarding the TDCs calculated with linear measures of correlation, I decided to present the results based on Pearson's product-moment correlation in this chapter and move the results for the other linear measures of correlation (Spearman's ρ and Kendall's τ) to appendix A.3. This is due to the better performance of Pearson's measure of correlation regarding the stability of the calculated TDCs (see Figure 3.13). If we compare the three different measures of linear correlation (for overview see Table 3.5), we can see that they seem to behave comparable to the other, non-linear measures of tail dependence.

Table 3.5 presents an overview of the rankings calculated by the various methods and we can see that they produce comparable results in the upper tail as well as in the lower tail and for the top as well as for the bottom group. The number of similar subindices in the same rank varies between 2 and 5, with a majority of 2 and 3 similar subindices. For rank 2 in the bottom group of the upper tail we even have seven times the same subindex (MAF). If we look at the number of different subindices per group, we have 4 or 5 different subindices for the upper tail and 7 different subindices for each of the groups in the lower tail. This is a rather consistent picture even though subindex EMN can be found in rank 3 in the top group of the lower tail (non-parametric method of Malevergne and Sornette) as well as in rank 3 of the bottom group in the lower tail (Schmidt and Stadtmüller and Pearson's r).

If we compare the various rankings of the TDC in the upper tail with a Kruskal-Wallis test, we get a p-value of 0.19 for the comparison between all eight methods applied. The comparison between the methods based on EVT yields a p-value of 0.93 and between those based on linear measures of correlation we get a p-value of 0.66. Based on this p-values, the null hypothesis that the medians come from the same population cannot be rejected. However, if we look at the p-values in the lower tail, we get a p-value of 0.002 for the comparison among all eight approaches, meaning that we can reject the null hypothesis on a significance level of 0.01. The p-value for comparison between the non-linear measures of tail dependence is 0.26 and between the linear measures of tail dependence 0.14. We can see that the results produced by the various methods differ more in the lower tail than in the upper tail, as it is not possible to reject the null hypothesis in the upper tail, i.e. that all TDCs come from the same population. In contrast to the upper tail, we can reject this assumption in the lower tail even on a significance level of 0.01.

To investigate this a little further, I calculated Spearman's rank correlation coefficient ρ among the various TDCs. The results for the upper tail are presented in Table 3.6, those for the lower tail in Table 3.7. We can see that for both tails a rather strong correlation among the various non-linear methods and Pearson's r exist. For the upper tail the correlation among all methods is relatively high (the lowest value being 0.67 between the non-parametric method of Malevergne and Sornette and Kendall's τ). This is supported by the findings of the Kruskal-Wallis test that found a p-value of 0.19 for the upper tail. Additionally the p-values are all significant on a 0.05 level, meaning that the null hypothesis that the correlations are equal to zero can be rejected. If we look at the lower tail, we can see that the non-parametric linear measures of correlation (Spearman's ρ and Kendall's τ) show a negative correlation with the other methods except with Pearson's r. When investigating the p-values, we can see that they are high, hence we cannot reject the null hypothesis that the correlations are zero. This finding is also supported by the low p-value (0.002) we found for the lower tail applying the Kruskal-Wallis test. Summarizing, we can state that the non-linear measures of tail dependence and Pearson's r produce comparable results regarding the rankings of the TDCs, whereas the behavior of the non-parametric linear measures of correlation is not always consistent with the other measures of tail dependence and seems to dependent upon the situation that is investigated.

The economic interpretation of the results is a bit more tricky, but it seems that the TDC is mainly influenced by the allocation percentages of the subindices within the THFI (see Table 2.1). Additionally one can see that hedge fund strategies that are primarily focused on shares (Emerging Markets and Long/Short Equity) show a rather high TDC in the upper tail, whereas Managed Futures seem to be rather independent of the index behavior. This actually makes sense, as these hedge funds generally employ systematic trading programs that are decoupled from the often mentioned herding behavior of traders in extreme situations.

Table 3.1.: This table shows $\hat{\lambda}$ for THFI vs. $\mathrm{THFI_{sub}}$ according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\mathrm{up}}$ and $\hat{\lambda}_{\mathrm{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
$\mathrm{THFI_{sub}}$	$\hat{\lambda}_{\mathrm{up}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,mean}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,90}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,95}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,99}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{up}}$	$\hat{\lambda}_{\mathrm{lo}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,mean}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,90}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,95}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,99}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{lo}}$
COA	0.2	0.21	0.32	0.35	0.42	0.59	0.08	0.14	0.16	0.27	0.31	0.4	0.62	0.08
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.77**	**0.77**	**1**	**1**	**1**	**1**	**0.17**	0.27	0.28	0.41	0.46	0.54	0.77	0.11
EMN	*0.03*	*0.03*	*0.05*	*0.06*	*0.09*	*0.14*	*0.02*	**0.62**	**0.64**	**1**	**1**	**1**	**1**	**0.29**
EVD	0.59	0.6	0.81	0.89	1	1	0.16	**0.64**	**0.67**	**1**	**1**	**1**	**1**	**0.24**
EDDI	0.35	0.37	0.53	0.57	0.68	0.99	0.12	0.57	0.6	0.95	1	1	1	0.21
EDMS	0.57	0.57	0.75	0.8	0.92	1	0.14	0.52	0.57	0.96	1	1	1	0.23
EDRA	0.1	0.12	0.18	0.21	0.26	0.42	0.05	0.24	0.27	0.42	0.49	0.66	0.98	0.13
FIA	0.26	0.27	0.41	0.45	0.54	0.82	0.1	0.2	0.22	0.39	0.46	0.62	1	0.13
GLM	**0.64**	**0.65**	**0.79**	**0.84**	**0.93**	**1**	**0.11**	**0.63**	**0.64**	**0.8**	**0.85**	**0.98**	**1**	**0.12**
LSE	**0.66**	**0.67**	**0.89**	**0.96**	**1**	**1**	**0.15**	0.56	0.57	0.76	0.83	0.98	1	0.15
MAF	*0.02*	*0.02*	*0.05*	*0.05*	*0.07*	*0.13*	*0.02*	*0.03*	*0.04*	*0.08*	*0.09*	*0.14*	*0.27*	*0.03*
MST	0.04	0.05	0.09	0.1	0.13	0.21	0.03	*0.08*	*0.11*	*0.21*	*0.27*	*0.39*	*0.95*	*0.08*

Table 3.2.: This table shows $\hat{\lambda}$ for THFI vs. THFI_{sub} according to the *parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	0.21	0.22	0.32	0.35	0.41	0.54	0.08	0.14	0.15	0.23	0.25	0.3	0.36	0.06
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.76**	**0.74**	**0.85**	**0.87**	**0.91**	**0.99**	**0.09**	0.21	0.22	0.32	0.34	0.4	0.47	0.08
EMN	*0.03*	*0.04*	*0.07*	*0.08*	*0.1*	*0.14*	*0.02*	0.37	0.37	0.56	0.61	0.7	0.87	0.14
EVD	0.6	0.6	0.7	0.73	0.79	0.97	0.08	**0.4**	**0.41**	**0.51**	**0.55**	**0.62**	**0.76**	**0.08**
EDDI	0.43	0.43	0.53	0.56	0.63	0.9	0.08	0.33	0.33	0.46	0.5	0.6	0.82	0.1
EDMS	0.67	0.66	0.75	0.78	0.84	0.97	0.08	0.39	0.39	0.5	0.53	0.61	0.72	0.09
EDRA	0.12	0.13	0.2	0.23	0.27	0.42	0.05	0.19	0.19	0.28	0.31	0.36	0.47	0.07
FIA	0.26	0.26	0.37	0.39	0.45	0.6	0.08	0.14	0.16	0.27	0.31	0.37	0.51	0.08
GLM	**0.83**	**0.81**	**0.88**	**0.9**	**0.93**	**0.99**	**0.06**	**0.51**	**0.51**	**0.61**	**0.64**	**0.68**	**0.77**	**0.08**
LSE	**0.77**	**0.76**	**0.86**	**0.88**	**0.92**	**0.98**	**0.08**	**0.45**	**0.45**	**0.54**	**0.57**	**0.62**	**0.75**	**0.08**
MAF	*0.02*	*0.03*	*0.05*	*0.06*	*0.08*	*0.12*	*0.02*	*0.03*	*0.04*	*0.07*	*0.09*	*0.12*	*0.16*	*0.03*
MST	0.06	0.06	0.1	0.12	0.15	0.2	0.03	*0.08*	*0.1*	*0.18*	*0.2*	*0.27*	*0.41*	*0.06*

Table 3.3.: This table shows $\hat{\lambda}$ for THFI vs. THFI_{sub} according to the non-parametric approach by *Schmidt and Stadtmüller* and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{U}}$ and $\hat{\lambda}_{\text{L}}$ are set italic and the three highest values bold. $\hat{\lambda}_{\text{U}}^{\text{EVT}}$ and $\hat{\lambda}_{\text{L}}^{\text{EVT}}$ are not shown in the table as the values are similar to those found for $\hat{\lambda}_{\text{U}}$ and $\hat{\lambda}_{\text{L}}$.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{U}}$	$\hat{\lambda}_{\text{U,mean}}^{\text{bs}}$	$\hat{\lambda}_{\text{U,90}}^{\text{bs}}$	$\hat{\lambda}_{\text{U,95}}^{\text{bs}}$	$\hat{\lambda}_{\text{U,99}}^{\text{bs}}$	$\hat{\lambda}_{\text{U,max}}^{\text{bs}}$	$\hat{\sigma}_{\text{U}}^{\text{bs}}$	$\hat{\lambda}_{\text{L}}$	$\hat{\lambda}_{\text{L,mean}}^{\text{bs}}$	$\hat{\lambda}_{\text{L,90}}^{\text{bs}}$	$\hat{\lambda}_{\text{L,95}}^{\text{bs}}$	$\hat{\lambda}_{\text{L,99}}^{\text{bs}}$	$\hat{\lambda}_{\text{L,max}}^{\text{bs}}$	$\hat{\sigma}_{\text{L}}^{\text{bs}}$
COA	0.33	0.29	0.43	0.48	0.52	0.62	0.1	0.43	0.41	0.52	0.57	0.67	0.81	0.1
DSB	*0*	*0.01*	*0*	*0.05*	*0.1*	*0.24*	*0.02*	*0.1*	*0.08*	*0.14*	*0.19*	*0.24*	*0.43*	*0.06*
EMM	0.52	0.5	0.62	0.67	0.71	0.81	0.1	0.33	0.4	0.52	0.57	0.64	0.76	0.1
EMN	0.43	0.39	0.52	0.57	0.62	0.81	0.11	*0.29*	*0.26*	*0.38*	*0.43*	*0.48*	*0.62*	*0.1*
EVD	0.48	0.47	0.62	0.62	0.69	0.81	0.1	**0.57**	**0.53**	**0.67**	**0.67**	**0.76**	**0.86**	**0.1**
EDDI	0.29	0.3	0.43	0.48	0.52	0.67	0.1	0.48	0.48	0.62	0.62	0.71	0.81	0.1
EDMS	**0.57**	**0.54**	**0.67**	**0.71**	**0.76**	**0.9**	**0.11**	0.48	0.47	0.62	0.62	0.71	0.86	0.11
EDRA	*0.24*	*0.22*	*0.33*	*0.38*	*0.48*	*0.62*	*0.1*	0.38	0.34	0.48	0.52	0.57	0.71	0.1
FIA	*0.24*	*0.24*	*0.38*	*0.38*	*0.48*	*0.62*	*0.09*	0.33	0.35	0.48	0.52	0.62	0.76	0.1
GLM	**0.71**	**0.7**	**0.81**	**0.86**	**0.9**	**1**	**0.1**	**0.57**	**0.6**	**0.71**	**0.76**	**0.81**	**0.95**	**0.1**
LSE	**0.57**	**0.59**	**0.71**	**0.74**	**0.81**	**0.9**	**0.09**	**0.62**	**0.57**	**0.71**	**0.76**	**0.81**	**0.9**	**0.11**
MAF	*0.14*	*0.18*	*0.29*	*0.33*	*0.38*	*0.52*	*0.09*	*0.14*	*0.16*	*0.29*	*0.29*	*0.38*	*0.48*	*0.08*
MST	0.33	0.31	0.43	0.48	0.52	0.67	0.1	0.38	0.36	0.48	0.52	0.62	0.67	0.1

Table 3.4.: This table shows $\hat{r}$ for THFI vs. $THFI_{sub}$ according to *Pearson*'s product-moment correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{r}$. The three lowest values for $\hat{r}_{up}$ and $\hat{r}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
$THFI_{sub}$	$\hat{r}_{up}$	$\hat{r}^{bs}_{up,mean}$	$\hat{r}^{bs}_{up,90}$	$\hat{r}^{bs}_{up,95}$	$\hat{r}^{bs}_{up,99}$	$\hat{r}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{r}_{lo}$	$\hat{r}^{bs}_{lo,mean}$	$\hat{r}^{bs}_{lo,90}$	$\hat{r}^{bs}_{lo,95}$	$\hat{r}^{bs}_{lo,99}$	$\hat{r}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
COA	0.09	0.11	0.43	0.51	0.64	0.67	0.25	0.21	0.18	0.53	0.62	0.72	0.83	0.3
DSB	*-0.26*	*-0.22*	*0.07*	*0.17*	*0.34*	*0.49*	*0.22*	*-0.28*	*-0.15*	*0.45*	*0.59*	*0.75*	*0.87*	*0.4*
EMM	**0.59**	**0.49**	**0.72**	**0.75**	**0.81**	**0.85**	**0.2**	**0.59**	**0.37**	**0.8**	**0.84**	**0.91**	**0.95**	**0.41**
EMN	0.24	0.24	0.55	0.62	0.73	0.86	0.24	*-0.08*	*-0.15*	*0.27*	*0.38*	*0.53*	*0.65*	*0.33*
EVD	0.49	0.49	0.69	0.73	0.8	0.88	0.18	0.57	0.34	0.81	0.85	0.92	0.96	0.46
EDDI	0.07	0.07	0.37	0.45	0.6	0.8	0.24	**0.63**	**0.42**	**0.84**	**0.88**	**0.94**	**0.97**	**0.42**
EDMS	**0.57**	**0.59**	**0.76**	**0.81**	**0.86**	**0.92**	**0.14**	0.52	0.3	0.77	0.82	0.89	0.96	0.45
EDRA	*-0.04*	*-0.03*	*0.29*	*0.36*	*0.48*	*0.62*	*0.25*	0.22	0.02	0.63	0.71	0.84	0.9	0.55
FIA	0.02	0.09	0.35	0.42	0.54	0.65	0.21	0.22	0.2	0.49	0.59	0.79	0.92	0.24
GLM	**0.79**	**0.78**	**0.88**	**0.9**	**0.94**	**0.97**	**0.09**	**0.65**	**0.7**	**0.86**	**0.89**	**0.93**	**0.97**	**0.11**
LSE	0.53	0.52	0.76	0.81	0.87	0.9	0.21	0.52	0.4	0.79	0.85	0.92	0.95	0.38
MAF	*-0.2*	*-0.16*	*0.12*	*0.2*	*0.33*	*0.5*	*0.21*	*-0.27*	*-0.21*	*0.26*	*0.33*	*0.46*	*0.7*	*0.31*
MST	0.05	0.11	0.36	0.41	0.52	0.68	0.21	0.05	0.04	0.36	0.46	0.63	0.86	0.24

Table 3.5.: This table shows the ranking of TDCs for THFI vs. THFI_{sub} for the top and the bottom group of the upper and lower tail of the return distribution. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Tail	Upper tail						Lower tail					
Group	Top group			Bottom group			Top group			Bottom group		
Ranking	1	2	3	1	2	3	1	2	3	1	2	3
Mal. and Sor., non-par, Gabaix	EMM	LSE	GLM	DSB	MAF	EMN	EVD	GLM	EMN	DSB	MAF	MST
Mal. and Sor., non-par, Hill	EMM	LSE	GLM	DSB	MAF	EMN	EVD	GLM	EMN	DSB	MAF	MST
Mal. and Sor., par, Gabaix	GLM	LSE	EMM	DSB	MAF	EMN	GLM	LSE	EVD	DSB	MAF	MST
Mal. and Sor., par, Hill	GLM	LSE	EMM	DSB	MAF	EMN	GLM	LSE	EVD	DSB	MAF	MST
Schmidt and Stadtmüller	GLM	LSE	EDMS	DSB	MAF	EDRA	LSE	GLM	EVD	DSB	MAF	EMN
Pearson's r	GLM	EMM	EDMS	DSB	MAF	EDRA	GLM	EDDI	EMM	DSB	MAF	EMN
Spearman's ρ	GLM	EMM	EDMS	EDRA	DSB	FIA	GLM	FIA	EMM	EDRA	EMN	EDMS
Kendall's τ	GLM	EDMS	EMM	EDRA	MAF	DSB	GLM	FIA	EMM	EDRA	EMN	EVD
# of different subindices (rank)	2	3	3	2	2	4	3	5	3	2	2	4
Max. # of same subindex (rank)	6	5	3	6	7	4	5	3	3	6	6	4
# of similar subindices (group)	4			5			7			7		
List of similar subindices (group)	EDMS			DSB	MAF		EDDI	FIA		DSB	EVD	
	EMM			EDRA			EMM	GLM		EDMS	MAF	
	GLM			EMN			EMN	LSE		EDRA	MST	
	LSE			FIA			EVD			EMN		

Table 3.6.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation THFI vs. THFI_{sub}. This values are calculated for the *upper tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{\text{np,G}}$	$\lambda_{\text{np,H}}$	$\lambda_{\text{p,G}}$	$\lambda_{\text{p,H}}$	λ_{S}	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{\text{np,G}}$)	1	1	0.9725	0.9725	0.7917	0.8407	0.7015	0.6667
Mal. and Sor., non-par, Hill ($\lambda_{\text{np,H}}$)	1	1	0.9725	0.9725	0.7917	0.8407	0.7015	0.6667
Mal. and Sor., par, Gabaix ($\lambda_{\text{p,G}}$)	0.9725	0.9725	1	1	0.8386	0.8626	0.7235	0.7052
Mal. and Sor., par, Hill ($\lambda_{\text{p,H}}$)	0.9725	0.9725	1	1	0.8386	0.8626	0.7235	0.7052
Schmidt and Stadtmüller (λ_{S})	0.7917	0.7917	0.8386	0.8386	1	0.971	0.8992	0.9046
Pearson's r	0.8407	0.8407	0.8626	0.8626	0.971	1	0.9519	0.9422
Spearman's ρ	0.7015	0.7015	0.7235	0.7235	0.8992	0.9519	1	0.9807
Kendall's τ	0.6667	0.6667	0.7052	0.7052	0.9046	0.9422	0.9807	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{\text{np,G}}$)	0	0	0	0	0.0013	0.0005	0.0075	0.0128
Mal. and Sor., non-par, Hill ($\lambda_{\text{np,H}}$)	0	0	0	0	0.0013	0.0005	0.0075	0.0128
Mal. and Sor., par, Gabaix ($\lambda_{\text{p,G}}$)	0	0	0	0	0.0003	0.0002	0.0052	0.0071
Mal. and Sor., par, Hill ($\lambda_{\text{p,H}}$)	0	0	0	0	0.0003	0.0002	0.0052	0.0071
Schmidt and Stadtmüller (λ_{S})	0.0013	0.0013	0.0003	0.0003	0	0	0	0
Pearson's r	0.0005	0.0005	0.0002	0.0002	0	0	0	0
Spearman's ρ	0.0075	0.0075	0.0052	0.0052	0	0	0	0
Kendall's τ	0.0128	0.0128	0.0071	0.0071	0	0	0	0

Table 3.7.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation THFI vs. THFI_{sub}. This values are calculated for the *lower tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{\text{np,G}}$	$\lambda_{\text{np,H}}$	$\lambda_{\text{p,G}}$	$\lambda_{\text{p,H}}$	λ_{S}	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{\text{np,G}}$)	1	0.9972	0.9271	0.9231	0.6685	0.7273	-0.1319	-0.1928
Mal. and Sor., non-par, Hill ($\lambda_{\text{np,H}}$)	0.9972	1	0.9462	0.9422	0.6801	0.7279	-0.1047	-0.1713
Mal. and Sor., par, Gabaix ($\lambda_{\text{p,G}}$)	0.9271	0.9462	1	0.9986	0.7981	0.7545	-0.0468	-0.1352
Mal. and Sor., par, Hill ($\lambda_{\text{p,H}}$)	0.9231	0.9422	0.9986	1	0.8066	0.7493	-0.0549	-0.1433
Schmidt and Stadtmüller (λ_{S})	0.6685	0.6801	0.7981	0.8066	1	0.741	0.0249	-0.0443
Pearson's r	0.7273	0.7279	0.7545	0.7493	0.741	1	0.2755	0.2445
Spearman's ρ	-0.1319	-0.1047	-0.0468	-0.0549	0.0249	0.2755	1	0.9807
Kendall's τ	-0.1928	-0.1713	-0.1352	-0.1433	-0.0443	0.2445	0.9807	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{\text{np,G}}$)	0	0	0	0	0.0125	0.0048	0.6693	0.5279
Mal. and Sor., non-par, Hill ($\lambda_{\text{np,H}}$)	0	0	0	0	0.0105	0.0048	0.7336	0.5758
Mal. and Sor., par, Gabaix ($\lambda_{\text{p,G}}$)	0	0	0	0	0.0011	0.0029	0.8794	0.6597
Mal. and Sor., par, Hill ($\lambda_{\text{p,H}}$)	0	0	0	0	0.0009	0.0032	0.8632	0.6406
Schmidt and Stadtmüller (λ_{S})	0.0125	0.0105	0.0011	0.0009	0	0.0038	0.9357	0.8857
Pearson's r	0.0048	0.0048	0.0029	0.0032	0.0038	0	0.3623	0.4208
Spearman's ρ	0.6693	0.7336	0.8794	0.8632	0.9357	0.3623	0	0
Kendall's τ	0.5279	0.5758	0.6597	0.6406	0.8857	0.4208	0	0

3.3. THFI vs. Hedge Funds of ISPartners ($\mathbf{HF_{ISP}}$)

In this section we investigate the tail dependence between the THFI and the various hedge funds (HF_{ISP}) that compose the FoHF of ISPartners. In this constellation the THFI acts as explaining factor for the i-th hedge fund, with the factor model looking as follows: $HF_{ISP,i} = \beta_i \cdot \text{THFI} + \epsilon_i$, with $i = 1, \ldots, 26$. What we want to calculate here, is the probability of large losses or gains of the various hedge funds of ISPartners conditional on large losses or gains of the THFI.

3.3.1. Moving Threshold Graphs

The calculation of the TDC for this relation is subject to large variations due to the very small samples. In this case I could only analyze 53 observations per hedge fund in total. If we look at Figure 3.14, we can see that values for the tail index estimators decrease with increasing k and seem to level off between a range of ca. 2 to 6 in the upper tail and 0.5 and 3 in the lower tail. This result is difficult to interpret, as it is assumed that the tails of return distributions show power-law behavior with a tail index of roughly 2 to 3.

Looking at Figure 3.15 we can see that the method of Poon et al. does not perform well. Either the values for the TDC lie close together, as is the case for χ in the lower and upper tail when applying the Hill estimator or, when applying the Gabaix estimator, the results for $\overline{\chi}$ are strongly fluctuating leading to subsequent fluctuations of χ. Based on this behavior I conclude that the method of Poon et al. is not suited to calculate the TDC for small samples.

The two methods developed by Malevergne and Sornette show a different behavior. When having a closer look at Figure 3.16 presenting the results based on the non-parametric approach, it is difficult to find a stable regime for any value of k. However, if we look at Figure 3.17 showing the results based on the parametric approach, we can see that λ starts to become stable between 0.2 and 0.3 for the ratio k/N. This more stable behavior of the parametric approach might be explained by the fact that the parametric approach uses the mean values for two of its parameters, namely C_Y and C_ϵ, whereas the non-parametric approach consists only of one parameter (l) that is smoothed by taking its mean over the whole tail.

Figure 3.18, presenting the results for the TDC based on the method developed by Schmidt and Stadtmüller, supports the finding that λ becomes more or less stable between values of 0.2 and 0.3 for the ratio k/N (similarly for λ^{EVT}).

Based on the above visual inspection of the figures and the trade off between a low bias (low k) vs. a low variance (high k) for the tail index estimates, I decided to calculate the detailed results for $k/N = 0.2$. This relates to 11 observations for the upper as well as for the lower tail of the return distribution.

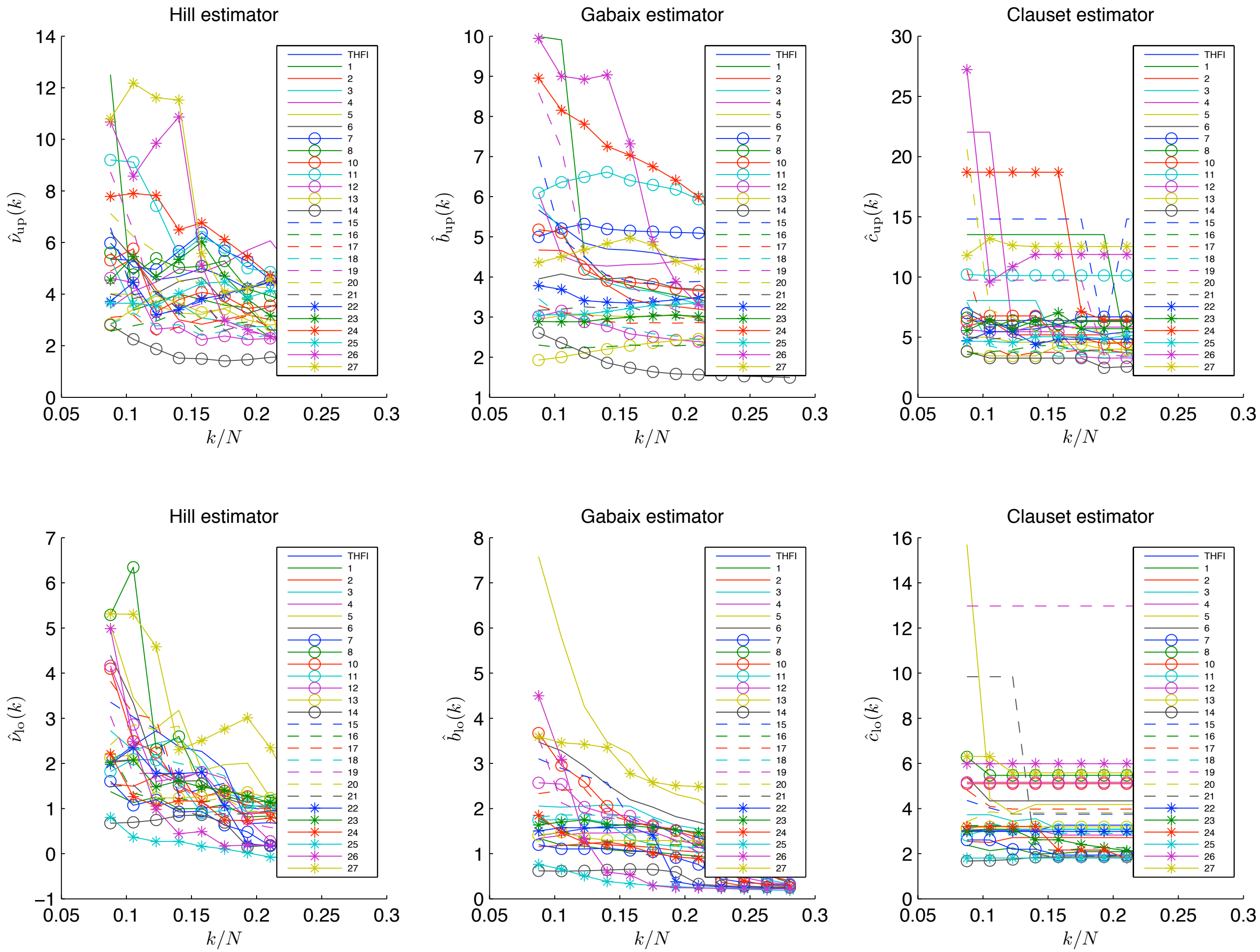
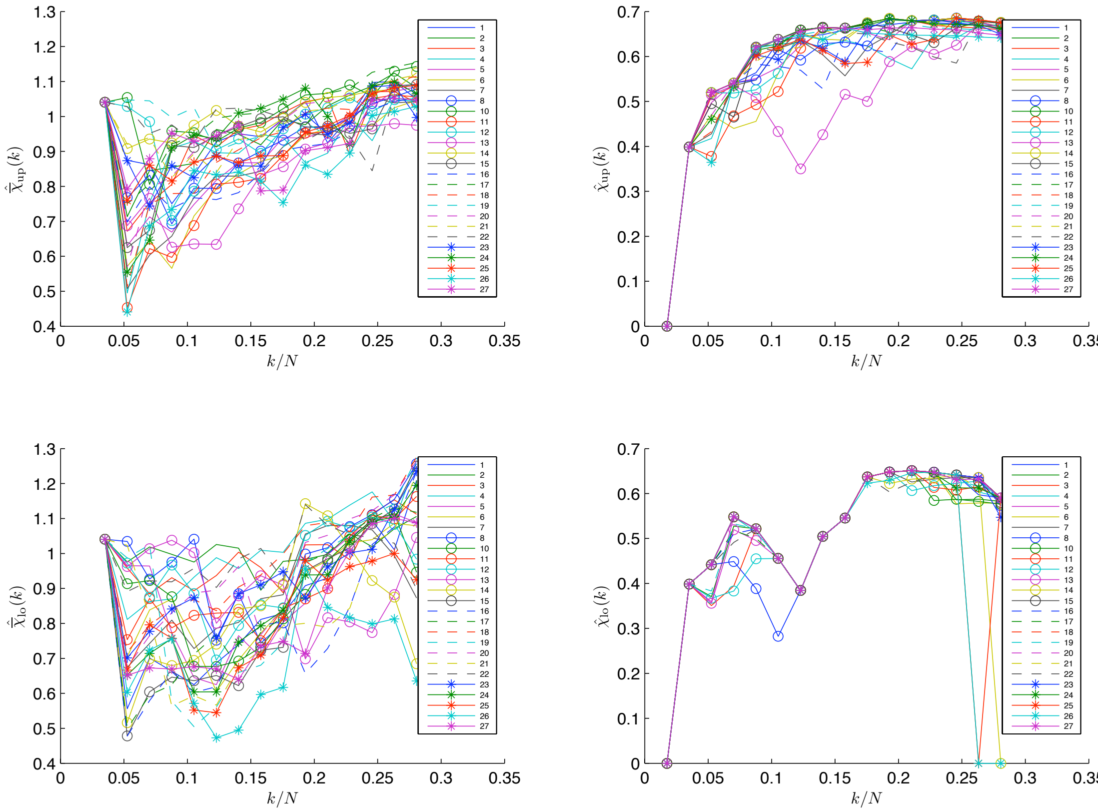
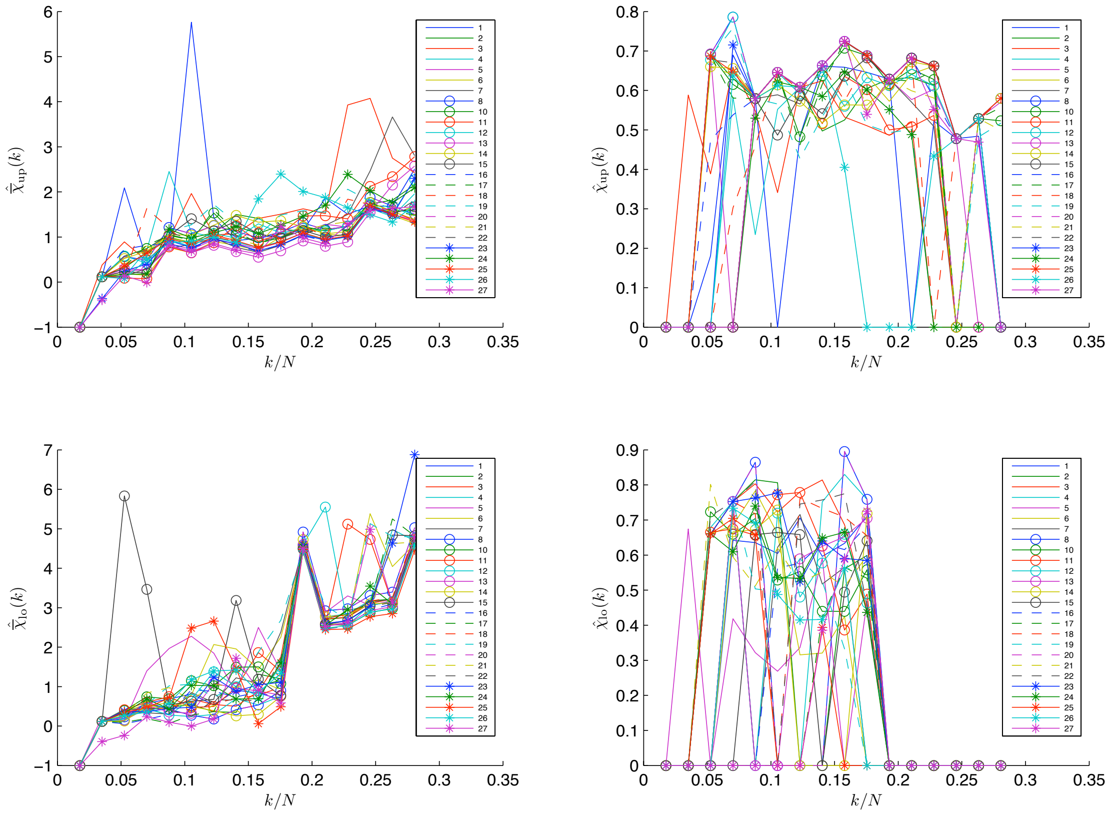


Figure 3.14.: The above figure shows the estimates for three *tail index* estimator for the THFI and the single hedge funds of the $\mathrm{FoHF}_{\mathrm{ISP}}$. From left to right we have the Hill, the Gabaix, and the Clauset estimator. The first row contains the tail indices for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) $\overline{\chi}$ and χ with Hill estimator



(b) $\overline{\chi}$ and χ with Gabaix estimator

Figure 3.15.: $\overline{\chi}$ and χ calculated for THFI vs. HF_{ISP} applying the Hill estimator in sub-figure (a) and the Gabaix estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

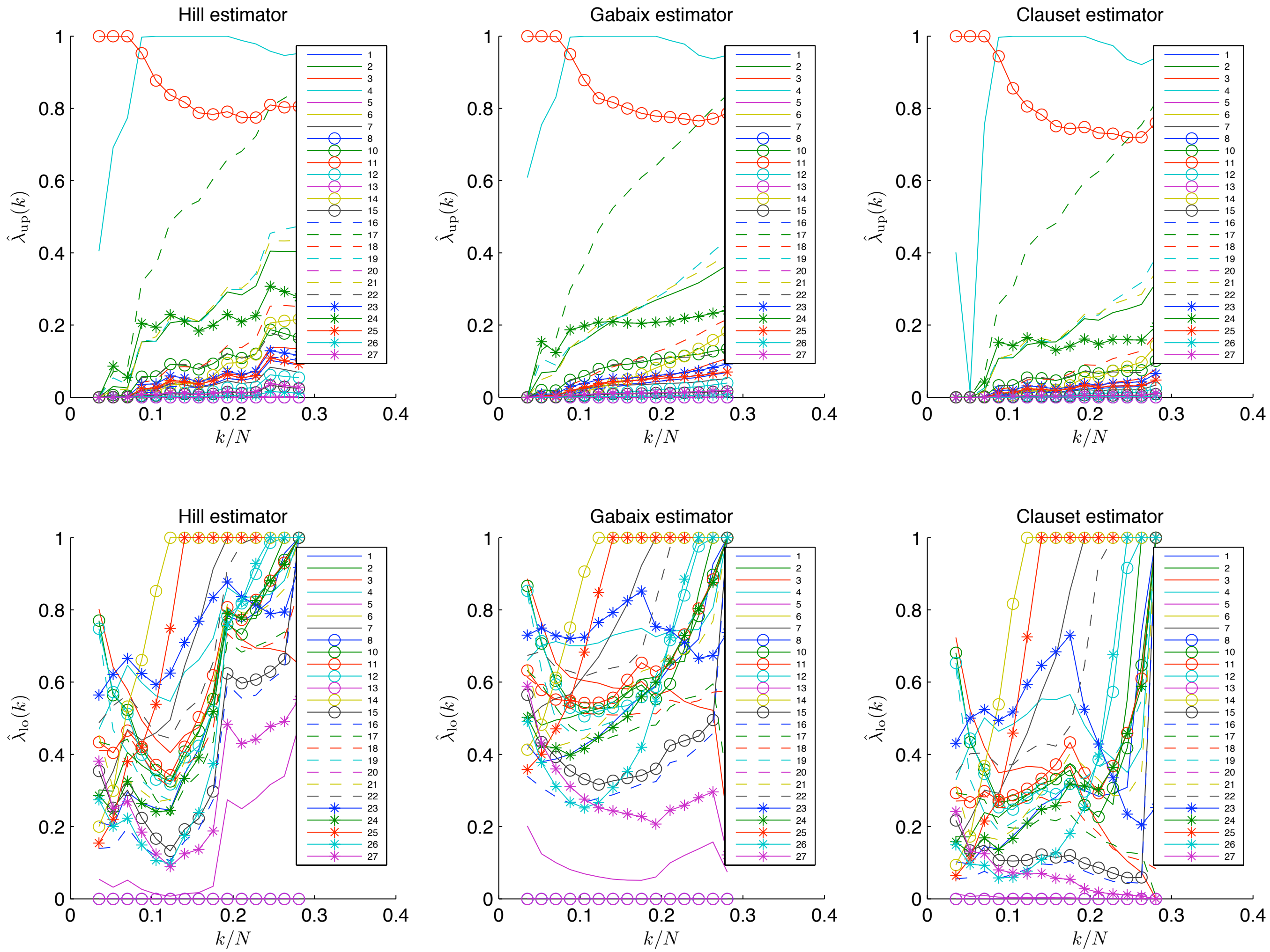


Figure 3.16.: λ for THFI vs. $\mathrm{HF}_{\mathrm{ISP}}$ with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

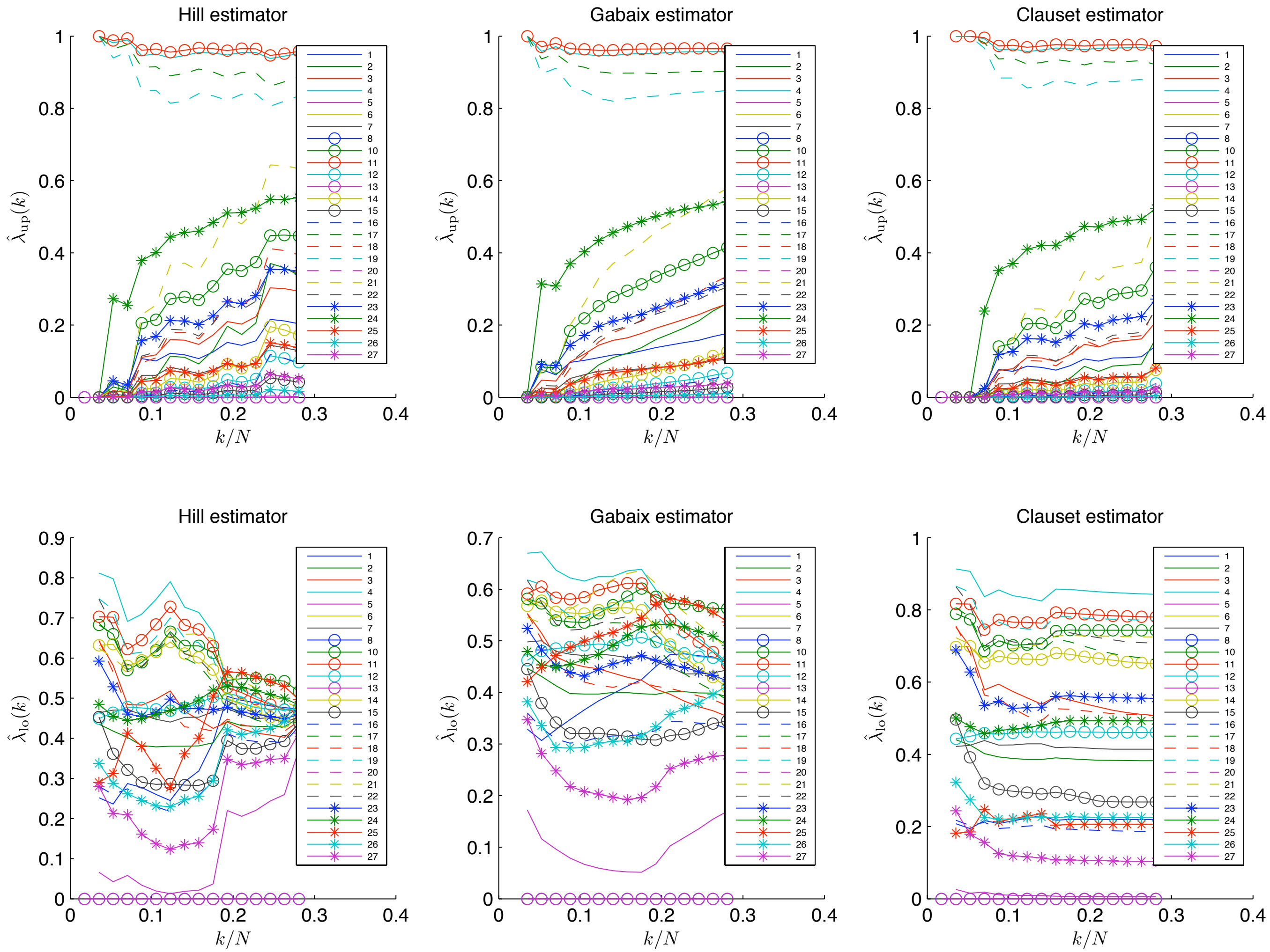
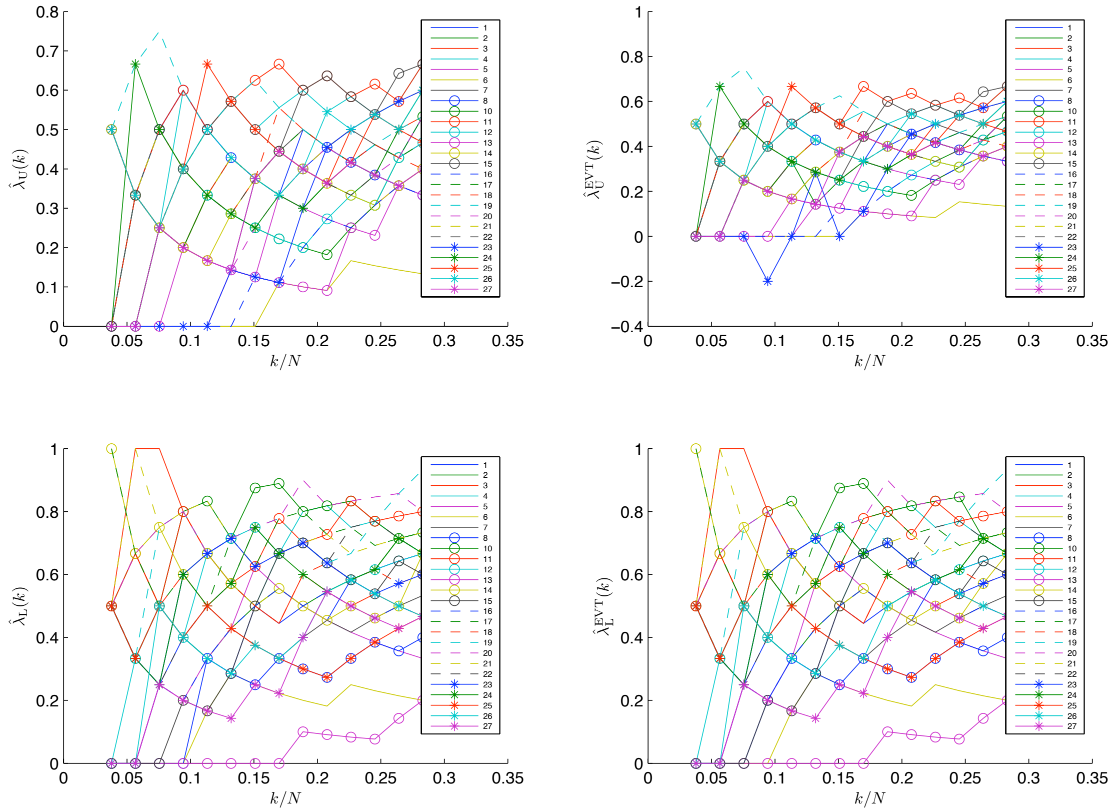
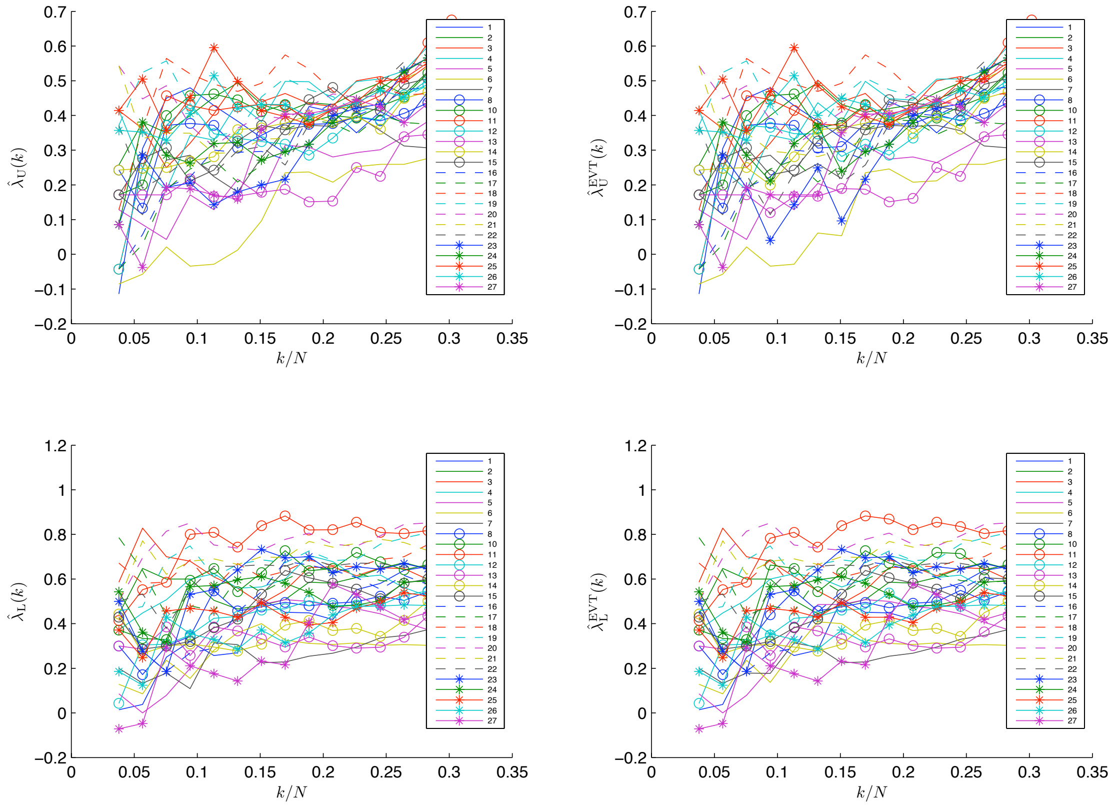


Figure 3.17.: λ for THFI vs. HF_{ISP} with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) original results



(b) smoothed results

Figure 3.18.: λ for THFI vs. HF_{ISP} by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

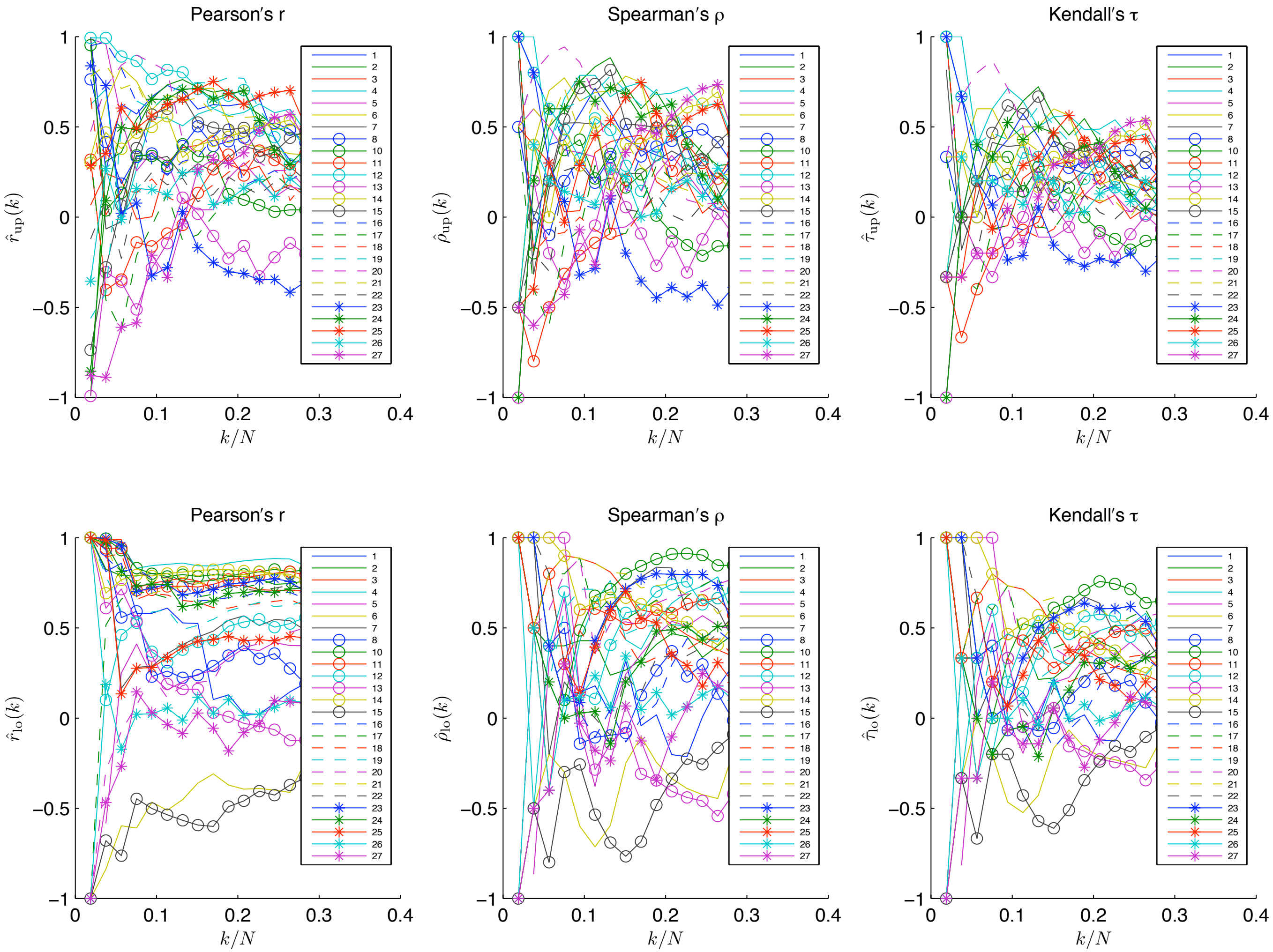


Figure 3.19.: TDC for THFI vs. HF_{ISP} based on linear measures of correlation. From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

3.3.2. Detailed Results

In this section the detailed results are presented and their uncertainty is tested with a bootstrap procedure as described in section 2.4. To make the tables easier to read, I again highlighted the three (if there were more similar TDCs, e.g. zeros, sometimes also more) highest and lowest values of the TDCs for the upper and lower tail. In the following paragraphs I will focus my analysis on these highest (top group) and lowest (bottom group) values, as I assume that they are of most interest.

The detailed results in this section are all calculated for the ratio $k/N = 0.2$, corresponding to 11 data points for k as $N = 53$. As the estimates of λ are almost similar for the parametric and the non-parametric approach of Malevergne and Sornette when applying the Hill or the Gabaix estimator, I moved the results that are based on the use of the Hill estimator in appendix A.4. I do not present detailed results for the Clauset and Huisman estimator, as I do not expect to gain additional insight from them. Due to the findings that the method of Poon et al. does not produce useful results, I do not present them here.

If we compare the results obtained by applying the non-parametric and the parametric method of Malevergne and Sornette with the Gabaix (Table 3.8) estimator, we can see that the top and the bottom group in the upper tail are similar. Looking at the lower tail we find that two subindices of the top group are different but the bottom group is similar (for overview see Table 3.12).

Having a closer look at the results derived with the method by Schmidt and Stadtmüller (Table 3.10), we can see some similarities with the results presented above (e.g. HF_{ISP}-4 in the top group of the upper tail), but there are also remarkable difference, as for example HF_{ISP}-26 that is in the top group of the upper tail but has very low values for the TDC when calculated with the methods of Malevergne and Sornette.

If we compare the above results with results based on linear measures of correlations, we can see that the top and bottom groups are partly similar, but also that there seem to be some differences between non-linear and linear measures of tail dependence. In Table 3.11, the results for the Pearson product-moment correlation r are given. I decided to show only the results for Pearson's r in the results and discussion chapter, as the behavior of r is more stable than the behavior of Spearman's ρ or Kendall's τ.

To compare the different results and investigate whether they differ significantly I performed a Kruskal-Wallis test. The resulting p-value for the upper tail is $1 \cdot 10^5$, meaning that the null hypothesis that all medians are the same can be reject. The situation is similar for the lower tail, where the p-value is $2 \cdot 10^{-8}$. Also among the non-linear measures of tail dependence the null hypothesis can be rejected as the p-value for the upper tail is 0.0001 and for the lower tail $1 \cdot 10^{-7}$. The situation looks a little bit different for the linear measures of tail dependence where the p-value for the upper tail is 0.05 and for the lower tail 0.04. Hence, we have a rather different situation compared to section 3.2, where the null hypothesis could not be rejected in the upper tail and the p-value in the lower tail was rather large within the same kind of tail dependence measures.

To better understand what the above finding means, I calculated Spearman's rank correlation among the various measures of tail dependence. The results for the upper tail are displayed in Table 3.13. Here we can see that ρ is high among the methods from Malevergne and Sornette, but rather small, if compared with the other measures of tail dependence. The method of Schmidt and Stadtmüller has low correlation with all other measure of tail dependence. The correlation among the linear measures of tail dependence is comparably high, but the p-values are high as well. Hence, the null hypothesis that the correlations are zero cannot be rejected. The results for the lower tail (Table 3.14) are slightly different, as the correlations among all methods are higher than in the upper tail; with the exception of the correlations related to the method by Schmidt and Stadtmüller. When looking at the single TDCs calculated with the method by Schmidt and Stadtmüller, we can see that there are many TDCs that have similar values (e.g. 0.45). I assume that this is the reason for the low p-value in the Kruskal-Wallis test and the comparably low correlation coefficients found here. However, it is important to keep in mind that all the calculations performed here are subject to large uncertainties due to small samples.

The economic interpretation of the above findings is again rather challenging. As almost all strategies can be found in any of the four groups (top and bottom group of upper tail, and top and bottom group of lower tail, respectively), I would like to leave the economic interpretation of the results up to the respective experts.

Table 3.8.: This table shows $\hat{\lambda}$ for THFI vs. HF_{ISP} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.04	0.05	0.1	0.13	0.18	0.36	0.04	0.64	0.63	1	1	1	1	0.28
2	0.31	0.36	0.65	0.78	1	1	0.21	0.7	0.69	1	1	1	1	0.22
3	0.05	0.08	0.17	0.23	0.38	0.85	0.08	0.63	0.59	0.82	0.91	1	1	0.17
4	**1**	**0.87**	**1**	**1**	**1**	**1**	**0.19**	0.78	0.74	1	1	1	1	0.2
5	*0*	*0*	*0*	*0*	*0.01*	*0.02*	*0*	*0.1*	*0.13*	*0.25*	*0.29*	*0.33*	*1*	*0.1*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	0.03	0.04	0.09	0.11	0.16	0.25	0.04	**0.95**	**0.78**	**1**	**1**	**1**	**1**	**0.25**
8	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.05*	*0.01*	0.35	0.37	0.61	0.78	1	1	0.2
10	0.11	0.14	0.27	0.32	0.45	0.95	0.1	0.64	0.69	1	1	1	1	0.22
11	**0.77**	**0.75**	**1**	**1**	**1**	**1**	**0.22**	0.69	0.69	1	1	1	1	0.22
12	0.02	0.03	0.08	0.09	0.14	0.46	0.03	0.67	0.69	1	1	1	1	0.25
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	0.09	0.15	0.36	0.51	0.98	1	0.17	**1**	**0.96**	**1**	**1**	**1**	**1**	**0.11**
15	0.01	0.02	0.04	0.05	0.07	0.13	0.02	0.43	0.44	0.81	1	1	1	0.25
16	0.04	0.08	0.17	0.22	0.35	0.98	0.08	0.41	0.44	0.72	0.9	1	1	0.21
17	**0.61**	**0.63**	**1**	**1**	**1**	**1**	**0.29**	0.55	0.53	0.75	0.83	1	1	0.18
18	0.12	0.17	0.34	0.44	0.73	1	0.14	0.58	0.56	0.85	1	1	1	0.21
19	0.29	0.35	0.6	0.73	1	1	0.19	0.69	0.66	1	1	1	1	0.23
20	0.1	0.12	0.21	0.25	0.33	0.6	0.07	0.65	0.6	0.86	0.96	1	1	0.21
21	0.31	0.37	0.65	0.79	1	1	0.21	0.72	0.69	1	1	1	1	0.21
22	0.12	0.14	0.25	0.3	0.43	1	0.09	0.88	0.78	1	1	1	1	0.23
23	0.05	0.07	0.15	0.19	0.3	1	0.07	0.75	0.72	1	1	1	1	0.21
24	0.23	0.26	0.43	0.48	0.6	1	0.12	0.68	0.67	1	1	1	1	0.25
25	0.06	0.07	0.13	0.16	0.24	0.35	0.05	**1**	**0.9**	**1**	**1**	**1**	**1**	**0.22**
26	*0*	*0.01*	*0.02*	*0.03*	*0.05*	*0.14*	*0.01*	0.63	0.64	1	1	1	1	0.33
27	0.01	0.01	0.03	0.04	0.08	0.18	0.02	0.25	0.27	0.45	0.53	0.76	1	0.15

Table 3.9.: This table shows $\hat{\lambda}$ for THFI vs. HF_{ISP} according to the *parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.13	0.15	0.28	0.33	0.42	0.63	0.09	0.48	0.43	0.64	0.71	0.85	0.99	0.18
2	0.17	0.29	0.68	0.78	0.9	0.99	0.25	0.42	0.41	0.52	0.56	0.65	0.83	0.09
3	0.18	0.21	0.37	0.43	0.56	0.87	0.12	0.41	0.41	0.55	0.59	0.65	0.82	0.11
4	**0.95**	**0.93**	**0.99**	**0.99**	**1**	**1**	**0.06**	**0.57**	**0.56**	**0.72**	**0.77**	**0.86**	**0.98**	**0.12**
5	*0*	*0*	*0*	*0.01*	*0.02*	*0.04*	*0*	*0.1*	*0.13*	*0.24*	*0.27*	*0.33*	*0.46*	*0.08*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	0.07	0.08	0.16	0.2	0.28	0.5	0.06	0.45	0.43	0.61	0.67	0.79	1.04	0.15
8	*0*	*0.01*	*0.01*	*0.02*	*0.03*	*0.07*	*0.01*	0.25	0.26	0.4	0.45	0.59	0.32	0.12
10	0.35	0.37	0.63	0.7	0.81	0.98	0.19	**0.57**	**0.56**	**0.76**	**0.81**	**0.9**	**0.99**	**0.15**
11	**0.96**	**0.95**	**0.99**	**1**	**1**	**1**	**0.05**	0.56	0.53	0.7	0.74	0.81	0.97	0.13
12	0.04	0.07	0.17	0.23	0.36	0.94	0.08	0.5	0.48	0.66	0.71	0.79	0.93	0.14
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	0.08	0.12	0.27	0.36	0.6	0.97	0.12	0.51	0.49	0.62	0.66	0.79	0.93	0.1
15	0.01	0.03	0.06	0.09	0.13	0.29	0.03	0.33	0.32	0.46	0.5	0.61	0.86	0.12
16	0.03	0.08	0.2	0.31	0.54	0.9	0.11	0.36	0.34	0.47	0.5	0.57	0.87	0.12
17	**0.89**	**0.87**	**0.95**	**0.97**	**0.98**	**1**	**0.08**	0.48	0.47	0.6	0.63	0.67	0.79	0.1
18	0.23	0.3	0.64	0.73	0.85	0.97	0.22	0.42	0.41	0.56	0.61	0.68	0.88	0.12
19	0.84	0.81	0.93	0.95	0.98	0.99	0.11	0.53	0.51	0.67	0.71	0.78	0.99	0.12
20	0.27	0.29	0.53	0.61	0.74	0.94	0.17	0.51	0.48	0.63	0.67	0.73	0.86	0.11
21	0.48	0.51	0.89	0.94	0.98	1	0.28	**0.59**	**0.56**	**0.73**	**0.76**	**0.85**	**1**	**0.13**
22	0.25	0.28	0.52	0.6	0.74	0.89	0.18	0.51	0.5	0.64	0.68	0.75	0.92	0.11
23	0.19	0.22	0.42	0.49	0.63	0.82	0.14	0.46	0.45	0.62	0.68	0.78	0.91	0.14
24	0.53	0.51	0.76	0.81	0.89	0.99	0.19	0.54	0.5	0.69	0.74	0.83	0.98	0.16
25	0.09	0.1	0.19	0.23	0.32	0.53	0.07	**0.59**	**0.51**	**0.73**	**0.8**	**0.92**	**0.99**	**0.19**
26	0.01	0.01	0.03	0.04	0.06	0.15	0.01	0.37	0.36	0.52	0.57	0.69	1.21	0.14
27	0.02	0.02	0.05	0.07	0.11	0.24	0.02	0.23	0.24	0.36	0.39	0.47	0.68	0.1

Table 3.10.: This table shows $\hat{\lambda}$ for THFI vs. HF_{ISP} according to the non-parametric approach by *Schmidt and Stadtmüller* and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_U$ and $\hat{\lambda}_L$ are set italic and the three highest values bold. $\hat{\lambda}_U^{EVT}$ and $\hat{\lambda}_L^{EVT}$ are not shown in the table as the values are similar to those found for $\hat{\lambda}_U$ and $\hat{\lambda}_L$.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_U$	$\hat{\lambda}^{bs}_{U,mean}$	$\hat{\lambda}^{bs}_{U,90}$	$\hat{\lambda}^{bs}_{U,95}$	$\hat{\lambda}^{bs}_{U,99}$	$\hat{\lambda}^{bs}_{U,max}$	$\hat{\sigma}^{bs}_U$	$\hat{\lambda}_L$	$\hat{\lambda}^{bs}_{L,mean}$	$\hat{\lambda}^{bs}_{L,90}$	$\hat{\lambda}^{bs}_{L,95}$	$\hat{\lambda}^{bs}_{L,99}$	$\hat{\lambda}^{bs}_{L,max}$	$\hat{\sigma}^{bs}_L$
1	0.45	0.46	0.64	0.73	0.82	1	0.14	0.45	0.52	0.73	0.73	0.82	1	0.14
2	0.36	0.35	0.55	0.64	0.73	0.91	0.15	0.55	0.54	0.73	0.73	0.82	1	0.14
3	0.36	0.47	0.64	0.73	0.82	1	0.15	0.55	0.49	0.64	0.73	0.91	1	0.15
4	**0.55**	**0.56**	**0.73**	**0.82**	**0.91**	**1**	**0.14**	**0.82**	**0.71**	**0.91**	**0.91**	**1**	**1**	**0.14**
5	0.27	0.28	0.45	0.55	0.64	0.82	0.13	0.45	0.43	0.64	0.73	0.82	1	0.17
6	*0.09*	*0.09*	*0.18*	*0.27*	*0.36*	*0.55*	*0.1*	*0.18*	*0.19*	*0.36*	*0.36*	*0.55*	*0.73*	*0.12*
7	0.45	0.45	0.64	0.73	0.82	1	0.15	0.45	0.43	0.64	0.64	0.82	1	0.15
8	0.45	0.4	0.64	0.64	0.73	0.91	0.15	*0.27*	*0.32*	*0.45*	*0.55*	*0.64*	*0.82*	*0.14*
10	*0.18*	*0.29*	*0.45*	*0.55*	*0.64*	*0.91*	*0.15*	**0.82**	**0.78**	**0.91**	**1**	**1**	**1**	**0.13**
11	**0.64**	**0.63**	**0.82**	**0.82**	**0.91**	**1**	**0.14**	0.73	0.71	0.91	0.91	1	1	0.13
12	0.27	0.31	0.55	0.55	0.73	0.91	0.15	0.64	0.63	0.82	0.82	0.91	1	0.14
13	*0.09*	*0.2*	*0.36*	*0.45*	*0.55*	*0.73*	*0.13*	*0.09*	*0.08*	*0.18*	*0.27*	*0.36*	*0.55*	*0.09*
14	0.36	0.38	0.55	0.64	0.73	0.91	0.15	0.45	0.5	0.73	0.73	0.82	1	0.15
15	**0.64**	**0.57**	**0.73**	**0.82**	**0.91**	**1**	**0.14**	0.64	0.56	0.73	0.82	0.91	1	0.15
16	0.27	0.26	0.45	0.55	0.64	0.73	0.15	0.55	0.48	0.64	0.73	0.82	0.91	0.14
17	0.45	0.45	0.64	0.73	0.82	1	0.15	0.73	0.7	0.82	0.91	1	1	0.13
18	0.45	0.45	0.64	0.73	0.82	0.91	0.15	0.64	0.55	0.73	0.73	0.82	1	0.14
19	0.45	0.51	0.73	0.73	0.91	1	0.16	0.64	0.72	0.91	0.91	1	1	0.14
20	0.45	0.39	0.55	0.64	0.73	0.91	0.15	**0.82**	**0.78**	**0.91**	**1**	**1**	**1**	**0.12**
21	0.45	0.46	0.64	0.73	0.82	1	0.15	0.73	0.67	0.82	0.91	0.91	1	0.13
22	0.36	0.39	0.55	0.64	0.73	0.91	0.14	0.64	0.67	0.82	0.91	0.91	1	0.14
23	0.45	0.41	0.64	0.73	0.82	1	0.18	0.64	0.6	0.82	0.82	0.91	1	0.15
24	0.36	0.41	0.64	0.64	0.82	0.91	0.16	0.55	0.6	0.82	0.82	0.91	1	0.14
25	0.36	0.47	0.64	0.73	0.82	1	0.15	*0.27*	*0.36*	*0.55*	*0.64*	*0.73*	*0.91*	*0.14*
26	**0.55**	**0.48**	**0.64**	**0.73**	**0.82**	**0.91**	**0.14**	0.55	0.47	0.64	0.73	0.82	1	0.15
27	0.36	0.39	0.55	0.64	0.73	0.91	0.15	0.55	0.37	0.55	0.64	0.73	0.91	0.15

Table 3.11.: This table shows $\hat{r}$ for THFI vs. HF_{ISP} according to *Pearson*'s product-moment correlation and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{r}$. The three lowest values for $\hat{r}_{up}$ and $\hat{r}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{r}_{up}$	$\hat{r}^{bs}_{up,mean}$	$\hat{r}^{bs}_{up,90}$	$\hat{r}^{bs}_{up,95}$	$\hat{r}^{bs}_{up,99}$	$\hat{r}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{r}_{lo}$	$\hat{r}^{bs}_{lo,mean}$	$\hat{r}^{bs}_{lo,90}$	$\hat{r}^{bs}_{lo,95}$	$\hat{r}^{bs}_{lo,99}$	$\hat{r}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.62	0.54	0.8	0.84	0.9	1	0.28	0.13	0.14	0.66	0.75	0.86	0.94	0.39
2	**0.71**	**0.61**	**0.8**	**0.84**	**0.9**	**1**	**0.2**	0.78	0.53	0.97	0.98	0.99	1	0.53
3	0.3	0.25	0.7	0.77	0.88	0.99	0.36	0.73	0.66	0.9	0.92	0.96	1	0.27
4	0.56	0.55	0.79	0.83	0.9	1	0.2	**0.86**	**0.71**	**0.97**	**0.98**	**0.99**	**1**	**0.33**
5	0.22	0.14	0.54	0.62	0.75	0.97	0.33	0.33	0.39	0.65	0.75	0.85	0.99	0.21
6	0.52	0.49	0.8	0.85	0.9	0.97	0.28	*-0.37*	*-0.35*	*0*	*0.12*	*0.35*	*0.94*	*0.26*
7	0.33	0.31	0.59	0.65	0.79	0.99	0.24	0.54	0.49	0.88	0.94	0.99	1	0.29
8	0.42	0.4	0.74	0.8	0.89	0.98	0.29	0.35	0.23	0.64	0.74	0.86	0.99	0.35
10	0.13	0.15	0.53	0.6	0.74	0.93	0.32	**0.79**	**0.78**	**0.93**	**0.95**	**0.98**	**1**	**0.18**
11	0.35	0.23	0.64	0.73	0.88	0.99	0.33	0.77	0.69	0.94	0.96	0.98	1	0.24
12	0.67	0.5	0.86	0.9	0.94	0.99	0.37	0.51	0.55	0.76	0.8	0.88	0.98	0.16
13	*-0.23*	*-0.15*	*0.24*	*0.34*	*0.55*	*0.92*	*0.29*	0.01	-0.06	0.25	0.35	0.54	0.86	0.24
14	0.44	0.42	0.78	0.85	0.92	0.99	0.31	**0.82**	**0.75**	**0.95**	**0.96**	**0.98**	**1**	**0.25**
15	0.48	0.47	0.73	0.79	0.86	0.99	0.23	*-0.49*	*-0.39*	*0.14*	*0.32*	*0.68*	*0.94*	*0.35*
16	0.52	0.27	0.7	0.76	0.87	0.98	0.35	0.67	0.49	0.9	0.93	0.97	1	0.48
17	0.11	0.12	0.5	0.6	0.77	0.98	0.29	0.72	0.74	0.89	0.92	0.95	0.99	0.15
18	0.25	0.2	0.72	0.84	0.92	0.99	0.39	0.61	0.53	0.85	0.89	0.95	1	0.3
19	**0.77**	**0.63**	**0.88**	**0.9**	**0.94**	**0.98**	**0.25**	0.59	0.59	0.8	0.85	0.92	1	0.19
20	0.45	0.33	0.77	0.84	0.92	0.98	0.33	0.36	0.45	0.81	0.86	0.91	0.99	0.26
21	0.56	0.46	0.79	0.83	0.88	0.96	0.36	0.78	0.79	0.92	0.94	0.97	1	0.14
22	0.22	0.13	0.62	0.71	0.83	0.95	0.37	0.74	0.55	0.89	0.92	0.96	1	0.4
23	*-0.3*	*-0.3*	*0.08*	*0.21*	*0.45*	*0.78*	*0.28*	0.74	0.65	0.92	0.96	0.98	1	0.28
24	**0.68**	**0.57**	**0.82**	**0.85**	**0.9**	**1**	**0.26**	0.69	0.51	0.88	0.91	0.95	0.99	0.44
25	**0.68**	**0.64**	**0.88**	**0.92**	**0.96**	**0.99**	**0.21**	0.45	0.37	0.85	0.9	0.97	1	0.34
26	*0.09*	*0.15*	*0.6*	*0.72*	*0.89*	*0.97*	*0.35*	0.1	0.06	0.34	0.43	0.65	0.88	0.24
27	0.3	0.31	0.69	0.77	0.86	0.99	0.34	*-0.18*	*0.01*	*0.43*	*0.55*	*0.74*	*0.95*	*0.3*

Table 3.12.: This table shows the ranking of TDCs for THFI vs. HF_{ISP} for the top and the bottom group of the upper and lower tail of the return distribution. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Tail	Upper tail						Lower tail					
Group	Top group			Bottom group			Top group			Bottom group		
Ranking	1	2	3	1	2	3	1	2	3	1	2	3
Mal. and Sor., non-par, Gabaix	4	11	17	6	13	5	14	25	7	6	13	5
Mal. and Sor., non-par, Hill	4	11	17	6	13	5	14	25	7	6	13	5
Mal. and Sor., par, Gabaix	11	4	17	6	13	5	21	25	10	6	13	5
Mal. and Sor., par, Hill	11	4	17	6	13	5	25	21	10	6	13	5
Schmidt and Stadtmüller	11	15	4	6	13	10	10	20	4	13	6	8
Pearson's r	19	2	25	23	13	26	4	14	10	15	6	27
Spearman's ρ	19	2	4	23	13	10	10	7	23	15	13	27
Kendall's τ	2	19	4	23	13	10	10	7	23	15	27	13
# of different hedge funds (rank)	4	5	3	2	1	3	5	5	4	3	3	4
Max. # of same hedge fund (rank)	3	2	4	5	8	4	3	3	3	4	5	4
# of similar hedge funds (group)	7			6			8			6		
List of similar hedge funds (group)	2	17		5	23		4	20		5	15	
	4	19		6	26		7	21		6	27	
	11	25		10			10	23		8		
	15			13			14	25		13		

Table 3.13.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation THFI vs. HF_{ISP}. This values are calculated for the *upper tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	0.9959	0.9427	0.9551	0.3332	0.2861	0.3126	0.3239
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0.9959	1	0.9455	0.9572	0.3613	0.2698	0.292	0.3044
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.9427	0.9455	1	0.9979	0.393	0.1738	0.1754	0.1729
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.9551	0.9572	0.9979	1	0.3994	0.1967	0.2002	0.2004
Schmidt and Stadtmüller (λ_S)	0.3332	0.3613	0.393	0.3994	1	0.0187	0.1497	0.1918
Pearson's r	0.2861	0.2698	0.1738	0.1967	0.0187	1	0.8128	0.7648
Spearman's ρ	0.3126	0.292	0.1754	0.2002	0.1497	0.8128	1	0.9791
Kendall's τ	0.3239	0.3044	0.1729	0.2004	0.1918	0.7648	0.9791	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0.0963	0.1565	0.12	0.1065
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0.0698	0.1826	0.1478	0.1305
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0.047	0.3958	0.3915	0.3984
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0.0432	0.3355	0.3269	0.3263
Schmidt and Stadtmüller (λ_S)	0.0963	0.0698	0.047	0.0432	0	0.9276	0.4655	0.3479
Pearson's r	0.1565	0.1826	0.3958	0.3355	0.9276	0	0	0
Spearman's ρ	0.12	0.1478	0.3915	0.3269	0.4655	0	0	0
Kendall's τ	0.1065	0.1305	0.3984	0.3263	0.3479	0	0	0

Table 3.14.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation THFI vs. HF_{ISP}. This values are calculated for the *lower tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	0.9995	0.7798	0.7673	0.2686	0.6497	0.6036	0.6235
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0.9995	1	0.7874	0.7749	0.2745	0.6462	0.6074	0.6238
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.7798	0.7874	1	0.9947	0.5675	0.6655	0.6498	0.6297
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.7673	0.7749	0.9947	1	0.5335	0.6319	0.6346	0.6119
Schmidt and Stadtmüller (λ_S)	0.2686	0.2745	0.5675	0.5335	1	0.5159	0.4951	0.4938
Pearson's r	0.6497	0.6462	0.6655	0.6319	0.5159	1	0.657	0.6993
Spearman's ρ	0.6036	0.6074	0.6498	0.6346	0.4951	0.657	1	0.9882
Kendall's τ	0.6235	0.6238	0.6297	0.6119	0.4938	0.6993	0.9882	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0.1846	0.0003	0.0011	0.0007
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0.1748	0.0004	0.001	0.0007
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0.0025	0.0002	0.0003	0.0006
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0.005	0.0005	0.0005	0.0009
Schmidt and Stadtmüller (λ_S)	0.1846	0.1748	0.0025	0.005	0	0.007	0.0101	0.0104
Pearson's r	0.0003	0.0004	0.0002	0.0005	0.007	0	0.0003	0.0001
Spearman's ρ	0.0011	0.001	0.0003	0.0005	0.0101	0.0003	0	0
Kendall's τ	0.0007	0.0007	0.0006	0.0009	0.0104	0.0001	0	0

3.4. S&P 500 vs. HF_{ISP}

In this section we compare the various hedge funds that are part of FoHF of ISPartners with the S&P 500. In this setup the S&P 500 is treated as explaining factor in the factor model: $\mathrm{HF}_{\mathrm{ISP},i} = \beta_i \cdot \mathrm{S\&P\ 500} + \epsilon_i$, with $i = 1, \ldots, 26$. In this setting we want to know the probability that the hedge funds of ISPartners incur a large loss or gain conditional on large losses or gains of the S&P 500.

3.4.1. Moving Threshold Graphs

The behavior of the various tail indices is already investigated further up (see Figures 3.1 and 3.14), therefore I do not show this information here again. I moved the figure related to the method of Poon et al. to appendix A.10, as the previous sections have revealed that their measure of tail dependence performs poorly in the situation investigated here.

If we look at Figures 3.20 and 3.21, we can see that the values for the TDCs seem to be significantly lower in the upper compared to the lower tail. It is also difficult to see some sort of stability for the upper tail. The situation looks better for the lower tail, especially for the parametric approach.

If we investigate Figure 3.22 more closely, it is obvious that the results for the TDCs calculated according to equations (2.74) and (2.76), and (2.75) and (2.77), respectively, are similar. This is interesting, as Schmuki [20] found results that also are quite similar but still slightly differing, dependent on the indicator function applied. The only small difference I found between $\hat{\lambda}$ and $\hat{\lambda}^{\mathrm{EVT}}$ can be seen in Figure 3.18 for the upper tail. As the results for both estimators are similar in this section, I only present the detailed results for the TDC calculated according to equations (2.74) and (2.75).

When looking at Figure 3.23 with the stability analysis for the linear measures of correlation, we can see that again Pearson's r seems to be the most stable TDC.

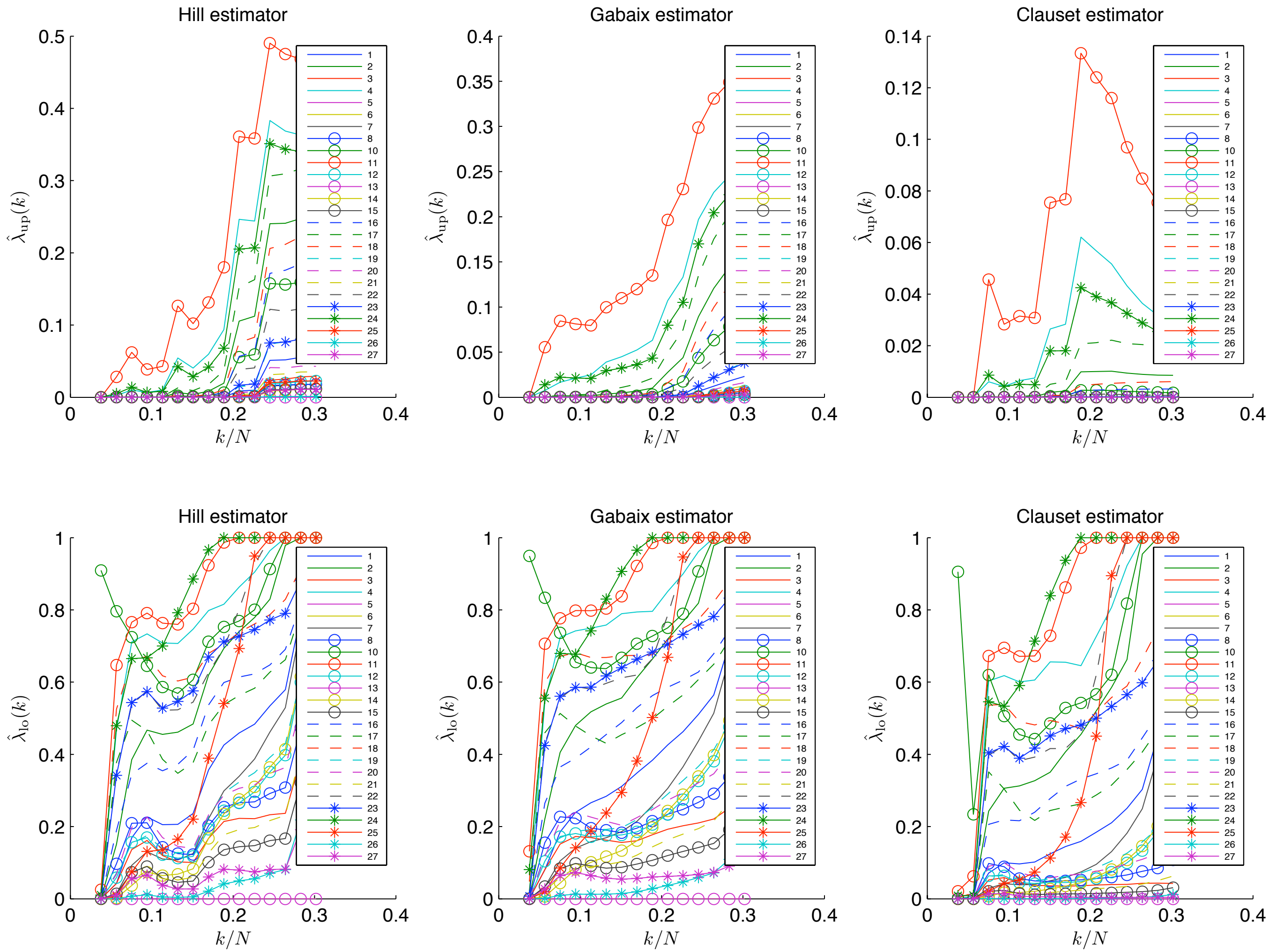


Figure 3.20.: λ for S&P 500 vs. $\mathrm{HF_{ISP}}$ with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

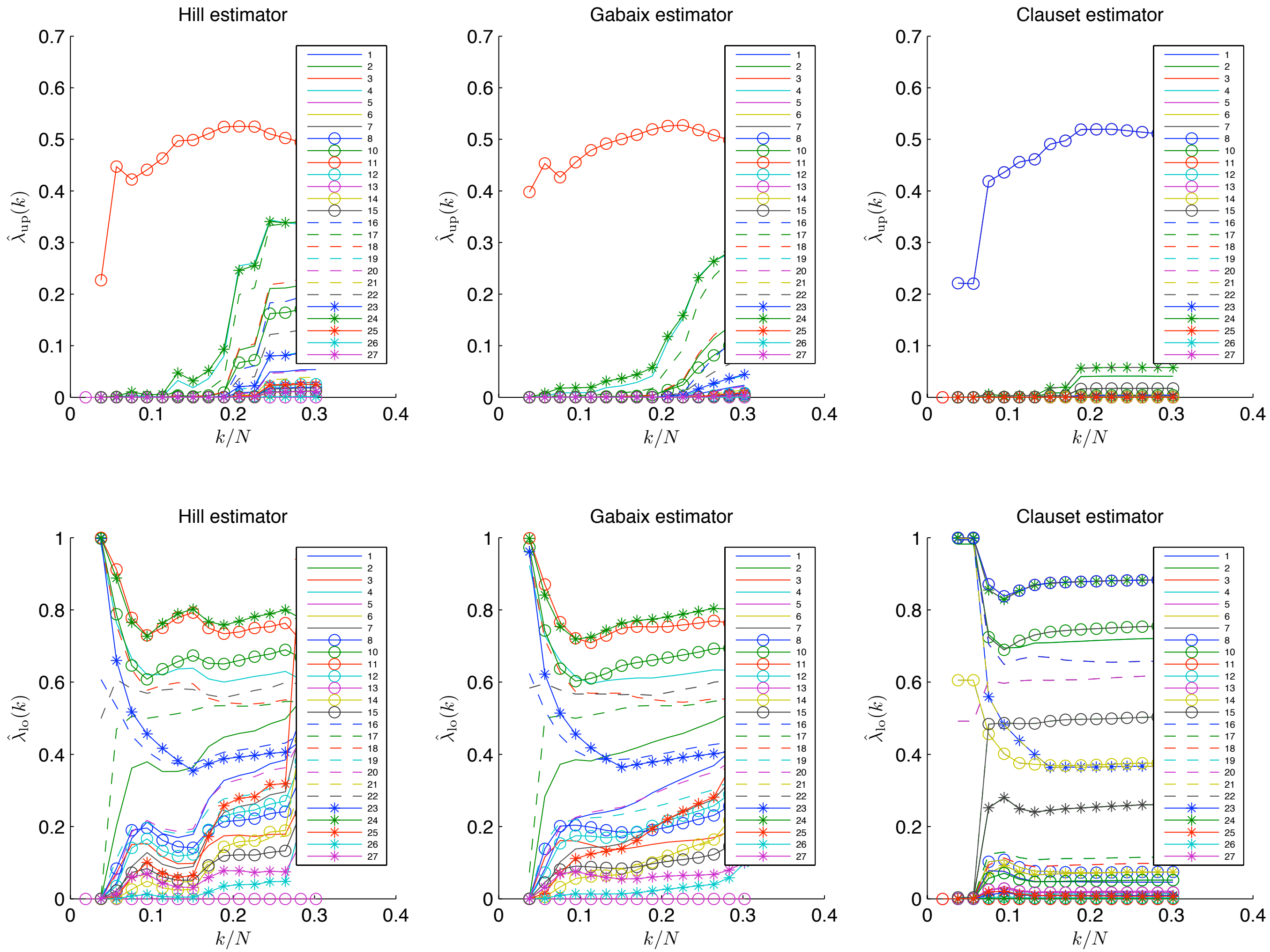
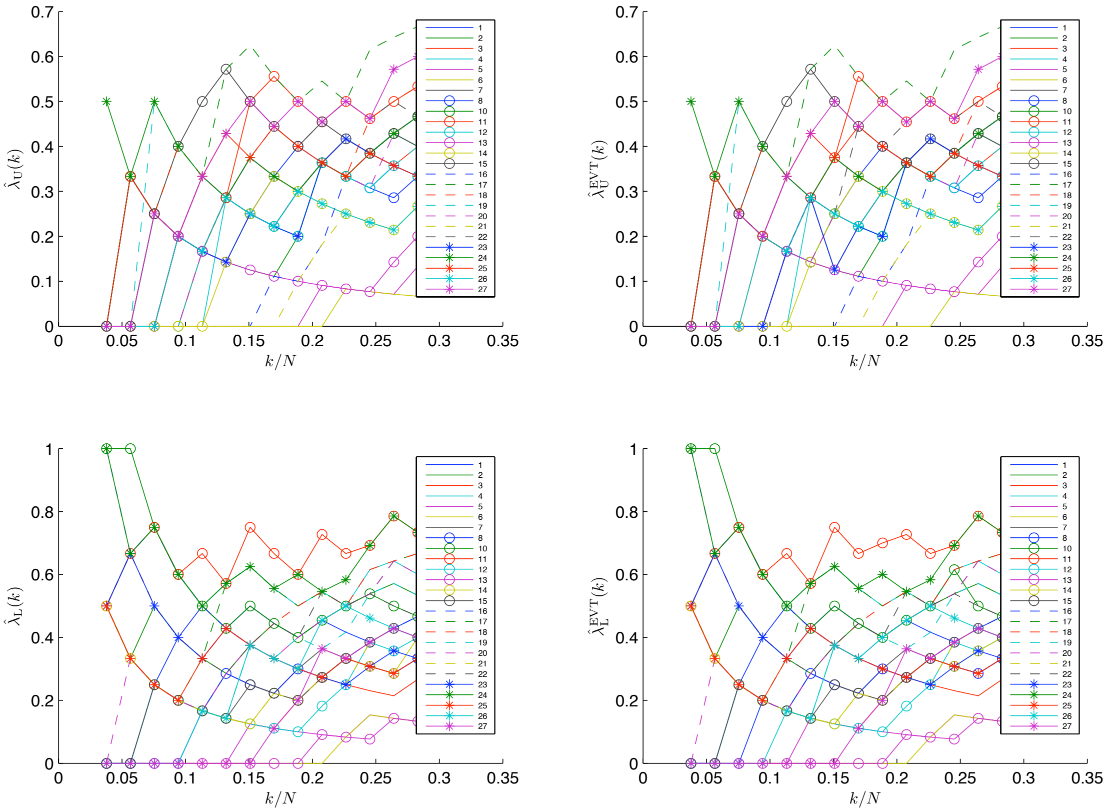
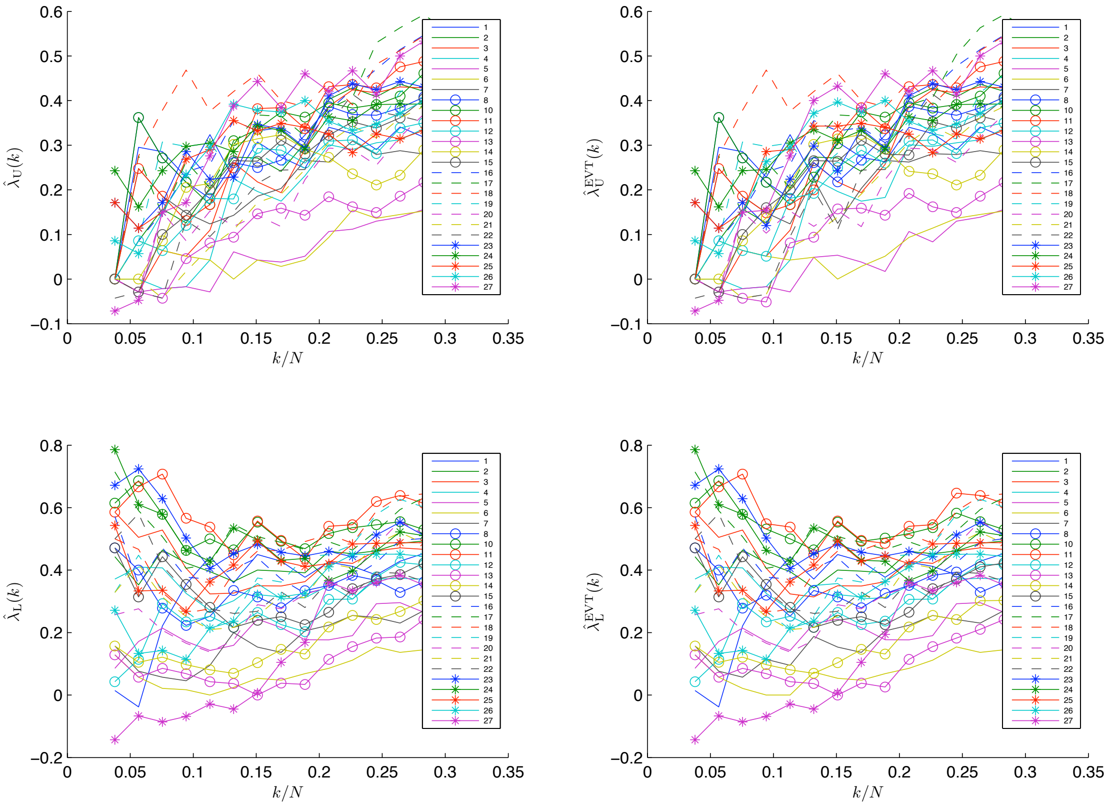


Figure 3.21.: λ for S&P 500 vs. HF_{ISP} with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) original results



(b) smoothed results

Figure 3.22.: λ for S&P 500 vs. HF_{ISP} by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

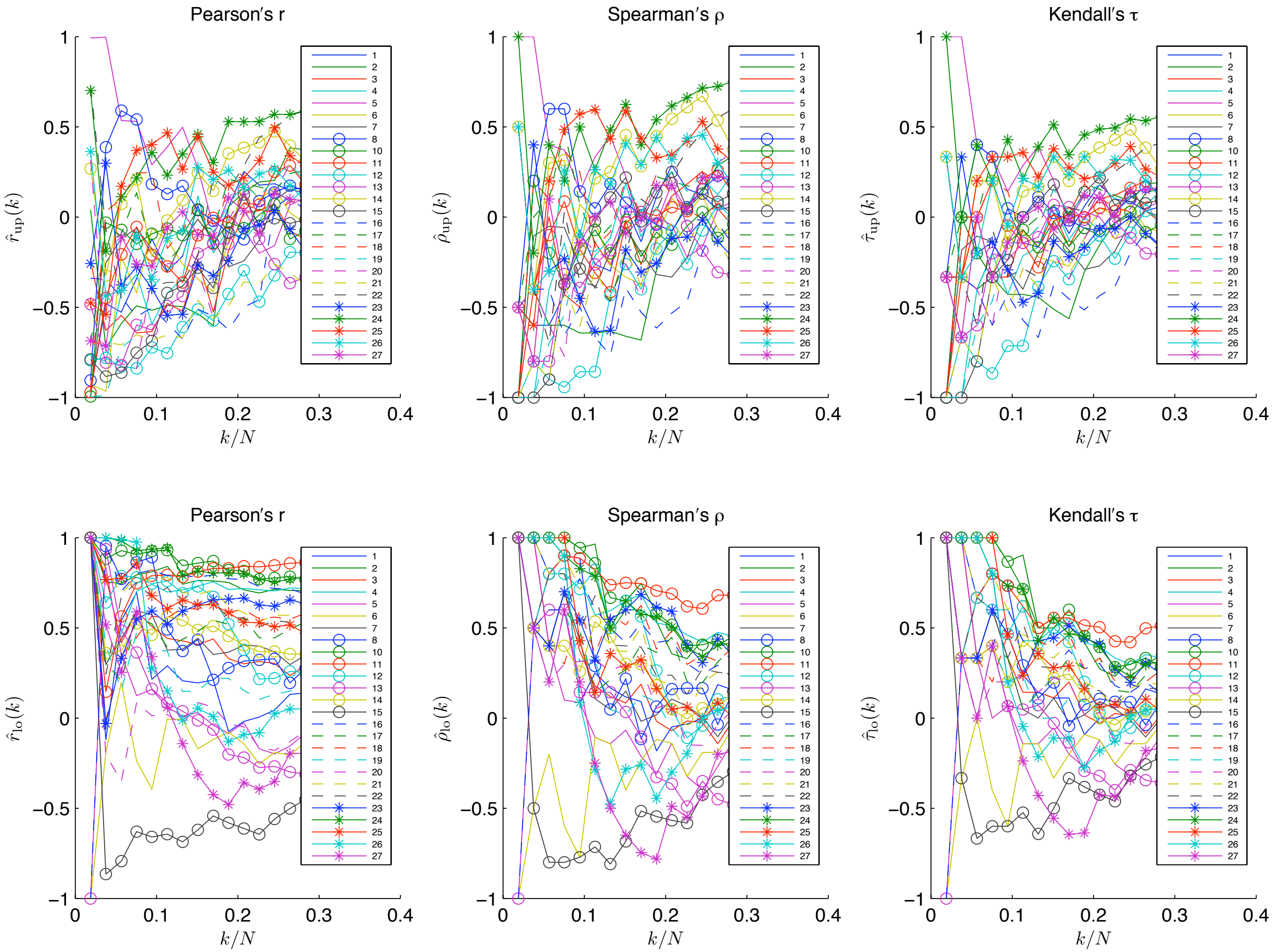


Figure 3.23.: TDC for S&P 500 vs. HF_{ISP} based on linear measures of correlation. From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

3.4.2. Detailed Results

This section contains the detailed results and their uncertainty analysis that is performed with a bootstrapping procedure as described in section 2.4 of the methodology chapter. To make the tables easier to read, I highlighted the three (if there were more similar TDCs, e.g. zeros, sometimes also more) highest and lowest values of the TDCs for the upper and lower tail. As I assume that theses highest (top group) and lowest (bottom group) values are of most interest, I will focus my analysis on them.

The detailed results in this section are all calculated for the ratio $k/N = 0.2$, corresponding to 11 data points for k as $N = 53$. This makes the results comparable with the other section and seems to be a reasonable choice according to the stability analysis performed in the section before. As the estimates of λ are almost similar for the parametric and the non-parametric approach of Malevergne and Sornette when applying the Hill or the Gabaix estimator, I moved the results that are based on the use of the Hill estimator in appendix A.5. I decided to discard the detailed results for the Clauset and Huisman estimator, as I do not expect to gain additional insight from them. As the method of Poon et al. does not seem to work well in this setting, I will not present any detailed results.

If we look at Table 3.15 and 3.16, we see immediately that there are many zeros in the upper tail and almost no large TDCs. The situation is different for the lower tail, where the picture looks rather normal, despite three zero in the bottom group. It seems that large gains of the hedge funds of ISPartners are almost independent of large gains of the S&P 500. Interestingly this is not the case for large losses, here the hedge funds seem to be correlated with the movements of the S&P 500.

The situation is different when looking at the results based on Schmidt and Stadtmüller's method (see Table 3.17). Here, the TDCs are in the same range independent of upper or lower tail. This either nourishes the suspicion that the findings from above are spurious, comparable to the ones we already saw in section 3.1 for the comparison between the THFI and the S&P 500 or that Schmidt and Stadtmüller's method is not as sensitive as it should be.

If we investigate the results based on Pearson's r (Table 3.18), we can see that there are many negative values in the upper tail (this holds also for the other linear measures of tail dependence as can be seen in Tables A.11 and A.12). This, in contrast to the results of Schmidt and Stadtmüller, supports the findings based on Malevergne and Sornette's methods that hedge funds do not seem to be correlated with the S&P 500 in the extremes of the upper tail. Additionally, the results in the lower tail look comparable to what we saw for the results based on Malevergne and Sornette's approaches.

The overview in Table 3.19 supports what I have just stated above: Malevergne and Sornette's methods produce very consistent results, independent of the tail index estimator applied. The top and the bottom groups in the upper and lower tails for Schmidt and Stadtmüller's method are different when compared with the groups from the other methods. The linear measures of tail dependence show a rather consistent picture for all groups.

To investigate whether the calculated TDCs differ between the various methods, I performed a Kruskal-Wallis test to compare their medians. For the upper tail we get a p-value of $4.9 \cdot 10^{-9}$, indicating that at least one of the methods produces significantly different results compared to the others. If we compare the non-linear methods among each other the p-value results in $1.5 \cdot 10^{-11}$. This is comprehensible, as the TDC estimates based on Schmidt and Stadtmüller's method are clearly higher than those based on Malevergne and Sornette's methods. When comparing the results among the linear methods, we get a p-value of 0.88, indicating that they behave very similarly. In the lower tail we have a p-value of 0.002 for the comparison between all methods. This value is small enough to reject the null hypothesis and to assume that at least one of the methods produces different results. However, the p-value for the non-linear methods is 0.47, meaning that their medians could be similar. For the linear methods the p-value is 0.03 and therefore already below the significance level of 0.05 but above one of 0.01. The following analysis based on the Spearman rank correlation might help to shed light on this partly inconclusive situation.

When looking at the correlation coefficients for the upper tail in Table 3.20, it is interesting to see that the coefficients are high within the methods of Malevergne and Sornette but rather small for all correlations that include the method of Schmidt and Stadtmüller. The correlations between Malevergne and Sornette's methods and the linear measures of tail dependence are small. If we also consider the large p-values, the correlation might even be zero, as the null hypothesis cannot be rejected. Within the linear measures of tail dependence the correlations are large and the p-values zero, indicating that they seem to have strong and significant correlations. This can be expected, as these methods are closely related to each other.

The economic interpretation is again very difficult, but it seems that there are some patterns recognizable. We have many event driven funds in the top groups of the upper and lower tail and several Global Macro funds in the bottom groups of the upper and lower tails. It seems that this event driven funds have a relatively high exposure to equity that makes them dependent on extreme movements of the S&P 500. This actually makes sense, as event driven funds try to profit from potential mispricings of securities related to specific corporate or market events such as mergers or bankruptcies [24]. On the other the Global Macro funds seem to be more or less independent of the index movement as they might have low exposure to equity in general or low exposure to equity indexed in the S&P 500. For further economic analysis please be referred to a hedge fund expert and keep in mind that the results presented here are based on very small samples (53 observations in total).

Table 3.15.: This table shows $\hat{\lambda}$ for S&P 500 vs. HF_{ISP} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	*0*	*0.01*	*0.02*	*0.03*	*0.06*	*0.13*	*0.01*	0.43	0.5	1	1	1	1	0.3
2	0.03	0.06	0.16	0.2	0.29	0.82	0.07	0.72	0.73	1	1	1	1	0.23
3	*0*	*0*	*0.01*	*0.01*	*0.03*	*0.09*	*0.01*	0.19	0.23	0.38	0.46	0.72	1	0.13
4	**0.11**	**0.15**	**0.29**	**0.34**	**0.44**	**0.66**	**0.1**	**0.85**	**0.82**	**1**	**1**	**1**	**1**	**0.17**
5	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.06*	*0*	0.31	0.42	1	1	1	1	0.3
8	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.07*	*0*	0.23	0.3	0.57	0.76	1	1	0.21
10	0.01	0.03	0.09	0.11	0.17	0.29	0.04	0.75	0.78	1	1	1	1	0.2
11	**0.2**	**0.24**	**0.43**	**0.5**	**0.65**	**1**	**0.14**	**1**	**0.9**	**1**	**1**	**1**	**1**	**0.14**
12	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.08*	*0.01*	0.23	0.35	0.78	1	1	1	0.26
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	*0*	*0*	*0*	*0*	*0.01*	*0.04*	*0*	0.24	0.36	1	1	1	1	0.29
15	*0*	*0*	*0*	*0.01*	*0.01*	*0.05*	*0*	0.12	0.18	0.38	0.51	1	1	0.17
16	0.01	0.04	0.1	0.14	0.22	0.8	0.05	0.59	0.62	1	1	1	1	0.2
17	0.05	0.1	0.24	0.29	0.39	0.68	0.1	0.53	0.58	1	1	1	1	0.25
18	0.02	0.05	0.14	0.19	0.29	0.65	0.07	0.72	0.72	1	1	1	1	0.21
19	*0*	*0.01*	*0.03*	*0.05*	*0.08*	*0.23*	*0.02*	0.29	0.38	0.81	1	1	1	0.26
20	*0*	*0.01*	*0.02*	*0.02*	*0.04*	*0.12*	*0.01*	0.27	0.33	0.62	0.79	1	1	0.21
21	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.09*	*0.01*	0.16	0.22	0.42	0.56	1	1	0.18
22	0.01	0.02	0.06	0.08	0.13	0.24	0.03	0.79	0.76	1	1	1	1	0.23
23	*0*	*0.01*	*0.04*	*0.05*	*0.09*	*0.17*	*0.02*	0.7	0.7	1	1	1	1	0.21
24	**0.08**	**0.12**	**0.24**	**0.28**	**0.36**	**0.5**	**0.08**	**1**	**0.92**	**1**	**1**	**1**	**1**	**0.13**
25	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.08*	*0*	0.67	0.65	1	1	1	1	0.35
26	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.04	0.13	0.34	0.68	1	1	0.22
27	*0*	*0*	*0*	*0*	*0.01*	*0.06*	*0*	0.06	0.09	0.19	0.23	0.36	1	0.08

Table 3.16.: This table shows $\hat{\lambda}$ for S&P 500 vs. HF_{ISP} according to the *parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	*0*	*0.01*	*0.02*	*0.03*	*0.06*	*0.13*	*0.01*	0.31	0.34	0.59	0.67	0.81	1.25	0.19
2	0.02	0.05	0.14	0.17	0.24	0.44	0.06	0.45	0.45	0.63	0.7	0.83	0.97	0.13
3	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.14*	*0.01*	0.16	0.17	0.28	0.32	0.4	0.76	0.08
4	**0.11**	**0.16**	**0.31**	**0.36**	**0.48**	**0.76**	**0.12**	0.62	0.61	0.77	0.81	0.88	0.98	0.12
5	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.06*	*0*	0.22	0.26	0.47	0.56	0.73	1.1	0.16
8	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.06*	*0*	0.2	0.23	0.39	0.45	0.6	1.02	0.13
10	0.01	0.04	0.1	0.12	0.18	0.34	0.04	**0.67**	**0.64**	**0.84**	**0.88**	**0.93**	**0.99**	**0.16**
11	**0.53**	**0.52**	**0.73**	**0.78**	**0.87**	**0.97**	**0.15**	**0.75**	**0.73**	**0.88**	**0.91**	**0.95**	**1**	**0.12**
12	*0*	*0*	*0.01*	*0.01*	*0.03*	*0.08*	*0.01*	0.22	0.24	0.43	0.5	0.64	1.03	0.14
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	*0*	*0*	*0*	*0*	*0.01*	*0.05*	*0*	0.12	0.18	0.39	0.52	0.75	1.1	0.16
15	*0*	*0*	*0*	*0.01*	*0.01*	*0.04*	*0*	0.1	0.13	0.26	0.31	0.43	0.7	0.1
16	*0*	*0.04*	*0.11*	*0.15*	*0.24*	*0.49*	*0.06*	0.4	0.41	0.55	0.59	0.68	0.89	0.11
17	0.06	0.12	0.29	0.36	0.49	0.77	0.12	0.53	0.52	0.71	0.76	0.84	0.98	0.15
18	0.02	0.06	0.16	0.21	0.28	0.71	0.07	0.55	0.54	0.7	0.74	0.82	0.95	0.12
19	*0*	*0.01*	*0.04*	*0.05*	*0.09*	*0.19*	*0.02*	0.26	0.28	0.46	0.52	0.63	0.94	0.13
20	*0*	*0.01*	*0.02*	*0.03*	*0.05*	*0.12*	*0.01*	0.31	0.32	0.55	0.62	0.76	1	0.17
21	*0*	*0*	*0.01*	*0.02*	*0.04*	*0.1*	*0.01*	0.12	0.16	0.29	0.36	0.49	0.98	0.11
22	*0*	*0.02*	*0.06*	*0.09*	*0.13*	*0.2*	*0.03*	0.57	0.57	0.73	0.77	0.87	0.97	0.12
23	*0*	*0.01*	*0.04*	*0.06*	*0.09*	*0.19*	*0.02*	0.38	0.39	0.54	0.59	0.68	0.82	0.12
24	**0.12**	**0.16**	**0.3**	**0.34**	**0.47**	**0.73**	**0.11**	**0.78**	**0.75**	**0.9**	**0.93**	**0.97**	**1.01**	**0.12**
25	*0*	*0*	*0.01*	*0.01*	*0.03*	*0.07*	*0.01*	0.22	0.3	0.61	0.75	1	3.49	0.28
26	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.03	0.06	0.16	0.25	0.49	1.36	0.11
27	*0*	*0*	*0*	*0.01*	*0.01*	*0.04*	*0*	0.06	0.08	0.16	0.19	0.27	0.42	0.06

Table 3.17.: This table shows $\hat{\lambda}$ for S&P 500 vs. HF_{ISP} according to the non-parametric approach by *Schmidt and Stadtmüller* and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_U$ and $\hat{\lambda}_L$ are set italic and the three highest values bold. $\hat{\lambda}_U^{EVT}$ and $\hat{\lambda}_L^{EVT}$ are not shown in the table as the values are similar to those found for $\hat{\lambda}_U$ and $\hat{\lambda}_L$.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_U$	$\hat{\lambda}^{bs}_{U,mean}$	$\hat{\lambda}^{bs}_{U,90}$	$\hat{\lambda}^{bs}_{U,95}$	$\hat{\lambda}^{bs}_{U,99}$	$\hat{\lambda}^{bs}_{U,max}$	$\hat{\sigma}^{bs}_U$	$\hat{\lambda}_L$	$\hat{\lambda}^{bs}_{L,mean}$	$\hat{\lambda}^{bs}_{L,90}$	$\hat{\lambda}^{bs}_{L,95}$	$\hat{\lambda}^{bs}_{L,99}$	$\hat{\lambda}^{bs}_{L,max}$	$\hat{\sigma}^{bs}_L$
1	0.36	0.33	0.55	0.55	0.73	0.91	0.14	0.45	0.42	0.64	0.64	0.73	1	0.15
2	0.36	0.31	0.55	0.55	0.73	0.91	0.15	0.45	0.44	0.64	0.64	0.73	0.91	0.14
3	0.36	0.35	0.55	0.55	0.73	0.91	0.14	0.27	0.21	0.36	0.45	0.55	0.73	0.12
4	0.36	0.34	0.55	0.55	0.64	0.82	0.14	**0.55**	**0.55**	**0.73**	**0.77**	**0.82**	**1**	**0.14**
5	*0.09*	*0.05*	*0.18*	*0.18*	*0.36*	*0.55*	*0.08*	*0.09*	*0.11*	*0.27*	*0.27*	*0.36*	*0.55*	*0.1*
6	*0*	*0.04*	*0.18*	*0.18*	*0.36*	*0.64*	*0.08*	*0*	*0.05*	*0.18*	*0.27*	*0.36*	*0.55*	*0.09*
7	0.36	0.33	0.55	0.55	0.73	0.91	0.15	0.36	0.28	0.45	0.45	0.64	0.73	0.13
8	0.36	0.31	0.45	0.55	0.64	0.82	0.14	0.27	0.28	0.45	0.55	0.64	0.82	0.13
10	0.36	0.32	0.55	0.55	0.64	0.82	0.14	0.45	0.45	0.64	0.64	0.77	1	0.14
11	**0.45**	**0.48**	**0.64**	**0.73**	**0.82**	**1**	**0.15**	**0.73**	**0.66**	**0.82**	**0.91**	**0.91**	**1**	**0.13**
12	0.36	0.3	0.45	0.55	0.64	0.82	0.14	0.18	0.21	0.36	0.45	0.55	0.82	0.14
13	*0.09*	*0.13*	*0.27*	*0.36*	*0.45*	*0.73*	*0.11*	*0.09*	*0.05*	*0.18*	*0.18*	*0.27*	*0.45*	*0.08*
14	0.27	0.25	0.45	0.45	0.64	0.91	0.14	0.27	0.24	0.45	0.45	0.55	0.73	0.13
15	**0.45**	**0.45**	**0.64**	**0.73**	**0.82**	**0.91**	**0.15**	0.27	0.29	0.45	0.55	0.64	0.91	0.15
16	0.18	0.23	0.45	0.55	0.64	0.82	0.16	0.27	0.35	0.55	0.64	0.73	0.91	0.14
17	**0.55**	**0.58**	**0.73**	**0.82**	**0.91**	**1**	**0.14**	**0.55**	**0.54**	**0.73**	**0.73**	**0.82**	**1**	**0.14**
18	0.36	0.41	0.64	0.64	0.73	1	0.15	**0.55**	**0.51**	**0.73**	**0.73**	**0.82**	**0.91**	**0.14**
19	0.36	0.33	0.55	0.55	0.64	0.82	0.14	0.36	0.41	0.55	0.64	0.73	0.91	0.14
20	0.36	0.33	0.55	0.55	0.64	0.82	0.14	0.45	0.45	0.64	0.73	0.82	0.91	0.15
21	0.18	0.17	0.36	0.36	0.55	0.64	0.13	0.27	0.25	0.45	0.45	0.55	0.73	0.14
22	**0.45**	**0.43**	**0.64**	**0.64**	**0.82**	**1**	**0.15**	0.55	0.45	0.64	0.73	0.73	0.91	0.14
23	0.36	0.31	0.55	0.55	0.64	0.82	0.14	0.27	0.31	0.45	0.55	0.64	0.91	0.14
24	0.36	0.38	0.55	0.64	0.73	1	0.15	**0.55**	**0.59**	**0.73**	**0.82**	**0.91**	**1**	**0.14**
25	0.36	0.36	0.55	0.64	0.73	0.82	0.15	0.27	0.3	0.45	0.55	0.64	0.82	0.14
26	0.27	0.26	0.45	0.45	0.64	0.73	0.14	0.45	0.37	0.55	0.64	0.73	0.82	0.14
27	**0.45**	**0.49**	**0.64**	**0.73**	**0.82**	**1**	**0.14**	0.36	0.25	0.45	0.55	0.64	0.91	0.16

Table 3.18.: This table shows $\hat{r}$ for S&P 500 vs. HF_{ISP} according to *Pearson*'s product-moment correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{r}$. The three lowest values for $\hat{r}_{up}$ and $\hat{r}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{r}_{up}$	$\hat{r}^{bs}_{up,mean}$	$\hat{r}^{bs}_{up,90}$	$\hat{r}^{bs}_{up,95}$	$\hat{r}^{bs}_{up,99}$	$\hat{r}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{r}_{lo}$	$\hat{r}^{bs}_{lo,mean}$	$\hat{r}^{bs}_{lo,90}$	$\hat{r}^{bs}_{lo,95}$	$\hat{r}^{bs}_{lo,99}$	$\hat{r}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.06	0.05	0.48	0.6	0.79	0.9	0.33	-0.07	0.11	0.54	0.62	0.81	0.96	0.31
2	0.06	-0.12	0.48	0.55	0.68	0.93	0.48	0.75	0.64	0.9	0.92	0.97	1	0.27
3	-0.19	-0.16	0.36	0.54	0.76	0.83	0.39	0.33	0.17	0.73	0.82	0.93	0.99	0.48
4	-0.07	-0.05	0.42	0.57	0.73	0.94	0.35	0.71	0.67	0.93	0.96	0.99	1	0.24
5	0.01	0.06	0.54	0.66	0.8	1	0.36	-0.09	-0.16	0.46	0.67	0.91	0.98	0.49
6	-0.13	-0.11	0.38	0.48	0.73	0.92	0.35	-0.08	-0.15	0.26	0.38	0.55	0.83	0.31
7	-0.28	-0.1	0.33	0.44	0.59	0.84	0.32	0.36	0.27	0.81	0.87	0.95	0.99	0.44
8	-0.09	0.05	0.41	0.5	0.67	0.82	0.26	0.28	0.28	0.7	0.82	0.93	0.97	0.32
10	-0.03	-0.05	0.35	0.49	0.65	0.91	0.3	**0.82**	**0.77**	**0.94**	**0.95**	**0.97**	**0.99**	**0.21**
11	-0.03	0.02	0.49	0.57	0.75	0.83	0.35	**0.83**	**0.81**	**0.96**	**0.97**	**0.98**	**0.99**	**0.14**
12	*-0.43*	*-0.37*	*0.13*	*0.31*	*0.6*	*0.85*	*0.35*	0.52	0.38	0.8	0.85	0.93	0.99	0.38
13	-0.15	-0.19	0.31	0.49	0.66	0.98	0.36	*-0.2*	*-0.18*	*0.31*	*0.5*	*0.67*	*0.9*	*0.38*
14	**0.35**	**0.31**	**0.65**	**0.73**	**0.85**	**0.96**	**0.29**	0.45	0.29	0.84	0.9	0.95	0.97	0.47
15	-0.09	-0.08	0.49	0.65	0.83	0.94	0.41	*-0.58*	*-0.55*	*-0.3*	*-0.11*	*0.54*	*0.81*	*0.25*
16	*-0.62*	*-0.39*	*0.2*	*0.36*	*0.55*	*0.73*	*0.37*	0.77	0.68	0.92	0.95	0.97	0.99	0.3
17	0.21	0.1	0.49	0.58	0.7	0.92	0.3	0.52	0.44	0.83	0.88	0.94	0.99	0.32
18	0.18	0.06	0.4	0.52	0.71	0.9	0.28	0.59	0.54	0.86	0.9	0.95	0.98	0.28
19	0.07	0.07	0.53	0.64	0.81	0.92	0.36	0.22	0.11	0.66	0.75	0.88	0.96	0.41
20	-0.02	-0.06	0.5	0.6	0.77	0.9	0.41	-0.03	-0.01	0.59	0.69	0.79	0.91	0.43
21	*-0.49*	*-0.35*	*0.28*	*0.43*	*0.64*	*0.81*	*0.39*	0.62	0.51	0.83	0.87	0.92	0.96	0.32
22	0.15	0.18	0.65	0.72	0.8	0.88	0.39	0.56	0.49	0.82	0.87	0.92	0.97	0.31
23	-0.24	-0.2	0.18	0.27	0.54	0.78	0.29	0.66	0.58	0.92	0.94	0.97	1	0.3
24	**0.53**	**0.47**	**0.75**	**0.8**	**0.88**	**0.97**	**0.25**	**0.81**	**0.73**	**0.95**	**0.96**	**0.99**	**1**	**0.26**
25	0.18	0.3	0.61	0.69	0.79	0.89	0.26	0.58	0.42	0.8	0.86	0.91	0.97	0.36
26	**0.26**	**0.16**	**0.48**	**0.55**	**0.69**	**0.85**	**0.27**	-0.13	-0.03	0.32	0.46	0.78	0.99	0.28
27	0.11	0.03	0.47	0.58	0.72	0.94	0.32	*-0.48*	*-0.34*	*0.05*	*0.21*	*0.53*	*0.82*	*0.31*

Table 3.19.: This table shows the ranking of TDCs for S&P 500 vs. HF_{ISP} for the top and the bottom group of the upper and lower tail of the return distribution. 'Mal.' stands for Malevergne and 'Sor.' for Sornette. (The TDCs for the bottom groups in the upper and lower tail are all equal to zero in all columns for the methods of Malevergne and Sornette. For the non-parametric method of Malevergne and Sornette applying the Hill estimator the TDCs for 14 and 26 are also equal to zero in all columns in the bottom group of the upper tail.)

Tail	Upper tail						Lower tail					
Group	Top group			Bottom group			Top group			Bottom group		
Ranking	1	2	3	1	2	3	1	2	3	1	2	3
Mal. and Sor., non-par, Gabaix	11	4	24	5	6	13	24	11	4	5	6	13
Mal. and Sor., non-par, Hill	11	4	24	5	6	13	24	11	4	5	6	13
Mal. and Sor., par, Gabaix	11	24	4	5	6	13	24	11	10	5	6	13
Mal. and Sor., par, Hill	11	4	24	5	6	13	24	11	10	5	6	13
Schmidt and Stadtmüller	17	27	11	6	5	13	11	24	4	6	13	5
Pearson's r	24	14	26	16	21	12	11	10	24	15	27	13
Spearman's ρ	24	14	26	16	7	2	11	23	10	27	15	26
Kendall's τ	24	14	26	16	2	7	11	10	24	27	15	13
# of different subindices (rank)	3	3	3	3	5	4	2	4	3	4	3	2
Max. # of same subindex (rank)	4	3	4	4	4	5	4	4	3	4	5	7
# of similar subindices (group)	7			8			5			6		
List of similar subindices (group)	4	24		2	12		4	24		5	26	
	11	26		5	13		10			6	27	
	14	27		6	16		11			13		
	17			7	21		23			15		

Table 3.20.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation S&P 500 vs. HF_{ISP}. This values are calculated for the *upper tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	0.9075	0.9092	0.9065	0.3702	0.265	0.1222	0.0682
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0.9075	1	0.8298	0.9982	0.3974	0.2483	0.0626	0.0163
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.9092	0.8298	1	0.8348	0.3673	0.3299	0.2278	0.1721
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.9065	0.9982	0.8348	1	0.4014	0.2666	0.0864	0.0405
Schmidt and Stadtmüller (λ_S)	0.3702	0.3974	0.3673	0.4014	1	0.2766	0.2695	0.2504
Pearson's r	0.265	0.2483	0.3299	0.2666	0.2766	1	0.8511	0.8362
Spearman's ρ	0.1222	0.0626	0.2278	0.0864	0.2695	0.8511	1	0.9865
Kendall's τ	0.0682	0.0163	0.1721	0.0405	0.2504	0.8362	0.9865	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0.0626	0.1908	0.5521	0.7405
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0.0444	0.2213	0.7613	0.937
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0.0649	0.0998	0.2631	0.4004
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0.0421	0.1881	0.6749	0.8443
Schmidt and Stadtmüller (λ_S)	0.0626	0.0444	0.0649	0.0421	0	0.1713	0.183	0.2173
Pearson's r	0.1908	0.2213	0.0998	0.1881	0.1713	0	0	0
Spearman's ρ	0.5521	0.7613	0.2631	0.6749	0.183	0	0	0
Kendall's τ	0.7405	0.937	0.4004	0.8443	0.2173	0	0	0

Table 3.21.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation S&P 500 vs. HF_{ISP}. This values are calculated for the lower tail of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	1	0.9645	0.9736	0.7452	0.8423	0.8424	0.8477
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	1	1	0.9645	0.9736	0.7452	0.8423	0.8424	0.8477
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.9645	0.9645	1	0.9985	0.7881	0.8152	0.8404	0.8472
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.9736	0.9736	0.9985	1	0.7887	0.8166	0.8356	0.8439
Schmidt and Stadtmüller (λ_S)	0.7452	0.7452	0.7881	0.7887	1	0.4554	0.463	0.4965
Pearson's r	0.8423	0.8423	0.8152	0.8166	0.4554	1	0.9567	0.9681
Spearman's ρ	0.8424	0.8424	0.8404	0.8356	0.463	0.9567	1	0.9921
Kendall's τ	0.8477	0.8477	0.8472	0.8439	0.4965	0.9681	0.9921	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0	0	0	0
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0	0	0	0
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0	0	0	0
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0	0	0	0
Schmidt and Stadtmüller (λ_S)	0	0	0	0	0	0.0194	0.0172	0.0099
Pearson's r	0	0	0	0	0.0194	0	0	0
Spearman's ρ	0	0	0	0	0.0172	0	0	0
Kendall's τ	0	0	0	0	0.0099	0	0	0

3.5. S&P 500 vs. $THFI_{sub}$

In this section I present the results for the dependence structure between the S&P 500 and the various hedge fund subindices ($THFI_{sub}$) of the THFI. This is of interest, as we should see a stronger dependence between those subindices and the S&P 500 that consist mainly of shares. The underlying factor model for this section is: $HF_{THFI,i} = \beta_i \cdot \text{S\&P 500} + \epsilon_i$, with $i = 1, \ldots, 13$.

3.5.1. Moving Threshold Graphs

I do not again show the results for the tail index estimators, as they have already been presented before in Figures 3.1 and 3.8. The results based on the method of Poon et al. are not presented here, but moved to appendix A.6, as previous sections have shown that the performance of the TDC estimators is poor.

If we look at Figures 3.24 and 3.25, we see a comparable picture to what we have seen in the previous section, i.e. rather small TDCs in the upper tail. This is comprehensible, as we again try to explain the extreme behavior of hedge funds based on extreme movements of a stock index.

When looking at Figures 3.24,3.25, and 3.26, it is difficult to find a plateau of stability. However, as the results should be comparable with those presented in the sections above, it seems reasonable to chose 0.12 for the ratio k/N.

In Figure 3.27 we can see that the behavior of Pearson's r is again the most stable one and that 0.12 seems to be an acceptable choice for the ratio k/N in this setting.

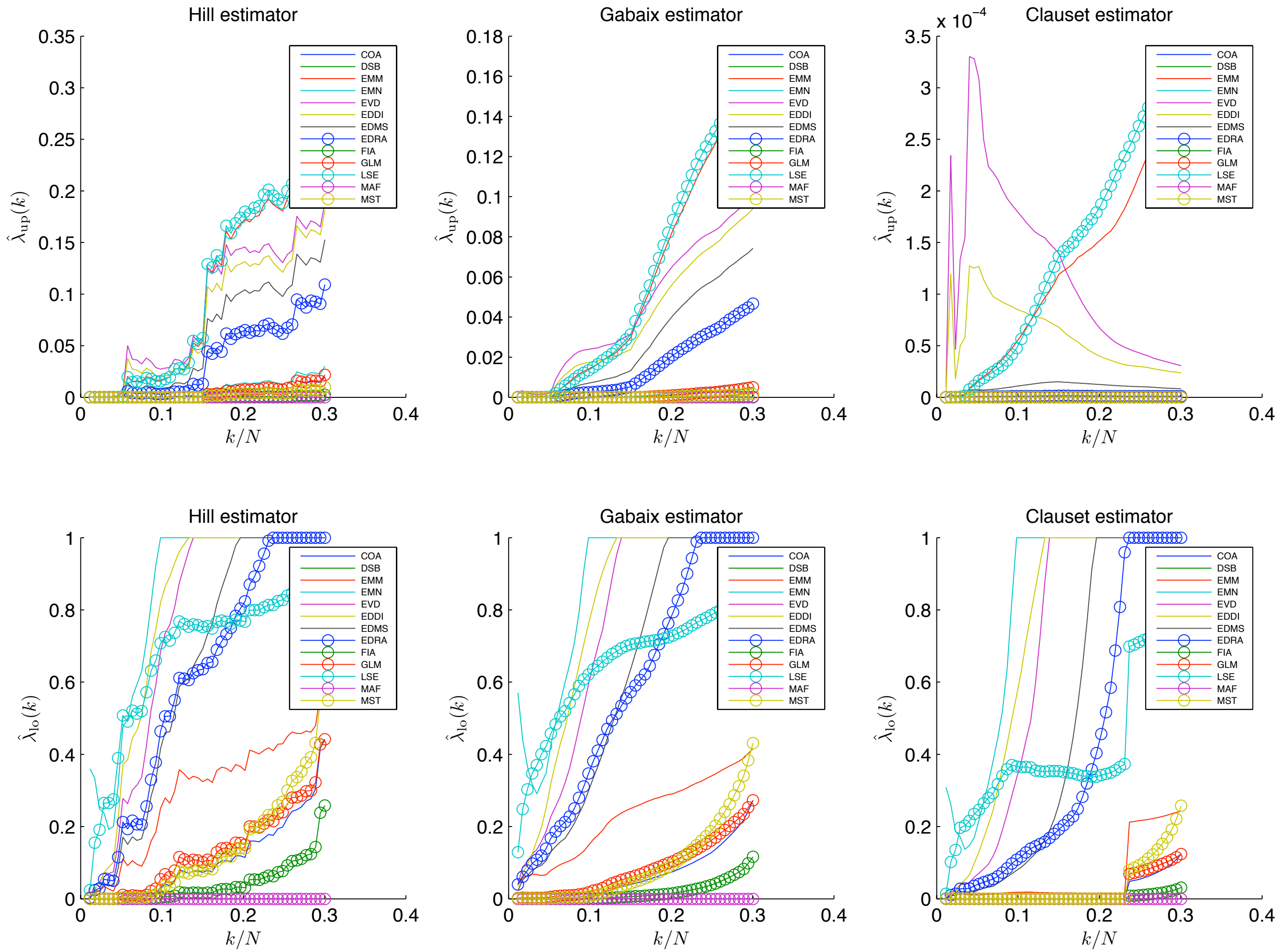


Figure 3.24.: λ for S&P 500 vs. $\mathrm{THFI}_{\mathrm{sub}}$ with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

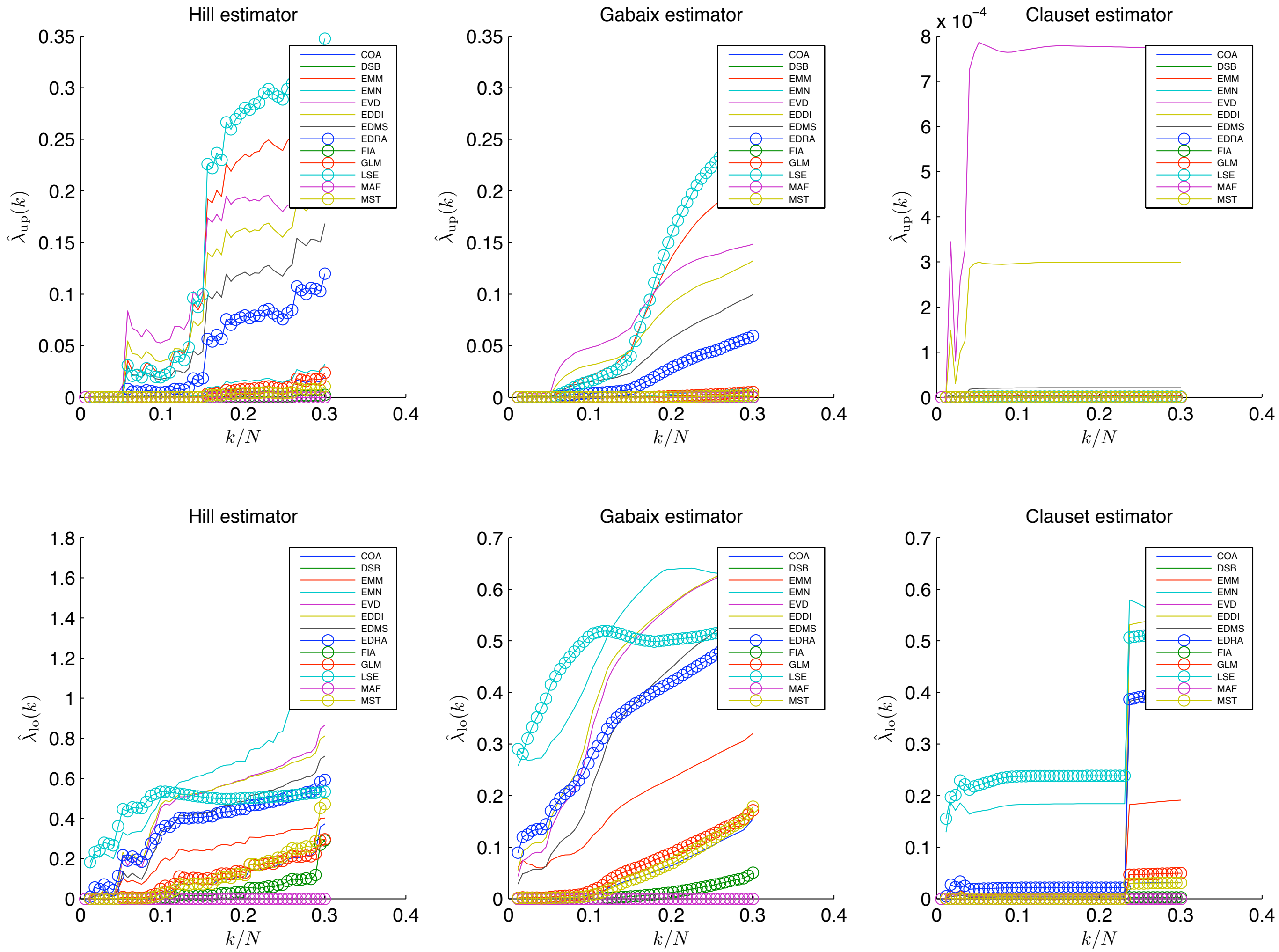
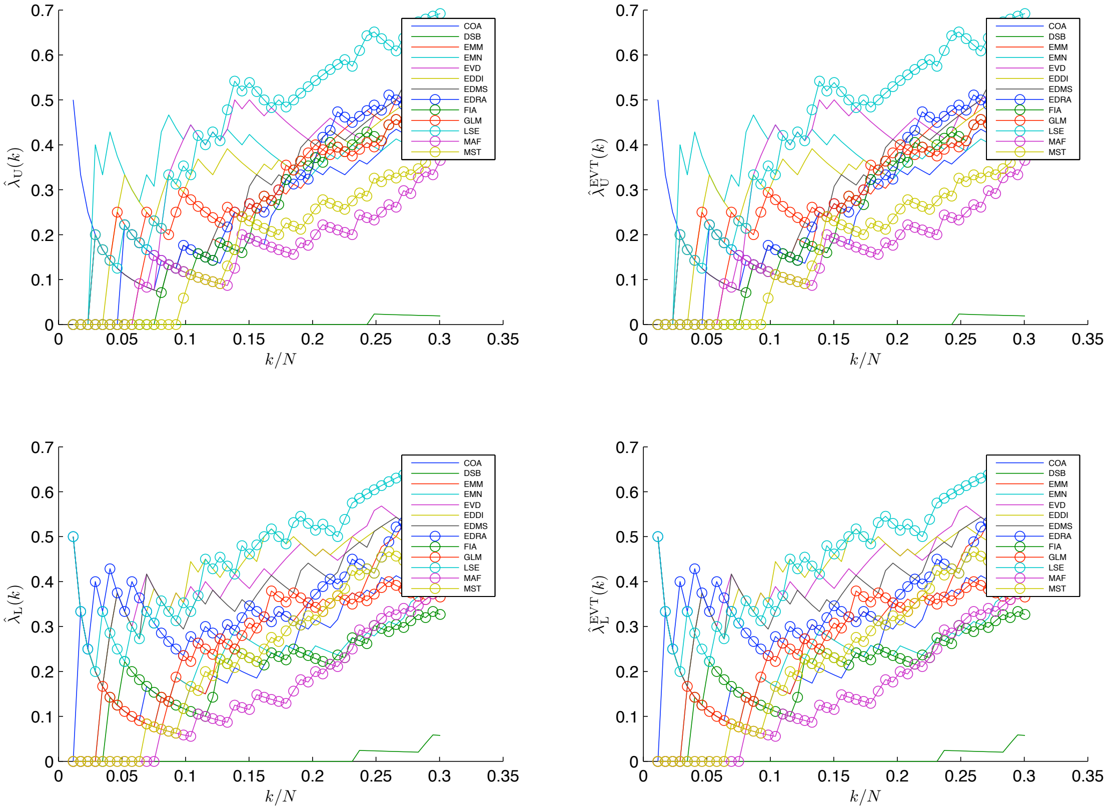
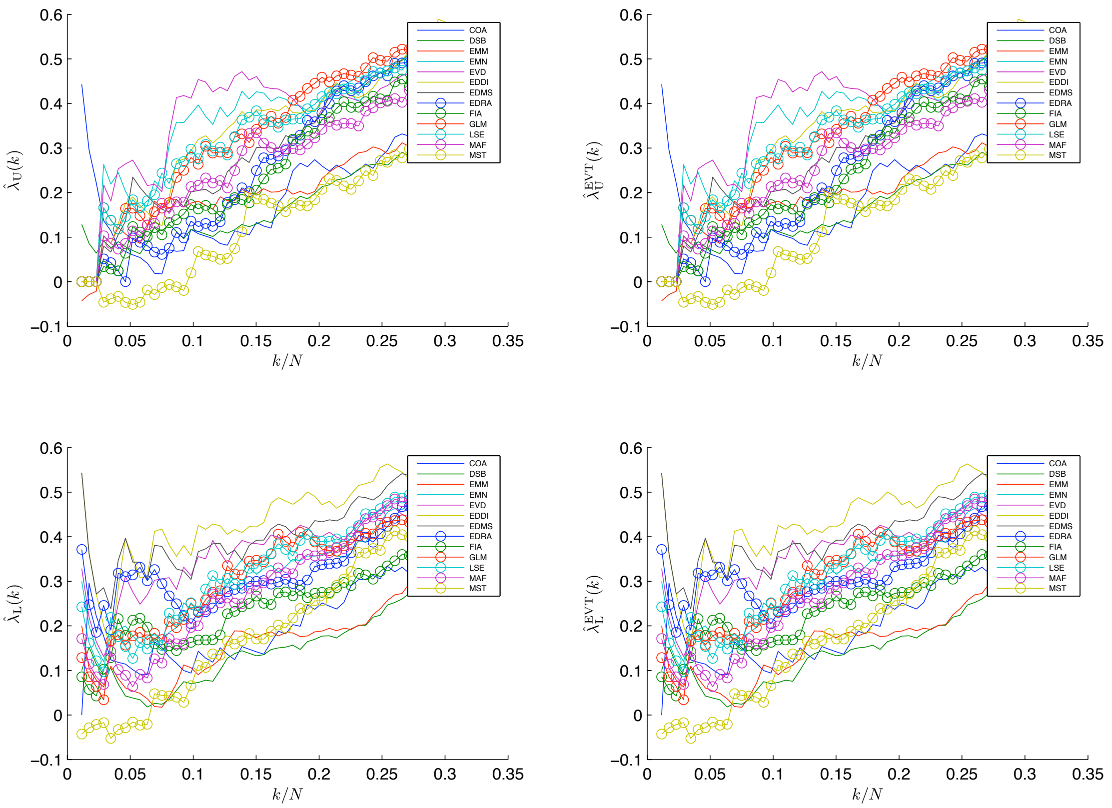


Figure 3.25.: λ for S&P 500 vs. THFI_{sub} with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.



(a) original results



(b) smoothed results

Figure 3.26.: λ for S&P 500 vs. THFI_{sub} by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

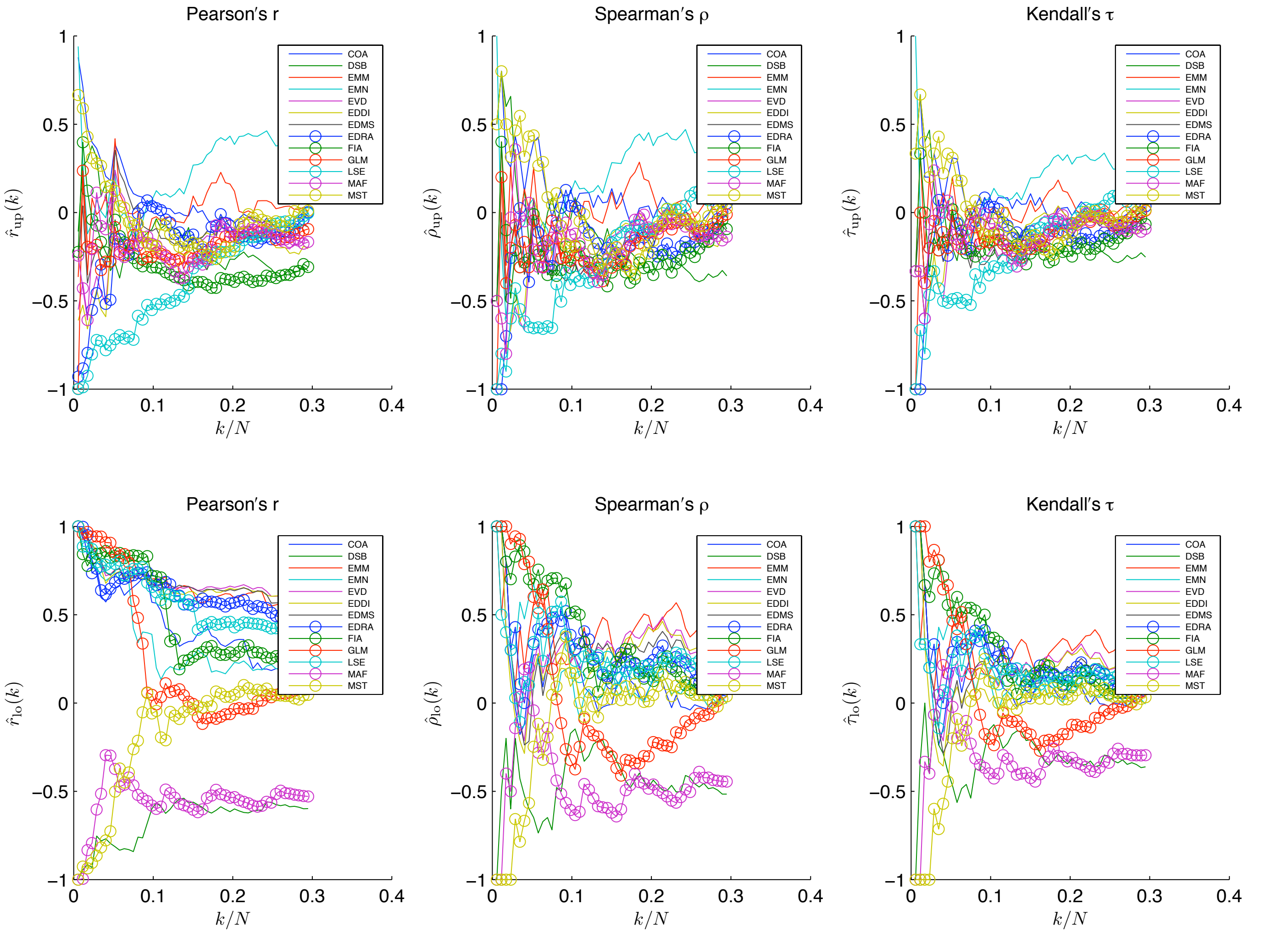


Figure 3.27.: TDC for S&P 500 vs. $THFI_{sub}$ based on linear measures of correlation. From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

3.5.2. Detailed Results

In this section the details are presented and their uncertainty is tested with a bootstrap procedure as described in section 2.4. To make the tables easier to read, the three (if there were more similar TDCs, e.g. zeros, sometimes also more) highest and lowest values of the TDCs for the upper and lower tail are highlighted. In the following paragraphs I will focus my analysis on these highest (top group) and lowest (bottom group) values, as I assume that they are of most interest (for an overview see Table 3.26).

The detailed results in this section are all calculated for the ratio $k/N = 0.12$, corresponding to 21 data points for k as $N = 173$. As the estimates of λ derived with the parametric and the non-parametric method of Malevergne and Sornette are very similar for the Hill and the Gabaix estimator, I placed the results that are based on the Hill estimator in appendix A.6. I do not present detailed results for the Clauset and Huisman estimator, as I do not expect to gain additional insight from them. Due to the findings that the method of Poon et al. does not produce useful results, I decided not to present them here.

When looking at Tables 3.22 and 3.23, the similarity between the previous section is obvious. There are many zeros in the upper tail and the subindices of the top group have all very small values (max = 0.05). The situation in the lower tail is also similar, as the TDCs for the top group are in a comparably normal range and only three TDCs have a value of zero. It seems that the probability of extreme upward movements of hedge funds is not conditional on extreme upward movements of the S&P 500.

The situation looks different for the results based on the method of Schmidt and Stadtmüller (although again very similar to the previous section). The values of the top and bottom group in the upper as well as in the lower tail are in the lower range but there is only one zero in the bottom group. These results imply that there is a tail dependence between the subindices of the THFI and the S&P 500.

When investigating Pearson's r (see Figure 3.25) we have support for the results based on Malevergne and Sornette's methods. There are many values for TDCs in the upper tail that are below zero and even one value of the top group is negative. The picture is again different for the lower tail where we find 'normal' behavior of the TDCs.

To see if the TDCs are really different, I applied a Kruskal-Wallies test that compares the medians. The p-value for upper tail is $2.5 \cdot 10^{-8}$, indicating that the null hypothesis can be rejected and at least one method has a median significantly different from the others. If we compare the non-linear measures of tail dependence we still have a very low p-value of $1.2 \cdot 10^{-5}$, meaning that at least one median is different from the others. In this case it must be the median of the TDCs calculated with the method of Schmidt and Stadtmüller. The p-value for the linear measures of tail dependence is 0.5. Hence, the null hypothesis cannot be rejected and the values are likely to be from the same population. If we calculate the p-value for the lower tail we get a different picture as the p-value is 0.15. Hence, we cannot reject the assumption that the medians are similar, i.e. from the same population. For the non-linear measures of tail dependence we get a p-value of 0.74 and for the linear measures a p-value of 0.07. This indicates that the results calculated with all methods are rather similar.

If we compare the correlations among the methods in the upper tail (Table 3.27), we can see high correlation coefficients among the methods of Malevergne and Sornette, intermediate values for the correlations related to the results based on Schmidt and Stadtmüller's approach and negative values for correlations between non-linear measures of tail dependence and linear measures. However, the p-values are comparably large (especially for Pearson's r) and the null hypothesis that the coefficients are zero cannot be reject for most of the relations. If we look at the correlation coefficient within the group of linear measures we find high values supporting the findings from above (p-value of 0.5 in the Kruskal-Wallis test). In the lower tail, as can be expected, the coefficients are all positive and higher than for the upper tail. The highest correlation coefficients can be found again between TDCs based on the the methods of Malevergne and Sornette and among those based on linear measures of correlation. The correlation coefficients for the TDCs calculated with the method of Schmidt and Stadtmüller and for those with Pearson's product moment correlation are in the middle field.

The economic interpretation of the above presented findings is difficult, but we can see some patterns. The dedicated short bias-strategy is only present in the bottom group, indicating that this sort of hedge fund is independent from extreme movements of the S&P 500 index. This makes sense, as these hedge funds are mainly focused on detailed company research that makes them relatively independent from market movements. A similar pattern can be found for the Managed Futures (CTAs). This is comprehensible, as CTA hedge funds often employ trading software that does not show the typical herding behavior that traders might lapse into during extreme situations. For further interpretations please be referred to hedge fund experts.

Table 3.22.: This table shows $\hat{\lambda}$ for S&P 500 vs. THFI_{sub} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.02	0.03	0.06	0.07	0.1	0.17	0.02
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.02**	**0.03**	**0.07**	**0.09**	**0.14**	**0.26**	**0.03**	0.21	0.22	0.33	0.37	0.47	0.99	0.08
EMN	*0*	*0*	*0*	*0*	*0*	*0.02*	*0*	**1**	**0.9**	**1**	**1**	**1**	**1**	**0.19**
EVD	**0.03**	**0.04**	**0.07**	**0.08**	**0.11**	**0.22**	**0.02**	**0.82**	**0.78**	**1**	**1**	**1**	**1**	**0.23**
EDDI	**0.02**	**0.03**	**0.05**	**0.06**	**0.09**	**0.18**	**0.02**	**0.94**	**0.83**	**1**	**1**	**1**	**1**	**0.2**
EDMS	0.01	0.01	0.03	0.04	0.06	0.15	0.01	0.44	0.51	0.95	1	1	1	0.24
EDRA	*0*	*0.01*	*0.01*	*0.02*	*0.03*	*0.08*	*0.01*	0.47	0.51	0.82	0.99	1	1	0.21
FIA	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0.01*	*0.01*	*0.03*	*0.06*	*0.01*
GLM	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.04	0.05	0.09	0.11	0.15	0.33	0.03
LSE	**0.02**	**0.03**	**0.07**	**0.09**	**0.14**	**0.27**	**0.03**	0.67	0.68	1	1	1	1	0.2
MAF	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
MST	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.02	0.03	0.07	0.09	0.15	0.31	0.03

Table 3.23.: This table shows $\hat{\lambda}$ for S&P 500 vs. $\mathrm{THFI}_{\mathrm{sub}}$ according to the *parametric* approach by *Malevergne and Sornette* applying the *Gabaix* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\mathrm{up}}$ and $\hat{\lambda}_{\mathrm{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
$\mathrm{THFI}_{\mathrm{sub}}$	$\hat{\lambda}_{\mathrm{up}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,mean}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,90}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,95}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,99}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{up,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{up}}$	$\hat{\lambda}_{\mathrm{lo}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,mean}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,90}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,95}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,99}}$	$\hat{\lambda}^{\mathrm{bs}}_{\mathrm{lo,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{lo}}$
COA	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.02	0.03	0.05	0.06	0.09	0.17	0.02
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.03**	**0.05**	**0.12**	**0.16**	**0.22**	**0.41**	**0.05**	0.15	0.16	0.24	0.26	0.33	0.58	0.06
EMN	*0*	*0*	*0*	*0*	*0.01*	*0.02*	*0*	**0.51**	**0.51**	**0.72**	**0.77**	**0.87**	**0.97**	**0.16**
EVD	**0.05**	**0.07**	**0.12**	**0.14**	**0.18**	**0.34**	**0.04**	0.42	0.43	0.61	0.68	0.79	0.94	0.13
EDDI	**0.04**	**0.05**	**0.08**	**0.1**	**0.13**	**0.24**	**0.03**	**0.44**	**0.45**	**0.62**	**0.68**	**0.77**	**0.92**	**0.12**
EDMS	0.02	0.03	0.05	0.06	0.08	0.16	0.02	0.3	0.31	0.45	0.51	0.65	0.84	0.11
EDRA	*0*	*0.01*	*0.02*	*0.03*	*0.04*	*0.1*	*0.01*	0.33	0.33	0.44	0.48	0.57	0.87	0.08
FIA	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0.01*	*0.01*	*0.03*	*0.07*	*0.01*
GLM	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.03	0.04	0.09	0.1	0.14	0.25	0.03
LSE	0.02	0.06	0.16	0.22	0.32	0.55	0.07	**0.52**	**0.51**	**0.65**	**0.69**	**0.78**	**0.95**	**0.11**
MAF	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
MST	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.02	0.03	0.06	0.08	0.12	0.32	0.03

Table 3.24.: This table shows $\hat{\lambda}$ for S&P 500 vs. THFI_{sub} according to the non-parametric approach by *Schmidt and Stadtmüller* and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{U}}$ and $\hat{\lambda}_{\text{L}}$ are set italic and the three highest values bold. $\hat{\lambda}_{\text{U}}^{\text{EVT}}$ and $\hat{\lambda}_{\text{L}}^{\text{EVT}}$ are not shown in the table as the values are similar to those found for $\hat{\lambda}_{\text{U}}$ and $\hat{\lambda}_{\text{L}}$.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{U}}$	$\hat{\lambda}^{\text{bs}}_{\text{U,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{U,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{U,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{U,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{U,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{U}}$	$\hat{\lambda}_{\text{L}}$	$\hat{\lambda}^{\text{bs}}_{\text{L,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{L,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{L,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{L,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{L,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{L}}$
COA	0.14	0.16	0.29	0.29	0.38	0.48	0.08	0.19	0.17	0.29	0.33	0.38	0.52	0.08
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	0.19	0.19	0.29	0.33	0.38	0.52	0.09	0.19	0.22	0.33	0.38	0.43	0.62	0.09
EMN	**0.43**	**0.41**	**0.52**	**0.57**	**0.67**	**0.76**	**0.11**	0.24	0.22	0.33	0.38	0.48	0.57	0.09
EVD	**0.43**	**0.43**	**0.57**	**0.62**	**0.67**	**0.86**	**0.1**	**0.38**	**0.38**	**0.52**	**0.52**	**0.62**	**0.71**	**0.1**
EDDI	0.33	0.35	0.48	0.52	0.57	0.76	0.1	**0.43**	**0.42**	**0.52**	**0.57**	**0.67**	**0.81**	**0.1**
EDMS	0.19	0.2	0.33	0.38	0.43	0.62	0.09	**0.38**	**0.34**	**0.48**	**0.52**	**0.57**	**0.71**	**0.1**
EDRA	0.14	0.2	0.33	0.33	0.43	0.57	0.09	0.29	0.29	0.38	0.43	0.52	0.62	0.09
FIA	0.14	0.17	0.29	0.33	0.38	0.52	0.09	*0.14*	*0.17*	*0.29*	*0.33*	*0.38*	*0.52*	*0.09*
GLM	0.24	0.26	0.38	0.43	0.48	0.67	0.1	0.24	0.23	0.33	0.38	0.48	0.62	0.09
LSE	**0.43**	**0.44**	**0.57**	**0.62**	**0.67**	**0.76**	**0.1**	**0.43**	**0.41**	**0.52**	**0.57**	**0.62**	**0.71**	**0.1**
MAF	*0.1*	*0.13*	*0.24*	*0.24*	*0.33*	*0.38*	*0.07*	*0.1*	*0.09*	*0.19*	*0.19*	*0.24*	*0.43*	*0.06*
MST	*0.1*	*0.12*	*0.24*	*0.29*	*0.33*	*0.48*	*0.08*	0.19	0.18	0.29	0.33	0.38	0.48	0.08

Table 3.25.: This table shows $\hat{r}$ for S&P 500 vs. $\mathrm{THFI}_{\mathrm{sub}}$ according to *Pearson*'s product-moment correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{r}$. The three lowest values for $\hat{r}_{\mathrm{up}}$ and $\hat{r}_{\mathrm{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
$\mathrm{THFI}_{\mathrm{sub}}$	$\hat{r}_{\mathrm{up}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{up,mean}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{up,90}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{up,95}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{up,99}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{up,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{up}}$	$\hat{r}_{\mathrm{lo}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{lo,mean}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{lo,90}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{lo,95}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{lo,99}}$	$\hat{r}^{\mathrm{bs}}_{\mathrm{lo,max}}$	$\hat{\sigma}^{\mathrm{bs}}_{\mathrm{lo}}$
COA	**0.03**	**0.04**	**0.45**	**0.54**	**0.71**	**0.88**	**0.29**	0.42	0.37	0.77	0.82	0.9	0.94	0.35
DSB	-0.07	-0.13	0.16	0.23	0.38	0.63	0.21	*-0.63*	*-0.53*	*-0.15*	*-0.01*	*0.22*	*0.45*	*0.26*
EMM	**-0.03**	**-0.03**	**0.22**	**0.28**	**0.39**	**0.53**	**0.18**	**0.71**	**0.53**	**0.86**	**0.89**	**0.94**	**0.96**	**0.33**
EMN	**0.11**	**0.11**	**0.43**	**0.53**	**0.64**	**0.75**	**0.25**	0.18	0.22	0.59	0.66	0.82	0.89	0.29
EVD	-0.21	-0.21	0.06	0.15	0.3	0.42	0.21	0.65	0.53	0.85	0.88	0.94	0.97	0.31
EDDI	*-0.28*	*-0.23*	*0.07*	*0.19*	*0.35*	*0.55*	*0.22*	0.61	0.5	0.83	0.87	0.92	0.95	0.33
EDMS	-0.15	-0.17	0.07	0.15	0.31	0.47	0.21	**0.68**	**0.55**	**0.85**	**0.88**	**0.93**	**0.97**	**0.3**
EDRA	-0.04	-0.07	0.19	0.27	0.4	0.54	0.21	**0.69**	**0.55**	**0.84**	**0.87**	**0.92**	**0.96**	**0.27**
FIA	*-0.34*	*-0.36*	*-0.13*	*-0.03*	*0.11*	*0.29*	*0.17*	0.56	0.46	0.84	0.88	0.91	0.93	0.33
GLM	-0.24	-0.24	0.03	0.12	0.3	0.6	0.2	0.11	0.01	0.46	0.6	0.85	0.93	0.35
LSE	*-0.49*	*-0.46*	*-0.15*	*-0.01*	*0.25*	*0.37*	*0.23*	0.62	0.48	0.82	0.86	0.92	0.98	0.32
MAF	-0.25	-0.24	0.01	0.08	0.31	0.52	0.19	*-0.49*	*-0.55*	*-0.33*	*-0.24*	*-0.1*	*0.2*	*0.16*
MST	-0.07	-0.14	0.18	0.3	0.53	0.76	0.25	*-0.21*	*-0.06*	*0.23*	*0.35*	*0.53*	*0.87*	*0.23*

Table 3.26.: This table shows the ranking of TDCs for S&P 500 vs. $THFI_{sub}$ for the top and the bottom group of the upper and lower tail of the return distribution. 'Mal.' stands for Malevergne and 'Sor.' for Sornette. (The TDCs for the bottom group in the upper tail are all equal to zero in all columns for the methods of Malevergne and Sornette. For the non-parametric method of Malevergne and Sornette applying the Hill estimator the TDCs for COA, GLM, and MST are also equal to zero in all columns in the bottom group of the upper tail.)

Tail	Upper tail						Lower tail					
Group	Top group			Bottom group			Top group			Bottom group		
Ranking	1	2	3	1	2	3	1	2	3	1	2	3
Mal. and Sor., non-par, Gabaix	EVD	LSE	EMM	DSB	FIA	MAF	EMN	EDDI	EVD	DSB	MAF	FIA
Mal. and Sor., non-par, Hill	EVD	EDDI	EMM	DSB	FIA	MAF	EMN	EDDI	EVD	DSB	MAF	COA
Mal. and Sor., par, Gabaix	EVD	EDDI	EMM	DSB	FIA	MAF	LSE	EMN	EDDI	DSB	MAF	FIA
Mal. and Sor., par, Hill	EVD	EDDI	LSE	DSB	FIA	MAF	EMN	LSE	EDDI	DSB	MAF	FIA
Schmidt and Stadtmüller	LSE	EVD	EMN	DSB	MST	MAF	EDDI	LSE	EVD	DSB	MAF	FIA
Pearson's r	EMN	COA	EMM	LSE	FIA	EDDI	EMM	EDRA	EDMS	DSB	MAF	MST
Spearman's ρ	EMN	COA	EMM	LSE	EVD	EDMS	EMM	EDRA	FIA	MAF	DSB	GLM
Kendall's τ	EMN	COA	EMM	LSE	EDMS	EVD	EDRA	FIA	EMN	MAF	DSB	GLM
# of different subindices (rank)	3	4	3	2	4	4	5	5	5	2	2	4
Max. # of same subindex (rank)	4	3	6	5	5	5	3	2	3	6	6	4
# of similar subindices (group)	6			8			8			6		
List of similar subindices (group)	COA	EVD		DSB	FIA		EDDI	EMN		COA	MAF	
	EDDI	LSE		EDDI	LSE		EDMS	EVD		DSB	MST	
	EMM			EDMS	MAF		EDRA	FIA		FIA		
	EMN			EVD	MST		EMM	LSE		GLM		

Table 3.27.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation S&P 500 vs. $THFI_{sup}$. This values are calculated for the *upper tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	0.9388	0.9874	0.9482	0.6398	-0.3001	-0.526	-0.5758
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0.9388	1	0.9252	0.9935	0.5969	-0.2261	-0.5005	-0.551
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.9874	0.9252	1	0.9482	0.6093	-0.2671	-0.4886	-0.5491
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.9482	0.9935	0.9482	1	0.6021	-0.2516	-0.5152	-0.5759
Schmidt and Stadtmüller (λ_S)	0.6398	0.5969	0.6093	0.6021	1	-0.152	-0.3905	-0.3869
Pearson's r	-0.3001	-0.2261	-0.2671	-0.2516	-0.152	1	0.8154	0.8303
Spearman's ρ	-0.526	-0.5005	-0.4886	-0.5152	-0.3905	0.8154	1	0.9862
Kendall's τ	-0.5758	-0.551	-0.5491	-0.5759	-0.3869	0.8303	0.9862	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0.0185	0.3191	0.0648	0.0395
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0.0313	0.4575	0.0815	0.051
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0.0271	0.3777	0.0902	0.052
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0.0294	0.4069	0.0716	0.0394
Schmidt and Stadtmüller (λ_S)	0.0185	0.0313	0.0271	0.0294	0	0.62	0.187	0.1916
Pearson's r	0.3191	0.4575	0.3777	0.4069	0.62	0	0.0007	0.0004
Spearman's ρ	0.0648	0.0815	0.0902	0.0716	0.187	0.0007	0	0
Kendall's τ	0.0395	0.051	0.052	0.0394	0.1916	0.0004	0	0

Table 3.28.: This table shows the correlation between the eight methods that have been applied to calculate the TDCs for the relation S&P 500 vs. $THFI_{sup}$. This values are calculated for the *lower tail* of the return distribution with Spearman's rank correlation coefficient ρ. 'Mal.' stands for Malevergne and 'Sor.' for Sornette.

Correlation coefficients	$\lambda_{np,G}$	$\lambda_{np,H}$	$\lambda_{p,G}$	$\lambda_{p,H}$	λ_S	r	ρ	τ
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	1	0.9958	0.9666	0.9792	0.8548	0.5284	0.3116	0.3071
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0.9958	1	0.9626	0.9834	0.8595	0.5565	0.3628	0.3581
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0.9666	0.9626	1	0.9792	0.8939	0.5505	0.3532	0.3513
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0.9792	0.9834	0.9792	1	0.879	0.5565	0.371	0.3664
Schmidt and Stadtmüller (λ_S)	0.8548	0.8595	0.8939	0.879	1	0.5964	0.3792	0.3523
Pearson's r	0.5284	0.5565	0.5505	0.5565	0.5964	1	0.9243	0.8956
Spearman's ρ	0.3116	0.3628	0.3532	0.371	0.3792	0.9243	1	0.9849
Kendall's τ	0.3071	0.3581	0.3513	0.3664	0.3523	0.8956	0.9849	1
p-values								
Mal. and Sor., non-par, Gabaix ($\lambda_{np,G}$)	0	0	0	0	0.0002	0.0634	0.3	0.3075
Mal. and Sor., non-par, Hill ($\lambda_{np,H}$)	0	0	0	0	0.0002	0.0483	0.2231	0.2296
Mal. and Sor., par, Gabaix ($\lambda_{p,G}$)	0	0	0	0	0	0.0512	0.2365	0.2392
Mal. and Sor., par, Hill ($\lambda_{p,H}$)	0	0	0	0	0.0001	0.0483	0.212	0.2182
Schmidt and Stadtmüller (λ_S)	0.0002	0.0002	0	0.0001	0	0.0314	0.2013	0.2378
Pearson's r	0.0634	0.0483	0.0512	0.0483	0.0314	0	0	0
Spearman's ρ	0.3	0.2231	0.2365	0.212	0.2013	0	0	0
Kendall's τ	0.3075	0.2296	0.2392	0.2182	0.2378	0	0	0

4. Conclusions

The task of this work has been to investigate the behavior of various tail dependence measures when applied to hedge fund data and assess which one of the methods performs best. Ideally, a best practice guide for the choice of the most suitable method can be derived based on this work. If we recapitulate the results presented and discussed in the above chapter, we can draw various conclusions that might help to fulfill, at least partly, the above task.

As the size of the investigated samples is small (53 or 173 observations), the right choice of the tail index estimator is crucial. We could see in section 3.1 that the Huisman estimator performs poorly. This has been surprising as this estimator has been developed to improve the quality of the Hill estimator for small samples. Hence, I decided not to use this estimator for the subsequent sections anymore. The Clauset estimator performed comparably well, but showed some lock-in effects due to the min max procedure applied. Therefore, I did not present detailed results that are based on the application of this estimator. The Hill estimator performed comparably well and proofed to be a useful tail index estimator, despite its strong bias for small samples. Therefore, I calculated detailed results applying this estimator and present them in the appendix. The best performance, however, showed the Gabaix estimator. That was the reason that I focused my analysis on those results that were gained when the Gabaix estimator has been applied.

If we have a closer look at the methods to estimate the TDCs, we can see that the method developed by Poon et al. does not perform well for the given tasks (see section 3.1.1) and can therefore be excluded as a good candidate for the reliable estimation of TDCs. This is surprising, as in the paper of Poon et al. [18] the method performs well in the empirical tests. The reason for the poor performance of Poon et al.'s method might be the small samples used in this work. However, the performance of Poon et al.'s method in the thesis of Schmuki [20] has been unfruitful as well, leaving questions open about the suitability of the method to estimate TDCs in general.

Having a closer look at the results obtained with the methods developed by Malevergne and Sornette, we can see that the choice of the tail index estimator (i.e. Hill or Gabaix) did not have a remarkable influence on the results gained. This holds for the non-parametric as well as for the parametric approach. We can also see that the non-parametric and the parametric approach yielded comparable and sometimes almost similar results. This is supported by the finding that all correlation coefficients comparing the methods among each other are higher than 0.78 and most often higher than 0.9. However, there seems to be a general tendency that the TDCs calculated with the parametric approach behave smoother than those calculated with the non-parametric approach. This might be due to the fact that for the non-parametric approach only one parameter (l) is smoothed by taking its mean over the whole tail (see equation (2.48)), whereas for the parametric approach two parameters (C_{Y} and C_{ϵ}) are smoothed (see equations (2.55) and (2.56)).

Looking at specific results derived with methods from Malevergne and Sornette, we have the interesting finding that the S&P 500 reacts stronger to extreme upward than downward movements of the THFI. To check whether this reflects reality or is just a spurious result, I performed the calculations with an adjusted factor model as proposed by Yannick Malevergne. This adjusted model should produce the same results as the original model, with exchanged return vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. However, the results based on the adjusted model are not the same (compare Figure 3.7 with Figures 3.3, 3.4, and 3.5), leaving room open for speculations about the 'true' tail dependence between the S&P 500 and the THFI. If we investigate the TDCs for the above relation with the method developed by Schmidt and Stadtmüller, we cannot find such an asymmetry between upper and lower tail. In fact, the values for λ_{U} and λ_{L} are equal (0.29) for $k/N = 0.12$. It might be that the asymmetry between the upper and the lower tail is due to the β used for the estimation of λ in the methods of Malevergne and Sornette. The method of Schmidt and Stadtmüller, however, applies indicator functions as estimators (see equations (2.74), (2.75), (2.76), and (2.77)) that are independent of the order of the return vectors $\boldsymbol{x}$ and $\boldsymbol{y}$.

The method developed by Schmidt and Stadtmüller produces similar (most often equal) results for both estimators ($\hat{\lambda}$ and $\hat{\lambda}^{\mathrm{EVT}}$). Hence, only the detailed results for λ are presented in the thesis. This is somehow surprising, as the estimators are different and and have been derived differently. The TDC estimates calculated with the method of Schmidt and Stadtmüller are often independent of the other results. However, the correlation coefficients between Schmidt and Stadtmüller's method and the methods of Malevergne and Sornette are in a range of 0.33 to 0.89 and in a range of -0.39 and 0.97 for the relation with the linear measures of dependence. This implies, as can be expected, that the results of Schmidt and Stadtmüllers method are closer related to the other nonlinear measures of tail dependence than to the linear measures of tail dependence.

When investigating the linear measures of correlation, namely Pearson's r, Spearman's ρ, and Kendall's τ, we can see that they produce often comparable results. This is also supported by the finding that the p-values calculated with the Kruskal-Wallis test vary between 0.03 and 0.85, and the correlation coefficients among the three methods are all larger than 0.66 and most often higher than 0.9.

To come to a final conclusion and trying to answer the question which measure of tail dependence suits best for our purpose, I have to admit that the picture gained during this work is too heterogeneous as that it is possible to make a simple statement pro or contra a single method. The following three surprises are part of this heterogeneous picture: the first was how badly the method of Poon et al. performed, the second was the poor performance of the Huisman estimator, and the third was the finding that results derived with linear methods of tail dependence are sometimes comparable to the ones derived with nonlinear methods. However, this holds only for those calculations where 53 observations were available. In the other cases, the results for the linear and the nonlinear methods are clearly different. Therefore, I would propose that the nonlinear methods generally are superior to the linear methods. It also seems that the precision of the nonlinear methods is better than for the linear methods, as the standard deviations calculated with bootstrapping are by trend smaller for the nonlinear methods than for the linear ones. However, we need to keep in mind that the range for the nonlinear TDCs is from 0 to 1, whereas the range for the linear methods is from -1 to 1. To decide whether the methods of Malevergne and Sornette are better or worse than the method of Schmidt and Stadtmüller, we need to know whether Malevergne and Sornette's methods produce spurious results for small data sets (as might be the case for the relation THFI vs. S&P 500) or not. If the results are not spurious, I would prefer the methods of Malevergne and Sornette, as the moving threshold graphs are smoother (especially for the parametric approach), the separation between the different TDCs is generally better than for Schmidt and Stadtmüller's method, and the underlying factor model makes the concept more comprehensible. However, the beauty of the method developed by Schmidt and Stadtmüller is the symmetry of its results, i.e. the independence of the order of return vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, its independence from assumptions regarding the investigated return distribution (no need to assume a specific power-law behavior, i.e. no tail index estimation needed), the consistent results obtained with both estimators, and its robustness.

Conclusions regarding the economic interpretation of the results are difficult, as only few tendencies could be extracted. However, it seems that managed futures (CTAs) have small tail dependencies with the market, as they often apply trading software that does not show herding behavior in extreme situations. This is also valid for hedge funds that do not consist, or only to a very small extent, of equity. On the other hand, we have hedge funds that are mainly equity based (e.g event driven funds) and, hence, show a higher tail dependence with the market movements. Further economic interpretations are beyond the scope of this work and should be left to a hedge fund expert.

Although the above findings are interesting and hopefully helpful for the better understanding of tail dependencies of hedge funds, they are still subject to large uncertainties. Therefore, I would propose to tackle the following questions/tasks as next steps:

- Prolongation of data sets based on benchmarks, as the amount of data points is crucial for the quality of the TDC estimation,
- Implementation of GARCH filters to test whether the return data reveals autoregressive behavior,
- Test the quality of the methods applied with synthetic data and test which tail index estimator performs best,
- Test the found TDCs on real data by composing portfolios of hedge funds with low TDCs and high TDCs,
- Implement confidence intervals in the moving threshold charts to ease the choice of an optimal threshold value k,

To conclude and put everything into a broader perspective, I found the world of hedge funds to be multifaceted and almost as diverse as real nature. There are so many different strategies and substrategies, different ways to combine funds and hedge funds, different means of investment that one can hardly draw any conclusion valid for all hedge funds. Some of them seem to follow risky strategies by using their legitimate freedom to invest in almost anything one can think of, while others actually try to hedge away risks by elaborate diversification strategies. All in all this work gave me a interesting insight into the fascinating world of hedge funds.

Bibliography

[1] A. Clauset, C.R. Shalizi, and MEJ Newman. Power-law distributions in empirical data. *arXiv*, 706, 2007.

[2] S. Coles, J. Heffernan, and J. Tawn. Dependence Measures for Extreme Value Analyses. *Extremes*, 2(4):339–365, 1999.

[3] H. Drees and X. Huang. Best Attainable Rates of Convergence for Estimators of the Stable Tail Dependence Function. *Journal of Multivariate Analysis*, 64(1):25–47, 1998.

[4] B. Efron. Bootstrap methods: another look at the jackknife. *Breakthroughs in Statistics*, 1997.

[5] B. Efron and R.J. Tibshirani. An Introduction to the Bootstrap. *Monographs on Statistics and Applied Probability*, 57:1–177, 1993.

[6] P. Embrechts, C. Kluppelberg, and T. Mikosch. *Modelling Extremal Events*. Springer, 1997.

[7] X. Gabaix and R. Ibragimov. Rank-1/2: A Simple Way to Improve the Ols Estimation of Tail Exponents. *NBER Working Paper*, pages 1–31, 2008.

[8] J.E. Heffernan. A Directory of Coefficients of Tail Dependence. *Extremes*, 3(3):279–290, 2000.

[9] B.M. Hill. A simple general approach to inference about the tail of a distribution. *Annals of Statistics*, 3(5):1163–1174, 1975.

[10] X. Huang. *Statistics of bivariate extreme values*, volume 22. Thesis Publishers and Tinbergen Institute, Rotterdam, 1992.

[11] R. Huisman, KG Koedijk, CJM Kool, and F. Palm. Tail-Index Estimates in Small Samples. *Journal of Business & Economic Statistics*, 19(2):208–216, 2001.

[12] A.W. Ledford and J.A. Tawn. Statistics for near independence in multivariate extreme values. *Biometrika*, 83(1):169–187, 1996.

[13] A.W. Ledford and J.A. Tawn. Concomitant tail behaviour for extremes. *Adv. in Appl. Probab*, 30(1):197–215, 1998.

[14] Y. Malevergne and D. Sornette. Minimising extremes. *Risk*, 15(11):129–133, November 2002.

[15] Y. Malevergne and D. Sornette. How to account for extreme co-movements between individual stocks and the market. *The Journal of Risk*, 6(3):71–116, 2004.

[16] Y. Malevergne and D. Sornette. *Extreme Financial Risks: From Dependence to Risk Management*. Springer, 2006.

[17] L. Peng. *Second Order Condition and Extreme Value Theory*, volume 178. Thesis Publishers and Tinbergen Institute, Rotterdam, 1998.

[18] S.H. Poon, M. Rockinger, and J. Tawn. Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications. *Review of Financial Studies*, 17(2):581–610, 2004.

[19] R. Schmidt and U. Stadtmüller. Non-parametric Estimation of Tail Dependence. *Scandinavian Journal of Statistics*, 33(2):307–335, 2006.

[20] S. M. Schmuki. Tail Dependence: Implementation, Analysis, and Study of the most Recent Concepts. Master's thesis, Swiss Federal Institute of Technology, 2008.

[21] W.F. Sharpe. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, 19(3):425–442, 1964.

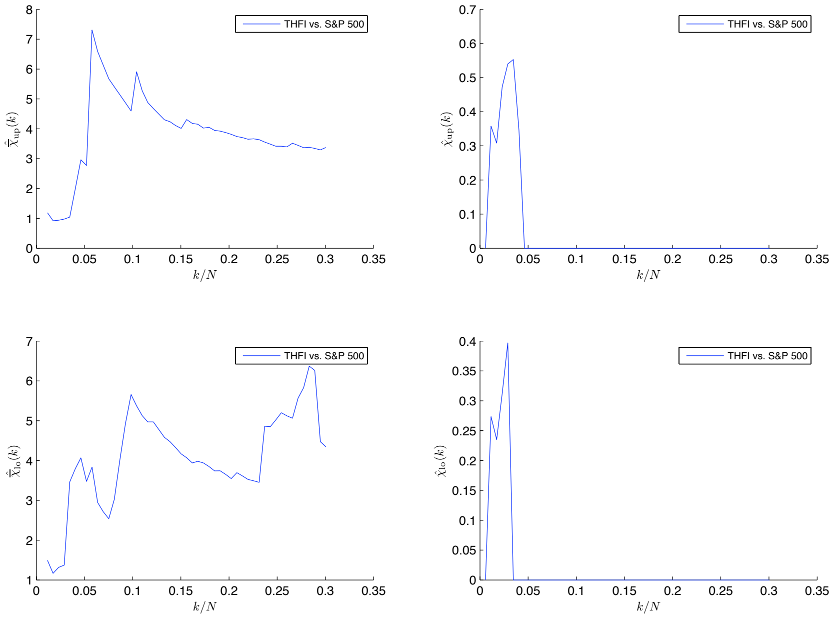
[22] M. Sibuya. Bivariate extreme statistics, I. *Annals of the Institute of Statistical Mathematics*, 11(2):195–210, 1959.

[23] A. Sklar. Fonction de répartition à n dimensions et leurs marges. *Publication de l'Institute de Statistique de l'Université de Paris*, 8:229–231, 1959.

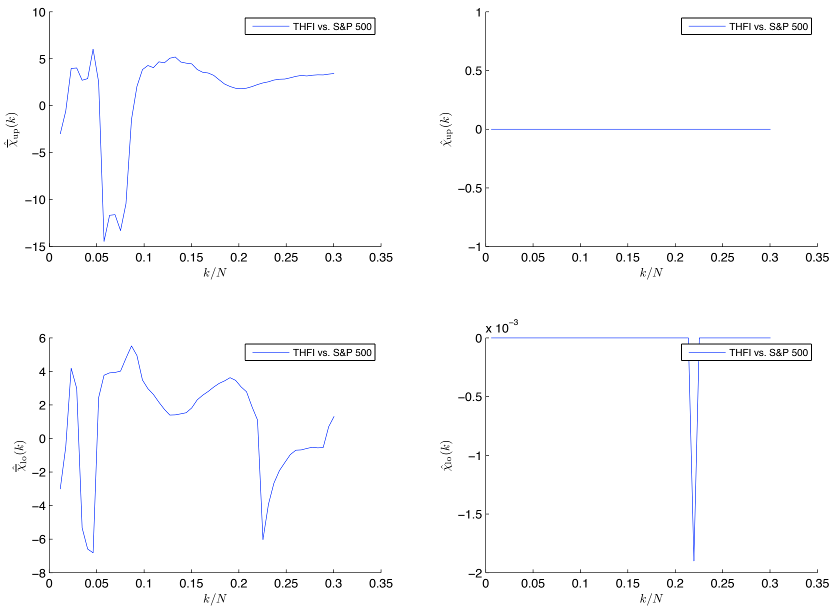
[24] Credit Suisse and Tremont. Tremont hedge fund index (www.hedgeindex.com). Internet.

A. Appendix

A.1. THFI vs. S&P 500



(a) With Clauset estimator



(b) With Huisman estimator

Figure A.1.: $\overline{\chi}$ and χ calculated for THFI vs. S&P 500 applying the Clauset estimator in subfigure (a) and the Huisman estimator in subfigure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

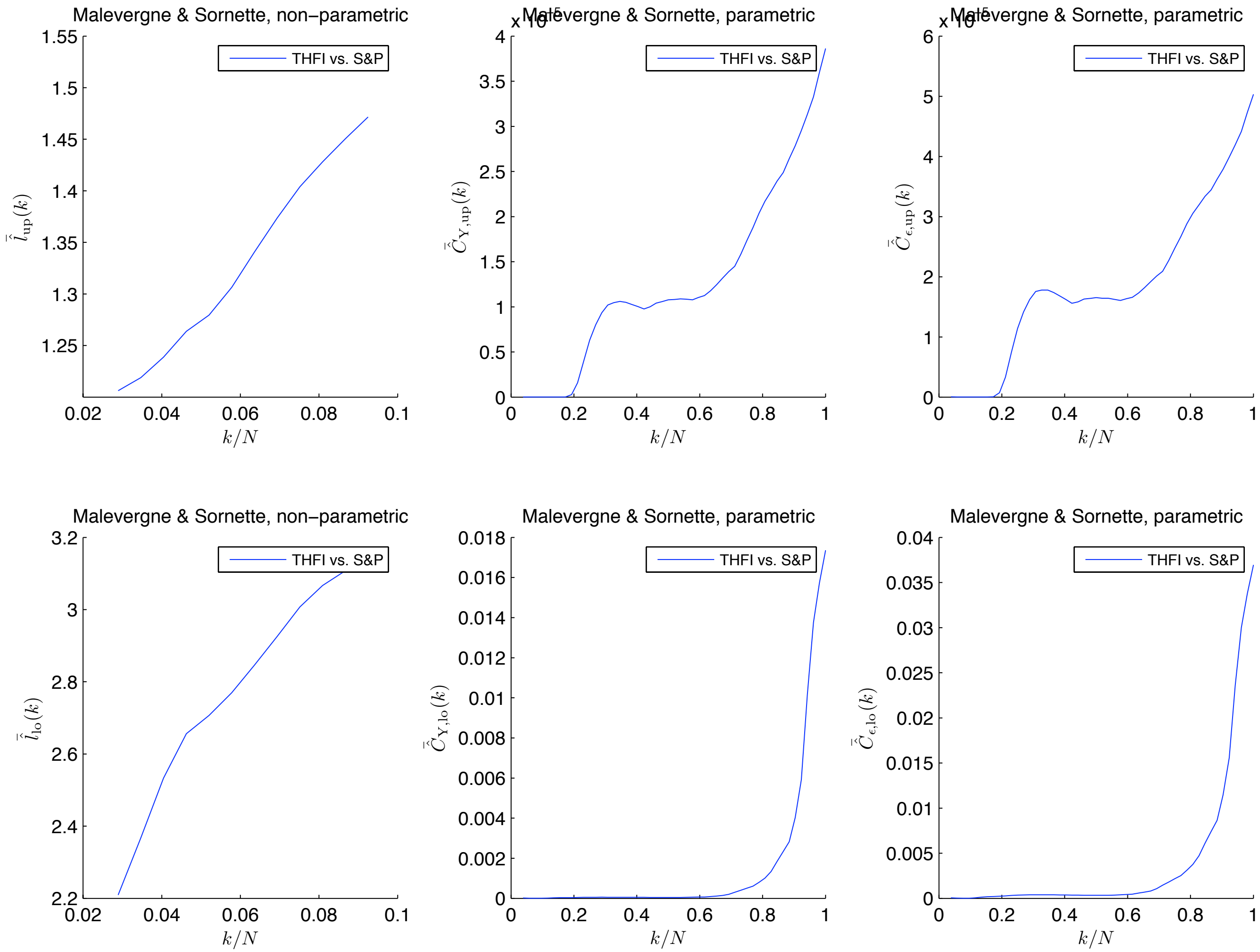
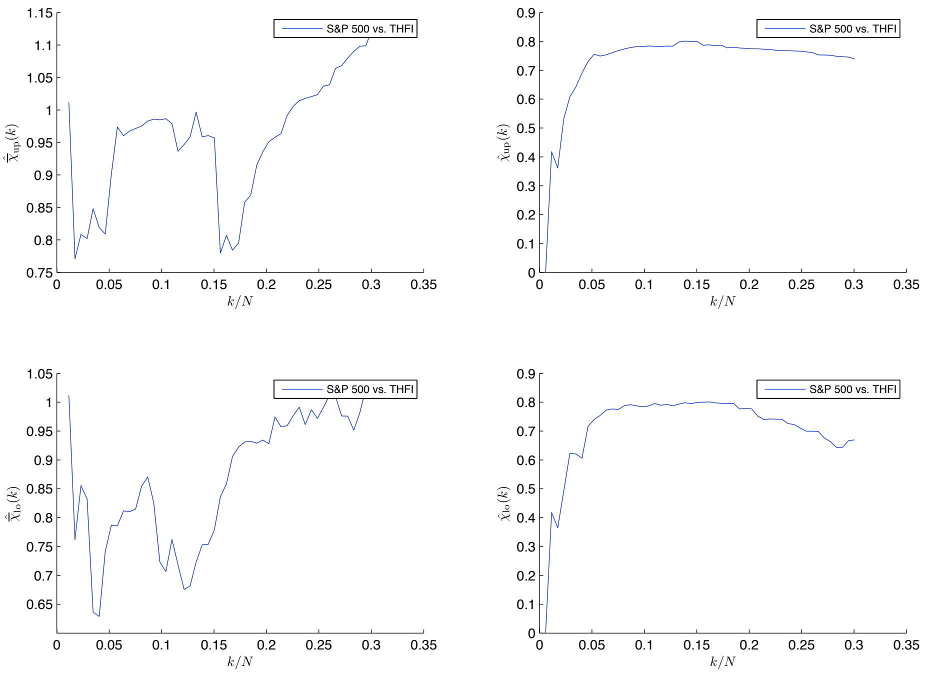
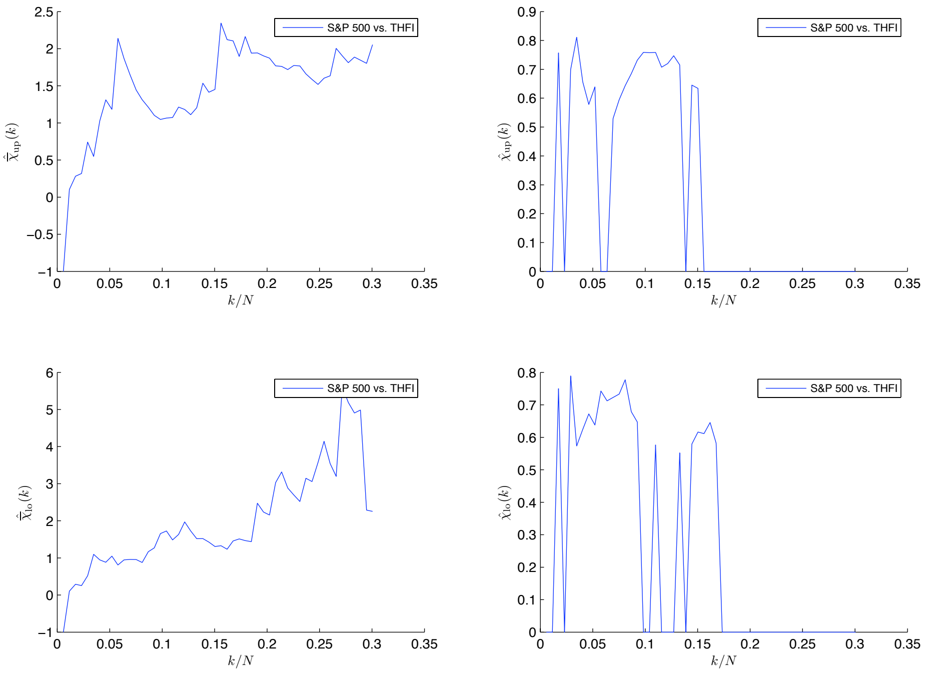


Figure A.2.: Parameters used to calculate the non-parametric and parametric method of Malevergne and Sornette. From left to right: $\bar{\hat{l}}(k)$ for the non-parametric, and the scale factor $\bar{\hat{C}}_Y$ and $\bar{\hat{C}}_\epsilon$ for the non-parametric approach. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

A.2. S&P 500 vs. THFI



(a) $\overline{\chi}$ and χ with Hill estimator



(b) $\overline{\chi}$ and χ with Gabaix estimator

Figure A.3.: $\overline{\chi}$ and χ calculated for S&P 500 vs. THFI applying the Hill estimator in sub-figure (a) and the Gabaix estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

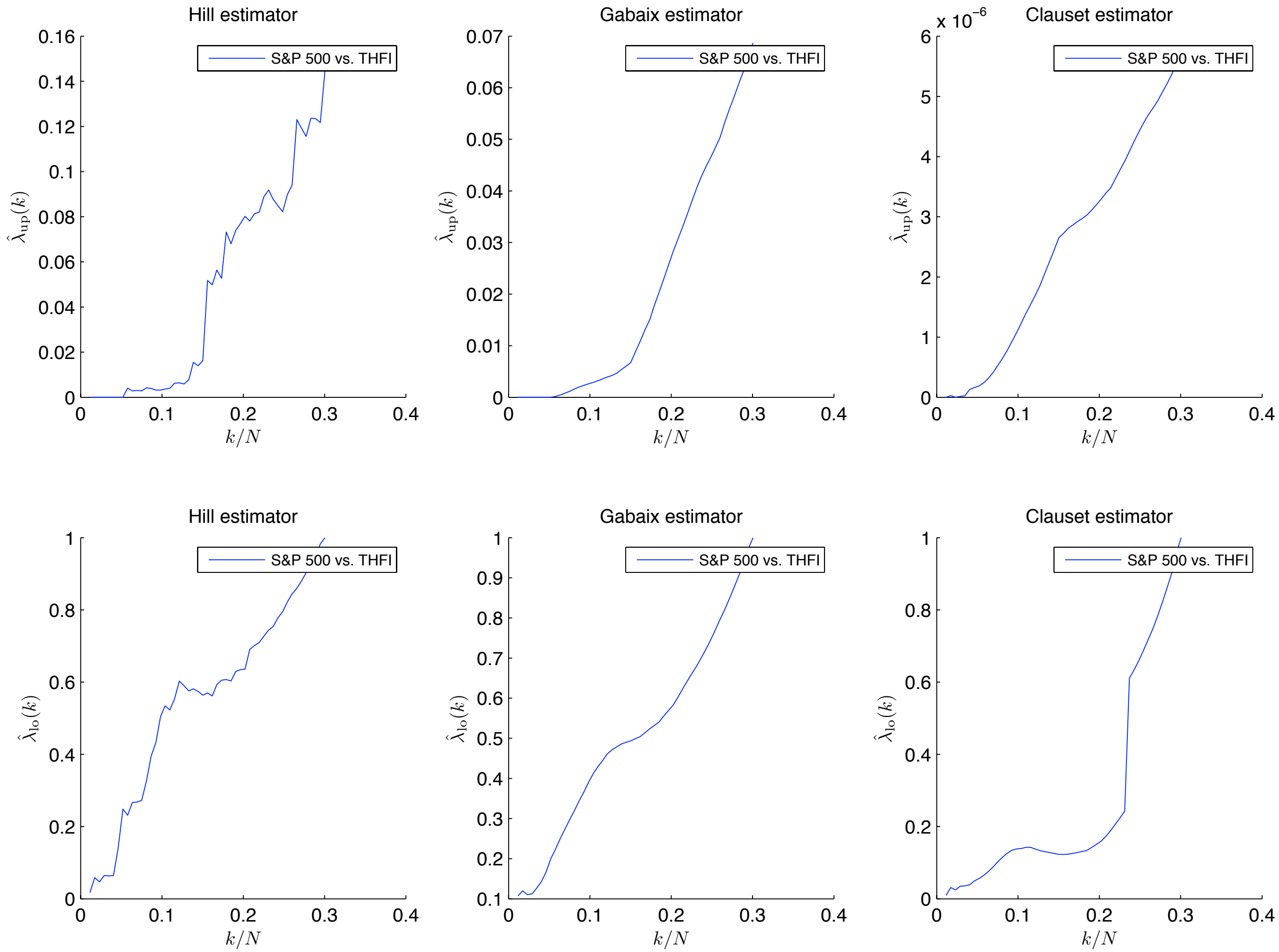


Figure A.4.: λ for S&P 500 vs. THFI with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *non-parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

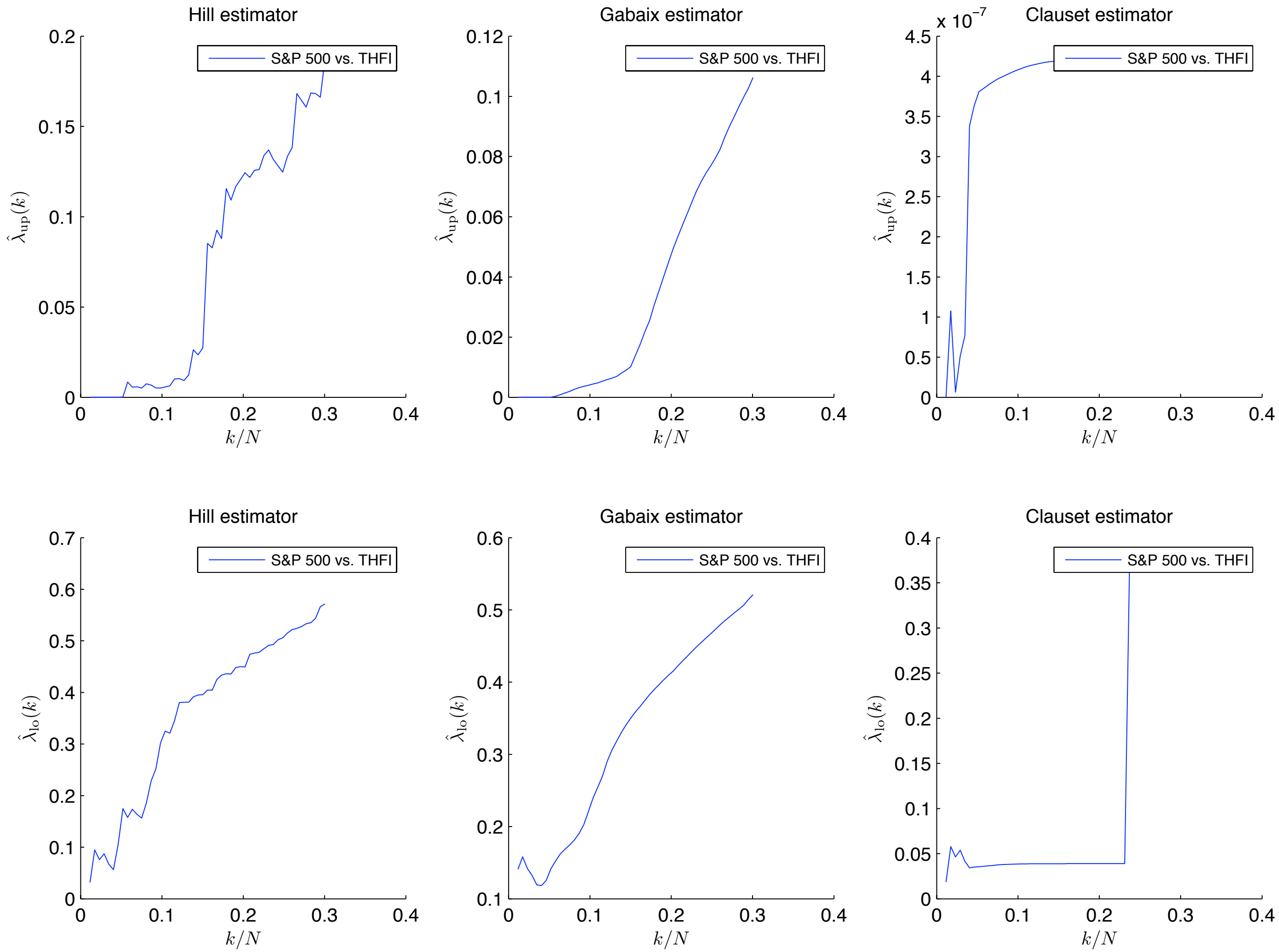


Figure A.5.: λ for S&P 500 vs. THFI with Hill, Gabaix, and Clauset estimator from left to right. The TDC has been calculated applying the *parametric* method by *Malevergne and Sornette*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

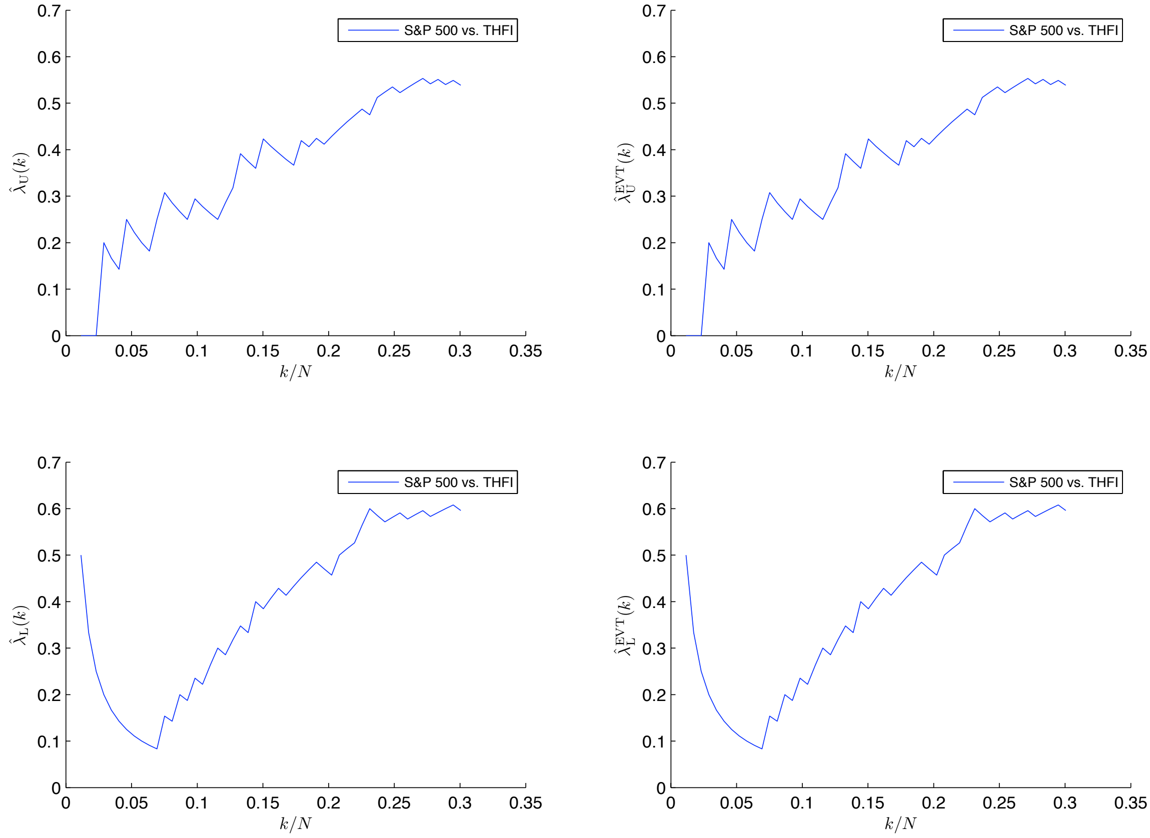


Figure A.6.: λ for S&P 500 vs. THFI by applying the non-parametric method developed by *Schmidt and Stadtmüller*. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

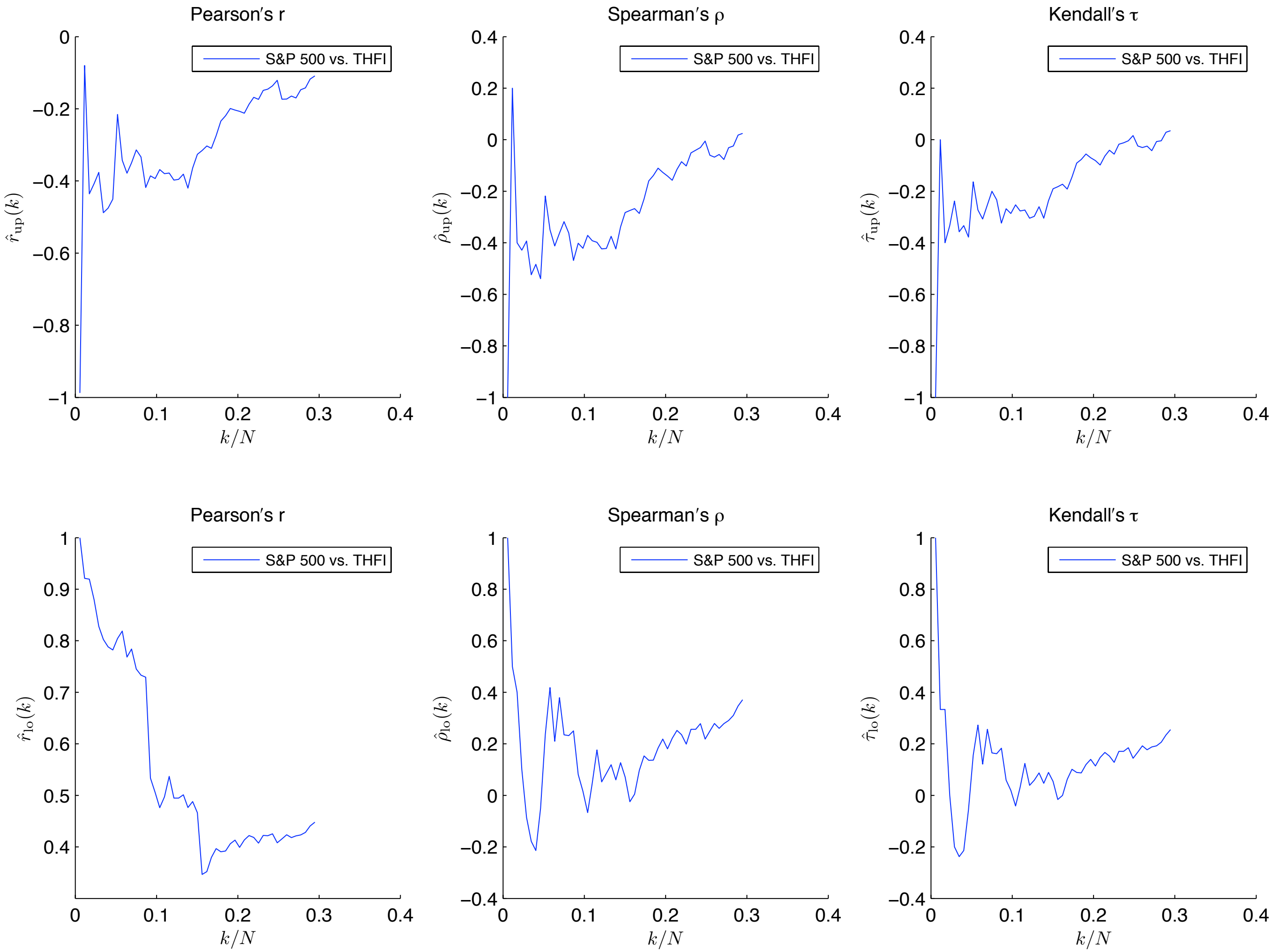
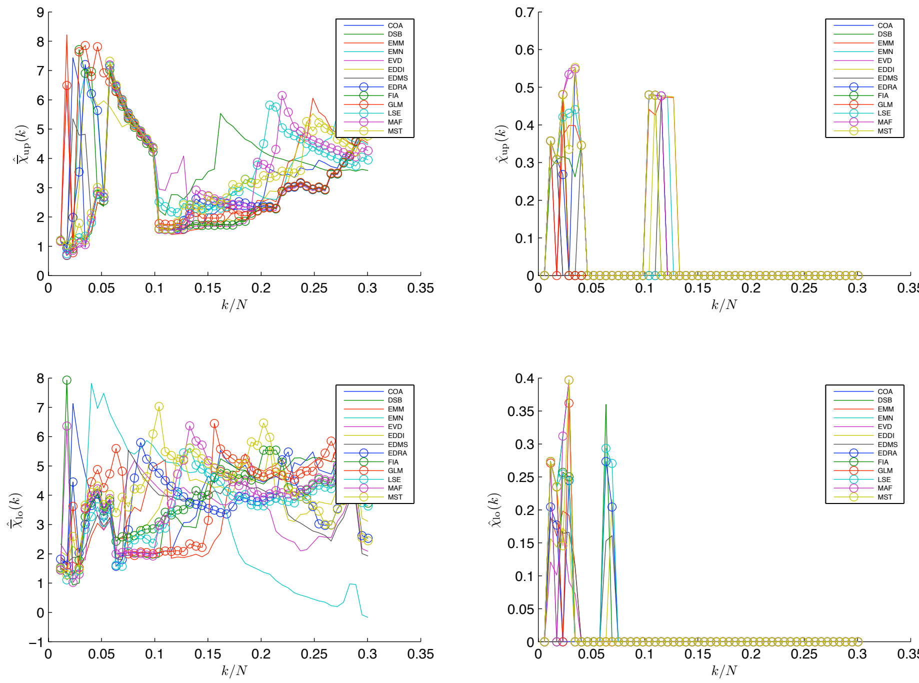
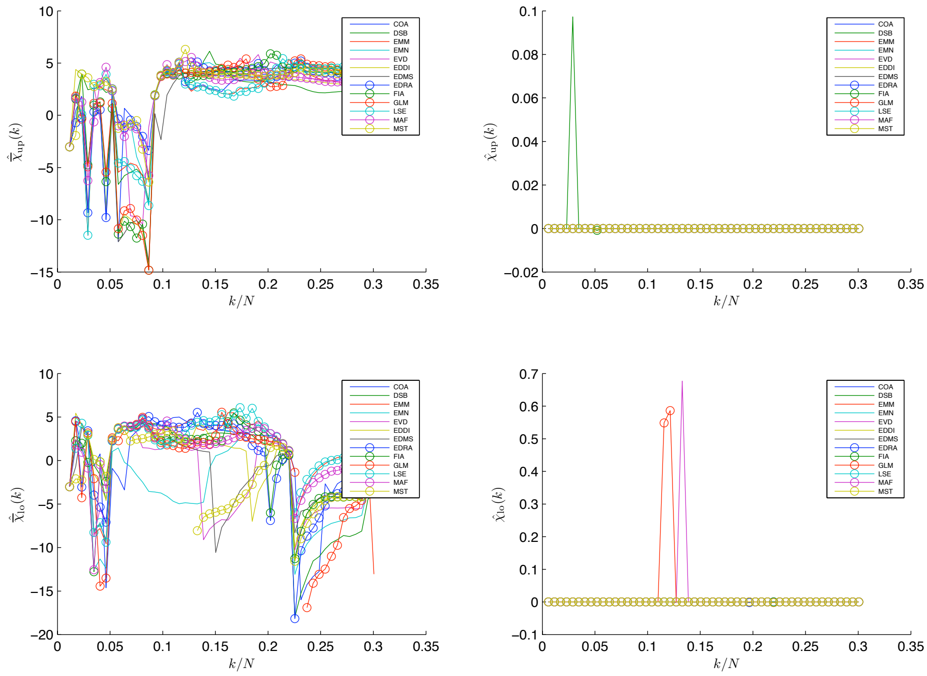


Figure A.7.: λ for S&P 500 vs. THFI calculated with *linear measures of correlation*. From left to right we have Pearson's r, Spearman's ρ, and Kendall's τ. The first row contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

A.3. THFI vs THFI$_{\text{sub}}$



(a) With Clauset estimator



(b) With Huisman estimator

Figure A.8.: $\overline{\chi}$ and χ calculated for THFI vs. THFI$_{\text{sub}}$ applying the Clauset estimator in sub-figure (a) and the Huisman estimator in sub-figure (b).

Table A.1.: This table shows $\hat{\lambda}$ for THFI vs. THFI_{sub} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	0.17	0.17	0.27	0.29	0.35	0.42	0.07	0.12	0.11	0.18	0.2	0.24	0.31	0.05
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.75**	**0.74**	**0.85**	**0.87**	**0.92**	**1**	**0.09**	0.23	0.22	0.32	0.35	0.39	0.48	0.08
EMN	*0.02*	*0.02*	*0.04*	*0.06*	*0.08*	*0.11*	*0.02*	**0.59**	**0.59**	**0.74**	**0.79**	**1**	**1**	**0.12**
EVD	0.57	0.55	0.72	0.76	0.84	0.97	0.14	**0.61**	**0.61**	**0.72**	**0.75**	**0.84**	**1**	**0.08**
EDDI	0.32	0.32	0.46	0.5	0.56	0.68	0.11	0.53	0.53	0.6	0.62	0.66	0.7	0.06
EDMS	0.54	0.52	0.66	0.69	0.75	0.83	0.11	0.49	0.48	0.57	0.6	0.64	0.7	0.07
EDRA	0.09	0.09	0.15	0.17	0.21	0.26	0.05	0.21	0.2	0.28	0.3	0.34	0.39	0.06
FIA	0.23	0.23	0.35	0.38	0.45	0.55	0.09	0.17	0.16	0.22	0.24	0.27	0.31	0.05
GLM	**0.61**	**0.6**	**0.68**	**0.69**	**0.73**	**0.8**	**0.07**	**0.61**	**0.59**	**0.67**	**0.69**	**0.72**	**0.76**	**0.07**
LSE	**0.64**	**0.62**	**0.72**	**0.75**	**0.8**	**0.9**	**0.09**	0.53	0.51	0.63	0.66	0.71	0.77	0.1
MAF	*0.01*	*0.02*	*0.04*	*0.04*	*0.06*	*0.09*	*0.01*	*0.02*	*0.02*	*0.05*	*0.06*	*0.08*	*0.14*	*0.02*
MST	0.03	0.04	0.07	0.09	0.12	0.17	0.03	*0.07*	*0.07*	*0.12*	*0.13*	*0.16*	*0.22*	*0.04*

Table A.2.: This table shows $\hat{\lambda}$ for THFI vs. THFI_{sub} according to the *parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	0.19	0.2	0.26	0.28	0.3	0.33	0.05	0.13	0.13	0.2	0.21	0.26	0.35	0.05
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.77**	**0.77**	**0.83**	**0.86**	**0.9**	**0.98**	**0.05**	0.2	0.2	0.27	0.29	0.32	0.38	0.06
EMN	*0.03*	*0.03*	*0.06*	*0.07*	*0.1*	*0.14*	*0.02*	0.35	0.35	0.41	0.46	0.54	0.82	0.06
EVD	0.62	0.62	0.64	0.65	0.67	0.87	0.02	**0.39**	**0.4**	**0.45**	**0.46**	**0.48**	**0.52**	**0.05**
EDDI	0.43	0.42	0.48	0.49	0.51	0.65	0.05	0.3	0.3	0.37	0.4	0.45	0.51	0.06
EDMS	0.68	0.68	0.72	0.73	0.78	0.87	0.03	0.38	0.38	0.42	0.43	0.45	0.47	0.04
EDRA	0.1	0.11	0.17	0.19	0.22	0.26	0.04	0.17	0.17	0.23	0.25	0.29	0.37	0.05
FIA	0.24	0.24	0.31	0.32	0.34	0.38	0.05	0.12	0.12	0.19	0.22	0.29	0.38	0.06
GLM	**0.84**	**0.84**	**0.89**	**0.91**	**0.95**	**0.99**	**0.04**	**0.52**	**0.52**	**0.56**	**0.57**	**0.59**	**0.68**	**0.03**
LSE	**0.78**	**0.78**	**0.85**	**0.87**	**0.91**	**0.98**	**0.05**	**0.46**	**0.45**	**0.49**	**0.5**	**0.52**	**0.58**	**0.04**
MAF	*0.02*	*0.02*	*0.05*	*0.06*	*0.07*	*0.11*	*0.02*	*0.02*	*0.03*	*0.06*	*0.07*	*0.1*	*0.21*	*0.02*
MST	0.04	0.05	0.1	0.11	0.13	0.18	0.03	*0.07*	*0.07*	*0.13*	*0.15*	*0.21*	*0.33*	*0.04*

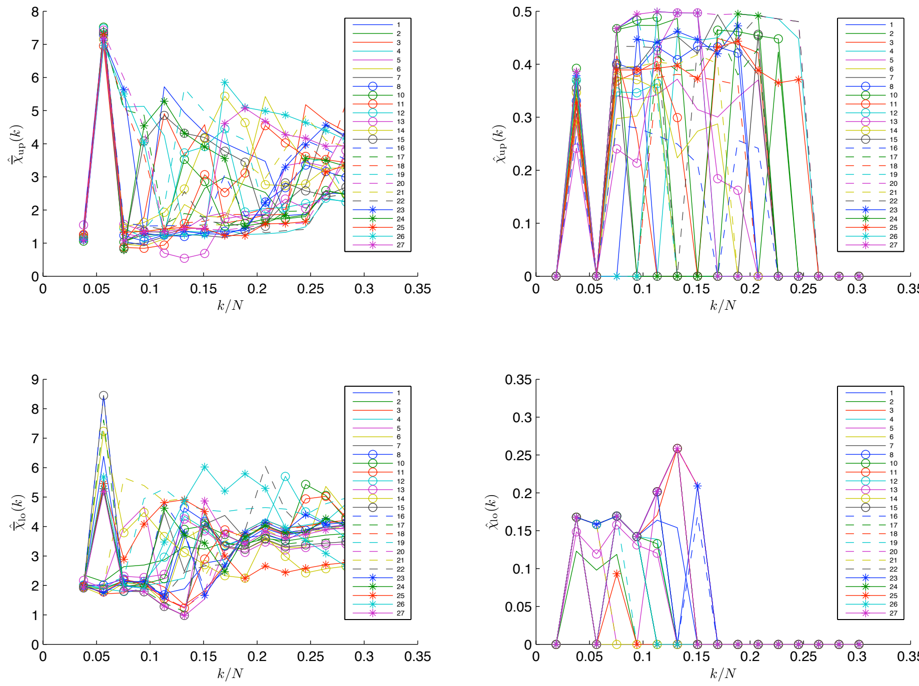
Table A.3.: This table shows $\hat{\rho}$ for THFI vs. THFI_{sub} according to *Spearman*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\rho}$. The three lowest values for $\hat{\rho}_{\text{up}}$ and $\hat{\rho}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\rho}_{\text{up}}$	$\hat{\rho}^{\text{bs}}_{\text{up,mean}}$	$\hat{\rho}^{\text{bs}}_{\text{up,90}}$	$\hat{\rho}^{\text{bs}}_{\text{up,95}}$	$\hat{\rho}^{\text{bs}}_{\text{up,99}}$	$\hat{\rho}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\rho}_{\text{lo}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,90}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,95}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,99}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	0.04	0.07	0.41	0.51	0.64	0.78	0.28	0.1	0.12	0.49	0.59	0.73	0.94	0.28
DSB	*-0.21*	*-0.17*	*0.11*	*0.21*	*0.44*	*0.56*	*0.22*	0.13	0.12	0.44	0.52	0.69	0.88	0.25
EMM	**0.54**	**0.42**	**0.67**	**0.72**	**0.81**	**0.87**	**0.21**	**0.23**	**0.1**	**0.44**	**0.51**	**0.66**	**0.74**	**0.26**
EMN	0.39	0.37	0.67	0.73	0.85	0.92	0.24	*-0.21*	*-0.22*	*0.12*	*0.19*	*0.42*	*0.69*	*0.26*
EVD	0.36	0.4	0.67	0.73	0.86	0.94	0.22	-0.12	-0.05	0.3	0.38	0.6	0.73	0.27
EDDI	0.06	0.08	0.4	0.49	0.63	0.8	0.25	0.07	0.12	0.42	0.5	0.66	0.87	0.25
EDMS	**0.53**	**0.53**	**0.78**	**0.84**	**0.91**	**0.96**	**0.21**	*-0.13*	*-0.09*	*0.26*	*0.36*	*0.54*	*0.66*	*0.26*
EDRA	*-0.23*	*-0.22*	*0.11*	*0.2*	*0.39*	*0.63*	*0.26*	*-0.47*	*-0.37*	*-0.01*	*0.1*	*0.25*	*0.46*	*0.26*
FIA	*-0.19*	*-0.06*	*0.28*	*0.38*	*0.53*	*0.67*	*0.26*	**0.34**	**0.19**	**0.55**	**0.63**	**0.78**	**0.92**	**0.28**
GLM	**0.71**	**0.65**	**0.81**	**0.85**	**0.9**	**0.95**	**0.14**	**0.76**	**0.72**	**0.88**	**0.9**	**0.94**	**0.96**	**0.16**
LSE	0.33	0.39	0.64	0.72	0.8	0.84	0.21	0.09	0.1	0.43	0.54	0.64	0.79	0.27
MAF	-0.19	-0.12	0.18	0.26	0.44	0.61	0.23	-0.03	-0.1	0.22	0.33	0.49	0.64	0.25
MST	-0.07	0.03	0.37	0.44	0.56	0.74	0.26	-0.01	-0.02	0.28	0.37	0.54	0.9	0.23

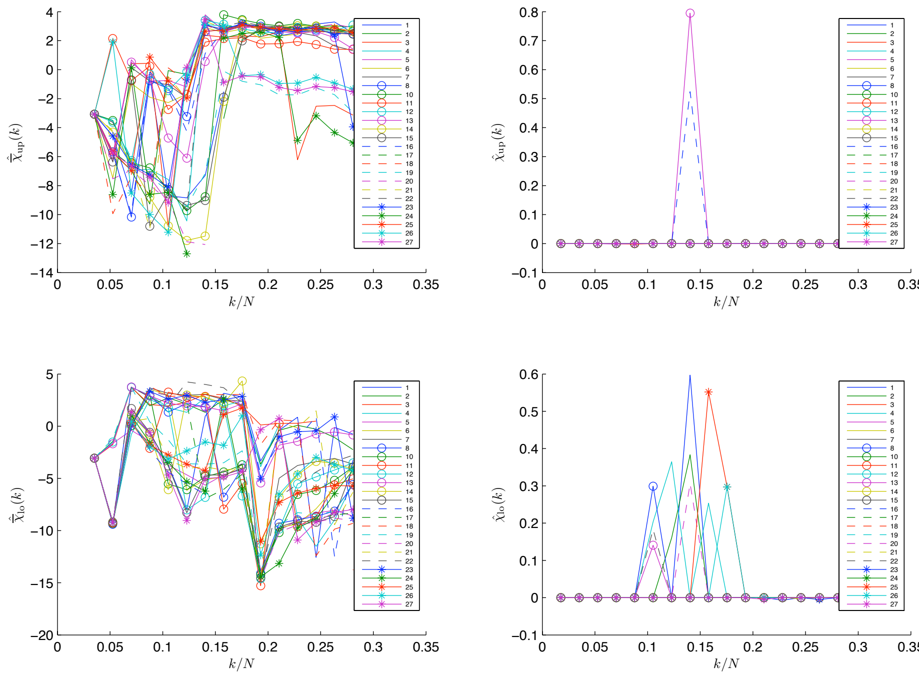
Table A.4.: This table shows $\hat{\tau}$ for THFI vs. THFI_{sub} according to *Kendall*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\tau}$. The three lowest values for $\hat{\tau}_{\text{up}}$ and $\hat{\tau}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\tau}_{\text{up}}$	$\hat{\tau}^{\text{bs}}_{\text{up,mean}}$	$\hat{\tau}^{\text{bs}}_{\text{up,90}}$	$\hat{\tau}^{\text{bs}}_{\text{up,95}}$	$\hat{\tau}^{\text{bs}}_{\text{up,99}}$	$\hat{\tau}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\tau}_{\text{lo}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,90}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,95}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,99}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	0.02	0.05	0.31	0.36	0.47	0.66	0.21	0.07	0.09	0.36	0.43	0.57	0.81	0.21
DSB	*-0.13*	*-0.1*	*0.13*	*0.19*	*0.32*	*0.54*	*0.17*	0.1	0.1	0.35	0.41	0.55	0.68	0.19
EMM	**0.38**	**0.28**	**0.49**	**0.54**	**0.63**	**0.7**	**0.17**	**0.19**	**0.09**	**0.35**	**0.41**	**0.53**	**0.66**	**0.19**
EMN	0.32	0.31	0.54	0.58	0.72	0.83	0.19	*-0.16*	*-0.15*	*0.1*	*0.18*	*0.34*	*0.58*	*0.2*
EVD	0.28	0.3	0.51	0.57	0.66	0.79	0.17	*-0.1*	*-0.04*	*0.21*	*0.28*	*0.4*	*0.6*	*0.2*
EDDI	0.02	0.04	0.3	0.35	0.44	0.57	0.2	0.06	0.1	0.34	0.4	0.53	0.61	0.19
EDMS	**0.4**	**0.42**	**0.63**	**0.69**	**0.77**	**0.87**	**0.17**	-0.1	-0.06	0.18	0.25	0.4	0.53	0.19
EDRA	*-0.19*	*-0.19*	*0.06*	*0.14*	*0.25*	*0.39*	*0.19*	*-0.38*	*-0.28*	*-0.03*	*0.04*	*0.19*	*0.4*	*0.19*
FIA	-0.13	-0.04	0.22	0.28	0.41	0.55	0.19	**0.3**	**0.18**	**0.45**	**0.52**	**0.67**	**0.77**	**0.21**
GLM	**0.52**	**0.5**	**0.64**	**0.68**	**0.77**	**0.85**	**0.12**	**0.55**	**0.55**	**0.72**	**0.76**	**0.8**	**0.87**	**0.14**
LSE	0.25	0.29	0.51	0.56	0.69	0.82	0.17	0.02	0.07	0.32	0.4	0.52	0.75	0.2
MAF	*-0.14*	*-0.11*	*0.11*	*0.17*	*0.29*	*0.41*	*0.17*	0	-0.06	0.18	0.25	0.42	0.6	0.19
MST	-0.02	0.04	0.28	0.34	0.46	0.57	0.19	0.06	0.03	0.23	0.31	0.45	0.73	0.17

A.4. THFI vs. HF_{ISP}



(a) With Clauset estimator



(b) With Huisman estimator

Figure A.9.: $\overline{\chi}$ and χ calculated for THFI vs. HF_{ISP} applying the Clauset estimator in sub-figure (a) and the Huisman estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

Table A.5.: This table shows $\hat{\lambda}$ for THFI vs. HF_{ISP} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.05	0.04	0.08	0.09	0.12	0.18	0.03	0.77	0.67	0.84	0.87	0.93	1	0.18
2	0.33	0.29	0.38	0.4	0.43	0.49	0.07	0.81	0.72	0.83	0.85	0.89	1	0.13
3	0.06	0.05	0.1	0.11	0.14	0.18	0.03	0.76	0.65	0.79	0.8	0.83	0.88	0.15
4	**1**	**0.99**	**1**	**1**	**1**	**1**	**0.03**	0.87	0.79	0.88	0.89	0.91	0.92	0.1
5	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	*0.27*	*0.17*	*0.3*	*0.31*	*0.36*	*0.47*	*0.11*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	0.04	0.03	0.06	0.08	0.11	0.16	0.02	**0.97**	**0.91**	**1**	**1**	**1**	**1**	**0.13**
8	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.03*	*0*	0.54	0.39	0.57	0.59	0.64	0.72	0.18
10	0.13	0.11	0.18	0.2	0.24	0.3	0.05	0.77	0.66	0.81	0.83	0.87	1	0.15
11	**0.78**	**0.75**	**0.89**	**0.92**	**0.98**	**1**	**0.12**	0.81	0.71	0.84	0.87	0.92	1	0.14
12	0.03	0.02	0.04	0.05	0.07	0.1	0.02	0.79	0.67	0.83	0.85	0.88	0.93	0.17
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	0.1	0.08	0.12	0.12	0.13	0.15	0.03	**1**	**0.99**	**1**	**1**	**1**	**1**	**0.05**
15	0.01	0.01	0.03	0.03	0.05	0.08	0.01	0.61	0.47	0.65	0.67	0.71	0.8	0.19
16	0.05	0.04	0.08	0.09	0.11	0.15	0.02	0.6	0.45	0.63	0.65	0.69	0.76	0.18
17	**0.63**	**0.62**	**0.71**	**0.76**	**0.85**	**1**	**0.08**	0.71	0.57	0.74	0.76	0.8	0.89	0.18
18	0.14	0.11	0.17	0.18	0.21	0.26	0.04	0.73	0.6	0.75	0.77	0.8	0.86	0.17
19	0.31	0.27	0.37	0.4	0.44	0.5	0.08	0.81	0.7	0.87	0.9	0.96	1	0.19
20	0.11	0.09	0.16	0.19	0.23	0.31	0.05	0.78	0.67	0.81	0.83	0.86	0.91	0.16
21	0.33	0.29	0.37	0.39	0.43	0.52	0.07	0.83	0.74	0.92	0.97	1	1	0.17
22	0.13	0.11	0.17	0.19	0.22	0.27	0.05	0.93	0.84	1	1	1	1	0.17
23	0.06	0.05	0.09	0.11	0.14	0.2	0.03	0.85	0.78	0.88	0.9	0.96	1	0.1
24	0.25	0.22	0.34	0.37	0.44	0.53	0.09	0.8	0.7	0.87	0.91	0.99	1	0.18
25	0.07	0.05	0.1	0.11	0.14	0.19	0.03	**1**	**0.98**	**1**	**1**	**1**	**1**	**0.07**
26	0.01	0.01	0.01	0.02	0.03	0.06	0.01	0.76	0.66	0.99	1	1	1	0.25
27	0.01	0.01	0.02	0.03	0.05	0.09	0.01	0.44	0.31	0.48	0.5	0.55	0.66	0.15

Table A.6.: This table shows $\hat{\lambda}$ for THFI vs. HF_{ISP} according to the *parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.14	0.13	0.21	0.23	0.27	0.42	0.06	0.5	0.41	0.51	0.52	0.55	0.9	0.13
2	0.21	0.27	0.54	0.6	0.73	1	0.19	0.45	0.41	0.45	0.46	0.53	0.91	0.07
3	0.2	0.18	0.28	0.3	0.35	0.79	0.07	0.44	0.42	0.49	0.53	0.64	0.96	0.09
4	**0.95**	**0.95**	**0.99**	**1**	**1**	**1**	**0.04**	0.53	0.59	0.76	0.81	0.92	1	0.13
5	*0*	*0*	*0*	*0*	*0.01*	*0.05*	*0*	*0.22*	*0.15*	*0.28*	*0.35*	*0.52*	*0.9*	*0.13*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	0.08	0.07	0.13	0.15	0.19	0.3	0.04	0.47	0.44	0.47	0.48	0.49	0.66	0.04
8	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.06*	*0.01*	0.34	0.26	0.39	0.42	0.44	0.55	0.12
10	0.38	0.36	0.49	0.53	0.6	0.9	0.11	**0.54**	**0.58**	**0.66**	**0.7**	**0.81**	**0.97**	**0.07**
11	**0.96**	**0.96**	**0.99**	**1**	**1**	**1**	**0.03**	0.53	0.56	0.7	0.75	0.86	1	0.1
12	0.05	0.05	0.11	0.14	0.18	0.58	0.04	0.5	0.48	0.51	0.53	0.61	1.02	0.04
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	0.1	0.09	0.19	0.22	0.31	0.95	0.07	0.5	0.52	0.64	0.67	0.75	0.97	0.09
15	0.02	0.02	0.05	0.06	0.09	0.22	0.02	0.39	0.32	0.42	0.43	0.45	0.77	0.1
16	0.04	0.05	0.11	0.15	0.23	0.96	0.05	0.41	0.33	0.42	0.43	0.5	0.93	0.11
17	**0.88**	**0.89**	**0.96**	**0.98**	**0.99**	**1**	**0.05**	0.48	0.49	0.61	0.66	0.8	0.99	0.1
18	0.27	0.27	0.55	0.61	0.71	0.92	0.16	0.45	0.41	0.48	0.52	0.62	0.95	0.09
19	0.83	0.85	0.92	0.94	0.98	1	0.06	0.51	0.54	0.67	0.72	0.85	1	0.09
20	0.29	0.27	0.38	0.41	0.47	0.82	0.09	0.5	0.51	0.62	0.66	0.79	0.98	0.09
21	0.52	0.53	0.84	0.88	0.98	1	0.23	**0.55**	**0.58**	**0.71**	**0.75**	**0.84**	**1**	**0.09**
22	0.27	0.26	0.39	0.41	0.46	0.73	0.1	0.5	0.51	0.64	0.69	0.82	0.99	0.1
23	0.21	0.19	0.29	0.31	0.35	0.72	0.07	0.48	0.45	0.51	0.55	0.63	0.93	0.08
24	0.53	0.52	0.61	0.64	0.78	0.99	0.07	0.53	0.51	0.56	0.57	0.6	0.86	0.06
25	0.1	0.09	0.16	0.18	0.21	0.37	0.05	**0.57**	**0.53**	**0.72**	**0.81**	**0.91**	**1**	**0.14**
26	0.01	0.01	0.02	0.03	0.05	0.17	0.01	0.42	0.35	0.44	0.46	0.47	0.5	0.11
27	0.02	0.02	0.05	0.06	0.09	0.19	0.02	0.33	0.24	0.36	0.39	0.48	0.89	0.13

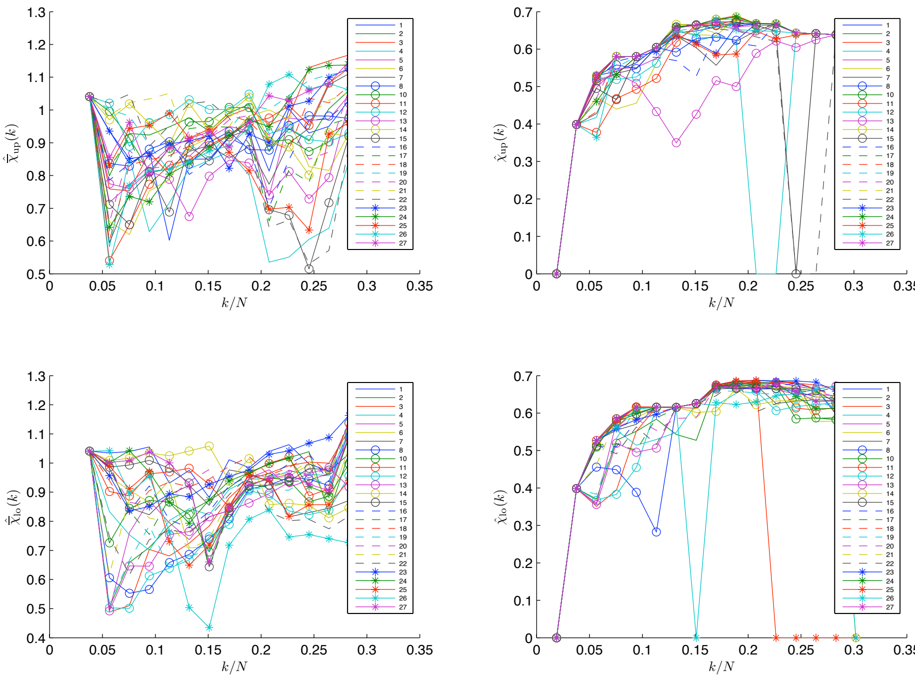
Table A.7.: This table shows $\hat{\rho}$ for THFI vs. HF_{ISP} according to *Spearman*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\rho}$. The three lowest values for $\hat{\rho}_{up}$ and $\hat{\rho}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\rho}_{up}$	$\hat{\rho}^{bs}_{up,mean}$	$\hat{\rho}^{bs}_{up,90}$	$\hat{\rho}^{bs}_{up,95}$	$\hat{\rho}^{bs}_{up,99}$	$\hat{\rho}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\rho}_{lo}$	$\hat{\rho}^{bs}_{lo,mean}$	$\hat{\rho}^{bs}_{lo,90}$	$\hat{\rho}^{bs}_{lo,95}$	$\hat{\rho}^{bs}_{lo,99}$	$\hat{\rho}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.34	0.3	0.7	0.78	0.89	0.98	0.35	0.02	0.03	0.54	0.65	0.85	0.99	0.39
2	**0.69**	**0.53**	**0.87**	**0.92**	**0.99**	**1**	**0.3**	0.41	0.38	0.85	0.93	1	1	0.37
3	0.2	0.2	0.71	0.79	0.92	1	0.38	0.58	0.51	0.87	0.92	0.99	1	0.34
4	**0.68**	**0.63**	**0.87**	**0.92**	**1**	**1**	**0.23**	0.54	0.56	0.87	0.92	0.99	1	0.28
5	0.24	0.15	0.58	0.66	0.81	0.96	0.35	0.46	0.36	0.76	0.83	0.91	0.98	0.34
6	0.41	0.41	0.82	0.87	0.96	1	0.36	-0.25	-0.31	0.14	0.32	0.56	0.99	0.33
7	0.31	0.25	0.68	0.77	0.89	0.98	0.33	**0.84**	**0.63**	**0.9**	**0.94**	**0.97**	**1**	**0.26**
8	0.39	0.36	0.76	0.84	0.94	0.99	0.31	0.26	0.14	0.64	0.74	0.87	0.97	0.37
10	*0.01*	*0.02*	*0.5*	*0.6*	*0.83*	*0.96*	0.37	**0.88**	**0.78**	**0.94**	**0.97**	**1**	**1**	**0.19**
11	0.54	0.26	0.74	0.86	0.96	1	0.4	0.6	0.53	0.83	0.89	0.95	1	0.27
12	0.34	0.26	0.72	0.8	0.93	0.98	0.37	0.7	0.6	0.85	0.89	0.97	1	0.25
13	*-0.27*	*-0.09*	*0.38*	*0.5*	*0.74*	*0.88*	*0.36*	*-0.35*	*-0.29*	*0.13*	*0.39*	*1*	*1*	*0.33*
14	0.46	0.47	0.79	0.84	0.93	1	0.28	0.62	0.53	0.87	0.93	0.99	1	0.31
15	0.5	0.45	0.81	0.88	0.96	1	0.3	*-0.48*	*-0.38*	*0.15*	*0.29*	*0.65*	*0.84*	*0.37*
16	0.5	0.24	0.69	0.79	0.92	1	0.34	0.08	0.18	0.68	0.77	0.89	0.97	0.4
17	0.29	0.14	0.58	0.68	0.81	0.96	0.34	0.65	0.6	0.87	0.91	0.96	1	0.24
18	0.12	0.11	0.57	0.78	0.94	1	0.38	0.34	0.37	0.73	0.81	0.91	1	0.31
19	**0.71**	**0.49**	**0.81**	**0.89**	**0.98**	**1**	**0.32**	0.52	0.46	0.8	0.83	0.93	0.99	0.3
20	0.34	0.19	0.71	0.8	0.97	1	0.39	0.64	0.6	0.86	0.9	0.95	1	0.24
21	0.22	0.25	0.67	0.76	0.87	0.93	0.35	0.74	0.69	0.94	0.97	1	1	0.26
22	0.13	0.07	0.53	0.68	0.8	0.91	0.35	0.34	0.28	0.72	0.81	0.94	1	0.37
23	*-0.45*	*-0.29*	*0.15*	*0.33*	*0.64*	*0.85*	*0.32*	**0.8**	**0.66**	**0.92**	**0.94**	**1**	**1**	**0.26**
24	0.6	0.45	0.78	0.82	0.87	0.96	0.31	0.48	0.33	0.76	0.83	0.92	0.96	0.38
25	0.57	0.52	0.83	0.88	0.97	0.99	0.27	0.53	0.39	0.73	0.8	0.91	1	0.3
26	0.02	0.16	0.65	0.74	0.89	0.97	0.36	0.14	0.1	0.52	0.64	0.78	0.97	0.33
27	0.46	0.39	0.79	0.84	0.92	0.99	0.38	*-0.35*	*-0.04*	*0.46*	*0.58*	*0.75*	*0.97*	*0.39*

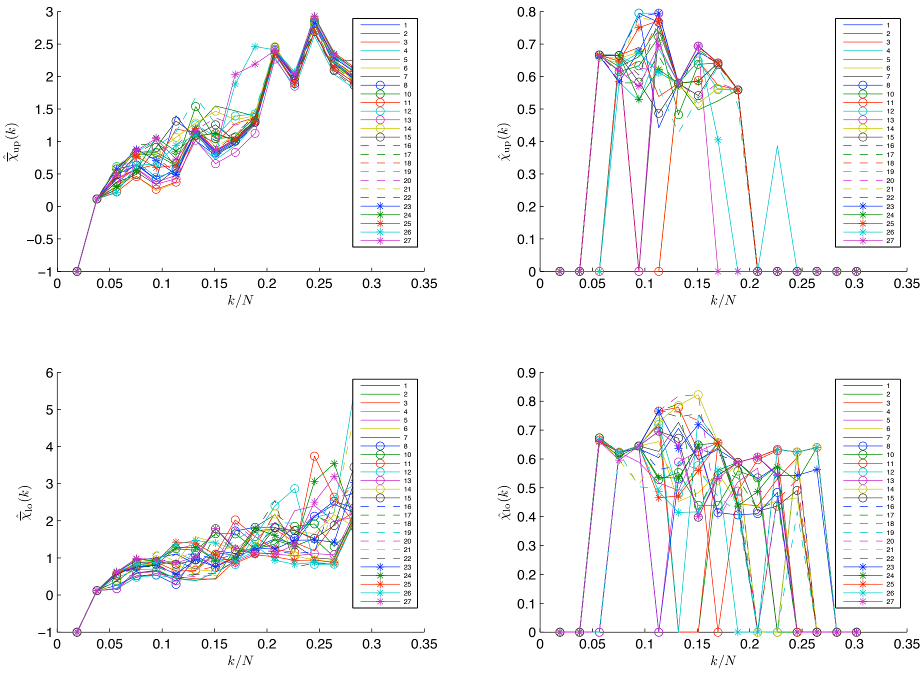
Table A.8.: This table shows $\hat{\tau}$ for THFI vs. HF_{ISP} according to *Kendall*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\tau}$. The three lowest values for $\hat{\tau}_{up}$ and $\hat{\tau}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\tau}_{up}$	$\hat{\tau}^{bs}_{up,mean}$	$\hat{\tau}^{bs}_{up,90}$	$\hat{\tau}^{bs}_{up,95}$	$\hat{\tau}^{bs}_{up,99}$	$\hat{\tau}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\tau}_{lo}$	$\hat{\tau}^{bs}_{lo,mean}$	$\hat{\tau}^{bs}_{lo,90}$	$\hat{\tau}^{bs}_{lo,95}$	$\hat{\tau}^{bs}_{lo,99}$	$\hat{\tau}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.24	0.24	0.57	0.64	0.79	0.92	0.27	0.02	0.03	0.43	0.55	0.74	0.96	0.31
2	**0.55**	**0.41**	**0.73**	**0.83**	**0.93**	**1**	**0.27**	0.38	0.32	0.75	0.85	1	1	0.33
3	0.12	0.14	0.51	0.58	0.8	0.92	0.29	0.38	0.38	0.75	0.82	0.95	1	0.31
4	**0.48**	**0.5**	**0.75**	**0.81**	**0.93**	**1**	**0.22**	0.42	0.44	0.73	0.8	0.92	1	0.26
5	0.12	0.06	0.42	0.49	0.66	0.82	0.28	0.33	0.28	0.6	0.69	0.86	0.97	0.27
6	0.33	0.31	0.68	0.73	0.84	0.93	0.29	-0.16	-0.22	0.14	0.25	0.45	1	0.28
7	0.26	0.22	0.54	0.64	0.78	0.89	0.25	**0.67**	**0.52**	**0.77**	**0.83**	**0.92**	**1**	**0.22**
8	0.3	0.3	0.62	0.7	0.86	1	0.25	0.2	0.11	0.49	0.58	0.75	1	0.29
10	*0*	*0*	*0.38*	*0.5*	*0.72*	*0.89*	*0.3*	**0.71**	**0.64**	**0.88**	**0.92**	**1**	**1**	**0.21**
11	0.41	0.24	0.61	0.72	0.86	1	0.31	0.42	0.4	0.68	0.76	0.88	1	0.24
12	0.21	0.16	0.55	0.64	0.79	1	0.32	0.56	0.48	0.71	0.76	0.91	1	0.2
13	*-0.2*	*-0.07*	*0.32*	*0.43*	*0.66*	*1*	*0.3*	*-0.18*	*-0.17*	*0.2*	*0.35*	*1*	*1*	*0.29*
14	0.33	0.38	0.65	0.72	0.8	0.93	0.23	0.48	0.42	0.77	0.85	0.96	1	0.29
15	0.36	0.37	0.68	0.76	0.89	1	0.24	*-0.35*	*-0.31*	*0.09*	*0.21*	*0.44*	*0.92*	*0.31*
16	0.35	0.16	0.52	0.64	0.75	0.93	0.29	0.02	0.1	0.52	0.62	0.78	0.91	0.33
17	0.21	0.15	0.46	0.55	0.69	0.86	0.26	0.49	0.48	0.75	0.8	0.88	0.96	0.22
18	0.15	0.15	0.53	0.67	0.87	1	0.29	0.24	0.28	0.58	0.66	0.81	1	0.25
19	**0.52**	**0.38**	**0.69**	**0.77**	**0.89**	**1**	**0.28**	0.35	0.31	0.62	0.7	0.84	1	0.26
20	0.24	0.13	0.53	0.65	0.86	1	0.32	0.45	0.46	0.73	0.8	0.88	1	0.22
21	0.11	0.16	0.47	0.55	0.69	1	0.26	0.6	0.57	0.84	0.91	0.98	1	0.24
22	0.12	0.05	0.39	0.52	0.69	0.93	0.27	0.24	0.2	0.54	0.64	0.8	1	0.29
23	*-0.27*	*-0.21*	*0.11*	*0.25*	*0.43*	*0.69*	*0.25*	**0.64**	**0.55**	**0.83**	**0.88**	**1**	**1**	**0.24**
24	0.39	0.31	0.57	0.63	0.79	0.93	0.25	0.31	0.18	0.58	0.65	0.79	0.96	0.34
25	0.42	0.4	0.66	0.73	0.86	1	0.21	0.35	0.26	0.57	0.65	0.79	1	0.25
26	0.03	0.12	0.48	0.58	0.76	0.93	0.28	0.07	0.05	0.35	0.41	0.55	0.78	0.24
27	0.3	0.33	0.66	0.74	0.86	0.96	0.28	*-0.27*	*-0.04*	*0.36*	*0.43*	*0.67*	*0.83*	*0.3*

A.5. S&P 500 vs. HF_{ISP}



(a) With Hill estimator



(b) With Gabaix estimator

Figure A.10.: $\overline{\chi}$ and χ calculated for S&P 500 vs. HF_{ISP} applying the Hill estimator in sub-figure (a) and the Gabaix estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

Table A.9.: This table shows $\hat{\lambda}$ for S&P 500 vs. HF_{ISP} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.01	0	0.01	0.01	0.02	0.03	0	0.46	0.43	0.49	0.5	0.53	0.74	0.06
2	0.11	0.06	0.12	0.12	0.14	0.16	0.04	0.74	0.75	0.96	1	1	1	0.14
3	*0*	*0*	*0*	*0*	*0.01*	*0.01*	*0*	0.22	0.2	0.3	0.33	0.38	0.49	0.08
4	**0.25**	**0.17**	**0.27**	**0.28**	**0.3**	**0.34**	**0.08**	**0.86**	**0.85**	**1**	**1**	**1**	**1**	**0.09**
5	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.34	0.32	0.4	0.42	0.5	0.96	0.07
8	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.27	0.24	0.36	0.4	0.45	0.55	0.09
10	0.06	0.03	0.07	0.07	0.09	0.12	0.02	0.77	0.75	0.87	0.89	0.94	1	0.1
11	**0.36**	**0.27**	**0.39**	**0.41**	**0.44**	**0.49**	**0.11**	**1**	**0.97**	**1**	**1**	**1**	**1**	**0.05**
12	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.27	0.24	0.35	0.38	0.44	0.54	0.09
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0.28	0.31	0.46	0.68	1	1	0.17
15	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.14	0.13	0.21	0.24	0.28	0.35	0.06
16	0.06	0.03	0.06	0.07	0.08	0.09	0.02	0.61	0.59	0.68	0.7	0.73	0.8	0.08
17	0.15	0.1	0.16	0.17	0.18	0.19	0.06	0.56	0.53	0.65	0.69	0.73	0.79	0.11
18	0.08	0.05	0.08	0.09	0.1	0.11	0.03	0.74	0.71	0.8	0.83	0.86	0.99	0.08
19	0.01	0.01	0.02	0.02	0.03	0.05	0.01	0.32	0.3	0.42	0.45	0.5	0.59	0.09
20	0.01	0	0.01	0.01	0.02	0.03	0	0.31	0.27	0.39	0.41	0.46	0.53	0.09
21	*0*	*0*	*0.01*	*0.01*	*0.01*	*0.02*	*0*	0.19	0.17	0.25	0.27	0.3	0.36	0.06
22	0.04	0.02	0.05	0.05	0.06	0.08	0.02	0.8	0.79	1	1	1	1	0.14
23	0.02	0.01	0.02	0.03	0.03	0.05	0.01	0.73	0.7	0.78	0.8	0.83	0.93	0.07
24	**0.21**	**0.14**	**0.23**	**0.25**	**0.28**	**0.34**	**0.08**	**1**	**0.99**	**1**	**1**	**1**	**1**	**0.04**
25	*0*	*0*	*0*	*0*	*0.01*	*0.01*	*0*	0.69	0.71	1	1	1	1	0.26
26	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0.05	0.05	0.09	0.1	0.13	0.16	0.03
27	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.08	0.07	0.14	0.16	0.21	0.28	0.05

Table A.10.: This table shows $\hat{\lambda}$ for S&P 500 vs. HF_{ISP} according to the *parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{up}$ and $\hat{\lambda}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\lambda}_{up}$	$\hat{\lambda}^{bs}_{up,mean}$	$\hat{\lambda}^{bs}_{up,90}$	$\hat{\lambda}^{bs}_{up,95}$	$\hat{\lambda}^{bs}_{up,99}$	$\hat{\lambda}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\lambda}_{lo}$	$\hat{\lambda}^{bs}_{lo,mean}$	$\hat{\lambda}^{bs}_{lo,90}$	$\hat{\lambda}^{bs}_{lo,95}$	$\hat{\lambda}^{bs}_{lo,99}$	$\hat{\lambda}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.01	0.02	0.06	0.07	0.09	0.13	0.02	0.34	0.33	0.49	0.55	0.69	0.94	0.13
2	0.09	0.09	0.22	0.23	0.26	0.32	0.09	0.46	0.45	0.55	0.59	0.66	-1	0.09
3	*0*	*0.01*	*0.03*	*0.04*	*0.06*	*0.1*	*0.01*	0.18	0.17	0.28	0.32	0.4	1.44	0.09
4	**0.25**	**0.19**	**0.34**	**0.36**	**0.38**	**0.42**	**0.13**	0.61	0.62	0.68	0.7	0.78	-1.37	0.05
5	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
6	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
7	*0*	*0.01*	*0.02*	*0.03*	*0.04*	*0.07*	*0.01*	0.26	0.25	0.46	0.54	0.73	1.7	0.15
8	*0*	*0.01*	*0.02*	*0.03*	*0.04*	*0.07*	*0.01*	0.22	0.21	0.35	0.4	0.47	-0.14	0.1
10	0.07	0.07	0.17	0.19	0.22	0.27	0.07	**0.66**	**0.68**	**0.76**	**0.79**	**0.87**	**1.57**	**0.07**
11	**0.53**	**0.51**	**0.55**	**0.56**	**0.6**	**0.71**	**0.04**	**0.74**	**0.76**	**0.85**	**0.88**	**0.94**	**-1.37**	**0.07**
12	*0*	*0.01*	*0.02*	*0.03*	*0.05*	*0.08*	*0.01*	0.24	0.23	0.38	0.44	0.53	0.27	0.11
13	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
14	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.06*	*0.01*	0.16	0.18	0.44	0.55	0.76	1.92	0.18
15	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.06*	*0.01*	0.12	0.13	0.25	0.32	0.42	0.41	0.09
16	0.05	0.07	0.2	0.22	0.25	0.31	0.08	0.41	0.4	0.47	0.48	0.49	-1.82	0.07
17	0.2	0.16	0.34	0.36	0.39	0.43	0.13	0.53	0.53	0.58	0.6	0.64	-1.07	0.05
18	0.09	0.09	0.23	0.24	0.28	0.34	0.09	0.54	0.56	0.6	0.62	0.67	0.85	0.04
19	0.02	0.03	0.09	0.1	0.13	0.18	0.04	0.28	0.27	0.39	0.43	0.49	-0.25	0.09
20	0.01	0.02	0.05	0.06	0.08	0.13	0.02	0.33	0.31	0.44	0.47	0.52	0.87	0.1
21	*0*	*0.01*	*0.04*	*0.05*	*0.06*	*0.11*	*0.02*	0.15	0.15	0.33	0.42	0.49	0.92	0.12
22	0.04	0.05	0.13	0.14	0.18	0.23	0.05	0.57	0.59	0.63	0.66	0.73	1.2	0.05
23	0.02	0.03	0.09	0.1	0.13	0.18	0.04	0.39	0.39	0.46	0.48	0.5	-0.47	0.07
24	**0.25**	**0.19**	**0.34**	**0.35**	**0.37**	**0.41**	**0.12**	**0.77**	**0.79**	**0.88**	**0.91**	**0.96**	**1.25**	**0.08**
25	*0*	*0.01*	*0.03*	*0.03*	*0.05*	*0.08*	*0.01*	0.28	0.29	0.44	0.91	1.2	7.76	0.83
26	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	0.04	0.06	0.2	0.34	0.38	-0.18	0.13
27	*0*	*0*	*0.01*	*0.02*	*0.03*	*0.05*	*0.01*	0.08	0.08	0.17	0.21	0.32	1.53	0.07

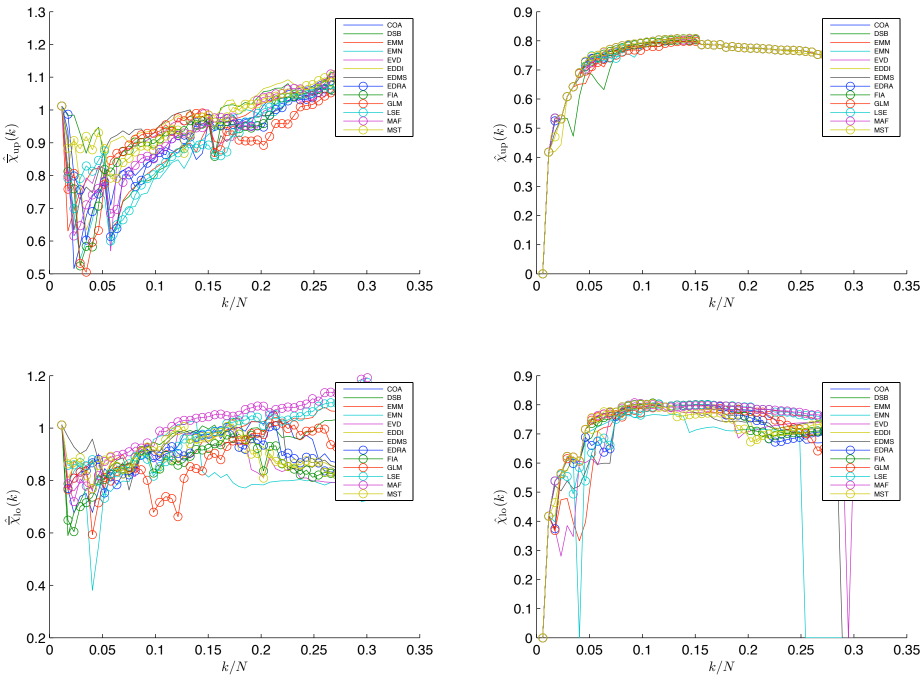
Table A.11.: This table shows $\hat{\rho}$ for S&P 500 vs. HF_{ISP} according to *Spearman*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\rho}$. The three lowest values for $\hat{\rho}_{up}$ and $\hat{\rho}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\rho}_{up}$	$\hat{\rho}^{bs}_{up,mean}$	$\hat{\rho}^{bs}_{up,90}$	$\hat{\rho}^{bs}_{up,95}$	$\hat{\rho}^{bs}_{up,99}$	$\hat{\rho}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\rho}_{lo}$	$\hat{\rho}^{bs}_{lo,mean}$	$\hat{\rho}^{bs}_{lo,90}$	$\hat{\rho}^{bs}_{lo,95}$	$\hat{\rho}^{bs}_{lo,99}$	$\hat{\rho}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.11	0.04	0.51	0.64	0.77	0.96	0.36	-0.18	0.05	0.49	0.65	0.88	1	0.35
2	*-0.29*	*-0.22*	*0.43*	*0.54*	*0.68*	*0.82*	*0.46*	0.54	0.48	0.94	0.97	1	1	0.35
3	-0.04	-0.02	0.49	0.61	0.78	0.89	0.36	-0.02	0.03	0.53	0.65	0.86	1	0.38
4	0.06	0.03	0.5	0.65	0.78	0.92	0.37	0.53	0.52	0.85	0.92	0.98	1	0.29
5	0	0.06	0.57	0.67	0.87	1	0.37	-0.31	-0.21	0.32	0.49	0.75	1	0.39
6	-0.14	-0.1	0.36	0.45	0.64	0.85	0.33	-0.12	-0.21	0.26	0.38	0.6	0.83	0.34
7	*-0.32*	*-0.16*	*0.33*	*0.46*	*0.69*	*0.88*	*0.35*	0.02	0.06	0.53	0.64	0.85	1	0.36
8	-0.04	0.06	0.47	0.58	0.75	0.98	0.32	0.06	0.14	0.59	0.76	0.95	1	0.36
10	-0.08	-0.09	0.34	0.48	0.67	0.84	0.33	**0.58**	**0.53**	**0.92**	**0.97**	**1**	**1**	**0.34**
11	0.01	0.03	0.49	0.6	0.79	0.89	0.36	**0.71**	**0.66**	**0.92**	**0.94**	**0.98**	**1**	**0.26**
12	-0.17	-0.22	0.32	0.45	0.71	0.93	0.4	0.26	0.24	0.82	0.86	0.94	1	0.45
13	-0.04	-0.06	0.43	0.57	0.71	1	0.37	-0.33	-0.23	0.28	0.43	0.64	1	0.39
14	**0.45**	**0.42**	**0.77**	**0.82**	**0.92**	**0.97**	**0.3**	0.18	0.17	0.67	0.77	0.91	1	0.4
15	0.14	0.12	0.64	0.75	0.88	1	0.38	*-0.55*	*-0.52*	*-0.07*	*0.11*	*0.46*	*0.7*	*0.32*
16	*-0.62*	*-0.41*	*0.1*	*0.27*	*0.55*	*0.7*	*0.36*	0.42	0.36	0.75	0.83	0.93	1	0.35
17	0.22	0.13	0.54	0.62	0.73	0.87	0.32	0.36	0.29	0.74	0.83	0.94	1	0.35
18	0.24	0.08	0.5	0.61	0.78	0.96	0.32	0.38	0.4	0.79	0.86	0.97	1	0.33
19	0.16	0.13	0.61	0.72	0.89	1	0.37	0.09	0.03	0.54	0.63	0.85	1	0.39
20	-0.11	-0.04	0.49	0.61	0.76	0.83	0.4	0.05	0.09	0.64	0.76	0.88	1	0.42
21	-0.2	-0.18	0.29	0.46	0.73	0.93	0.37	0.22	0.26	0.79	0.9	0.99	1	0.41
22	0.03	0.14	0.67	0.74	0.85	0.94	0.41	0.24	0.29	0.71	0.79	0.9	1	0.35
23	-0.26	-0.21	0.22	0.36	0.56	0.91	0.33	**0.62**	**0.48**	**0.82**	**0.87**	**0.95**	**1**	**0.3**
24	**0.54**	**0.56**	**0.85**	**0.89**	**0.96**	**1**	**0.27**	0.56	0.5	0.86	0.93	1	1	0.32
25	0.33	0.4	0.69	0.76	0.86	0.94	0.28	0.16	0.16	0.61	0.68	1	1	0.35
26	**0.44**	**0.31**	**0.66**	**0.73**	**0.85**	**1**	**0.29**	*-0.45*	*-0.2*	*0.26*	*0.4*	*0.85*	*1*	*0.35*
27	0.17	0.1	0.54	0.65	0.78	0.94	0.33	*-0.78*	*-0.49*	*0.01*	*0.16*	*0.64*	*1*	*0.35*

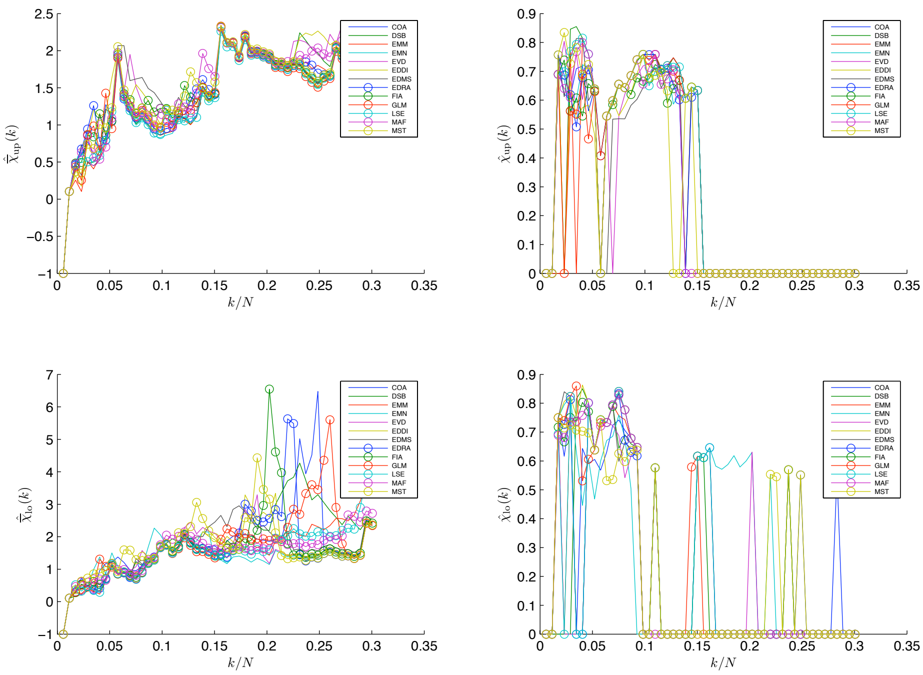
Table A.12.: This table shows $\hat{\tau}$ for S&P 500 vs. HF_{ISP} according to *Kendall*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\tau}$. The three lowest values for $\hat{\tau}_{up}$ and $\hat{\tau}_{lo}$ are set italic and the three highest values bold.

	upper tail							lower tail						
HF_{ISP}	$\hat{\tau}_{up}$	$\hat{\tau}^{bs}_{up,mean}$	$\hat{\tau}^{bs}_{up,90}$	$\hat{\tau}^{bs}_{up,95}$	$\hat{\tau}^{bs}_{up,99}$	$\hat{\tau}^{bs}_{up,max}$	$\hat{\sigma}^{bs}_{up}$	$\hat{\tau}_{lo}$	$\hat{\tau}^{bs}_{lo,mean}$	$\hat{\tau}^{bs}_{lo,90}$	$\hat{\tau}^{bs}_{lo,95}$	$\hat{\tau}^{bs}_{lo,99}$	$\hat{\tau}^{bs}_{lo,max}$	$\hat{\sigma}^{bs}_{lo}$
1	0.06	0.05	0.39	0.48	0.67	0.88	0.27	-0.09	0.07	0.42	0.53	0.7	1	0.26
2	*-0.3*	*-0.21*	*0.3*	*0.4*	*0.57*	*0.76*	*0.37*	0.42	0.42	0.84	0.96	1	1	0.3
3	0	0.01	0.38	0.47	0.69	0.87	0.29	0.02	0.06	0.41	0.51	0.7	0.96	0.28
4	0	0	0.35	0.45	0.63	0.75	0.28	0.42	0.41	0.72	0.8	0.92	1	0.25
5	0.03	0.06	0.46	0.56	0.76	1	0.3	-0.27	-0.18	0.24	0.33	0.64	0.82	0.32
6	-0.12	-0.1	0.23	0.32	0.51	0.79	0.25	-0.09	-0.18	0.18	0.28	0.53	0.7	0.27
7	*-0.29*	*-0.15*	*0.22*	*0.32*	*0.51*	*0.76*	*0.29*	0.02	0.04	0.42	0.54	0.73	0.9	0.3
8	-0.06	0.04	0.34	0.42	0.59	0.77	0.24	0.07	0.12	0.51	0.62	0.85	1	0.3
10	-0.06	-0.06	0.27	0.39	0.54	0.84	0.26	**0.45**	**0.43**	**0.8**	**0.91**	**1**	**1**	**0.3**
11	0	0.05	0.38	0.47	0.65	0.92	0.28	**0.51**	**0.51**	**0.8**	**0.87**	**0.96**	**1**	**0.24**
12	-0.18	-0.22	0.19	0.32	0.53	0.8	0.33	0.2	0.18	0.65	0.74	0.91	1	0.37
13	-0.02	-0.03	0.35	0.46	0.64	0.89	0.29	*-0.29*	*-0.22*	*0.18*	*0.27*	*0.41*	*0.81*	*0.29*
14	**0.33**	**0.32**	**0.61**	**0.68**	**0.8**	**0.93**	**0.25**	0.13	0.13	0.54	0.67	0.86	1	0.33
15	0.14	0.11	0.5	0.61	0.8	0.89	0.3	*-0.38*	*-0.4*	*0*	*0.12*	*0.37*	*0.62*	*0.29*
16	*-0.52*	*-0.38*	*0.02*	*0.15*	*0.34*	*0.6*	*0.29*	0.27	0.27	0.59	0.67	0.8	0.91	0.27
17	0.18	0.12	0.41	0.5	0.69	0.9	0.24	0.27	0.24	0.58	0.69	0.84	0.96	0.28
18	0.18	0.09	0.4	0.5	0.69	0.89	0.25	0.27	0.31	0.64	0.72	0.87	1	0.27
19	0.12	0.11	0.48	0.6	0.75	0.9	0.3	0.09	0.04	0.4	0.51	0.7	0.91	0.28
20	-0.12	-0.07	0.32	0.42	0.56	0.68	0.3	0.02	0.05	0.49	0.61	0.75	1	0.33
21	-0.18	-0.17	0.2	0.28	0.47	0.76	0.28	0.2	0.25	0.64	0.75	0.92	1	0.3
22	0.03	0.11	0.49	0.56	0.68	0.77	0.32	0.16	0.22	0.57	0.65	0.83	0.95	0.28
23	-0.17	-0.14	0.17	0.28	0.42	1	0.25	0.44	0.36	0.67	0.75	0.87	1	0.26
24	**0.45**	**0.47**	**0.73**	**0.79**	**0.92**	**1**	**0.22**	**0.45**	**0.41**	**0.76**	**0.84**	**1**	**1**	**0.3**
25	0.21	0.28	0.53	0.61	0.75	0.86	0.22	0.16	0.16	0.5	0.62	0.92	1	0.28
26	**0.33**	**0.25**	**0.53**	**0.61**	**0.72**	**0.93**	**0.23**	-0.27	-0.09	0.25	0.4	0.63	1	0.27
27	0.12	0.07	0.41	0.5	0.69	0.92	0.26	*-0.64*	*-0.41*	*0.02*	*0.12*	*0.45*	*0.81*	*0.31*

A.6. S&P 500 vs. $THFI_{sub}$



(a) With Hill estimator



(b) With Gabaix estimator

Figure A.11.: $\overline{\chi}$ and χ calculated for S&P 500 vs. $THFI_{sub}$ applying the Clauset estimator in sub-figure (a) and the Huisman estimator in sub-figure (b). $\overline{\chi}$ and χ are measures of asymptotic independence and dependence, respectively, developed by *Poon* et al. The first row of a subfigure contains the TDCs for the upper tail, the second row for the lower tail of the return distribution. The variable k represents the threshold number and N the total number of observations of the investigated sample.

Table A.13.: This table shows $\hat{\lambda}$ for S&P 500 vs. THFI_{sub} according to the *non-parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0.08*	*0.06*	*0.1*	*0.1*	*0.12*	*0.15*	*0.02*
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.03**	**0.03**	**0.04**	**0.05**	**0.06**	**0.08**	**0.01**	0.36	0.32	0.38	0.4	0.42	0.46	0.05
EMN	*0*	*0*	*0*	*0*	*0*	*0.01*	*0*	**1**	**1**	**1**	**1**	**1**	**1**	**0.02**
EVD	**0.04**	**0.04**	**0.06**	**0.07**	**0.1**	**0.15**	**0.02**	**0.88**	**0.86**	**1**	**1**	**1**	**1**	**0.13**
EDDI	**0.03**	**0.03**	**0.05**	**0.06**	**0.08**	**0.12**	**0.02**	**0.96**	**0.92**	**1**	**1**	**1**	**1**	**0.11**
EDMS	0.01	0.01	0.03	0.03	0.04	0.07	0.01	0.59	0.57	0.67	0.69	0.73	0.84	0.08
EDRA	0.01	0.01	0.01	0.01	0.02	0.04	0	0.61	0.59	0.67	0.69	0.72	0.78	0.06
FIA	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0.02	0.01	0.02	0.03	0.03	0.04	0.01
GLM	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0.12	0.09	0.14	0.14	0.16	0.19	0.03
LSE	**0.03**	**0.03**	**0.04**	**0.05**	**0.06**	**0.08**	**0.01**	0.77	0.75	0.81	0.82	0.85	0.88	0.05
MAF	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
MST	*0*	*0*	*0*	*0*	*0*	*0*	*0*	0.08	0.06	0.09	0.1	0.11	0.14	0.02

Table A.14.: This table shows $\hat{\lambda}$ for S&P 500 vs. THFI_{sub} according to the *parametric* approach by *Malevergne and Sornette* applying the *Hill* estimator and results obtained by bootstrapping (bs) the whole data set 5000 times with $k/N = 0.12$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\lambda}$. The three lowest values for $\hat{\lambda}_{\text{up}}$ and $\hat{\lambda}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\lambda}_{\text{up}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\lambda}_{\text{lo}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,90}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,95}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,99}}$	$\hat{\lambda}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	*0*	*0*	*0*	*0*	*0.01*	*0.03*	*0*	0.08	0.05	0.09	0.1	0.12	0.18	0.03
DSB	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
EMM	**0.04**	**0.06**	**0.15**	**0.19**	**0.22**	**0.26**	**0.05**	0.25	0.21	0.27	0.28	0.3	0.33	0.05
EMN	*0*	*0*	*0.01*	*0.01*	*0.02*	*0.04*	*0*	**0.58**	**0.56**	**0.66**	**0.69**	**0.76**	**0.9**	**0.08**
EVD	**0.07**	**0.08**	**0.15**	**0.17**	**0.21**	**0.26**	**0.04**	0.51	0.48	0.56	0.58	0.62	0.72	0.09
EDDI	**0.05**	**0.06**	**0.11**	**0.14**	**0.18**	**0.23**	**0.04**	**0.52**	**0.49**	**0.57**	**0.59**	**0.62**	**0.71**	**0.08**
EDMS	0.03	0.03	0.07	0.1	0.13	0.18	0.03	0.42	0.37	0.44	0.46	0.49	0.57	0.08
EDRA	0.01	0.01	0.04	0.06	0.08	0.13	0.02	0.4	0.37	0.41	0.42	0.44	0.5	0.05
FIA	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0.02*	*0.01*	*0.02*	*0.03*	*0.04*	*0.09*	*0.01*
GLM	*0*	*0*	*0*	*0*	*0.01*	*0.02*	*0*	0.11	0.08	0.12	0.13	0.15	0.21	0.04
LSE	**0.04**	**0.07**	**0.19**	**0.23**	**0.28**	**0.32**	**0.07**	**0.52**	**0.52**	**0.55**	**0.56**	**0.59**	**0.67**	**0.03**
MAF	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
MST	*0*	*0*	*0*	*0*	*0*	*0.02*	*0*	0.07	0.05	0.09	0.1	0.12	0.19	0.03

Table A.15.: This table shows $\hat{\rho}$ for S&P 500 vs. THFI_{sub} according to *Spearman*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{\rho}$. The three lowest values for $\hat{\rho}_{\text{up}}$ and $\hat{\rho}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\rho}_{\text{up}}$	$\hat{\rho}^{\text{bs}}_{\text{up,mean}}$	$\hat{\rho}^{\text{bs}}_{\text{up,90}}$	$\hat{\rho}^{\text{bs}}_{\text{up,95}}$	$\hat{\rho}^{\text{bs}}_{\text{up,99}}$	$\hat{\rho}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\rho}_{\text{lo}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,90}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,95}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,99}}$	$\hat{\rho}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	**0.11**	**0.09**	**0.4**	**0.49**	**0.62**	**0.81**	**0.24**	0.09	0.15	0.52	0.64	0.8	1	0.28
DSB	-0.03	-0.08	0.22	0.29	0.45	0.54	0.23	*-0.33*	*-0.28*	*0.05*	*0.14*	*0.32*	*0.6*	*0.26*
EMM	**0.06**	**-0.02**	**0.26**	**0.34**	**0.46**	**0.57**	**0.22**	**0.43**	**0.32**	**0.62**	**0.69**	**0.79**	**0.91**	**0.24**
EMN	**0.14**	**0.13**	**0.46**	**0.55**	**0.68**	**0.76**	**0.26**	0.04	0.07	0.41	0.5	0.66	0.8	0.27
EVD	*-0.33*	*-0.26*	*0.05*	*0.13*	*0.33*	*0.63*	*0.23*	0.25	0.26	0.55	0.61	0.74	0.81	0.23
EDDI	-0.27	-0.21	0.11	0.19	0.39	0.6	0.24	0.09	0.16	0.48	0.56	0.68	0.82	0.25
EDMS	*-0.32*	*-0.31*	*-0.04*	*0.05*	*0.25*	*0.45*	*0.21*	0.34	0.28	0.56	0.63	0.73	0.81	0.22
EDRA	-0.09	-0.1	0.22	0.3	0.44	0.58	0.24	**0.41**	**0.3**	**0.65**	**0.71**	**0.83**	**0.93**	**0.26**
FIA	-0.27	-0.28	0.01	0.14	0.31	0.61	0.22	**0.35**	**0.34**	**0.72**	**0.78**	**0.87**	**0.96**	**0.3**
GLM	-0.28	-0.26	0.02	0.11	0.29	0.46	0.21	*-0.09*	*-0.21*	*0.17*	*0.3*	*0.57*	*0.95*	*0.29*
LSE	*-0.34*	*-0.35*	*-0.04*	*0.07*	*0.3*	*0.57*	*0.25*	0.26	0.25	0.56	0.63	0.75	0.84	0.25
MAF	-0.24	-0.22	0.08	0.16	0.36	0.63	0.23	*-0.46*	*-0.53*	*-0.29*	*-0.17*	*0.08*	*0.33*	*0.18*
MST	-0.01	-0.14	0.22	0.33	0.51	0.66	0.27	-0.08	0.05	0.38	0.48	0.59	0.84	0.26

Table A.16.: This table shows $\hat{r}$ for S&P 500 vs. THFI_{sub} according to *Kendall*'s rank correlation and results obtained by bootstrapping (bs) the whole data set 1000 times with $k/N = 0.2$. The numbers 90, 95, and 99 correspond to the according quantile of the bootstrapping distribution of $\hat{r}$. The three lowest values for $\hat{r}_{\text{up}}$ and $\hat{r}_{\text{lo}}$ are set italic and the three highest values bold.

	upper tail							lower tail						
THFI_{sub}	$\hat{\tau}_{\text{up}}$	$\hat{\tau}^{\text{bs}}_{\text{up,mean}}$	$\hat{\tau}^{\text{bs}}_{\text{up,90}}$	$\hat{\tau}^{\text{bs}}_{\text{up,95}}$	$\hat{\tau}^{\text{bs}}_{\text{up,99}}$	$\hat{\tau}^{\text{bs}}_{\text{up,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{up}}$	$\hat{\tau}_{\text{lo}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,mean}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,90}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,95}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,99}}$	$\hat{\tau}^{\text{bs}}_{\text{lo,max}}$	$\hat{\sigma}^{\text{bs}}_{\text{lo}}$
COA	**0.08**	**0.06**	**0.28**	**0.36**	**0.48**	**0.56**	**0.18**	0.09	0.14	0.45	0.53	0.67	0.86	0.23
DSB	-0.03	-0.08	0.14	0.2	0.3	0.38	0.17	*-0.26*	*-0.23*	*0.01*	*0.07*	*0.2*	*0.28*	*0.19*
EMM	**0.03**	**-0.02**	**0.18**	**0.24**	**0.36**	**0.49**	**0.16**	0.31	0.25	0.47	0.53	0.64	0.69	0.17
EMN	**0.09**	**0.1**	**0.33**	**0.39**	**0.52**	**0.62**	**0.19**	**0.04**	**0.07**	**0.31**	**0.4**	**0.57**	**0.76**	**0.2**
EVD	*-0.23*	*-0.19*	*0.02*	*0.08*	*0.23*	*0.59*	*0.17*	0.15	0.17	0.4	0.46	0.53	0.73	0.18
EDDI	-0.2	-0.15	0.08	0.14	0.27	0.51	0.18	0	0.08	0.32	0.41	0.52	0.67	0.19
EDMS	*-0.23*	*-0.24*	*-0.04*	*0.02*	*0.14*	*0.41*	*0.16*	0.2	0.16	0.38	0.43	0.53	0.7	0.17
EDRA	-0.06	-0.06	0.17	0.23	0.36	0.47	0.17	**0.32**	**0.25**	**0.5**	**0.56**	**0.68**	**0.77**	**0.2**
FIA	-0.18	-0.19	0.01	0.06	0.19	0.42	0.15	**0.24**	**0.26**	**0.55**	**0.61**	**0.72**	**0.8**	**0.23**
GLM	-0.18	-0.18	0.03	0.09	0.21	0.4	0.16	*-0.06*	*-0.13*	*0.14*	*0.23*	*0.43*	*0.73*	*0.21*
LSE	*-0.27*	*-0.28*	*-0.02*	*0.05*	*0.25*	*0.44*	*0.2*	0.19	0.2	0.42	0.49	0.63	0.82	0.18
MAF	-0.16	-0.16	0.06	0.12	0.28	0.4	0.18	*-0.3*	*-0.37*	*-0.16*	*-0.09*	*0.03*	*0.27*	*0.15*
MST	0	-0.11	0.13	0.23	0.39	0.5	0.19	-0.05	0.06	0.33	0.4	0.52	0.65	0.2

A.7. Source Code

The following sections contain the most important Matlab codes that have been used to calculate the results presented above.

A.7.1. Help Scripts

This scripts have been used to load and prepare the data for the calculations.

Loading of Data

```
%% Reading of data from Excel
% Prices_rel=xlsread('C:\Users\...\xy.xls','Data','B5:AB57');
Prices=xlsread('C:\Users\...\xy.xls','Data','B2:C175');

%% Calculation of return matrix
r=174;%number of rows in prices matrix
c=2;%number of columns in prices matrix
Returns=log(Prices(1:r-1,:)./Prices(2:r,:)); %if prices are given directly
% Returns=log(Prices_rel+1); %if price changes in percentages are given

%% Definition of date parameters (definition of how many rows of
% the return matrix shall be used for the calculations)
a=1;
b=r-1; %if prices are given directly
% b=r; %if price changes in percentages are given
```

Definition of Tail

```
%% Definition of tail
p=20/100;%Percentage of return distribution treated as tail
Z=roundn(p*(b-a+1),0);
```

Sorting of Data

```
%% Sorting of return data
A=Returns;
A_up=sort(A(a:b,1:c),'descend');
A_lo=sort(-A(a:b,1:c),'descend');
A_up_k=A_up(1:Z,:); % from 1 to Z;
A_lo_k=A_lo(1:Z,:);
A_up_k_1=A_up_k(:,1);
A_lo_k_1=A_lo_k(:,1);
```

A.7.2. Calculation of β

```
function bbeta=beta(A,c)% Calculation of beta
    for q=1:c-1
        coeff=polyfit(A(:,1),A(:,q+1),1);
        bbeta(q)=coeff(1);
    end
end
```

A.7.3. Tail Index Estimators

Hill Estimator

```
function v=hill(A,k,q)% Calculation of Hill estimator
   A=sort(A,'descend');
   for j=1:q;
       v(j)=(1/(k)*sum(log(A(1:k,j)/A(k,j))))^-1;
   end
end
```

Gabaix Estimator

```
function [bn_up,bn_lo]=gabaix(A,k,q)

   %% Calculation of rank matrix
   A_up=sort(A,'descend');
   A_lo=sort(-A,'descend');
   s=size(A);
   C_up=NaN(s(1,1),q);
   C_lo=NaN(s(1,1),q);
   for j=1:q;%go along columns
       for i=1:s(1,1);%go along rows
           v_up=find(A(:,j)==A_up(i,j));%positive tail;
           v_lo=find(-A(:,j)==A_lo(i,j));%negative tail;
           t_up=size(v_up);
           t_up=t_up(1,1);
           t_lo=size(v_lo);
           t_lo=t_lo(1,1);
           if t_up>1;
               for z=1:t_up;
                   C_up(v_up(z),j)=0.5-t_up/2+i;%positive tail;
               end
           else C_up(v_up,j)=i;
           end
           if t_lo>1;
               for z=1:t_lo;
                   C_lo(v_lo(z),j)=0.5-t_lo/2+i;%negative tail;
               end
           else C_lo(v_lo,j)=i;
           end
       end
   end

   %% Calculation of bn
   A_up_log=log(A_up(1:k,1:q));
   A_lo_log=log(A_lo(1:k,1:q));
   C_up_log=log(sort(C_up)-0.5);
   C_lo_log=log(sort(C_lo)-0.5);
   for i=1:q;
       p_up=polyfit(A_up_log(:,i),C_up_log(1:k,i),1);%pos. tail
       p_lo=polyfit(A_lo_log(:,i),C_lo_log(1:k,i),1);%neg. tail
       bn_up(i)=-(p_up(1,1));
       bn_lo(i)=-(p_lo(1,1));
   end

end
```

Clauset Estimator

The code for this estimator can be downloaded from the following homepage: `http://www.santafe.edu/~aaronc/powerlaws/`. The name of the file is plfit.m.

Huisman Estimator

```
function b_wls=hillwt(A,k,q)% Calculation of Huisman estimator
   A=sort(A,'descend');
   W=zeros(k,k);
    global V;
    for i=1:q
        for j=1:k
            g(j,i)=1/(j)*sum(log(A(1:j,i))-log(A(j,i)));
            W(j,j)=sqrt(j);
            V(j,:)=[1 j];
        end
        b(:,i)=inv(V'*W'*W*V)*V'*W'*W*g(:,i);
        b_wls(i)=b(1,i)^-1;
   end
end
```

A.7.4. Tail Dependence Coefficients (TDCs)

Poon et al., Hill Estimator

```
% This script calculates the upper and lower tail dependence using a
% non-parametric approach by Poon et al. and the Hill estimator;
%% Clear variables and close open windows
clear all;
close all;

%% Reading of data, calc of return matrix, and define date param.
read_data;

%% Definition of tail
def_tail;

%% Sorting of return data
sort_data;

%% Calculation of v
v_up=hill(A_up,Z,n);
v_lo=hill(A_lo,Z,n);

%% Calculation of L(x) and L(y) (slowly varying function)
t=linspace(1,Z,Z);
s=size(t);
s=s(2);
for q=1:c;
    for i=1:s;
        f=t(i);
        d=@(f)(f/(b-a)*(A_up(f,q))^v_up(q));
        dd=@(f)(f/(b-a)*(A_lo(f,q))^v_lo(q));
        l_up(i,q)=d(f);
        l_lo(i,q)=dd(f);
    end
end
lmk_up=mean(l_up(1:Z,:));
```

```
lmy_up=mean(l_up(Y:Z,:));
lmk_lo=mean(l_lo(1:Z,:));
lmy_lo=mean(l_lo(Y:Z,:));

%% Calculation of S and T (transformation to unit Fréchet marginals)
for q=1:c-1;
    for i=1:Z;
        S_k_up(i,1)=-1/log(1-lmk_up(1)*A_up(i,1)^(-v_up(1)));
        T_k_up(i,q)=-1/log(1-lmk_up(q+1)*A_up(i,q+1)^(-v_up(q+1)));
        S_y_up(i,1)=-1/log(1-lmy_up(1)*A_up(i,1)^(-v_up(1)));
        T_y_up(i,q)=-1/log(1-lmy_up(q+1)*A_up(i,q+1)^(-v_up(q+1)));
        S_k_lo(i,1)=-1/log(1-lmk_lo(1)*(A_lo(i,1))^(-v_lo(1)));
        T_k_lo(i,q)=-1/log(1-lmk_lo(q+1)*(A_lo(i,q+1))^(-v_lo(q+1)));
        S_y_lo(i,1)=-1/log(1-lmy_lo(1)*(A_lo(i,1))^(-v_lo(1)));
        T_y_lo(i,q)=-1/log(1-lmy_lo(q+1)*(A_lo(i,q+1))^(-v_lo(q+1)));
    end
end

%% Calculation of Z=min(S,T)
for q=1:c-1;
    for i=1:Z;
        z_k_up(i,q)=min(S_k_up(i,1),T_k_up(i,q));
        z_y_up(i,q)=min(S_y_up(i,1),T_y_up(i,q));
        z_k_lo(i,q)=min(S_k_lo(i,1),T_k_lo(i,q));
        z_y_lo(i,q)=min(S_y_lo(i,1),T_y_lo(i,q));
    end
end

%% Calculation of chi-cross (ovchi) and respective standard
% deviation (std_ovchi)
for q=1:c-1;
    ovchi_k_up(q)=2/Z*(sum(log(z_k_up(:,q)./z_k_up(Z,q))))-1;
    std_ovchi_k_up(q)=(ovchi_k_up(q)+1)/sqrt(Z);
    ovchi_y_up(q)=2/Z*(sum(log(z_y_up(:,q)./z_y_up(Z,q))))-1;
    std_ovchi_y_up(q)=(ovchi_y_up(q)+1)/sqrt(Z);
    ovchi_k_lo(q)=2/Z*(sum(log(z_k_lo(:,q)./z_k_lo(Z,q))))-1;
    std_ovchi_k_lo(q)=(ovchi_k_lo(q)+1)/sqrt(Z);
    ovchi_y_lo(q)=2/Z*(sum(log(z_y_lo(:,q)./z_y_lo(Z,q))))-1;
    std_ovchi_y_lo(q)=(ovchi_y_lo(q)+1)/sqrt(Z);
end

%% Calculation of chi in case chi-cross cannot be rejected to be equal to one
for q=1:c-1;
    if abs(ovchi_k_up(q)-1)<=std_ovchi_k_up(q);
        chi_k_up(q)=z_k_up(Z,q)*Z/(b-a+1);
        std_chi_k_up(q)=sqrt(z_k_up(Z,q)^2*Z*(b-a+1-Z)/(b-a+1)^3);
    else
        chi_k_up(q)=0;
        std_chi_k_up(q)=0;
    end
    if abs(ovchi_y_up(q)-1)<=std_ovchi_y_up(q);
        chi_y_up(q)=z_y_up(Z,q)*Z/(b-a+1);
        std_chi_y_up(q)=sqrt(z_y_up(Z,q)^2*Z*(b-a+1-Z)/(b-a+1)^3);
    else
        chi_y_up(q)=0;
        std_chi_y_up(q)=0;
    end
    if abs(ovchi_k_lo(q)-1)<=std_ovchi_k_lo(q);
        chi_k_lo(q)=z_k_lo(Z,q)*Z/(b-a+1);
        std_chi_k_lo(q)=sqrt(z_k_lo(Z,q)^2*Z*(b-a+1-Z)/(b-a+1)^3);
    else
        chi_k_lo(q)=0;
        std_chi_k_lo(q)=0;
```

```
    end
    if abs(ovchi_y_lo(q)-1)<=std_ovchi_y_lo(q);
        chi_y_lo(q)=z_y_lo(Z,q)*Z/(b-a+1);
        std_chi_y_lo(q)=sqrt(z_y_lo(Z,q)^2*Z*(b-a+1-Z)/(b-a+1)^3);
    else
        chi_y_lo(q)=0;
        std_chi_y_lo(q)=0;
    end
end
```

Malevergne & Sornette, Non-parametric, Bootstrapping, Gabaix Estimator

```
% This script bootstraps the upper and lower tail dependence using
% a non-parametric approach of Malevergne and Sornette applying
% the Hill estimator;

%% Clear variables and close open windows
clear all;
close all;

%% Reading of data, calc of return matrix, and define date param.
read_data;

%% Definition of tail
def_tail;

%% Sorting of data
sort_data;

%% Calculation of beta
bbeta=beta(A,c);

%% Bootstrapping of v(k)
C=@(A_up_k_1)(A_up_k_1);
D=@(A_lo_k_1)(A_lo_k_1);
bs=5000;
[bs_y_up,index_up]=bootstrp(bs,C,A_up_k_1);
[bs_y_lo,index_lo]=bootstrp(bs,D,A_lo_k_1);
bs_y_up=bs_y_up';
bs_y_lo=bs_y_lo';
for i=1:bs;
    v_up(i)=hill(bs_y_up(:,i),Z,1);
    v_lo(i)=hill(bs_y_lo(:,i),Z,1);
end

%% Bootstrapping of l(k)
for k=1:c-1;
    for j=1:bs;
        for i=1:Z;
            bs_x_up(i,j,k)=A_up_k(index_up(i,j),k+1);
            bs_x_lo(i,j,k)=A_lo_k(index_lo(i,j),k+1);
        end
    end
end
bs_x_up=sort(bs_x_up,'descend');
bs_x_lo=sort(bs_x_lo,'descend');
bs_y_up=sort(bs_y_up,'descend');
bs_y_lo=sort(bs_y_lo,'descend');
for k=1:c-1;
    for j=1:bs;
        for i=1:Z;
            bs_l_up(i,j,k)=bs_x_up(i,j,k)/bs_y_up(i,j);
```

```
                bs_l_lo(i,j,k)=bs_x_lo(i,j,k)/bs_y_lo(i,j);
            end
        end
end
bs_l_up_m=mean(bs_l_up);
bs_l_lo_m=mean(bs_l_lo);

%% Calculation of lambda
for k=1:c-1;
    for j=1:bs;
        if(bbeta(k)>0)
            bs_L_np_k_up(j,k)=1/(max(1,bs_l_up_m(1,j,k)/bbeta(k)))^v_up(j);
            bs_L_np_k_lo(j,k)=1/(max(1,bs_l_lo_m(1,j,k)/bbeta(k)))^v_lo(j);
        elseif(bbeta(k)<=0)
            bs_L_np_k_up(j,k)=0;
            bs_L_np_k_lo(j,k)=0;
        end
    end
end
bs_L_np_k_up_m=mean(bs_L_np_k_up);
bs_L_np_k_lo_m=mean(bs_L_np_k_lo);

%% Calculation of relative error
% for i=1:c-1;
%     rel_k_up(i)=(L_nonpar_k_up(i)-bs_L_np_k_up_m(i))/L_nonpar_k_up(i);
%     rel_k_lo(i)=(L_nonpar_k_lo(i)-bs_L_np_k_lo_m(i))/L_nonpar_k_lo(i);
%     rel_y_up(i)=(L_nonpar_y_up(i)-bs_L_np_y_up_m(i))/L_nonpar_y_up(i);
%     rel_y_lo(i)=(L_nonpar_y_lo(i)-bs_L_np_y_lo_m(i))/L_nonpar_y_lo(i);
% end

%% Calculation of quantiles for upper and lower tail and k=4%
for i=1:c-1;
    bs_L_quant_90_k_up(i)=roundn(quantile(bs_L_np_k_up(:,i),0.90),-2);
    bs_L_quant_95_k_up(i)=roundn(quantile(bs_L_np_k_up(:,i),0.95),-2);
    bs_L_quant_99_k_up(i)=roundn(quantile(bs_L_np_k_up(:,i),0.99),-2);
    bs_L_max_k_up(i)=roundn(max(bs_L_np_k_up(:,i)),-2);
    bs_L_std_k_up(i)=roundn(std(bs_L_np_k_up(:,i)),-2);
    bs_L_quant_90_k_lo(i)=roundn(quantile(bs_L_np_k_lo(:,i),0.90),-2);
    bs_L_quant_95_k_lo(i)=roundn(quantile(bs_L_np_k_lo(:,i),0.95),-2);
    bs_L_quant_99_k_lo(i)=roundn(quantile(bs_L_np_k_lo(:,i),0.99),-2);
    bs_L_max_k_lo(i)=roundn(max(bs_L_np_k_lo(:,i)),-2);
    bs_L_std_k_lo(i)=roundn(std(bs_L_np_k_lo(:,i)),-2);
end

%% Calculate L_nonpar_k
lambda_Malevergne_nonpar_v;

%% Output to Excel
Z1={'Lambda, nonparametric, Malevergne and Sornette, Hill-estimator, bootstrapping'};
Z2={'COA','DSB','EMM','EMN','EVD','EDDI','EDMS','EDRA','FIA','GLM',...
    'LSE','MAF','MST'};
% Z2={'1','2','3','4','5','6','7','8','10','11','12','13','14','15','16',...
%     '17','18','19','20','21','22','23','24','25','26','27'};
Z3={'L_nonpar_k_up';'bs_L_np_k_up_m';'bs_L_quant_90_k_up';...
    'bs_L_quant_95_k_up';'bs_L_quant_99_k_up';'bs_L_max_k_up';'bs_L_std_k_up'};
Z4=[L_nonpar_k_up;roundn(bs_L_np_k_up_m,-2);bs_L_quant_90_k_up;...
    bs_L_quant_95_k_up;bs_L_quant_99_k_up;bs_L_max_k_up;bs_L_std_k_up];
Z5={'L_nonpar_k_lo';'bs_L_np_k_lo_m';'bs_L_quant_90_k_lo';...
    'bs_L_quant_95_k_lo';'bs_L_quant_99_k_lo';'bs_L_max_k_lo';'bs_L_std_k_lo'};
Z6=[L_nonpar_k_lo;roundn(bs_L_np_k_lo_m,-2);bs_L_quant_90_k_lo;...
    bs_L_quant_95_k_lo;bs_L_quant_99_k_lo;bs_L_max_k_lo;bs_L_std_k_lo];
Z11={'N';'k';'p';'y';'# of bs samples'};
Z12=[b;Z;p;Y;bs];
```

```
Hour=num2str(hour(now));
Minute=num2str(minute(now));
Second=num2str(fix(second(now)));
DATE=strcat(date, '_', Hour, '.', Minute, '.', Second);
xlswrite('C:\Users\...\xy.xls', Z1, DATE, 'A1');
xlswrite('C:\Users\...\xy.xls', Z2, DATE, 'B2');
xlswrite('C:\User\...\xy.xls', Z3, DATE, 'A3');
xlswrite('C:\User\...\xy.xls', Z4, DATE, 'B3');
xlswrite('C:\User\...\xy.xls', Z5, DATE, 'A10');
xlswrite('C:\User\...\xy.xls', Z6, DATE, 'B10');
xlswrite('C:\User\...\xy.xls', Z11, DATE, 'A31');
xlswrite('C:\User\...\xy.xls', Z12, DATE, 'B31');
```

Malevergne & Sornette, Parametric, Varying k's, Huisman Estimator

```
% This script calculates the upper and lower tail dependence using a
% parametric approach by Malevergne and Sornette applying the Hill estimator
% and varying k;

%% Clear variables and close open windows
% clear all;
% close all;

%% Reading of data, calc of return matrix, and define date param.
read_data;

%% Definition of tail
def_tail_vark;

%% Sorting of return data
sort_data;

%% Calculation of beta
bbeta=beta(A,c);

%% Calculation of v(k)
for k=1:Z;
    c_up(k)=plfit(A_up_k_1(1:k,:))';
    c_lo(k)=plfit(A_lo_k_1(1:k,:))';
end

%% Calcuation of Cy
for k=1:Z;
    t=linspace(1,k,k);
    s=size(t);
    s=s(2);
    for i=1:s;
        f=t(i);
        d_up=@(f)(f/(b-a)*(A_up(f,1))^c_up(k));
        d_lo=@(f)(f/(b-a)*(A_lo(f,1))^c_lo(k));
        cy_up(i,1)=d_up(f);
        cy_lo(i,1)=d_lo(f);
    end;
    cymk_up(k)=mean(cy_up(1:k));
    cymk_lo(k)=mean(cy_lo(1:k));
end

%% Calculation of epsilons
for q=1:c-1;
    eps=Returns(:,q+1)-(bbeta(1,q)*Returns(:,1));
    Eps(:,q)=eps;
end
```

```

%% Calculation of Ceps%%
eps_up=sort(Eps(a:b,:),'descend');
eps_lo=sort(-Eps(a:b,:),'descend');
for k=1:Z;
    for q=1:c-1;
        for j=1:s;
            f=t(j);
            d_up=@(f)(f/(b-a)*(eps_up(f,q))^c_up(k));
            d_lo=@(f)(f/(b-a)*(eps_lo(f,q))^c_lo(k));
            ce_up(j,q)=d_up(f);
            ce_lo(j,q)=d_lo(f);
        end
        cemk_up(q,k)=mean(ce_up(1:k,q));
        cemk_lo(q,k)=mean(ce_lo(1:k,q));
    end
end

%% Calculation of lambda
for k=1:Z;
    for q=1:c-1;
        if(bbeta(q)>0)
            L_par_k_up(q,k)=(1+(bbeta(q)^-c_up(k))*cemk_up(q,k)/cymk_up(k))^-1;
            L_par_k_lo(q,k)=(1+(bbeta(q)^-c_lo(k))*cemk_lo(q,k)/cymk_lo(k))^-1;
        elseif(bbeta(q)<=0)
            L_par_k_up(q,k)=0;
            L_par_k_lo(q,k)=0;
        end
    end
end

%% Plotting of lambda in dependence of k
set(0,'DefaultAxesLineStyleOrder','-|-o|--|-*')
% TEXT={'COA','DSB','EMM','EMN','EVD','EDDI','EDMS','EDRA','FIA','GLM',...
%       'LSE','MAF','MST'};
% TEXT={'1','2','3','4','5','6','7','8','10','11','12','13','14','15',...
%       '16','17','18','19','20','21','22','23','24','25','26','27'};
% TEXT={'THFI vs. S&P'};
TEXT={'S&P 500 vs. THFI'};
FONTSIZE=8;
figure(1);
x=linspace(1,Z,Z);
s=size(Returns);
for q=1:c-1;
    b=x./s(1,1);
    subplot(2,3,3)
    title('Clauset estimator');
    xlabel('$k/N$','interpreter','latex');
    ylabel('$\hat{\lambda}_{\rm{up}}(k)$','interpreter','latex');
    hold all;
    plot(b,L_par_k_up(q,:))
    if q==c-1
        legend(TEXT','FontSize',FONTSIZE);
    end;
    subplot(2,3,6)
    title('Clauset estimator');
    xlabel('$k/N$','interpreter','latex');
    ylabel('$\hat{\lambda}_{\rm{lo}}(k)$','interpreter','latex');
    hold all;
    plot(b,L_par_k_lo(q,:))
    if q==c-1
        legend(TEXT','FontSize',FONTSIZE);
    end;
```

```
end

% Print figure 1 to pdf
orient('tall');
orient('landscape');
print -depsc C:\Users\...\xy;
```

Schmidt & Stadtmüller, Bootstrapping

```
% This script bootstraps the upper and lower tail dependence using a
% non-parametric approach by Schmidt and Stadtmüller;

%% Clear variables and close open windows
clear all;
close all;

%% Reading of data, calc of return matrix, and define date param.
read_data;

%% Definition of tail
def_tail;

%% Bootstrapping of first column of return data
A=Returns(a:b,:);
q=1;%column number
A_1=A(:,q);
G=@(A_1)(A_1);
bs=5000;
[bs_y,index]=bootstrp(bs,G,A_1);
bs_y=bs_y';
bs_y_s=sort(bs_y);
s=size(A);
C=zeros(s(1,1),s(1,2));

%% Calculation of rank matrix for first column
for m=1:bs;
    for i=1:s(1,1);%move along rows
        v=find(bs_y(:,m)==bs_y_s(i,m));
        t=size(v);
        t=t(1,1);
        if t>1;
            for z=1:t;
                C(v(z),m,1)=0.5-t/2+i;
            end
        else C(v,m,1)=i;
        end
    end
end

%% Calculation of rank matrix for remaining columns
for q=1:c-1;%move along columns
    for m=1:bs;
        for i=1:b;%move along rows
            bs_x(i,m,q)=A(index(i,m),q+1);
        end
    end
end
bs_x_s=sort(bs_x);
for q=1:c-1;%move along columns
    for m=1:bs;
        for i=1:s(1,1);%move along rows
            v=find(bs_x(:,m,q)==bs_x_s(i,m,q));
```

```
                t=size(v);
                t=t(1,1);
                if t>1;
                    for z=1:t;
                        C(v(z),m,q+1)=0.5-t/2+i;
                    end
                else C(v,m,q+1)=i;
                end
            end
        end
    end

%% Calculation of indicator function as approximation for lambda
sum_lo(1:bs,1:c-1)=0;
sum_up(1:bs,1:c-1)=0;
sum_lo_EVT(1:bs,1:c-1)=0;
sum_up_EVT(1:bs,1:c-1)=0;
for q=1:c-1;
    for m=1:bs;
        for i=1:b-a+1;
            if(C(i,m,1)<=Z) && (C(i,m,q+1)<=Z);
                if sum_lo(m,q)<Z;%to avoid values above k due to similar ranks
                    sum_lo(m,q)=sum_lo(m,q)+1;
                end
            end
            if(C(i,m,1)>(b-a+1)-Z) && (C(i,m,q+1)>(b-a+1)-Z);
                if sum_up(m,q)<Z;%to avoid values above k due to similar ranks
                    sum_up(m,q)=sum_up(m,q)+1;
                end
            end
            if(C(i,m,1)<=Z) || (C(i,m,q+1)<=Z);
                if sum_lo_EVT(m,q)<2*Z;%to avoid values above 2*k due
                                                    %to similar ranks
                    sum_lo_EVT(m,q)=sum_lo_EVT(m,q)+1;
                end
            end
            if(C(i,m,1)>(b-a+1)-Z) || (C(i,m,q+1)>(b-a+1)-Z);
                if sum_up_EVT(m,q)<2*Z;%to avoid values above 2*k due
                                                      %to similar ranks
                    sum_up_EVT(m,q)=sum_up_EVT(m,q)+1;
                end
            end
        end
    end
end
for m=1:bs;
    bs_L_lo(m,:)=sum_lo(m,:)./Z;
    bs_L_up(m,:)=sum_up(m,:)./Z;
    bs_L_lo_EVT(m,:)=2-sum_lo_EVT(m,:)./Z;
    bs_L_up_EVT(m,:)=2-sum_up_EVT(m,:)./Z;
end

%% Calculation of quantiles for upper and lower tail and k=4%
for q=1:c-1;
    bs_L_up_m(q)=roundn(mean(bs_L_up(:,q)),-2);
    bs_L_quant_90_up(q)=roundn(quantile(bs_L_up(:,q),0.90),-2);
    bs_L_quant_95_up(q)=roundn(quantile(bs_L_up(:,q),0.95),-2);
    bs_L_quant_99_up(q)=roundn(quantile(bs_L_up(:,q),0.99),-2);
    bs_L_max_up(q)=roundn(max(bs_L_up(:,q)),-2);
    bs_L_std_up(q)=roundn(std(bs_L_up(:,q)),-2);
    bs_L_lo_m(q)=roundn(mean(bs_L_lo(:,q)),-2);
    bs_L_quant_90_lo(q)=roundn(quantile(bs_L_lo(:,q),0.90),-2);
    bs_L_quant_95_lo(q)=roundn(quantile(bs_L_lo(:,q),0.95),-2);
```

```
    bs_L_quant_99_lo(q)=roundn(quantile(bs_L_lo(:,q),0.99),-2);
    bs_L_max_lo(q)=roundn(max(bs_L_lo(:,q)),-2);
    bs_L_std_lo(q)=roundn(std(bs_L_lo(:,q)),-2);
    bs_L_up_m_EVT(q)=roundn(mean(bs_L_up_EVT(:,q)),-2);
    bs_L_quant_90_up_EVT(q)=roundn(quantile(bs_L_up_EVT(:,q),0.90),-2);
    bs_L_quant_95_up_EVT(q)=roundn(quantile(bs_L_up_EVT(:,q),0.95),-2);
    bs_L_quant_99_up_EVT(q)=roundn(quantile(bs_L_up_EVT(:,q),0.99),-2);
    bs_L_max_up_EVT(q)=roundn(max(bs_L_up_EVT(:,q)),-2);
    bs_L_std_up_EVT(q)=roundn(std(bs_L_up_EVT(:,q)),-2);
    bs_L_lo_m_EVT(q)=roundn(mean(bs_L_lo_EVT(:,q)),-2);
    bs_L_quant_90_lo_EVT(q)=roundn(quantile(bs_L_lo_EVT(:,q),0.90),-2);
    bs_L_quant_95_lo_EVT(q)=roundn(quantile(bs_L_lo_EVT(:,q),0.95),-2);
    bs_L_quant_99_lo_EVT(q)=roundn(quantile(bs_L_lo_EVT(:,q),0.99),-2);
    bs_L_max_lo_EVT(q)=roundn(max(bs_L_lo_EVT(:,q)),-2);
    bs_L_std_lo_EVT(q)=roundn(std(bs_L_lo_EVT(:,q)),-2);
end
```

Linear Measures of Correlation, Bootstrapping

```
% This script calculates the bootstrap statistics for the linear measures
% of correlations (Spearman, Pearson, and Kendall)

%% Clear variables and close open windows
clear all;
close all;

%% Reading of data, calc of return matrix, and define date param.
read_data;

%% Definition of tail
def_tail;

%% Bootstrapping of first column of return data
A=Returns(a:b,:);
q=1;%column number
A_1=A(:,q);
G=@(A_1)(A_1);
bs=1000;
[bs_y,index_bs]=bootstrp(bs,G,A_1);
bs_y=bs_y';

%% Bootstrapping of the rest of the return data while conserving
% the dependence structure
for m=1:bs;
    for q=1:c;
        for i=1:b;
            bs_yx(i,q,m)=A(index_bs(i,m),q);
        end
    end
end

%% Sorting of return data
for m=1:bs;
    [bs_y_s(:,m),index_sort(:,m)]=sortrows(bs_y(:,m));
end
for m=1:bs;
    for q=1:c;
        for i=1:b;
            bs_yx_s(i,q,m)=bs_yx(index_sort(i,m),q,m);
        end
    end
end
```

```

%% Calculation of correlations
bs_r=zeros(c,c,bs);
bs_r_k_up=zeros(c,c,bs);
bs_r_k_lo=zeros(c,c,bs);
% bs_rho=zeros(c,c,bs);
% bs_rho_k_up=zeros(c,c,bs);
% bs_rho_k_lo=zeros(c,c,bs);
% bs_tau=zeros(c,c,bs);
% bs_tau_k_up=zeros(c,c,bs);
% bs_tau_k_lo=zeros(c,c,bs);
for m=1:bs
    bs_r(:,:,m)=corr(bs_yx_s(:,:,m));
    bs_r_k_up(:,:,m)=corr(bs_yx_s(b-Z:b,:,m));
    bs_r_k_lo(:,:,m)=corr(bs_yx_s(1:Z,:,m));
%     bs_rho(:,:,m)=corr(bs_yx_s(:,:,m),'type','Spearman');
%     bs_rho_k_up(:,:,m)=corr(bs_yx_s(b-Z:b,:,m),'type','Spearman');
%     bs_rho_k_lo(:,:,m)=corr(bs_yx_s(1:Z,:,m),'type','Spearman');
%     bs_tau(:,:,m)=corr(bs_yx_s(:,:,m),'type','Kendall');
%     bs_tau_k_up(:,:,m)=corr(bs_yx_s(b-Z:b,:,m),'type','Kendall');
%     bs_tau_k_lo(:,:,m)=corr(bs_yx_s(1:Z,:,m),'type','Kendall');
end
bs_r=squeeze(bs_r(1,2:c,:))';
bs_r_k_up=squeeze(bs_r_k_up(1,2:c,:))';
bs_r_k_lo=squeeze(bs_r_k_lo(1,2:c,:))';
% bs_rho=squeeze(bs_rho(1,2:c,:))';
% bs_rho_k_up=squeeze(bs_rho_k_up(1,2:c,:))';
% bs_rho_k_lo=squeeze(bs_rho_k_lo(1,2:c,:))';
% bs_tau=squeeze(bs_tau(1,2:c,:))';
% bs_tau_k_up=squeeze(bs_tau_k_up(1,2:c,:))';
% bs_tau_k_lo=squeeze(bs_tau_k_lo(1,2:c,:))';

%% Rounding of figures
bs_r_m=roundn(mean(bs_r),-2);
bs_r_k_up_m=roundn(mean(bs_r_k_up),-2);
bs_r_k_lo_m=roundn(mean(bs_r_k_lo),-2);
% bs_rho_m=roundn(mean(bs_rho),-2);
% bs_rho_k_up_m=roundn(mean(bs_rho_k_up),-2);
% bs_rho_k_lo_m=roundn(mean(bs_rho_k_lo),-2);
% bs_tau_m=roundn(mean(bs_tau),-2);
% bs_tau_k_up_m=roundn(mean(bs_tau_k_up),-2);
% bs_tau_k_lo_m=roundn(mean(bs_tau_k_lo),-2);

%% Calculation of quantiles for upper and lower tail
for q=1:c-1;
    bs_r_quant_90_k_up(q)=roundn(quantile(bs_r_k_up(:,q),0.90),-2);
    bs_r_quant_95_k_up(q)=roundn(quantile(bs_r_k_up(:,q),0.95),-2);
    bs_r_quant_99_k_up(q)=roundn(quantile(bs_r_k_up(:,q),0.99),-2);
    bs_r_max_k_up(q)=roundn(max(bs_r_k_up(:,q)),-2);
    bs_r_std_k_up(q)=roundn(std(bs_r_k_up(:,q)),-2);
    bs_r_quant_90_k_lo(q)=roundn(quantile(bs_r_k_lo(:,q),0.90),-2);
    bs_r_quant_95_k_lo(q)=roundn(quantile(bs_r_k_lo(:,q),0.95),-2);
    bs_r_quant_99_k_lo(q)=roundn(quantile(bs_r_k_lo(:,q),0.99),-2);
    bs_r_max_k_lo(q)=roundn(max(bs_r_k_lo(:,q)),-2);
    bs_r_std_k_lo(q)=roundn(std(bs_r_k_lo(:,q)),-2);
%     bs_rho_quant_90_k_up(q)=roundn(quantile(bs_rho_k_up(:,q),0.90),-2);
%     bs_rho_quant_95_k_up(q)=roundn(quantile(bs_rho_k_up(:,q),0.95),-2);
%     bs_rho_quant_99_k_up(q)=roundn(quantile(bs_rho_k_up(:,q),0.99),-2);
%     bs_rho_max_k_up(q)=roundn(max(bs_rho_k_up(:,q)),-2);
%     bs_rho_std_k_up(q)=roundn(std(bs_rho_k_up(:,q)),-2);
%     bs_rho_quant_90_k_lo(q)=roundn(quantile(bs_rho_k_lo(:,q),0.90),-2);
%     bs_rho_quant_95_k_lo(q)=roundn(quantile(bs_rho_k_lo(:,q),0.95),-2);
%     bs_rho_quant_99_k_lo(q)=roundn(quantile(bs_rho_k_lo(:,q),0.99),-2);
```

```
%           bs_rho_max_k_lo(q)=roundn(max(bs_rho_k_lo(:,q)),-2);
%           bs_rho_std_k_lo(q)=roundn(std(bs_rho_k_lo(:,q)),-2);
%           bs_tau_quant_90_k_up(q)=roundn(quantile(bs_tau_k_up(:,q),0.90),-2);
%           bs_tau_quant_95_k_up(q)=roundn(quantile(bs_tau_k_up(:,q),0.95),-2);
%           bs_tau_quant_99_k_up(q)=roundn(quantile(bs_tau_k_up(:,q),0.99),-2);
%           bs_tau_max_k_up(q)=roundn(max(bs_tau_k_up(:,q)),-2);
%           bs_tau_std_k_up(q)=roundn(std(bs_tau_k_up(:,q)),-2);
%           bs_tau_quant_90_k_lo(q)=roundn(quantile(bs_tau_k_lo(:,q),0.90),-2);
%           bs_tau_quant_95_k_lo(q)=roundn(quantile(bs_tau_k_lo(:,q),0.95),-2);
%           bs_tau_quant_99_k_lo(q)=roundn(quantile(bs_tau_k_lo(:,q),0.99),-2);
%           bs_tau_max_k_lo(q)=roundn(max(bs_tau_k_lo(:,q)),-2);
%           bs_tau_std_k_lo(q)=roundn(std(bs_tau_k_lo(:,q)),-2);
end
```