

Relation between  
interest rates and inflation

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## Declaration

I hereby declare that this thesis was performed and written on my own and that references and resources used within this work have been explicitly indicated.

I am aware that making a false declaration may have serious consequences.

Zürich, 15/03/2010

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(Signature)



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# Chapter 1

## Introduction

This thesis was developed within the financial and risk modeling group of Scor - a reinsurance company with worldwide operations. Interest rates and inflation are very important risk factors for the insurance industry due to the fact that insurance is a long-term business where premiums are collected today and claims are paid out in the future. High and unpredicted inflation can seriously distort the ability of insurance company to cover the insured losses of the counterparties. Interest rates influence the ability of the company to generate positive returns on the money invested. If inflation is higher than interest rates than the insurance company instead of saving and increasing money loses them. Wrong expectations about the risks associated with inflation and interest rates can eventually lead to the insolvency of the company.

The goal of this thesis is to check and if necessary to modify the key assumptions of the original models developed in [Müller et al., 2010] for the interest rates and inflation simulation. These models are used in the Economic Scenario Generator (ESG) - a tool being developed within Scor for the simulation of the risks associated with key macroeconomic factors. Inflation and interest rates are assumed to be weak mean-reverting processes. The interdependencies between interest rates and inflation are modeled via the introduction of the real interest rates (interest rates adjusted for inflation) which influence both of them.

The real interest rates behavior suggest that they are mean-reverting processes which exhibit stronger mean-reversion than interest rates or inflation alone. Thus, the interest rate process is governed by two mean-reverting forces: the first comes from interest rates and the second one from real interest rates. The inflation process is simulated in a similar way, i.e. as a process driven by two mean-reversions: inflation mean-reversion and real interest rate mean-reversion.

Preliminary analyses showed that inflation rises or falls occur slightly in advance compared to the interest rates rise or fall. It means that interest rate

can be partially predicted by inflation. This fact is reflected in the ESG modeling.

We aim to check the assumption of strong mean-reversion of real interest rate and calibrate the parameters of this mean-reversion. Another goal is the estimation of the mean-reversion parameters of inflation and interest rates and the analysis of the relevant lead-lag effects.

This report is organized in the following way. The first two chapters are devoted to the description of the data and the data sources used for the analysis. The third chapter briefly describes the ESG and the original models used for its simulations. We have to emphasize that aim of the third chapter of this report is not to give a detailed description of the models but rather to establish the direction of the research. The problem of seasonality in inflation is addressed in the fourth chapter. While not being one of the goals of this work seasonality is an inherent property of the inflation and its impact on the results of analysis should be minimized. The lead-lag analyses are presented in the fifth chapter. Finally, we focus on the mean-reversion analyses in the sixth chapter. The detailed description of the Ornstein-Uhlenbeck model, its parameters, calibration technique as well as the results of the tests on various time series are outlined in the six chapter. As a main result of this work we are able to give some recommendations on the parameter choice for the ESG. At the end, some concluding remarks are made in the seventh chapter.



## Chapter 2

# Data and sources of data used in the analysis of inflation

In this chapter we discuss inflation and the Consumer Price Index (later referred as CPI).

In the first paragraph the brief description of inflation rate and the Consumer Price index is given together with the sources of data used in the analysis. The second paragraph gives the formal definition of inflation rate. The Eurozone CPI reconstruction technique originally implemented by Fabio Sigrist for SCOR in 2008 (see [Sigrist, 2008]) and used in our analysis is presented in the third paragraph.

The Consumer Price Index construction is outlined in the section A.1 of the Appendix of this report.

### 2.1 Inflation and Consumer Price Index

#### 2.1.1 General information

The inflation is defined as the rate at which the general level of prices for goods and services rises and deflation, consequently, is the rate at which the general level of prices for goods and services falls. In case of inflation the unit of currency loses its purchasing power with the time while in case of deflation the unit of currency gains the purchasing power.

One of the biggest questions is how to measure inflation. Usually, the Consumer Price Index is used as the general proxy for inflation. The CPI is a measure of average level of prices for goods and services consumed by the average household in a given area. It measures changes in price level for the same set of goods and services from one period to another. The CPI is the relative rather than exact measure of price changes. It is not valued in

currency units of the country. Usually, the price level of one of the years is taken as the reference (for example 100 points for year 2005) and the rest is measured with respect to this reference.

There are several major problems related to an adequate CPI evaluation. In order to ensure the comparability of the CPI among the years it has to gauge prices of the same items. However, the set of goods and services captured by the CPI is not constant but slowly changing together with the changing structure of consumption. People had different preferences and economy produced different goods one hundred years ago. As a consequence, the CPI is a slowly changing measure.

There are also several kinds of the CPI assessed for different consumption baskets and different regions. For example, there is a CPI which measures price level excluding food and oil which is called the *core CPI*. There are also CPIs measured separately for urban and rural areas, etc. All of those different CPIs could be regarded as different proxies for the price level and raise an important question of choosing the one which serves the purpose best.

CPIs of different countries are usually measured by the National Statistical Bureaus and published every month. National Statistical Bureaus of different countries have different statistical methods for calculating their CPIs. These differences are caused by dissimilar consumption baskets of nations as well as by the number of other reasons. The comparability of the CPIs across the countries is an essential issue too.

We use the so-called *general CPI* for analysis. It usually contains costs of food, beverages, clothing, fuel, transportation, medical care, education, telecommunication, leisure goods and other services. Sometimes, it also includes rents for housing and owner-occupied housing costs.

Inflation is the change of CPI from one period to another and is usually calculated as the return of CPI.

### **2.1.2 Why measuring inflation is important**

High or unpredictable inflation can have a negative impact on the economy. It distorts the firms' long-term plans. The uncertainty about future real value of money can significantly discourage investment decisions. Pension funds, insurance companies and other financial service firms closely watch inflation in order to ensure positive real return on investments.

Having clear expectation about inflation is important for the appropriate allocation of the investment portfolio preventing it from losing its real value in case of returns being inferior to the inflation.

### 2.1.3 Data used

The analysis is performed with the historical CPI data of five countries: the USA, the UK, the Eurozone, Japan and Switzerland, downloaded from Bloomberg. Bloomberg takes data from Eurostat for the Eurozone and from the OECD (Organization for Economic Cooperation and Development) for the rest. The OECD, in turn, gets the data from the National Statistical Bureaus of the corresponding countries. The more detailed description of CPIs by country is given in A.1 paragraph of the appendix.

For the analysis of inflation we use the general CPI Index. The longest period of time that is available in Bloomberg is almost 50 years: starting from January 1960 till July 2009 for all the countries except the Eurozone.

The Eurozone CPI data exists only from January 1996 which is too short for a reliable statistical analysis. For this reason, we build the synthetic Eurozone CPI based on the German and French CPIs (see 2.3 for the details) which allows us to overcome the problem of data scarcity. The reconstruction goes back to January 1988 because of the minimum data length requirement of 20 years for the ESG.

We searched for the alternative to Bloomberg sources of data to obtain the longer history for the study of the mean-reversion of inflation rates. We found the longer history for the USA CPI in [Shiller, 2000]. The series downloaded from Shiller's homepage spans the period of time of around 140 years starting on January 1871.

We decided to conduct our study on monthly rather than quarterly data to get more statistically reliable results, despite the fact that only quarterly data is employed for the ESG simulations at the moment.

## 2.2 Logarithmic inflation

Inflation in our analysis is defined as the logarithmic return of the CPI. In fact, we use two different types of inflation, depending on the analysis performed. The reason for this is data artifacts such as seasonality and short-term mean-reverting noise, which are discussed later in this report.

Let  $I_t$  be the annualized monthly logarithmic inflation at time  $t$ , this variable is used in the analysis of seasonality and defined as follows:

$$I_t = \ln \left( \frac{CPI_t}{CPI_{t-1}} \right) \cdot 12, \quad (2.1)$$

Annual logarithmic inflation  $I_t^y$  is used in the mean-reversion analysis and is calculated in the following way:

$$I_t^y = \ln \left( \frac{CPI_t}{CPI_{t-12}} \right). \quad (2.2)$$

X	a	$b_1$	$b_2$	R-squared
Annual	0.0042	0.7070	0.2872	0.9324
Monthly	0.0048	0.8360	0.0967	0.6491

Table 2.1: Results of linear regressions for monthly and annual inflation series.

## 2.3 Eurozone CPI Reconstruction

The monthly CPI data for the Eurozone obtained from Bloomberg dates back to January 1996. The ESG uses data from January 1988 for its runs, this is why we had to reconstruct the Eurozone CPI.

The synthetic CPI is build to extend the history for another 8 years using the data of German and French CPIs. The linear regression model is fit to the yearly inflation (eq. 2.3) and monthly inflation data (eq. 2.4). Yearly inflation model and monthly inflation model are defined as follows:

$$\ln\left(\frac{\text{Eurozone } CPI_t}{\text{Eurozone } CPI_{t-12}}\right) = a + b_1 \cdot \ln\left(\frac{\text{French } CPI_t}{\text{French } CPI_{t-12}}\right) + b_2 \cdot \ln\left(\frac{\text{German } CPI_t}{\text{German } CPI_{t-12}}\right) + \varepsilon_t, \quad (2.3)$$

$$\ln\left(\frac{\text{Eurozone } CPI_t}{\text{Eurozone } CPI_{t-1}}\right) = a + b_1 \cdot \ln\left(\frac{\text{French } CPI_t}{\text{French } CPI_{t-1}}\right) + b_2 \cdot \ln\left(\frac{\text{German } CPI_t}{\text{German } CPI_{t-1}}\right) + \varepsilon_t. \quad (2.4)$$

Goodness-of-fit statistics (R-squared) is examined in order to determine the best model. The results of the analysis are shown in table 2.1. According to the R-squared statistics, the model (2.3) is the one that better describes the data, thus it becomes a natural choice for the CPI data reconstruction.

## Chapter 3

# Data and sources of data used in the interest rate analysis

In this chapter we discuss interest rates.

We use a variety of different interest rates and interest rates data sources for the research. Different data sources are considered mainly for two reasons: first, we are aiming to test our hypothesis 4.3 on data from different countries and, second, one particular source does not always contain data of required quality (i.e. there can be missing values, or the data series might be too short).

In the first paragraph the detailed description of the interest rate data and its origin is given. The formal mathematical definition of interest rate as it is understood in the context of this report can be found in the second section. Third paragraph is devoted to the description of interest rate data reconstruction procedures used for the extension of some of our data series. The Eurozone interest rate reconstruction technique follows the work of Fabio Sigrist on the data reconstruction for the Economic Scenario Generator (see [Sigrist, 2008]) while the U.S. 10 years interest rate interpolation is the one used in [Shiller, 2000].

### 3.1 Interest rates. Data used

The government yields for the USA, the UK, Eurozone, Japan and Switzerland are used as proxies for the risk-free interest rates. We have chosen three different maturities for the research: 3 months, 1 year and 10 years. Our choice is based on the fact that 3 months interest rates are the ones that are most influenced by the Central banks' regulatory actions, while 10 years interest rates mostly reflect the expectations of the market and 1 year interest rates are important benchmark rates for the economy and can be

regarded as a sort of compromise between long-term and short-term rates. Our analysis is conducted only with three maturities stated above.

We employed Bloomberg as a primary source of data. Data for the USA and the UK interest rates exist from April 1991, Japanese interest rates time series date back to September 1992, Swiss interest rates start in February 1994 and the Eurozone interest rate time series start on December 1998. The longest data series have 18 years of data and the shortest ones only 10 years.

Mean-reversion analyses usually require sufficiently long series to catch periodic movements of the underlying process. We made a research on the alternative data sources which could provide us with longer data series. When they could not be found, we used a data reconstruction models described in paragraph 3.3, taking Bloomberg data as a base for reconstruction. Reconstructed series go back to January 1988.

The alternative sources of data used are described below:

**The USA.** USA interest rates are provided by the U.S. department of Treasury. These rates are known as Constant Maturity Treasury(CMT) rates. Yields are interpolated by the Treasury from the daily yield curve which is based on the closing market bid yields on actively traded Treasury securities in the OTC market. The Treasury Yield Curve Methodology is published on the U.S. Treasury website <sup>1</sup>. The 3 months interest rate dates back to January 1982, 1 year and 10 years - to April 1953.

The International Monetary Fund (IMF) database also contains data for 3-month Treasury bills, obtained from secondary market and calculated on a discount basis. These data series start on January 1934. [Shiller, 2000] uses the long-term interest rate dating back to January 1871. The series is interpolated from the annual data series which starts on January 1871. For the details of interpolation refer to section 3.3.2 of paragraph 3.3. Note, that we have got the U.S. CPI series starting on January 1871 as well.

**The UK.** The UK nominal spot curves are estimated by the Monetary Instruments and Markets Division of the Bank of England. The 1 year and 10 years curves are provided by the Bank of England from January 1970. They are based on yields on UK government bonds (gilts). The methodology used to construct yield curves is described in the Bank of England Quarterly Bulletin article [Anderson and Sleath, 1999a], and a detailed technical description can be found in the [?].

The 3 month interest rate data provided by Bank of England has a

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<sup>1</sup><http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yieldmethod.html>

lot of missing values, for that reason we proceed with the 3 months Bloomberg data and extend the history till January 1988.

**The Eurozone.** Any alternative source of the Eurozone interest rates would provide us with the reconstructed data, so we made the reconstruction by ourselves. Eurozone interest rates are extended for more than 10 years to January 1988. For details refer to subsection 3.3.1 of the paragraph 3.3.

**Japan.** We selected the Bank of Japan yield on 10 years TSE bonds as the long-term interest rate. The series begins in 1986.

As the benchmark for 3 months and 1 year interest rates we chose 'Average Interest Rates on Certificates of Deposit/60 days - 89 days' and 'Average Interest Rates on Certificates of Deposit/180 days - less than 1 year' respectively. They date from April 1985 and April 1986 respectively <sup>2</sup>. Japanese Treasury Bills rate as well as Japan 10 years government rates are provided by the International Monetary Fund dating back to January 1957 and October 1966 respectively. These series are used for our analysis too.

**Switzerland.** The Swiss National Bank presents information on yields on Swiss Confederation bonds with maturities from 2 to 10 years, all the series initiate on January 1988 <sup>3</sup>.

Swiss Franc 3 months and 12 months LIBOR rates serve as the benchmarks for short term interest rates in our analysis. The publicly available series start on January 1989 and can be found on the Swiss National Bank web page.

Swiss Treasury Bills rates as well as Swiss government rates can be downloaded from the International Monetary Fund database. These two series start on January 1980 and January 1964 respectively.

We use data from different sources and of different length for our analysis. The table which summarizes the key information about interest rate time series is shown in Table 3.1.

## 3.2 Logarithmic interest rate

Interest rate in our analysis is defined as the logarithmic interest rates

$$R_t = \ln(1 + r_t), \quad (3.1)$$

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<sup>2</sup>You can find these data series in the following address  
[http://www.stat-search.boj.or.jp/index\\_en.html](http://www.stat-search.boj.or.jp/index_en.html)

<sup>3</sup>For details on calculation refer to  
[http://www.banque-nationale.ch/en/mmr/reference/renditen\\_book/source](http://www.banque-nationale.ch/en/mmr/reference/renditen_book/source)

Country	Maturity	Source	Rate	Start	End	Size
USA	3 months	IMF	Treasury bill	01.1934	11.2009	911
		U.S. Treasury	CMT rate	01.1982	10.2009	334
	1 year	U.S. Treasury	CMT rate	04.1953	10.2009	679
	10 years	Shiller	Treasury bonds Rec	01.1871	11.2009	1667
U.S. Treasury		CMT rate	04.1953	10.2009	679	
UK	3 months	Bloomberg	Government bonds Rec	01.1988	07.2009	259
		Bloomberg	Government bonds	01.1991	07.2009	220
	1 year	Bank of England	UK Nominal spot rate	01.1970	10.2009	478
10 years	Bank of England	UK Nominal spot rate	01.1970	10.2009	478	
Eurozone	3 months	Bloomberg	Government bonds Rec	01.1988	07.2009	259
		Bloomberg	Government bonds	01.1998	07.2009	128
	1 year	Bloomberg	Government bonds Rec	01.1988	07.2009	259
		Bloomberg	Government bonds	01.1998	07.2009	128
	10 years	Bloomberg	Government bonds Rec	01.1988	07.2009	259
		Bloomberg	Government bonds	01.1998	07.2009	128
Japan	3 months	IMF	Treasury bills, bid	01.1957	12.2009	636
		Bank of Japan	Certificate of Deposit	04.1985	10.2009	295
	1 year	Bank of Japan	Certificate of Deposit	04.1986	10.2009	283
	10 years	IMF	Gvt Benchmarks, bid	01.1966	12.2009	520
		Bank of Japan	Yields on TSE Bonds	01.1986	10.2009	286
Switzerland	3 months	IMF	Treasury bills, bid	01.1980	12.2009	360
		Swiss National Bank	CHF LIBOR	01.1989	09.2009	249
	1 year	Swiss National Bank	CHF LIBOR	01.1989	09.2009	249
	10 years	IMF	Gvt Benchmarks, bid	01.1964	12.2009	551
		Swiss National Bank	Spot rate, CH bonds	01.1988	09.2009	261

Table 3.1: Summary of interest rates time series used in the analysis.



where  $r_t$  is the spot interest rate at time  $t$ .

### 3.3 Data reconstruction

#### 3.3.1 Data reconstruction of interest rates for the Eurozone.

The Eurozone data reconstruction is described in this section, more details can be found in 'Government Bond Yields' subchapter of the 'EUR' chapter from [Sigrist, 2008].

The synthetic Eurozone end of month data reconstruction for the period from 1988 to 2008 is based on the monthly average Eurozone data from the ECB and the German end of month data from the Bundesbank. For all the maturities for which the ECB provides data ( $\geq 2$  years), we calculate the factor

$$f_t^m = \frac{\bar{y}_{t-2}^m + \bar{y}_{t-1}^m + \bar{y}_t^m + \bar{y}_{t+1}^m + \bar{y}_{t+2}^m}{z_{t-3}^m + z_{t-2}^m + z_{t-1}^m + z_t^m + z_{t+1}^m + z_{t+2}^m}, \quad (3.2)$$

where  $\bar{y}^m$  denotes the monthly average yields from the ECB of bonds with maturity of  $m$  years and  $z^m$  denotes the end of month yield of the corresponding bond from the Bundesbank. We then estimate the Eurozone end of month yield  $y^m$  as

$$y_t^m = f_t^m \cdot z_t^m. \quad (3.3)$$

For yields of bonds with maturities of three months, six months, and one year, we do not have monthly average data from the ECB. Nonetheless, we calculate synthetic Eurozone yield data  $y_t^{1/4}$ ,  $y_t^{1/2}$  and  $y_t^1$  using the same factor  $f_t^2$  as for the yields of bonds with two years maturity:

$$y_t^m = f_t^2 \cdot z_t^m, \quad m = 1/4, \quad 1/2 \text{ and } 1 \text{ year}. \quad (3.4)$$

The next step is reconstruction of synthetic yield data from Bloomberg data. In summary, to reconstruct the yield data from Bloomberg data we use the following model.

$$b_t = a + b * y_t + \varepsilon_t, \quad (3.5)$$

where  $b_t$  is the Bloomberg yield data, and  $y_t$  is the synthetic reconstructed data.

The models are fitted by least squares with data from 1999 to 2008. The constants in 3.5 for the different maturities are reported in Table 3.2, however, for this work we are only interested in reconstruction of bonds with maturities of 3 months .

Maturity	$a$	$b$
3 months	0.0009	0.9402
1 year	0.0006	0.9779
10 years	0.0021	0.9310

Table 3.2: Linear regression (eq. 3.5) parameters estimated by the OLS .

### 3.3.2 Interpolation of the U.S. long-term interest rate

[Shiller, 2000] performs the interpolation of the U.S. long-term interest rate. The original data set contains monthly data starting from January 1953 and annual data from January 1871. In order to obtain monthly time series from January 1871 till January 1953, the calculations described below are performed.

Assume that  $y_t$ , for  $t = 1, \dots, 83$  is the annual interest rate time series starting on January 1871 and finishing on January 1953. The 11 data points between  $y_t$  and  $y_{t+1}$  are computed as follows,

$$y_t^1 = y_t, \quad t = 1, \dots, 83, \quad (3.6)$$

$$y_t^i = \frac{12 - (i - 1)}{12} \cdot y_t^1 + \frac{i - 1}{12} \cdot y_{t+1}^1, \quad i = 2, \dots, 12, \quad (3.7)$$

where  $y_t^i$  is the observation of the  $i^{th}$  month of year  $t$ . This means that

$$y_t^2 = \frac{11}{12} \cdot y_t^1 + \frac{1}{12} \cdot y_{t+1}^1, \quad t = 1, \dots, 82, \quad (3.8)$$

$$y_t^3 = \frac{10}{12} \cdot y_t^1 + \frac{2}{12} \cdot y_{t+1}^1, \quad t = 1, \dots, 82, \quad (3.9)$$

$$\dots \dots \dots \quad (3.10)$$

$$y_t^{12} = \frac{1}{12} \cdot y_t^1 + \frac{11}{12} \cdot y_{t+1}^1, \quad t = 1, \dots, 82. \quad (3.11)$$

Thus, the linear interpolation is performed in order to get interest rates on monthly basis from yearly time series. Linear interpolation is enough for our purposes cause we aim to study long-term interest rates behaviour and short-term (less than 1 year) variation is not so important for us.

## Chapter 4

# Economic Scenario Generator

Economic Scenario Generator (or the ESG) is the project developed at SCOR. The purpose of the ESG is to provide SCOR with tools for modeling key macroeconomic variables behaviors in the future. Economic variables include interest rates, inflation, GDP, foreign exchange rates, equity indices. The modeling of the economic scenarios is based on bootstrapping. The bootstrap observations are drawn randomly from the empirical distribution of the simulated variables. Dependencies, random shocks and GARCH effects are introduced into simulations depending on the variable. For more details on the particular model for each economic variable refer to the main document [Müller et al., 2010]. The resulting simulations allow SCOR to assess risks associated with the macroeconomic factors.

We focus on the interest rates and inflation modeling in this study. In order to formulate the direction of research of the current thesis we briefly introduce the assumptions and the models of interest rates and inflation used in the ESG.

### 4.1 Assumptions of the inflation and interest rates models

The general idea of bootstrapping is to randomly pick past time  $t_i$  and assume that the observation of variable  $x_i$  at this time is used as its value in the future time  $t_j$ . The drawback of this method is that variable  $x_i$  will never reach any unobserved in its history value. To overcome this problem the ESG for its simulations uses innovations (i.e. changes) of the variables rather than variables themselves. Thus, instead of picking  $x_i$  at random time  $t_i$ , the change of the variable  $\Delta x_i$  is chosen.

The change of the variable (innovation) is defined as its deviation from its

conditional expectation, i.e.

$$\Delta x_i = x_i - \mathbb{E}_{i-1} [x_i]. \quad (4.1)$$

The motivation to define it in this way is to be able to model stochastic part of the innovation, i.e. deviation from its expected value.

The forecasted value at future point of time  $t_j$  is defined as

$$x_j = \mathbb{E}_{j-1} [x_j] + \Delta x_i, \quad (4.2)$$

where  $\Delta x_i$  is the historical innovation at random point of time  $t_i$  in the past. In order to be able to calculate the innovations of all the variables and the forecasted values, the expectation of the variable at time  $t_i$  conditional on the information available at the time  $t_{i-1}$ , i.e.  $\mathbb{E}_{i-1} [x_i]$ , should be defined. Modeling of interest rates and inflation is based on several important empirical observations.

First, neither inflation nor interest rates are drifting off some interval and, whenever extreme values are achieved, they eventually come back to normal levels. This is an evidence of the mean-reversion property of the underlying processes. However, the mean-reversion is not so strong to give a clear answer on how fast these variables revert to normal levels. Interest rates and inflation are assumed to be the mean-reverting processes with very low mean-reversion speed. The figures of interest rates and inflation are given in the chapter 7 on the figures 7.4 and 7.3.

Second, inflation and interest rates do not develop independently of each other. Graphs of the inflation and interest rate together can be found on the figure 7.5 of chapter 7, from which it can be seen that they drift together and difference between them stays in some range. This fact led to an assumption that real interest rate, defined as difference between interest rate and inflation, is exposed to mean-reverting forces on its own and these forces are stronger than in the case of interest rates and inflation alone.

Third, inflation is leading interest rates, i.e. the positive (negative) change in inflation leads to positive (negative) change in interest rate with a time delay of several months. This fact can also be observed on the figure 7.5 to some extent.

One of the main tasks of this thesis is to check the hypothesis formulated above and first presented in the [Müller et al., 2010] and calibrate some relevant parameters. The hypothesis we are aiming to check are:

- 1) Interest rates and inflation are weak mean-reverting processes.
- 2) Real interest rate is mean-reverting process with higher speed of mean reversion than in the case of inflation and interest rate alone.

3) Inflation leads interest rates.

## 4.2 Inflation and interest rates models

Let us denote the real interest as  $\rho_i^{adj}$  at time point  $t_i$  and define it as<sup>1</sup>

$$\rho_i^{adj}(T) = \rho_i(T) - I_i^*, \quad (4.3)$$

where  $\rho_i(T)$  is the forward interest rate at time  $t_i$  for time interval  $[T, T + \Delta T]$ , where  $\Delta T = 3$  months, and  $I_i^*$  is the seasonally adjusted inflation.

Assume, that  $\overline{\rho^{adj}}$  is the long-term target for  $\rho_i^{adj}$ , or in other words, that the real interest rate process reverts to  $\overline{\rho^{adj}}$ , which is constant over time.

After defining a long-term target for real interest rate, we can draw a conclusion about long-term targets of interest rate and inflation. The inflation and interest rate targets should be defined in a way that their difference is long-term real interest rate target. The long-term target for inflation is defined as follows

$$\overline{I_i^*} = I_i^* - \mu \cdot (\overline{\rho^{adj}} - \rho_i^{adj}), \quad (4.4)$$

and the long-term target for interest rate is

$$\overline{\rho_i} = \rho_i + (1 - \mu) \cdot (\overline{\rho^{adj}} - \rho_i^{adj}). \quad (4.5)$$

Note, that the difference between 4.5 and 4.4 is

$$\overline{\rho_i} - \overline{I_i^*} = \overline{\rho^{adj}}, \quad \text{for every } i. \quad (4.6)$$

The hypothesis that inflation is leading interest rates is modeled in the ESG by choosing  $\mu < 1 - \mu$ .

As indicated above, inflation is influenced by two mean-reverting forces: inflation mean-reversion and real interest rate mean-reversion. Its conditional expectation formula should link two mean-reverting processes: inflation weak mean-reversion and real interest rate stronger mean-reversion, i.e.

$$\mathbb{E}_{i-1} [I_i^*] = I_{i-1}^* + \eta_I (m_I - I_{i-1}^*) + \xi (\overline{I_{i-1}^*} - I_{i-1}^*), \quad (4.7)$$

where  $\eta_I$  is the speed of inflation weak mean-reversion,  $m_I$  is the long-term constant mean towards which inflation reverts driven

---

<sup>1</sup>Note, that real interest rate defined here is different from the real interest rate  $R_i^{Real}$  that is used in our analysis later in 7.5, because it is defined through forward interest rates and not spot interest rate.

by weak mean-reverting forces.

$\xi$  is the speed of real interest rate mean-reversion and

$\bar{I}_i^*$  is the target determined by real interest rate mean-reverting forces.

Similarly, interest rate conditional expectation should take into account the weak mean-reversion of interest rate itself and the real interest rate stronger mean-reversion, i.e.<sup>2</sup>

$$\mathbb{E}_{i-1} [\rho_i] = \rho_{i-1} + \eta_\rho (m_\rho - \rho_{i-1}) + \xi (\bar{\rho}_{i-1} - \rho_{i-1}), \quad (4.8)$$

where  $\eta_\rho$  is the speed of interest rate weak mean-reversion,  $m_\rho$  is the long-term constant mean towards which interest rate reverts driven by weak mean-reverting forces.  $\xi$  is the speed of real interest rate mean-reversion and  $\bar{\rho}_i$  is the target determined by real interest rate mean-reversion.

Note that

$$\mathbb{E}_{i-1} [\rho_i - I_i^*] \approx \rho_{i-1} - I_{i-1}^* + \xi (\bar{\rho}^{adj} - \rho_{i-1}^{adj}), \quad (4.9)$$

### 4.3 Conclusion. Hypothesis to be checked

In conclusion, we would like to summarize the main three hypothesis to be checked in this work.

- 1) Interest rates and inflation are weak mean-reverting processes.
- 2) Real interest rate is mean-reverting process and the speed of mean reversion is higher than in the case of inflation and interest rate alone.
- 3) Inflation predicts interest rates.

We also aim to calibrate mean-reversion parameters and understand the extent of lead-lag effects in inflation and interest rates.

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<sup>2</sup>In fact, model for interest rates is different and a simplified version of it is presented here. The original model is as follows, let  $z_i(T) = Z [\rho_i(T), \sigma_z^2(T)]$  be non-linear transformation of  $\rho_i$ , then

$\mathbb{E}_{i-1} [z_i] = z_{i-1} + \eta_z (m_z - z_{i-1}) + \xi \sqrt{\frac{m\Delta T}{T}} (Z [\bar{\rho}_{i-1}, \sigma_z^2] - z_{i-1})$ , for details refer to [Müller et al., 2010]

## Chapter 5

# Seasonality in inflation and seasonal adjustment techniques

This chapter is devoted to the seasonality of inflation time series.

The proofs of seasonality in inflation time series are presented in the first paragraph. The second paragraph contains a brief overview of the seasonal adjustment techniques to give the reader an introduction to the existing methods. The method of seasonal adjustment used in the ESG together with its strong and weak sides is discussed in the third paragraph. The last paragraph of this chapter describes our improvement of the method via more effective seasonal adjustment technique. The fifth paragraph provides the reader with the definition of the moving average operator employed in the improved method of deseasonalization . For more information on this topic see [Dacorogna et al., 2001]. The X-12-ARIMA method of seasonal adjustment developed by the U.S. Census Bureau and employed by several National Statistical Bureaus for the seasonal adjustment is discussed in the fourth paragraph of this chapter.

### 5.1 Seasonality

It is well recognized that inflation is subject to cyclical movements. It is normally higher in winter and spring and lower in summer and autumn. For example, more fresh vegetables and fruits are available in summer and autumn lowering the prices of the food. On the other hand, heating season in winter pushes the prices of fuel up. These cyclical effects lead to seasonality. While being an important fact, seasonality can seriously distort the analysis of the long-term inflation behavior. In order to catch important dependencies and trends in the time series evolution the inflation time series needs to be first seasonally adjusted, i.e. the seasonal components of the

time series have to be eliminated. To observe that inflation is subject to seasonal variations and to study its seasonal pattern we use autocorrelation plot for the inflation of several data series. If there are significant periodic autocorrelations this will tell us that data series exhibit seasonal variations.

**Definition 5.1.1.** The *autocorrelation* is defined as a correlation of the time series with itself over successive time lags. Let  $x_t$  be a time series,  $x_{t+\tau}$  time series shifted by the time  $\tau$ , and  $\bar{x}$  and  $\sigma$  be the time series mean and standard deviation respectively. Then the autocorrelation with the lag  $\tau$  is defined as

$$\text{Autocorr}(\tau) = \frac{\mathbb{E}(x_t - \bar{x}) \cdot \mathbb{E}(x_{t+\tau} - \bar{x})}{\sigma^2} \quad (5.1)$$

**Definition 5.1.1.** The *autocorrelation plot* is the plot of the time series autocorrelations over successive time lags.

The autocorrelation plots of the USA inflation, the UK inflation, the Eurozone inflation, Japanese inflation and Swiss inflation are presented in figure 5.1. The underlying CPI time series are from April 1991.

For every monthly time series we can observe significant autocorrelation at the 12<sup>th</sup> lag. This indicates the existence of one year cycle. It means that inflation tends to rise (or fall) in the same months of the year. There are other specific effects which differ from the one time series to the other.

The USA inflation demonstrates significant autocorrelation at 1 and 11 months lag in addition to 12<sup>th</sup>. The British, Eurozone, Swiss inflation autocorrelation plots exhibit significant half of the year autocorrelations. In addition to this, Eurozone inflation shows negative 4 and 8 months autocorrelations, while Swiss and British inflation demonstrates negative 3 and 9 months autocorrelations. Japanese inflation autocorrelation plot has negative values for the 2<sup>nd</sup>, 3<sup>rd</sup>, 9<sup>th</sup> and 10<sup>th</sup> lags.

We can see some common and uncommon cyclical behaviors amongst the different countries which suggest that the seasonal adjustment techniques chosen should be able to deal with the different cyclical patterns. The difference in the seasonal patterns can be explained by different consumer behaviors as well as by different methods of evaluation of the CPI used by National Statistical Bureaus.

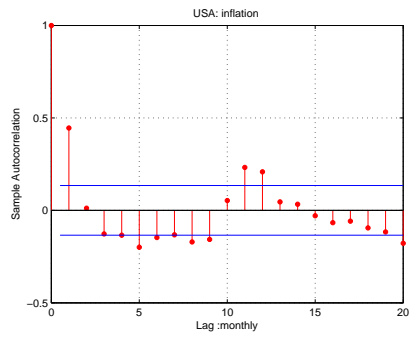
The findings described below will further prove the presence of seasonality in inflation data. We conduct a statistical test to identify whether the samples consisting of the observations from different quarters of the year have the same distribution.

**Definition 5.1.1.** The two-sample Kolmogorov-Smirnov test is used to compare the empirical distributions of two samples  $X_1, X_2$  to test the following hypothesis:

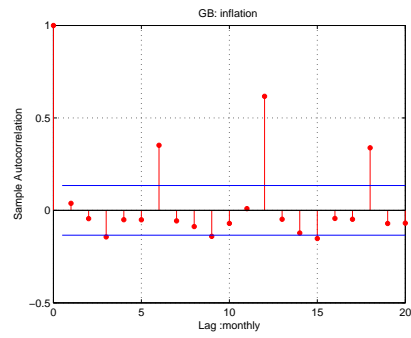
$H_0$ :  $X_1$  and  $X_2$  are drawn from the same continuous distribution.

$H_1$ :  $X_1$  and  $X_2$  are drawn from different continuous distributions.

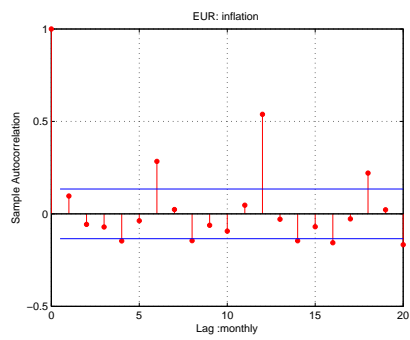




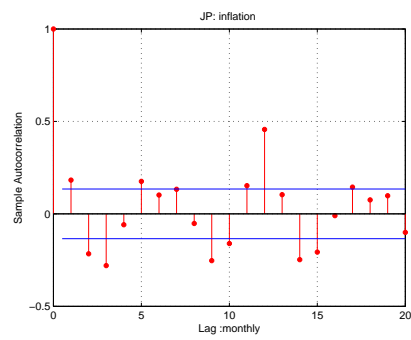
(a) USA



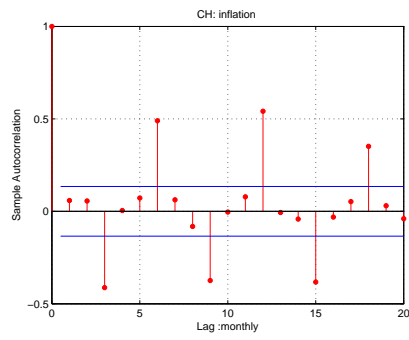
(b) UK



(c) the Eurozone



(d) Japan



(e) Switzerland

Figure 5.1: Autocorrelation plots of inflation.

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.00	0.00	0.00	0.44	0.00	0.00
K.-S. statistic	0.41	0.40	0.74	0.16	0.41	0.4
UK						
p-value	0.01	0.92	0.04	0.02	0.00	0.03
K.-S. statistic	0.31	0.10	0.26	0.28	0.42	0.27
Euro zone						
p-value	0.29	0.00	0.00	0.07	0.02	0.70
K.-S. statistic	0.20	0.39	0.35	0.24	0.29	0.13
Japan						
p-value	0.01	0.15	0.72	0.72	0.01	0.03
K.-S. statistic	0.29	0.21	0.13	0.13	0.31	0.27
Switzerland						
p-value	0.60	0.15	0.01	0.04	0.00	0.43
K.-S. statistic	0.14	0.21	0.30	0.26	0.35	0.16

Table 5.1: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonal inflation.

We form 4 samples of inflation data for each country:

- the first sample comprises the observations coming from the first quarter of the year,
- the second sample - from the second quarter,
- the third and the fourth samples contain the observations of inflation from the third and fourth quarters of the year respectively.

We chose quarterly samples rather than monthly because of the quarterly character of seasonality and the possibility to get more reliable statistical results with bigger samples. The two-sample Kolmogorov-Smirnov test is performed on these four data samples.

It is clear from the table 5.1 that at least three out of six tests reject the null hypothesis at the 5% significance level for every country. Significant repeating autocorrelations together with the results of the Kolmogorov-Smirnov test prove the existence of seasonal pattern in inflation time series and show the need for efficient seasonal adjustment technique.

## 5.2 Seasonal adjustment techniques overview

Seasonality is usually considered to be an undesired feature of the time series which can seriously distort the analysis. Seasonal adjustment is the process

of identifying and removing seasonal components from the time series. Several seasonal adjustment techniques have been developed until now. We are going to discuss some of them.

The easiest way to perform seasonal adjustments of the series with serial dependency in lag  $j$  is to build  $j$  subseries  $X_{k+(i-1)*j}$ ,  $k = 1, \dots, j$ ,  $i = 1, 2, \dots$ , then subtract from the subseries their own mean and add back the mean of the whole series, i.e.

$$X_{k+(i-1)*j}^{SA} = X_{k+(i-1)*j} - \overline{X_k} + \overline{X}, \quad (5.2)$$

where  $X^{SA}$  is the seasonally adjusted series,  $\overline{X_k}$  is the mean of the  $k^{th}$  subseries and  $\overline{X}$  is the mean of the whole series  $X$ .

If a subseries has higher (lower) mean than the rest of the series, this excess (resp. lack) is removed from it, thus getting rid of seasonal raise (resp. fall). This method does not perform so well if seasonal excess or lack changes over time. It can be improved by allowing the mean of the subseries and the series to vary with time, thus suggesting to substitute the arithmetic averages by moving averages.

Another method of seasonal adjustment is based on the use of dummy variables. Dummy variable for the seasonal component can be used in case of the latter being constant:

$$Y_t = \alpha + \beta X_t + \gamma_1 D_t^1 + \gamma_2 D_t^2 + \dots + \gamma_{j-1} D_t^{j-1}, \quad (5.3)$$

where  $X_t$  is the explanatory variable and  $D_t^k$ ,  $k = 1, \dots, j-1$  is the seasonal dummy variable which is 1 if  $t$  is multiple of  $k$  and 0 otherwise.

Another big class of seasonal-adjustment procedures is based on ARIMA (Autoregressive Integrated Moving Average) models. The general form of the  $ARIMA(p, d, q)(P, D, Q)$  model is given below:

$$\phi(B) \Phi(B^s) (1 - B^d) (1 - B^D) x_t = \theta(B) \Theta(B^s) a_t, \quad (5.4)$$

where  $x_t$  is the time series,  $B$  is the backshift operator, i.e.  $Bx_t = x_{t-1}$ ,  $s$  is the seasonal period,  $\phi(B) = (1 - \phi_1 - \dots - \phi_p B^p)$  is the nonseasonal autoregressive operator,  $\Phi(B^s) = (1 - \Phi_1 B^s - \dots - \Phi_P B^{Ps})$  is the seasonal autoregressive operator,  $\theta(B) = (1 - \theta_1 B - \dots - \theta_q B^q)$  is the nonseasonal moving average operator,  $\Theta(B^s) = (1 - \Theta_1 B^s - \dots - \Theta_Q B^{Qs})$  is the seasonal moving average operator and  $a_t \sim N(0, \sigma^2)$ .

We will not provide more details for ARIMA based seasonal adjustment techniques because they were not considered to be suitable for the ESG model due to the complexity of their implementation.

### 5.3 Seasonal adjustment method used in the ESG

A simple seasonal adjustment of inflation is implemented in the ESG. The whole sample of seasonal inflation  $I^{original}$  is split into four quarterly samples  $I^h$ ,  $h = \overline{1,4}$  (the ESG uses quarterly inflation series), then the seasonal adjustment is performed by deducting the mean of the quarter and adding back the mean of the whole sample.

Since we use monthly rather than quarterly samples for our analysis, an extension of the method described above is applied to our monthly inflation series. The seasonally adjusted inflation  $I^*$  is

$$I_k^{*h} = I_{h+12*(k-1)}^{original} - \overline{I^h} + \overline{I^{original}}, \quad (5.5)$$
$$h = 1, \dots, 12, \quad k = 1, \dots, N^h,$$

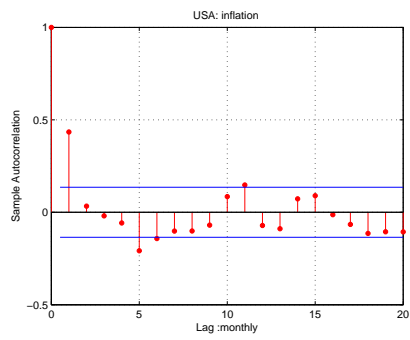
where  $\overline{I^h}$  is the mean of the  $I^h$  subsample,  $\overline{I^{original}}$  is the mean of the original sample of inflation and  $N^h$  is the number of elements in the  $h^{th}$  subsample. This method, while being computationally efficient, does not solve the task of the deseasonalization entirely. The autocorrelation graphs 5.2 of the seasonally adjusted series together with the two-sample Kolmogorov-Smirnov test of figure 5.2 prove the insufficiency of the applied technique. The analysis shows significant autocorrelations at the 12<sup>th</sup> lag in the U.K. inflation, at the 6<sup>th</sup> and 12<sup>th</sup> lags in the Eurozone inflation and at the 3<sup>rd</sup>, 6<sup>th</sup>, 9<sup>th</sup> and 12<sup>th</sup> lags in the Swiss inflation. Some of the Kolmogorov-Smirnov tests reject the null hypothesis at the 5% significance level (see Japanese and American data). Not all the seasonal components were removed from the data. It can be partially explained by the dynamic nature of the seasonal component of the series.

We were seeking to improve the existing technique or to find an alternative one. The idea for an improvement was to use moving averages instead of arithmetic averages in the eq. 5.5 for the purpose of taking into account changing seasonal component.

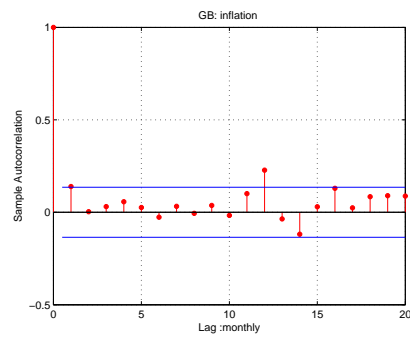
### 5.4 X-12-ARIMA model

The X-12-ARIMA was used as alternative technique for seasonal adjustment in order to benchmark the results and the efficiency of our deseasonalization methods.

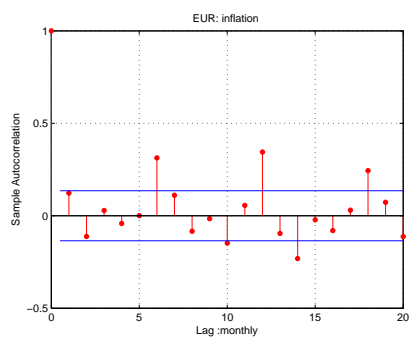
The X-12-ARIMA is the software developed by the U.S. Census Bureau. It is used for all official seasonal adjustments produced by the Census Bureau. The X-12-ARIMA software can be used for forecasting seasonal series as well as for identifying seasonal, irregular and trend components of the data



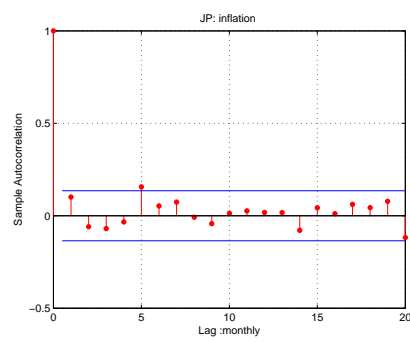
(a) USA



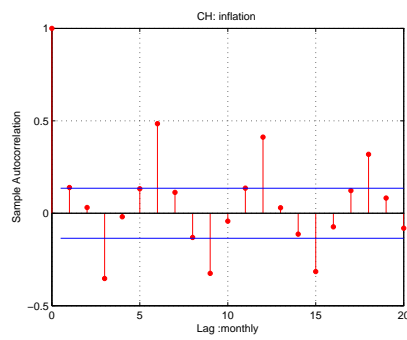
(b) UK



(c) the Eurozone



(d) Japan



(e) Switzerland

Figure 5.2: Autocorrelation plots of seasonally adjusted inflation. The adjustment was performed with the ESG seasonal adjustment technique.

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.82	0.36	0.00	0.16	0.00	0.01
K.-S. statistic	0.12	0.17	0.33	0.20	0.35	0.30
UK						
p-value	0.76	0.76	0.88	0.80	0.31	0.81
K.-S. statistic	0.13	0.13	0.11	0.12	0.18	0.12
Eurozone						
p-value	0.52	0.78	0.81	0.76	0.88	0.97
K.-S. statistic	0.17	0.14	0.14	0.14	0.13	0.11
Japan						
p-value	0.00	0.14	0.04	0.66	0.31	0.17
K.-S. statistic	0.33	0.21	0.26	0.13	0.18	0.21
Switzerland						
p-value	0.64	0.50	0.72	0.48	0.88	0.24
K.-S. statistic	0.14	0.15	0.13	0.16	0.11	0.19

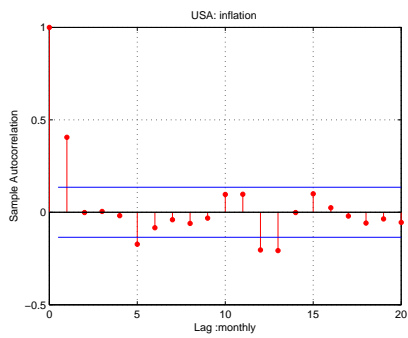
Table 5.2: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted with the ESG method inflation.

series. The X-12-ARIMA model also performs all the necessary statistical tests to control for the adequacy of the model fit. The Census Bureau uses the ARIMA( $p, d, q$ ) ( $P, D, Q$ ) (see eq. 5.4 for details) model as the basis for their seasonal adjustment and forecasting software. For the details refer to the X-12-ARIMA Reference Manual<sup>1</sup>.

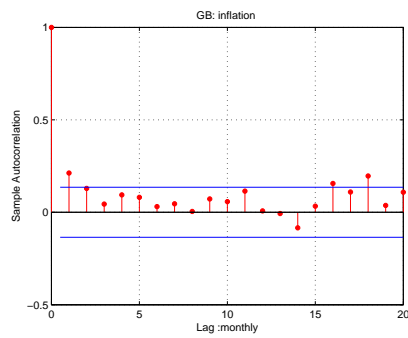
We use the X-12-ARIMA software as a benchmark to assess the performance of our seasonal adjustment techniques. We applied the X-12-ARIMA to our five series of inflation. The autocorrelation graphs and the Kolmogorov-Smirnov test results for the deseasonalized series are shown in figure 5.3 and table 5.3 respectively.

We can see that the X-12-ARIMA model was able to remove the significant positive 12<sup>th</sup> lag autocorrelation for all the 5 samples. The 1<sup>st</sup> lag autocorrelation of the U.S. inflation still remains and we can also see the overcorrection effect in the significant negative 13<sup>th</sup> and 14<sup>th</sup> lag autocorrelations which did not exist in the original sample. The significant 6<sup>th</sup> lag autocorrelation of the British inflation was removed together with the negative 3<sup>rd</sup> and 9<sup>th</sup> lag autocorrelations. The Eurozone seasonal 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> lag autocorrelations were eliminated. At the same time the 1<sup>st</sup> lag autocorrelation (around 0.2) was introduced into the British and the Eurozone samples. The autocorrelation function graph of Japanese inflation shows that the Japanese seasonal components were removed. The Swiss inflation

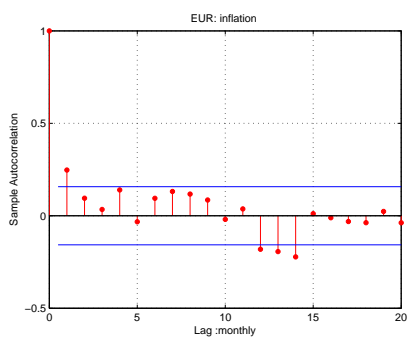
<sup>1</sup><http://www.census.gov/ts/x12a/final/temp/x12adocV03.pdf>



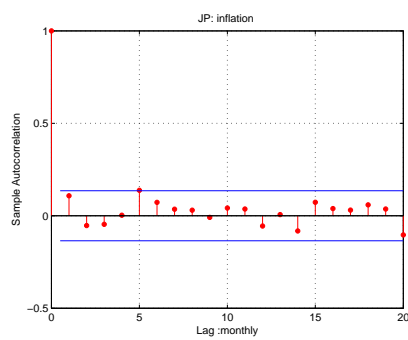
(a) USA



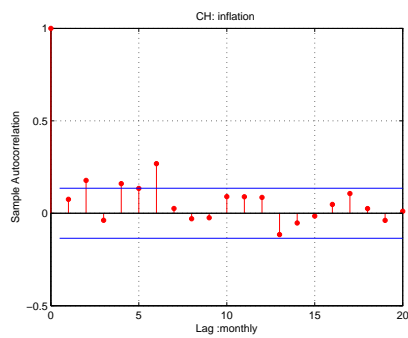
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 5.3: Autocorrelation plots of seasonally adjusted inflation. The adjustment was performed with the X-12-ARIMA software.

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.51	0.94	0.19	0.46	0.23	0.78
K.-S. statistic	0.15	0.10	0.20	0.16	0.19	0.12
UK						
p-value	0.56	0.68	0.56	0.99	0.24	0.37
K.-S. statistic	0.15	0.13	0.15	0.08	0.19	0.17
Eurozone						
p-value	0.33	0.33	1.00	0.73	0.47	0.67
K.-S. statistic	0.20	0.20	0.09	0.15	0.18	0.16
Japan						
p-value	0.38	0.61	0.07	0.69	0.52	0.82
K.-S. statistic	0.17	0.14	0.24	0.13	0.15	0.12
Switzerland						
p-value	0.34	0.37	0.41	0.50	0.29	0.70
K.-S. statistic	0.17	0.17	0.17	0.15	0.18	0.13

Table 5.3: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted with the X-12-ARIMA inflation.

still exhibits the significant 6<sup>th</sup> lag autocorrelation as the original data.

The two-sample Kolmogorov-Smirnov tests performed on the seasonally adjusted data showed that  $H_0$  was not rejected at the 5% significance level in all the cases, which means that after X-12-ARIMA adjustment there is no statistical evidence that quarterly samples come from different distributions.

In conclusion, we can say that the X-12-ARIMA model managed to remove most of the seasonal components of the data.

However, X-12-ARIMA is the product of the U.S. Census Bureau and cannot be used for the seasonal adjustment in the ESG because of several reasons. First of all, there is very limited way to influence and control the process, second, the ESG does not need such a universal tool for seasonal adjustment and by implementing something simpler, a higher computational efficiency could be achieved.

The X-12-ARIMA fit the following  $ARIMA(p, d, q)(P, D, Q)$  models (see eq. 5.4) to our data:  $ARIMA(0, 0, 1)(0, 1, 1)$  to the U.S. inflation,  $ARIMA(0, 0, 0)(0, 1, 1)$  to the U.K.inflation,  $ARIMA(0, 0, 0)(0, 1, 1)$  to the Eurozone inflation,  $ARIMA(0, 0, 0)(0, 1, 1)$  to the Japanese inflation, and  $ARIMA(0, 0, 3)(0, 1, 1)$  to the Swiss inflation. We notice that there are no autoregressive terms in the models and, as a result, we decide to employ Moving Average in order to perform seasonal adjustment. The definition of the chosen Moving Average operator and some related theory is given below in paragraph 5.5.



## 5.5 Moving Average operator introduction

### 5.5.1 Some important facts about operators

2

Let us consider an operator  $\Omega$  with kernel  $K(t)$ , defined as

$$\Omega[I](t) = \int_{-\infty}^t K(t-t^*) \cdot I(t^*) dt^* = \int_0^{\infty} K(t^*) \cdot I(t-t^*) dt^*, \quad (5.6)$$

where  $I(t)$  is a continuous function of time.

An *average operator* has a kernel which is nonnegative and normalized to unity, i.e.

$$\int_0^{\infty} K(t) dt = 1, \quad K(t) > 0. \quad (5.7)$$

The *range* of the operator is defined in the following way:

$$R[\Omega] = \int_0^{\infty} K(t) t dt. \quad (5.8)$$

The *width* of the operator is defined as

$$\omega^2[\Omega] = \int_0^{\infty} K(t) (t - R[\Omega])^2 dt. \quad (5.9)$$

The *Exponential Moving Average* operators have an exponentially decaying kernel.

The evaluation of the operator at initial time  $T$  is subject to an *initialization error*. The *build-up time interval* is the time which passes before the initialization error is less or equal to a given value  $\epsilon$ .

Assume, the process  $I(t)$  is known since time  $-T$  and modeled before as a random walk with no drift,

$$\Omega[I](t) = \int_{-T}^t K(t-t^*) \cdot I(t^*) dt^*. \quad (5.10)$$

The average build-up error  $\epsilon$  at time  $t = 0$  is given by

$$\epsilon^2 = \mathbb{E} \left[ \left( \int_{-\infty}^{-T} K(-t^*) I(t^*) dt^* \right)^2 \right]. \quad (5.11)$$

For a given build-up error  $\epsilon$  this is the implicit definition of the build-up time  $T$ .

---

<sup>2</sup>The original source of the theory on Moving Average operator is [Dacorogna et al., 2001].

Assuming that  $I$  has a constant volatility  $\sigma$ , after some transformations (refer for details to [Dacorogna et al., 2001]) we arrive to the following formula for  $\epsilon$ ,

$$\epsilon^2 = 2\sigma \cdot \int_T^\infty K(t) dt \int_T^t K(t^*) I(t^* - T) dt^*. \quad (5.12)$$

The build-up interval  $T$  should be inferred from this equation. In most of the times, the analytical solution of the equation 5.12 does not exist. However, there is a simple rule of thumb: the heavier the tail of the kernel the longer the build-up time is.

As a measure of the tail we can take the *aspect ratio*

$$AR[\Omega] = \frac{(\int_0^\infty K(t) \cdot t^2 dt)^{1/2}}{\int_0^\infty K(t) \cdot t dt}. \quad (5.13)$$

### 5.5.2 EMA, iterated EMA and MA operators

Assume that we have a time series  $I_k$ ,  $k = 1, \dots, N$  of length  $N$ . The basic Exponential Moving Average model (referred as EMA) is the averaging operator with an exponentially decaying kernel:

$$K_{EMA}(k; n) = \frac{e^{-k/n}}{n}, \quad k = 1, \dots, N, \quad (5.14)$$

where  $n$  is the range of the operator.

The exponential form of the kernel leads to the efficient discrete evaluation with the following simple iterative formula

$$EMA_1(k; n) = \mu \cdot EMA_1(k-1; n) + (1-\mu) \cdot I_k, \quad k = 2, \dots, N, \quad (5.15)$$

where  $\mu = e^{-1/n}$ .

With the above formula, the convolution can be calculated in a computationally efficient way. The simple EMA has to be initialized with the initial value

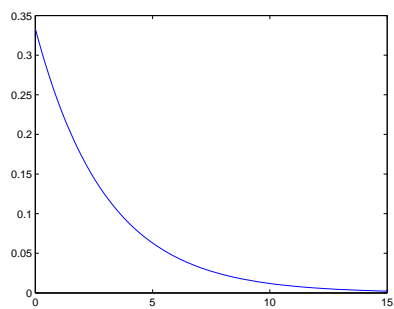
$$EMA_1(1; n) = EMA_{INIT}. \quad (5.16)$$

The range and the aspect ratio of the simple EMA are

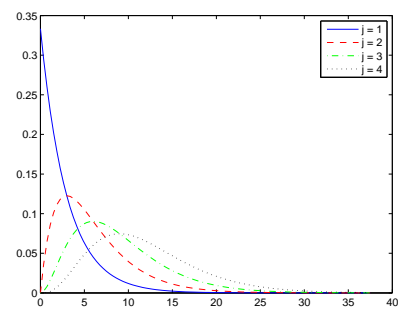
$$R_{EMA} = n, \quad (5.17)$$

and

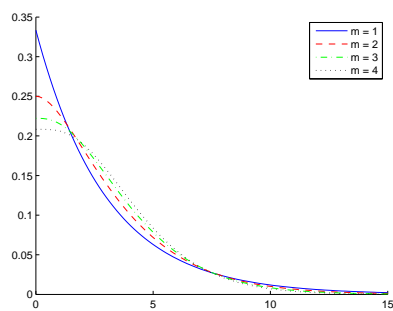
$$AR_{EMA} = \sqrt{2}. \quad (5.18)$$



(a) EMA operator kernel.



(b) Iterated EMA operator kernel.



(c) Moving Average operator kernel.

Figure 5.4: Exponentially decaying kernels of EMA, Iterated EMA and MA operators. The range is equal to 3 for EMA and all of the MA operators. For iterated EMA the range is equal to 3, 6, 9, 12 for  $j=1, 2, 3, 4$  respectively.

The build-up time of the simple EMA is

$$\frac{T}{n} = -\ln \epsilon + \frac{1}{2} \ln \left( \frac{\sigma}{2} \cdot n \right), \quad (5.19)$$

where  $\sigma$  is the volatility of the process  $I_k$ .

The basic EMA operator is iterated to provide for a family of iterated operators with kernel  $K_{EMAI}$  and a shorter build-up time

$$EMA_j(k; n) = \mu EMA_j(k-1; n) + (1-\mu) EMA_{j-1}(k; n), \quad (5.20)$$

$$k = 2, \dots, N; \quad j = 1, 2, \dots. \quad (5.21)$$

The kernel of the iterated EMA is

$$K_{EMAI}^j(k; n) = \frac{1}{(j-1)!} \left( \frac{k}{n} \right)^{j-1} \frac{e^{-k/n}}{n}. \quad (5.22)$$

The iterated EMA operator has the following characteristics:

$$R_{EMAI}^j = j \cdot n, \quad (5.23)$$

$$\omega_{EMAI}^j = j \cdot n^2, \quad (5.24)$$

$$AR_{EMAI}^j = \sqrt{\frac{j+1}{j}}. \quad (5.25)$$

The iterated EMA with the range  $j \cdot n$  has a shorter build-up time than the simple EMA with the range  $n$ . It is proved by the aspect ratio, which is close to 1 for the large enough  $j$ .

We need to construct the operator which has the shorter build-up time than the simple EMA and at the same time shorter range than the iterated EMA operator.

The Moving Average (MA) operator that is used for seasonal adjustment has shorter build-up time than the simple EMA and the same range as the simple EMA

$$MA^m(k; n) = \frac{1}{m} \sum_{j=1}^m EMA_j(k; n^*), \quad m \in \mathbb{Z}, \quad (5.26)$$

$$n^* = \frac{2n}{m+1}, \quad (5.27)$$

$$\mu = e^{-1/n}. \quad (5.28)$$

The parameter  $n^*$  is chosen so that the range of  $MA^m$  is  $n$  independent of  $m$ ,

$$R_{MA} = n. \quad (5.29)$$

MA is a family of rectangular-shaped kernels with the relative weight of the distant path controlled by  $m$

$$K_{MA^m}(k; n) = \frac{m+1}{m} \cdot \frac{e^{-k/n^*}}{2n} \sum_{j=0}^{m-1} \frac{1}{j!} \left(\frac{k}{n^*}\right)^j, \quad k = 1, \dots, N. \quad (5.30)$$

The *aspect ratio* of the MA operator is

$$AR_{MA}^m = \sqrt{\frac{4(m+2)}{3(m+1)}} \longrightarrow \frac{2}{\sqrt{3}}. \quad (5.31)$$

## 5.6 Improved method of seasonal adjustment

The Moving Average operator is used to deseasonalize the time series of inflation. The method of deseasonalization is described below in detail.

Monthly observations of inflation start from 05.1991 until 07.2009 which gives  $N = 219$  observations of inflation  $I^{original}$  in total. We split the sample into 12 samples according to the month of the year in the following way:

$$I_k^h = I_{h+12*(k-1)}^{original}, \quad (5.32)$$

$$h = 1, \dots, 12, \quad k = 1, \dots, N^h,$$

where  $N^h$  is the number of observations in  $h^{th}$  subsample.

We have 12 samples of, in average, 18 observations and we estimate the moving average  $MA_h^m$  for each of the 12 samples. The initial value is set to the mean over a number of years equal to a given range  $n$ , i.e.

$$EMA_1^h(1; n) = \frac{1}{n} \sum_{j=1}^n I_j^h. \quad (5.33)$$

In order to obtain seasonally adjusted inflation  $I^{*h}$  we subtract the moving average of the monthly sample  $MA_h^m$  with the range  $n$  from the original inflation sample, thus making a seasonal adjustment, and add back the moving average of the whole sample  $MA^m$  with the range  $n \cdot 12$  to preserve non-seasonal information. The ranges of  $MA_h^m$  and  $MA^m$  are chosen so that two time intervals over which averaging is performed coincide. The ranges  $n$  and  $n \cdot 12$  of  $MA_h^m$  and  $MA^m$  correspond to the same time interval length. The adjusted inflation  $I^{*h}$  is calculated with the following formula:

$$I_k^{*h} = I_k^h - MA_h^m(k; n) + MA^m(k; 12 \cdot n), \quad (5.34)$$

$$h = 1, \dots, 12, \quad k = 1, \dots, N^h.$$

Finally, we build one sample of seasonally adjusted inflation  $I^*$  from twelve samples of seasonally adjusted inflation  $I^{*h}$  as follows:

$$I_{h+12(k-1)}^* = I_k^{*h}, \quad (5.35)$$

$$h = 1, \dots, 12, \quad k = 1, \dots, N^h.$$

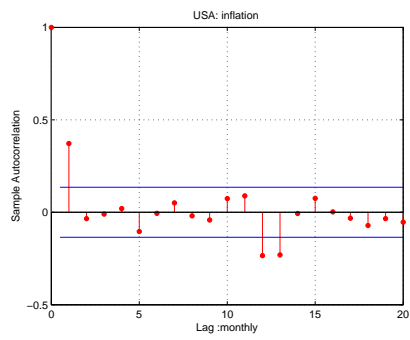
The moving average operator has two parameters:  $m$ , determining flatness of the kernel, and  $n$ , determining the range of the kernel. These parameters have a big impact on the resulting series and their correct estimation is a crucial task of the thesis. We chose the autocorrelation function (ACF) and two-sample Kolmogorov-Smirnov test as criteria for the parameter evaluation. We are aiming to pick one set of parameters for the model, such that we could perform the deseasonalization for all the inflation series with the same  $m$  and  $n$ . We would rather sacrifice very good performance of a set of parameters for one particular series if we can choose another set which suits on average better for all the series.

The outcomes of the tests are presented in figure 5.5 and table 5.4.

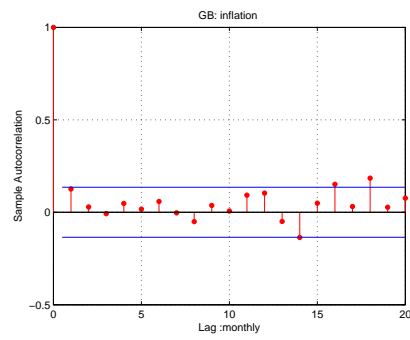
The performance of the method using Moving Averages with the parameters  $m = 4$ ,  $n = 3$  is very similar to the performance of the X-12-ARIMA method. The U.S. inflation ACF shows an overcorrection effect by introducing negative 12<sup>th</sup> and 13<sup>th</sup> lag autocorrelations. The significant autocorrelations demonstrated by the seasonal British, Eurozone and Japanese inflation series ACF graphs are removed by the method. The 3<sup>rd</sup> and 6<sup>th</sup> lag significant autocorrelations indicate the presence of residual seasonality in the Swiss inflation data. The same type of behavior was demonstrated by the same time series deseasonalized with X-12-ARIMA model (the difference is that the 3<sup>rd</sup> lag autocorrelation was not significant in the case of the X-12-ARIMA.) The 1<sup>st</sup> lag autocorrelation is introduced in the Eurozone inflation data, similar to X-12-ARIMA deseasonalization.

The null hypothesis of the two-sample Kolmogorov-Smirnov test was rejected at 5% level only for the U.S. 2<sup>nd</sup> and 4<sup>th</sup> quarters samples.

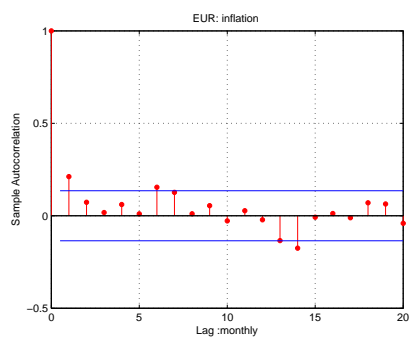
The results of the Kolmogorov-Smirnov test and ACF led to the choice of  $m = 4$ ,  $n = 3$ . The results of the tests for  $m = 4, 8$  and  $N = 2, 4$  are presented in the appendix A.2.



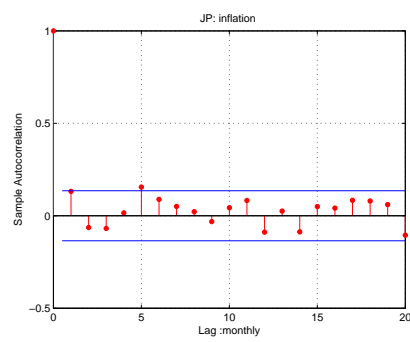
(a) USA



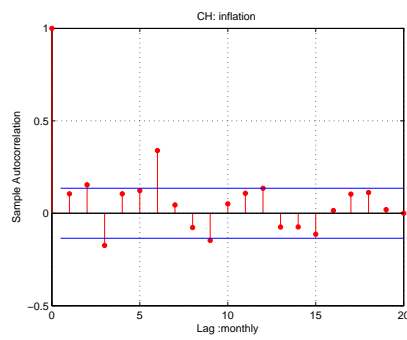
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 5.5: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^4(\cdot; 3)$ ,  $m=4$ ,  $n=3$ .

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.50	0.32	0.07	0.34	0.02	0.60
K.S. statistic	0.15	0.18	0.24	0.17	0.28	0.14
UK						
p-value	0.37	0.60	0.28	0.86	0.73	0.64
K.S. statistic	0.11	0.09	0.14	0.13	0.17	0.12
Eurozone						
p-value	0.20	0.85	0.56	0.48	0.52	0.33
K.S. statistic	0.20	0.11	0.15	0.16	0.15	0.18
Japan						
p-value	0.60	0.43	0.19	0.98	0.65	0.52
K.S. statistic	0.14	0.16	0.20	0.09	0.14	0.15
Switzerland						
p-value	0.09	0.46	0.72	0.54	0.08	0.36
K.S. statistic	0.23	0.16	0.13	0.15	0.24	0.17

Table 5.4: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation deseasonalized with  $MA^4(\cdot; 3)$ , ( $m=4$ ,  $n=3$ ).



## Chapter 6

# Cross-correlation analysis of interest rates and inflation

**Cross-correlation 1.** The cross-correlation is a lagged correlation of two time series. Let  $x_t$  and  $y_t$  be time series,  $y_{t-\tau}$  time series shifted by the time  $\tau$ , then the cross-correlation is a function of the lag  $\tau$ , defined as

$$CCorr(\tau) = Corr(x_t, y_{t-\tau}), \quad \tau = 0, \pm 1, \pm 2, \dots, \pm l. \quad (6.1)$$

Cross-correlation is also called lead-lag correlation.

We study the cross-correlation function of interest rates and inflation in order to determine whether there is a casual relation between them. Asymmetry of the cross correlation function around the zero lag suggests that one time series predicts or leads the other time series. If  $CCorr(\tau) > CCorr(-\tau)$ , then the first series ( $x_t$ ) leads the second series ( $y_t$ ), if  $CCorr(\tau) < CCorr(-\tau)$  then the second series ( $y_t$ ) leads the first series ( $x_t$ ).

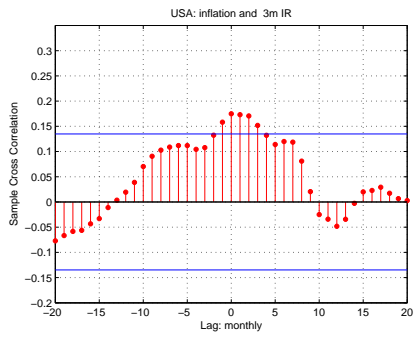
Inflation in our study is the annualized logarithmic seasonally adjusted inflation as defined in 2.1 and interest rate is the logarithmic interest rate 3.1. The graphs of the cross-correlation function of the 3 months interest rates and inflation are presented in figure 6.1. The interest rate data series being analyzed are taken from Bloomberg (data also used in the ESG). The U.S. and British data are from April 1991, Eurozone is from December 1998, Japanese data is from September 1992 and Swiss data are from February 1994.

The asymmetry of the cross-correlation function<sup>1</sup> provides evidence that the British inflation leads British 3 month interest rates, the same applies to the Eurozone interest rates and inflation. U.S., Swiss and Japanese inflation and interest rates do not show any lead-lag effects.

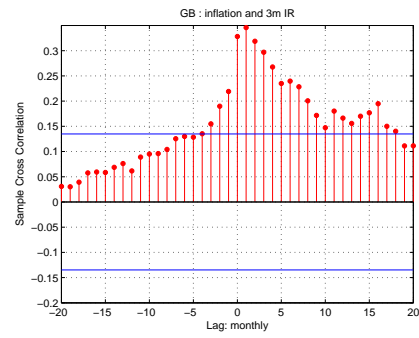
We should underline that the asymmetry of the cross-correlation function

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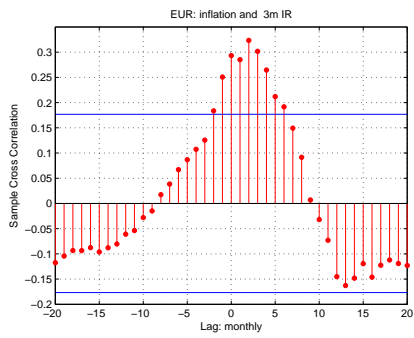
<sup>1</sup>Refer to [Dacorogna et al., 2001] for the details on interpreting lead-lag effects with cross-correlation function.



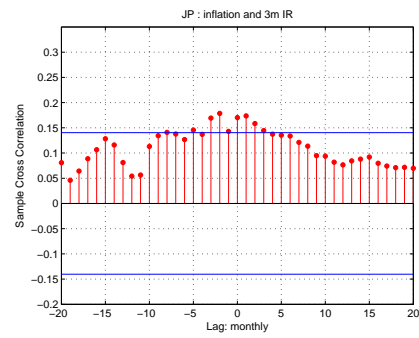
(a) USA



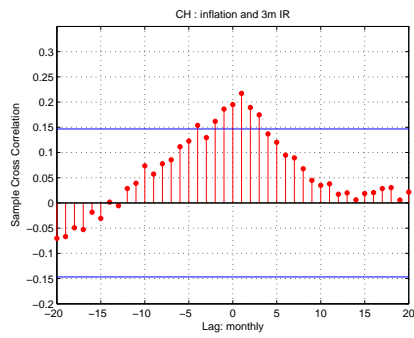
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 6.1: Cross-correlation of inflation and 3 months interest rates.

doesn't hold for the period longer than 20 years. If we perform the same test on the data series longer than described above the cross-correlation function becomes nearly symmetrical. It means that that lead-lag effect doesn't hold for longer periods.

The causality between interest rates and inflation is still an open question in economical science. There is a work [Fama, 1975] which shows that for 1953-71 period of time the nominal interest rate summarize all the information about future inflation rates which contradicts the result obtained here. We should draw the attention of the reader that the causality between interest rates and inflation is not stable. We also have to remember that CPI is not a market proxy for inflation but a statistical measure which cannot provide the consistency of results across all the periods of time.

## Chapter 7

# Mean-reversion model

In this chapter we study the mean-reverting property of interest rates, inflation and real interest rates. The mean-reverting property suggests the presence of a long-term average towards which the process tends to revert whenever it has significantly deviated from it.

A mean-reversion property of interest rates and inflation is suggested by economic intuition. Inflation and interest rates take values in some predefined range. Interest rate cannot reach either negative values or zero. Inflation can be negative but normally it does not reach too big negative values (e.g. -10%) and tends to stay at some positive level, i.e. 2%-10%. It means that the underlying process does not develop independently of its previous state and should have some kind of memory indicating the direction of the development. Whenever the value of the process is too low it should tend to increase and whenever it is too high it should tend to decrease. This type of behavior can be reproduced by the mean-reversion models.

We will consider the Ornstein-Uhlenbeck model for our analysis. In the first paragraph the description of this model is given. The switch from monthly to annual inflation is explained in the second section. The results of the Ornstein-Uhlenbeck model fit to interest rates, inflation and real interest rates are discussed in the third, fourth and fifth paragraphs respectively. Finally, the application of our analysis to the ESG is given in the last paragraph of this chapter.

### 7.1 Ornstein-Uhlenbeck model

#### 7.1.1 Description of the Ornstein-Uhlenbeck model

<sup>1</sup> The Ornstein-Uhlenbeck model is a mean-reversion stochastic model, given by

$$dx = \eta \cdot (m - x) dt + \sigma dW_t, \quad (7.1)$$

---

<sup>1</sup>more information on this topic can be found in [Dixit and Pindyck, 1994]

where  $m$  is the mean towards which the process  $x_t$  reverts,  $\eta$  is the speed of mean-reversion (the higher  $\eta$  the faster process comes back to its average level),  $\sigma$  is the process volatility and  $W_t \sim N(0, t)$ .

The integral representation yields

$$x(T) = x(0)e^{-\eta T} + m(1 - e^{-\eta T}) + \sigma e^{-\eta T} \int_0^T e^{\eta t} dW_t. \quad (7.2)$$

Taking the expectation of (7.2) we get the following result for the  $\mathbb{E}[x(T)]$

$$\mathbb{E}[x(T)] = x(0)e^{-\eta T} + m(1 - e^{-\eta T}), \quad (7.3)$$

and the long-term mean

$$\lim_{T \rightarrow \infty} \mathbb{E}[x(T)] \rightarrow m.$$

We obtain the following expression for variance by integrating the square of the stochastic term in (7.2)

$$\text{Var}[x(T)] = (1 - e^{-2\eta T}) \frac{\sigma^2}{2\eta}. \quad (7.4)$$

The long-term variance limit of the distribution of  $x_t$  is found by taking the limit of (7.4)

$$\lim_{T \rightarrow \infty} \text{Var}[x(T)] \rightarrow \frac{\sigma^2}{2\eta}. \quad (7.5)$$

The variance is bounded and  $x_t$  has a stationary distribution, i.e.  $x_t \sim N\left(m, \frac{\sigma^2}{2\eta}\right)$ .

The area of our interest is the calibration of the model 7.1 to the time series of interest rates and inflation. In order to estimate the parameters  $m$ ,  $\eta$  and  $\sigma$  we need to get the discrete-time representation of the (7.1) model.

If  $dt \rightarrow 0$  then (7.1) can be rewritten as

$$dx = m(1 - e^{-\eta dt}) - x(1 - e^{-\eta dt}) + \sigma dW_t. \quad (7.6)$$

The equation (7.6) is the continuous case of the AR(1) process

$$x_t - x_{t-1} = m(1 - e^{-\eta \Delta t}) - x_{t-1}(1 - e^{-\eta \Delta t}) + \epsilon_t, \quad (7.7)$$

where  $\epsilon \sim N(0, \sigma_\epsilon^2)$

The (7.7) is reduced to the following linear regression

$$x_t - x_{t-1} = a + bx_{t-1} + \epsilon_t. \quad (7.8)$$

The parameters  $m$ ,  $\eta$  and  $\sigma$  of (7.1) can be obtained knowing  $a$ ,  $b$  and  $\sigma_\epsilon$  from equations (7.9), (7.10) and (7.11), i.e.

$$m = -\frac{a}{b}, \quad (7.9)$$

$$\eta = -\ln(1+b), \quad (7.10)$$

$$\sigma = \sigma_\epsilon \cdot \sqrt{\frac{2 \ln(1+b)}{(1+b)^2 - 1}}. \quad (7.11)$$

### 7.1.2 Half-life of the Ornstein-Uhlenbeck process

The half-life of the process is defined as the average time until the expected value of the process reaches the middle point between the current state  $x(0)$  and the long-term mean  $m$ .

The other way to get the expected value of the Ornstein-Uhlenbeck process is by integrating the deterministic part of equation (7.1).

$$\frac{dx}{m-x} = \eta dt. \quad (7.12)$$

Integrating (7.12) from  $x(0)$  to  $x(T)$  yields

$$\ln(m-x) \Big|_{x(0)}^{x(T)} = -\eta T, \quad (7.13)$$

$$\ln\left(\frac{m-x(T)}{m-x(0)}\right) = -\eta T. \quad (7.14)$$

For  $T = \text{half-life}$ , we have  $(m-x(T)) = 0.5(m-x(0))$ .  
The half-life  $T$  is

$$T = \frac{\ln(2)}{\eta}. \quad (7.15)$$

## 7.2 Presence of short-term mean-reverting noise in the monthly inflation time series

This section is devoted to the justification of the choice of annual inflation and not annualized monthly inflation. Let us recall here the definition of annualized monthly inflation  $I_t$  and annual inflation  $I_t^y$ ,

$$I_t = \ln\left(\frac{CPI_t}{CPI_{t-1}}\right) \cdot 12, \quad (7.16)$$

$$I_t^y = \ln\left(\frac{CPI_t}{CPI_{t-12}}\right). \quad (7.17)$$

The analysis on seasonality is performed on  $I_t$  while mean-reversion analysis is performed on  $I_t^y$ .

The graphs of the annualized monthly inflation are shown in figure 7.1. After examining them we can conclude that the annualized monthly inflation experience spurious oscillation along its long-term trend.

Country	Start	Mean	Volatility	Skewness	Kurtosis
USA	01.1961	4.05%	8.53%	1.2172	4.3555
UK	01.1961	5.53%	14.98%	1.4185	4.5146
Eurozone Rec	01.1989	2.35%	2.50%	0.0627	3.2966
Eurozone	01.1996	1.89%	2.11%	0.0381	4.3791
Japan	01.1961	3.48%	12.79%	1.5122	6.3650
Switzerland	01.1961	2.9%	6.76%	0.8160	3.4488

Table 7.1: Statistics for the seasonally adjusted annual inflation data series.

The mean-reversion analysis performed on these data shows that the half-life of the inflation is between 0.5 and 1 months, which corresponds to the short-term cyclical movements observed in figure 7.1. While being an essential characteristic of inflation data series this short-term noise can distort the results of long-term inflation behavior analyses. The annual inflation presented in figure 7.3 doesn't have this distorting property and this is the main reason why we have to switch to annual inflation. However, we need to keep in mind that annual inflation spans overlapping intervals of time, thus its elements are not independent. Moreover annual inflation incorporates price trends with a delay compared to monthly inflation.

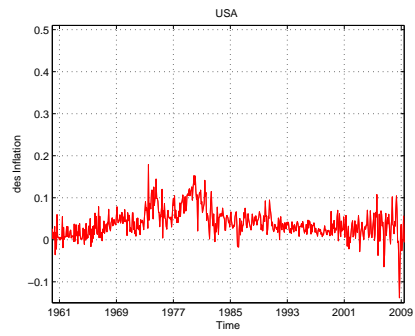
The simple statistics for the seasonally adjusted inflation series used in mean-reversion analysis is given in table 7.1.

### 7.3 Inflation model calibration

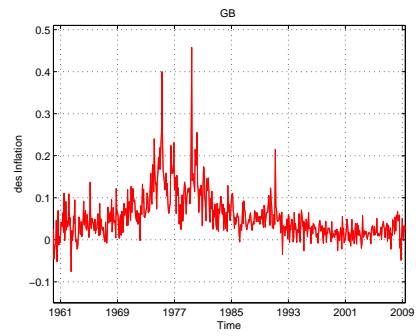
The second inflation variable (annual logarithmic inflation)  $I^y$  is used in the mean-reversion analysis because of the presence of the short-term mean-reverting noise in the monthly logarithmic inflation  $I$  which distorts the model calibration. The inflation is seasonally adjusted with the method described in the section 5.6.

We applied the same method of a seasonal adjustment for the series with a much longer history than the one used in analysis of the seasonality. The analyses of seasonality were conducted for series which are used for the ESG simulations. Unfortunately, such short series is not always enough to get reliable statistical estimates in the mean-reversion analysis. However, the changing with the time pattern of seasonality in inflation time series would make it more difficult to find the appropriate deseasonalization technique for time interval as long as 50 or even more (140 -USA CPI) years.

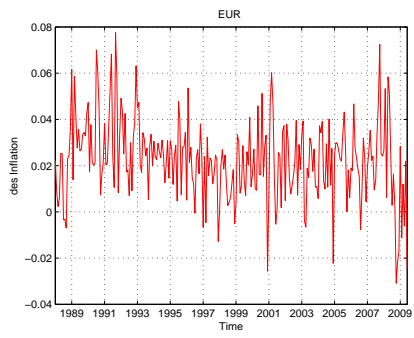
The calibration of the Ornstein-Uhlenbeck model involves fitting linear regression 7.8 to the data and identifying the model mean, volatility and mean-reversion speed with equations 7.9, 7.11 and 7.10 respectively. After that the conclusion of the data distribution variance can also be made



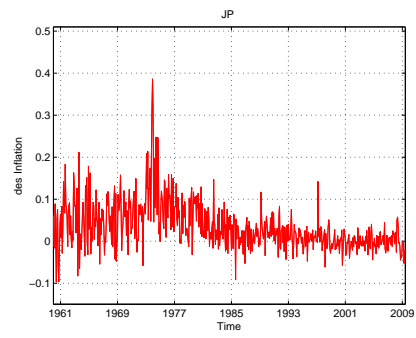
(a) USA



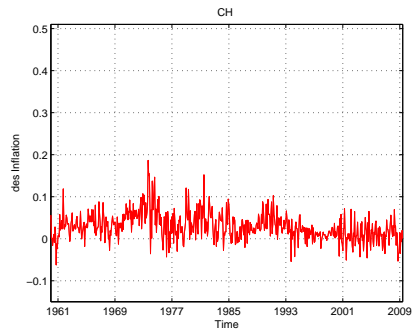
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 7.1: Short-term noise in monthly annualized inflation time series.



through the use of the result 7.5. The results are shown in table 7.2. More than a half of results are statistically insignificant which can be explained by the insufficiency of data as well as by not appropriately chosen data model. The Ornstein-Uhlenbeck model does not describe inflation behavior well. However, we obtained some significant results for the U.S. inflation and we can state that the longer the U.S. inflation time series the lower the half-life of the process. This can be explained by the fact that U.S. inflation series behavior changed over time (see fig. 7.2). Between years 1872 and 1915 the inflation experienced noticeable oscillations around its mean and later its behavior changed, which is reflected in the results obtained with the series dating back to more recent periods of time. The shorter is the U.S. inflation series the longer is the mean-reversion half-time (around 2 years for 140 years of data, around 4 years for 80 years of data and around 9 years for 50 years of data) and the less significant results are. The dynamic mean-reversion behaviour of the inflation time series is demonstrated by the U.S. inflation, which proves, that the Ornstein-Uhlenbeck model assumptions of constant speed of mean-reversion, constant mean and volatility are not suitable for solving that type of problem. The graphs of the inflation can be found on the figure 7.3.

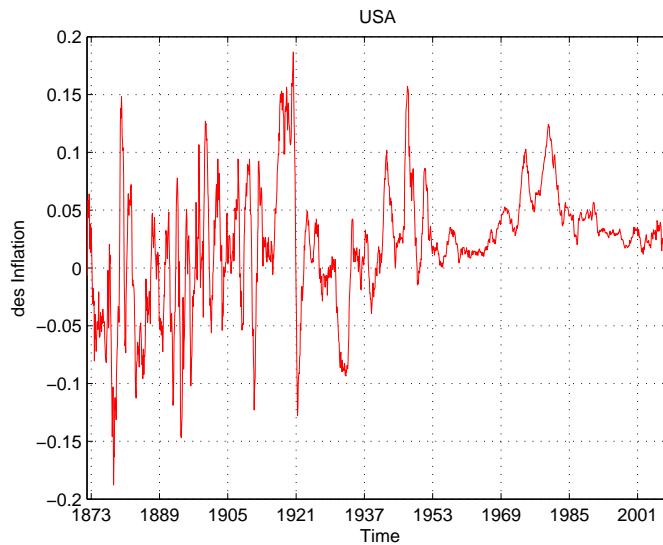
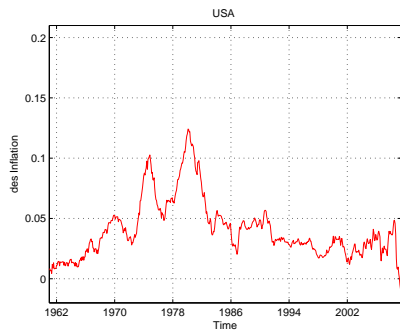


Figure 7.2: USA inflation time series from year 1872.

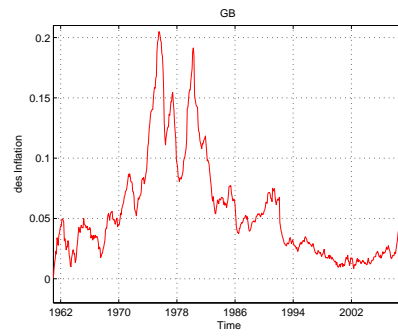
## 7.4 Interest rates model calibration

Interest rates in our analysis are defined as logarithmic interest rates

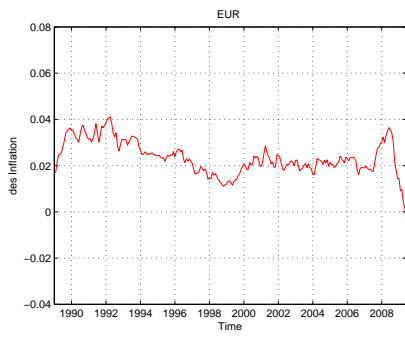
$$R_t = \ln(1 + r_t) \quad (7.18)$$



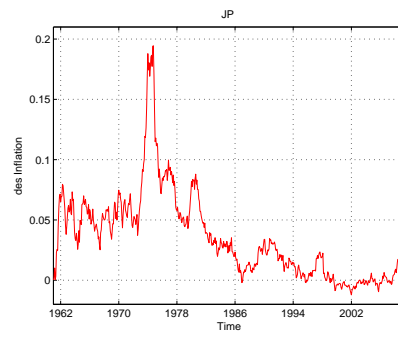
(a) USA



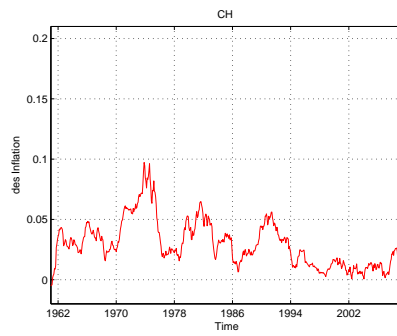
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 7.3: Annual logarithmic seasonally adjusted inflation.

	Start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
USA	01.1872	2.03%	0.0294	1.24%	17.66%	1.96	2.47	1.63	0.0145	0.00%
	01.1935	3.46%	0.0142	0.51%	10.41%	4.07	7.05	2.86	0.0063	1.76%
	01.1961	3.48%	0.0065	0.34%	10.32%	8.89	33.98	5.61	0.0023	25.21%
UK	01.1961	6.00%	0.006	0.45%	14.10%	9.61	33.55	5.60	0.0034	16.13%
Eurozone	01.1989	1.78%	0.0133	0.20%	4.20%	4.35	-12.37	1.83	0.0022	46.10%
	01.1996	1.36%	0.018	0.20%	3.65%	3.21	-5.66	1.23	0.0027	52.00%
Japan	01.1961	3.18%	0.0111	0.58%	13.43%	5.22	12.68	3.28	0.0050	8.89%
Switzerland	01.1961	2.87%	0.0133	0.33%	6.93%	4.34	9.14	2.84	0.0063	5.66%

Table 7.2: Results of the Ornstein-Uhlenbeck model parameter estimation for annual logarithmic seasonally adjusted inflation. Start - start date of the series,  $m$  - long-term mean,  $\eta$  - speed of mean-reversion,  $\sigma$  -volatility,  $\sigma^y$  - annualized long-term volatility of the series, i.e. series  $\sim N(m, \sigma^y)$ ,  $T$  - half-life of the process,  $T + \sigma_T$  and  $T - \sigma_T$  - standard error bounds of the half-life estimate,  $R^2$  and  $p$  -R-squared and p-values of the linear regression respectively.

where  $r_t$  is the spot interest rate at time  $t$ .

We calibrate the model 7.1 to the data series presented in the table 3.1. The simple statistics for these data is summarized in the table 7.4.

We proceed in the same way as in the section 7.3. The results presented in the table 7.4 show that Ornstein-Uhlenbeck model does not describe the time series well. 80% of the coefficients estimated are not statistically significant at 5% level, suggesting that some arbitrary trends in the data were caught by the Optimal Least Squares estimation. Only the results for the 3 months U.S. interest rates and 3 months Japanese interest rates are reliable from a statistical point of view.

There could be several reasons why these results are obtained. First, the mean-reversion of the logarithmic interest rates can be so slow that it is impossible to estimate it with the time series of the length we have. Second, the level of the mean-reversion, the speed of the mean reversion and the volatility of the time series could be non-stationary which implies that a different model should be used. We could propose to use so-called CIR-CEV model as a direction of further research of interest rates and inflation behaviour.

The graphs of the 10 years logarithmic interest rates are given in figure 7.4. The graphs of the 3 months, 1 year logarithmic interest rates can be found in the appendix of this report on figures A.6, A.7 respectively.

## 7.5 Real interest rates model calibration

**Real interest rate 1.** The real interest rate is the difference between nominal interest rate and inflation.

In the context of this report, the real interest rate is the logarithmic real risk-free interest rate. The inflation used in calculation of the real interest rate is annual seasonally adjusted inflation.

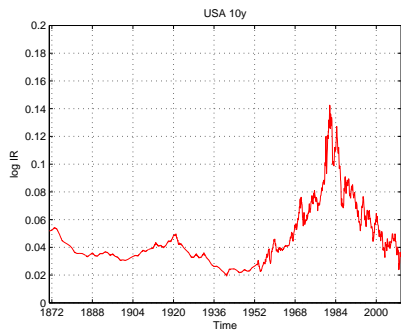
The real interest rate is defined in the following way

$$R_t^{real} = \ln \left( 1 + r_t^{real} \right), \quad (7.19)$$

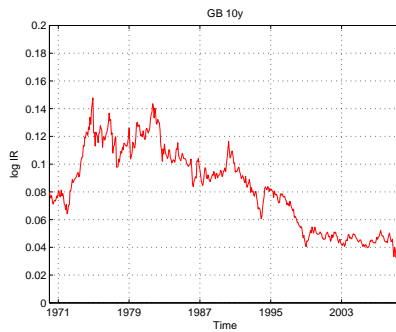
$$r_t^{real} = r_t - i_t^{*y}, \quad (7.20)$$

where  $r_t$  is the nominal interest rate and  $i_t^{*y}$  is the simple(non-logarithmic) annual seasonally adjusted inflation.

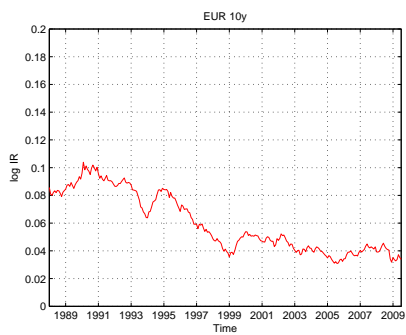
The simple annual seasonally adjusted inflation was obtained in the following way: first, the annualized logarithmic inflation  $I_t$  was seasonally adjusted and the annualized logarithmic seasonally adjusted inflation  $I_t^*$  was obtained, then the annualized logarithmic inflation was transformed into the annual logarithmic seasonally adjusted inflation using the well-known property of



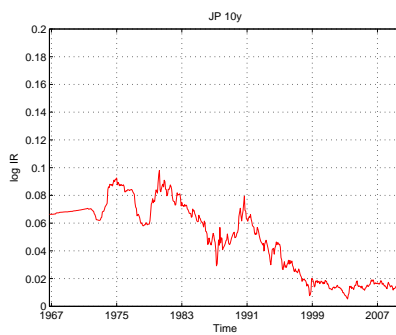
(a) USA



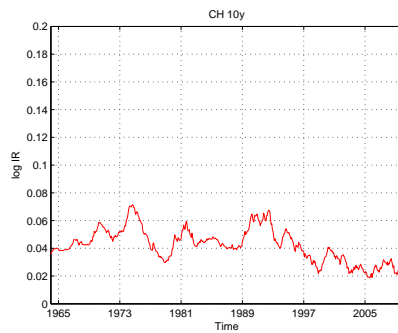
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 7.4: 10 years logarithmic interest rates.

Country	Maturity	Rate	Start	Mean	Volatility	Skewness	Kurtosis
USA	3 months	Treasury bill	01.1934	3.73%	10.34%	0.8683	3.7384
		CMT rate	01.1982	4.96%	8.97%	0.3013	3.1177
	1 year	CMT rate	04.1953	5.34%	9.78%	0.8800	4.0146
	10 years	Treasury bonds Rec	01.1871	4.57%	7.43%	1.7663	6.3694
CMT rate		01.1953	6.14%	8.67%	0.9309	3.5167	
UK	3 months	Government bonds Rec	03.1988	6.22%	10.21%	1.0064	3.5495
		Government bonds	03.1991	5.25%	6.34%	0.469	4.9899
	1 year	UK Nominal spot rate	01.1970	7.58%	10.19%	0.0458	2.2241
10 years	UK Nominal spot rate	01.1970	8.30%	10.15%	0.0295	1.8218	
Eurozone	3 months	Government bonds Rec	01.1988	4.92%	9.61%	0.7462	2.3058
		Government bonds	12.1998	2.94%	3.57%	-0.2343	2.6247
	1 year	Government bonds Rec	01.1988	5.07%	9.32%	0.6809	2.2239
Government bonds		12.1998	3.10%	3.50%	-0.1499	2.304	
Japan	3 months	Treasury bills, bid	01.1957	5.00%	12.62%	0.1275	2.3631
		Certificate of Deposit	04.1985	2.15%	8.63%	0.9063	2.4132
	1 year	Certificate of Deposit	04.1986	1.96%	8.00%	1.0486	2.8218
Switzerland	3 months	Gvt Benchmarks, bid	10.1966	4.90%	9.03%	-0.1588	1.6092
		Yields on TSE Bonds	01.1986	3.04%	6.34%	0.5868	1.9696
	1 year	Treasury bills, bid	01.1980	3.46%	8.31%	0.6326	2.5194
CHF LIBOR		01.1989	3.13%	8.94%	0.977	2.7269	
10 years	CHF LIBOR	01.1989	3.26%	8.33%	0.9725	2.7621	
	Gvt Benchmarks, bid	01.1964	4.22%	4.14%	0.1389	2.5390	
		Spot rate, CH bonds	01.1988	3.82%	4.43%	0.5771	2.2919

Table 7.3: Statistics for the logarithmic interest rates data series.

USA	Start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1934	3.63%	0.0067	0.35%	10.55%	8.68	21.05	5.46	0.0032	8.89%
	01.1982	2.13%	0.0128	0.29%	6.28%	4.51	8.75	3.02	0.0126	4.09%
	04.1953	5.00%	0.0085	0.40%	10.58%	6.82	19.01	4.15	0.0036	11.88%
10 years	01.1871	4.18%	0.0029	0.16%	7.53%	20.21	58.92	12.18	0.0014	12.80%
	04.1953	6.29%	0.0056	0.26%	8.51%	10.31	33.98	6.02	0.0030	15.33%
UK	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1988	-13.19%	0.0015	0.38%	24.35%	38.51	-8.62	5.95	0.0001	85.50%
	03.1991	2.84%	0.019	0.33%	5.86%	3.04	9.03	1.82	0.0104	13.19%
	01.1970	5.91%	0.0095	0.51%	12.82%	6.08	38.51	3.28	0.0029	23.77%
10 years	01.1970	6.83%	0.006	0.38%	12.02%	9.63	N/A	4.77	0.0021	31.68%
Eurozone	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1988	17.04%	-0.0016	0.25%	N/A	-36.1	-8.14	14.44	0.0003	77.40%
	12.1998	5.24%	-0.0091	0.20%	N/A	-6.35	-2.19	6.64	0.0021	60.85%
	01.1988	92.55%	-0.0002	0.26%	N/A	-288.81	-9.17	9.96	0.0000	96.00%
1 year	12.1998	-11.61%	0.0012	0.22%	15.56%	48.14	-3.17	2.74	0.0000	94.97%
	01.1988	1.77%	0.0048	0.21%	7.42%	12.03	-41.26	5.30	0.0001	96.00%
10 years	12.1998	4.11%	0.0444	0.18%	2.09%	1.3	3.68	0.78	0.0191	12.17%
Japan	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1957	4.36%	0.0188	0.72%	12.95%	3.08	5.34	2.16	0.0088	1.78%
	04.1985	-1.23%	0.0060	0.18%	5.69%	9.63	33.98	5.61	0.0068	15.96%
	04.1986	-1.81%	0.0042	0.16%	5.95%	13.85	472.70	7.02	0.0038	30.38%
1 year	10.1966	1.71%	0.0033	0.26%	10.92%	17.62	-56.10	7.60	0.0011	44.73%
	01.1986	1.84%	0.0128	0.26%	5.63%	4.51	13.75	2.69	0.0077	13.89%
Switzerland	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1980	2.38%	0.0127	0.41%	8.96%	4.54	16.01	2.63	0.0054	16.29%
	01.1989	-0.22%	0.0065	0.30%	9.11%	8.89	-57.76	4.10	0.0031	38.64%
	01.1989	-0.02%	0.0062	0.27%	8.43%	9.28	-60.60	4.29	0.0030	38.69%
1 year	01.1964	3.68%	0.0056	0.16%	5.16%	10.39	-891.47	5.15	0.0018	32.34%
	01.1988	2.60%	0.0059	0.17%	5.42%	9.79	-22.22	3.98	0.0019	48.70%

Table 7.4: Results of the Ornstein-Uhlenbeck model parameter estimation for the logarithmic government interest rates. Start - start date of the series,  $m$  - long-term mean,  $\eta$  - speed of mean-reversion,  $\sigma$  -volatility,  $\sigma^y$  - annualized long-term volatility of the series, i.e. series  $\sim N(m, \sigma^y)$ ,  $T$  - half-life of the process,  $T + \sigma_T$  and  $T - \sigma_T$  - standard error bounds of the half-life estimate,  $R^2$  and  $p$  - R-squared and p-values of the linear regression respectively.

logarithm that  $\ln(x_t/x_{t-1}) = \ln(x_t) - \ln(x_{t-1})$

$$I_t^{*y} = \frac{1}{12} \sum_{j=t-11}^t I_j^*. \quad (7.21)$$

Finally, the simple inflation was calculated as

$$i_t^{*y} = \exp(I_t^{*y}) - 1. \quad (7.22)$$

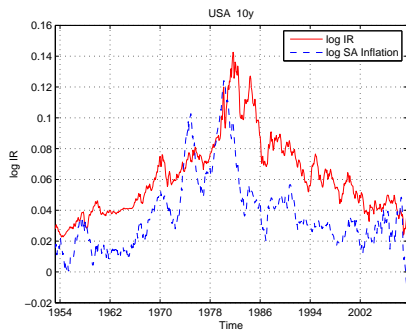
Real interest rate is the real return not affected by inflation that investors get for their investments. It reflects the growth(or decline) of the purchasing power of the capital unlike nominal interest rate. Real interest rate is an important characteristic affecting investors decisions. Investors, in general, aim to get positive return on their investments above the inflation level rather than just some predefined level of return without inflation taken into consideration. Real interest rates unlike nominal interest rates can fall below zero.

Our mean-reversion analysis for nominal interest rates and inflation did not lead to any significant results. The Ornstein-Uhlenbeck model is not a suitable model for interest rates and inflation on their own. However, there is one interesting observation concerning the joint behavior of interest rates and inflation. It can be seen in the picture 7.5 that interest rate and inflation tend to rise/fall together, so that the difference between them stays in some interval. If it becomes too big/small than it decreases/increases during the next period of time. The difference between interest rate and inflation is exactly the real interest rate. We can conclude from the picture 7.5 that inflation and nominal interest rate are not independent: they move together so that real interest rate values fluctuate in some range and whenever they are close to the bounds of this range they are forced to get back to its middle. Indeed, it makes economical sense because investors should closely watch returns over inflation level. Whenever inflation is high, they ask for higher interest rates to compensate for it.

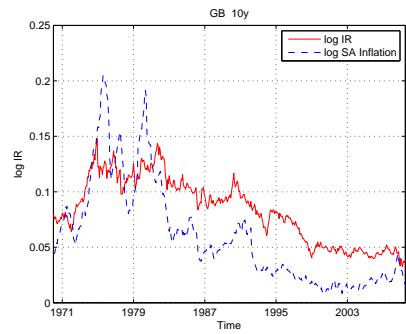
Our hypothesis is that real interest rates are subject to stronger mean-reverting forces than nominal interest rates on their own. We proceed in the same way as in the sections 7.4 and 7.3 in order to check the validity of the hypothesis stated above. The simple statistics for the real interest rates data series is given in the table 7.5. The results of the Ornstein-Uhlenbeck model fit are shown in the table 7.6.

All the coefficients except few for UK and Eurozone real interest rates are significant. The mean of the real interest rate depends on maturity: it is lower for 3 months rates and higher for 10 years, which corresponds to upward slopping yield curves. The mean of 3 months real interest rate is

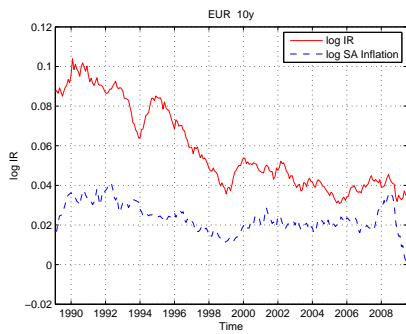




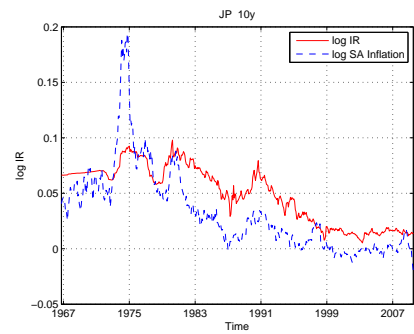
(a) USA



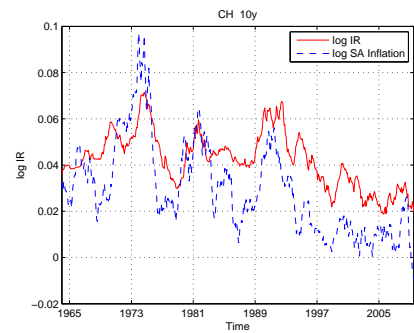
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure 7.5: 10 years logarithmic interest rate and annual seasonally adjusted logarithmic inflation.

Country	Maturity	Rate	Start	Mean	Volatility	Skewness	Kurtosis
USA	3 months	Treasury bill	01.1934	0.09%	11.11%	-2.1881	10.4714
		CMT rate	01.1982	1.72%	6.44%	-0.301	2.7824
	1 year	CMT rate	04.1953	1.69%	6.35%	-0.0885	3.6052
		CMT rate	01.1961	1.80%	6.55%	-0.1999	3.5592
10 years	Treasury bonds Rec	01.1872	2.35%	16.87%	-0.4591	4.8319	
	CMT rate	01.1961	2.65%	6.58%	-0.2283	4.0146	
UK	3 months	Government bonds Rec	03.1988	3.44%	6.86%	0.1023	4.6637
		Government bonds	03.1991	2.90%	5.13%	-1.2687	6.9039
	1 year	UK Nominal spot rate	01.1970	1.59%	13.35%	-1.6656	6.0471
10 years	UK Nominal spot rate	01.1970	2.37%	10.53%	-1.6656	6.0471	
Eurozone	3 months	Government bonds Rec	01.1988	2.59%	8.30%	0.7825	2.3058
		Government bonds	12.1998	0.97%	3.19%	0.1907	1.8640
	1 year	Government bonds Rec	01.1988	2.72%	8.01%	0.7131	2.2405
		Government bonds	12.1998	1.13%	3.29%	0.2650	1.8614
10 years	Government bonds Rec	01.1988	3.51%	5.97%	0.3138	1.9730	
	Government bonds	12.1998	2.23%	2.74%	-0.0093	2.1968	
Japan	3 months	Treasury bills, bid	01.1961	1.21%	9.07%	-0.0819	6.9469
		Certificate of Deposit	04.1985	1.41%	6.03%	0.5895	2.3382
	1 year	Certificate of Deposit	04.1986	1.29%	5.60%	0.6972	2.5839
10 years	Gvt Benchmarks, bid	10.1966	1.6%	9.15%	-2.8836	14.3759	
	Yields on TSE Bonds	01.1986	2.36%	4.30%	0.1583	2.5732	
Switzerland	3 months	Treasury bills, bid	01.1980	1.27%	4.95%	0.4718	2.8964
		CHF LIBOR	01.1989	1.39%	5.23%	0.7492	2.7204
	1 year	CHF LIBOR	01.1989	1.53%	4.76%	0.6868	2.6783
	10 years	Gvt Benchmarks, bid	01.1964	1.31%	4.68%	-0.7969	3.5521
Spot rate, CH bonds		01.1988	2.06%	2.81%	0.1968	3.3499	

Table 7.5: Statistics for the real interest rate data series.

around 1% on average for all the countries, the mean of 1 year real interest rate is on average around 1.3% and the mean of 10 years real interest rate is around 2.25%. Note that Japanese interest rates starting on April 1986 are the certificates of deposit rates, so they still contain some risk premium and risk-free rate should be lower in this case. The Eurozone results should be taken with caution because they are obtained with the reconstructed data and not the actual one.

The half-life of the real interest rate is on average 2 years, it means that it takes on average 2 years for the real interest rate to reduce its deviation from its mean by half. We also observe that half-life is on average lower for Swiss real interest rates, which can be explained by the smaller volume of the Swiss market compared to the others and its stronger regulation by the Swiss National Bank.

The annual long-term volatility depends on the time series. In particular, it varies a lot depending on the length of the time interval spanned by the time series. As a result, we are unable to draw a uniform conclusion about its value. It could mean that volatility changes over time rather than stays constant as implied by the Ornstein-Uhlenbeck model.

The hypothesis of the stronger mean reversion of the real interest rate compared to the nominal interest rate was proved by the results of the Ornstein-Uhlenbeck model fit.

## 7.6 Results of the mean-reversion analysis used in the ESG

The analysis of the mean-reversion of the real interest rate provide us with results that can be used in the Economic Scenario Generator for the simulations of interest rates and inflation. We choose the average half-life of the real interest rate which is approximately 2 years and calculate the speed of mean-reversion corresponding to the chosen half-life, i.e.

$$T = 2 \text{ years} = 8 \text{ quarters} = \frac{\ln(2)}{\eta}, \quad (7.23)$$

$$\eta = \frac{\ln(2)}{8} = 0.0866. \quad (7.24)$$

USA	Start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1934	0.21%	0.0221	0.68%	11.14%	2.61	3.84	1.98	0.0109	0.17%
	01.1982	1.42%	0.0273	0.41%	6.08%	2.12	3.83	1.45	0.0152	2.52%
	04.1953	1.73%	0.0379	0.50%	6.35%	1.52	2.12	1.18	0.0187	0.04%
1 year	01.1961	1.79%	0.0395	0.53%	6.53%	1.46	2.09	1.12	0.0194	0.08%
	01.1872	2.33%	0.0318	1.23%	16.86%	1.82	2.26	1.52	0.0157	0.00%
10 years	01.1961	2.79%	0.0228	0.42%	6.81%	2.53	4.25	1.80	0.0107	1.26%
UK	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	03.1988	2.04%	0.0146	0.45%	9.12%	3.96	144.41	1.98	0.0036	30.58%
	01.1991	1.66%	0.0161	0.39%	7.53%	3.59	1.68	-28.81	0.0036	37.55%
	01.1970	1.07%	0.0199	0.77%	13.37%	2.90	5.45	1.98	0.0098	3.12%
10 years	01.1970	2.24%	0.0232	0.65%	10.45%	2.49	4.34	1.73	0.0116	1.92%
Eurozone	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1989	0.34%	0.0124	0.26%	5.72%	4.66	10.90	2.96	0.0126	7.95%
	01.1999	0.65%	0.0359	0.24%	3.10%	1.61	4.04	0.92	0.094	11.88%
	01.1989	0.94%	0.0152	0.28%	5.56%	3.80	7.81	2.50	0.0154	5.26%
1 year	01.1999	0.91%	0.0406	0.26%	5.56%	1.42	3.25	0.84	0.022	9.59%
	01.1989	2.84%	0.0194	0.27%	4.75%	2.86	5.89	1.98	0.0168	4.30%
10 years	01.1999	2.23%	0.0414	0.25%	3.01%	1.40	4.11	0.76	0.0165	15.06%
Japan	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1961	1.02%	0.0635	0.87%	8.49%	0.91	1.17	0.74	0.0351	0.00%
	04.1985	1.22%	0.0227	0.36%	5.85%	2.54	5.40	1.65	0.0123	5.93%
	04.1986	1.16%	0.0237	0.34%	5.41%	2.44	5.23	1.58	0.0126	6.08%
1 year	10.1966	1.67%	0.0276	0.62%	9.21%	2.10	3.39	1.51	0.0135	0.85%
	01.1986	2.32%	0.0548	0.41%	4.29%	1.05	1.66	0.77	0.0273	0.54%
Switzerland	start	$m$	$\eta$	$\sigma$	$\sigma^y$	$T$	$T + \sigma_T$	$T - \sigma_T$	$R^2$	$p$
3 months	01.1980	1.34%	0.0634	0.50%	4.91%	0.91	1.31	0.70	0.0314	0.08%
	01.1989	1.16%	0.0367	0.39%	4.99%	1.57	2.87	1.08	0.0200	2.66%
	01.1989	1.34%	0.0394	0.37%	4.52%	1.47	2.61	1.02	0.0215	2.15%
1 year	01.1988	2.11%	0.0734	0.32%	4.74%	1.63	2.43	1.22	0.0338	0.30%
	01.1964	1.44%	0.0354	0.36%	2.89%	0.79	1.20	0.58	0.0170	0.23%

Table 7.6: Results of the Ornstein-Uhlenbeck model parameter estimation for logarithmic real interest rates.

Start - start date of the series,  $m$  - long-term mean,  $\eta$  - speed of mean-reversion,  $\sigma$  -volatility,  $\sigma^y$  - annualized long-term volatility of the series, i.e. series  $\sim N(m, \sigma^y)$ ,  $T$  - half-life of the process,  $T + \sigma_T$  and  $T - \sigma_T$  - standard error bounds of the half-life estimate,  $R^2$  and  $p$  -R-squared and p-values of the linear regression respectively.

## Chapter 8

# Conclusion

In this report we have revised and quantified the assumptions of the models for interest rates and inflation developed in [Müller et al., 2010].

The first assumption is that interest rates and inflation are mean-reverting processes with very low speed of mean-reversion.

The second assumption is that real interest rates are mean-reverting processes and exhibit stronger mean-reversion than interest rates and inflation alone. The third assumption is that inflation slightly leads interest rates.

The Ornstein-Uhlenbeck model was used for the mean-reversion analysis of the interest rates, inflation and the real interest rates. The model was calibrated to time series from five countries: USA, UK, Eurozone, Japan and Switzerland. We have performed the analysis of time series of different length: 20 years, 50 years and 140 years (only for the U.S.). The results were consistent across different data samples.

Our analysis showed that the half-life of real interest rate is on average 2 years, its long-term mean is between 0.5% and 2% depending on the maturity. The same parameters estimates for interest rate and inflation are statistically insignificant suggesting that low mean-reversion speed does not allow us to get statistically reliable estimates.

We have encountered the problem of seasonality in inflation which was solved in a computationally efficient way with the moving average operator. The efficiency of the model was tested against the X-12-ARIMA model <sup>1</sup> and it showed a similar performance with less computational effort.

In conclusion we can say that this thesis has successfully accomplished the tasks assigned by the Financial and Risk Modelling team of SCOR,

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<sup>1</sup>X-12-ARIMA is a software tool developed by the U.S. Bureau of Census for the time series deseasonalization.

within all the three main areas of investigation:

It has introduced and tested a new technique for seasonal adjustments.

It has confirmed with good statistical significance two of the main assumptions of the inflation and interest rates models of the ESG.

Wherever possible, it has quantified the relevant mean reversion parameters.

# Appendix A

## Appendix

### A.1 CPI collection and aggregation

We describe the details of the CPI collection and aggregation process for every country under consideration.

There are several statistical issues which arise during the CPI measuring process. Different goods can disappear because of natural and seasonal reasons in the consumption of the households, for example fruits and vegetables or some sport equipment. The prices of these products cannot be observed during certain periods of time. Other products can just naturally become obsolete due to, for example, outdated technology. Statisticians need to find the replacement for an obsolete item which wouldn't affect the statistical properties of CPI. There are several ways to deal with missing prices, i.e. direct comparison, direct quality adjustment, imputation. If the new and old items are essentially the same, the commodity analyst assumes no quality difference exists, and the price comparison between the items is used in the index. The estimate of the quality difference is calculated in direct quality adjustment method. When statistician are unable to estimate the value of the quality changes, imputation (special procedure for handling missing information) is used.

Rents, mortgages and owner-occupied housing can be dealt in different ways, depending on the structure of housing ownership of a country.

The way how elementary quotes are aggregated into an index is an important characteristic of CPI. The aggregation process involves three major components: elementary indexes, aggregation weights and a price index number formula that uses the weights and elementary indexes to compile them into a published index.

In this section we discuss the treatment of missing prices and of quality changes, the treatment of seasonal items, the treatment of owner-occupied housing and rentals for housing, aggregation process and a few other issues concerning CPI measurement.

The CPI data is provided by the OECD (Organization for Economic Cooperation and Development) and can be found on their web page, i.e.

<http://www.oecd.org>.

The details which were not summarized by the OECD were taken directly from the official documents of the countries.

### **A.1.1 USA**

**Source.** US Bureau of Labor Statistics <sup>1</sup>.

**Source Periodicity.** Monthly.

**Base period.** The national base year is 1984 = 100.

**Type of prices.** Transaction prices. The prices include any applicable taxes, discounts and rebate.

**Data Characteristics.** The data is collected on the following categories:

- 1) food at home-nonmeat staples, 2) food at home-meat, poultry, fish,
- 3) food at home-fruits and vegetables, 4) other food at home, plus beverages (alcoholic and nonalcoholic), 5) food away from home, 6) fuels and utilities, 7) household furnishings and operations, 8) apparel and upkeep, 9) transportation excluding motor fuel, 10) motor fuel, 11) medical care, 12) education and communication, 13) recreation and other commodities and services,

#### **Sampling.**

**Sample size.** Each month 78500 price quotations are obtained from approximately 25500 outlets. About 48000 housing units are contacted to collect data on rents.

**Sampling method.** Prices for the goods and services used to calculate the CPI are collected in 87 urban areas throughout the country and from about 23,000 retail and service establishments. Data on rents are collected from about 50,000 landlords or tenants.

**Method of collection.** Most pricing is conducted by personal interview; some pricing takes place by telephone. Prices are usually collected throughout the month. Food, rent, utilities, and some other items are priced monthly in all urban areas. Most other items are priced monthly in three large urban areas, and every second month in other areas.

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<sup>1</sup><http://www.bls.gov/cpi/cpiovrwv.htm>



**Geographic coverage.** All urban areas of 2500 or more population within the 50 states; U.S. territories are not included. The current CPI geographic sample is based on the 1990 Census of Population.

**Population coverage.** Residents in urban areas who include about 87 percent of the total civilian non institutional population, including wage earners and clerical workers, professional, managerial, and technical workers, short-term workers, the self-employed, the unemployed, retirees, and others not in the labor force.

**Item coverage.** The Consumer Price Index is calculated on the basis of a market basket of 305 entry level items representing all goods and services purchased for everyday living by all residents in urban areas.

#### **Aggregation and consolidation.**

**Elementary aggregates.** For most item categories, representing approximately 61 percent of the total expenditure weight, basic indexes are compiled using a geometric mean formula.

**Index formula.** The Laspeyres price index is used to aggregate elementary indexes into published CPI indexes.

**Weights.** The weights for the CPI are derived from the Consumer Expenditure Surveys for 2003-04, and the average for those 2 years. Historically weights have been revised once every 10 years; however, starting in 2002, weights have been revised every other year.

#### **Housing.**

**Owner-occupied housing.** The rent and REQ indexes measure the change in the cost of shelter for renters and owners, respectively. Price change data for these two indexes come from the CPI Housing survey. Each month, BLS field representatives gather information from renter units on the rent for the current and previous months and on what services are provided.

**Rentals for housing** The rent estimates used in the CPI are "contract rents." They are the payment for all services the landlord provides in exchange for the rent. For example, if the landlord provides electricity, it is part of the contract rent. The price is adjusted for quality change as well as for aging.

**Medical care.** The CPI covers only that part of health care commodities, services, and health insurance premiums that consumers pay for "out of pocket."

**Interest, credit charges and taxes.** Government taxes, social security payments are excluded from the CPI.

**Missing prices and Quality changes.** The commodity analyst chooses one of the following three methods to handle the replacement: direct comparison, direct quality adjustment, imputation.

**Seasonal items.** During the period when a seasonal item is unavailable, its price is imputed following standard imputation procedures. When an item returns at the beginning of its season several months later, the price is directly compared with the item's last price, as it has been imputed forward. When an item becomes permanently unavailable, the standard procedure is to substitute the most similar item sold in the outlet.

### A.1.2 UK

**Source.** Office for National Statistics, United Kingdom (ONS) <sup>2</sup>.

**Source Periodicity.** Monthly.

**Base period.** National base year 2005=100.

**Type of prices.** Price used in the CPI calculation includes taxes such as Value Added Tax (VAT) and insurance tax, as well as duties, including air passenger duty.

**Data Characteristics.** The data is collected on the following categories: 1) food and non-alcoholic beverages, 2) alcohol and tobacco, 3) clothing and footwear, 4) housing, fuel, light and household services, (excluding mortgage interest payments, depreciation, council tax, ground rent and building insurance), 5) furniture and household equipment, 6) health, personal goods and services (health-related items), 7) transport, motoring expenditure, fares and other travel costs, 8) communication, 9) recreation and culture, leisure goods, leisure services, 10) education fees and subscriptions, 11) restaurants and hotels, 12) miscellaneous goods and services, personal goods and services (non health-related items).

### Sampling

**Sample Size.** Sample size: About 120,000 prices are collected each month from 20,000 outlets in around 150 randomly selected areas throughout the United Kingdom.

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<sup>2</sup>[http://www.statistics.gov.uk/downloads/theme\\_economy/CPI\\_Technical\\_Manual.pdf](http://www.statistics.gov.uk/downloads/theme_economy/CPI_Technical_Manual.pdf)

**Sample method.** Prices are collected in around 150 locations across the 12 government regions within the UK. Location selection takes place separately within each region, with the probability of a particular location being selected proportional to the number of employees in the retail sector in that location.

Until 1994, the sample of outlets chosen within a location was purely judgmental. Since 1995 the different method has been employed. Refer for details to

**Method of collection.** Local collection is used for most items; prices are obtained from outlets in about 150 locations around the country. Some 110,000 quotations are obtained by this method. Local collectors should try to collect all prices every month, except for seasonal items.

**Geographic coverage.** The whole of the UK i.e. England, Scotland, Wales and Northern Ireland, is covered.

**Population coverage.** All private UK households, foreign visitors to the UK and residents of institutional households.

**Item coverage.** All monetary expenditure on goods and services bought within the domestic territory and covered by Household Final Consumption Expenditure (HHFCE) as defined for the UK's National Accounts. The CPI includes more than 650 items.

#### **Aggregation and consolidation.**

**Elementary aggregates.** The CPI generally uses the geometric mean.

**Index formula.** Within each year the CPI is a Laspeyres-type index.

**Weights** The weights are updated annually. Given the focus on 'monetary' expenditures, imputed expenditures, such as imputed rents and company cars in kind, are excluded. The data used to produce the weights comes from a variety of sources, the most important of which is the Expenditure and Food Survey (EFS).<sup>3</sup>

#### **Housing.**

**Owner-Occupied Housing.** The CPI excludes a measure of Owner-Occupied Housing.

**Rentals for housing.** The CPI includes a measure of rented housing.

**Medical Care.** The CPI includes healthcare services.

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<sup>3</sup>This is a survey of the expenditure patterns of private households based on a sample of around 7,000 households; it is conducted continuously with reports issued annually.

**Interest, credit charges and taxes.** The CPI does not include council tax, mortgage interest payments, house depreciation, buildings insurance, ground rent, savings and direct taxes, national insurance contributions, repayment of loans.

**Missing prices and quality adjustments.** If temporarily unavailable the base price is temporarily removed from index calculation so that the weight for that product is redistributed among other products in the item index; if permanently unavailable then the replacement is selected. Price collectors select products with significant market share and, where possible, of the same quality.

Estimation of price change using price change for similar products is used in most cases where an adjustment is required. Hedonic regression which relates the price of an item to its measurable characteristics is used for personal computers, laptops, mobile phone handsets and digital cameras, with option costing used for the quality adjustments of new cars.

**Seasonal items.** Seasonal clothing prices collected for pre-specified months and last available price carried forward for months with no collection; weights held constant throughout year. Similar applies to seasonal fruit and vegetables, but item weights vary from month to month within fixed class weights.

**Quality comments.** CPI data published before January 1998 are OECD estimated.

### A.1.3 Eurozone

**Source.** Eurostat <sup>4</sup> evaluates the CPI for the Eurozone.

**Source Periodicity.** Monthly

**Base period** Base period is 1996 = 100 .

**Type of prices.** The prices measured are those actually faced by consumers, so for example they include sales taxes on products, such as Value Added Tax, and they reflect end-of-season sales prices.

**Data Characteristics.** The CPIs calculated by Eurostat are harmonized indices of consumer prices (HICPs). The European Union's Harmonized Indices of Consumer Prices (HICPs) were developed as a response to the need for comparable CPIs to measure the convergence

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<sup>4</sup>Eurostat is the Statistical Office of the European Communities situated in Luxembourg, [http://epp.eurostat.ec.europa.eu/cache/ITY\\_OFFPUB/KS-BE-04-001/EN/KS-BE-04-001-EN.PDF](http://epp.eurostat.ec.europa.eu/cache/ITY_OFFPUB/KS-BE-04-001/EN/KS-BE-04-001-EN.PDF)

of inflation of EU Member States as a criterion for entry to the Monetary Union. The HICPs are a new family of consumer price indices calculated according to a harmonized approach and a regulated set of definitions. The HICPs provide the best statistical basis for international comparisons of consumer price inflation in the European perspective, covering virtually all areas of household final monetary consumption expenditure. The most prominent among HICPs is the Monetary Union Index of Consumer Prices (MUICP = Eurozone). The MUICP is calculated as a weighted average of the HICPs of the countries<sup>5</sup> of the Eurozone. The MUICP is the key indicator of price stability for the European System of Central Banks (ESCB) and the European Central Bank (ECB).

The HICPs aim to cover the full range of final consumption expenditure for all types of households. The data is collected on the following categories: 1) food, 2) alcohol and tobacco, 3) clothing, 4) housing, household equipment, 5) health, 6) transport, 7) communications, 8) recreation and culture, 9) education, 10) hotels and restaurants, 11) miscellaneous .

## Sampling

**Sample Size.** The sample is an aggregate of the samples of each country in the Eurozone.

**Sample method.** The sample technique is specific for each country in the Eurozone.

**Method of collection.** The price collection in the Member States is typically carried out by a combination of visits to local retailers and service providers and central collection (via mail, telephone, email and the internet).

**Geographic coverage.** The HICPs cover all expenditures within the territory, whether by residents or visitors.

**Population coverage.** The index covers all purchases by households within the territory of a country, those by both resident and non-resident households. All sections of the population are covered in principle, including the extremes of the income distribution and including the institutional population.

**Item coverage.** The coverage of the HICPs is defined in terms of 'household final monetary consumption expenditure'.

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<sup>5</sup>the Eurozone countries covered by the MUICP include Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Slovenia and Spain.

## **Aggregation and consolidation.**

**Elementary aggregates.** The HICPs use ratios of arithmetic mean prices or of geometric means, forbidding the use of arithmetic means of price relatives except where this can be shown not to affect comparability.

**Index formula.** The HICPs are Laspeyres-type price indices.

**Weights.** There is no uniform basket applying to all countries. The HICPs are based on the prices and expenditures which are representative in each country and not on an average 'euro-basket'. The weights should relate to a period of not more than seven years before the index year. Checks should also be made each year to see whether any important changes have taken place and selective adjustments introduced when necessary.

**Aggregation across countries.** The aggregation across countries uses country weights for 'household final monetary consumption expenditure'. The MUICP is compiled as a weighted average of the countries in the Eurozone. The country weights are derived from national accounts data for 'household final monetary consumption expenditure'.

## **Housing.**

**Owner-Occupied Housing.** The imputed prices for the consumption of the service provided by owner-occupied housing are currently excluded from the HICPs.

**Rentals for housing.** The rented housing is included in the HICP.

**Medical care.** The HICP includes the net price paid by consumers (after reimbursements), while some national CPIs exclude these purchases or record the gross price.

**Interest, credit charges and taxes.** The HICPs exclude interest and credit charges, regarding them as financing costs rather than consumption expenditure.

**Missing prices and quality changes** Carrying forward the most recent observation for more than two months is not permitted. Currently, there is no regulation for the member states about the replacement mechanism, so this problem is left to local statistical bureaus.

For the HICPs there are minimum standards for quality adjustment - explicit quality adjustments must be made whenever possible and the whole of a price change should never be ascribed to quality differences without justification. In practice all of the Member States make adjustments for the changing quality of goods and services in their HICPs - using a range of direct and indirect methods.

**Seasonal items.** HICPs includes end-of-season sales prices.

#### A.1.4 Japan

**Source.** The series is directly provided by Statistics Bureau of Japan <sup>6</sup>.

**Source Periodicity.** Monthly.

**Data Characteristics.** The data is collected on the following categories: 1)food, 2)housing, 3)furniture and household utilities, 4)clothes and footwear, 5)medical care, 6)transportation and communication, 7)education, 8)reading and recreation, 9)miscellaneous.

**Base period** The national reference year is 2005 = 100.

**Type of prices.** Prices are transaction prices, excluding temporary reductions, special sales, etc.

#### **Sampling.**

**Sample Size** . Each month, 584 representative items are priced in about 34000 goods and service outlets resulting in about 233000 monthly price quotations. Prices for fresh food items are collected three times per month. Rents are surveyed monthly through a sample covering both the public and private sectors.

**Sample techniques.** Municipalities and household selection: Random multistage sampling. The first stage is the selection of 168 municipalities (120 cities and 48 towns and villages) from the 3,230 municipalities that comprise Japan. The selection of municipalities depends on a number of criteria, including that each capital city of a prefecture be included. Selected cities are then stratified into four groups according to size: 49 major cities (cities with prefectural government or populations of 1 million or more), 22 middle-sized cities (150,000 to 1 million), 28 small cities A (50,000 to 150, 000), and 21 small cities B (50,000 or less). From each sampled municipality, survey areas are selected: 16 (or more) areas are selected from major cities, 6 areas from middle-sized cities, 6 areas from small cities A, 4 areas from small cities B, and 2 areas from towns and villages. The selection of areas is with probability proportionate to population size. Finally, in each selected area, six two-or-more person households are selected and surveyed for six months. The responding households are rotated every six months. The sample allocation is revised every five years.

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<sup>6</sup><http://www.stat.go.jp/english/data/cpi/1586.htm>

**Method of collection.** Approximately 750 price collectors visit 30,000 outlets to collect prices of reselected representative items. There are 509 items and 719 item specifications

**Geographic coverage.** The whole country which is divided into 167 strata, one municipality is selected from each stratum by using probability sampling method to represent the entire country.

**Population coverage** Households with two or more persons are included. One-person households are not included.

**Item coverage.** Items are selected only from those accounting for more than 1/10000 of household consumption expenditures. 585 items are classified into 10 major groups. The items are selected from all goods and services normally purchased for consumption. The shelter service provided by owned houses is incorporated in the index through the imputed rent approach.

The following items are not included: non-consumption expenditures (such as income taxes and social security payments) or outgoings other than expenditures (such as savings including deposits, security purchases, and property purchases), remittances, money gifts, religious contributions (donations and offerings to temples, churches, and of-fertory) and obligation fees (fees paid to neighborhood association, alumni and union due.

#### **Aggregation and consolidation.**

**Elementary aggregates.** The first stage of aggregation is performed on nearly 100,000 quotations (each of 585 items in 167 municipalities). The price relative for an item in a municipality is its average price in the current month divided by its average price in the base year. The elementary aggregate index is therefore a ratio of averages or Dutot index<sup>7</sup>. The "elementary aggregate" price relatives are averaged over municipalities using the number of multiperson households in each municipality, as a ratio of the total number of such households, as weights.

**Index formula.** A Laspeyres index using relative expenditure shares as weights is used at the higher level of aggregation.

**Weights.** The weights are calculated on the basis of average household living expenditures by municipality, derived from the Family Income and Expenditure Survey in the base year of the CPI. Weights are revised every year.

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<sup>7</sup> Dutot index- price index defined as the ratio of the unweighted arithmetic average of the prices in the current period to the unweighted arithmetic average of the prices in the base period.



## **Housing.**

**Owner-Occupied Housing.** Owner-occupied housing is incorporated in the index through the imputed rent approach.

**Rentals for housing.** The index includes a measure of rented housing.

**Medical care.** Since high-cost medical care exceeding a certain amount is refunded, prices considering the refund (not the amount paid at the hospital, but the actual share of patients after the deduction of benefits by the social security fund) are applied.

**Interest, credit charges and taxes.** Expenditures other than the living expenditure (e.g., direct taxes, social insurance premiums, security purchases, land and housing purchases) are not included in the scope of the index.

**Missing prices and quality changes** For temporarily unavailable, seasonal, perishable items, such as fresh fruit and fish, the overall weight is held fixed at the annual level. There is an implied imputation for the price change of the missing items based on the long-run price change of existing items.

Explicit quality adjustments are made, when applicable. The option cost method is applied to automobiles and hedonic indices are used for digital cameras and personal computers.

**Seasonal items.** For fresh fish and shellfish, fresh vegetables and fresh fruits the monthly variable weights are used for compiling the index. For seasonal goods excluding fresh foods, the average prices of the month when the survey is conducted are substituted for the prices of the month when the survey is not conducted.

### **A.1.5 Switzerland**

**Source.** Federal Statistical Office, Switzerland (OFS) <sup>8</sup>.

**Source Periodicity.** Monthly.

**Base period.** The national base period is December 2005 = 100.

**Type of prices.** Prices are transaction prices, including indirect taxes (VAT) and subsidies and excluding credit and interest payments. The reduction of prices such as promotions and sales is taken into account. Only the prices for final consumption are registered. The national

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<sup>8</sup><http://www.bfs.admin.ch/bfs/portal/fr/index/infothek/publ.html?publicationID=2787>

direct taxes, social security payments as well as investments are not considered to be consumption expenses.

**Data Characteristics.** The data are grouped in the following main categories : 1)food and non-alcoholic beverages, 2)alcoholic beverages and tobacco, 3)clothing and shoes, 4)housing and energy, 5)furniture and maintenance of the house, 6)medical expenses, 7)transports, 8)communications, 9)leisure and cultural visits, 10)education and training, 11)restaurants and hotels, 12)miscellaneous.

### **Sampling.**

**Sample Size.** In total about 400000 prices are collected every year.

**Sample method.** The frequency of collecting prices for the products which have regular short-term variations (perishable products) is monthly. The prices of petroleum are registered two times per months. The prices of other goods of the CPI basket are collected every three or four months. The prices which are known in advance, such as telecommunication and transport, are collected non-periodically.

**Method of collection.** The price collection is done by visits to the outlets, telephone interviews, internet price surveys. Data collection through barcode scanning is a part of the data aggregation process too.

**Geographic coverage.** The regions were chosen according to the following criteria: Switzerland is divided into 7 big regions, between one and three smaller regions are chosen inside of these big regions and maximum one per canton. Only big urban areas are taken into account to reflect the areas with the highest consumption.

**Population coverage.** The population under consideration includes private households permanently residing in Switzerland. Tourists, foreign students, temporary workers are excluded from the survey.

**Item coverage.** The basket of products is the representative basket of goods and services consumed by the private households.

### **Aggregation and consolidation.**

**Elementary aggregates.** Elementary aggregate (the same region and the same channel of distribution) is constructed as the geometric mean. The intermediate aggregation (aggregation of different regions and distribution channels) is calculated via the arithmetic average.

**Index formula.** The index is calculated using the chained-Laspeyres formula.

**Weights.** Weights are mainly derived from Households Budget Survey 2006. The source of the weighting is the survey on the revenues and consumption of citizens permanently residing in Switzerland. It is annually conducted by the Federal Statistical Office. 12 samples are randomly drawn from the electronic telephone directory from the 7 big regions in Switzerland. The household randomly chosen are interviewed during one month about their periodic and non-periodic expenses as well as their revenues. 11 regions were chosen to reveal the prices from 2006 till 2010. The weights are brought up to date every December.

### **Housing.**

**Owner-Occupied Housing.** The rental equivalence method is used. The weight for owners is included in rents. The mortgage interest payments constitute about one third of the total housing weight in the CPI.

**Rentals for housing.** About 5000 of rented housing throughout Switzerland are drawn randomly from the data base. The questionnaire is completed by the landlords and the prices of the housing are estimated on its base after adjusting for quality and age of the housing.

**Medical care.** The CPI includes medical expenses, i.e. treatment, medications, dental care, hospitalization.

**Interest, credit charges and taxes.** Income taxes and social security payments, pension fund payments, medical insurance payments are not the part of the CPI.

**Missing prices and quality adjustments.** The last registered price of missing good is reported until the new one appears. This technique aims to reduce the volatility of the CPI.

There are four techniques for replacement of the articles which completely disappear from the consumption basket, 1) Direct comparison. 2) Chain method, it is mostly applied to the technological items. Statisticians need to find the item which is close in functionality to the one having disappeared. 3) Direct quality adjustment. This method is applied to the vehicles and computers. 4) If none of three above methods works the product is removed from the basket and the new one is introduced.

**Seasonal items.** The last registered price of seasonal good is reported until the new one appears.

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.70	0.32	0.07	0.34	0.02	0.48
K.S. statistic	0.13	0.18	0.24	0.17	0.28	0.16
UK						
p-value	0.50	0.76	0.28	0.27	0.72	0.60
K.S. statistic	0.15	0.12	0.19	0.18	0.13	0.14
Eurozone						
p-value	0.24	0.70	0.41	0.16	0.81	0.63
K.S. statistic	0.19	0.13	0.17	0.21	0.12	0.14
Japan						
p-value	0.60	0.22	0.19	0.84	0.47	0.54
K.S. statistic	0.14	0.20	0.20	0.11	0.16	0.15
Switzerland						
p-value	0.02	0.35	0.87	0.39	0.03	0.49
K.S. statistic	0.29	0.17	0.11	0.17	0.27	0.16

Table A.1: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation deseasonalized with  $MA^8(\cdot; 4)$ , ( $m=8$ ,  $n=4$ ).

**Quality comments.** The index covers all households from May 1993. Prior to this date, urban households of wage and salary earners were covered.

## A.2 Tests of the parameters of MA operator

### A.2.1 $m=8$ , $n=4$ .

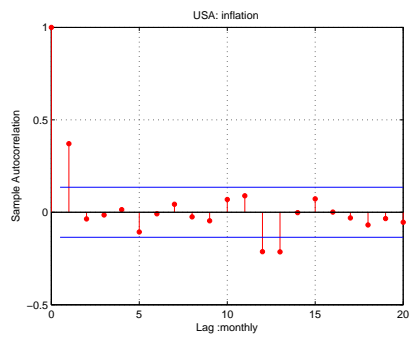
The model has a similar performance to the model with parameters  $m = 4$ ,  $n = 3$  if only ACF A.1 is considered. the Kolmogorov-Smirnov test results A.1 are slightly worse than for the model chosen for seasonal adjustment and the computational complexity is higher.

### A.2.2 $m=8$ , $n=3$

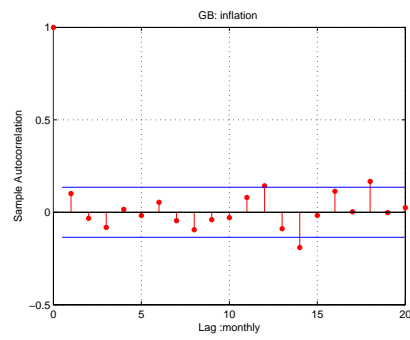
This model is recognized to be inferior to the model with parameters  $m = 4$ ,  $n = 3$  mainly due to the higher computational complexity of the model.  $m = 8$  implies more iterations for calculating the convolution. The ACF graph and the Kolmogorov-Smirnov tests performance is very similar (see figure A.2 and table A.2 for details).

### A.2.3 $m=8$ , $n=2$

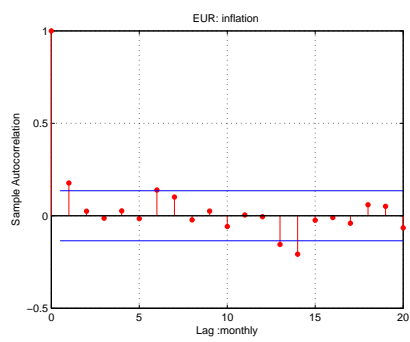
There are some significant autocorrelations introduced by seasonal adjustment (for example for Swiss inflation, see figure A.3). Introducing new



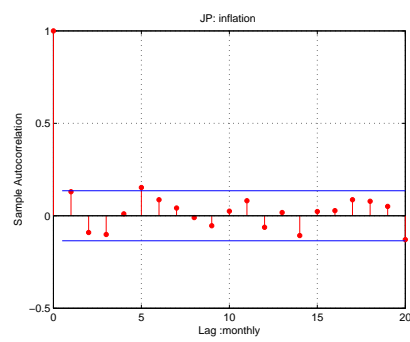
(a) USA



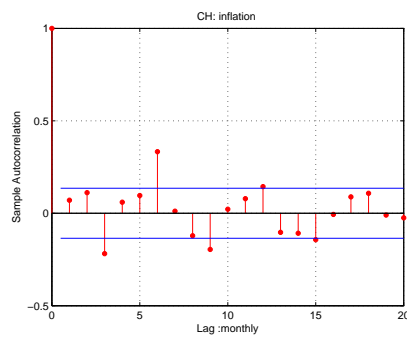
(b) UK



(c) Eurozone

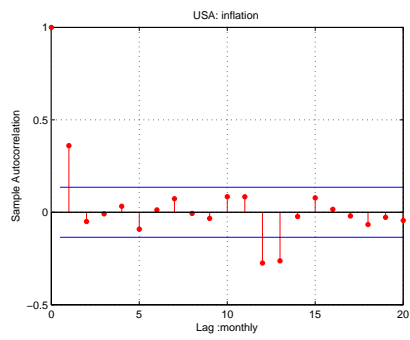


(d) Japan

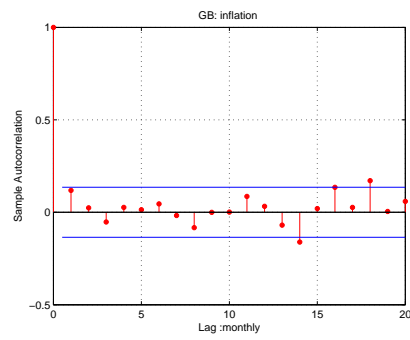


(e) Switzerland

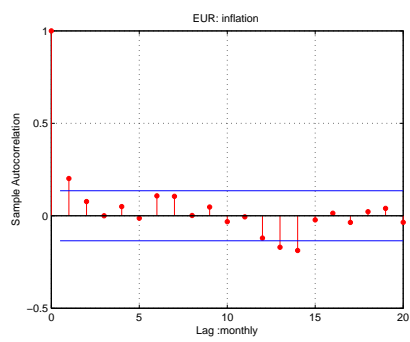
Figure A.1: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^8(\cdot; 4)$ , ( $m=8, n=4$ ).



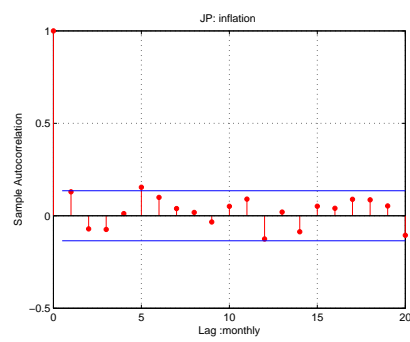
(a) USA



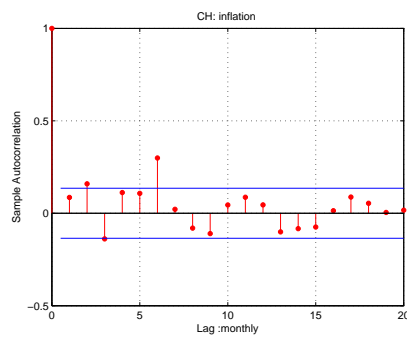
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure A.2: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^8(\cdot; 3)$  ( $m=8, n=3$ ).

USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.65	0.45	0.12	0.32	0.04	0.44
K.S. statistic	0.14	0.16	0.22	0.18	0.26	0.16
UK						
p-value	0.50	0.60	0.28	0.99	0.90	0.59
K.S. statistic	0.15	0.14	0.19	0.08	0.11	0.14
Eurozone						
p-value	0.42	0.67	0.41	0.24	0.39	0.63
K.S. statistic	0.16	0.13	0.17	0.19	0.17	0.14
Japan						
p-value	0.60	0.33	0.41	0.96	0.65	0.52
K.S. statistic	0.14	0.18	0.17	0.09	0.14	0.15
Switzerland						
p-value	0.09	0.63	0.87	0.70	0.09	0.50
K.S. statistic	0.23	0.14	0.11	0.13	0.23	0.15

Table A.2: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation, deseasonalized with  $MA^8(\cdot; 3)$ , ( $m=8$ ,  $n=3$ ).

features into the data series is undesirable and as for all the models with  $m = 8$ , the computational complexity is higher compared to the ones with  $m = 4$ .

#### A.2.4 $m=4$ , $n=4$

There are some significant autocorrelations left after seasonal adjustment, which are removed by the model with  $m = 4$ ,  $n = 3$ , see figure A.4. They include the 12<sup>th</sup> lag autocorrelation of the British and Swiss inflation and the 6<sup>th</sup> lag autocorrelation of the Eurozone inflation. In addition, the value of ACF of the Swiss inflation is greater for the 3<sup>rd</sup> and the 6<sup>th</sup> lags compared to the ones obtained after seasonal adjustment with the  $m = 4$ ,  $n = 3$  model. The two-sample Kolmogorov-Smirnov test results show that the null hypothesis was rejected at 5% significance level in two cases for the U.S. and in two cases for Japanese inflation time series.

#### A.2.5 $m=4$ , $n=2$

There are some significant autocorrelations introduced by the seasonal adjustment (for example for Swiss and Japanese inflation, see figure A.5), which allows us to exclude this model from the list of models that could be used for seasonal adjustment.

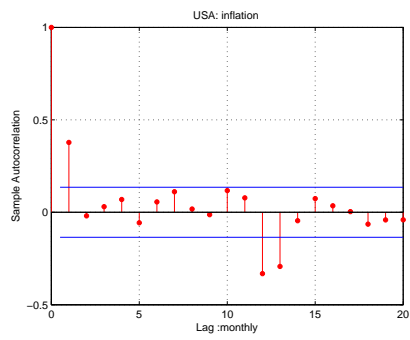
USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.66	0.78	0.41	0.15	0.04	0.81
K.S. statistic	0.14	0.12	0.17	0.21	0.26	0.12
UK						
p-value	0.38	0.52	0.56	0.93	0.64	0.31
K.S. statistic	0.17	0.15	0.15	0.10	0.14	0.18
Eurozone						
p-value	0.19	0.65	0.28	0.49	0.51	0.80
K.S. statistic	0.20	0.14	0.19	0.15	0.15	0.12
Japan						
p-value	0.41	0.58	0.28	0.50	0.48	0.84
K.S. statistic	0.17	0.14	0.19	0.15	0.16	0.12
Switzerland						
p-value	0.17	0.46	0.56	0.70	0.44	0.92
K.S. statistic	0.21	0.16	0.15	0.13	0.16	0.10

Table A.3: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation deseasonalized with  $MA^8(\cdot; 2)$ , ( $m=8$ ,  $n=2$ ).

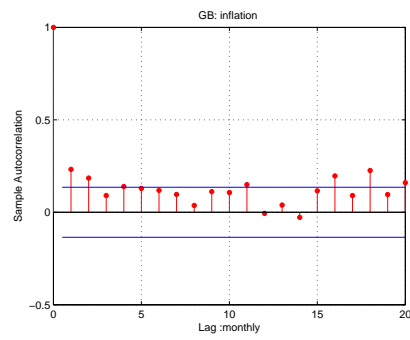
USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.52	0.15	0.01	0.32	0.02	0.44
K.S. statistic	0.15	0.21	0.30	0.18	0.28	0.16
UK						
p-value	0.32	0.45	0.19	0.39	0.72	0.60
K.S. statistic	0.18	0.16	0.20	0.17	0.13	0.14
Eurozone						
p-value	0.06	0.70	0.41	0.24	0.70	0.33
K.S. statistic	0.24	0.13	0.17	0.19	0.13	0.18
Japan						
p-value	0.60	0.14	0.07	0.53	0.45	0.36
K.S. statistic	0.14	0.21	0.24	0.15	0.16	0.17
Switzerland						
p-value	0.02	0.25	0.87	0.17	0.03	0.50
K.S. statistic	0.29	0.19	0.11	0.21	0.27	0.15

Table A.4: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation, deseasonalized with  $MA^4(\cdot; 4)$ , ( $m=4$ ,  $n=4$ ).

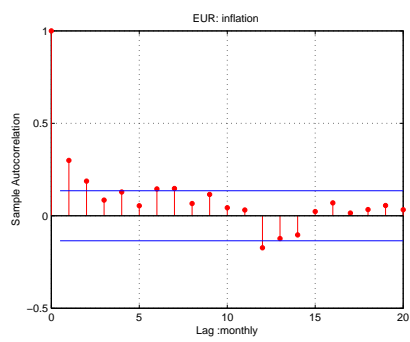




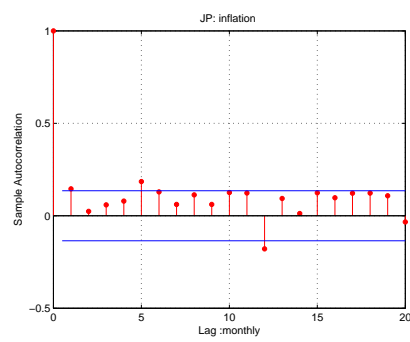
(a) USA



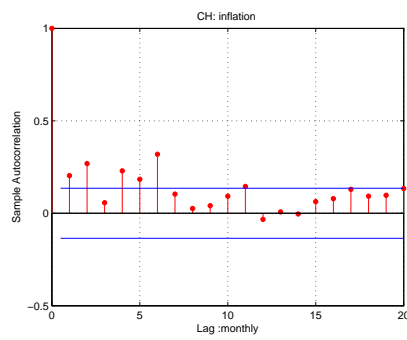
(b) UK



(c) Eurozone

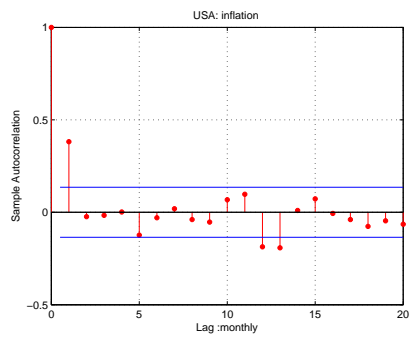


(d) Japan

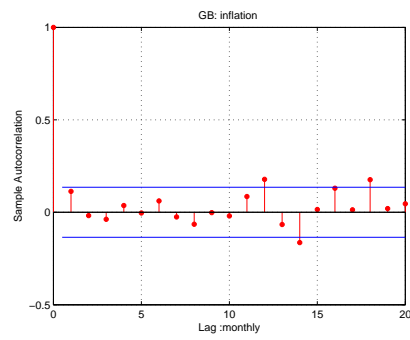


(e) Switzerland

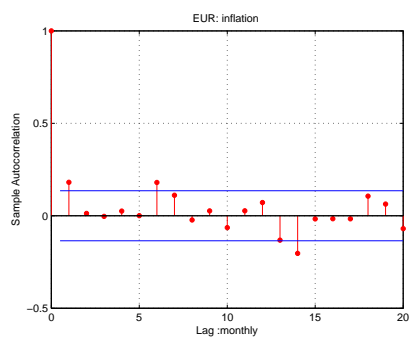
Figure A.3: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^8(\cdot; 2)$ , ( $m=8$ ,  $n=2$ ).



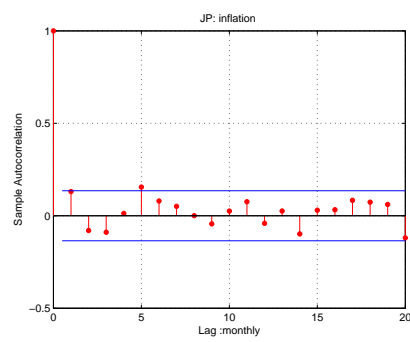
(a) USA



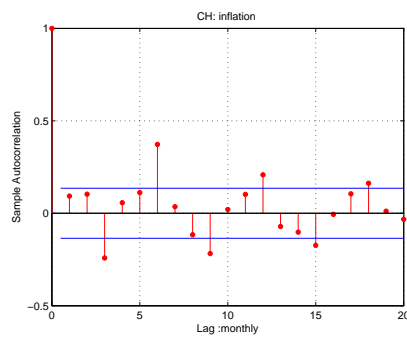
(b) UK



(c) Eurozone

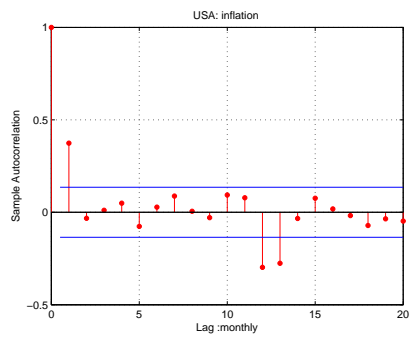


(d) Japan

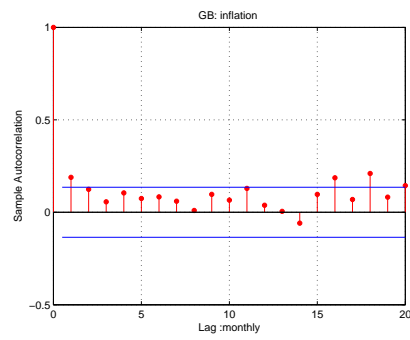


(e) Switzerland

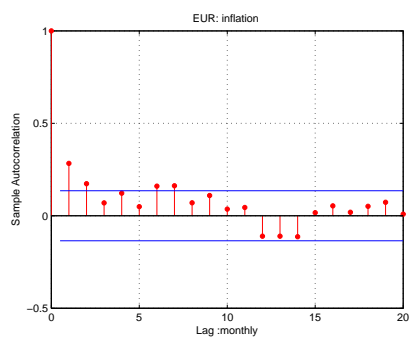
Figure A.4: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^4(\cdot; 4)$ , ( $m=4$ ,  $n=4$ ).



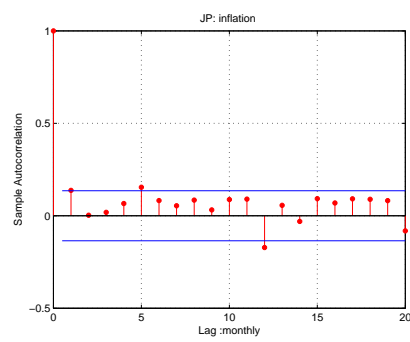
(a) USA



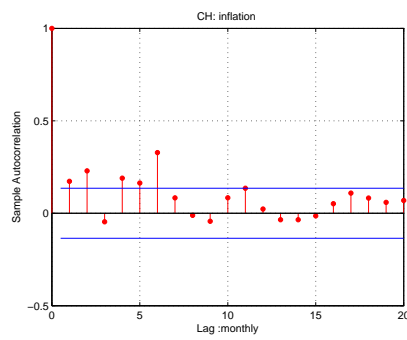
(b) UK



(c) Eurozone



(d) Japan



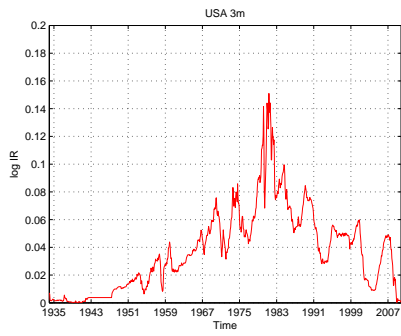
(e) Switzerland

Figure A.5: Autocorrelation plots of seasonally adjusted inflation deseasonalized with  $MA^4(\cdot; 2)$ , ( $m=4$ ,  $n=2$ ).

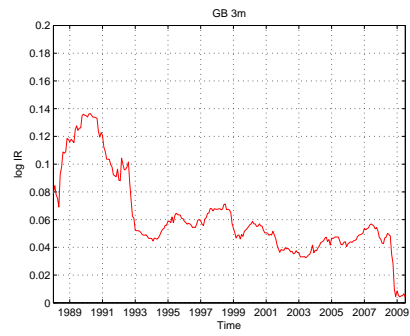
USA	q1,q2	q1,q3	q1,q4	q2,q3	q2,q4	q3,q4
p-value	0.65	0.77	0.19	0.15	0.02	0.82
K.-S. statistic	0.41	0.39	0.74	0.16	0.41	0.39
UK						
p-value	0.38	0.62	0.41	0.99	0.91	0.60
K.-S. statistic	0.17	0.14	0.17	0.08	0.10	0.14
Euro zone						
p-value	0.20	0.81	0.41	0.36	0.62	0.81
K.-S. statistic	0.20	0.12	0.17	0.17	0.14	0.12
Japan						
p-value	0.68	0.43	0.41	1.00	0.24	0.68
K.-S. statistic	0.13	0.16	0.17	0.08	0.19	0.13
Switzerland						
p-value	0.09	0.33	0.72	0.70	0.20	0.93
K.-S. statistic	0.23	0.18	0.13	0.13	0.20	0.10

Table A.5: Results of the two-sample Kolmogorov-Smirnov test performed on quarterly samples of seasonally adjusted inflation deseasonalized with  $MA^4(\cdot; 2)$ , ( $m=4$ ,  $n=2$ ).

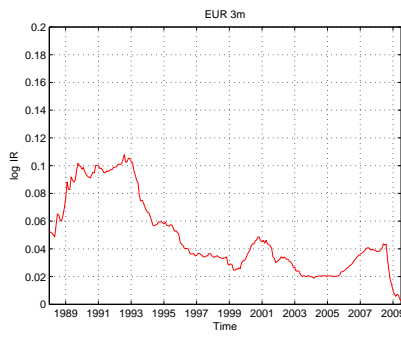
### A.3 Graphs of the inflation and interest rates time series



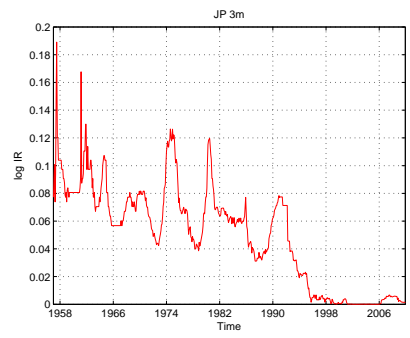
(a) USA



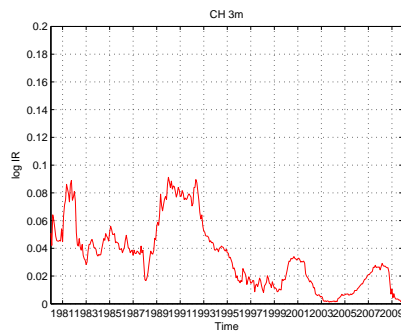
(b) UK



(c) Eurozone

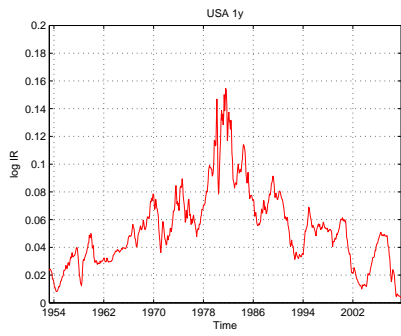


(d) Japan

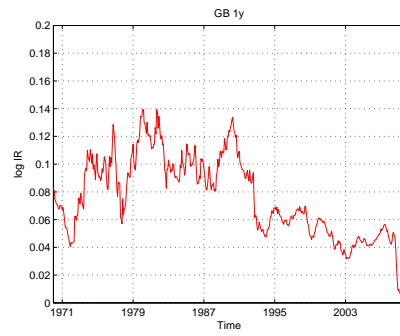


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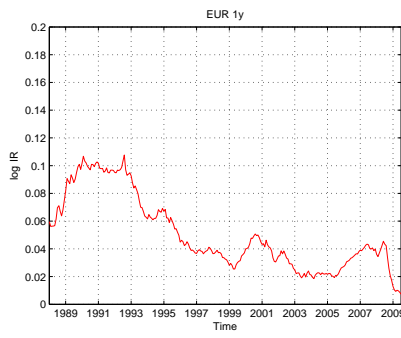
Figure A.6: 3 months logarithmic interest rates.



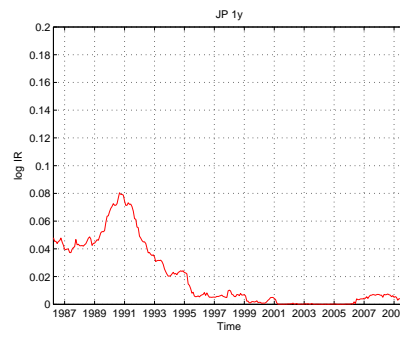
(a) USA



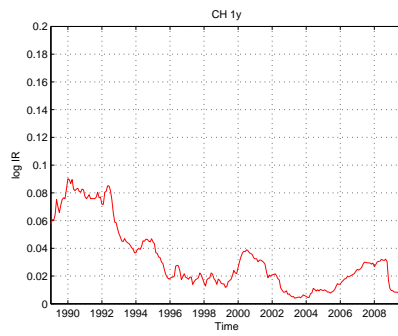
(b) UK



(c) Eurozone



(d) Japan



(e) Switzerland

Figure A.7: 1 year logarithmic interest rates.

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