ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Influence of LPPL traders on financial markets

Master Thesis

Daniel Philipp

February 2, 2015

Advisors: Prof. Dr. Didier Sornette, Matthias Leiss

Department of Management, Technology and Economics, ETH Zürich

Abstract

We have studied an agent-based model proposed by Kaizoji et al. [16] that was designed to investigate bubbles on financial markets. The model assumes two types of agents: rational traders that invest a constant fraction of their wealth in stocks and noise traders that rely in their investment decision on social imitation and past price performance. We found that, using this model, it is possible to simulate markets that exhibit important statistical regularities that resemble those of real markets, namely volatility clustering and a slowly decaying autocorrelation function of absolute returns. Furthermore, the emergence of bubbles and their connection to the parameters that control noise trader behavior have been established and verified in simulations. We also found that the model is able to generate bubbles with a realistic duration, i.e. a length of roughly one year.

Building upon this model, we have motivated and proposed another trader type whose agents use log-periodic power laws (LPPL) to predict bubbles. A simple bubble detection algorithm was developed with a corresponding investment strategy balancing full information and computation time.

The main goal of this work was to quantify the impact of an LPPL-powered strategy employed by some market participants on the whole market and in particular on bubble development. To this end, we simulated markets with LPPL investors of three different initial wealth levels. For all wealth levels, the LPPL investors on average increased the height of the price bubble peak whilst having little to no influence on the point in time that the peaks emerged. The increase was of the same order of magnitude as the fraction of the overall wealth on the market that was allocated to the dragon hunters. The dragon hunters were unable to systematically outperform the other traders. Actually, the larger their share of the overall wealth, the poorer they performed. This may be explained by the simplicity of their investment strategy.

0. Contents

Contents **iii**

1 Introduction **1**

2 Theory **4**
2.1 Stylized facts of financial markets 4
2.2 Agent-based model for financial markets 5
2.3 Log-periodic power law (LPPL) 10
2.3.1 General idea 10
2.3.2 Short derivation of the log-periodic power law . . . 12
2.3.3 Fitting procedure 14
2.4 Extension of the agent-based model with LPPL trader . . . 14
2.4.1 Bubble detection & investment decision 15
2.4.2 Modified dynamical equations 16

3 Methods & Results **20**
3.1 Implementation & setup 20
3.1.1 `MATLAB` implementation 21
3.1.2 Chosen simulation parameters 24
3.2 Influence of κ on bubble formation 27
3.3 Filtering of simulations 29
3.3.1 Original model 29
3.3.2 Extended model 35
3.4 Immediate impact of investment 36
3.5 Comparison of bubbles 39
3.5.1 Bubble measures 39
3.5.2 Comparison results 41

4 Conclusion & Outlook 46

5 Acknowledgements 49

Bibliography 50

1. Introduction

Financial bubbles are occur naturally on most financial markets. They can be characterized as a moment in time in which an asset prize widely exceeds its intrinsic value. Throughout history, there have been many famous examples; the earliest well-documented bubble, the so-called *tulip mania*, dates back to the Dutch Golden Age in the 17th century where the prices for tulips rose to about 2000 % of their previous value and crashed again in a matter of months [29]. Even though the effect of this crash on the general Dutch economy at the time is debated [9, 10], this steep rise in price remains unbelievable: at the peak of the bubble, certain kinds of tulips were worth more than a house. However, there are many other examples where the burst of a bubble in one market led to considerable consequences for the whole economy. As a recent famous example, the US sub-prime mortgage crisis which is generally considered to have triggered the Global Financial Crisis of 2007-2008 springs to mind.

According to mainstream thinking, bubbles are often recognized only after they have burst and cannot be predicted. In the end of the 1990s, the *Johansen-Ledoit-Sornette (JLS) model* was developed to mathematically (using log-periodic power laws (LPPL)) describe the price evolution prior to a crash and provide a method to predict the time that the bubble bursts. The idea is that a market undergoing unsustainable growth (i.e. a market in a bubble) follows a certain type of oscillations superposed on faster-than-exponential growth which can be picked up by fitting financial data to the model. Both the unsustainable (or *super-critical*) growth and the oscillations are artifacts of social imitation in the hierarchy of peer groups [40]. This model allows a detection of a bubble using the price dynamics *before* the burst of the bubble.

In principle[1], it is possible to predict the time of the crash of financial markets using the JLS model. If one knows whether an asset is overvalued or not and the time until which its value will continue to grow, one can easily imagine an investment strategy that could possibly make a lot of money: buy as soon as the bubble starts and go short before the bubble will (most likely) burst. Investors could exploit the knowledge they gain from bubble analysis to beat the market and make money. But how would this affect the market as a whole and the formation, length and height of a bubble?

The first idea that springs to mind to answer this question would be to use real market data, develop an investment strategy for these investors and observe the investment decisions. This however, is a little short-sighted: the influence of the investment decisions would not be included in the pricing of the traded asset and no impact can be observed.

Another approach is to use *agent-based modeling (ABM)* to find a model to simulate a financial market. In agent-based modeling – as the name suggests – many agents interact with each other according to some predefined rules. Additional investors can then be added by formulating how they interact with the other investors. This makes ABMs especially suitable to answer our question. Kaizoji et al. [16] proposed an agent-based model for financial markets with a special focus on bubble formation. The main task is then to extend this model by the new trader type, simulate the original and the extended model with the same parameters and compare the resulting asset prices. The answers that this analysis might give will remain a nice gimmick if they are not somehow connected to real markets. But how do we know whether the resulting simulations are anywhere close to reality?

Financial markets are very complicated systems with many drivers. However, there are statistical regularities that have been observed on most financial markets in the past, so-called *stylized facts*. A simulation of a financial market should at least exhibit these stylized facts to allow inference for real markets.

In this thesis, we will try to get an understanding of the impact that investors using the JLS model for their investment decision have on the

[1]In fact, this has already been achieved. The US real estate bubble as an example among many has been identified as such with the JLS model before the crash. The Financial Crisis Observatory, established to observe and predict financial bubbles, publishes reports of the monitored assets every couple of months (http://www.er.ethz.ch/research/fco).

frequency, length and height of price bubbles. Sections 2.1 and 2.2 will give a summary of the most important points of the agent-based model and its applicability to financial markets, followed by a short overview of the JLS model and the fitting procedure of the resulting log-periodic power law in section 2.3. The proposed algorithm with which the new investors make their investment decision is covered in section 2.4. The implementation of the model and the conducted simulations are described in section 3.1. Section 3.3 covers the obstacles that were encountered in the simulations and how they were mitigated before the actual bubble comparison in section 3.5. We will conclude in chapter 4.

2. Theory

First we will explain the stylized facts of financial markets and the implication this has for market simulations in section 2.1. The agent-based model that is used to simulate the financial market has been outlined by Kaizoji et al. [16] and will be introduced in the following section 2.2. It relies on the assumption of two types of traders or investment strategies. For this thesis, the model was extended to include a third trader that uses the *Johansen-Ledoit-Sornette (JLS) model* to detect times when the stock is overpriced and tries to exploit this information to maximize their profits. The JLS model is described in section 2.3. Details on the implementation of the model extension can be found in section 2.4.

2.1 Stylized facts of financial markets

According to the semi-strong form of the efficient market hypothesis, the autocorrelation of asset returns should be zero for time lags greater than zero [7]. Non-zero autocorrelation of a value for a certain time lag Δt generally implies that it is possible at time t_0 to know with a probability higher than chance what the value is at time $t_0 + \Delta t$. This would enable investors to consistently beat the market. In other words, all information available at a time t is incorporated in the asset price and it is thus impossible to correctly guess the price at time $t+1$. Formally, the autocorrelation of returns is defined as follows:

$$\rho_\tau = \frac{\mathrm{Cov}(r_t, r_{t+\tau})}{\sigma_t \sigma_{t+\tau}} = \frac{\mathrm{E}\left[(r_t - \mu_t)(r_{t+\tau} - \mu_{t+\tau})\right]}{\sigma_t \sigma_{t+\tau}}. \tag{1}$$

Here r_t denotes the returns at time t, σ_t the standard deviation, E the expected value operator and μ the mean value of returns. Even if asset returns are nearly uncorrelated (there are some statistically significant correlations that however cannot be exploited with trading strategies [3]), they are not i.i.d. stochastic processes. The degree of fluctuations is not uniform and there are more turbulent and more tranquil periods.

This lack of i.i.d. properties is furthermore reflected in powers of absolute autocorrelations which tend to exhibit much higher and longer-lasting autocorrelations than expected. This means that the extent of fluctuations is somehow connected, i.e. that periods of large fluctuations tend to be followed by periods of small fluctuations and vice versa [23]. Note that this only connects the *magnitude* of fluctuations and not their sign. Autocorrelations of absolute returns can thus be used as a measure of volatility and the volatility can be predicted well. This phenomenon is known as *volatility clustering*.

The hyperbolic decay of absolute autocorrelations has been found to be a stylized fact in financial markets [3] and therefore a good model for a financial market should be able to reproduce this behavior. It was found that the agent-based model described in the next section exhibits this behavior for some simulations [26]. We will discuss this further in section 3.3.

2.2 Agent-based model for financial markets

The idea of defining microscopic interactions between autonomous agents and studying the macroscopic implications these interactions have on the whole system dates back to the 1940s when mathematicians Stanislaw Ulam and John von Neumann proposed what is now known as a *cellular automaton* [38]. With the increase of computational power, agent-based modelling found its way into other sciences in the 1970s with Thomas Schelling's *segregation model* [27].

In finance, the concept of *noise traders* or *positive-feedback investors*, introduced by Black [1] and Kyle [19] in the 1980s, was one of the first attempts of using agent-based modeling for financial markets. They introduced the term to describe investors that do not base their buy and sell decisions on fundamental data but rather on price patterns and trends [32].

In 1990, de Long et al. [4, 5] were the first to propose an agent-based model which used the concept of noise traders and their positive feedback to study the formation of bubbles. This has been followed by numerous other work in the past 20 years. An overview of agent-based models in economics can be found in [32].

Kaizoji et al. [16] propose a model in which investors are assumed to invest their wealth in either a risky asset with dividends d_t at time t (e.g. stocks) or a risk-free asset with a constant return rate R_f (e.g. bonds). Furthermore, it is assumed that there are two types of traders.

The first group consists of rational investors that aim to maximize their expected utility of next period wealth (cf. [3, 22]). It is shown that under certain assumptions (i.a. constant relative risk aversion) this leads to a strategy of investing a constant fraction x^{r} (x in the original paper) of the rational investors' wealth in the risky asset.

To account for the effect of social imitation in stock markets the second trader type, the noise traders, makes their decision to go long/short by evaluating the stocks' past performance (price momentum H_t) and the opinion of the other noise traders (opinion index s_t). The influence these quantities have on the noise traders' decision making is governed by κ_t which follows a stochastic process to account for the ever-changing macroeconomic and sociopolitical context of the market.

The detailed mathematical implementation of both traders, their interaction and explanation of all parameters can be found in Kaizoji et al. [16]. Unless explicitly stated, we will use the same naming conventions for all variables. We will only reiterate the dynamical equations that follow from balancing excess demand for the risky asset from both investor types to satisfy the market clearing condition:

Dynamics of noise traders' opinion index

$$s_t = \frac{1}{N_{\mathrm{n}}} \left(\sum_{k=1}^{\frac{1+s_{t-1}}{2}N_{\mathrm{n}}} \left[1 - 2\zeta_k(p^+_{t-1})\right] + \sum_{j=1}^{\frac{1-s_{t-1}}{2}N_{\mathrm{n}}} \left[2\xi_j(p^-_{t-1}) - 1\right] \right), \tag{2}$$

$$p^{\pm}_{t-1}(s_{t-1}, H_{t-1}) = \frac{1}{2}\left[p \mp \kappa\left(s_{t-1} + H_{t-1}\right)\right], \tag{3}$$

Dynamics of risky asset price

$$\frac{P_t}{P_{t-1}} = \frac{\sum\limits_{j=\mathrm{n,r}} x^j_t \left[\left(1+R_f\right)\left(1 - x^j_{t-1}\right) + \left(r + \sigma_r u_t\right) x^j_{t-1}\right] W^j_{t-1}}{\sum\limits_{j=\mathrm{n,r}} x^j_{t-1}\left(1 - x^j_t\right) W^j_{t-1}}, \tag{4}$$

Wealth dynamics (rational investors & noise traders)

$$\frac{W_t}{W_{t-1}} = x^{\mathrm{r}} \left(\frac{P_t}{P_{t-1}} + (r + \sigma_r u_t) \right) + (1 - x^{\mathrm{r}}) \left(1 + R_f \right), \tag{5}$$

$$\frac{W_t^n}{W_{t-1}^n} = \frac{1 + s_{t-1}}{2} \left(\frac{P_t}{P_{t-1}} + (r + \sigma_r u_t) \right) + \frac{1 - s_{t-1}}{2} \left(1 + R_f \right), \tag{6}$$

Momentum of risky asset price (price momentum)

$$H_t = \theta H_{t-1} + (1 - \theta) \left(\frac{P_t}{P_{t-1}} - 1 \right). \tag{7}$$

Table 1 list the meaning of all quantities in detail. The lower index t indicates that the quantity changes with time t. These equations can be used to simulate the market dynamics for a number of time steps.

Even though the resulting model relies on random number generation, some parameters in the model can be singled out to control the emergence, frequency and length of bubbles. Since the formation and dynamics of bubbles is very much connected to the parameter κ_t [16, 26] we shall examine it closer. It is given by

$$\kappa_t = \kappa_{t-1} + \underbrace{\eta(\mu_\kappa - \kappa_{t-1})}_{\text{mean-reverting}} + \underbrace{\sigma_\kappa v_t}_{\text{random walk}}, \tag{8}$$

which describes an Ornstein-Uhlenbeck process, a mean-reverting Wiener process. The last summand in the equation is white noise with σ_κ and v_t being the standard deviation and standard i.i.d. random variables with distribution $N(0,1)$, respectively. The mean is denoted by μ_κ and the mean reversion rate by $\eta > 0$,

$$\eta = \frac{1}{\Delta T} \log \left(\frac{\kappa_0 - \mu_\kappa}{p - \mu_\kappa} \right). \tag{9}$$

The parameter κ_0 quantifies the long term behavior of κ, a stationary probability distribution $\sim N(\mu_\kappa, \sigma_\kappa / \sqrt{2\eta(\kappa_0)}) = N(\mu_\kappa, \sigma_\kappa^{\text{stat}}(\kappa_0))$. If we set it to $\kappa_0 = \mu_\kappa + 2\sigma_\kappa^{\text{stat}} = \mu_\kappa + 2 \cdot 0.1p$, we can assure that the Ornstein-Uhlenbeck process has a standard deviation of $0.1p$ and that a deviation of κ_t two standard deviations above μ_κ will revert within ΔT days [16]. We see that the average reversion time ΔT and the mean reversion level μ_κ play an important role in determining κ (cf. [26] for more details). Figure 1 visualizes the connection between the parameters and η.

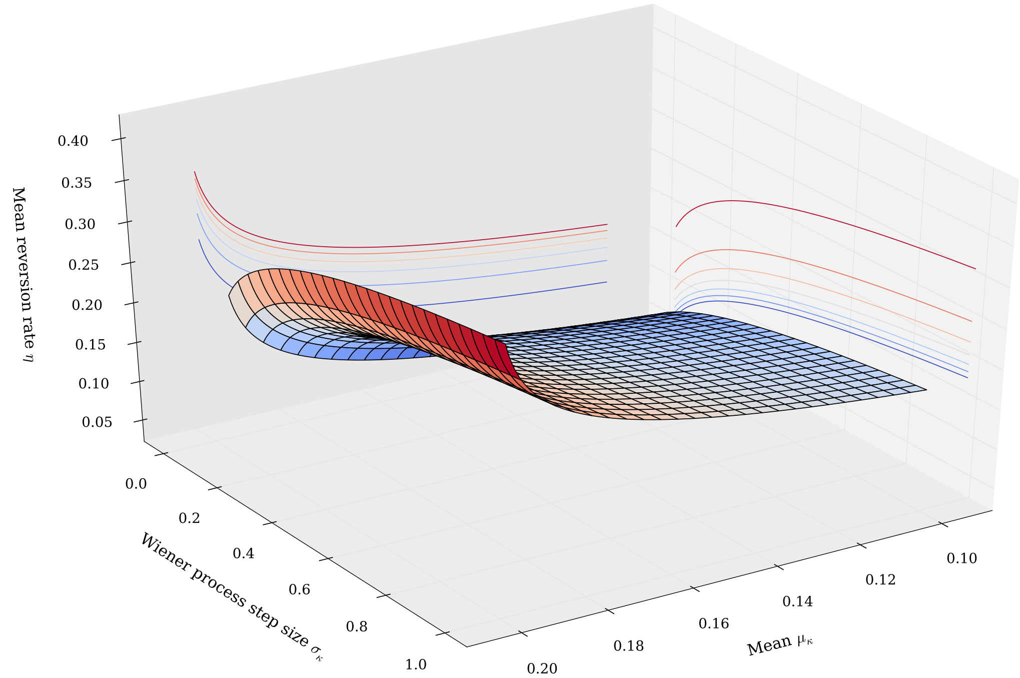

Figure 1: This plot shows the dependence of the mean reversion rate η on the mean μ_κ and Wiener process step size σ_κ for $\Delta T = 20$. As expected, choosing μ_κ close to p leads to a large mean reversion rate η which means shorter critical κ sequences. Note that changing ΔT changes the z-scale linearly; the qualitative behavior.

Using the above construction, the Wiener process step size σ_κ is connected to ΔT and will thus be not considered here as an independent parameter controlling κ (also see [16]). This is done because the effect of changing μ_κ is much easier visualized than the effect of changing of the Wiener process step size.

As can be seen in equation (3), κ reflects the impact that herding and price momentum have on the probability $p^{\pm}$ for bullish/bearish noise traders, respectively. Following the theoretical considerations in [16], we can approximate the price equation (4) for special cases during a bubble to:

$$\frac{P_t}{P_{t-1}} = 1 + \text{const} \cdot (s_t - s_{t-1}) + \mathcal{O}\left(r, R_f, (s - s_0)^2\right). \tag{10}$$

This is equivalent to exponential growth of the price with s. Bubble regimes in financial markets have been found to exhibit super-exponential growth in time in the formation phase of a bubble. This would be the case if s increases with time as well. The opinion index s along with the price momentum H influence the dynamics via equation (3) through the probability $p_t^{\pm}$ of an noise trader to be invested in the risky/risk-free asset.

The expected value of s is given by (see ch. 2.2.3 in [16])

$$\mathbb{E}[s_t] = (1 + \kappa - p)s_{t-1} + \kappa H_{t-1}. \tag{11}$$

For $\kappa > p$, this indeed means that the opinion index grows with time. The stochastic fluctuations in the latter term in equation (8) can drive κ into this regime, while the η-term drives it back to $\mu_\kappa < p$. In the unstable regime for κ above p, the price thus grows super-exponentially.

The mathematical description of the noise traders' social herding is based on the well-known *Ising model*[2] that is used in physics to describe spin systems and magnetic phenomena. Thus, it is also possible to understand the role that κ plays in the formation of bubbles by looking at results that have been well-established in physics. The opinion index s is analogous to the magnetization in the Ising model which moves in a U-shaped potential well in the sub-critical phase. The emergence of bubbles is then the same as a phase transition where the shape of the potential changes from U-shaped to double-U-shaped [28]. Figure 2 shows a schematic plot of the potential in the different regimes and its connection to the presented agent-based model. The experimental validation of these theoretical considerations is described in section 3.2.

Since ΔT controls the speed with which the process reverts to its mean μ_κ, we can interpret this parameter as the average bubble length. The frequency or ease with which bubbles will emerge is controlled by setting the mean μ_κ further or closer to p. Figure 3 shows an example of an Ornstein-Uhlenbeck process with the quantities p and μ_κ plotted as well.

Using this knowledge, it is possible to explore whether this model is suitable to obtain realistic price data. It was found that the stylized facts of real financial markets mentioned in chapter 1 were fulfilled for certain parameters [26]. Thus, the model can be used to study the impact of the new investors on bubble dynamics.

[2]Named after physicist Ernst Ising (1900-1998) [18], the Ising model is a simple model in statistical mechanics to model ferromagnetism. Its simplicity and analytical solvability in two dimensions led to applications in many other fields of study.

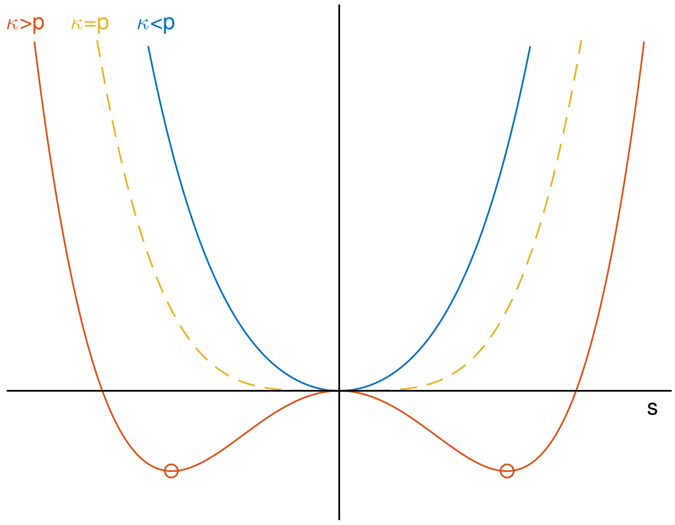

Figure 2: Schematic plot of the potential of the opinion index in critical (orange), sub-critical (blue) and transition (yellow) regimes. Note that at the phase transition, the number of minima changes from 1 to 2. For a stationary state, s moves to either the positive or negative minimum (circled in plot) thus giving rise to positive or negative bubbles.

2.3 Log-periodic power law (LPPL)

The Johansen-Ledoit-Sornette model [13, 15, 12, 30] was developed to describe the dynamics of financial markets during bubbles and crashes. Similar to the agent-based model it is assumed that there are rational traders and noise traders who exhibit herding behavior that can destabilize the asset price [8]. In the following, we will shortly summarize the general idea behind the model in section 2.3.1, give a brief derivation of the log-periodic power law equation and describe how to fit the model to data in section 2.3.3.

2.3.1 General idea

Generally, a bubble can be understood as a deviation from the fundamental value of an asset. During a positive bubble there is an excessive demand and during a negative bubble there is disproportionate selling [33]. The fundamental value of an asset however is difficult to precisely define and thus this only shifts the problem of defining a bubble instead of solving it.

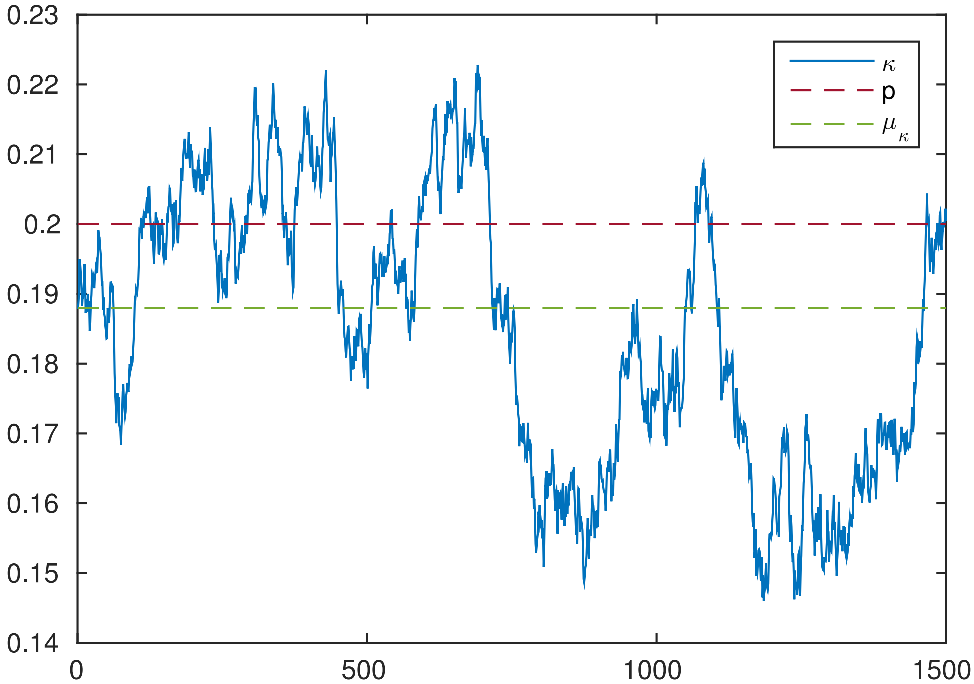

Figure 3: Graph showing κ, an Ornstein-Uhlenbeck process. Here the parameters are $\Delta T = 180$, $\mu_\kappa = 0.188$. The mean μ_κ to which the process reverts to is plotted in green, the red line denotes p, the border between critical and sub-critical regime. Values $\kappa > p$ are correlated with formation of price bubbles [26].

A different approach is to think about the mechanism behind the rise in price. As innovation or ground-breaking new technology leads to investments by smart investors the price goes up and more investors follow. This positive feedback leads to a super-exponential growth in price instead of a fixed growth rate (proportional growth). Super-exponential growth is not sustainable and is bound to undergo a regime change before the singularity in finite time since an infinitely large price is not sensible in reality [17].

The Johansen-Ledoit-Sornette model (JLS model) has been developed to describe the evolution of the price in this regime of unsustainable growth [35]. In a bubble, the price undergoes certain oscillations that reflect human grouping patterns [33]. It was shown [40] that these social hierarchies manifest themselves in log-periodic oscillations of the price with decreasing amplitudes.

These oscillations are superposed onto the super-exponential growth in a bubble. However, the model aims to describe only the regime of unsustainable growth due to social imitation. At the singularity (or critical time t_c), the mathematical description has to break down and we enter a new regime.

All these considerations can be summed up by the following *log-periodic power law (LPPL)* for the expected value of the log-price up to t_c:

$$\mathbb{E}\left[\log P_t\right] = \underbrace{A + B(t_c - t)^m}_{\text{positive feedback}} + \underbrace{C(t_c - t)^m}_{\text{decreasing amplitude}} \underbrace{\cos\left(\omega \log(t_c - t) - \phi\right)}_{\text{log-periodic oscillations}}. \tag{12}$$

We can plug in $t = t_c$ to see that A gives the expected log-price at the time of the crash. The parameters B and C quantify the magnitude of the power law acceleration and the amplitude of the log-periodic oscillations, respectively. The exponent m specifies the super-exponential growth and it holds that $0 < m < 1$ [15]. The angular frequency ω is related to the aforementioned social hierarchies while ϕ quantifies the oscillation time scale. We will give a short derivation in the next section.

Equation (12) by construction only applies to times of unsustainable growth. For fitting of financial market data it is thus important to choose the time interval $[t_1, t_2]$ to find suitable parameters. We will address obstacles or troubles one encounters while fitting in section 2.3.3.

2.3.2 Short derivation of the log-periodic power law

The underlying assumption of the JLS model [15] is that during a bubble the price P_t follows a jump diffusion process:

$$\mathrm{d}P_t = \mu(t)P_t\mathrm{d}t - \sigma(t)P_t\mathrm{d}W_t - \kappa P_t\mathrm{d}j. \tag{13}$$

Here, $\mu(t)$ is a time-varying drift, $\sigma(t)$ the volatility and $\mathrm{d}W_r$ is the infinitesimal standard Wiener process step size. The last term basically models the crash. The quantity j denotes a jump process with its value jumping from 0 to 1 at the crash while $\kappa \in (0,1)$ quantifies the amplitude of the jump. The jump process is governed by the crash hazard rate $h(t)$ which denotes the probability per unit time that a crash will happen under the condition that it has not yet happened:

$$\mathrm{d}j = \begin{cases} 1, & \text{with probability } h(t)\mathrm{d}t \\ 0, & \text{with probability } (1 - h(t))\mathrm{d}t. \end{cases} \tag{14}$$

For the case that the crash has not yet occured, we can calculate the expectation to

$$\begin{aligned}\mathbb{E}_t\left[\frac{\mathrm{d}P_t}{P_t}\right] &= \mu(t)\mathrm{d}t - \kappa\mathbb{E}_t[\mathrm{d}j] \\ &= \mu(t)\mathrm{d}t - \kappa(1\cdot h(t) + 0\cdot(1-h(t))) \\ &= (\mu(t) - \kappa h(t))\mathrm{d}t\end{aligned} \tag{15}$$

The no-arbitrage condition [2] states that the price process should be a martingale, i.e. $\mathbb{E}_t[\mathrm{d}P_t] = 0$. It follows that

$$\mu(t) = \kappa h(t), \tag{16}$$

relating volatility μ with the weight of the jump κ times the crash hazard rate $h(t)$. This can intuitively be understood as a compensation of higher risk with higher returns. We can rewrite equation (13) before the crash as

$$\mathrm{d}P_t = \kappa h(t) P_t \mathrm{d}t + \sigma(t) P_t \mathrm{d}W_t. \tag{17}$$

Integrating by separation of variables and taking the expectation yields:

$$\mathbb{E}\left[\log\left(\frac{P_t}{P_{t_0}}\right)\right] = \kappa\int_{t_0}^{t} h(t')\ \mathrm{d}t'. \tag{18}$$

Here we used that W_t describes a standard Brownian motion. The higher the probability of a crash in the above equation, the faster the price must increase to satisfy the martingale condition. So far, we have not made any assumptions about $h(t)$. To have power law growth in price, the simplest form is

$$h(t) = B(t_c - t)^{m-1}, \tag{19}$$

with $B > 0$ a constant and the critical point or theoretical date of the bubble death $t_c > 0$. The parameter m must satisfy $0 < m < 1$; otherwise the price would diverge when $t \to t_c$ without the bubble bursting. At time t_c, the bubble has the highest probability to burst. Plugging this form of $h(t)$ into equation (18), we can obtain the power law behavior of the price during the bubble. However, the crash hazard rate can be generalized by allowing complex exponents m which can be justified by considering hierarchical structures in financial markets. In Johansen et al. [15], the first-order expansion of the general solution for the crash hazard rate is given as:

$$h(t) \approx B(t_c - t)^{m-1} + C(t_c - t)^{m-1}\cos\left[\omega\log(t_c - t) + \phi\right]. \tag{20}$$

Plugging this expression into equation (18) leads to the log-periodic power law for the price

$$\mathbb{E}[\log(P_t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos\left[\omega \log(t_c - t) + \phi\right] \quad (21)$$

which is indeed equation (12).

2.3.3 Fitting procedure

Fitting equation (12) to financial data in a time window $[t_1, t_2]$ boils down to a multivariate non-linear optimization problem with a total of 7 parameters (3 linear and 4 non-linear). Solving these kinds of problems is non-trivial since local optimization algorithms may get trapped in local minima. The state of the art for these problems are meta-heuristic algorithms such as taboo search [12, 34] or genetic algorithm [11] which are often not stable. Filimonov & Sornette proposed a more robust and less resource-intensive calibration scheme for the LPPL model [8] which reduces the number of non-linear parameters to three (t_c, ω, m). To this end, equation (12) was rewritten and the resulting four linear parameters were slaved to the non-linear ones. Details can be found in [8]. This scheme was also used in this thesis to do all LPPL fitting.

The parameters in equation (12) are subject to some constraints which follow from the assumptions made in the model or emerge from analysis of many historical bubbles [14, 20, 21, 34, 37, 39]. These are among others:

$$0.1 \leq m \leq 0.9, \qquad 6 \leq \omega \leq 13, \qquad |C| < 1, \qquad B < 0. \quad (22)$$

Figure 4 shows an example of a fit of a log-periodic power law to real data from the Shanghai composite index using the above constraints on the parameters where the dotted lines in the plot mark the time window $[t_1, t_2]$ that was used for fitting. The fitting procedure is highly sensitive towards the choice of this window. This will be reflected in the strategy chosen by the LPPL trader in section 2.4.

2.4 Extension of the agent-based model with LPPL trader

The agent-based model (ABM) introduced in section 2.2 consists of two types of traders. The influence of traders using the LPPL model in their

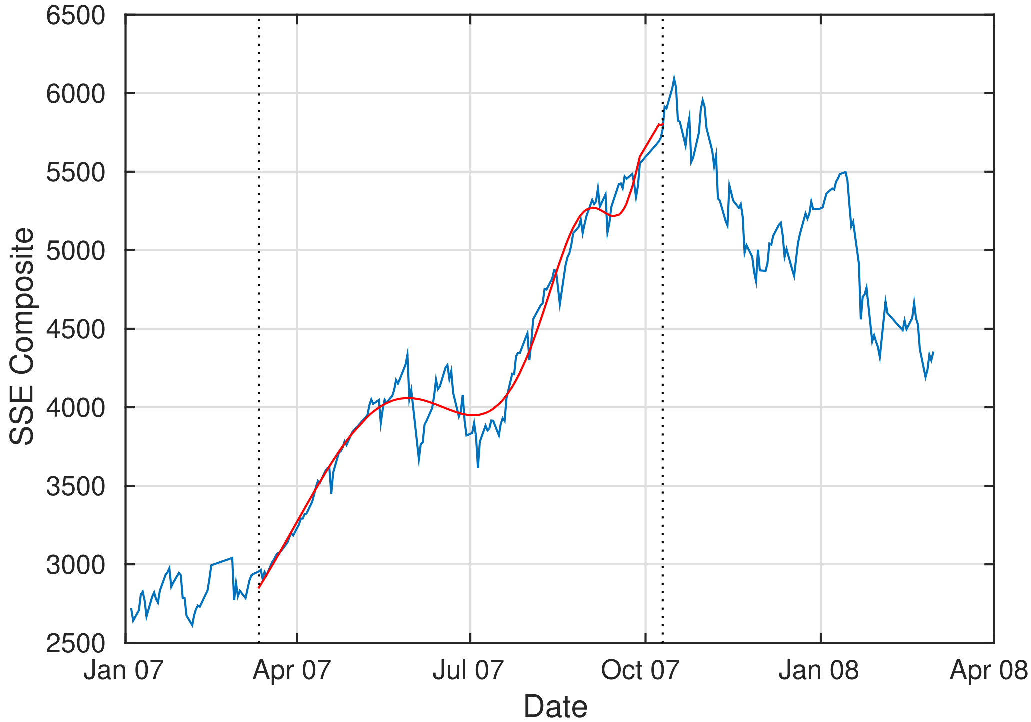

Figure 4: Real data from the Shanghai Composite Index from January 2007 to April 2008. The dotted lines mark the beginning and end of the fitting window (t_1 & t_2). The fit to the LPPL equation (12) is shown in red. The plot was reproduced using the same time windows as in [8]).

investment strategy can be investigated by extending the ABM. Since the simulated markets are reproducible by setting the random seed, the impact can be studied by simulating the same market with and without the LPPL investors. In the following, we will introduce this third investor type, their strategy, and the influence on the dynamic equations stated before. We will speak of *dragon hunters* following the idea that these traders try to "hunt down" dragon kings [31, 36], i.e. the emerging bubbles and potential crashes.

2.4.1 Bubble detection & investment decision

We choose as default strategy for non-bubble times that the dragon hunters partially behave like the rational investors in the current model, i.e. investing a fixed percentage in the risky asset[3]. In contrast to the rational investors, however, dragon hunters use their knowledge of the LPPL model to try finding and exploiting emerging bubbles.

A dragon hunter considers an asset overpriced as soon as fitting of the price to the LPPL model in any time window yields parameters that satisfy the constraints in equation (22). Since fitting every possible time window in every time step would prove to be too resource-intensive, dragon hunters only look at several intervals with different lengths. Furthermore, they check whether the time of the crash predicted by the model is far enough into the future to be safe from errors.

If they detect an LPPL signal, i.e. an emerging bubble, dragon hunters increase their investment in the risky asset to 100% of their wealth to profit from the superexponential growth in price before the bubble bursts. As soon as the signal is not picked up anymore in a re-evaluation of the strategy, dragon hunters revert to their previous behavior of investing a fixed percentage of their wealth in the risky asset. We will go into more detail regarding the actual implementation and chosen parameters in chapter 3.

2.4.2 Modified dynamical equations

Due to the two possible states of dragon hunters (bubble detected and no bubble), we need to find the dynamic equations (2) to (7) for these two cases. We define the dragon hunters' relative wealth in the risky asset at time t as x_t^{d} (recall that the rational traders relative wealth in the risky asset is x^{r}) and their wealth at time t as W_t^{d}. Basically, dragon hunters alternate their invested relative wealth between two values; this means that we have to keep track of the time step in the derivation. Other than that, they are the same as rational traders.

We can thus follow the derivation of the dynamical equations in Kaizoji et al. [16] and calculate the dragon hunters excess demand $\Delta D_t^{\mathrm{d}} := P_t X_t - P_t X_{t-1}$ analogously to the rational traders. Here X_t denotes the wealth invested in the risky asset at time t. If we denote the dividends

[3] In general, dragon hunters may also be prone to imitate, which translates to reacting to a mixed signal of opinion index s and the LPPL signal. This corresponds to noise traders working on nonlinear momentum as compared to the previous setup, where the noise traders use the opinion index and the price momentum to guide their decision. For simplicity, we shall start by neglecting social imitation.

at time t with d_t we find:

$$\Delta D_t^{\mathrm{d}} = x_t^{\mathrm{d}} W_{t-1}^{\mathrm{d}} \frac{(1 - x_{t-1}^{\mathrm{d}})(P_{t-1}(1 + R_f) - P_t) + x_{t-1}^{\mathrm{d}} d_t}{P_{t-1}}. \tag{23}$$

This equation is structurally identical to the excess demand for the rational traders (cf. equation 19 in [16]), however there is a time dependence in x^{d} due to the nature of the dragon hunter strategy. The dividends are modeled as a stochastic process, $d_t/P_{t-1} = r + \sigma_r u_t$, with u_t a series of standard i.d.d. random variables with distribution $N(0,1)$, σ_r the step size and r the expectation. Employing the market clearing condition $\Delta D_t^{\mathrm{r}} + \Delta D_t^{\mathrm{n}} + \Delta D_t^{\mathrm{d}} = 0$ leads to the price equation for the extended model:

Dynamics of risky asset (extended)

$$\frac{P_t}{P_{t-1}} = \frac{\sum\limits_{j=\mathrm{n,r,d}} x_t^j \left[(1 + R_f) \left(1 - x_{t-1}^j \right) + (r + \sigma_r u_t)\, x_{t-1}^j \right] W_{t-1}^j}{\sum\limits_{j=\mathrm{n,r,d}} x_{t-1}^j \left(1 - x_t^j \right) W_{t-1}^j} \tag{24}$$

This equation is the same as the original equation (4) except that there is one more term in the sum in both numerator and denominator accounting for the dragon hunters. Setting $x_t^{\mathrm{d}} = x_{t-1}^{\mathrm{d}} = 0$ in equation (24) we can recover equation (4). This also means that excluding dragon hunters from shaping the risky asset price is as simple as setting $x_t^{\mathrm{d}} = 0$ for all times t. Technically, this does not mean that we exclude the dragon hunters from the model; rather we do not let them participate in the pricing of the risky asset since they have all of their wealth invested in the risk-free asset.

The dragon hunters' wealth equation is the analogue of the rational traders':

Wealth dynamics (dragon hunters)

$$W_t^{\mathrm{d}} = (P_t + d_t) x_{t-1}^{\mathrm{d}} \frac{W_{t-1}^{\mathrm{d}}}{P_{t-1}} + (1 - x_{t-1}^{\mathrm{d}}) W_{t-1}^{\mathrm{d}} (1 + R_f). \tag{25}$$

Since the dragon hunters' wealth at time t depends on the wealth that was invested in the step before, we have a dependence on x_{t-1}^{d}, i.e. the fraction of wealth in the risky asset at the time step *before* t. The price momentum H_t retains its previous form since the updated dynamics is accounted for by the modified price equation. The opinion index s_t governs the behavior of noise traders; we did not couple the dragon hunters via social herding (see footnote on page 15) which leaves this equation untouched as well. The new dynamics however is incorporated via the influence of H on the probabilities $p_t^{\pm}$.

With these theoretical considerations in mind we can now proceed to the implementation of the extended model.

Table 1: Overview of quantities used in the model and their meaning. More details can be found in [16].

Quantity	Description
s_t	noise trader opinion index
N_{n}	number of noise traders
$\zeta_k(y)$, $\xi_k(y)$	random variables that take value 1 with probability y and 0 otherwise
$p_t^{\pm}$	probability for a noise trader to become bullish or bearish (i.e. invests in risky/risk-free asset)
H_t	momentum of risky asset price
p	$2/p$ days is the average holding time of positions for noise traders, enters in equation (3)
κ	quantifies the influence of social interactions s_t and price momentum H
P_t	risky asset price
R_f	return rate of risk-free asset
r	mean of stochastic process for dividend-price-ratio d/P
σ_r	standard deviation of stochastic process for dividend-price ratio d/P
u_t	series of standard i.i.d. random variables with distribution $N(0,1)$
θ	$1/(1-\theta)$ is the length of noise traders' memory of past stock performance
W_t^{r}	wealth of rational investors
W_t^{n}	wealth of noise traders
x^{r}	fraction of rational investors' wealth invested in risky asset
x_t^{n}	fraction of noise trader wealth in risky asset, $x_t^{\mathrm{n}} = (1+s_t)/2$
W_t^{d}	wealth of dragon hunters, introduced in section 2.4.2
x_t^{d}	fraction of dragon hunter wealth in risky asset, introduced in section 2.4.2

3. Methods & Results

Using the dynamical equations described in sections 2.2 and 2.4, the proposed extended model can be implemented. The dragon hunters' contribution depends on the relative amount of wealth x^{d} that they invest. They rely on the LPPL fitting procedure for their investment decision. We will give a general overview over the implementation in Matlab and the chosen parameters in section 3.1. The influence of κ on bubble formation in the original agent-based model is investigated in section 3.2. For various reasons described in section 3.3, some of the data needed to be discarded before the analysis. The immediate dragon hunter impact will be discussed in section 3.4 before the bubble impact and the means to measure it are described in section 3.5.

3.1 Implementation & setup

Due to the length of the computations involved, all simulations were done on the Brutus computation cluster at ETH Zürich. Brutus (**B**etter **R**eliability and **U**sability **T**hanks to **U**nified **S**ystem) is a high performance cluster with over 900 computing nodes with a total of more than 19,000 processor cores and a peak performance of slightly over 200 teraflops[4,5]. The Chair of Entrepreneurial Risks is one of the largest shareholders financing 768 cores that are at disposal to all members for high performance computing.

[4] www.brutuswiki.ethz.ch/brutus/Brutus_cluster

[5] http://www.nzz.ch/aktuell/digital/brutus-laesst-die-rechenkerne-gluehen-1.17293424

3.1.1 MATLAB implementation

The MATLAB code for one simulation run can roughly be separated into three parts:

1. Initialization
2. Simulation loop
3. Plotting & saving

To do multiple runs, the simulation script was wrapped by a loop script which triggered multiple simulations with a different random seed each time. We shall describe all parts in the following.

Initialization

In the first part, the fixed simulation parameters are set. This includes among others the initial wealth assigned to each investor type (W_0^{n}, W_0^{r}, W_0^{d}), the dragon hunter strategy update interval, the LPPL fitting windows $[t_1, t_2]$, the Ornstein-Uhlenbeck process parameters (ΔT, μ_κ), the expected value of dividends r and risk-free return rate R_f.

Some values (d_t, κ, s_t) are dependent on a (pseudo-)random number generator. In MATLAB 2013 and onwards, this is by default [24] implemented using the *Mersenne twister* algorithm [25] which generates pseudorandom numbers using a starting value or *random seed*. Setting the seed to a fixed value in the initialization assures that the (pseudo-)randomly generated values can be reproduced. This is crucial to be able to compare the results with and without dragon hunters.

Simulation loop

The actual simulation is done in a loop where all values necessary to describe the dynamics are calculated for every time step. This includes the social interaction strength κ_t which is calculated using equation (8). This is needed to calculate the noise traders' opinion index $s_t(s_{t-1}, p_t^+, p_t^-, H_{t-1})$ via equation (2). Equation (3) gives probabilities for bullish/bearish behavior of noise traders $p_t^{\pm}(\kappa_t, s_{t-1}, H_{t-1})$. The price equation for the extended model, $P_t(P_{t-1}, W_{t-1}^{\mathrm{n,d,t}}, s_t, s_{t-1}, d_t, x_t^{\mathrm{d}}, x_{t-1}^{\mathrm{d}})$, depends on the dragon hunters' relative invested wealth x^{d} at times t and $t-1$. Following the algorithm described in section 2.4, the dragon hunters base their investment decision, essentially described by x_t^{d}, on the LPPL fitting result.

To this end, they choose a fitting interval $[t_1, t_2]$ in which the price is fitted to the log-periodic power law in equation (12). The right bound of the interval t_2 is always chosen as the current time step t. LPPL fitting generally needs at least one month of price data for a sensible result; hence, the dragon hunters do not start assessing their behavior immediately but only after a time $t_2 - t_1$.

In practice, the quality of the fit strongly depends on the chosen time window. Also, LPPL behavior can be observed over different time scales. One would need to fit every possible time window in every step for a complete analysis. The fitting procedure, however, is quite resource intensive. To strike a balance between LPPL signal search quality and computation time, we decided to pick different intervals $[t_1^{(i)}, t2]$ and vary the starting points $t^{(i)}$ by $-\delta t, \ldots, 0, \ldots, \delta t$. By choosing different $t^{(i)}$ we can search in different time scales. To mitigate the sensibility on the actual chosen day, we look for LPPL signals around $t^{(i)}$ with a range $\pm\delta t$. Furthermore, we decided to let the dragon hunters re-evaluate their strategy only every couple of days. From the computational point of view the simulation was sped up by using a fit function written in C++ and compiled for use in MATLAB.

We define that an LPPL signal is picked up by the dragon hunters if the fit parameters in a time step lie in the acceptable ranges mentioned in section 2.4 in *any* of the time windows. The critical time t_c, which gives the time of the crash predicted by the model, is evaluated as well: if $t_c - t > t_\Delta$ (for $t_\Delta > 0$), the dragon hunters increase their relative investment to $x^{\mathrm{d}} = 1$. This is equivalent to investing all of their wealth in the risky asset. In case the predicted crash is too close, the dragon hunters refrain from investing and stay with their default $x^{\mathrm{d}} = 0.3$. The parameter t_Δ thus is a measure of the risk aversion of the dragon hunters: a large t_Δ means that they fear the risk of an earlier crash than is predicted by the model and want to be on the safe side while dragon hunters with t_Δ close to zero do not want to miss any further increase in price.

In the last part of one time step, the wealth of the respective traders $W_t^{\mathrm{n,r,t}}$ and the price momentum H_t are calculated according to equations (5), (6) and (25) and equation (7), respectively.

Plotting & saving

To get a quick overview over each simulation, the results were directly plotted and saved on Brutus. Plotted quantities were price, price momentum, opinion index, absolute wealth levels, dividend-price ratio, returns, social interaction strength κ and the signed and absolute auto-

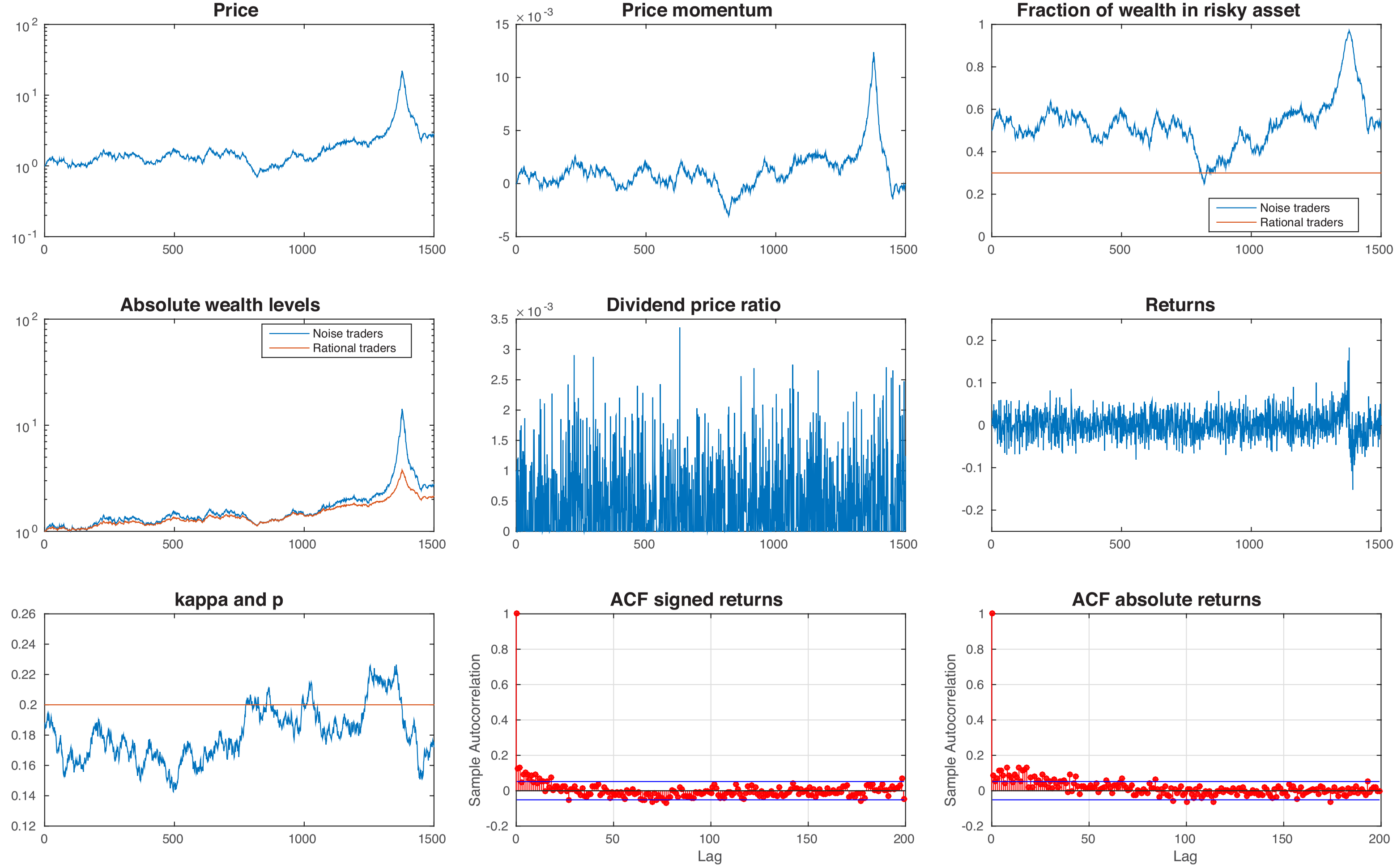

Figure 5: This figure shows a typical output from Brutus for the original model. *Top:* risky asset price P_t, price momentum H_t and the fraction of the trader's wealth in the risky asset ($= (1+s_t)/2$ for noise traders and $= x$ for rational traders). *Middle:* wealth levels of all traders included in the model normalized at $t = 1$; dividend-price ratio d_t/P_{t-1}; returns $R_t = (P_t - P_{t-1})/P_{t-1}$. *Bottom:* social interaction strength κ with the red line denoting p, the border between sub- and supercritical regime; the autocorrelation function of signed and absolute returns as a function of time lag. In this case, we can see volatility clustering in the returns towards the end. This leads to the non-zero autocorrelation for small time lags that are shown in the bottom right.

correlations of the returns. The latter two are, in combination with the returns, important to spot volatility clustering. Figure 5 shows an example plot. Furthermore, all calculated values were saved as a Matlab workspace file to enable analysis after the calculations.

Wrapping script

For the analysis of the dragon hunter influence multiple simulations with different random seeds were needed. This was achieved by wrapping the simulation script with another script which iterates over different random seeds. Since the dragon hunters did not pick up an LPPL signal in every simulation the uninteresting seeds could be sorted out this way.

3.1.2 Chosen simulation parameters

Generally, all simulations were done for 1500 trading days, i.e. 6 years. This ensures that there is enough time for bubbles to emerge. Naturally, the numerical values for the parameters (see table 2) greatly influence the outcome of the simulation. They were chosen such that a number of criteria were fulfilled. We will briefly discuss those criteria in the following.

Bubble behavior

As argued in the theory section, the Ornstein-Uhlenbeck process behind the social interaction strength κ is crucial for the noise trader behavior. To obtain a sensible frequency of bubbles and a realistic length ($\sim$ 1 year) we found $\mu_\kappa = 0.94$ for the mean and $\Delta T = 180$ for the mean reversion time to be good values. The expected dividends r and the risk-free return rate R_f were chosen such that we can expect 4% and 2% p.a., respectively.

Reproducibility

Since the noise traders are responsible for the formation of bubbles, their parameters have a great influence on the price dynamics. The aim of this thesis is to quantify the impact that a dragon hunter type investor behavior has on the market. It is thus convenient that "by default" (i.e. in times without an LPPL signal), the behavior of the dragon hunters is the same as the rational traders'. If we fix the initial wealth ratio $W_0^\mathrm{n}/(W_0^\mathrm{d}+W_0^\mathrm{r}) = 2$, for both the simulations with and without dragon

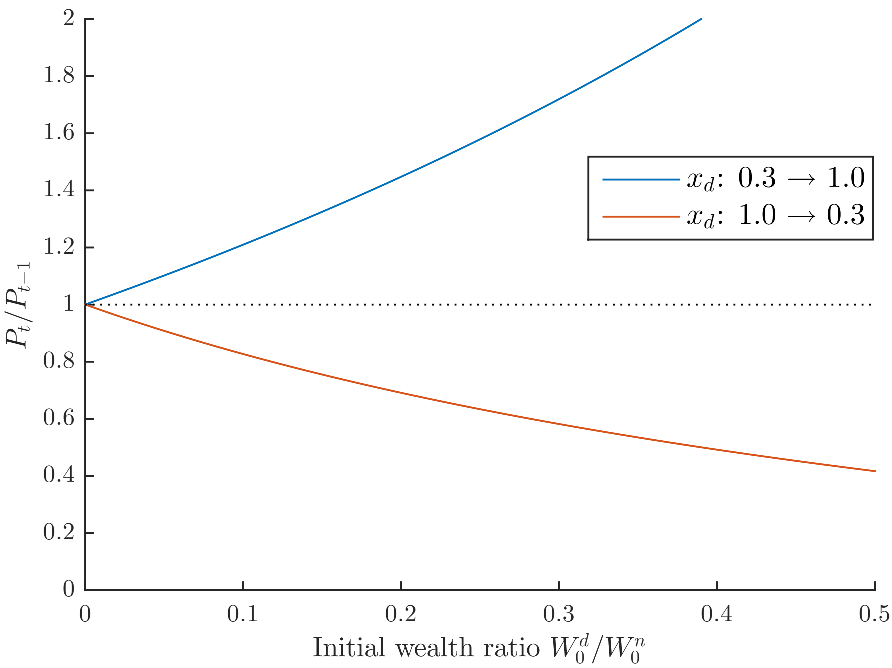

Figure 6: Price ratio P_t/P_{t-1} (following equation (24)) plotted against the initial wealth ratio of dragon hunter and noise trader $W_0^{\mathrm{d}}/W_0^{\mathrm{n}}$ when the dragon hunters decide to invest ($x_{\mathrm{d}} = 0.3 \rightarrow 1$). The stochastic quantities u_t and s_t have been set to zero to study the impact of the dragon hunter's change in investment on the price. Note that the rational traders' initial wealth W_0^{r} is chosen such that the wealth ratio $\nu = 2$ remains constant. The immediate effect of dragon hunter investment is quite large. This is why we chose their initial wealth to be rather small ($[0.01, 0.05, 0.1] \cdot W_0^{\mathrm{n}}$). From this plot, the price increase should be around 2%, 10% and 21%, respectively. A similar but reverse effect is observable as soon as the dragon hunters pull out again. The drops should be by 2%, 9% and 17%. Note that this plot only gives an idea of the magnitude of the dragon hunters' immediate impact. In the simulations, all stochastic quantities are of course not set to zero and play an important role.

Table 2: Overview over the chosen parameter values

Parameter	Value	Description
ΔT	180	Mean reversion time [days]
p	0.2	Controls average holding time of noise traders
μ_κ	0.94p	Mean of social interaction strength
r	$0.04/250$	Expected value of dividends, corresponds to 4% annualized
R_f	$0.02/250$	Risk-free return rate, corresponds to 2% annualized
N_n	1000	Number of noise traders
W_0^n	100	Initial noise trader wealth
ν	2	Initial wealth ratio $W_0^\mathrm{n}/(W_0^\mathrm{d}+W_0^\mathrm{r})$
Check frequency	3	Time between DH re-evaluations
$[t_1^{(i)}, t_2]$	$[50, 100, 200, 400]$	Scale of time windows for fitting
δt	10	Maximum absolute variation of $t_1^{(i)}$
t_Δ	5	Time to predicted crash

hunters, we preserve the influence of the noise traders on the market and can solely observe the dragon hunters' impact. In other words: when simulating with dragon hunters, we reduce the rational traders' initial wealth and give a certain fraction to the dragon hunters. This also means that the price will be different for the two simulations only if the dragon hunters pick up any signal in the duration of the simulation.

Since the numerical values of wealth are arbitrary and only influence the magnitude of the outcome but not the overall behavior, we can fix the noise traders' initial wealth to $W_0^\mathrm{n} = 100$ and choose the LPPL traders' initial wealth as a fraction. The sudden investment and withdrawal of the dragon hunters will lead to a jump in price. To make this jump not too large, we chose $W_0^\mathrm{d}/W_0^\mathrm{n} = [0.01, 0.05, 0.1]$ which corresponds to $W_0^\mathrm{r}/W_0^\mathrm{n} = [0.49, 0.45, 0.4]$. On investment or removal of wealth from the risky asset, the dragon hunters' impact on the price can be estimated by removing the stochastic quantities from the price equation. Figure 6 shows this estimate and gives an intuition for the impact of a sudden noise trader investment with these initial wealth ratios.

LPPL fitting

Setting $\Delta T = 180$ essentially translates to an average of 180 days that κ typically takes to return to the mean after it has been two standard deviations away from its mean (cf. section 2.2). This is also a measure for the typical duration of bubbles in the model and thus the fitting windows chosen by the dragon hunters should reflect that. We chose 4 different time windows $t_1^{(i)} - t_2$, namely 50, 100, 200 and 400 days. The lower bound was varied by up to $\delta t = \pm$ 10 days. The choice of these parameters means that some time has to pass before the dragon hunters have acquired enough price data to be able to make an investment choice. The earliest possible time for the dragon hunter to increase their investment in our case is $t = t_2 = 50 + 10 = 60$.

The minimum distance between predicted crash and the current day, $t_\Delta < t_c - t$, was set to 5 days. This means that dragon hunters refrain from investing when the crash is predicted by the model in less than one week. This signifies a certain risk aversion as well as precaution since a more sustainable strategy would be to use all possible fit windows and create a confidence interval for the crash. Setting $t_\Delta = 5$ is an attempt to mitigate this.

At every evaluation of the investment strategy, the dragon hunter thus needs to make at most[6] 84 LPPL fits. With 3 days between every re-evaluation, this amounts to 28 evaluations on average for every time step. By spreading the fit windows over different time scales, we attempted to catch as many LPPL signals as possible while maintaining a reasonable computation time.

3.2 Influence of κ on bubble formation

In section 2.2 we argued that bubbles form when $\kappa > p$. To test these considerations, we replaced the Ornstein-Uhlenbeck process in the original agent-based model with a rectangular pulse (subcritical $\rightarrow$ critical $\rightarrow$ subcritical)

$$\kappa(t) = \begin{cases} 5p/4 & : \; t_0 \leq t \leq t_0 + \Delta t \\ 3p/4 & : \; \text{else} \end{cases} \tag{26}$$

[6] If a signal is found in *at least one* window, the dragon hunter stops looking in other windows and starts investing directly. This can decrease the amount of fits that need to be done.

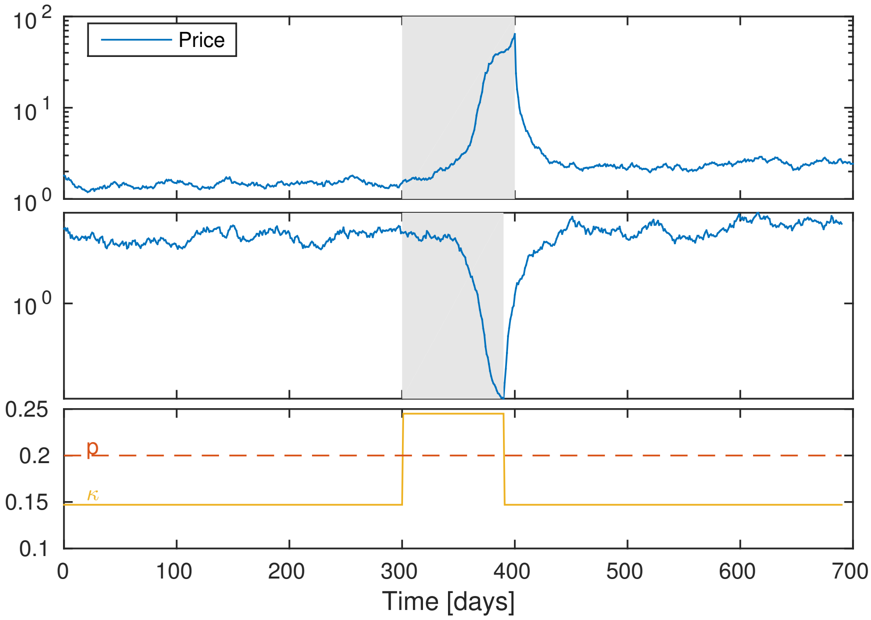

Figure 7: Different influences of rectangular κ pulse with plateau of 90 days in the critical regime on the simulated price. *Top:* supercritical κ sequences lead to superexponential growth in price. *Middle:* supercritical κ sequences lead to a crash at another point in time. *Bottom:* κ pulse with values described in equation (26). Whether there is a crash or a bubble depends on the noise traders' opinion index. Note that the visible change is not immediate but can rather take some time, but as long as the pulse is long enough, superexponential behavior is bound to happen. Also note that the time axes do not coincide; the two price plots have been centered around the time the κ pulse was "sent".

where $p = 0.2$ is the value separating critical ($\kappa > p$) and subcritical ($\kappa < p$) regions of κ. The time t_0 denotes the time the pulse is sent, Δt denotes the length of the pulse. This way, the regime change can be studied without the noise from the Ornstein-Uhlenbeck process. Whether the pulse produces bubbles or crashes depends on whether the opinion index at the start of the pulse is growing or sinking.

Figure 7 nicely shows the two effects a rectangular κ pulse can have on the price. If the pulse length Δt is chosen too long, the noise traders fully polarize and the price may diverge. Very short pulses did not yield a visibly different price dynamics. This can be explained by the lower initial slope of superexponential vs. exponential growth. Note that in the case of a crash the price cannot go down to zero as there will always be a demand from the rational traders.

3.3 Filtering of simulations

The simulation parameters described in the previous section were carefully chosen and based on plausible assumptions whenever possible. The search windows for LPPL signals were chosen with the goal of balancing performance with computation time. However, the resulting simulations were not always usable to quantify the impact of dragon hunters on a financial market for a number of reasons. Out of 2000 simulations with the initial wealth ratios $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = [0.01, 0.05, 0.1]$ 12.8%, 11.3% and 10.8%, respectively, were usable for analysis. The reasons can be distinguished as issues with the original and with the extended model and will be described in the following. Figure 10 shows the counter-example of a simulation that fit all criteria and was included in the simulation.

3.3.1 Original model

First, a part of the simulations did not yield a satisfactory amount of volatility clustering. This means that the absolute autocorrelations of the returns did not exhibit a power law decay but rather an exponential one. The correlations fell into the 95% confidence limits of a normal distribution for time lags $\Delta t > 0$, i.e. did not exhibit the slow decay usually found in financial markets [6]. Random seeds that led to these autocorrelations were hence discarded as they did not suit our purpose of studying the impact on a "realistic" market.

As we have seen in section 3.2, κ plays an important role in the formation of bubbles. We mentioned that long periods of κ in the supercritical regime above p can lead to an exploding price. The simulation itself was terminated as soon as the price diverged. However, the long mean reversion time ΔT sometimes meant that κ spent time in the supercritical regime long enough to completely polarize the noise traders to $s \simeq 1$ which most of the time led to unrealistic bubbles (e.g. price grows to one hundred times the amount at the beginning). Figure 8 shows an example of such a polarized state. For the same reasons as stated above, these types of simulations were excluded from the analysis.

The case $s \simeq -1$, i.e. bearish noise traders can not be treated symmetrical to bullish noise traders. The price can never go to zero as there is always a demand from the rational traders. Simulations that showed longer periods of negatively polarized noise traders were discarded as well. Figure 9 showcases these types of discarded simulations.

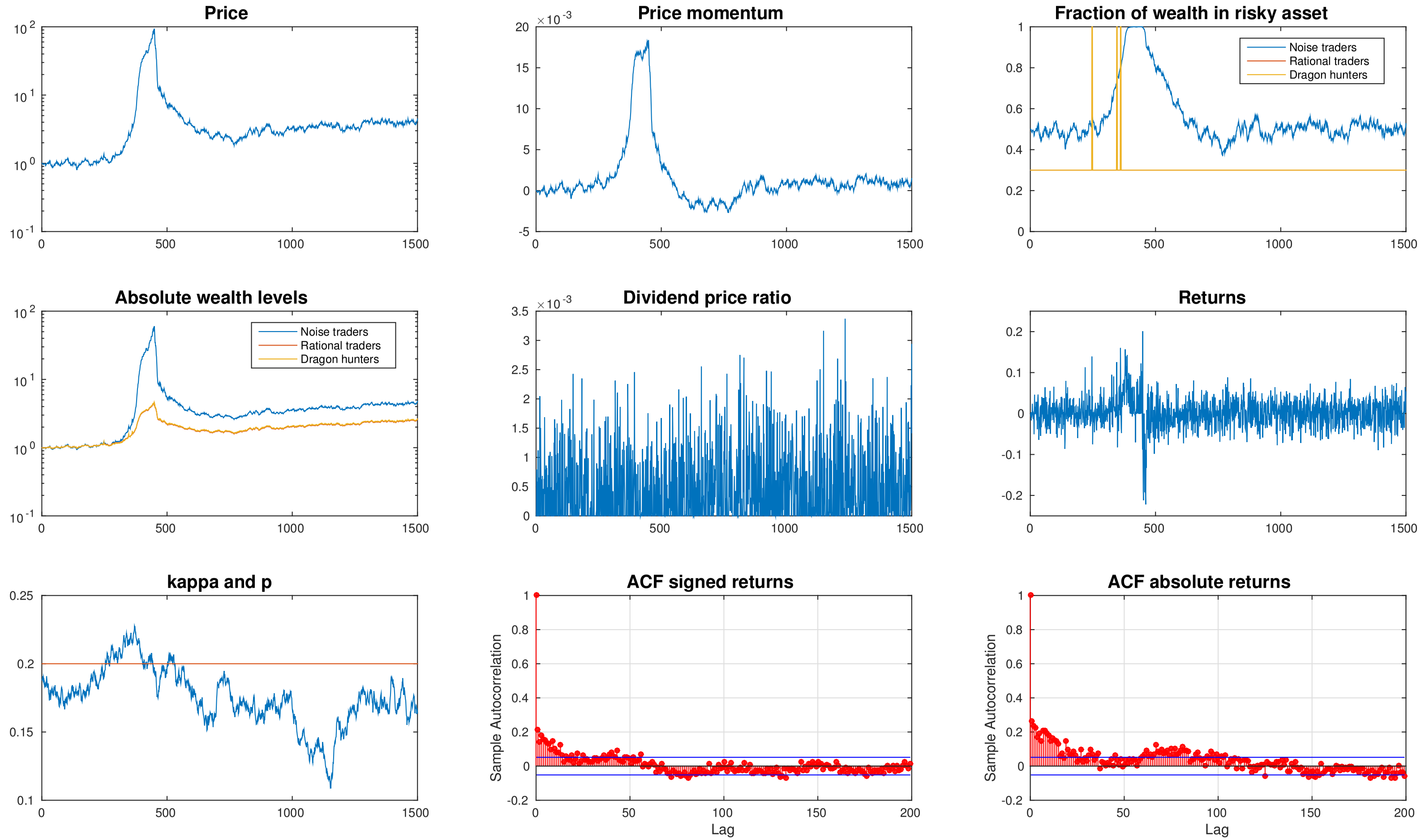

Figure 8: Simulation output with initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.05$ showing completely polarized noise traders that trigger a excessively large bubble (price grows 100-fold in a matter of months). In the top right the long duration of polarization is shown which is caused by a long sequence of κ above p. Note that in principle this would otherwise have been a simulation suitable for analysis: the dragon hunters invested before the peak. Also note that the rational traders wealth is plotted as well but due to the large y-scale the difference between rational traders and dragon hunters is not visible.

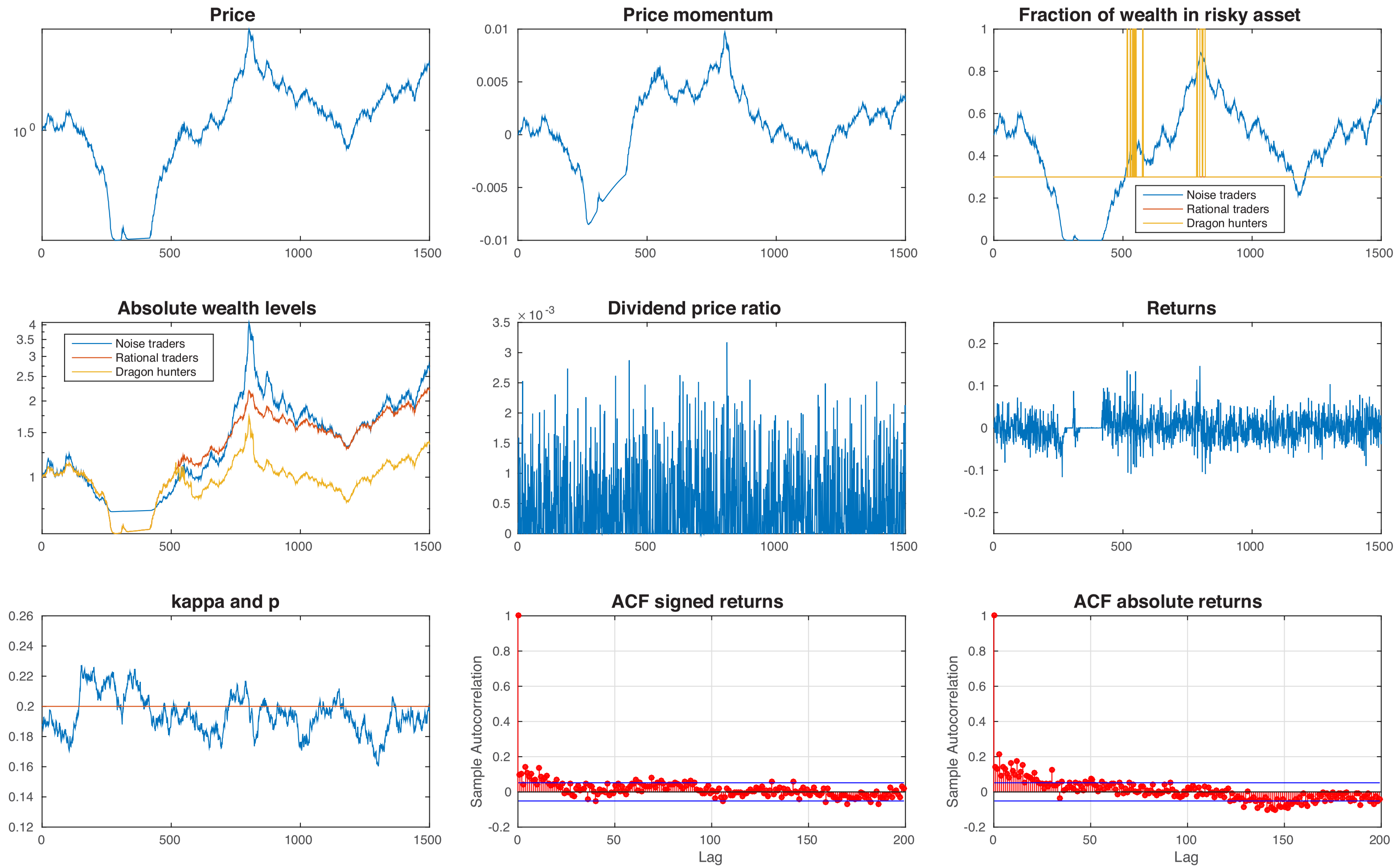

Figure 9: Simulation output with initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.05$ showing completely polarized noise traders that trigger a crash in price. Again, this is triggered by the social interaction strength κ which is above p for the duration of the crash. This is an example of a simulation that was not chosen even though the dragon hunters detected a bubble and invested. In about 30% of the simulations, long supercritical sequences of κ triggered a bubble which let the price explode (above 10^2 for an initial price of 10^0). These simulations were of course also excluded in the analysis (cf. figure 8). Note that here the dragon hunters are outperformed by the other traders even though they invest before the bubbles. Since the dragon hunters buy at a high price and sell at a cheap price, they always suffer wealth losses (cf. section 3.4). They perform well if this effect is compensated by noise traders drastically driving up the price before the dragon hunters sell again.

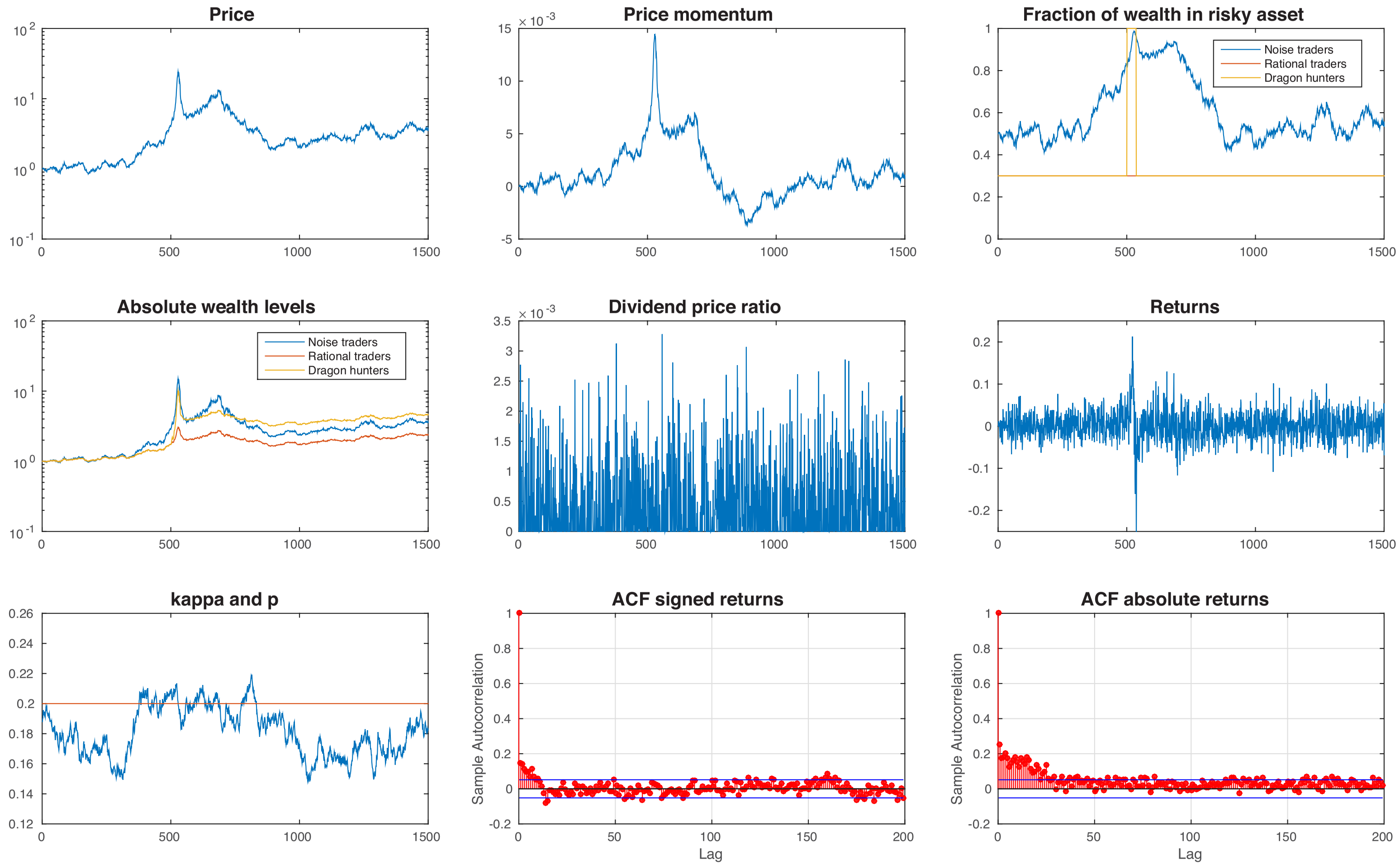

Figure 10: Simulation output exemplifying a good simulation for initial wealth ratio $W_0^d / W_0^n = 0.05$. There is a pronounced peak in price with a dragon hunter investment directly beforehand which makes them outperform the other traders. The peak coincides with a supercritical κ. Despite the high peak there is no polarization of noise traders. There is visible volatility clustering and non-zero absolute autocorrelations for lags close to zero. However the signed autocorrelations did not immediately go down to zero.

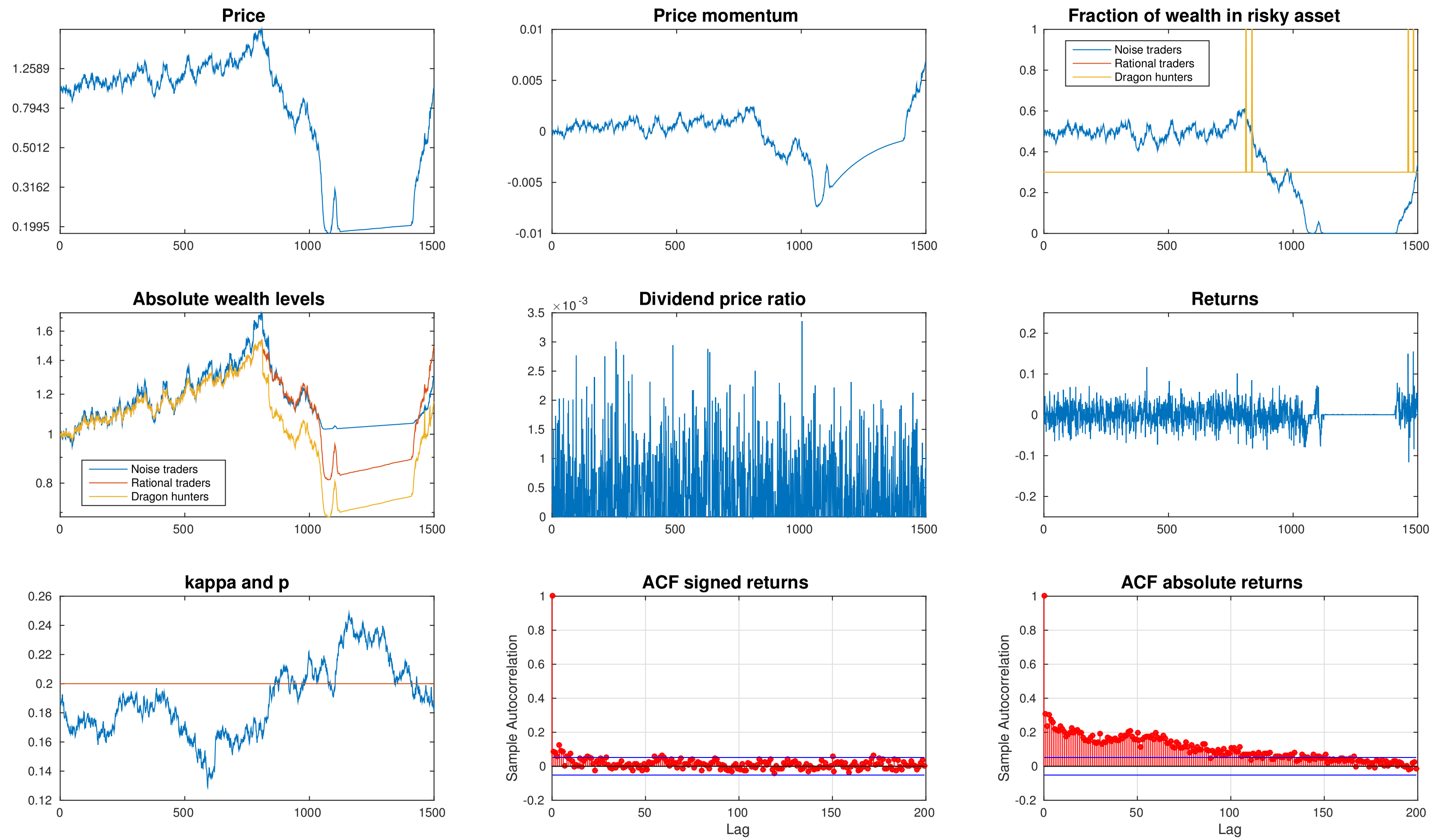

Figure 11: Simulation output with initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.05$ showing an example of dragon hunter investment at an inconvenient time. The dragon hunters start investing when the price is starting to crash. Note that we also find noise trader polarization which further contributes to the dismissal of above simulation.

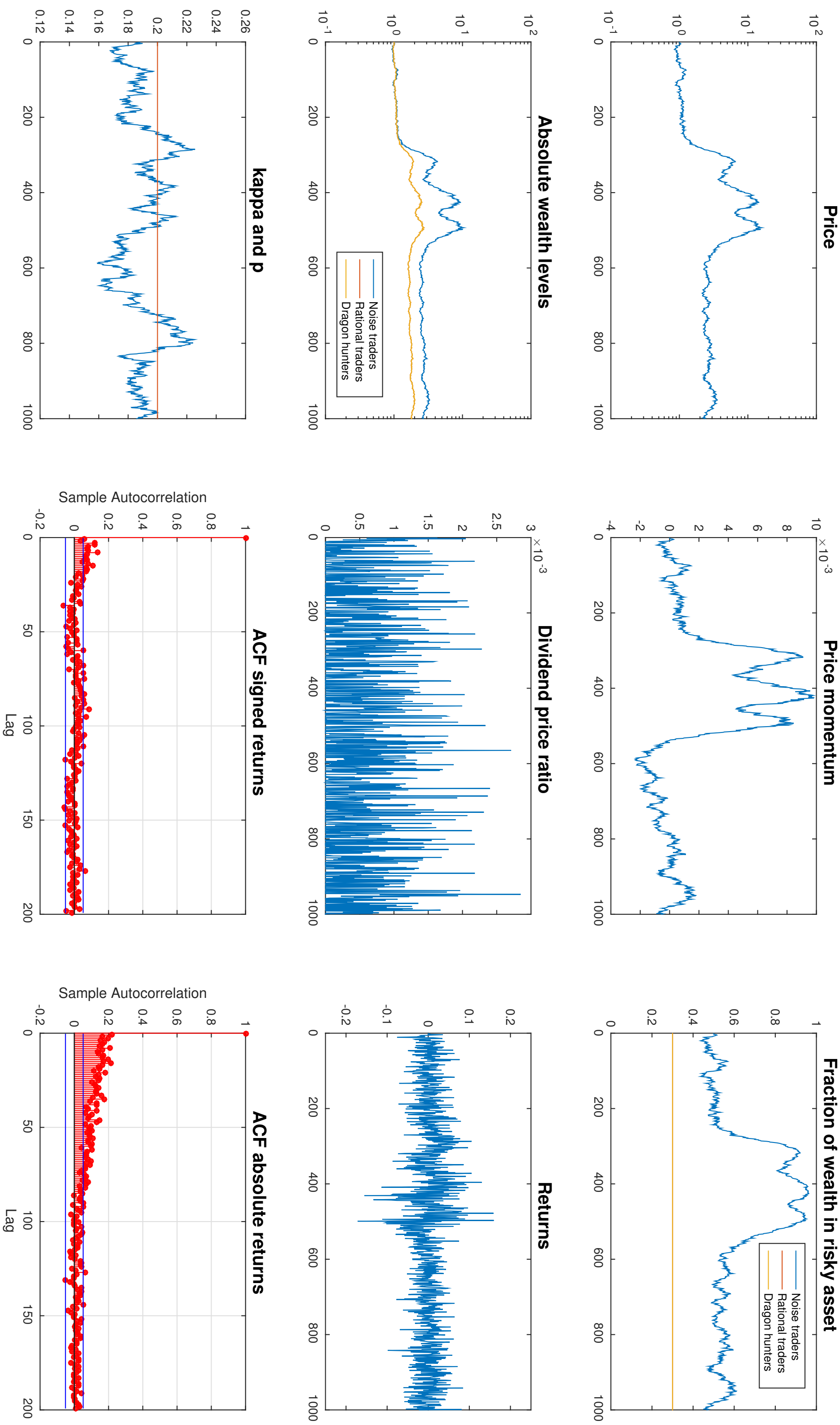

Figure 12: Simulation output with initial wealth ratio $W_0^d / W_0^n = 0.05$ where dragon hunters miss a bubble visible by eye. Missing the bubble is most likely due to the incomplete scanning of available time windows since the price already starts to visibly increase at around $t = 200$.

3.3.2 Extended model

The dragon hunters did not pick up a signal in every simulation. This should not be surprising since not every market necessarily produces a bubble. Only some of the time clearly visible areas of superexponential growth were missed. This reaffirms our approach to search for bubbles in different time scales. We attributed the missed bubbles to the fact that, as explained in section 3.1.2, the bubble search could not be done in a fully comprehensive manner. Figure 12 visualizes a case where the dragon hunters did not invest even though a very large bubble appears.

In some cases, dragon hunters started to invest in a seemingly "wrong" time, i.e. during a drop in price. Of course, it is easy to know ex post what the "wrong" time is; the dragon hunters picked up an LPPL signal and made a poor decision. As above, we attribute this to the non-ideal fitting procedure[7]. We basically assume that the LPPL model is fully capable to describe the most important aspects of a bubble before the crash with a high probability. Failing to recognize the wrong investment is thus simply an inadequate use of mathematical model by the dragon hunter brought about by the simplifications that had to be made in the bubble detection process. Figure 11 shows an example of dragon hunters investing at the beginning of a crash.

In real life, dragon hunters would want to make sure that every possible information is incorporated into the investment decision. However this does not mean that we discarded all simulations with bad dragon hunter decisions. Possible poor performances of the dragon hunters in some parts of a simulation are not necessarily detrimental to the goal to quantify the dragon hunter impact: there might just as well be other bubbles in the same simulation before which a better investment decision was made.

In order to analyze bubbles, one needs to be able to recognize them as such. Some of the time, dragon hunters recognized an LPPL signal in the absence of a clearly visible bubble. The time spent by κ in the supercritical regime was too short to lead to a large deviation of the price from the ground noise. This made the investment unusable for analysis as our analyzing method depends on pronounced peaks in price (cf. section 3.5). Again, these simulations were only discarded when no other interesting investment before a bubble was found.

[7] We will discuss ideas to further improve the bubble detection or the dragon hunter strategy in the future in chapter 4.

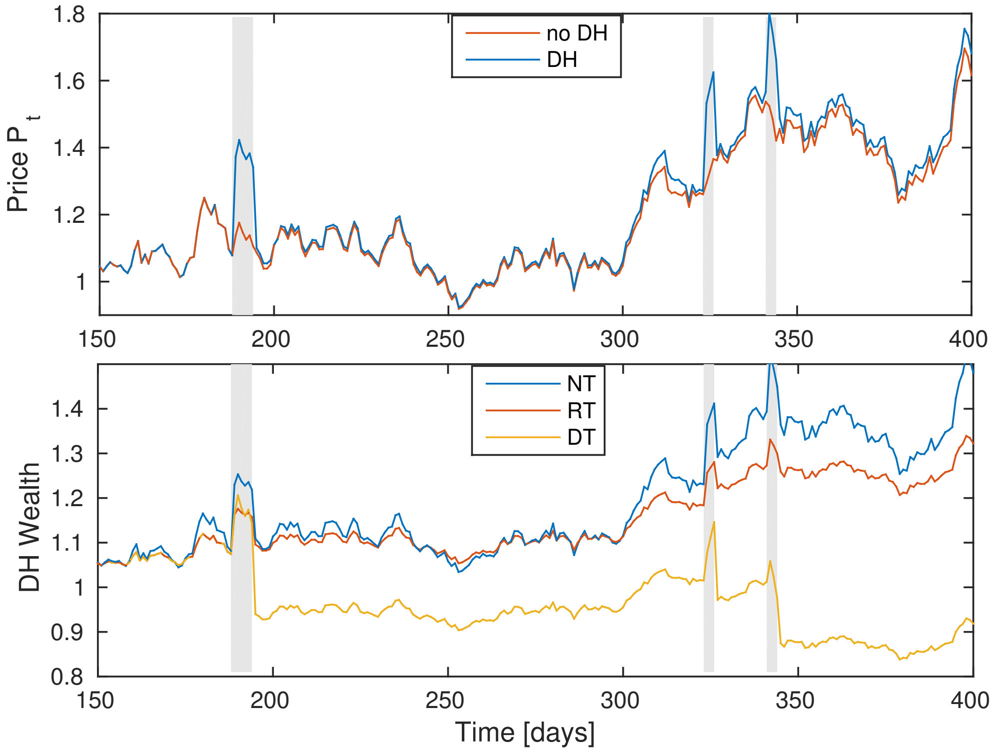

Figure 13: *Top:* comparison of price with (blue) and without (orange) dragon hunters for an example simulation with $W_0^d / W_0^n = 0.1$. Shaded in gray are periods where the dragon hunter decided to invest. The bump in price and the drop directly after are directly related to dragon hunter investment. *Bottom:* resulting wealth for noise trader (blue), rational trader (orange) and dragon hunter (yellow). The asymmetrical drop in wealth is clearly visible every time the dragon hunter picks up a signal.

3.4 Immediate impact of investment

Figure 6 gives an estimate of the expected impact of a full dragon hunter investment. Due to the very sudden change in demand, the price should increase; for the chosen initial wealth ratios $W_0^d / W_0^n = [0.01, 0.05, 0.1]$ we expect a price increase of $[1\%, 10\%, 21\%]$ under the assumption that the initial wealth ratios have not changed too much. When the dragon hunters decide to partially short their stocks, a drop similar to the bump on full investment is to be expected. An example dragon hunter impact for $W_0^d / W_0^n = 0.1$ is shown in figure 13 which illustrates the observed bumps and following drops nicely. Figure 16 compares the price for different wealth ratios.

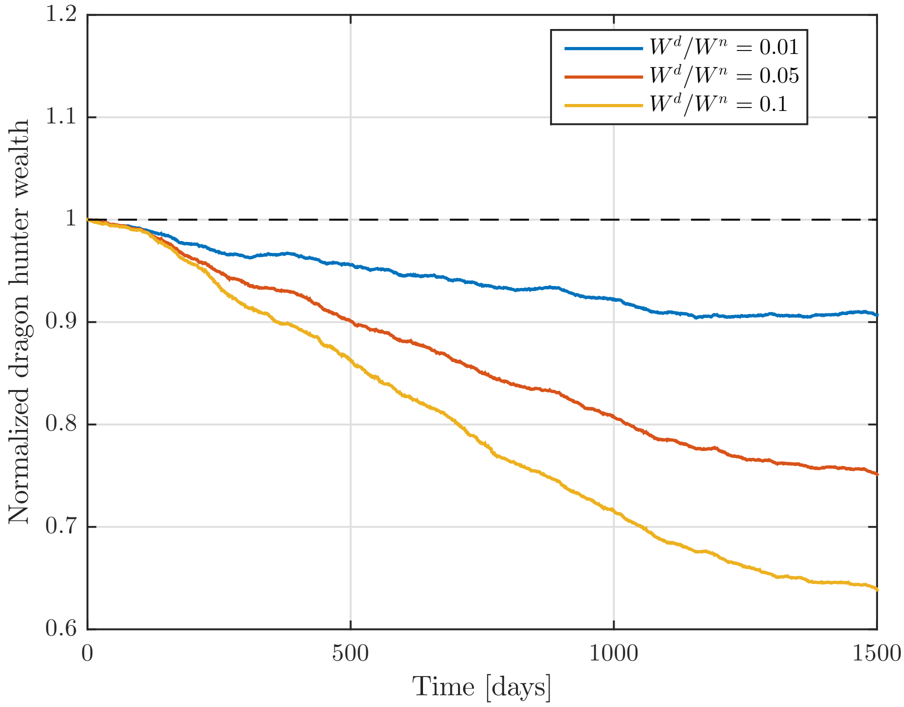

Figure 14: Dragon hunter wealth over the course of the simulation time (1500 days) relative to the overall market wealth. The wealth of all analyzed simulations were averaged, divided by the total wealth in the market at time t, $W_t^\text{d} + W_t^\text{r} + W_t^\text{n}$, and normalized to 1 at $t = 0$. The higher the dragon hunters' initial wealth, the greater the impact on the price on investment and the worse the wealth performance. On average, the dragon hunters perform worse than the rest of the market (marked by the dashed black line). This can be attributed to the imperfect investment algorithm. Having many dragon hunters (or, equivalently, giving each dragon hunter more money) thus leads to a worse performance for all dragon hunters. Note that this is *relative to the market*, i.e. the dragon hunters still increase their wealth. For the raw averages see figure 15.

Curiously, we noticed that after selling, the dragon hunters' wealth in some cases decreased and settled below the wealth level of the rational traders, i.e. bump and drop were not symmetrical (cf. figure 13). This effect was observed to be stronger when the dragon hunters start with a larger amount of wealth. The positive impact on the dragon hunters' wealth at the time of investment is outweighed by the selling the shares. Also, we noticed that the negative effect on dragon hunter wealth decreases with lower relative wealth. Figure 14 shows the dragon hunter wealth averaged over all analyzed simulations for different initial wealth ratios relative to the overall market wealth $W_t^\text{d} + W_t^\text{r} + W_t^\text{n}$. The trend for worse performances with increasing initial wealth is clearly visible.

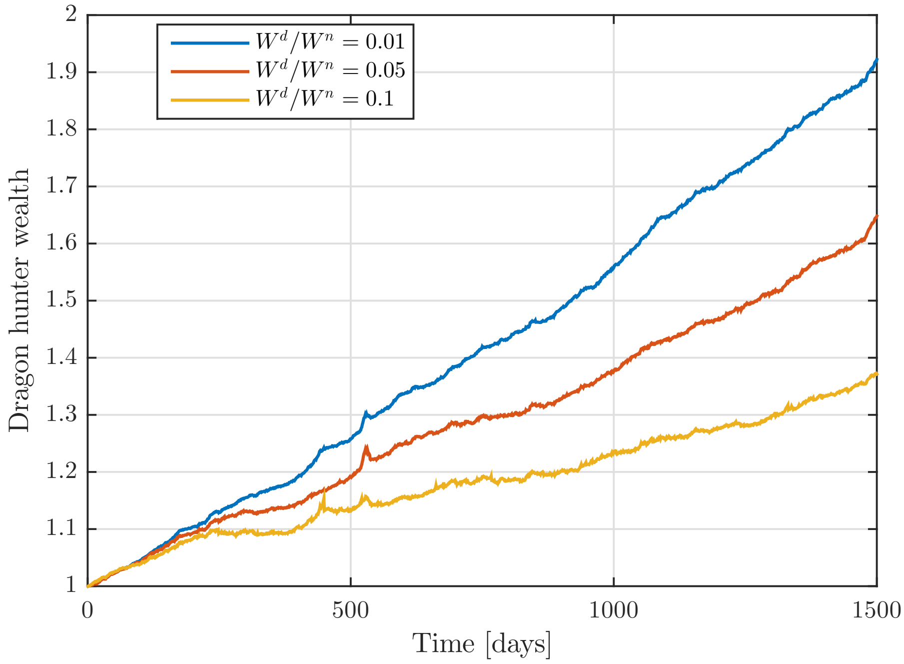

Figure 15: Dragon hunter wealth over the course of the simulation time (1500 days), normalized at $t = 0$, for different initial wealth ratios. The dragon hunters increase their wealth as the market evolves but do so less when the wealth ratio is greater. The wealth growth rate is however less than the overall market growth rate (cf. figure 14).

We can understand this effect by looking at the LPPL trader wealth equation when $x^{\mathrm{d}}_{t-1} = 0.3 \to 1 = x^{\mathrm{d}}_t$ and $x^{\mathrm{d}}_{t-1} = 1 \to 0.3 = x^{\mathrm{d}}_t$.

Recall that the dragon hunters' wealth at time t is given by

$$W^{\mathrm{d}}_t = \left[\frac{(P_t + d_t)}{P_{t-1}} x^{\mathrm{d}}_{t-1} + (1 - x^{\mathrm{d}}_{t-1})(1 + R_f) \right] W^{\mathrm{d}}_{t-1}.$$

Most of the time, the dividends d_t are small compared to the price P_t. The risk-free return rate R_f is also very small compared to 1. For a general understanding of the wealth dynamics, we can neglect both quantities in the wealth equation. As argued above, we can approximate the price at time t by $P_t \simeq \gamma P_{t-1}$ with $\gamma > 0$[8]. If the dragon hunters increase their invested wealth $\gamma = \gamma_0 > 1$ holds. In the reverse case, we have $\gamma = 1/\gamma_0 < 1$. For an initial wealth ratio of 0.1 for example, γ_0 would

[8]Note that γ is generally time-dependent, otherwise we could not have superexponential growth during bubbles. We are merely looking the price at one moment t *without* the influence of the noise traders who drive the bubble dynamics.

be 1.21 and $1/\gamma_0 = 0.83$ (cf. figure 6). For the case $x^{\mathrm{d}}_{t-1} = 0.3 \to 1 = x^{\mathrm{d}}_t$ we can write $P_t \simeq \gamma_0 P_{t-1}$ which leads us to

$$W^{\mathrm{d}}_t \approx [0.3\gamma_0 + 0.7]\, W^{\mathrm{d}}_{t-1}.$$

For the reverse case, we obtain

$$W^{\mathrm{d}}_t \approx \frac{W^{\mathrm{d}}_{t-1}}{\gamma_0}.$$

Since $\gamma_0 > 1$, the jump $1 \to 0.3$ has a larger impact on wealth than vice versa. Another way to understand this is to remember that the price directly reacts to the demand but the wealth is measured by the assets held at time t. When selling their risky assets, the dragon hunters trigger a drop in price which reduces their wealth in the next step. Naturally, this effect becomes more prominent the larger γ_0 is. For small initial wealth ratios the factor γ_0 is smaller and the overall difference is not as pronounced.

We have to note, however, that these considerations completely leave out any noise trader dynamics and are only valid for rather tranquil prices. The noise traders cause the formation of bubbles. Indeed, when the dragon hunters invested during superexponential growth, they naturally outperformed both noise and rational traders.

3.5 Comparison of bubbles

So far, we have established a framework and chosen the right parameters to enable us to quantify the impact of dragon hunters. The impact will be analyzed by comparing price data for the same random seed with and without dragon hunters. Recall that the wealth is distributed between the traders such that simulating *with* dragon hunters that remain in their "default" state $x^{\mathrm{d}} = 0.3$ yields the *exact same data* as simulating *without* our third investor type. If there is any difference between the price or wealth evolution it *must* be due to the dragon hunters.

3.5.1 Bubble measures

Comparing price data from our extended model is not a trivial task. Since dragon hunters can invest at any moment that they find an LPPL signal, the price will necessarily not be very smooth. To even this out, we performed averages to get an understanding over the impact

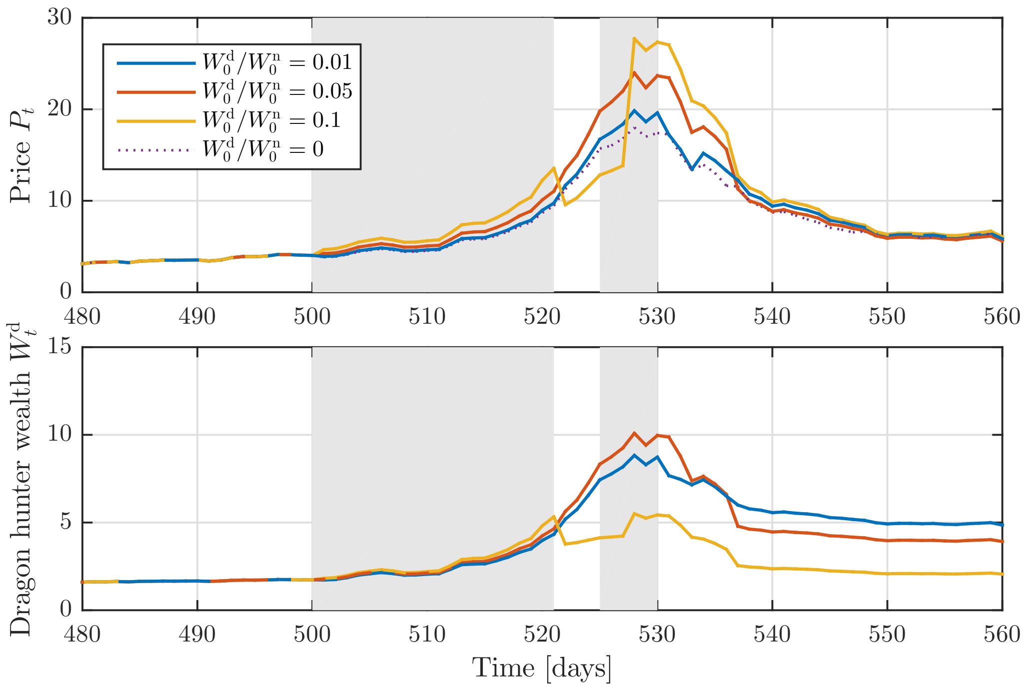

Figure 16: Comparison of price and wealth for the chosen initial wealth ratios around a rather prominent bubble *without* full dragon hunter investment before $t = 500$. Dragon hunter investment is shaded in gray. *Top:* Price P_t on a linear scale around a bubble. The price increases with chosen initial wealth; a trend that is also found in most other bubbles (cf. figures 17 to 19). Note the impact of dragon hunter investment on the price. *Bottom:* Dragon hunter wealth W_t^d normalized at $t = 1$ plotted on a logarithmic scale. While high initial wealth ratios drive the price to higher values, the dragon hunters perform worse in terms of relative wealth.

of the dragon hunters. We looked at the relative price differences, i.e. $(P_t^{DH} - P_t)/P_t$ (P_t : price without dragon hunters, P_t^{DH} : price with dragon hunters), in an environment around a peak to compare the results.

In more detail, we searched for the highest price peak in every simulation that passed the filtering criteria mentioned in section 3.3 and checked whether the dragon hunter invested in a time window of 200 days before the peak. This window was chosen in relation to the average bubble length ΔT which had been set to 180 days in the simulations. This way we could make sure that possible investments were due to a signal triggered by the bubble.

Bubbles do not appear and crash immediately, but rather develop, rise fast, crash and decay. To capture this behavior, we considered the time window 200 days before and after the no-dragon-hunter peak for the price difference analysis. This meant that bubbles that were too close to the beginning ($t_{\text{bubble}} < 200$) or end ($T - t_{\text{bubble}} < 200$) of the simulation time could not be considered and were discarded. To find the peaks, the built-in MATLAB function `findpeaks` was used. The relative price difference $(P_t^{\text{DH}} - P_t)/P_t$ was computed for all usable simulations and averaged. Exactly 2000 simulations were done with the same random seeds with and without dragon hunters. The constrictions left us with 196 (wealth ratio 0.01), 168 (wealth ratio 0.05) and 146 (wealth ratio 0.1) peaks to analyze.

3.5.2 Comparison results

Figures 17 to 19 show the results of price difference averaging around peaks for the three different wealth ratios. For better visibility, the x-axis was shifted such that all peaks are at $t = 0$. Strikingly, all average differences are positive which indicates that the dragon hunter investment strategy leads to an increase in price. All three averages have a prominent peak at $t = 0$ which means that the dragon hunters on average did not move the peak to an earlier or later point in time.

But why is the time of the crash fixed? In the original model, bubbles were introduced only by the inclusion of the noise traders. Their social imitation (quantified by the opinion index s_t) and regard for past prices drive the formation of bubbles. Supercritical κ then allow for social interaction strengths that trigger a positive feedback mechanism. Adding a third type of investors that do not directly interact with the noise traders will not change this dynamics. Their technical interaction is merely indirect via (i) the price which changes the noise traders' wealth that they can reinvest or (ii) the price momentum. In the current implementation of the model, the price momentum only impacts noise traders via the probability for bullish or bearish noise trader behavior, $p_t^{\pm} = [p + \kappa \cdot (s_t + H_t)]$ (equation (3)). Usually, we have $H_t \sim 10^{-3}$, so H_t is negligible compared with $s_t \in [-1, 1]$ in most cases, the effect is very small. The dependence on dragon hunter behavior is limited to the very indirect interaction via the price. This means that the peak is not very likely to change and – if it does – it is only shifted by a few days triggered by the indirect price interaction.

The peak averages for the three initial wealth ratios are 0.4%, 3% and 9%, respectively. They reflect the different starting wealth of the rational traders which is directly connected to the gravitas with which

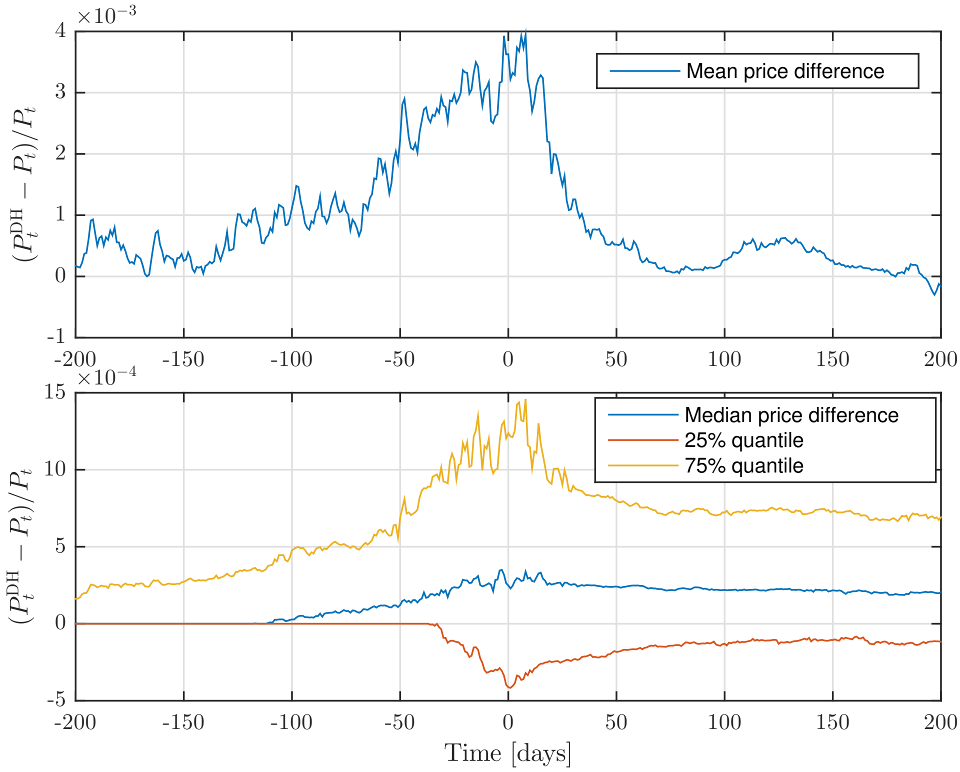

Figure 17: The mean and median relative price difference of all suitable peaks for initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.01$. The highest peaks in each simulation have been aligned to coincide with $t = 0$ and the average/median of the $(P_t^{\mathrm{DH}} - P_t)/P_t$ over all suitable simulations was computed. The 25% quantile indicates a considerable amount of negative differences that cannot be seen in the average. This might explain why – in contrast to the other wealth ratios in figures 18 and 19 – the average price difference peak (≈ 0.004) is one order of magnitude lower than initial wealth ratio 0.01.

the dragon hunters can invest. Recall that a wealth ratio of $W_t^{\mathrm{d}}/W^{\mathrm{n}} = [0.01, 0.05, 0.1]$ corresponds to a percentage of total wealth $W_t^{\mathrm{d}}/(W^{\mathrm{d}} + W^{\mathrm{r}} + W^{\mathrm{n}}) = [0.7\%, 3\%, 7\%]$. The average peaks are in the same order of magnitude and nicely confirm the limit of dragon hunter impact.

We saw in section 3.4 that the dragon hunter wealth was negatively affected by the initial wealth ratio. Figure 20 shows the average wealth of noise and rational traders as a function of time plotted for different initial wealth ratios. As in the original model, the noise traders do better than the rational traders since the price largely follows the investment behavior of the noise traders.

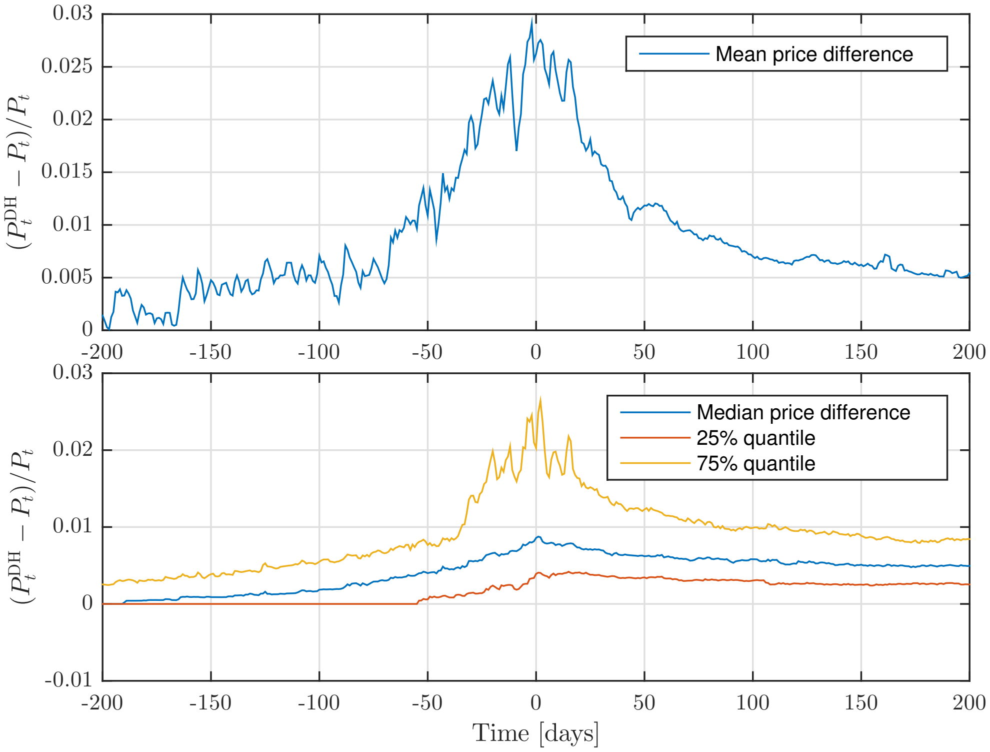

Figure 18: The same plot as in figure 17 for initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.05$. Note that the peak price difference is in the same order of magnitude as the chosen fraction of wealth that the dragon hunters hold initially.

The impact of the dragon hunters on the price was found to be clearly correlated to the wealth they were provided with in the beginning. The larger the initial wealth, the larger the observed affirming effect on the price. We could not observe any kind of shift of the crash or a "smoothing out" effect on the bubbles due to the dragon hunters.

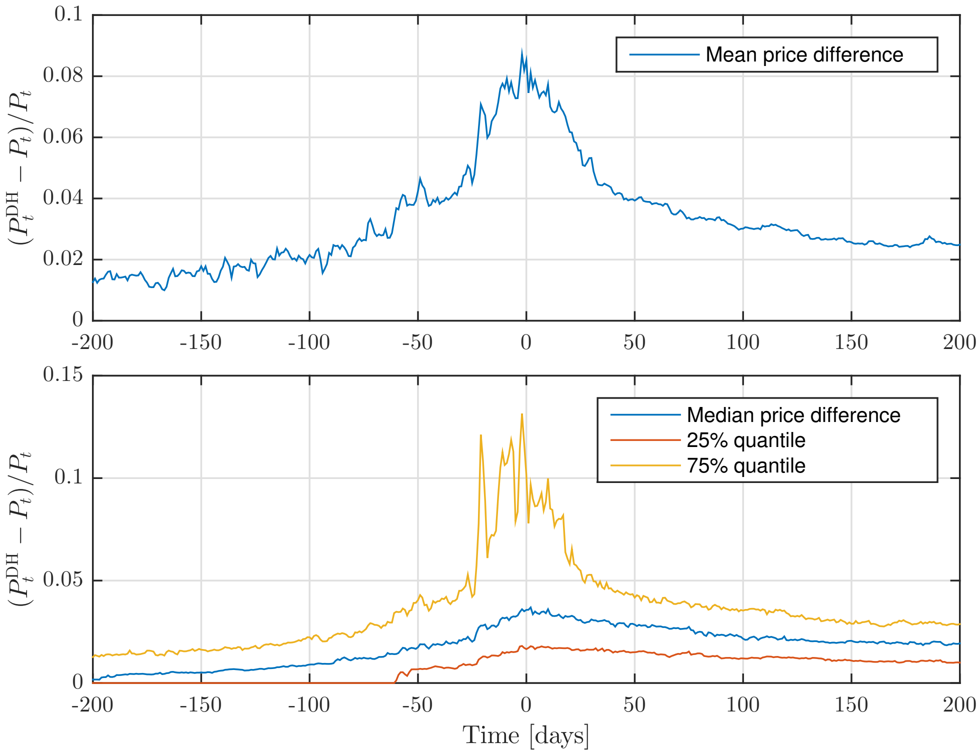

Figure 19: The same plot as in figure 17 for initial wealth ratio $W_0^{\mathrm{d}}/W_0^{\mathrm{n}} = 0.1$. Similar to figure 18 the peak price difference (≈ 0.09) is close to the initial wealth ratio of the dragon hunters.

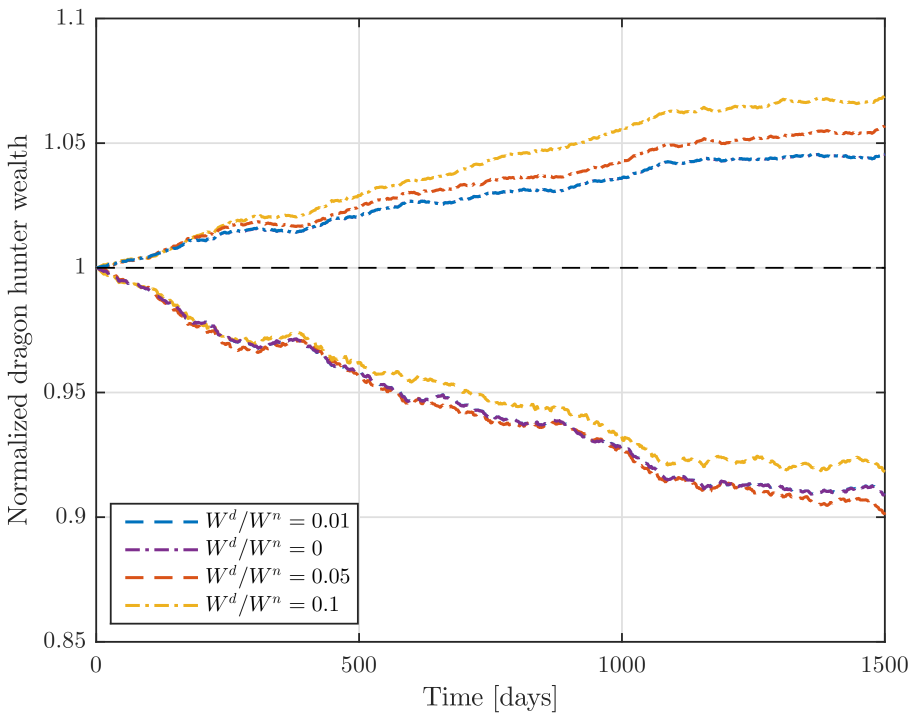

Figure 20: Average normalized wealth of rational traders (dashed) and noise traders (dash-dotted). The rational traders, similar to the dragon hunters, are performing worse than the market for all initial wealth ratios. In contrast, the noise traders do well and take the lion's share of the market yields. Both rational and noise traders however benefit from a larger relative initial wealth of the dragon hunters (for the dragon hunters, it is the opposite, cf. figure 14). The plot for the lowest initial wealth ratio hardly differs from the plot without dragon hunters; they appear to be the same but they are slightly different following the discovered low average price difference.

4. Conclusion & Outlook

In the previous chapters, we have studied an agent-based model proposed by Kaizoji et al. [16] that was designed to investigate bubbles on financial markets. In the original model, two types of agents are assumed: rational traders that invest a constant fraction of their wealth in stocks and the rest in bonds and noise traders that rely in their investment decision on social imitation and past price performance. The social herding by the noise traders can lead to a positive feedback which permits the emergence of price bubbles. The frequency, length and height of the bubbles can be explained by a variety of parameters. We found that, using this model, it is possible to simulate markets that exhibit important stylized facts of real financial markets, namely volatility clustering and slowly-decaying autocorrelations functions of absolute returns. We also found that it is possible to generate bubbles using this model that last realistic time scales of $\sim$1 year. All these findings suggest that the agent-based model can be used to study some aspects of real markets.

Building upon this model, we have motivated and proposed an additional trader type whose agents use log-periodic power laws to predict bubbles and exploit this knowledge by investing until the crash happens. A simple bubble detection algorithm was developed with a corresponding investment strategy. The LPPL traders fit the price data to their model and invest if the fit parameters lie in a certain range indicating a bubble. Since the fitting procedure is resource intensive, we tried to strike a balance between completeness of bubble information and computation time when designing the strategy. The simulation parameters were carefully chosen and the choices were – whenever possible – motivated by estimates of the model behavior.

The main goal of this work was to quantify the impact of an LPPL-powered strategy employed by some market participants on the whole market with special focus on bubble development. To this end, we simulated markets with three different initial wealth levels allocated to the LPPL investor.

For all three wealth levels, we observed that the price around bubbles rises on average when dragon hunters are involved. Obviously, the price at the peak is also higher but we observed the largest price difference at the actual price peaks. The position of the peaks remained the same; the bubbles did not form or burst earlier or later and the dragon hunters had little to no influence on the point in time that the peaks emerge. The increase at the peak was of the same order of magnitude as the fraction of the overall wealth on the market that was allocated to the dragon hunter.

Due to the chosen strategy of the dragon hunters and some misread bubbles they were unable to systematically outperform the noise traders with their strategy. Actually, the larger their share of the overall wealth, the poorer they performed relative to the market. This may be explained by the simplicity of their investment strategies.

Most of the shortcomings of the dragon hunters can be attributed to the sacrifices that were made in the LPPL signal detection. A thorough analysis might improve the individual performance. An easy fix would be to evaluate more time windows more frequently. Also, the LPPL strategy could be modified such that the dragon hunters do not invest as soon as they detect a signal but rather wait until the number of signals reaches a certain threshold. Then, a confidence interval for the expected critical time t_c might make the investment decision more secure and prevent "wrong" investments. However, these fixes should not change the result regarding the dragon hunter impact as the shortcomings were addressed in this thesis by filtering of the simulations.

The dragon hunters' performance compared to the noise traders is very hard to improve. The dragon hunters have to compete with a trader type whose investment decisions heavily influence the price and basically "make" the market. We found that the dragon hunters practically hurt themselves with their abrupt buying and shorting. To smooth out the bumps, one might think of a linear change in investment. It is hard to predict whether the advantage of a less jerky price prevails over the more inertial investment decision, but it would certainly be worth exploring.

Another angle of attack could be to look at the interaction between noise traders and dragon hunters. The interaction in the extended model discussed in this thesis is restricted to the price and the price momentum. A quantitative analysis of the interaction, perhaps by comparing the opinion index with and without dragon hunters, might give further insight into how the model can be modified. So far, the opinion index s_t and the price momentum H_t are weighted by the same social interaction strength κ even though the opinion index is usually much larger. It could be interesting to introduce a separate interaction strength for the price momentum to upgrade its influence.

A more complicated but conceptually more interesting way could be to directly couple noise traders with dragon hunters via social imitation. This way the investment decisions triggered by LPPL signals have a larger impact on the whole system. Actually, it is quite unrealistic to assume that the dragon hunters secretly invest without attracting imitators or listening to the prevailing opinion. An "upgrade" of the dynamical equations similar to this thesis will most likely not be possible. The equations would have to be redeveloped but this approach presents a good starting point for future research.

5. Acknowledgements

First, I would like to express my gratitude to Prof. Dr. Didier Sornette for the opportunity to write my thesis under his supervision and the access to ongoing research and material. Furthermore, I would like to sincerely thank Matthias Leiss for the excellent guidance, the various inputs and great discussions during the writing of this thesis.

Also, I would like to thank Dr. Vladimir Filimonov, Dr. Qunzhi Zhang and Zalàn Forrò for the valuable help and guidance regarding LPPL fitting and real-time bubble detection for the LPPL trader.

My deepest gratitude goes to my girlfriend not only for all the patience and support throughout but also for valuable auxiliary work done in connection with the filtering of results. Furthermore, I would like express my sincere gratitude to my family for the emotional support all throughout my studies.

Last but not least I would like to thank the other masters students at the Chair of Entrepreneurial Risks, especially Paweł Morzywołek, Joel Bloch, Stefan Rustler and Maximilian Seyrich, for the interesting debates, valuable inputs and overall good spirit that fueled this thesis.

6. Bibliography

[1] Fischer Black. Noise. *The Journal of Finance*, 41(3):529–543, 1986.

[2] Olivier J. Blanchard and Mark W. Watson. Bubbles, rational expectations and financial markets. Working Paper 945, National Bureau of Economic Research, July 1982.

[3] Carl Chiarella, Roberto Dieci, and Xue-Zhong He. *Heterogeneity, Market Mechanisms, and Asset Price Dynamics*, chapter 5, pages 277–344. Handbook of Financial Markets: Dynamics and Evolution. Elsevier, January 2009.

[4] James B. De Long, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. Noise trader risk in financial markets. *Journal of Political Economy*, 98(4):703–738, August 1990.

[5] James B. De Long, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395, 1990.

[6] William Egan. Testing for autocorrelation in the stock market, January 2008.

[7] Eugene F. Fama. Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2):383–417, May 1970.

[8] Vladimir Filimonov and Didier Sornette. A stable and robust calibration scheme of the log-periodic power law model. *Physica A:*

Statistical Mechanics and its Applications, 392(17):3698–3707, September 2013.

[9] Peter M. Garber. *Famous First Bubbles: The Fundamentals of Early Manias*. MIT Press, Cambridge, 2000.

[10] Anne Goldgar. *Tulipmania: Money, Honor, and Knowledge in the Dutch Golden Age*. The University of Chicago Press, 2007.

[11] Emilie Jacobsson. How to predict crashes in financial markets with the log-periodic power law. Master's thesis, Department of Mathematical Statistics, Stockholm University, 2009.

[12] Anders Johansen, Olivier Ledoit, and Didier Sornette. Crashes as critical points. *International Journal of Theoretical and Applied Finance*, 3(1):219–255, Jan 2000.

[13] Anders Johansen and Didier Sornette. Critical crashes. *Risk*, 5(1):91–94, 1999.

[14] Anders Johansen and Didier Sornette. Shocks, crashes and bubbles in financial markets. *Brussels Economic Review*, 53(2):201–253, 2010.

[15] Anders Johansen, Didier Sornette, and Olivier Ledoit. Predicting financial crashes using discrete scale invariance. *Journal of Risk*, 1(4):5–32, 1999.

[16] Taisei Kaizoji, Matthias Leiss, Alexander Saichev, and Didier Sornette. Super-exponential endogenous bubbles in an equilibrium model of rational and noise traders. *Journal of Economic Behavior and Organization (in press)*, 2015.

[17] Taisei Kaizoji and Didier Sornette. *Bubbles and Crashes*. Encyclopedia of Quantitative Finance. 2010.

[18] Sigismund Kobe. Ernst Ising 1900-1998. *Brazilian Journal of Physics*, 30:649–653, March 2000.

[19] Albert S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335, 1985.

[20] Li Lin, Ruo En Ren, and Didier Sornette. A consistent model of 'explosive' financial bubbles with mean-reversing residuals. *International Review of Financial Analysis*, 33:210–225, 2014.

[21] Li Lin and Didier Sornette. Diagnostics of rational expectation financial bubbles with stochastic mean-reverting termination times. *The European Journal of Finance*, 19(5):344–365, 2013.

[22] Thomas Lux and Michele Marchesi. Scaling and criticality in a stochastic multi-agent model of a financial market. *Nature*, 397:498–500, 1999.

[23] Benoît B. Mandelbrot. The variation of certain speculative prices. *The Journal of Business*, 36(4):394–419, 1963.

[24] MATLAB. *Version 8.2 (R2013b) Documentation*. The MathWorks Inc., Natick, Massachusetts, 2013.

[25] Makoto Matsumoto and Takuji Nishimura. Mersenne twister: A 623-dimensionally equidistributed uniform pseudo-random number generator. *ACM Transactions on Modeling and Computer Simulation*, 8(1):3–30, January 1998.

[26] Daniel Philipp. Long super-exponential bubbles in an agent-based model (semester thesis). Technical report, ETH Zurich, July 2014.

[27] Thomas C. Schelling. Models of segregation. *The American Economic Review*, 59(2):488–493, May 1969.

[28] Franz Schwabl. *Statistical Mechanics*. Springer-Verlag, 2nd edition, 2006.

[29] Robert J. Shiller. *Irrational Exuberance*. Princeton University Press, 2nd edition, 2005.

[30] Didier Sornette. *Why Stock Markets Crash: Critical Events In Complex Financial Systems*. Princeton University Press, 2004.

[31] Didier Sornette. Dragon-kings, black swans and the prediction of crises. *International Journal of Terraspace Science and Engineering*, 2(1):1–18, July 2009 (http://ssrn.com/abstract=1470006).

[32] Didier Sornette. Physics and financial economics (1776–2014): puzzles, Ising and agent-based models. *Reports on Progress in Physics*, 77(6):062001, June 2014.

[33] Didier Sornette and Peter Cauwels. Financial bubbles: Mechanism, diagnostic and state of the world. *Review of Behavioral Economics (in press)*, 2015 (http://ssrn.com/abstract=2423790).

[34] Didier Sornette and Anders Johansen. Significance of log-periodic precursors to financial crashes. *Quantitative Finance*, 1(4):452–471, 2001.

[35] Didier Sornette, Anders Johansen, and Jean-Philippe Bouchaud. Stock market crashes, precursors and replicas. *Journal de Physique*, 6(1):167–175, January 1996.

[36] Didier Sornette and Guy Ouillon. Dragon-kings: Mechanisms, statistical methods and empirical evidence. *The European Physical Journal Special Topics*, 205(1):1–26, 2011.

[37] Didier Sornette, Ryan Woodard, Wanfeng Yan, and Wei-Xing Zhou. Clarifications to questions and criticisms on the Johansen–Ledoit–Sornette financial bubble model. *Physica A: Statistical Mechanics and its Applications*, 329(19):4417–4428, October 2013.

[38] John von Neumann. *The General and Logical Theory of Automata*, volume 5 of *John von Neumann - Collected Works*. Pergamom.

[39] Wei-Xing Zhou and Didier Sornette. Statistical significance of periodicity and log-periodicity with heavy-tailed correlated noise. *International Journal of Modern Physics C*, 13(2), 2002.

[40] Wei-Xing Zhou, Didier Sornette, Russell A. Hill, and Robin I. M. Dunbar. Discrete hierarchical organization of social group sizes. *Proceedings of the Royal Society of London B: Biological Sciences*, 272(1561):439–444, 2005.

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Declaration of originality

The signed declaration of originality is a component of every semester paper, Bachelor's thesis, Master's thesis and any other degree paper undertaken during the course of studies, including the respective electronic versions.

Lecturers may also require a declaration of originality for other written papers compiled for their courses.

I hereby confirm that I am the sole author of the written work here enclosed and that I have compiled it in my own words. Parts excepted are corrections of form and content by the supervisor.

Title of work (in block letters):

Authored by (in block letters):
For papers written by groups the names of all authors are required.

Name(s):	**First name(s):**

With my signature I confirm that
- I have committed none of the forms of plagiarism described in the 'Citation etiquette' information sheet.
- I have documented all methods, data and processes truthfully.
- I have not manipulated any data.
- I have mentioned all persons who were significant facilitators of the work.

I am aware that the work may be screened electronically for plagiarism.

Place, date	**Signature(s)**

For papers written by groups the names of all authors are required. Their signatures collectively guarantee the entire content of the written paper.