Eidgenössische Technische Hochschule Zürich
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Meet-or-Beat: A Savings Model of Discretionary Accruals

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A thesis submitted to attain the degree of

MASTER OF SCIENCE

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December 2019

Abstract

We study a dynamic model of earnings management in which asymmetric stock market behavior around earnings benchmarks arises endogenously as a consequence of constrained managerial earnings manipulation. Our infinite horizon model reveals an equilibrium that is characterized by managers leveraging precautionary savings ("cookie jars") to meet investors' expectations in future periods. We show that for sufficiently poor economic results, the manager underreports earnings by the maximum, hence preferring to take a "big bath" in hope of prompting a more favorable stock price reaction in future periods. For high firm earnings, the manager's reporting decision is ambiguous and shifts from precautionary saving towards overstating earnings as firm savings and the manager's degree of myopia increase. In addition to providing comparative statics for the asymmetric stock market behavior around benchmarks, we structurally estimate our model. Our results provide a first estimate for the constraints on a manager's reporting discretion relative to firm size and indicate a limit of 0.5% of total assets. We complement empirical studies in the accounting literature which estimate the average level of discretionary accruals on firms' balance sheets.

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Chapter 1

Introduction

A significant stream of accounting literature focuses on explaining the prevalence of earnings management. While firms' reasons for managing earnings may vary widely, managers' incentives to manage earnings are frequently attributed to managerial compensation being linked to a firm's stock price. Since periodic earnings announcements by managers serve as an important source of information for investors, managers thus have an incentive to manage earnings.[1] In addition, generally accepted accounting principles (GAAP) provide managers with significant flexibility in their earnings reports. Recognition of sales not yet shipped, delay in maintenance expenditures and the recognition of losses on assets that have a fair market value below the current book value are just a few examples of the managerial leeway in reporting earnings.[2] Managers therefore not only have the incentives but also the ability to manipulate earnings in hope of prompting a more favorable stock price reaction.[3] An earnings management pattern commonly discussed in the literature is the tendency of managers manipulating earnings to meet or beat certain benchmarks. For example, both survey evidence (Graham et al. [2005]) and discontinuities in earnings distributions around benchmarks (Burgstahler and Dichev [1997]) are consistent with earnings management to meet or beat certain earnings targets. More specifically, last period's earnings, zero profits, and analysts' forecasts have been proven to be significant benchmarks (Degeorge et al. [1999]) for reported earnings. Chen et al. [2003] finds that the stock market's reaction to reports around these bench-

[1] See, for instance, Basu et al. [2013] and May [1971].

[2] Shilit [2002] provides an extensive analysis of means to manipulate earnings.

[3] For a review of empirical literature on earnings management, see Healy and Wahlen [1999].

marks is asymmetric: Earnings reports failing to meet a benchmark are penalized by a drop in stock price that is significantly larger than the increase in stock price for a corresponding report that beats the benchmark. We rely on the empirical results in Barton and Simko [2002] to provide a potential explanation for the asymmetric stock price reaction in a rational expectations framework. To this end, we rely on Barton and Simko [2002] that finds that "a manager's ability to optimistically bias earnings decreases with the extent to which net assets are already overstated on the balance sheet". As a result, when firms fail to meet a benchmark, investors infer that not only the current period's earnings are low but also that the firm has exhausted its reporting discretion. If a manager's discretionary behavior is indeed constraint by the history of earnings manipulations, this suggests that earnings have been overstated in prior periods, causing investors to revise downward their beliefs about past earnings. This in turn triggers a larger stock price reaction for earnings that fall short of the benchmark than for earnings that meet or beat the benchmark.

We suggest a rational expectations model in which asymmetric stock market behavior around benchmarks arises endogenously. In equilibrium, investors believe that the manager uses the available slack in the balance sheet to meet an earnings target whenever possible. Since such beliefs result in large penalties when failing to meet the benchmark, it is optimal for the manager to manipulate earnings consistent with investors' beliefs. Because investors revise their beliefs not only about concurrent earnings based on the manager's report but also about past earnings, stock prices are not linear in earnings but follow a recursive Bayesian filtering process instead.[4] To the best of our knowledge, this is the first theoretical model in the literature in which the concept of precautionary savings in a dynamic setting is applied to firms' earnings management decision.

We study two models: a discrete two-period model and a continuous infinite-horizon model which generalizes the findings of the parsimonious model. In the parsimonious two-period model, the manager of a firm privately observes the firm's true earnings before issuing a report to investors on the stock market. The manager is not confined to report truthfully but can instead engage in earnings management and leverage leeway within accounting conventions to manipulate reported earnings. The amount of

[4]Linear stock prices are prevalent in prior literature. For instance, see Beyer et al. [2018], Fischer and Verrecchia [2000] and Stein [1989].

accumulated accruals is however constrained which precludes sustained overstating of periodic earnings by the manager. Consistent with the theoretical literature on earnings management, we assume that the manager's utility is linked to the firm's stock price, encouraging him to anticipate the stock market's reaction to his earnings announcements. A firm's stock price is set by investors on the stock market, based on the publicly available reported earnings.

We conjecture that the following is a Perfect Bayesian Equilibrium: For positive firm earnings, the manager decides to understate earnings but only to the extent that reported earnings still meet or beat the benchmark. This allows him to build up so called "cookie jars", hoping to use the additional savings to meet the benchmark in a future period. For negative earnings, the manager overstates earnings to meet the benchmark unless his reporting discretion is exhausted. In this case, he decides to take a "big bath", i.e., accepts to report bad economic results hoping that the additional savings can be used to meet the benchmark in a future period. We find that our conjectured long-sighted manager behavior imposes equilibrium conditions on the manager's assumed utility of saving, i.e., his expected utility from increasing the likelihood that he will be able to meet benchmarks in future periods by building up cookie jars today. When the utility of saving is low, which corresponds to myopic manager behavior, the utility that follows from overstating earnings in the current period exceeds the manager's assumed benefit from saving for future periods. By contrast, if the utility of saving is high, the asymmetry of stock price reactions provides sufficient incentives for the manager to meet the benchmark whenever possible and to save for future periods otherwise. We also find that negative earnings surprises trigger a larger stock market reaction for all equilibrium states consistent with the empirical findings in Chen et al. [2003].

The second model we study extends the parsimonious model to a more general setting while maintaining the same economic forces at its core. All distributions are continuous and we consider an infinite horizon. The manager considers all future stock prices for his reporting decision, but myopic behavior causes him to discount future stock prices. Similar to the parsimonious model, the amount of accumulated discretionary accruals the manager can use to optimistically bias earnings is constrained. The reporting discretion available to the manager in a given period hence depends on all prior reporting decisions. The restriction is bi-directional and therefore limits not only the manager's ability to meet or beat the earnings benchmark in this period but also his ability to pessimistically bias reported earnings and build up cookie jars. Investors use recursive Bayesian filtering to update their beliefs about firm earnings after an earnings announcement. To increase tractability, we limit investors' memory. Specifically, we assume that investors summarize information between earnings announcements. We thus account for limited memory. We conjecture that the following is a Perfect Bayesian Equilibrium: Whenever possible, the manager reports earnings that just meet the benchmark. If the discretionary constraints prohibit him from doing so, he understates earnings as much as possible. Our conjectured equilibrium is the extension of the parsimonious model's equilibrium to continuous distributions and hence includes both "big bath" behavior and the build up of "cookie jars". In equilibrium, positive reported earnings allow investors to infer the current period's true firm earnings but do not provide information about past earnings. Negative reported earnings reveal the current period's true earnings and provide information about past earnings. More specifically, they lead investors to revise their beliefs about past earnings downward. The magnitude of the effect is higher if reported earnings are closer to the benchmark than if reported earnings are very low. The intuition is as follows. When reported earnings fall short of the benchmark but are not very low, investors infer that the manager did not even build up a small cookie jar in the past that would have allowed him to meet the benchmark this period when earnings only fell short of the benchmark by a small amount. Thus, investors revise their expectations about past earnings down by more than if reported earnings in the current period are so low that even if the manager has built up a reasonable cookie jar in the past, the benchmark is out of reach. If reported earnings just meet the benchmark, investors increase their expectation of past earnings since meeting the benchmark is a positive signal about the cookie jars the manager has built up in the past.

Numerical analysis shows that the conjectured equilibrium holds for low discounting factors, i.e., for a low degree of myopia. As the manager increasingly discounts the utility obtained from future stock prices, he prefers overstating today's earnings to saving reporting discretion for the future. In the extreme case of perfectly myopic behavior, i.e., if the manager only considers the current period's stock price, overstating earnings by the maximum amount is the manager's optimal choice for all earnings realizations. In addition to the discounting factor, the size of the firm's current cookie jars affects the manager's equilibrium reporting strategy. If firm savings are high, additional savings provide a low additional utility since the probability that they increase the likelihood of meeting the benchmark in the near future is low. As firm savings increase, overstating earnings hence replaces precautionary saving as the manager's optimal reporting choice for positive earnings reports. These two main findings suggest a refined equilibrium reporting strategy which specifies overstating earnings as the manager's optimal reporting choice for earnings above a threshold determined by firm savings and discounting factor.

The maximum size of "cookie jars" that managers are able to build up is naturally unobservable. As a result, we use structural estimation to recover the latent variable that captures the constraint the balance sheet places on managers' ability to build up cookie jars. Based on quarterly data on stock prices and reported net income from WRDS, we structurally estimate the manager's ability to understate earnings relative to total assets. The manager's ability to optimistically bias earnings is assumed to be constrained by previous savings which eliminates the need for the estimation of an additional latent parameter.[5] We first estimate the parameter at the industry level which requires the assumption that the parameter is constant both across firms of the same industry and over time. We find that managers can understate earnings by around 0.44% of total assets. The results are fairly consistent among industries, ranging from 0.32% to 0.87%. An analysis based on firm size indicates that the estimate is also consistent across firms of different sizes.

Related Literature Earnings management has been studied extensively in both empirical and theoretical accounting literature. The prevalence of discretionary reporting behavior has been established by an overwhelming amount of empirical evidence while theoretical studies provided models to help understand firms' and investors' behavior in different settings.

Theoretical models of earnings management are models in which the manager must disclose a signal but is not confined to report truthfully. However, misreporting is either costly to the manager or firm or is constraint in some other fashion. Early models of earnings management in a capital market setting include Dye [1988] and Titman and

[5] In addition, we allow for discretion even in the absence of savings. However, the impact of this parameter converges to zero and is hence excluded from our structural estimation.

Trueman [1986]. Both papers study investors' inference problems when the manager's optimization problem is not entirely known to investors and establish the conditions under which earnings management arises. Fischer and Verrecchia [2000] also assumes that the manager's objective is not perfectly known to investors but focuses on the amount of information earnings management destroys from investors' perspective. Similar to many models of earnings management, Fischer and Verrecchia [2000] analyzes a linear equilibrium. As a result, it does not capture managers' incentives related to earnings targets. In contrast, Degeorge et al. [1999] focuses on the relevance of earnings thresholds and shows that an exogenous bonus for meeting or exceeding a threshold induces the manager to meet the threshold whenever possible, assuming a two-period setting. Our model differs from Degeorge et al. [1999] in so far as managers' incentives to meet or beat a benchmark are not exogenously given as the result of some bonus contract but rather arise endogenously as a result of investors' inferences. This is similar to Guttman et al. [2006] that studies equilibria in which discontinuities in the distribution of reported earnings may emerge despite exclusively smooth distributions in the single period model's setup. Since Guttman et al. [2006] considers a single-period model, managers' consideration of building up "cookie jars" is absent from the model. In contrast, Kirschenheiter and Melumad [2002] demonstrates managerial big bath behavior in a two-period setting and thus extends our understanding of the multi-period nature of corporate reporting. Similar to other two-period models of earnings management (see, e.g., Ewert and Wagenhofer [2005]), Kirschenheiter and Melumad [2002] assumes that any accrual-based earnings management in the first period must reverse in the second period. Beyer et al. [2018] extends previous models and considers both a finite and infinite-horizon setting. Instead of a forced exogenous reversal of bias after the last period, prominent in two-period models, earnings manipulation is assumed to be unbounded but costly. Similar to Beyer et al. [2018], we also study an infinite horizon setting in which reporting bias is not forced to reverse at a specific point in time. However, in contrast to Beyer et al. [2018], our model does not study linear equilibria but rather considers managements' incentives to meet or beat benchmarks that arise endogenously in the capital market setting. This causes the equilibrium to be non-linear. To summarize, we complement the existing theoretical literature by applying ideas from precautionary savings models to an earnings management setting and suggest an infinite horizon model in which asymmetric stock market behavior around benchmarks arises endogenously. Specifically, we focus on an equilibrium in which constraints on a firm's reporting discretion cause investors' expectations to act as an important benchmark in the interaction between firm managers and investors.

Earnings management is also the subject of numerous empirical studies. For a review, see Dechow et al. [2010]. Our motivation to extend the existing theoretical literature stems most notably from the following two empirical findings. First, Barton and Simko [2002] finds that "a manager's ability to optimistically bias earnings decreases in the extent to which net assets on the balance sheet are already overstated" and therefore confirms the intuition that a firm's balance sheet constrains the amount of discretionary accruals available to its manager. Second, empirical studies such as Chen et al. [2003] find that the stock market's reaction to positive and negative earnings surprises is asymmetric with negative earnings surprises triggering a larger response. Consistent with Chen et al. [2003], we find that the stock market's reaction to negative news is approximately 1.69 times larger than its reaction to positive news.

In addition, we structurally estimate our infinite horizon model and provide a first validation of the results in Hribar and Nichols [2007] and Gerakos and Kovrijnykh [2013] based on a theoretical economic rather than statistical model. In both papers, the authors conducted an empirical estimation of the magnitude of discretionary accruals on firms' balance sheets relative to firm size. In contrast to Hribar and Nichols [2007] and Gerakos and Kovrijnykh [2013], we do not estimate the actual magnitude of discretionary accruals on firms' balance sheets at any point in time but rather estimate the maximum possible magnitude of discretionary accruals capturing the idea of the balance sheet as a constraint to cumulative earnings management.

Chapter 2

Discrete Two-Period Model

The multi-agent problem we are trying to model poses many challenges and requires extensive mathematical efforts. For his decision on what earnings to report, a firm's manager engages in non-cooperative sequential decision-making with the firm's investors on the stock market who set the firm's stock price based on their belief about true firm value. Consequently, modeling their interaction requires the introduction of an equilibrium concept. But even solving for an equilibrium turns out to be challenging. We therefore first develop a simplified model with the ambition to illustrate the main economic forces of our model. In the next chapter, we will then generalize our model to a more general setting.

This chapter consists of three sections. We first describe the model setup. The second section includes both a description of the equilibrium concept we use as well as our approach to solving for an equilibrium. In the third and last part of this chapter we describe and discuss the results.

2.1 Setup

In this section, we outline our model and discuss its assumptions as well as their economic intuition. The model consists of three elements: a firm which generates economic earnings, a manager who can engage in earnings management in order to maximize his expected utility and the stock market which sets the firm's stock price based on the publicly available information.

In each period, the manager privately observes the firm's true earnings. He then issues a report without being confined to report truthfully. Instead, he can overstate or understate earnings. The reporting discretion is however constraint by his previous reporting history. As his utility is assumed to be linked to the firm's stock price, the manager anticipates the stock market's reaction when choosing the amount of discretion included in reported earnings. The investors set the price based on the publicly available reported earnings. We consider discrete time periods and analyze a two-period model.

Firm

We consider a firm whose true earnings e_t are uniformly distributed but can only take the five discrete integer values between -2 and 2. The probability density function, hereafter referred to as PDF, for the true earnings at time t is therefore given by

$$\mathrm{f}_{e_t}(\bar{e}_t) = \begin{cases} \frac{1}{5} & \forall \bar{e}_t \in \{-2, -1, 0, +1, +2\} \\ 0 & \text{otherwise} \end{cases} \qquad \forall t. \tag{2.1}$$

Since we chose a PDF that is symmetric around $e_t = 0$, the expected value of true earnings at any time t is $\mathbb{E}[e_t] = 0$ which will serve as the benchmark for our model. We will show that the manager tries to meet this benchmark and failing to do so is penalized by the stock market.

Manager

At every time t, the manager decides on what earnings to report. His reported earnings r_t are equal to the sum of true earnings e_t and the discretion δ_t, i.e., $r_t = e_t + \delta_t$. We consider a rational manager who optimizes his current period's utility. The manager's utility in turn is assumed to be tied to the firm's stock price which is set by investors on the stock market. For the manager's reporting decision at time t, he considers the current period's stock price, which will be set by investors after the report has been issued, as well as all expected future stock prices. Even though we only consider a two-period model we are interested in obtaining results that may also occur in a model with more than two periods. As the number of reporting periods increases, the manager's strategy set of possible reporting decisions increases exponentially. To maintain tractability, we therefore assume that the manager considers the future in the form of a constant expected utility of saving. This allows us to express the manager's objective function without taking expectations along every possible price path. Let u denote the manager's expected future utility from having "saved" or underreported earnings by 1 in the current period t. The manager's maximization problem at time t is then given by

$$\max_{\delta_t} u_t = \begin{cases} P_t(r_t = e_t + 1) & \delta_t = 1 \\ P_t(r_t = e_t) + u & \delta_t = 0 \\ P_t(r_t = e_t - 1) + 2u & \delta_t = -1 \end{cases} \tag{2.2}$$

Barton and Simko [2002] showed that a firm's ability to overstate reports decreases in the extent to which previous balance sheets were overstated. To account for this result we impose a constraint on the manager's reporting discretion. We assume the manager can only overstate by the amount saved in the past, i.e., to the extent to which previous reported earnings were understated, which gives us the constraint

$$\delta_t \leq \sum_{i=1}^{t-1} e_i - r_i. \tag{2.3}$$

We further assume exogenously given upper and lower bounds for the discretion to account for an additional limitation of the discretion as a manipulation equal to or greater than the true earnings appears to be unrealistic. We therefore require $\delta_t \in \{-1, 0, +1\}$. As described in the next section, some values of r_t will not be part of the manager's reporting strategy in equilibrium. In order to avoid having to specify off-equilibrium beliefs, we therefore assume that the manager may strategically choose δ_t only with some probability $1-\alpha$ while with probability $\alpha \in (0,1)$, the manager is forced to report truthfully. We define x_t as a binary variable that describes whether the manager can engage in earnings management in period t. x_t is independent across periods and independent of all other random variables such that its PDF is given by

$$f_{x_t}(\bar{x}_t) = \begin{cases} \alpha & \bar{x}_t = T \\ 1-\alpha & \bar{x}_t = S \end{cases} \tag{2.4}$$

where T is an abbreviation for 'truthful reporting', S indicates the manager's option for 'strategic reporting' and $\alpha \in (0,1)$. In case of $x_t = T$, the manager's report is equal to true earnings, i.e., $r_t = e_t$. This is equivalent to

$$f_{\delta_t|x_t}(\bar{\delta}_t|T) = \begin{cases} 1 & \bar{\delta}_t = 0 \\ 0 & \text{otherwise} \end{cases} . \tag{2.5}$$

Stock Market

Our assumption that the manager's payoff is linked to the firm's stock price requires us to specify how the stock price is set by investors. We assume that the firm does not make any payouts to shareholders, e.g., in the form of dividends. The firm's true equity at time t is thus given by the sum of all true earnings up to period t. Investors form expectations about the true firm value based on reported earnings which are the only publicly available information. Since true earnings are not serially correlated and the expected value $\mathbb{E}[e_t]$ is equal to zero, the expected change in firm value equals zero in each period. When estimating the firm's true value, investors therefore only need to price all previous as well as the current period's true earnings. If we were to assume perfect rationality of investors, at time t, they would consider all previous reported earnings $r_{1..t}$. For a longer horizon, this assumption appears to be implausible. We will therefore assume limited memory of investors for our parsimonious model despite its two-period setup. More specifically, we assume that investors do not remember the earnings reported in the previous period. We would like to remark that this assumption can be dropped without changing the results of the model yet severely simplifies the notation and interpretation of the model. To summarize, stock prices are set by investors who form their beliefs using

$$P_t(r_t) = \mathbb{E}\left[\sum_{i=1}^{t} e_i \mid r_t\right]. \tag{2.6}$$

2.2 Equilibrium

In this section, we introduce our equilibrium notion and describe an approach to derive an equilibrium tuple that includes a reporting strategy and a pricing function. We will consider a Perfect Bayesian Equilibrium in which investors correctly infer the manager's discretion strategy. Given the resulting stock price function, the manager has no incentive to deviate from the discretion strategy the investors inferred. Mathematically speaking, a Perfect Bayesian Equilibrium is defined as a discretion strategy δ_t^* for the manager, together with a stock price function P_t^*, such that for any t:

i) $P_t^* = \mathbb{E}[\sum_{i=1}^{t} e_i \mid r_t^*]$ and

ii) $\delta_t^*(e_t) \in \arg\max_{\delta_t} u_t \quad \forall e_t$,

where $r_t^* = e_t + \delta_t^*$. The two conditions guarantee sequential rationality and consistency, both of which are required for a Perfect Bayesian Equilibrium. To derive an equilibrium, we proceed as follows. We start by conjecturing an equilibrium discretion strategy for the manager. We then calculate investors' pricing strategy assuming they correctly infer the manager's discretion strategy. This allows us to derive the equilibrium conditions for the manager's assumed utility of saving u that need to be satisfied for the conjectured equilibrium to hold.

In the previous section, we defined reporting discretion such that reported earnings r_t at time t are given by $r_t = e_t + \delta_t$. We can interpret δ_t as the difference between reported and true earnings. Positive values of δ_t occur in case of overstating earnings while negative values describe understating earnings, i.e., saving in hope of a higher utility in the future. We proceed by conjecturing the equilibrium discretion strategy as

$$\delta_t(e_{1..t}, r_{1..t-1} \mid x_t = S) = \begin{cases} -1 & e_t \in \{1, 2\} \\ 0 & e_t = 0 \\ +1 & e_t = -1, \sum_{i=1}^{t-1} e_i - r_i \geq 1 \\ -1 & e_t = -1, \sum_{i=1}^{t-1} e_i - r_i < 1 \\ 0 & e_t = -2 \end{cases} \quad (2.7)$$

for all t.[6] Though being a discrete function, the conjectured discretion strategy is based on the results in Degeorge et al. [1999] where the asymmetric stock market behavior arose through an exogenous premium for meeting the benchmark. Our aim is to

[6] The uniqueness of our conjectured equilibrium is yet to be proved or disproved.

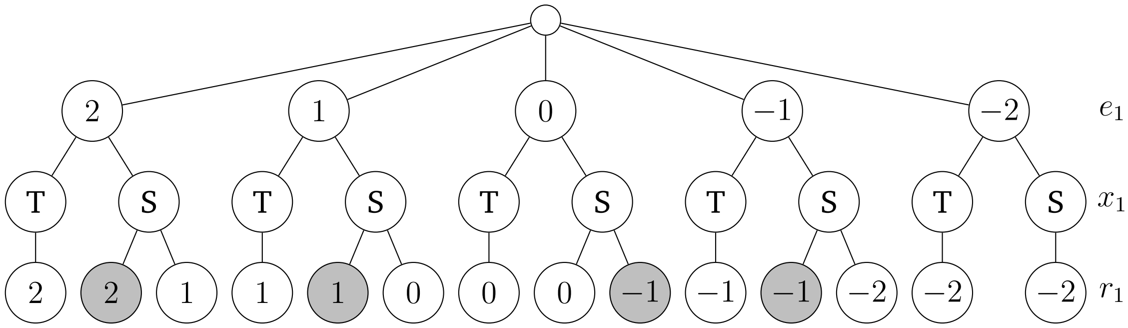


Figure 2.1: The extensive form of the manager's reporting strategy at $t = 1$ illustrates the potential outcomes for true earnings in the first period e_1 and the binary variable x_1 which describes whether or not the manager can engage in earnings management in the first period. For each combination, the network shows the feasible choices of reported earnings r_1. Light gray nodes correspond to off-equilibrium paths which we use to derive the equilibrium conditions.

provide evidence for similar discretionary behavior for a model where the asymmetric stock market behavior arises endogenously. For positive earnings, the manager decides to understate reported earnings allowing him to build up so called "cookie jars" so the benchmark of $r_t = 0$ can be met in future periods. For true earnings of $e_t = 0$, the manager reports truthfully as true earnings just meet the benchmark. The case of $e_t = -1$ is of special interest as meeting the benchmark is a crucial element of our analysis. We conjecture that the manager will report $r_t = 0$ unless the discretion constraint is binding. If he did not save in previous periods and is not able to overstate reported earnings, he will instead engage in understating and report $r_t = -2$. He accepts to take a big hit in the current period hoping for a bigger payoff in the future. This phenomenon is commonly referred to as "big bath" strategy and has been studied by exisiting literature, e.g. in Kirschenheiter and Melumad [2002]. The higher reward for meeting a benchmark further incentivizes the manager to save in case of positive earnings. Our model hence includes both big bath behavior as well as the build up of cookie jars to meet benchmarks in future periods.

Figure 2.1 illustrates the extensive form of the manager's reporting strategy in the first period and shows the possible outcomes for true earnings e_1 and x_1. The bottom row of nodes shows the feasible choices for the reported earnings r_1. Light gray nodes indicate paths that do not correspond to our conjectured equilibrium and that are used to derive the equilibrium conditions for u.

For every period t, our conjectured discretion strategy imposes four conditions on u for the equilibrium to hold. First, saving must be more attractive for positive true earnings, i.e., for both $e_t = 1$ and $e_t = 2$. Second, meeting the benchmark $r_t = 0$ needs to be more attractive than saving and reporting $r_t = -1$. Furthermore saving and reporting $r_t = -2$ must be preferred to reporting $r_t = -1$ which corresponds to taking a "big bath".[7]

Lemma 1. *There exists a Perfect Bayesian Equilibrium for the conjectured discretion strategy specified in Equation 2.7 if and only if the stock price function fulfills the following four conditions*

i) $P_t(2) < P_t(1) + u$

ii) $P_t(1) < P_t(0) + u$

iii) $P_t(-1) + u < P_t(0)$

iv) $P_t(-1) < P_t(-2) + u$

for all t*.*

Before deriving the stock price functions, we note that linear price functions, which have been the predominant focus of previous literature as in Beyer et al. [2018], Fischer and Verrecchia [2000] and Stein [1989], do not illustrate the empirical findings we are interested in. Instead of simplifying the price function by assuming linearity, we will therefore calculate the price function as defined in section 2.1 using investors' expectation of all true firm earnings $\{e_1, \ldots, e_t\}$ given the earnings reports $\{r_1, \ldots, r_t\}$. For the discrete two-period model we consider in this chapter, this turns out to be a straightforward process. For the generalized continuous and infinite horizon model in the next chapter, this requires some additional mathematical effort since the number of potential paths is no longer finite.

To simplify our notation we introduce $f(x)$ as an abbreviation for the PDF $f_x(\bar{x})$. Additionally, we will use $\{x_{1..t}\}$ to denote the set $\{x_1, \ldots, x_t\}$. Stock prices are determined by the expected value of the sum of true earnings based on the report at time t, i.e., $P_t(r_t) = \mathbb{E}[\sum_{i=1}^{t} e_i \mid r_t]$. Some analysis allows us to derive the stock price as a function of known PDFs. For the proof, we refer to the appendix as with all other lemmas, theorems and corollaries.

[7] For $t > 1$, a fifth condition $P_t(2) < P_t(0) + 2u$ arises which can, however, be proven to be redundant using the first two conditions.

Lemma 2. *In equilibrium, the imperfectly rational investors set the stock price according to the equation*

$$P_t(r_t) = \sum_{e_1} \cdots \sum_{e_t} \left[\left(\sum_{i=1}^{t} e_i \right) * \frac{\sum_{x_1} \cdots \sum_{x_t} \left[f(r_t|e_{1..t}, x_{1..t}) * \prod_{i=1}^{t} f(x_i) \right]}{\sum_{e_1} \cdots \sum_{e_t} \sum_{x_1} \cdots \sum_{x_t} \left[f(r_t|e_{1..t}, x_{1..t}) * \prod_{i=1}^{t} f(x_i) \right]} \right]$$

where the probability mass function

$$f(r_t|e_{1..t}, x_{1..t}) = \begin{cases} 1 & r_t = e_t, x_t = T \\ 1 & r_t = e_t + \delta_t(e_{1..t-1}, r_{1..t-1}(e_{1..t-1}, x_{1..t-1}), e_t), x_t = S \\ 0 & \textit{otherwise} \end{cases}$$

can be evaluated using the decision network in Figure 2.1.[8]

Substitution of the stock prices using the equation above yields the equilibrium conditions as univariate functions of α.

2.3 Proof of Existence for Equilibrium and Results

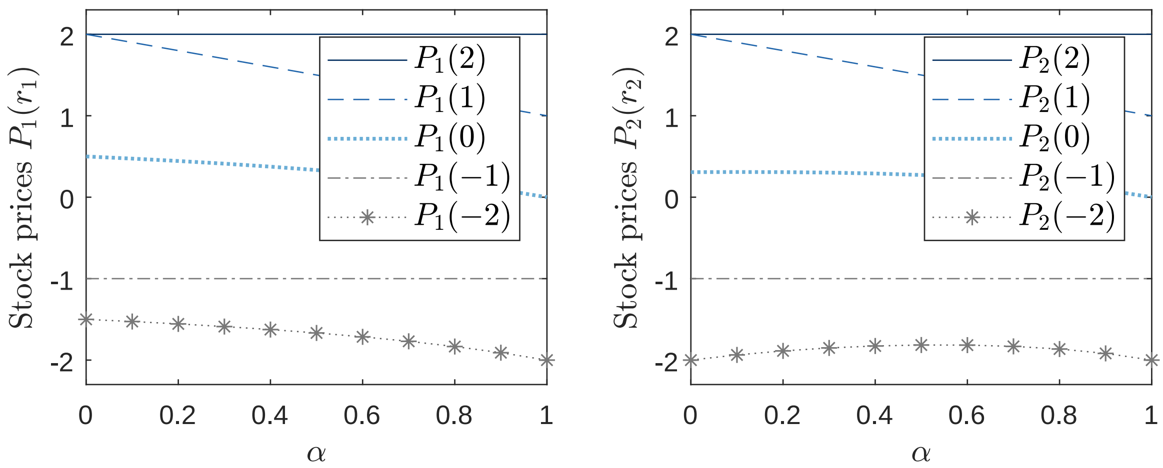
We proceed by calculating the stock price functions and analyzing the equilibrium conditions and try to provide economic intuition for the results. We further characterize the (a)symmetry of the stock price reaction to reported earnings which is one of our main points of interest.

Stock Price Functions

Using Lemma 2 and the decision network illustrated in Figure 2.1, we can derive the stock price functions for $t = 1$ and $t = 2$ which are shown in Figure 2.2. The explicit functional forms are specified in the appendix.

$P_t(2)$ and $P_t(-1)$ are independent of α as investors can perfectly infer the true earnings e_t. The reason is that both values will only be reported if the manager reports

[8] The resulting functional forms for $P_1(r_1)$ and $P_2(r_2)$ are listed in the Appendix.



(a) $t = 1$ (b) $t = 2$

Figure 2.2: Equilibrium stock price reactions $P_t(r_t)$ to reported earnings r_t for both periods as a function of the parameter α which denotes the probability of the manager being forced to report truthfully instead of engaging in earnings management

truthfully. In contrast, $r_t = 1$ is reported in two cases: when the manager reports truthfully for $e_t = 1$ and when the manager reports strategically for $e_t = 2$. $P_t(1)$ therefore decreases from 2 to 1 as α increases and the manager is more and more forced to report truthfully. With $r_t = 0$ being the benchmark in our model, $P_t(0)$ takes on a special report as it may result from underreporting, overreporting or truthful reporting. However, under- and overreporting are not equally likely. To see this, consider the manager's reporting strategy for $e_t = 1$ and $e_t = -1$. When $e_t = 1$ and $x_t = S$, the manager will always report $r_t = 1$. In contrast, when $e_t = -1$ and $x_t = S$, the manager can report $r_t = 0$ only if he has saved in the past. This asymmetry of the discretion strategy explains why the expected value of true earnings for $r_t = 0$ is strictly positive.

Figure 2.3 provides an alternative illustration of the stock price reactions to reported earnings for different values of α. The visualization of the asymmetry in stock prices around the benchmark will be used to better understand the equilibrium conditions in the following section.

Equilibrium Conditions

Inserting the stock price functions into the equilibrium conditions yields the range of u for which our conjectured equilibrium holds, depending on α. To understand our results, it is worthwhile taking a closer look at the equilibrium conditions. Each condition consists

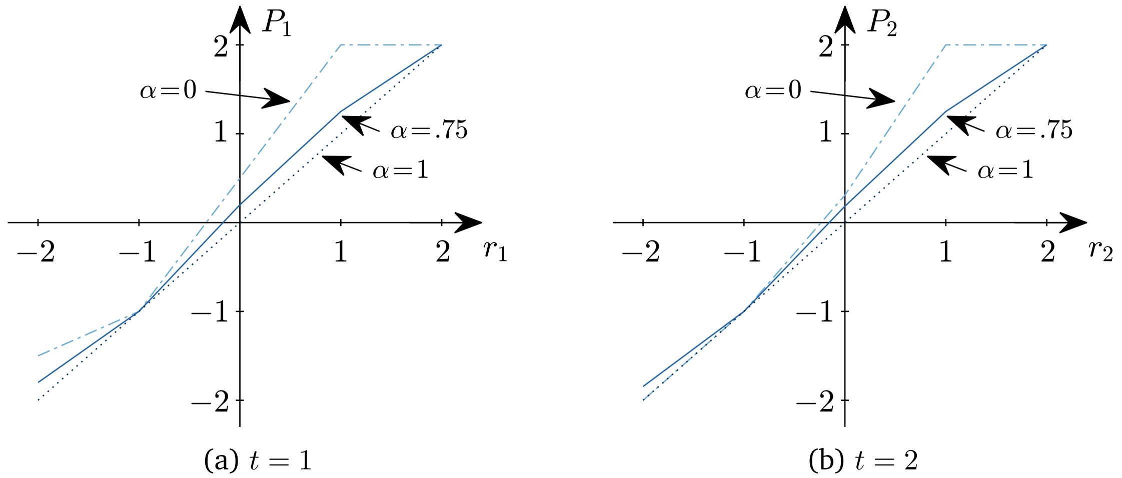


Figure 2.3: Equilibrium stock price reactions $P_t(r_t)$ to reported earnings r_t for both periods for different values of the parameter α which denotes the probability of forced truthful reporting

of two stock prices and the utility of saving u. The conjecture of the manager using his discretion to report the benchmark $r_t = 0$ when true earnings equal -1, imposes an upper limit on u: if u were too high, the manager would always prefer to save instead of using his discretion to meet the target. As discussed above, $P_t(0) > 0$ while $P_t(-1) = -1$. As a result, the upper bound on u, $P_t(0) - P_t(-1)$, always exceeds 1 and decreases in α. We conjecture that the manager underreports earnings when the earnings e_t are $\in \{-1, 1, 2\}$. Based on our discussion of $P_t(r_t)$ above, we can see that the manager's foregone price premiums from reporting truthful when $e_t = -2$ and $e_t = -1$, $P_t(2)-P_t(1)$ and $P_t(-1) - P_t(-2)$, are always $\in (0, 1)$. In contrast, the managers price premium from reporting truthfully rather than saving is always greater than 1 (but less than 1.5) when $e_t = 1$, i.e., $P_t(1) - P_t(0) \in (1, 1.5)$. It is decreasing in α and equals 1 for $\alpha = 0$. The binding lower bound on u therefore arises from the manager saving when $e_t = 1$ and always exceeds 1. From the above discussion, we can see that for $\alpha = 0$, the set of equilibrium sustaining values of u is empty. The argument extends to small values of α. In particular, for all $\alpha \leq 0.427$, our model results in an empty solution space for u as high values of $P_t(1)$ make reporting $r_t = 1$ more attractive than saving and reporting $r_t = 0$, which contradicts our conjectured reporting strategy. In contrast for $\alpha > 0.427$, our model results in a non-empty solution space for u for which our conjectured discretion strategy actually provides an equilibrium. For a visual explanation, consider Figure 2.3. In the previous paragraph, we established that saving for $e_t = 1$ and overstating for $e_t = -1$ constitute the binding lower and upper bounds for the utility of saving u. In other words, the price difference $P_t(0) - P_t(-1)$ must exceed $P_t(1) - P_t(0)$ for our conjectured equilibrium to hold. Graphically, this corresponds to a steeper slope below zero. At time $t = 1$, the slope below zero is greater or equal to the slope above zero for all α. All equilibrium conditions are thus satisfied. The limited equilibrium range for α arises at time $t = 2$. While the stock price decrease between $r_2 = 0$ and $r_2 = -1$ exceeds $P_2(1) - P_2(0)$ for α close to 1, the equilibrium conditions are violated for sufficiently small values of α. For reporting $r_2 = 0$ to be more attractive than saving and reporting $r_2 = -1$, the utility of saving needs to exceed a value that is higher than its upper bound, imposed by the condition that saving and reporting $r_2 = 0$ is more attractive than reporting $r_2 = 1$.

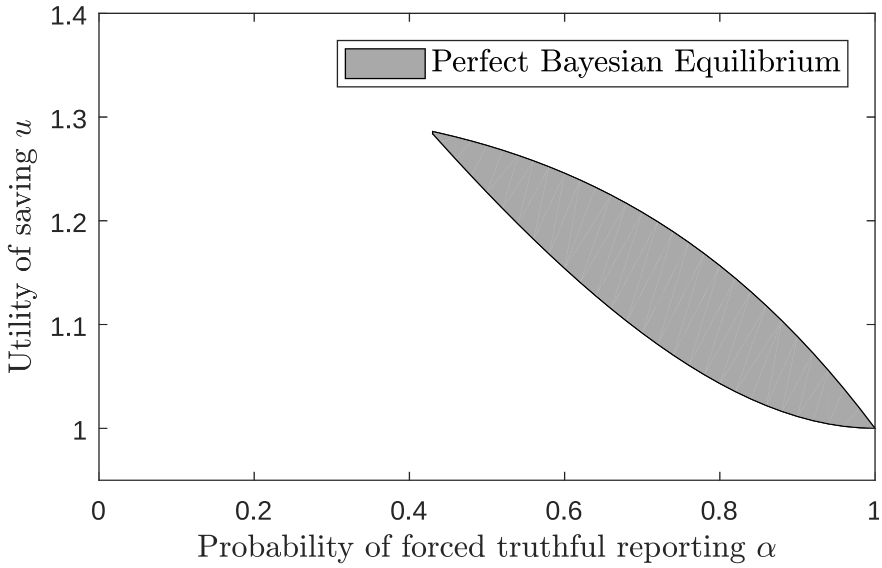


Figure 2.4: Equilibrium conditions for the utility of saving u as a function of the parameter α which denotes the probability of forced truthful reporting

Figure 2.4 illustrates the range of u for which the conjectured equilibrium holds. The first observation we make is the convergence of the solution range to 1. As α approaches 1, the manager is more and more likely to report truthfully. For $\alpha = 1$ we get a fully revealing model in which reported earnings are always equal to true earnings. Consequently, $P_t(r_t) = r_t$ for all r_t which explains why $u = 1$ is the only viable solution to the equilibrium conditions as the price difference between any two neighboring price functions is equal to 1. A discretion of 1 or -1 causes a change in the stock price by 1 and -1, respectively, which is why $u = 1$ is the only equilibrium solution.

For all values $\alpha < 1$ the equilibrium range lies above 1. For $e_t \in \{-2, 0\}$ increasing current period's reported earnings by 1 leads to a price increase higher than 1 which implies that the expected utility of saving needs to be higher than 1. This result is consistent with our intuition. For a continuous earnings distribution and an earnings realization far off the benchmark, we expect that increasing the reported earnings by 1 should increase the stock price by approximately 1. The manager would hence only save if the expected utility of saving is greater than 1. This in turn could be realized by a price difference between $P_t(0)$ and $P_t(-1)$ that is greater than 1 in future periods.

Asymmetry of Stock Price Reactions to Reported Earnings

After establishing that our conjectured discretion strategy, which includes both big bath behavior and "cookie jars", does indeed yield an equilibrium, we will now look at the stock market's reaction to reported earnings. More specifically, we are interested in the (a)symmetry of the stock price sensitivity to positive and negative earnings surprises. In our model, reported earnings of 0 equal the expected value of true earnings. Positive and negative reported earnings can therefore be seen as positive and negative earnings surprises, respectively. Empirical research as in Barth et al. [1999] and Bhojraj et al. [2009] has shown that stock market reactions to positive and negative earnings surprises differ in the form of a higher stock price decrease for negative earnings surprises compared to the stock price increase for an equivalent positive earnings surprise. We are therefore interested in the symmetry of stock price reactions in our model. Since our model includes a discretion constraint which is known to investors, we're hoping for our model to illustrate an asymmetric stock price reaction. As the stock prices in proximity of the benchmark are of particular interest, we consider the stock price sensitivity to positive surprises $\Delta P = P_t(1) - P_t(0)$ and negative surprises $\Delta P = P_t(0) - P_t(-1)$ and define an asymmetric stock price reaction by the condition

$$P_t(1) - P_t(0) < P_t(0) - P_t(-1). \tag{2.8}$$

Corollary 1. *The stock price reaction to reported earnings is asymmetric for all equilibrium values of α. Negative earnings surprises are penalized by a price decrease more stringent than the price increase for positive earnings surprises.*

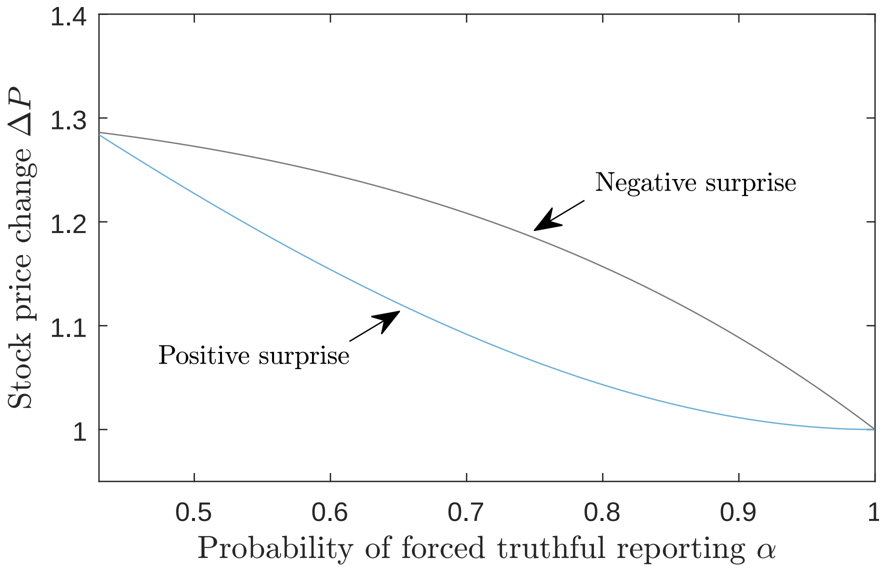


Figure 2.5: Stock price sensitivity to positive and negative earnings surprises as a function of the parameter α which denotes the probability of forced truthful reporting

Figure 2.5 describes the results for all equilibrium values of α for $t = 2$. The price difference $P_2(0) - P_2(-1)$ is strictly greater than $P_2(1) - P_2(0)$ for all equilibrium values of α. The penalty for failing to meet the benchmark is hence higher then the reward for beating it. The results of our model confirm the empirical results in Barth et al. [1999].

Chapter 3

Continuous Infinite-Horizon Model

The parsimonious model we developed in the previous chapter provided an example in which the main economic forces of interest, namely earnings management and asymmetric stock market behavior, occur in a fairly simple setting. We showed that under certain circumstances a constraint on discretionary accruals can cause asymmetric stock market behavior for positive and negative earnings surprises. Our ambition for this chapter is now to generalize the setting. The model in this chapter will be mainly based on continuous variables and will further consider an infinite horizon. We will, again, consider a manager who can engage in earnings management in hope of causing a more favorable stock market reaction when he meets the benchmark. The manager's reporting discretion is however constraint by previous overstatements. The stock market, on the other hand, tries to estimate the true firm value based on the publicly available earnings reports. This two-agent model is a case of non-cooperative sequential decision making. The manager anticipates the stock market's reaction when deciding on what earnings to report, a fact investors on the stock market are well aware of. For their estimation of the true firm value, they hence need to consider the manager's strategy which in turn depends on their own strategy. The interdependence of the two agents' strategies causes the system to be intractable and makes an analysis of the general system dynamics impossible. We are therefore required to limit our analysis to equilibrium states. By looking at equilibria with constant strategies, we can transform the stock market's estimation problem into a case of state uncertainty. Given the manager's discretion strategy, investors estimate the true firm value recursively. After an earnings announcement, the new information is used to update the belief about the current period's and all previous earnings. Since accumulated savings determine the constraint on managerial reporting discretion, the stock price updating process satisfies the Markov property and can thus be expressed by recursive Bayesian filtering.[9] The Markov property will further be leveraged to structurally estimate our model in chapter 4.

This chapter is structured as follows. First, we describe the setup of our model. We proceed by conjecturing an equilibrium and deriving the system equations for the conjectured equilibrium. In the third section, we provide a numerical proof of existence for the equilibrium and discuss the results.

3.1 Setup

Our model consists of three elements: (i) a firm with randomly distributed earnings, (ii) a manager who can engage in earnings management but whose discretion is limited by both the amount saved in the past and an exogenous lower limit, (iii) and the stock market that prices the firm based on publicly available information. In each period, the manager privately observes the firm's true earnings. He then issues a report but is not confined to reporting truthfully. Instead, he can overstate or understate earnings hoping for a more favorable stock market reaction. An upper and lower bound constrain the manager's discretion. We assume that the manager's utility is linked to the firm's stock price. For his reporting decision, he thus anticipates the stock market's reaction. The investors on the stock market use the publicly available earnings reports to estimate the true firm value, taking into account the manager's option to engage in earnings management. We consider continuous distributions for all variables and look at an infinite horizon model.

Firm

In order to choose a suitable probability density function for true earnings e_t, we analyze data on earnings reports to derive a representative firm earnings distribution. The data set we use contains 15.000 firms taken from the Compustat/CRSP merged database.

[9] A stochastic process satisfies the Markov property if the updating process depends only on the current state of the system and not on the sequence of prior events.

Accounting for growth, we consider the changes in reported earnings compared to the corresponding quarter in the previous year. Minimizing the mean squared error for a set of common distributions suggests that a Laplace distribution is the best fit to our data. We therefore choose a Laplace distribution for the firm's true earnings. More specifically, we assume the probability density function

$$\mathrm{f}_{e_t}(\bar{e}_t) = \begin{cases} \frac{1}{2b_e} \exp^{\frac{\bar{e}_t}{2b_e}} & \forall \bar{e}_t \leq 0 \\ \frac{1}{2b_e} \exp^{-\frac{\bar{e}_t}{2b_e}} & \forall \bar{e}_t > 0 \end{cases} \qquad \forall t. \tag{3.1}$$

The parameter b_e is the scaling parameter which is related to the variance by $\mathrm{Var}(e_t) = 2b_e^2$.[10] We assume that no dividends are paid out to shareholders. Aggregate earnings which we will from now on denote by true equity θ_t are then given by $\theta_t = \theta_{t-1} + e_t$ for all t.

Manager

At every time t, the manager privately observes the true earnings e_t and decides on what earnings r_t to report. His reported earnings r_t equal the sum of true earnings e_t and the discretion δ_t, i.e., $r_t = e_t + \delta_t$. We consider a rational and risk neutral manager who optimizes his current period's utility. The manager's utility is assumed to be linked to the firm's stock price. He considers both current period's stock price and all expected future stock prices. The prevalence of myopic manager behavior has been well established by existing literature (Graham et al. [2005]). All stock prices in the future are therefore discounted by a discounting factor β. The objective function for the manager's optimization problem in period t can be denoted by

$$\max_{\delta_t} u_t = \sum_{i=t}^{\infty} \beta^{i-t} \, \mathbb{E}[P_t \mid \delta_t]. \tag{3.2}$$

Barton and Simko [2002] showed that a firm's ability to overstate reports decreases in the extent to which previous balance sheets were overstated. To account for this result we impose a constraint on the manager's discretion. We assume the manager can overstate

[10] For our distribution we assume a mean of zero, thereby neglecting any kind of growth and serial correlation of earnings. We will account for this when setting up our structural estimation and relate earnings results of our model to changes in earnings in the data set.

by the amount saved in the past plus a constant term δ_0. We thereby allow for earnings management even in the absence of prior savings.[11] The discretion constraint follows as

$$\delta_t \leq \delta_0 + \sum_{i=1}^{t-1} e_i - r_i. \tag{3.3}$$

Our constraint provides an upper limit to the manager's discretion choice. As an unlimited negative discretion appears to be unrealistic, we introduce an additional constraint on the discretion which sets a lower limit to big bath type behavior. We introduce $\delta_{\min} \in \mathbb{R}_{\leq 0}$ and denote the constraint by

$$\delta_t \geq \delta_{\min}. \tag{3.4}$$

Stock Market

The manager considers the stock market's reaction to earnings announcements when deciding on what earnings to report. We are therefore required to specify the stock market's behavior and formulate how the stock price is set by investors depending on reported earnings. We start by introducing the variable γ_t which we define as the sum of all reported earnings, i.e., $\gamma_t := \sum_{i=1}^{t} r_i$. Between two consecutive periods t and $t+1$, investors learn the new reported earnings r_{t+1} and update their beliefs and the stock price to incorporate the additional information. Figure 3.1 illustrates the sequence of events between t and $t+1$.

We introduce an intermediate time step $t+\varepsilon$ to accommodate limited recall. Between t and $t+\varepsilon$, investors forget about the explicit values of γ_{t-1} and r_t and only remember the sum γ_t. They summarize the two pieces of information γ_{t-1} and r_t in one piece of information γ_t. We assume that the forgetting process is a passive process that happens unconsciously. Setting a new stock price, at the contrary, is considered to be an active process as the investors need to actively engage in the purchase or sale of shares. Consequently, in our model, investors don't update the stock price until new information appears in the form of the reported earnings r_{t+1}. Using the prior belief about firm eq-

[11] The impact of δ_0 on the system dynamics can be shown to converge to zero. For our structural estimation we can hence analyze the system without having to consider the impact of δ_0 by excluding the first periods from our analysis.

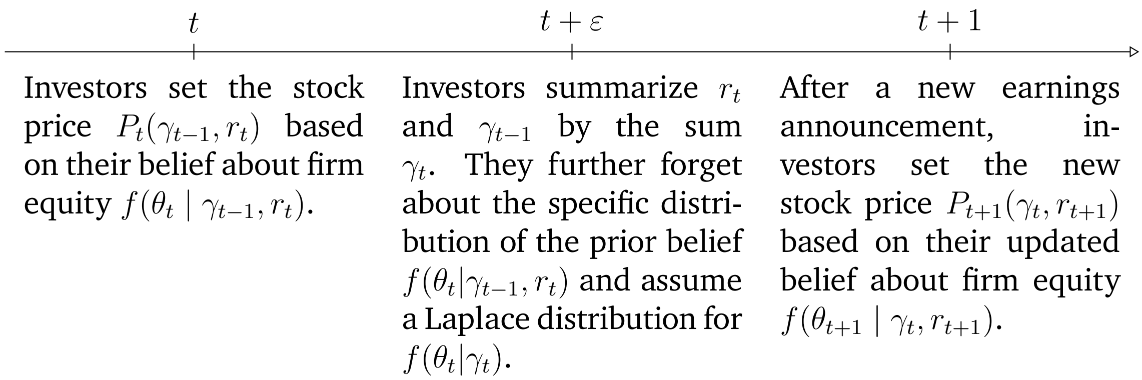


Figure 3.1: Sequence of events between two consecutive time steps. Over time, investors forget about the specific distribution used for the choice of the stock price P_t and summarize reported equity γ_{t-1} and reported earnings r_t by γ_t. When learning about r_{t+1}, they update their belief and set the new stock price P_{t+1} accordingly.

uity $f(\theta_t|\gamma_t)$, investors now incorporate r_{t+1} into their belief and update the stock price accordingly. The new values $P_{t+1}(\gamma_t, r_{t+1})$ and $f(\theta_{t+1}|\gamma_t, r_{t+1})$ are the updated versions of $P_t(\gamma_{t-1}, r_t)$ and $f(\theta_t|\gamma_{t-1}, r_t)$ at time t. We can now derive a first expression to structure the remainder of our calculations.

Lemma 3. *For any two consecutive periods t and $t+1$, the stock price change can be expressed as*

$$P_{t+1}(\gamma_t, r_{t+1}) - P_t(\gamma_{t-1}, r_t) = \mathbb{E}\Big[e_{t+1} \mid \gamma_t, r_{t+1}\Big] + \mathbb{E}\Big[\sum_{i=1}^{t} e_i \mid \gamma_t, r_{t+1}\Big] - \mathbb{E}\Big[\sum_{i=1}^{t} e_i \mid \gamma_t\Big].$$

The first summand on the right-hand side of the equation describes the expected value of true earnings at $t+1$. Investors try to infer the true value of e_{t+1}, given the two pieces of information available, γ_t and r_{t+1}. The difference between the second and third summand describes the change in expectation about all past earnings after reported earnings r_{t+1} are announced. The earnings announcement r_{t+1} contains information about both the current period's earnings e_{t+1} and all past earnings $e_{1..t}$. Some analysis shown in Appendix A allows us to express all three summands as a function of the prior belief about firm equity $f(\theta_t|\gamma_t)$ and the probability mass function $f(r_{t+1}|e_{t+1}, \theta_t, \gamma_t)$. The latter follows immediately from $r_{t+1} = e_{t+1} + \delta_{t+1}(e_{t+1}, \theta_t, \gamma_t)$. The stock price change in the following lemma is therefore fully specified, given the manager's reporting strategy $\delta_{t+1}(e_{t+1}, \theta_t, \gamma_t)$.

Lemma 4. *Following the structure from Lemma 3, the stock price change for any two consecutive periods is given by*

$$\begin{aligned} P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right) = & \frac{\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}{\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)} \\ & + \frac{\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})}{\int_{\theta_t} f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})} \\ & - \int_{\theta_t} \theta_t f\left(\theta_t|\gamma_t\right), \end{aligned}$$

where $f(\theta_t|\gamma_t)$ corresponds to investors' prior belief about firm equity, and $f(r_{t+1}|e_{t+1}, \theta_t, \gamma_t)$ follows from $r_{t+1} = e_{t+1} + \delta_{t+1}(e_{t+1}, \theta_t, \gamma_t)$.

The decision rules for firm manager and investors are specified in Equation 3.2 and Lemma 4, respectively, and we proceed by analyzing an equilibrium for their interaction.

3.2 Equilibrium

Similar to the parsimonious model, we will consider a Perfect Bayesian Equilibrium in which investors correctly conjecture the manager's reporting strategy. Given the resulting stock price function, the manager has no incentives to deviate from the reporting strategy the investors conjectured. Mathematically speaking, a Perfect Bayesian Equilibrium is defined as a discretion strategy δ_t^* for the manager, together with a stock price function P_t^* for the stock market investors, such that:

i) P_t^* is set by the equation in Lemma 4 and is updated after every earnings announcement.

ii) $\delta_t^*(e_t) \in \arg\max_{\delta_t} u_t \quad \forall e_t$.

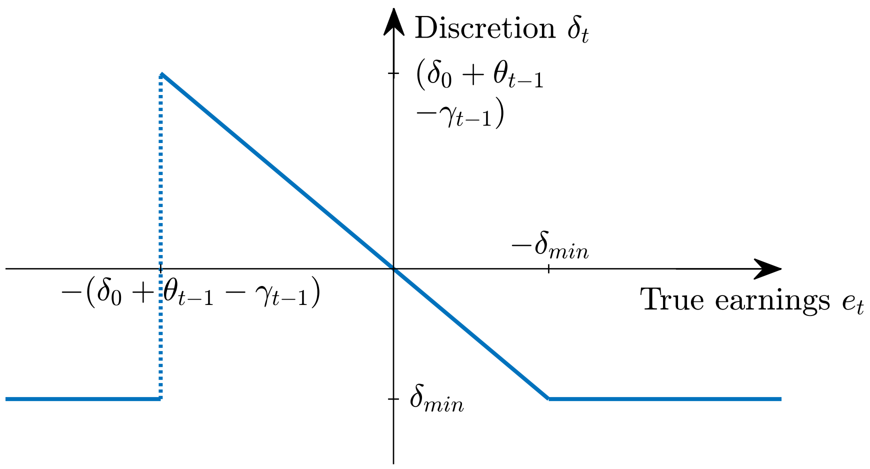
The two conditions guarantee sequential rationality and consistency, both of which are required for a Perfect Bayesian Equilibrium. To solve for an equilibrium, we first conjecture an equilibrium discretion strategy for the manager. Second, we solve the stock price equation assuming the conjectured discretion strategy. In our final step, we conduct a numerical analysis to provide a proof of existence for the conjectured equilibrium.

The reporting discretion is defined by $r_t = e_t + \delta_t$. Positive values of δ_t describe overstated earnings while negative values occur for understatements. We conjecture that, in equilibrium, the manager manages earnings according to

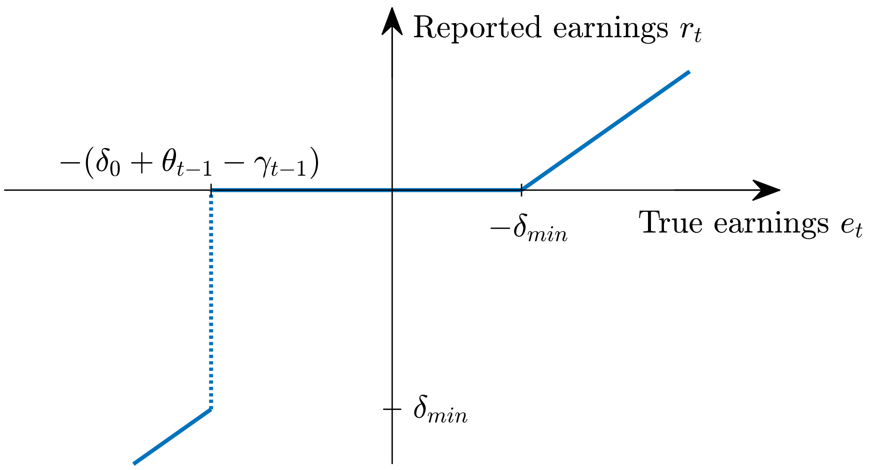
$$\delta_t(e_t, \theta_{t-1}, \gamma_{t-1}) = \begin{cases} \delta_{\text{min}} & e_t < -(\delta_0 + \theta_{t-1} - \gamma_{t-1}) \\ -e_t & -(\delta_0 + \theta_{t-1} - \gamma_{t-1}) \leq e_t \leq -\delta_{\text{min}} \\ \delta_{\text{min}} & e_t > \delta_{\text{min}} \end{cases} . \tag{3.5}$$

The empirical results in Chen et al. [2003] and the results of our simple model suggest that the stock market penalizes companies for not meeting a benchmark. The manager is therefore encouraged to meet the benchmark whenever he can. In our model, $\mathbb{E}[e_t] = 0$, which acts as the benchmark. We conjecture the manager meeting the benchmark, i.e., $\delta_t = -e_t$ and $r_t = 0$, for small negative earnings. For lower negative earnings, the discretion constraint is eventually binding. Earnings below the savings force the manager to report negative earnings. We conjecture that he will use what is commonly referred to as big bath strategy and understate by the maximum amount possible. The manager accepts bad economic results in the current period, also known as "taking a big bath", in hope of using the additional savings to meet the benchmark in a future period. We conjecture similar behavior for earnings results above the benchmark. The manager understates and hence saves to build up so called "cookie jars". His understatements are limited by the lower discretion bound δ_{min}. The conjectured discretion strategy is illustrated in Figure 3.2a. Figure 3.2b shows the corresponding reporting strategy $r_t(e_t)$. The lower limit for true earnings e_t that still allows the manager to report $r_t = 0$ depends on the amount saved and therefore varies in time.

Assuming the conjectured equilibrium discretion strategy holds, we can now derive the equilibrium stock price function using the approach laid out in Lemma 4. The double integrals in both numerator and denominator of the first two summands require extensive analysis to be solved. Integrating over θ_t turns out to be particularly challenging as θ_t only appears in the definition of the sub-domains. For the details, the reader is invited to consult Appendix A. Throughout the solution of the integrals, case distinctions among two dimensions arise. It is no surprise that the stock price function is piecewise-defined for different ranges of r_{t+1}. The fact that the discretion strategy is a piecewise function suggests that different reported earnings are interpreted differently by investors. First, investors' reaction differs for positive reported earnings $r_{t+1} > 0$, reported earnings just



(a) Conjectured equilibrium reporting discretion strategy $\delta_t(e_t)$



(b) Conjectured equilibrium reporting strategy $r_t(e_t)$

Figure 3.2: (a) Conjectured equilibrium reporting discretion strategy $\delta_t(e_t)$ as a function of true firm earnings e_t (specified in Equation 3.5), (b) and the corresponding reporting strategy $r_t(e_t)$. The savings $\delta_0 + \theta_{t-1} - \gamma_{t-1}$ available to the manager at time t depend on the difference between true equity θ_{t-1} and reported equity γ_{t-1}. δ_0 denotes the leeway in managing earnings in the absence of savings. δ_{min} denotes the lower limit on understating earnings.

meeting the benchmark $r_{t+1} = 0$, and negative reports $r_{t+1} < \delta_{\min}$. Second, observe that $\gamma_t - \theta_t$ measures cumulative discretionary accruals up to time t and, consequently, $\delta_0 - (\gamma_t - \theta_t)$ measures the maximum discretion the manager can exploit to overreport earnings in $t+1$. Investors cannot observe true cumulative earnings θ_t and therefore conjecture that the manager's maximum discretion equals $\delta_0 - (\gamma_t - \mathbb{E}[\theta_t|\Omega_t])$ where Ω_t is investors' information set. Given investors' prior belief, $\mathbb{E}[\theta_t|\Omega_t]$ translates into μ_{θ_t}. Combining the case distinctions along the two dimensions results in seven sub-domains for the stock price change. The resulting stock price functions are summarized in the following lemma. Due to their length and since the following corollaries will focus on the second case, some of the functions for the first case are abbreviated.

Lemma 5. *When assuming the manager acts according to the strategy specified in Equation 3.5, investors update the firm's stock price according to a piecewise function, defined on seven sub-domains. Suppose $\gamma_t - \delta_0 - \mu_{\theta_t} < 0$. Then, the stock price change is of form*

$$
\begin{aligned}
&P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right) \\
&= \begin{cases}
r_{t+1} - \delta_{\min} + f_1(b_{\theta_t}, \delta_0, \delta_{\min}, \gamma_t, \mu_{\theta_t}) & r_{t+1} < \delta_{\min} + \gamma_t - \delta_0 - \mu_{\theta_t} \\
r_{t+1} - \delta_{\min} + f_2(r_{t+1}, \gamma_t, b_{\theta_t}, \delta_{\min}, \delta_0) & \delta_{\min} + \gamma_t - \delta_0 - \mu_{\theta_t} \leq r_{t+1} \leq \delta_{\min} \\
f_3(b_e, b_{\theta_t}, \delta_0, \delta_{\min}, \gamma_t, \mu_{\theta_t}) + f_4(b_e, b_{\theta_t}, \delta_0, \delta_{\min}, \gamma_t, \mu_{\theta_t}) & r_{t+1} = 0 \\
r_{t+1} - \delta_{\min} & r_{t+1} > 0
\end{cases}
\end{aligned}
$$

Let now $\gamma_t - \delta_0 - \mu_{\theta_t} \geq 0$. The stock price change then follows as

$$
\begin{aligned}
&P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right) \\
&= \begin{cases}
r_{t+1} - \delta_{\min} + \dfrac{2\mu_{\theta_t} - (\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} + b_{\theta_t}) \exp^{-\frac{\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}}}{2 - \exp^{-\frac{\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}}} - \mu_{\theta_t} & r_{t+1} < \delta_{\min} \\
\dfrac{-\frac{1}{b_e}\left(\frac{b_e b_{\theta_t}}{b_e + b_{\theta_t}}\right)^2 \exp^{-\frac{\gamma_t - \delta_0 - \mu_{\theta_t}}{b_{\theta_t}}} + 2\left((\delta_{\min} - b_e)\exp^{\frac{\delta_{\min}}{b_e}} + b_e\right)}{\frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{\gamma_t - \delta_0 - \mu_{\theta_t}}{b_{\theta_t}}} + 2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right)} & \\
\quad + \dfrac{2\mu_{\theta_t}\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{\gamma_t - \delta_0 - \mu_{\theta_t}}{b_{\theta_t}}}\left[\gamma_t - \delta_0 + \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}} b_{\theta_t}\right]}{2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{\gamma_t - \delta_0 - \mu_{\theta_t}}{b_{\theta_t}}}} - \mu_{\theta_t} & r_{t+1} = 0 \\
r_{t+1} - \delta_{\min} & r_{t+1} > 0
\end{cases}
\end{aligned}
$$

Despite their complexity, the stock price functions provide intuitive results consistent with the model setup. We will characterize stock prices for the different sub-domains in the following corollaries. For the proofs, we refer to the appendix. For simplicity, the following corollaries only consider the case $\gamma_t - \delta_0 - \mu_{\theta_t} \geq 0$. The stock price reaction is then defined for three sub-domains: $r_{t+1} < \delta_{\min}$, $r_{t+1} = 0$, and $r_{t+1} > 0$. We first consider the simple case of $r_{t+1} > 0$, followed by $r_{t+1} < \delta_{\min}$, and finally $r_{t+1} = 0$ which turns out to be the most complex case.

Corollary 2. *Positive reported earnings $r_{t+1} > 0$ allow investors to infer true earnings e_{t+1} but do not provide information about past earnings.*

Careful consideration of the equilibrium discretion strategy explains the simplicity of the stock price reaction for positive earnings reports. In our model, $r_{t+1} > 0$ are only reported for positive true earnings $e_{t+1} > \delta_{\min}$. Investors can therefore perfectly infer true earnings by reconstructing the discretion choice as $e_{t+1} = r_{t+1} - \delta_{\min}$. We now turn to the more interesting case of $r_{t+1} < \delta_{\min}$.

Corollary 3. *Let $r_{t+1} < \delta_{\min}$. Investors can infer true earnings e_{t+1}. Investors' updated beliefs about past earnings satisfy the following properties:*

i) Expectations of past earnings are lowered for all values of r_{t+1}.

ii) The magnitude of the downward update of beliefs strictly increases in r_{t+1}.

A report of $r_{t+1} < \delta_{\min}$ contains two pieces of information. First, it provides the investors with deterministic information about true earnings e_{t+1}. For the same reasons as outlined above, investors can infer the true value of earnings e_{t+1}. Second, negative earnings reports provide an upper limit for the amount saved in the past. More specifically, the firm savings cannot exceed the value of $r_{t+1} - \delta_{\min}$. If the manager had saved an amount $\delta_0 + \theta_t - \gamma_t > |r_{t+1} - \delta_{\min}|$, his reporting strategy would have consisted of overstating earnings to meet the benchmark $r_{t+1} = 0$. Investors can hence infer an upper bound for the amount saved. This additional piece of information motivates investors to revise downward their belief about past earnings. The reason is that learning about an upper bound of past savings (i.e., understatement of equity) must always result in investors revising their belief about past earnings downward. Part (i) of the corollary formalizes this effect. Next, we are interested in the magnitude of the effect as a function of r_{t+1}. To provide intuition for the result in part (ii) of the corollary, we consider two extreme examples. First, assume the manager reports earnings of $r_{t+1} \ll 0$ significantly below zero. The fact that the manager's savings did not allow a report of zero only provides a high upper bound on past savings and therefore only a small amount of information. Meeting the benchmark would have required extremely high savings. The limit on savings the investors can infer is hence high and does not lead to a significant negative surprise. On the contrary, an earnings report just below zero contains a significantly higher level of information about the firm savings. It proves that the manager did not even have sufficient savings to counteract the barely negative earnings. As a conclusion, we expect that the magnitude of the downward updating of beliefs increases in r_{t+1}. Reports closer to zero are penalized by a more severe decline in belief about past earnings. The proof in the appendix formally confirms our intuition.

Corollary 4. *Let $r_{t+1} = 0$. Investors' update of beliefs about past earnings satisfies the following properties:*

i) Expectations of past earnings are raised for all values of γ_t and μ_{θ_t}.

ii) Suppose b_e and b_{θ_t} satisfy

$$b_{\theta_t} < 2b_e \left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right).$$

The magnitude of the upward revision of beliefs about past earnings increases as investors' belief about firm savings approaches zero, for all $(\mu_{\theta_t}, \gamma_t)$. For all (b_e, b_{θ_t}) that do not satisfy this condition, there exists a combination $(\mu^*_{\theta_t}, \gamma^*_t)$ such that the effect occurs for all $\mu_{\theta_t} - \gamma_t > \mu^*_{\theta_t} - \gamma^*_t$. The combination $(\mu^*_{\theta_t}, \gamma^*_t)$ satisfies the equation

$$\frac{\gamma^*_t - \delta_0 - \mu^*_{\theta_t}}{b_{\theta_t}} - \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \frac{1}{2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right)} \exp^{-\frac{\gamma^*_t - \delta_0 - \mu^*_{\theta_t}}{b_{\theta_t}}} = -\frac{b_e}{b_e + b_{\theta_t}}.$$

Earnings of zero are reported in two cases: first, in case of positive earnings $e_{t+1} \in [0, -\delta_{\min}]$. For slightly positive earnings the manager understates and reports zero, independently of savings. This case therefore does not provide any information to investors about past earnings. Earnings of zero are further reported for negative earnings above the discretion constraint, i.e., for $e_{t+1} \in [-(\delta_0+\theta_t-\gamma_t), 0)$. If investors believed the firm savings were extremely high, they assigned a high probability to a report of $r_{t+1} = 0$ and are not surprised. If they expected the firm savings to be low, they deemed a report of $r_{t+1} = 0$ unlikely and are likely to update to a more positive belief about past earnings. Two effects can be derived. First, a report of $r_{t+1} = 0$ can only prompt a positive update of investors' beliefs about past earnings. For high expected savings, investors revise their beliefs only slightly. For low expected savings, they significantly revise their beliefs resulting in an increased stock price. Second, this effect is stronger when investors conjecture the firm's savings, $\mu_{\theta_t} - \gamma_t$, to be low, as the report is more surprising. Corollary 4 (i) establishes the first result. Since we only consider $\gamma_t - \delta_0 - \mu_{\theta_t} \geq 0$, which corresponds to a negative belief about firm savings, we cannot proof the second effect but analyze the behavior for a negative belief about firm savings instead. We show that the magnitude of the upward revision of beliefs about past earnings increases as the belief about firm savings approaches zero. Depending on the parameters b_e and b_{θ_t}, the effect either holds for all values of $(\mu_{\theta_t}, \gamma_t)$ or for all $(\mu_{\theta_t}, \gamma_t)$ that exceed a threshold such that $\mu_{\theta_t} - \gamma_t > \mu^*_{\theta_t} - \gamma^*_t$. The first term of the stock price function for $r_{t+1} = 0$ corresponds to investors' belief about e_{t+1}. It is the best estimate for e_{t+1} given their belief about savings and knowledge of the distribution of true earnings.

3.3 Proof of Existence for Equilibrium

After conjecturing an equilibrium strategy for the manager and deriving the corresponding stock market behavior, we now turn towards proving that the conjectured equilibrium exists as a Perfect Bayesian Equilibrium. The complexity of the system equations prohibits us from providing an analytic proof. We are hence limited to a numerical proof of existence. We first describe our approach, followed by a discussion of the results.

Approach We will provide numerical evidence that no alternative discretion choice provides a higher utility for the manager given investors' stock price reactions. To this end, we develop a MATLAB implementation to simulate the system dynamics. The code follows the following structure.

i) We simulate the system for a certain number of periods $t^* - 1$ assuming both manager and investors follow the equilibrium strategies.

ii) We want to show that at time t^*, the manager has no incentive to deviate from the conjectured discretion strategy. To this end, we define a two-dimensional grid for true earnings e_{t^*} and the discretion δ_{t^*}. For every point on the grid, i.e., for every combination of e_{t^*} and δ_{t^*}, we want to calculate the corresponding utility u_{t^*}.

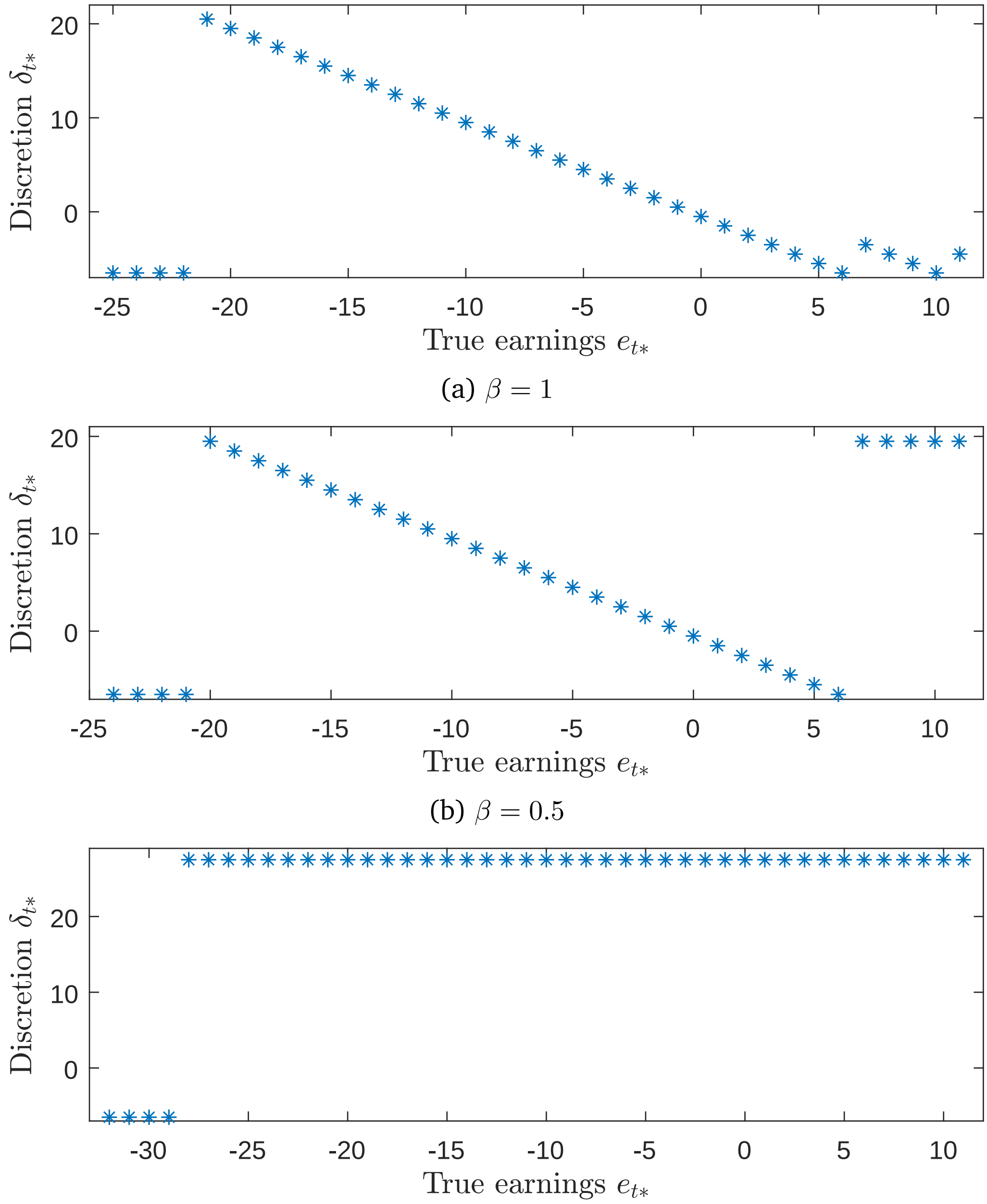
iii) As the utility u_{t^*} depends on all future stock prices, we simulate the system for a sufficiently large number of periods following t^* for every point on the grid. To increase stability, we simulate the future multiple times.

iv) Using both the immediate stock price reaction in period t^* and all future stock prices, we calculate the manager's utility u_{t^*} for every point on the grid.

v) The graphs illustrate the discretion choice δ_{t^*} that yields the highest utility u_{t^*} for every value of e_{t^*}.

Results Similar to the future utility from saving, u, in our parsimonious model, the discounting factor β plays an important role in the manager's decision making. For lower values of β, he cares less about the economic results of future periods. For this myopic behavior, we expect a decrease of the likelihood of understated earnings. A low value of β causes potential utility gains in the future by meeting the benchmark to be discounted significantly. Figure 3.3 shows the optimal discretion strategy at t^* for different values of β.

We first look at the results for $\beta = 1$. The manager values all periods equally and shows no myopic behavior. The fact that the current period and all future periods are valued equally suggests that the conjectured understating does indeed occur. The numerical results confirm our expectation. The numerically optimal discretion strategy corresponds to our conjectured strategy with some numerical imprecision. Figure 3.3b shows the numerical results for $\beta = 0.5$. The manager now discounts the future at a rate of $1/2$, making overstating in the current period more attractive. The optimal discretion strategy consequentially deviates from our conjectured strategy. For high values of true earnings e_{t*}, the manager is best off by reporting the maximum amount possible



(a) $\beta = 1$

(b) $\beta = 0.5$

(c) $\beta = 0$

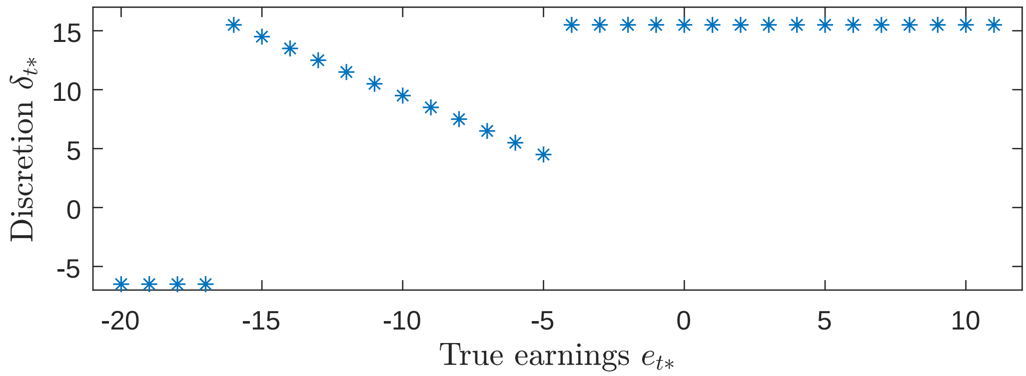
Figure 3.3: Numerical results for optimal discretion strategy $\delta_{t^*}(e_{t^*})$ for different discounting factors β. We consider period $t^* = 20$, run $1,000$ iterations per grid point, and choose $\delta_{\min} = -6.5$.

instead of saving for future periods. The existing firm savings are apparently sufficient to make additional savings not worth the current period's sacrifice over the manager's "shorter horizon." In Figure 3.3c, we can observe the results for the extreme case of $\beta = 0$. The manager only considers the current period's stock prices which implies that the utility function directly reflects the stock price functions. As expected the manager hence chooses to overstate instead of saving for the majority of true earnings. For values of e_{t^*} below the amount saved, the manager cannot meet the benchmark and is forced to report negative earnings. Looking at the stock price functions in Lemma 5 shows that there are two opposite effects. The belief about current period's earnings is strictly increasing in r_t. Reporting higher earnings is thus more beneficial. The belief about past earnings, however, is strictly decreasing in r_t as formulated in Corollary 3. For the result in Figure 3.3c, the latter effect prevails, causing the manager to understate despite the fact that savings are worthless to him.

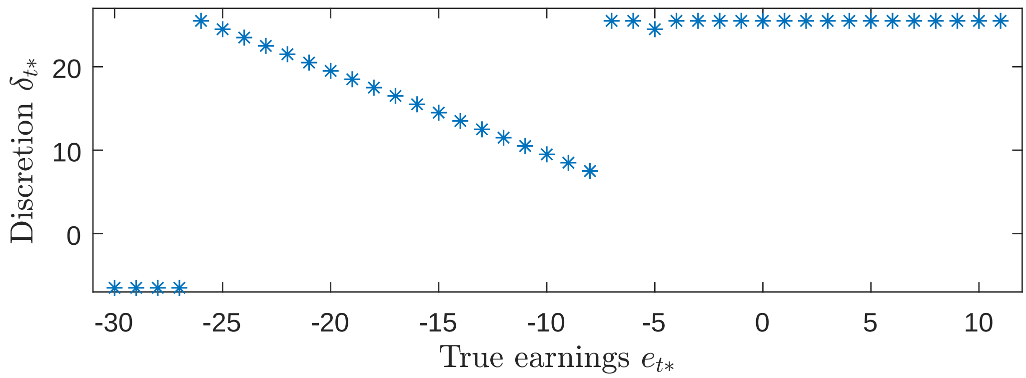
The analysis of the optimal discretion strategy for different discounting factors indicated the importance of another variable we did not consider explicitly in the previous graphs, the firm savings at time t^*. Just like a lower discounting factor makes saving less attractive, higher savings decrease the probability of an additional saving being used to meet the benchmark over the course of the "remaining horizon".[12] We compare the results for different savings at time t^* while keeping the discounting factor constant at $\beta = 0.8$. The results are shown in Figure 3.4.

The firm savings increase from (a) to (c). The results show that the range for which the manager prefers overstating to saving increases in savings. For the lowest level of savings in (a), the manager only deviates from the conjectured strategy for $e_{t*} > -5$. For the two graphs with higher savings, the limit decreases to $e_{t*} > -8$ and $e_{t*} > -30$, respectively. When savings are already high, the manager has a lower incentive to save as the probability that the additional savings can be used to meet the benchmark in a future period is low. We conclude that the limit up to which the manager follows our conjectured savings strategy decreases both for a lower discounting factor β and for higher savings $\delta_0 + \theta_t - \gamma_t$.

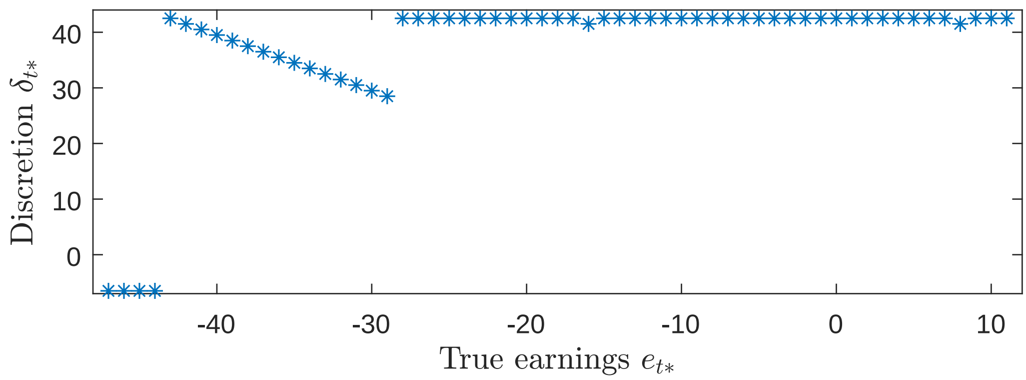
[12]Mathematically speaking, the manager's horizon is infinite. The expression "remaining horizon" refers to the fact that periods in the distant future are discounted to a value close to zero.



(a) Savings $\delta_0 + \theta_{t^*-1} - \gamma_{t^*-1} = 16$



(b) Savings $\delta_0 + \theta_{t^*-1} - \gamma_{t^*-1} = 26$



(c) Savings $\delta_0 + \theta_{t^*-1} - \gamma_{t^*-1} = 43$

Figure 3.4: Numerical results for optimal discretion strategy $\delta_{t^*}(e_{t^*})$ for different firm savings at time $t^* = 20$. The discounting factor is held constant at $\beta = 0.8$. We choose $\delta_{\min} = -6.5$ and run $1,000$ iterations per grid point.

Refined Equilibrium Discretion Strategy Based on the results of the equilibrium analysis in the previous section, we propose a refined equilibrium discretion strategy. To simplify notation we introduce the variable s_t to denote the savings at time t as defined by $s_t := \delta_0 + \theta_t - \gamma_t$. The results in the previous section suggest that there exists an amount of earnings such that for earnings above the threshold, the manager decides to overstate by the maximum amount possible. Let $f : \mathbb{R}^2 \mapsto \mathbb{R}$ be a function that maps the discount factor β and the savings s_t to a real valued number. Without specifying a functional form for $f(\beta, s_t)$, we formulate the following proposition for a refined equilibrium reporting strategy.

Proposition 1. *There exists a Perfect Bayesian Equilibrium in which the manager's reporting strategy satisfies*

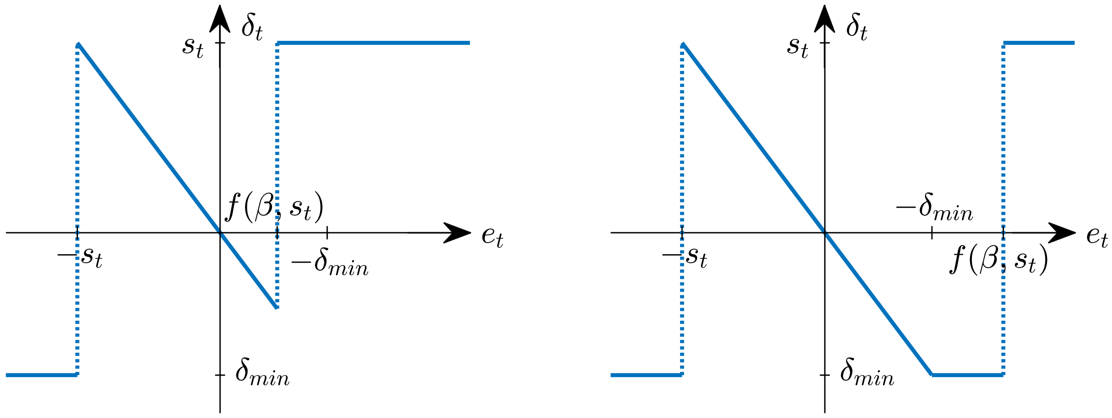
$$\delta_t(e_t, \theta_{t-1}, \gamma_{t-1}) = \begin{cases} \delta_{\min} & e_t < \min\{-s_t, f(\beta, s_t)\} \\ -e_t & -s_t \leq e_t \leq \min\{-\delta_{\min}, f(\beta, s_t)\}\,,\ f(\beta, s_t) > -s_t \\ \delta_{\min} & -\delta_{\min} < e_t < f(\beta, s_t)\,,\ f(\beta, s_t) > -\delta_{\min} \\ s_t & e_t > f(\beta, s_t) \end{cases}$$

for some real valued function $f : (\beta, s_t) \mapsto \mathbb{R}$ *with the properties:*

i) $\frac{\partial f(\beta, s_t)}{\partial \beta} > 0$,

ii) $\frac{\partial f(\beta, s_t)}{\partial s_t} < 0$.

As β increases, the manager cares increasingly about future periods, thus making saving more attractive. Higher savings s_t make additional savings less attractive, thereby lowering the threshold of overstating. In Figure 3.5, we illustrate the discretion strategy for the two cases (a)$-s_t < f(\beta, s_t) < -\delta_{\min}$ and (b) $f(\beta, s_t) > -\delta_{\min}$. For low earnings, the manager follows a big bath strategy and accepts to take a big hit in the current period, hoping to use the savings in a future period to meet the benchmark. Whenever possible, the manager reports zero, thereby just meeting the benchmark. In contrast to our previously conjectured strategy, the manager's choice to understate for all positive earnings is bounded by a function that depends on both savings and the manager's discounting factor. If savings are already high, or the future is severely discounted, the



(a) $-s_t < f(\beta, s_t) < -\delta_{\min}$ (b) $f(\beta, s_t) > -\delta_{\min}$

Figure 3.5: Refined equilibrium discretion strategy $\delta_t(e_t)$ as a function of true earnings e_t and firm savings s_t. β denotes the manager's discounting factor and $\delta_{\min}$ the lower bound for the manager's discretion choice.

manager prefers to overstate in the current period instead of saving for future periods. Similar behavior is to be expected for a finite horizon model as the manager approaches the end of its tenure at the firm, a phenomenon commonly referred to as “CEO horizon problem”.

Chapter 4

Structural Estimation

4.1 Estimation Method and Identification

The objective of this chapter is to estimate the model parameters for the continuous infinite-horizon model of the previous chapter: the maximum discretionary savings $\delta_{\min}$, the available discretion in the absence of savings δ_0, and the variance of true earnings b_e. To simplify the estimation process, we estimate b_e outside of the model relying on reported rather than true earnings. We regress firm size on variance of earnings to establish a functional relationship between the two firm characteristics. To reflect this relationship we will scale all relevant data by a proxy for firm size. By close inspection, we further notice that the impact of δ_0 on the system dynamics converges to zero for large t. Initially, a higher value of δ_0 causes a shift of the discretion constraint and thereby alters the manager's discretionary behavior. After a sufficient number of periods, however, the savings for different values of δ_0 converge. The manager's subsequent reporting decisions are thus independent of δ_0. We conclude that the parameter δ_0 is not essential to the dynamics of our model. Instead of estimating the vector $(\delta_{\min}, \delta_0)$, we estimate $\delta_{\min}$, set $\delta_0 = 0$ and only consider the system dynamics in steady-state. For the estimation, we overidentify our model and specify two moments. The complexity of the stock price functions prevents us from finding closed form solutions for our identified moments, making a generalized method of moments unfeasible. We conduct a simulated method of moments instead. A MATLAB implementation allows us to simulate the model for different parameters, calculate the moments for each parameter choice, and choose the parameter value that best fits the moments from the data. The extensive model illustrates both asymmetric stock market behavior and earnings management, causing the earnings distribution to be discontinuous at the benchmark. We construct two moments based on these fundamental properties. In the previous chapter, we established an equilibrium in which the manager engages in earnings management and uses discretionary accruals to meet the benchmark as frequently as possible. Consequently, we observe both a discontinuity around the benchmark and an imbalance of reported earnings above and below the benchmark. In the following, we describe the two moments we use in the estimation and provide intuition for the identification strategy.

Moment 1 Recall that we want to estimate the parameter $\delta_{\min}$ which specifies a bound for the manager's discretionary behavior. Its magnitude describes by how much the manager can understate earnings in any given period. Higher savings in periods of significantly positive and negative earnings enhance the manager's chances of meeting the benchmark in subsequent periods. The magnitude of $\delta_{\min}$ is therefore expected to affect the likelihood of positive earnings. More specifically, the model predicts a higher probability of an earnings report to meet the benchmark for higher values of $\delta_{\min}$. To reflect this intuition, we define our first moment as

$$m_1 \equiv \Pr\left(r_t - \mathbb{E}\left[r_t \mid r_{1..t-1}\right] \geq 0\right). \tag{4.1}$$

In other words, the moment describes the probability of a given earnings report to meet or beat the benchmark and we expect a value in excess of fifty percent. Implementing the moment for our model is intuitive. Let $\mathbf{x} \in \mathbb{R}^2$ be the data set including reported earnings and stock prices and let $\hat{\mathbf{x}} \in \mathbb{R}^2$ denote the simulated data set. Since $\mathbb{E}[e_t = 0]$, we want the moment to be a measure for $\Pr(r_t \geq 0)$ and define

$$\hat{m}_1(\hat{\mathbf{x}} \mid \delta_{\min}) := \frac{n_{\text{pos}}}{n_{\text{pos}} + n_{\text{neg}}} \tag{4.2}$$

where n_{pos} and n_{neg} describe the number of positive and negative earnings reports in the simulated data set $\hat{\mathbf{x}}$. For the data moment, we closely follow the method of Ball and Brown [1968]. Under a priori reasoning, current period's earnings are predicted to equal last period's earnings, i.e., $\mathbb{E}\left[r_t \mid r_{1..t-1}\right] = r_{t-1}$. We implement our notion of $\Pr(r_t - r_{t-1} \geq 0)$ in the form of

$$m_1(\mathbf{x}) := \frac{n_{\text{beat}}}{n_{\text{beat}} + n_{\text{fail}}} \tag{4.3}$$

where n_{beat} describes the number of reported earnings that meet or beat last period's reported earnings and n_{fail} describes the number of reported earnings below the benchmark of r_{t-1} in the data set $\mathbf{x}$.

Moment 2 Given investors' knowledge of the manager's discretion constraints, positive and negative earnings surprises trigger different responses by the stock market. Negative surprises cause the stock price to decrease by more than positive surprises cause it to increase. A failure to meet the benchmark indicates to investors not only that this period's earnings fall short but also that the manager has overstated earnings in the past such that his available discretion is insufficient to make up for the current short-fall. The latter prompts investors to revise downward their beliefs about past earnings. The discretion limit $\delta_{\min}$ directly impacts the manager's saving behavior. A change in $\delta_{\min}$ is therefore expected to affect the extent to which investors update their belief about past earnings when reported earnings fall short of the benchmark. We are thus interested in the asymmetry of the stock price reaction and develop a notion closely related to the earnings response coefficient. More specifically, let $\Delta P := P_t - P_{t-1}$ and $\Delta r := r_t - \mathbb{E}\left[r_t \mid r_{1..t-1}\right]$. We define the sensitivity to negative earnings surprises as

$$S_n \equiv \lim_{a \to -\infty} \frac{1}{a} \int_a^0 \frac{\partial \Delta P_t - \mathbb{E}[\Delta P_t]}{\partial \Delta r_t} \, d\Delta r_t. \tag{4.4}$$

At the core of its definition is the stock price sensitivity to an earnings surprise $\partial \Delta P_t / \partial \Delta r_t$. To account for general stock market growth, we include $\mathbb{E}\left[\Delta P_t\right]$ so as to only consider the stock price change related to the firm's earnings announcement. Finally, we integrate the sensitivity to derive a measure for the mean sensitivity to negative earnings surprises. We spare the reader the definition of a sensitivity to positive earnings surprises which is symmetric and only differs in the integration range. After laying out the intuition of the second moment, we proceed by deriving a specific definition for our model. Since

$\mathbb{E}[e_t] = 0$, an intuitive measure for the stock price sensitivity to negative earnings surprises is given by the expression

$$\hat{S}_n := \frac{1}{n_n} \sum_{\mathcal{R}_n} \frac{P_{t+1} - P_t}{r_{t+1}} \tag{4.5}$$

where $\mathcal{R}_n$ is the set of all simulated periods in $\hat{\mathbf{x}}$ with $r_t \in (-\infty, \delta_{\min}]$, and n_n denotes the number of elements in $\mathcal{R}_n$. The positive stock price sensitivity is defined equivalently by

$$\hat{S}_p := \frac{1}{n_n} \sum_{\mathcal{R}_p} \frac{P_{t+1} - P_t}{r_{t+1}} \tag{4.6}$$

where $\mathcal{R}_p$ is the set of all simulated periods with $r_t \in [-\delta_{\min}, \infty)$, and n_p denotes the number of elements in $\mathcal{R}_p$. We combine the two sensitivities and define

$$\hat{m}_2(\hat{\mathbf{x}} \mid \delta_{\min}) := \frac{\hat{S}_n}{\hat{S}_p}. \tag{4.7}$$

Related to the definition of the second data moment, we make three observations. First, we note that under a priori reasoning, $\mathbb{E}[r_t \mid r_{1..t-1}] = r_{t-1}$. Second, the data shows an increase of average stock prices over time. To account for that fact as well as firms' diverging exposure to systematic risk, we benchmark firm i's change in stock price to $\beta\Delta\bar{P}_t$ where $\bar{P}_t$ denotes the average stock return in period t. Third, we account for seasonality by comparing reported earnings to the corresponding quarter in the previous year, i.e., we consider $\Delta r_t = r_t - r_{t-4}$. As a result, we can express the stock price sensitivities by

$$S_n := \frac{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{n_i^n} \sum_{t \in \mathcal{R}_i^n} (P_{i,t} - P_{i,t-1} - \beta\,\Delta\bar{P}_t)}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{n_i^n} \sum_{t \in \mathcal{R}_i^n} (r_{i,t} - r_{i,t-4})} \tag{4.8}$$

and

$$S_p := \frac{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{n_i^p} \sum_{t \in \mathcal{R}_i^p} (P_{i,t} - P_{i,t-1} - \beta\,\Delta\bar{P}_t)}{\frac{1}{n}\sum_{i=1}^{n} \frac{1}{n_i^p} \sum_{t \in \mathcal{R}_i^p} (r_{i,t} - r_{i,t-4})}, \tag{4.9}$$

which fully describe the moment

$$m_2(\mathbf{x}) := S_n / S_p. \tag{4.10}$$

$\mathcal{R}_i^n$ and $\mathcal{R}_i^p$ denote the sets of all periods in which firm i reported positive and negative earnings surprises, respectively. n_i^n and n_i^p describe the number of elements in $\mathcal{R}_i^n$ and $\mathcal{R}_i^p$ while n denotes the number of firms in the data set $\mathbf{x}$.

Optimization In our SMM estimation, we choose $\delta_{\min}$ such that some distance measure of the data moments $m_i(\mathbf{x})$ from the simulated moments $\hat{m}_i(\hat{\mathbf{x}} \mid \delta_{\min})$ is minimized. Let

$$\vec{e}\,(\mathbf{x}, \hat{\mathbf{x}} \mid \delta_{\min}) := \begin{pmatrix} \frac{\hat{m}_1(\hat{\mathbf{x}} \mid \delta_{\min}) - m_1(\mathbf{x})}{m_1(\mathbf{x})} & \frac{\hat{m}_2(\hat{\mathbf{x}} \mid \delta_{\min}) - m_2(\mathbf{x})}{m_2(\mathbf{x})} \end{pmatrix}^\intercal \tag{4.11}$$

denote the relative error of simulated moments. Moreover, let $\mathbf{W} \in \mathbb{R}^2$ be the weighting matrix. We can express our minimization problem as

$$\min_{\delta_{\min}} \vec{e}\,(\mathbf{x}, \hat{\mathbf{x}} \mid \delta_{\min})^\intercal \,\mathbf{W}\, \vec{e}\,(\mathbf{x}, \hat{\mathbf{x}} \mid \delta_{\min}). \tag{4.12}$$

We want $\mathbf{W}$ to produce precise estimates. To minimize the asymptotic variance, we choose $\mathbf{W}$ using a two-step variance covariance estimator. We first derive an initial estimate $\hat{\delta}_{\min}$ using the identity matrix $\mathbf{I}_2$ as weighting matrix. The resulting error vector provides a new estimate for the variance covariance matrix. More specifically, we calculate

$$\hat{\mathbf{\Omega}} = \frac{1}{2}\vec{e}\,(\mathbf{x}, \hat{\mathbf{x}} \mid \hat{\delta}_{\min})\vec{e}\,(\mathbf{x}, \hat{\mathbf{x}} \mid \hat{\delta}_{\min})^\intercal \tag{4.13}$$

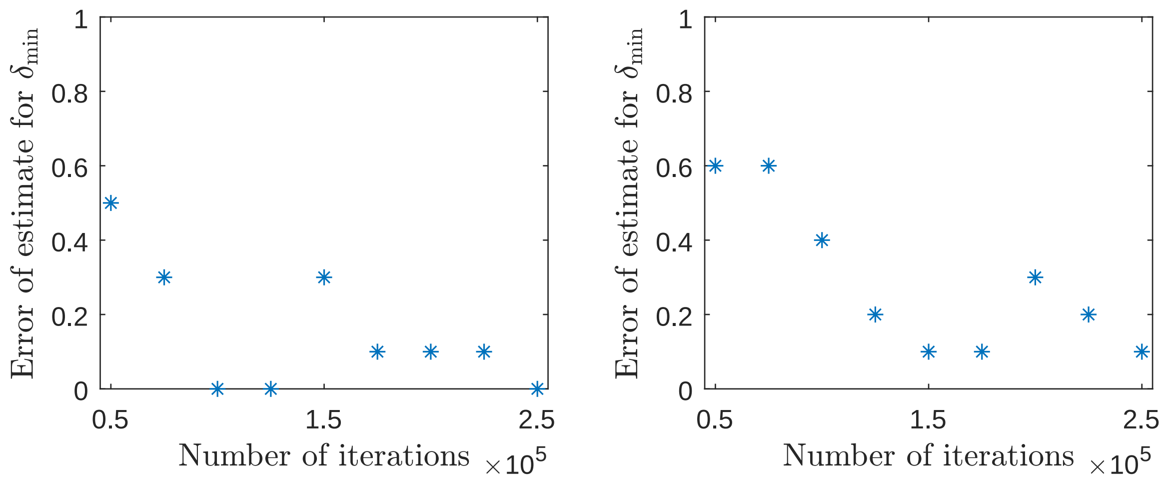
and use the inverse to choose $\mathbf{W}$, i.e., $\mathbf{W} = \hat{\mathbf{\Omega}}^{-1}$.

Proof of Convergence We prove the convergence of our structural estimation numerically in three steps. First, we choose a "true" value for $\delta_{\min}$. Next, we simulate the model for the given parameter and derive simulated data moments. Based on the resulting moments, we can now estimate $\delta_{\min}$. We analyze the accuracy of the estimator depending on the number of iterations.

Our model suggests a deterministic relation between reported earnings and stock prices, an assumption that clearly does not hold for the data set. We add the assumption that the price is further affected by unobserved random variables and prove that the estimator converges to the true parameter value despite the noise. We assume this "noise" term ε follows a Laplace distribution with mean 0 and is independent of all other random variables. The independence of ε allows us to estimate the variance of noise σ_ε^2 using data on stock prices and reported earnings. Assuming $\Delta P = \Delta r + \varepsilon$, we derive

$$\mathrm{Var}\left[\varepsilon\right] = \mathrm{Var}\left[\Delta P\right] - \mathrm{Var}\left[\Delta r\right]. \tag{4.14}$$

Based on the data set we describe in detail in the next section, we calculate $\mathrm{Var}[\Delta P]$ and $\mathrm{Var}[\Delta r]$ and use the resulting variance $\mathrm{Var}[\varepsilon] = 2b_e^2$ for the proof of convergence. Figure 4.1 illustrates the results for the structural estimation for two values of $\delta_{\min}$. Both figures describe the error between estimated and true value of $\delta_{\min}$ as a function of the number of iterations. Despite some numerical imprecision, the estimate appears to converge to the true value of $\delta_{\min}$. Even for low numbers of iterations, the estimate's error quickly falls below 0.3 which corresponds to a relative error of 6% and 3%.



(a) $\delta_{\min} = -5$ (b) $\delta_{\min} = -10$

Figure 4.1: Error between estimated and true value of $\delta_{\min}$, which describes the lower discretion limit for the manager. The results are illustrated as a function of the number of iterations of the simulated method of moments.

4.2 Data

The structural estimation requires data on market value and reported earnings. Data on market value, reported net income, total assets, total revenue and SIC code are obtained from the Compustat/CRSP merged database for the years from 2009 to 2019. Since earnings announcements and the corresponding stock market reactions are an essential part of this thesis, we use quarterly data. With the objective of removing all erroneous data

	Industry portfolio	No. of firms	No. of observations
1	Consumer Non-Durables	289	7,884
2	Consumer Durables	152	4,087
3	Manufacturing	547	15,952
4	Energy	404	9,579
5	Chemicals	148	4,123
6	Business Equipment	1,400	32,960
7	Telecom	210	5,523
8	Utilities	183	5,367
9	Wholesale and Retail	555	15,120
10	Healthcare	1,032	18,531
11	Finance	2,393	68,161
12	Other	979	24,927
	Total	8,292	212,214

Table 4.1: Descriptive statistics for French-Fama industry portfolios. Data set includes quarterly data from 2009 to 2019, taken from the Compustat/CRSP merged database.

entries, we drop entries with negative revenues, a negative market value, or a net income higher than the total revenue. In addition, we ignore all firms with a total revenue below USD 1 million per year. We further truncate both relative stock price change $\Delta P/P$ and $\Delta r/P$ at the 1% level. We expect both the stock price sensitivities to earnings reports as well as the magnitude of discretionary behavior to vary across industries. Consequently, we classify firms into industry portfolios following the Fama-French 12 industry classification. Table 4.1 provides an overview of the split of the data set into the twelve industry portfolios. Our data set includes $8,292$ firms and a total of $212,214$ quarterly data entries. Table 4.2 provides descriptive statistics for market value, net income, total revenue and total assets. While data on total revenue is primarily used to clean the data set, total assets will serve as proxy for firm size. Table 4.3 provides descriptive statistics for the relative market value change and changes in reported earnings relative to market value. Since both variables are truncated at the 1% level, the most extreme market value changes in the sample consist of a stock price decline by 50% and an increase by 106.5%. We note that, on average, stock prices increase, which motivates our consideration of the average market growth for the second moment in our structural estimation.

	mean	median	std. dev.
Market value	$5,590.6$	692.8	$24,270.3$
Net income	82.2	4.6	543.0
Total revenue	$1,268.0$	149.5	$5,249.0$
Total assets	$17,132.5$	$1,096.4$	$121,780.7$

Table 4.2: Descriptive statistics for quarterly data on fundamentals. All values are in millions of USD.

	mean	median	std. dev.	min	max
$\frac{P_{t+1}-P_t}{P_t}$	0.063	0.027	0.783	-0.508	1.065
$\frac{r_{t+1}-r_{t-3}}{P_t}$	-0.019	-0.029	115.673	-0.310	0.333

Table 4.3: Descriptive statistics for the relative change in market value P_t and the change in reported net income r_t relative to market value P_t. Variables are based on quarterly data and the data is truncated at the 1% level for both variables.

4.3 Findings

Based on the French-Fama 12 industry classification, we calculate the two data moments and use the results to estimate the lower discretion limit $\delta_{\min}$ using simulated method of moments. Recall that $\delta_{\min}$ is defined as the maximum amount by which a manager can understate earnings. We therefore expect estimates of $\delta_{\min}$ to be negative. In the model, $\delta_{\min}$ represents an absolute dollar amount. As a result, we expect its magnitude to be a function of firm size. More specifically, a firm with a larger balance sheet is expected to have greater leeway in reporting earnings. To account for $\delta_{\min}$'s dependence on firm size, we scale all variables by total assets as a measure of the size of a firm's balance sheet. Both the two data moments defined in Equation 4.3 and Equation 4.10 in section 4.1 and the estimate for $\delta_{\min}$ that minimizes the objective function in Equation 4.12 are summarized by industry in Table 4.4. We first consider the results for the two data moments. Both moments are fairly consistent between industry portfolios. The first moment, in particular, only varies marginally suggesting that the probability of meeting a benchmark is independent of a firm's industry. The first data moment's consistency further implies that differences in the result of our estimation will largely be caused by differences in the second moment. The first moment $m_1 = 0.65$ for the entire data set reflects that, in our sample, firms meet or beat last period's earnings in 65% of all quarters. Similarly, the second moment $m_2 = 1.69$ for the entire data set reflects that a firm's failure to meet the benchmark by a small amount yields a 69% stronger market reaction than if the firm beats the benchmark by the same amount. The last column in Table 4.4 shows the results of our estimation. All estimates take values just below zero and are therefore consistent with our model. Their range from -0.87% to -0.32% of total assets is consistent with the idea that manipulating earnings by more than a few percent of total assets seems unrealistic. To the best of our knowledge, there is no prior evidence for the magnitude of a parameter similar to $\delta_{\min}$. However, while they do not provide evidence for the discretion limits on a periodical basis, Hribar and Nichols [2007] and Gerakos and Kovrijnykh [2013] estimate the level of discretionary accruals at a given moment in time. They find that the absolute value of discretionary accruals amounts to 5.2% and 0.7% of total assets, respectively. Even though the different definitions of our parameter $\delta_{\min}$ and the level of discretionary accruals prevent a direct comparison, their results provide a rough validation of our estimates. A close inspection of the results further indicates that the magnitude of the estimate decreases in the magnitude of the

	Industry portfolio	$m_1(\mathbf{x})$	$m_2(\mathbf{x})$	$\frac{\delta_{\min}}{\text{Total Assets}}$
1	Consumer Non-Durables	0.62	1.43	−0.0065
2	Consumer Durables	0.64	1.46	−0.0061
3	Manufacturing	0.63	1.59	−0.0056
4	Energy	0.64	1.33	−0.0073
5	Chemicals	0.63	1.25	−0.0087
6	Business Equipment	0.63	1.83	−0.0032
7	Telecom	0.61	1.69	−0.0041
8	Utilities	0.66	1.50	−0.0058
9	Wholesale and Retail	0.64	1.56	−0.0052
10	Healthcare	0.64	1.60	−0.0049
11	Finance	0.69	1.78	−0.0038
12	Other	0.64	1.39	−0.0067
	All	0.65	1.69	−0.0044

Table 4.4: Results for the data moments $m_1(\mathbf{x})$ and $m_2(\mathbf{x})$ of our simulated method of moments (specified in Equation 4.3 and Equation 4.10), as well as the resulting estimate for the the lower discretion bound $\delta_{\min}$ relative to firm size, approximated by total assets. $m_1(\mathbf{x})$ denotes the probability of an earnings report meeting or beating last period's earnings in the data set $\mathbf{x}$. $m_2(\mathbf{x})$ captures the asymmetry of stock price reactions to positive and negative earnings surprises. The measure for firm size in our estimation process b_e is set to 50. Results are classified following the French-Fama 12 industry classification. All values are dimensionless quantities.

second moment. In other words, the stock price asymmetry decreases as the manager's discretionary slack increases. To understand this observation, consider the impact of $\delta_{\min}$ on the manager's reporting strategy and the resulting stock price reaction. If the manager can significantly understate earnings, i.e., for low values of $\delta_{\min}$, he will only report significantly negative reports. The probability of meeting the benchmark consequently increases. In Corollary 2, we established that the magnitude of the downward update of beliefs about past earnings strictly increases in r_{t+1} for all negative reports. In other words, the asymmetry of the stock price reaction decreases as r_{t+1} decreases. Consequently, a lower value of $\delta_{\min}$ implies a lower asymmetry in the stock price reaction to positive and negative earnings surprises.

The estimation results in Table 4.4 were created using the same constant value for b_e, the scale parameter of the Laplace distribution describing the firm's volatility of economic earnings. In particular, we assume $b_e = 50$. The value of b_e describes the level of variation in a firm's economic and true earnings and needs to be specified in our estimation model. Using a regression of the variance in reported earnings and total assets, we establish a functional relationship between variation in earnings, b_e, and firm size. We can thereby map any choice of b_e to the corresponding firm size. To understand the effect of the choice of b_e on our estimates, we conduct a comparative statics analysis. More specifically, we run the estimation for different values of b_e while considering the same data moments. Table 4.5 provides the estimates we obtain for different values of b_e while keeping the data moments $m_1(\mathbf{x}) = 0.65$ and $m_2(\mathbf{x}) = 1.69$ constant. First, remember that the standard deviation of a Laplace distribution is given by $\sigma_e = \sqrt{2}b_e$. Our choices for b_e hence translate into standard deviations of earnings between $\$7\text{M}(b_e = 5)$ and $\$350\text{M}(b_e = 250)$. Despite the significant differences in size, all estimates are almost identical. We conclude that our choice of b_e in the estimation process does not significantly alter the estimation results.

In addition to the comparative statics in Table 4.5, we proceed by further examining the impact of firm size on the estimate. We use total assets as proxy variable for firm size and split the data set into four equally sized categories such that each group accounts for 25% of firms in the sample. The results are summarized in Table 4.6. Similar to our analysis per industry portfolio in Table 4.4, the variation in the first data moment is insignificant suggesting that the probability of meeting or beating a benchmark is

b_e	$\frac{\delta_{\min}}{\text{Total Assets}}$
5	-0.0043
25	-0.0043
50	-0.0044
150	-0.0043
250	-0.0044

Table 4.5: Estimation results for the lower discretion bound $\delta_{\min}$ relative to firm size, approximated by total assets. We consider different values for b_e, which is our choice of firm size in the estimation process. We consider the mean moments in our data set $m_1(\mathbf{x}) = 0.65$ and $m_2(\mathbf{x}) = 1.69$.

independent of firm size. The asymmetry of stock price reactions to negative and positive earnings surprises, which we denote by $m_2(\mathbf{x})$, decreases in firm size. The magnitude of the effect is however small. Similar to the results in Table 4.4, the estimate for $\delta_{\min}$ decreases marginally in firm size, caused by the decrease of $m_2(\mathbf{x})$. This suggests that larger firms can use more discretionary accruals relative to total assets. The magnitude of the effect is however small.

Total assets TA	$m_1(\mathbf{x})$	$m_2(\mathbf{x})$	$\frac{\delta_{\min}}{\text{TA}}$
TA $<$ \$306	0.644	1.71	-0.0041
\$306 $<$ TA $<$ \$1,216	0.650	1.69	-0.0041
\$1,216 $<$ TA $<$ \$4,507	0.649	1.69	-0.0044
TA $>$ \$4,507	0.650	1.64	-0.0045

Table 4.6: Estimation results after splitting the sample into four groups, according to total assets TA. All four categories contain 25% of the sample. $m_1(\mathbf{x})$ and $m_2(\mathbf{x})$ are the data moments, specified in Equation 4.3 and Equation 4.10. The last column contains the estimate for the lower discretion bound $\delta_{\min}$ relative to firm size, approximated by total assets. Total assets are in millions of USD. The remaining values are dimensionless quantities.

Chapter 5

Conclusion

We study a dynamic model of earnings management in which asymmetric stock market behavior around earnings benchmarks arises endogenously as a consequence of constrained managerial earnings manipulation. We conjecture an equilibrium reporting strategy and analyze its validity in an infinite-horizon setting. Our results confirm the hypothesis that managers use precautionary savings to meet investors' expectations in future periods. We show that the prevalence of both big bath behavior for poor economic results and the build up of cookie jars for positive firm earnings can be explained by asymmetric stock price reactions around earnings benchmarks. However, the equilibrium reporting strategies that evolved from two-period models in the accounting literature do not perfectly generalize to an infinite horizon setting. Myopic manager behavior, e.g., towards the end of a manager's tenure, and high existing firm savings cause the manager's optimal reporting choice to shift from precautionary saving towards overstating earnings. Future research may consider alternative equilibrium reporting strategies to analyze the equilibrium's uniqueness.

Based on the asymmetry of stock price reactions around earnings benchmarks and the discontinuity in earnings distributions, we estimate the limit for downward earnings management relative to firm size. Our results indicate a limit of 0.5% of total assets and complement empirical studies in which the average level of discretionary accruals on firms' balance sheets is estimated. The structural estimation can be extended by picking up the constraint on upwards earnings management and estimating both upper and lower limit for a manager's leeway in reporting earnings. In addition, further empirical research can estimate measures more closely related to our notion of reporting constraints.

Appendix A

Mathematical Proofs

Proof of Lemma 1

Lemma 1 follows directly from the decision network in Figure 2.1. □

Proof of Lemma 2

By Bayes' theorem and the independence of all $f(e_t)$, stock prices follow as

$$\begin{aligned}
P_t(r_t) &= \mathbb{E}\Big[\sum_{i=1}^{t} e_i | r_t\Big] \\
&= \sum_{e_1} \cdots \sum_{e_t} \Big[(\sum_{i=1}^{t} e_i) * f(e_{1..t}|r_t) \Big] \\
&= \sum_{e_1} \cdots \sum_{e_t} \Big[(\sum_{i=1}^{t} e_i) * \frac{f(r_t|e_{1..t}) * f(e_{1..t})}{f(r_t)} \Big] \\
&= \sum_{e_1} \cdots \sum_{e_t} \Big[(\sum_{i=1}^{t} e_i) * \frac{f(r_t|e_{1..t}) * \prod_{i=1}^{t} f(e_i)}{f(r_t)} \Big] \\
&= \sum_{e_1} \cdots \sum_{e_t} \Big[(\sum_{i=1}^{t} e_i) * \frac{f(r_t|e_{1..t}) * \prod_{i=1}^{t} f(e_i)}{\sum_{e_1} \cdots \sum_{e_t} f(r_t|e_{1..t}) * \prod_{i=1}^{t} f(e_i)} \Big].
\end{aligned}$$

We note that $f(e_t) = 1/5$ for all e_t and obtain

$$P_t(r_t) = \sum_{e_1} \cdots \sum_{e_t} \Big[(\sum_{i=1}^{t} e_i) * \frac{f(r_t|e_{1..t})}{\sum_{e_1} \cdots \sum_{e_t} f(r_t|e_{1..t})} \Big]$$

where the distribution $f(r_t|e_{1..t})$ remains as the only unknown. To this end, note that all $f(x_t)$ are independent which, in combination with the total probability theorem, yields

$$f(r_t|e_{1..t}) = \sum_{x_1} \cdots \sum_{x_t} \Big[f(r_t|e_{1..t}, x_{1..t}) * \prod_{i=1}^{t} f(x_i) \Big].$$

Inserting the distribution into the stock price function gives

$$P_t(r_t) = \sum_{e_1} \cdots \sum_{e_t} \left[(\sum_{i=1}^{t} e_i) * \frac{\sum_{x_1} \cdots \sum_{x_t} \left[f(r_t|e_{1..t}, x_{1..t}) * \prod_{i=1}^{t} f(x_i) \right]}{\sum_{e_1} \cdots \sum_{e_t} \sum_{x_1} \cdots \sum_{x_t} \left[f(r_t|e_{1..t}, x_{1..t}) * \prod_{i=1}^{t} f(x_i) \right]} \right].$$

To specify the remaining probability mass function $f(r_t|e_{1..t}, x_{1..t})$, we note that

$$r_t = \begin{cases} e_t & x_t = T \\ e_t + \delta_t(e_{1..t-1}, r_{1..t-1}(e_{1..t-1}, x_{1..t-1}), e_t) & x_t = S \end{cases}$$

which we can translate into the PMF

$$f(r_t|e_{1..t}, x_{1..t}) = \begin{cases} 1 & r_t = e_t, x_t = T \\ 1 & r_t = e_t + \delta_t(e_{1..t-1}, r_{1..t-1}(e_{1..t-1}, x_{1..t-1}), e_t), x_t = S \\ 0 & \text{otherwise} \end{cases}.$$

This PMF can easily be evaluated using the decision network in Figure 2.1. □
Table A.1 provides the resulting functional forms for stock prices at both $t = 1$ and $t = 2$.

	$P_t(2)$	$P_t(1)$	$P_t(0)$	$P_t(-1)$	$P_t(-2)$
$t = 1$	2	$2 - \alpha$	$\frac{1-\alpha}{2-\alpha}$	-1	$\frac{\alpha-3}{2-\alpha}$
$t = 2$	2	$2 - \alpha$	$\frac{-\alpha^2-3\alpha+4}{3\alpha^2-11\alpha+13}$	-1	$\frac{\alpha^2+3\alpha-14}{-3\alpha^2+\alpha+7}$

Table A.1: Stock price reactions $P_t(r_t)$ to earnings reports r_t for discrete two-period model as a function of α which denotes the probability of forced truthful reporting

Proof of Lemma 3

By our model setup, the stock price change between any two consecutive periods t and $t+1$ follows as

$$\begin{aligned} P_{t+1}(\gamma_t, r_{t+1}) - P_t(\gamma_{t-1}, r_t) &= P_{t+1}(\gamma_t, r_{t+1}) - P_t(\gamma_t) \\ &= \mathbb{E}\left[\sum_{i=1}^{\infty} e_i \mid \gamma_t, r_{t+1}\right] - \mathbb{E}\left[\sum_{i=1}^{\infty} e_i \mid \gamma_t\right] \\ &= \mathbb{E}\left[\sum_{i=1}^{t+1} e_i \mid \gamma_t, r_{t+1}\right] - \mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t\right] \\ &= \mathbb{E}\Big[e_{t+1} \mid \gamma_t, r_{t+1}\Big] + \mathbb{E}\Big[\sum_{i=1}^{t} e_i \mid \gamma_t, r_{t+1}\Big] - \mathbb{E}\Big[\sum_{i=1}^{t} e_i \mid \gamma_t\Big]. \end{aligned}$$

Note that reported earnings before period t do not provide any information about future earnings $e_{t+1\ldots\infty}$. All subsequent earnings can thus be removed from the two sums. $\square$

Proof of Lemma 4

The tripartite structure laid out in Lemma 3 defines our proceeding for the search of an explicit stock price function.

First summand Note that e_{t+1} and γ_t are independent. In addition, consider the conditional independence of θ_t and e_{t+1} given γ_t. Apply Bayes' theorem to obtain the expected value of new earnings e_{t+1}

$$\begin{aligned}
\mathbb{E}\left[e_{t+1} \mid \gamma_t, r_{t+1}\right] &= \int_{e_{t+1}} e_{t+1} f(e_{t+1}|\gamma_t, r_{t+1}) \\
&= \int_{e_{t+1}} e_{t+1} \frac{f(r_{t+1}|e_{t+1}, \gamma_t) f(e_{t+1}|\gamma_t)}{f(r_{t+1}|\gamma_t} \\
&= \frac{\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}{\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}.
\end{aligned}$$

The Laplace distribution of true earnings $f(e_{t+1})$ is specified in the model setup. $f(\theta_t|\gamma_t)$ denotes investors' prior belief about past earnings, given the reported equity γ_t. The univariate degenerate distribution $f(r_{t+1}|e_{t+1}, \theta_t, \gamma_t)$ follows from $r_{t+1} = e_{t+1} + \delta_{t+1}(e_{t+1}, \theta_t, \gamma_t)$.

Second summand We apply Bayes' theorem to obtain

$$\begin{aligned}
\mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t, r_{t+1}\right] &= \mathbb{E}\left[\theta_t \mid \gamma_t, r_{t+1}\right] \\
&= \int_{\theta_t} \theta_t f(\theta_t|\gamma_t, r_{t+1}) \\
&= \int_{\theta_t} \theta_t \frac{f(\theta_t|\gamma_t) f(r_{t+1}|\gamma_t, \theta_t)}{f(r_{t+1}|\gamma_t)} \\
&= \frac{\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1}|\gamma_t, \theta_t)}{\int_{\theta_t} f(r_{t+1}|\theta_t, \gamma_t) f(\theta_t|\gamma_t)} \\
&= \frac{\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})}{\int_{\theta_t} f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})}.
\end{aligned}$$

Note that the second summand comprises the same distributions that specify the first summand.

Third summand By definition of the expected value, the price difference's third summand follows directly as

$$\mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t\right] = \mathbb{E}\left[\theta_t \mid \gamma_t\right] = \int_{\theta_t} \theta_t f\left(\theta_t|\gamma_t\right)$$

and hence only depends on investors' prior belief.

Price difference equation We combine the three expressions derived above to obtain the price difference for any two consecutive periods as

$$\begin{aligned} P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right) = & \frac{\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}{\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)} \\ & + \frac{\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})}{\int_{\theta_t} f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(e_{t+1})} \\ & - \int_{\theta_t} \theta_t f\left(\theta_t|\gamma_t\right). \end{aligned}$$ □

Proof of Lemma 5

We derive the equilibrium stock price functions by following the structure in Lemma 4. Solving the three summands sequentially will show that the first and second summand are piecewise functions over the same sub-domains. The resulting stock price function will hence be defined by a piecewise function.

First summand Recall that the first summand is given by

$$\frac{\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}{\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)}.$$

Both numerator and denominator include two integrations over true equity θ_t and true earnings e_{t+1}. By inspection, they only differ by the variable e_{t+1} and share the same inner integration. We first solve the common integral $\int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)$ (i), and use the solution to derive expressions for denominator (ii) and numerator (iii).

i. Common integral The term $\int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)$ contains two distributions: the degenerate univariate distribution $f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t)$ that follows from $r_{t+1} = e_{t+1} + \delta_{t+1}(e_{t+1}, \theta_t, \gamma_t)$, as well as investors' prior belief about true firm equity $f(\theta_t|\gamma_t)$. Limited rationality causes investors to assume a Laplace distribution for the prior belief $f(\theta_t|\gamma_t)$. Values of γ_t that exceed $\delta_0 + \theta_t$ are thus assigned a probability strictly greater than zero, making hypothetical negative savings mathematically possible. To account for this, we adapt the discretion strategy to obtain

$$\delta_{t+1}(e_{t+1},\theta_t,\gamma_t)=\begin{cases}\delta_{\min} & e_{t+1}<\min\{-(\delta_0+\theta_t-\gamma_t),0\}\\ -e_{t+1} & e_{t+1}\in[\min\{-(\delta_0+\theta_t-\gamma_t),0\},-\delta_{\min}]\\ \delta_{\min} & e_{t+1}>\delta_{\min}\end{cases}.$$

The solution of the common integral requires integrating over true equity θ_t which occurs in the discretion strategy only in the definition of the sub-domains. Before solving the common integral, we introduce the delta function g. Let g be a real function $g:\mathbb{R}\mapsto\mathbb{R}$ of form

$$g(x)=\begin{cases}1 & x=0\\ 0 & \text{otherwise}\end{cases}.$$

Some analysis yields the solution to the common integral.

$$\int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(\theta_t|\gamma_t)=\int_{\theta_t} g(r_{t+1}-e_{t+1}-\delta_{t+1}(e_{t+1},\theta_t,\gamma_t))f(\theta_t|\gamma_t)$$

$$=\begin{cases}\displaystyle\int_{-\infty}^{\gamma_t-\delta_0-e_{t+1}} g(r_{t+1}-e_{t+1}-\delta_{\min})f(\theta_t|\gamma_t)\,d\theta_t + \int_{\gamma_t-\delta_0-e_{t+1}}^{\infty} g(r_{t+1})f(\theta_t|\gamma_t)\,d\theta_t & e_{t+1}<0\\ \displaystyle\int_{-\infty}^{\infty} g(r_{t+1})f(\theta_t|\gamma_t)\,d\theta_t & 0\le e_{t+1}\le-\delta_{\min}\\ \displaystyle\int_{-\infty}^{\infty} g(r_{t+1}-e_{t+1}-\delta_{\min})f(\theta_t|\gamma_t)\,d\theta_t & e_{t+1}>-\delta_{\min}\end{cases}$$

$$=\begin{cases} g\left(r_{t+1}-e_{t+1}-\delta_{\min}\right)\dfrac{1}{2}\exp^{\frac{\gamma_t-\delta_0-e_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}} + g(r_{t+1})\dfrac{1}{2}\left(2-\exp^{\frac{\gamma_t-\delta_0-e_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}\right) & e_{t+1}<0,\gamma_t-\delta_0-e_{t+1}\le\mu_{\theta_t}\\ g\left(r_{t+1}-e_{t+1}-\delta_{\min}\right)\dfrac{1}{2}\left(2-\exp^{-\frac{\gamma_t-\delta_0-e_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}\right) + g(r_{t+1})\dfrac{1}{2}\exp^{-\frac{\gamma_t-\delta_0-e_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}} & e_{t+1}<0,\gamma_t-\delta_0-e_{t+1}>\mu_{\theta_t}\\ g(r_{t+1}) & 0\le e_{t+1}\le-\delta_{\min}\\ g(r_{t+1}-e_{t+1}-\delta_{\min}) & e_{t+1}>-\delta_{\min}\end{cases}$$

ii. Denominator Using the solution for the common integral, we solve the denominator of the first summand. Substitution of the common integral gives

$$\int_{e_{t+1}} f(e_{t+1})\int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(\theta_t|\gamma_t)$$

$$= \begin{cases} \int_{-\infty}^{\gamma_t - \delta_0 - \mu_{\theta_t}} \Big[g\left(r_{t+1} - e_{t+1} - \delta_{\min}\right) \frac{1}{2} \left(2 - \exp^{-\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}}\right) \\ \quad + g(r_{t+1}) \frac{1}{2} \exp^{-\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}} \Big] \frac{1}{2b_e} \exp^{\frac{e_{t+1}}{b_e}} \; de_{t+1} \\ \quad + \int_{\gamma_t - \delta_0 - \mu_{\theta_t}}^{0} \Big[g\left(r_{t+1} - e_{t+1} - \delta_{\min}\right) \frac{1}{2} \exp^{\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}} & \gamma_t - \delta_0 - \mu_{\theta_t} < 0 \\ \quad + g(r_{t+1}) \frac{1}{2} \left(2 - \exp^{\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}}\right) \Big] \frac{1}{2b_e} \exp^{\frac{e_{t+1}}{b_e}} \; de_{t+1} \\ \int_{-\infty}^{0} \Big[g\left(r_{t+1} - e_{t+1} - \delta_{\min}\right) \frac{1}{2} \left(2 - \exp^{-\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}}\right) & \gamma_t - \delta_0 - \mu_{\theta_t} \geq 0 \\ \quad + g(r_{t+1}) \frac{1}{2} \exp^{-\frac{\gamma_t - \delta_0 - e_{t+1} - \mu_{\theta_t}}{b_{\theta_t}}} \Big] \frac{1}{2b_e} \exp^{\frac{e_{t+1}}{b_e}} \; de_{t+1} \end{cases}$$

$$+ \int_{0}^{-\delta_{\min}} g(r_{t+1}) \frac{1}{2b_e} \exp^{-\frac{e_{t+1}}{b_e}} \; de_{t+1} + \int_{-\delta_{\min}}^{\infty} g(r_{t+1} - e_{t+1} - \delta_{\min}) \frac{1}{2b_e} \exp^{-\frac{e_{t+1}}{b_e}} \; de_{t+1}.$$

Let $a := \gamma_t - \delta_0 - \mu_{\theta_t}$ to simplify notation of the case distinctions for the remainder of this proof.

$$\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)$$

$$= \begin{cases} \frac{1}{4b_e} \left(2 - \exp^{-\frac{a + \delta_{\min} - r_{t+1}}{b_{\theta_t}}}\right) \exp^{\frac{r_{t+1} - \delta_{\min}}{b_e}} & r_{t+1} < \delta_{\min} + a, a < 0 \\ \frac{1}{4b_e} \exp^{\frac{a + \delta_{\min} - r_{t+1}}{b_{\theta_t}} + \frac{r_{t+1} - \delta_{\min}}{b_e}} & \delta_{\min} + a \leq r_{t+1} \leq \delta_{\min}, a < 0 \\ \frac{1}{4} \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{\frac{a}{b_e}} + \frac{1}{4} \Big[2(1 - \exp^{\frac{a}{b_e}}) & r_{t+1} = 0, a < 0 \\ \quad - \frac{b_{\theta_t}}{b_{\theta_t} - b_e} \exp^{\frac{a}{b_{\theta_t}}} (1 - \exp^{a(\frac{1}{b_e} - \frac{1}{b_{\theta_t}})}) \Big] \\ \frac{1}{4b_e} \left(2 - \exp^{-\frac{a + \delta_{\min} - r_{t+1}}{b_{\theta_t}}}\right) \exp^{\frac{r_{t+1} - \delta_{\min}}{b_e}} & r_{t+1} < \delta_{\min}, a \geq 0 \\ \frac{1}{4} \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} & r_{t+1} = 0, a \geq 0 \end{cases}$$

$$+ \frac{1}{2} g(r_{t+1}) \left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right) + \begin{cases} 0 & r_{t+1} \leq 0 \\ \frac{1}{2b_e} \exp^{-\frac{r_{t+1} - \delta_{\min}}{b_e}} & r_{t+1} > 0 \end{cases}$$

The denominator is a piecewise function with seven sub-domains which are defined by two factors: reported earnings r_{t+1} and the term $\gamma_t - \delta_0 - \mu_{\theta_t}$, abbreviated by a, which we discussed in the discussion of Lemma 5. Let $a < 0$. The denominator is given by

$$\int_{e_{t+1}} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1}, \gamma_t, \theta_t) f(\theta_t|\gamma_t)$$

$$= \begin{cases} \frac{1}{4b_e}\left(2-\exp^{-\frac{a+\delta_{\min}-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} < \delta_{\min}+a \\ \frac{1}{4b_e}\exp^{\frac{a+\delta_{\min}-r_{t+1}}{b_{\theta_t}}+\frac{r_{t+1}-\delta_{\min}}{b_e}} & \delta_{\min}+a \leq r_{t+1} \leq \delta_{\min} \\ \frac{1}{4}\frac{b_{\theta_t}}{b_e+b_{\theta_t}}\exp^{\frac{a}{b_e}}+\frac{1}{4}\Bigg[2\left(1-\exp^{\frac{a}{b_e}}\right)-\frac{b_{\theta_t}}{b_{\theta_t}-b_e} & \\ \quad \exp^{\frac{a}{b_{\theta_t}}}\left(1-\exp^{a\left(\frac{1}{b_e}-\frac{1}{b_{\theta_t}}\right)}\right)\Bigg]+\frac{1}{2}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) & r_{t+1}=0 \\ \frac{1}{2b_e}\exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1}>0 \end{cases}$$

For $a \geq 0$, we obtain

$$\int_{e_{t+1}} f(e_{t+1})\int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(\theta_t|\gamma_t)$$

$$= \begin{cases} \frac{1}{4b_e}\left(2-\exp^{-\frac{a+\delta_{\min}-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} \leq \delta_{\min} \\ \frac{1}{4}\frac{b_{\theta_t}}{b_e+b_{\theta_t}}\exp^{-\frac{a}{b_{\theta_t}}}+\frac{1}{2}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) & r_{t+1}=0 \\ \frac{1}{2b_e}\exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1}>0 \end{cases}$$

iii. Numerator By inspection, numerator and denominator only differ by the variable e_{t+1}. The derivation of the numerator is therefore similar to the previous calculations but includes additional integrations by parts.

$$\int_{e_{t+1}} e_{t+1}f(e_{t+1})\int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(\theta_t|\gamma_t)$$

$$= \begin{cases} \int_{-\infty}^{a}\left[g\left(r_{t+1}-e_{t+1}-\delta_{\min}\right)\frac{1}{2}\left(2-\exp^{-\frac{a-e_{t+1}}{b_{\theta_t}}}\right)+\frac{g(r_{t+1})}{2}\exp^{-\frac{a-e_{t+1}}{b_{\theta_t}}}\right] & \\ \quad \frac{1}{2b_e}\exp^{\frac{e_{t+1}}{b_e}}e_{t+1}\,de_{t+1}+\int_{a}^{0}\left[g\left(r_{t+1}-e_{t+1}-\delta_{\min}\right)\frac{1}{2}\exp^{\frac{a-e_{t+1}}{b_{\theta_t}}}\right. & a<0 \\ \quad \left.+\frac{g(r_{t+1})}{2}\left(2-\exp^{\frac{a-e_{t+1}}{b_{\theta_t}}}\right)\right]\frac{1}{2b_e}\exp^{\frac{e_{t+1}}{b_e}}e_{t+1}\,de_{t+1} & \\ \int_{-\infty}^{0}\left[g\left(r_{t+1}-e_{t+1}-\delta_{\min}\right)\frac{1}{2}\left(2-\exp^{-\frac{a-e_{t+1}}{b_{\theta_t}}}\right)\right. & \\ \quad \left.+\frac{g(r_{t+1})}{2}\exp^{-\frac{a-e_{t+1}}{b_{\theta_t}}}\right]\frac{1}{2b_e}\exp^{\frac{e_{t+1}}{b_e}}e_{t+1}\,de_{t+1} & a \geq 0 \end{cases}$$

$$+\int_{0}^{-\delta_{\min}} g(r_{t+1})\frac{e_{t+1}}{2b_e}\exp^{-\frac{e_{t+1}}{b_e}}\,de_{t+1}+\int_{-\delta_{\min}}^{\infty} g(r_{t+1}-e_{t+1}-\delta_{\min})\frac{e_{t+1}}{2b_e}\exp^{-\frac{e_{t+1}}{b_e}}\,de_{t+1}$$

$$= \begin{cases} \frac{1}{4b_e}\left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}<\delta_{\min}+a, a<0 \\ \frac{1}{4b_e}\exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}+\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & \delta_{\min}+a\le r_{t+1}\le\delta_{\min}, a<0 \\ \frac{1}{4}\Bigg[\frac{b_{\theta_t}}{b_e+b_{\theta_t}}\exp^{\frac{a}{b_e}}\left(a-\frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}}\right)-2\Bigg(a\exp^{\frac{a}{b_e}} & \\ \quad +b_e\left(1-\exp^{\frac{a}{b_e}}\right)\Bigg)+\frac{b_{\theta_t}}{b_{\theta_t}-b_e}\exp^{\frac{a}{b_{\theta_t}}} & r_{t+1}=0, a<0 \\ \quad \left(a\exp^{a\left(\frac{1}{b_e}-\frac{1}{b_{\theta_t}}\right)}+\frac{b_e b_{\theta_t}}{b_{\theta_t}-b_e}\left(1-\exp^{a\left(\frac{1}{b_e}-\frac{1}{b_{\theta_t}}\right)}\right)\right)\Bigg] & \\ \frac{1}{4b_e}\left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}<\delta_{\min}, a\ge 0 \\ -\frac{1}{4b_e}\left(\frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}}\right)^2\exp^{-\frac{a}{b_{\theta_t}}} & r_{t+1}=0, a\ge 0 \end{cases}$$

$$+\,g(r_{t+1})\frac{1}{2}\left((\delta_{\min}-b_e)\exp^{\frac{\delta_{\min}}{b_e}}+b_e\right)+\begin{cases}0 & r_{t+1}\le 0\\ \frac{1}{2b_e}\exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}>0\end{cases}$$

Let $a<0$.

$$\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(\theta_t|\gamma_t)$$

$$= \begin{cases} \frac{1}{4b_e}\left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}<\delta_{\min}+a \\ \frac{1}{4b_e}\exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}+\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & \delta_{\min}+a\le r_{t+1}\le\delta_{\min} \\ \frac{1}{4}\Bigg[\frac{b_{\theta_t}}{b_e+b_{\theta_t}}\exp^{\frac{a}{b_e}}\left(a-\frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}}\right)-2\Bigg(a\exp^{\frac{a}{b_e}}+b_e & \\ \quad \left(1-\exp^{\frac{a}{b_e}}\right)\Bigg)+\frac{b_{\theta_t}}{b_{\theta_t}-b_e}\exp^{\frac{a}{b_{\theta_t}}}\Bigg(a\exp^{a\left(\frac{1}{b_e}-\frac{1}{b_{\theta_t}}\right)} & r_{t+1}=0 \\ \quad +\frac{b_e b_{\theta_t}}{b_{\theta_t}-b_e}\left(1-\exp^{a\left(\frac{1}{b_e}-\frac{1}{b_{\theta_t}}\right)}\right)\Bigg)\Bigg]+\frac{1}{2}\left((\delta_{\min}-b_e)\exp^{\frac{\delta_{\min}}{b_e}}+b_e\right) & \\ \frac{1}{2b_e}\exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}>0 \end{cases}$$

For $a\ge 0$, we obtain

$$\int_{e_{t+1}} e_{t+1} f(e_{t+1}) \int_{\theta_t} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(\theta_t|\gamma_t)$$

$$= \begin{cases} \frac{1}{4b_e}\left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right)\exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}<\delta_{\min} \\ -\frac{1}{4b_e}\left(\frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}}\right)^2\exp^{-\frac{a}{b_{\theta_t}}}+\frac{1}{2}\left((\delta_{\min}-b_e)\exp^{\frac{\delta_{\min}}{b_e}}+b_e\right) & r_{t+1}=0 \\ \frac{1}{2b_e}\exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}}(r_{t+1}-\delta_{\min}) & r_{t+1}>0 \end{cases}$$

Second summand We derive a solution for

$$\frac{\int_{\theta_t}\theta_t f(\theta_t|\gamma_t)\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(e_{t+1})}{\int_{\theta_t} f(\theta_t|\gamma_t)\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(e_{t+1})}$$

in three steps. We first solve the common integral (i) $\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(e_{t+1})$. The solution is then used to calculate the denominator (ii) and numerator (iii).

i. Common integral The common integral follows directly from the equilibrium reporting strategy defined in Equation 3.5 and we obtain

$$\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\theta_t,\gamma_t)f(e_{t+1}) = \begin{cases} f_{e_{t+1}}(r_{t+1}-\delta_{\min}) & r_{t+1}<\delta_{\min}-\max\{0,\delta_0+\theta_t-\gamma_t\} \\ \int_{\min\{0,-(\delta_0+\theta_t-\gamma_t)\}}^{-\delta_{\min}} f(e_{t+1})\,de_{t+1} & r_{t+1}=0 \\ f_{e_{t+1}}(r_{t+1}-\delta_{\min}) & r_{t+1}>0 \end{cases}.$$

ii. Denominator Given the solution of the common integral, we proceed by solving the denominator. Substitution of the common integral yields

$$\int_{\theta_t} f(\theta_t|\gamma_t)\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(e_{t+1})$$
$$= \begin{cases} \int_{-\infty}^{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}} \frac{1}{2b_{\theta_t}}\exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}}\frac{1}{2b_e}\exp^{-\frac{|r_{t+1}-\delta_{\min}|}{b_e}}\,d\theta_t & r_{t+1}\le\delta_{\min} \\ \int_{-\infty}^{\gamma_t-\delta_0}\frac{1}{2b_{\theta_t}}\exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}}\left(\int_0^{-\delta_{\min}}\frac{1}{2b_e}\exp^{-\frac{|e_{t+1}|}{b_e}}\,de_{t+1}\right)d\theta_t & \\ \quad+\int_{\gamma_t-\delta_0}^{\infty}\frac{1}{2b_{\theta_t}}\exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}}\left(\int_{-(\delta_0+\theta_t-\gamma_t)}^{-\delta_{\min}}\frac{1}{2b_e}\exp^{-\frac{|e_{t+1}|}{b_e}}\,de_{t+1}\right)d\theta_t & r_{t+1}=0 \\ \int_{-\infty}^{\infty}\frac{1}{2b_{\theta_t}}\exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}}\frac{1}{2b_e}\exp^{-\frac{|r_{t+1}-\delta_{\min}|}{b_e}}\,d\theta_t & r_{t+1}>0 \end{cases}$$

which evaluates to

$$\int_{\theta_t} f(\theta_t|\gamma_t)\int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t)f(e_{t+1})$$

$$
= \begin{cases}
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \begin{cases} \exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a \leq r_{t+1} \\ 2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a > r_{t+1} \end{cases} & r_{t+1} \leq \delta_{\min} \\
\begin{cases} \frac{1}{4}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right)\exp^{\frac{a}{b_{\theta_t}}} + \frac{1}{4}\Bigg[\left(2-\exp^{\frac{\delta_{\min}}{b_e}}\right)\left(2-\exp^{\frac{a}{b_{\theta_t}}}\right) \\ \quad - \exp^{\frac{\gamma_t-\delta_0}{b_e}-\frac{\mu_{\theta_t}}{b_{\theta_t}}} \frac{b_e}{b_e - b_{\theta_t}}\Bigg(\exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} \\ \quad - \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)}\Bigg) - \frac{b_e}{b_e+b_{\theta_t}} \exp^{\frac{\gamma_t-\delta_0-\mu_{\theta_t}}{b_e}}\Bigg] & a < 0 \\ \frac{1}{4}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right)\left(2-\exp^{-\frac{a}{b_{\theta_t}}}\right) + \frac{1}{4}\Bigg[\left(2-\exp^{\frac{\delta_{\min}}{b_e}}\right) \\ \quad \exp^{-\frac{a}{b_{\theta_t}}} - \frac{b_e}{b_e+b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}}\Bigg] & a \geq 0 \end{cases} & r_{t+1} = 0 \\
\frac{1}{2b_e} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} > 0
\end{cases}
$$

Let $a < 0$. The denominator admits as its solution

$$
\int_{\theta_t} f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1})
$$

$$
= \begin{cases}
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right) & r_{t+1} < \delta_{\min}+a \\
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a \leq r_{t+1} \leq \delta_{\min} \\
\frac{1}{4}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right)\exp^{\frac{a}{b_{\theta_t}}} + \frac{1}{4}\Bigg[\left(2-\exp^{\frac{\delta_{\min}}{b_e}}\right)\left(2-\exp^{\frac{a}{b_{\theta_t}}}\right) & \\
\quad - \exp^{\frac{\gamma_t-\delta_0}{b_e}-\frac{\mu_{\theta_t}}{b_{\theta_t}}} \frac{b_e}{b_e - b_{\theta_t}}\Bigg(\exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} & r_{t+1} = 0 \\
\quad - \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)}\Bigg) - \frac{b_e}{b_e+b_{\theta_t}} \exp^{\frac{a}{b_e}}\Bigg] & \\
\frac{1}{2b_e} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} > 0
\end{cases}
$$

Let $a \geq 0$. The solution evaluates to

$$
\int_{\theta_t} f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1})
$$

$$
= \begin{cases}
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \left(2-\exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right) & r_{t+1} \leq \delta_{\min} \\
\frac{1}{2}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{1}{4}\frac{b_{\theta_t}}{b_e+b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} & r_{t+1} = 0 \\
\frac{1}{2b_e} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} > 0
\end{cases}
$$

iii. Numerator Substitution of the common integral provides the integrations we need to solve for the numerator.

$$\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1})$$
$$= \begin{cases} \displaystyle\int_{-\infty}^{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}} \theta_t \frac{1}{2b_{\theta_t}} \exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}} \frac{1}{2b_e} \exp^{-\frac{|r_{t+1}-\delta_{\min}|}{b_e}} \, d\theta_t & r_{t+1} \leq \delta_{\min} \\ \displaystyle\int_{-\infty}^{\gamma_t-\delta_0} \theta_t \frac{1}{2b_{\theta_t}} \exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}} \left(\int_0^{-\delta_{\min}} \frac{1}{2b_e} \exp^{-\frac{|e_{t+1}|}{b_e}} \, de_{t+1} \right) d\theta_t & \\ \quad + \displaystyle\int_{\gamma_t-\delta_0}^{\infty} \theta_t \frac{1}{2b_{\theta_t}} \exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}} \left(\int_{-(\delta_0+\theta_t-\gamma_t)}^{-\delta_{\min}} \frac{1}{2b_e} \exp^{-\frac{|e_{t+1}|}{b_e}} \, de_{t+1} \right) d\theta_t & r_{t+1} = 0 \\ \displaystyle\int_{-\infty}^{\infty} \theta_t \frac{1}{2b_{\theta_t}} \exp^{-\frac{|\theta_t-\mu_{\theta_t}|}{b_{\theta_t}}} \frac{1}{2b_e} \exp^{-\frac{|r_{t+1}-\delta_{\min}|}{b_e}} \, d\theta_t & r_{t+1} > 0 \end{cases}$$

Consider $r_{t+1} \leq \delta_{\min}$. The integral evaluates to

$$\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1})$$
$$= \frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \begin{cases} (\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-b_{\theta_t}) \exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a \leq r_{t+1} \\ 2\mu_{\theta_t} - (\delta_{\min}+\gamma_t-\delta_0-r_{t+1}+b_{\theta_t}) \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a > r_{t+1} \end{cases}.$$

For $r_{t+1} = 0$, we obtain

$$\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1})$$
$$= \begin{cases} \frac{1}{4}\left(2\mu_{\theta_t} - (\gamma_t-\delta_0-b_{\theta_t}) \exp^{\frac{a}{b_{\theta_t}}}\right) + \frac{1}{2}\mu_{\theta_t}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) - \frac{1}{4}\exp^{\frac{\gamma_t-\delta_0}{b_e}} & \\ \quad \left[\frac{b_e}{b_e-b_{\theta_t}} \exp^{-\frac{\mu_{\theta_t}}{b_{\theta_t}}} \left(\mu_{\theta_t} \exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} - (\gamma_t-\delta_0) \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} \right.\right. & \\ \quad\quad \left.\left. - \frac{b_e b_{\theta_t}}{b_e-b_{\theta_t}} \left(\exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} - \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} \right) \right) \right. & a < 0 \\ \quad \left. + \frac{b_e}{b_e+b_{\theta_t}} \exp^{\frac{\mu_{\theta_t}}{b_{\theta_t}}} \left(\mu_{\theta_t} \exp^{-\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} + \frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}} \exp^{-\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} \right) \right] & \\ \frac{1}{2}\mu_{\theta_t}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{1}{4}\frac{b_{\theta_t}}{b_e+b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} \left[\gamma_t-\delta_0+\frac{b_{\theta_t}}{b_e+b_{\theta_t}}(2b_e+b_{\theta_t}) \right] & a \geq 0 \end{cases}$$

We used L'Hôpital's rule to solve integration results of form

$$\lim_{\theta_t\to-\infty} \theta_t \exp^{\theta_t} = \lim_{\theta_t\to-\infty} \frac{1}{-\exp^{-\theta_t}} = 0.$$

Finally, consider the case $r_{t+1} > 0$. Note that $f_{e_t}(r_{t+1}-\delta_{\min})$ is independent of θ_t. By definition of the expected value, the remaining integral evaluates to the mean μ_{θ_t}. Consequently, the numerator equals

$$\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1}) = \frac{1}{2b_e}\mu_{\theta_t} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}}.$$

We combine the three cases and note that the sub-domains correspond to the denominator's structure. Let $a < 0$.

$$\begin{aligned}
&\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1}) \\
&= \begin{cases}
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \left[2\mu_{\theta_t} - (\delta_{\min}+\gamma_t-\delta_0-r_{t+1}+b_{\theta_t}) \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right] & r_{t+1} < \delta_{\min}+a \\
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} (\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-b_{\theta_t}) \exp^{\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}} & \delta_{\min}+a \leq r_{t+1} \leq \delta_{\min} \\
\frac{1}{4}\left(2\mu_{\theta_t} - (\gamma_t-\delta_0-b_{\theta_t}) \exp^{\frac{a}{b_{\theta_t}}}\right) + \frac{1}{2}\mu_{\theta_t}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) - \frac{1}{4}\exp^{\frac{\gamma_t-\delta_0}{b_e}} & \\
\quad \left[\frac{b_e}{b_e-b_{\theta_t}} \exp^{-\frac{\mu_{\theta_t}}{b_{\theta_t}}} \left(\mu_{\theta_t} \exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} - (\gamma_t-\delta_0) \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} \right.\right. & \\
\qquad \left. - \frac{b_e b_{\theta_t}}{b_e-b_{\theta_t}} \left(\exp^{\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} - \exp^{(\gamma_t-\delta_0)\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)}\right)\right) & r_{t+1} = 0 \\
\quad \left. + \frac{b_e}{b_e+b_{\theta_t}} \exp^{\frac{\mu_{\theta_t}}{b_{\theta_t}}} \left(\mu_{\theta_t} \exp^{-\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)} + \frac{b_e b_{\theta_t}}{b_e+b_{\theta_t}} \exp^{-\mu_{\theta_t}\left(\frac{1}{b_{\theta_t}}-\frac{1}{b_e}\right)}\right)\right] & \\
\frac{1}{2b_e}\mu_{\theta_t} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} > 0
\end{cases}
\end{aligned}$$

Let $a \geq 0$.

$$\begin{aligned}
&\int_{\theta_t} \theta_t f(\theta_t|\gamma_t) \int_{e_{t+1}} f(r_{t+1}|e_{t+1},\gamma_t,\theta_t) f(e_{t+1}) \\
&= \begin{cases}
\frac{1}{4b_e} \exp^{\frac{r_{t+1}-\delta_{\min}}{b_e}} \left[2\mu_{\theta_t} - (\delta_{\min}+\gamma_t-\delta_0-r_{t+1}+b_{\theta_t}) \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}\right] & r_{t+1} \leq \delta_{\min} \\
\frac{1}{2}\mu_{\theta_t}\left(1-\exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{1}{4}\frac{b_{\theta_t}}{b_e+b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} \left[\gamma_t-\delta_0+\frac{b_{\theta_t}}{b_e+b_{\theta_t}}(2b_e+b_{\theta_t})\right] & r_{t+1} = 0 \\
\frac{1}{2b_e}\mu_{\theta_t} \exp^{-\frac{r_{t+1}-\delta_{\min}}{b_e}} & r_{t+1} > 0
\end{cases}
\end{aligned}$$

Third summand By definition, the Laplace distribution is symmetric around the mean. The third summand immediately evaluates to

$$\mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t\right] = \int_{\theta_t} \theta_t f(\theta_t|\gamma_t) = \mu_{\theta_t}.$$

Results The three solutions we derived fully specify the equilibrium stock price behavior. The stock price difference equation is a piecewise function defined for seven sub-domains. Let $a < 0$. The stock price reaction P_{t+1} to any reported earnings r_{t+1}

follows

$$
P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right)
= \begin{cases}
r_{t+1} - \delta_{\min} + \dfrac{2\mu_{\theta_t} - (\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} + b_{\theta_t}) \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}}{2 - \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}} - \mu_{\theta_t} & r_{t+1} < \delta_{\min} + a \\
r_{t+1} - \delta_{\min} + \delta_{\min} + \gamma_t - \delta_0 - r_{t+1} - b_{\theta_t} & \delta_{\min} + a \leq r_{t+1} \leq \delta_{\min} \\
f_1(b_e, b_{\theta_t}, \delta_0, \delta_{\min}, \gamma_t, \mu_{\theta_t}) + f_2(b_e, b_{\theta_t}, \delta_0, \delta_{\min}, \gamma_t, \mu_{\theta_t}) & r_{t+1} = 0 \\
r_{t+1} - \delta_{\min} & r_{t+1} > 0
\end{cases}
$$

Due to its length, the reaction to $r_{t+1} = 0$ is abbreviated. Let $a \geq 0$.

$$
P_{t+1}\left(\gamma_t, r_{t+1}\right) - P_t\left(\gamma_{t-1}, r_t\right)
= \begin{cases}
r_{t+1} - \delta_{\min} + \dfrac{2\mu_{\theta_t} - (\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} + b_{\theta_t}) \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}}{2 - \exp^{-\frac{\delta_{\min}+a-r_{t+1}}{b_{\theta_t}}}} - \mu_{\theta_t} & r_{t+1} < \delta_{\min} \\
\dfrac{-\frac{1}{b_e}\left(\frac{b_e b_{\theta_t}}{b_e + b_{\theta_t}}\right)^2 \exp^{-\frac{a}{b_{\theta_t}}} + 2\left((\delta_{\min} - b_e)\exp^{\frac{\delta_{\min}}{b_e}} + b_e\right)}{\frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} + 2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right)} & \\
\quad + \dfrac{2\mu_{\theta_t}\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}} \left[\gamma_t - \delta_0 + \frac{b_{\theta_t}}{b_e + b_{\theta_t}}(2b_e + b_{\theta_t})\right]}{2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right) + \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \exp^{-\frac{a}{b_{\theta_t}}}} - \mu_{\theta_t} & r_{t+1} = 0 \\
r_{t+1} - \delta_{\min} & r_{t+1} > 0
\end{cases}
$$

□

Proof of Corollary 2

Corollary 2 follows directly from the manager's equilibrium reporting strategy specified in Equation 3.5. □

Proof of Corollary 3

For part (i) of the corollary to hold, we need to show that the change in beliefs about past earnings is negative for all $r_{t+1} < \delta_{\min}$. By definition of the second case, $\gamma_t - \delta_0 - \mu_{\theta_t} \geq 0$. Lemma 5 specifies the update of beliefs as

$$
\mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t, r_{t+1}\right] - \mathbb{E}\left[\sum_{i=1}^{t} e_i \mid \gamma_t\right] =
\frac{2\mu_{\theta_t} - (\delta_{\min} + \gamma_t - \delta_0 - r_{t+1} + b_{\theta_t}) \exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}}{2 - \exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}} - \mu_{\theta_t}.
$$

It remains to verify that the term is negative for all values of γ_t and μ_{θ_t}. Let $a := (\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1})/b_{\theta_t}$. The inequality simplifies to

$$\frac{2\mu_{\theta_t}-(\delta_{\min}+\gamma_t-\delta_0-r_{t+1}+b_{\theta_t})\exp^{-a}}{2-\exp^{-a}}-\mu_{\theta_t}<0 \quad \forall r_{t+1}\leq\delta_{\min},$$

or equivalently

$$\frac{\exp^{-a}}{2-\exp^{-a}}(\mu_{\theta_t}-\delta_{\min}-\gamma_t+\delta_0-b_{\theta_t}+r_{t+1})<0.$$

By definition of the exponential function, the numerator $\exp^{-a}$ is strictly positive. We conjecture that the term in brackets is strictly negative and prove that the inequality

$$r_{t+1}<\gamma_t-\delta_0-\mu_{\theta_t}+\delta_{\min}+b_{\theta_t}$$

holds. Recall that $\gamma_t-\delta_0-\mu_{\theta_t}\geq 0$. The scaling parameter of the prior Laplace distribution b_{θ_t} is per definition positive. Since we consider the case $r_{t+1}<\delta_{\min}$, the inequality of form $r_{t+1}<\delta_{\min}+c^2$ is thus always satisfied. It remains to verify that the denominator $2-\exp(-a)$ is strictly positive. After substitution of a, the inequality

$$2-\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}>0$$

can be rewritten as

$$r_{t+1}<b_{\theta_t}\ln 2+\gamma_t-\delta_0-\mu_{\theta_t}+\delta_{\min}.$$

Both elements of the first summand on the right-hand side of the inequality are positive. We further note that $\gamma_t-\delta_0-\mu_{\theta_t}$ is positive. Once again, we obtain an equation of form $r_{t+1}<\delta_{\min}+c^2$ which is satisfied for all $r_{t+1}<\delta_{\min}$. Two out of the three expressions in our main inequality are positive, one expression is negative. We thus proved our initial claim. □

Part (ii) of the corollary conjectures that the magnitude of the downward update of beliefs strictly increases in r_{t+1}. In other words, negative reports closer to the benchmark are penalized more severely, as they provide a lower upper bound on firm savings. Mathematically speaking, we need to show that

$$\frac{\partial}{\partial r_{t+1}}\left(\frac{2\mu_{\theta_t}-(\delta_{\min}+\gamma_t-\delta_0-r_{t+1}+b_{\theta_t})\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}}{2-\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-r_{t+1}-\mu_{\theta_t}}{b_{\theta_t}}}}-\mu_{\theta_t}\right)<0 \quad \forall r_{t+1}\leq\delta_{\min}.$$

The inequality evaluates to

$$\left(2-\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}\right)^{-2}\left[\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}\left(1-\frac{1}{b_{\theta_t}}(\delta_{\min}+\gamma_t-\delta_0+b_{\theta_t}-r_{t+1})\right)\right.$$
$$\left(2-\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}\right)+\frac{1}{b_{\theta_t}}\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}(2\mu_{\theta_t}-(\delta_{\min}+\gamma_t-\delta_0+b_{\theta_t}-r_{t+1})$$
$$\left.\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}})\right]<0.$$

Let $a := (\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1})/b_{\theta_t}$. Some analysis yields

$$\frac{\exp^a}{b_{\theta_t}}\left(2(\mu_{\theta_t}-\delta_{\min}-\gamma_t+\delta_0+r_{t+1})-b_{\theta_t}\exp^a\right)\left(2-\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}-r_{t+1}}{b_{\theta_t}}}\right)^{-2}<0.$$

The fact that $\exp(a)$ and b_{θ_t} are strictly positive implies that the fraction is strictly positive. The squared term in brackets is positive. For our initial claim to hold, it remains to show that the remaining term in brackets is negative, i.e.,

$$2(\mu_{\theta_t}-\delta_{\min}-\gamma_t+\delta_0+r_{t+1})-b_{\theta_t}\exp^a<0 \quad \forall\, r_{t+1}\le\delta_{\min}.$$

Some analysis and substitution of a provides

$$r_{t+1}-\frac{b_{\theta_t}}{2}\exp^{-\frac{\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t}}{b_{\theta_t}}}\exp^{\frac{r_{t+1}}{b_{\theta_t}}}<-\mu_{\theta_t}+\delta_{\min}+\gamma_t-\delta_0.$$

Let $b := b_{\theta_t}/2\,\exp(-(\delta_{\min}+\gamma_t-\delta_0-\mu_{\theta_t})/b_{\theta_t})$ and $c := -\mu_{\theta_t}+\delta_{\min}+\gamma_t-\delta_0$. We note that $b>0$. Our final claim follows as

$$r_{t+1}-b\exp^{\frac{r_{t+1}}{b_{\theta_t}}}<c.$$

We prove the inequality in two steps. We first show that the left hand side of the inequality is monotonically increasing for all $r_{t+1}<\delta_{\min}$. We then establish our claim by showing that the inequality holds for $r_{t+1}=\delta_{\min}$. To establish monotonicity of the left hand side, we first derive

$$\frac{\partial}{\partial r_{t+1}}\left(r_{t+1}-b\exp^{\frac{r_{t+1}}{b_{\theta_t}}}\right)=1-\frac{b}{b_{\theta_t}}\exp^{\frac{r_{t+1}}{b_{\theta_t}}}>0.$$

Rearranging the inequality and taking the natural logarithm on both sides yields

$$r_{t+1}<b_{\theta_t}\ln 2+\gamma_t-\delta_0-\mu_{\theta_t}+\delta_{\min}.$$

The inequality of form $r_{t+1}<\delta_{\min}+c^2$ holds for all $r_{t+1}<\delta_{\min}$. We therefore established monotonicity. It remains to show that the inequality holds for $r_{t+1}=\delta_{\min}$ where the left hand side takes its largest value. We substitute $\delta_{\min}$ for r_{t+1} and obtain

$$\delta_{\min}-b\exp^{\frac{\delta_{\min}}{b_{\theta_t}}}<c.$$

After substitution of b and c, the inequality reads as

$$\gamma_t-\delta_0-\mu_{\theta_t}+\frac{b_{\theta_t}}{2}\exp^{-\frac{\gamma_t-\delta_0-\mu_{\theta_t}}{2}}>0.$$

Recall that $\gamma_t-\delta_0-\mu_{\theta_t}\ge 0$. Since, in addition, the exponential function is strictly greater than zero for all real numbers, this concludes our proof. $\square$

Proof of Corollary 4

For part (i) of Corollary 4 to hold, we need to show that the change in beliefs about past earnings is negative for all values of γ_t and μ_{θ_t}. Let $a := 2\,(1-\exp(\delta_{\min}/b_e))$ and let $b := b_{\theta_t}/(b_e+b_{\theta_t})\exp(-(\gamma_t-\delta_0-\mu_{\theta_t})/b_{\theta_t})$. We note that $b>0$ and since $\delta_{\min}<0$, it follows that $\exp(\delta_{\min}/b_e)<1$ and consequently $a>0$. We need to prove that

$$\frac{\mu_{\theta_t}a+\left(\gamma_t-\delta_0+\frac{b_{\theta_t}}{b_e+b_{\theta_t}}(2b_e+b_{\theta_t})\right)b}{a+b}-\mu_{\theta_t}<0.$$

The fact that $b>0$ and $a+b>0$ simplifies the inequality and we obtain

$$\mu_{\theta_t}<\gamma_t-\delta_0+\frac{b_{\theta_t}}{b_e+b_{\theta_t}}(2b_e+b_{\theta_t})$$

which is equivalent to

$$\gamma_t-\delta_0-\mu_{\theta_t}>-\frac{2b_e+b_{\theta_t}}{b_e+b_{\theta_t}}b_{\theta_t}.$$

By definition of the sub-domain, the left hand side is greater than zero while the right hand side is smaller than zero for all values of b_e and b_{θ_t}. This concludes our proof for the first part of Corollary 4. □

For part (ii) of the corollary, we derive the conditions for which the magnitude of the positive update increases in investors' belief about firm savings. We use $\mu_{\theta_t}-\gamma_t$ as a measure for investors' belief about firm savings. Instead of using a change of variables, we calculate the partial derivatives for both μ_{θ_t} and γ_t and derive the conditions for the former to be positive and the latter to be negative for all values of μ_{θ_t} and γ_t.

(a) We differentiate the update with respect to μ_{θ_t} and solve for the range for which the magnitude of the update increases. We solve

$$\frac{\partial}{\partial\mu_{\theta_t}}\left(\frac{\mu_{\theta_t}a+b(\mu_{\theta_t})\left(\gamma_t-\delta_0+\frac{2b_e+b_{\theta_t}}{b_e+b_{\theta_t}}b_{\theta_t}\right)}{a+b(\mu_{\theta_t})}-\mu_{\theta_t}\right)>0$$

where a and b are defined in the first part of this proof. Let $b^{'}(\mu_{\theta_t})$ denote the first partial derivative with respect to μ_{θ_t}. The derivative evaluates to

$$\frac{\left(a+b^{'}(\mu_{\theta_t})\left(\gamma_t-\delta_0+\frac{2b_e+b_{\theta_t}}{b_e+b_{\theta_t}}b_{\theta_t}\right)\right)(a+b(\mu_{\theta_t}))-\left(\mu_{\theta_t}a+b(\mu_{\theta_t})\left(\gamma_t-\delta_0+\frac{2b_e+b_{\theta_t}}{b_e+b_{\theta_t}}b_{\theta_t}\right)\right)b^{'}(\mu_{\theta_t})}{(a+b(\mu_{\theta_t}))^2}>1$$

which, after some analysis, gives the inequality

$$a\frac{\gamma_t-\delta_0-\mu_{\theta_t}}{b_{\theta_t}}+a\frac{b_e}{b_e+b_{\theta_t}}-b(\mu_{\theta_t})>0.$$

Let $x := (\gamma_t-\delta_0-\mu_{\theta_t})/b_{\theta_t}$ be our measure for investors' belief about firm savings. Substituting $b(\mu_{\theta_t})$ yields

$$ax + a\frac{b_e}{b_e + b_{\theta_t}} - \frac{b_{\theta_t}}{b_e + b_{\theta_t}}\exp^{-x} > 0,$$

or equivalently

$$x - c\exp^{-x} > -\frac{b_e}{b_e + b_{\theta_t}}$$

where $c := (1/a)b_{\theta_t}/(b_e + b_{\theta_t})$ and $c > 0$. The left-hand side is monotonically increasing for all values of x which follows from

$$\frac{\partial}{\partial x}\left(x - c\exp^{-x}\right) = 1 + c\exp^{-x} > 0.$$

For an increase of the magnitude to hold for all $(\mu_{\theta_t}, \gamma_t)$, we thus require

$$x - c\exp^{-x}\Big|_{x=0} > -\frac{b_e}{b_e + b_{\theta_t}}$$

since the definition of the case we consider implies $x \geq 0$. The condition follows as

$$b_{\theta_t} < 2b_e\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right).$$

(b) We differentiate the update with respect to γ_t and solve for the range for which the magnitude of the update decreases. We solve

$$\frac{\partial}{\partial \gamma_t}\left(\frac{\mu_{\theta_t}a + b(\gamma_t)\left(\gamma_t - \delta_0 + \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}}b_{\theta_t}\right)}{a + b(\gamma_t)} - \mu_{\theta_t}\right) < 0$$

which evaluates to

$$\frac{\left(b'(\mu_{\theta_t})\left(\gamma_t - \delta_0 + \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}}b_{\theta_t}\right) + b(\gamma_t)\right)(a + b(\gamma_t)) - \left(\mu_{\theta_t}a + b(\gamma_t)\left(\gamma_t - \delta_0 + \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}}b_{\theta_t}\right)\right)b'(\gamma_t)}{(a + b(\gamma_t))^2} < 0.$$

Since $(a + b(\gamma_t))^2 > 0$, the numerator must be negative. In addition, $b'(\gamma_t) = -(1/b_{\theta_t})b(\gamma_t)$ which we can use to derive

$$-ab(\gamma_t)\frac{\gamma_t - \delta_0 - \mu_{\theta_t}}{b_{\theta_t}} + ab(\gamma_t)\left(1 - \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}}\right) + b(\gamma_t)^2 < 0.$$

Using the same abbreviations x and c, we get

$$x - c\exp^{-x} > 1 - \frac{2b_e + b_{\theta_t}}{b_e + b_{\theta_t}}$$

which is redundant, given the inequality in (a). We derived the condition required for our conjectured effect to occur for all values of $x > 0$, or equivalently for all beliefs about firm savings $\mu_{\theta_t} - \gamma_t$. The monotonicity of the function $x - c\exp^{-x}$ implies the existence of an additional case. Consider the case where the first condition is not satisfied. More specifically, suppose

$$b_{\theta_t} > 2b_e \left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right).$$

It follows that the magnitude of the update strictly increases in $\mu_{\theta_t} - \gamma_t$ for all $(\mu_{\theta_t}, \gamma_t)$ that satisfy $\mu_{\theta_t} - \gamma_t > \mu^*_{\theta_t} - \gamma^*_t$ where $(\mu^*_{\theta_t}, \gamma^*_t)$ satisfies

$$\frac{\gamma^*_t - \delta_0 - \mu^*_{\theta_t}}{b_{\theta_t}} - \frac{b_{\theta_t}}{b_e + b_{\theta_t}} \frac{1}{2\left(1 - \exp^{\frac{\delta_{\min}}{b_e}}\right)} \exp^{-\frac{\gamma^*_t - \delta_0 - \mu^*_{\theta_t}}{b_{\theta_t}}} = -\frac{b_e}{b_e + b_{\theta_t}}. \qquad \square$$

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