Systematic Foreign Exchange Hedger for Multi-Currency Portfolios using Genetic Algorithms

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Abstract

This master thesis investigates the impact of currency hedging on the portfolio return and volatility. We adopt the static hedging approach. The mathematics of currency hedging is laid out and an empirical case study is presented to give an initial flavor of the problem and the motivation for a more robust currency hedging solution.

The main task of the thesis is to construct a systematic portfolio currency hedging optimization algorithm for international portfolios, which is then solved by genetic algorithms (GAs). Under the traditional Markowitz mean-variance portfolio framework, we propose a new measure called relative volatility reduction factor and derive a revised objective function for our hedging optimization. Our aim is to search for the "best" combination of currency hedge ratios that maximizes volatility reduction effect with a controlled variability, subject to a minimum return threshold. Two variants of the benchmark are analyzed and three application cases are implemented to evaluate the effectiveness of GA hedging optimization. The hedging model is validated by time series cross-validation technique in each of the application case.

Moreover, we give a detailed analytical description of currency hedging using foreign exchange options. Different from previous research, the focus in this thesis is not about differentiating the two approaches, but more about connecting option strategies with forward contracts, as options are merely a special kind of forwards. One final application case is implemented using European vanilla options.

Keywords: currency risk, hedging, foreign exchange derivatives, systematic portfolio optimization

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Introduction

International diversification has become an essential practice for modern asset managers. A legacy of Harry Markowitz's Modern Portfolio Theory introduced more than 60 years ago (*Harry Markowitz, 1952*), portfolio diversification advocates a mixture of investments in geographically-diverse markets and in a wide spectrum of asset classes so as to smooth out unsystematic risks. Cross-border investments tend to have lower correlation with domestic assets and the foreign currency exposures might add extra diversification benefit to the portfolio. According to a *report on global pension fund released by Association of the Luxembourg Fund Industry (ALFI),* in 2008 oversea investment by the pension funds of the majority of OECD countries (excluding the US) amounted to about 25% out of total investments on average. The same figure jumped to approximately 31% in 2014.

On the other side of the coin, the rising trend also exposes investors to increasing currency risks, which can evolve into disasters during financial turmoil. The global financial crisis in 2007-2008 witnessed the huge volatility swings of foreign exchange rates that derailed numerous multi-currency portfolios. Let's have a look at two examples to grab a feeling of how currency risks can work against us.

Suppose a Japanese investor entered into a leveraged carry trade in Turkish Lira (TRY) before crisis, the period when carry trades were extremely lucrative. Starting from the end of 2007 to the beginning of 2009 (during the peak of the crisis), TRY plunged more than 45% while the funding currency Japanese Yen (JPY) appreciated more than 28%. The lucrative dynamics of carry trades was totally reversed, i.e. she had to covert a slumping currency (TRY) back to a rapidly appreciating funding currency (JPY) to cover the debt. Without any protection, the investor would have suffered a double loss amounting to 73% from currency risks alone. The magnitude of portfolio damage from currency fluctuations can be astonishing in high volatility market regime.

Let's now consider a US investor who made an investment in S&P/TSX Composite Index (Canadian equity market index) back at the beginning of 2002. The total (holding period) return for him from the beginning of 2002 to May 2008 summed up to 93% (roughly 11% annually). In the same period, US equity market (S&P 500) produced only 23% return (about 3.3% annually). The strong CAD appreciation cycle against USD contributed to significant currency profits to the portfolio, magnifying the investment return to 165% in USD term (16% annually). Exchange risks made a common return into an extraordinary one. If the investor decided to fully hedge CAD risk instead over the period because of the weakness of the currency before 2002, she would have forgone the additional 73% profit resulting from the favorable currency movement. In this case, with the advantage of hindsight, it is better not to hedge the currency exposure. The same story would have had a totally different ending for a Canadian investor investing in US equity market. The 23% return from S&P 500 would have turned into a negative 23% portfolio return because of the USD depreciation against CAD. In this case, it is definitely beneficial to hedge away currency risk.

The purpose of this thesis is to extend the scope of traditional currency hedging methodologies using forward contracts and construct a systematic currency hedging algorithm that is applicable to both forward and option hedging instruments. The structure is as follows: Section 1 presents a string of important academic literature related to the topic and their key findings. In Section 2, we lay out the mathematical description of portfolio currency hedging and its impact on the portfolio return and volatility. Section 3 presents an empirical case study to further deepen our understanding of hedging on portfolio performance and the impact of macroeconomic theme and base currency strength on the hedging decision. Section 4-6 are dedicated to building up a systematic currency hedger for international portfolios, where the optimization algorithm is formulated, parameterized and implemented via Genetic Algorithms in R. Two benchmark scenarios for three application cases are analyzed and the hedging model is validated using time series cross-validation technique. We will analyze the results and present relevant conclusions in Section 7.

1 Literature Review and Main Assumptions

1.1 Literature Review

Two questions are being actively debated: Should we hedge portfolio currency risk or not? What is the best approach to hedging if the answer to the previous question is yes? The traditional view of hedging defines the process in which an investor with a predetermined investment portfolio makes use of financial instruments as an effort to avoid exchange rate uncertainties (Ronald W. Anderson and Jean-Pierre Danthine, 1979). It is a transformation of cash flows or market value that the investor regards as reducing the risk of a position (Peter A. Abken and Milind M. Shrikhande, 1997). Under this context, currency hedging is a defensive way to protect against potential risks and portfolio performance deterioration. A second definition expands the role of hedging to be both risk minimization and speculative motives (Glen and Jorion, 1993). When the hedge ratio is positive, it is for defensive purpose and when it is negative, it consists of a speculative component and a hedging component.

On the theoretical realm of the debate, two main strands of opinions split. Perold and Schulman (1988) argued that currency hedging is a “free lunch”, as currency has zero expected return in the long-term and its correlations with other asset classes are nearly zero. Therefore, full hedging is recommended to reduce portfolio risk without sacrificing any reward. Similarly, Eun and Resnick (1988) found that exchange rate risk is to a large extent non-diversifiable and it is necessary to hedge away the adverse impact from currency exposures. They confirmed that currency hedging can indeed bring down portfolio risk without negatively impacting return. Forward hedging strategy was proposed and studied.

On the contrary, Froot (1993) took a more cautious stance with respect to currency hedging. He pointed out that currency hedging can only be effective in the short-term horizons. At long-term horizons, hedging is less useful in reducing portfolio variance because of mean-reversion of real exchange rates to purchasing power parity.

As the flexible exchange rate regimes mature and the rapid development and sophistication of derivatives markets for foreign exchange, there is an increasing consensus that currency hedging is helpful, especially for the bond portfolios, where currency risks dominate the total portfolio volatility and account for 95% of the unhedged portfolio risk (Jochen M. Schmittmann, 2010). Different approaches towards hedging are studied and compared. Guo and Ryan (2016), who investigated the effectiveness of three types of hedge ratios: uniform hedge ratio across all currencies, asset-specific hedge ratio and currency-specific hedge and found out that currency-by-currency hedging is the optimal choice among all. Unlike the traditional mean-variance approach, they adopted Conditional Value-at-Risk as the primary risk measure to more accurately account for tail risks for non-normally distributed financial returns.

1.2 Main Assumptions

In this thesis, we take the traditional stance of currency hedging and analyze the hedging optimization problem under mean-variance approach. Though we are aware of the fact that the left-tail of the return distribution of financial time series is usually underestimated in the mean-variance framework, we think it is still a good starting point to build and implement a currency hedging optimization program and help us understand the dynamics behind more clearly.

The purpose of this thesis is to construct a portfolio currency hedging algorithm and build up a systematic currency hedger to be at asset manager's disposal potentially. The main-stream analytical framework is adopted and two kinds of hedging strategies are involved: using foreign exchange forwards to hedge and using foreign exchange options to hedge. We extend the scope of previous research by studying and implementing currency option hedging.

Before going into the main sections of the thesis, we need to list a few assumptions that underpin the entire flow of the research.

1. We adopt the traditional Markowitz mean-variance portfolio framework in this thesis.
2. The hedging purpose is only defensive, i.e. we don't take on additional currency positions in order to speculate on the direction of movements of the currency.
3. We assume there are no market frictions such as short-selling restrictions or transaction costs involved in the currency forward market. We don't take any margin requirements or any potential counterparty default risk into consideration.
4. We endorse that currency hedging plays a more important role to the portfolio performance in a short-run horizon.
5. We consider an investor with an exogenous allocation of portfolio assets (equities or/and bonds).
6. We assume that the investor's domestic riskless interest rate is zero ($r_f = 0$).

2 Hedging Foreign Exchange Risks in Multi-Currency Portfolios

2.1 Hedging Methodology and Notation

Currency hedging has impact on both return and volatility profiles of internationally-diversified portfolios. When an investor invests in a foreign asset, the position essentially consists of two components. On the one hand, there is a position in the asset itself, which is exposed to the uncertainty of its local return (the return that is measured in its denominated currency). On the other hand, there exists a cash position in the foreign currency exposed to spot exchange risk. As the performance of the portfolio is eventually measured in the base currency of the investor, currency fluctuations can make a huge difference to the result when the investor wants to exit the investment and convert the foreign investment proceeds back to home currency, especially during a financial turmoil when foreign exchange rates can be notoriously volatile.

However, the effectiveness of currency hedging is highly dependent on the base currency of the portfolio. For instance, the case of currency hedging for a Japanese investor is totally different from that of an Australia-based investor. Without taking the behaviors of the base currency into consideration might bring big troubles. In the following, we will categorize two types of currencies sitting at the extremes of the risk spectrum. The first category of currencies is safe-haven currencies, represented by Japanese Yen (JPY) and Swiss Franc (CHF). Safe-haven currencies have a few common characteristics that make them "safe" and resilient, tending to remain strong or strengthen against major peer currencies in periods of market distress or during a financial crisis. Those developed countries usually have a very stable government and a strong financial system. They enjoy high export demands and international trade surplus that in consequence lead to a constant upward pressure for the currency. Safe-haven currencies typically exhibit negative correlations with equities and positive correlations with bond returns and foreign exchange market volatility. Therefore, they are especially sought after in risk-off market regimes (a market condition when investors reduce their risk appetite in response to uncertainties or investment environment deterioration).

The second category of currencies at the other extreme is commodity currencies, represented by Australian Dollar (AUD) and New Zealand Dollar (NZD) as commodity currencies, and Canadian Dollar (CAD) and Russian Ruble (RUB) as petrocurrencies. They derive their names from the fact that Australian and New Zealand's economies have a big proportion dedicated to commodity exporting whereas Canadian and Russian economies are highly oil-dependent. Given such intrinsic macroeconomic nature, commodity currencies are highly volatile during risk-off modes or crisis, displaying positive correlations to global equity markets, i.e. when the equity market crashes, these currencies are prone to collapse against major peers.

Apart from the above mentioned two extreme categories of currencies in the developed-market (DM) currency space, there are also currencies lying in between, exhibiting partially risk-off and partially risk-on characteristics depending on the periods. For example, Euro used to be a currency that is more safe-haven-like before the burst of financial crisis and the subsequent Eurozone sovereign debt crisis that further exacerbated its vulnerability.

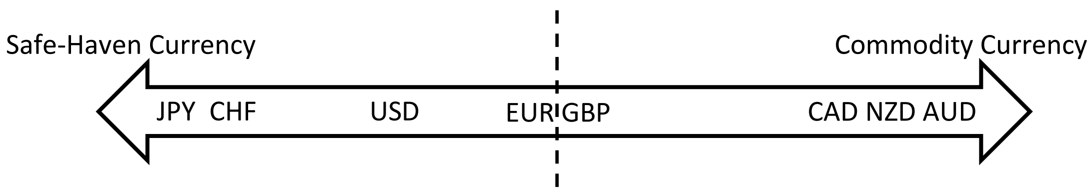


Fig 2.1 Major Developed-Market Currencies on Risk Spectrum by FX Market Convention

Source: FX market practitioner

Note: This risk spectrum of major developed-market (DM) currencies demonstrates the general perception within the foreign exchange market, under normal market conditions. Under the influence of unconventional monetary policies, e.g. the large-scale Quantitative Easing programs in US and Eurozone, there can be potential adjustments to the USD and EUR positions on the spectrum, which will not be reflected in this diagram.

Therefore, in the case of a Japanese investor, it makes sense to hedge currency risks to avoid further portfolio performance deterioration caused by currency losses. For an Australian investor, on the other hand, there may not be big motivation or even no need to do hedging because the property of the base currency itself acts as a natural hedge in turmoil times. The hedging case for a EUR-based or GBP-based investor is more complicated and needs to be more closely analyzed.

2.2 Hedging with Foreign Exchange Forwards

Currency forward contracts are one of the most widely used instruments to hedge against currency risks of multi-currency portfolios. The contract obligates both parties to transact a certain amount of underlying currency at a predetermined forward exchange rate at a specific future time point. Forward exchange rate refers to the exchange rate for the transaction happening at the future point of time, as opposed to spot exchange rate. Under the Assumption 3, it is free to enter such a financial contract with the counterparty, however, in reality it may involve transaction costs and embedded risk premium.

2.2.1 Covered Interest Rate Parity

Covered Interest Parity, which is based on a no-arbitrage condition, demonstrates the relationship between forward and spot exchange rates as well as the relationship between foreign and domestic interest rates. The equation is as follows:

$$\frac{F_{t-1}}{S_{t-1}} = \frac{(1+i_d)^T}{(1+i_f)^T} \qquad (1)$$

where the forward contract has a maturity of T expressed as the proportion to a year; F_{t-1} and S_{t-1} represent forward and spot exchange rates at the initiation of the contract, in units of base currency per foreign currency, and i_d and i_f represent domestic and foreign short-term interest rates respectively. We use a simple example below to illustrate this equilibrium. Suppose a US-based investor borrows 1 USD at a short-term interest rate of i_{USD}. She is supposed to pay back 1+i_{USD} at the maturity of the loan. She converts this 1 USD immediately to Turkish Lira (TRY) at the spot exchange rate S_0 and gets $1/S_0$ TRY to invest in the Turkish government bond market. At maturity, she will receive a guaranteed amount of (1+i_{TRY})/ S_0 TRY from her investment. In order to transfer the proceeds back to home currency USD, she will then have to sell (1+i_{TRY})/ S_0 TRY at the unknown market spot rate S_1 in exchange for USD, which exposes her investment to exchange rate fluctuations. In this case, the investor is given a possibility to replace the uncertain future spot exchange rate with a known forward exchange rate F_0 to eliminate the exchange rate risk. By selling forward TRY to buy back USD through a forward contract at the outset, the investor locks in the rate of conversion between two currencies and avoid any foreign exchange uncertainties. In the end, she will receive $\frac{F_0(1+i_{TRY})}{S_0}$ USD back.

Borrowing in base currency at home and lending the proceeds abroad in a foreign currency must lead to equal USD outcomes on both sides of the transaction. Otherwise, there exists an arbitrage opportunity where investor can make profits out of no risk. In consequence, the equation must hold for $1 + i_{USD} = \frac{F_0(1+i_{TRY})}{S_0}$. Therefore, the forward exchange rate is determined by the spot exchange rate and the domestic and foreign interest rates with the same maturity as the forward contract.

2.2.2 Foreign Exchange Forward Valuation

During each hedging period, we buy a foreign exchange forward with maturity T equal to the hedging horizon and mark-to-market it on a regular basis. The profit and loss of the contract is the hedging return on that specific valuation date, which can significantly impact the volatility pattern of the hedged portfolio. In order to value the forward at time t $(0 < \mathrm{t} < \mathrm{T})$, we need to derive the expected future cash flow payoff at maturity T if the position were to be balanced off, and discount this payoff back to today t. We will illustrate the valuation process via a detailed example below.

Suppose a US investor wants to lock in the exchange rate to sell 1 million Turkish Lira (TRY) in exchange for home currency USD at time T, i.e. she is short a one-million TRYUSD forward with forward price F_T and maturity T. The TRY and USD short-term riskless interest rates are i_{TRY} and i_{USD} respectively. At expiry T, her TRY account will be debited by 1 million in order to exchange for F_T million USD back into her USD account.

Table 2.1: The expected future cash flows at expiry T of the TRYUSD forward at initiation.

The cash flows of currency transaction at contract expiry T is depicted as follows:

Forward Position	Cash Flow at Expiry T	
	USD Account	TRY Account
Short TRYUSD forward at initiation (t=0)	+F_T mio	-1 mio

Now at time t, TRYUSD spot becomes S_t and we assume here no change in interest rates. For the purpose of completely cancelling out the initial forward position, the investor has to settle a new forward contract with maturity T-t, notional equal to the exact amount of the original contract with the prevailing forward exchange rate at time t, denoted as F_t.

$$F_t = \frac{(1 + i_{USD})^{T-t}}{(1 + i_{TRY})^{T-t}} \times S_t \tag{2}$$

With an exact offsetting position, the expected future cash flows at expiry T are altered. The updated cash flow table is displayed below:

Table 2.2: The altered expected future cash flows from two offsetting forward positions, at the same maturity T.

Forward Position	Cash Flow at Expiry t=T	
	USD Account	TRY Account

Short TRYUSD forward at initiation(t=0)	$+F_T$ mio	-1 mio
Offsetting(long) forward at valuation date(t=t)	$-F_t$ mio	+1 mio
Total position at expiry (t=T)	$(F_T - F_t)$	0

At expiry T, she will have a net zero position in foreign currency TRY and a net non-zero position in USD. If we discount the net difference in USD back to time t using the US risk-free interest rate, we will get the value of the forward at time t equal to $\frac{F_t - F_T}{(1+i_{USD})^{T-t}} = \frac{\frac{(1+i_{USD})^{T-t}}{(1+i_{TRY})^{T-t}} \times S_t - F_T}{(1+i_{USD})^{T-t}} = \frac{S_t}{(1+i_{TRY})^{T-t}} - \frac{F_T}{(1+i_{USD})^{T-t}}$. The generalized forward value during its life V_t $(0 < t < T)$ is formulated as

$$V_t = \frac{S_t}{(1+i_f)^{T-t}} - \frac{F_T}{(1+i_d)^{T-t}} \quad (3)$$

where i_f and i_d are foreign riskless interest rate and domestic riskless interest rate respectively.

2.2.3 Unhedged and Hedged Single Asset Return and Volatility

We use subscription 0 and T to indicate initial condition and the maturity condition. P_0^{base} and P_T^{base} represent the portfolio values in base currency at the beginning of the period and at the end of the period respectively, while $P_0^{foreign}$ and $P_T^{foreign}$ represent the portfolio values in foreign currency at the beginning of the period and at the end of the period respectively. We denote the local return of the asset as R^{local} and the foreign exchange spot return over the period as R^{FX}. The net base currency return of the fully forward-hedged foreign asset during the period R_{Fwd}^{base} can thus be expressed as:

$$R_{Fwd}^{base} = \frac{P_T^{base}}{P_0^{base}} - 1 = \frac{\frac{P_0^{base}}{S_0}(1+R^{local})S_T + \frac{P_0^{base}}{S_0}(F_0 - S_T)}{P_0^{base}} - 1$$

$$= \frac{S_T}{S_0}(1+R^{local}) + \frac{F_0 - S_0 - S_T + S_0}{S_0} - 1$$

$$= (1+R^{FX})(1+R^{local}) + \frac{F_0 - S_0}{S_0} - \frac{S_T - S_0}{S_0} - 1$$

$$= (1+R^{FX})(1+R^{local}) + FP - R^{FX} - 1$$

$$\approx R^{local} + FP \quad (4)$$

Here we define the forward premium/discount FP for a foreign currency as $= \frac{F_0 - S_0}{S_0}$. When a foreign currency has a higher interest rate than that of the base currency, it is said to have forward discount. If we sell this kind of foreign currency forward, we are essentially paying the interest rate differential to perform the hedge, and the expected return of the investment is reduced by such interest rate differential. In the case of forward premium, i.e. when the interest rate of the foreign currency is lower than that of the home currency, we gain the interest rate differential out of hedging. Therefore, carry cost can have a huge impact on the hedged portfolio performance, which will be revisited in the empirical hedging case in Section 3.

The variance of the fully-hedged foreign asset return in base currency can be expressed as the sum of the variance of the foreign asset local return, the variance of the forward premium/discount and the covariance between the two.

$$\sigma_{Fwd}^2 = \sigma_{R^{local}}^2 + \sigma_{FP}^2 + 2\text{cov}\,(R^{local}, FP\,) \tag{5}$$

2.2.4 Unhedged and Hedged Portfolio Return and Volatility

Suppose we have an internationally diversified portfolio consisting of N assets and M foreign currency exposures (M is not necessarily equal to N). According to Modern Portfolio Theory (Harry Markowitz, 1952), the unhedged portfolio return in base currency $R_{Unhedged}^{base}$ can be derived by summing the weighted base currency returns of N portfolio assets.

$$R_{Unhedged}^{base} = \sum_{i=0,j=1}^{M,N} \omega_{i,j} R_{i,j}^{base} \tag{6}$$

where $R_{i,j}^{base}$ represents the base currency return of asset j denominated in currency i ($i = 0, 1, \dots, M$) with $i = 0$ denoting base currency itself. Therefore, $\sum_{i=0,j=1}^{M,N} \omega_{i,j} = 1$.

When there exist foreign investments in the portfolio, the unhedged portfolio return can also be decomposed into two components - local return of the foreign asset (return that is denominated in the respective foreign currency) and the foreign exchange spot return.

$$R_{Unhedged}^{base} = \sum_{i=0,j=1}^{M,N} \omega_{i,j} R_{i,j}^{local} + \sum_{i=1}^{M} \omega_i R_i^{FX} \tag{7}$$

where $R_{i,j}^{local}$ and R_i^{FX} denote local return of the asset j in currency $i = 0, 1, \dots, M$ and the spot return for currency $i = 1, \dots, M$ respectively.

For a hedged portfolio, the total return in base currency is formulated as the sum of the unhedged portfolio return and the hedging return.

$$R_{Hedged}^{base} = R_{Unhedged}^{base} + R_{Hedging}^{base} \tag{8}$$

The foreign exchange hedging overlay return over the single hedging period is calculated by

$$R_{Hedging}^{base} = \sum_{i=1}^{M} h_i \omega_i R_i^{Hedging} \tag{9}$$

where $R_i^{Hedging}$ stands for the payoff on a unit of notional short position in the forward contract on currency i, which is determined by both the forward premium/discount of the currency and the spot return. Here h_i is defined as the hedge ratio, indicating how much of exposure to currency i is hedged.

$$R_i^{Hedging} = (-1) \times (S_T - F_0) = \frac{F_0 - S_0}{S_0} - \frac{S_T - S_0}{S_0} = FP_i - R_i^{FX} \tag{10}$$

The one-period forward-hedged portfolio return in base currency can be derived as follows:

$$\begin{aligned} R_{Hedged}^{base} = R_{Unhedged}^{base} + R_{Hedging}^{base} &= \sum_{i=0,j=1}^{M,N} \omega_{i,j} R_{i,j}^{local} + \sum_{i=1}^{M} \omega_i R_i^{FX} + \sum_{i=1}^{M} h_i \omega_i R_i^{Hedging} \\ &= \sum_{i=0,j=1}^{M,N} \omega_{i,j} R_{i,j}^{local} + \sum_{i=1}^{M} \omega_i R_i^{FX} + \sum_{i=1}^{M} h_i \omega_i (FP_i - R_i^{FX}) \end{aligned}$$

$$\xrightarrow[\quad]{100\%\ Hedging\ (h_i = 1)} \sum_{i=0,j=1}^{M,N} \omega_{i,j} R_{i,j}^{local} + \sum_{i=1}^{M} \omega_i FP_i \tag{11}$$

As Equation (11) shows, the forward-hedged portfolio return in base currency can be approximated by the sum of the weighted average of asset local returns and the weighted average of forward premium/discount of foreign currencies. By hedging, we transform the uncertain spot exchange rate fluctuations into the certain currency forward premium/discount.

We can continue to derive the variance of the unhedged portfolio return in base currency:

$$\begin{aligned} \sigma_{Unhedged}^2 = &\sum_{p=1}^{N} \sum_{q=1}^{N} \omega_p\, \omega_q \rho_{pq}^{L} \sigma_p^{L} \sigma_q^{L} \\ &+ \sum_{p=1}^{M} \sum_{q=1}^{M} \omega_p\, \omega_q \rho_{pq}^{FX} \sigma_p^{FX} \sigma_q^{FX} \\ &+ 2 \sum_{p=1}^{N} \sum_{q=1}^{M} \omega_p\, \omega_q \rho_{pq}^{L,FX} \sigma_p^{L} \sigma_q^{FX} \end{aligned} \tag{12}$$

where the first term represents the weighted covariances of local returns of N foreign assets that make up the total portfolio with superscript L for local return. The second term is the weighted covaraınces of spot exchange returns of different foreign currencies with FX representing exchange rate spot return. The third term stands for the weighted cross-covariances between local returns and spot returns.

For a fully-hedged portfolio, the variance formula can be extended in the same manner.

$$\sigma_{Hedged}^{2} = \sum_{p=1}^{N}\sum_{q=1}^{N}\omega_p\,\omega_q\rho_{pq}^{L}\sigma_p^{L}\sigma_q^{L} + \sum_{p=1}^{M}\sum_{q=1}^{M}\omega_p\,\omega_q\rho_{pq}^{FP}\sigma_p^{FP}\sigma_q^{FP} + 2\sum_{p=1}^{N}\sum_{q=1}^{M}\omega_p\,\omega_q\rho_{pq}^{L,FP}\sigma_p^{L}\sigma_q^{FP} \tag{13}$$

Compared to the unhedged portfolio variance in Equation (12), the forward premium standard deviation σ^{FP} replaces the exchange rate spot return standard deviation $\sigma^{FX}$and the correlation between forward premium and local return replaces the correlation between FX spot return and local return. This becomes the crucial justification for the effectiveness of forward hedging in bringing down the portfolio risk because by nature the forward premium tends to be small and stable exhibits much smaller volatility compared to foreign exchange rates, and its correlation with equity markets is substantially subdued.

3 An Empirical Currency Hedging Case Study

Given the mathematical description of currency hedging using forwards that is detailed in Section 2, we hereby demonstrate the impact of hedging on an international equity portfolio through a simple empirical hedging case study. First, we will introduce the data used. Then, we will move on to the analysis by computing the performance metrics of the portfolio with and without currency hedging and visualize the efficient frontiers of both portfolios.

3.1 Data

In this case study, we take the perspective of a U.S.-based investor (USD as portfolio base currency). We consider equally-weighted equity investments in two foreign countries – Japan and Eurozone (using German equity market as a proxy. The hedging horizon is 1 year and the hedging performance is evaluated on a monthly basis. The dataset stretches across 2005-2016 and is obtained from Bloomberg.

The equity returns are computed from the equity index of the respective market. We choose the following two flagship equity indices: Nikkei 225 (Nikkei) for Japan, a market index for Tokyo Stock Exchange and DAX as a proxy for Eurozone stock market, a German blue-chip stock index comprising 30 major German companies trading at Frankfurt Stock Exchange. The returns and standard deviations are both annualized.

We hold equity index investment positions in Japanese and Eurozone (German) markets from 2005 to 2016. But for hedging, we use 1-year currency forward contracts to hedge the foreign exchange exposures in the portfolio and readjust the hedging instruments annually. The holding period is longer than the tenor of the hedging instrument. In reality, investors rebalance the hedges only periodically in order to reduce potential hedging costs (quarterly and annually are recommended). In addition, as described in Assumption 3, we will not take any transaction costs or risk premium priced in forward contracts into consideration.

Under the context of full hedging (also called unitary hedging), we sell 100% of the initial foreign currency exposure forward in exchange for base currency USD at the beginning-of-period forward rates (ex-ante hedging). This is used to hedge a corresponding long position in the equity index of a foreign country. The underlying currency pairs are quoted in units of base currency per unit of foreign currency, i.e. to execute the hedging for the three long equity index positions, we need to sell a JPYUSD, a EURUSD and a CHFUSD forward.

3.2 Hedging Analysis

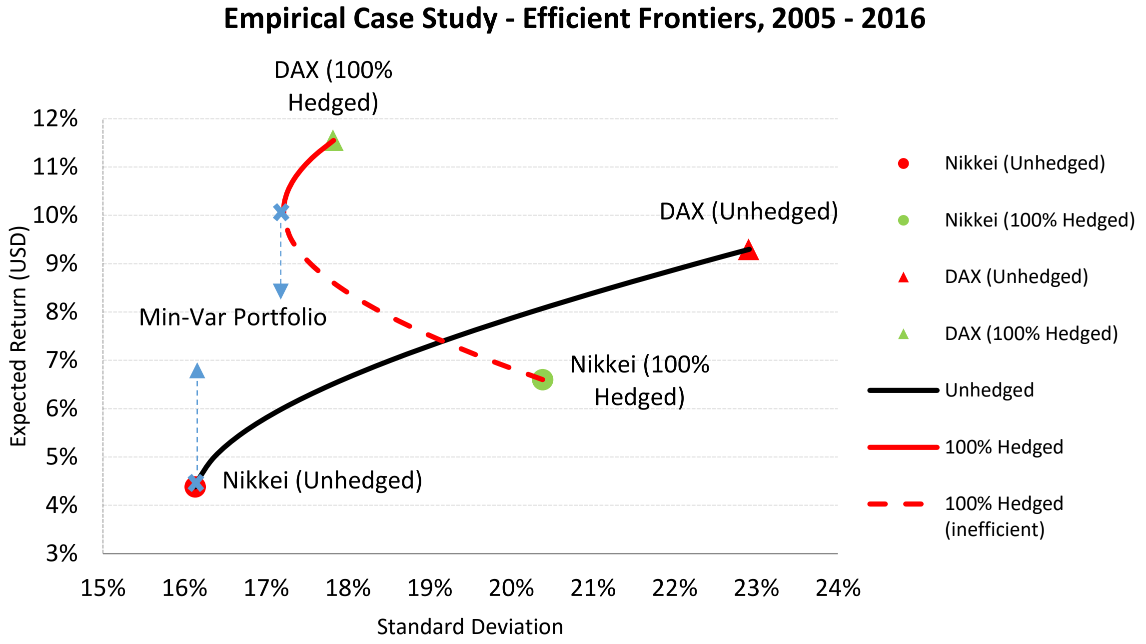


Fig. 3.1 Efficient Frontiers of Unhedged Portfolio and 100%-Hedged Portfolio

Notes: The expected returns and standard deviations of optimal portfolios are annualized results.

Figure 3.1 plots the set of optimal portfolios for unhedged and fully-hedged international equity-index portfolio, with the solid part being the efficient frontier of the respective portfolio and the dashed part being inefficient. Given different optimal weights to the individual (country-specific) equity-index portfolio, the unhedged efficient frontier is computed through unhedged USD returns during the sample period, whereas the hedged frontier is derived from the 100% hedged USD returns of equity indices. The left-most point of the efficient frontier represents the minimum-variance portfolio in the respective case (highlighted in blue cross in Figure 3.1). The curve stretches towards northeast, which intuitively explains that investors demand more return when bearing more risk. The frontier maps all possible efficient combinations of the two individual index portfolio, i.e. given a certain level of expected portfolio return, it shows the combination with minimal portfolio standard deviation. Only the part above the minimum-variance portfolio (the solid line) can be called efficient, as given a certain level of portfolio volatility, the portfolios on the solid part of the frontier always have higher returns than those on the dashed part of the frontier.

Figure 3.1 shows that currency hedging can help improve portfolio risk-adjusted performance to some extent. Though the minimum-variance portfolio of the unhedged portfolio is to the left of the minimum-variance portfolio of the fully-hedged portfolio, indicating that unhedged index portfolio can provide a risk-averse investor with a portfolio with least risk (but with least return as well), the efficient frontier of the fully-hedged portfolio is on top of the unhedged efficient frontier. At any given level of standard deviation within the fully-hedged portfolio volatility range, the hedged portfolio always has higher return than the unhedged portfolio. Moreover, the curvature of the hedged efficient frontier shows a higher risk-return trade-off effect compared to the flatter unhedged efficient frontier. With a small increment of risk, the hedged portfolio will yield a much larger return. We can also observe the locations of unhedged country-specific equity-index portfolios on the plot. Japanese (JP) portfolio and German (EU) portfolio sit at the two extremes of the unhedged and fully-hedged optimal portfolio plots. The circle labels the Japanese equity-index portfolio and the triangle labels the German equity-index portfolio. In both frontiers, JP portfolio serves as the starting point, with lower return, and German portfolio is the ending point, with higher return.

When we have a closer look at the optimal weights assigned to the individual equity-index portfolios, we find out that for the unhedged frontier, it starts with 100% Nikkei investment with a low risk - low return profile. The weight of DAX investment is gradually increasing and the weight of Nikkei is decreasing accordingly as the frontier stretches northeast, bringing up the portfolio return and volatility as DAX is a high return – high risk asset. For the fully-hedged frontier, it is a different picture. It starts with 100% investment in Nikkei as well, but the 100% hedged Nikkei has a much higher volatility than that of the unhedged. At the same time, the fully-hedged DAX investment has a lower standard deviation, even smaller than that of Nikkei, therefore the frontier exhibits more curvature.

Table 3.1 International Equity-Index Portfolios, Unhedged and Unitarily Hedged Performance Summary Statistics, January 2005 to December 2016

	Japan	**Eurozone (Germany)**	**Equal-Weight-Portfolio**
	Unhedged Equity-Index Portfolio USD Returns		
Mean Return (%)	4.4	9.3	6.8
Standard Deviation (%)	16.2	23.0	19.0
	100%-Hedged Equity-Index Portfolio USD Returns		
Mean Return (%)	6.6	11.5	9.3
Standard Deviation (%)	20.5	17.9	17.3
Change in σ_{hedged} relative to $\sigma_{unhedged}$(%)	26.4	-22.2	-9.0
	Forward Premium		
Mean Return (%)	1.8	0.5	-
Standard Deviation (%)	1.6	0.8	-
$\rho^{L,FP}$	-0.28	0.40	-
	FX Returns		
Mean Return (%)	-0.4	-1.8	-
Standard Deviation (%)	10.0	10.4	-
$\rho^{L,FX}$	-0.89	0.31	-

Table 3.1 displays the summary statistics of the two individual equity-index portfolios as well as the equally-weighted international portfolio consisting of these two assets. We calculate mean returns and standard deviations of unhedged and 100% hedged portfolios, correlations between equity-index local returns and currency spot returns/currency forward premiums, and percentage changes of hedged portfolio volatility relative to the unhedged during the sample period of 2005-2016.

The reason for large volatilities of unhedged USD returns of individual equity-index portfolios becomes obvious if we look at the high standard deviations of foreign exchange spot returns. By contrast, the volatilities of forward premium/discount are substantially lower, only about 12% of volatility level of exchange rate return on average. This helps to massively reduce currencies' contribution to portfolio risk in a hedged portfolio.

As described in Equation (12) and Equation (13), the correlations between spot return/forward premium and asset local return also impact the overall portfolio volatility. In our case study, we can see that Japanese Yen exhibits a substantial negative correlation with Japanese stock index Nikkei, which confirms their categorization of safe-haven currencies and the associate safe-haven properties. Euro, on the other hand, shows a positive correlation with local equity index DAX.

In the case of JP portfolio, as the currency's power of counter-movement in event of market downturns is so strong (its correlation to local stock market is close to perfectly negative correlated, i.e. -0.9), it is the best natural hedge a portfolio can have. If we instead hedge away JPY spot exposure, we will throw away this advantage and nonideally lift the portfolio risk. As per the results shown in Table 3.1, the standard deviation of hedged JP portfolio is 20.5%, increasing from the level of 16.2% in the unhedged case, which leads to an over 26% jump in portfolio risk. Currency hedging helps to reduce the equal-weight portfolio volatility by almost 10% and also increase the return level to a great extent. In the case of EU portfolio, there is no big difference between the correlations between EUR spot return/EUR forward premium and the DAX local return, but the variance of forward premium is dramatically smaller than that of Euro spot return. Therefore, hedging is effective and brings down over 20% of the portfolio risk while improving portfolio return at the same time.

The data in Table 3.1 also confirms the relationship between foreign exchange rate return, forward premium/discount and asset local return, as described in Equation (4). If fully hedged, the currency forward premium/discount will replace the return of spot exchange rate. Therefore, in both individual portfolio, the hedged portfolio return is equal to the unhedged portfolio return, plus the currency loss that is added back (in case of currency gain, it needs to be deducted), and plus the forward premium/discount.

On an equal-weight-portfolio level, fully hedging both currency exposures increases the portfolio return while lowering portfolio volatility by nearly 10%.

3.3 Three Sub-Periods Hedging Analysis

Fig. 3.2 Three Market Regimes within the Sample Period of 2005-2016. The figure shows the price history of Dollar Index (DXY), a measure of the value of USD relative to a basket of major trade partners' currencies, including Euro, Japanese Yen, British Pound, Canadian Dollar, Swedish Krona and Swiss Franc. The market theme in sub-period 1 is a USD depreciation cycle despite strength of economic growth during that period. The second crisis sub-period features high market volatilities and severe asset return plunges. Sub-period 3 depicts a steep USD strengthening cycle.

Motivated by the previous sections of this thesis, we know that portfolio base currency strength can have big impact on the currency hedging decision. According to Abken and Shrikhande (1997), since currency strength or weakness is closely related to the overall market regime, therefore macroeconomic factors will affect portfolio currency hedging decision as well. They found in their paper that during a typical USD depreciation cycle, leaving portfolio unhedged will yield a better result than performing currency hedge, while in a USD appreciation cycle, hedged portfolio outperforms the unhedged.

The sample period of our case study covers three different market regimes, i.e. economic boom in sub-period 2005-2007, an unprecedented global financial crisis and the subsequent European sovereign debt crisis in sub-period 2008-2012 and the gradual economic recovery (in an unconventionally low-interest rate environment due to large-scale Quantitative Easing programs from world's major central banks) in sub-period 2013-2016. Now, we treat the three sub-periods as independent datasets and perform full hedging to each of them to examine the macroeconomic environment's implications for the currency hedging decision.

Panel A: Sub-period 1 2005-2007

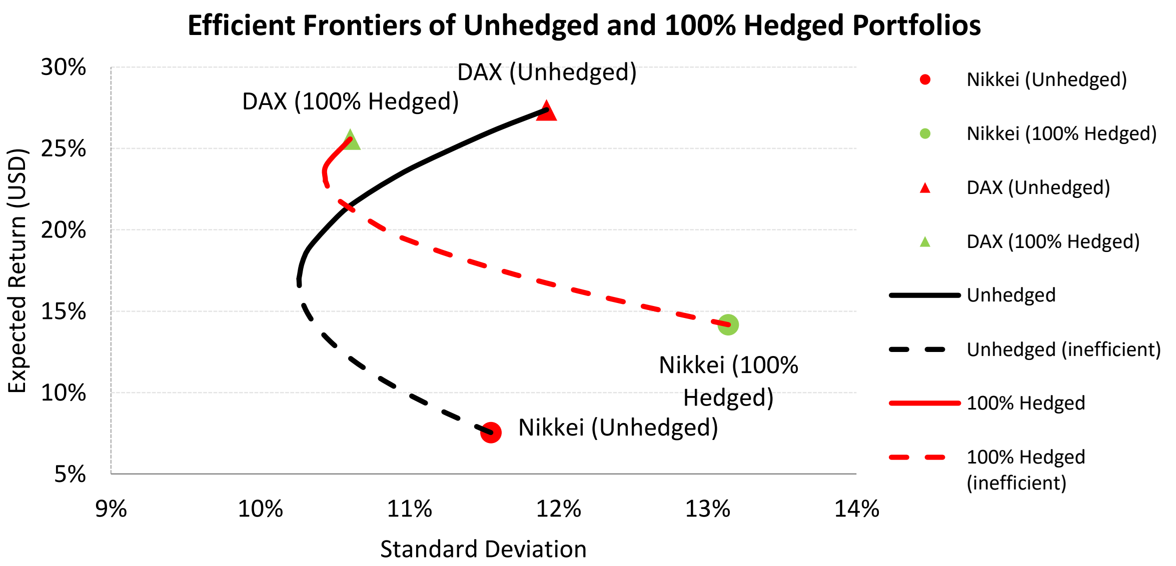


Figure 3.3 Efficient Frontiers of Unhedged Portfolio and Unitarily Hedged Portfolio in Three Subperiods. Panel A: Sub-period 1, 2005-2007. We assume U.S. domestic riskless interest rate is zero ($r_f = 0$) when generating efficient frontiers.

Table 3.2 International Equity-Index Portfolios, Unhedged and Unitarily Hedged Performance Summary Statistics, 3 Sub-Periods

Panel A. 2005-2007 Boom Regime

	Japan	Eurozone (Germany)	Equal-Weight-Portfolio
	Unhedged Equity-Index Portfolio USD Returns		
Mean Return (%)	7.5	27.4	17.7
Standard Deviation (%)	11.7	12.1	10.5
	100%-Hedged Equity-Index Portfolio USD Returns		
Mean Return (%)	14.2	25.6	20.2
Standard Deviation (%)	13.3	10.8	11.0
Change in σ_{hedged} relative to $\sigma_{unhedged}$(%)	13.8	-11.1	4.8
	Forward Premium		
Mean Return (%)	4.2	1.3	-
Standard Deviation (%)	0.9	0.6	-
$\rho^{L,FP}$	-0.91	-0.86	-
	FX Returns		
Mean Return (%)	-2.5	3.1	-
Standard Deviation (%)	6.9	7.0	-
$\rho^{L,FX}$	-0.999	-0.9997	-

Note: Returns and standard deviations are both annualized.

Figure 3.3 Panel A – Panel C show the efficient frontiers of unhedged and 100%-hedged diversified equity-index portfolios in three sub-periods respectively. The unhedged and 100% hedged USD returns of country-specific index portfolios are computed in the same manner as before.

In Panel A of Figure 3.3, the efficient frontiers for sub-period 2005-2007 are plotted as well as the individual country portfolios. One general expectation is that when base currency USD weakens, foreign investments are worth more when converted back to base currency. Therefore, keeping the spot currency exposures in this case acts as a natural hedge and currency hedging on the other hand is not ideal. From the plot, we can see that the result is mixed. Unhedged portfolio can supply the safest combination of the two equity-index assets (global minimum-variance portfolio). However, the two frontiers cross at such a point, below which the unhedged frontier can provide more risk-controlled portfolio combinations and the hedged frontier consists of portfolios with a little bit higher volatility but much higher returns. Beyond the crossing point, the hedged efficient frontier dominates that of the unhedged, indicating hedging is a better choice, as for any given level of portfolio return, the portfolios on the hedged efficient frontier always have lower risks.

Summary statistics in Panel A Table 3.2 gives us more insights. We can see that during this period, as economic growth is stable, the general market volatility is healthy. According to Abken and Shrikhande (1997), positive market conditions will make USD stronger, which is not the case in 2005-2007. The reason can be that currency strength is relative, even in a general upward cycle, if foreign currencies strengthen even more than USD, it will on the contrary depreciate. In our case, Japanese Yen depreciated against USD during this period because huge volume of Yen-related carry trades constantly pressured the currency downwards. This flow is also reflected in the extremely high forward premium of Japanese Yen. Euro appreciated against USD in the same period. Therefore, the relationship between base currency strength and the macroeconomic cycles is not entirely fixed. It still depends on the composition of foreign currencies in the portfolio.

Another observation is that hedging JPY currency exposure results in higher portfolio volatility, as the negative correlation between JPY spot exchange rate and the local equity market is eliminated. On contrary, hedging EUR exposure is ideal and it helps to bring down the risk by 11%.

Panel B: Subperiod 2 2008-2012

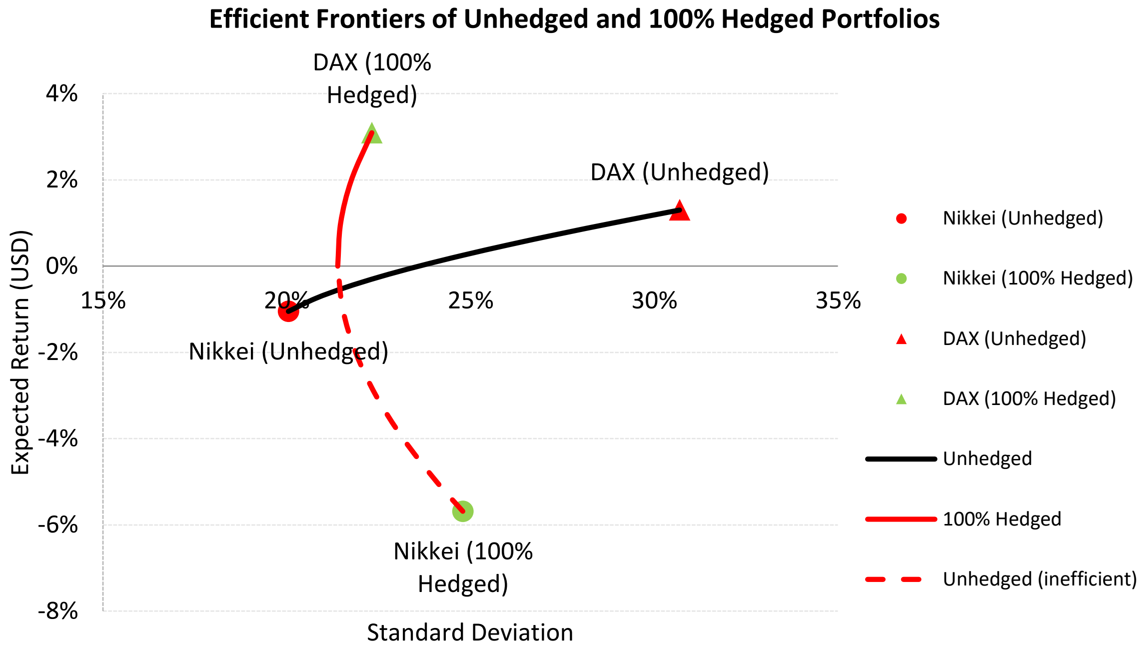


Figure 3.3 (cont.) Efficient Frontiers of Unhedged Portfolio and Unitarily Hedged Portfolio in Three Subperiods. Panel B: Sub-period 2, 2008-2012. We assume U.S. domestic riskless interest rate is zero ($r_f = 0$) when generating efficient frontiers.

Table 3.2 International Equity-Index Portfolios, Unhedged and Unitarily Hedged Performance Summary Statistics, 3 Sub-Periods (cont.)

Panel B. 2008-2012 Crisis Regime

	Japan	Eurozone (Germany)	Equal-Weight-Portfolio
	Unhedged Equity-Index Portfolio USD Returns		
Mean Return (%)	-1.1	1.3	-0.2
Standard Deviation (%)	20.2	30.9	23.9
	100%-Hedged Equity-Index Portfolio USD Returns		

Mean Return (%)	-5.7	3.1	-1.6
Standard Deviation (%)	25.0	22.5	21.7
Change in σ_{hedged} relative to $\sigma_{unhedged}$(%)	23.8	-27.2	-9.2
	Forward Premium		
Mean Return (%)	1.3	-0.1	-
Standard Deviation (%)	1.1	0.4	-
$\rho^{L,FP}$	-0.70	0.31	-
	FX Returns		
Mean Return (%)	5.9	-1.9	-
Standard Deviation (%)	10.9	13.5	-
$\rho^{L,FX}$	-0.89	0.54	-

Note: Returns and standard deviations are both annualized.

Figure 3.3 Panel B and Table 3.2 Panel B display the unhedged and hedged efficient frontiers and the summary statistics in sub-period 2, from 2008-2012. This period marks the worst financial crisis in history when the financial systems collapsed globally and asset prices experienced plunges and huge volatility swings. We can see from Table 3.2 Panel B that both volatilities of equities and foreign exchange rates soared up. Not long after that, the European Sovereign Debt Crisis burst out in Europe, which worsened the situation. To combat the crisis, U.S. Federal Reserve (Fed) and European Central Bank (ECB) released large-scale quantitative easing programs and other non-conventional monetary policies to stimulate the economies. In such unusual macro-environment, currency movements are hugely uncertain and pose significant challenges to the international portfolios.

Judging from the efficient frontier plot in Figure 3.3 Panel B, hedging is effective in controlling the portfolio volatility in sub-period 2, but the portfolio return would be dragged down even further. This is because, as the strongest safe-haven currency, Japanese Yen appreciated 6% during the crisis period. If we hedge it away, we will give away such big currency gain it could have brought to the portfolio if left unhedged. In addition, due to large-scale unwinding of Yen-related carry trades, JPY forward premium diminished, which leaves little benefit in hedging the currency.

The results from sub-period 2 generally confirms the risk mitigation function of currency hedging, which is an extremely valuable portfolio protection method during stress periods.

Panel C: Sub-period 3, 2013-2016

Efficient Frontiers of Unhedged and 100% Hedged Portfolios

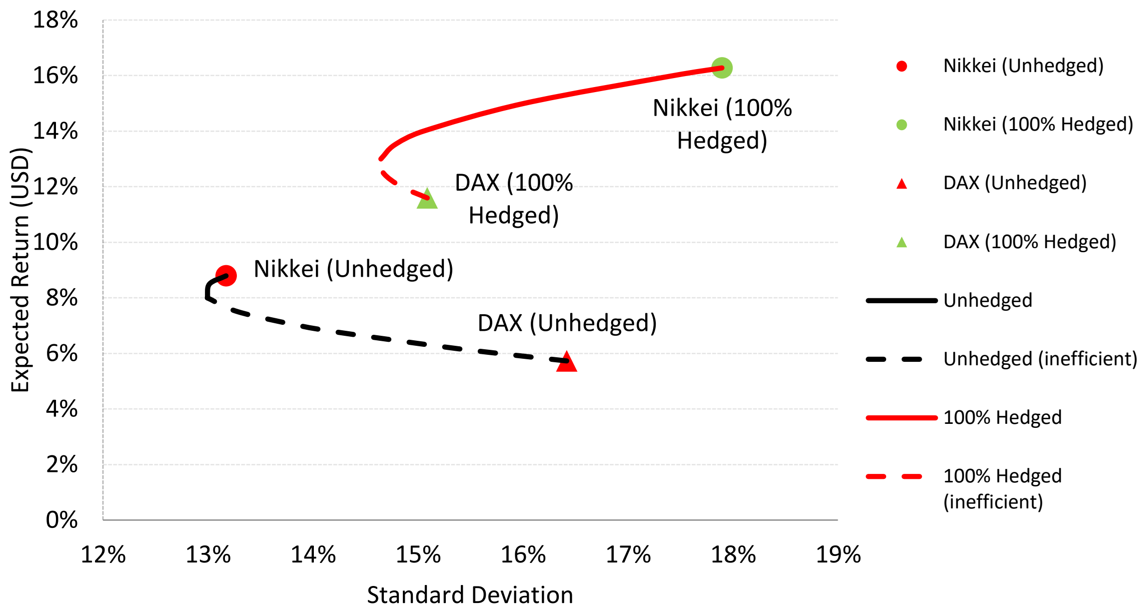


Figure 3.3 (cont.) Efficient Frontiers of Unhedged Portfolio and Unitarily Hedged Portfolio in Three Subperiods. Panel B: Sub-period 3, 2013-2016. We assume U.S. domestic riskless interest rate is zero ($r_f = 0$) when generating efficient frontiers.

Table 3.2 International Equity-Index Portfolios, Unhedged and Unitarily Hedged Performance Summary Statistics, 3 Sub-Periods (cont.)

Panel C. 2013-2016 Recovery Regime

	Japan	**Eurozone (Germany)**	**Equal-Weight-Portfolio**
	Unhedged Equity-Index Portfolio USD Returns		
Mean Return (%)	8.8	5.7	7.3
Standard Deviation (%)	13.3	16.6	13.6
	100%-Hedged Equity-Index Portfolio USD Returns		
Mean Return (%)	16.3	11.6	13.9
Standard Deviation (%)	18.1	15.2	15.2
Change in σ_{hedged} relative to $\sigma_{unhedged}$(%)	36.1	-8.4	11.8
	Forward Premium		
Mean Return (%)	0.7	0.6	-
Standard Deviation (%)	0.5	0.5	-
$\rho^{L,FP}$	-0.46	-0.05	-

	FX Returns		
Mean Return (%)	-6.8	-5.3	-
Standard Deviation (%)	9.6	7.4	-
$\rho^{L,FX}$	-0.78	0.87	-

Note: Returns and standard deviations are both annualized.

Figure 3.3 Panel C and Table 3.2 Panel C display the unhedged and hedged efficient frontiers and the summary statistics in sub-period 3, from 2013-2016. As the global economy steadied its path to recovery, this sub-period features a steep USD appreciation cycle. U.S. Federal Reserve halted the quantitative easing programs that were meant to weaken USD to stimulate economies and entered into a slow but steady upward cycle of U.S. interest-rate. The general underlying theme for USD is also disturbed by a few major risk events, e.g. Brexit referendum in 2016, U.S. election in 2016, among other geopolitical risk events. This explains the volatility spikes we see around 2015-2016 in Figure 3.2. The efficient frontiers in Panel C Figure 3.3 demonstrate some interesting results. The entire hedged frontier lies above the unhedged frontier. For most of the attainable portfolio volatility level, hedged frontier always has a much higher return that almost doubles, compared to the unhedged. For a risk-averse investor, the unhedged frontier can still provide a least risky combination of assets. Hedging currency risks in this sub-period expands the opportunity set, making it possible to reap extraordinary returns under a moderate portfolio risk.

The summary statistics in Panel C Table 3.2 also characterizes this sub-period. USD strengthened a lot against both JPY and EUR. Currency hedging helps to wipe out big currency losses, significantly improving the return profile of the hedged portfolio.

3.4 Empirical Hedging Case Study Summary

Through the empirical hedging case study analyzed above, we come to a mixed picture of whether of not to perform currency hedging. As the macroeconomic dynamics in the new era, especially after the catastrophic financial crisis in 2007-2008, need to be redefined, there is no clear-cut signal of currency hedging anymore. Therefore, we see a strong necessity to come up with a systematic currency hedging methodology and tool, which is capable of more robust hedging strategies that is independent of market themes, for modern asset managers' disposal.

4 Currency Hedging Optimization Algorithm

Now that we have a better intuition of how currency hedging works mathematically and empirically on the portfolio level, in this section we will formulate the hedging optimization problem and develop the currency hedging algorithm that solves the optimization problem. It will provide a generic model to deal with a majority of currency hedging issues for all types of portfolios. In addition, we will give a brief introduction to Genetic Algorithms (refered as "GA" henceafter) that we select as our optimization tool in this thesis.

4.1 Formulation of currency hedging optimization problem

Portfolio currency hedging optimization problems are different from the traditional portfolio optimization problems. The latter addresses the problem of how to assign weight to each portfolio asset so that a certain portfolio objective is achieved, e.g. portfolio variance is minimized or portfolio return is maximized, under such optimal allocation. The currency hedging optimization problem in our set-up focuses on getting an optimal combination of hedge ratios, so that the extra overlay of currency derivatives can contribute to certain portfolio objective optimally. In other words, it can be essentially viewed as a search problem in terms of how much percentage of the respective foreign currency exposure should be hedged away by what combination of hedging instruments that jointly lead to the best outcome.

As described in Assumption 5, for the purpose of this thesis, we assume exogenous portfolio asset allocation, i.e. we are given a predetermined portfolio that consists of N assets $(A_1, A_2, \ldots, A_N)$ with returns $(R_1, R_2, \ldots, R_N)$ and weights $(\omega_1, \omega_2, \ldots, \omega_N)$, exposed to M types of foreign currencies $(C_1, C_2, \ldots, C_M)$. The initial value of the portfolio is P_0^{base} measured in base currency.

Under the Markowitz mean-variance portfolio framework, to search for an optimal hedging strategy to be applied in addition to the existing portfolio, firstly we need to define what means "optimal" for us. As motivated in Introduction (Section 1) and in Empirical Currency Hedging Case Study (Section 3), we highly value the risk reduction effect of currency hedging that sustains portfolio stability and its capability of protection in financial turmoil. This is also in line with our defensive stance towards currency hedging. Therefore, we formulate our optimization objective in such a way that it is able to control and reduce portfolio volatility and the variability of the volatility reduction. We aim for our model to perform well overall no matter which macroeconomic cycle we are in and no matter what direction the foreign currencies are heading to in the future, which is the essence of a robust and stable solution to the portfolio currency hedging optimization. Given the fact that portfolio volatilities can be very different across different years, it makes more sense to talk about the relative volatility reduction by the hedging model compared to the no hedging benchmark, rather than the absolute amount of volatility reduction. Moreover, we care about the stability of hedging's effectiveness over different years, i.e. the variability of volatility reduction over the years in the sample period needs to be controlled and reduced to an acceptable level to enable a stable solution. Therefore, in this thesis, we follow the classical Markowitz mean-variance portfolio optimization framework, but define a new variable that is tailored to our currency hedging optimization problem - the relative portfolio volatility reduction ($\Delta\sigma_{\downarrow}$). We rewrite the terms in the mean-variance optimization objective to be the mean and variance of $\Delta\sigma_{\downarrow}$.

At the same time, we need to ensure that the reduced portfolio volatility is not at the cost of too much portfolio return. No hedging and full hedging are two extreme hedging methods, which we call in this thesis naïve hedging strategies. While the volatility benefit from hedging is measured against no hedging benchmark to determine the value of hedging, the return outcome is compared against the smaller return of the two naïve hedging strategies, as the return performance out of hedging optimization is acceptable as long as it is not the worst performer. Therefore, we constrain the hedged portfolio return to be at least as high as the lower return of unhedged and fully-hedged portfolios, which is shown in Constraint 1.

The decision variable h_i $(i = 1, \dots, M)$ refers to the hedge ratio of foreign currency exposure C_i and is searched in the domain of [0, 1] (see Constraint 2). We don't allow negative hedge ratios, i.e. adding extra exposure to the foreign currencies is not allowed, as our purpose of hedging is restricted to be defensive instead of speculative. Naturally, a positive hedge ratio means the investor needs to sell that proportion of the foreign currency exposure forward in order to protect herself against the foreign exchange risks involved in the foreign asset position.

Objective function:

$$\max.\ \mathrm{Z} = \mathrm{E}(\Delta\sigma_{\downarrow}) - \frac{1}{2}\mathrm{E}[(\Delta\sigma_{\downarrow} - \mathrm{E}(\Delta\sigma_{\downarrow}))^2)]$$

where

$$\Delta\sigma_{\downarrow} = \frac{\sigma_{\mathrm{uh}} - \sigma_{\mathrm{gah}}}{\sigma_{\mathrm{uh}}}$$

subject to

Constraint 1: Minimum portfolio return

$$\mathrm{R}_{\mathrm{gah}} \geq \min(\mathrm{R}_{\mathrm{uh}}, \mathrm{R}_{\mathrm{fh}})$$

Constraint 2: Defensive hedge ratio domain

$$0 \leq h_i \leq 1, \qquad \mathrm{i} \in [1,2, \dots, \mathrm{M}]$$

where the subscription of gah, uh and fh refer to GA-hedged, unhedged and fully-hedged respectively. GA-hedged portfolio is the optimization solution from the hedging model. We will keep the notation throughout the rest of the thesis. σ_{uh} and σ_{gah} stand for yearly unhedged and GA-hedged portfolio volatility respectively and the difference between the two is the yearly volatility reduction amount. If the difference is positive, it means GA hedging is bringing the portfolio standard deviation below that of unhedged benchmark, which is a sign of outperformance. Otherwise the portfolio risk is higher and GA hedging produces a negative effect. The first term in the objective function represents that we want to maximize the average of the relative volatility reduction amounts over the years in the given period. The second term is used to control and penalize bigger variability of such volatility reduction effects over the years.

4.2 A Brief Introduction to Genetic Algorithms

In this thesis, we select genetic algorithms as the technique to solve our currency hedging optimization problems. Genetic algorithms are adaptive stochastic search algorithms that derive their roots from biological evolution and natural selection. It exploits intelligent search mechanisms that are capable of dealing with various kinds of optimization problems. Their emerging usage in currency hedging field can be seen in the paper by Álvarez-Díez, Alfaro-Cidb and Fernández-Blanco (2016), in which they use GA to implement a cross-hedging strategy to mitigate currency risks. GAs work on a randomized basis, but they are not at all random. Instead they use past information to direct themselves to explore the better-performing area. The optimization process relies on biology-inspired operators including selection, crossover and mutation.

4.2.1 Genetic Algorithm Methodology

The way how Genetic Algorithms work involve randomization and iterations combined with various genetic operators that mimic the biological evolutionary processes. It requires a genetic representation of the solution space and a fitness function to value the fitness of the solutions in the space. The basic procedure is outlined below.

Phase 1 Initialization

A GA procedure starts with a randomly generated population of candidate solutions (called "individuals") to the optimization problem. Thus, users need little knowledge to start such a procedure. Each candidate solution contains a set of properties (called "chromosomes") which can adapt and evolve over the next generations. The randomness in this initial phase ensures the entirety of search space so that a whole universe of possibilities is incorporated.

Phase 2 Selection

In the next phase, GAs calculate the fitness value for each candidate solution according to the fitness function fed in. Individuals with higher fitness values perform better than those with lower fitness values because GAs are typically preset to maximize the fitness function. By principal they will be awarded the privilege of parenthood in the next generation, i.e. they are given a higher probability to be selected to be parents of the next generation.

Phase 3 Genetic Operators

A second generation of individuals (children) is produced out of the parent pool from the first generation. The production involves a few types of genetic operations. One of the most powerful genetic operators is crossover, where chromosomes from the parent solutions are taken and recombined to produce the child. In this process, random selection may take place several times. The child solution will thus share many characteristics from its parents. Crossover can increase genetic diversity from generation to generation and extend the search in many directions. Another powerful feature of crossover is that, by being attracted to the high fitness value signals in the generation and lean toward more sampling in that specific region, it is able to cut through the search space more efficiently.

Mutation is another important genetic operator, where one or more of the gene values in chromosomes are altered away from their initial states. It is similar to the biological mutation, which results in possibly a complete change of the solution instead of a gradual adaption. Intuitively, mutation takes place with a much lower probability just like that in the biological evolutionary process.

In addition, there are many other heuristics that are deployed to make the search process more efficient and robust.

Phase 4 Termination

Through hundreds of thousands of iteration rounds that repeat Phase 2-3, GA algorithms will stop when a certain termination condition is met. For instance, a user-specified maximum number of iterations is reached or a solution is found that meets the minimum criteria.

One end note of applying genetic algorithms to solve currency hedging problems is that we need to translate the original optimization objective function into a proper fitness function that GA can understand. Mostly we will have a constrained optimization problem and we may have a minimization problem that GA cannot directly solve. One way of transformation is through the introduction of penalty functions which can help convert constrained problems into unconstrained ones by adding an artificial penalty term if the constraint is violated.

4.3 Portfolio Hedging Optimization Algorithm

The generic portfolio currency hedging algorithm can be parameterized and implemented by the following computational procedure (in R):

Table 4.1 Hedging Optimization Algorithm

Inputs	
$\boldsymbol{N}$	number of assets in the portfolio (given)
$\boldsymbol{T}$	number of years in the given period (given)
$\boldsymbol{P}$	asset price time series, vector (given)
$\boldsymbol{R_{local} \leftarrow monthlyreturn(P^{n})}$	asset monthly local return series, vector
$\boldsymbol{P^{FX}}$	exchange rate time series, vector (given)
$\boldsymbol{R_{FX} \leftarrow monthlyreturn(P^{FX})}$	exchange rate monthly return series, vector
$\boldsymbol{V_0^{base}}$	initial portfolio value in base currency (given)
$\boldsymbol{\omega}$	asset weights, vector (given)
$\boldsymbol{F_0}$	transacted forward exchange prices in hedging instruments
$\boldsymbol{F_t}$	interim monthly forward prices for hedging valuation, (interpolated from Bloomberg)
Computations	
$\boldsymbol{V_0^{local} \leftarrow \omega V_0^{base}/P_0^{FX}}$	initial asset local values, vector (P_0^{FX} as initial exchange rate of the period)
$\boldsymbol{V_t^{local} \leftarrow V_0^{local}(1 + R_{local})}$	asset local value series, vector
$\boldsymbol{V_t^{base} \leftarrow V_t^{local} P^{FX}}$	asset base currency value series, vector
$\boldsymbol{V_{uh.Port}^{base} \leftarrow rowSums(V_t^{base})}$	unhedged portfolio values in base currency, vector
$\boldsymbol{R_{uh.Port}^{base} \leftarrow monthlyreturn(V_{uh.Port}^{base})}$	unhedged portfolio monthly base currency returns, vector
$\boldsymbol{R_{uh,holding}^{base} \leftarrow \frac{last(V_{uh.Port}^{base})}{V_0^{base}} - 1}$	unhedged portfolio holding period base currency return

$\boldsymbol{R^{base}_{uh,annualized} \leftarrow (1 + R^{base}_{uh,holding})^{\frac{1}{T}} - 1}$	unhedged portfolio annualized base currency return
$\boldsymbol{h.notional_t \leftarrow V^{local}_{t_0}}$ *($\boldsymbol{t_0}$ =beginning of each year)*	hedging notional at the beginning of each year, vector
$\boldsymbol{h.return_t \leftarrow -h.notional_t(F_t - F_0)}$	full hedging return in year t, list
$\boldsymbol{V^{base}_{h.Port}}$	fully-hedged portfolio values in base currency, vector
$\boldsymbol{R^{base}_{h.Port} \leftarrow monthlyreturn(V^{base}_{h.Port})}$	fully-hedged portfolio monthly base currency returns, vector
$\boldsymbol{R^{base}_{h,holding} \leftarrow \frac{last(V^{base}_{h.Port})}{V^{base}_0} - 1}$	fully-hedged portfolio holding period base currency return
$\boldsymbol{R^{base}_{h,annualized} \leftarrow (1 + R^{base}_{h,holding})^{\frac{1}{T}} - 1}$	fully-hedged portfolio annualized base currency return
Decision Variables	
$\boldsymbol{h_i}$	hedge ratio for foreign currency i
Objective Function	
$\mathbf{max.\ mean}(\frac{\boldsymbol{\sigma_{uh} - \sigma_{gah}}}{\boldsymbol{\sigma_{uh}}}) - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{var}(\frac{\boldsymbol{\sigma_{uh} - \sigma_{gah}}}{\boldsymbol{\sigma_{uh}}})$	
Constraint	
Minimum portfolio return	$R_{gah} \geq \min(R^{base}_{uh,annualized}, R^{base}_{h,annualized})$
Defensive hedge ratio domain	$0 \leq h_i \leq 1, \quad \mathrm{i} \in [1,2,\ldots,\mathrm{M}]$
Genetic Algorithm Output	
$\mathbf{h}^*$	optimal hedge ratio vector

Notes: The hedging optimization algorithm is to accommodate currency hedging using currency forward contracts. The preset hedging horizon is 1 year and the hedging valuation is performed on a monthly basis. The exchange rates are quoted as units of base currency per foreign currency. Subscription 0 refers to the very beginning of the entire sample period and t refers to each year within the sample period.

5 Currency Hedging Optimization Implementation and Cross-Validation

Given the power of genetic algorithms in tackling general optimization problems, in Section 5 we will implement three currency hedging cases with increasing complexity using foreign exchange forwards. The generic portfolio currency hedging algorithm presented in Section 4 will be used and tested.

In all cases, we take the perspective of a US-based investor (USD as portfolio base currency). We begin with a case consisting of two foreign assets A_1 and A_2 and two foreign currency exposures C_1 and C_2. The hedging toolbox has forward contracts of currency C_1 and C_2 against base currency USD, i.e. the hedging currencies are matching the portfolio currency exposures and there is no "redundancy" in the hedging toolbox. In order to examine the value of currency hedging (measured by the difference between the performance of the portfolio with and without hedging) from different perspectives, two benchmark scenarios are selected to be evaluated against our hedging model. The first benchmark is the unhedged base currency return and volatility of the portfolio, which provides direct insights regarding the hedging decision for the U.S. investor. The second benchmark is the unhedged portfolio local return and volatility computed as the weighted average of unhedged asset local returns and its corresponding volatility. The currency effects are stripped out already to demonstrate the baseline case of how well the portfolio could have performed if there were no currency effects. This benchmark is relevant when we take the stance that we only value the investment returns, without considering any foreign exchange influence no matter whether it is good or bad. The two benchmarks are different performance measurement methods and are both relevant.

After the model output is analyzed, we will use the time series cross-validation technique to evaluate the model accuracy in generating the right hedging strategy. Apart from the volatility benefits, we will also assess the Sharpe ratio performance which measures the risk and return trade-off of portfolios.

In Case 2, we expand the asset classes and increase the hedging dimension. It will be a five-currency portfolio with both equity and bond investments, four of which are foreign investments. The hedging toolbox in this case is still matching.

In Case 3, we reuse the portfolio configuration in Case 1 (2-asset and 2-foreign-currency portfolio) and increase the dimension of our hedging toolbox to contain more than 2 hedging currencies. We will test our hedging algorithm's capability in dealing with dimensionality reduction issues and compare the hedging result with that of Case 1.

5.1 Case 1: 2-asset, 2-foreign-currency-exposure

In Case 1, a U.S.-based investor takes investment positions in two overseas equity markets ($N = M = 2$) - Japanese equity market (benchmark index "Nikkei") and European equity market (German benchmark index "DAX" as proxy), and splits wealth equally between the two, i.e.$\omega = (0.5, 0.5)$. The initial investment is 1 million USD ($V_0^{base} = 1'000'000\ USD$). We implement our hedging optimization program within the sample period of 2005-2016 ($T = 12$).

Implementation Inputs

Nikkei, DAX, JPYUSD, EURUSD price time series data are obtained from Bloomberg. We rebalance the forward hedging instruments annually during the sample period of 2005-2016 and the forward hedging valuation is performed on a monthly basis, i.e. at the end of each month, we use the forward price of the same forward contract but with a time to maturity equal to the days left between the valuation date and the original settlement date of the contract, to value against the original forward price of the contract. These interim JPYUSD and EURUSD forward prices are interpolated from Bloomberg and fed into the hedging optimization algorithm.

Implementation Parameters

The decision variables h_1 and h_2 refer to hedge ratio of JPY exposure and EUR exposure respectively and are searched in the domain of [0,1]. A positive hedge ratio means this portion of foreign currency exposure is recommended to be sold forward to achieve better portfolio performance.

The benchmark here is the unhedged portfolio base currency (USD) return and volatility, against which the GA hedging strategy is measured. As mentioned in Section 4, the return threshold in the GA constraint is chosen to be the minimum of the returns from the two naïve hedging strategies - no hedging (in base currency term) and 100% hedging.

We run the GA optimization with a population size of 200 candidate solutions. The probability of crossover between the parent chromosomes is set to be 0.8 and the probability of mutation in a parent chromosome is 0.1. Elitism, a parameter controlling how many fittest individuals can survive in each generation, is determined to be 5%. We restrict the number of iterations to be 200 at maximum for better computational efficiency and optimization will terminate after 30 rounds of unimproved iterations. The configuration has a satisfactory balance between adequate optimality in outputs and the computation costs.

Benchmark 1 - Unhedged Portfolio Base Currency Performance

Hedging Analysis

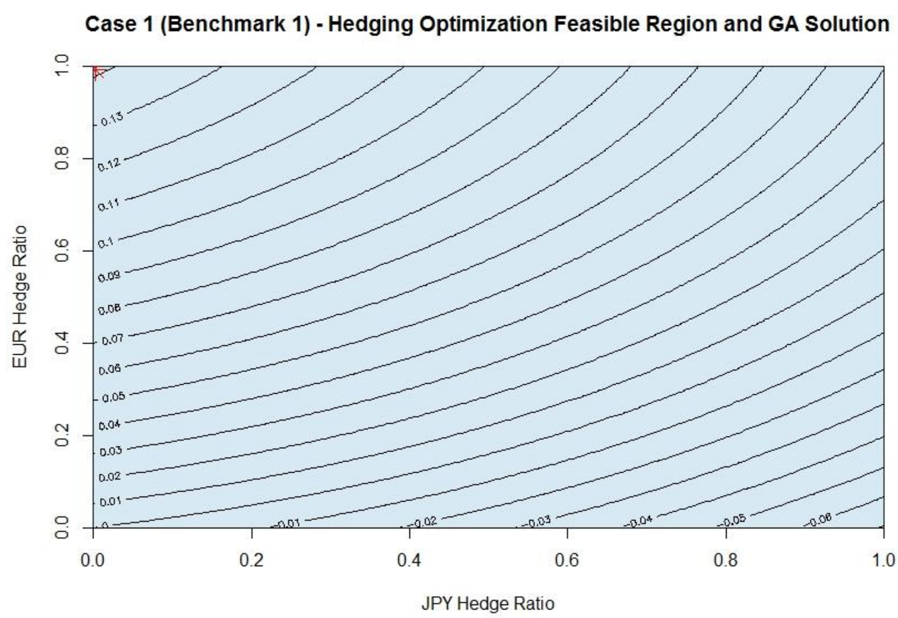


Fig. 5.1 Hedging Optimization Feasible Region and GA Solution for Benchmark 1 (Unhedged Portfolio Performance in USD term), Case 1. The X-axis labels the JPY hedge ratio and Y-axis labels the EUR hedge ratio. The unit square is the entire search space, but only the area highlighted in light blue is the feasible region to the optimization problem. The contours represent different objective function values and the red asterisk at the top-left corner marks the final GA solution.

Figure 5.1 displays the entire 2-dimensional search space of our hedging optimization problem (in the form of a unit square), with h_1 (JPY hedge ratio) and h_2 (EUR hedge ratio) both spanning from 0 to 1. The feasible region of the optimization problem is highlighted in light blue color and in this case the entire search space is feasible. The contours represent different levels of fitness values which are gradually rising along the northwest direction and reach maximum at the northwest corner of the search space. The output GA solution is marked as a red asterisk in the top-left corner. The plot gives us an intuitive image of how GA algorithm directs its search actions to locate the optimal combination of hedge ratios in an efficient manner – by cutting through and more or less following the northwest passage.

GA hedging outputs the following hedge ratios for JPY and EUR:

Table 5.1 GA Output, Case 1 (Benchmark 1)

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio
6.79%	0.003	0.994

The unhedged and 100%-hedged portfolio returns are 6.79% and 9.29% respectively. Therefore, the return threshold for the GA hedging is the smaller one of the two, 6.79%. GA outputs a valid solution. A close-to-zero hedge ratio for JPY and a close-to-one hedge ratio for EUR suggest it is recommended to sell nearly all the EUR exposure in the portfolio forward while keeping JPY exposure nearly intact to achieve the optimal portfolio performance.

Table 5.2 summarizes the portfolio performance metrics in base currency USD under each hedging strategy over the sample period of 2005-2016: no hedging, 100% hedging and GA hedging retrospectively applied to 2005-2016.

Table 5.2 Summary Statistics of Portfolio Performance under Three Hedging Strategies

	Unhedged Portfolio	100% Hedged Portfolio	GA Solution min. ret = 6.79%
Portfolio Return (%)	6.8	9.3	8.2
Portfolio Volatility (%)	19.0	17.3	14.6
Sharpe Ratio	0.357	0.537	0.563
	GA Hedging vs. No Hedging		
Volatility Reduction (%)	23.3		
Sharpe Ratio Improvement (%)	57.6		

Notes: Table 5.2 shows the performance metrics summary of unhedged, fully-hedged and GA-hedged portfolios, including Sharpe ratio performance. The bottom half of the table presents the relative volatility reduction and relative Sharpe ratio improvement effects by GA hedging. As per our definition, a positive value in volatility reduction means GA hedging achieves a lower volatility; a positive value in Sharpe ratio improvement means GA hedging achieves a higher Sharpe ratio.

GA hedging works effectively as expected and lowers the portfolio volatility to 14.6% from 19% in the unhedged portfolio. A 23% volatility drop is not at any cost of return compromise, i.e. return is even 21% better than the unhedged benchmark. The double gain justifies a significant increase in the measure of the risk-adjusted return (Sharpe ratio) by nearly 58%.

Table 5.3 and Table 5.4 list and comapre the yearly volatility reduction and Sharpe ratio improvement effects from GA hedging in the sample period of 2005-2016.

Table 5.3 Yearly Performance Metrics of Benchmark Unhedged Portfolio and GA-Hedged Portfolio

	No Hedging			**GA Hedging**		
	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
2005	16.7	12.7	1.312	23.3	11.3	2.057
2006	20.1	10.9	1.838	15.6	10.4	1.503
2007	16.5	8.5	1.951	11.5	7.7	1.495
2008	-37.6	30.2	-1.244[2]	-35.1	20.9	-1.679[2]
2009	22.3	29.8	0.748	20.5	24.7	0.830
2010	9.6	21.9	0.438	13.3	14.1	0.947
2011	-15.5	24.2	-0.641[2]	-13.8	21.0	-0.656[2]
2012	21.8	17.6	1.242	21.2	13.9	1.520
2013	30.1	11.6	2.591	27.8	9.2	3.018
2014	-8.2	10.2	-0.806[2]	-0.7	8.1	-0.092[2]
2015	2.4	16.0[1]	0.148	8.9	18.2[1]	0.489
2016	3.4	17.2	0.196	6.0	15.5	0.386

Notes: [1] marks a worse volatility performance by the GA hedging strategy. [2] marks the occasions when the Sharpe ratios of both portfolios are within negative territory. They need to be analyzed separately.

Table 5.4 Yearly GA Hedging Effectiveness in Return, Volatility and Sharpe Ratio, 2005-2016

	Relative Return Increase	Relative Volatility Reduction	Relative Sharpe Ratio Improvement
2005	40%	11%	57%
2006	-22%	5%	-18%
2007	-30%	9%	-23%
2008	6%	31%	35%
2009	-8%	17%	11%
2010	39%	36%	116%
2011	11%	13%	2%
2012	-3%	21%	22%
2013	-8%	21%	16%
2014	91%	21%	89%
2015	278%	-14%	231%
2016	77%	10%	97%
Mean	39.3%	14.9%	53.0%
Variance	0.7052	0.0164	0.5169

Notes: Return, volatility and Sharpe ratio of GA-hedged portfolio are compared to the benchmark unhedged portfolio each year in the sample period of 2005-2016 and the visualization shows the relative increase/decrease. Green bars represent outperformance while red bars represent underperformance.

We can have a detailed sense of GA hedging effectiveness from the data presented in Table 5.3 and Table 5.4. In terms of volatility benefit, GA produces an outstanding result, lowering portfolio risk in most of the years in sample period and the average reduction amounts to 15% over the years. The underperformance only occurs in 2015 and the magnitude is relatively minor compared to the benefits it brings to other years. In terms of portfolio Sharpe ratio performance, the outcome is equally satisfactory, with underperformance only happening in 2 out of 12 years. The dashed boxes highlight the three years when the Sharpe ratio of both portfolios lie in the negative territory. Sharpe ratio has limitations in interpreting risk-adjusted return in the negative domain, hence we have to compare the returns and volatilities separately before drawing any conclusions. For all three years, GA hedging strategy boosts portfolio return and decreases portfolio volatility, claiming a better performer between the two. Another observation worth noting is that the magnitude of Sharpe ratio improvement is much higher than that of volatility reduction, highlighting an extraordinary performance from GA hedging strategy.

In order to explore the reason why GA hedging is underperforming the unhedged benchmark in certain years, we will do a simple portfolio variance decomposition analysis. Figure 5.3 shows the decomposition of unhedged and GA-hedged portfolio variances and compare each of the portfolio variance components

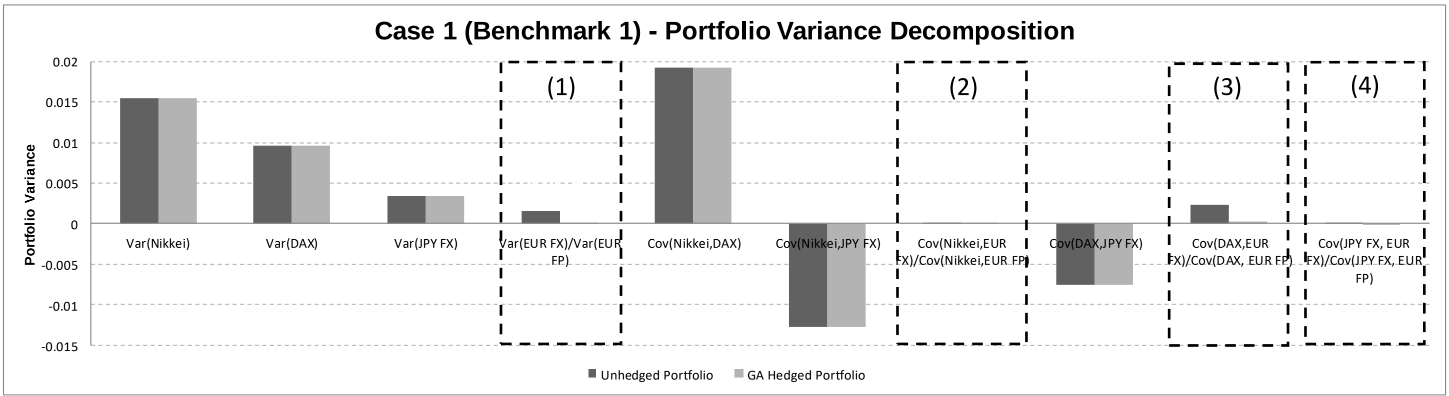


Figure 5.2 Comparison of Unhedged and GA-Hedged Portfolio Variance Components. The figure compares different variance components of unhedged and GA-hedged portfolios, so as to demonstrate the roots of volatility reduction by currency hedging using forwards.

Figure 5.2 compares the variances and covariances of different asset and currency components in unhedged portfolio and GA-hedged portfolio. The GA hedging result of roughly keeping all JPY exposure and selling off all EUR exposure helps to reduce the portfolio volatility, partly by replacing the variance of EURUSD spot return with the variance of the EUR forward premium (see dashed box labeled (1)). The latter is dramatically smaller. In addition, the covariance between EUR forward premium and DAX, between EUR forward premium and JPY spot return are substantically smaller than those in the unhedged portfolio (see dashboxes labeled (3) and (4)). Only EUR spot exchange rate is less corelated to Nikkei than EUR forward premium (see dashed box labeled (2)), which cuts down a little bit variance reduction benefit. In addition, the rationale for leaving JPY exposure unhedged is japanese yen exchange rate has a much lower correlation with two equity assets (both negative) than the JPY forward premium, making it a good natural hedge for the portfolio. The negative correlation with equity markets can easily be explained by its safe-haven currency property introduced in Section 2. Therefore, hedging JPY exposure would otherwise give away this natural protection. GA hedging program hence produces a reasonable solution.

Time Series Cross-Validation

As explained in Section 4, the objective of our portfolio currency hedging optimization not only concerns the overall volatility reduction effect over a given period of time, but also its variability in order to ensure the robustness of the solution. By applying the GA hedge ratios back to the data of 2005-2016, we confirm the contribution of the GA hedging strategy to a general portfolio performance improvement. On the one hand, it reduces portfolio risk and the risk mitigation effect over the years is quite stable. On the other hand, it increases Sharpe ratio by a large degree on average. Moreover, when compared to 100% hedging, GA hedging strategy delivers a 3-time larger average relative volatility reduction and the variance of volatility reduction almost halves that of full hedging at the same time, indicating it is producing a more efficient and more resilient and stable hedging solution than naïve hedging methods.

In the next step, we will use a model validation technique called cross-validation to more accurately assess how well the hedging algorithm can be generalized to an independent data set or to the unknown future. Specifically for financial time series data with inherent temporal dependencies, the traditional k-fold cross-validation technique needs to be modified and conducted with a rolling prediction origin. It is different from the traditional method in that it avoids random sampling processes that would disturb the order of data. The occurrence of applying a model that is derived from a set of forward-looking data to the past is logically incorrect. However, the revised procedure has the same fundamental idea as the traditional technique and can serve well the purpose of checking the model validity.

In the k-fold cross validation, there are two groups of dataset – training dataset and test dataset. Each fold is a process where we train a model out of training data and apply this model to the test data. The performance measures of the test dataset are recorded on file. The whole procedure repeats k times, with enlarging training dataset and a changing test dataset. In our case, the training data span starts from 5 years and the test data span remains to be 1 year, i.e. in fold 1, the process begins with training the model from the initial five years (2005-2009) and then testing the trained model on the next subsequent year (2010). In the next fold, the previous test data joins the original training dataset and a new model gets trained out of the updated training set (2005-2010). The subsequent year (2011) will take the turn as the new test data. With the increasing number of folds of validation, the training data gets more and more lengthy while the test data is always one year, until the final fold of T-1: 1 (T-1 years as training set and the final year available as test data).

The following Figure 5.3 intuitively illustrates how this time series cross-validation procedure is implemented over the whole dataset.

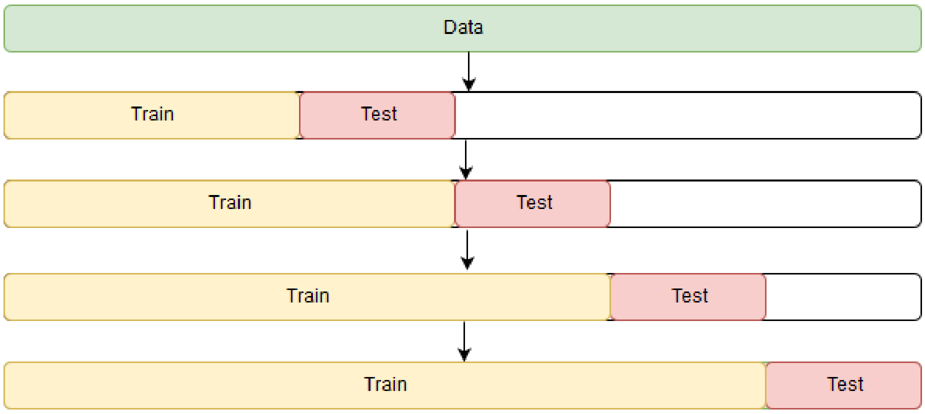


Figure 5.3 Time Series Cross-Validation Procedure Diagram. The diagram shows the relationship between training data and test data and how it evolves between fold to fold. (Source:https://stats.stackexchange.com/questions/14099/using-k-fold-cross-validation-for-time-series-model-selection)

In our case, the 7-fold time series cross-validation will be executed in the following way: We take 2005 to 2009 as our first set of training data and run the GA hedging algorithm to obtain a set of optimal hedge ratios, which is then applied to the data of 2010. The performance measures (return, volatility and Sharpe ratio of the test data) of the test year are then recorded. In the same manner, we perform the remaining six rounds of cross-validation. Table 5.5 below aggregates the test results and Table 5.6 demonstrates the performance of GA hedging solution in terms of relative volatility reduction and relative Sharpe ratio improvement.

Table 5.5 7-Fold Time Series Cross-Validation Summary Statistics

	GA Output		**Test Data Performance**					
			GA Hedging			No Hedging		
	h_1	h_2	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
Fold 1	0.512	0.980	10.4	15.3	0.680	9.6	21.9	0.438
Fold 2	0.035	0.980	-13.9	21.0	-0.659(2)	-15.5	24.2	-0.641(2)
Fold 3	0.009	0.985	21.2	14.0	1.519	21.8	17.6	1.242
Fold 4	0.006	0.991	27.8	9.2	3.019	30.1	11.6	2.591
Fold 5	0.004	0.996	-0.7	8.1	-0.090(2)	-8.2	10.2	-0.806(2)
Fold 6	0.004	0.996	8.9	18.2(1)	0.489	2.4	16.0(1)	0.148
Fold 7	0.004	0.963	5.9	15.5	0.380	3.4	17.2	0.196

Notes: (1) marks a worse volatility performance by the GA hedging strategy. (2) marks the occasions when the hedge ratios of both portfolios are within negative territory. They need to be analyzed separately.

Table 5.6 Yearly GA Hedging Effectiveness in Time Series Cross-Validation, 2005-2016

	Relative Return Increase for Test Data	Relative Volatility Reduction for Test Data	Relative Sharpe Ratio Improvement for Test Data
Fold 1	*8%*	30%	55%
Fold 2	*11%*	13%	3%
Fold 3	*-3%*	21%	22%
Fold 4	*-8%*	21%	16%
Fold 5	*91%*	21%	89%
Fold 6	*278%*	-14%	232%
Fold 7	*75%*	10%	94%
Mean	64.6%	14.4%	73.0%
Variance	1.0362	0.0200	0.6138

Judging from the GA hedge ratio outputs on the left part of Table 5.5, except for fold 1, GA hedging optimization suggests keeping all Japanese Yen exposure in all other rounds of validation. On the other hand, EUR hedge ratios in all folds of cross-validation indicate a nearly full hedge of the currency.

On average, we see an outstanding risk reduction and Sharpe ratio elevation result, averaging 15% and 73% respectively over 7 folds of validation. Fold 6 stands out as the only round of experiment that produces worse volatility result than no hedging benchmark. Over the 10 years of training data, EURUSD spot exchange rate has been positively correlated with stocks most of the time, thus adding to portfolio risk. In consequence, GA output suggests hedging away all EUR exposure. However, in the test year of 2015 (the test year of fold 6), Euro spot moves against equity assets, making itself a natural hedge if left unhedged. The effectiveness of GA hedging using forward contracts is to some extent confined by the condition that the market situations (e.g. asset correlations and overall market volatility) remain more or less the same as the years from which the hedge ratios are derived. When opposite movements or abnormal disturbances are present in the market, GA is likely to underperform doing nothing to currency exposures. However, this effectiveness deviation from the average does not alter the case for GA hedging strategy by any means. From a bigger picture over many years, it does produce satisfactory outcomes overall and makes currency risk hedging worthwhile.

Benchmark 2 – Unhedged Portfolio Local Currency Performance

The benchmark here is the unhedged portfolio local currency performance, against which the GA hedging strategy is measured. Same as before, the return threshold in the GA constraint is chosen to be the minimum of the returns from the two naïve hedging strategies.

Hedging analysis

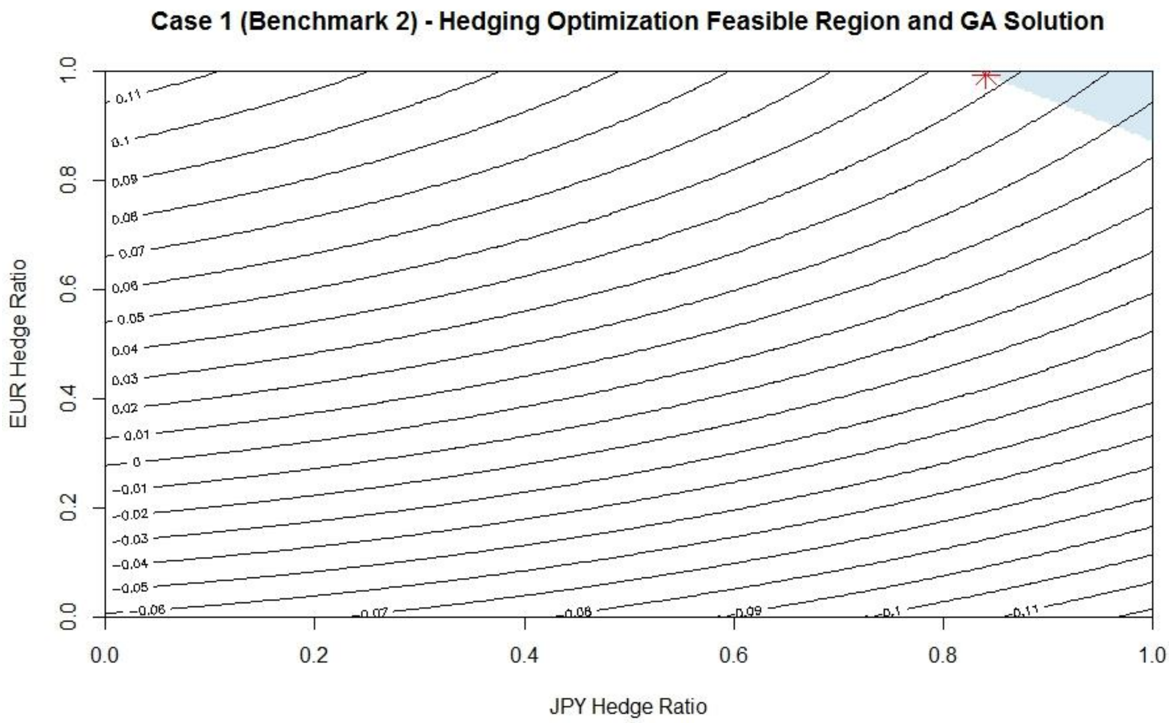


Fig. 5.4 Hedging Optimization Feasible Region and GA Solution for Benchmark 2 (Unhedged Portfolio Performance in Local Currency), Case 1. The X-axis labels the JPY hedge ratio and Y-axis labels the EUR hedge ratio. The unit square is the entire search space, but only the area highlighted in light blue is the feasible region to the optimization problem. The contours represent different objective function values and the red asterisk at the top-right corner marks the final GA solution.

Figure 5.4 displays search space of our hedging optimization with local currency performance benchmark. h_1 (JPY hedge ratio) and h_2 (EUR hedge ratio) are both spanning from 0 to 1. Different from the USD benchmark scenario, the feasible region (highlighted in light blue color) now moves to the top right corner and the size of area is substantially reduced. The contours represent different levels of fitness values which are still gradually rising along the northwest direction and reach maximum at the northwest corner of the search space. The output GA solution is marked as a red asterisk at the top-right corner.

The GA outputs are listed in Table 5.7 below.

Table 5.7 GA Output, Case 1 (Benchmark 1)

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio
9.11%	0.841	0.994

The unhedged and 100% hedged portfolio returns are 9.11% and 9.29% respectively. Therefore, the return threshold for the GA hedging is the smaller one of the two, 9.11%. GA outputs a valid solution. The result suggests it is recommended to sell a large percentage of JPY exposure (84%) and nearly all the EUR exposure (99%) in the portfolio forward in order to achieve optimal volatility benefits.

Table 5.8 summarizes the portfolio local currency performance metrics under each hedging strategy over the period of 2005-2016: no hedging, 100% hedging and GA hedging retrospectively applied to the sample period.

Table 5.8 Summary Statistics of Portfolio Performance under Three Hedging Strategies

	Unhedged Portfolio	100% Hedged Portfolio	GA Solution (min. ret = 9.11%)
Portfolio Return (%)	9.1	9.3	9.1
Portfolio Volatility (%)	17.7	17.3	15.7
Sharpe Ratio	0.514	0.538	0.580
	GA Hedging vs. No Hedging		
Volatility Reduction (%)	11.3		
Sharpe Ratio Improvement (%)	12.8		

Notes: Table 5.8 shows the performance metrics summary of unhedged, fully-hedged and GA-hedged portfolios, including Sharpe ratio performance. The bottom half of the table presents the relative volatility reduction and relative Sharpe ratio improvement effects by GA hedging. As per our definition, a positive value in volatility reduction means GA hedging achieves a lower volatility; a positive value in Sharpe ratio improvement means GA hedging achieves a higher Sharpe ratio.

We can see from Table 5.8 that GA hedging results in a moderate volatility reduction and Sharpe ratio improvement effect, reading 11.3% and 12.8% respectively. During the sample period, Japanese Yen and Euro plunged 12% and 22% respectively. By excluding such huge currency losses, the benchmark return is increased substantially to 9.11% from 6.8% in the USD benchmark scenario. The much higher minimum return constraint not only squeezes the feasible region in which the optimal hedge ratios can be searched (as is shown by the tiny feasible region in Figure 5.4), it also creates challenge to the Sharpe ratio improvement of the GA algorithm as the return hurdle to overcome is much higher.

Table 5.9 lists the yearly volatility reduction and Sharpe ratio improvement effects from GA hedging in the sample period of 2005-2016. Table 5.10 illustrates the yearly relative return increase, relative volatility reduction and relative Sharpe ratio improvement effects of GA hedging, compared to the benchmark unhedged portfolio.

Table 5.9 Yearly Performance Metrics of Benchmark Unhedged Portfolio and GA-Hedged Portfolio

	No Hedging			**GA Hedging**		
	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
2005	33.7	12.7	2.655	30.0	11.4	2.617
2006	14.4	11.3(1)	1.276	18.2	11.4(1)	1.602
2007	5.6	10.0	0.560	10.8	9.8	1.105
2008	-41.2	27.2	-1.516(2)	-41.4	24.1	-1.718(2)
2009	21.4	23.9	0.898	21.7	22.9	0.948

2010	6.5	17.2	0.379	8.5	16.0	0.533
2011	-16.0	19.0[1]	-0.843[2]	-15.5	21.1[1]	-0.731[2]
2012	26.0	16.4	1.583	25.6	15.5	1.644
2013	41.1	10.9	3.771	33.5	9.6	3.478
2014	4.9	10.2	0.477	3.2	9.3	0.342
2015	9.3	20.3	0.458	9.2	19.1	0.484
2016	3.6	18.1	0.202	5.5	18.1	0.303

Notes: [1] marks a worse volatility performance by the GA hedging strategy. [2] marks the occasions when the hedge ratios of both portfolios are within negative territory. They need to be analyzed separately.

Table 5.10 Yearly GA Hedging Effectiveness in Return, Volatility and Sharpe Ratio, 2005-2016

	Relative Return Increase	Relative Volatility Reduction	Relative Sharpe Ratio Improvement
2005	*-11%*	10%	-1%
2006	*26%*	0%	25%
2007	*93%*	2%	97%
2008	*0%*	11%	13%
2009	*1%*	4%	6%
2010	*31%*	7%	41%
2011	*4%*	-11%	-13%
2012	*-2%*	5%	4%
2013	*-18%*	12%	-8%
2014	*-35%*	9%	-28%
2015	*-1%*	6%	6%
2016	*51%*	0%	50%
Mean	11.5%	4.5%	15.9%
Variance	0.1185	0.0042	0.1136

Notes: Return, volatility and Sharpe ratio of GA-hedged portfolio are compared to the benchmark unhedged portfolio each year in the sample period of 2005-2016 and the visualization shows the relative increase/decrease. Green bars represent outperformance while red bars represent underperformance.

We can have a detailed sense of GA hedging effectiveness from the data presented in Table 5.9 and Table 5.10. In terms of volatility benefit, GA hedging still produces a satisfactory result, lowering portfolio risk in most of the years in the sample period. The underperformance in 2006 and 2011 are relatively acceptable compared to the benefits it brings to other years. In terms of portfolio Sharpe ratio performance, the outcome is more mixed, with deterioration happening in 4 out of 12 years. For years 2005, 2013 and 2014, given the positive volatility mitigation effects by GA hedging , the decrease of Sharpe ratio is rooted in the lower returns. In year 2011, the case is opposite. The return benefit is not enough to compensate the volatility increase, thus leading to a lower Sharpe ratio.

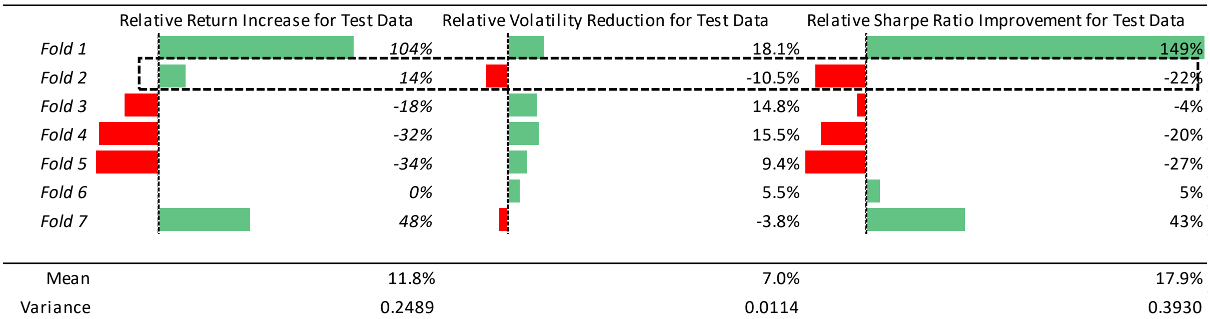
Time Series Cross-Validation

We apply the same time series cross-validation procedure explained before. Table 5.11 aggregates the results and Table 5.12 demonstrates the performance of GA hedging solution in terms of relative volatility reduction and relative Sharpe ratio improvement:

Table 5.11 7-Fold Time Series Cross-Validation Summary Statistics

GA Output			Test Data Performance					
			GA Hedging			No Hedging		
	h_1	h_2	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
Fold 1	0.004	0.994	13.3	14.1	0.946	6.5	17.2	0.379
Fold 2	0.017	0.995	-13.8	21.0[(1)]	-0.657[(2)]	-16.0	19.0[(1)]	-0.843[(2)]
Fold 3	0.032	0.988	21.3	14.0	1.524	26.0	16.4	1.583
Fold 4	0.020	0.994	27.9	9.2	3.029	41.1	10.9	3.771
Fold 5	0.849	0.996	3.2	9.3	0.348	4.9	10.2	0.477
Fold 6	0.973	0.995	9.3	19.2	0.483	9.3	20.3	0.458
Fold 7	0.977	0.993	5.4	18.8[(1)]	0.289	3.6	18.1[(1)]	0.202

Notes: [(1)] marks a worse volatility performance by the GA hedging strategy. [(2)] marks the occasions when the hedge ratios of both portfolios are within negative territory. They need to be analyzed separately.



The cross-validation result shows that GA hedging for benchmark 2 scenario underperforms the benchmark 1 scenario, but overall still generates satisfactory outcomes.

Case 1 Summary

In summary, through two different ways of performance measurement, with benchmark 1 more relaxing and benchmark 2 stricter, we successfully build a strong case for the effectiveness of our portfolio currency hedging algorithm, especially its risk mitigation function. Both of the scenarios validate the reliability of its performance

5.2 Case 2: 5-asset, 4-foreign-currency-exposure

In Case 2, the U.S. investor invests both domestically and internationally in multi-asset classes, which more resembles portfolios in reality. Similarly, we assume wealth is split equally between the five assets: S&P 500 (U.S. equity market index); S&P Japan Government Bond Index, a tracker for the performance of Japanese Yen denominated government bonds issued by Japanese issuers; S&P Eurozone Sovereign Bond Index, a measure of the performance of Euro-denominated government bond markets in both developed and emerging Eurozone countries; Swiss Market Index (SMI), the benchmark equity index of Switzerland and FTSE 100, the benchmark equity index of U.K..

The initial investment is 1 million USD. We implement our hedging optimization program within the sample period of 2008-2016, as the bond indices data are only available from 2008.

Implementation Inputs

S&P 500, SMI, FTSE 100, JPYUSD, EURUSD, CHFUSD and GBPUSD time series data are obtained from Bloomberg. S&P Japan Government Bond Index and S&P Eurozone Sovereign Bond Index data are both downloaded from S&P Dow Jones Indices database. The investor has an equal split of wealth between the five assets, i.e. $\boldsymbol{\omega} = (0.2, 0.2, 0.2, 0.2, 0.2)$. We rebalance the forward hedging instruments annually during 2008-2016 and the hedging valuation is on a monthly basis, same as in Case 1. The interim JPYUSD, EURUSD, CHFUSD, GBPUSD forward prices are interpolated from Bloomberg.

Implementation Parameters

The decision variables (h_1, h_2, h_3, h_4) refer to hedge ratios of JPY, EUR, CHF and GBP exposure respectively and are searched in the domain of [0, 1]. The GA hedging optimization is carried out in a 4-dimensional space with the same implementation parameters as in Case 1. As the search dimension increases, the computation takes longer time and we are no longer able to visualize the feasible region of the optimization.

Benchmark 1 – Unhedged Portfolio Base Currency Performance

Hedging Analysis

GA outputs the following hedge ratios for JPY, EUR, CHF and GBP:

Table 5.12 GA Output, Case 2 (Benchmark 1)

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio	CHF Hedge Ratio	GBP Hedge Ratio
1.48%	0.545	0.993	0.997	0.975

The portfolio returns for two naïve strategies are 1.48% and 3.32 % respectively. Therefore, the unhedged portfolio return 1.48% is chosen as the minimum return level of the optimization. Result suggests that the investor needs to sell 55% of JPY exposure, nearly all EUR, CHF and GBP exposures.

Table 5.13 below summarizes the portfolio performance metrics in base currency USD under each hedging strategy over the period of 2008-2016: no hedging, 100% hedging and GA hedging retrospectively applied to 2008-2016.

Table 5.13 Summary Statistics of Portfolio Performance under Three Hedging Strategies

	Unhedged Portfolio	100% Hedged Portfolio	GA Solution min. ret = 1.48%
Portfolio Return (%)	1.5%	3.3%	3.1%
Portfolio Volatility (%)	9.9%	6.5%	6.4%
Sharpe Ratio	0.149	0.513	0.489
	GA Hedging vs. No Hedging		
Volatility Reduction (%)	34.9		
Sharpe Ratio Improvement (%)	227		

Notes: Table 5.13 shows the performance metrics summary of unhedged, fully-hedged and GA-hedged portfolios, including Sharpe ratio performance. The bottom half of the table presents the relative volatility reduction and relative Sharpe ratio improvement effects by GA hedging. As per our definition, a positive value in volatility reduction means GA hedging achieves a lower volatility; a positive value in Sharpe ratio improvement means GA hedging achieves a higher Sharpe ratio.

As we increase the number of assets in the portfolio, we can see an obvious diversification benefit by allocating to diverse kinds of asset classes and spreading investments to more geographically-diversified markets. The overall portfolio risk without hedging drops to below 10% as opposed to 17.6% of its counterpart in Case 1, which is almost a 50% drop. Traditionally bonds tend to be negatively correlated to stock markets, offsetting some of the investment risks in the portfolio. In our case, Japanese and European bond markets tend to move in opposite direction to the U.S., Swiss and British equities markets. However, the huge risk mitigation trades away the portfolio rewards, partly because partial portfolio wealth is allocated to bond assets with lower return profiles.

GA hedging solution successfully reduces portfolio volatility by 35% and increases the Sharpe ratio by more than two folds, which are remarkable outcomes.

Table 5.14 below aggregates the yearly performance of GA hedging strategy in terms of return, volatility and Sharpe ratio.

Table 5.14 Yearly GA Hedging Effectiveness in Return, Volatility and Sharpe Ratio, 2008-2016

	Relative Return Increase	Relative Volatility Reduction	Relative Sharpe Ratio Improvement
2008	14%	32%	26%
2009	-15%	38%	38%
2010	-26%	47%	39%
2011	0%	44%	77%
2012	16%	41%	96%
2013	13%	22%	45%
2014	539%	40%	829%
2015	88%	-3%	-88%
2016	338%	42%	650%
Mean	107.5%	33.6%	190.2%
Variance	3.8692	0.0236	10.1521

From Table 5.14, we can see a stable volatility reduction effect over the years, with an average of 34% and low variability. 2015 stands out as the only year that GA hedging strategy brings volatility disadvantage (with only minor deviation). In the same manner in Case 1, we perform a detailed variance decomposition analysis to give a deeper understanding of the results.

Generally speaking, bond assets have negative correlations with currency exchange rates, except that Japanese Yen has a small positive correlation with its local bond market (which can be explained by the fact that Japanese government bonds and Japanese currencies are two main safe-haven assets. They will more or less move in the same direction when the market is in risk-off mode). Therefore, all the currency hedging for the bond assets are making a negative impact. For the equity assets, it is generally better to hedge currency risks as forward premiums have lower correlation with equity markets than the spot exchange rates, except that JPY has a natural hedge advantage to SMI, GBP is less positively correlated to FTSE 100 and S&P 500 than its forward premium.

The three dashed boxes in Table 5.14 highlight the three occasions when Sharpe ratio of both portfolios are negative. For the first two years, both return and volatility performances are positive, hence the Sharpe ratio should be increasing as well. For year 2015, the return advantage cannot justify the volatility deterioration. Therefore a negative result is produced. However, overall speaking, GA hedging leads to stunning overall outperformance over the benchmark portfolio.

Time Series Cross-Validation

In Case 2, we continue with the same time series cross-validation procedure as in Case 1 to gain a better understanding of how well our hedging model can be generalized to independent datasets. Similarly, we will perform 5-fold cross-validation, starting with 4 initial years as training dataset and the fifth year as test dataset in fold 1. The remaining four folds will be done in the same manner.

Table 5.15 below lists the GA hedging output hedged ratios (h_1 for JPY hedge ratio, h_2 for EUR hedge ratio, h_3 for CHF hedge ratio and h_4 for GBP hedge ratio) and the performance metrics summary of the GA-hedged portfolio and unhedged benchmark 1 portfolio.

Table 5.15 5-Fold Time Series Cross-Validation Summary Statistics

	GA Output				**Test Performance**					
					GA Hedging			No Hedging		
	h_1	h_2	h_3	h_4	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
Fold 1	0.708	0.961	0.961	0.963	7.8	4.4	1.776	6.2	7.1	0.862
Fold 2	0.493	0.974	0.885	0.949	10.4	5.2	1.993	9.4	6.6	1.409
Fold 3	0.498	0.955	0.964	0.914	5.3	4.1	1.306	-1.3	6.7	-0.195
Fold 4	0.491	0.979	0.990	0.987	-0.5	8.3 (1)	-0.061 (2)	-3.9	8.2 (1)	-0.480 (2)
Fold 5	0.274	0.977	0.945	0.934	4.6	5.4	0.866	1.1	8.4	0.128

Notes: (1) marks a worse volatility performance by the GA hedging strategy. (2) marks the occasions when the hedge ratios of both portfolios are within negative territory. They need to be analyzed separately.

Table 5.16 5-Fold Time Series Cross-Validation Summary Statistics

	Relative Return Increase for Test Data	Relative Volatility Reduction for Test Data	Relative Sharpe Ratio Improvement for Test Data
Fold 1	26%	39%	106%
Fold 2	11%	22%	42%
Fold 3	512%	39%	771%
Fold 4	87%	-2%	-87%
Fold 5	331%	36%	579%
Mean	193.6%	26.7%	282.1%
Variance	4.8299	0.0306	13.8162

The 5-fold cross-validation results demonstrate a stable volatility benefit of the GA hedging algorithm over unhedged portfolio, with only a slight deviation in the fourth round of validation. The Sharpe ratio results are more volatile.

Benchmark 2 – Unhedged Portfolio Local Currency Performance

Hedging Analysis

GA outputs the following hedge ratios for JPY, EUR, CHF and GBP:

Table 5.17 GA Output, Case 2

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio	CHF Hedge Ratio	GBP Hedge Ratio
3.32%	0.948	0.993	0.235	0.989

A noticeable difference between GA output in benchmark 2 and that in benchmark 1 scenario is that the investor needs to double JPY hedge amount and decrease CHF hedge ratio to 24%.

Table 5.18 below summarizes the portfolio local currency performance metrics under each hedging strategy over the period of 2008-2016: no hedging, 100% hedging and GA hedging retrospectively applied to 2008-2016.

Table 5.18 GA Output, Case 2

	Unhedged Portfolio	100% Hedged Portfolio	GA Solution min. ret = 3.32%
Portfolio Return (%)	3.4%	3.3%	3.3%
Portfolio Volatility (%)	7.5%	6.5%	7.0%
Sharpe Ratio	0.453	0.513	0.471
	GA Hedging vs. No Hedging		
Volatility Reduction (%)	6.4		
Sharpe Ratio Improvement (%)	4.0		

Notes: Table 5.18 shows the performance metrics summary of unhedged, fully-hedged and GA-hedged portfolios, including Sharpe ratio performance. The bottom half of the table presents the relative volatility reduction and relative Sharpe ratio improvement effects by GA hedging. As per our definition, a positive value in volatility reduction means GA hedging achieves a lower volatility; a positive value in Sharpe ratio improvement means GA hedging achieves a higher Sharpe ratio.

In this case, the volatility of unhedged portfolio local return (benchmark 2) is lower than the unhedged portfolio USD return (benchmark 1), confirming the fact that currency fluctuations add to a big portion of entire portfolio risk. As for the GA hedging performance, it effectively decreases the portfolio risk, but it is not able to exceed the minimum return threshold. The possible reason can be that JPY and CHF are currencies bearing big forward premiums (1% and 0.95% respectively) over the sample period, which means the investor can reap the interest rate differential when she performs the hedge. An increase in the JPY hedge ratio, however, is offset by an even bigger decrease in the CHF hedge ratio. Given the nearly equal forward premiums of the two currencies, the net effect would be a small shrinkage of return for the GA portfolio in benchmark 2 scenario. In addition, stripped off the negative currency movements, the unhedged portfolio in benchmark 2 scenario exhibits a much higher return level that more than doubles that of benchmark 1 scenario, hence pushing up the Sharpe ratio of the unhedged portfolio by more than three folds.

Table 5.19 Yearly GA Performance

	Relative Return Increase	Relative Volatility Reduction	Relative Sharpe Ratio Improvement
2008	14%	12%	3%
2009	-15%	18%	4%
2010	21%	7%	30%
2011	33%	4%	30%
2012	-3%	1%	-3%
2013	-7%	9%	3%
2014	-25%	-12%	-34%
2015	32%	7%	27%
2016	-11%	-15%	-22%
Mean	4.3%	3.5%	4.2%
Variance	0.0455	0.0117	0.0506

Time Series Cross-Validation

Table 5.20 below lists the GA hedging output hedged ratios (h_1 for JPY hedge ratio, h_2 for EUR hedge ratio, h_3 for CHF hedge ratio and h_4 for GBP hedge ratio) and the performance metrics summary of the GA-hedged portfolio and unhedged benchmark 1 portfolio.

Table 5.20 5-Fold Time Series Cross-Validation Summary Statistics

	GA Output				**Test Performance**					
					GA Hedging			No Hedging		
	h_1	h_2	h_3	h_4	Return (%)	Volatility (%)	Sharpe Ratio	Return (%)	Volatility (%)	Sharpe Ratio
Fold 1	0.719	0.988	0.969	0.947	7.8	4.4	1.795	9.2	5.4	1.693
Fold 2	0.609	0.975	0.985	0.956	10.9	5.2	2.090	13.7	6.1	2.241
Fold 3	0.479	0.974	0.824	0.928	5.0	4.2	1.197	6.7	4.2	1.590
Fold 4	0.531	0.970	0.967	0.973	-0.5	8.3	-0.065[2]	-1.0	8.5	-0.117[2]
Fold 5	0.903	0.985	0.438	0.986	4.2	5.1[1]	0.816	4.5	4.6[1]	0.983

Notes: [1] marks a worse volatility performance by the GA hedging strategy. [2] marks the occasions when the hedge ratios of both portfolios are within negative territory. They need to be analyzed separately.

	Relative Return Increase for Test Data	Relative Volatility Reduction for Test Data	Relative Sharpe Ratio Improvement for Test Data
Fold 1	-15%	20%	6%
Fold 2	-20%	15%	-7%
Fold 3	-25%	0%	-25%
Fold 4	45%	2%	44%
Fold 5	-7%	-12%	-17%
Mean	-4.4%	4.9%	0.4%
Variance	0.0817	0.0154	0.0739

The cross-validation in this case shows a more mixed picture of GA hedging performance. It is producing negative impact on returns, though the volatility reduction is still robust.

5.3 Forward Hedging with a Generic Toolbox

Built upon Case 1 and Case 2 in Section 5.1 and Section 5.2, we further expand the scope of our portfolio currency hedging methodology. In Case 3, we hedge the sample portfolio currency risks using a generic forward hedging toolbox with a basket of currencies available as hedging currencies. The hedging instruments are the forward contracts of foreign currencies against base currency USD. In reality, in order to deal with all kinds of portfolios in question, we need to prepare a generic hedging toolbox with a wider universe of hedging currencies ready. It is beneficial especially when the portfolio involves illiquid currencies, whose uncertainties you want to hedge away but there is no or an immature derivatives market for them. In this case, asset managers usually need to cross-hedge the currency using derivatives of other closely correlated currency pairs. When the currency exposures of the portfolio are not exactly matching the currency list in the hedging toolbox (normally less than), there is likely to be a “redundancy” issue existing in the hedging process, i.e. the search space has a higher dimension than that of the problem solution space. Depending on the correlations between hedging currencies and assets in the portfolio, between hedging currencies and portfolio currencies and between hedging currencies themselves, GA can assign an arbitrary weight to each of the hedging currency so that they jointly contribute to the hedging optimization goal most efficiently. We will revisit Case 1 and expand its scope.

5.3.1 Case 1 Revisited: 2 Assets – 2 Foreign Currencies – 3 Hedging Currencies

Case 1 Revisited: 2 asset s, 2 foreign currencies, 3 hedging currencies

We revisit the 2 asset-2 foreign currency case discussed in Section 5.1. Instead of having only JPYUSD and EURUSD forward contracts available as hedging instruments, we add a third currency GBP into the toolbox, i.e. GBPUSD forward is also ready to take over some of the hedging tasks. We denote the hedge ratio of GBP to cover JPY exposure as h_3 and hedge ratio of GBP to cover EUR exposure as h_4 and don’t allow the combined JPY hedging by JPYUSD forward and GBPUSD forward respectively to exceed its original exposure:

$$\mathrm{h}_1 \times \mathrm{E}_{\mathrm{JPY}} + \frac{(\mathrm{E}_{\mathrm{JPY}} \times \mathrm{FX}_{\mathrm{JPYGBP}}) \times \mathrm{h}_3}{\mathrm{FX}_{\mathrm{JPYGBP}}} \leq \mathrm{E}_{\mathrm{JPY}}$$

$$\mathrm{h}_1 + \mathrm{h}_3 \leq 1 \tag{14}$$

where E_{JPY} refers to the total exposure of JPY and FX_{JPYGBP} refers to the exchange rate of JPYGBP. Equation (14) shows us that the sum of the hedge ratios coming from JPYUSD and GBPUSD hedging instruments should be no bigger than 1. The same constraint applies to the EUR exposure.

$$\mathrm{h}_2 + \mathrm{h}_4 \leq 1 \tag{15}$$

Thus, the upgraded optimization problem can be formulated as:

Objective function:

$$\max.\ \mathrm{Z} = \mathrm{E}(\Delta\sigma_{\downarrow}) - \frac{1}{2}\mathrm{E}[(\Delta\sigma_{\downarrow} - \mathrm{E}(\Delta\sigma_{\downarrow}))^2)]$$

where

$$\Delta\sigma_{\downarrow} = \frac{\sigma_{\mathrm{uh}} - \sigma_{\mathrm{gah}}}{\sigma_{\mathrm{uh}}}$$

subject to

Constraint 1: Minimum portfolio return

$$R_{\mathrm{gah}} \geq \min(R_{\mathrm{uh}}, R_{\mathrm{fh}})$$

Constraint 2: Maximum combined hedging notional for JPY exposure

$$h_1 + h_3 \leq 1$$

Constraint 3: Maximum combined hedging notional for EUR exposure

$$h_2 + h_4 \leq 1$$

Constraint 4: Defensive hedge ratio domain

$$0 \leq h_i \leq 1, \qquad i \in [1,2,3,4]$$

In Case 3, we only implement one benchmark scenario - the no hedging portfolio local currency return and volatility. GA algorithm is expanded to incorporate constraints (2) and (3). Each constraint must be transformed into a penalty function, breaking which will lead to the objective being penalized. Therefore, the fitness function in this case involves three penalty terms. GA hedging output is listed below:

Table 5.21 GA Output, Case 3

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio	GBP for JPY Hedge Ratio	GBP for EUR Hedge Ratio
9.11%	0.179	0.662	0.744	0.318

The result shows that altogether 92% JPY exposure and 98% of EUR exposure need to be hedged. Compared to Case 1 with only two hedging currencies available (84%% hedging for JPY and 99% hedging for EUR as the hedging output), the results match each other in general, with a slightly bigger difference in JPY hedge ratio. The time series validation also demonstrates the reliability of the hedging algorithm in solving the currency hedging optimization problem, despite the fact that the unhedged local currency benchmark is the more difficult one to outperform, when compared to the USD benchmark.

When we analyze the correlation matrix between currency and asset returns, we can get a hint of why EURUSD and JPYUSD hedging instruments are giving away some portion of the hedging to GBPUSD. For EUR exposure, though the spot exchange rate of EURUSD only has a slightly positive correlation with Nikkei, DAX and JPYUSD spot, it itself exhibits much larger volatility than the EUR forward premium, which will replace the spot exchange return to be a source of portfolio risk if EUR were to be hedged away by the forward. In addition to that is a negative correlation between EUR forward premium and Yen spot rate that justifies hedging against EUR exposure. GBP is useful here because it significantly moves opposite to Yen spot exchange rate, even more so for its forward premium, despite the fact that GBP has a tremendous positive correlation with the assets. Therefore, a small part of the EUR exposure is given to the GBPUSD forward hedging.

The comparison of the two hedging approaches below signifies that there is no distinct difference between the two. However, we are surprised to see that the hedging toolbox with three currencies even slightly outperform the one with only two matching currencies. It definitely enlightens the path ahead to further expand the hedging toolbox space to leverage more out of this versatile alternative.

Table 5.22 Hedging Performance Summary Statistics Comparison, between 2- Currency Toolbox and 3-Currency Toolbox.

Hedging Performance Comparison (Unhedged Local Currency Benchmark)

2 Hedging Currencies (JPY, EUR) vs. 3 Hedging Currencies (JPY, EUR, GBP)

	Unhedged Portfolio	**100% Hedged Portfolio**	**GA Solution (JPY, EUR)**	**GA Solution (JPY, EUR, GBP)**
Portfolio Return(%)	9.1	9.3	9.1	9.14
Portfolio Volatility(%)	17.7	17.3	15.7	15.3
Sharpe Ratio	0.514	0.538	0.580	0.597
	GA Solution Performance (JPY, EUR)		**GA Solution Performance (JPY, EUR, GBP)**	
Volatility Reduction(%)	11.3		13.6	
Sharpe Ratio Improvement(%)	12.8		16.1	

6 Currency Hedging using Foreign Exchange Options (Analytical Description)

Apart from foreign exchange forward contracts, another widely used hedging instrument is foreign exchange options, which give the buyer the right not the obligation to buy or sell a certain amount of currency pair at a specific exchange rate (strike price K) on or before a fixed future point of time. On the one hand, FX options require initial cash overlay called option premium, which seems costly compared to forward hedging. On the other hand, options give investors more flexibility with regard to when, whether and how much to exercise. In this section, we will mainly develop an analytical understanding of currency hedging using option contracts and compare it with forward contracts.

6.1 Foreign Exchange Options versus Forwards

FX options differ from forwards in two aspects: Firstly in terms of risk protection, since options have a non-linear pay-off profile, the maximum loss is bounded by the option premium the investor pays upfront to obtain such right. As a result, the currency risk could be completely eliminated. In terms of profit potential, the investor enjoys an unlimited upside profit potential, but it is simultaneously cancelled out by the currency loss in the original foreign investment position. When the currency movement is favorable, the investor could throw away the option and reap the spot exchange rate return when converting back to home currency.

Most research papers so far focused mainly on the difference between the two hedging instruments. However, forwards and options are in essence closely related. The engineering of different kinds of options and option strategies can be viewed as a way to replicate forwards, to different extents. Forwards have a linear payoff diagram on the contract delivery date. If investor is long, the slope of the line is positive whereas the slope is negative for a short position. Options display a more complicated non-linear payoff profile. Let’s take a simple vanilla call option as an example. At extremes, when the call option is deep in the money, the option is behaving exactly like a forward (100%), making profit out of the magnitude of price increase above the strike price; When the call option is deep out of the money, the option is not at all like a forward (0%) because forward will display unlimited loss whereas option only loses its premium. Delta, one of the most important greeks in option theory, describes how much an option is behaving like a forward. Therefore, option delta is equal to 1 in deep-in-the-money area, 0 in deep-out-of-the-money area and approximately 0.5 at the strike. This connection between foreign exchange options and foreign exchange forwards gives us a fresh perspective and rich possibilities to replicate a forward hedging instrument using different combinations of options to achieve different hedging objectives, thus enlarging the potential hedging toolbox space.

6.2 Option Strategies

In this section, we will lay out a few most common option strategies that can be used to replicate forward contracts. Their mechanics will be explained and payoff diagrams will also be drawn.

Vanilla Call/Put

The simplest option structure is the vanilla call/put option on the currency pair, i.e. investor buys put on the foreign currency and buy the call on the base currency. In order to get an unbiased option price based on the market convention of currency pair quotations, it is possible for us to buy a vanilla call option or put option depending on different currency pairs. For example, the market convention is USDJPY, so to hedge against JPY exposure, we need to buy a call option on USDJPY (call on USD and put on JPY); However, for EURUSD pair, we buy a put option on EURUSD (put on EUR and call on USD). The vanilla call/put option, as the payoff diagram shows, gains nothing (expires) when the spot exchange rate ends up below/above the option strike price and benefits from the magnitude of the currency movement when it ends up above/below the strike. Therefore, compared to the forward hedging, vanilla call/put option hedging expires useless on the downside with an option premium loss but a currency profit only limited by the magnitude when the foreign currency strengthens. On the upside, the option has the same functional behavior as forward.

In our option hedging case study, we will use at-the-money-forward (ATMF) call/put option whose strike price is same as the 1y forward price.

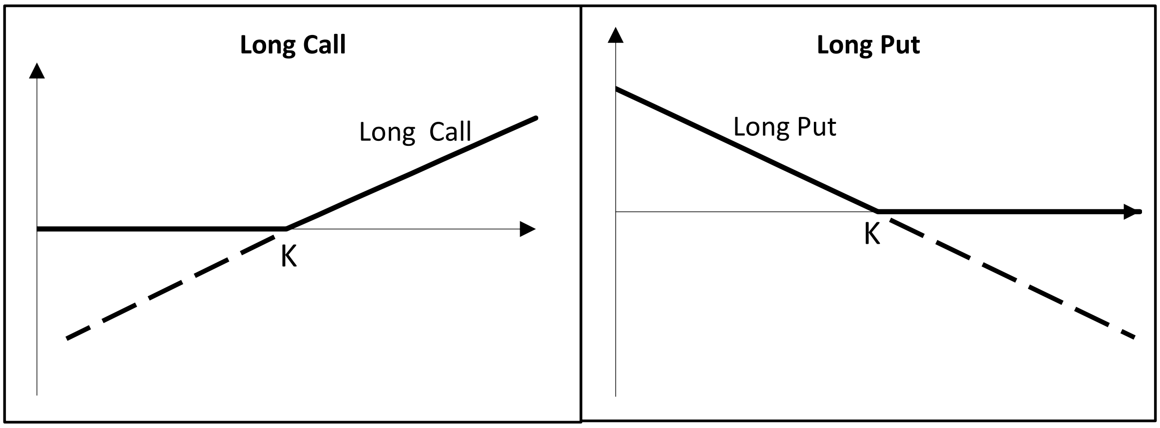


Fig. 6.1 Vanilla Call/Put Payoff Diagrams

Risk Reversal

Risk reversal is an option strategy that is composed of buying (selling) a call and selling(buying) a put with the same maturity simultaneously, normally both are out-of-money. By market convention, when we are long the call and short the put, we are long the risk reversal, vice versa. Risk reversal is used to replicate a long position of the underlying asset, therefore it is also called a synthetic long of the asset. In terms of cost, risk reversal is cheaper than simple vanilla option as it earns money back by selling a put or call option. In terms of payoff profile, in the case of long risk reversal, it behaves in the same way as forward in the above call strike region and below put strike region, and expires useless when the spot is stuck in between.

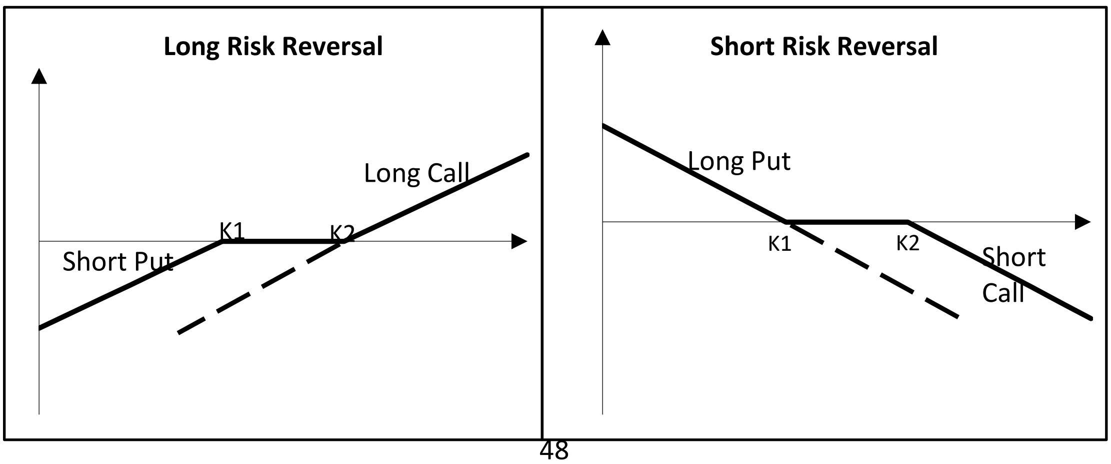


Fig. 6.2 Risk Reversal Payoff Diagrams

Call/Put Spread

Call/Put spread is an option strategy that involves purchasing and writing an equal number of call/put options on the same underlying asset with same maturity date, however the strike prices are different. Bull call spread consists of selling call options that have higher strike price than that of the long call options. With a higher strike price (all other parameters equal), the short call options are cheaper than the long call options, which means an initial outlay is needed for the structure. The bull call spread option structure is used to express the view that the underlying asset price will rise but will only experience moderate price fluctuations. It reduces the cost of the overall structure by limiting the upside potential to the short call's strike price. If the spot price goes past short call's strike price, the profit remains stagnant. The investor pays the premium on the other hand to limit the downside risk to be maximum the long call option's strike price.

On the contrary, the bear call spread involves buying call options that have higher strikes than the ones sold short. It is a bet against the asset price, but only within a small range of price fluctuation. The maximum profit that can be gained from this option strategy is the difference between the price paid to buy the call and the price received for writing the call.

Payoff diagram is as follows.

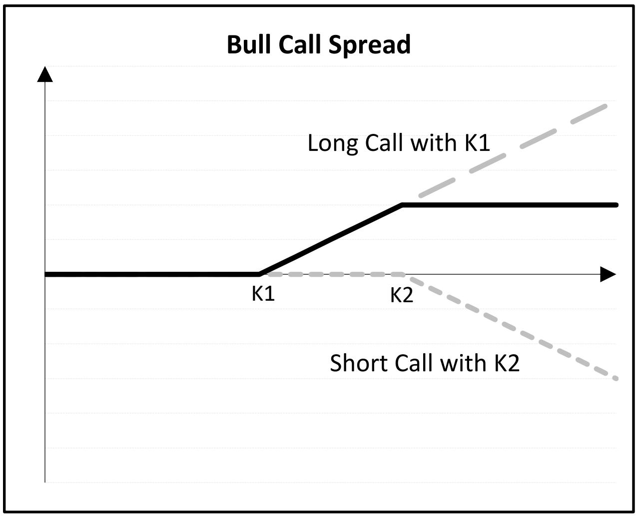


Fig. 6.3 Bull Call Spread Payoff Diagram

Call and Put Synthetic Long/Short

Synthetic option is an option strategy that mimics the behavior of long or short the underlying asset. A synthetic long is a bullish strategy on the underlying asset whereas a synthetic short is a bearish bet on the asset. It involves buying a call and selling a put with the same strike price and same expiration date (for synthetic long) and selling a call and buying a put with the same strike price and same expiration date (for synthetic short).

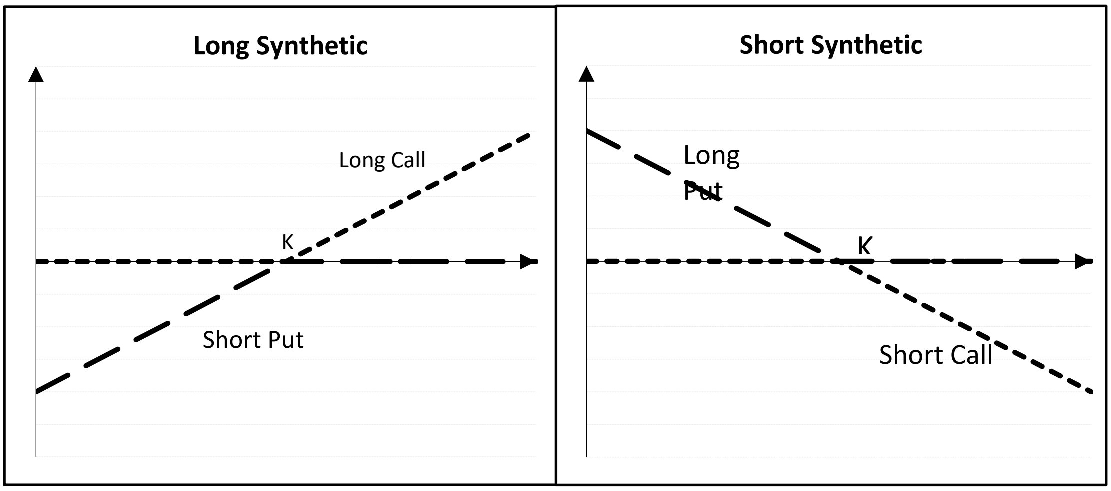


Fig. 6.4 Synthetic Long/Short Payoff Diagrams

6.3 Portfolio Currency Hedging using FX Options Implementation

In this part of the analysis, we will use FX options to hedge against portfolio currency risks and compare its performance to that of forward hedging. As discussed above, options differ significantly from forward hedging as it gives more flexibility in terms of when, whether and how much to exercise, at a cost. In this case, we follow the setups of the forward hedging cases analyzed in the previous sections: the 2-asset portfolio denominated in USD consists of equity investments in Japan and Eurozone (Nikkei and DAX respectively). The rebalancing is performed annually and the hedging valuation is on a monthly basis. We use the foreign currency exposure at the beginning of the year as the full hedge notional. The option instruments are European style. Similar to the forward hedging case, for the option valuation we do at the end of each month during the year, we need to reprice the same option with the new time to maturity (equal to the time difference between the current valuation date and the option expiry date) and the corresponding volatility, forward rate and interest rates.

According to Garman-Kohlhagen model for foreign exchange option valuation, there are six parameters that will affect the FX option price, namely the spot price of the exchange rate (S), the strike price of the option (K), the domestic risk-free interest rate (r_d), the foreign risk-free interest rate (r_f), the volatility of the currency pair (σ) and the time-to-maturity (T). Extended from Black-Scholes model that is based on the assumption that borrowing and lending riskless rates are the same, the Garman-Kohlhagen model takes the interest rate differential between base and foreign currencies into consideration as they will jointly affect the value of FX option value. The theoretical value of a call and a put FX option can be computed by the formulas:

$$C_t(S,T,K) = Se^{-r_f(T-t)}N(d_1) - Ke^{-r_d(T-t)}N(d_2)$$

$$P_t(S,T,K) = -Se^{-r_f(T-t)}N(-d_1) + Ke^{-r_d(T-t)}N(-d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r_d - r_f + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + (r_d - r_f - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

where T-t represents the time to maturity. C_t and P_t are the prices of a European call and a European put option in the base currency term to sell one unit of the foreign currency exposure.

The Garman-Kohlhagen model assumes that the volatility of the exchange rates is constant and the prices follow a lognormal distribution, which are both not true in reality. Therefore, instead of calculating the option premiums via theoretical pricing models, we use a professional option pricing tool called "SuperDerivatives" which supplies real market data (volatility, forward rate, etc.) and incorporate market conventions when pricing options.

We start with the simplest option strategy - using vanilla call/put options to hedge against portfolio currency risk. For JPY exposure, the market quotation convention is USDJPY, so we need to buy a USDJPY call option to obtain the right to sell JPY at expiration. For EUR exposure, the market quotation convention is EURUSD, so we need to do the opposite - to buy a put option to gain the right to sell EUR at maturity and buy base currency USD back. Therefore, option strategy 1 combines buying a 1-year USDJPY call option and buying a 1-year EURUSD put option.

The strike of the call and put options is set to be at-the-money-forward (ATMF), which is equal to the 1-year forward rate on the day of purchase. This forward rate will theoretically be the spot exchange rate on the day of expiry and is exactly equal to the forward price transacted in the forward hedging case. However, in option hedging, we have to round up the ATMF strikes as per option market convention, for each currency pair, only options with a certain number of decimals in strikes can be quoted. For example, for USDJPY, 0.5 is the smallest increment in option strikes while for EURUSD it is 0.005. ATMF option is the most liquid FX option contract and is the easiest to value, which makes it more popular than at-the-money-spot (ATMS) option. In our case, if on the expiration date, USDJPY spot exchange rate is trading above the strike (USD strengthens against JPY), the option will be exercised to offset the currency loss resulting from JPY depreciation. At expiry, the exercised option will become a spot transaction, in which the investor can buy the notional amount of USDJPY at the strike price and sell the same amount of USDJPY at a higher price in the spot market, thus making a profit corresponding to the magnitude of the difference between spot exchange rate and the strike. If USDJPY goes under the strike at expiry, the option will be abandoned and expire useless with the option premium lost. The investor is able to keep the profit from foreign currency appreciation without giving it away to a hedging loss, which will be the case if we hedge using forward. However, it depends on how the spot exchange rate ends up, whether the hedging loss in forward hedging can justify the amount of option premium paid to eliminate this uncertain exposure. In other words, we can find a break-even point (BEP) between the two approaches. Another important factor to notice is the notional of the option contract we will be buying. The delta of an ATMF option is roughly 50%, which means it behaves only 50% like a forward. In the full hedging scenario, if we want to achieve the same hedging exposure as the forward hedging, we need to double the notional of the options bought. For example, if the total JPY exposure is worth 500,000 USD at the purchase date, in case of forward hedging we buy 500,000 worth of forward but in case of options, we need to buy 1,000,000 worth of contract to get the same position.

For each year, we buy a new USDJPY call and a new EURUSD put with new ATMF strikes and new notional amounts. To perform monthly option valuation, we use SuperDerivatives in combination with Bloomberg to get the option prices between 2005-2016. For simplification, we don't take bid-ask spread of the option price into consideration, though it might make a difference to the option price. Instead, we directly use the mid-price. We feed all the data into GA currency hedging algorithm in R. The objective of the option hedging optimization remains same as in the forward hedging case.

The table below lists the contract details of all the option hedging instruments across years.

Table 6.1 Contract Details of Option Hedging Instruments, 2005 -2016

	USDJPY Call Notional (USD)	USDJPY Call Premium (USD)	USDJPY Call Premium (in %)	EURUSD Put Notional (USD)	EURUSD Put Premium (USD)	EURUSD Put Premium (in %)
2005	1'000'000	33'528	3.4%	1'000'000	40'204	4.0%
2006	1'222'290	36'850	3.0%	1'110'867	37'561	3.4%
2007	1'292'455	32'480	2.5%	1'509'173	36'540	2.4%
2008	1'223'674	37'816	3.1%	2'040'223	62'943	3.1%
2009	873'159	49'458	5.7%	1'164'966	86'657	7.4%
2010	1'012'708	57'810	5.7%	1'478'955	77'548	5.2%
2011	1'126'427	58'170	5.2%	1'604'168	89'062	5.6%
2012	982'087	40'616	4.1%	1'325'232	79'936	6.0%
2013	1'070'443	39'838	3.7%	1'740'954	60'727	3.5%
2014	1'381'935	60'716	4.4%	2'275'652	75'936	3.3%
2015	1'301'462	54'052	4.2%	2'056'403	69'476	3.4%
2016	1'414'321	48'618	3.4%	2'022'826	82'675	4.1%

Table 6.1 above presents the yearly notional amounts of the USDJPY call options and EURUSD put options that are bought as hedging instruments. The option premiums are derived from SuperDerivatives. From the data, we can observe a clear jump of the insurance prices from beginning of 2009 for both the contracts as the global financial crisis culminated in autumn 2008 and investors flooded to seek portfolio protection from the option derivatives. For Japanese Yen option, it calmed down after 2012 while EURUSD option continued its high risk premium due to the burst of Eurozone sovereign debt crisis. Option premiums give us a clear intuition of the market sentiment of risk regimes. During the peak of the crisis, option becomes an extremely expensive asset to purchase and failing to break-even such high cost might result in cost-inefficiency of option contracts compared to forward contracts.

6.3.1 Case 4 2-asset portfolio hedging with European vanilla options

Benchmark – Unhedged Portfolio Local Currency Performance

In the similar fashion as the three hedging cases using forward contracts presented in Section 5, we will implement one hedging case using options, but only with the local currency benchmark. GA outputs the following results:

Table 6.2 GA Output, Case 4, Local Currency Benchmark

Constraint Level	JPY Hedge Ratio	EUR Hedge Ratio
8.20%	0.999	0.953

Both hedge ratios are close to 1, suggesting that we would be better off if we conduct full hedging for both currencies. However, the time series cross-validation results show a mixed picture of its effectiveness. Compared to the forward hedging approach, it is underperforming in most of the years.

Table 6.3 7-Fold Time Series Cross-Validation, Case 4

	Relative Return Increase for Test Data	Relative Volatility Reduction for Test Data	Relative Sharpe Ratio Improvement for Test Data
Fold 1	63%	34%	148%
Fold 2	-19%	-18%	-1%
Fold 3	-42%	21%	-26%
Fold 4	-37%	14%	-26%
Fold 5	3%	-7%	-3%
Fold 6	-29%	-4%	-32%
Fold 7	-74%	14%	-70%
Mean	-19.1%	7.9%	-1.4%
Variance	0.1875	0.0327	0.4871

In order to understand the underlying dynamics of forward hedging and option hedging, we select year 2015 as a sample year to study the currency movements' impact on the hedging P&L profile.

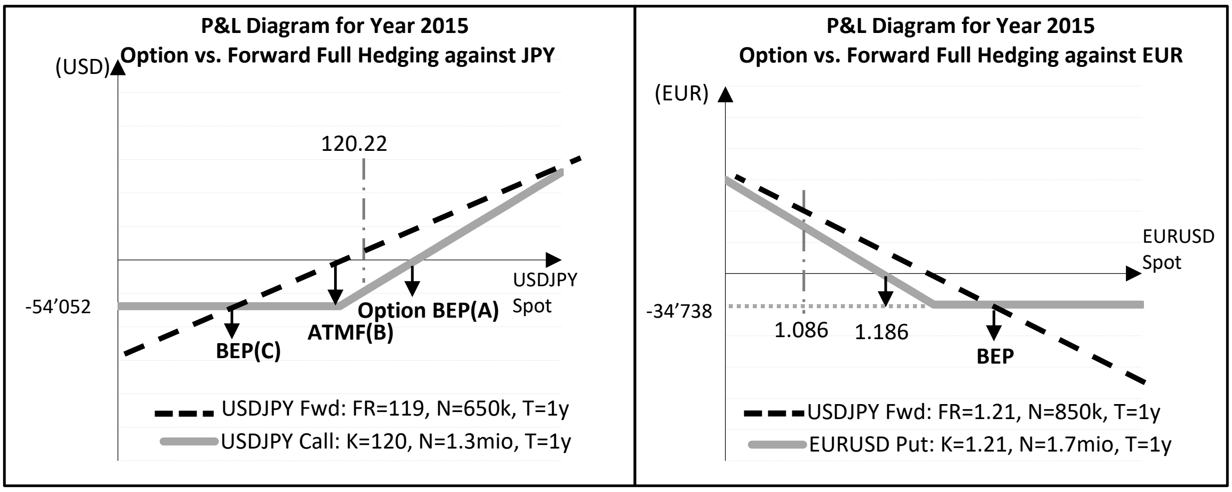


Fig. 6.5 P&L Diagram for USDJPY Call Option and EURUSD Put Option and Corresponding Forwards in Year 2015. BEP is an abbreviation for break-even price and ATMF refers to at-the-money-forward price. The details of the forward contracts and option contracts are listed.

Let's have a closer look at the above P&L Diagram and take the JPY one (left diagram in the above figure) as the example. At the break-even point of the USDJPY call option (Point A), the option gain is just enough to cover the option cost. Above point A, the payoff of the option is twice as large as that of the forward, as given the equal spread of the exchange rates, the option notional doubles that of the forward. But the expensive premium has to be deducted to calculate the option profit and it makes the comparison uncertain. In that area, JPY depreciates against USD, resulting in a currency loss and this loss is compensated by both forward and option hedging strategies, but to different extents. Point B is the ATMF point, the 1y forward rate of the USDJPY forward and the strike of the option. Theoretically they should be the same, but as explained earlier, due to the market convention of option strikes for currency pairs, there is a slight deviation in our case. Between point A and point B, option is gaining but not yet able to cover the cost, while forward is making a bit profit which is offset by the spot currency loss. The break-even point between option hedging and forward hedging lies on point C, where the forward is losing as much money as the option premium. Between point B and point C, the option expires useless and the forward is also suffering hedging loss. But in this area, JPY is strengthening and making a spot currency profit. Below point C is when options are particularly useful because forward hedging gives away all the gain from the JPY appreciation while the option gives us the possibility to fully capture the benefit of the favorable currency movement, despite a fixed premium loss.

Comparing the two hedging strategies, they both agree on no hedging of Japanese Yen. As we can see from the option, the final USDJPY spot of 120.22 doesn't even earn back the premium, having a negative effect on the hedging return. In the forward hedging, it makes a little bit profit but the more important reason to leave it unhedged is because of JPY's counter correlation with assets that helps to reduce portfolio variance. For Euro, the forward hedging hedges 98% and option hedging hedges 72% away. Euro depreciated more than 10% in 2015 and it would result in huge currency loss if left unhedged in the portfolio. Both hedging strategies are effective in offsetting against it. However, option was bought at a cost of 3.5% of the portfolio value, which is a huge monetary outlay compared to forward hedging (excluding the potential transaction costs, EUR forward premium positive for this year). Therefore, option hedging significantly deteriorates the hedged portfolio performance.

7 Results and Future Work

In this thesis, we studied the impact of currency hedging on the risk and return profiles of multi-currency portfolios. Through the mathematical representation of portfolio return and volatility with and without hedging, it builds up an initial case for the effectiveness of currency hedging. In a fully-hedged portfolio, forward premium or discount will replace the role of exchange rate spot return, thus eliminating the impact from foreign exchange fluctuations. We are also aware of the importance of the categories of portfolio base currency, which is an essential factor to consider in the hedging strategy.

A simple 2-asset empirical case study presents a mixed picture for the currency hedging decision. Our traditional expectation with regards to macroeconomic cycle's and base currency strength's influence on the hedging outcome is not confirmed. Possible explanation is that, firstly the base currency strength is not an absolute guarantee for the hedging decision. It still largely depends on the foreign currency composition of the portfolio and its dynamics with those currencies. Secondly, there is no doubt that the general market themes drive currency movements. However, there is no fixed relationship between the two since exchange rates are influenced by a large universe of factors. We can draw the conclusion that during a financial stress period when uncertainties prevail, market volatilities surge and asset prices plunge, currency hedging can provide sufficiently good risk protection to the international portfolios.

The core part of this thesis is dedicated to the construction of a systematic portfolio currency hedging algorithm. Our revised objective function to the hedging optimization problem can well maximize the risk mitigation effect by the GA hedging solution and limit the variability of such benefit over the years. Through the implementation and cross-validation of three application cases, we see that there is an on average excellent performance of the GA hedging model in achieving better portfolio performance, in terms of return, volatility and Sharpe ratio. When we use two different measurement perspectives, the results might differ. In most cases, the USD benchmark hedging results outperform the local currency benchmark, as the negative currency movements in the sample period make the local currency benchmark return much higher and a more difficult hurdle to overcome. The hedging results also show a robust performance over different cases, confirming its applicability in multi-currency portfolios and multi-asset portfolios. The time series cross validation procedure also validates our hedging model in each of the application case. However, there remains much to be expanded, especially in terms of the third application case, where the investor can hedge the portfolio currency risk with a more generic toolbox.

Currency hedging with options gives us another fresh perspective to look at the currency hedging issue. Options have different P&L profiles than forwards, but option strategies are more versatile and flexible. Another future direction of research lies in the option domain, to study the strike space of vanilla options or the currency hedging with various option strategies.

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