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A comprehensive reliability test of technical trading strategies

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Abstract

This thesis attempts to study the reliability of technical trading strategies in the recent decade. It has tested strategies with more than $15,000$ trading setups. We propose a randomized trading strategy as one of the benchmarks, which randomly selects and trades stocks. We study the empirical distributions of risk adjusted performances resulted from technical trading and randomized trading. None of the tested strategies are expected to consistently generate skill-based excess profits compared to the market. However, We do find the existence of technical trading setups that consistently generate significantly worse performance compared to the market and luck. Additionally, we have explored the predictability of technical strategies on direction of stock price movement, as well as the importance of stock selection using technical trading strategies.

Keywords trading strategy, technical analysis, randomized trading, hypothesis test, empirical distribution, financial market, efficient market hypothesis, time series

Contents

List of Tables

List of Figures

Chapter 1

Introduction

Technical trading strategies aim to predict security prices of financial markets based on the analysis of the historical price movement, without consideration of fundamental and economic information[10]. Technical indicators are quantitative measures of the past price data[40]. The well-known Efficient Market Hypothesis (EMH) suggests that the financial markets are efficient if its prices fully reflect all relevant information. Under this hypothesis, it is unlikely to consistently exceed market average returns using technical analysis[25][11]. Many researchers in economics and finance have empirically tested EMH. Some agree with the weak form of EMH, saying that technical analysis cannot be effectively helpful in making trading decisions[40][25]. The others argue that technical analysis may have been an important determination factor in forecast of the financial markets, such as international stock markets [40][25][24][17]. The professional investors have long-standing interests in technical trading strategies and their indicators when making investment decisions in real markets. Technical analysis only require historical stock data such as price movement and volume. Compared to the fundamental analysis, technical analysis save more research cost and time of the investors.

Much recent empirical research of technical trading show evidence of profitability with 3 types of technical trading strategies: price momentum, moving average crossover and pair trading [27][13][12][20]. C.H. Park and S.H. Irwin have reviewed 95 modern studies, out of which 56 studies conclude that at least 1 type of technical strategies may generate profitability in stock markets. However, C.H. Park and S.H. Irwin point out that most of empirical studies are subject to data snooping bias and ex-post selection problem [29]. Furthermore, investors are concerned about whether the market average excess returns generated by technical strategies are mostly due to luck. Researchers such as Barras, Laurent, et al. have developed technique to measure “skill” of investment funds with controls for false discoveries. The study finds skilled investments existing prior to 1996, but almost none by 2016[1]. Such research tests “skill” of outperforming strategies. However, if we accept the existence of “skill”, skill is not only there when the tested strategy significantly outperforms the market but also when it significantly underperforms. It is at our interest to test whether a technical strategy that consistently outperforms or underperforms the market by trading “skill” exists in the recent decade.

In this paper, we revisit and backtest the above mentioned technical strategies in U.S. stock market in recent decade. We do not focus on the best results performed by the testing strategies. We study the general statistics and distributions of all results of the strategies. We discuss whether an optimized result can be achieved by changing setups (i.e. indicators and parameters) of the strategy. We try to identify trading “skill” by studying significantly good and bad performances of the strategies. We test strategies with long and short positions, in order to study whether the tested strategy can consistently predict the direction of price movement. We discuss the importance of stock selection. We explore the sensitivity of risk adjusted performance of strategies by changing indicators and parameters. In addition, we explore the time series of stock prices in the sampling period.

The thesis contains 3 parts which are organized in 6 chapters. Besides introduction, Chapter 2 lists the key notations used in the thesis. Chapter 3 presents the first part: exploratory analysis of retrieved financial data. It describes the data cleaning process, statistics of daily stock prices and test the normality of the price movement. We also explore the features of financial time series through ARFIMA/GARCH model. The second part, setup of tested technical strategies, is introduced in Chapter 4. It also introduces the setup of benchmark strategies. The last part backtests the strategies. The backtesting methodology is demonstrated in Chapter 4 and the testing results are demonstrated in Chapter 5. Chapter 6 summarises the conclusion and thoughts for the future research.

Chapter 2

Notation

Exploratory analysis

r_t is logarithm stock return at day t

p_t is stock price (in U.S.dollar) at day t and $t \in [2, N]$

$ARFIMA(p, d, q)$ is the Auto-regressive Fractionally Integrated Moving Average model

p is the number of autoregressive terms (AR) and $p \in Z^+$

d is the order of differences and $d \in R$

q is the number of moving averages (MA) and $q \in Z^+$

GARCH(p, q) is the Generalized Autoregressive Conditional Heteroskedasticity model with $p \leq 3$ and $q \leq 3$

Price-moment strategy

m is the calibration length in month and $m \in \mathbb{Z}^+$

STK total number of stocks that are available to be selected in the trading portfolio of price-momentum investment and $STK \in Z^+$ and $STK \leq 505$

$P_{i(M)}$ is the closing price of stock i at the last trading day of month M

$r_{i(M)}$ is the monthly return of stock i on month M, where $i \in [stock_1, stock_2, ..., stock_{STK}]$

$r^{cum}_{im(M)}$ is the cumulative monthly return of stock i on month M with calibration length of m months

$r^{mean}_{im(M)}$ is the monthly mean return of stock i from month M_0 to $M_0 + m - 1$ with calibration length m

$r^{risk}_{im(M)}$ is the risk adjusted mean return of stock i on month M calibrated by m months

$\sigma_{im(M)}$ is the unbiased standard deviation of monthly returns of stock i from month M_0 to month $M_0 + m - 1$

$\lfloor \theta STK \rfloor$ is number of best (or worst) performing stocks that are selected to take the long (or short) position in the investment portfolio

$I_{\eta\theta(M)}$ as the trading account value in thousand U.S. dollars with η_s, η_l and θ taken by trading rule generated on month M

η_l is the percentage of trading account value that we invest into buying stocks per month

η_s is the percentage of trading account value that we invest into shorting stocks per month

k, j are respectively one of the selected stocks in long and short trading, and $k, j \in \{stock_1, stock_2, ..., stock_{\lfloor \theta STK \rfloor}\}$

$w_{\eta_l\theta k(M)}$ is the amount of money invested to buy the selected stock k on month M

$w_{\eta_s\theta j(M)}$ is the amount of money invested to short the selected stock j on month M

s is holding period in month

$R_{k(M)}$ is the simple monthly trading return of the selected stock k on month M

$\Delta I_{\eta_l\theta(M)}$ is the change in trading account value that generated from the long position investment on month $M - s$ and liquidated on month M

$\Delta I_{\eta_s\theta(M)}$ is accordingly the change in account value generated from the short position on month M

Moving average crossover strategy

$SMA(\lambda)_t$ is Simple Moving Average at trading day t with calibration length of λ days

$EMA(\lambda)_t$ is Exponential Moving Average at trading day t with calibration length λ days

X is the EMA multiplier

$MA(\lambda)_t$ is the general notation of moving average indicator, $MA(\lambda)_t \in \{ EMA(\lambda)_t, SMA(\lambda)_t\}$

POS_t is the trading position taken according to the trading signals generated at the end of day t where $POS_t \in \{-1, 0, 1\}$

$I_{MA,\lambda(t+1)}$ is the trading account value on day $t + 1$, resulted from crossover strategy with indicator $MA(\lambda)_t$

R_{t+1} is the simple daily price return at $t+1$ day

$(MA(\lambda_1)_t, MA(\lambda_2)_t)$ is the combination of 2 indicators with different length of λ and $(MA(\lambda_1)_t, MA(\lambda_2)_t) \in \{(EMA(\lambda_1)_t, EMA(\lambda_2)_t),(SMA(\lambda_1)_t, SMA(\lambda_2)_t)\}$

sl is the stop loss ratio and $sl \in \{0.00, 0.01, \ldots, 0.03\}$

Pair trading strategy

κ is the calibration length, also explained as number of trading days in the pair formation period

$(stock_x, stock_y)$ is any combination of 2 stocks in the same sector

(x, y) is the selected pair to be invested in the trading period

$\rho_{x'y'}$ is the estimated correlation of stock prices of pair $(stock_x, stock_y)$ within κ days

$p_{x't}$ is closing price of $stock_x$ at trading day t

$\overline{p}_{x'}$ is sample mean of closing prices of $stock_x$ within κ days

Pr_t is the price ratio of pair (x, y) at trading day t

$\overline{Pr}_{\kappa t}$ is sample mean of price ratio of pair (x, y) estimated within the calibration period of κ days

$\sigma_{\kappa t}$ is unbiased standard deviation of price ratio of (x, y) estimated within the calibration period of κ days

$TS_{\mathrm{x}t}$ is the general form of trading signals generated for stock x at day t

$POS_{\mathrm{x}t}$ is the position we hold for stock x at day t

$I_{\kappa\alpha\beta\eta'(t+1)}$ is the trading account value obtained from pair trading strategy at day $t+1$

η'_x is the percentage of the trading account value that we daily invest into stock x

η'_y is the percentage of the trading account value that we daily invest into stock y

$R_{\mathrm{x}(t+1)}$ is the daily simple return generated from investment of stock x at day $t+1$

Benchmark strategies

$R_{longandhold(t+1)}$ is daily simple return of SP500 index

$I_{longandhold(t+1)}$ is the value of trading account generated by long and hold of SP500 at day

$t+1$

$p_{index(t)}$ is the closing price of SP500 index at day t *stock* is the stock randomly selected among the available SP500 stocks at trading day t

$p_{stock(t)}$ is the closing price of *stock* at day t

$I_{random(t)}$ is the trading account value resulted from randomized trading at day t

Measures of financial performance

I_s is a general notion of trading account values at trading day s and

$I_s \in \{I_M, I_{MA,\lambda(t+1)}, I_{\kappa\alpha\beta\eta'(t+1)}\}$

AR is annualised return of trading strategy

CR is the cumulative return on initial trading capital

ϵ_T is annualised volatility

R_s^{acc} is the simple return of trading account value in the tested period s

σ_s is the sample standard deviation of simple returns of account value in the tested period s

SH is Sharpe ratio

RF_s is the annual risk free rate of return

SR is Sortino ratio

DR is the downside standard deviation

C is Calmar ratio $MDD(\mathrm{T})$ is the maximum drawdown until T

pred is frequency that the trading position gains profitability

w is the number of trading days that have achieved a positive simple investment return

v is total number of trading days in the testing period

Backtesting of strategies

$\hat{F}_m(x)$ is the unbiased estimator of $F(x)$

$F_0(x)$ is the empirical distribution function of the benchmark randomized trading strategy

$F_{pair}(x)$ is the empirical distribution of pair trading strategy

$F_i(x)$ and $F_j(x)$ are all combination of the empirical distribution functions $F_{mom1}(x), ..., F_{mom6}(x)$

$F_k(x)$ and $F_n(x)$ are all combination of the distribution functions $F_{MA1}(x), F_{MA2}(x), ..., F_{MA4}(x)$

$G_{mom1}(x), G_{mom2}(x)..., G_{momN}(x)$ are the empirical distribution functions of the momentum trading strategy

$G_{MA1}(x), ..., G_{MAK}(x)$ are the distributions of moving average crossover strategy

A^2_{mn} is Anderson-Darling test statistic (also AD distance)

$CUMR$ is cumulative return of the trading strategy

Chapter 3

Exploratory Analysis

3.1 Data description

In the exploratory analysis, we analyse the performance of trading strategies based on daily stock prices of SP500 index and its constituents in the past 10 years from 01 January 2009 to 06 March 2019. Our data are sourced from Yahoo Finance. The dataset contains $1,239,540$ observations (rows) and 4 variables (columns), namely "symbol","sector","date" and "close". Stock symbols of 504 constituents and the index are recorded in column "symbol". The number of the constituents is more than 500 because companies such as Facebook were not counted as SP500 until the recent years. Some other companies were eliminated from the SP500 due to its unfavorable performance. The stock prices of the companies are recorded as NA when these companies are not included in the SP500 at the corresponding dates. The corresponding industries of the constituents are recorded in column "sector". We define closing price as daily adjusted closing price with dividend and split adjustment. Due to the computational complexity and limited data availability, we assume the opening price at day t the same as the closing price at day $t-1$. The closing prices are recorded in column "close". Minimum 195 and maximum $2,559$ trading days are identified for stocks during the past 10 years period. On average, there are 251 trading days per year. We clean the dataset and explore missing values, extreme values and unusual observations. $1,983$ missing values (shown as "NA") are identified in data of 3 stocks between year 2009 and 2018. With the investigation of these stocks, we omit all NAs since either no price records exist for the correspondent stock and date, or the stock was not included in the SP500 at the correspondent date. The possible reasons of no price records are that 1) the stocks were de-listed and 2) the company was under a significant change such as mergers and acquisition. No stock price is observed to be 0 or below 0. Neither do we find abnormal value of the dates. 52 (equivalent to 10.30%) stocks have less than $2,559$ observations of daily closing price. Figure 3.1 shows the histogram of the number of stocks with less than $2,559$ records of close. The reason is that some companies such as Facebook have not been listed in the SP500 since 10 years ago. The other companies such as Google have history of de-listing and dual-listing. Among these stocks, 4 have less than 500 closing prices. Table 3.1 demonstrates number of daily closings of these stocks. We consider the number of observations sufficient for daily trading. We hence keep these stocks in the dataset.

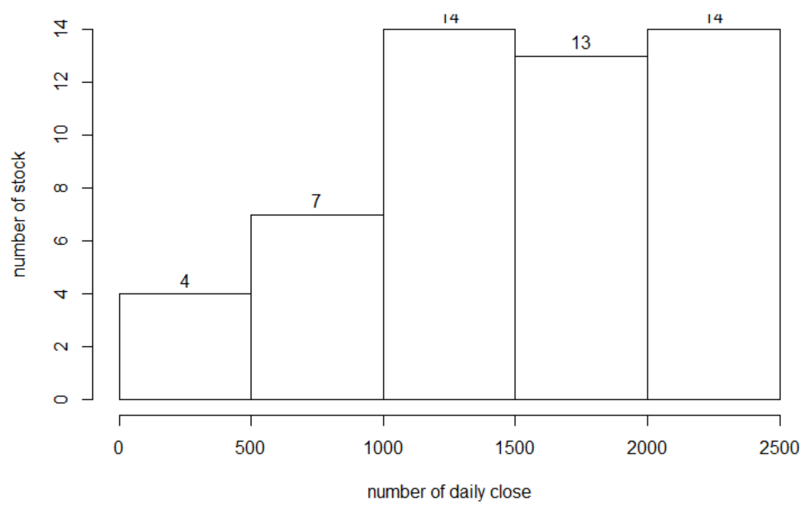

Figure 3.1: Summary of the number of stocks with less than $2,559$ records of daily closing price

Symbol	Number of daily close
DGX	285
EVRG	195
LIN	195
PPG	286

Table 3.1: Summary of stocks with less than 500 daily closing prices recorded in the analysed dataset

3.2 Price description

We explore the stock closing prices of SP500 index and its constituents in the past 10 years. Table 3.2 summarises the statistics.

Price	**SP500 index**	**SP500 constituents**
No. of observations	2559	1 236 981
Min.	676.530	0.001
1st Qu.	1283.630	25.724
Median	1836.250	43.880
Mean	1781.223	65.108
3rd Qu.	2145.205	74.160
Max.	2930.850	2206.090
Standard deviation	573.093	92.100

Table 3.2: Descriptive statistics for daily closing prices of SP500 index and its constituents in the past 10 years from 01 January 2009 to 06 March 2019

According to the findings of S.I. Ivanov et al.(2013) that explored the SP500 stock prices from 01 July to 18 March 2011, the observed closeness of mean and median price implies the normality of SP500 index [19]. However, we do not observe this closeness of median and mean continues in our sampling period. The median and mean of constituents also vary from 43.880 to 65.108. As S.I. Ivanov et al.(2013) stated, the closeness does not formally test the normality, we apply several normality tests in the next section. Additionally, the price range of SP500 constituents varies from 0.001 to 2206.090. The index has price range between 676.530 and 2930.850.

Figure 3.2 illustrates the historical closing prices of SP500 index. We observe an uptrend in the first 9 years. This trend seems to vanish from year 2018. While the variance seems consistent in the first 9 years, it increases from year 2018. We also plot the historical prices of SP500 constituents. Figure 3.3 present 32 stocks as the examples. The graphs of these stocks are randomly selected. We identify parts of stocks with similar price behavior to the index in the past 10 years. Nevertheless, prices of more than half of stocks do not follow a continuous uptrend. Moreover, similar price behaviors of SP500 stocks imply the existence of a positive correlation.

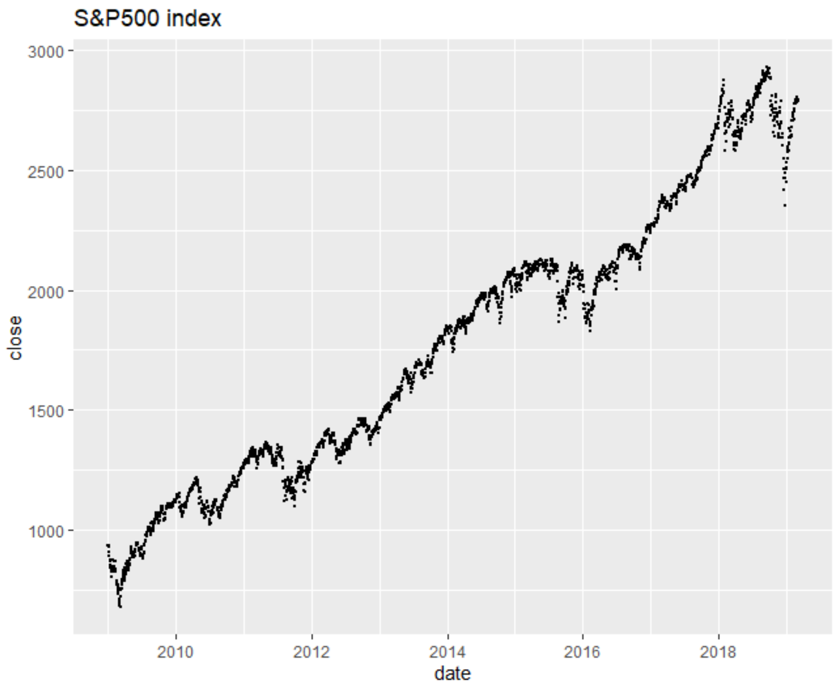

Figure 3.2: Stock price behavior of SP500 index in the past 10 years from 01 January 2009 to 06 March 2019

3.3 Normality test of price change

The normality of stock price changes has been widely studied by researchers. B. Mandelbrot and H. M. Taylor(1967) summarise thoughts of distribution to be 1) Gaussian and 2) stable Paretian[26]. J. Teichmoeller(1971) argues that stock prices do not appear to be distributed as a simple mixture of normal distributions[41]. We test the normality of daily price change in the past 10 years, by similar statistical methods implemented in the study of S.I. Ivanov et al.(2013). We conduct the parametric test (Jarque Bera test) and non-parametric test (Shapiro Wilk test) on daily stock price change. Compared

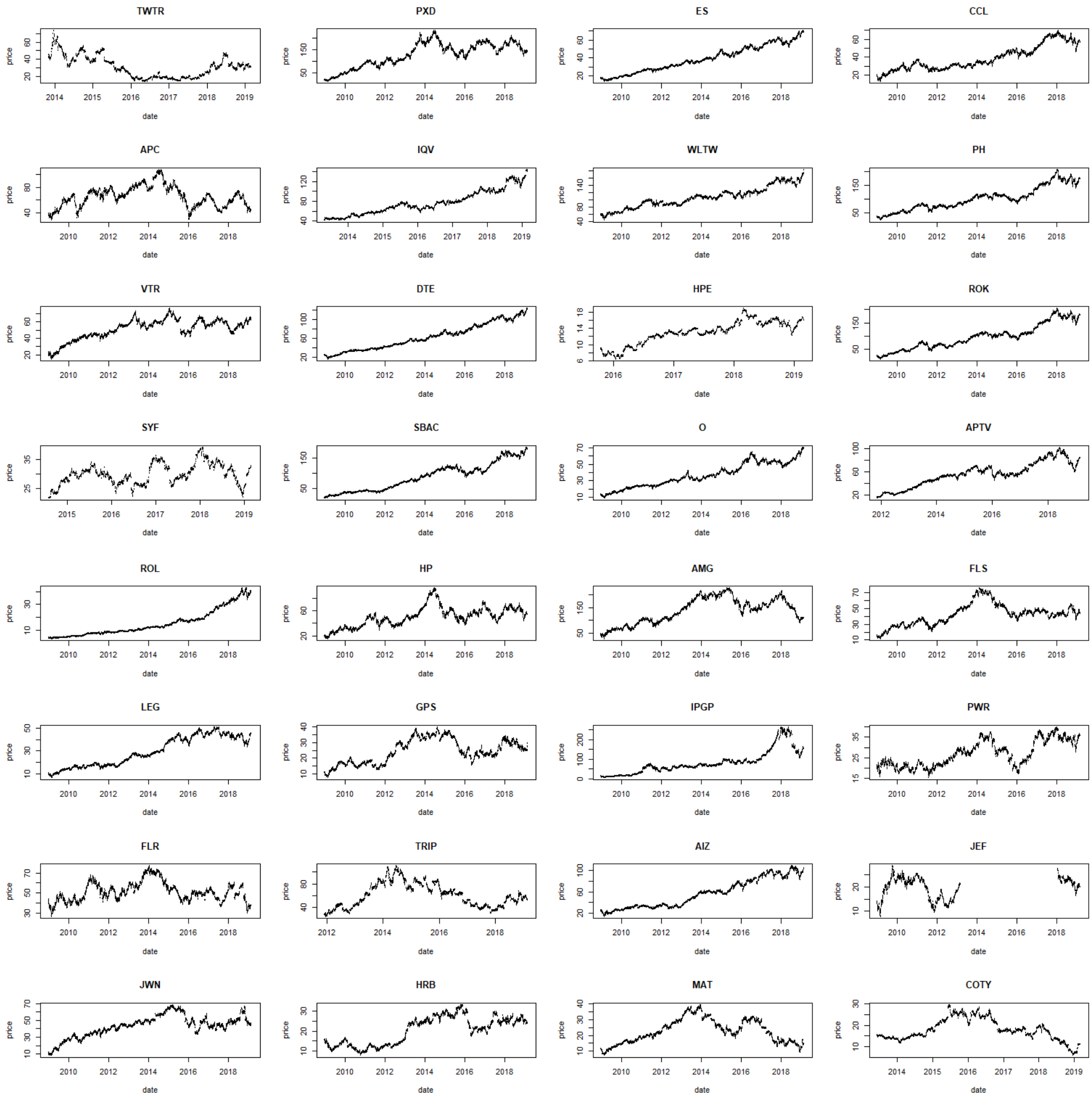

Figure 3.3: Examples - historical stock prices of SP500 constituents in the past 10 years from 01 January 2009 to 06 March 2019

to Kolmogorov Smirnov test conducted by S.I. Ivanov et al.(2013), Shapiro Wilk test is found more powerful for a test with sample size larger than 1,000, according to N.M. Razali and Y.B. Wah(2011)[32]. In the study of B.W. Yap and C.H. Sim(2011), Shapiro Wilk test also shows good power properties in a wide range of normality tests for both asymmetric and symmetric distributions[43]. We have null hypothesis: the differences of daily stock prices follows a normal distribution.

Table 3.3 summarises the results of the normality tests. For SP500 index and 503 constituents, the null hypotheses are rejected by both Shapiro Wilk test and Jarque Bera test. Table 3.4 shows details of the stock whose price changes cannot reject the null hypothesis. We hereby conclude that except for stock LIN, the daily price difference of tested stocks do not follow normal distribution.

	SP500 index	**SP500 index and constituents**	
Test	p-value	p-value>0.05	p-value<0.05
Shapiro Wilk	8.7e-32	1	504
Jarque Bera	<2.2e-16	1	504

Table 3.3: Summary of normality test results for daily stock price change of SP500 index and its constituents in the past 10 years from 01 January 2009 to 06 March 2019

Stock	Shapiro Wilk test	Jarque Bera test
LIN	0.152	0.054

Table 3.4: Test results of daily stock price change of the stock with the normality

We further visualize the sample quantiles against normal quantiles of stock price changes in the past 10 years. Figure 3.4 shows the qq-plot and histogram of daily price changes of SP500 index. Figure 3.5 shows exampled qq-plots of price changes of randomly selected constituents. Not all stocks have a symmetric distribution of price changes. The heavy tailed property is observed in the most of plots. The finding is aligned with many existing studies. V. Chavez-Demoulin, P. Embrechts and S. Sardy(2014) describe the heavy tails as the ”well-known statistical feature” exhibited in time series of financial asset values[6].

Figure 3.6 plots the normality of the stock LIN. It confirms the results of statistic tests.

3.4 Modeling stock returns with ARFIMA/GARCH

Prior to the backtesting of trading strategies, we aim to understand the feature of our 10-years length time series of SP500 stocks. With the observed heavy tails feature, we define daily stock return in definition (3.1). The logarithm brings extreme values closer to the mean. G. Dorfleitner(2002) states that compared to the “simple return”, “log return (due to its time additivity) is suited for time series models”[9].

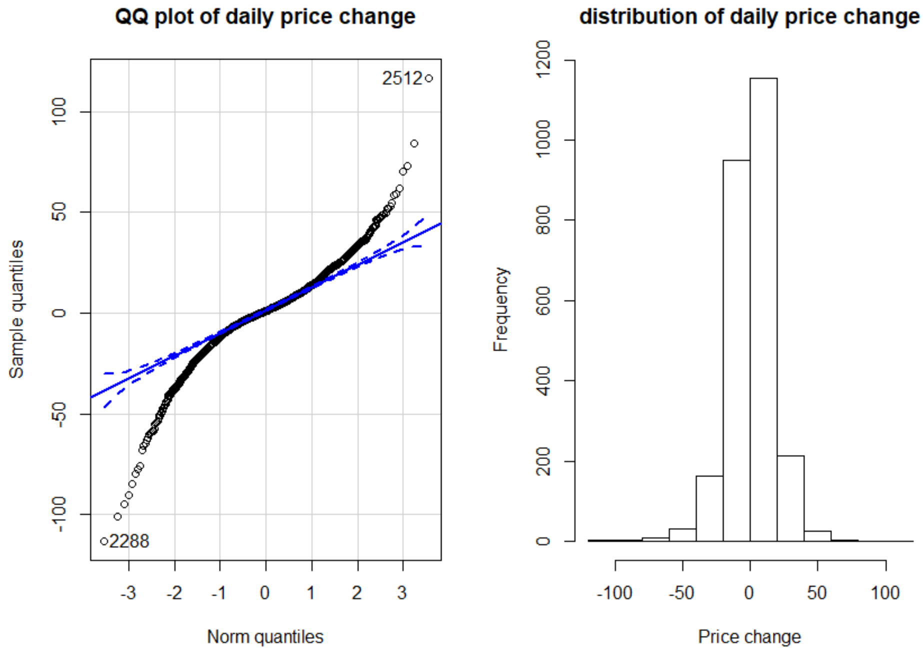

Figure 3.4: Price change qq-plot and histogram of SP500 index in the past 10 years from 01 January 2009 to 06 March 2019

$$r_t := ln(\frac{p_t}{p_{t-1}}) \tag{3.1}$$

where r_t is stock return at day t. p_t is stock closing price (in U.S. dollar) at day t, and $t \in [\,2,\, \mathbf{N}\,]$.

We fit time series of stock returns with the Auto-regressive Fractionally Integrated Moving Average (ARFIMA) model developed by C. Granger and R. Joyeux(1980) and J. Hosking (1981)[14][15][18]. Compared to Auto-regressive Integrated Moving Average (ARIMA) model, ARFIMA model allows order of differences d to be non-integer. Some existing studies (P. Bagavathi Sivakumar and V.P. Mohandas(2009),G. Bhardwaj and N.R.Swanson(2006)) exhibit that ARFIMA models have better empirical modeling ability on financial returns than traditional Box and Jenkins ARIMA models since the financial time series are believed featuring with "long memory and both short term and long term influences"[38][2]. The general form of ARFIMA(p, d, q) is expressed in equation (3.2), where p is the number of autoregressive terms (AR) and $p \in Z^+$; d is the order of differ-

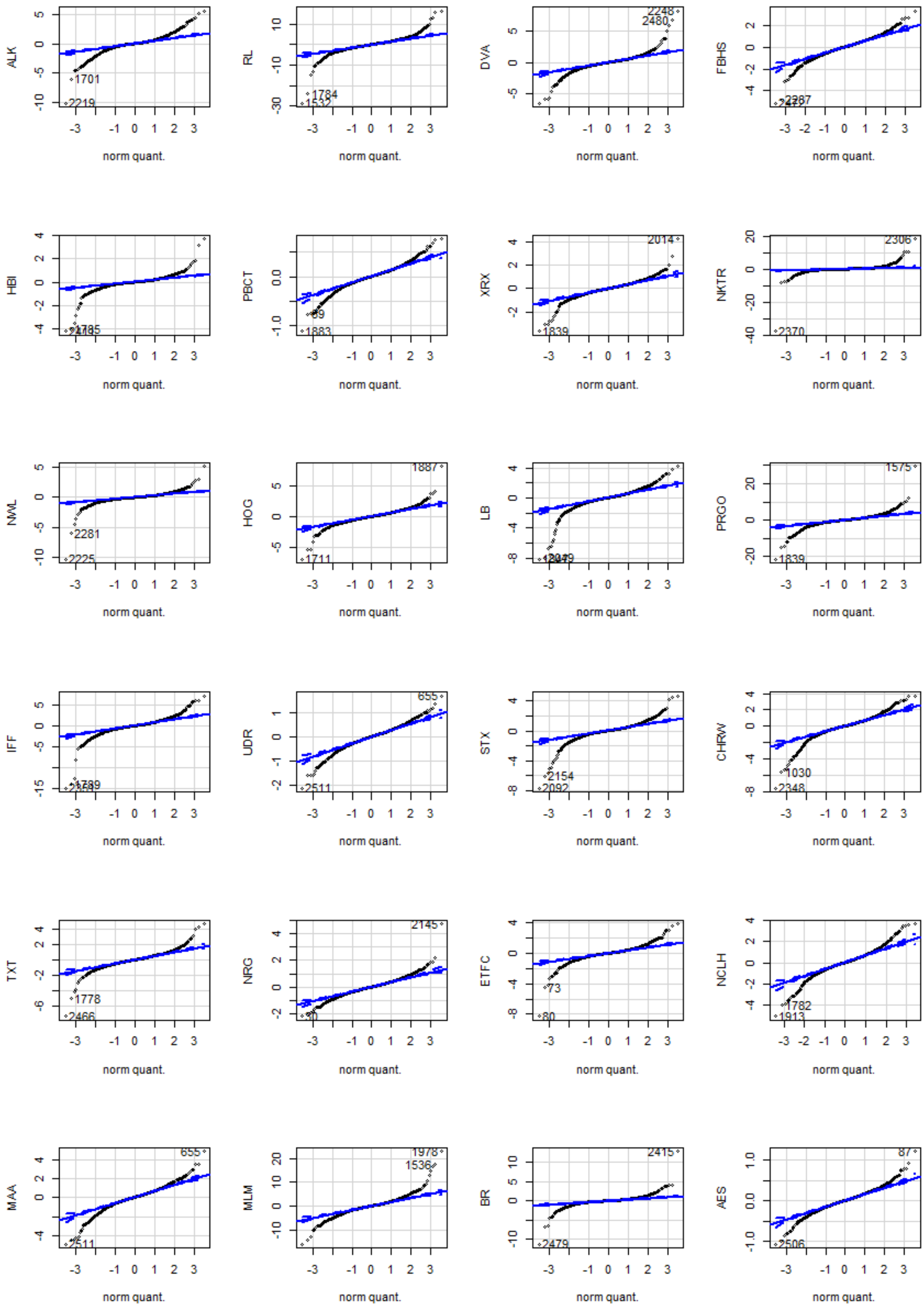

Figure 3.5: Examples - qq-plots of SP500 constituents price changes in the past 10 years from 01 January 2009 to 06 March 2019

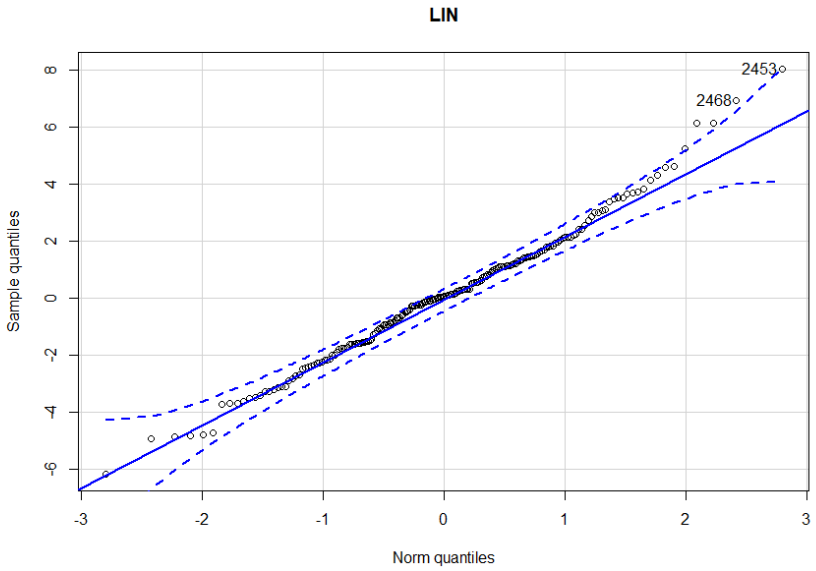

Figure 3.6: Empirical distribution of price change for the stock with normality hypothesis not rejected

ences and $d \in R$; and q is the number of moving averages (MA) and $q \in Z^+$. The vector X_t refers to the set of time series data, $t \in \mathbb{Z}^+$; ϕ_i and ϑ_i are respectively parameters of AR and MA , $\phi_i, \theta_i \in R$; B is lag operator and ε_t are error terms assumed to be i.i.d from zero-mean normal distribution[14][15][18].

$$(1 - \sum_{i=1}^{p} \phi_i B^i)(1 - B)^d X_t = (1 + \sum_{i=1}^{q} \vartheta_i B^i)\ \varepsilon_t \tag{3.2}$$

We assume the innovations are empirically t-distributed in the ARFIMA models due to the observed heavy tails. The fitted models are selected by Akaike information criterion(AIC). The summary of fitted model for the stocks is shown in table 3.5. All fitted models have 0 order of differences. It means an ARMA(p, q) can sufficiently describe the time series. 44 time series of stock returns are fitted by ARFIMA$(0, 0, 0)$. It shows no evidence of the dependent structure. We assume them to be independent. We also assume such time series to be identically distributed. Additional 57 fitted models have $AR = 0$, and the other 40 models have $MA = 0$. ARFIMA$(1, 0, 1)$ is fitted to describe the time series of 119 stocks (approximately 24% of total number of models) and 145 stocks have ARFIMA$(2, 0, 2)$ (approximately 29%). The time series of stock returns of SP500 index is fitted with ARFIMA$(2, 0, 2)$.

ARFIMA(p,d,q)	0,0,0	0,0,1	0,0,2	1,0,0	1,0,1	1,0,2	2,0,0	2,0,1	2,0,2
Number of model	44	47	10	37	119	44	3	56	145

Table 3.5: Summary of fitted ARFIMA models for stock returns of SP500 index and constituents from 01 January 2009 to 06 March 2019

We test whether the auto-correlations of residuals are 0 by Ljung-Box test and their normality by the above mentioned tests. Table 3.6 summarises the statistic test results. The Ljung-Box tests of 66 stocks reject the null hypothesis that residuals of the fitted model are independently distributed. All 505 models have at least one normality test with the null hypothesis of normality rejected.

	Auto-correlation	**Normality test**	
Test	Ljung-Box	Shapiro Wilk	Jarque Bera
p-value>0.05	439	0	1
p-value<0.05	66	505	504

Table 3.6: Summary of normality tests and auto-correlation tests for ARFIMA fitted residuals on stock return time series of SP500 index and constituents from 01 January 2009 to 06 March 2019

Due to the observed heteroskedasticity of the residuals, We apply Generalized Autoregressive Conditional Heteroskedasticity (GARCH) on the residuals of ARFIMA models. Due to the computational complexity, we limit GARCH(p, q) with $p \leq 3$ and $q \leq 3$. The theoretical student's t-distribution is assumed as the conditional distribution. Fitted models are selected by the lowest AIC. The selected GARCH models are summarised in table 3.7. 268 out of 505 models (approximately 53%) are fitted with GARCH$(1, 1)$. 464 fitted models have AR term equal to 1, which counts for around 92% of total number of models.

GARCH(p,q)	1,0	1,1	1,2	1,3	2,0	2,1	2,2	2,3	3,0	3,1
Number of model	2	268	93	101	1	2	14	21	1	2

Table 3.7: Summary of GARCH models fitted for ARFIMA residuals of SP500 index and constituents from 01 January 2009 to 06 March 2019

The test results of normality and serial correlation of GARCH residuals are the same as these of ARFIMA residuals shown in table 3.6. In figure 3.7, we visualize the sample quantiles of residuals against normal quantiles by qq-plots. The exampled GARCH models of constituents are randomly selected. Compared to the qq-plots of stock price change, the same features are observed. It shows that ARFIMA/GARCH models do not sufficiently capture the heavy tails. The model's goodness of fit is far from a reasonable expectation. More sophisticated studies are required.

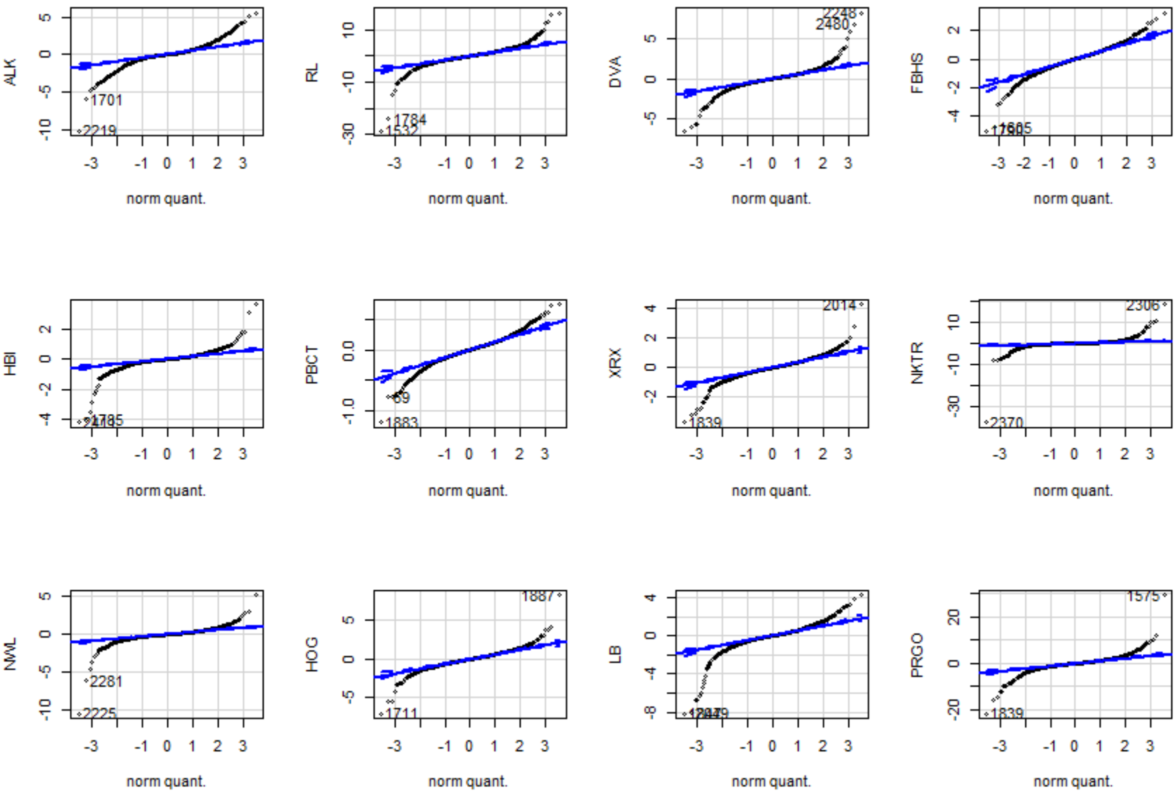

Figure 3.7: Examples - qq-plots of ARFIMA/GARCH residuals of time series of SP500 index and constituents from 01 January 2009 to 06 March 2019

Chapter 4

Methodology

In this chapter, we describe the setups of technical trading strategies and backtesting methodology. We trade SP500 stocks respectively by price-momentum, moving average crossover and pair trading strategies from 31 March 2011 to 06 March 2019. This starting date is selected because the listed companies have released the annual report of the previous year and the financial performance of $Q1$ or $H1$ (for companies with financial year ended in October) of the recent year. All positions are liquidated at the opening price of 06 March 2019, the date that we start writing this thesis. As we aim to find a trading strategy with consistent profitability, we take the length of trading period to be 8 years.

In order to simulate the equity trades conducted by professional investors, we set up the strategies based on the understanding and adjustment of trading rules introduced by the book "151 Trading Strategies" and "Technical Analysis of the Financial Markets: A Comprehensive Guide to Trading Methods and Applications"[21][27]. All of the trading strategies can take long and short positions indicated by the trading signals. They are dollar neutral without consideration of volatility weight on the positions. The trading lasts for $1,994$ days. In order to simulate the investment from professional investors, we assume that we have 1 million U.S. dollars of equity in cash or cash equivalents. We have a credit line from a bank that allows us to borrow maximum 9 million U.S. dollars. We can hence leverage our capital to a maximum 10 times. Therefore, the initial amount in our trading account is assumed to be 10 million U.S. dollars. We allow buying and selling maximum 10 million U.S. dollars per trading day. We hence allow tradings to be all-in trading. We only trade stocks at the opening per day. We assume that no intra-day tradings are possible due to the limited data availability. If the account value becomes 0 or negative, "break" is recorded at the trading day when 0 is reached. As we are interested in a consistently profitable trading strategy, no further trades are allowed and the account value remains as "break" until the last trading day of the testing period.

4.1 Price-momentum strategy

The price-momentum trading rules are trend following strategies believing that the observed trend continues in the next short period. Z. Kakushadze and J.A. Serur (2018) describe it as "buy the best performed stocks and sell the worst performed stocks"[21]. During a calibration period m months where $m \in [3, 4, ..., 12]$, the stock performance is measured by one of the 4 indicators summarised below. We omit stocks with insufficient records of daily closing price in the calibration period. We notate STK the total number of stocks that are available to be selected as the trading stocks of price-momentum trading, where $STK \in Z^+$ and $STK \leq 505$.

Monthly return Let $P_{i(M)}$ be the closing price of stock i at the last trading day of month M. $r_{i(M)}$ is the monthly return of stock i on month M, where $i \in [stock_1, stock_2, ..., stock_{STK}]$.

$$r_{i(M)} = \frac{P_{i(M)}}{P_{i(M-1)}} - 1 \tag{4.1}$$

Cumulative return Let $P_{i(M-m)}$ be the closing price of stock i at the last trading day of month $M - m$. $r^{cum}_{im(M)}$ notates the cumulative monthly return of stock i on month M with calibration length of m months.

$$r^{cum}_{im(M)} = \frac{P_{i(M)}}{P_{i(M-m)}} - 1 \tag{4.2}$$

Mean return Let $r^{mean}_{im(M)}$ be the monthly mean return of stock i from month M_0 to $M_0 + m - 1$ with calibration length m.

$$r_{im(M)}^{mean} = \frac{1}{m} \sum_{M=M_0}^{M_0+m-1} r_{i(M)} \tag{4.3}$$

Risk adjusted mean return We notate $r_{im(M)}^{risk}$ as the risk adjusted mean return of stock i on month M calibrated by m months. In equation (4.5), $\sigma_{im(M)}$ is the unbiased standard deviation of monthly returns of stock i from month M_0 to month $M_0 + m - 1$.

$$r_{im(M)}^{risk} = \frac{r_{im(M)}^{mean}}{\sigma_{im(M)}} \tag{4.4}$$

$$\sigma_{im(M)} = \sqrt{\frac{1}{m-1} \sum_{M=M_0}^{M_0+m-1} (r_{i(M)} - r_{im(M)}^{mean})^2} \tag{4.5}$$

We rank the stock performance represented by the indicator from the highest to the lowest. As for the selection of investment stocks, We choose $\lfloor \theta STK \rfloor$ number of stocks with the best performance (in the highest rank) and the same amount of stocks with the worst performance (in the lowest rank) where the parameter $\theta \in [0.01, 0.02, ..., 0.25]$. Therefore, we have $2\lfloor \theta STK \rfloor$ stocks selected for the investment. When taking $\theta = 0.01$, it represents that the top 1% best performed stocks are believed to follow the uptrend and the worst 1% stocks are expected to continue the bad performance by the investor. $max(\theta) = 0.25$ is taken to ensure that at most the first quantile of the best and the worst performed stocks are selected for the investment because more than 25% comparably good or bad stock performance could be caused by noise rather than a captured trend in the short period.

We notate $I_{\eta\theta(M)}$ as the trading account value in thousand U.S. dollars with η and θ taken by the trading rule on month M. The initial account value is $I_0 = 10,000$. Let η_l be the percentage of trading account value that we invest into buying stocks per month. Let η_s be the percentage of the trading account value that we sell stocks to enter a short position. We invest $\eta_l I_{\eta\theta(M)}$ thousand U.S. dollars for the long trading and $(\eta_s) I_{\eta\theta(M)}$ for the short trading on month M. We are interested in comparison between trades with long and short positions and trades with only long position. We hence take two combinations $\{\eta_l = 0.5, \eta_s = 0.5\}$ and $\{\eta_l = 1, \eta_s = 0\}$.

We would like to invest the equal amount of money buying or selling each selected stock. k, j are respectively one of the selected stocks in long and short trading. Given $k, j \in \{stock_1, stock_2, ..., stock_{\lfloor \theta STK \rfloor}\}$, we buy the selected stock k with $w_{\eta_l \theta k(M)}$ thousand U.S. dollars and short stock j with $w_{\eta_s \theta j(M)}$ formulated in equation (4.6) and (4.7).

$$w_{\eta_l \theta k(M)} = \frac{\eta_l I_{\eta\theta(M)}}{\lfloor \theta STK \rfloor} \tag{4.6}$$

$$w_{\eta_s \theta j(M)} = \frac{(\eta_s) I_{\eta\theta(M)}}{\lfloor \theta STK \rfloor} \tag{4.7}$$

We trade at the opening price of the first trading day on month $M+1$, which is considered as the closing price on the last trading day of month M. We hold the stocks for a holding length of s months and liquidate the positions at the closing price on the last trading day of month $M+s$. We always take $s = 1$ since we do not discuss the impact of holding length on the trading results in the scope of this thesis.

Equation (4.8) formulates the simple monthly return of the selected stock k on month M, denoted $R_{k(M)}$. The return of stock j with the short position is denoted $R_{j(M)}$.

$$R_{k(M)} = \frac{P_{k(M)}}{P_{k(M-s)}} - 1 \tag{4.8}$$

Let $\Delta I_{\eta_l \theta(M)}$ be the change in trading account value generated from the long position investment on month $M - s$ and liquidated on month M. $\Delta I_{\eta_s \theta(M)}$ is accordingly the change in account value generated from the short position.

$$\Delta I_{\eta_l \theta(M)} = w_{\eta_l \theta k(M)} \Big(\sum_{k=1}^{k=\lfloor \theta STK \rfloor} R_{k(M)} \Big) \tag{4.9}$$

$$\Delta I_{\eta_s\theta(M)} = -w_{\eta_s\theta j(M)}\Big(\sum_{j=1}^{j=\lfloor\theta STK\rfloor} R_{j(M)}\Big) \tag{4.10}$$

Therefore, $I_{\eta\theta(M)}$ is formulated in equation (4.11).

$$I_{\eta\theta(M)} = I_{\eta\theta(M-s)} + \Delta I_{\eta_l\theta(M)} + \Delta I_{\eta_s\theta(M)} \tag{4.11}$$

Table 4.1 summarises the setup of indicators and parameters for the price-momentum strategy. We define trading rule as a trading strategy with a set of indicator and parameter. In total, $1,550$ price-momentum trading rules are tested. The indicator monthly return is equivalent to the cumulative return with $m = 1$. We take calibration length m starting from 3 because data from at least 3 months are required to calculate the indicator risk adjusted mean return. With θ varying from 0.01 to 0.25, minimum $\lfloor 0.02STK\rfloor$ and maximum $\lfloor 0.5STK\rfloor$ stocks are selected to be invested per month. We test equally 775 rules with the combinations $\{\eta_l = 0.5, \eta_s = 0.5\}$ and $\{\eta_l = 1, \eta_s = 0\}$ in order to compare the performance of the long only positioned strategy and the strategy with long and short positions.

Indicator and parameter	**Description**
Indicator	$r_{i(M)}, r^{cum}_{im(M)}, r^{mean}_{im(M)}, r^{risk}_{im(M)}$
Calibration length (in month)	$m \in \{3, 4, ..., 12\}$
Holding period (in month)	$s = 1$
Percentage of stock number to be selected in the investment	$\theta \in \{0.01, 0.02, ..., 0.25\}$
Weight of trading account value to be invested in a long and short position	$\{\eta_l = 0.5, \eta_s = 0.5\}, \{\eta_l = 1, \eta_s = 0\}$

Table 4.1: Setup of indicators and parameters of price-momentum trading rules

4.2 Moving average crossover strategy

Moving averages are widely used by technical traders to smooth the price fluctuation in order to measure the trends. As the setup of crossover strategy does not involve a trading stock selection, we trade SP500 index with the crossover trading rules. We respectively use two well known indicators of crossover strategy described by J. Murphy (1999): the Simple Moving Average (SMA) and the Exponential Moving Average (EMA) [27]. We interpret the definitions in (4.12) and (4.13).

Let $SMA(\lambda)_t$ be SMA at trading day t with calibration length of λ days. As $1,994$ trading days are identified in the backtesting period, $\lambda, t \leq 1994$. p_w is the closing price at trading day w.

$$SMA(\lambda)_t = \frac{1}{\lambda} \sum_{w=t-\lambda+1}^{t} p_w \tag{4.12}$$

Compared to SMA, EMA gives higher weight on recent price changes than the past data[27]. Let $EMA(\lambda)_t$ be EMA at trading day t with calibration length λ days. $EMA(\lambda)_t$ is formulated in (4.13). X is the EMA multiplier defined in (4.14).

$$EMA(\lambda)_t = \begin{cases} SMA(\lambda)_t, & \text{if } t = \lambda \\ X\Big(p_t - EMA(\lambda)_{t-1}\Big) + EMA(\lambda)_{t-1}, & \text{otherwise} \end{cases} \tag{4.13}$$

$$X := \frac{2}{\lambda + 1} \tag{4.14}$$

To simplify the notation in the trading rules, we notate the indicator as $MA(\lambda)_t \in \{EMA(\lambda)_t, SMA(\lambda)_t\}$. We implement two types of trading rules - price crossover and double crossover.

Price crossover trading rule generates a long signal when stock price cross above the single indicator $MA(\lambda)_t$ and generates a short signal when stock price cross below the indicator[27]. We notate POS_t the trading position taken at day t where $POS_t \in \{-1, 0, 1\}$. Table 4.2 summarises trading signals generated by the comparison of the indicator and closing price at day t, as well as POS_t.

Trading signal generated at day t triggers the execution of entering or liquidating posi-

Condition	**Trading signal**	POS_t
$p_t > MA(\lambda)_t$	liquidate short position and enter long position	1
$p_t < MA(\lambda)_t$	liquidate long position and enter short position	-1
$p_t = MA(\lambda)_t$	liquidate the current position	0

Table 4.2: Summary of trading signals generated at day t and trading position taken at day t by price crossover method

tions at the opening price of day $t+1$. $I_{MA,\lambda(t+1)}$ is denoted as the trading account value at day $t+1$, resulted from crossover strategy with indicator $MA(\lambda)_t$. Equation (4.15) formulates $I_{MA,\lambda(t+1)}$, known that I_0 is 10 million U.S. dollars. R_{t+1} in definition (4.16) is the simple daily price return at $t+1$ day.

$$I_{MA,\lambda(t+1)} = \begin{cases} I_0(1 + POS_t \cdot R_{t+1}), & \text{if } t = 1 \\ I_{MA,\lambda(t)}(1 + POS_t \cdot R_{t+1}), & \text{otherwise} \end{cases} \tag{4.15}$$

where

$$R_{t+1} := \frac{p_{t+1}}{p_t} - 1 \tag{4.16}$$

We summarise the setup of price crossover trading rules in table 4.3. $1,002$ trading rules are tested. λ is taken from 2 to 502 because at least 2 data points are required to calculate the mean. 502 trading days represents a 2 years calibration period with averagely 251 trading days per year. Equally, 501 rules are tested with indicator $SMA(\lambda)_t$ and with $EMA(\lambda)_t$.

Indicator and parameter	**Description**
Indicator	$SMA(\lambda)_t, EMA(\lambda)_t$
Calibration length (in day)	$\lambda \in \{2, 3, \ldots, 502\}$

Table 4.3: Summary of setup of price crossover trading rules

Double crossover trading rule compares two indicators with different calibration lengths. We hereby notate $(MA(\lambda_1)_t, MA(\lambda_2)_t)$ as the combination of 2 indicators with different length of λ. $(MA(\lambda_1)_t, MA(\lambda_2)_t) \in \{(EMA(\lambda_1)_t, EMA(\lambda_2)_t),(SMA(\lambda_1)_t, SMA(\lambda_2)_t)\}$

where $\lambda_1, \lambda_2 \in \mathbb{Z}^+$ and $\lambda_1 < \lambda_2$. A long signal is generated when $MA(\lambda_1)_t$ is greater than $MA(\lambda_2)_t$ and a short signal is generated when it becomes smaller than $MA(\lambda_2)_t$[27]. Table 4.4 demonstrates the signals generated by $MA(\lambda_1)_t$ and $MA(\lambda_2)_t$, as well as the trading position taken according to the closing price at day t. The correspondent value of trading account $I_{MA,\lambda(t+1)}$ refers to equation (4.15).

Condition	**Trading signal**	POS_t
$MA(\lambda_1)_t > MA(\lambda_2)_t$	liquidate short position and enter long position	1
$MA(\lambda_1)_t < MA(\lambda_2)_t$	liquidate long position and enter short position	-1
$MA(\lambda_1)_t = MA(\lambda_2)_t$	liquidate the current position	0

Table 4.4: Summary of trading signals generated at day t and holding position at day t by double crossover method

According to Z. Kakushadze and J.A. Serur (2018), a "stop-loss" ratio, denoted as sl, can be added to the double crossover method in order to stop the profit loss from the unexpected price change. The rule requires the liquidation of trading position when a daily price change at day t that leads to a profit loss reaches sl in percentage compared to the price at day $t-1$ [21]. Table 4.5 describes the trading signals generated at day t and trading position taken at day t.

Condition	**Trading signal**	POS_t
$MA(\lambda_1)_t > MA(\lambda_2)_t$ $p_t < (1-sl)p_{t-1}$	liquidate position	0
$MA(\lambda_1)_t > MA(\lambda_2)_t$ $p_t \geq (1-sl)p_{t-1}$	liquidate short position and enter long position	1
$MA(\lambda_1)_t < MA(\lambda_2)_t$ $p_t > (1+sl)p_{t-1}$	liquidate position	0
$MA(\lambda_1)_t < MA(\lambda_2)_t$ $p_t \leq (1+sl)p_{t-1}$	liquidate long position and enter short position	-1
$MA(\lambda_1)_t = MA(\lambda_2)_t$	liquidate the current position	0

Table 4.5: Summary of trading signals generated at day t and trading position taken at day t by double crossover method with a stop loss ratio sl applied

Table 4.6 indicates the setup of the double crossover trading rules. Due to computational complexity, we are not able to test all possible combinations of λ_1 and λ_2. We limit the calibration lengths to be a multiple of 5. We are interested in whether the application of stop loss ratio could improve the trading performance. We hereby test trading rules respectively with and without a stop loss ratio. sl is set to be maximum 3% in order to avoid the extreme case of a continuous profit loss that will cause less than half of the investment capital left in the trading account in 23 trading days (approximately a month). $14,640$ trading rules are tested in total.

Indicator and parameter	**Description**
Indicator	$(SMA(\lambda_1)_t, SMA(\lambda_2)_t), (EMA(\lambda_1)_t, EMA(\lambda_2)_t)$
Calibration length λ_1 (in day)	$\lambda_1 \in \{5, 10, \ldots, 150\}$
Calibration length λ_2 (in day)	$\lambda_2 \in \{200, 205, \ldots, 500\}$
Stop loss ratio sl	$sl \in \{0.00, 0.01, \ldots, 0.03\}$

Table 4.6: Setup of indicators and parameters of double crossover trading rules

4.3 Pair trading strategy

The last type of strategy we test is pair trading strategy, a well known Wall Street investment strategy retrieved back to mid-1980s according to the study of E. Gatev, W.N. Goetzmann, et al. (2006)[13]. This strategy trades two stocks whose prices are empirically found to be highly correlated. If a price spread is observed, professional investors believe that there is a mis-pricing of the pair because the market is overconfident about the price of the better performed stock and has underestimated the value of the worse stock [13]. We should hence enter the short position by selling the comparably better performed stock and enter the long position by buying the worse performed stock.

The setup of pair trading rules is based on the methodology described by E. Gatev, W.N. Goetzmann, et al. (2006), which splits the testing time series into “pair formation period” and “trading” period[13]. In the “pair formation period”, we aim select a pair of stocks for the backtesting trade. We notate κ as the calibration length, also explained as number of trading days in the pair formation period. First of all, we cluster the available stocks by sector. As the companies with similar business activities face the same opportunity and challenge, investors expect the change of their stock prices following the same trend. We notate pair $(stock_x, stock_y)$ as any combination of 2 stocks in the same sector, so that $stock_x, stock_y \in \{stock_1, stock_2, ..., stock_n\}$. We then calculate Pearson's sample correlation coefficient of the stock prices of $(stock_x, stock_y)$ within κ days. Equation (4.17) formulates the correlation coefficient [5]. We randomly select a pair (x, y) with $\rho_{x'y'} > 0.95$ and check the background of the companies to ensure that both companies conduct similar business activities and serve similar customer segments. The selected pairs within various calibration length are demonstrated in Appendix A.

$$\rho_{x'y'} = \frac{\sum_{t=1}^{\kappa}(p_{x't} - \overline{p}_{x'})(p_{y't} - \overline{p}_{y'})}{\sqrt{\sum_{t=1}^{\kappa}(p_{x't} - \overline{p}_{x'})^2}\sqrt{\sum_{t=1}^{\kappa}(p_{y't} - \overline{p}_{y'})^2}} \tag{4.17}$$

where $\rho_{x'y'}$ is the estimated correlation of stock prices of pair $(stock_x, stock_y)$ given calibration length κ. $p_{x't}$ and $p_{y't}$ are respectively the closing prices of $stock_x$ and $stock_y$ at trading day t. $\overline{p}_{x'}$ and $\overline{p}_{y'}$ are respectively the sample means of closing prices of $stock_x$ and $stock_y$ within κ days.

The trading period starts from 31 March 2011. We trade the selected stocks (x, y) when a "spread" is identified. we identify the "spread" by measuring the distance of the pair price ratio from its observed sample mean [13]. In definition (4.18), equation (4.19) and (4.20), Pr_t, $\overline{Pr}_{\kappa t}$ and $\sigma_{\kappa t}$ accordingly denote the price ratio of pair (x, y) at trading day t, its sample mean and unbiased standard deviation estimated within the calibration period of κ days.

$$Pr_t := \frac{p_{\mathrm{xt}}}{p_{\mathrm{yt}}} \tag{4.18}$$

where p_{xt} and p_{yt} are the closing prices of selected stock x and y at day t.

$$\overline{Pr}_{\kappa t} = \frac{1}{\kappa}\sum_{i=t-\kappa+1}^{t} Pr_i \tag{4.19}$$

$$\sigma_{\kappa t} = \sqrt{\frac{1}{\kappa - 1} \sum_{i=t-\kappa+1}^{t} (Pr_i - \overline{Pr}_{\kappa t})^2} \tag{4.20}$$

Let $TS_{\mathrm{x}t}$ and $TS_{\mathrm{y}t}$ be the general form of trading signals respectively generated for stock x and y at day t. We notate the options of the signals in table 4.7.

Notation	**Description**
EL	enter long position
LL	liquidate long position
ES	enter short position
LS	liquidate short position

Table 4.7: Summary of trading signals and notations

Table 4.8 demonstrates the pair trading rules. Given parameters $\alpha, \beta \in \mathbb{Z}^+$ and $\beta < \alpha$, we enter positions when Pr_t is further than α standard deviations away from the mean $\overline{Pr}_{\kappa t}$. We liquidate the positions when it returns to β standard deviation from the mean.

Condition	TS_{xt}	TS_{yt}
$Pr_t \leq \overline{Pr}_{\kappa t} - \alpha\sigma_{\kappa t}$	EL	ES
$Pr_t \geq \overline{Pr}_{\kappa t} + \beta\sigma_{\kappa t}$	LL	LS
$Pr_t \geq \overline{Pr}_{\kappa t} + \alpha\sigma_{\kappa t}$	ES	EL
$Pr_t \leq \overline{Pr}_{\kappa t} - \beta\sigma_{\kappa t}$	LS	LL

Table 4.8: Summary of trading signals generated based on the closes at day t by pair trading strategy

The trading signals are generated at the end of day t. We execute the trade at the opening of day $t+1$. We notate $POS_{\mathrm{x}t}$ and $POS_{\mathrm{y}t}$ as the position respectively taken for stock x and y according to the trading signals generated at day t.

$$POS_{\mathrm{y}(t)} = -POS_{\mathrm{x}(t)} \tag{4.21}$$

Figure 4.1 illustrates the trading position of stock x taken according to the signal generated at the end of day t under conditions of $POS_{\mathrm{x}(t-1)}$ and $TS_{\mathrm{x}t}$. With the trading account value denoted as $I_{\kappa\alpha\beta\eta'(t+1)}$, we invest η'_x of the account value into stock x and η'_y in stock y at the opening price of day $t+1$. We generate the value of trading account in equation

(4.23) where $R_{\mathrm{x}(t+1)}$ is the daily simple return generated from investment of stock x at day $t+1$ and formulated in equation (4.22). $R_{\mathrm{y}(t+1)}$ is accordingly the daily simple return of stock y at day $t+1$.

$$R_{\mathrm{x}(t+1)} := \frac{p_{\mathrm{x}(t+1)}}{p_{\mathrm{x}t}} - 1 \tag{4.22}$$

$$I_{\kappa\alpha\beta\eta'(t+1)} = \begin{cases} I_0 + \eta'_x I_0 POS_{\mathrm{x}t} \cdot R_{\mathrm{x}(t+1)} + \eta'_y I_0 POS_{\mathrm{y}t} \cdot R_{\mathrm{y}(t+1)}, & \text{if } t = 0 \\ I_{\kappa\alpha\beta\eta(t)} + \eta'_x I_{\kappa\alpha\beta\eta(t)} POS_{\mathrm{x}t} \cdot R_{\mathrm{x}(t+1)} + & \\ \eta'_y I_{\kappa\alpha\beta\eta'(t)} POS_{\mathrm{y}t} \cdot R_{\mathrm{y}(t+1)}, & \text{otherwise} \end{cases} \tag{4.23}$$

We summarise the setup of pair trading rules in table 4.9. The calibration length κ varies from 100 days (approximately a half trading year) to 500 days (about 2 trading years). We invest equal amount of account value into stock x and stock y since we do not consider the difference in volatility of long and short positions in the scope of this study. A total of 252 pair trading rules are tested.

Indicator and parameter	**Description**
Calibration length κ (in day)	$\kappa \in \{100, 150, \ldots, 500\}$
Indicator parameter α	$\alpha \in \{1.50, 1.75, \ldots, 3.00\}$
Indicator parameter β	$\beta \in \{0.25, 0.50, \ldots, 1.00\}$
Weight of account value to be invested in a long and short position	$\eta'_x = \eta'_y 0.5$

Table 4.9: Summary of pair trading rule setup

4.4 Benchmark strategies

We compare the performance of the above mentioned strategies with that of two benchmark strategies - "long and hold strategy" and "randomized trading strategy".

Long and hold strategy has widely been used by academic researchers and financial professionals. It is described as "a passive investment strategy in which an investor buys stocks and holds them for a long period regardless of fluctuations in the market"[8]. In

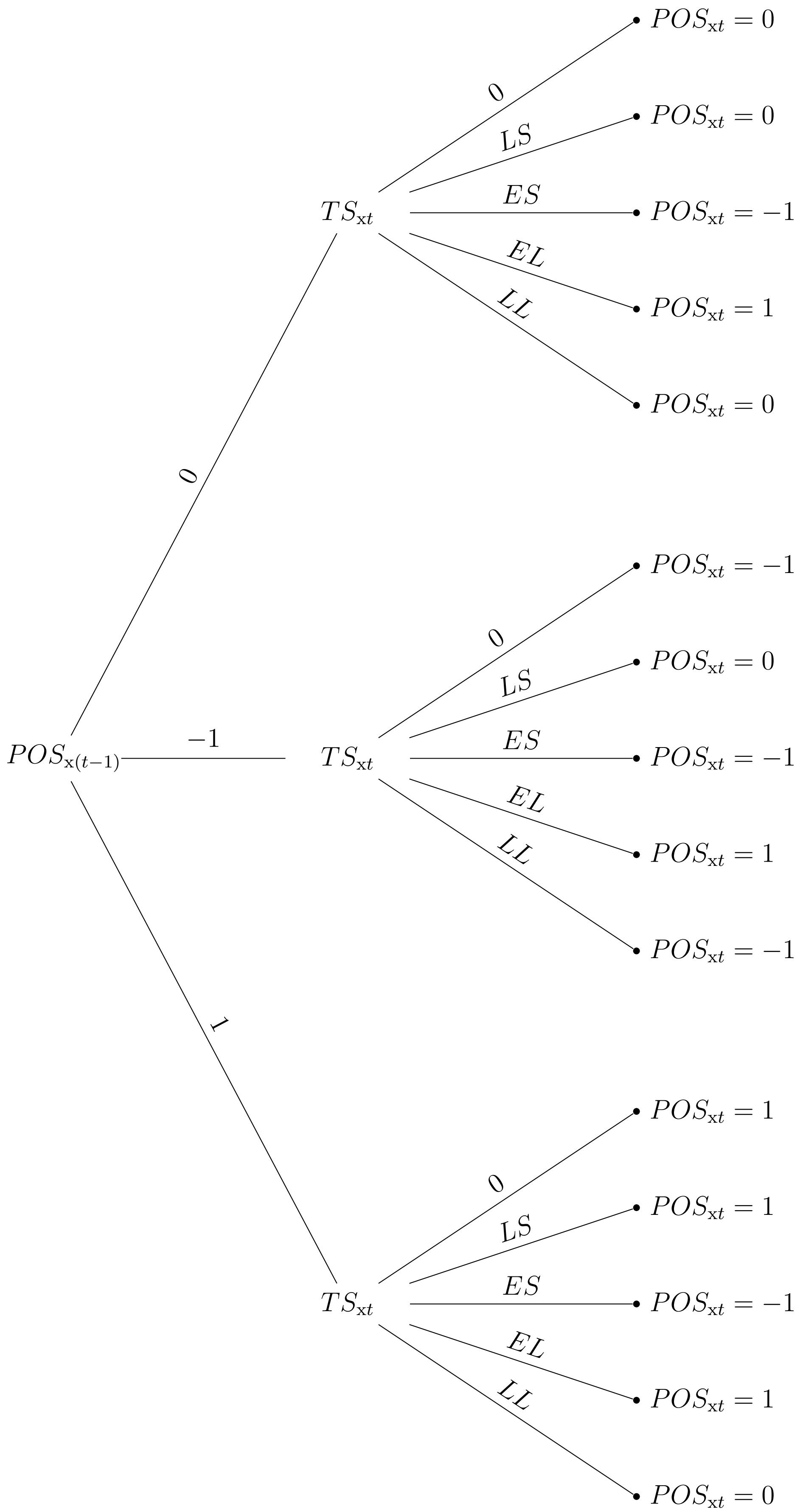

Figure 4.1: Possibilities of trading positions taken by pair trading strategy

equation (4.24) and (4.25), we notate the daily simple return $R_{longandhold(t+1)}$ and value of trading account $I_{longandhold(t+1)}$ at the end of day $t+1$. $p_{index(t)}$ is the closing price of SP500 index at day t.

$$R_{longandhold(t+1)} = \frac{p_{index(t+1)}}{p_{index(t)}} - 1 \tag{4.24}$$

$$I_{longandhold(t+1)} = (1 + R_{longandhold(t+1)})I_{longandhold(t)} \tag{4.25}$$

To achieve a better performance than long and hold strategy is a long-standing goal of the traders and professional investors. A better result indicates that it is useful to trade stocks using the technical strategy in the selected period.

Randomized trading strategy is a strategy that we use to simulate a trading that select and invest in stocks by luck. We randomly choose a stock on trading day t. Every stock has the same probability to be selected. We buy this stock at its closing price $p_{stock(t)}$ with all capital in the trading account at the opening of day $t+1$. We sell all holding stocks at the end of day $t+1$. Our trading account value at day $t+1$ is denoted as $I_{random(t+1)}$ and is formulated in equation (4.26).

$$I_{random(t+1)} = (1 + R_{random(t+1)})I_{random(t)} \tag{4.26}$$

where

$$R_{random(t+1)} = \frac{p_{stock(t+1)}}{p_{stock(t)}} - 1 \tag{4.27}$$

4.5 Measures of financial performance

We use 5 financial metrics to measure the annualised performance of trading strategies. We introduce I_s as a general notion of trading account values at trading day s so that $I_s \in \{I_{\eta\theta(M)}, I_{MA,\lambda(t+1)}, I_{\kappa\alpha\beta\eta'(t+1)}\}$.

Annualised return is the geometric mean of annualised investment gain or loss estimated from the given period[7]. The formula is indicated in equation (4.28) where AR notates annualised return. AR has not taken into consideration the investment volatility.

$$AR := (1 + CR)^L - 1 \tag{4.28}$$

where L is average trading days per year divided by account held days so that $L = \frac{251}{1994}$ Let CR be the cumulative return formulated in equation (4.29)

$$CR = \frac{I_s}{I_0} - 1 \tag{4.29}$$

Annualised volatility is measured to estimate risk of the investment. It argues that "relationship between time and standard deviation increases with the square root of time". This is under assumption that daily returns follow an independent and normal distribution[4]. In equation (4.30), annualised volatility is notated as ϵ_T. $T = 12$ when the trading frequency is 1 month and $T = 251$ when the frequency is daily. The simple return of trading account value in the tested period s is denoted as R_s^{acc}. Let σ_s be the sample standard deviation of R_s^{acc}. σ_s is formulated in the same form of equation (4.5).

$$\epsilon_T := \sqrt{T} \cdot \sigma_s \tag{4.30}$$

$$R_s^{acc} = \frac{I_s}{I_{s-1}} - 1 \tag{4.31}$$

Sharpe ratio measures risk-adjusted investment return. In equation (4.32), (4.33) and (4.34), we use Ex Post Sharpe Ratio defined by William F. Sharpe in 1994 [37]. We notate Sharpe ratio as SH.

$$SH = \frac{\overline{D}}{\sigma_D} \tag{4.32}$$

where $\overline{D}$ is the expected annual excess return or the average differentiation between total return and risk free rate of return

$$\overline{D} := \frac{1}{T}\sum_{s=1}^{T}(R_s^{acc} - RF_s) \tag{4.33}$$

and σ_D is the standard deviation of excess returns

$$\sigma_D := \sqrt{\frac{1}{T-1}\sum_{s=1}^{T}\left(R_s^{acc} - RF_s - \overline{D}\right)^2} \tag{4.34}$$

We let RF_s be the annual risk free rate of return. As we are testing U.S. based stocks, 1-year U.S. Treasury bill is used as risk free rate. Table 4.10 indicates yearly return rate from 2011 to 2018[3].

Date	Rate (r_b)
Dec. 31, 2011	0.17%
Dec. 31, 2012	0.17%
Dec. 31, 2013	0.13%
Dec. 31, 2014	0.11%
Dec. 31, 2015	0.30%
Dec. 31, 2016	0.60%
Dec. 31, 2017	1.17%
Dec. 31, 2018	2.25%

Table 4.10: 1 Year Treasury Bill Rate from year 2011 to 2018

Sortino ratio differentiates from Sharpe ratio since it uses the so-called "downside standard deviation" instead of sample standard deviation of excess returns[23]. The "downside standard deviation" is the volatility that "penalizes only returns falling below a user-specified target or required rate of return"[33]. In our case, we use RF_s as the required rate of return since investors will not be interested in trading stocks if the return cannot exceed the risk free rate. This metric is formulated under assumption of normality. In definition (4.35), SR refers to Sortino ratio and DR is the downside standard deviation[33][39].

$$SR := \frac{\overline{D}}{DR} \tag{4.35}$$

$$DR \in [\sigma_D \mid R_s^{acc} - RF_s < 0] \tag{4.36}$$

Calmar ratio uses annual rate of return divided by "maximum drawdown risk"[22]. The maximum drawdown measures the maximum decline from the historical peak of the portfolio. Let C be Calmar ratio. Let $MDD(\mathrm{T})$ be the maximum drawdown until time T. Equation (4.37) and (4.38) formulate C and $MDD(\mathrm{T})$, where $R_{(t)}$ and $R_{(\tau)}$ are simple trading returns at trading day t and τ. $\mathrm{T}, t, \tau \in \mathbb{Z} \cap [1, 1994]$ [16]. As we aim to find technical strategy with consistent profitability or loss, we are simulating tradings conducted by investors with an investment horizon of 8 to 10 years. We hence take T = 8 in order to measure the maximum loss the investors could unfortunately suffer in the entire investment period.

$$C := \frac{AR}{MDD(T)} \tag{4.37}$$

$$MDD(\mathrm{T}) := \max_{\tau \in (0,\mathrm{T})} \left[\max_{t \in (0,\tau)} R_{(t)} - R_{(\tau)} \right] \tag{4.38}$$

Prediction accuracy measures the frequency that a profit is realised by trading in the test period. Let *pred* denote frequency that the trading position gains profitability. In equation (4.39), w is the number of trading days that have achieved a positive simple investment return and v is total number of trading days in the testing period. This financial metric is measured for moving average crossover strategies and long and hold strategy.

$$pred = \frac{w}{v} \tag{4.39}$$

4.6 Backtesting of strategies

We backtest strategies with the historical data from the past 8 years from 31 March 2011 to 06 March 2019. With the financial metrics of strategies and the benchmarks, we understand the profitability of the strategies through descriptive statistics of financial measures with comparison to the benchmarks.

We study the Sharpe ratio of the trading strategy and of the random benchmark by the empirical estimators of their 4 moments - mean, variance, skewness and kurtosis[28]. We plot the probability density functions of the Sharpe ratio of the trading strategy and of the randomized trading benchmark. The density function is smoothed by Gaussian kernel. The Sharpe ratio of the tested trading rules by various indicators and η are further explored graphically and statistically.

We would like to understand if the risk adjusted results of the testing strategy are mostly due to luck. The general definition of the empirical distribution function is

$$\hat{F}_m(x) := \frac{1}{m}\sum_{i=1}^{m}\mathbf{1}_{X_i \le x} \tag{4.40}$$

where $\hat{F}_m(x)$ is the unbiased estimator of distribution $F(x)$[42][36]. $X_1, X_2, ..., X_m$ are the samples of Sharpe ratio.

We define $F_0(x)$ as the empirical distribution function of the benchmark randomized trading strategy which represents luck. Let $F_{pair}(x)$ be the distribution of pair trading strategy. Notations of the other empirical distribution functions and their corresponding rules in price-momentum strategy and moving average strategy are summarised in table 4.11 and 4.12.

Dist. function	**Description of trading rules**
$F_{mom1}(x)$	Indicator= $r^{cum}_{i(M)}$, $\eta = 1.0$
$F_{mom2}(x)$	Indicator= $r^{cum}_{i(M)}$, $\eta = 0.5$
$F_{mom3}(x)$	Indicator= $r^{mean}_{i(M)}$, $\eta = 1.0$
$F_{mom4}(x)$	Indicator= $r^{mean}_{i(M)}$, $\eta = 0.5$
$F_{mom5}(x)$	Indicator= $r^{risk}_{i(M)}$, $\eta = 1.0$
$F_{mom6}(x)$	Indicator= $r^{risk}_{i(M)}$, $\eta = 0.5$

Table 4.11: Summary of distribution function of price-momentum trading rules by indicator and η

Anderson-Darling test can be used to test if several sets of observed data can be described as coming from a common population whose distribution function is not required

Dist. function	Description of trading rules
$F_{MA1}(x)$	Indicator= $SMA(\lambda)_t$
$F_{MA2}(x)$	Indicator= $EMA(\lambda)_t$
$F_{MA3}(x)$	Indicator= $\{SMA(\lambda_1)_t, SMA(\lambda_2)_t\}$, $sl = 0$
$F_{MA4}(x)$	Indicator= $\{EMA(\lambda_1)_t, EMA(\lambda_2)_t\}$, $sl = 0$
$F_{MA5}(x)$	Indicator= $\{SMA(\lambda_1)_t, SMA(\lambda_2)_t\}$, $sl \neq 0$
$F_{MA6}(x)$	Indicator= $\{EMA(\lambda_1)_t, EMA(\lambda_2)_t\}$, $sl \neq 0$

Table 4.12: Summary of distribution function of moving average trading rules by indicator

to be specified[36]. We firstly test the homogeneity of samples with the hypothesis that Sharpe ratio of the trading rules from the same trading strategy with various indicators and η follow the same population distribution. Here, we have 2 individual hypothesis tests. For the momentum trading strategy, $H_0 : F_i(x) = F_j(x)$. $F_i(x)$ and $F_j(x)$ are all combination of the empirical distribution functions $F_{mom1}(x), F_{mom2}(x), ..., F_{mom6}(x)$. For the moving average trading strategy, $H_0 : F_k(x) = F_n(x)$. $F_k(x)$ and $F_n(x)$ are all combination of the distribution functions $F_{MA1}(x), F_{MA2}(x), ..., F_{MA4}(x)$. With the acceptance of the H_0, we can pool $F_i(x)$ and $F_j(x)$ (the same for $F_k(x)$ and $F_n(x)$) into a single distribution. We hence define $G_{mom1}(x), G_{mom2}(x)..., G_{momN}(x)$ as the empirical distribution functions of the momentum trading strategy and $G_{MA1}(x), ..., G_{MAK}(x)$ as the distributions of moving average crossover strategy, where $N, K \in Z^+$, $1 \leq N \leq 6$ and $1 \leq K \leq 4$.

We are then interested in whether the distance between the Sharpe ratio of randomized trading benchmark and the tested trading rules is sufficiently close so that we can model them from an unspecified common distribution function. $N + K + 1$ individual hypothesis tests are conducted. Table 4.13 summarises the null hypotheses.

The two-sample Anderson-Darling test statistic is used to estimate the distance [36][30]. (4.39) presents the general formula of distance A^2_{mn} to test $H_0 : F(x) = G(x)$.

$$A^2_{mn} = \frac{mn}{N} \int \frac{(F_(x) - G_(x))^2}{H_N(x)(1 - H_N(x))} dH_N(x) \tag{4.41}$$

where m are number of samples $X_1, X_2, ..., X_m$ from empirical distribution $F(x)$ and n

Trading strategy	**Null hypothesis**
Price-momentum	$G_{mom1}(x) = F_0(x)$ $G_{mom2}(x) = F_0(x)$ ... $G_{momN}(x) = F_0(x)$
Moving average	$G_{MA1}(x) = F_0(x)$ $G_{MA2}(x) = F_0(x)$ ... $G_{MAK}(x) = F_0(x)$
Pair trading	$F_{pair}(x) = F_0(x)$

Table 4.13: Summary of null hypotheses to test if the risk adjusted performance results of the tested trading strategy are mostly caused by luck

are the number of samples from $Y_1, Y_2, ..., Y_n$ following the distribution $G(x)$ [36][30].

$$N := m + n \tag{4.42}$$

$$H_N(x) := \frac{m \cdot F(x) + n \cdot G(x))}{N} \tag{4.43}$$

Further analysis of the performance of the trading rules are implemented. We notate trading rules with 1% largest Sharpe ratio as the best performed trading rules, and these with 1% lowest Sharpe ratio as the worst performed rules. We visualize the log ratio of cumulative trading account value resulted from the SP500 long and hold strategy and the best and the worst performed tested trading rules. The ratio of the trading strategy, denoted as $CUMR$, is formulated in equation (4.43) where I_t is the trading account value at day $t(t \in \{0, 1, ..., 1994\})$ and I_0 is the initial investment value.

$$CUMR = ln(\frac{I_t}{I_0}) \tag{4.44}$$

In addition, we explore the sensitivity of the risk-adjusted performance results when we change the parameters. We separately visualize the value of Sharpe ratio and parameters of the tested trading strategy with various indicators in 3 dimensional surface plots.

Chapter 5

Backtesting results

This chapter presents the backtesting results of the strategies. We aim to find strategies that lead to a consistent profitability in the past 8 years. We also study strategies that consistently outperform and underperform the market not mostly due to luck.
are interested in the strategies with significantly bad performance compared to the performance mostly caused by luck, because these strategies also imply the existence of skillful trading.

5.1 Price momentum strategy

Table 5.1 summarises the performance of the price-momentum strategy and the benchmarks measured by financial metrics. The median and the mean of the tested strategy are larger than 0 in terms of annualised return, annualised risk adjusted excess return and maximum drawdown adjusted return. Out of $1,550$ tested trading rules, 992 (equivalent to 64.00%) have reached positive annualised return and drawdown adjusted return. 865 (equivalent to 55.81%) rules have reached profitable risk adjusted excess returns represented by SH and SR. Such result shows the possibility of a price-momentum strategy generating consistent profitability in the past 8 years.

In comparison to the performance of benchmarks, the median and mean of the momentum strategy do not surpass those of the benchmarks in terms of all financial metrics. Table

5.2 shows the number and quantile of trading rules outperformed long and hold strategy. In table 5.1, we observe that the median and mean of randomized trading are greater than 0 in terms of all financial metrics. It means that the excess return widely believed to be generated by skills is also consistently achieved in purely randomized trading strategy, i.e. by luck.

Metric	AR	ϵ_T	SH	SR	C
Min.	−0.08	0.05	−0.66	−0.91	−0.13
1st Qu.	−0.01	0.07	−0.17	−0.23	−0.04
Median	0.03	0.09	0.17	0.28	0.13
Mean	0.04	0.09	0.26	0.48	0.33
3rd Qu.	0.07	0.10	0.71	1.22	0.71
Max.	0.25	0.26	1.14	2.25	1.54
Benchmark1: long and hold strategy					
	0.10	0.15	0.63	0.79	0.50
Benchmark2: randomized trading					
Min.	−0.17	0.27	−0.55	−0.74	−0.21
1st Qu.	0.05	0.31	0.13	0.18	0.10
Median	0.11	0.31	0.32	0.43	0.25
Mean	0.11	0.32	0.33	0.45	0.29
3rd Qu.	0.17	0.32	0.51	0.70	0.43
Max.	0.47	0.43	1.54	2.39	1.88

Table 5.1: Summary of financial metrics of 1,550 price-momentum trading rules backtested from 31 March 2011 to 06 March 2019

Metric	AR	ϵ_T	SH	SR	C
Number of rules	116	1466	550	684	615
Quantile in %	7.48	94.58	35.48	44.13	39.68

Table 5.2: Summary of price-momentum trading rules that outperformed long and hold strategy in the backtest from 31 March 2011 to 06 March 2019

Figure 5.1 illustrates the smoothed probability density of the Sharpe ratio of the price-momentum strategy and benchmarks. The density function of the benchmark is approximately symmetric, while density function of the momentum strategy is asymmetric with 2 clearly observed humps.

Table 5.3 describes the Sharpe ratio generated from the momentum trading rules by 4 moments. We do not identify similar patterns in Sharpe ratio of trading rules with various

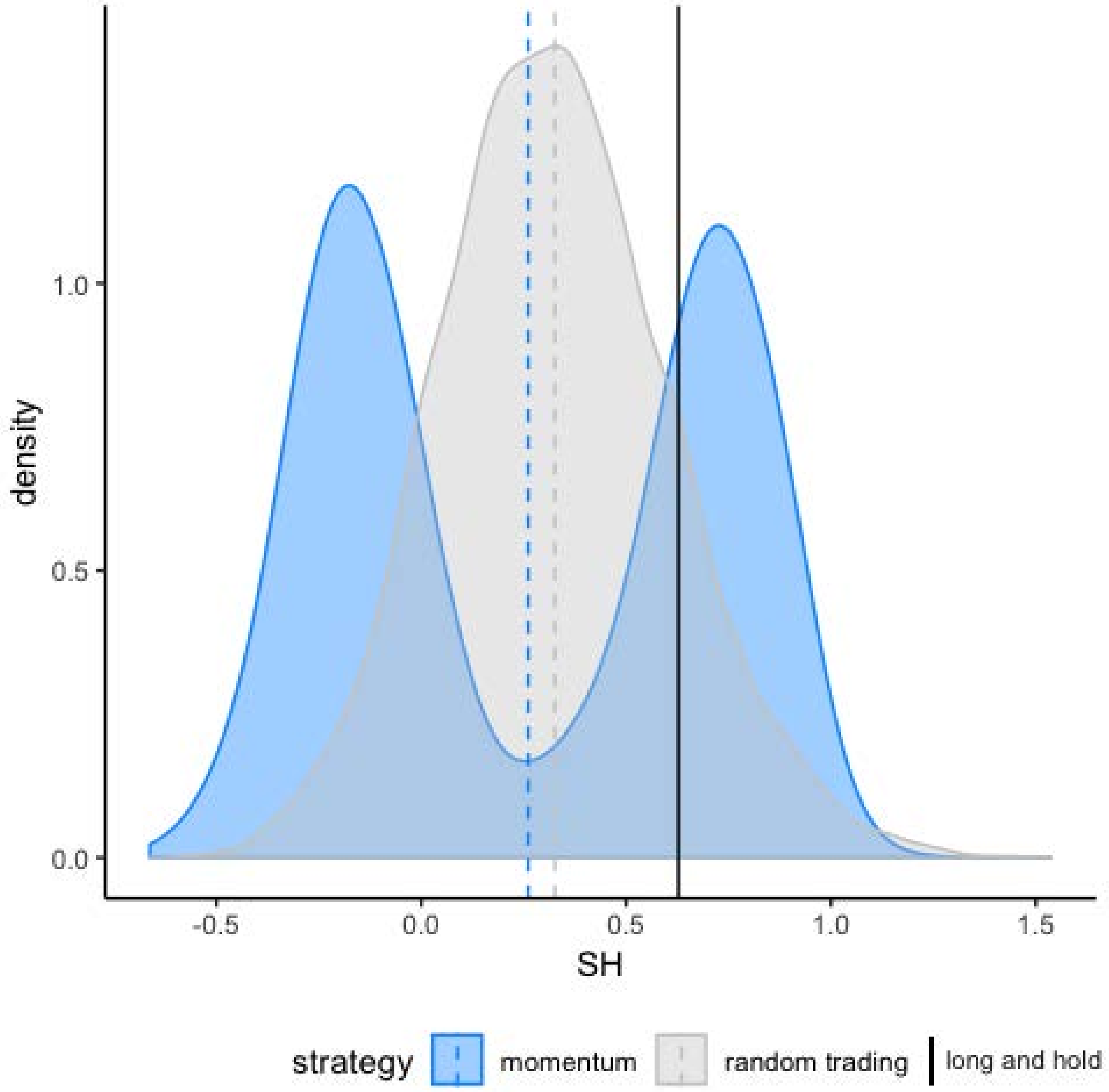

Figure 5.1: Probability density of Sharpe ratio of price-momentum strategy(blue), randomized trading strategy(grey) and long and hold strategy(black)

indicators and η. They are also different from the statistics of randomized trading strategy. Although mean of the tested strategy is smaller than mean of randomized trading, its variance is about $\frac{1}{3}$ greater than the benchmark. Momentum rules with $\eta_l = 0.5$ have a negative mean of Sharpe ratio while rules with $\eta_l = 1.0$ have more than twice mean of randomized trading. The variance of Sharpe ratio of rules with long and short position is as small as 0.050, while the variance of rules with only long position reaches 0.507, which is more than twice variance of the benchmark.

Strategy	Mean	Variance	Skewness	Kurtosis
Randomized trading	0.326	0.186	0.118	0.089
Price-momentum strategy	0.261	0.278	0.186	0.155
Momentum rules($\eta_l = 0.5$)	−0.167	0.050	−0.015	0.006
Momentum rules($\eta_l = 1.0$)	0.689	0.507	0.387	0.305
Momentum rules($r^{cum}_{i(SM)}$,$\eta_l = 1.0$)	0.701	0.522	0.401	0.316
Momentum rules($r^{cum}_{i(SM)}$,$\eta_l = 0.5$)	−0.162	0.050	−0.016	0.007
Momentum rules($r^{mean}_{i(SM)}$,$\eta_l = 1.0$)	0.760	0.600	0.486	0.403
Momentum rules($r^{mean}_{i(SM)}$,$\eta_l = 0.5$)	−0.114	0.034	−0.007	0.003
Momentum rules($r^{risk}_{i(SM)}$,$\eta_l = 1.0$)	0.604	0.396	0.272	0.193
Momentum rules($r^{risk}_{i(SM)}$,$\eta_l = 0.5$)	−0.226	0.066	−0.022	0.008

Table 5.3: Statistic description of the Sharpe ratio generated from the price momentum trading rules

Figure 5.2 and 5.3 respectively visualize the density of Sharpe ratio of momentum trading rules categorised by η and by indicators and η. We observe that the density functions of momentum trading rules are different from each other and from function of the randomized trading strategy. It implies that 1) careful selection of the indicator and parameter could lead to a better trading performance, and 2) the results obtained from the price-momentum strategy may not mostly be caused by luck. These two assumptions need to be confirmed through statistical tests.

As of the hypothesis test $H_0 : F_i(x) = F_j(x)$, 15 individual tests have been conducted. The largest p-value generated from the tests is approximately 0.001 with $A^2_{mn} = 5.664$. Therefore for all individual tests, we reject null hypothesis that the empirical distributions of Sharpe ratio of price momentum trading rules with any different indicators or η come from the same population distribution function.

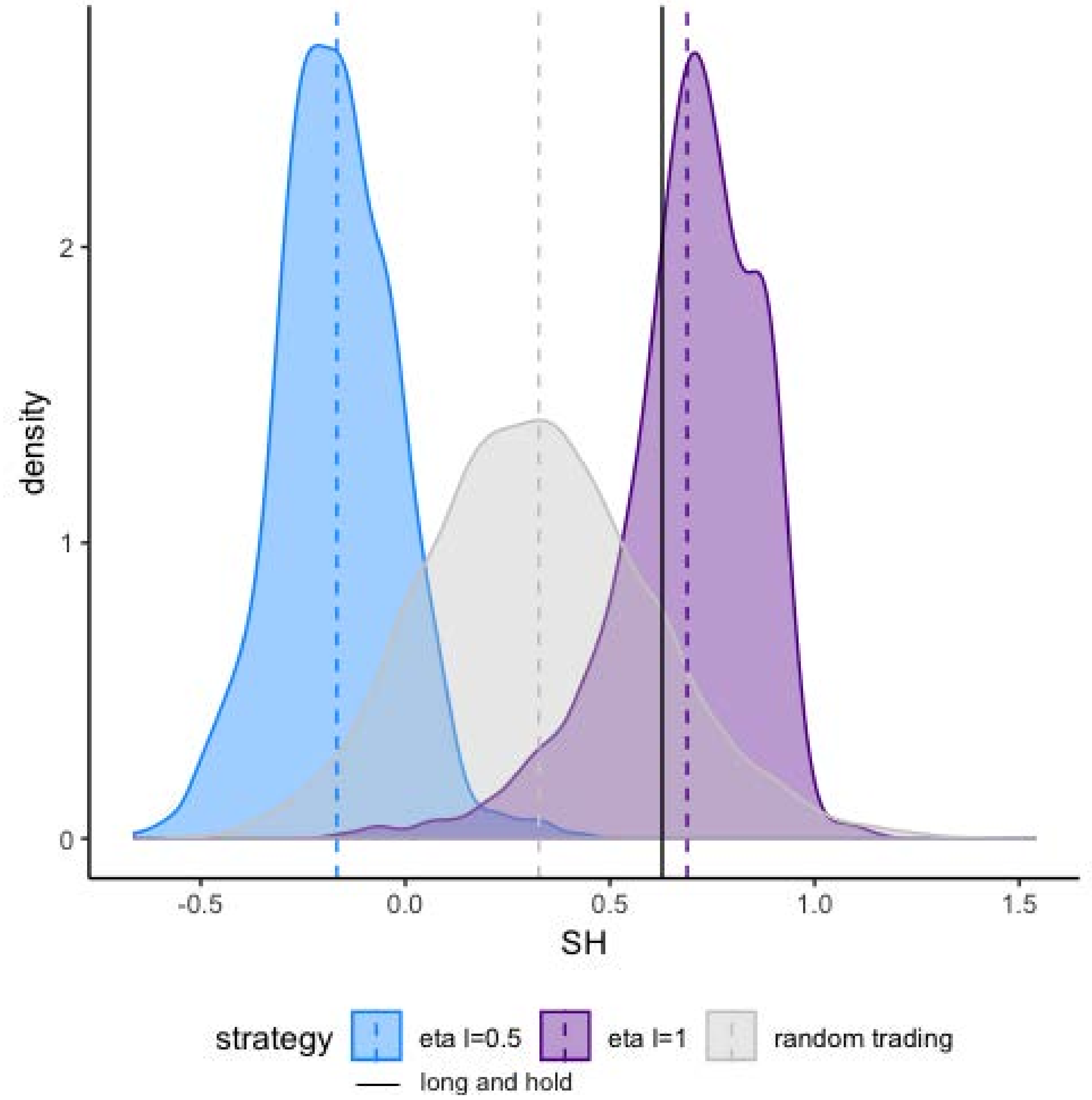

Figure 5.2: Probability density of Sharpe ratio of the randomized trading rules and momentum trading rules with $\eta_l = \eta_s = 0.5$(long and short) and $\eta_l = 1.0, \eta_s = 0.0$(long only)

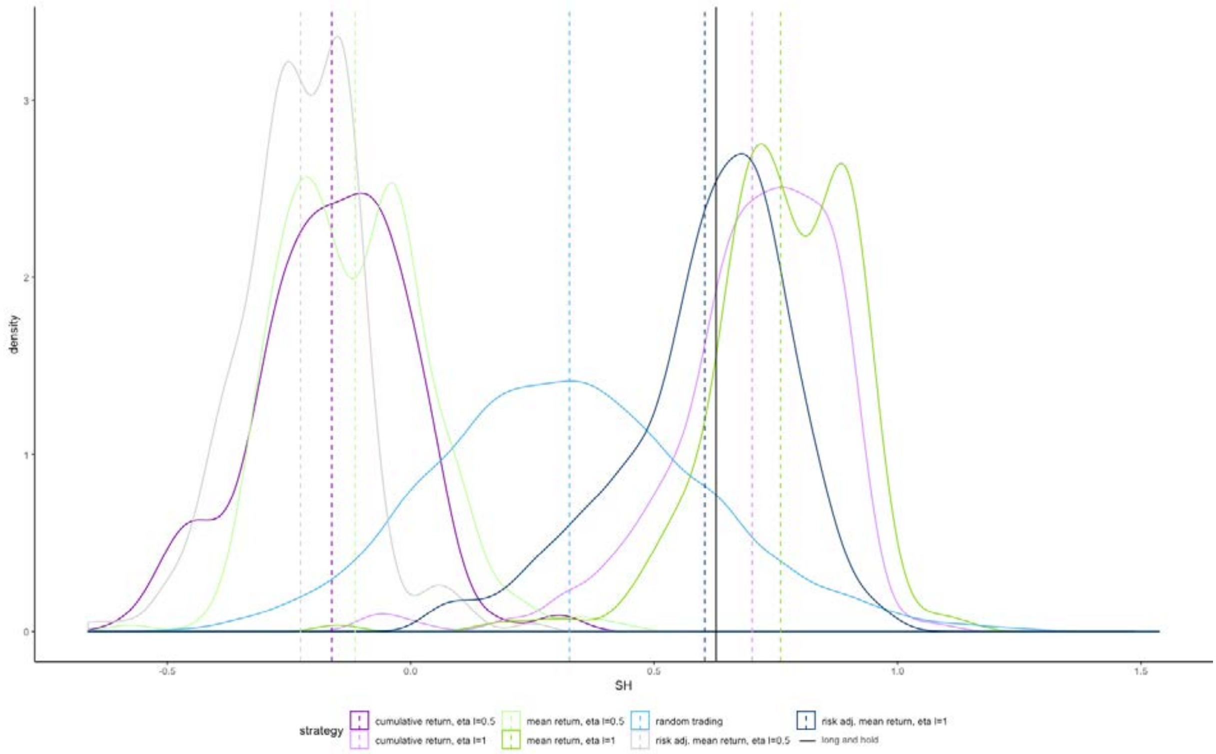

Figure 5.3: Probability density of Sharpe ratio of the benchmarks and momentum trading rules with different indicators and η_l

Table 5.4 demonstrates the results of Anderson-Darling tests under null hypotheses that the Sharpe ratio of tested momentum trading rules and of randomized trading rules are obtained from a common distribution function. All null hypotheses are rejected. It is aligned with our observation in the graphs of the density function. We cannot conclude that the risk adjusted trading results of the price-momentum strategy are mostly due to luck. In addition, the distribution of momentum trading rules indicated by risk adjusted mean return with $\eta_l = \eta_s = 0.5$ is the most different from the distribution of randomized strategy. However, the distribution of momentum rules with the same indicator and $\eta_l = 1.0, \eta_s = 0.0$ is the most similar to the distribution of randomized strategy. Generally as for the tested rules indicated by the same indicator, rules with long and short positions have the performance distributions further from the randomized strategy than the rules with only long position.

Null hypothesis	AD distance(A^2_{mn})	p-value
$G_{mom1}(x) = F_0(x)$	272.50	7.200×10^{-150}
$G_{mom2}(x) = F_0(x)$	445.00	1.384×10^{-244}
$G_{mom3}(x) = F_0(x)$	319.10	1.857×10^{-175}
$G_{mom4}(x) = F_0(x)$	347.90	2.964×10^{-191}
$G_{mom5}(x) = F_0(x)$	148.90	5.791×10^{-82}
$G_{mom6}(x) = F_0(x)$	526.2	3.446×10^{-289}

Table 5.4: Summary of Anderson-Darling test results under H_0: Sharpe ratio of tested momentum trading rules and of randomized trading are from a common distribution function

We further analyse the profits and loss of the best and the worst performed rules in the backtesting period. Figure 5.4 and 5.5 visualize the log ratio of the cumulative account value resulted from 16 best and 16 worst performed rules as well as the returns of SP500 long and hold strategy. All of the best performed rules hold only long position ($\eta_l = 1.0$, $\eta_s = 0.0$), while all of the worst rules hold equally long and short positions ($\eta_l = \eta_s = 0.5$). This suggests that price-momentum strategy do not consistently and correctly predict the direction of the daily change of stock prices. The best performed rules are indicated by monthly cumulative return, monthly mean return or risk adjusted mean return. The worst performed rules are generated by one of the 4 indicators. We observe that the best performed rules hardly surpass the benchmark between year 2012 and 2014, while the worst rules generally outperform the benchmark in the first trading year. This may due to the fact that the market was recovering from a financial crisis. The observation implies the changing structure of U.S. stock market. The majority of the best performed rules have achieved a ratio higher than that of long and hold strategy after 2014, while the ratio of the worst performing rules hardly surpass 0 after year 2014.

Figure 5.6, 5.7 and 5.8 illustrate the sensitivity of the Sharpe ratio of momentum trading rules with the change in parameters and indicators. First of all, the sensitivities of the performance of trading rules with different indicators are different. Trading rules with long only position trading and with long and short position seem to share similar changing pattern of Sharpe ratio when the other numeric parameters change. Momentum trading rules only with long position always have a better risk adjusted results than rules with long

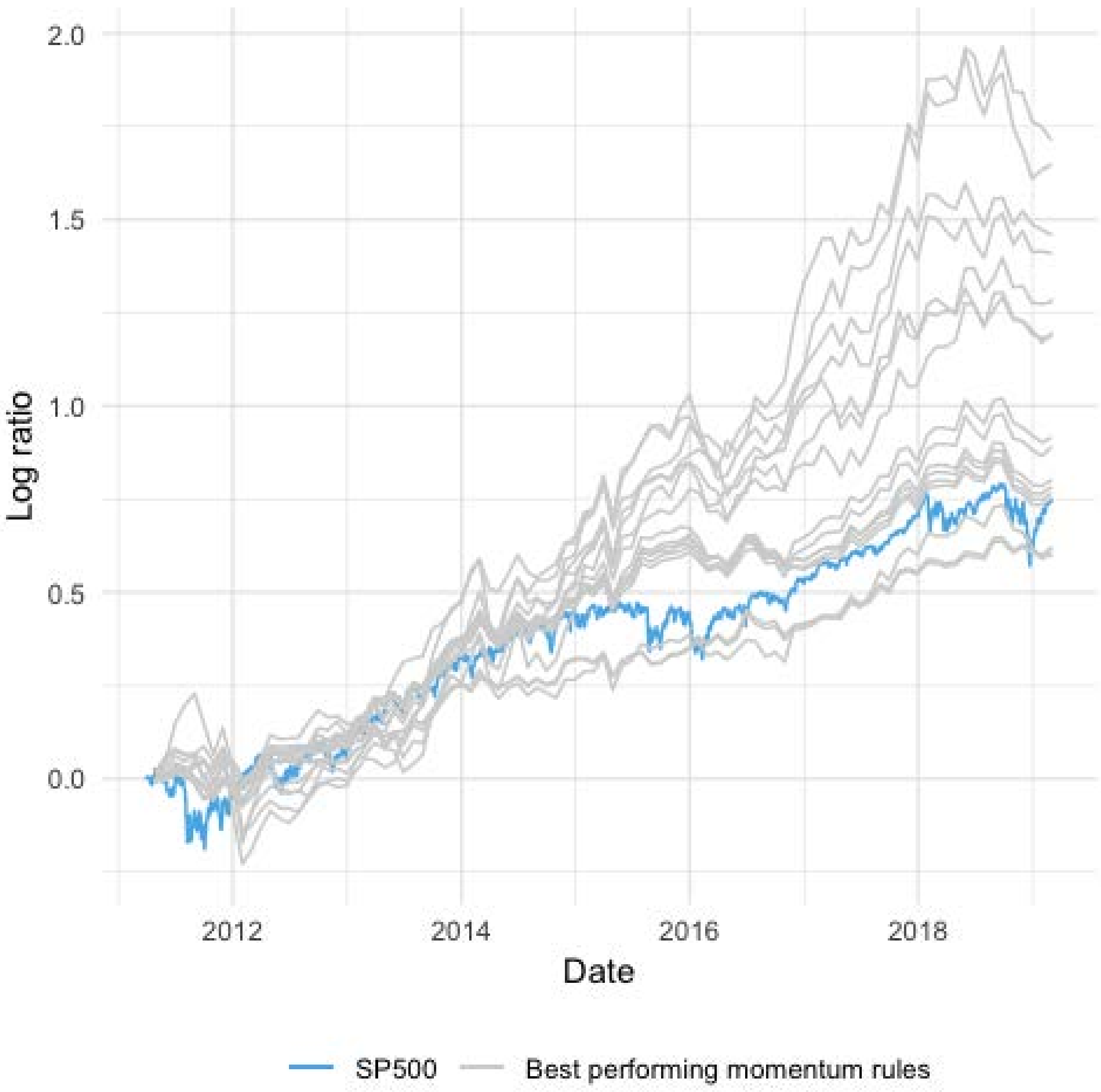

Figure 5.4: Log ratio of the cumulative account value resulted from the best performed momentum rules and SP500 long and hold trading strategy from 31 March 2009 to 06 March 2019 (No transaction cost applied)

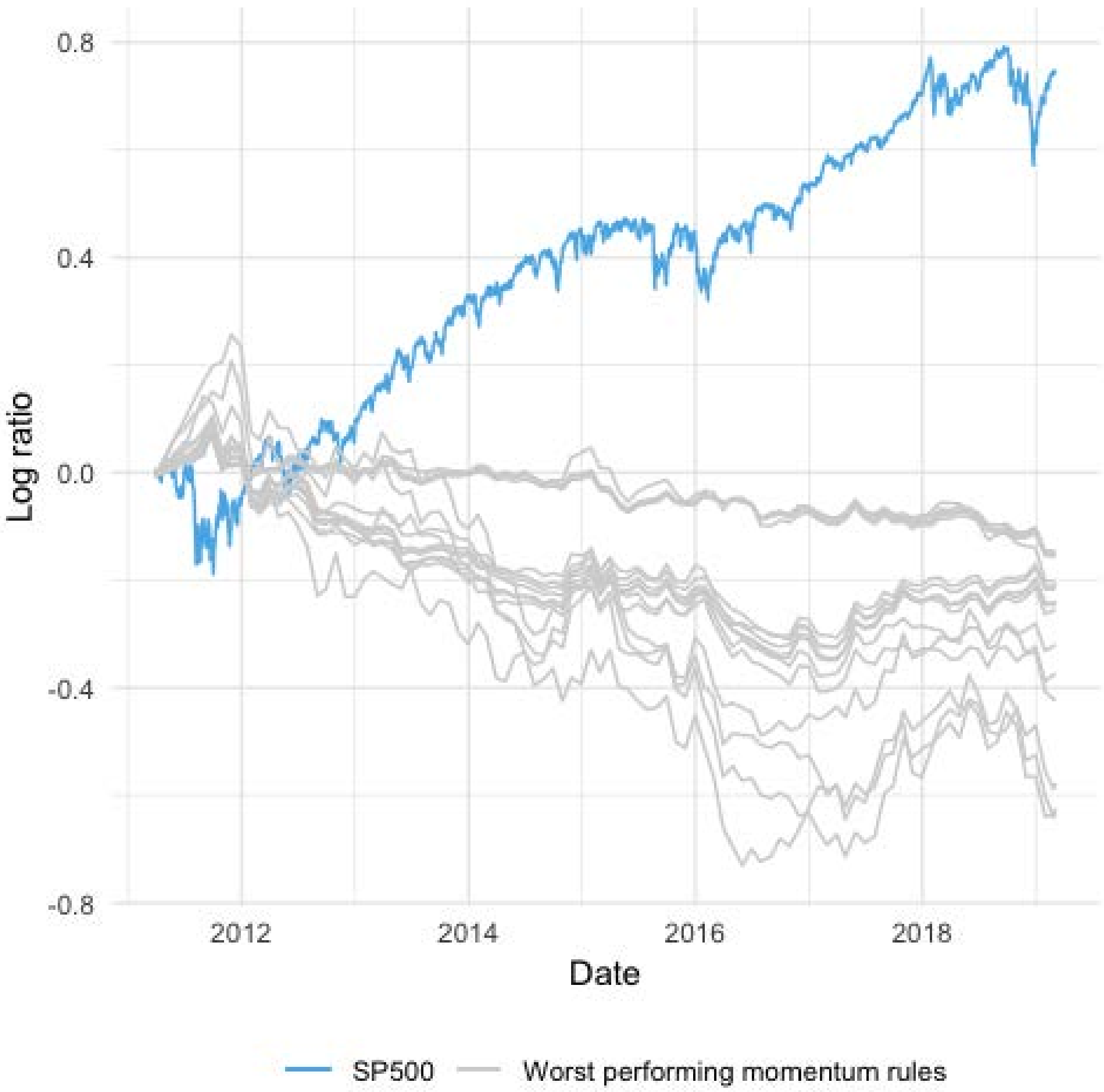

Figure 5.5: Log ratio of the cumulative account value resulted from the worst performed momentum rules and SP500 long and hold trading strategy from 31 March 2009 to 06 March 2019 (No transaction cost applied)

and short position, regardless of the change in other parameters. When we take monthly mean return and cumulative return as the indicators, Sharpe ratio increases with the increasing number of selected stocks θ until $\theta = 5$ and then decreases with the increasing θ. However, when the risk adjusted monthly mean return is used as the trading indicator, the performance continue to increase with the increasing θ. In addition, regardless of the indicators, the risk adjusted performance of the momentum rules generally improves with the longer calibration length, especially if we move the calibration length from $m = 3$ to $m = 10$.

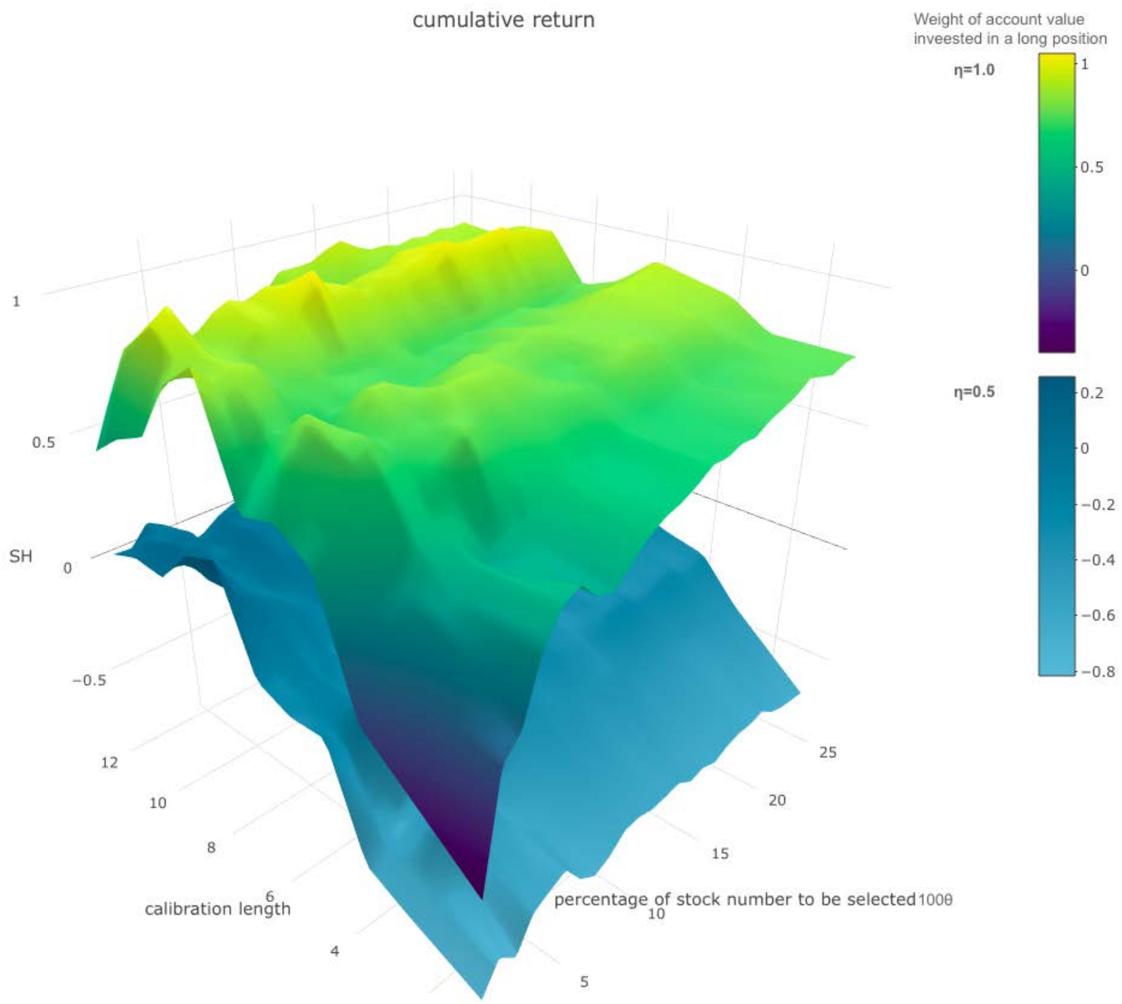

Figure 5.6: Sensitivity of risk adjusted performance of price momentum trading rules indicated by cumulative return

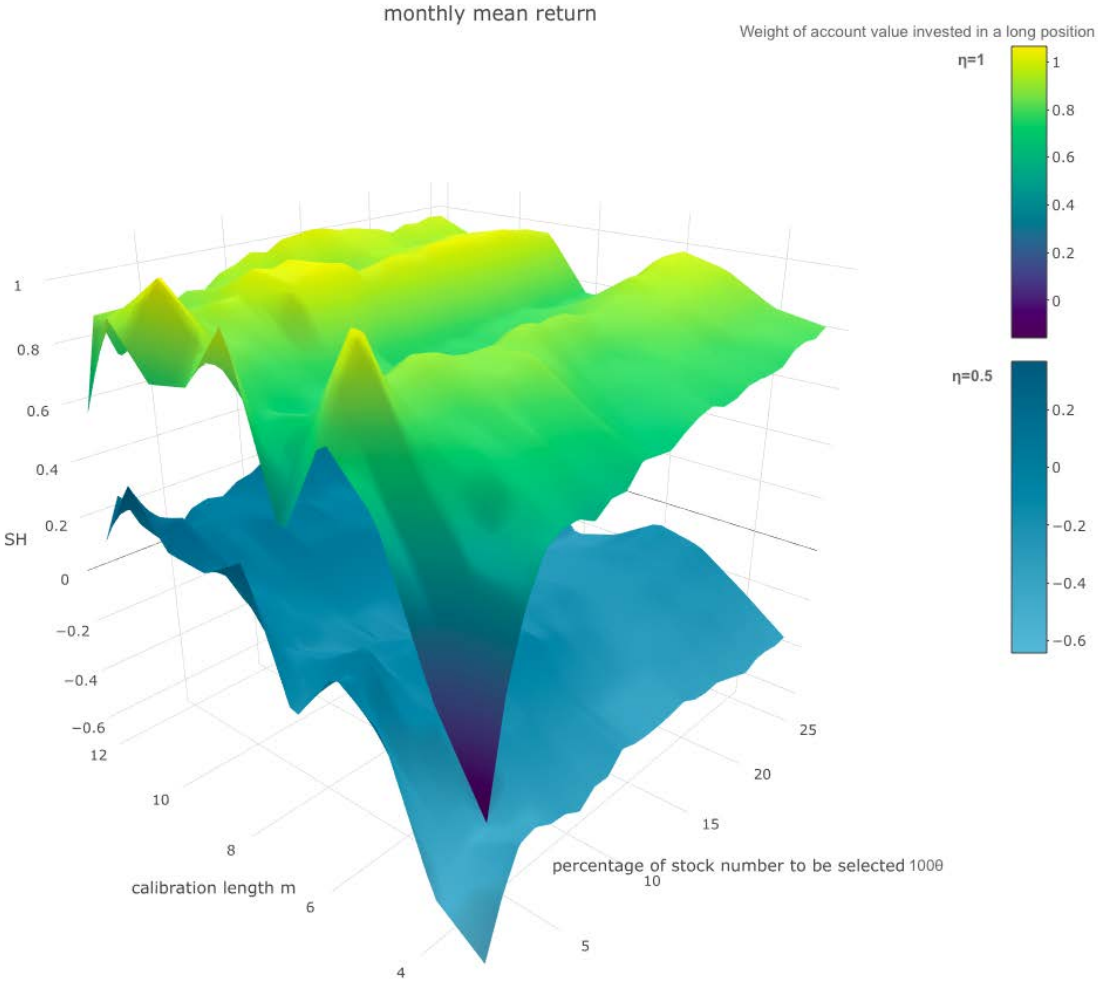

Figure 5.7: Sensitivity of risk adjusted performance of price momentum trading rules indicated by mean return

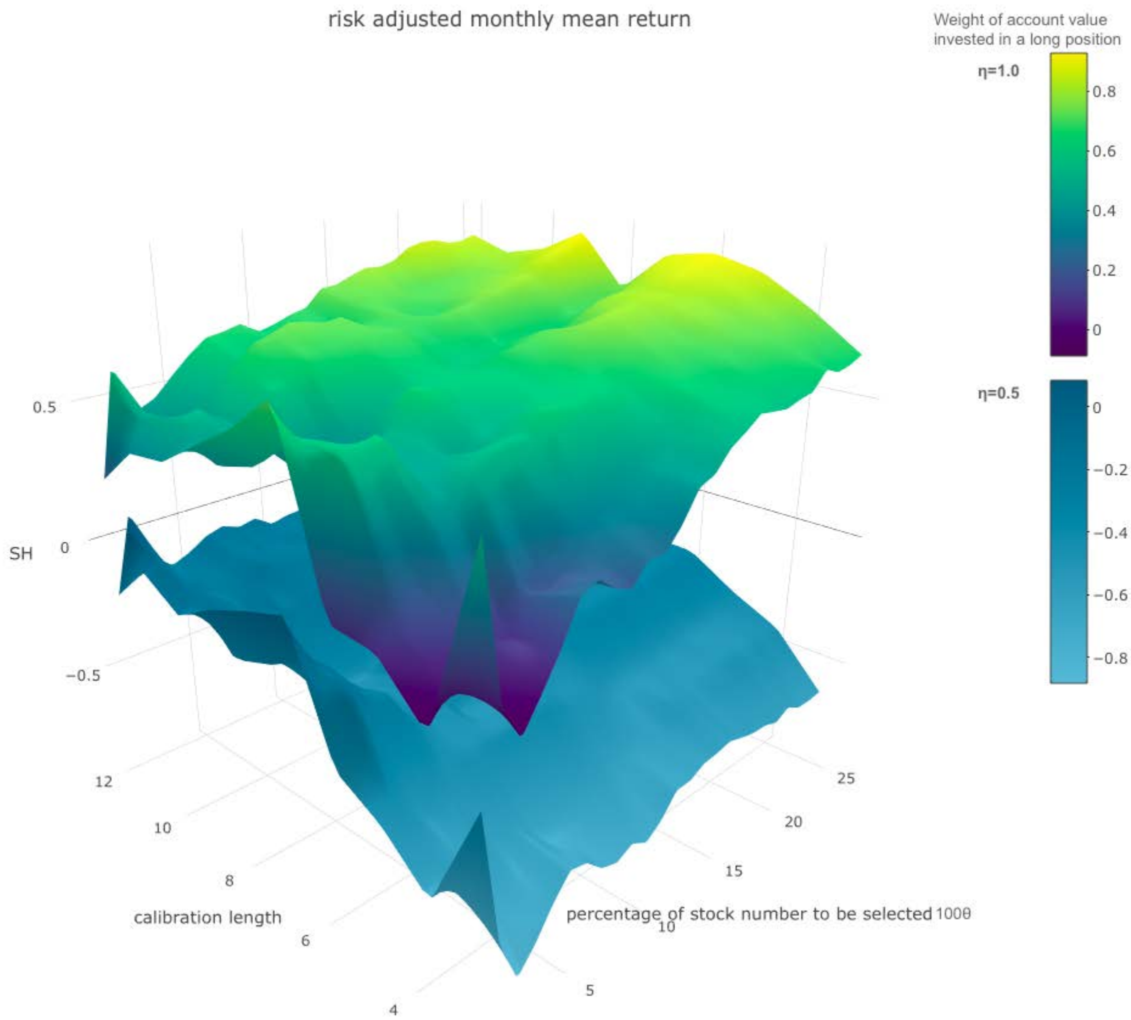

Figure 5.8: Sensitivity of risk adjusted performance of price momentum trading rules indicated by risk adjusted mean return

5.2 Moving average crossover strategy

The performing results of the crossover strategy are summarised in table 5.5 and 5.6. $11,541$ out of $15,642$ tested crossover rules (equivalent to 73.78%) achieve the positive mean and median of the annualised return and Calmar ratio. $10,244$ (equivalent to 65.49%) rules have generated positive risk adjusted excess returns represented by SH and SR. The mean and median of crossover strategy are lower than those of price-momentum strategy. While each maximum financial metric of crossover strategy is lower than the correspondent maximum metric of price-momentum strategy, the minimum metrics of crossover strategy is close to those of momentum strategy.

In comparison to the benchmark I, the crossover strategy generally underperforms since no more than 2.00% of the rules have outperformed in terms of all financial metrics. The expected prediction accuracy of the tested strategy is 3% lower than the benchmark 1. The crossover strategy also under-performs randomized trading strategy since its mean, median and maximum results are all lower than the results of benchmark2. In addition, the minimum results of randomized trading are better than the minimum results of crossover rules in terms of risk adjusted excess returns. It implies that crossover strategy possibly generates significantly bad performance. We apply statistical tests to explore if the worst performed rules are mostly due to bad luck.

Figure 5.9 demonstrates the smoothed probability density function of crossover strategy. The pattern of crossover strategy is different from the symmetric pattern of the randomized trading. 3 humps are observed in its density function. Figure 5.10 and 5.11 illustrate the density functions of crossover rules breakdown by indicators and whether a stop loss ratio is applied. Table 5.7 describes the moments of tested crossover rules. We do not find the same moments among rules listed in the table. Crossover rules indicated by EMA have larger mean, variance, skewness and kurtosis than rules indicated by SMA. Price crossover rules generate negative mean of Sharpe ratios, while the double crossover rules are expected to achieve the profitability. None of the crossover rules have the moments

Metric	AR	ϵ_T	SH	SR	C	pred
Min.	-0.09	0.13	-0.66	-0.90	-0.16	0.46
1st Qu.	< -0.01	0.14	-0.05	-0.06	< -0.01	0.50
Median	0.02	0.14	0.09	0.11	0.06	0.52
Mean	0.02	0.14	0.13	0.16	0.10	0.51
3rd Qu.	0.05	0.15	0.33	0.40	0.21	0.52
Max.	0.10	0.15	0.63	0.79	0.50	0.54
Benchmark1: long and hold strategy						
	0.10	0.15	0.63	0.79	0.50	0.54
Benchmark2: randomized trading						
Min.	-0.17	0.27	-0.55	-0.74	-0.21	0.48
1st Qu.	0.05	0.31	0.13	0.18	0.10	0.52
Median	0.11	0.31	0.32	0.43	0.25	0.52
Mean	0.11	0.32	0.33	0.45	0.29	0.52
3rd Qu.	0.17	0.32	0.51	0.70	0.43	0.53
Max.	0.47	0.43	1.54	2.39	1.88	0.56

Table 5.5: Statistics for financial performance of 15,630 moving average crossover trading rules tested from 01 January 2009 to 06 March 2019

Metric	AR	ϵ_T	SH	SR	C	pred
Number of rules	275	0	275	275	0	275
Quantile in %	1.76	0	1.76	1.76	0	1.76

Table 5.6: Summary of moving average crossover trading rules that outperformed long and hold strategy in the backtest from 31 March 2011 to 06 March 2019

larger than those of the randomized trading. In addition, double crossover rules with the stop loss ratio do not generate a larger mean than rules without the stop loss ratio. As observed in the figure, rules without stop loss ratio even perform a larger maximum SH and lower minimum SH compared to the rules with the ratio. We hence conclude that adding a stop loss ratio does not improve the risk adjusted trading performance.

In the results of multiple tests under $H_0 : F_k(x) = F_n(x)$, the maximum p-value is 2.340×10^{-23} with $A^2_{mn} = 42.04$. The null hypotheses are rejected. Table 5.8 summarises the results from Anderson-darling tests under null hypothesis that the empirical distributions of crossover rules and randomized trading strategy can be obtained from a common population distribution function. Again, the null hypotheses are rejected. We do not find strong evidence showing that the crossover trading strategy obtains results mostly by luck through Anderson-Darling tests. The empirical distributions of crossover rules

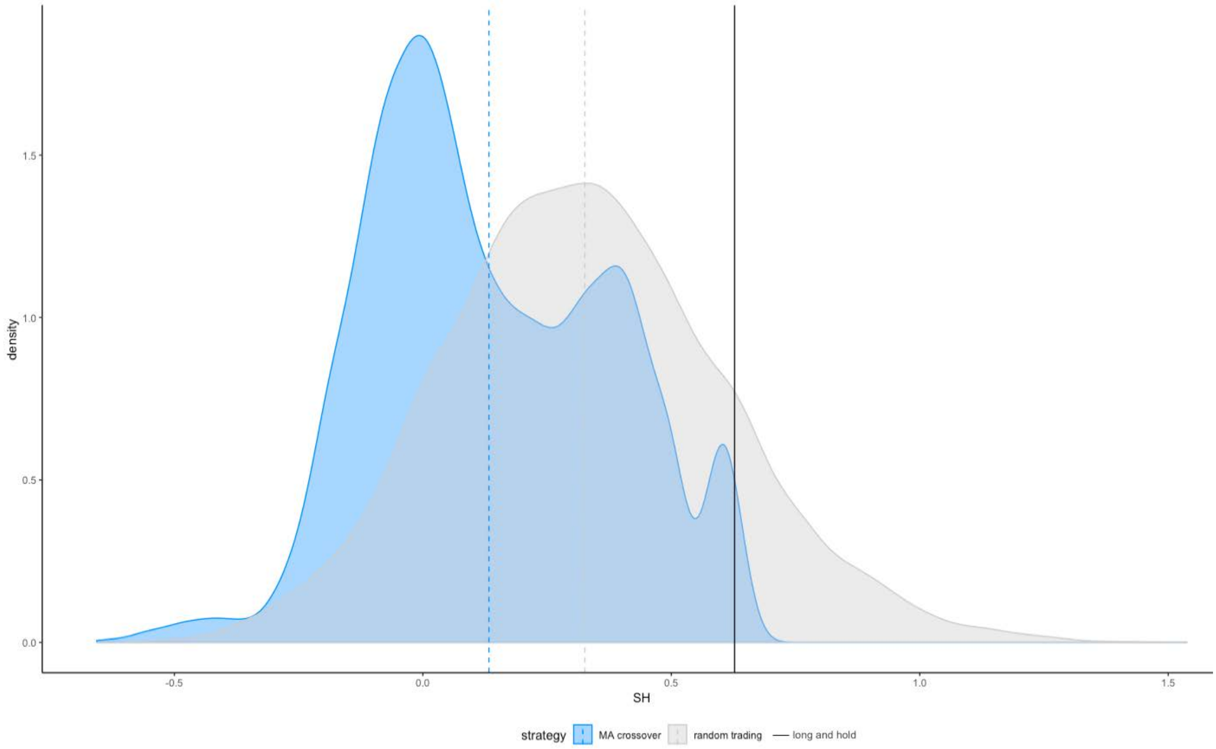

Figure 5.9: Probability density of Sharpe ratio of moving average crossover strategy(blue), randomized trading strategy(grey) and long and hold strategy(black)

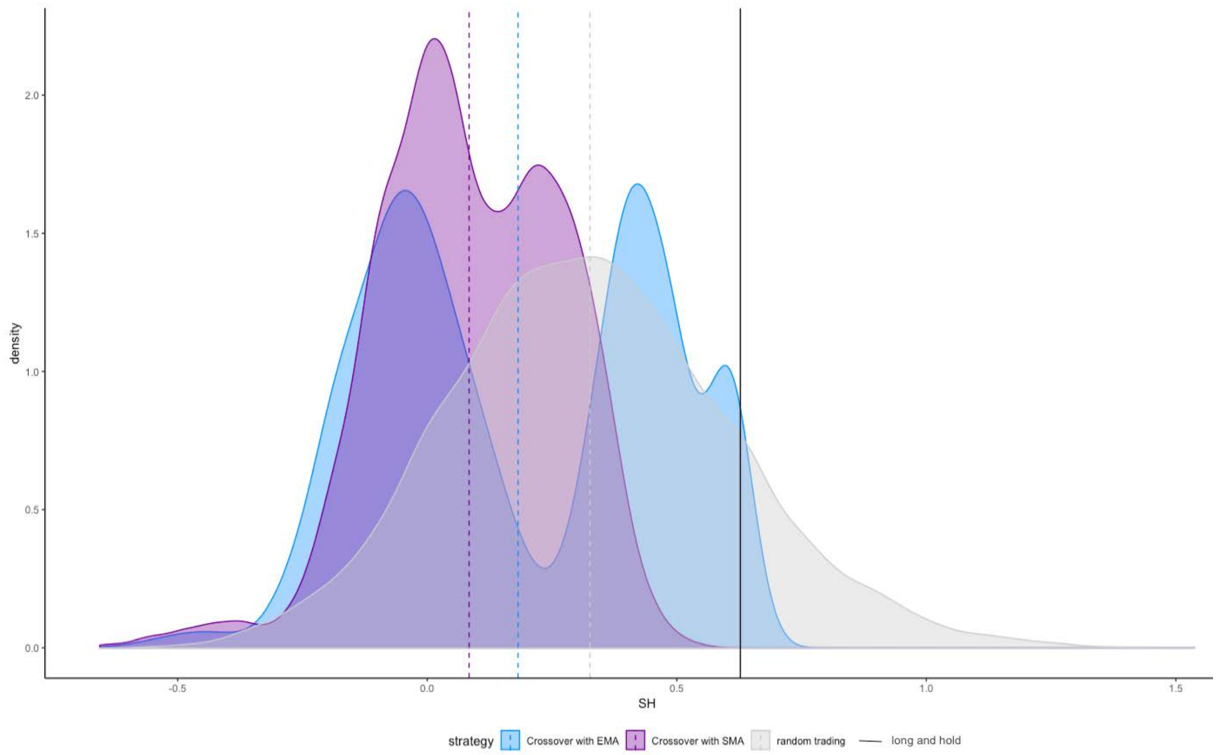

Figure 5.10: Probability density of Sharpe ratio of the benchmarks and crossover rules with indicators SMA and EMA

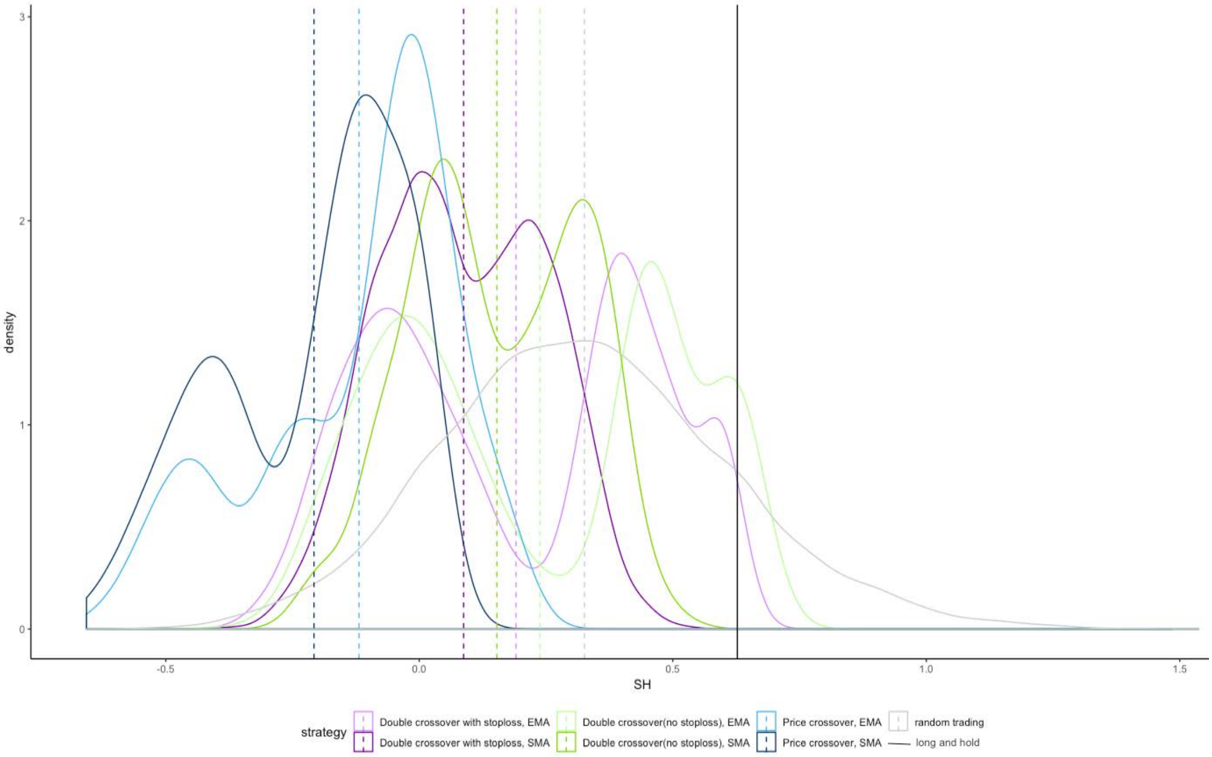

Figure 5.11: Probability density of Sharpe ratio of the benchmarks and crossover rules with different indicators, $sl = 0$ and $sl \neq 0$

Strategy	Mean	Variance	Skewness	Kurtosis
Randomized trading	0.326	0.186	0.118	0.089
Moving average crossover strategy	0.133	0.076	0.028	0.016
Crossover rules(Indicator SMA)	0.084	0.040	0.007	0.004
Crossover rules(Indicator EMA)	0.189	0.113	0.049	0.028
Price crossover($SMA(\lambda)_t$)	−0.208	0.076	−0.033	0.015
Price crossover($EMA(\lambda)_t$)	−0.118	0.053	−0.021	0.010
Double crossover($\{SMA(\lambda1)_t, SMA(\lambda2)_t\}$,$sl = 0$)	0.153	0.052	0.016	0.005
Double crossover($\{EMA(\lambda1)_t, EMA(\lambda2)_t\}$,$sl = 0$)	0.238	0.138	0.070	0.040
Double crossover($\{SMA(\lambda1)_t, SMA(\lambda2)_t\}$,$sl \neq 0$)	0.087	0.033	0.007	0.002
Double crossover($\{EMA(\lambda1)_t, EMA(\lambda2)_t\}$,$sl \neq 0$)	0.191	0.110	0.048	0.025

Table 5.7: Statistic description of the Sharpe ratio generated from the moving average crossover trading rules

indicated by SMA are further away from the distribution of randomized strategy than those indicated by EMA. The smallest AD distance is found in the test of EMA double crossover rules without a stop loss ratio. The largest distance is identified in the test of SMA double crossover rules with stop loss ratio.

Null hypothesis	AD distance(A^2_{mn})	p-value
$SMA(\lambda)_t$	824.80	$<1.000 \times 10^{-300}$
$EMA(\lambda)_t$	594.40	$<1.000 \times 10^{-300}$
$\{SMA(\lambda1)_t, SMA(\lambda2)_t\}$,$sl = 0$	368.70	9.972×10^{-203}
$\{EMA(\lambda1)_t, EMA(\lambda2)_t\}$,$sl = 0$	136.20	4.612×10^{-75}
$\{SMA(\lambda1)_t, SMA(\lambda2)_t\}$,$sl \neq 0$	1545.00	$<1.000 \times 10^{-300}$
$\{EMA(\lambda1)_t, EMA(\lambda2)_t\}$,$sl \neq 0$	451.80	2.154×10^{-248}

Table 5.8: Summary of Anderson-Darling test results under H_0: empirical distribution of Sharpe ratio of tested moving average crossover rules and randomized trading strategy can be obtained from a common population distribution function

Figure 5.12 and 5.13 respectively plot log ratio of the account value resulted from the best and worst performed crossover rules. Only 2 best performed rules outperform long and hold strategy until 2016. No crossover rules have generated greater cumulative returns than the benchmark after year 2016. The majority of the worst performed rules have outperformed the benchmark in the first trading year. These rules have hardly generated profits from the initial account value since year 2012. This is aligned to the findings in price-momentum strategy. It suggests the changing market structure before year 2012.

Figure 5.14 illustrates how Sharpe ratio is changed with the changing parameters and indicators by price crossover trading method. Sharpe ratio hardly becomes positive when SMA is used in the price crossover method. The profitable excess return can be achieved when the trade is indicated by EMA with calibration length taken between 200 and 425 days. Under the same calibration length, the absolute values of SH that are generated from price crossover rules indicated by EMA are mostly higher than that of rules indicated by SMA. Additionally, rules with calibration length below 200 days do not generate a positive SH. Perhaps the calibration window is too short to capture the trend. Figure 5.15 shows the sensitivity of the Sharpe ratio of double crossover rules without stop loss

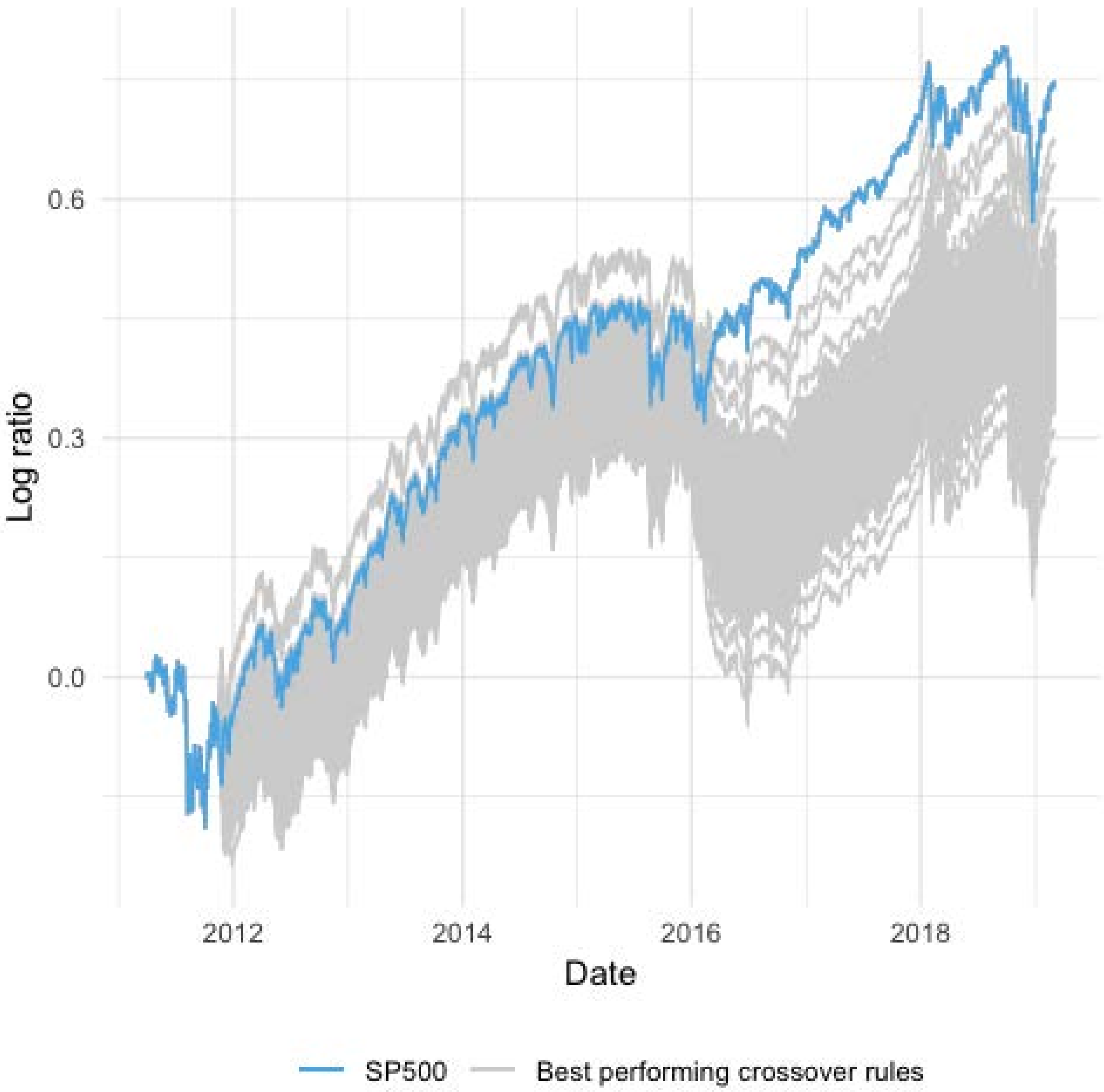

Figure 5.12: Log ratio of the trading account value resulted from the best performed crossover rules and SP500 long and hold trading strategy from 31 March 2011 to 06 March 2019 (No transaction cost applied)

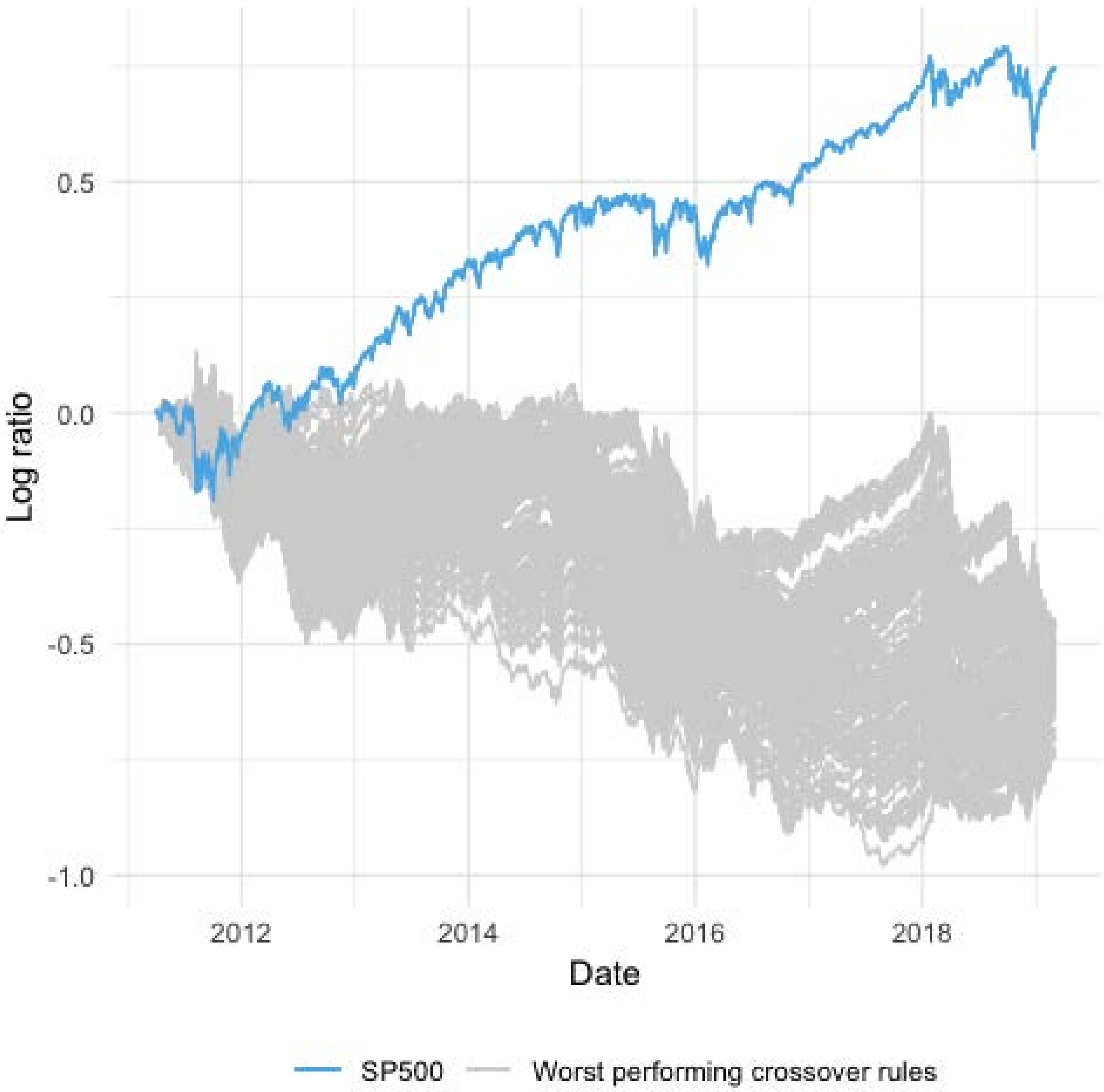

Figure 5.13: Cumulative returns of the worst performed crossover rules and SP500 long and hold trading strategy from 31 March 2011 to 06 March 2019 (No transaction cost applied)

ratio. We find that SH is likely to be larger when both calibration lengths λ_1 and λ_2 increase. Additionally, Sharpe ratio decreases as the difference between λ_1 and λ_2 decreases.

Figure 5.14: Sensitivity of risk adjusted performance of price crossover trading rules

5.3 Pair trading strategy

Table 5.9 summarises the financial performances of the pair trading strategy. 6 (2.38%) out of 252 trading rules have achieved a positive annualised return in the backtesting period. Only 1 trading rule generates annualised Sharpe ratio larger than 0. 8 rules have reached a positive drawdown adjusted annualised return. Therefore, no strong evidence of consistent profitability resulted from pair trading strategy is found. The maximum performances of the pair trading are much lower than those of the benchmarks in terms

Figure 5.15: Sensitivity of risk adjusted performance of double crossover trading rules without stop loss ratio

of AR, SH, SR and C. The minimum risk adjusted excess returns generated from pair trading strategy are even lower than the minimum of randomized trading. We conduct hypothesis tests to identify whether pair trading obtains significantly bad results that are not caused by bad luck.

Metric	AR	ϵ_T	SH	SR	C
Min.	−0.11	0.04	−0.93	−1.03	−0.15
1st Qu.	−0.03	0.07	−0.45	−0.42	−0.09
Median	−0.02	0.09	−0.32	−0.31	−0.07
Mean	−0.03	0.09	−0.36	−0.37	−0.07
3rd Qu.	−0.01	0.11	−0.23	−0.24	−0.05
Max.	0.01	0.13	0.01	0.11	0.05
Benchmark1: long and hold strategy					
	0.10	0.15	0.63	0.79	0.50
Benchmark2: randomized trading					
Min.	−0.17	0.27	−0.55	−0.74	−0.21
1st Qu.	0.05	0.31	0.13	0.18	0.10
Median	0.11	0.31	0.32	0.43	0.25
Mean	0.11	0.32	0.33	0.45	0.29
3rd Qu.	0.17	0.32	0.51	0.70	0.43
Max.	0.47	0.43	1.54	2.39	1.88

Table 5.9: Summary of financial metrics of 252 pair trading rules backtested from 31 March 2011 to 06 March 2019

Figure 5.16 plots the probability density function of Sharpe ratio. The density function of pair trading is asymmetric with a single observed hump. It is different from the function of the randomized trading. Table 5.10 describes the Sharpe ratio in moments. Mean of pair trading is twice lower than that of the randomized trading. No big difference is found between variances of pair trading and the benchmark.

Strategy	Mean	Variance	Skewness	Kurtosis
Randomized trading	0.326	0.186	0.118	0.089
Pair trading strategy	−0.359	0.164	−0.090	0.056

Table 5.10: Statistic description of the Sharpe ratio generated from the pair trading rules

Only one Anderson-darling test has been conducted to test whether the empirical distributions of risk adjusted excess returns of pair trading strategy and of randomized trading strategy can be obtained from the same population distribution. As the result, we obtain $A^2_{mn} = 653.50$ and p-value smaller than 0.01×10^{300}. H_0 is rejected. This suggests that the

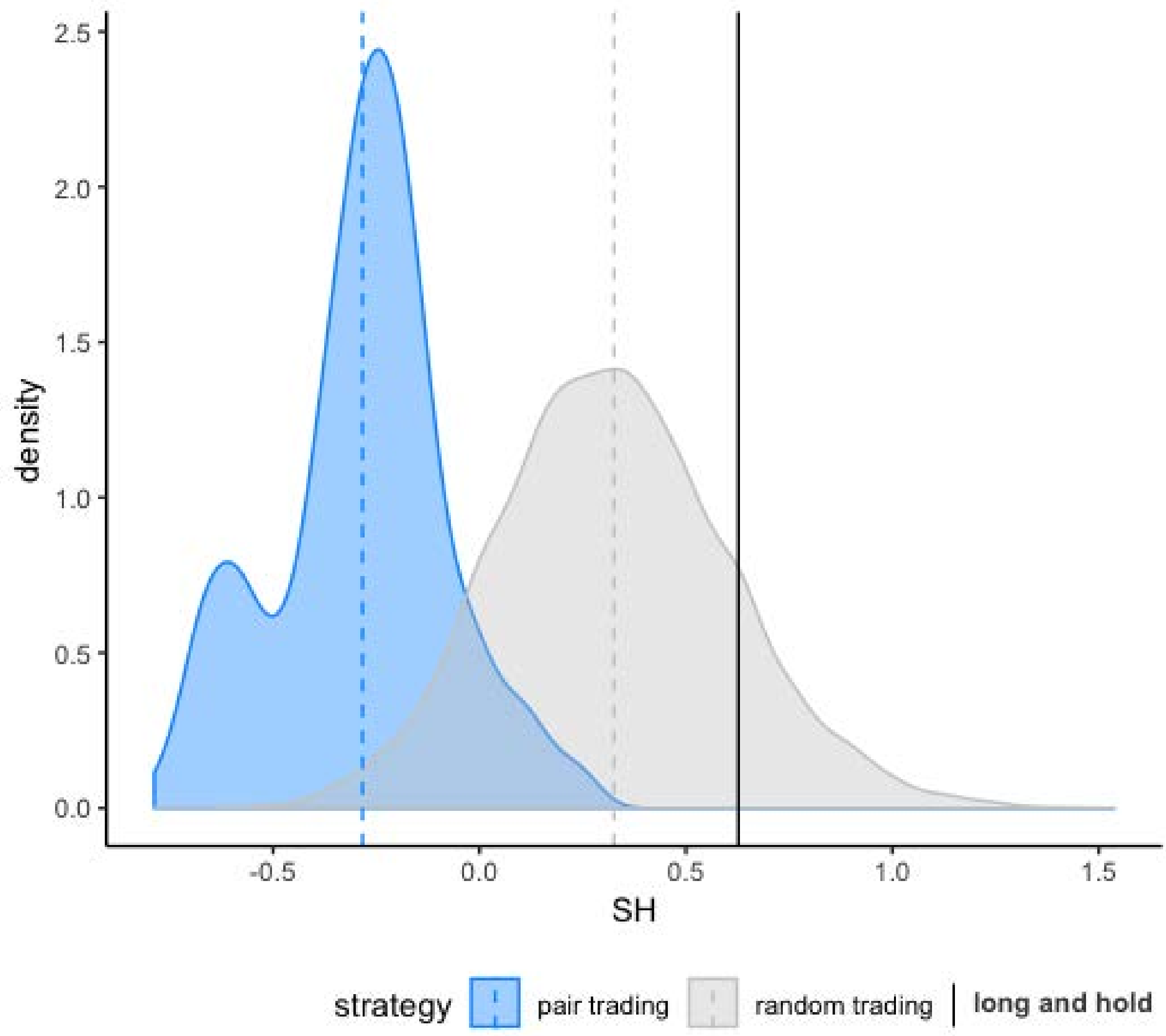

Figure 5.16: Probability density of Sharpe ratio of pair trading strategy(blue), randomized trading strategy(grey) and long and hold strategy(black)

distribution of Sharpe ratio of pair trading strategy is different from that of randomized strategy, i.e. luck.

Figure 5.17 and 5.18 respectively demonstrate the log ratio of the best and the worst performed pair trading rules. The best performed rules hardly realise profits in the trading account through out the entire backtesting period. Compared to SP500 long and hold strategy, they outperform in the first trading year when more loss has occurred in trading account of long and hold strategy than of best performed pair trading rules. The worst performed rules neither generate profits in the trading account, nor outperform long and hold strategy in the backtesting period.

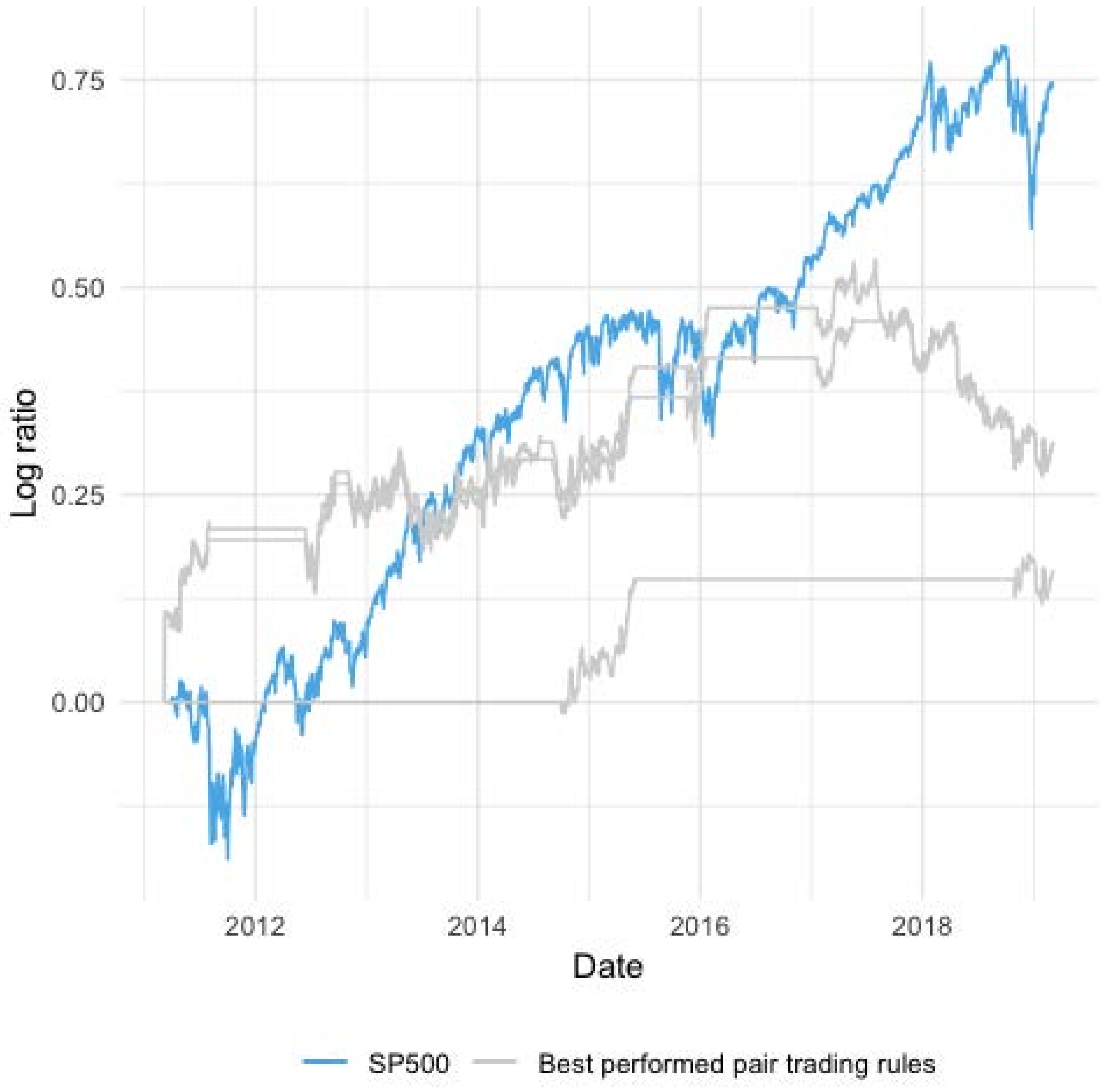

Figure 5.17: Log ratio of the trading account value resulted from the best performed pair trading rules and SP500 long and hold strategy from 31 March 2011 to 06 March 2019 (no transaction cost applied)

Figure 5.19 illustrates a boxplot of Sharpe ratio of pair trading rules with various tested calibration length, regardless of other parameters. Overlaps are respectively found in rules with $\kappa = \{150, 200, 250, 350\}$ and $\kappa = \{400, 450, 500\}$. The Sharpe ratio is likely to be the lowest when 300 days are taken as the calibration length.

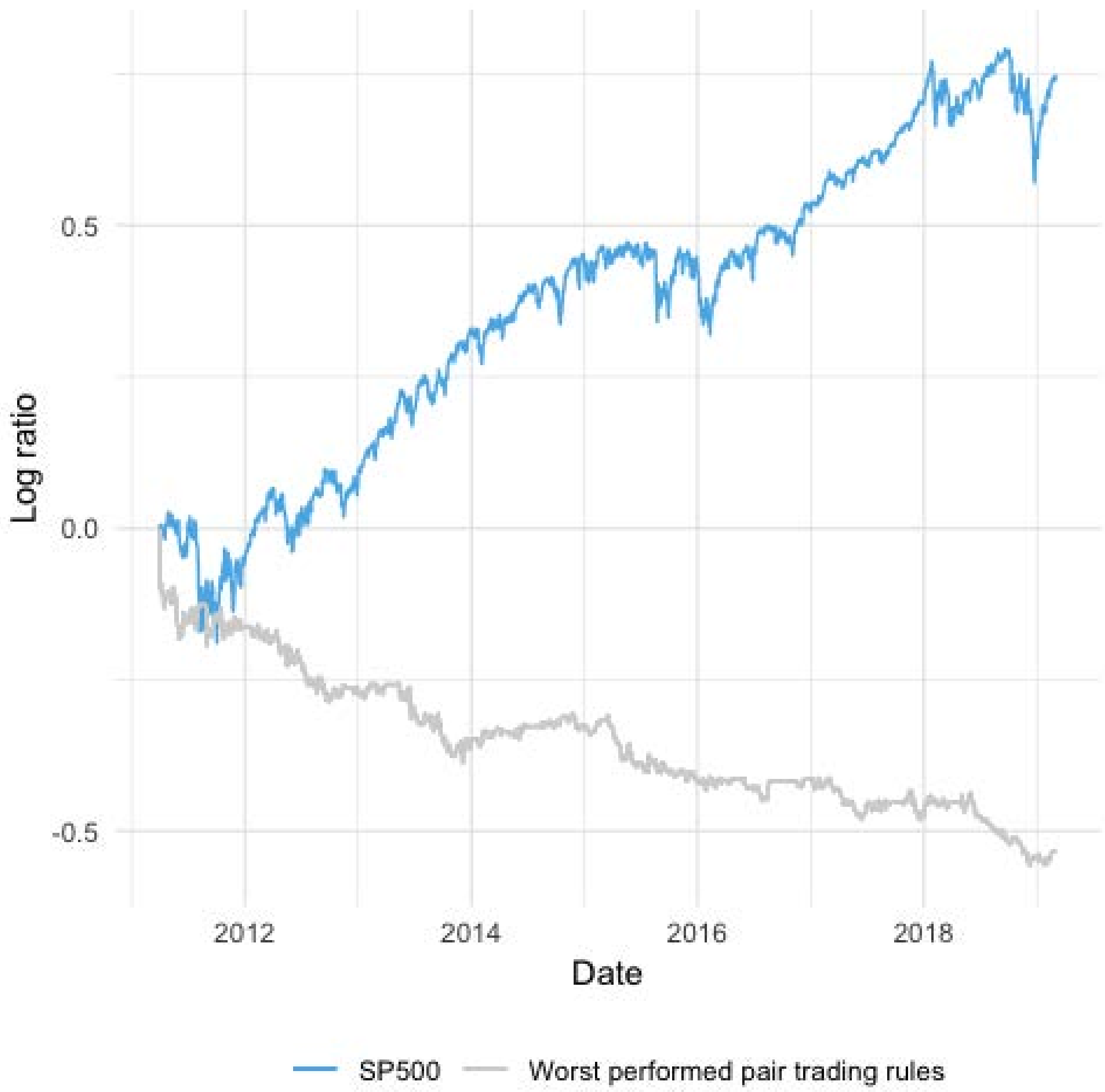

Figure 5.18: Log ratio of the trading account value resulted from the worst performed pair trading rules and SP500 long and hold strategy from 31 March 2011 to 06 March 2019 (no transaction cost applied)

We take rules with $\kappa = \{250, 300\}$, whose Sharpe ratio are expected to be the highest and lowest compared to rules with other calibration lengths. We illustrate the sensitivity of the Sharpe ratio of those rules in figure 5.20. We find that SH reacts differently to the changing α and β when calibration length varies from 250 to 300. Given $\kappa = 300$, SH increases with the increasing α and β. Under condition of $\kappa = 250$, SH increases with the increasing β but decreases with increasing α.

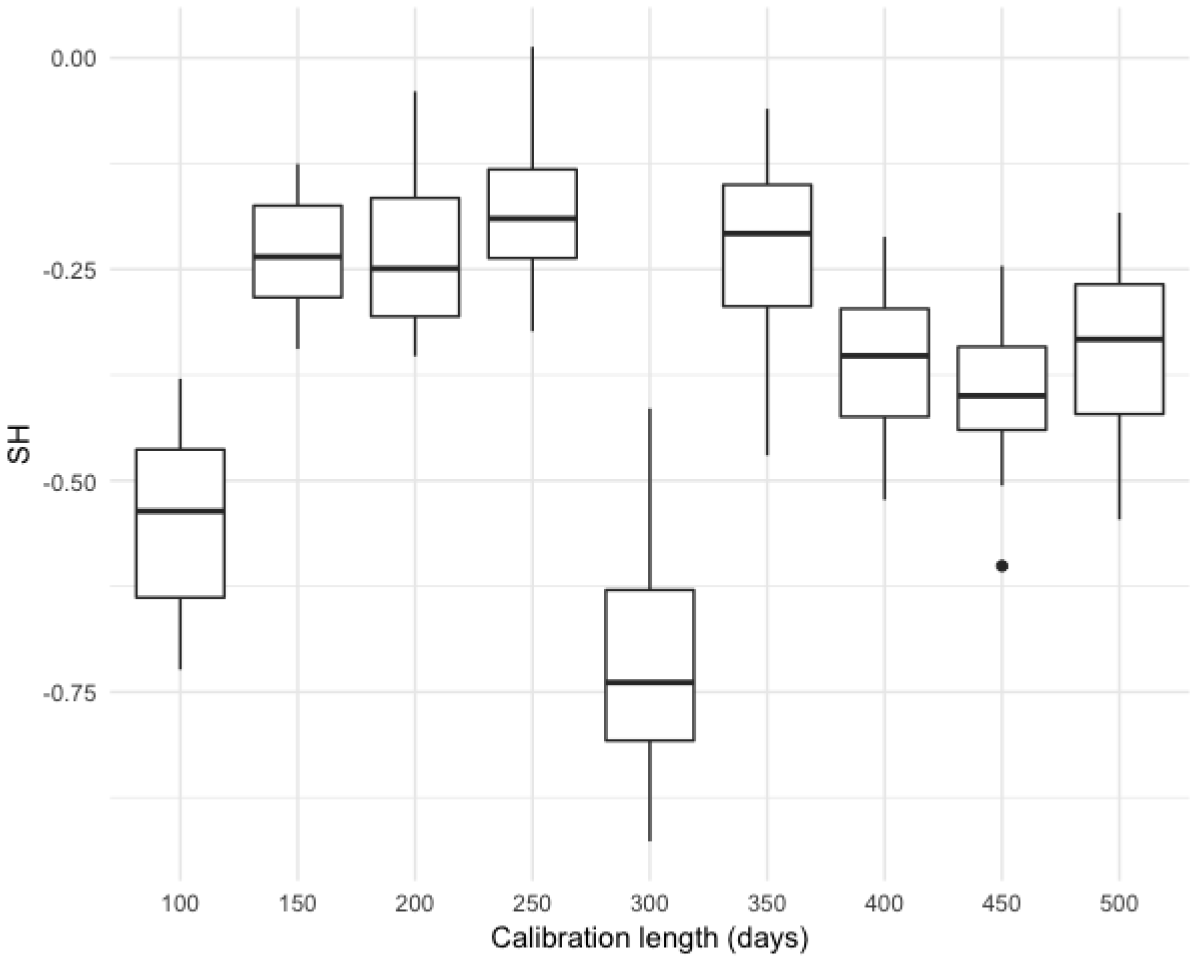

Figure 5.19: Boxplot of Sharpe ratios of pair trading strategy with different calibration length κ

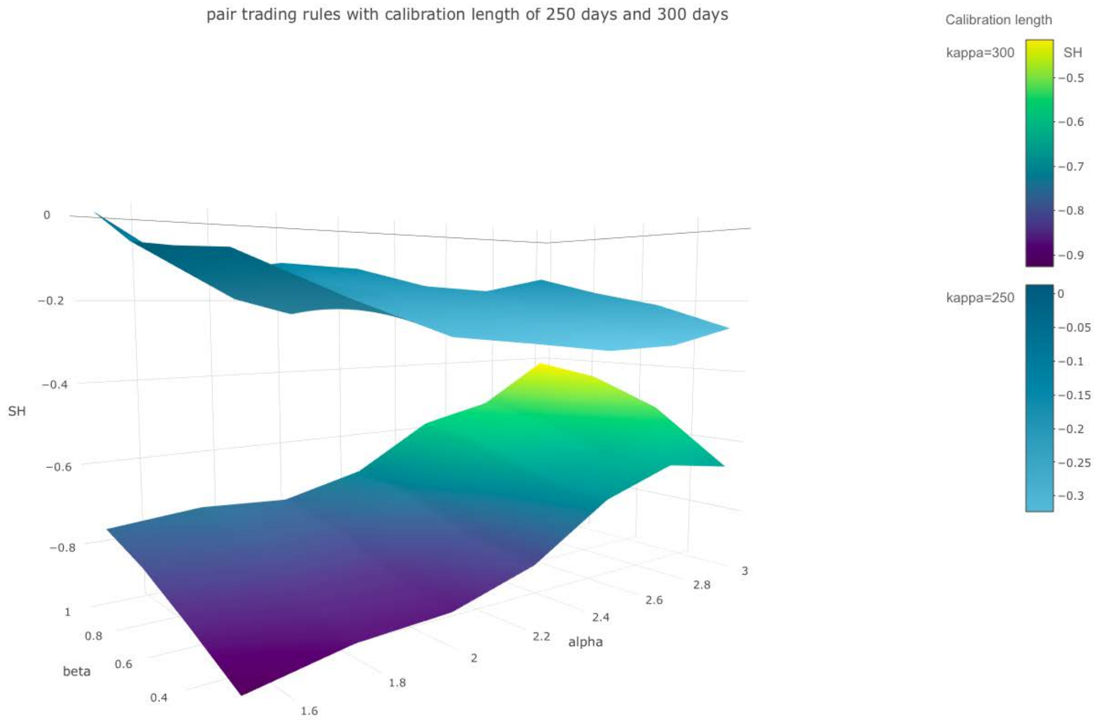

Figure 5.20: Sensitivity of risk adjusted performance of pair trading rules with calibration length $\kappa \in \{250, 300\}$

Chapter 6

Conclusion

This thesis provides detailed methodology to set up technical trading strategies and to backtest strategy performances. It gives insights on performances of technical strategies widely used by professional investors.

As of the exploratory analysis, we find that the daily prices of SP500 index enjoy an uptrend in the past 10 years, while prices of its constituents do not necessarily follow the same trend. The daily price movements of SP500 stocks most likely do not follow a normal distribution. The observed heavy tailed feature supports prior researches of financial time series.

With the backtests of technical strategies, none of the tested strategies outperform the market on average. Price-momentum strategy with only long position is likely to consistently beat the strategy that buy and hold SP500 index in the past 8 years. None of the tested strategies have achieved better annualised risk adjusted returns than the maximum risk adjusted returns of randomized trading strategy, i.e. luck. However, the minimum risk adjusted returns of all three tested strategies are worse than those of randomized strategy. The probability density functions and empirical distributions of Sharpe ratios resulted from the tested strategies are found to be different from those of randomized trading. It suggests the possible existence of trading rules that consistently generate significantly bad performance by "skill". These trading rules are found in all 3 types of tested strategies.

Price-momentum strategy with long and short position has severely underperformed price-momentum strategy with only long position. It implies that such strategy cannot correctly predict the direction of daily price change in a consistent way. The maximum prediction accuracy of moving average crossover strategy is not higher than accuracy of long and hold strategy. The same implication applies on moving average crossover strategy.

In addition, the results of Anderson-Darling tests show that Sharpe ratios of tested strategy with different setups (i.e. indicators and parameters) follow different population distributions. As the density functions of these Sharpe ratios also vary from each other, we conclude that the optimized results can be achieved by selecting indicator and tuning parameters of a technical strategy.

We do not find strong evidence determining the importance of stock selection using technical trading strategies, since the tested strategies involving stock selection do not outperform the randomized trading on average.

6.1 Future research

Due to the time limitation and data availability, the findings in this thesis are limited. Further important discussions require to be considered in the future research. Although the findings suggest a change in market from the financial crisis in the first trading year, the U.S. stock market generally has an uptrend in the backtesting period without any financial crisis. This empirical study likely only reflects and compares the trading results in a stock market with growing bubble. Further backtests of technical tradings in the period of financial crisis is required. Secondly, the trading activities in reality require transaction costs based on the trading volume and frequency. For the simplicity of computation, no transaction cost has been applied in our backtests. No evidence is found in this study that rejects EMH. However, the trading signals generated in this study are only based on daily closing price of the stock and single indicator. The setups of the technical tradings can be much more sophisticated. Trading prediction based on intra-day prices and combinations of several indicators can be of interest of the future researchers. Since we limit stock selection among SP500 constituents, the backtest is still subject to a pre-selection bias. Future research should consider more stocks in various financial markets.

Furthermore, our empirical study has rejected all null hypotheses of Anderson-Darling tests. We find that Anderson-Darling test does not reject its null hypothesis only when the distance between two distributions are extremely small. It remains unknown whether the top performed price-momentum trading rules outperform long and hold strategy mostly due to luck. A resampling-based stepdown multiple testing method developed by J.P. Romano and M. Wolf tests can be considered for the future study[35][34]. It is recommended to use this method to test:

$H_0 : E[SH_{testedstrategy}] = E[SH_{longandhold}]$

$H_A : E[SH_{testedstrategy}] > E[SH_{longandhold}]$

The test statistic can be estimated as the difference between empirical Sharpe ratio of the tested strategy and benchmark (long and hold strategy). Let test statistic be S.

$$S = SH_{testedstrategy} - SH_{longandhold} \tag{6.1}$$

With the implementation of block bootstrap introduced by Politis and Romano in 1992, we suggest resampling the observed daily returns of benchmark with various block size denoted as $b \in \{1, 2, ...B\}$ for r times ($r \geq 1000$)[31]. We can hence obtain r empirical SH of the benchmark measured from the resampled time series. We denote these Sharpe ratios respectively as $SH_{b,1}, SH_{b,2}, ..., SH_{b,r}$. In equation (6.1), $S_{b,r}$ is the test statistics measured from the resampled time series when block size is b.

$$S_{b,r} = SH_{testedstrategy} - SH_{b,r} \tag{6.2}$$

Equation (6.2) formulates the unadjusted p-value given block size b. The research of J.P. Romano and M. Wolf (2016) introduces the method to generate the adjusting p-values[35].

$$\hat{p}_b := \frac{\#\{S_{b,r} \geq S\} + 1}{r + 1} \tag{6.3}$$

It can be interesting to discuss the test results, to compare the results under condition of different b, and to explore the reliability of findings suggested by adjusted p-value and unadjusted p-value.

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Appendix A

Selected stocks of pair trading strategy with various calibration length

The descriptions of the stocks are cited from the official page of the company and Wikipedia.

κ	**Stock** x	**Description**	**Stock** y	**Description**
100	TROW	T. Rowe Price Group, Inc. is an American publicly owned global asset management firm that offers funds, advisory services, account management, and retirement plans and services for individuals, institutions, and financial intermediaries	SCHW	The Charles Schwab Corporation is a bank and stock brokerage firm based in San Francisco, California

150	EIX	Edison International is a public utility holding company based in Rosemead, California	DTE	DTE Energy is a Detroit-based diversified energy company involved in the development and management of energy-related businesses and services nationwide
200	FRT	Federal Realty Investment Trust is a real estate investment trust that invests in shopping centers in the Northeastern United States	BXP	Boston Properties is a real estate investment trust that owns and manages Class A office properties in the United States
250	PXD	Pioneer Natural Resources Company is a company engaged in hydrocarbon exploration in the Cline Shale, which is part of the Spraberry Trend of the Permian Basin, where the company is the largest acreage holder	CXO	Concho Resources Inc. is a company engaged in hydrocarbon exploration
300	GWW	W. W. Grainger, Inc. is an American industrial supply company founded in 1927 in Chicago. It provides consumers with access to a consistent supply of motors	AOS	A. O. Smith Corporation is an American manufacturer of both residential and commercial water heaters and boilers and the largest manufacturer and marketer of water heaters in North America. It also supplies water treatment products in the Asian market

350	GWW	W. W. Grainger, Inc. is an American industrial supply company founded in 1927 in Chicago. It provides consumers with access to a consistent supply of motors	CAT	Caterpillar Inc. is an American Fortune 100 corporation which designs, develops, engineers, manufactures, markets and sells machinery, engines, financial products and insurance to customers via a worldwide dealer network. It is the world's largest construction equipment manufacturer
400	VNO	Vornado Realty Trust is a real estate investment trust formed in Maryland, with its primary office in New York City. The company invests in office buildings and street retail in Manhattan.	FRT	Federal Realty Investment Trust is a real estate investment trust that invests in shopping centers in the Northeastern United States
450	O	Realty Income Corporation is a real estate investment trust that invests in free-standing, single-tenant commercial properties in the United States, Puerto Rico, and the United Kingdom.	HCP	Healthpeak Properties, Inc. formerly HCP, Inc. is a diversified real estate investment trust that owns and develops healthcare real estate within the United States for Life Science, Senior Housing and Medical Office.

500	MMM	The 3M Company, formerly known as the Minnesota Mining and Manufacturing Company, is an American multinational conglomerate corporation operating in the fields of industry, worker safety, health care, and consumer goods.	FLS	The Flowserve Corporation is an American multinational corporation and one of the largest suppliers of industrial and environmental machinery such as pumps, valves, end face mechanical seals, automation, and services to the power, oil, gas, chemical and other industries.

Table A.1: Summary of stocks selected for pair trading strategy

Appendix B

Example codes: setups of technical trading strategies in R

```r
###############
### momentum strategy
###############
### rank monthly returns by stock
### theta 1:25, eta 0.5, M=S=1
########################
trade_mom_mreturn<-list()
for (k in 1:25) { # eta
  for (i in 1:nrow(df_date)) {
    a<-df_mr_rank[df_mr_rank$yrmth==df_date$yrmth[i], ]
    # select best stocks
    rank_max<-max(a$rank)
    rank_90<-quantile(a$rank, probs=(100-k)/100)
    b<-a[a$rank<=rank_max & a$rank>=rank_90, ]
    select_stk<-b$symbol
    # select worst stocks
    rank_min<-min(a$rank)
    rank_10<-quantile(a$rank, probs=k/100)
    c<-a[a$rank>=rank_min & a$rank<=rank_10, ]
    select_stk2<-c$symbol
    # apply to actual trading
    if (i+1>nrow(df_date)) break
    else
      actual<-df_mr_rank[df_mr_rank$yrmth==df_date$yrmth[i+1], ]
    rank_buy<-actual[actual$symbol %in% select_stk, ]
    total_buy<-sum(rank_buy$return)/length(select_stk) #final change of bought stocks
    rank_sell<-actual[actual$symbol %in% select_stk2, ]
    total_sell<-sum(rank_sell$return)/length(select_stk2) #final change of sold stocks
    # save to df
    trade_moment[i, "buy"]<-total_buy
    trade_moment[i, "sell"]<-total_sell
  }
  ### account value
  trade_moment$account<-NA
  trade_moment[1, "account"]<-I_0+trade_moment[1, "buy"]*I_0*w_buy-trade_moment[1, "sell"]*I_0*w_sell
  for (i in 2:nrow(trade_moment)) {
    trade_moment[i, "account"]<-trade_moment[(i-1), "account"]+trade_moment[i, "buy"]*trade_moment[(i-1), "account"]*w_buy-trade_moment[i, "sell"]*
    trade_moment[(i-1), "account"]*w_sell
  } #account value
  trade_mom_mreturn[[k]]<-trade_moment
  names(trade_mom_mreturn)[k]<-paste("ls_M_1_S_1_eta", k, sep = "_")
}
```

```
###############
### MA indicators
###############
trade_ma_ind<-list()
### SMA
for (k in 2:502) {
  trade_SMA<-data.frame()
  for (i in k:nrow(tradeMA)) {
    a<-tradeMA[(i-k+1):i,"close"]
    ma<-mean(a)
    trade_SMA<-rbind(trade_SMA,ma)
  }
  trade_SMA$date<-tradeMA[k:length(tradeMA$date),"date"]
  trade_SMA$open<-tradeMA[k:length(tradeMA$open),"open"]
  trade_SMA$close<-tradeMA[k:length(tradeMA$close),"close"]
  colnames(trade_SMA)<-c("SMA","date","open","close") #final df:tradeMA

### EMA5
  Multiplier_MA<-(2/(k+1))
  trade_SMA$EMA<-NA

  #day1:
  trade_SMA[1,"EMA"]<-trade_SMA[1,"SMA"]
  #dayi:
  for(i in 2:length(trade_SMA$SMA)){
    close<-tradeMA[tradeMA$date %in% trade_SMA[i,"date"],"close"]
    EMA_prev<-trade_SMA[(i-1),"EMA"]
    trade_SMA[i,"EMA"]<-EMA_prev+(close-EMA_prev)*Multiplier_MA
  }
  trade_ma_ind[[k-1]]<-trade_SMA
  names(trade_ma_ind)[k-1]<-paste("MA", k, sep = "_")
}
```

```
### price crossover
trade_ma_res<-list()
trade_ma_res_acc<-list()
for (j in 2:502) {
  x<-trade_ma_ind[[j-1]]
  x$close<-tradeMA[j:length(tradeMA$close),"close"] #add closing price
  x$date <- as.Date(unlist(x$date)) #change format: unlist
  x$open <- as.character(unlist(x$open)) #change format: unlist
  x$close <- as.numeric(unlist(x$close)) #change format: unlist
# add price change
  trade_return_MA <- matrix(NA, ncol = 2, nrow = length(x$close)-1)
  for (i in 2:length(x$close)){
    trade_return_MA[i-1,1]<-x$close[i]-x$close[i-1]
  } #pchange=Pt-P(t-1)
  trade_return_MA<-as.data.frame(trade_return_MA)
  trade_return_MA$date<-x[-1,"date"] #add date
  colnames(trade_return_MA)<-c("pchange","preturn","date")
#add return
  for (i in 2:length(x$close)){
    trade_return_MA[i-1,2]<-x$close[i]/x$close[i-1]-1
  } #preturn=Pt/P(t-1)-1
###add signal SMA5
  trade_return_MA$changeSMA<-NA
  for (i in 1:(length(x$SMA)-1)){
    if (x[i,"close"] > x[i,"SMA"]) trade_return_MA[i,"changeSMA"]<-trade_return_MA[i,"preturn"]
    else ifelse(x[i,"close"] < x[i,"SMA"], trade_return_MA[i,"changeSMA"]<-c(-trade_return_MA[i,"preturn"]),
                trade_return_MA[i,"changeSMA"]<-0)
  } #signal change
### add signal EMA5
  trade_return_MA$changeEMA<-NA
  for (i in 1:(length(x$EMA)-1)){
    if (x[i,"close"] > x[i,"EMA"]) trade_return_MA[i,"changeEMA"]<-trade_return_MA[i,"preturn"]
    else ifelse(x[i,"close"] < x[i,"EMA"], trade_return_MA[i,"changeEMA"]<-c(-trade_return_MA[i,"preturn"]),
                trade_return_MA[i,"changeEMA"]<-0)
  } #signal change
### add start point
  row_ma<-which(trade_return_MA$date=="2011-03-31")
  trade_return_MA_acc<-trade_return_MA[row_ma:nrow(trade_return_MA), ]
```

```
### price crossover
### account SMA
  trade_return_MA_acc$PSMA<-NA #portf. value
  trade_return_MA_acc[1,"PSMA"]<-c(trade_return_MA_acc[1,"changeSMA"]*I_0+I_0) #1st day portf. value
  for(i in 2:length(trade_return_MA_acc$PSMA)){
    if(NA %in% trade_return_MA_acc[(i-1),"PSMA"]) {a<-trade_return_MA_acc[i,"changeSMA"]*I_0+I_0}
    else {a<-trade_return_MA_acc[i,"changeSMA"]*trade_return_MA_acc[(i-1),"PSMA"]+trade_return_MA_acc[(i-1),"PSMA"]}
    ifelse(a>0,trade_return_MA_acc[i,"PSMA"]<-a,
           trade_return_MA_acc[i,"PSMA"]<-NA)}
### account EMA
  trade_return_MA_acc$PEMA<-NA #portf. value
  trade_return_MA_acc[1,"PEMA"]<-c(trade_return_MA_acc[1,"changeEMA"]*I_0+I_0) #1st day portf. value

  for(i in 2:length(trade_return_MA_acc$PEMA)){
    if(NA %in% trade_return_MA_acc[(i-1),"PEMA"]) {a<-trade_return_MA_acc[i,"changeEMA"]*I_0+I_0}
    else {a<-trade_return_MA_acc[i,"changeEMA"]*trade_return_MA_acc[(i-1),"PEMA"]+trade_return_MA_acc[(i-1),"PEMA"]}
    ifelse(a>0,trade_return_MA_acc[i,"PEMA"]<-a,
           trade_return_MA_acc[i,"PEMA"]<-NA)
  }
  trade_ma_res[[j-1]]<-trade_return_MA
  trade_ma_res_acc[[j-1]]<-trade_return_MA_acc
}
```

```
##### double crossover
### lambda
lambda1<-5*seq(from=1,to=30) #30
lambda2<-5*seq(from=40,to=100) #61

trade_dble_res<-list()
for (a in 1:30) for (b in 1:61) { #30 61
  j<-lambda2[b]
  k<-lambda1[a]
  res<-trade_ma_res[[j-1]]
  x<-res[ ,c("preturn","date")]
#add signal SMA
  ind<-trade_ma_ind[[j-1]]
  ind_short<-trade_ma_ind[[k-1]]
  x$changeSMA<-NA
  for (i in 1:(nrow(ind)-1)){
    long<-ind[i,"SMA"]
    short<-ind_short[j-k+i, "SMA"]
    if (short > long) {x[i,"changeSMA"]<-x[i,"preturn"]}
    else {ifelse(short < long, x[i,"changeSMA"]<-c(-x[i,"preturn"]),
                 x[i,"changeSMA"]<-0)}
  } #signal change
#add signal EMA
  x$changeEMA<-NA
  for (i in 1:(nrow(ind)-1)){
    long<-ind[i,"EMA"]
    short<-ind_short[j-k+i, "EMA"]
    if (short > long) x[i,"changeEMA"]<-x[i,"preturn"]
    else ifelse(short < long, x[i,"changeEMA"]<-c(-x[i,"preturn"]),
                x[i,"changeEMA"]<-0)
  } #signal change
### starting point
  row_ma<-which(x$date=="2011-03-31")
  x<-x[row_ma:nrow(x), ]
```

```
##### double crossover
### account value
  x$PSMA<-NA #portf. value
  x[1,"PSMA"]<-c(x[1,"changeSMA"]*I_0+I_0) #1st day portf. value
  for(i in 2:nrow(x)){
    if(NA %in% x[(i-1),"PSMA"]) {print("break")}
    else {a2<-x[i,"changeSMA"]*x[(i-1),"PSMA"]+x[(i-1),"PSMA"]}
    ifelse(a2>0,x[i,"PSMA"]<-a2,
           x[i,"PSMA"]<-NA)
  }
  x$PEMA<-NA #portf. value
  x[1,"PEMA"]<-c(x[1,"changeEMA"]*I_0+I_0) #1st day portf. value
  for(i in 2:nrow(x)){
    if(NA %in% x[(i-1),"PEMA"]) {print("break")}
    else {a3<-x[i,"changeEMA"]*x[(i-1),"PEMA"]+x[(i-1),"PEMA"]}
    ifelse(a3>0,x[i,"PEMA"]<-a3,
           x[i,"PEMA"]<-NA)
  }
  list_x<-list(x)
  names(list_x)<-paste("MA_lambda1", k, "lambda2", j, sep = "_")
  trade_dble_res<-c(trade_dble_res,list_x)
}
```

```
####################
### STOPLOSS - double crossover
###############
### lambda
trade_stop_res<-list()
for (sl in c(0.01,0.02,0.03)) for (a in 1:30) for (b in 1:61) {
  k<-lambda1[a]
  j<-lambda2[b]
  res<-trade_ma_res[[j-1]]
  x<-res[ ,c("preturn","date")]
#add signal SMA
  ind<-trade_ma_ind[[j-1]]
  ind_short<-trade_ma_ind[[k-1]]
  x$sig_SMA<-NA
  for (i in 1:(nrow(ind)-1)){
    long<-ind[i,"SMA"]
    short<-ind_short[j-k+i, "SMA"]
    if (short > long) {x[i,"sig_SMA"]<-1}
    else {ifelse(short < long, x[i,"sig_SMA"]<-c(-1),
                 x[i,"sig_SMA"]<-0)}
  } #add t day SMA signal
  x$sig_liq<-NA
  for (i in (j-k+1):(nrow(ind_short)-1)){
    pt<-ind_short[i,"close"]
    Pl<-c((1+sl)*ind_short[i-1,"close"])
    ps<-c((1-sl)*ind_short[i-1,"close"])
    if (pt > Pl) x[i-j+k,"sig_liq"]<-1
    else ifelse(pt < ps, x[i-j+k,"sig_liq"]<-c(-1),
                x[i-j+k,"sig_liq"]<-0)
  }# add liquidation ratio
```

```
### STOPLOSS - double crossover
#Add change position
  x$pos<-NA
  for(i in 1:nrow(x)){
    if(i==1){
      if(x[i,"sig_SMA"]==1) {ifelse(x[i,"sig_liq"]==-1,
                                    x[i,"pos"]<-0,
                                    x[i,"pos"]<-1)}
      else {ifelse(x[i,"sig_liq"]==1,
                   x[i,"pos"]<-0,
                   x[i,"pos"]<-c(-1))}
    }
    else{
      if(x[i-1,"pos"]==1 && x[i,"sig_SMA"]==-1 && x[i,"sig_liq"]==1)
      {
        x[i,"pos"]<-1
      }
      else{
        if(x[i-1,"pos"]==-1 && x[i,"sig_SMA"]==1 && x[i,"sig_liq"]==-1)
        {
          x[i,"pos"]<-c(-1)
        }
        else{
          if(x[i,"sig_SMA"]==1) {ifelse(x[i,"sig_liq"]==-1,
                                        x[i,"pos"]<-0,
                                        x[i,"pos"]<-1)}
          else {ifelse(x[i,"sig_liq"]==1,
                       x[i,"pos"]<-0,
                       x[i,"pos"]<-c(-1))}
        } } }}
#daily return
  x$changeSMA<-x$preturn*x$pos
### add signal EMA
  x$sig_EMA<-NA
  for (i in 1:(nrow(ind)-1)){
    long<-ind[i,"EMA"]
    short<-ind_short[j-k+i, "EMA"]
    if (short > long) {x[i,"sig_EMA"]<-1}
    else {ifelse(short < long, x[i,"sig_EMA"]<-c(-1),
                 x[i,"sig_EMA"]<-0)}
  } #add t day EMA signal
```

```
### STOPLOSS - double crossover
  x$sig_liq<-NA
  for (i in (j-k+1):(nrow(ind_short)-1)){
    pt<-ind_short[i,"close"]
    Pl<-c((1+sl)*ind_short[i-1,"close"])
    ps<-c((1-sl)*ind_short[i-1,"close"])
    if (pt > Pl) {x[i-j+k,"sig_liq"]<-1}
    else ifelse(pt < ps, x[i-j+k,"sig_liq"]<-c(-1),
                x[i-j+k,"sig_liq"]<-0)
  }# add liquidation ratio
#Add change position
  x$pos<-NA
  for(i in 1:nrow(x)){
    if(i==1){
      if(x[i,"sig_EMA"]==1) {ifelse(x[i,"sig_liq"]==-1,
                                    x[i,"pos"]<-0,
                                    x[i,"pos"]<-1)}
      else {ifelse(x[i,"sig_liq"]==1,
                   x[i,"pos"]<-0,
                   x[i,"pos"]<-c(-1))}
    }
    else{
      if(x[i-1,"pos"]==1 && x[i,"sig_EMA"]==-1 && x[i,"sig_liq"]==1)
      {
        x[i,"pos"]<-1
      }
      else{
        if(x[i-1,"pos"]==-1 && x[i,"sig_EMA"]==1 && x[i,"sig_liq"]==-1)
        {
          x[i,"pos"]<-c(-1)
        }
        else{
          if(x[i,"sig_EMA"]==1) {ifelse(x[i,"sig_liq"]==-1,
                                        x[i,"pos"]<-0,
                                        x[i,"pos"]<-1)}
          else {ifelse(x[i,"sig_liq"]==1,
                       x[i,"pos"]<-0,
                       x[i,"pos"]<-c(-1))}}}}}
#daily return
  x$changeEMA<-x$preturn*x$pos
### starting point
  row_ma<-which(x$date=="2011-03-31")
  x<-x[row_ma:nrow(x), ]
```

```
### STOPLOSS - double crossover
### account
  x$PSMA<-NA #portf. value
  x[1,"PSMA"]<-c(x[1,"changeSMA"]*I_0+I_0) #1st day portf. value

  for(i in 2:nrow(x)){
    if(NA %in% x[(i-1),"PSMA"]) {print("break")}
    else {a2<-x[i,"changeSMA"]*x[(i-1),"PSMA"]+x[(i-1),"PSMA"]}
    ifelse(a2>0,x[i,"PSMA"]<-a2,
           x[i,"PSMA"]<-NA)
  }


  x$PEMA<-NA #portf. value
  x[1,"PEMA"]<-c(x[1,"changeEMA"]*I_0+I_0) #1st day portf. value

  for(i in 2:nrow(x)){
    if(NA %in% x[(i-1),"PEMA"]) {print("break")}
    else {a2<-x[i,"changeEMA"]*x[(i-1),"PEMA"]+x[(i-1),"PEMA"]}
    ifelse(a2>0,x[i,"PEMA"]<-a2,
           x[i,"PEMA"]<-NA)
  }

  ###
  list_x<-list(x)
  names(list_x)<-paste("MA_sl", sl, "lambda1", k, "lambda2", j, sep = "_")
  trade_stop_res<-c(trade_stop_res,list_x)
}
```

```
#####################
### pair trading ###
#####################
trade_pair_AB<-list()
for (k in 1:9) for (al in alpha[1:length(alpha)]) for (be in beta[1:length(beta)]) {
  stock.A<-pair_total[k,1]
  stock.B<-pair_total[k,2]
  kappa<-pair_total[k,3]
# Pair-price ratio
  pair.A<-SP_noBHF[SP_noBHF$symbol %in% stock.A,c("symbol","date","close")]
  colnames(pair.A)<-c("symbolA","date","closeA")
  pair.B<-SP_noBHF[SP_noBHF$symbol %in% stock.B,c("symbol","date","close")]
  colnames(pair.B)<-c("symbolB","date","closeB")

  pair.AB<-data.frame()
  if(identical(pair.A$date,pair.B$date)==TRUE){
    pair.AB<-join(pair.A,pair.B,by="date")
    pair.AB$pr<-pair.AB$closeA/pair.AB$closeB
  } else{print("not identical date")}
# add indicator rules
  pair.AB$e.long<-NA
  pair.AB$e.short<-NA
  pair.AB$l.long<-NA
  pair.AB$l.short<-NA

  for(i in 1:(nrow(pair.AB)-(kappa-1))){
    #calculate mean and sd
    mean<-mean(pair.AB$pr[i:(i+(kappa-1))])
    sd<-sd(pair.AB$pr[i:(i+(kappa-1))])
    #rules
    pair.AB$e.long[i+(kappa-1)]<-mean-al*sd
    pair.AB$e.short[i+(kappa-1)]<-mean+al*sd
    pair.AB$l.long[i+(kappa-1)]<-mean+be*sd
    pair.AB$l.short[i+(kappa-1)]<-mean-be*sd}
   pair.AB<-pair.AB[kappa:nrow(pair.AB), ]
# add enter signal
  pair.AB$sigA<-NA
  #str(pair.AB)
  for(i in 1:nrow(pair.AB)){
    if (pair.AB$pr[i]<=pair.AB$e.long[i]){
      pair.AB$sigA[i]<-1
    } else{
      ifelse(pair.AB$pr[i]>=pair.AB$e.short[i],
             pair.AB$sigA[i]<-c(-1),
             pair.AB$sigA[i]<-0)}}
```

```
### pair trading ###
# add liquidation signal
  pair.AB$liqA<-NA
  for(i in 1:nrow(pair.AB)){
    if (pair.AB$pr[i]>=pair.AB$l.long[i]){
      pair.AB$liqA[i]<-c(-1)
    } else{
      ifelse(pair.AB$pr[i]<=pair.AB$l.short[i],
             pair.AB$liqA[i]<-1,
             pair.AB$liqA[i]<-0) }}
# add daily price change
  pair.AB$pchangeA<-NA
  pair.AB[-1,"pchangeA"]<-pair.AB[-1,"closeA"]-pair.AB[-nrow(pair.AB),"closeA"]
# add return A
  pair.AB$returnA<-NA
  pair.AB[1,"returnA"]<-pair.A[pair.A$date=="2009-10-15","closeA"]/pair.AB[1,"closeA"]-1
  pair.AB[-1,"returnA"]<-pair.AB[-1,"closeA"]/pair.AB[-nrow(pair.AB),"closeA"]-1
# add price change B
  pair.AB$pchangeB<-NA
  pair.AB[-1,"pchangeB"]<-pair.AB[-1,"closeB"]-pair.AB[-nrow(pair.AB),"closeB"]
# add return B
  pair.AB$returnB<-NA
  pair.AB[1,"returnB"]<-pair.B[pair.B$date=="2009-10-15","closeB"]/pair.AB[1,"closeB"]-1
  pair.AB[-1,"returnB"]<-pair.AB[-1,"closeB"]/pair.AB[-nrow(pair.AB),"closeB"]-1
```

```
### pair trading ###
# add position (trading final signal A)
  pair.AB$posA<-NA
  for(i in 1:length(pair.AB$posA)){
    #DAY 1 position
    if(i==1){
      if(pair.AB[i,"sigA"]==1) {
        pair.AB[i,"posA"]<-1}
      else {ifelse(pair.AB[i,"sigA"]==-1,
                   pair.AB[i,"posA"]<-c(-1),
                   pair.AB[i,"posA"]<-0)}}
    else{#Day n position
      if(pair.AB[i-1,"posA"]==1) #current position(t-1) is long
      {ifelse(pair.AB[i,"liqA"]==-1 && pair.AB[i,"sigA"]==-1,
              pair.AB[i,"posA"]<-c(-1),
              ifelse(pair.AB[i,"liqA"]==-1 && pair.AB[i,"sigA"]!=-1,
                     pair.AB[i,"posA"]<-0,
                     pair.AB[i,"posA"]<-1))}
      else{ if(pair.AB[i-1,"posA"]==-1) #current position(t-1) is short
        {ifelse(pair.AB[i,"liqA"]==1 && pair.AB[i,"sigA"]==1,
                pair.AB[i,"posA"]<-1,
                ifelse(pair.AB[i,"liqA"]==1 && pair.AB[i,"sigA"]!=1,
                       pair.AB[i,"posA"]<-0,
                       pair.AB[i,"posA"]<-c(-1)))}
        else{if(pair.AB[i,"sigA"]==1) {
            pair.AB[i,"posA"]<-1}
          else {ifelse(pair.AB[i,"sigA"]==-1,
                       pair.AB[i,"posA"]<-c(-1),
                       pair.AB[i,"posA"]<-0)}}}}}

  # add position B
  pair.AB$posB<-c(-pair.AB$posA)
  # actual CHANGE PRICE
  pair.AB$changef<-NA
  for(i in 2:nrow(pair.AB)){
    pair.AB[i,"changef"]<-pair.AB[i-1,"posA"]*pair.AB[i,"pchangeA"]+pair.AB[i-1,"posB"]*pair.AB[i,"pchangeB"]} #absolute price change of portfolio
  ### starting point
  row.AB<-which(pair.AB$date=="2011-03-31")
  pair.AB<-pair.AB[row.AB:nrow(pair.AB), ]
  ### account value
  pair.AB$account<-NA
  pair.AB$account[1]<-I_0+I_0*w_buy*pair.AB$returnA[1]*pair.AB$posA[1]+I_0*w_sell*pair.AB$returnB[1]*pair.AB$posB[1]
  for (i in 2:nrow(pair.AB)){
    pair.AB$account[i]<-pair.AB$account[i-1]+pair.AB$account[i-1]*w_buy*pair.AB$returnA[i]*pair.AB$posA[i]+pair.AB$account[i-1]*w_sell*pair.AB$returnB[i]*pair.AB$posB[i]
```

```
### pair trading ###
  }
  list_AB<-list(pair.AB)
  names(list_AB)<-paste("pair_kappa", kappa, "alpha", al, "beta", be, sep = "_")
  trade_pair_AB<-c(trade_pair_AB,list_AB)
}
```

Appendix C

Example codes: financial metrics in R

```
###############
### financial measures
###############
### annualised return
f.AR<-function(x){
  CR<-x[nrow(x),"account"]/I_0-1
  return(AR<-(1+CR)^(251/1994)-1)
}

#############
### annualised volatility
f.delta<-function(x){
  x$rchange<-NA
  x$rchange[1]<-(x$account[1]/I_0)-1
  for (i in 2:nrow(x)){
    x$rchange[i]<-(x[i,"account"]/x[i-1,"account"])-1
  }
  STD<-sd(x$rchange)
  return(Delta<-sqrt(12)*STD)
}
```

```
###############
### financial measures
###############
### sharpe ratio
Rf<-c(0.17,0.17,0.13,0.11,0.3,0.6,1.17,2.25)
Rf<-Rf/100
Rf_mean<-mean(Rf) #0.00565

#Rf
f.sharpe<-function(x, mrf=Rf_mean){
  #rf
  x$rf<-rep(0,nrow(x))
  ifelse(length(x[x$yrmnth=="2011-12","yrmnth"])==0, x[x$yrmnth=="2012-12","rf"]<-0.17/100,
         x[x$yrmnth=="2011-12","rf"]<-0.17/100)
  x[x$yrmnth=="2012-12","rf"]<-0.17/100
  x[x$yrmnth=="2013-12","rf"]<-0.13/100
  x[x$yrmnth=="2014-12","rf"]<-0.11/100
  x[x$yrmnth=="2015-12","rf"]<-0.3/100
  x[x$yrmnth=="2016-12","rf"]<-0.6/100
  x[x$yrmnth=="2017-12","rf"]<-1.17/100
  x[x$yrmnth=="2019-03","rf"]<-2.25/100
  #AR
  AR<-f.AR(x)
  #dash_D
  dash_D<-AR-mrf
  #vol
  x$rchange<-NA
  x$rchange[1]<-(x$account[1]/I_0)-1
  for (i in 2:nrow(x)){
    x$rchange[i]<-(x[i,"account"]/x[i-1,"account"])-1
  }
  STD<-sd(x$rchange-x$rf)
  sig_D<-sqrt(12)*STD
  #sharpe
  return(SH<-dash_D/sig_D)
}
```

```
###############
### financial measures
###############
### Sartino ratio
f.sartino<-function(x,mrf=Rf_mean){
  #rf
  x$rf<-rep(0,nrow(x))
  ifelse(length(x[x$yrmnth=="2011-12","yrmnth"])==0, x[x$yrmnth=="2012-12","rf"]<-0.17/100,
         x[x$yrmnth=="2011-12","rf"]<-0.17/100)
  x[x$yrmnth=="2012-12","rf"]<-0.17/100
  x[x$yrmnth=="2013-12","rf"]<-0.13/100
  x[x$yrmnth=="2014-12","rf"]<-0.11/100
  x[x$yrmnth=="2015-12","rf"]<-0.3/100
  x[x$yrmnth=="2016-12","rf"]<-0.6/100
  x[x$yrmnth=="2017-12","rf"]<-1.17/100
  x[x$yrmnth=="2019-03","rf"]<-2.25/100
  #rchange
  x$rchange<-NA
  x$rchange[1]<-(x$account[1]/I_0)-1
  for (i in 2:nrow(x)){
    x$rchange[i]<-(x[i,"account"]/x[i-1,"account"])-1
  }
  DR<-x$rchange-x$rf
  DR<-DR[DR<0]
  #dash_D
  AR<-f.AR(x)
  dash_D<-AR-mrf
  #SR
  return(SR<-dash_D/(sqrt(12)*sd(DR)))
}
```

```
###############
### financial measures
###############
### Calmar ratio
f.raccount<-function(x){
  x$rchange<-NA
  x$rchange[1]<-(x$account[1]/I_0)-1
  for (i in 2:nrow(x)){
    x$rchange[i]<-(x[i,"account"]/x[i-1,"account"])-1
  }
  return(x)
}

trade_mom_mreturn2<-lapply(trade_mom_mreturn,f.raccount)

f.calmar<-function(x){
  rownames(x)<-x$date
  C<-CalmarRatio(x[ ,"rchange",drop=FALSE],scale=12)
  return(C)
}
```