Modelling Uncertainty & Flexibility in the Financial Analysis of a Real Estate Development Project in Switzerland

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Abstract

Real estate development inherits uncertainty & flexibility to a large degree. Traditional financial valuation techniques fail to account for this and thus we employ a real options model to study uncertainty & flexibility in real estate development.

Real option analysis has not found wide application in the real estate development practice yet and we argue that this is due to a lack of pragmatism and comprehensibility of the commonly used models.

We therefore employ the so called "Engineering Approach" introduced by de Neufville and Scholtes (2011) that is based on Monte-Carlo simulations to better analyse and communicate uncertainty & flexibility in real estate development projects.

We develop a Monte-Carlo simulation model based on past data of identified risk drivers that determine the value of a real estate development project. We employ copula and vector autoregressive modelling techniques to account for interdependencies among these risk drivers and apply the resulting simulation model to a large-scale development project in the region of Zurich. Our analysis reveals the risk structure of this real estate development project and shows how downside risk can be reduced by the use of flexibility.

Keywords: Real Options Analysis; Real Estate Development; Monte-Carlo Simulation; Vector Autoregression; Copula ; Engineering Approach; Flexibility; Valuation

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Johannes Peter

Content

1 Introduction

Real estate development is an important activity: It is the production factory of our cities that transforms unproductive land to urban space for people to live, work and enjoy. It shapes the character of the places we spend so much time in and if well done, can substantially improve the quality of life. Real estate development is also an entrepreneurial activity: it involves the decision to take the future of an area in one's hands, to shape it to the needs of upcoming generations and the aspiration to make it a truly successful product on the real estate market.

As such it is also a risky activity: Development binds large amounts of financial capital into a fixed asset while it is not granted that the proceedings will be higher than the investment. The financial success depends both on uncertain cost and uncertain revenues and in order to be successful, one must assess these uncertainties to a high degree. Proper investment analysis is crucial to deal with these uncertainties, to reduce and anticipate them whenever possible.

For more than ten years now real estate prices have been rising continuously and one might be inclined to think that this trend will continue for another ten years. But among the many things we can learn from history, one lesson is that the future is uncertain: just a few years ago the global financial system was close to a meltdown and few would have predicted then the strong growth in real estate prices that we observed in the past few years. Quo Vadis?

Real estate developers need to cope with uncertainty, think about it and find strategies to deal with it. One way of doing so has always been flexibility. If demand for office space is low a developer can wait and build when markets have recovered, or he can change the project to another use. When looking into practice however, the employed financial analysis tools neither properly account for flexibility, nor uncertainty when dealing with real estate development projects.

One way of financial analysis that deals with flexibility & uncertainty is real options analysis. This approach is closely linked to the valuation of financial options and is thus based on assumptions that are not necessarily applicable in the world of real estate development.

Also we argue that due to a lack of accessibility and applicability, real options analysis has not yet gained much attention in the Swiss real estate development practice. We thus introduce a more intuitive, simulation based real options model grounded on the works of de Neufville, Scholtes and Geltner. This so called "Engineering Approach" works with "classic" net present value calculations and combines them with simulations of future market scenarios. Based on these scenarios, decision rules are implemented and optimised to find the optimal behaviour for the simulated future.

Thereby we make flexibility & uncertainty more accessible in the analysis of real estate development projects.

The thesis is structured in the following way: After a short review of the literature on real options in Chapter 2, we give an overview of the real estate development process and the role of risk, uncertainty & flexibility within this process in Chapter 3. In Chapter 4 we discuss the current valuation practice in Switzerland and introduce real options analysis. In Chapter 5 we present a simulation-based model and then show the applicability and the results of this method on a case study in Chapter 6. Chapter 7 concludes this thesis and discusses possible improvements of the model and directions for further research.

2 Literature Review

In this chapter we present the evolvement of real option theory applied to real estate development in the recent past. This helps to understand the balancing act between academic theory and applicability when working with real options in practice. Many terms that are mentioned in this chapter are dealt with in more detail in subsequent chapters.

Real options are a popular topic in the recent literature: A search on the Web of Science[1] yields more than 100 publications related to real options every year over the last decade. The origin of this popularity is closely linked to advances related to the pricing of financial options. Black and Scholes (1973) opened up the field with their revolutionary equilibrium-pricing model for financial options. Six years later Cox, Ross and Rubinstein (1979) already mention the possibility to apply the approach on other problems than financial options in their famous work on financial option pricing. In this article they introduce the binomial approach that is still the basis for many real option valuations nowadays. Since then, real options have been the subject of diverse studies and been applied to various fields of economic research such as capital budgeting problems, valuations, micro-economic-decision-making and more. Whenever dealing with uncertainty and/or flexibility thereby one is prone to come over real options sooner or later. This has also led to some unintended fame of real options when they were widely used as justification for the sky-high valuations of internet stocks before the burst of the dotcom bubble in 2000 (Mauboussin 1999).

In the field of real estate research it took until 1985 for real options to find their applications. Titman (1985) was the first to introduce a real option approach that is closely linked to the binomial model of Cox, Ross and Rubinstein (1979) to value vacant land. With his model he was able to explain the behaviour of many land owners who wait with construction in order to profit from higher expected prices. He showed mathematically that higher uncertainty in the future value of built property leads to a higher option value and thereby to a delay in the exercise of the option. Eight years later Quigg (1993) gave evidence of these findings with the first empirical study on the real option value of vacant land in Seattle. Her findings indicate an average premium of 6% that is paid for the option to wait. Real options theory applied to real estate and other fields of practice remained however a niche and only slowly gained more attention with further publications. The theory was notably made more public by works of Trigeorgis (1993) and later by Copeland (2001).

[1] Web of Science is a large multi disciplinary knowledge database: http://isiknowledge.com/wos

[2] Other developers in Switzerland such as Allreal, Mobimo and Priora have their own investment portfolio and develop mainly

Their work was thereby directed towards bridging the gap between academia and practice. After publishing on the use of financial option models for the valuation of vacant land in 1989 (Geltner 1989), Geltner and Miller further introduced the Samuelson McKean Formula as the "Black-Scholes formula of real estate" in their standard textbook on real estate finance in 2001 (Geltner and Miller 2001).

Thereafter many publications were made on the topic of real options in real estate development at the MIT Center for Real Estate under the supervision of Geltner: Among them there is Hengels (2005) who introduced a model to evaluate large-scale real estate development projects using binomial trees. The goal of his work was to make real options theory using binomial trees accessible by practitioners. But one of the main drawbacks of this approach became highly visible in his work: While the computational effort to analyse a project becomes high very fast, the ability to conduct meaningful conclusions from the analysis becomes more and more difficult. This is due to the fact that results are not easily retraced in the model. We discuss this issue in Chapter 4.2 on the choice of our method. Barman and Nash (2007) tried to overcome this shortcoming by using a combination of the Samuelson McKean formula and Monte-Carlo simulations and Masunaga (2007) made a comparative study of Monte-Carlo simulations and binomial trees to value real estate development projects. As he concluded, there is a significant difference between the results obtained by Monte-Carlo simulation and binomial tree evaluation. As Hodder, Mello et al. (2001) pointed out before, this issue can be addressed by using risk adjusted discount rates depending on the time to expiration of the option and the actual value of the underlying asset, but the problem of accessibility would still not be addressed.

During that time de Neufville (2006) was working on real options using Monte-Carlo simulations while focusing on the value of flexibility. This method was further developed into the "Engineering Approach" and published in 2011 (de Neufville and Scholtes 2011). The advantage of this approach is the specific development for practitioners and we thus make extensive use of it within this study. Geltner and de Neufville (2012) further propose to use both the "Engineering Approach" and the binomial real options model to make financial analysis of real estate development projects. In this thesis we focus on the business applicability of real options and the application of the "Engineering Approach" to a real project in Switzerland, we therefore do not employ both methods.

Most of the proposed real option methods in the context of real estate development deal with the "Anglo-Saxon" real estate development process (see e.g. Guthrie (2009)). However, there is hardly anything on the Swiss real estate development practice (except for Maurer (2006)). But why is this important? As we will see the Swiss real estate development practice is different from the "Anglo-Saxon" way. This has an influence on the specification of the real options model. A second reason is that real options analysis depends on the market where it is applied. It is important to adapt the model to the specifics of this market especially when dealing with Monte-Carlo simulations. Thirdly it is important to apply a theoretical approach to real life situations, involve practitioners and analyse with them the benefits and merits of the solutions "scientists" came up with. We handle these issues subsequently in the following chapters.

3 Real Estate Development Process

This section gives a short overview of the typical processes involved in real estate development, while highlighting some specialities of the Swiss developer environment. By understanding the process of real estate development the reader should become aware of the importance of uncertainty & flexibility within the field of real estate development. We first look at the steps that have to be taken for a successful development and at the risks involved in this process. Finally we highlight the role of flexibility & uncertainty in this context.

3.1 Insights from Professionals

In the course of this study various professionals in the field of real estate were met to discuss the valuation of uncertainty & flexibility in the context of real estate development. We gained insights into the professional real estate valuation domain thanks to Werner Ramseyer from *Sproll & Ramseyer AG*. He explained to us in detail the current valuation practice for real estate in Switzerland and kindly provided case studies. Marc Petitjean from *Helbling Beratung und Bauplanung AG* gave us insights into the real estate development practice and Stephan Fahrländer from *Fahrländer & Partner AG* provided us with a rich data set for the development of the real estate market in Switzerland. Furthermore Alain Chaney from *IAZI AG* gave very valuable input for modelling the real estate market. Finally Othmar Ulrich and his team at *Steiner AG* were a very rich source of knowledge when discussing the applicability of financial models, the valuation practice of real estate development projects and the importance of uncertainty & flexibility in the real estate development process. Whenever possible we try to incorporate insights from these professionals into this thesis.

3.2 Definition

Geltner and Miller (2007) define real estate development as the process where financial capital becomes fixed as physical capital, or more broadly the process of transforming an idea to reality. It includes all the steps from conceptualisation to realisation and sometimes sale of a real estate project.

3.3 Steps in the Development Process

The steps to be taken in real estate development are similar internationally and among projects. The specialities of the Swiss vs. the “Anglo-Saxon” development environment arise from the different parties that are involved in these steps and thus the different risks that they bear. We first go through the steps and then discuss the specialities of the Swiss development environment.

1. **Project Initiation**

Schulte (2002) defines three initial states that lead to the initiation of a real estate development project:

/ **Site looking for use**: When the piece of land to be developed is already given, it is the task of the developer to come up with a project idea and a concept. The developer defines a suitable use for the site that generates the highest possible value. Value in this context does not mean economic value only, but also social and ecological value. Only by taking all three aspects into account, developers will generate sustainable solutions that guarantee long term success. For this he needs to have a "feeling" for the real estate market and know what kind of uses will be in demand at the given location not now but in the future. Therefore he needs to be visionary and anticipate what works and what does not at a given location.

/ **Use looking for site**: When the project idea is already given, the task is to find a suitable location. The developer will need to have information sources that help him find promising sites. A well-established network of people involved in the land transaction market is crucial but also databases and newspapers are of use. Since land acquisition cost is one of the main capital expenditures of a project development, a detailed financial feasibility analysis of the identified selection is of high importance. The phase is concluded with the successful purchase of a plot or the obtainment of a right to build on it in the future.

/ **Capital looking for site and use:** The third initial state involves the processes of finding a site and a use. The needed capabilities are largely the same as with the two other initial states and not repeated here.

2. **Project Conception**

The second phase starts with an in-depth feasibility study regarding location, market, usage, competition, risk and profitability. Based on these studies a detailed project concept is prepared that serves as a basis for the communication with stakeholders. Involved parties such as neighbours, politicians, investors and tenants are contacted and the project is further specified in consideration of them. When the project is feasible and well defined, architects and planners are hired for drawing designs of the future buildings and planning the environment.

The final goal of this phase is the receipt of the building permit that authorises the start of construction. Once construction has started, the project becomes very capital intense and major changes of plans are very difficult and expensive to achieve. To reduce the risk of vacancies it is therefore essential that future tenants are identified, involved and committed to the project already before construction starts. Developers will thus look for prospective tenants already during the project conception phase and not proceed until the risk of vacancies is reduced substantially.

3. **Project Realisation**

   A contractor, who manages the construction process and construction works, executes construction until completion, often for a prearranged price. Thereby the contractor takes over construction cost risk from the developer. The role of the developer in this phase is that of controlling and managing the work executed by the contractor. Also he will focus on finding additional tenants and buyers for the project.

4. **Lease-Up & Tenant Finishes**

   During the lease-up period the project enters the market, additional tenants or buyers commit themselves to the project and often a customized finish for the tenants is made. Depending on how many tenants committed themselves to the project already in the preceding phases, the risk of not leasing up the whole project can be substantial. Large vacancies and or lower than expected rent contracts can have a severe impact on the profitability of the project. Depending on the project and the market environment it can take several months up to years until a project is fully leased out.

5. **Stabilised Operation**

   Once the project is fully leased out, it enters the phase of stabilised operation where it produces a hopefully stable and increasing cash flow for the lifetime of the object. Subsequent investments for maintenance and sometimes redevelopments are necessary in order to sustain the long term profitability of the object on the market.

Specialties of the Swiss Developer Environment

As pointed out before, specialities of the Swiss vs. the “Anglo-Saxon” real estate environment arise not from the steps in the development but from the players that are involved in these steps. Geltner et al. (2007), taking the viewpoint of the “Anglo-Saxon” development environment, describe Step 1 and 2 of the project as the preliminary phase of a project. This phase is often conducted by an “entrepreneurial” developer, who may or may not continue with the project until completion. The construction and lease-up phase together represent the “development” project where large amounts of capital are committed to the project.

In the “Anglo-Saxon” environment there are large developers, sometimes in the form of real estate investment trusts, specialised only on this part of real estate development. Due to their access to funding they can commit the large financial funds needed for construction and either sell the project once it is stabilised, or keep it in their own portfolio. These developers would however not invest in a project without a construction permit in place. The discussed literature on real estate development focuses on Step 3 to Step 5 of the development, taking the viewpoint of these developers.

In Switzerland however, the business environment is market by “entrepreneurial” developers, such as Halter, Implenia, Steiner, Losinger and HRS[2] who pursue the project from Step 1 to Step 4. For them the investment starts already before a construction permit is achieved often with the purchase or optioning[3] of land. These developers will often sell the project to an institutional investor with the receipt of the construction permit at the end of Step 2, the “Project Conception” phase. Depending on the contract, the developer is responsible for leasing up the project and delivering it for a specified price. After completion and the lease-up period investors operating in Switzerland keep the project as a stabilised asset in their portfolio. We therefore have institutional investors on the buy side that commit themselves very early to the project and “entrepreneurial” developers that take the project through all steps, but sell the project early on. Due to the sale of the project with the receipt of the construction permit, the Swiss developer transfers some of the risk to the investor, which has to be taken into account in the financial analysis of the project. Additionally developers in Switzerland are often construction contractors, managing the construction work themselves, thereby taking over construction cost risk. In this thesis we focus on the entrepreneurial developer and investment analysis from his viewpoint.

3.4 Risks in the Development Process

Except for the case where the investor or final user is already committed to the project from the beginning, real estate development is among the riskiest entrepreneurial activities (Schulte 2002). This comes from the fact that capital investments for product creation are high, products are fixed to their location, served market segments are often small and demand is highly uncertain. The correct assessment and management of risk is therefore indispensable and belongs to the main capabilities of successful developers. Schulte (2002) defines the main risks of real estate development as:

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[2] Other developers in Switzerland such as Allreal, Mobimo and Priora have their own investment portfolio and develop mainly for themselves. While the model in this thesis can be applied to their investment perspective as well, we do not explicitly focus on them.

[3] Optioning in this context refers to the widespread practice of developers to pay a prearanged price for the land to the owner on the condition that the construction permit is obtained.

/ **Development Risk**: The risk of not planning an adequate use for a specific location and the risk of planning financially non-feasible projects

/ **Time Risk**: Due to the financial leverage of most projects time risk is among the most important risk factors. Delays can harm the profit of developers substantially.

/ **Approval Risk**: All development projects need to be approved by the authorities. Neighbours can raise objections that can result in financially harmful project changes.

/ **Financing Risk**: Development projects are financially daring undertakings that require partners with corresponding financial power. Funding might not be achieved or might be stopped due to delays or other problems resulting in the failure of a project and severe financial consequences for the developer.

/ **Building Ground Risk**: The building ground bears high potential for additional cost and delays. This is due to the fact that building grounds bear uncertainty regarding supportable load and contamination that cannot be eliminated completely with preliminary studies.

/ **Cost Risk**: Cost risks arise mainly from the long time horizon of development projects and the uncertainty regarding the exact specification of the future product. Thus it is often very difficult to predict exactly the production cost of a large-scale development project and additional costs may arise from the other mentioned risk factors.

/ **Market Risk:** The final test of every project is when it comes to market. Are the potential tenants willing to pay the calculated rents resp. is demand high enough to meet the additional supply at the specified price? How much are investors willing to pay for real estate assets? Real estate value is driven by the space market that couples demand and supply for space and by capital markets (Geltner and Miller 2007). These two markets are already difficult to assess in the present and their behaviour is much more difficult to predict for multiple years ahead. Inevitably this leads to large uncertainty when dealing with the market risk of real estate developments. Besides that real estate markets behave in long lasting cycles that are characterised by periods of strong growth in prices and high construction activity followed by phases of stagnation and price decline (Dokko, Edelstein et al. 2001). Developers need to anticipate markets correctly and make the right preparations and decisions based on their estimations.

In the context of market risk, a short excurse on the Swiss real estate crisis of the 1990s gives us valuable insights into the harmful effects of such an event.

The Swiss Real Estate Crisis in the early 1990s

After the stock market crash in 1987, the Swiss National Bank (SNB) increased liquidity and decreased interest rates to counter negative effects of the crash and prevent a slackening economy. The economy however recovered faster than expected and the increased supply of liquidity together with an overheating economy lead to inflation rates of 5% by the end of 1989 (Jetzer 2007). At the same time investors shifted their interest due to the uncertainty in stock markets from stocks towards real estate that was assumed to be a more reliable and save asset class. This lead to a further increase in prices for real estate that were already at a high level in the mid 1980s. Due to the high inflation rates by the end of the 1989 the SNB increased short term interest rates drastically from 3.8% in July 1988 up to 9.5% in January 1990 (Jetzer 2007). Additionally the Swiss government passed a bill in October 1989 on land laws that hindered the speculative trade of land and increased capital requirements to purchase land (Meier 2009). The strong increase in interest rate brought the strong growth in real estate prices to an end, which was followed by a decrease in prices of 20% (Jetzer 2007). Especially regional banks that were highly active in the real estate mortgage market incurred large losses on their assets. In October 1991 the “Spar- und Leihkasse Thun” had to close down due to high write offs on mortgage loans, leading to the first bank run in Switzerland since the 1930s (Holderegger 2006). Between 1991 and 1996 more than half of the original 180 regional banks disappeared (Jetzer 2007). As Meier (2009) points out, among the causes for the real estate crisis were low interest rates, high liquidity in the finance system and also a lax mortgage granting policy of many banks during that time. This is an interesting aspect in the light of the current market environment were interest rates are lower than in the late 1980s (currently 1.5% for a 5 year fixed mortgage) and banks recently pledged to increase their lending standards based on self regulation (Chapman 2012). We see the effects of the early 1990s crisis on the real estate market in the data later on.

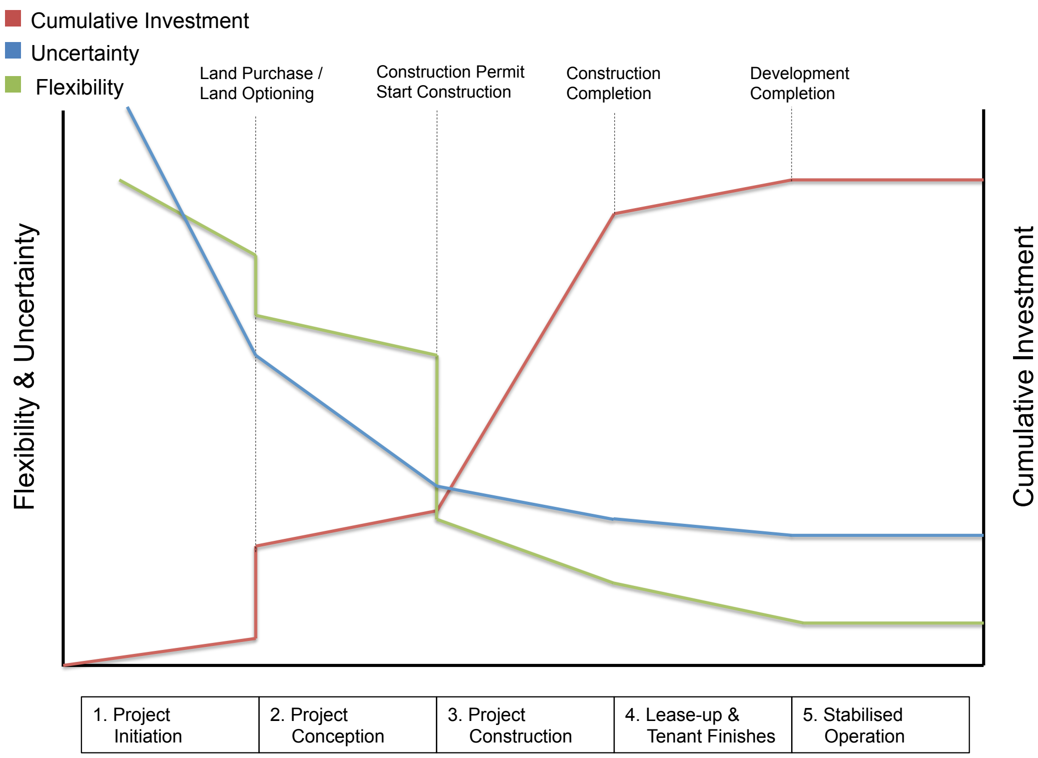
3.5 The Role of Flexibility & Uncertainty

As we have seen there are multiple risks/uncertainties involved in real estate development. The developer has several tools to deal with this and among the most important are the right analysis and the corresponding action towards these risks. A developer can e.g. decrease the approval risk by integrating authorities early on and he can reduce building ground risk through detailed analysis by specialists.

Another tool in dealing with risks is the use of flexibility: A developer can e.g. adapt the project to a changing market environment, postpone development until markets are more favourable, split the project up in multiple subprojects and realise them in succession, etc.

It is this skilful management of flexibility that often makes the difference between successful development and financial failure. But flexibility & uncertainty do not remain constant over the duration of a project.

Figure 1: Flexibility, Uncertainty and Cumulative Investment in the Development Process



Source: Own illustration based on Geltner et al (2007) and Schulte (2002)

As illustrated in Figure 1 flexibility & uncertainty decrease during the life time of a project. Uncertainty can thereby be reduced by analysis and commitments of prospective buyers/tenants, while flexibility decreases when decisions are made. At the same time more and more capital is bound to the project as illustrated by the red line in Figure 1. Only when uncertainty is reduced by a fair degree, one is willing to commit large amounts of capital.

At the beginning of the development, there is almost complete freedom on what to build or where to invest but then the project becomes more and more specified and things are not changed that easily anymore. It is thus of utmost importance to make the right adjustments to a project while flexibility is still high and possibly preserve certain flexibility that could be of value later on. In order to make best use of flexibility, a developer has to know where it is, how important/valuable it is, which one to preserve and when best to exercise it. It lies in the competence of the successful developer to know this due to his experience and intuition. While probably most developers would agree that they have a lot of experience and intuition, the question if there are not any quantifiable tools to deal with uncertainty & flexibility arises. This leads us to our next chapter about the valuation practice of development projects.

4 Valuation of Real Estate Development Projects

In the fist part of this chapter we describe the capitalised earnings method and the net present value method, which constitute the prevailing methods in valuation practice of real estate development in Switzerland. In addition, we highlight both their strengths and weaknesses and stress the need for better tools to deal with uncertainty & flexibility. In the second part of this chapter, we introduce real options analysis as a method that effectively deals with uncertainty & flexibility in the context of valuation. We discuss the reasons why the approach has not found wide application in the real estate development practice yet and stress the need for a more applicable and intuitive model. This leads us to a simulation based model that we introduce in detail in Chapter 5.

4.1 Common Practice

As shown by Müller (2007) in his study on the valuation practices in Switzerland, the capitalised earnings and the net present value method (NPV) prevail in the appraisal of real estate development projects. We thus introduce these two methods in this chapter and use them later on in our model. When discussing the valuation of real estate development projects it is important to differentiate between the valuation of existing, built real estate that already entered the phase of stabilised operation, and the valuation of a project that is in one of the preceding phases of the development process introduced in Chapter 3. This comes from the fact that the two cases inherit different risk structures: in the case of a development project, the future value of the asset can only be estimated and incurs a large degree of uncertainty. Additionally the process of obtaining the asset involves binding large amounts of financial capital that does not generate positive cash flow until tenants move in or the project is sold. Now how should we estimate the value of the development project? As we will see, the process is first to know what the asset would be worth if it existed today (value of the built, stabilised asset), and then to account for the risk of obtaining that asset. So we need to have the concept of valuing built assets and development projects in mind when discussing the valuation of real estate development projects.

4.1.1 Capitalised Earnings Method

The capitalised earnings method is foremost a method to value stabilised real estate assets. The developer can however use it to estimate the value of a real estate development project.

This is done by estimating the value of the project if it would exist today (value of the built project), using the capitalised earnings method, and then deducting all costs until completion of the project. The resulting value is the profit of the developer, hence this method is also known as the developer calculation. It is also used when the value of land has to be estimated: The developer then estimates the value of the built project and deducts costs for construction, financing and development. The difference between value and cost is the maximum value of land. The developer will however not want to pay this maximum value for the land, since then he has no profit. The value of a built project can be approximated by:

1. Calculating the annual Gross Operating Income (GOI) of the project by multiplying the leasable square meters with the expected average rent per square meter per year. Vacancies are deducted from the GOI. The result is the expected yearly cash flow from our building:

$$GOI = m^2 \cdot RentRevenues \cdot (1 - Vacancy) \tag{4.1}$$

2. Calculating the value of the project by dividing the GOI by the cap rate:

$$Value = \frac{GOI}{CapRate} \tag{4.2}$$

Equation (4.2) is equivalent to the dividend discount model introduced by Gordon (1962) developed to value an asset based on an infinite series of dividend payments D growing at rate g with constant cost of capital r :

$$Value = \sum_{t=1}^{\infty} D_0 \cdot \frac{(1+g)^t}{(1+r)^t} \tag{4.3}$$

Equation (4.3) is a converging geometric series with the partial sum:

$$Value = D_0 \cdot \frac{(1+g)}{(r-g)} \tag{4.4}$$

When we set growth to zero, (4.4) becomes to:

$$Value = \frac{D_0}{r} \tag{4.5}$$

(4.5) is the value of a constant dividend payment discounted at rate r into perpetuity. The growth rate g has a strong influence on the value and setting it to zero might seem misleading on first sight. This is however how the capitalised earnings equation is set up in the professional real estate valuation domain according to Canonica (2009) and Fierz (2005), two of the standard books on valuation in Switzerland. Doing otherwise would imply an estimation on rental growth from the valuation professional, which is not applicable since it is basically a speculation about the future. As pointed out by W. Ramseyer, the real estate valuation expert is foremost concerned with the value of an object at the present moment of time and neither in the future nor in the past. Seen from this view point a zero growth rate g as in equation (4.5) becomes justified. There are however other valuation standards, where a growth rate *g* is employed as described in Geltner et al. (2007). Other authors such as Hoesli, Jani et al. (2005) employ a model with growth rate to describe real estate values. We will however, stick to the concept of (4.5), since it captures the main drivers of real estate value and is easier to model since we do not have to make a rather difficult estimate on growth. Furthermore it is often employed in practice.

In (4.5) we calculate the value with net dividend income D_0. *GOI* in (4.2) however does not directly correspond to net dividend income since cost for operation and maintenance of the property are not accounted for yet. Therefore the cap rate in (4.2) consists of the cost of capital *r* plus an addition that accounts for operation and maintenance of the property. These additions are according to Fierz (2005) between 1.0 to 1.5% for new properties.

The cost of capital *r* consists of the weighted average cost of capital (WACC) of the investor. According to practitioners the WACC in real estate often consists of 60% to 70% of debt capital and 30% to 40% equity capital. The cost of debt capital is thereby determined by interest rates while the cost of equity capital is determined by the required return of the investor.

Especially in the current market environment with very low interest rates, we can observe the high influence of decreasing cap rates due to a reduced WACC on the value of real estate assets. Since cost of capital is decreasing, cap rates are decreasing as well and the value of real estate increases from a valuation standpoint. It can be seen how (4.2) brings together the two drivers of real estate value if we interpret the *GOI* as the result of the space market and the cap rate as the result of the capital market.

Now that the developer estimated the expected value of the project he estimates the projected cost of the project consisting of:

- Land acquisition costs
- Planning & development cost s
- Construction costs
- Financing costs

The projected profit is calculated by subtracting the costs from the prospective value of the project. If this profit is high enough to compensate the developer for the subjective risks, the developer will proceed with further analysis of the project. Often the feasibility analysis is used to determine if the acquisition costs of the project (in most cases the land cost) are at an acceptable level or if not stated, how much the developer is able to pay while still having an acceptable profit.

While the feasibility analysis can be done rather easily and without looking too much into details, it is important to understand that it is only useful as a first step in the financial analysis and that further analysis is needed to optimally assess the investment opportunity (Gelter et al. 2007) . This is due to the fact that the capitalised earnings method does not appropriately take into account opportunity cost of capital, the time it takes to achieve profits and the risk involved. One consequence of this is that it becomes very difficult to decide between mutually exclusive projects when only applying the capitalised earnings method. Müller (2007) points out however that many real estate professionals in Switzerland use the capitalised earnings method as their main method to assess a development project, which emphasises the importance of providing more sophisticated models for investment decisions in real estate development. This leads us to the next method: The net present value method.

4.1.2 Net Present Value Method

The basic idea behind the net present value method is to not only look at the absolute profit but also at the time it takes to acquire that profit, the risk involved and the opportunity cost of capital. Cash flows that lie in the future are discounted to the present using an appropriate discount rate. The discount rate accounts not only for the time value of money (e.g. the risk free rate) but also for the riskiness of the future cash flow by applying an appropriate risk adjusted discount rate. Thereby future cash flows are "valued" in the present, can be compared with each other and the overall project can be assessed. When we discount the net cash flow of each period of a project to the present we obtain the net present value (NPV). The NPV method is also used in the context of valuing stabilised real estate assets, where it is basically a more diligent estimation than with the capitalised earnings method underpinned by the same principles. In the context of real estate development the application is however different. In this study we take the viewpoint of the developer, who sells the project to an investor during or after the development process. Here the sales volume and the costs of the project are discounted to the present using an appropriate discount rate. The sales volume thereby is estimated using the capitalised earnings method. The NPV makes a statement about which project to choose among mutually exclusive projects. The decision rule thereby is:

- Maximize the NPV across all mutually exclusive alternatives
- Never choose an alternative that has: NPV < 0

With the equation:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} \tag{4.6}$$

with

CF_t : cash flow at time t

r : risk adjusted discount rate

The NPV valuation is the standard tool used in real estate development and other fields to value investment projects. The method is easy to use and intuitively comprehensible which is probably why it enjoys such large popularity. There are however some shortcomings that we need to discuss.

4.1.3 Shortcomings

Disadvantages of the capitalised earnings method are that it does not account properly for the risk and the opportunity cost of capital. The NPV method effectively deals with these by using a risk adjusted discount rate and the timing of cash flows, suffers however from other problems in correctly assessing the value of a development project. Especially in terms of flexibility & uncertainty the usual way of applying the NPV fails to account important aspects. As we know from the overview of the real estate development process done in Chapter 3, these are two very important factors and thus we discuss their role in the NPV method here.

Flexibility

One of the main problems in applying the "classic" NPV method is the inability to account for flexibility in the assessment of a project. NPV analysis assumes that all cash flows will occur according to the calculation, which neglects the fact that managers actually manage their projects and make adjustments when things do not go as planned or new opportunities arise. As we discussed earlier, the exercise and planning for flexibility is one of the major tools a developer has to manage risks in the development process.

When we only employ the classic NPV method we fail to account for the value of flexibility and possibly understate the value of a project or even worse, fail to recognise, plan and exploit flexibility in the development process.

Uncertainty

A second problem of the NPV method is firstly the consideration of uncertainty over the choice of an appropriate discount rate and secondly the consideration of uncertainty over the estimated cash flows. Let us first discuss the discount rate: The discount rate accounts for the time value of money and also for the riskiness of the cash flows. While the time value of money can be approximated by the risk free rate, how does one know which discount rate to choose for the risk involved? In Switzerland discount rates for real estate development range between 8 to 14% according to practitioners. In order to calculate conservative developers are inclined to use a higher discount rate, the larger the uncertainty in a project is, but larger uncertainty means higher upward and downward potential. The use of a higher discount rate then penalises foremost the larger downward potential, making them unattractive from an investment point of view. However this penalising might be unjustified, since there is also a higher upward potential in such a project, that one could make use of, while not having to face the downward outcome by e.g. using flexibility. The question to choose an appropriate discount rate then becomes difficult. The second problem with the NPV method is the assumption of deterministic fixed cash flows in the calculation. How does one know which values to choose if there is large uncertainty in these cash flows? One way to deal with this is scenario analysis where multiple possible cash flows are considered.

We discussed the currently used valuation methods and the problems arising from applying them. Since real estate development involves uncertainty & flexibility, we need better tools for the financial analysis and decision-making process of projects. We thus introduce real option analysis as a method that effectively deals with this issue in the next part of this chapter.

4.2 Real Option Analysis

Real option analysis (ROA) is an elegant way of handling flexibility & uncertainty in investment projects. Similar to financial options, Copeland (2001) defines a real option as "the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time – the life of the option."

The idea behind it is, that we have uncertainty in the outcome of projects and flexibility that allows us to take actions accordingly. For example when markets have a downturn during the lifetime of the project (uncertainty), we do not have to face the full downside, but can e.g. abandon the project (flexibility) thereby cutting our losses.

This is very similar to financial options traded on financial markets: By obtaining a financial call option, we get the right but not the obligation to buy a certain stock for a certain price in the future. If at maturity, the stock price is higher than the exercise price of the option, we win. If at maturity, the stock price is lower than the exercise price however, our option becomes worthless. The parallels of financial options and real world projects led to the application of the financial option theory to real world projects. This is an important causality to bear in mind when applying real option frameworks since this origin of the approach from financial options has some benefits but also some drawbacks.

A very elegant feature of real options as a valuation tool is that it can be used directly as an extension of the conventional NPV method. Mun (2002) proposed the concept of the expanded NPV as a combination of the option value and the conventional NPV method:

$$NPV = PV\ Benefits - PV\ Costs$$

$$Options\ Value = Benefits\ of\ Options - Cost\ of\ Acquiring\ Options$$

$$expanded\ NPV = NPV + Options\ Value$$

This formulation of a real option lets us define the value of the option in a straightforward way as the difference in value between a project with and without the option. When a project has no flexibility, the value of the option is simply zero. Also when a project is without uncertainty our option is worthless, since there is no flexibility we could employ. Only when there is both uncertainty and flexibility there is option value and the higher the uncertainty, the higher this option value becomes. This stands in contrast to the standard financial theory that tells us that uncertainty is something that decreases value. Uncertainty means up and downward potential however and with options we can make use out of this, hence real options like uncertainty (Geltner and de Neufville 2012).

The way to do real option analysis is to recognise uncertainty, think about flexibility and compare the case with flexibility against the one without. If this results in a positive number, we have found an option of value. Mun (2006) thereby derives the value of ROA from 50% thinking about it, 25% of number crunching and another 25% of interpretation of results. The thinking about flexibility & uncertainty, model and value it, and then designing the project accordingly is really the essence of the approach.

There are three main approaches to apply real option analysis:

/ Binomial Approach

/ Closed Form Solutions

/ Monte-Carlo Simulation

4.2.1 Binomial Approach

The binomial approach introduced by Cox, Ross and Rubinstein (1979) is a widely used method to value financial and real options.

The binomial approach is based on the assumption of:

- Perfect markets: Full information is available, therefore there are no arbitrage opportunities, there is no sure gain of money.
- Complete markets: Any risk can be replicated without transaction cost.
- Rational behaviour: Market participants act rationally and therefore exercise options in an optimal way.
- Geometric Brownian Motion: In most models the underlying asset follows a geometric brownian motion, also known as a random walk.

Under these assumptions investors will be indifferent in holding the actual option or of owning a replicated portfolio consisting of bonds and equity that results in the same payoff structure. We can then value this replicated portfolio using the risk free rate and risk free probabilities that account for the risky payoff structure.

The lifetime of an option is split up in multiple time steps, resulting in multiple up-or-down movements of the underlying asset, thereby creating a tree of possible states of the underlying asset. The value of the option is then calculated by working the tree backwards from the end to the beginning. At each node the value of the option is calculated taking into account the state of the underlying asset and possible states one step ahead. The advantage of this method compared to classic NPV valuation is that we do not account for the riskiness of the pay-out structure over the discount rate but with probabilities, thereby separating risk and the time value of money.

4.2.2 Closed Form Solutions

There are several closed-form solutions such as the Black-Scholes (Black et al. 1973) or the Samuelson McKean formula (Geltner et al. 2007) and many more abbreviations for specific options. These closed form solutions can often be interpreted as binomial models with infinitesimally small time steps, which was already showed for the Black-Scholes formula by Cox, Ross and Rubinstein (1979). The closed form solutions have the advantage that they are easy to implement and give a result very quickly once the parameters are estimated. However they are usually made only for valuing a very specific kind of option (e.g. American or European option) thus they do not allow for complex pay out structures. As with the binomial models they only allow for one kind of option within a project and do not account for interchanges of multiple options. However, as we know from real estate development, there are several options available to us, which makes the approach applicable only for certain cases (e.g. the value of vacant land). The underlying process of these solutions is in most cases a geometric Brownian motion that is based on the lognormal distribution with constant variance σ. While this might be somewhat close to what we observe in stock markets it is highly questionable that the value of development projects behave this way. After all there are multiple factors influencing the value. Furthermore it can be difficult to estimate or communicate the significance of the results, since these models give out one specific number that needs to be interpreted with great care. Also the models make it extremely important to understand the underlying processes and assumptions exactly since without it one is prone to using the wrong model and/or drawing the wrong conclusions.

4.2.3 Monte-Carlo Simulations

Other widely used methods for valuing real options are based on Monte-Carlo simulations. This approach uses thousands of randomly generated scenarios of possible future market outcomes and calculates the value of the project under these scenarios. As with the other real option frameworks, the value of the option is the difference between the project with and without the option but here we do not necessarily base the model on the idealised assumptions of financial option models (de Neufville and Scholtes 2011). As we will see this so called "Engineering Approach" gives us great flexibility in dealing with real options. One of the difficulties with this approach however is that we have to implement flexibility into the model manually so that it behaves under the market scenarios as we specify it. This can be seen as a disadvantage against the other two approaches that exercise options always in an optimal way. Another difficulty lies in the modelling of the underlying asset value. In contrast to the other models, we are free to choose how the value of our project is determined and we do not rely on the geometric Brownian motion. Depending on the required sophistication of the model, one will need to apply advanced statistical tools that are to be handled with great care, in order to get meaningful results. However, once the model is implemented it offers great possibilities for adaptation and analysis, which makes it a good tool for analysis.

4.2.4 Critique and Choice of Method

Although real option theory enjoys large popularity in academia and other industries, it is rarely used in the Swiss real estate practice (see (Müller 2007)).

Why is that so? One of the reasons comes from the theory itself: The valuation of an asset using a replicating portfolio and the non-arbitrage argument may be a very elegant way in theory, but in reality, although real estate is a traded asset, it is impossible to find a replicating portfolio with the same risk exposure as that of a development project. Also it is clear, that since the real estate market is rather intransparent and transactions are infrequent, it is far from the no arbitrage assumption. So when we work with these assumptions we have to be very careful with interpretation. De Neufville et al. (2011) put it as: "*We need to resist the temptation to apply the techniques of financial options blindly to such projects* [technical projects].

We need to handle such applications with great care. This is because the context of technological projects differs significantly from that of financial transactions. The assumptions underlying the theory of financial options are not generally valid for projects, and that theory is thus of limited value for the design and implementation of technological systems."

Other authors such as Copeland (2001) argue that the assumptions underlying financial options are no real problem since they are already implied in the risk-adjusted-discount-rate of the NPV. Also we have to acknowledge that of course all models are simplifications of the real world and assumptions help us to get closer to the truth in order to make meaningful decisions. Conversations with industry professionals however suggested other reasons more linked to the business world that real options have not made the leap into the Swiss real estate development practice yet:

1. Development projects in Switzerland often start with the securing of land. At that stage often no concrete building project exists, usage is only feebly defined and the threat of the public voting against a large construction project is always there. The risks involved in this first stage of the project are often of equal or higher importance than that of the market risk. The decision maker has to take care of all involved risks and cannot rely too much on the numerical result that comes out of a real option valuation.
2. Real options analysis values a project higher than with normal NPV analysis. This is inherent in the system, since the option adds value to the NPV. Decision makers however want to calculate conservatively, since they still want to have margin when things turn out differently than expected. So why should one bother to make the analysis only to get a small additional value, when he already knows that he has some security margin?
3. Decision makers take over large responsibility towards their superiors or shareholders. None of them wants to argue over the applicability of a replicating portfolio as the reason for an investment decision. Things need to be easily understood, defendable and communicated in order to have impact. This is not necessarily the case with real options so far.

This makes clear that we need to use an applicable approach that clearly shows the value of the analysis in order to have impact with real option analysis. In our view the mentioned "Engineering Approach" using Monte-Carlo simulations is the method of choice for such a goal.

We will go into the specifics of the model that we use into the subsequent sections. However, the main reasons for this choice are the following:

Add-on Character

As will be shown in the successive chapters, the approach can be implemented rather easily to the existing framework of a developer. Developers make their investment calculations mostly with Excel, therefore they are familiar with the basic functions of the program and are able to modify calculations according to their needs. The model we apply is basically an enhancement of the widely used spreadsheet calculations in Excel by overlapping the existing calculations with simulations of the future real estate market and implementing flexibility.

Comprehensibility

Users can follow the process from changes in the real estate market to changes in the value of the project and since they are familiar with the NPV method, they understand the valuations and their implications. Numbers can be tracked and manipulated giving confidence in the results and avoiding the black-box phenomenon. In contrast to the binomial approach we do not employ the assumptions of financial options theory but work with normal NPV. This is not perfect either of course but is easier to comprehend since practitioners are used to work with it.

Customisation

Customisation is easily possible by changing input parameters such as rent levels, vacancies and capitalisation rates. The framework can be adapted to the specific project or the simulations can be added to an existing NPV valuation framework. It is also possible (and strongly advised) to perform extensive sensitivity analysis on the specific project that helps to further understand and optimize market risk exposure of the project.

We discussed the current valuation practice of real estate development projects and advocated the need for incorporating uncertainty & flexibility into the valuation for a more profound analysis and decision-making process. We then introduced real option analysis as a promising way to effectively deal with uncertainty & flexibility and discussed reasons why the method has not yet found wide application in the real estate development practice. We came to the conclusion that we need a more applicable and intuitive approach to have impact in practice. This let us favour the "Engineering Approach" as an analysis framework for real estate development projects. In the next chapter we look at this framework in detail and show how we can apply and configure it for the Swiss real estate market.

5 Simulation Based Real Options Model

In the following chapter we first explain the "Engineering Approach" introduced by de Neufville and Scholtes (2011). This framework helps us to assess flexibility & uncertainty in a real estate development context and we use it extensively in our case study in Chapter 6. In the second part of this chapter we first look at the development of prices, vacancies and cap rates in the region of Zurich Unterland and then introduce two models for simulation based on this data. This simulation framework generates possible future scenarios of the real estate market, which allows analysing real estate development project under these scenarios.

5.1 Project Modelling Framework: The "Engineering Approach"

The "Engineering Approach" is designed to help practitioners design better systems by analysing flexibility & uncertainty. In order to be applicable, the approach is based on pragmatic and simplifying assumptions that differ from the binomial and the closed form real options models.

Instead of working with a replicating portfolio and risk neutral probabilities as used in the binomial and the closed form real options model, the "Engineering Approach" works with net present values with constant discount rates as we know them. We therefore do not value cash flows, based on the theoretical sound framework introduced in Chapter 4.2.1.

But should we not build up on this theoretical framework and present answers that are as correct as possible, that "get things right" as exactly as possible? De Neufville et al. (2011) describe this aspect of their approach as: *"…we believe that our modest aspiration to "get it better" is more likely to improve practice. Indeed the concept of "getting it right" is difficult to defend once we accept that modelling the performance of socio-technological systems is as much an art as science."*

We argue for an applicable approach in Chapter 4.2.4 and hypothesise that ROA with binomial trees or closed form solutions have not been widely adopted in practice because they lack the pragmatism that is needed "to get things better" instead of "getting things right". They are too closely linked to the "correct" valuation of financial options that they lack the flexibility to be easily adapted to real projects. We believe that by applying the "Engineering Approach" we are better able to catch the essence of real options analysis, which is to think about flexibility & uncertainty, model and value it, and then to make better project analysis and decision-making.

The “Engineering Approach” is divided into four steps:

/ Creating the most likely initial cash flow model

/ Incorporate uncertainty into the model

/ Incorporate flexibility into the model and

/ Maximize value by applying an optimal decision rule

We introduce the four steps in detail here and apply them to our case study later on.

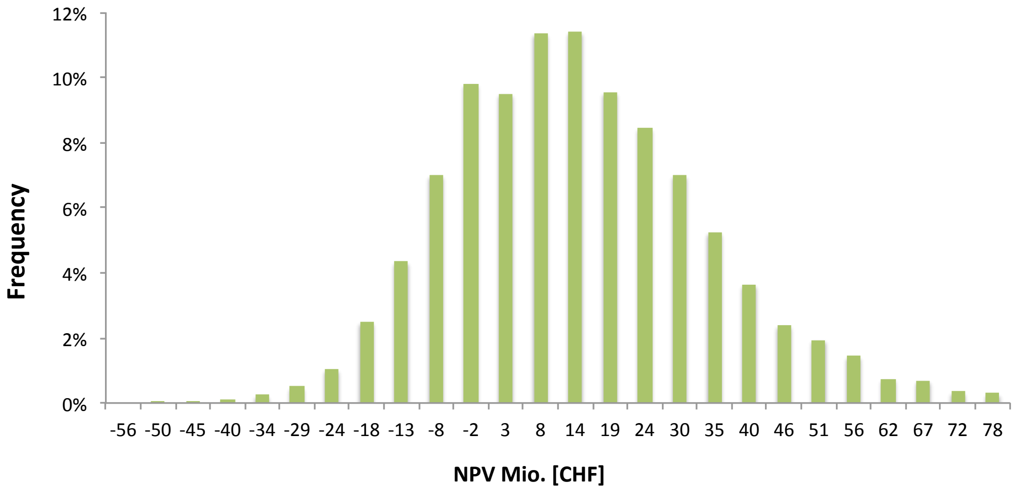
5.1.1 Step 1: Create the Most Likely Initial Cash Flow Model

The first step is to create a pro forma cash flow model incorporating development and construction cost, project scale, schedule and the estimated sales volume. The sales volume, the estimated value of the finished project on the market, is calculated using the capitalised earnings method. The cash flows are discounted to present values, using an appropriate discount rate, which results in the NPV of the project. As we know from Chapter 4.1.3 “Shortcomings”, the resulting NPV neither incorporates uncertainty in cash flows nor flexibility yet, but since it is the standard valuation tool it serves as a good benchmark against which we can compare our further calculations. We will therefore call the result of this calculation the “base case”.

5.1.2 Step 2: Incorporate Uncertainty into the Model

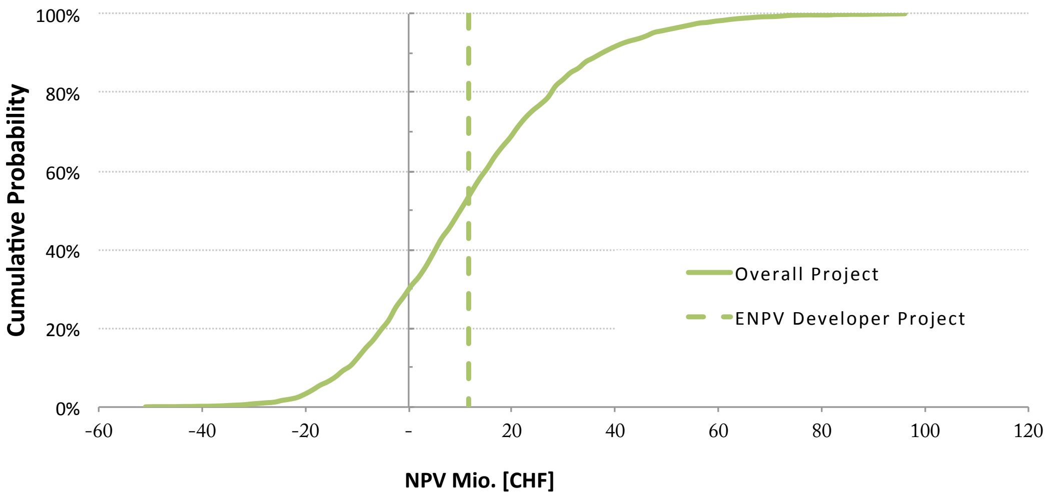
We enhance our initial cash flow pro forma by recognising uncertainty. We do this by identifying the risk drivers that determine our future cash flows and then try to model them. For our case we assume that the future cash flows only depend on the real estate market and therefore make a model of this real estate market. We will then use this model to simulate subsequent scenarios of this market. By doing so we can generate thousands of possible future scenarios and their corresponding cash flows. As shown in Figure 2, this results in a distribution of possible outcomes rather than one estimated value as we obtained it from the “base case”. Since we do not yet incorporate flexibility, we will call this the “static case”. We analyse the “static” case using histograms and cumulative probability curves of the resulting NPV distribution, as shown in Figure 2 and 3.

Figure 2: Example of a NPV Distribution



Source: Own illustration

Figure 3: Example of a Cumulative Probabilities Curve and Expected NPV



Source: Own illustration

With the help of these graphs we get a feeling for the range of possible outcomes and can better understand the risk structure of the project. The cumulative probabilities curve can also be described as the value at risk and gain (VARG) curve. It states the probability of an outcome below or above a certain NPV. A flat VARG curves thereby indicates a wide range of possible outcomes whereas a steep curve indicates a low range.

The vertical line depicts the mean or expected value of the distribution. We will call this the expected net present value (ENPV). The shape of these graphs depends on the simulation model that we use and the cash flow structure of the project. In order to draw right conclusions, it is crucial to use an appropriate simulation model. We will therefore give a detailed analysis on different approaches and how it can be done in Chapter 5.2 on the simulation framework.

5.1.3 Step 3: Incorporate Flexibility into the Model

Now that we have combined our project with a wide range of possible market scenarios, we can think of actions to take when certain scenarios occur. These will be our options, since we can always choose whether or not to exercise them. Our available options highly depend on the specific project and in real estate these often include:

/ Option to delay (e.g. wait with construction until market recovers from a downturn)
/ Option to switch (e.g. switch from an office to a housing use)
/ Option to abandon (e.g. sell the property for salvage value)
/ Option to phase (e.g. build only part of the project and the rest later)
/ Option to expand (e.g. build an extension of the project)

We can model these options by including decision rules into our spreadsheet model. For decision rules to work we need first a trigger value that triggers a certain action, and secondly an execution variable that leads to the actual change in the model.

A decision rule then has the form of: "if variable A (the trigger value, e.g. value of apartments) falls bellow a certain threshold, then variable B (the execution variable e.g. variable for delay of construction for one year) will be one and thus lead to a delay in the successive cash flow of one year ."

When we let the simulation of scenarios run, actions will be taken depending on the specific scenario and therefore a scenario sensitive distribution of NPV outcomes is obtained. When analysing the resulting distribution curves it is our goal to decrease the amount of negative outcomes and to increase the amount of positive ones. This corresponds to a shift of the VARG curve to the right. We will call this calculation model the "flexible case".

This third step forces us to think about our available options and what to do in advance before things turn out to be different than in the base case scenario. It is this thinking about options and how to preserve and use them that gives additional value due to flexibility. Figure 1: Flexibility, Uncertainty and Cumulative Investment in the Development Process on page 12 shows how flexibility decreases during the development process. Thanks to simulations we are now able to identify and quantify the most valuable options from the beginning, keep them alive during the development and exercise them if needed. This is one of the main advantages of this methodology.

5.1.4 Step 4: Maximize Value by Applying Optimal Decision Rules

The fourth step deals with finding the right set of decision rules that optimises the overall outcome. Additional sensitivity analysis is performed on the model. Which value to optimise, depends largely on the decision maker. In our study we focus on the NPV, but a decision maker might be interested in other profitability metrics such as the internal rate of return or the absolute return. This can be implemented as well. The optimal combination is found by experimenting with different combination of decision rules and project parameters.

5.2 Simulation Framework

After introducing the "Engineering Approach" we now look in detail at our data and how to model it. First we give an overview of what, why and how we are going to model the risk drivers of a real estate development project. Then we look at our data, the development of the observations depicted in the data in the past years and on correlations among the data. In the third part of this chapter we introduce two ways to model this data with the goal of obtaining a model for simulation.

5.2.1 Overview

Goal

We want to obtain a model for the simulation of risk drivers for real estate development projects. We need this model to analyse possible future values of a real estate development projects from the viewpoint of a developer.

What to Model

The first question to answer towards this goal is that of what we actually need to model. We are interested in the value of real estate assets in the future, since this is one of the main success drivers of a real estate development project. In Chapter 4 we discuss that the gross operating income and the capitalisation rate approximates the value of a finished real estate project. These two variables determine to a large degree how much an investor is willing to pay for a real estate asset to a developer or, if the asset remains in a portfolio, what the book value of the asset is. It is thus apparent that we need to model these two variables. Another important part of a real estate development project are condominiums. These are directly sold to private persons already before, during and after the construction phase. Depending on the project, they also have a large influence on the bottom line of the developer and thus we are interested in modelling them as well. Further, construction cost is an important aspect of a development project that inhibits uncertainty, so this is something we will want to model as well. We go into more detail on what we need to model in Chapter 5.2.2 on the risk drivers of the model and continue here with the overview of the model.

Time Horizon

Since real estate development takes time, we are interested in a rather long time horizon into the future. Development projects take from initiation to stabilised operation at least 4 to 5 years. Since we are interested in flexibility and therefore also consider the delay of construction we take into account a simulation horizon of 10 years.

How to Model

The second question we need to answer is that of how to model these variables. The method we use is to look at past data for the variables of interest, so called time series, and fit a model to this data. This model is then used for the simulation of future variables. A simple model is that of a trend plus noise model. The model takes the form of:

$$y_t = a \cdot t + b + \varepsilon_t \tag{5.1}$$

with

y_t : data point at time t (e.g. GOI)

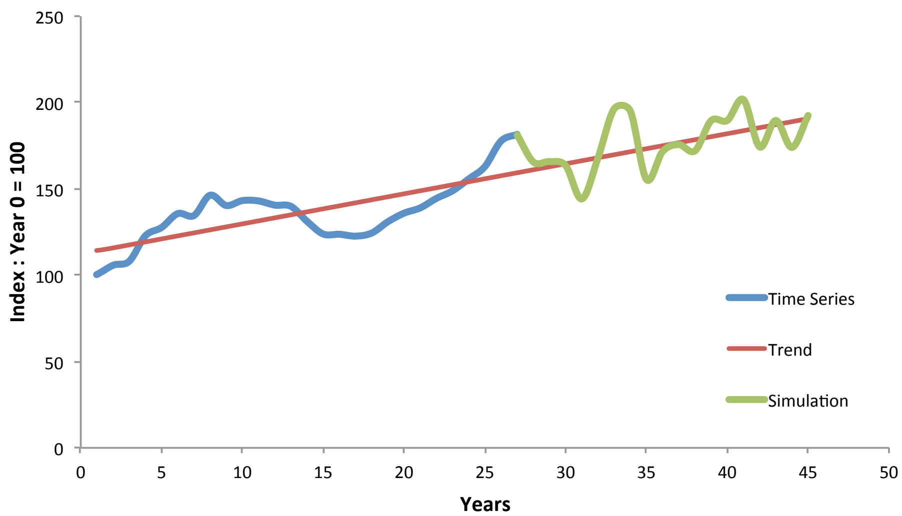
t : time

a,b: constant parameters to be estimated

ε_t: randomly distributed error terms

The term a∙t + b is the model trend line and ε_t the noise around the trend. We can estimate the model using ordinary least squares (OLS) method on the past data, thereby minimizing the sum of squared vertical distances between the data and the predicted model, in this case the trend line. By extrapolating the trend into the future and generating new noise around the trend, using random draws from e.g. a normal distribution, we generate simulations of the model into the future. This is illustrated in Figure 4 :

Figure 4 : Simulation with a Trend + Noise Model



Source: Own illustration

This rather simple approach neglects however that the time series of interest do not necessarily follow a constant trend and also that there might be dependence with other variables that need to be considered. Also we see already in Figure 4 that the generated simulation does not inherit the same characteristics as the past time series it is based on: The simulated line jumps above and below the trend line in shorter time intervals than in the past. We will thus have to employ more complex modelling techniques.

Stationarity

Additionally in order to use a model for simulations, we have to use the concept of stationarity. A stationary time series has the property that the probabilistic character of the series does not change over time, so that any section of the time series is "typical" for every other section with the same length (Dettling 2012). If any section of the time series is typical for every other section and if we can assume that this property holds also in the future, then we can use this time series also for forecasts of future time series. A stationary time series has constant expectation $E[X_t]$, constant variance $Var(X_t)$ and the covariance between the observations $Cov(X_{t1},X_{t2})$, i.e. the dependency structure, depends only on the lag between the observations. Now most time series are not stationary. Every time series with e.g. a trend or a deterministic seasonal pattern violates the concept of stationarity. By transforming and decomposing the data, we can however often find a stationary process. Actually the introduced trend + noise model (5.1) is already such a decomposition. The goal of such a decomposition is to find one that yields a stationary noise process ε_t. As we stated already we often cannot assume a trend that remains constant over time as in the trend + noise model. One way of dealing with this is to work with logarithmic returns rather than the raw time series. Thereby we detrend and normalise the data and hopefully obtain a stationary noise process.

Working with Logarithmic Returns

A common practice when modelling time series and analysing dependence among them is to work with logarithmic returns (log returns). By using log returns we focus on the relative change of the time series, which is directly comparable to the relative change in other time series. Additionally we remove piecewise a linear trend if there existed one. Throughout this chapter we thus work with logarithmic returns rather than discrete returns. Log returns have the convenient property that they are additive and easily obtained from time series by taking the natural logarithm of the series and differentiating by one time step:

$$r_{t+1} = \ln\left(\frac{y_{t+1}}{y_t}\right) = \ln(y_{t+1}) - \ln(y_t) \quad (5.2)$$

with

r_{t+1} : logarithmic return between time t and t+1

y_t,y_{t+1}: data point at time t

Independent on which model we use for simulation, we first transform the data to logarithmic returns, fit a model to it, and use this model for the simulation of a future horizon of 10 years. We thus generate future logarithmic returns that we can apply to the value we observe at present and thereby obtain our desired prediction of future time series:

$$y_{t+1} = y_t \cdot e^{r_{t+1}} \quad (5.3)$$

with

t : time of last observed data point

r_{t+1}: predicted return for one time step ahead

y_{t+1}: one time step ahead prediction of data point

Indices & Absolute Values

As we are going to see, most of the data used in this study is based on indices rather than absolute values. For our calculations on real estate development projects we need absolute values however. We address this by first estimating the model on the indexed data as described above and then using an absolute value currently observed in the market as the start value of our simulation. We thus assume that the absolute value will behave the same as the index it is based on. This is not necessarily true if we choose values that do not correspond to the index, so we should use only start values that actually correspond to the used index.

After this overview on what and how we are going to model, we now look into the details of the model.

5.2.2 Risk Drivers

The question is what exactly do we need to model to simulate the value of a real estate development project. As we discussed in Chapter 4 on the valuation practice, a real estate asset from the investment perspective is determined by the GOI and the capitalisation rate. Additionally, we often have a part of the development project consisting of condominiums that are directly sold to private owners.

Investment Asset

Let us first look at the part of the project that enters the market as a real estate investment asset (no condominiums) and estimate the value it would have today. In real estate development a project often consists of multiple uses such as housing, office and commercial. For simplicity we work with per m^2 values:

$$Value_{Asset} = \frac{\mathrm{GOI_{total}}}{\mathrm{CapRate}} \tag{5.4}$$

$$\mathrm{GOI_{total}} = \mathrm{GOI_{Housing}} + \mathrm{GOI_{Office}} + \mathrm{GOI_{Commercial}} \tag{5.5}$$

$$\mathrm{GOI_{Housing}} = \mathrm{Rent_{Housing}} \cdot (1 - \mathrm{Vacancy_{Housing}}) \tag{5.6}$$

$$\mathrm{GOI_{Office}} = \mathrm{Rent_{Office}} \cdot (1 - \mathrm{Vacancy_{Office}}) \tag{5.7}$$

$$\mathrm{GOI_{Commercial}} = \mathrm{Rent_{Commercial}} \cdot (1 - \mathrm{Vacancy_{Commercial}}) \tag{5.8}$$

For estimating the value of a real estate asset, we thus have to model rents, vacancies and cap rates for all uses.

Condominiums

Let us shortly discuss the condominiums that are also often a part of the development project. Buyers of condominiums do not take the investor perspective of calculating rents and cap rates, but base their decision to buy a condominium on their budget and preference. Since in Switzerland most condominiums are partly financed with mortgage loans, mortgage rates play an important role on their affordability. Condominium prices are measured on a per m^2 basis for specific regions in Switzerland which makes it possible to use the corresponding indices directly for modelling.

Deal Noise & Vacancy

When an developer sells a project to an investor, there is always a certain amount of uncertainty around the valuation price, depending on the bargaining power of the two parties negotiating (Geltner and Miller 2007). We will address this by introducing an independent variable “deal noise” to account for this. Additionally developers in Switzerland sell a project often before the receipt of the building permit with a certain amount of space already preleased to prospective tenants. There remains however often some vacancy risk that the investor will want to have compensated. We take this into account by applying twice the vacancy rate observed on the market on our sales price to the investor.

Also when selling condominiums, the price might differ from the current market price due to the location, marketing success and specifics of the project. We will thus use deal noise on condominiums as well.

Construction Cost

On the cost side we have the construction cost as our only risk driver. We are hereby looking at the market risk of construction cost, assuming that the developer estimated the cost of his project accurately and that the cost risk is therefore only driven by the uncertainty of variable prices for the same services and goods.

Table 1 gives a summary on the discussed risk drivers that we need to model:

Table 1: Risk Drivers

Earnings Side	Cost Side
Rents (Housing, Office, Commercial)	Construction Cost
Vacancies (Housing, Office, Commercial)	
Cap Rates	
Transaction Prices Condominiums	
Deal Noise	

The total sales price of a project to investors and private buyers can then be stated as:

$$Price_{total} = \delta_{Asset} \cdot Value_{Asset} \cdot \mathrm{A}_{Asset} + \delta_{Condo} \cdot P_{Condo} \cdot \mathrm{A}_{Condo} \tag{5.9}$$

with

δ_{Asset}, δ_{Condo} : deal noise asset and condominiums

A_{Asset}, A_{Condo} : saleable space in m^2

P_{Condo} : price condominiums per m^2

$Value_{Asset}$: value of the asset according to (5.4)

And the cost including construction cost risk as:

$$Cost_{total} = Cost_{Land} + Cost_{Development} + Cost_{Financing} + \sigma \cdot Cost_{Construction} \tag{5.10}$$

with

σ : construction cost uncertainty

5.2.3 Data

The independent real estate and urban development consulting company *Fahrländer & Partner Raumentwicklung AG* (FPRE) provided us with data specific for the region where our case study is located. The data is extracted from the Real Estate Scenario Cockpit (Fahrländer 2012) of FPRE and covers the time span from 1985 to 2011 in yearly time steps. The RESC is constructed with data from multiple rent indices, data from the Federal Statistics Office, the cantons and other sources. For replication of our results one is advised to use the freely available data for rents provided by the Swiss National Bank[4]. We used this data source also for the data on construction cost and gross domestic product.

Whenever analysing real estate data, it is important to clearly understand its nature. In our data set we look at the evolution of rents, vacancies and cap rates from 1985 to 2011. During that time the products on the space market have changed substantially: while a new apartment in 1985 with an automatic dishwasher was in an upper price range, it is standard equipment in new apartments nowadays. Similar examples can be made for windows, heating, elevator, etc. Changes in quality also occurred in terms of location: with the increased supply of mobility, locations further away from cities have become more attractive. This is especially the case with suburban areas, that became much more attractive due to increased mobility services. Furthermore the real estate market is very heterogeneous: objects differ from each other substantially in terms of size, location, age, etc. We therefore do not deal with a commodity like gold that does not change over the years and is the same wherever it is bought. We can account for this by using hedonic pricing models that take into account quality properties of the objects. The method used on our data set is described for rents by Wüst&Partner (2000) and for condominium transaction prices by Fahrländer (2012).

Table 2 gives an overview of the used data:

Table 2: Data Used in this Study

Data	Category	Time	Resolution	Region	Source
Rents, Vacancies	Housing, Office, Commercial	1985 - 2011	Yearly	Zurich Unterland	FPRE
Transaction Prices	Condominiums	1985 – 2011	Yearly	Zurich Unterland	FPRE
Cap Rates	Housing, Office, Commercial	1985 – 2012	Yearly	Zurich Unterland	FPRE
Construction Cost Index	Cost Index all Constructions	1998 – 2011	Yearly	Canton of Zurich	SNB
Gross Domestic Product	Economy	1990 - 2011	Yearly	Switzerland	SNB

We now describe the obtained data and highlight specifics of their behaviour in the past.

Rent Revenues

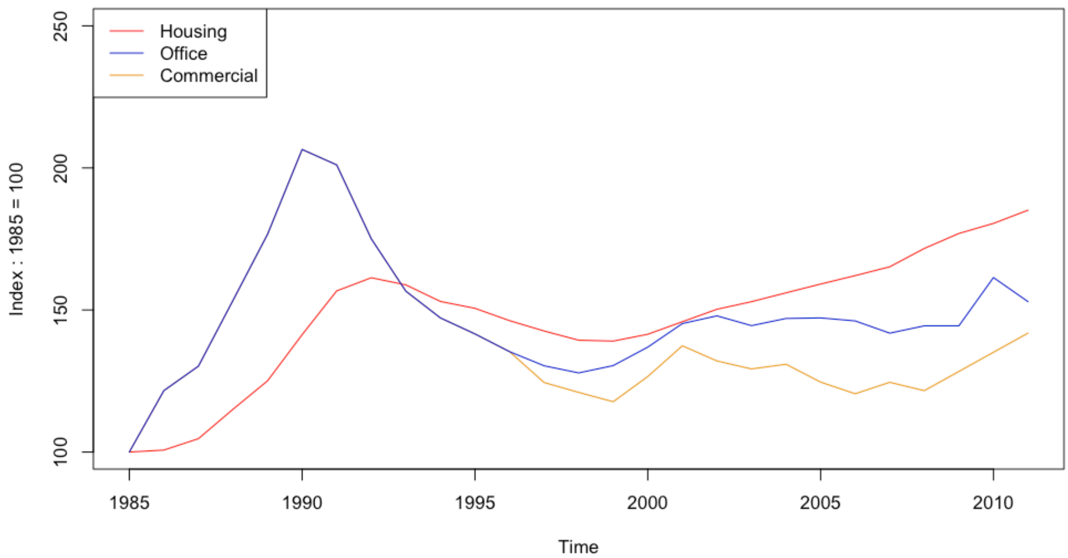
We use indices for the rent revenues of the different usages (housing, office and commercial). The indices correspond to rents observed in the MS (mobilité spaciale) region Zurich Unterland and do not include vacancies yet. For further information on the locality of this region one is advised to look at Schuler (2005). An important notion is that the rent indices

[4] www.snb.ch

are based on asking prices for rents and not on average revenues observed in actual properties. Due to restrictions and long-term contracts it is difficult for owners of real estate to immediately adjust rents to current asking prices and thus their revenues will tend to be lower on average. For real estate development however, the use of asking prices comes in handy, since development projects come new to market and we can assume that they will generate revenues close to current asking prices. For housing the used index corresponds to middle class multi-family dwellings, for office and commercial, it is the average asking price observed in the specified region.

Figure 5 shows the development of rent revenues during the observed time period. We see a sharp increase in rents from 1985 to 1990 with a peak in 1991, a sharp decline thereafter and since 1998/1999 a steady growth. The sharp decline in rents is associated with the real estate crisis in the early nineties we discussed in Chapter 3.4. From the figure we can already see that the three indices behave somewhat correlated. The indices of office and commercial space are identical until 1996 and behave strongly correlated thereafter. The equality of the two indices in the beginning of the series comes from the index construction that itself is based on multiple rent indices. Apparently rents for commercial and office use are based on the same data source during the beginning of the data. This will have further implications for our model later on.

Figure 5: Rent Revenue Index Zurich Unterland

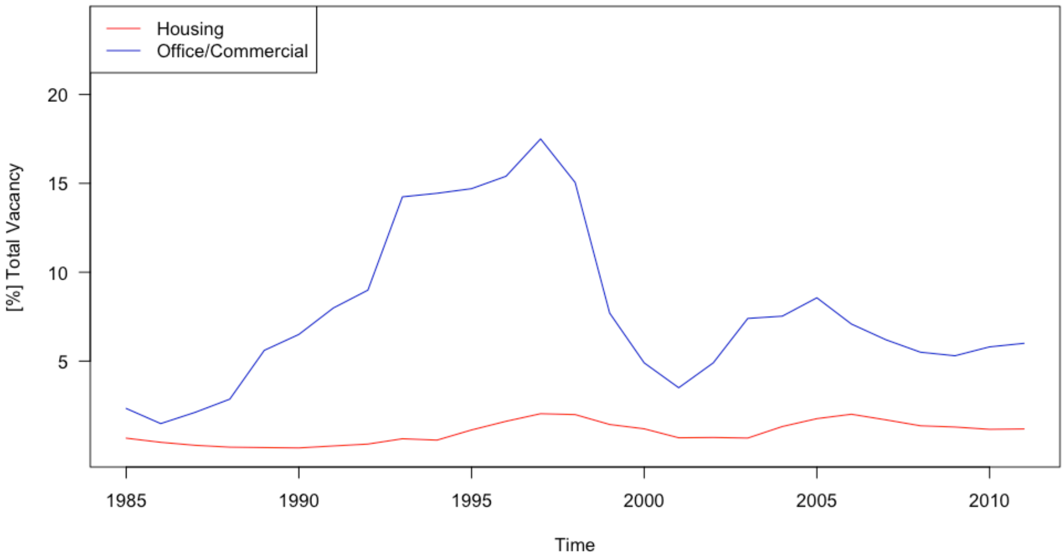


(Source: RESC Fahrländer & Partner)

Vacancies

For vacancies we have the same usages and data source as for rent revenues. These are the vacancies that we observe on the overall market. Here we see exactly identical values for office and commercial uses, which is why there are only two lines visible in Figure 6 instead of three. Values vary from 2% up to almost 18% in the middle of the 1990s for office & commercial uses while they stay between small 0.5 and 2% for housing use. Again we see the effect of the burst of the real estate bubble in the early 1990s that resulted in large vacancy rates for office & commercial uses. Interestingly it did not affect vacancies in the housing market very much. We can therefore legitimately assume that office and commercial uses had more risk of vacancies and therefore loss of earnings in the past than housing.

Figure 6: Vacancies Zurich Unterland

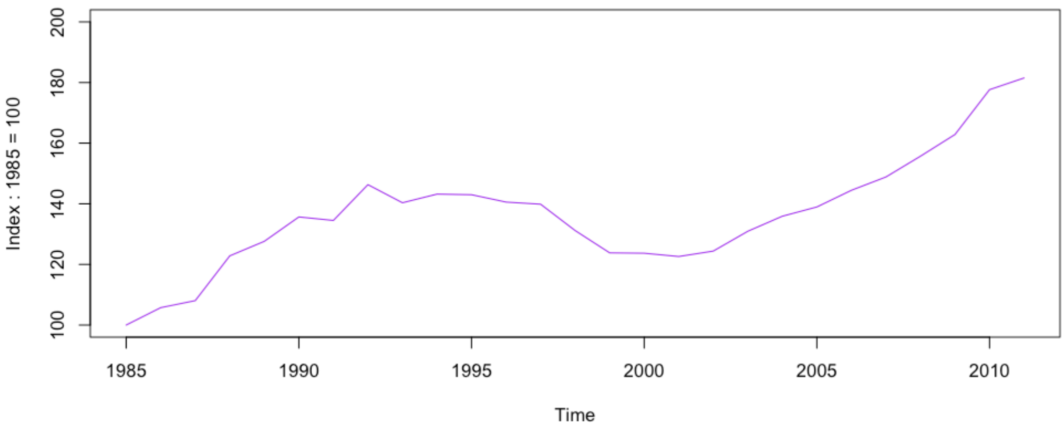


Source: RESC Fahrländer & Partner)

Transaction Prices Condominiums

We use the transaction price index of middle class condominiums for our analysis. Condominiums enjoy large popularity especially in recent years, which can be seen in the large price increase of almost 50% from 2000 to 2011. We can also observe that condominium prices have a very similar price development as that of housing rents.

Figure 7: Transaction Price Index Condominiums Zurich Unterland



Source: RESC Fahrländer & Partner

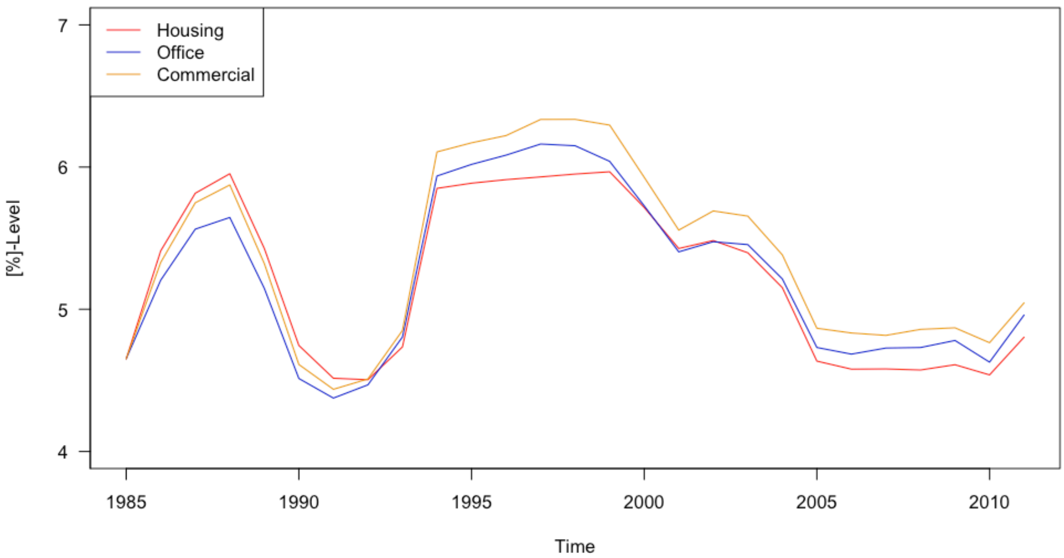
CapRates

From the RESC we obtained value indices for housing, office and commercial uses. These value indices were constructed by using the equation:

$$Value = Rent \cdot (1 - Vacancy)/CapRate \tag{5.11}$$

The cap rate thereby consists of an empirically measured premium plus a WACC consisting of 60% leverage, based on mortgage rates and 40% equity capital, based on bond rates and on an additional premium for real estate. This approach corresponds to the calculation of cap rates in Chapter 4. To obtain cap rates from the value indices, we solve equation (5.11) for cap rates using the already described rent and vacancy data and levelled them on 5% for the year 2011. We thereby obtain three time series of cap rates pictured in Figure 8:

Figure 8 : Cap Rates Zurich Unterland

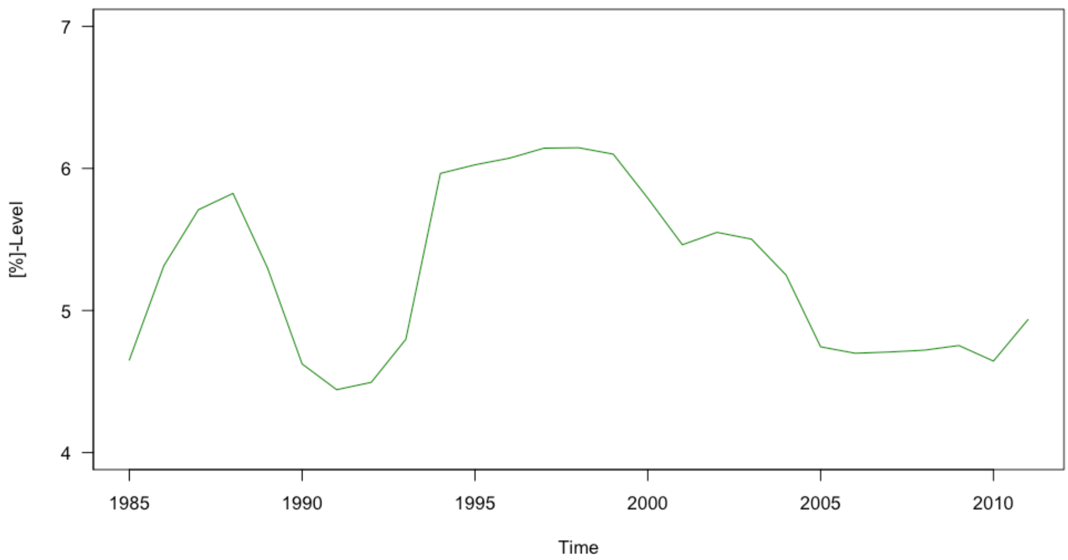


Source: Own calculation based on RESC Fahrländer & Partner

The levelling on 5% is a rather a simplified approach to estimate cap rates but since we are more interested in the relative change and the correlation with rents and vacancies, the absolute value should not matter too much. As seen in Figure 8, all three cap rate time series behave highly correlated and because we do not want to unnecessarily complicate our model, we aggregated cap rates into one single time series by taking the average of the previous three cap rates. For our model we are going to use this aggregated time series as our input. The result is illustrated in Figure 9.

An important thing to note with cap rates in a historic context is that they are currently at a very low level due to the interest rate policy of the SNB. As soon as interest rates start to rise again, this will have an influence on cap rates, which will result in decreasing real estate value if rent revenues do not rise simultaneously.

Figure 9: Average Cap Rates Zurich Unterland

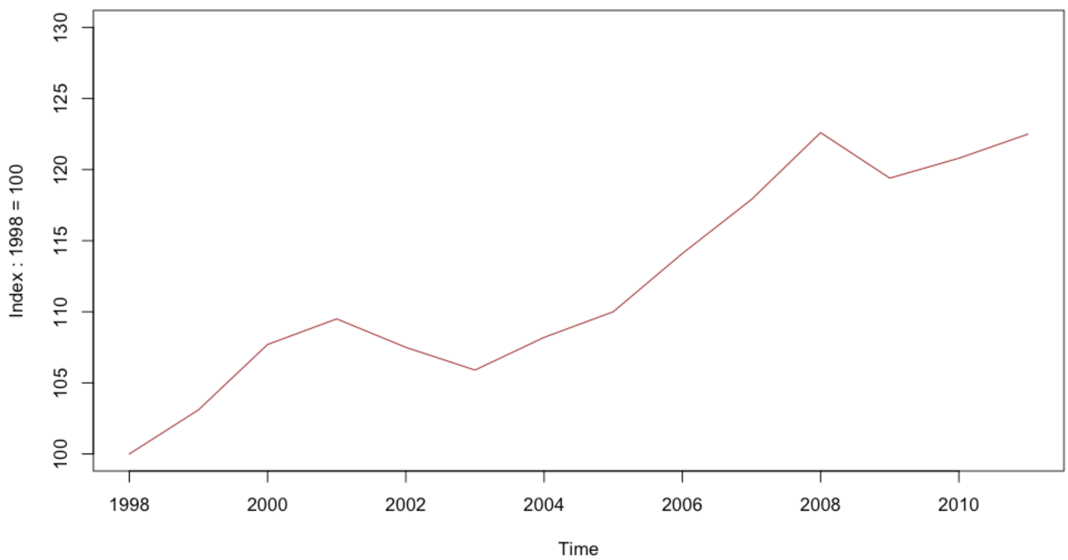


Source: RESC Fahrländer & Partner

Construction Cost Index

We use the construction cost index from the SNB for the region of Zurich. The index is quarterly available since 1998. From 1998 to 2011 construction cost have been rising on average by 1.6% per year.

Figure 10 : Construction Cost Index

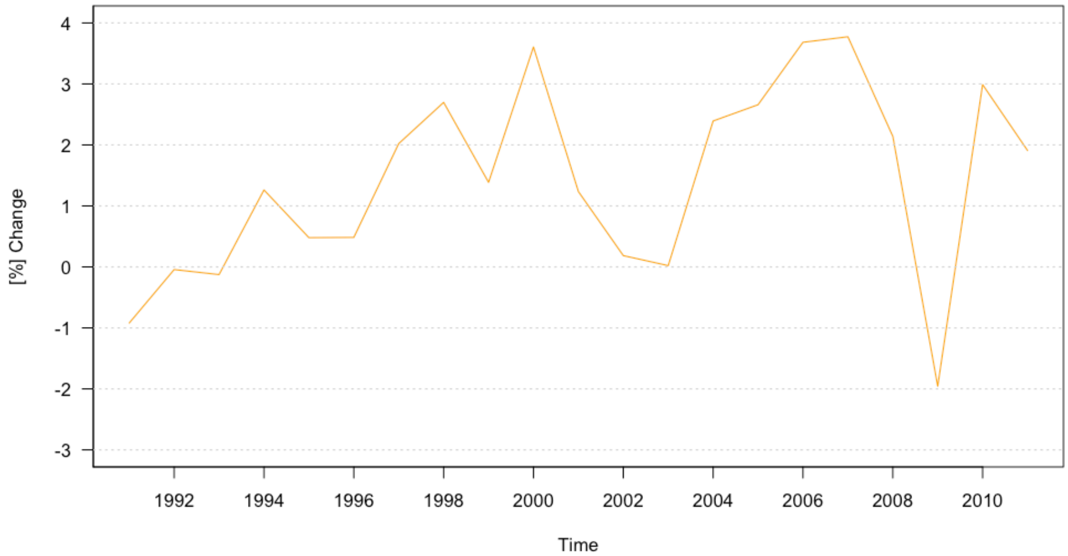


Source: SNB

Real Gross Domestic Product

We use the real gross domestic product (GDP) as our exogenous variable for the VAR-model we introduce later on. We work with the real GDP because we expect the economic activity to have an influence on expenditure on rents and condominiums, and also on vacancy rates especially in the office and commercial market. Unfortunately we have data only for the time period from 1990 until 2011 and not for the whole time period of the other data sets starting in 1985. The yearly percentage change is the growth in GDP and depicted in Figure 11. Since we look at the changes per year we lose one data point for 1990. We see a negative growth in GDP from 1991 to 1993. This corresponds to the time of the real estate crisis, so while growth was close to zero we had also decreasing prices. The second time when GDP growth was very low is 2002 and 2003. When we look at were GDP growth was close to zero and zero growth in 2002 and 2003 and again negative growth in 2009.

Figure 11 : Growth in Real GDP 1991 - 2011



Source: SNB

Interpretation of Data in the Light of Forecasting

Our data for office & commercial uses are almost identical and we use a very simplified method to calculate our cap rates. Furthermore we have only yearly data points and not the full time span for all data sets. To assume that we can conduct an exact forecast out of this data would not be very credible. But then again we have millions of data sets about financial markets and even with this huge data history it is apparently not possible to develop accurate forecasts. After all, the saying goes that forecasting using past data is like driving a car looking through the rear window. Something that might go fairly well when we already know the road ahead but gets extremely difficult when we face the unexpected (Dettling 2012). This is of course also true for the real estate market, especially when looking at the burst of the recent real estate bubble in the US.

We presume however, that by capturing important properties and relations of data in the past we can get a model that yields credible paths of the future. This does not presume that we know what is going to happen, which is impossible, but gives us a sense of what is possible.

We now look at relations that we observe in our data set and implications of this for our modelling framework.

Correlation & Trends

We see already by eye that rents and prices over all uses rise and fall in positive correlation during the observation period. We also observe that there seems to be a trend in the evolution of rents and condominium prices. While the office and commercial market had a very sharp correction in the early nineties, the decline in housing rents was much lower. We will therefore expect more volatility in the rents for office and commercial space than for housing space.

To study the dependence among variables we calculate the linear correlation between the log returns of the data. Correlation coefficients range between -1 and 1. A value close to 1 or -1 thereby indicates strong positive or negative correlation, while a coefficient close to 0 indicates no linear correlation. Looking at the correlation coefficients of the data in Table 3, we observe indeed a positive correlation of around 0.5 between housing, office, commercial rents and condominium prices. Further we see that the office and commercial rents are indeed very similar due to their high correlation coefficient of 0.9 and the similar correlation coefficients also with other time series. While there is small negative correlation of -0.2 between housing rents and housing vacancies we do not see this for office/commercial rents and office/commercial vacancies. We further observe negative correlation of -0.5 between housing rents and cap rates indicating that when cap rates go up, housing rents go down. This correlation is however less profound for office rents (-0.24) and commercial rents (-0.17).

Table 3 : Correlation Matrix Data Zurich Unterland

	Housing Rents	Office Rents	Comm. Rents	Housing Vacanc.	Office Vacanc.	Comm. Vacanc.	Condo. Prices	Cap Rates
Housing Rents	1.00							
Office Rents	0.54	1.00						
Comm. Rents	0.56	0.90	1.00					
Housing Vacanc.	-0.21	-0.64	-0.64	1.00				
Office Vacanc.	0.33	-0.06	-0.08	0.28	1.00			
Comm. Vacanc.	0.33	-0.06	-0.08	0.28	1.00	1.00		
Condo. Prices	0.50	0.43	0.43	-0.26	0.28	0.28	1.00	
Cap Rates	-0.52	-0.24	-0.17	-0.12	-0.10	-0.10	-0.04	1.00

Rents & Vacancy

One would expect that when vacancies in a region go up, that rents will start to fall. This is a question of balancing out supply and demand in the space market where vacancies indicate a high level of supply that is not absorbed by the market. In order to match supply and demand we would thus expect a decrease in prices. When looking at Figure 5 and Figure 6, we see by eye that while office & commercial vacancies went up during the nineties, prices decrease substantially. Looking at correlation coefficients however, we see only a small negative correlation between housing rents and housing vacancies and no correlation between office and commercial rents and office and commercial vacancies. It is however important to note here that we are currently looking at correlation among log returns at the same time t. However, there might be a lagged dependence between e.g. office rents and office vacancies so that rising vacancies have an effect on rents one or two years later. We discuss this further when modelling our data later on.

Cap Rates

Cap rates went down substantially before the early nineties crisis while rents were rising, resulting in very high values for real estate. Since 1999 cap rates are falling, which is reflected in the higher valuation of real estate all over Switzerland.

After discussing the data, let us shortly recapitulate what we did so far and what the next steps are towards reaching the goal of modelling the risk drivers for the financial analysis of a real estate development project: In Chapter 5.2.2 on the risk drives, we discussed what we have to model. We then looked at past data that describes the behaviour of these variables in Chapter 5.2.3. The next step is to find an appropriate model that we can fit to this data. After fitting a model to the data, we can then use it to generate simulation of future variables. This is a necessity to incorporate uncertainty into the “Engineering Approach”.

5.2.4 Modelling

We first introduce a Copula and then a Vector Autoregressive (VAR) model that we fit to the data. These two models have the property that they take into account interdependence among the multiple time series, which is something we want to consider when using the model for simulation later on.

We use the following techniques for modelling our risk drivers:

Table 4: Techniques used for Modelling

Risk Driver	Method
Rents	VAR- & Copula Model
Transaction Prices Condominiums	VAR- & Copula Model
Vacancies	VAR- & Copula Model
Cap Rate	VAR- & Copula Model
Construction Cost	Copula Model

Copula Model

In the overview in Chapter 5.2.1 we discuss that by taking log returns we wish to obtain a stationary process that we can model. Let us assume that this is true and the obtained log returns are stationary. We then have nine time series of stationary processes that we can fit a model to. Let us further assume that all these time series follow a normal Gaussian distribution each with constant mean and variance. We could then estimate mean and variance based on our data and use random draws from the calibrated Gaussian distributions to generate simulated returns. These we could use for the simulation of future outcomes.

Now there is a major flaw if we make simulation like this: we neglect an important property of our data, which is their dependence on each other. We already discussed that for example housing rents and condominium prices are correlated and thus it would makes sense to take this into account in our model. Otherwise the simulated time series would behave completely independent from each other, which will certainly not be very feasible in our case. Copulas are a way to account for correlation among multiple time series and thus we employ them here.

Definition

Copulas are mathematical tools that are useful for the simulation of linearly correlated data. They gained large attention and distribution in investment banks and insurance companies before the latest financial crisis. It was assumed to be possible to price the rather complex financial derivative class of collateralised debt obligations (CDOs) that were at the core of the late crisis, with the help of Gaussian copulas (Salmon 2009). In the context of blindly trusting in financial models, they therefore have some similarity to the Black-Scholes model that is blamed for the stock market crash in 1987. The reason for their large distribution however is that they have some very useful properties that we can make use of:

Copulas are joint distribution functions that link a multidimensional distribution to its one dimensional marginals. These marginals can be made standard uniform, which makes them ideal for further transformation and simulation. Embrechts (2009) explains the basics of copulas as:

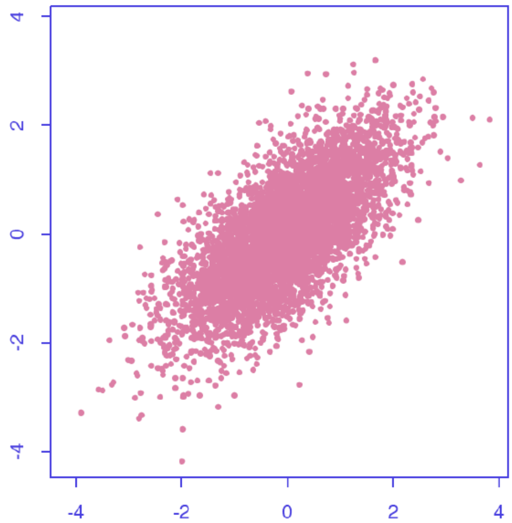
In the one dimensional case we have a random variable *X* with a continuous cumulative distribution function *F*. We have *U = F(X)* with *U* as a standard uniform distributed variable [0,1]. We can also transform this back by applying the inverse cumulative distribution function: $X = F^{-1}(U)$. When we have the multivariate case we can write the joint cumulative distribution function $F(x_1, x_2)$ as:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{5.12}$$

With *C* as our copula: A multivariate distribution function with standard uniform marginals (U_1, U_2). Formula (5.11) couples the marginals $F_1(x_1)$, $F_2(x_2)$ to the joint cumulative distribution function $F(x_1,x_2)$ via the copula. Sklar (1959) showed that there exists a unique copula for n-dimensional multivariate distribution functions if the marginals are continuous.

Let us now look on the joint realisations as obtained from the two dimensional Gaussian and t-copula with standard normal marginals as depicted in Figure 12 and Figure 13.

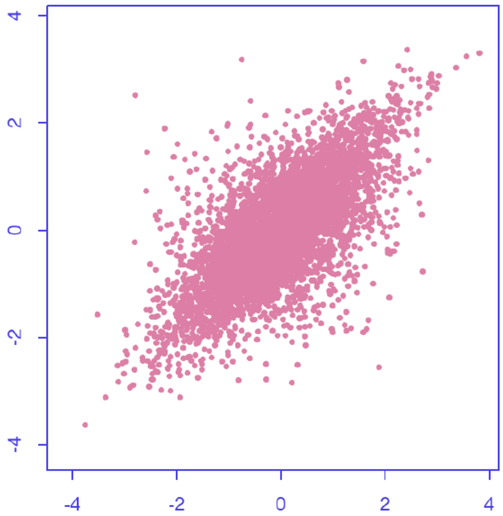
Figure 12: Gaussian Copula with Correlation of 0.7



Source: (Neslehova 2006)

We see that the Gaussian copula indeed produces linearly correlated marginals since realisations are clustered around a linear slope and not randomly distributed, as seen in the scatterplot in Figure 12. Now there are also different copulas than the Gaussian that yield different dependency structures. One of these is for example the t-copula as depicted in Figure 13:

Figure 13: t-Copula with Correlation of 0.7 and 4 df



Source: (Neslehova 2006)

While both copulas have the same correlation coefficient of 0.7, the t-copula yields more clustering of realisations among the tails compared to the Gaussian copula. We see this, when we compare e.g. the lower left corner of Figure 13 with the lower left corner of Figure 12. The t-copula is thus better suited to model dependence among extreme events than the Gaussian copula. There are however other copulas that take into account the asymmetry in correlation meaning that while there might be correlation among the right tail of the distribution, there is little or none on the left tail or vice versa. Modellers have a wide variety of choices available to choose among copulas and to find one that "best" suits their data. As Embrechts (2009) points out in this context, there is no obvious answer in the question on which copula to use. It really depends on the data and the dependence structure one wishes to model especially in the light of changing circumstances. While one might find a copula (or more generally a model) that fits the observed dependence perfectly, it remains questionable if the relations modelled will also hold in the future. If the market circumstances change, as in the case with the pricing of CDOs using Gaussian copulas, then previous valid models often become obsolete. This is something we have to be aware of when modelling dependencies.

Application

We use the above result to create joint distribution functions with the dependencies we specify. The first step is to choose a copula that suits the correlation structure of the data. If correlations among the tails are observed (tail dependence), then e.g. a t-copula might be applicable.

In our case we have a data set consisting of 27 data points for every time series. This is not a large data set for analysing the dependence structure of the data and we have to choose a copula that fits all dependencies. We thus make an estimate on what the dependence structure could possibly look like. Since we do not have sufficient data to study and model tail dependence of the variables, we choose the Gaussian copula to describe the dependence structure of our data.

Once we decide on a copula, we feed it with the dependence structure we found in our data. This is described by the linear correlation matrix depicted in Table 3. The copula then gives us the standard uniform marginals with the dependence structure still in place. We can use these marginals together with a distribution function that fits our data. This is the second convenient property of the copula: we can choose a distribution function that fits our data and this for every single time series. Let us look on the descriptive statistics of our data:

Table 5: Descriptive Statistics Log-Returns Data

	Housing Rents	Office Rents	Comm. Rents	Housing Vacanc.	Office Vacanc.	Comm. Vacanc.	Cond. Price	Cap Rates
Mean	0.02	0.02	0.01	0.02	0.04	0.04	0.02	0.00
Std. Dev.	0.04	0.08	0.09	0.38	0.30	0.30	0.04	0.07
Skewness	0.86	0.55	0.50	0.47	-0.29	-0.29	0.09	0.92
Kurtosis	0.66	-0.01	-0.33	-0.62	0.33	0.33	0.51	2.65

In Table 5 we depict the four moments of the risk drivers that describe their distribution. We see a positive mean for all risk drivers except for cap rates which is zero, meaning that the average log return of the risk drivers was positive in the past. Further we note that the vacancies have a high standard deviation of 0.3 indicating that log returns show a wide distribution of possible outcomes. Further we see skewness in the data. Skewness describes the asymmetry of the distribution. Thereby a negative value indicates a long left tail of the probability distribution while a positive value indicates a long right tail of the distribution. For rents we see positive skewness indicating that rents have a long right tail, while vacancies have a longer left tail. The kurtosis we see in the last line is shown here as excess kurtosis in comparison to the standard normal distribution. A positive value depicts higher kurtosis than the standard normal and a negative depicts lower kurtosis. Kurtosis can be interpreted as the peakedness of the distribution, meaning the width of the peak. Peakedness comes together with fat tails, meaning that the tails of the distribution still have a relatively high probability of occurrence. A high kurtosis indicates a narrow peak with fat tails, while a low kurtosis indicates a wide peak with thin tails in comparison to the standard normal distribution. Cap rates have a high kurtosis indicating that the distribution has a narrow peak and fat tails. Now since we observe skewness and kurtosis in our data it would be elegant to choose a distribution that can account for this. The normal Gaussian distribution that we use for the example in the beginning of this section does not serve us well in this case, since it is defined only by mean and variance. Luckily there are many other distributions in the toolbox of a financial modeller and a commonly used one that accounts for skewness and kurtosis is the Normal Inverse Gaussian (NIG) distribution. We thus use this distribution, calibrate it over the mean, variance, skewness and kurtosis from our data depicted in Table 5 and apply it to the standard uniform marginals obtained from the copula. We thereby create a joint multivariate distribution with the marginals following the NIG and a dependence structure still in place as we specified it. We can then use this distribution for simulations just as in the example in the beginning of this section but this time considering correlation and, due to the NIG, even skewness and kurtosis of the data.

While this process might seem rather tedious to do, it can be easily implemented in MATLAB together with the NIG-package by Werner (2006).

Results

We use the described method to simulate 10'000 correlated log returns and obtain the following descriptive statistics:

Table 6: Simulated vs. Original Data Copula Model

	Housing Rents	Office Rents	Comm. Rents	Housing Vacanc.	Office Vacanc.	Comm. Vacanc.	Condo. Prices	Cap Rates
Mean Simulation	0.02	0.02	0.01	0.02	0.04	0.04	0.02	0.00
Mean Data	0.02	0.02	0.01	0.02	0.04	0.04	0.02	0.00
Std .Dev. Simulation	0.04	0.08	0.09	0.38	0.30	0.30	0.04	0.07
Std. Dev. Data	0.04	0.08	0.09	0.38	0.30	0.30	0.04	0.07
Skewness Simulation	0.80	0.48	0.45	0.43	-0.24	-0.24	0.08	0.87
Skewness Data	0.86	0.55	0.50	0.47	-0.29	-0.29	0.09	0.92
Kurtosis Simulation	1.13	0.28	0.32	0.28	0.22	0.22	0.59	2.17
Kurtosis Data	0.66	-0.01	-0.33	-0.62	0.33	0.33	0.51	2.65

We can already see by eye that mean and standard deviation of the simulated variables are very much the same for the simulated variables and the original data. For skewness there are deviation in the region of 0.05 from the original data, which is not very much. Kurtosis on the other hand seems to be not captured that well by the NIG distribution. There are deviations from the original data ranging from 0.08 up to 0.9. Apparently the NIG distribution thus did not work so with the kurtosis of the distribution. Looking at the correlation matrix we see the following results:

Table 7: Correlation Matrix Simulated Log-Returns

	Housing Rents	Office Rents	Comm. Rents	Housing Vacanc.	Office Vacanc.	Comm. Vacanc.	Condo. Prices	Cap Rates
Housing Rents	1.00							
Office Rents	0.53	1.00						
Comm. Rents	0.56	0.90	1.00					
Housing Vacancies	-0.21	-0.62	-0.62	1.00				
Office Vacancies	0.33	-0.06	-0.08	0.27	1.00			
Comm. Vacancies	0.33	-0.06	-0.08	0.27	1.00	1.00		
Condo. Prices	0.50	0.43	0.43	-0.26	0.28	0.28	1.00	
Cap Rates	-0.51	-0.23	-0.17	-0.12	-0.10	-0.10	-0.03	1.00

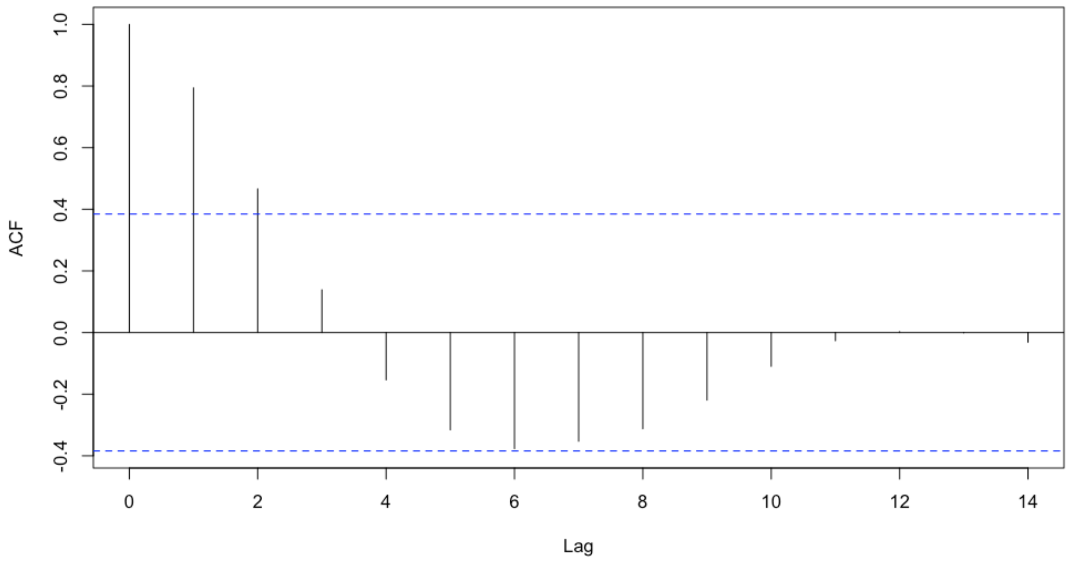
If we compare this correlation matrix to the one in Table 3 on page 40, we see by eye that they are very similar. Differences range in the region of 0.00 to 0.03, which is very little.

Thus we can assume that our copula model really captures important relations among the time series and is able to simulate new variables that inherit these dependencies. From the descriptive statistics we can further assume that except for the tails these simulated variables also follow a similar distribution as our original data. This is an excellent result so let us use this model for the simulation of future outcomes of our risk drivers, right?

Not so fast. So far we have only looked at dependence between the different time series at the same time t, but what about dependence of observations within one time series itself? What happens if observation r_{t+1} is dependent on r_t? The introduced copula model is only valid if the observations are independent from each other across time, if there is no dependence between r_t and r_{t+h} for all lags h. We then say that there is no serial correlation and that the observations within the time series are independent. We check for this by looking at the autocorrelation and the partial autocorrelation[5] by calculating the correlation coefficient across time with specific lags. The autocorrelation result for the log returns of housing rents is depicted in Figure 14:

[5] Partial autocorrelation measures remaining dependency after autocorrelation has been accounted for.

Figure 14: Autocorrelation Log-Returns Housing Rents



Source: Own illustration

We see significant autocorrelation of 0.8 for a lag of 1 year and of 0.4 for a lag of two years in the housing rent data. We also have significant autocorrelation for the log returns of the other time series as depicted in Figure 43 in Appendix I. This speaks against our copula model because there, we do not take autocorrelation into account. Luckily there is another tool available that considers autocorrelation and dependence among time series. This is the class of vector autoregressive models that we introduce next.

Vector Autoregressive Model (VAR)

Definition

Like the copula, a VAR model is a mathematical tool to capture linear interdependencies among multiple time series. A VAR model explains its evolution based on its own lags and the lags of the other variables in the model. This makes it a powerful and flexible tool for modelling multiple time series since the evolution of one time series is also dependent on the other ones. We further have the advantage to make the model dependent on exogenous variables. We can then use this exogenous variable in our forecast, adding a deterministic part to the model. This is of interest to make the model more stable and to analyse different scenarios. A VAR models takes the form of:

$$y_t = c_{Ex} y_{Ex,t} + A_1 y_{t-1} + \cdots + A_p y_{t-p} + e_t, \ \ t = 0, \pm 1, \pm 2, \dots \qquad (5.13)$$

where $y_t = (y_{1t},\dots,y_{Kt})'$ is a (K × 1) random vector, A_i is fixed (K × K) coefficient matrix, $c_{Ex} = (c_1, \dots, c_k)'$ is a (K × 1) fixed vector of coefficents allowing for the possibility of a exogenous term with observations $y_{Ex,t}$. $e_t = (e_{1t}, \dots, e_{Kt})'$ is a (K × 1) vector containing error terms with zero mean $E(e_t) = 0$, covariance matrix $E(u_t u_t') = \sum_u$ and no serial correlation $E(e_t e'_{t-k}) = 0$. p is the lag order of the model.

Written in matrix notation a VAR with lag p = 1, VAR(1), and two variables takes the form of:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \cdot y_{Ex,t} + \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \cdot \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \tag{5.14}$$

Application

When applying this model to our data then e.g. the condominium log return for a VAR(1) is determined by:

$$r_{Condo,t} = a_{Condo,Condo,t-1} \cdot r_{Condo,t-1} + a_{Condo,H\_R,t-1} \cdot r_{H\_R,t-1} + \cdots + a_{Condo,GDP} \cdot r_{GDP,t} + e_{Condo,t} \tag{5.15}$$

with:

H_R : Housing Rents

$r_{Condo,t}$: log return condominium at time t

$a_{Condo,Condo,t-1}$, $a_{Condo,H\_R,\ t-1}$: lag coefficients for lag t-1

$r_{Condo,t-1}$, $r_{H\_R,t-1}$: previous log returns at time t-1

$a_{Condo,GDP}$: coefficient for GDP at time t

$r_{GDP,t}$: log return GDP at time t

$e_{Condo,t}$: correlated error term for condominiums at time t

We estimate the coefficients of the model by using ordinary least square method on our data set of past time series. We chose a lag-2 model over a lag-1 model based on Akaike's Information Criterion (AIC), which is a standard tool in statistics for the selection of models. Further we use real GDP as an exogenous variable for our model. We do this with the intention of "gluing" the model to a deterministic variable once we use it for forecasting. Depending on the stability of the model, estimated variables otherwise go quickly out of bounds of what one would expect to be reasonable. We use the GDP as an exogenous variable, since it is an indication of the economic activity and we expect this to have an influence on our risk drivers. The drawback of using the GDP as exogenous variable is that we have only data from 1990 until 2011. This means that we must use this time span for the other variables as well. We thus loose 5 years of data in our already small data set, resulting in 21 observations of log returns. Further we cannot use the VAR model for simulations when two endogenous variables are very similar. The covariance matrix of the error terms then becomes singular, which is a property that makes it unsuitable for Cholesky decomposition. Cholesky decomposition is however something we need for simulation. As we discussed earlier, office and commercial rents are very similar or identical in the data set and thus we have to work with either the time series for office or the one for commercial use. We choose the time series for office rents and vacancies and will thus assume that commercial rents and vacancy behave the same as their office use counterparts. Considering that the correlation coefficient is 0.9 between office and commercial rents and that vacancies are identical in the data set, we can assume that this simplification does not alter results too much.

Once we have estimated the model, we test the significance of the estimated coefficients by looking at their p-values. We test the null hypothesis that coefficients are zero using t-statistics and consider coefficients with a p-value above 0.1 to be non significant. The statistic programming environment of R together with the “VARS” package by (Pfaff 2008) is used to estimate the model.

Results

The estimated coefficient matrix is shown below in Table 8. Further results on the estimation of coefficients are found in Appendix II:

Table 8: Lag Coefficient Matrix A

	Condo. Price (Condo)	Housing Rents (H_R)	Office/Com. Rents (O_R)	Housing Vacancy (H_V)	Office/Com. Vacancy (O_V)	Cap Rate (CAP)
Condo.l1	-	-	-	-	-	-
Condo.l2	0.41	-	-	-5.07	-	0.43
H_R.l1	1.47	0.92	-	13.75	-	-1.83
H_R.l2	-0.50	-0.26	-	-	3.80	1.52
O_R.l1	-	-	-0.44	-	-	-
O_R.l2	0.49	-	-	-	5.80	-
H_V.l1	-	-0.03	-0.09	-	0.36	-
H_V.l2	-	-	-0.08	0.36	-	-
O_V.l1	-	-	-	0.56	-	-0.04
O_V.l2	-	-	-	0.70	-	-0.06
CAP.l1	0.74	-	-0.72	-	5.45	0.50
CAP.l2	-	-	0.58	5.01	-	-
GDP	0.74	0.35	0.95	-	-	-0.38

The columns depict the estimated coefficients for one risk driver. So we see that e.g. housing rents depend on the lagged log returns of themselves (H_R.l1 and H_R.l2) on lagged housing vacancy (H_V.l1) and on GDP. GDP has a significant positive influence on rents and condominium prices and a negative influence on cap rates. That means that when the GDP log return is positive, this has a positive influence on rent log returns and a negative one on cap rates. We do however not see an influence of current GDP on vacancy. We did however not test for the influence of lagged GDP on current vacancy, since we could not implement

this into the model with the used statistical package. However, this surely would be an interesting aspect to look at in further research.
By looking at Table 8 we also see some lagged variables that have an extremely large influence on current returns. This is foremost the case for the estimation of vacancies e.g. for lag 1 housing rent (H_R.l1) on housing vacancy (H_V) the estimated coefficient is 13.8. Such large variables do not speak for the adequacy of the model to use it for forecasting since already small errors in the estimation of e.g. housing rents have a very strong influence on housing vacancy one time step ahead. Also we have some dependencies we would not expect to be so strong such as the negative influence of lag 2 condominium returns (Condo.l2) ($a_{H\_V,Condo.l2}$=-5.07) on current housing vacancies. One can hardly think of a rational explanation of why the return in condominium prices should influence housing vacancies that much two years later. We could correct for this by setting the corresponding coefficient to zero but then again this might not be adequate, since statistically there is an influence. Also setting a significant coefficient to zero leads to a chain reaction in the whole model since it is set up in way that everything is interdependent. As we will see when applying the model for forecasting, it is one thing to estimate a model that fits the data well and quite something else to create a model that is "realistic" in the sense of generating credible results.

We illustrate this in the next section, where we use the estimated model for the simulation of future scenarios.

5.2.5 Simulation of Future Scenarios

After fitting a model to our data we now use it for the simulation of scenarios. We can easily generate thousands of scenarios and analyse how the real estate development project performs under these. Assuming that the estimated dependency structure depicted in the model holds also in the future, we should be able to generate credible results. However, this assumption might not hold, since dependence among variables does not necessarily remain constant. Future events might alter the dependency among the variables, leaving our model invalid. This leads to the fact that our simulations will always include quite some inaccuracy and be a good deal away from accurate forecasts or even predictions. It is our intention, however, to estimate the range of possible outcomes and then to identify and quantify valuable options under these outcomes. It is thus not necessary to have a perfect, crystal ball like model for forecasting, which is impossible anyway. The model should help to be prepared for the uncertainty lying ahead and make the flexibility we have in dealing with it more tangible.

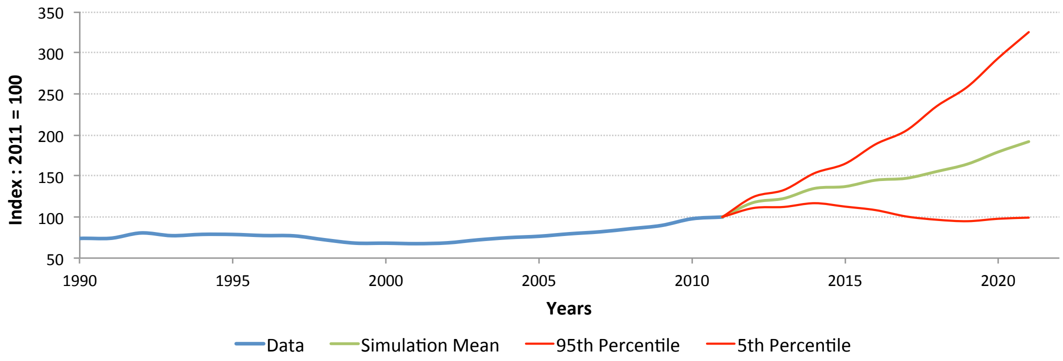
Simulations with a VAR Model in Excel

We use Excel to generate simulations of scenarios and to analyse them. This is convenient for the practitioner, since he can thus implement the simulation mechanism directly as an add-on to existing project calculations. The first step however, is to do simulations of the risk drivers and analyse, if these results are credible and thus, if they should be used in conjunction with the development project calculations.

The input of the VAR model is according to (5.13) a vector of correlated error terms. In our case we have six error terms at time t, one for every variable: $e_t = (e_{t,Condo}, e_{t,H\_R},..., e_{t,CAP})$. For simulations we assume these error terms to follow a normal distribution with zero mean, variance and correlation among these error terms being the same as the error terms from the estimated model. We call this a noise process and generate it in Excel using the implemented NORMSINV() and RAND() function. The NORMSINV() is the inverse of the standard normal Gaussian cumulative distribution function where we can add variance later on. We generate new independent error terms following a standard normal distribution by using $X = F^{-1}(U)$, with $F^{-1}()$ as the inverse cumulative distribution function and U[0,1] generated by RAND(). We generate multiple error terms by using "Data Tables" in Excel that allow for the storage of simulation results. Correlation among the error terms and variance is taken into account by using the Cholesky decomposition on the covariance matrix of error terms from the model estimation and multiplying the decomposed lower triangular matrix with the uncorrelated error terms. As the copula, this is a method to account for correlation among multiple time series. The result is the desired vector of correlated error terms, in our case $e_t = (e_{t,Condo}, e_{t,H\_R},..., e_{t,CAP})$, that is used together with the lag coefficient matrix A in Table 8 and the last two log returns of the data set (r_{t-1}, r_{t-2}) to generate simulations. For every time step ahead we get a set of randomly generated log returns following the properties of our model. Now to see the evolution of the future scenarios, we apply the simulated log returns to the last observations of our data set using (5.3): $y_t = y_{t-1} \cdot e^{r_t}$, for t = 1, 2, … n with n as the desired simulation horizon.

We simulate 2000 times a simulation horizon of 10 years, assuming a deterministic constant growth in GDP of 2%. Simulations yielded the following results:

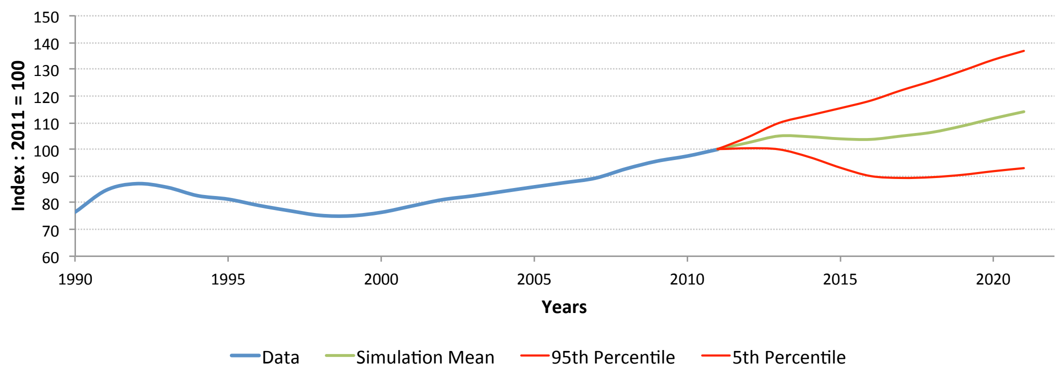
Figure 15: Simulation of Condominiums with a VAR(2) Model



Source: Own illustration

We see a strong upward slope in condominium prices estimated by the model with an average price 45% higher after five years and almost doubled after ten years. The 90% confidence interval depicted by the 95th percentile (upper red curve) and 5th percentile (lower red curve) indicate a high upward potential while the possibility to fall below the start price is very low.

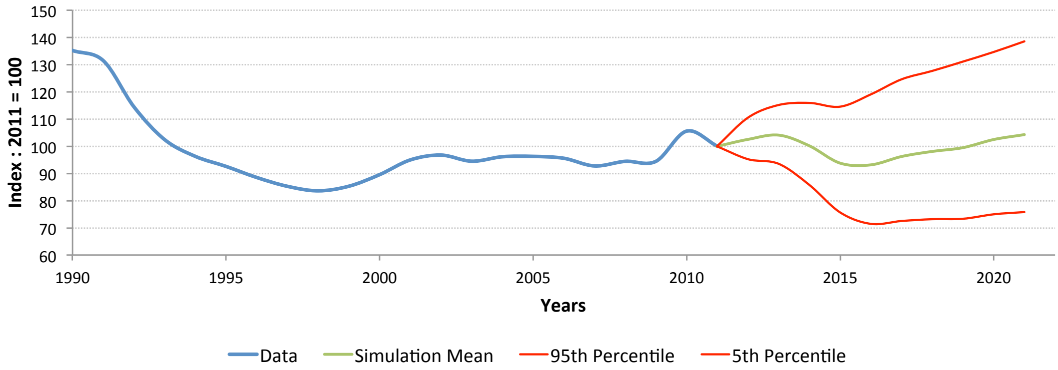
Figure 16: Simulation of Housing Rents with a VAR(2) Model



Source: Own illustration

Housing rents remain in a smaller confidence interval than the condominium prices and increase on average 14% compared to the start value. The 90% confidence interval has an upper bound of +36% and a lower bound of -7% after 10 years.

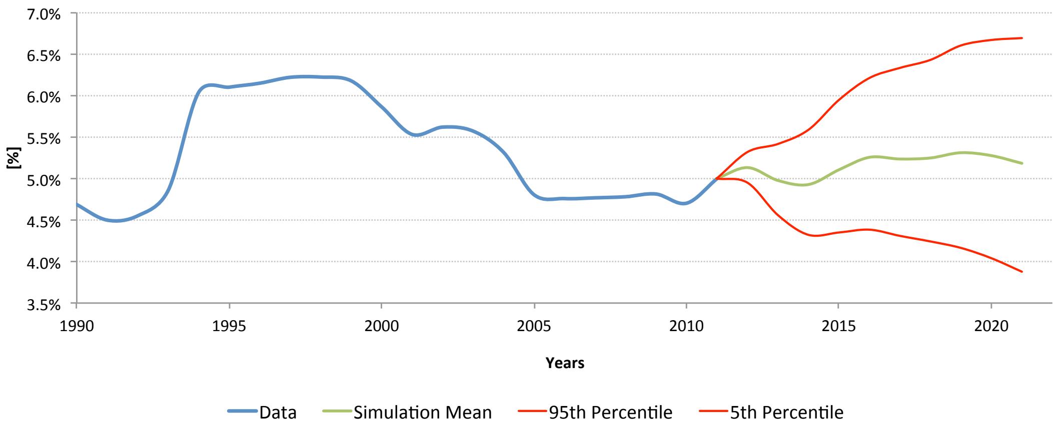
Figure 17: Simulation of Office Rents with a VAR(2) Model



Source: Own illustration

The simulation of office rents shows an average increase after two years close to the start value of 4%. The 90% confidence interval after ten years ranges between +39 and -24% compared to the start value of 100. Compared to the price decline in the early nineties of -38% within eight years and a price increase since then of + 20% results seem feasible. Since we use exactly the same model for computing the commercial rents, we do not show results here.

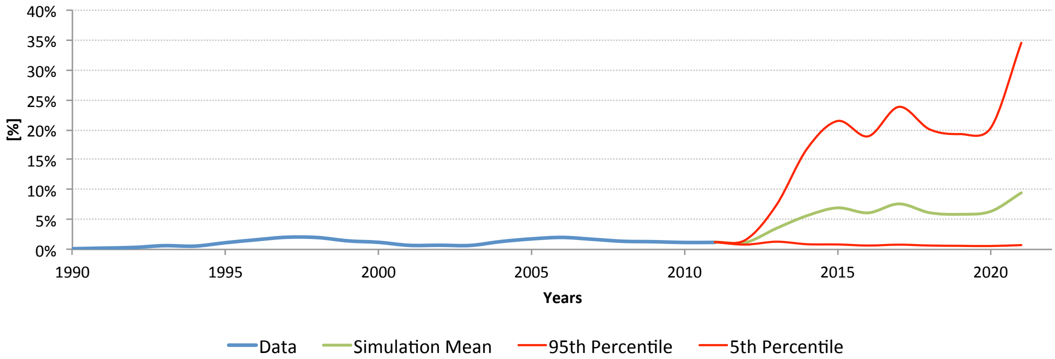
Figure 18: Simulation of Cap Rates with a VAR(2)-Model



Source: Own illustration

Simulation of cap rates depicted in Figure 18 yielded average cap rates of 5.2% after 10 years which is 0.2% above the start value of 5.0% percent. Cap rates increase on average after the first year and range in the 90% confidence region of 3.8 to 6.7%. We can however fairly assume that we are already at an exceptional low level of cap rates today and that results ranging 1.5% below the start value are rather unlikely, from a feasibility point of view.

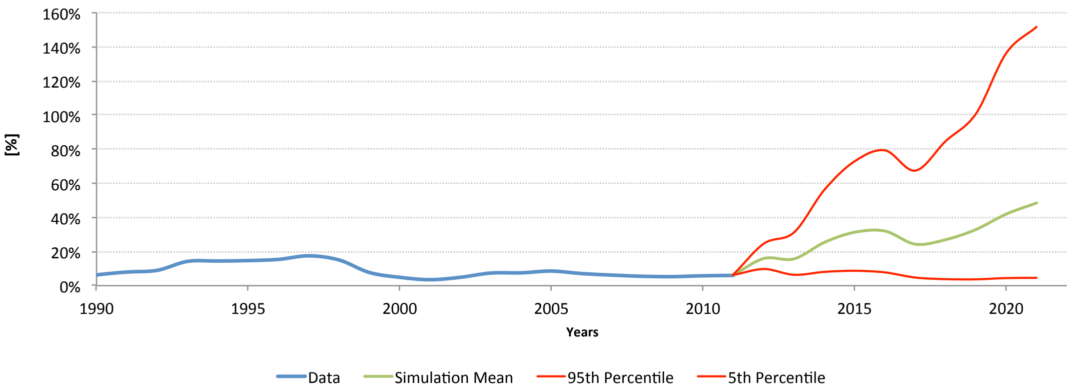
Figure 19: Simulation of Housing Vacancies with a VAR(2) Model



Source: Own illustration

While housing vacancies remain between zero and two per cent in the past they become quickly out of bounds of what one would expect to be reasonable in our model, as seen in Figure 19. This is because housing vacancy rates are in the region of zero and four per cent where a change of only half a percentage point leads to a log return in the area of 30%. The result is a large standard deviation of 0.38 as seen in the descriptive statistics of the data, back in Table 5 on page 45. In the VAR model the standard deviation of error terms is reduced to 0.15 but we have the strong influence of H_R.l1, Condo.l1 and CAP.l2 that amplify especially high or low corresponding lagged log returns, which results in the unrealistic simulation results for vacancies. Results are even worse for the simulation of office/commercial vacancies as depicted in Figure 20:

Figure 20 : Simulation of Office Vacancies VAR(2)-Model



Source: Own illustration

For office vacancies simulations go completely out of bounds and above a value of 100%, which is impossible.

Discussion

These discouraging results do not stand for the applicability of the introduced VAR(2) model for the simulation of the risk drivers. While the simulation of housing and office rents remain in a feasible region, they do less so for cap rates and condominium prices. For vacancies, the results are not feasible. The application of a VAR(1) model showed similar results and is thus not applicable as well. One of the main problems in modelling the data is the high standard deviation of the vacancy for housing, office and commercial use. Our approach in modelling is inapt to effectively deal with this, while still preserving the dependency structure among the variables. Also we have to consider that our simulation horizon in this case of 10 years is rather long in comparison to the 21 yearly observations that we use to estimate the model. The quality and quantity of the data is another issue. Quarterly data might be more adequate to capture lagged dependencies among the variables and in recent years, this kind of data becomes more and more available. A freely available, part half yearly, part quarterly index for real estate performance and price is available from IAZI[6] for example. Split up into the different risk drivers, as we model, it is however difficult to find higher resolution data. Also it is evident that our data set is in the case of office and commercial use an approximation to the real market environment, since we would not expect them to be almost identical. More accurate data on these uses might help to fit a better model.

However, the question remains, how one should introduce uncertainty of the risk drivers into the financial analysis. So far we stick as much as possible to the information in our data set and model dependency, as it is captured in the data. One can, however, also make own estimates on dependency as Hoesli, Jani et al. (2005) do in their Monte-Carlo simulation model on real estate values in Geneva. They e.g. assume a linear dependence between vacancy rates and rental growth of -0.75. The assumption of reasonable dependencies, when justified, might provide better results than the approach we used so far to model the data. Thereby, the introduced techniques can be of great use to model specifically as one desires to.

Modelling partly based on Assumptions

We have come so far to model the risk drivers that it would be a shame to give up now, after the drawbacks with autocorrelation and the VAR model. We could switch back to the Copula model but then we run the risk that dependencies are not captures well because we do not consider autocorrelation and lagged influences on the variables. Also we have the problem that using the Copula model, due to the high standard deviation of vacancies, the modelled risk drivers go out of reasonable bounds as well. In order to model the data with realistic

[6] www.iazicifi.ch/

results we thus need to leave the way of strictly staying with the obtained data and specify dependencies and parameters according to reasonable assumptions. We thereby employ the copula, to model dependencies based on estimates, but stick as much as possible to our obtained data set.

We base our model on the following simplifying assumptions:

- There is no serial correlation among the log returns of the risk drivers.
- Log returns of the risk drivers follow the dependencies and distributions we specify.
- Office and Commercial rents and vacancies are perfectly correlated.

We take office and commercial rents and vacancies together because we had to reduce computational efforts and because it should not matter that much for our case study later on.[7]

Let us look at the moments we specify for our distributions and the reasons we do so:

Table 9: Specified Moments Copula Model

	Housing Rents	Office/ Com. Rents	Housing Vacancies	Office Vacancies	Condo Prices	Cap Rates	Constr. Cost
Mean	0.02	0.02	0.00	0.00	0.02	0.01	0.02
Std. Dev.	0.04	0.08	0.20	0.20	0.04	0.04	0.02
Skewness	0.86	0.55	0.00	0.00	0.09	0.00	0.00
Kurtosis	0.66	-0.01	0.00	0.00	0.50	0.00	0.00

We change skewness and kurtosis of the vacancies parameters to zero thereby assuming the distributions to be symmetric and tails to be similar to a normal distribution, we further decrease their standard deviation to 0.2, so that absolute values for vacancies stay in a reasonable range. Further we put the mean to zero, so that we have an expected value for vacancy log returns of zero. This means that vacancies are as likely to go up or down, while being symmetric. For the same reason we increase the mean from cap rates to 0.01 since we expect on average raising cap rates in the future due to the low interest rate environment we observe at the moment. We also decrease standard deviation of cap rates from 0.07 to 0.04, so that they remain in a feasible range during our long simulation horizon of ten years. If we change the moments of the distribution in this way it is difficult to estimate the influence on skewness and kurtosis and we thus set them to zero, so that our distribution are similar to the normal Gaussian. We work with the Gaussian distribution because it is kind of the standard distribution to use, when there is no reason to do otherwise. We further introduce construction cost with mean and standard deviation according to data from the construction cost index of the SNB.

[7] One could however treat them separately by e.g. setting their correlation coefficient to 0.8 or 0.9. Thereby one could also create a different distribution for commercial vacancies than for office vacancies.

As we already stated, we neglect the existence of autocorrelation. We do this because setting dependencies based on judgment becomes very complicated to implement in a VAR model. While we could restrict the influence of the lagged variables easily, one would have to change the covariance matrix of the error terms to alter correlation among the variables at time t. Drawing the right conclusions from the error terms and change them after the model has been estimated, is however a difficult undertaking that we do not engage in here. The assumption of no autocorrelation has an influence on the validity of the model that we need to discuss later on. However, since we specify dependencies by ourselves, we can now also incorporate correlation between construction cost and the other parameters. We specify the following correlation matrix as an input for the copula:

Table 10: Specified Correlation Matrix Copula

	Housing Rents	Office/Com. Rents	Housing Vacancies	Office/Com. Vacancies	Condo	Cap Rate	Constr. Cost
Housing Rents	1.00						
Office/Com Rents	0.50	1.00					
Housing Vacancies	-0.50	-0.10	1.00				
Office Vacancies	-0.10	-0.50	0.50	1.00			
Condo	0.50	0.50	-0.50	-0.05	1.00		
Cap Rate	-0.10	-0.10	0.10	0.10	-0.10	1.00	
Construction Cost	0.30	0.30	-0.30	-0.30	0.30	-0.10	1.00

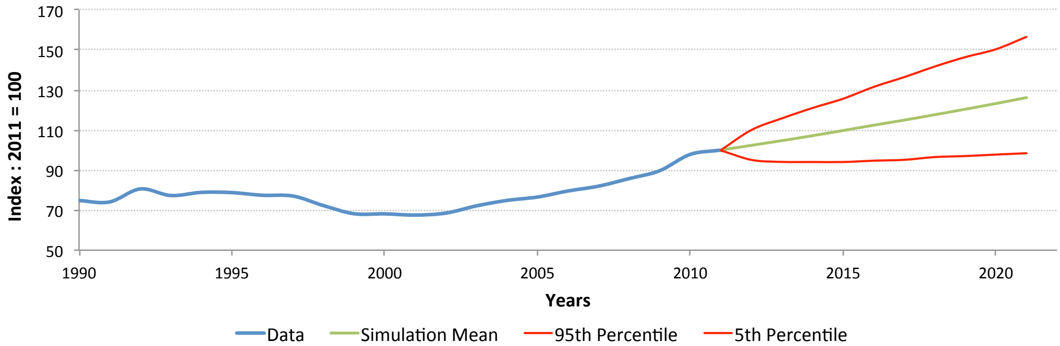
In the data we have seen positive correlation among the two rents and condominium prices of 0.4 to 0.5, we thus set their correlation coefficient to 0.5. As Hoesli et al. (2005) we assume vacancies to be negatively correlated with the corresponding rent (-0.5) and a bit less with the rent of the other time series (-0.1). As within our data we assume a negative correlation between housing vacancies and condominiums of -0.5 (in the data -0.3). Further we assume the two vacancies to be positively correlated but in our case with a larger correlation coefficient of 0.5. Indicating that vacancies tend to rise and fall together. In our data we saw negative correlation between cap rates and rents, which we keep here as well. We keep them at low -0.1 however. Deviating from our data, we choose a slightly positive correlation between cap rates and vacancies of 0.1 because we expect that when vacancies go up, real estate becomes less attractive from an investment perspective and thus cap rates go up as well. We chose a rather weak relation of 0.1 though. Finally we assume construction cost to be correlated with the other variables according to Table 10. It seems reasonable that while rents and prices are rising, construction cost does as well. We are thereby assuming that rents and prices are rising when the economy is growing and so construction companies have full orders and can raise prices due to low market pressure. When there are rising vacancies we expect construction cost to fall however, due to less construction activity and higher capacity in the construction sector. Correlation Matrix and descriptive statistics of the generated distributions are found in Appendix III.

Simulation of a Copula Model in Excel

From MATLAB we produce a table with 10'000 correlated random draws from the NIG distribution for our risk drivers with dependence as specified in Table 10. We import this table into Excel and draw random samples from it using the LOOKUP() function together with RAND(). Analogue to simulating the VAR-Model in Excel, we use "Data Tables" to simulate 2000 times a simulation horizon of 10 years.

Results look as follows:

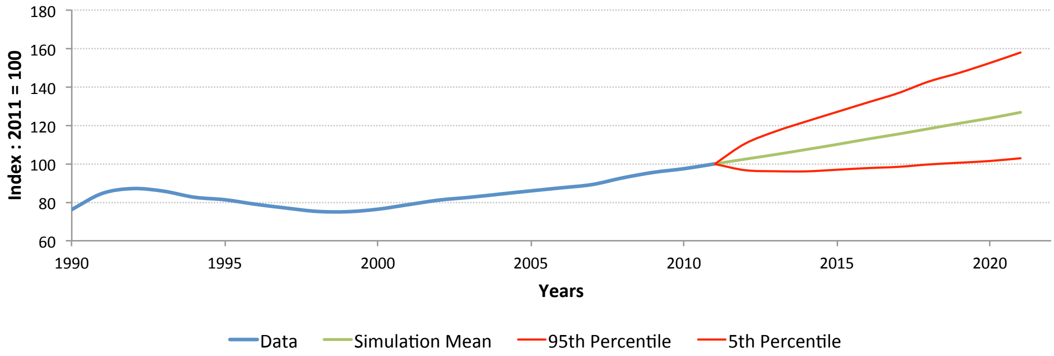
Figure 21: Simulation of Condominiums with a Copula Model



Source: Own illustration

Due to the mean log return of 2% we see an average increase of 26% in condominium prices over ten years. Compared to the VAR-Model, results are in a more feasible range. When we consider that prices rose by 48% in the last 10 years. Downward potential is with a minimum 5th percentile of -6% however very small.

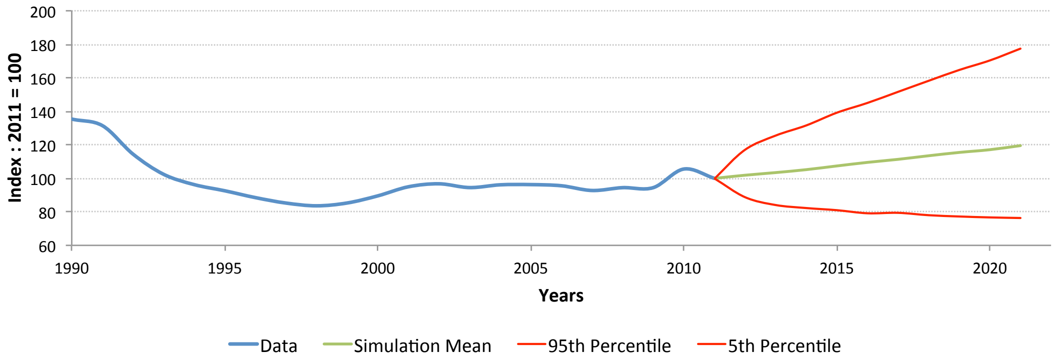
Figure 22: Simulation of Housing Rents with a Copula Model



Source: Own illustration

Since mean and variance are similar for housing rents as for condominium prices, we see a very similar development of rents with an average increase of 27% compared to 2011.

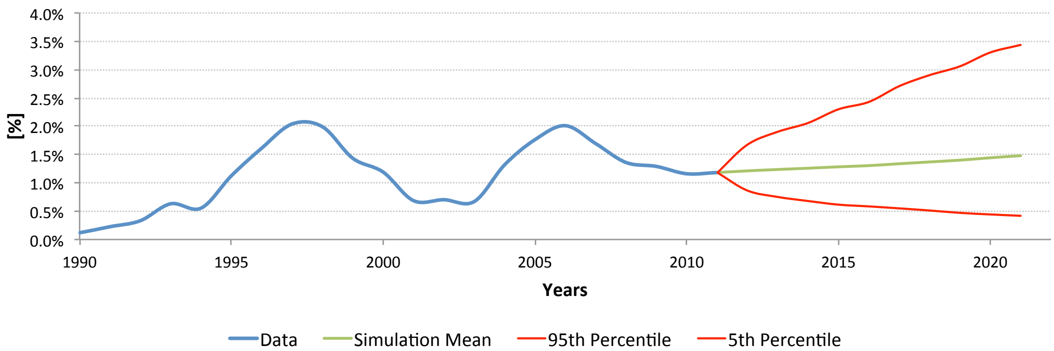
Figure 23: Simulation of Office Rents with a Copula Model



Source: Own illustration

Due to the higher standard deviation in office and commercial rents of 8% compared to 4% in housing and condominium uses, we see a wider distribution of possible outcomes. The 90% confidence interval between the 5th and 95th percentile ranges between -25 and + 79% after 10 years.

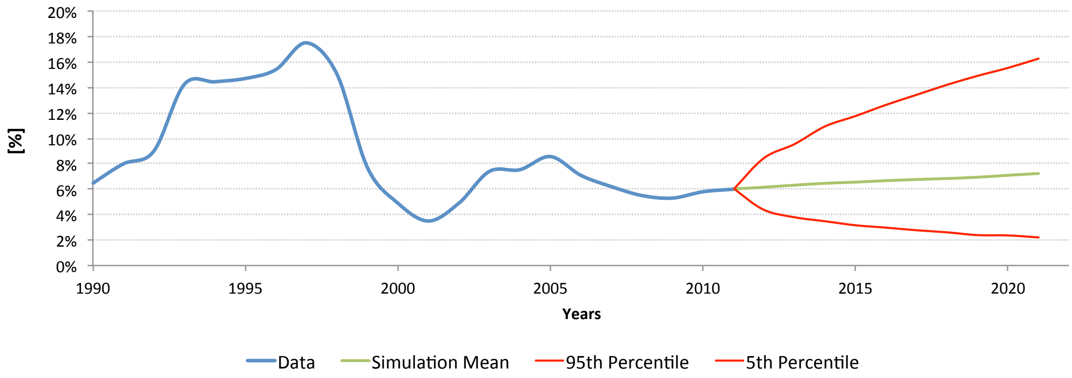
Figure 24: Simulation of Housing Vacancies with a Copula Model



Source: Own illustration

As we can clearly see in Figure 24, the simulated housing vacancies remain in a range that is feasible with absolute values for the 90% confidence interval of 0.4 and 3.5%.

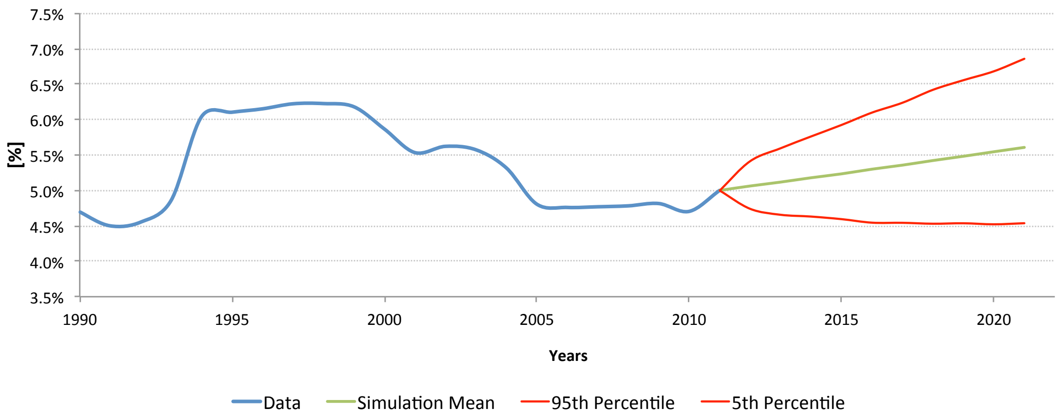
Figure 25: Simulation Office/Commercial Vacancies with a Copula Model



Source: Own illustration

Also for office and commercial vacancies we see a feasible range of the 90% confidence interval of 2 up to 16% after 10 years. Average vacancy rate is at 7.5% after 10 years.

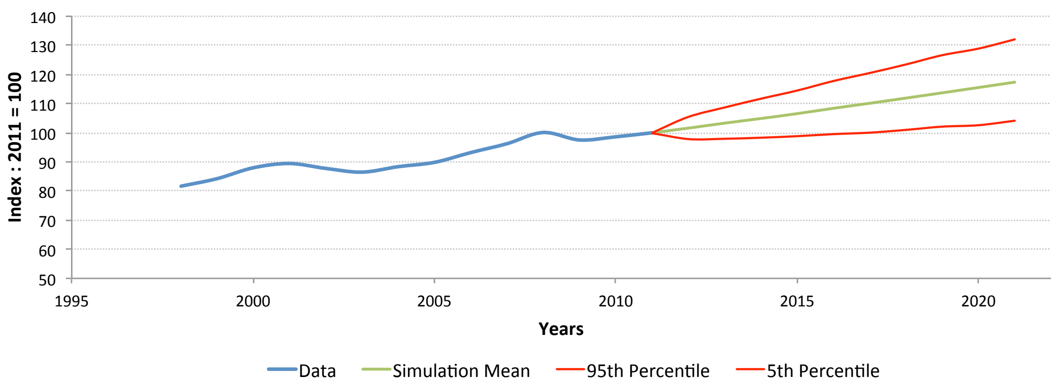
Figure 26: Simulation of Cap Rates with a Copula Model



Source: Own illustration

The introduction of a 1% average mean for log returns of cap rates leads to an average increase of cap rates to 5.6% after 10 years. The 90% confidence interval remains between 4.5 and 6.8%, which can be considered as feasible compared to the past development.

Figure 27: Simulation of Construction Cost with a Copula Model



Source: Own illustration

Also for construction cost we can now simulate dependent log returns. Since construction costs were almost constantly rising during the past years, this is also reflected in the simulations with an average increase in construction costs after ten years of +18%. The confidence interval ranges between +31 and +2% after 10 years.

Discussion

After studying the confidence intervals of our risk drivers we can state that they vary in regions that seem feasible for application to our real estate development project. During the last section we focused so much on confidence intervals and average forecasts that we must not forget that the individual scenarios within these intervals do behave correlated as we specified it. This means for example, that rents and condominium prices will tend to rise and fall together while vacancies behave the opposite. This speaks in favour for the applied model. We neglected autocorrelation however, which results in simulations that are not influenced by their past realisations. In real estate however, we often observe periods of rising prices followed by periods of falling prices and the same is true for vacancies. Autoregressive models better capture such processes because there, the probability for a positive observation is higher, when the previous observation was positive as well (assuming that the lag coefficient is positive). This adds momentum to the process, which is more realistic. However, further work and some reasonable assumptions on the VAR model would most definitely yield a realistic model capturing these features. Another drawback of the copula model is that we are not able to make simulations dependent on GDP as we did with the VAR model. The consideration of GDP gives us the possibility to influence results and add own expectations to the model by changing its rate of growth. This is highly desirable to perform further scenario analysis. The copula model on the other hand is able to specify dependencies, as we believe them to be reasonable. This is a large advantage, especially when data is rare. Additionally modellers can bring in their own expectations and experience into the modelling of the data, which could finally lead to better results than when modelling past data and believing that the future will behave accordingly without adding own expectations. One way to implement a more interactive simulation model while still considering dependencies of the variables would be to (1) generate dependent standard uniform marginal with the copula in MATLAB, (2) import these dependent marginals into Excel and (3) transform them there using inverse cumulative distributions implemented in Excel. We could then specify these distributions with the moments we receive from past data and which we can modify to study different scenarios, thereby creating a more interactive simulation model.

After discussing advantages and drawback of our model and giving input on how we could further improve it, we now turn our heads towards applying this model to a real life case. But first we shortly have to discuss how we are getting from the simulated indices to absolute values.

5.2.6 Simulation of Absolute Values

So far we have worked with indices for rents and condominium prices. For the analysis of a real estate development project we need absolute values however. We thus specify a start value for housing rents, office rents, commercial rents and condominium prices in accordance with the absolute prices we observe in the specific region. As with the indices, we employ (5.3): $y_t = y_{t-1} \cdot e^{r_t}$, for t = 1, 2, … n with r_t simulated by our model and y_{t-1} as the start value of our simulation. An example for a 5 years simulated scenario is given in Table 11.

Table 11: Example of a Five Year Scenario with Start Values

Years	Housing Rents	Housing Vacancy	Office Rents	Office Vacancy	Com. Rent	Com. Vacancy	Price Condo	Cap Rate	Constr. Cost
Start Values	250	1.2%	250	6.0%	200	5.0%	5000	5.0%	1.00
0	250	1.1%	255	6.3%	200	5.0%	5000	5.0%	1.00
1	258	0.9%	269	6.7%	215	6.7%	4874	5.5%	1.02
2	271	0.7%	280	5.2%	237	5.2%	5144	6.1%	1.05
3	268	0.7%	289	5.6%	222	5.6%	5312	6.5%	1.08
4	278	0.7%	300	7.2%	274	7.2%	5027	6.5%	1.11
5	286	0.5%	321	5.4%	281	5.4%	5363	6.1%	1.10

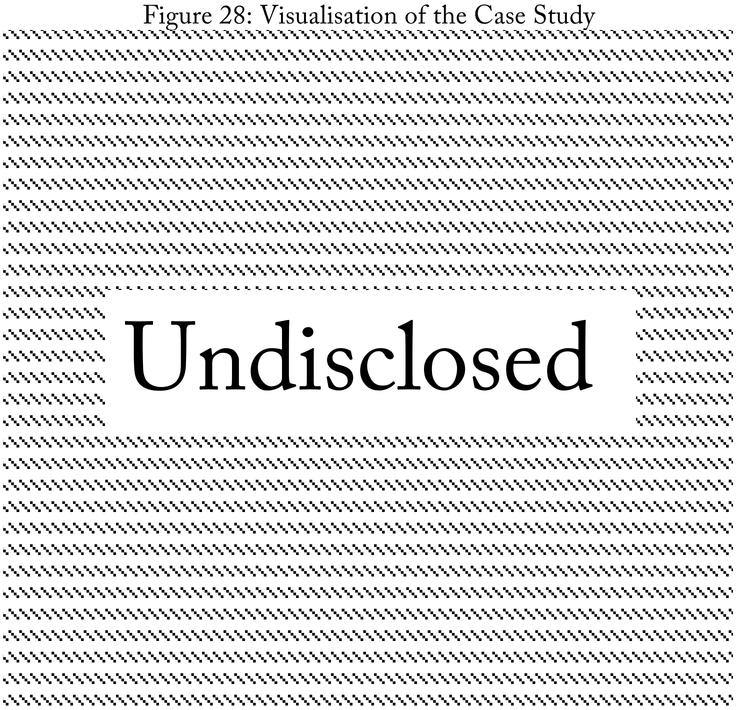
These values we can now directly apply to the cash flow statement of a development project using equations (5.4), (5.9) and (5.10) from page 33 and 33 to perform simulations of the development project. We do this in the next chapter on our case study.

Before going on to the case study, let us shortly recapitulate what we learned in this chapter. We started with introducing the “Engineering Approach” and its four steps of (1) creating the most likely cash flow model, (2) introducing uncertainty, (3) introducing flexibility and (4) maximising value by optimization. We then went on to the simulation framework that introduces uncertainty into the “Engineering Approach”. We first identified the risk drivers determining the value of a real estate development project, analysed data depicting these risk drivers and then fitted a model to this data. We then used a copula and a VAR approach for modelling and showed how these models can be used for simulations directly in Excel. Finally we decided on a copula model, specified it based on our assumptions and analysed results. After showing how we create absolute values with this model, we now apply it to our case study in the next chapter.

6 Application of the "Engineering Approach" to a Real Project - A Case Study in the Canton of Zurich

In this Chapter we apply the developed simulation model to a real life case study. First we give some background information on the project and then go through the four steps of the "Engineering Approach" while discussing results.

Figure 28: Visualisation of the Case Study



Source: undisclosed

6.1 Case Background

A real estate developer in Switzerland provided the case for this study. Numbers have been changed and detailed cash flow calculations are not published in order to honour the privacy of the involved companies. Assumptions for cap rates and prices do not correspond to assumptions made by the developer and involved parties.

The property we are analysing is a large former industrial area consisting of 40'000 m^2 in the canton of Zurich. It is planned to contain four different uses:

- Subsidised rental housing
- Condominiums
- Office space
- Commercial space

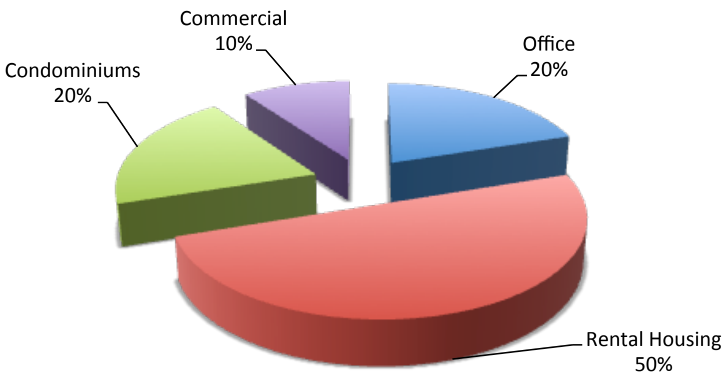
Once finished, it will consist of several building blocks retaining over ### apartments and ##'### m² of service- and retail space. Giving home to approximately ### people and workspace for another #### it will function like a micro city within the community. The location is close to a train station that connects it to the economic region of Zurich. The project is planned to finish by 20##. By its size it is one of the largest development projects in Switzerland in the upcoming years. The size and usage mix make this project an extremely interesting object for study.

Project Set-Up

The project is split up between two independent cooperative building associations (CBAs) and the developer. CBAs are very common in Switzerland and provide housing at preferred conditions to their tenants. The CBAs invest in rental housing units that they keep in their portfolio after completion, while the developer builds condominiums, office and commercial space. The CBAs settled on a fixed price for construction with a third-party construction company, so that they do not bear construction cost risk. It will be interesting to see how this reduces their risk exposure. Since the CBAs offer rents below the market rate, it reduces their risk of vacancies but also their potential cash flow from rent revenues. This has implications on the risk exposure of their project.

The complete area of 40’000 m² (equivalent to eight football fields) is separated into the following uses:

Figure 29: Project Land Use



Source: Own illustration

As we see in Figure 29, the developer uses 50% of land area for condominiums, office, and commercial uses. The two CBAs use the remaining 50% of land for rental housing.

Concept of Cooperative Building Associations

In Switzerland there are many CBAs that provide housing for their members. These associations are not profit oriented and offer rents that cover their cost (cost rent). Additionally they often get preferred funding or a default guarantee by the government that lowers their financing cost. Due to this, they can offer rents below market prices to their tenants, who are themselves members of the cooperative association. In the following analysis we refer to the rent offered by the CBAs as "subsidised rent" and to the usual rent charged on the market as "market rent".

Sale to Third Party Investor

As we have mentioned in Chapter 3.3, many developers sell their project with the receipt of the construction permit to an institutional real estate investor. After the sale of the project the developer is however responsible for delivering the finished project to the investor for a specified price, quality and time in the future. He will thus have to engage a contractor, or, if he is a contractor himself, build the project on his own, while receiving consecutive payments from the investor. We assume that the developer can sell the project under the given market conditions with the receipt of the construction permit. Before an investor buys a project, a part of it has to be preleased to future tenants. Otherwise investors perceive the risk as too high for investment and will not engage in the project. However, it is seldom possible to prelease the whole project, which is why the investor bears vacancy risk to some degree. We account for this by taking twice the vacancy observed on the market as a markdown on the project for the risk compensation of the investor. This results in high markdown on the sale price of the developer when vacancies are high due to the reduction in *GOI*.

Flexibility

The project consists of several building blocks, which gives the option to phase construction and realise buildings in conjunction. Phasing however is not for free, cost synergies in planning and construction are lost and a developer will want to know what the threshold is, so that phasing makes sense. Also the developer has the flexibility to delay construction or to not build at all and it will be interesting to see how this flexibility alters the risk exposure of the project.

Topics of Analysis

Summarized we have the following questions that we want to answered with this study:

/ What is the market risk exposure of the projects?

/ How does subsidised housing and a fixed construction cost contract alter the risk of the CBAs?

/ How much is the option to phase construction and potentially abandon the project worth and what is the influence on risk exposure?

A word of caution: When analysing this project, it is important to bear in mind that we focus on market risk and the flexibility in responding to it. As we have mentioned in Chapter 3.4, there are numerous other risks involved in real estate development that we should not forget when interpreting results. Among them there is e.g. the approval risk, which can delay a project substantially or the planning risk, that can increase cost substantially.

Let us now go through the steps of the "Engineering Approach" and analyse the results when we apply our model for the risk drivers from the last chapter.

6.2 Step 1: Create the Most likely initial Cash Flow Model

As we describe in Chapter 4.1.2, we construct the initial cash flow pro forma by (1) valuing the finished project using the capitalised earnings method, (2) estimating the cost for realising the project and (3) estimating the timing of cash flows. By discounting the cash flows to the present we obtain the NPV, our benchmark for all upcoming calculations. We make two separate cash flow statements for the developer and the CBAs. Since the CBA projects are identical, we only look at one of them that thus covers 25% of land and inherits half the investment volume of the two. Further we refer to CBA instead of CBAs indicating that we refer to only one of them.

Assumptions Developer

/ Land cost are spent at year 0 together with fees and cost for land improvement.

/ We use a risk adjusted discount rate that incorporates the cost of capital of 12%[8], thus all calculations are excluding financing cost.

/ Construction for the developer starts in year 4 and is finished for the whole project in year 7 (4 years).

/ The office & commercial part of the project are sold to an investor for the current market rates in year 4 at twice the current vacancy rate of 6%, payment schedule is according to construction progress.

/ The condominiums are subsequently sold to buyers starting in year 4.

/ Based on own estimates, the cap rate is assumed to be 5%

/ Based on own estimates, prices/rents are assumed to be:

- Office: CHF 250 / m^2 / year
- Commercial: CHF 200 / m^2 / year
- Condominiums: CHF 5000 / m^2

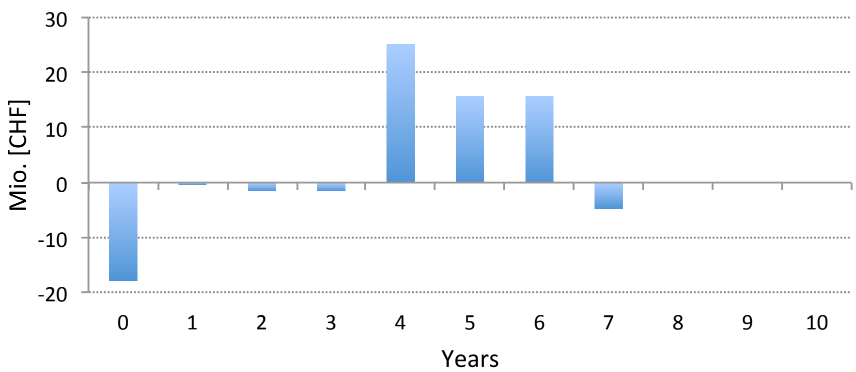
/ Based on the last observations in the data, vacancies are assumed to be:

- Office: 6% (twice = 12%)
- Commercial: 6% (twice = 12%)

Results

Since the developer sells the project with the start of construction, he receives payments parallel to the construction progress and does not suffer much negative cash flows during construction. This has a positive effect on NPV. The total investment sum of the project is CHF 154 Mio. and the value of the finished project is estimated to be CHF 185 Mio. This results in a net profit of CHF 31 Mio.

Figure 30: Net Cash Flows Developer



Source: Own illustration

The NPV for the base case of the developer project is CHF 10.3 Mio.

[8] A risk adjusted discount rate of 12% for real estate development projects is common in Switzerland according to practitioners.

Assumptions CBA

/ Land cost are spent at year 0 together with fees and cost for land improvement.

/ The discount rate is assumed to be 5% based on own estimates and includes the cost of financing. We chose a lower discount rate for the investor than for the developer, because the CBA has lower risk in his project as we see later on.

/ Construction starts already in year 3, one year earlier than the developer and is finished for the whole project in year 6 (4 years)

/ With the completion of construction the building migrates into the portfolio of the CBA with a value according to rents and cap rate

/ Based on own estimates, the cap rate is assumed to be: 5%

/ Rents are assumed to be:

- Subsidised Housing: CHF 210 / m^2 / year

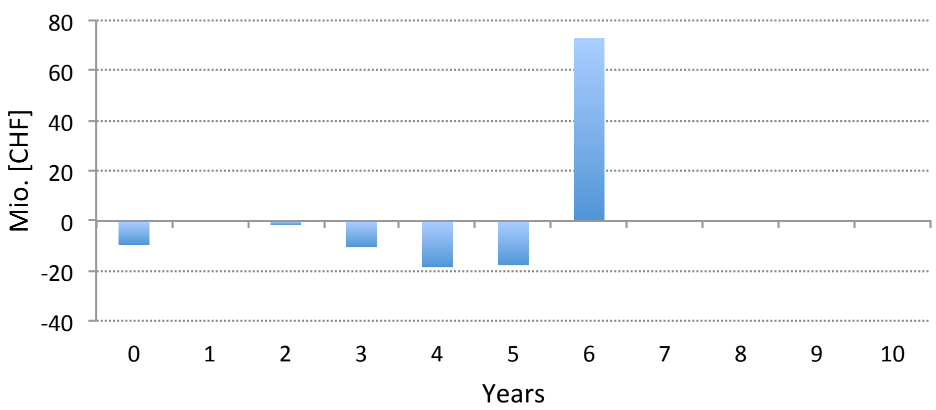
/ Vacancies:

- Subsidised Housing: 1%

Base Case CBA

The CBA has to come up with all cost until completion after which he receives the cash flow producing asset in his portfolio. Since the CBA has a contract for construction for a fixed price, there is no cost risk. Risk of vacancies is reduced due to below market rents and we assume the 5% cap rate to represent a sustainable market rate for capitalisation. Due to these risk-reducing factors of subsidised rents and pre negotiated construction contract, we assume a discount rate of 5% only. The investment sum for this project lies at CHF 77 Mio. and the value of the project is estimated to be CHF 88 Mio. as determined by capitalised earnings. Net profit of the investment is therefore CHF 11 Mio.

Figure 31: Net Cash Flows CBA



Source: Own illustration

The NPV for the base case of the CBA project is CHF 2.0 Mio.

6.3 Step 2: Incorporate Uncertainty into the Model

By incorporating uncertainty into the model we study the risk exposure of the project. We use the introduced Copula-Model to generate 2000 scenarios and study the different NPV outcomes for the developer and the CBA project.

Assumptions

We use our Copula-Model with the following starting values based on own estimates:

- Start Value Rents / Prices:
  - Housing:[9] CHF 240 / m^2 / year
  - Office: CHF 250 / m^2 / year
  - Commercial: CHF 200 / m^2 / year
  - Condominium: CHF 5000 / m^2
- Start Value Market Vacancies:
  - Housing: 1.2%
  - Office: 6.0%
  - Commercial: 6.0%
- Start Value Cap Rate:
  - 5%
- Deal noise δ is separate for condominiums, office and commercial:
  - We assume deal noise to follow a Normal Gaussian distribution with zero mean and 10% standard deviation. This is based on own estimates, since we do not have empirical data for this. Geltner et al. (2007) p. 273 cites multiple studies however in the US suggesting deal noise in the range of 5 to 15%.
  - The deal noise factor δ_{Asset} and δ_{Condo} from (5.9) on page 33 is thereby determined by: $\delta = (1+ \Phi)$ with Φ following the Normal Gaussian distribution as described above.
- We assume that the developer project inherits twice the vacancy observed in the market for office and commercial space at the point of sale to an institutional real estate investor.
- We assume that the CBA inherits half of the vacancy observed in the housing market, because they offer rents below market rents.

[9] This is the market rent which is above the subsidised rent of the CBAs.

Incorporating Uncertainty

We use the cashflow spreadsheet calculation from step 1 as the basis for our analysis. We can add multiple sheets to this spreadsheet, thereby building up our model. We add the copula simulation model from the last chapter as a separate sheet. There we generate future scenarios for our risk drivers. We then link these risk drivers directly with the cash flow statement, thereby adding uncertainty to the calculations. We then use “Data Tables” in another sheet to store and analyse the generated NPVs of the simulations.

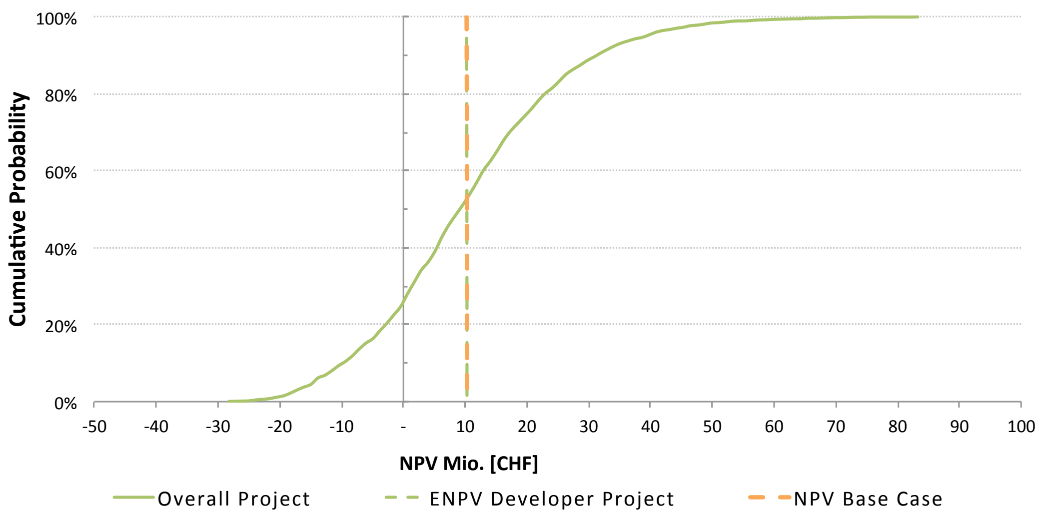
Results Developer

The overall project of the developer shows an ENPV of CHF 10.3 Mio with a standard deviation of CHF 17.2 Mio. The ENPV is therefore the same as in the base case scenario indicating that the average market scenario yields the same NPV as in the base case. The standard deviation is CHF 17.2 Mio indicating that there is indeed a lot of uncertainty in the cash flows of a real estate development project. The probability for a NPV of zero or below, lies at 26%. Since the developer project consists of condominiums and office & commercial use, it is interesting to see, how the different uses contribute to the overall risk structure of the project. We thus look into more detail on this.

Table 12: Developer Project "Static" Case

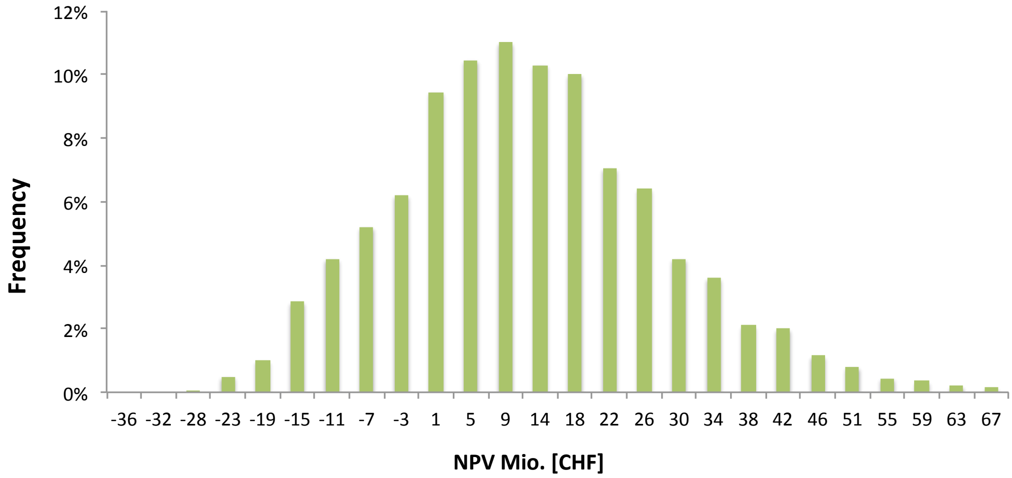
[CHF] Mio.	ENPV	Standard Deviation	5th Percentile	95th Percentile	Minimum	Maximum
Developer Project	10.3	16.8	-14.5	39.5	-35.5	86.6

Figure 32: Cumulative Distribution and ENPV Developer Project



Source: Own illustration

Figure 33: NPV Distribution Developer Project



Source: Own illustration

Risk Contribution of Subprojects

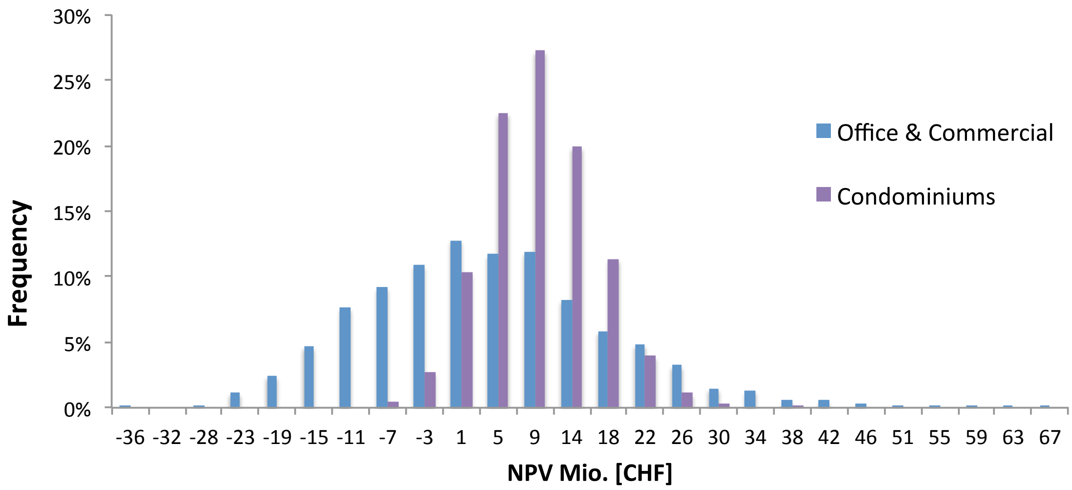
We split the developer project up into two subprojects: condominiums and office & commercial (O&C). The distribution of NPVs under uncertainty reveals a very interesting aspect of the project: while the condominium project has an ENPV of 7.8 Mio., the O&C project adds an ENPV of 2.5 Mio to the overall project. The standard deviation of the O&C subproject is however much higher than that of the condominium project reflecting a large up and downside potential of the O&C subproject as indicated in Table 13. The condominium subproject has less downside potential indicating that it is rather unlikely to loose money on it even if market conditions should turn out to be not that good. The O&C subproject on the other hand has a large up and downside potential due to higher volatility in the market for office space and its dependence on cap rates. We see the difference in the risk structure of the two subproject visualised in

Figure 34 and Figure 35. Note how the cumulative probability distribution is flatter in the case of the O&C subproject depicting a wider range of possible outcomes. This wide range of possible outcomes means that there is a lot of uncertainty in the O&C subproject. This should not be interpreted as a bad thing only, since this large uncertainty means both up and downside potential. If we can find ways e.g. by employing flexibility, we can make use of the high upside potential of the project while protecting us from the downside losses.

Table 13: Condominiums vs. O&C "Static" Case

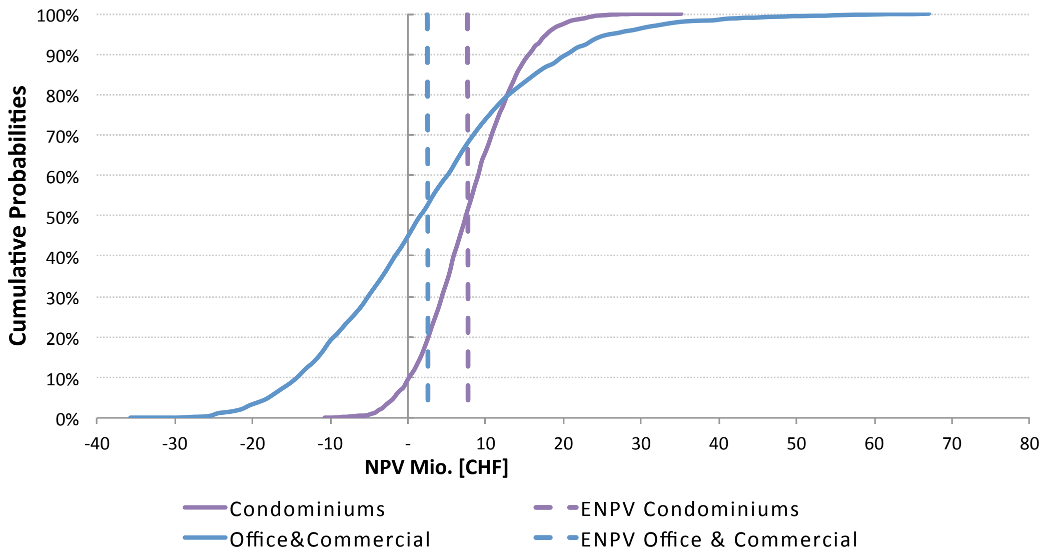
[CHF] Mio	ENPV	Standard Deviation	5th Percentile	95th Percentile	Minimum	Maximum
Condominiums	7.8	6.2	-1.9	-17.7	-9.8	31.3
Office & Commercial	2.5	14.2	-17.7	27.6	-33.1	70.0

Figure 34: "Static" NPV Distribution Condominiums and O&C



Source: Own illustration

Figure 35: "Static" Cumulative Probabilities Condominiums and O&C



Source: Own illustration

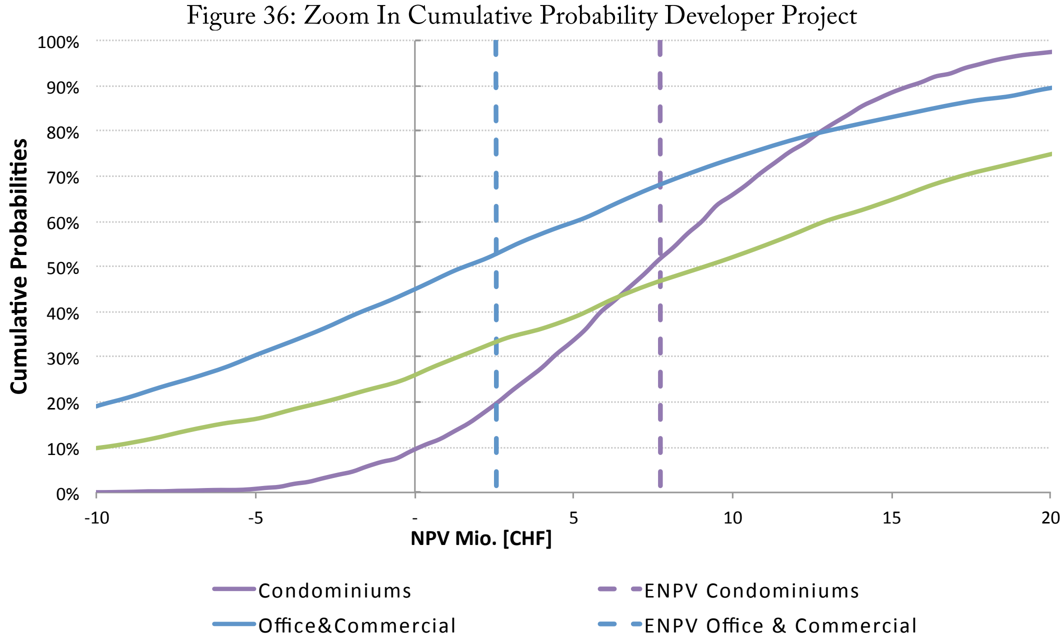
Diversification in Real Estate Development Projects

But if the condominiums have less risk and a higher ENPV why does the developer not build more condominiums? Successful development leads to attractive areas that combine living, work and leisure. The mistakes of the past where the idea was to separate all these functions led to unattractive urban areas and must not be repeated. Therefore it is important to build several uses especially on an area that large and with that location. There are also regulations that make sure that developers actually mix uses and do not just build the use with the highest pay off. However, the developer has quite some degree of freedom on what to build and as we see from Figure 35 the risk of building office and commercial space is quite substantial. An O&C only project would therefore be quite risky to do. By combining O&C with less risky condominiums, the developer can diversify and reduce the risk. This is illustrated on Figure

36 by the green cumulative probability curve of the overall project. There we see that the probability of a negative NPV is reduced by almost 20% compared to the O&C subproject. While these results help us to better understand the risk structure of the project, we should not forget how they came to be. From the analysis on the simulation of risk drivers in Chapter 5.2.5, we know that based on our modelling, condominiums tend to have higher prices of about 11% after 5 years. Additionally we make the office and commercial uses dependent on cap rates, vacancies and office rents, which results in three risk drivers, each with its own simulation path in every scenario. It is thus natural that office use will then have higher uncertainty than condominiums prices, which is mainly described by one risk driver with relatively low standard deviation and a positive mean value. The question then arises if we have to introduce more uncertainty into the condominium price and if so, how and on what basis. We could for example make condominium prices dependent on mortgage interest rates, since these determine the cost of owning a condominium to a large degree or we could introduce the risk of delayed sale in condominiums because of vacancies in the condominium market. One quickly sees that there are many possibilities to further improve the model and analyse the influence of different risk drivers on the development project.

We leave the developer project for now and continue with the analysis of the CBA project.

Figure 36: Zoom In Cumulative Probability Developer Project



Source: Own illustration

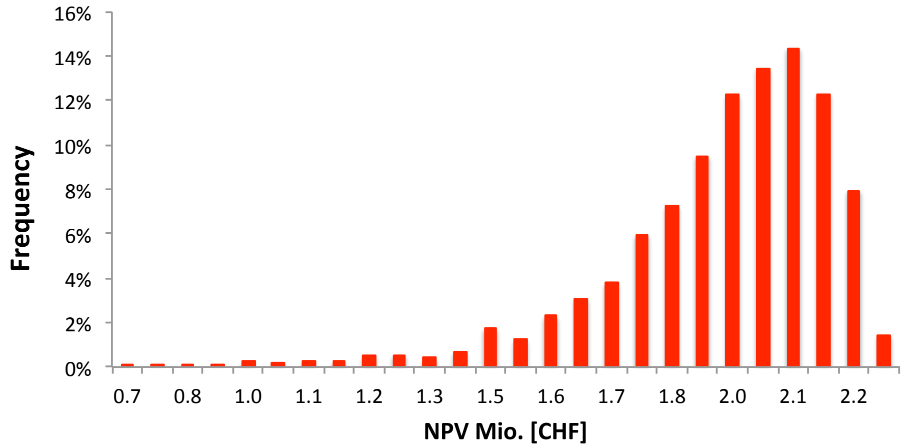
Results CBA

The CBA offers his apartment units for a rent below market rents. Therefore we would expect, that he faces less difficulty in renting out his apartments as long as market rents remain above his subsidised rents of CHF 210 per m^2 and vacancies stay at a low level. There is also no cap rate uncertainty since we assume that the CBA keeps the asset in his books with the fixed cap rate of 5.0%. Also cost uncertainty is removed due to a prearranged contract with a construction contractor. One would expect that the uncertainty due to these restrictions is rather low which is also concluded by our analysis:

Table 14: CBA Project "Static" Case

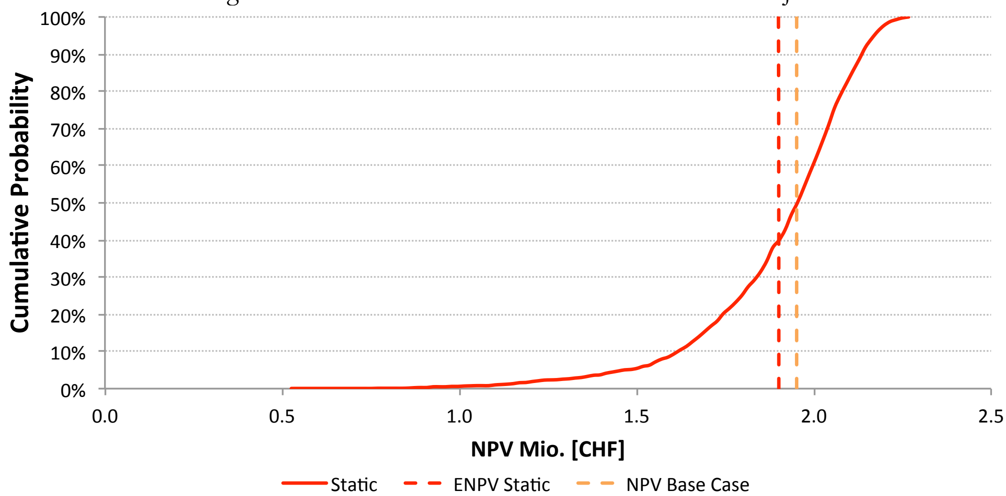
[CHF] Mio	ENPV	Standard Deviation	5th Percentile	95th Percentile	Minimum	Maximum
CBA Project	1.9	0.2	1.5	2.2	0.7	2.3

Figure 37: "Static" NPV Distribution CBA



Source: Own illustration

Figure 38: "Static" Cumulative Probabilities CBA Project



Source: Own illustration

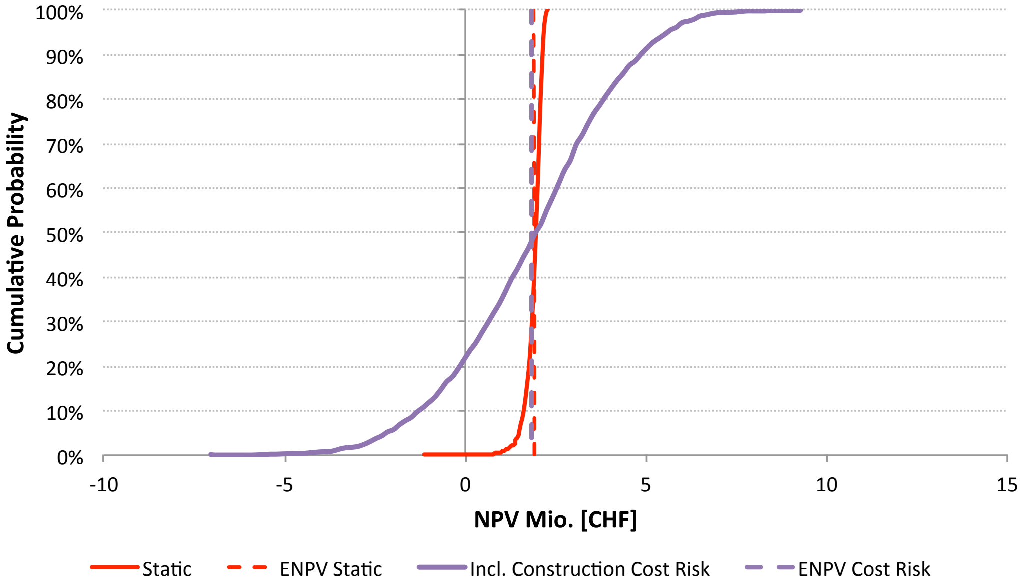
As we see from Figure 38 the risk of the CBA project is low as modelled by our simulations.

We see however that the ENPV of the project is slightly lower than in the "base case" were we had an NPV of CHF 2.0 Mio. This comes from the uncertainty in vacancies that reduce the value of the asset at completion. The model estimates that market rents for housing do not fall below the subsidised rent of CHF 210 / m^2 which is why we do not get negative NPVs for the project. We can also see that there is no upward potential in the project since the housing rent is fixed at CHF 210.

With such a risk profile, the flexibility to delay or abandon a project is not valuable since it will almost always result in a lower NPV due to the discount rate. Therefore we do not perform additional analysis on the value of flexibility for the CBA.

It is however interesting to see what would happen if the CBA did not pre negotiate construction cost and how this would alter the risk structure of the project. We know from Figure 27, the simulation of construction cost, that on average construction costs are about 7.5% higher between year four and five, when the project is in the middle of construction. We therefore have to deduct these 7.5% from construction cost of the CBA since they will already be included in the construction contact under this scenario. We further assume a premium charged for construction cost risk by the contractor of 1% that is also already included in the fixed contract. We have to deduct this 1% to study what the project would cost if the CBA had no fixed cost contract. Therefore we reduce construction cost by 8.5% in our spreadsheet model and simulate once more. Results are depicted in Figure 39. While the ENPV of the project remains the same, the up- and downward potential of the project increases. Since there is no increase in ENPV, this is not something to go for. If we would increase the deducted premium of 1% however, we would see higher gains in ENPVs for the CBA to be made. Therefore it really depends on how much premium has to be paid and how much the risk appetite of the CBA is until it makes sense to take over construction cost risk. We therefore see how the risk driver construction cost alters the risk structure of the project and how we can visualise and interpret results.

Figure 39: CBA Project incl. Construction Cost Risk



Source: Own illustration

Table 15: CBA Project incl. Cost Uncertainty "Static"

	ENPV	Standard Deviation	5th Percentile	95th Percentile	Minimum	Maximum
CBA Project incl. Cost Uncertainty	1.8	2.3	- 2.0	5.5	-6.2	9.5

We conclude the analysis of the CBA project here by stating, that due to the subsidised rents and due to the fixed construction cost contract, the risk exposure of the project is reduced. Therefore the low discount rate of 5% is indeed appropriate. The lack of uncertainty and the fixed subsidised rent in this project means however that there is also no upside potential and thus no value of flexibility. When interpreting results, we have to consider however, that our simulation model yields results with low downward market potential for housing rents as seen in our analysis on the outcomes of the model in Chapter 5.2.5. Thus it comes to no big surprise that rents do not fall below the reduced rent of the CBA. A different configuration of the model with e.g. higher standard deviation for housing rents would yield different results.

We now go on with the analysis of flexibility in the developer project. We do not do this for the CBA project, since uncertainty is low and we thus do not expect value of flexibility in this project.

6.4 Step 3: Incorporate Flexibility into the Model

From the analysis of the project under uncertainty we know that we have to take special care of the O&C part of the developer project. There is a rather high probability that we may face negative NPVs from this part of the project and thus we need to find ways on how to reduce this risk. We can further state that there is not much value in flexibility for the CBA, since there is little uncertainty in the value of the finished project and due to the restriction on rents, no upside potential. Also for the condominiums it seems rather unlikely to face a loss in NPV.

Delay Office & Commercial

To incorporate flexibility we set a minimum profit that the O&C subproject has to achieve in order to be executed. To do this we calculate the net profit per m^2 of the O&C subproject at the point of sale to a potential third party investor. This is simply the earnings per m^2 minus the cost of construction of the O&C subproject. We do not include land and development cost into this calculation because at this point in time the cost is already sunk in the project and we focus on the positive cash flows we generate with the upcoming decisions. In the base case we get a profit per m^2 of CHF 1'100.

We then set a rule, that the O&C subproject is only executed if the profit per m^2 is above a certain threshold value, e.g. CHF 400. Whenever the profit is bellow that value the sale of the project and start of construction will be delayed by one year and so on until the profit is above the threshold or year 10 is reached. If the sub project is not sold to a third party investor by year 10, the remaining land will be sold for its salvage value. We can thus study both the option to delay and the option to abandon the O&C project.

Assumptions

- / Remaining land can be sold for 80% of purchase price after 10 years
- / Phasing costs additional 2% of construction cost: CHF 1.6 Mio.
- / Condominiums are always sold and constructed for current prices.
- / O&C subproject is only executed if current profit per m^2 (excl. land) is above CHF 400 (the base case profit is CHF 1150)

Results

In Figure 40 and Figure 41 we see how the option to delay or abandon the O&C project reduces the downside risk of the project. Especially in Figure 41 we clearly see that there are fewer realisations with a very high negative NPV. The decision rule to delay construction when profit is below CHF 400 / m^2 proves to increase ENPV by 1.6 Mio. compared to the static case. This difference in ENPV is our option value. We also see that standard deviation decreases by CHF 1.4 Mio and that the 5th percentile is increased by CHF 6.3 Mio. in the flexible case. This is highly speaks for preserving the option to delay the O&C subproject, while executing the condominiums.

Table 16: Developer Project "Static" vs. "Flexible"

	ENPV	Standard Deviation	5th Percentile	95th Percentile	Minimum	Maximum
Developer Project "Static"	10.1	16.7	-14.3	39.2	-35.4	90.6
Developer Project "Flexible"	11.7	15.3	-8.0	39.2	-39.0	90.6

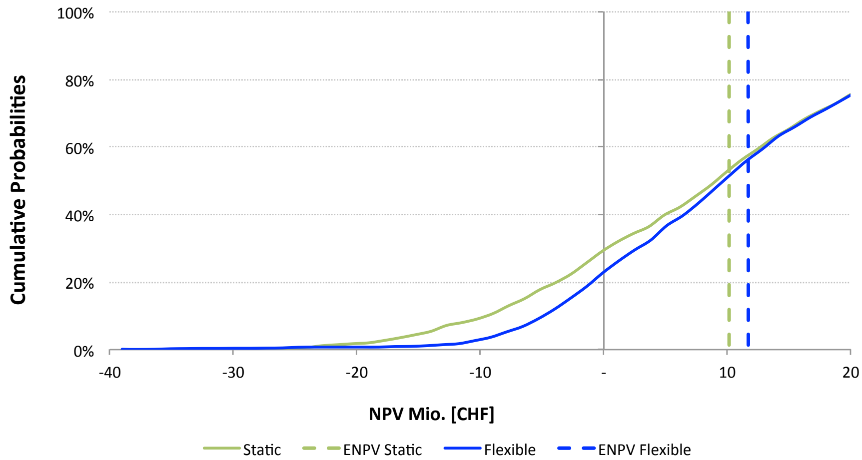
In Chapter 4.2 we define the value of an option as the difference between the value of the project with and without the option. We thus have:

Option Value = ENPV Flexible – ENPV Static

Option Value = CHF 1.6 Mio.

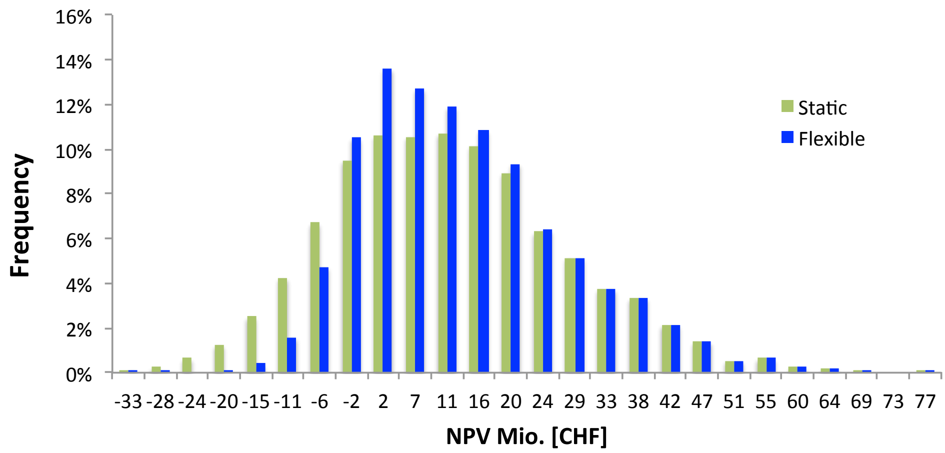
This equals an increase in ENPV of 16%.

Figure 40: Cumulative Probabilities Static vs. Flexible Developer Project



Source: Own illustration

Figure 41: NPV Distribution Flexible vs. Static Developer Project

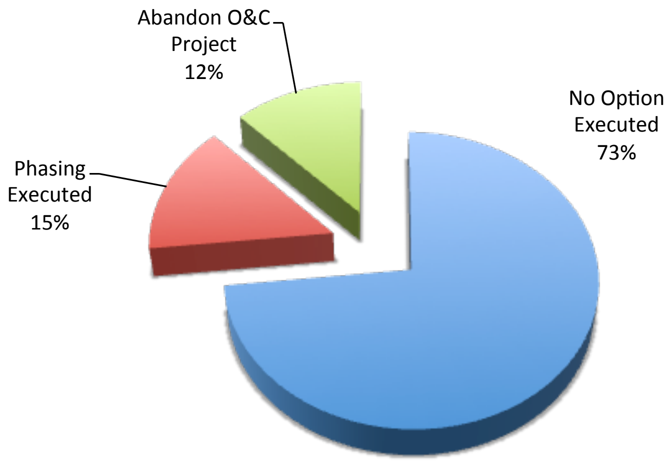


Source: Own illustration

In Figure 42 we see what decision were actually made within the spreadsheet model under the simulated scenarios. In 73% of the cases the option to delay construction was not executed. In 27% of the simulated scenarios however, the option to delay was executed whereas the O&C

subproject was completely abandoned in 12% of the scenarios. In the remaining 15% of cases however, the O&C subproject was delayed and executed at a later date. From our model, we know that if the subproject was delayed, we had a higher NPV than if we had not, in 98% of cases.

Figure 42: Decisions Made in Flexible Developer Project



Source: Own illustration

From these results we can conclude that the option to delay the project has value. Let us now go to the fourth step of the "Engineering Approach" and see if we can further improve this option value, by performing further analysis with the model.

6.5 Step 4: Maximize Value by Applying Optimal Decision Rules

This step of the "Engineering Approach" deals with finding the optimal configuration of the model to base decisions on. In step three we set a rough first guess on the threshold when delay should be initiated for the O&C subproject of CHF 400 / m^2. We can now further optimize this decision rule by increasing or decreasing the threshold at which the subproject is delayed. We thereby get the following results:

Table 17: Variation in Threshold Value

Threshold	200	400	600	800	1000	1200	1400	1600
Option Value	1.4	1.6	1.7	1.7	1.8	1.5	0.8	0.0

The option values for different thresholds remain rather close together. We see however that we have the highest option value for a threshold between 600 to 1000 CHF/m^2. Since figures and numbers do not differ much from the analysis of the last chapter we do not repeat them here but go over to a summary on this chapter.

Let us conclude this chapter with summarising the insights we gained from the analysis: (1) The "Engineering Approach" lets us better understand the uncertainty in a real estate development project by visualising potential outcomes. (2) By separating the developer project into subprojects we get valuable insights into their overall risk contribution. We see that the mix of condominiums and O&C project of the developer can reduce his risk exposure. (3) The CBAs face less uncertainty in their project because of a fixed cost contract and below market rents that reduce their risk of vacancies. Since there is little uncertainty, there is also no value of flexibility in this project. We can however use the "Engineering Approach" to study the effect of certain properties of the project such as a fixed construction cost contract and get valuable insights. (4) The option to delay the O&C subproject increases the ENPV of the project by CHF 1.7 to 1.8 Mio when the project is delayed at a net profit per built m^2 of CHF 600 to 800.

It is however important to note here that our results depend to a very large degree on the employed simulation mechanism. We need to be very careful to employ simulations that yield feasible results and also to model in a way that captures important properties of the market. The presented tools are a way to do so, but additional work on modelling is needed. We discuss this further in the next chapter where we conclude this thesis and give an outlook on further work to be done.

7 Conclusion

Using the "Engineering Approach" together with a copula based simulation process that we partly calibrated on real estate market data, we successfully analyse uncertainty & flexibility of a large-scale real estate development project. We visualise and quantify the risk structure of the project and draw meaningful conclusions for decision-makers and investors on the risk structure and the value of flexibility within a real estate development project. With our model we are able to show quantitatively that:

/ Real estate development projects inherit large uncertainty.

/ Risk can be diversified by the choice to build different uses such as the combination of office & commercial space with condominiums.

/ Risk can further be reduced by the flexibility to phase construction.

We further show how a fixed construction cost contract and subsidised housing rents alter the risk structure of a project. With this model we make uncertainty and flexibility tangible and make it possible to include this into the decision making process. Also we reveal highly interesting aspects on the influence of single risk drivers such as construction cost risk on the decisions made within a project. As such we make a contribution to the current analysis practice in real estate development in Switzerland.

The obtained results must however be seen in the light of the employed simulation mechanism. Certainly condominiums and housing uses incurred less downside risk in our analysis but this is mainly caused by the specifications of the simulation model. We constructed this model in the beginning on past data only and then went over to a model partly based on past data and partly on our own reasoning. The employed tools of copulas and VAR models thereby showed great promise in modelling the data, while accounting for interdependencies. Due to low resolution in data and high standard deviation in some part of the data, we were however not able to fit a VAR model that is practical for simulating multiple years ahead. We thus chose a copula model for simulations that is partly based on own assumptions on the dependency structure of the data. While this model takes into account dependencies among the variables it does not account for autocorrelation. This is an important drawback of the model and in further research it would be highly desirable to develop a model that accounts for autocorrelation as well. When working with the copula model, we found it to become rather static once it is fitted to the data. We cannot easily bring in own expectations of the future and manipulate the model interactively to perform in-depth analysis with different scenarios. This is however an important feature especially when dealing with simulations of a long simulation horizon as in the case of this thesis. Practitioners might want to implement simulation mechanisms that are on average rather conservative than too optimistic to make sure that they do not overstate the upside potential of a project. Further they will be interested to see how different assumptions on the risk drivers change outcomes. With the VAR model we have the possibility to analyse different scenarios to some degree by using the GDP as an exogenous variable. With the copula model we have to change the whole simulation process in MATLAB to perform analysis with different simulation scenarios. This is not very applicable. We gave ideas on how we can improve this by using dependent variables directly in Excel. Another interesting aspect in this context would be the study of tail dependence of the different risk drivers. With more data or analogue data from other markets one could study how the risk drivers behave in extreme situations like a crash and implement this into the model using tail dependent copulas.

However, while the modelling of data can be highly engaging, we need to bear in mind that the best model is of little use when there is no application. Practitioners need practical applications however to cope with the challenges they face in the correct analysis of real estate development projects. While the implementation of the "Engineering Approach" is highly applicable and reveals important aspects on the risk structure of a project, we have to spend further time on the development of an appropriate and applicable simulation mechanism for the risk drivers of a real estate development project. Thereby we should focus on important relations of the risk drivers, which is where the presented tools come in handy, but also on possibilities to manipulate outcomes and bring in own expectations. Also we should focus more on investigating further relations between risk drivers and their dependence on other variables. In this context interesting variables for consideration would be real GDP, since we saw a relation in our VAR analysis and interest rates, since they have an influence on cap rates and on the affordability of condominiums.

8 References

Barman, B. and K. E. Nash (2007). A Streamlined Real Options Model for Real Estate Development MSc in Real Estate Development, Massachusetts Institute of Technology.

Black, S. and F. Myron (1973). "The Pricing of Options and Corporate Liabilities." Journal of Political Economy **81**(3): 637-654.

Canonica, F. (2009). "Die Immobilienbewertung." Zürich/Chur, S **156**.

Chapman, M. (2012). "Immo-Hypotheken: Jetzt ziehen die Banken die Schraube an." Retrieved 22.12, 2012, from http://www.tagesanzeiger.ch/wirtschaft/konjunktur/ImmoHypotheken-Jetzt-ziehen-die-Banken-die-Schraube-an/story/22000081.

Copeland, T. V. A. (2001). Real Options: A Practitoner's Guide. New York, Texere.

Cox, J., S. Ross, et al. (1979). "Option pricing: A Simplified Approach." Journal of Financial Economics **7**(3): 229-263.

de Neufville, R. and S. Scholtes (2011). Flexibility in Engineering Design. Cambridge, MIT Press.

de Neufville, R., Scholtes, Stefan, Wang, T (2006). "Real Options by Spreadsheet: Parking Garage Case Example." Journal of Infrastructure Systems **12**(2): 107 - 111.

Dettling, M. (2012). Applied Time Series Analysis. Lecture Material ETH Zürich.

Dokko, Y., R. H. Edelstein, et al. (2001). "Real Estate Income and Value Cycles: A model of Market Dynamics." Global Perspective on Real Estate Cycles **6**: 63-87.

Embrechts, P. (2009). "Copulas: A personal view." Journal of Risk and Insurance **76**(3): 639-650.

Fahrländer, P. (2012, 3.12.2012). "Real Estate Scenario Cockpit RESC." from http://www.fpre.ch/de/01_doku/pd_RESC.pdf.

Fierz, K. (2005). Der Schweizer Immobilienwert, Schulthess.

Geltner, D. (1989). "On the Use of the Financial Option Price Model to Value and Explain Vacant Urban Land." Journal of the American Real Estate & Urban Economics Association **17**(2): 142-158.

Geltner, D. and R. de Neufville (2012). Uncertainty, Flexibility, Valuation & Design: How 21 Century Information & Knowledge Can Improve 21st Century Urban Development. RICS Global Symposium. Adelaide Australia.

Geltner, D. and N. Miller (2007). Commercial Real Estate Analysis and Investments, Second Edition. Ohio, South-Western.

Geltner, D. and N. G. Miller (2001). Commercial Real Estate Analysis and Investments, First Edition. Upper Saddle River, N.J., Prentice Hall.

Gordon, M. J. (1962). The Investment, Financing, and Valuation of the Corporation. Homewood, Ill.,, R.D. Irwin.

Guthrie, G. (2009). Evaluating Real Estate Development Using Real Options Analysis, Victoria University of Wellington.

Hengels, A. (2005). Creating a Practical Model Using Real Options to Evaluate Large-Scale Real Estate Development Projects, Massachusetts Institute of Technology.

Hodder, J. E., A. S. Mello, et al. (2001). "Valuing Real Options: Can Risk-Adjusted Discounting be Made to Work?" Journal of Applied Corporate Finance **14**(2): 90-101.

Hoesli, M. J., Elion; Bender, André (2005). "Monte Carlo Simulations for Real Estate Valuations." FAME - International Center for Financial Asset Management and Engineering(Research Papter N° 148).

Holderegger, U. (2006). "Schlussstrich unter Bankenpleite." NZZ Retrieved 16.12.2012, 2012, from http://www.nzz.ch/2006/01/03/wi/newzzEI09IATX-12.html.

Jetzer, J.-P. Z., Mathias (2007). Die Schweizerische Nationalbank, 1907 - 2007. Zürich, Schweizerische Nationalbank.

Masunaga, S. (2007). A Comparative Study of Real Options Valuation Methods : Economics-Based Approach vs. Engineering-Based Approach MSc in Real Estate Development, Massachusetts Institute of Technology.

Mauboussin, M., Hiler, B. (1999). "Rational Exuberance? Is There Method behind the Madness in Internet Stock Valuations?" Credit Suisse First Boston Equity Research.

Maurer, T. (2006). Anwendung von Real Options in der Immobilienpraxis. Master of Science, University of Zürich.

Meier, T. (2009). Ursachen und Folgen der Immobilienkrisen in der Schweiz, in Japan und in den USA und deren Vergleich, Universität Zürich.

Müller, V. (2007). Bewertung von Immobilien-Projektentwicklungen - Ermittlung der Best Practice-Methode. Master of Science, University of Zürich.

Mun, J. (2002). Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions, Wiley.

Mun, J. (2006). Modeling Risk: Applying Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization Techniques, Wiley.

Neslehova, J. (2006) "Eine Einführung in Copulas." ETH Zürich.

Pfaff, B. (2008). "VAR, SVAR and SVEC models: Implementation within R package vars." Journal of Statistical Software **27**(4): 1-32.

Quigg, L. (1993). "Empirical Testing of Real Option-Pricing Models." Journal of Finance **48**(2): 621-640.

Salmon, F. (2009). "Recipe for Disaster: The Formula That Killed Wall Street." Retrieved 1.12.2012, 2012, from http://www.wired.com/techbiz/it/magazine/17-03/wp_quant?currentPage=all.

Schuler, M. J., Dominique (2005). Die Raumgliederung der Schweiz. Bern, Bundesamt für Statistik.

Schulte, K.-W. B.-W., Stephan (2002). Handbuch Immobilien-Projektentwicklung. Köln.

Sklar, A. (1959). "Fonctions de répartition à n dimensions et leurs marges." Publ. Inst. Statist. Université Paris **8**(1): 11.

Titman, S. (1985). "Urban Land Prices under Uncertainty." American Economic Review **75**(3): 505-514.

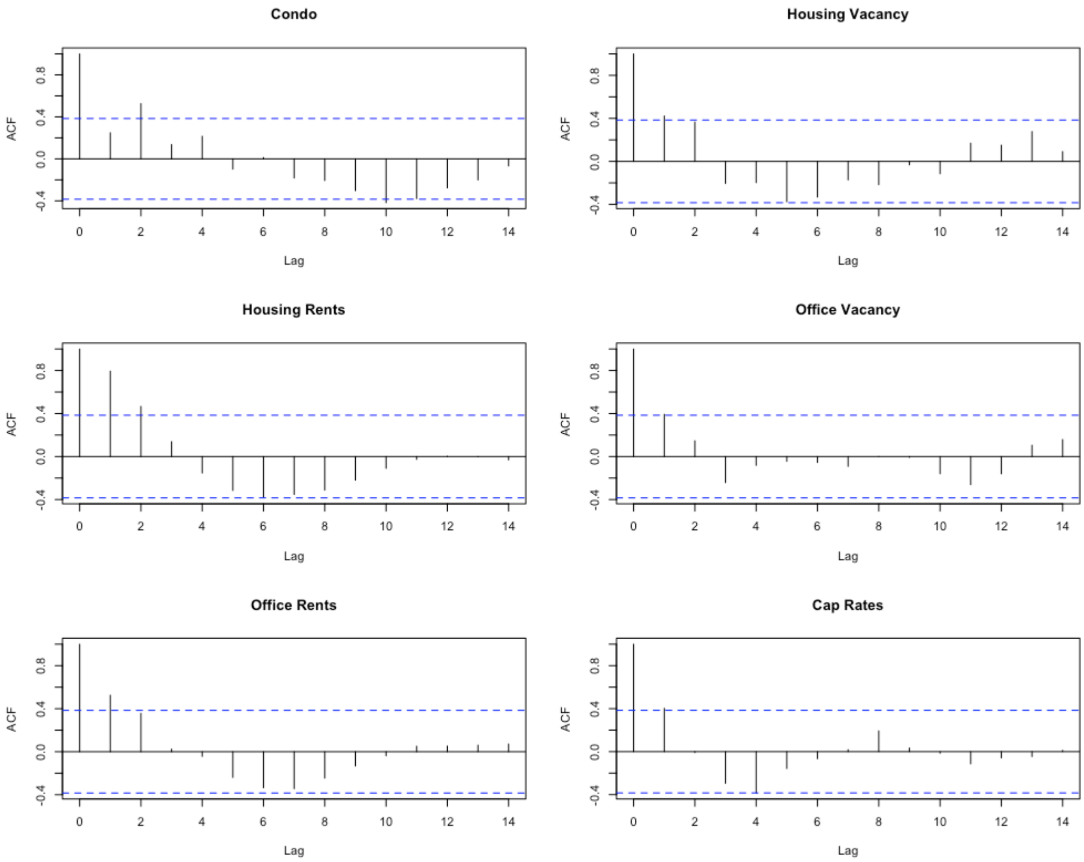
Trigeorgis, L. (1993). "Real Options and Interactions with Financial Flexibility." Financial Management **22**(3): 202-224.

Werner, R. (2006). "Normal Inverse Gaussian (NIG) distribution - updated version." Retrieved 1.12.2012, 2012, from http://www.mathworks.com/matlabcentral/fileexchange/10934-normal-inverse-gaussian-nig-distribution-updated-version.

Appendix

I. Autocorrelation

Figure 43 Autocorrelation Log-Returns Data 1986 - 2011



Source: Output from R

For the analysis of autocorrelation we do not separate between office and commercial use anymore because the data is almost identical, we thus depict the data for office only.

II. VAR-Model

VAR Estimation Results:

========================

Legend:

Condo	=	Condominiums
H_R	=	Housing Rents
O_R	=	Office Rents
H_V	=	Housing Vacancy
O_V	=	Office Vacancy
CAP	=	Cap Rates

Endogenous variables: Condo, H_R, O_R, H_V, O_V, CAP

Exogenous variable: GDP

Sample size: 19[10]

Log Likelihood: 247.322

Roots of the characteristic polynomial:[11]

0.891 0.891 0.8793 0.8793 0.8091 0.7681 0.7681 0.7273 0.7273 0.6797 0.6797 0.3078

p-Values Estimation Results:

Table 18 : p-Values and Residual Standard Errors

Estimated Variable	p-Value	Residual Standard Error
Condo	0.002	0.024
H_R	0.000	0.008
O_R	0.007	0.032
H_V	0.000	0.148
O_V	0.003	0.185
CAP	0.000	0.016

[10] First two observations are lost due to lag-2 model leaving 19 observations

[11] The VAR-process is said to be stable since the roots of the characteristic polynomial are less than one, which is the case here.

Estimation results for equation Condo:

===

Condo = H_R.l1 + CAP.l1 + Condo.l2 + H_R.l2 + O_R.l2 + GDP

	Estimate	Std. Error	t value	p-value coefficient
H_R.l1	1.4720	0.4856	3.031	0.00965 **
CAP.l1	0.7360	0.2488	2.958	0.01110 *
Condo.l2	0.4136	0.1726	2.397	0.03229 *
H_R.l2	-0.4983	0.2802	-1.779	0.09867 .
O_R.l2	0.4855	0.2275	2.134	0.05248 .
GDP	0.7419	0.3158	2.349	0.03529 *

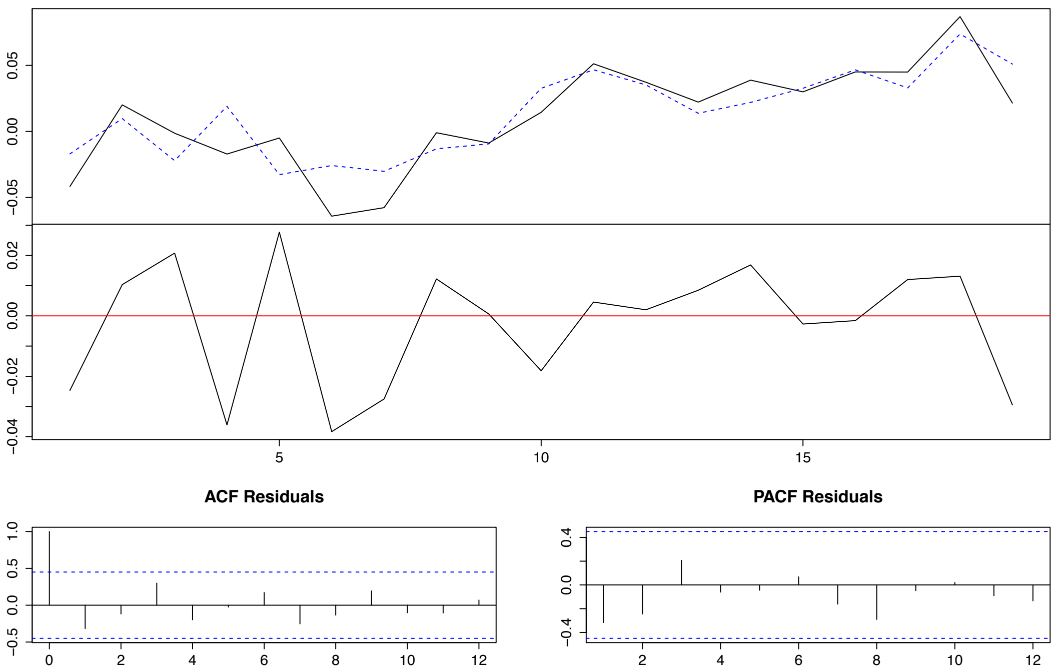
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02403 on 13 degrees of freedom

Multiple R-Squared: 0.7427, Adjusted R-squared: 0.6239

F-statistic: 6.254 on 6 and 13 DF, p-value estimation: 0.002837

Figure 44 : Diagram of Fit and Residuals Condominiums



Source: Output from R package "VARS"

Estimation results for equation H_R:

====================================

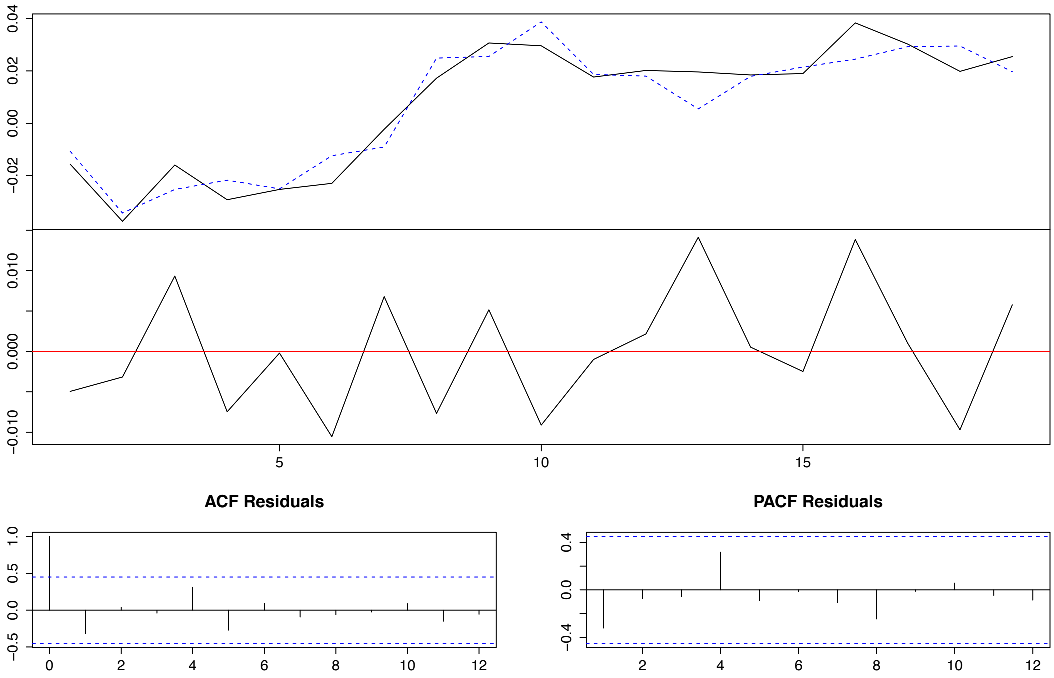
H_R = H_R.l1 + H_V.l1 + H_R.l2 + GDP

	Estimate	Std. Error	t value	p-value coefficient
H_R.l1	0.918131	0.126708	7.246	2.85e-06 ***
H_V.l1	-0.026074	0.006312	-4.131	0.000888 ***
H_R.l2	-0.259950	0.089372	-2.909	0.010804 *
GDP	0.345759	0.091988	3.759	0.001897 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.008306 on 15 degrees of freedom
Multiple R-Squared: 0.9081, Adjusted R-squared: 0.8836
F-statistic: 37.07 on 4 and 15 DF, p-value estimation: 1.307e-07

Figure 45 : Diagram of Fit and Residuals Housing Rents



Source: Output from R package "VARS"

Estimation results for equation O_R:

====================================

O_R = O_R.l1 + H_V.l1 + CAP.l1 + H_V.l2 + CAP.l2 + GDP

	Estimate	Std. Error	t value	p-value coefficient
O_R.l1	-0.44273	0.24467	-1.810	0.09354 .
H_V.l1	-0.09370	0.02867	-3.268	0.00612 **
CAP.l1	-0.72032	0.30359	-2.373	0.03377 *
H_V.l2	-0.07775	0.02880	-2.700	0.01820 *
CAP.l2	0.57607	0.24930	2.311	0.03790 *
GDP	0.94881	0.40905	2.320	0.03728 *

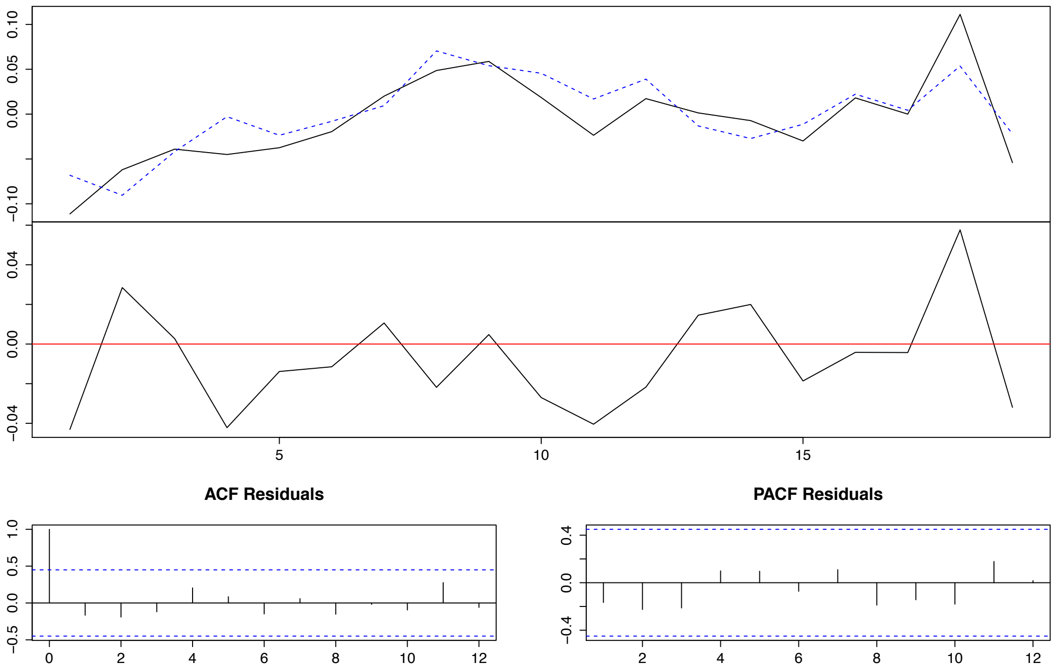
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0323 on 13 degrees of freedom

Multiple R-Squared: 0.7022, Adjusted R-squared: 0.5647

F-statistic: 5.108 on 6 and 13 DF, p-value estimation: 0.006699

Figure 46 : Diagram of Fit and Residuals Office Rents



Source: Output from R package "VARS"

Estimation results for equation H_V:

====================================

H_V = H_R.l1 + O_V.l1 + Condo.l2 + H_V.l2 + O_V.l2 + CAP.l2

	Estimate	Std. Error	t value	p-value coefficient
H_R.l1	13.7474	2.3779	5.781	6.37e-05 ***
O_V.l1	0.5609	0.1641	3.417	0.004586 **
Condo.l2	-5.0685	1.1956	-4.239	0.000966 ***
H_V.l2	0.3577	0.1127	3.175	0.007310 **
O_V.l2	0.6975	0.1723	4.047	0.001383 **
CAP.l2	5.0091	1.0397	4.818	0.000336 ***

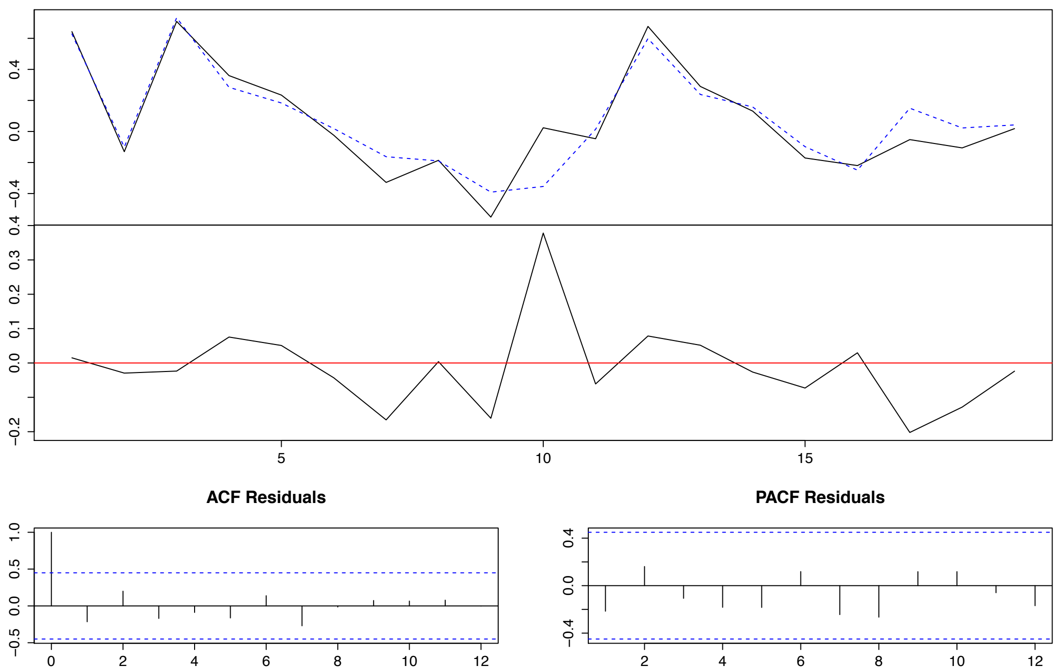
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1482 on 13 degrees of freedom

Multiple R-Squared: 0.8713, Adjusted R-squared: 0.812

F-statistic: 14.67 on 6 and 13 DF, p-value estimation: 4.094e-05

Figure 47 : Diagram of Fit and Residuals Housing Vacancy



Source: Output from R package "VARS"

Estimation results for equation O_V:

=====================================

O_V = H_V.l1 + CAP.l1 + H_R.l2 + O_R.l2

	Estimate	Std. Error	t value	p-value coefficient
H_V.l1	0.3620	0.1529	2.368	0.03175 *
CAP.l1	5.4477	1.5839	3.439	0.00365 **
H_R.l2	3.7960	1.2940	2.933	0.01027 *
O_R.l2	5.7984	1.6268	3.564	0.00283 **

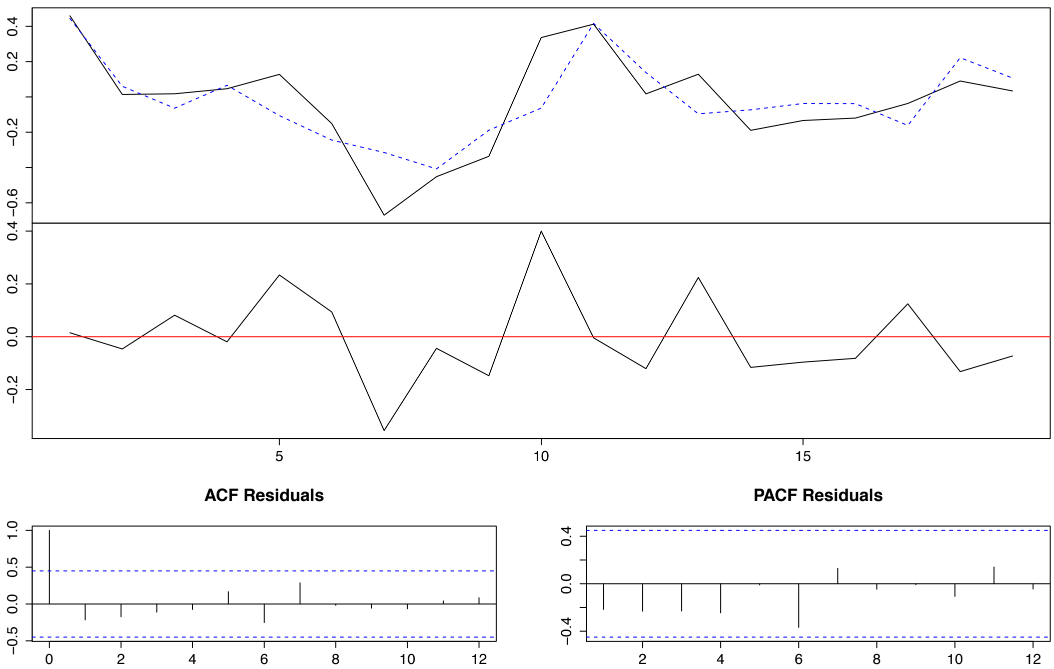
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1854 on 15 degrees of freedom

Multiple R-Squared: 0.6313, Adjusted R-squared: 0.533

F-statistic: 6.422 on 4 and 15 DF, p-value estimation: 0.003223

Figure 48 : Diagram of Fit and Residuals Office Vacancy



Source: Output from R package "VARS"

Estimation results for equation CAP:

===================================

CAP = H_R.l1 + O_V.l1 + CAP.l1 + Condo.l2 + H_R.l2 + O_V.l2 + GDP

	Estimate	Std. Error	t value	p-value coefficient
H_R.l1	-1.83039	0.37793	-4.843	0.000403 ***
O_V.l1	-0.04024	0.01993	-2.019	0.066438 .
CAP.l1	0.50006	0.11613	4.306	0.001021 **
Condo.l2	0.42730	0.13341	3.203	0.007592 **
H_R.l2	1.52242	0.21769	6.994	1.45e-05 ***
O_V.l2	-0.05936	0.01756	-3.381	0.005457 **
GDP	-0.37948	0.18665	-2.033	0.064781 .

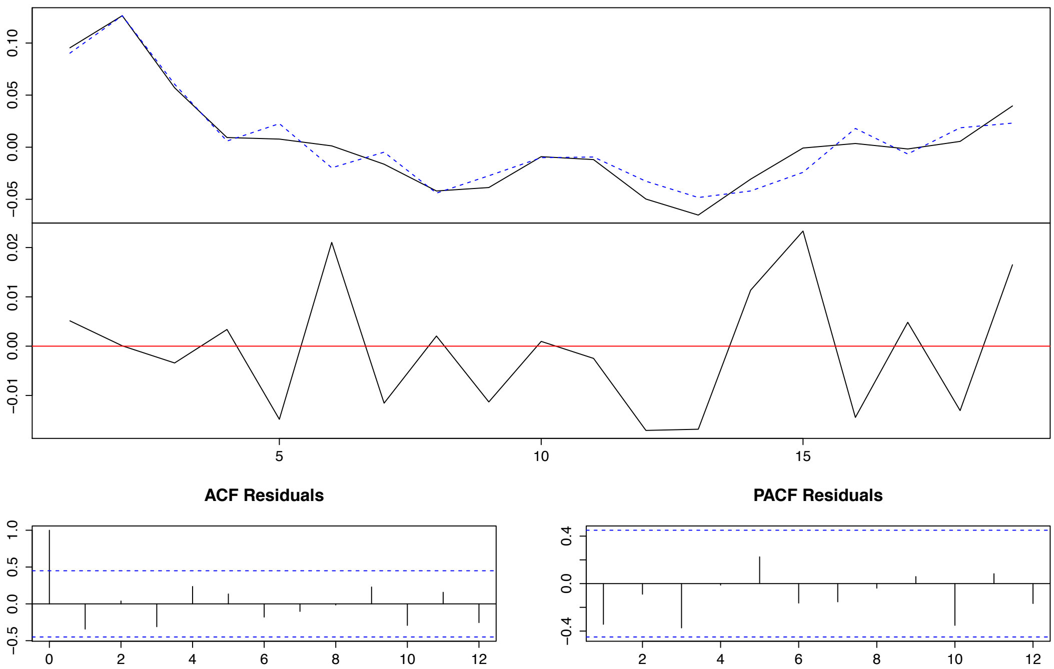
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01558 on 12 degrees of freedom

Multiple R-Squared: 0.9298, Adjusted R-squared: 0.8889

F-statistic: 22.71 on 7 and 12 DF, p-value: 5.103e-06

Figure 49 : Diagram of Fit and Residuals Cap Rates



Source: Output from R package "VARS"

III. Copula Model Own Assumptions

Table 19: Correlation Matrix Copula Model Own Assumptions

	Housing Rents	Office/Com. Rents	Housing Vacancies	Office/Com. Vacancies	Condo	Cap Rate	Constr. Cost
Housing Rents	1.00						
Office/Com Rents	0.49	1.00					
Housing Vacancies	-0.49	-0.10	1.00				
Office Vacancies	-0.10	-0.50	0.50	1.00			
Condo	0.49	0.50	-0.50	-0.05	1.00		
Cap Rate	-0.10	-0.10	0.10	0.10	-0.10	1.00	
Construction Cost	0.30	0.30	-0.30	-0.30	0.30	-0.10	1.00

Table 20: Descriptive Statistics Copula Model Own Assumptions

	Housing Rents	Office/ Com. Rents	Housing Vacancies	Office Vacancies	Condo Prices	Cap Rates	Constr. Cost
Mean	0.02	0.02	0.00	0.00	0.02	0.01	0.02
Std. Dev.	0.04	0.08	0.20	0.20	0.04	0.04	0.02
Skewness	0.87	0.87	-0.01	0.02	0.08	0.01	0.00
Kurtosis	1.39	0.12	0.10	0.08	0.50	0.09	0.09