Calibration of Bubble Indicators for Foreign Exchange Markets

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Abstract

This thesis examines the applicability of the Johansen-Ledoit-Sornette Log-Periodic Power Law model to currency markets as a means to detect the presence of bubbles and forecast their collapse. Previous research into the model has primarily focused on equity markets where different market dynamics may prevail. This study is the first to apply the model to a broad range of currencies and examine its predictive performance. Such a bubble alarm would be valued by many market participants, from central banks monitoring their domestic currency to asset managers managing their forex exposures. The model is calibrated on a range of developed market currencies, both on cross rates and exchange rate indices. The performance of bubble alarms suggests the model has predictive ability for both indices and cross rates, with greater predictive power for the latter. A Lattice Alarm is introduced which aggregates market information from multiple cross rates. The performance of this alarm would suggest that monitoring a currency for bubbles exclusively through an exchange rate index may be suboptimal. We also examine macroeconomic factors which may affect economic agents’ willingness and ability to herd and crowd into a bubble. We find little evidence that funding availability contributes to bubble activity. However, we find that bubble activity is greater during times when the returns earned from currency risk factor portfolios are depressed. This supports the argument that herding behaviour is greater when economic agents face unattractive return alternatives. Furthermore, results suggest that currency bubbles may directly affect the returns of these investment strategies, specifically in the case of carry trade.
Acknowledgements

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CHAPTER 1

Introduction

1.1 Motivation and Background

The topic of bubbles in financial markets has often led to heated debate in academics, the media, and industry. The financial crisis, ensuing economic stagnation, and monetary policy response has rekindled interest in the subject. Today, questions are arising regarding the prudence of easy monetary policy around the globe and the likelihood that expansive asset purchasing schemes and easy credit will culminate in giant asset bubbles. However economic analysis of the topic is fraught with difficulty. For every economic model that identifies a bubble type situation, another dismisses it based on fundamental explanations or alternative model specifications. Superficially, bubbles are often seen as an upward acceleration of price above fundamentals however an exact definition of fundamental value is often illusive.

The Johansen-Ledoit-Sornette Log-Periodic Power Law model offers a potential solution to this difficulty as it does not rely on challenging definitions of fundamental asset price value. The model defines a bubble as a period of faster than exponential growth resulting from positive feedbacks. The growth accelerates towards a critical point and is ultimately unsustainable which must result in a changing market dynamic. Indeed, the log-periodic power law framework provides a means to detect bubbles in real time and predict changes in the current market regime, be it a crash or a smooth plateauing.

From the first discovery of log-periodic patterns in financial markets in 1996 the model has been applied extensively to equity, real estate, and commodity markets among others. However, to date, currency markets have received limited coverage in the literature. Therefore the goal of this thesis project is to explore the applicability of the log-periodic power law bubble model to currency markets. Such a bubble alarm would be valuable to policy makers monitoring the value of their domestic currency and investment managers managing currency exposure among others.

A number of questions arise concerning the application of the model to these markets: should the model be calibrated differently on currencies than on other assets such as equities, real estate, or commodities; how do we distinguish bubbles in the base currencies from negative bubbles in the quote currencies in the case of currency pairs; is the model more suitably applied to exchange rate indices, commonly used by central banks and others to monitor the value of a currency; is the model actually relevant to currency markets where central banks have mandates to intervene and dampen currency overvaluation and overshooting? Furthermore we wish to explore those economic regimes which may affect agents’ willingness and ability to herd and crowd into a currency bubble. In this regard we consider funding availability and the alternative
return opportunities available to investors in the currency markets.

1.2 Chapter Overview

The thesis is organised as follows. Chapter 2 provides an overview of the foreign exchange market including participants and their objectives. The chapter also reviews some of the major academic models which attempt to explain exchange rate movements with consideration given to recent work on liquidity spirals and risk based explanations for carry trade and currency excess returns. Chapter 3 introduces the log-periodic power law model and gives an overview of research to date. We explore and illustrate the model in detail and motivate its application to the foreign exchange markets. The data and research methodology is outlined in Chapter 4. In our research we attempt to follow the recommendations from proponents of the model and avoid the pitfalls highlighted by the critics. We review the results in Chapter 5 including alarm calibration on currency markets, alarm predictive performance, and robustness. We also explore novel ways to distinguish bubbles in the base currency from negative bubbles in the quote currency. By reviewing news articles from the financial media we provide an economic backdrop to some of the more prominent bubble alarms. Macroeconomic bubble drivers are examined with consideration given to funding availability and currency risk factor portfolios. We conclude in Chapter 6 and suggest future research opportunities.
CHAPTER 2

The Foreign Exchange Markets

2.1 Introduction to the FX markets

The original economic agents in need of the foreign exchange markets were international traders and merchants. As most countries had their own currencies, international trade and the buying and selling of goods required exchange of currencies. Currency market makers or “dealers” naturally emerged in order to ease the search process involved in finding a suitable counter-party to trade currencies with. They held an inventory of currencies and stood ready to meet traders’ FX needs when called upon. Exchange rates fluctuated and by holding currency these dealers faced risk of loss in cases when exchange rates adversely moved and reduced the value of their portfolios. They therefore demanded compensation from their clients in the form of a bid-ask spread - they bought currency at prices lower than they demanded to sell them. While dealers were not required to quote bids and asks for currencies, failing to do so, especially when clients were most in need of liquidity, would result in reputational damage and loss of market share. In this setting, the traders who willingly buy currency at the ask price and sell at the bid price are liquidity takers while the dealers are known as liquidity makers, deemed to be providing liquidity to the market. This market structure, with direct contact between participants and without an organised exchange on which participants are matched, is known as an OTC or over-the-counter market. Although the modern FX market has evolved with technology it still largely retains this OTC characteristic. For an introductory text on the evolution of FX markets, please see King et al. (2011).

The complete history of the international FX market is expansive however today’s market has its direct roots in the system that emerged after the collapse of the Bretton Woods system. The Bretton Woods agreement of 1944/1945 recognised the failures in monetary and economic policy which contributed to the outbreak of war and it attempted to craft a better global foreign exchange system. The accord established a gold based value for the U.S. dollar and the British pound with other currencies linked to the value of the dollar. In this way, all currencies had a fixed value measured in gold. Currency exchange rates could only fluctuate in narrow 1% bands thereby forcing prudent monetary policy on member states and preventing competitive currency devaluations. Arguably the need to hedge and the opportunity to profit from currency movements were limited. With the collapse of the Bretton woods system in 1971, nations adopted a host of different currency arrangements with many moving toward more freely floating exchange rates. In many cases, countries gave their central banks mandates to intervene in markets by buying and selling currencies. In such systems currencies exchange rates and returns are driven by the collective force of all market participants.
In the early post Bretton Woods decades, the bulk of currency trading was concentrated in the inter-dealer market, a market of global international banks. The end users who demanded FX liquidity were corporations, governments, central banks, and non-dealer financial institutions. As technology progressively lowered the barriers to accessing the markets, retail clients, hedge funds, and high frequency trading firms have entered the market. Market microstructure research suggests that the price discovery process in asset markets involves two distinct categories of market participants: liquidity takers and liquidity makers. Therefore, in order to gain a complete picture of the market we first dissect global FX turnover by dividing market players into liquidity takers and makers. We discuss their characteristics and objectives alongside the strategies they employ when trading in the FX market.

2.2 Market Participants

2.2.1 Breakdown by market share

Unlike equity and bond markets, the FX market is largely unregulated and until recently this was not deemed to be a major point of concern for regulators or market participants. With little regulation there is minimal reporting from market participants and limited data available on the aggregate market. A key source on market activity is the triennial market survey released by the Bank for International Settlements (BIS, 2013). Figure 2.1 presents a breakdown of market turnover by counterparty based on the survey as of 2013. We then discuss these counterparties individually starting with the liquidity takers.

Figure 2.1: Foreign Exchange Turnover Market Shares - 2013

![Turnover by counterparty](image1)

![Breakdown of "Other Financial Institutions"](image2)

Figure 2.1 2013 FX Market turnover is broken down into counterparty shares. "Other Financial Institutions" is further dissected into its constituent components. Datasource: BIS (2013)

2.2.2 Liquidity Takers

Institutional investors

This category includes pension funds, mutual funds, endowments, and insurance companies. They are commonly referred to as real money investors as their motivation for trading currencies is driven by the need to manage their real asset (e.g. equity and bond) investment portfolios. The money they manage is usually denominated in a currency chosen by their clients e.g. Euros, and in order to construct a diverse portfolio they buy foreign assets e.g. US government

\[1\] As of November 2014, US, UK, and Swiss regulators have fined investment banks $4.3bn in response to a global FOREX manipulation ring. Source: FT.com
bonds, which requires them first to change Euros to Dollars. Currency movements are estimated
to account for 65% of the volatility of non-domestic bonds and 40% of the volatility of non-
domestic equities (James, 2013) and although investment managers have not always concerned
themselves deeply with the exchange rate component of returns when choosing international
investments (Taylor and Farstrup, 2006), currency risk has become increasingly important to
investment managers since the 2008 financial crisis (Melvin and Prins, 2010). Indeed, the
management of the currency component of these asset portfolios is increasingly delegated to a
separate overlay manager (Pojarliev and Levich, 2012) who may passively hedge risk according
to a predefined hedge ratio (perhaps a 50% ratio which minimises embarrassment) or actively
deviate from the benchmark hedge ratio by underweighting and overweighting currencies in
order to provide additional returns and diversification. Rime and Schrimpf (2013) argue that
increasingly international portfolios, currency overlay, and the recognition of FX as separate
asset class may have been among the key drivers of the growth in FX trading in the last decade.

Hedge Funds & Proprietary Trading Firms
Hedge fund is a name given to a special class of investment vehicle which follow exotic investment
strategies and to which access is generally restricted for the general public given the potential
risk and complexity involved. Those whose strategies involve currencies include global macro
funds and commodity trading advisors (CTAs) among others. They may use both derivatives
and large amounts of leverage when taking speculative positions in different currencies. As
an investment class, hedge funds have grown dramatically since the 1990s and they generally
create their strategies based on a number of different factors: fundamentals, interest differentials,
momentum, and volatility (King et al., 2011). Proprietary Trading firms follow similar
strategies however they generally do not accept money from external clients. Included in the
category are High Frequency Trading firms (HFTs) who first emerged in the early 2000s as the
currency trading infrastructure became ever more computerised. Their strategies are purely al-
gorithmic meaning computer programs execute trades and humans simply monitor the trading
systems and develop the computer code. They can trade thousands of times per second and
the success of their strategies is largely dependent on their speed. These strategies include clas-
sical arbitrage such as covered interest rate arbitrage or triangular arbitrage among currencies.
Additional strategies are closely tailored to the market microstructure of the currency markets
and include latency arbitrage, which exploits speed difference among market participants, or
liquidity redistribution, which detects imbalances in the order book or across different trading
platforms (BIS, 2011). In the case of liquidity redistribution, some HFTs can be seen to provide
liquidity to the market.

Corporations (within “Non-Financial Customers”)
Through their corporate treasury function, large global businesses access the FX markets to
facilitate those business activities which span multiple geographies and currencies including in-
ternational trade, mining, shipping, and manufacturing. They generally do not enter the market
to speculate in currency movements and aside from directly managing daily currency needs, they
may engage in hedging to limit their exposure to unfavourable currency fluctuations. Bodnar
et. al (1998) survey such corporates reliant on hedging and find that they typically have hedge
ratios in the 40-50% range with maturities under 6 months. As these large corporates increas-
ingly centralize their corporate treasury functions, they are internalizing their currency trading
and hedging, relying less on the markets (Rime and Schrimpf, 2013).

Central Banks (within “Official Sector”)
A central bank manages a state’s currency, money supply, and interest rates. The role of a cen-
central bank as it relates to exchange rates is dependent on the exchange rate regime of the state. Broadly defined, a state’s exchange rate regime may be fixed, floating, or intermediate. Under a fixed regime, the state’s currency is pegged against another currency or a basket of currencies and the role of the central bank is passive - the central bank will automatically clear any excess demand or supply for the currency. If demand for domestic currency increases, the central bank will sell additional units in exchange for foreign currency. If supply for the domestic currency increases, the central bank automatically buys the increase using its reserves of foreign currency. In this way the bank ensures that the domestic currency does not appreciate or depreciate beyond acceptable thresholds. Under a floating regime, the currency is free to fluctuate as the market dictates. While many countries have moved towards floating exchange rates following the collapse of the Bretton Woods system, they maintain the discretion to intervene in the foreign exchange markets to fulfill a number of objectives:

1. **Minimise Overshooting**: correcting excessive strengthening or weakening of the home currency when the market has over or under-reacted to new economic information

2. **Reduce Volatility**: excessive volatility can impact foreign direct investment into a country, international trade, domestic financial markets, or general market sentiment, which can all potentially be detrimental to the domestic economy

3. **Leaning-against-the-wind**: short term trends in the exchange rates may be deemed unfavourable by the central bank and therefore resisted

4. **Misalignment Correction**: the exchange rate may unfavourably alter economic fundamentals of the economy e.g. inflation projections, requiring suitable central bank intervention

An example of such market intervention was the ceiling set on the value of the Swiss Franc by Swiss National Bank in August 2011. After sustained strengthening of the CHF, the SNB stated that “it will no longer tolerate a EUR/CHF exchange rate below the minimum rate of CHF 1.20” (SNB, 2011). The Swiss Franc rose almost 30% in value when the ceiling was removed in January 2015.

**Retail Clients (within “Others”)**
Prior to the technological revolution of the late 1990s, retail investors incurred prohibitively high transaction costs to trade currencies. Retail aggregators (discussed below) opened the markets to small private traders and their trading now accounts for 3.5% of total turnover (BIS, 2013). Other market innovations like Deutsche Bank’s “G10 Currency Harvest” exchange traded fund has opened popular carry trade strategies to retail investors. Research suggests that retail traders are not informed and their trades do not anticipate exchange rate returns (Nolte and Nolte, 2009) and that retail traders tend to be unprofitable (Heimer et. al, 2011).

**2.2.3 Liquidity Makers**

**Reporting Dealers**
FX dealers earn income by taking speculative positions on their own account and providing liquidity to customers i.e. trading with customers when they arrive at the market. In the early 1970s following the cessation of Bretton Woods, FX trading took place over-the-counter via telephone with the bulk of trading occurring on the interdealer market. On it, large international banks traded with each other directly or anonymously through voice brokers. These brokers voiced the best bid-ask prices over multi-party phone lines known as squawk boxes (King et al., 2011). Interdealer trading exceeded 60% of trading and the market was opaque and difficult to access for smaller banks, investment firms, or retail clients.
Electronic trading began transforming the interdealer market beginning with Thomson Reuters Dealing in 1987 and Electronic Broking Service in 1993 - known as multi-bank platforms. These electronic brokers allowed dealers to trade anonymously and electronically and dominated the interdealer market for liquid currencies by the late 1990s. Voice brokers remained important for less liquid currencies.

In response to competition from independent ECNs (discussed below), FX dealing banks began to offer trading services to clients via electronic portals (single-bank platforms or proprietary trading platforms) and prime brokerage arrangements (BIS, 2011). Single-bank trading systems such as UBS’s FX Trader, Barclays’ BARX, and Deutsche Bank’s Autobahn are proprietary forex trading platforms owned by dealer banks giving clients access to liquidity and trading counterparties. Through prime brokerage relationships, dealer banks offer institutional clients like hedge funds and HFTs credit and access to the interdealer market. Today many types of clients can participate in the over-the-counter FX market on a more or less equal footing as the large FX dealing banks (King et al., 2011).

Global Custodians (within “Non-reporting banks”)
Large asset managers typically hire administrators or custodians who track their assets, calculate portfolio values, process dividends, and settle trades among other tasks. When they need to trade foreign currencies, real-money investors in some cases do not trade with the major dealer banks or on electronic platforms. Instead, they trade with their custodian, motivated largely by administrative efficiency (DuCharme, 2007).

Retail Aggregators (within “Others”)
Retail aggregators like OANDA and FXCM expanded access to the market to retail traders in the 2000s offering online margin accounts which streamed prices from Electronic Broking Service (EBS) and major banks. They bundle retail trades before offloading them to the interbank market thereby lowering trading costs for retail clients.

Electronic Communication Networks and Multi-bank platforms
Electronic trading improved efficiency and price transparency while the elimination of price discrimination in the interbank market reduced trading costs and bid ask spreads. However end customers did not begin reaping benefits from electronic trading until the turn of the millennium. At this point, independent non-bank firms entered the market and launched electronic communication networks (ECNs) which allowed customers to trade electronically with a range of dealers.

Exchanges
The Chicago Mercantile Exchange (CME) is a major player in foreign exchange markets holding a virtual monopoly on FX futures trading. While many electronic platforms discussed above offer limit order book features, the CME is the only official exchange playing a major role in FX markets.

2.2.4 Market Structure
By pooling together the liquidity takers and liquidity makers we can map the relationships that tie together all the market participants. Figure 2.2 identifies the market players within today’s foreign exchange market and highlights the new electronic platforms which have emerged in recent years.

High frequency trading firms and hedge funds typically access the inter-dealer market
Figure 2.2 By considering all market participants we can visualise the network which defines their relationships. Magenta denotes liquidity taker, blue liquidity maker, and green electronic communication platform. Graphic adapted from BIS (2011).

Through prime brokerage relationships with the large dealer banks like Morgan Stanley or J.P. Morgan, though multi-bank platforms like Reuters and EBS offer direct access through their infrastructure. They may also trade directly on the CME or ECNs like Currenex. Financial institutions and asset managers may settle currency trades with their custodian asset manager e.g. Blackrock or State Street, or go to the interdealer market through a dealer bank. Corporates and high net worth retail clients may access markets through a single bank platform like Citi Velocity or Barclays Barx, though many small retail clients trading online might opt for a retail aggregator like OANDA. FX dealers like to foster strong relationships with central banks and thereby ensure they receive market intervention trades directly.

Ultimately the interdealer market lies at the core of the FX market, but multi-bank platforms like EBS and Reuters, and electronic communication networks like Hotspot FX, FXall, FX Connect and Currenex play a major role facilitating electronic trade in this segment and open access beyond the major dealer banks. In 2013 electronic execution accounted for above 50% of all trade by volume, with voice execution making up the residual. Algorithmic non-human execution accounted up to 25% of all spot trade (BIS, 2013).

2.2.5 Currencies Traded

The US Dollar is the dominant currency and is involved in one side of over 80% of spot transactions. This reflects the position of the US Dollar as an international reserve currency which arose after the second world war. It has led to the perception that the US benefits from exorbitant privilege, with proponents of the idea arguing that the US is less prone to balance of payment difficulties and can borrow at lower costs in international markets (Eichengreen, 2012). In the currency market it is common to trade minor currencies via a major currency known as a vehicle currency and for many cross rates the US Dollar acts as the vehicle currency. For example, a trade from Brazilian Real to Mexican Peso would typically involve two separate trades - the first to convert the Real to US Dollars and the second to convert Dollars to Pesos. This concentrates
market liquidity into fewer currency pairs reducing transaction costs. However this practice may be less common among retail traders who frequently trade relatively illiquid currency pairs directly rather than via a vehicle currency (Rime and Schrimpf, 2013). In the Eurozone the Euro frequently acts as the vehicle currency and is involved in over one third of all spot trades. Together with the Japanese Yen and the British Pound these four currencies comprise the G4 currencies or the “majors”. In 2013, trade exclusively involving these currencies e.g. EUR/JPY, accounted for over 55% of all spot turnover.

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Table 1: Share of total spot turnover by currency (%) (BIS, 2013)

Below the majors, the commodity currencies of Australia, Canada, Norway, and New Zealand are the next largest group. Commodities represent a large share of exports from these countries and global demand for different natural resources like iron ore from Australia or hydrocarbons from Canada is seen to have a impact on their currencies. From 2001 the collective market share of these currencies has grown over 50%. Emerging markets currencies have been grouped into the BRICs and the MINTs, terms coined by industry experts which identify those countries likely to grow in dominance in the world economy. Both groups have seen trade in their currencies expand over 5x since 2001.

When trading a currency pair on the market, most exchange rates are quoted as the units of a currency required to purchase one US dollar. For example, the market for Mexiccan Pesos and US dollars is presented as USD/MXN where the dollar is known as the base currency and the peso is the quote currency. The exceptions are the EUR, GBP, AUD, and NZD which are given as the base currency and the dollar is the quote currency. In the news and on trading desks many of the heavily traded currency pairs have also received nicknames such as Cable for GBP/USD, Aussie for AUD/USD, Kiwi for NZD/USD, and Swissie for USD/CHF.

2.2.6 Instruments Traded

Global FX turnover grew 35% from 2010 to 2013 with spot turnover accounting for 37% of turnover in 2013, or $2tn per day. Turnover in outright forwards totaled $680bn or 12% of activity. An expansive range of derivatives and option contracts account for 7% of daily turnover. Foreign exchange swaps represent the largest component of daily turnover at $2.2tn (44%) however they are used primarily for overnight position management by banks and are generally seen to have little influence on the exchange rates (King et al., 2011). In total, the daily

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2BRICs currencies - Brazilian Real, Russian Ruble, Indian Rupee, Chinese Yuan, MINTs currencies- Mexican Peso, Indonesian Rupiah, Turkish Lire, Nigerian Naira
turnover in the foreign exchange market swells to $5.2tn per day (BIS, 2013). This compares
with a yearly turnover in global equity markets of $60tn as of 2013 (WFE, 2014).

2.3 Explaining the Exchange Rate

2.3.1 Purchasing Power Parity

One of the oldest concepts in long run exchange rate determination is known as Purchasing
Power Parity (PPP) and may have emerged as early as the 16th century (Rogoff, 1996). The
concept is based on the Law of One Price which states that a good must sell for the same price
regardless of location. In an international context where countries all have their own currencies,
a good should have the same price when expressed in the domestic currency of the home agent
regardless of its location, assuming no market frictions prevent goods arbitrage and goods are
perfect substitutes for one another. Absolute PPP posits that, in equilibrium, the exchange
rate between two countries will be identical to the ratio of the two countries’ price levels. It is
written as

\[ P_{d,i,t} = S_t P_{f,i,t} \quad i = 1, 2, ..., N \quad (2.1) \]

where \( P_{d,i,t} \) and \( P_{f,i,t} \) are the prices of good \( i \) in the domestic and foreign country respectively at
time \( t \) and \( S_t \) is the nominal exchange rate, being the price of one unit of foreign currency in
domestic currency at time \( t \). \( S^* \) is the PPP exchange rate that equates the purchasing power
of one unit of currency in the two different countries. Relative Purchasing Power Parity is a
weaker condition which says that the exchange rate will change to compensate for the inflation
differential between the two countries. If the domestic economy has a higher rate of inflation,
the price of a basket of goods there will rise and therefore its currency will depreciate versus
the foreign currency. The conditional can be written as

\[ \frac{P_{f,i,t+1} S_{t+1}}{P_{d,i,t+1}} = \frac{P_{f,i,t} S_t}{P_{d,i,t}} \quad i = 1, 2, ..., N \quad (2.2) \]

Influential papers such as Krugman (1978) and Frenkel (1978) wholly rejected both the absolute
and relative PPP as a short term relationship. However, later papers from Taylor (1988) and
Taylor and McMahon (1988) argued that the relationships may hold over longer horizons.
Rogoff (1996) suggests that the consensus among economists is that PPP deviations have a
half-life of three to five years. Thus while it may not be deemed a useful indicator in the short
run, at a horizon of several years or decades it may be valid.

2.3.2 Market Efficiency

In an efficient market, prices should reflect all the information available to the market partici-
pants and no agent should be able to earn excess returns from speculation (Sarno and Taylor,
2002). Academic literature presents three forms of market efficiency (Sarno and Taylor, 2002;
Fama, 1970).

1. Weak form: The current price of an asset reflects all the information contained in historical
   prices
2. Semi-strong form: The current price reflects not only historical prices, but also all publicly
   available information
3. Strong form: The price reflects all available information, both public and private (not
   released to the market) information.
Testing the efficiency of foreign exchange markets involves either uncovered interest rate parity (UIP) or covered interest rate parity (CIP). UIP says that, “if the risk-neutral efficient market hypothesis holds, then the expected exchange rate change - must just be offset by the opportunity cost of holding this currency rather than the other - the interest rate differential” (Sarno and Taylor, 2002). These parity conditions are no-arbitrage profit conditions for capital. Assuming a forward market exists, an investor can receive an interest rate \( i_d \) in the domestic market or convert currency at the exchange rate \( S \) and receive an interest rate \( i_f \) in the foreign market, then convert back to the home currency at time \( t + 1 \) using the forward rate \( F \) locked in at time \( t \). With infinite amounts of capital flowing between countries in search of the highest return, disregarding potential risk, the following covered interest rate parity condition is reached

\[
\frac{i_d - i_f}{1 + i_t} = \frac{F_{t,t+1} - S_t}{S_t}
\]

(2.3)

It is covered in the sense that investors are not subject to exchange rate fluctuations as they have secured the forward rate at the point of initiation. If the forward rate is equal to the future spot rate we arrive at uncovered interest rate parity

\[
\frac{i_d - i_f}{1 + i_t} = \frac{S'_{t,t+1} - S_t}{S_t}
\]

(2.4)

where \( S'_{t,t+1} \) denotes the expected spot rates at time \( t + 1 \) at the current time \( t \).

If foreign exchange market were efficient, forecasting the exchange rates would be possible by applying either the CIP or UIP. However, numerous authors such as Hansen and Hodrick (1980), Fama (1984), Engel (1996), and Chaboud and Wright (2003) have found violations in these conditions. The forward rates do not fully forecast future spot rates and high interest paying currencies tend to appreciate and low interest rate currencies tend to depreciate. The latter is known as the “Forward Premium Puzzle” and a large body of research has attempted to offer an explanation for why the phenomenon persists. Its existence has led to a currency investment strategy known as carry trade. This consists of borrowing in low interest rate currencies and investing in high interest rate currencies (Burnside et. al , 2006).

### 2.3.3 Structural Monetary Models

Following the end of the Bretton Woods system, monetary models emerged as the dominant exchange rate determination models during the 1970s. These models recognised the role that central banks and central bank policy could play in determining exchange rates and specifically consider the influence that supply and demand for money has on bilateral exchange rates. In all monetary models, the money supply and any variable that affects money demand, e.g. economic output, affects exchange rates (Barnett et. al , 2005). In the Flexible-price monetary model of Frenkel (1976), the exchange rate between two countries is based on their relative money supplies, incomes, and interest rates. Dornbusch (1976) introduces the Sticky-Price Monetary model which accounts for the failing of PPP by assuming prices in an economy are rigid in the short run. This is known as the classical exchange rate over-shooting model as monetary policy changes can cause the exchange rate to overshoot its equilibrium value temporarily. A third structural model from Hooper and Morton (1982) also incorporates the trade balance of countries when determining the exchange rates.

The seminal paper of Meese and Rogoff (1983) tested the out-of-sample performance of the Frenkel, Dornbusch, and Hooper-Morton models and concluded that none of the models performed as well as a random walk model at horizons of one to twelve months. Indeed, their
conclusion has remained largely undisputed. Rogoff (1996) found that currency fluctuations are large and volatile and difficult to explain even on an *ex post* basis. Clarida et. al (2001) said that, “from the early 1980s onward, exchange rate forecasting in general became increasingly seen as a hazardous occupation, and this remains largely the case”. Generally a consensus emerged that the standard models which relate exchange rates to economic fundamentals like inflation, output, or monetary supplies have difficulty predicting exchange rates or explaining past movements.

### 2.3.4 Summary of Exchange Rate Determinants

An exhaustive discussion of the commonly cited drivers of the FX markets is beyond the scope of this thesis, however we present the numerous factors assembled by Rosenberg (2003) as a summary. Determinants have been divided into short run, medium run, and long run forces. Technical factors such as investor positioning and order flow are seen to be short term drivers of exchange rate returns whereas macroeconomic factors such as purchasing power parity are seen to exert an influence over the long run.

**Figure 2.3:** Exchange Rate Determinants

![Figure 2.3](above.png)

**Figure 2.3** Above we illustrate a broad range of drivers that are commonly argued to affect and drive the currency markets. The influencing factors have been organised into short, medium, and long run drivers. Graphic adapted from Rosenberg (2003).

### 2.3.5 Recent Advances

While the conclusions of Meese and Rogoff (1983) have held up well over time, a number of advances have been made by considering risk-based approaches and funding availability:

**Risk-based approaches**

Recent research has adopted a risk-based approach to understanding carry trade returns and currency excess returns. Grounded in the arbitrage pricing theory of Ross (1976), these models argue that variations in the cross section of returns of exchange rates is driven by different exposures to a small number of risk factors. Lustig et. al (2011) identify a carry trade risk factor which they suggest captures global risk. This factor accounts for most of the cross sectional variation in average excess returns between high and low interest rate currencies. A market risk factor or Dollar risk factor captures the excess returns available to U.S. investors who go
long foreign currencies. Mancini, Ranaldo, and Wrampelmeyer (2012) introduce a liquidity risk factor which contributes to global risk. Investors likely suffer higher costs during long and illiquid environments and consequently require a premium for this risk. High interest rate currencies offer positive exposure to this risk while low interest rate currencies provide a hedge. Menkhoff et. al (2012) similarly identify a volatility risk factor. High interest rate currencies are negatively related to global FX volatility and deliver low returns in times of unexpected high volatility whereas low interest rate currencies provide a hedge.

**Liquidity Spirals and Funding Availability**

Brunnermeier and Pedersen (2008) present a model of liquidity spirals which addresses market liquidity and funding liquidity jointly. Traders provide market liquidity for which they earn a premium and the ability to do so is dependent on funding availability and *vice versa.* Securities have positive returns and negative skewness which is driven by an asymmetric response to fundamental shocks. Unlike positive shocks, negative shocks lead to funding difficulties for leveraged investors which decreases market liquidity further depressing prices resulting in a downward spiral. In the context of currency markets, Brunnermeier et. al (2008) shows that sudden currency crashes unrelated to economic news can be due to unwinding of carry trades when leveraged speculators face funding constraints. As margins and capital requirements are raised and limited funding squeezes market participants, liquidity dries up, market volatility increases, and there is a flight to safe haven currencies. This may offer an explanation for movements related to carry trade and the violent unwinding of such positions.

### 2.4 Summary

This chapter offers a broad introduction to foreign exchange markets and the factors which affect exchange rates. Market participants are divided into liquidity takers and liquidity makers depending on whether they demand from or supply liquidity to the markets. Objectives and strategies are discussed and a network of relationships which binds participants together is presented. We also introduce some of the major academic models which attempt to explain exchange rate movements and highlight some of the difficulties experienced within empirical research. Recent innovations in the literature may offer new insight into market drivers and a brief overview is provided. In chapter 3, we discuss the Log-Periodic Power Law model in detail and motivate its application to FX markets.
CHAPTER 3

Log-Periodic Power Law Model

3.1 Introduction

Financial crises cripple economies, lead to social unrest, and put a stopper on economic development and progress. One need only study the recent global recession to identify the discontent and disruption following such unfavourable episodes. Many economic commentators have proclaimed the years following 2007 as a lost decade, where economies worldwide have moved in reverse and have not yet fully recovered their previous vitality. While academic debate on the subject continues, there are many studies which adopt a view that the recent financial crises and many others were the culmination of asset bubbles and overly exuberant investor behaviour. Indeed Kindleberger (2000) and Galbraith (1994), present arguments that such periods of mania have frequently been seen throughout history. This has lead academics to study the precursors to such crises and policy makers to search for the tools which can diagnose bubbles early in order to take appropriate counter measures. Proper identification of bubble periods is however extremely difficult and periods of rapid growth in asset prices can often be justified by changing or improving fundamentals. While superficially bubbles can be defined as an upward acceleration of price above fundamental value, a precise definition of fundamental value remains largely illusive. The Johansen-Ledoit-Sornette Log-Periodic Power Law model, hereafter LPPL, presents a solution to this difficulty by defining a bubble as a period of faster than exponential growth resulting from positive feedbacks. It therefore accounts for periods when fundamentals are arguably leading to exponential asset price growth. The super-exponential growth accelerates towards a critical point and is ultimately unsustainable which must result in a changing market dynamic. The LPPL framework provides a means to detect bubbles in real time and predict changes in the current market regime, be it a crash or a smooth plateauing, and therefore is of interest to academics, market participants, and policy makers alike. To date, research into the model has primarily focused on equity markets while applications to foreign exchange markets have been sparse. Indeed the applicability of the model to these markets is yet to be determined. Given the challenges of forecasting in foreign exchange markets, as detailed in Section 2.3, the LPPL model may offer an innovative method to predict currency movements. Furthermore, the model’s bubble diagnostic properties could likely aid policy makers and central bankers who at times intervene in the markets when their currencies overshoot reasonable values. In the following sections we provide a more detailed literature review of the LPPL model, describe and illustrate the model and fitting procedure, and motivate its application to foreign exchange markets.
3.2 Literature Review

3.2.1 LPPL background

At its core, the LPPL framework argues that there are well defined patterns that exist in asset prices before critical points, also known as phase transitions, by analogy to extreme events in physical systems. As the market approaches this point, imitation among traders begins to dominate the investment decision and positive feedback results in a faster than exponential growth rate where the growth rate itself increases over time (Johansen et. al., 2000). This upward trend is decorated with log-periodic oscillations which reflects the competition between higher growth expectations and negative feedback spirals of crash expectations. As the critical point approaches the peaks and valleys of these oscillations decrease in amplitude and increase in frequency. Closer to this point the system becomes more sensitive to external influences and even a small disturbance in the market may trigger a crash or regime change (Johansen et. al., 1999).

Section 3.6 provides a visual illustration of these features using the US Dollar as a case study.

The Log-periodic structures in financial markets were independently discovered by Sornette et. al (1996) and Feigenbaum and Freund (1996) while studying the October 1987 US stock market crash, however heated debate arose over the economic mechanisms underlying the patterns (Feigenbaum, 2001; Sornette and Johansen, 2001). Recent literature continues to investigate if these patterns are simply the result of random fluctuations in the markets (Wosnitza and Leker, 2014). A number of papers have identified *ex post* the LPPL structures prior to crashes in a range of markets including equities (Drozdz et. al., 1999; Johansen et. al., 2000). Many papers have also identified crashes in market *ex ante* including real estate (Zhou and Sornette, 2003), and oil (Sornette et. al, 2009). Indeed the *Financial Bubble Experiment*, launched within the Financial Crisis Observatory at ETH Zurich, published advanced diagnostics and forecasts of bubble terminations across numerous markets (Sornette et. al, 2010; Sornette, Woodard et. al, 2010; Woodard et. al, 2011). Further evidence of LPPL crash precursors has been found in corporate bond spreads by Clark (2004) and credit default swaps by Wosnitza and Denz (2013). Application of the model was expanded to *anti-bubbles*, which are characterised by decelerating devaluations in the market over time. Johansen and Sornette (1999) successfully identify *anti-bubbles* in gold and equity markets with further evidence in stock markets presented by Sornette and Zhou (2002). A newer stand of literature focuses on forecasting the termination of bubbles and establishing the effectiveness of bubble alarms and trading strategies based on the model. Sornette and Zhou (2006) calibrate the LPPL model on the Dow Jones Industrial average index and create an early bubble alarm indicator. Yan et. al (2010) calibrate the model on the S&P500 index and similarly evaluate the performance of an early warning system based on the model. Later work by Yan et. al (2012) extend this alarm system to a trading model and evaluate performance across 10 different equity markets with attractive performance. In their work the concept of *negative bubbles* was also introduced, characterised by accelerating price falls. The most recent work on the topic by Vakhtina and Wosnitza (2015) tests an early warning system for bubbles in credit default swap markets. While the statistical significance of the precursors and their predictive power remains controversial (Chang and Feigenbaum, 2006) with a strong rebuttal from Lin et. al (2014), an increasing number of case studies in a wide variety of markets continue to reinforce the LPPL hypothesis.

The LPPL model has been applied extensively to equity markets yet the literature covering foreign exchange markets is limited in comparison. Johansen and Sornette (1999) study the USD/CHF and USD/DM (Deutchemark) exchange rate in the 1985 episode. Matsushita et. al. (2006) fit the model to the BRL/USD exchange rate from 2000 to 2005 identifying bubble

3.2.2 Academic Literature on Bubbles

In academic literature there remains little consensus on a common bubble definition or in fact on whether bubbles can actually exist in the markets. Gurkaynak (2005) surveys a large range of econometric approaches which test for the existence of bubbles and concludes that for every paper finding evidence of bubbles in asset prices, another exists which disputes the claim. Practitioners find the topic equally challenging with Alan Greenspan, former chairman of the U.S. Federal reserve, stating in 2002, following the dot-com bubble: “...We ... recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence” (Greenspan, 2002). As noted above, an attractive feature of the LPPL framework is the clear definition of a bubble period which does not rely on challenging definitions of fundamental asset price value.

The early literature on bubbles was based on the rational bubbles view where agents were assumed to be rational and perfectly informed and yet bubbles could exist. Empirical tests (Blanchard and Watson, 1982) had limited success in identifying bubbles prior to large price falls. Later research explored how incentives, market frictions, and non-standard preferences could contribute to bubble formation and prolongation. In this literature, herding can have a large effect on both individual investors and institutional fund managers. Investor whose utility depends on their relative wealth rather than its absolute level may prefer to herd in order to “keep up with the Joneses” (DeMarzo et. al , 2008). Fund managers may lack the time and resources to fully investigate an investment so copying other managers is preferable in some cases (Shiller, 2002). They may be compensated for relative rather than absolute performance and so are happy to follow the crowd (Lux, 1995). They may be forced to allocate investments to assets favoured by their clients who choose to invest in funds according the fund’s holdings - the so called “dumb money effect” (Lamont and Frazzini, 2008). Allen and Gorton (1993) argue that the limited liability of fund managers affects their willingness to ride bubbles. Unskilled managers hope to exit before the crash but even if they fail their downside is limited. Behavioural models relax the assumption of perfect rationality and a number of models consider positive feedback and momentum trading. Kaizoji (2010) demonstrates that crashes originate primarily from herding behaviour of noise traders who in turn are followed by momentum investors. Indeed, (Shiller, 2002) argues that media attention amplifies this feedback trading tendency. As more investors become interested in an asset, news coverage expands, attracting the attention of more investors.

A number of studies have considered bubbles in the context of foreign exchange markets. Jarrow and Protter (2010) argue that foreign exchange bubbles can arise from central bank intervention which causes a currency to deviate from its fundamental value or from traders acting competitively and in a concerted fashion. They observe that unlike asset price bubbles in financial securities and commodities, bubbles in currencies can be both positive and negative given that a pair of currencies is involved in each cross exchange rate. Several papers have undertaken an empirical approach with a focus on the USD peak of 1985. Evans (1986) defines a speculative bubble as a period of nonzero median excess returns and finds evidence of a negative bubble in the excess returns of holding GBP prior to the 1985 peak. Bettendorf and Chen (2013) likewise study the GBP/USD exchange rate but cannot reject the hypothesis that
the explosion in the exchange rate was driven by explosions in the economic fundamentals. Woo (1987) applies a portfolio balance model and finds evidence of a bubble whereas Wu (1995) uses a Kalman filter when testing for rational bubbles but finds no evidence of a bubble.

3.3 Motivation for research in foreign exchange markets

Given the current state of research, further exploration into the LPPL model’s application to currency markets can be motivated on a number of points:

1. Research into the LPPL model to date has been limited in terms of the breadth of currencies studied and the time period spanned. In a universe with potentially thousands of cross rates, in total only 6 cross rates have received attention with limited application to the recent past. Furthermore, the LPPL research has yet to expand from simply identifying log-periodic structures prior to pre-identified currency crashes to creating bubble alarm indicators as have been developed for equity and credit default swap markets.

2. As noted by Jarrow and Protter (2010), currency bubbles can be positive or negative, however to date there exists no framework to distinguish a bubble in the base currency from a negative bubble in the quote currency. Each exchange rate is comprised of two currencies which can each experience positive or negative bubble periods. Therefore identifying the driving force behind the price action is by no means trivial.

3. Policy makers and market participants often examine the movements of a currency in a currency basket or index which give a broad overview of how a currency is performing. Such baskets define a starting index for a single currency and evolve according to a weighted average of the performance of a number of cross rates of the currency in question, often assigning greater weight to the cross rates of more important trading partners. Questions arise over whether the LPPL model should be calibrated on these basket rates or on tradable cross rates. To date, this has received no attention in the literature.

4. The LPPL bubble definition necessarily depends on positive feedback effects to drive the price acceleration. While overly exuberant investor behaviour has long been studied in the context of asset markets such as equities and real estate, currency markets structure differ in a number of respects. Not only are they vast in size, with numerous market participants and deemed highly liquid, there are not subject to short sale constraints which has been cited as a bubble instigator in other markets. Furthermore, central banks play a considerably larger role in currency markets often with explicit mandates to intervene in the markets to ensure values remain in favourable territory. This may extinguish any bubble before it becomes a pervasive force.

3.4 Deriving the LPPL model

The JLS LPPL model extends the rational bubble model of Blanchard and Watson (1982). The dynamics of the asset price can be described by

$$\frac{dp}{p} = \mu(t)dt + (t)dW + \kappa dj$$

where $p$ is the market price of the asset, $\mu$ is the drift term and $dW$ is the increment of a Wiener process of zero mean and unit variance. The $dj$ accounts for the discontinuous jump in the asset price which is $dj=0$ before a crash and $dj=1$ after. The mean jump size is given by
κ. Of significance is the hazard rate $h(t)$ which determines the dynamics of the jumps. $h(t)dt$ is the probability that the crash occurs between $t$ and $t+dt$ conditioned on the fact is has not occurred yet. This gives $E_t[dj] = 1 \times h(t)dt + 0 \times (1 - h(t)dt)$ and therefore

$$E_t[dj] = h(t)dt$$

In order to describe the hazard rate, Johansen and Sornette (1999) apply the result that a system of variables close to a critical point can be described by a power law and the susceptibility of the system diverges as follows:

$$\chi \approx A(K_c - K)^{-\lambda}$$

where $A$ is a positive constant and $\lambda$ is the critical exponent. However such a model would only account for investors interconnected in a uniform way, while in real markets participants differ in size and interconnectedness. As Johansen and Sornette (1999) argue, models of imitative interactions based on hierarchical lattice structure more accurately capture the nature of real markets. The exponent is given by $\lambda = (\xi - 1)^{-1}$, where $\xi$ is the number of traders in the market network. A typical trader needs to be connected to at least one other hence $2 < \xi < \infty$. This ensure the exponent varies between 0 and 1. These models of imitative interaction exhibit the power law behaviour and predict that the critical exponent $\lambda$ can be a complex number. With a complex exponent, the first order expansion of the system is then given by

$$\chi \approx Re[A_0(K_c - K)^{-\lambda} + A_1(K_c - K)^{-\lambda+i\omega}]$$

$$\approx A_0'(K_c - K)^{-\lambda} + A_1'(K_c - K)^{-\lambda}cos(\omega ln(K_c - K) + \psi)$$

where $A_0$, $A_1$, and $\omega$ are real numbers and $Re[*]$ represents the real part of a complex number. The power law now has oscillations which are periodic in the logarithm of $(K_c - K)$ and $\omega$ in their log-angular frequency. Using this equation and considering a hierarchical structure for the market, the hazard rate behaves as follows

$$h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha}cos(\omega ln(K_c - K) + \psi)$$

where $t$ denotes the time and $t_c$ gives the most probable time of the regime change. Importantly, this is not necessarily a crash and there exists a finite probability that a bubble will land smoothly.

$$1 - \int_{t_0}^{t_c} h(t)dt > 0$$

A crash is not certain in the model which is crucial to ensure that traders remain in the market. In this rational expectation framework, the expected price rise must compensate for risk. Furthermore the price process is a martingale where the conditional expected value of the asset at time $t + 1$, given all previous data up to $t$, is equal to the price at time $t$ as follows

$$E_t[p(t')] = p(t), \text{ for all } t' > t$$

The no arbitrage condition reads $E_t[dp] = 0$. Taking the expectation of the asset price given by equation (3.1) under the filtration until time $t$ reads

$$E_t[dp] = \mu(t)p(t)dt + (t)p(t)E_t[dW] + \kappa p(t)E_t[dj]$$

Given $E_t[dW] = 0$, $E_t[dj] = h(t)dt$, and $E_t[dp] = 0$, this yields

$$\mu(t) = \kappa h(t)$$
Thus the return is determined by the risk of the crash given by the hazard rate $h(t)$. Conditioned on the fact that no crash has occurred, Equation 3.1 can now be written as

$$\frac{dp}{p} = \mu(t)dt + (t)dW = \kappa h(t)dt + (t)dW$$

(3.10)

and its conditional expectation is given by

$$E_t \left[ \frac{dp}{p} \right] = \kappa h(t)dt$$

(3.11)

Substituting expression (3.5) for $h(t)$ and integrating yields the log periodic power law equation

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi)$$

(3.12)

Note, the model does not specify what happens beyond the critical point $t_c$. Here the bubble terminates and there is a transition to a new regime. The critical value $t_c$ tends to overshoot the time of the crash as the parameter only gives the most probably time of the regime change. The probability increases as $t$ approaches $t_c$ however the probability is naturally skewed and we expect $t_c$ to be systematically later than the real time of the crash, consistent with rational expectations. The concept of a Negative bubble discussed in Section 4.3.2.

### 3.5 Fitting Procedure

The seven parameters $t_c, m, \omega, \phi, A, B, C$ given in equation (3.12) are estimated from the asset price series $y_t$ within a given time window $t \in [t_1, t_2]$. They are chosen such that the root mean square error (RMSE) between the data and the LPPL function are minimized. The objective function is given by

$$\min_\theta F(\theta) = \sum_{t=t_1}^{t_n} (y_t - \hat{y}_t)^2$$

$$= \sum_{t=t_1}^{t_n} (y_t - A - B(t_c - t)^m - C\cos(\omega \ln(t_c - t) + \phi))^2,$$

where $\theta = (t_c, \phi, \omega, m)$

(3.13)

Here $y_t$ is the price point and $\hat{y}_t$ is the price point predicted by the model. The function of the residual sum of squares $F(\theta)$ is not well behaved with many local minima which means local search algorithms can become trapped and fail to find global solutions. The complexity of the optimization problem is reduced by slaving the linear parameters $A,B,C$ to the four nonlinear parameters $t_C, m, \omega, \phi$. The search method applied in early studies by Johansen & Sornette first made a grid of points for the parameters $\omega$ and $t_c$ and applied a Taboo search to find the best $m$ and $\phi$. 10 equally spaced points were chosen from the grid and a Taboo search results was performed to find the local best solutions for each point. The results were used as initial conditions for a Levenberg-Marquardt algorithm. The solution with the lowest RMSE was then chosen as the best global solution. Later approaches applied a Genetic algorithm for a global search followed by a Nelder-Mean simplex search for local optimization. However, these approaches do not guarantee convergence to the best solution. Figure 3.1 below illustrates the complexity of the optimisation problem and highlights the risk of reaching a local rather than a global solution. The USD Bank of England (BoE) Effective Exchange Rate Index (ERI) prior to 1985 was chosen as a suitable case study where $t_1=24$-Nov-82 and $t_2=12$-Mar-85. The cross section of the complex cost function is generated by searching across two nonlinear parameters and holding the remaining two nonlinear parameters fixed.

A new fitting method proposed by Filimonov and Sornette (2013) greatly simplifies the optimization problem and therefore is used throughout this thesis. The approach reduces the
Figure 3.1: Cross Sections of the cost function for USD BoE ERI where \( t_1 = 24\text{-Nov-82} \) and \( t_2 = 12\text{-Mar-85} \) with the old fitting method applied

(a) \( F_1(t_c=13\text{-Mar-85}, \phi, m = 0.3, \omega) \)

(b) \( F_1(t_c, \phi = 1.5, m, \omega = 8) \)

(c) \( F_1(t_c, \phi, m = 0.3, \omega = 8) \)

(d) \( F_1(t_c, \phi = 1.5, m = 0.3, \omega) \)

Figure 3.1 Fixing two of the nonlinear parameters \((t_c, m, \omega, \phi)\) we can plot the shape of the cost function by varying the remaining two parameter. From figure (a) and (d) it is clear that local search methods would likely become trapped and fail to identify the global solution to the problem.

number of nonlinear parameters and removes the interdependence between the phase \( \phi \) and the angular log frequency \( \omega \). The cosine term in the LPPL formula is expanded as follows

\[
\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi) \\
= A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t)) \cos(\phi) + C(t_c - t)^m \sin(\omega \ln(t_c - t)) \sin(\phi) 
\]

Two new parameters are introduced, \( C_1 = C \cos(\phi) \) and \( C_2 = C \sin(\phi) \), and the LPPL equation is rewritten as

\[
\ln E[p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t)) 
\]

This new LPPL equation now has 3 nonlinear parameters \((t_c, \omega, m)\) and 4 linear parameters \((A, B, C_1, C_2)\) with \( C_1 \) and \( C_2 \) containing \( \phi \). The 4 linear parameters are slaved to the nonlinear parameters in the fitting procedure and are calculated from the obtained values of the nonlinear parameters. Using this new procedure the dimensionality of the nonlinear optimization problem
is reduced from a 4-dimensional space to a 3-dimensional space. For each choice of nonlinear parameters we obtain unique linear parameters from the matrix equation

\[
\begin{pmatrix}
\sum_{i} N \\
\sum_{i} f_i \\
\sum_{i} g_i \\
\sum_{i} h_i \\
\sum_{i} f_i^2 \\
\sum_{i} f_i g_i \\
\sum_{i} f_i h_i \\
\sum_{i} g_i^2 \\
\sum_{i} g_i h_i \\
\sum_{i} h_i^2 \\
\end{pmatrix}
\begin{pmatrix}
\hat{A} \\
\hat{B} \\
\hat{C}_1 \\
\hat{C}_2 \\
\end{pmatrix}
= \begin{pmatrix}
\sum_{i} y_i \\
\sum_{i} y_i f_i \\
\sum_{i} y_i g_i \\
\sum_{i} y_i h_i \\
\end{pmatrix}
\] (3.16)

where \(y_i = \ln(p(\tau_i))\), \(f_i = (t_c - \tau_i)^m\), \(g_i = (t_c - \tau_i)^m \cos(\omega \ln(t_c - \tau_i))\) and \(h_i = (t_c - \tau_i)^m \sin(\omega \ln(t_c - \tau_i))\). Representing the above in a compact way as \((X'X)b = X'y\), the general solution is given as \(b = (X'X)^{-1}X'y\).

From Figure 3.2 below we can see that the cost function now enjoys a much smoother structure with few local minima. This eliminates the need for metaheuristic searches like the Taboo search or Genetic algorithm. Optimisation can be performed using local search methods like the Levenberg-Marquardt nonlinear least squares algorithm or the Nelder-Mead simplex method.

**Figure 3.2:** Cross Sections of the cost function for USD BoE ERI where \(t_1 = 24\text{-Nov-82}\) and \(t_2 = 12\text{-Mar-85}\) with the new fitting method applied

(a) \(F_1(t_c, m = 0.3, \omega)\)  
(b) \(F_1(t_c=13\text{-Mar-85}, m, \omega)\)

(c) \(F_1(t_c, m, \omega = 8)\)

**Figure 3.2** By fixing one of the nonlinear parameters \((t_c, m, \omega)\) we can explore the shape of the cost function. In comparison to Figure 3.1 the cost function is considerably smoother reducing optimization to a single step of applying the Levenberg-Marquardt algorithm.
3.6 Illustrating the model

We now present LPPL curve fitted to the USD BoE ERI, illustrate the role of each parameter, and highlight some additional aspects of the model. As above, the window used to fit the curve is given by \( t_1 = 24\text{-Nov-82} \) and \( t_2 = 12\text{-Mar-85} \). There are two primary characteristics of the equation

- A faster than exponential growth captured by the power law \( A + B(t_c - t)^m \)
- An accelerating log-periodic oscillation decorating the power law growth given by \( C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t)) \)

Log periodicity means that the ratio of the distances between the consecutive peaks or troughs, as indicated in Figure 3.3, is constant.

**Figure 3.3:** LPPL fit to USD BoE ERI with \( t_1 = 24\text{-Nov-82} \) and \( t_2 = 12\text{-Mar-85} \)

The oscillations increase as \( t \) approaches \( t_c \) and the local maxima of the LPPL function are separated by time periods that tend to zero in a geometric fashion. The ratio of consecutive time intervals is given by the scaling ratio \( \lambda \). The relationship between the oscillation and this scaling ratio is given as

\[
\lambda \equiv e^{\frac{2\pi}{\omega}}
\] (3.17)

From the optimal fit for the USD time series above and for a fixed crash date \( t_c \), we then vary each parameter one-by-one to more clearly illustrate their role in defining the LPPL curve. **Figure 3.4 (a)** illustrates the effect of the oscillations by setting \( C_1 \) and \( C_2 \) to zero leaving only the power law to define the curve. By varying the exponent \( m \) we see it controls the steepness of the faster than exponential growth. \( \omega \) affects the frequency of the oscillations as illustrated by **Figure 3.4 (b)**. The linear parameter \( A \) gives the value the price would pass through were it to last until the critical time \( t_c \) (Note: \( t_c > t_2 \)). Increasing the value shifts the entire LPPL curve
Figure 3.4: The role of each parameter in the LPPL equation

(a) $m$ determines the shape of the power law
(b) $\omega$ drives the frequency of oscillations
(c) $A$ shifts the curve, $B$ drives dropoff from $A$
(d) $C_1$ and $C_2$ affect both the phase and amplitude

Figure 3.4 As in Figure 3.3, we fit the LPPL equation to the USD BoE ERI with $t_1 = 24$-Nov-82 and $t_2 = 12$-Mar-85. By holding $t_c$ fixed, we can explore the LPPL curve by varying the remaining parameters.

up as shown in Figure 3.4 (c). The $B$ parameter is the decrease in the price per unit time prior to the crash, providing the oscillation amplitude is sufficiently small. $C_1$ and $C_2$ control this oscillation amplitude but also affect the phase of the oscillations as these parameters contain the parameter $\phi$ in the new model definition. As illustrated in Figure 3.4 (d), an increase in $C_1$ increases the amplitude and also shifts the oscillations to the right. An increase in $C_2$ increases the amplitude but shifts the curve to the left.

3.7 Summary

This chapter introduces the LPPL model and reviews its application to different markets to date. Having received limited coverage in the LPPL literature, the motivation for studying currency markets is outlined. Following this, the derivation of the LPPL model is covered step-by-step. The difficulty involved in fitting the curve to real price data is clearly illustrated and the improvements from the new fitting procedure introduced by Filimonov and Sornette (2013) validate this method for use in this research project. The LPPL curve is also visualised and the parameters which drive the shape of the fit are explored in detail. In the chapter 4, we outline the data under consideration and methodology of the research.
4.1 Introduction

Having reviewed the LPPL model and motivated its application to currency markets in Chapter 3, the research goal broadly defined is to explore the applicability of the model in these markets. In this chapter we first outline the data under consideration followed by the research methodology. In accordance with Sornette and Zhou (2006) in their study of equity markets, we approach this study in a number of steps: (a) collect bubble samples from currency indices and currency pairs; (b) test for commonalities and distinguishing features to create a classification of bubble and negative bubble phenomena; (c) develop an alarm system which accounts for this classification; (d) test the predictive power of the alarms. Furthermore we investigate the macroeconomic factors which may drive economic agents’ willingness and ability to participate in currency bubbles.

4.2 Data

4.2.1 Investment Universe

The investment universe under consideration in this study spans 11 developed market currencies. The United States Dollar (USD), Euro (EUR), Japanese Yen (JPY), and British Pound (GBP) comprise the major G4 currencies. The commodity currencies include the Australian Dollar (AUD), the New Zealand Dollar, the Canadian Dollar (CAD), and the Norwegian Krone (NWK). The Swiss Franc (CHF), Swedish Krona (SEK), and Danish Krone (DNK) complete the set.

With 11 different currencies, 55 unique currency pairs exist \( N \times (N-1)/2 \), however we focus on the USD currency pairs, as they dominate trade and are typically the subject of academic research. All the currency pairs have been arranged with the USD as the base currency e.g. USD/CHF. The exchange rate time series were obtained from the Bank of England online database.

Between the Bretton Woods Agreement in 1944 and the Smithsonian Agreement in 1971, the USD provided one benchmark against which changes in the value of other currencies could be measured. However, since 1970 exchange rates were seen to become more volatile and it is no longer assumed that changes in exchange rates against the USD completely capture the overall exchange rate for a currency. Therefore, to measure the overall change in the exchange value of a given currency, a weighted average of the movements of a basket of cross rates can be used.
to create a currency index. With an index starting at 100 at inception, any increase reflects a strengthening of the currency and a decrease reflects a weakening. While many weighting approaches are conceivable, the weights typically reflect the relative importance of the other currencies based on trade flows of goods between the respective countries. For example, the United States trade with Europe accounts for approximately 40% of its total trade, and therefore the USD/EUR cross rate should receive 40% of the weight when constructing a basket index for the USD. As trading patterns evolve this weight is updated accordingly.

Central banks across the world construct such Effective Exchange Rate Indices in accordance with the need to monitor the value of their currencies, with slightly varying construction methodologies across banks. The Bank of England constructs indices for each of the 11 currencies listed above following an identical construction method for each, based on weights calculated by the International Monetary Fund\(^1\). Furthermore, unlike some central banks, they update each ERI on a daily basis and therefore their Effective Exchange Rate Indices, henceforth BoE ERIs/ERIs, were chosen as a suitable candidate for this study. Like the currency pairs data, the BoE ERI time series are also readily available on the Bank of England online interactive database.

### Table 4.1: Bank of England ERIs and USD currency pairs

<table>
<thead>
<tr>
<th>Currency</th>
<th>BoE ERI</th>
<th>Currency Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australian Dollar</td>
<td>AUD ERI</td>
<td>USD/AUD</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>CAD ERI</td>
<td>USD/CAD</td>
</tr>
<tr>
<td>Danish Krone</td>
<td>DKR ERI</td>
<td>USD/DKR</td>
</tr>
<tr>
<td>Euro</td>
<td>EUR ERI</td>
<td>USD/EUR</td>
</tr>
<tr>
<td>British Pound</td>
<td>GBP ERI</td>
<td>USD/GBP</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>JPY ERI</td>
<td>USD/JPY</td>
</tr>
<tr>
<td>Norweigan Krone</td>
<td>NWK ERI</td>
<td>USD/NWK</td>
</tr>
<tr>
<td>New Zealand Dollar</td>
<td>NZD ERI</td>
<td>USD/NZD</td>
</tr>
<tr>
<td>Swedish Krona</td>
<td>SEK ERI</td>
<td>USD/SEK</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>CHF ERI</td>
<td>USD/CHF</td>
</tr>
<tr>
<td>United States Dollar</td>
<td>USD ERI</td>
<td>-</td>
</tr>
</tbody>
</table>

### 4.2.2 Data Sample

The time frame in which any market indicator is tested is not without significance. Market behaviour changes over time and indeed some informative indicators, which have predictive ability, may suffer long periods of poor performance (likewise, some nonsense indicators may have periods of apparent strong performance). There are also arguments that the market becomes more efficient over time reducing the power that indicators may have once had. Furthermore, a sparse indicator, necessarily requires a suitably long look-back period in order to accumulate a sufficient number of signals to assess statistical significance.

With these points in mind, the data sample for each BoE ERI and currency pair listed above consists of 10,008 daily data points from 02 January 1975 to 01 August 2014. The start date was\(^1\) the construction of the Euro index prior to the Euro’s introduction on 1 January 1999 is based on weighting of the currencies which were supplanted by the Euro, with the German Deutschmark, French Franc, and Italian Lire receiving approximately 70% of the weight.
based on data availability, and indeed it gives a suitably long buffer period following the collapse of the Bretton Woods system. The end date marked the first day of the thesis project. The data has not been split into an in-sample and out-of-sample period (For robustness discussion, see Section 5.7).

<table>
<thead>
<tr>
<th>Table 4.2: Data sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In-sample</strong></td>
</tr>
<tr>
<td>Start-date</td>
</tr>
<tr>
<td>End-date</td>
</tr>
<tr>
<td>Data points</td>
</tr>
<tr>
<td>Interval</td>
</tr>
</tbody>
</table>

We note that not all currencies listed above were freely floating by January 1975, in some cases currency exchange rates remained fixed against the USD for many years. Indeed, the Danish krone / Euro exchange rate at the date of writing is pegged within a narrow band by the Danish Central Bank. Nonetheless, we have included the entire sample from 1975 for each currency in this study. A central bank’s intervention in the currency market with the intention of influencing its value may reflect their perception of currency overshoot, overvaluation, or bubble type situations. Indeed it is possible that an intervention may burst a bubble. Therefore excluding periods where central banks were more actively managing their currency’s value may inadvertently remove the scenarios the LPPL model claims to predict.

4.3 Methodology

4.3.1 Identifying Peaks & Troughs

While the LPPL model provides a framework to detect the presence of bubbles and forecast their collapse, we must first identify such periods in history in order to extract commonalities and features which distinguish them from the norm. As noted in Chapter 3, academic literature is not even in agreement on whether bubbles theoretically emerge in asset markets at all and within empirical literature, for every study that identifies a potential bubble period, another presents a fundamental explanation. Therefore, there does not exist a readily available record of bubble periods upon which the LPPL model can be calibrated.

Underlying the LPPL hypothesis is the notion that the power law and log-periodic oscillations are precursors to critical points in time, often crashes in the asset price in question. As such, ex post research into the model typically first identifies major crashes and then studies the applicability of the LPPL framework to the preceding period. Crashes have been identified as the largest drawdowns in the market. The magnitude of the drawdown can be measured in many different ways: the cumulative effect of successive price drops (von Bothmer and Meister, 2003; Jacobs, 2014), the cumulative effect of successive price drops ignoring increases of a certain fixed magnitude (Johansen and Sornette, 2010), or the % change between a peak $t_{max}$ and trough $t_{min}$ (Johansen and Sornette, 1999). In these studies the largest of the drawdowns, e.g. top 10, are chosen for study.

We follow the approach of Yan et. al (2012). We have identified crashes and rebounds in exchange rates as the major peaks and troughs in the prices series. It is important to note,
we do not claim each of these peaks or troughs were preceded by bubbles or negative bubbles, though for our method to be successful some must be. Furthermore, the LPPL model does not argue that every bubble must be followed by a crash, and therefore we do not account for bubbles which conclude with a smooth plateau by following this method.

In this thesis, we say a day \( d \) begins a crash (rebound) if the price on that day is the maximum (minimum) price in a window of 200 days before and 200 days after. Under this definition, the identification of a crash (rebound) is a non-causal process. In this way we avoid identifying two crashes (rebounds) in very close succession which both potentially resulted from the same bubble or negative bubble period. The dates of crashes and rebounds are given by,

\[
\text{Peak} = \{d | P_d = \max \{P_x\}, \forall x \in [d-200, d+200]\} \tag{4.1}
\]

\[
\text{Trough} = \{d | P_d = \min \{P_x\}, \forall x \in [d-200, d+200]\} \tag{4.2}
\]

The results of this algorithm when applied to the USD BoE ERI are presented below with 13 peaks and 14 troughs highlighted. This method is repeated on the remaining BoE ERI indices and currency pairs resulting in 147 peaks and 150 troughs in the indices and 116 peaks and 129 troughs in the cross rates.

**Figure 4.1:** Peaks & Troughs in the USD BoE Exchange Rate Index

In some cases following a quantitative identification method, authors have excluded certain crashes (rebounds) from analysis (Johansen and Sornette, 2010) based on external drivers or shocks which arguably led to a crash in the asset under study. Given the LPPL hypothesis claims to only identify endogenously driven bubbles, exogenous factors may still cause market crashes and so do not fall within the realm of the LPPL bubble prediction framework. Indeed, Johansen and Sornette (2010) claim that 25 of 49 major drawdowns (statistical outliers referred to as “dragon kings”) across equity, FX, bonds, and gold were caused by endogenously driven bubbles while 22 were caused by exogenous factors. The problem with such an approach is its subjective nature, especially in regards to FX markets, where explaining movements based on fundamental
arguments or exogenous factors is challenging. Furthermore it potentially creates a pre-selection problem, where we choose to study the periods which we qualitatively judge to exhibit LPPL-type structures thereby biasing results in favour of the LPPL hypothesis. Therefore we have not excluded any crashes or rebounds from the study. The disadvantages of this approach is that our classification system of bubbles or negative bubbles may be contaminated by crashes or rebounds which are exogenously driven.

4.3.2 Bubble Identification

Research into the model has expanded since its first application in 1996. With each additional study, the bubble prediction framework has been refined and improved, while critics have identified challenges and pitfalls. To give a fair and accurate accounting of the model we consider both recommendations of the model proponents and the observations made by critics.

The LPPL model, given that is has such a large number of parameters, will naturally produce a low root mean square error (RMSE) regardless of the time series to which it is fit. This is an clear criticism of the model and indeed von Bothmer and Meister (2003) show that only 3.9% of crash predictions based on RMSE alone are successful with many false alarms produced.

A second difficulty with the model is that it can be fit to markets in non-bubble periods and will predict a crash date \( t_c \) irregardless. von Bothmer and Meister (2003) find that crash prediction based on the \( t_c \) alone has a success rate of just 6.7%. Indeed, Bree et. al (2013) demonstrate the sloppiness of the \( t_c \) parameter. Varying the \( t_c \) parameter only slightly changes the RMSE, however changing the input time series only slightly may greatly change the \( t_c \) value, hazardous given noisy financial time series. While they recommend the Levenberg-Marquard algorithm for optimisation, they argue that crash predictions made on the basis of this parameter are likely to be unreliable even when bootstrap techniques are used for robustness. Based on synthetic data, they estimate the standard deviation of \( \sigma(t) \) is roughly half \( t_c - t_2 \). Recently, Vakhtina and Wosnitza (2015) find evidence supporting the latter point, with the discriminative power of \( t_c \) increasing as the actual crash date approaches.

Statistically testing of the goodness of fit of the LPPL in order to qualify or disqualify fits and generate effective bubble alarms is also difficult, though Gazola et. al (2008) introduce an AR(1)-GARCH(1,1) model which may be amenable to such approaches. Indeed, Bree et. al (2013) conclude that most of the assumptions underlying the mechanism from which the LPPL model is derived are untestable, though Sornette et. al (2013) offer a reply to many of the criticisms.

With these points in mind, we therefore focus on the Stylized Features of the LPPL (Filimonov and Sornette, 2013) when constructing a classification system to identify bubble periods. These are criteria against which a given LPPL function fit is judged in order to qualify a fit as a bubble or negative bubble. They supply acceptable ranges for the exponent parameter \( m \), the linear parameter \( B \), the oscillation frequency parameter \( \omega \), and the hazard rate.

As highlighted in Section 3.4 the exponent parameter should meet the following constraint

\[
0 < m < 1 \tag{4.3}
\]

While investors may be aware a bubble exists in a market, it is rational for these agents to remain invested in the market as a crash is not certain. They receive an excess return to compensate for this risk given by \( u(t) = \kappa h(t) \). In order to keep the probability of a crash finite and less than 1, the exponent \( m \) should remain less than 1 for all times \( t \leq t_c \). A further constraint \( m \geq 0 \) ensures that the asset price remains finite at all times, including \( t_c \). Moreover,
a bubble exists when the crash hazard rate is accelerating over time, reaching its maximum at $t_c$. The acceleration cannot occur if $m \geq 1$ or when $B > 0$. Therefore a second constraint for positive bubbles is given by

$$B < 0$$  \hspace{1cm} (4.4)$$

A third constraint is proposed for the $\omega$ parameter based on historical studies of bubbles.

$$6 < \omega < 13$$  \hspace{1cm} (4.5)$$

As argued by Filimonov and Sornette (2013) this ensures the oscillations are not too fast (or they may fit to the random component of the data) or too slow (in which case they would simply contribute to the trend). However, with limited application of the model to FX it remains unclear if bubbles in these markets display similar characteristics.

The fourth constraint concerns the hazard rate. As noted in Section 3.5, we have adopted the use of the new LPPL reformulation as presented by Filimonov and Sornette (2013) in order to overcome optimisation problems associated with the complex search space of the old model, as this was often a source of criticism for the LPPL framework (Bree et. al, 2013). The old model is given by

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi)$$  \hspace{1cm} (4.6)$$

and the new reformulation is given by

$$\ln E[p(t)] = A + B(t_c - t)^m + C_1(t_c - t)^m \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^m \sin(\omega \ln(t_c - t))$$  \hspace{1cm} (4.7)$$

In earlier research, von Bothmer and Meister (2003) presented a parameter constraint which ensures that the hazard rate (but not the price change) remains positive at all times. For bubbles this is given by

$$b \equiv -Bm - |C|\sqrt{m^2 + \omega^2} \geq 0$$  \hspace{1cm} (4.8)$$

It effectively ensures that the power law dominates and the amplitude of the oscillations are not excessive and accounts for the theoretical aspect of the model in regards to the hazard rate. As noted by Sornette et. al (2013), the hazard rate constraint is key to any bubble monitoring framework and is used by Yan et. al (2012) in their alarm system for equity markets and Vakhtina and Wosnitza (2015) when creating an alarm for Credit Default Swaps. Filimonov and Sornette (2013) highlight that the new formulation of the LPPL equation affects the constraint on the hazard rate as $C$ is replaced with $C_1$ and $C_2$ (these new parameters also contain the phase $\phi$). They give a hazard rate constraint as follows

$$|C| \leq 1 \quad \text{(old model)} \quad \text{or} \quad C_1^2 + C_2^2 \leq 1 \quad \text{(new model)}$$  \hspace{1cm} (4.9)$$

In contrast with Equation 4.8, this only constrains the oscillations to within acceptable amplitudes. While $C$ is the amplitude adjustment parameter of the old LPPL equation it can be shown that $\sqrt{C_1^2 + C_2^2}$ is the amplitude of the oscillations in the new model. Given,

$$f(x) = a \sin(x) + b \cos(x) = \sqrt{a^2 + b^2} \cos(x + y - \frac{\pi}{2})$$  \hspace{1cm} (4.10)$$

This is a cos function of amplitude $\sqrt{a^2 + b^2}$ and phase $y - \frac{\pi}{2}$.

With both the amplitude parameters $C_1$ and $C_2$ typically less than 1, we can formulate a more restrictive constraint on the hazard rate by accounting for the square root. The square
root of any number \(0 < x < 1\) will never reach or exceed 1, and therefore the constraint offered by Filimonov and Sornette (2013) from a practical aspect is no different.

\[ C_1^2 + C_2^2 \leq 1 \quad \text{is equivalent to} \quad \sqrt{C_1^2 + C_2^2} \leq 1 \quad (4.11) \]

However, there may be an effect if we account for noise in accordance with Jacobs (2014) by reducing 1 to 0.95 or lower. Furthermore, the square root will affect the constraint of von Bothmer and Meister (2003) when we extend it to the new model specification,

\[ b = -Bm - \sqrt{C_1^2 + C_2^2 \sqrt{m^2 + \omega^2}} \geq 0 \quad (4.12) \]

In both these cases, the constraints are now more restrictive. Recognising that \(0 < m < 1\) and \(\omega\) typically ranges from 6 to 13 for bubbles we see that

\[ \omega \gg m \quad \text{implying} \quad \sqrt{m^2 + \omega^2} \approx \omega \quad (4.13) \]

We can therefore simplify and re-arrange the positive hazard rate restriction into a more intuitive form as follows

\[ \frac{-Bm}{\omega \sqrt{C_1^2 + C_2^2}} \geq 1 \quad (4.14) \]

A higher value for this parameter reflects a lower amplitude of oscillations. Application of the constraint is illustrated in Figure 4.2. By cycling through different \(m\) and \(\omega\) values we can illustrate those combinations which meet the positive hazard rate restriction given by Equation 4.14 and those which fail to meet the restriction. As in Chapter 3 we have used the USD BoE Effective Exchange rate with \(t_1=24\text{-Nov-82}\) and \(t_2=12\text{-Mar-85}\) as a case study. Note: Many “acceptable” fits are not the best fit as we are cycling through \(m\) and \(\omega\) parameters not minimising RMSE. As outlined above, the restriction ensures the power law dominates the oscillations.

**Figure 4.2:** Hazard Rate acceptance or rejection - USD BoE ERI

![Figure 4.2](image-url)
When considering negative bubbles two of the above constraints need to be updated. Yan et. al (2012) define the rally hazard rate as the probability per unit time that the market rebounds in a strong rally. Whereas the upward accelerating price compensates investors for exposure to crash risk during a bubble, the downward accelerating bearish price in a negative bubble is interpreted as the cost that rational agents accept in return for a potential strong rally. Therefore the constraint on $B$ in the case of a negative bubble is given by

$$B > 0$$ (4.15)

The hazard rate constraint can be account for both positive and negative bubbles as follows

$$\frac{|B|m}{\omega \sqrt{C_1^2 + C_2^2}} \geq 1$$ (4.16)

Were $|B|$ accounts for negative $B$ values during bubbles and positive $B$ values during negative bubbles.

### 4.3.3 Fitting the LPPL model

In order to apply the model to the currency indices and currency pairs we need to determine the inspection dates where we would like to investigate the existence of a bubble. Yan et. al (2012) refit the model every 50 days in their study while von Bothmer and Meister (2003) refit the model every 5 trading days. A smaller interval between each fit reduces the likelihood that the model mistimes a bubble however we must balance this with the additional processing time required to fit the model. As such, we have chosen to refit the model every 10 days.

At each inspection date the LPPL equation needs to be fit to a data window defined by $(t_1, t_2)$. The inspection date defines $t_2$ whereas $t_1$ is determined by the length of the data window to which we choose to fit the LPPL model. Sornette and Zhou (2006) visually identify the bubble preceding a crash as the price trajectory which starts at the most obvious well-defined minimum before the ramp-up to the tipping point. However, the choice of $t_1$, remains somewhat challenging. Intuitively we could consider this the start of the bubble however in a live bubble detection environment this would necessarily be unknown. Furthermore, even in ex post bubble analysis the correct choice is unclear.

Sornette et. al (2013) argue that a single $t_1$ corresponding to a single window and single fit is unreliable and an ensemble of fits should be used to make the prediction more statistically reliable. Indeed they argue the key to forecasting performance is the combination of bubble diagnostics at multiple time scales, with common parameters across all scales indicative of robustness. In addition to this robustness argument, Drozdz et. al (1999) argue that log-periodic oscillations also occur at smaller time scales and Matsushita et. al (2006) find evidence of this at intra-day time scales. This may suggest a crash occurs at the culmination of a range of log periodic oscillations, each occurring at different time scales.

With this in mind, Jacobs (2014) applies 4 windows with lengths of between 1,008 trading days (4 years) and 252 trading days (1 year). Sornette and Zhou (2006) likewise employ a multiscale analysis with 9 windows ranging from 60 - 1680 trading days while Yan et. al (2012) use 29 windows ranging from 110 - 1500 days. At each inspection point along the currency pair and ERI time series, we have chosen to fit the model to 65 windows ranging from 110-750 trading days.

Johansen and Sornette (1999) argue that the curve should be fit to the log-price when the amplitude of the expected crash is proportional to the price increase during the bubble. The curve can also be fit to the real price however the data values may change over several orders of
magnitude and a normalized least squares minimisation is recommended in this case. Geraskin and Fantazzini (2013) conclude that estimating the model with log prices is simpler and the \( t_1 \) choice needed to be made more carefully when using real prices. Therefore, for both currency pairs and BoE ERIs we have fit the LPPL curve to log prices.

Finally, it is possible to use constrained optimisation when minimising the cost function as per Vakhtina and Wosnitza (2015) or Yan (2012) where bounds of the search space for a bubble were given as

\[
B < 0 \\
-B \geq \sqrt{1 + \frac{\omega^2}{m^2}|C|} \quad \text{(positive hazard rate, old model)} \\
m \in [0.001, 0.999] \\
\omega \in [0.01, 40] \\
t_c \in [t_2, t_2 + 0.375(t_2 - t_1)]
\]

As Bree et. al (2013) note, an alternative approach is to use unconstrained optimisation and reject or accept the fits which meet the stylized facts of the LPPL outlined above. Applying bounds to the search space likely biases results in favour the LPPL hypothesis and furthermore may increase the number of false positive alarm signals. Therefore we have followed unconstrained optimisation using the Levenberg-Marquardt algorithm. However, we have set an upper bound on \( t_c - t_2 \) at 200 days given the evidence regarding the sloppiness of \( t_c \).

### 4.3.4 Discriminative Power of the Classification system

Until this point, we have outlined the peak & trough algorithm we apply to identify the crashes and rebounds which may be preceded by bubbles or negative bubbles, the key characteristics or Stylized Facts of the LPPL which will serve as classification parameters that may help distinguish bubble periods from the norm, and the multiscale method used when fitting the LPPL curve to the time series under study.

In order to calibrate the LPPL model on currency markets we must identify the parameters, if any, which discriminate between bubble periods and non-bubble periods. Therefore for bubbles, we must compare LPPL parameters and constraints from the “pre-peak” LPPL fits with those from the “non-pre-peak” equation fits. Likewise, parameters and constraints from the “pre-trough” fits are compared with the “non-pre-trough” fits for negative bubbles.

As noted above, we identify between 116-150 peaks and troughs in total for currency pairs and BoE ERIs and fit the LPPL curve to windows of length 110-750 on each inspection date. As an inspection date is every 10 days, the peak (trough) may not necessary coincide with the day of inspection. Therefore we identify the inspection day which coincides with or immediately precedes the peak date. Having identified the appropriate date we can select the 65 LPPL fits made on this day. Then we can aggregate all the “pre-peak” (“pre-trough”) fits and determine remaining “non-pre-peak” (“non-pre-trough”) fits as shown in Table 4.3.

<table>
<thead>
<tr>
<th># Peaks</th>
<th># Pre-peak fits</th>
<th># Non-pre-peak fits</th>
<th># Troughs</th>
<th># Pre-trough fits</th>
<th># Non-pre-trough fits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency BoE ERIs (10)</td>
<td>147</td>
<td>9,555</td>
<td>592,234</td>
<td>150</td>
<td>9,750</td>
</tr>
<tr>
<td>Currency pairs (11)</td>
<td>116</td>
<td>7,540</td>
<td>654,550</td>
<td>129</td>
<td>8,385</td>
</tr>
</tbody>
</table>

Table 4.3: Grouped LPPL fits

In order to identifying parameters which can discriminate between bubble and non-bubble periods, we must classify all the LPPL fits into pre-peak (pre-trough) and non-pre-peak (non-pre-trough) fits for further study.
With suitably large samples, we can compare and contrast the distributions of the LPPL parameters from the “pre-peak” periods with those from the “non-pre-peak” periods, and likewise for the troughs. To estimate the probability density function of a parameter we apply a kernel smoothing function in accordance with Yan et. al (2012) and Sornette and Zhou (2006). This is a non-parametric way to estimate and visualise the density function of a random variable. Johansen and Sornette (2010) document approximate Gaussian distributions for $m$ and $\omega$ and therefore we adopt a normal kernel for smoothing. With a sample $(x_1, x_2, ..., x_n)$ drawn from an unknown density of $f$, the kernel density estimator is given by

$$
\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)
$$

where $K(\bullet)$ is the kernel, a non-negative weighting function which sums to one with mean zero. The normal kernel is given by

$$
K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}
$$

$h > 0$ is a free parameter called the bandwidth which determines the degree of smoothing. Effectively, at each data point, we place a normal kernel and sum all the kernels to create the kernel density estimate. A smaller bandwidth enables a more granular inspection of the data. The process is illustrated below in Figure 4.3 along with the effects of a lower / higher bandwidth. After constructing the kernel density estimates for the parameters we can visualise and contrast the distributions of the “pre-peak” and “pre-trough” parameters with the remaining fits. We confine this analysis to the non-linear parameters $t_c, \omega, m$. As noted above, in a bubble prediction framework, the $B$ and the hazard rate are only subject to positive or negative constraints and therefore we simply count the number of “pre-peak” or “pre-trough” fits which conform to these constraints to give an indication of their discriminative ability.

Following the univariate analysis outlined above, we assess the multivariate discriminative power of the model by construction of an appropriate alarm. When building an alarm system in equities and credit default swap market we note that Sornette and Zhou (2006), Yan et. al (2012), and Vakhtina and Wosnitza (2015) applied a Cora-3 algorithm. This was originally developed by Gelfand et. al (1976) for earthquake prediction systems. It is trained on an in-sample set of bubbles and may extract subtle patterns repeatedly exhibited in the LPPL fits before crashes or rebounds. In this research we have adopted a simple rules based alarm.

Figure 4.3: Kernel Density Estimation with Normal Kernel

![Figure 4.3](image)

Figure 4.3 In order to construct the kernel density estimate, the normal kernel (red) is first applied to the sample data points (black) and weighted to create the density. The bandwidth parameter $h$ smooths noise.
The focus of this thesis is on the applicability of the LPPL model in currency markets and the Cora-3 pattern recognition systems may obscure the results and indeed introduce new questions regarding the efficacy of the recognition algorithms. Indeed we observe this problem in the case of Vakhtina and Wosnitza (2015) where the alarm they build is significant only at the 70% level leaving the reader uncertain regarding the effectiveness of both the LPPL model and the Cora-3 algorithm.

Accordingly, we build a rules based alarm as follows:

1. We first study and verify the Stylized Facts of the LPPL for bubbles (negative bubbles) by contrasting the LPPL fits prior to peaks (troughs) with the LPPL fits during non-peak (non-trough) periods as previously outlined. In this way we determine suitable ranges for these parameters for currency pairs and currency indices. We do this in an aggregate fashion, grouping all the fits of different window lengths and all the currency pairs or ERIs together. As noted above, this is based on the assumption that some of the peaks (troughs) reflected the end of a bubble (negative bubble) period.

2. At each inspection day, the LPPL curve is fitted to 65 data windows varying from 110-750 days in length

3. Each of the 65 fits is inspected in order to determine if it meets the 4 Stylized Facts of the LPPL

4. An alarm is defined as the percentage of fits on an inspection day which meet the criteria and therefore has a strength which varies between 0 and 1.

5. We roll forward 10 days to the next inspection day and repeat the fitting process.

Unlike the Cora-3 algorithm we may not identify complex patterns preceding a crash however the approach may also be less susceptible to overfitting where a model captures random noise instead of an underlying structure behind data (discussed below). The performance of this alarm should give an indication of the applicability and effectiveness of the LPPL model when applied to currencies.

4.3.5 Alarm Performance Measurement

Evaluating trading strategies and testing for statistical significance is an extensive and growing field. Typically it is done in a backtest. This is a historical simulation and critique of an algorithmic signal or investment strategy. It attempts to answer a number of conceptually different questions: does the signal have predictive power / is it imbued with information not reflected in market prices, can a profitable strategy be created after accounting for trading costs, how does the strategy perform compared to alternatives?

In recent literature, a popular theme within this field considers the problem of overfitting or data snooping. This is a problem which occurs with increasing probability when a set of data is used more than once for the purpose of inference and model selection. When a good model is obtained after extensive searching there is always the danger that the observed good performance is not from good forecasting ability but from good luck (White, 2000). Indeed continued searching only increases the probability of finding what appears to be a good strategy but is effectively nonsense. This problem is practically unavoidable when considering currency time series - for example the USD / GBP daily time series has likely been studied many thousands of times by different researchers. Compounding this problem, Harvey and Liu
(2014) note that individual researchers explore many variations of a model and often chose to present the one with the most favourable results.

A number of strategies can be followed to adjust backtest statistics to account for multiple backtests and model iterations, for example Deflated Sharpe Ratios (Bailey and de Prado, 2014), Minimum Backtest lengths (Bailey et. al., 2014), and upward adjusted hurdle rates (Harvey and Liu, 2014). Ultimately this reflects the classic tension between Type 1 and Type 2 errors. More demanding significance tests reduce the likelihood of falsely identifying a bad strategy as good, while at the same time increases the likelihood of rejecting a true positive and thereby missing a truly good strategy. This problem is faced by most market participants: investors trying to separate good investment managers from financial charlatans, practitioners critiquing their own strategies, and academics testing different hypotheses. While we must be wary of spurious results, the strategy we have followed is to apply the LPPL model to multiple markets (21 total) with the assumption that, were a nonsense strategy applied, perhaps 1-2 markets would exhibit apparent strong performance. While we would expect perhaps 1 in 20 random strategies to appear statistically significant when applied to 1 market (given \( p = 0.05 \)), or likewise 1 strategy applied to 20 different markets, the probability of finding a single nonsense strategy which appears successful across a number of markets is much lower.

With these points in mind we first assess the information content of the bubble alarm signals by applying a Monte Carlo Permutation Test in accordance with Aronson (2006). When following this approach we note that a bubble signal should precede negative market returns and a negative bubble signal should precede positive market returns. The null hypothesis of this test says that the bubble or negative bubble signals are effectively random and do not provide information on the future market direction. The alternative hypothesis says the signal gives performance beyond a purely random strategy applied to the market in question. To perform the test, a large number of random signals are generated, the subsequent cumulative return of the market between 1 - 100 days is calculated, and the fraction of signals whose total return exceeds that of the candidate model is counted. This fraction is the probability that the supposedly superior return of the candidate model could have been obtained by nothing more than luck. Note, this method does not test the null hypothesis that expected returns are zero, rather if the market has an upward bias for example, the random signals will also have a higher mean. This approach is best illustrated by taking the CHF BoE ERI as an example, see Figure 4.4. Over the entire sample period, this index rose from 57 in January 1975 to 147 in August 2014, therefore there is an upward bias in the returns generated by Monte Carlo sampling, effectively lowering the hurdle rate for bubble alarms and raising it for negative bubble alarms. A hypothetical bubble signal return is also illustrated which represents the average market returns following 20 random bubble signals. It is important that the number of random signals in the Monte Carlo test be the same as the number of signals being tested. For example, in the case of the CHF ERI above, we are testing 20 bubble signals. Therefore each Monte Carlo Permutation also has 20 signals, and like the bubble signals, we take the average cumulative return of these in each permutation. In total this is repeated 500 times resulting in 500 mean cumulative returns. Significant regions shaded in grey are determined by the p-value. In our study we illustrate p-values of 0.1 and 0.05. The quantiles are adjusted as the permutation distributions are approximated in accordance with Conover (1999). Therefore for a two tailed test, more conservative quantiles are given in Table 4.4.

When applying this approach we have set a holding period of 100 days. The LPPL model does not claim perfect accuracy regarding the date of a crash or rebound, indeed \( t_c \) has a right skewed
Figure 4.4: Hypothetical Bubble Signals for CHF BoE ERI

Figure 4.4 The mean hypothetical bubble signal returns are represented in blue while the Monte Carlo return percentiles are given in grey. Each Monte Carlo sampling, in this example, simulates 100 days of cumulative returns for 20 random signals (given 20 hypothetical bubble signals) and calculates the mean cumulative return. The process is repeated 500 times. Significant regions are shaded in grey. In this example, the hypothetical signal was not significant (significant days are highlighted in red should they occur).

Table 4.4: Adjusted Quantiles

<table>
<thead>
<tr>
<th>q = 5% → ̂q = 3%</th>
<th>q = 95% → ̂q = 97%</th>
</tr>
</thead>
<tbody>
<tr>
<td>q = 2.5% → ̂q = 1%</td>
<td>q = 97.5% → ̂q = 99%</td>
</tr>
</tbody>
</table>

Figure 4.3 We must adjust the significant quantiles of the Monte Carlo Permutation test (See Figure 4.4) as the permutation distributions are approximated in accordance with Conover (1999).
4.3.6 Macroeconomic Bubble Drivers

The theory behind the LPPL model argues that herding behaviour drives the super-exponential growth prior to a crash. Therefore we also investigate possible macroeconomic factors which may affect investors’ willingness and ability to herd and expose themselves to crash risk. Theory on the Leverage Cycle (Geanakoplos, 2010) suggests that leverage is too high in boom times driving asset prices upward. Subsequent crashes follow a regular pattern: bad news leads to uncertainty and price volatility, lenders become nervous and ask for increased margins on loans, leveraged buyers are forced out of their positions and large price losses follow. As noted in Section 2.3.4 Brunnermeier et. al (2008) demonstrate that sudden currency crashes unrelated to economic news can be due to unwinding of carry trades driven by leveraged speculators facing funding constraints. They note the VIX option implied volatility of the S&P 500 equity index is a useful proxy for global risk appetite and funding availability. Furthermore, the TED spread (the spread between eurodollar LIBOR and 3 month USD Treasury Bill rates) can proxy the level of credit risk and funding liquidity in the interbank markets (Mancini, Ranaldo, and Wrampelmeyer, 2012). We therefore examine the relationship between the levels of the VIX and TED spread and marketwide bubble activity. These time series were obtained from the Federal Reserve Economic Data Online Database (FRED).

We also consider common return opportunities available to investors in currency markets. Market participants may have greater incentives to follow the crowd into a currency bubble when returns available from alternative currency return strategies are depressed or unfavourable. Conversely, currency bubbles may contribute or affect the returns of different currency investment strategies. For example, bubble crashes may offer an alternative explanation to liquidity spirals for the rapid unwinding of carry trade strategies.

As outlined in Section 2.3.4, a number of risk factors have been identified in currency markets which arguably should receive a return compensation: Carry Risk (Lustig et. al, 2011), Dollar Risk (Lustig et. al, 2011), Liquidity Risk (Mancini, Ranaldo, and Wrampelmeyer, 2012), and Volatility Risk (Menkhoff et. al, 2012). Investors obtain exposure to a risk through a zero cost portfolio which goes long those currencies most exposed to the risk and short those least exposed. We outline the portfolio construction process below. The requisite spot exchange rates and 1-month forward exchange rates were obtained from Reuters (via Datastream). For comparison purposes, we build portfolios from the same 10 currency pairs under investigation in this thesis as outlined in Section 4.2.1. The data set covers June 1990 to August 2014. Portfolio returns are examined before deducting transaction costs.

Firstly, as in Lustig et. al (2011), we use $s$ to denote the log of the spot exchange rate and $f$ for the log of the forward exchange rate, both in units of foreign currency per USD. The log excess returns $rx$ on buying a foreign currency in the forward market and selling it one month later in the spot market is given by

$$rx_{t+1} = f_t - s_{t+1} \quad (4.19)$$

**Carry Risk Portfolio:** At the end of each period $t$, this portfolio goes long the two currencies with the highest interest rate and short the two currencies with the lowest interest rate. The portfolio is rebalanced at the end of each month. In accordance with Lustig et. al (2011) we allocate according to forward discounts $f - s$, with a larger discount indicative of higher foreign interest rates. Under normal conditions, forward rates satisfy the covered interest rate parity condition - the forward discount is equal to the interest rate differential $f_t - s_t \approx i_f^t - i_d^t$, where $i_f^t$ and $i_d^t$ represent the nominal risk-free rates in the foreign and domestic market over the
maturity of the contract. We compute the log currency excess returns for the portfolio \( r_{x_{t+1}} \) as the average log excess returns of the 4 positions in the portfolio. This portfolio is referred to as \( HML_{FX} \). As Lustig et. al (2011) argue, this portfolio explains the common variation in carry trade portfolio returns and captures a global risk for which carry traders earn a premium.

**Dollar Risk Portfolio:** The Dollar risk factor \( DOL \) is the average excess return for a US investor who goes long all foreign currencies available in the forward market. It can be interpreted as the “market” return in USD available to US investors and is driven by fluctuations of the USD against a broad basket of currencies.

**Liquidity Risk Portfolio:** Mancini, Ranaldo, and Wrampelmeyer (2012) introduce a tradable liquidity risk factor \( IML \) (illiquid minus liquid) given by a currency portfolio which is long in the two most illiquid currencies and short in the two most liquid currencies. With evidence for liquidity spirals and declines in market wide FX liquidity (see Appendix E), investors likely demand a premium for being exposed to liquidity risk as investors suffer higher costs during long and unexpected illiquid environments. Indeed, illiquidity may aggravate carry trade losses during market turmoil by more than 25%. We define illiquidity shocks as the residuals of an AR(1) for global FX illiquidity. High interest rate currencies like AUD, CAD, NZD exhibit the highest correlations to shocks meaning they depreciate with decreases in liquidity. In contrast low interest rate currencies like JPY tend to appreciate during times of illiquidity providing a hedge against the risk. As Mancini, Ranaldo, and Wrampelmeyer (2012) argue, the liquidity risk factor has a high correlation with \( HML_{FX} \) and the risk of liquidity spirals captured by IML appears to contribute to global risk. In accordance with Karnaukh, Ranaldo, and Soderlind (2014), we first gauge the liquidity of each currency using the three low frequency measures: Corwin-Schultz, Gibbs sampler estimate of Roll’s model, and volatility. We then construct the global FX liquidity as the first principle component across the three measures and across all currencies. We then sort currencies based on their past betas to innovations in global FX illiquidity (illiquidity = -liquidity) using a rolling window of 36 months and go long the two most illiquid currencies and short the two most liquid, as measured by their betas. The portfolio is rebalanced monthly. See Appendix E for further details.

**Volatility Risk Portfolio:** Menkhoff et. al (2012) argue that high returns to currency speculation can partly be understood as a compensation for volatility risk. High interest rate currencies are negatively related to innovations in global FX volatility and deliver low returns in times of unexpected high volatility. Likewise, low interest rate currencies provide a hedge to this risk. As a result carry trade performs especially poorly during times of market turmoil and the high returns can partly be rationalized by this risk. In order to gauge global FX volatility Menkhoff et. al (2012) calculate the absolute daily log return \( |r_{\tau}^k| \) for each currency \( k \) on each day \( \tau \) in the sample. They then average over all currencies available on any given day and average daily values up to the monthly frequency. The global FX volatility proxy in month \( t \) is given by

\[
\sigma_{t}^{FX} = \frac{1}{T_t} \sum_{\tau \in T_t} \left[ \sum_{k \in K_{\tau}} \left( \frac{|r_{\tau}^k|}{K_{\tau}} \right) \right]
\]

(4.20)

where \( K_{\tau} \) gives the number of available currencies on day \( \tau \) and \( T_t \) gives the total number of trading days in month \( t \). Volatility innovations are then proxied by the residuals from an AR(1) model fitted to this global FX volatility measure. If volatility is a priced factor then we should expect a spread in the mean returns of currencies sorted according to their exposure to volatility movements. In accordance with Menkhoff et. al (2012) we sort currencies based on their past...
beta to innovations in global FX volatility using a rolling window of 36 months. The portfolio goes long the two currencies with the lowest betas to FX volatility and short those with the two highest betas (those offering a hedge against volatility risk). The portfolio is rebalanced monthly.

**Momentum Portfolio** In addition to the above risk based investment portfolios, we also consider returns available through a momentum strategy. At the end of each month, currencies are sorted according to their excess returns over the previous three months. The portfolio then goes long the two currencies with the highest past excess returns and short those two with the lowest. Momentum returns offer profitable returns to investors and are difficult to explain by means of standard risk factors (Asness et. al., 2013). Indeed, Menkhoff et. al (2011) show that currency momentum has different properties to carry trade. The portfolio is rebalanced monthly.

**Figure 4.5** illustrates the cumulative log returns for each of the five portfolios described above. We can see large drawdowns in the Carry Risk $HML_{FX}$, Liquidity Risk IML, and Volatility Risk VOL portfolios in 2008/2009. Furthermore, while the momentum portfolio is correlated with $HML_{FX}$, it does not suffer from similar drawdowns during the financial crisis period. With these portfolio returns at hand, we can judge the performance of alternative investment opportunities available to currency investors at different points in time. We then consider the relationship between these returns and bubble activity in Section 5.6.

**Figure 4.5**: FX Risk Factor Portfolios

![Currency Risk Factor Portfolios](image)

**Figure 4.5** Cumulative Log Returns for the currency risk portfolios are illustrated above. HML captures Carry Risk, DOL gives the average excess returns, IML captures liquidity risk, VOL gives volatility risk. MOM measures the returns of the momentum portfolio.

**4.3.7 Summary**

In this chapter we detail the currency pair and the Bank of England Exchange Rate indices under consideration. Following this we detail our methodology. We first identify peaks & troughs in the time series, design a classification system and bubble alarm based on the Stylized Features of the LPPL, and critique the predictive power of the bubble alarms with a Monte Carlo Permutation Test and other performance measures. We also explore the link between bubble activity and measures of funding availability and furthermore consider the link between bubbles and common currency investment strategies. Results are presented in Chapter 5.
CHAPTER 5

Results & Discussion

5.1 Introduction

As highlighted in Chapter 3 the LPPL literature to date has placed limited emphasis on currency markets and a number of questions remain unanswered regarding the application of the model to these markets: should the model be calibrated differently for currencies than for assets such as equities, bonds, or real estate; is the model more suitably applied to currency indices or currency pairs; can we distinguish between positive bubbles in the base currency and negative bubbles in the quote currency; can a successful alarm be created which detects and forecasts the collapse of bubbles?

Having outlined the methodology in Chapter 4 the results are organised as follows: we examine the discriminative power of the parameters and constraints in Section 5.2, then having identified the appropriate classification scheme for bubbles we present a rules based alarm and review its effectiveness in Section 5.3. We outline the USD Lattice alarm in Section 5.4 as a novel method to aggregate market information from multiple currency pairs. With this information in hand we review some of the major bubbles and negative bubbles identified in the markets in Section 5.5. We consider macroeconomic factors which may lead to increased bubble activity and the relationship between currency bubbles and carry trade in Section 5.6. Finally, in Section 5.7 we explore the robustness and economic profitability of the alarms.

5.2 Discriminative Power of the the LPPL parameters

As prescribed in Chapter 4, we have identified the major peaks and troughs in each time series and contrast the aggregate kernel densities of the “pre-peak” (“pre-trough”) parameters with “non-pre-peak” (“non-pre-trough”). While not all peaks (troughs) will necessarily be preceded by bubbles (negative bubbles), provided some are, in sufficient prevalence, we would expect a divergence in the contrasted kernel densities.

Given our alarm system is based on the 4 Stylized Features of the LPPL a key question concerns the appropriate constraint to apply to the $\omega$ parameter. The remaining 3 constraints are determined on a theoretical basis. As illustrated in Figure 5.1 we can discern a number of distinguishing features in the pre-peak and pre-trough densities however the kernel densities are typically quite similar with the exception of Figure 5 (c). Suggested windows are highlighted with vertical lines. In the case of currency indices Figure 5.1 (a & b), we identify density peaks between 4.6 and 12.6 in pre-peak fits and 3.5 and 9.5 in the case of pre-trough fits. Angular
log frequency observed in other markets typically range between 6 and 13. A lower $\omega$ means the oscillations are lower in frequency and contribute more to the growth trend. This may suggest that currency bubbles are less affected by the tension between higher growth and crash expectations and that pure herding plays a larger role in the bubble. We suggest an $\omega$ window ranging from 4-13.5 for currency indices based on these results. There is also some evidence of secondary harmonics at 22.1 in the data in the case of the pre-prek fits in Figure 5.1 (a).

For currency pairs Figure 5.1 (c & d), the data seems to suggest appropriate ranges of 2.5-11. Recalling that low oscillations will simply contribute to the power law trend and with two clear maxima at 4.3 and 9.5 we suggest an appropriate $\omega$ range between 3.5 - 11 thereby dismissing the local maxima at 2.74. As highlighted in Section 4.3.3 we have not used bounds during the fitting procedure and therefore the fitting yields values below zero in some cases. Given $\cos(-x) = \cos(x)$ we account for this accordingly in our bubble alarm.

**Figure 5.1: $\omega$ - Log-angular frequency Kernel Densities**

(a) BoE ERIs Pre-Peak Densities

(b) BoE ERIs Pre-Trough Densities

(c) Currency Pair Pre-Peak Densities

(d) Currency Pair Pre-Trough Densities

In the $m$ exponent kernel densities presented in Figure 5.2, we arguably see good discriminative
power in the “pre-peak” and “pre-trough” fits as indicated by the percentage of fits contained within the theoretical 0-1 window. For example, 47% of pre-trough fits in currency pairs meet the constrain as compared to 28% of the non-pre-trough fits in Figure 5.2 (d). In no kernel density does the 0-1 window capture more than 48% of the fits suggesting that the choice between bounded optimisation, where m is bounded between 0-1, and unconstrained optimisation is relevant. Whether or not false positive alarms can be reduced by applying unconstrained optimisation remains unclear as we do not know how constrained optimisation will jointly affect all the LPPL parameters and the fits qualified as bubbles.

Figure 5.2: m - Power Law Exponent Kernel Densities

(a) BoE ERIs Pre-Peak Densities

(b) BoE ERIs Pre-Trough Densities

(c) Currency Pairs Pre-Peak Densities

(d) Currency Pairs Pre-Trough Densities

**Figure 5.2** Green Kernel density estimates (pre-peak & pre-trough) are contrasted with Blue Kernel density estimates (non-pre-peak & non-pre-trough) in order to develop a system to classify bubble periods. If pre-bubble periods differ from the norm, and exist in the periods prior to peaks (troughs), we would expect some divergence in the kernel densities. Vertical windows reflect theoretical constraints. Percentages, colour in Green and Blue give the percentage of fits falling between the suggested windows. Red points highlight local maxima.

*Figure 5.3* illustrates the distributions of \(t_c - t_2\), effectively the number of days in which a crash is predicted to occur. We emphasize that the pre-peak (pre-trough) kernel densities reflect LPPL fits on the date of the peak (trough) or closest inspection date preceding it and so we would expect the distributions concentrated to the left. As noted, we applied a bound of 200 when \(t_c - t_2\) when optimising the cost function. In all four market we identify only slight discriminative power in the \(t_c\) parameter which would support the argument that crash predic-
tions reliant solely on the $t_c$ parameters may be unreliable (See Section 4.3.2 which describes the sloppiness of $t_c$).

**Figure 5.3:** $t_c$ - Crash date Kernel Densities

(a) BoE ERIs Pre-Peak Densities

(b) BoE ERIs Pre-Trough Densities

(c) Currency Pairs Pre-Peak Densities

(d) Currency Pairs Pre-Trough Densities

Table 5.1 illustrates the discriminative power of the constrains by counting the percentage of fits in a given group which are qualified by a given constraint. Here, we can identify some discriminative power in the crash date parameter $t_c$. For example, 38% of Pre-peak fits have a crash date within 50 days versus 34% of non-pre-peak fits. We can also see the constraint on the B parameter exhibits good discriminative power. As outlined in Section 4.3.2 the positive hazard rate constraint $\sqrt{C_1^2 + C_2^2} < 1$ is less restrictive than the updated Bothmer & Meister restriction. Indeed, we notice this new restriction is the most restrictive (with the lowest percentage of fits qualifying) across all groups. Furthermore, we highlight the two different $\omega$ constraints for both markets and typically the percentage of qualifying fits within a given group does not vary greatly. In this way, our subjective decision on the correct windows for this parameter may have limited impact on our results.
Table 5.1: Discriminative Power of constraints %

<table>
<thead>
<tr>
<th>Constraint</th>
<th>BoE ERI</th>
<th>Non-Pre-Peak</th>
<th>Pre-Peak</th>
<th>Δ %</th>
<th>Non-Pre-Trough</th>
<th>Pre-Trough</th>
<th>Δ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; m &lt; 1</td>
<td>35%</td>
<td>39%</td>
<td>6%</td>
<td>32%</td>
<td>47%</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>4 &lt; ω &lt; 13.5</td>
<td>58%</td>
<td>58%</td>
<td>6%</td>
<td>58%</td>
<td>55%</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>3.5 &lt; ω &lt; 11</td>
<td>57%</td>
<td>51%</td>
<td>-6%</td>
<td>57%</td>
<td>54%</td>
<td>-3%</td>
<td></td>
</tr>
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<td>t ≤ 50</td>
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<td>38%</td>
<td>4%</td>
<td>34%</td>
<td>36%</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>B &lt; 0</td>
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<td>71%</td>
<td>9%</td>
<td>-</td>
<td>-</td>
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<td></td>
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<td>√C₁ + C₂ &lt; 1</td>
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<td>91%</td>
<td>7%</td>
<td>84%</td>
<td>83%</td>
<td>-1%</td>
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<tr>
<td>√C₁ + C₂ ≥ 1</td>
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<td>12%</td>
<td>-2%</td>
<td>14%</td>
<td>17%</td>
<td>3%</td>
<td></td>
</tr>
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Cross Rates

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Non-Pre-Peak</th>
<th>Pre-Peak</th>
<th>Δ %</th>
<th>Non-Pre-Trough</th>
<th>Pre-Trough</th>
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<td>19%</td>
</tr>
<tr>
<td>4 &lt; ω &lt; 13.5</td>
<td>54%</td>
<td>62%</td>
<td>8%</td>
<td>54%</td>
<td>53%</td>
<td>-1%</td>
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<td>3.5 &lt; ω &lt; 11</td>
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<td>83%</td>
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<td>-19%</td>
<td>35%</td>
<td>25%</td>
<td>-10%</td>
</tr>
</tbody>
</table>

Table 5.1 Above, the discriminative power of the constraints are judged based on the percentage of fits which meet the constraint in a given group. Non-pre-peak should be contrasted with pre-peak values and likewise for the troughs. A constraint has discriminative power for bubble detection if pre-extrema % > non-pre-extrema %.

5.3 Bubble Alarm & Negative Bubble Alarm Performance

Having explored the Stylized Features of the LPPL we now test the bubble classification system through a rules based alarm. As outlined in Section 4.3.4, we count the number of LPPL fits on each inspection day which meet all the 4 following constraints:

1. 0 < m < 1
2. 4 < ω < 13.5 (BoE ERIs) or 3.5 < ω < 11 (Currency Pairs)
3. B < 0 (Bubbles) or B > 0 (Negative Bubbles)
4. \( \frac{|B|m}{\sqrt{C_1 + C_2}} \geq 1 \)

We have a separate alarm for Bubbles and for Negative Bubbles. The strength of the alarm is simply the percentage of fits that meet the constraints required for a bubble or negative bubble. It is important to note, the time series reflect end of day exchange rates and therefore the results of this alarm construction can only be known at the end of the inspection day. Furthermore, LPPL is fitted to data which does not extend into the future beyond the inspection day, rather backwards, 110 - 750 days into the past.

The Bubble alarm and Negative bubble alarm for the USD BoE ERI are presented below. The Bubble alarm is measured in red, while the Negative Bubble alarm is colored in green. The strength of the alarms is measured on the respective y axes. From a cursory visual inspection the alarms appear in close proximity to peaks or troughs. The alarm is relatively sparse despite no minimum threshold being applied. For visual inspection, all the BoE ERI alarms have been presented in Appendix A.1 while the currency pair alarms are given in Appendix A.2. If taken as accurate, and the LPPL bubble definition accepted, the alarms would suggest the Bubbles and Negative bubbles are perhaps quite prevalent in currency markets.
Figure 5.4: USD BoE ERI - Bubble Alarm (Red) & Negative Bubble Alarm (Green)

As outlined in Section 4.3.5, we primarily assess the predictive performance of the bubble and negative bubble alarms with a Monte Carlo Permutation test. To do so we must specify how we interpret the alarms. Ideally we would like to test independent alarms and from the USD BoE ERI alarms above we can see several occasions where the alarms last for a number of consecutive inspection dates (e.g. the first alarm). Indeed, across all currencies, 55% of alarms persisted for more than 1 inspection day. This may suggest the alarms indicate the presence, not the end, of a bubble, or may simply be the result of our suboptimal methodology which is inaccurate regarding the exact crash date of a bubble.

Therefore, in cases where an alarm persisted for several consecutive days, we only test the final day where the alarm was active (alarms which only lasted for 1 inspection day were also tested). We emphasize, this gives the most favourable performance results for the bubble alarms and the last day in a series of inspection days with alarms is not known ex ante in a live setting. Henceforth, we refer to these alarm days as the bubble Signals. In Section 5.7 we review the performance of alternative bubble signal definitions which are knowable ex ante.

Figure 5.5 below presents the results of the Monte Carlo permutation for the USD BoE ERI. As described in Section 4.3.5, the mean returns following the Signals are represented in blue while significant regions are shaded in grey. Red Markers indicate significant days. We can see, based on 11 bubble signals, there are 50 significant days from 100 in total. In the case of negative bubble signals, based on 13 signals, 4 days are significantly positive. The results of the Monte Carlo Permutation tests for the remaining BoE ERIs are presented in Appendix B.1, while the tests for the currency pairs are given in Appendix B.2. For convenience we summarise the results of these tests, along with the success ratio of the alarms, and the mean returns in Table 5.2 below. In order to calculate the success ratios and mean returns following the Signals we must define the holding period. As noted in Section 4.3.5, if we assume a long holding period, of perhaps 100 days, we may not correctly isolate the returns which are due to a crash or rebound. After visually inspecting the Monte Carlo Permutation Test graphs we set a holding period of 30 trading days. Many, though not all, of the significant days occurred within the 30 days following the Signal. Note, success ratios assumes negative returns should follow a bubble signal and positive returns should follow a negative bubble signal. By defining
Figure 5.5: USD BoE ERI Monte Carlo Permutation Test - Bubble & Negative Bubble Signals

The significance of the Bubble and Negative Bubbles Signals for the USD BoE ERI are tested with a Monte Carlo Permutation test. The mean returns are colored blue. We intuitively expect negative returns from the bubble signal and likewise positive returns following a negative bubble signal. Significant days are marked in red. Significant quantiles are shaded in gray. In total 11 bubble signals are tested with 50 days significantly negative. 13 negative bubble signals are tested with 4 significant days.

We can summarise the results from Table 5.2 as follows:

- From a total of 22 Boe ERI Signals tested, 11 Bubbles and 11 Negative Bubble, 10 had at least 1 significant day. From 20 currency pair signals, 10 bubbles and 10 negative bubbles, 14 had at least 1 significant day. Bubble alarms and negative bubble alarms were equally represented.

- Success ratios for the Signals ranged from 0% to 89%. As noted, the bubble signal is successful if the 30 day subsequent returns are negative whereas a negative bubble signal is successful if the returns are positive. The average bubble signal for BoE ERIs had a success ratio of 56% whereas negative bubble signals had a success ratio of 55%. Average success ratios for the currency pair signals was higher at 57% and 59%.

- Average returns following the bubble signals for the BoE ERIs was -0.38% and 0.30% for negative bubble signals. Average returns following currency pair signals was -0.94% for bubbles and 0.78% following negative bubble signals.

- Assuming an agent goes short following a bubble signal and long following a negative bubble signal, 7 of 11 Boe ERI aggregated signals have positive cumulative returns over the entire backtest. 8 of 10 currency pair aggregated signals have positive cumulative returns. Among the BoE ERIs, indicators for the most traded G4 currencies (USD, GBP, EURO, JAP) had noticeably better performance.

These results would favour both the applicability and the predictive ability of the LPPL framework when applied to currency markets. Predictive performance appears to be stronger for
### Table 5.2: Performance measurement for Bubble & Negative Bubble Signals

<table>
<thead>
<tr>
<th>Currency</th>
<th># Significant Days</th>
<th># Signals</th>
<th>Success Ratio</th>
<th>Mean Return</th>
<th># Significant Days</th>
<th># Signals</th>
<th>Success Ratio</th>
<th>Mean Return</th>
<th>Cumulative Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD ERI</td>
<td>-</td>
<td>14</td>
<td>64%</td>
<td>-0.35%</td>
<td>-</td>
<td>5</td>
<td>40%</td>
<td>-0.88%</td>
<td>-0.46%</td>
</tr>
<tr>
<td>CAD ERI</td>
<td>1</td>
<td>9</td>
<td>56%</td>
<td>-0.79%</td>
<td>-</td>
<td>17</td>
<td>41%</td>
<td>-0.24%</td>
<td>1.74%</td>
</tr>
<tr>
<td>DNK ERI</td>
<td>-</td>
<td>21</td>
<td>38%</td>
<td>0.06%</td>
<td>-</td>
<td>9</td>
<td>33%</td>
<td>-0.25%</td>
<td>-4.14%</td>
</tr>
<tr>
<td>EUR ERI</td>
<td>22</td>
<td>10</td>
<td>80%</td>
<td>-1.06%</td>
<td>20</td>
<td>9</td>
<td>80%</td>
<td>0.94%</td>
<td>19.04%</td>
</tr>
<tr>
<td>GBP ERI</td>
<td>-</td>
<td>8</td>
<td>38%</td>
<td>0.02%</td>
<td>78</td>
<td>10</td>
<td>60%</td>
<td>1.60%</td>
<td>16.07%</td>
</tr>
<tr>
<td>JAP ERI</td>
<td>-</td>
<td>8</td>
<td>63%</td>
<td>0.50%</td>
<td>10</td>
<td>8</td>
<td>75%</td>
<td>1.44%</td>
<td>8.17%</td>
</tr>
<tr>
<td>NZD ERI</td>
<td>-</td>
<td>8</td>
<td>63%</td>
<td>-0.68%</td>
<td>-</td>
<td>15</td>
<td>40%</td>
<td>-0.50%</td>
<td>-3.68%</td>
</tr>
<tr>
<td>NOK ERI</td>
<td>-</td>
<td>7</td>
<td>43%</td>
<td>0.19%</td>
<td>-</td>
<td>1</td>
<td>0%</td>
<td>-0.15%</td>
<td>-2.27%</td>
</tr>
<tr>
<td>CHF ERI</td>
<td>1</td>
<td>11</td>
<td>55%</td>
<td>0.73%</td>
<td>-</td>
<td>10</td>
<td>67%</td>
<td>1.42%</td>
<td>0.26%</td>
</tr>
<tr>
<td>SEK ERI</td>
<td>12</td>
<td>4</td>
<td>75%</td>
<td>-1.10%</td>
<td>-</td>
<td>8</td>
<td>75%</td>
<td>0.17%</td>
<td>6.23%</td>
</tr>
<tr>
<td>USD ERI</td>
<td>50</td>
<td>11</td>
<td>64%</td>
<td>-1.36%</td>
<td>13</td>
<td>62%</td>
<td>0.81%</td>
<td>25.37%</td>
<td></td>
</tr>
<tr>
<td>Aggregate</td>
<td>111</td>
<td>56%</td>
<td>-0.38%</td>
<td></td>
<td>101</td>
<td>55%</td>
<td>0.36%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Currency</th>
<th># Significant Days</th>
<th># Signals</th>
<th>Success Ratio</th>
<th>Mean Return</th>
<th>Cumulative Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD / AUD</td>
<td>8</td>
<td>38%</td>
<td>-0.47%</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>USD / CAD</td>
<td>12</td>
<td>58%</td>
<td>-0.22%</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>USD / DKR</td>
<td>2</td>
<td>10</td>
<td>60%</td>
<td>-0.22%</td>
<td>3</td>
</tr>
<tr>
<td>USD / EUR</td>
<td>2</td>
<td>9</td>
<td>56%</td>
<td>-1.95%</td>
<td>24</td>
</tr>
<tr>
<td>USD / GBP</td>
<td>5</td>
<td>8</td>
<td>38%</td>
<td>0.49%</td>
<td>1</td>
</tr>
<tr>
<td>USD / JAP</td>
<td>8</td>
<td>6</td>
<td>67%</td>
<td>-1.86%</td>
<td>-</td>
</tr>
<tr>
<td>USD / NZD</td>
<td>16</td>
<td>14</td>
<td>57%</td>
<td>-1.15%</td>
<td>-</td>
</tr>
<tr>
<td>USD / NOK</td>
<td>-</td>
<td>11</td>
<td>64%</td>
<td>-1.30%</td>
<td>95</td>
</tr>
<tr>
<td>USD / CHF</td>
<td>5</td>
<td>8</td>
<td>63%</td>
<td>-1.24%</td>
<td>6</td>
</tr>
<tr>
<td>USD / SEK</td>
<td>13</td>
<td>13</td>
<td>62%</td>
<td>-1.40%</td>
<td>27</td>
</tr>
<tr>
<td>Aggregate</td>
<td>99</td>
<td>57%</td>
<td>-0.94%</td>
<td></td>
<td>123</td>
</tr>
</tbody>
</table>

**USD Lattice**

<table>
<thead>
<tr>
<th># Significant Days</th>
<th># Signals</th>
<th>Success Ratio</th>
<th>Mean Return</th>
<th>Cumulative Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>12</td>
<td>66.67%</td>
<td>1.26%</td>
<td>99</td>
</tr>
</tbody>
</table>

**Table 5.2** The performance of the alarms are tested using a Monte Carlo Permutation test, success ratios, and mean returns, for each alarm in each currency. #Significant days are determined by the Monte Carlo Permutation Tests, with the number of signals tested in each currency given by #Signals. The Success ratios are based on whether the 30 returns following a bubble (negative bubble) signals were negative (positive). Mean return measures the average return over the 30 day period following a signal. Cumulative Returns aggregates the returns following positive and negative bubble signals assuming a 30 day holding period. Aggregate statistics for success ratios and mean returns are signal weighted.
currency pair signals. This may reflect good USD prediction performance. The cumulative returns (before costs) for the USD BoE ERI are displayed in Figure 5.6 which provide an insight into the cumulative directional accuracy of the alarm signals over the entire backtest. The graphs for the remaining BoE ERIs and currency pairs are presented in Appendix C.1 and Appendix C.2. We note, the USD BoE ERI alarm signal conceivably had the best performance while in some cases, other currencies had very large drawdowns. For example, the CHF BoE ERI Appendix C.1 miscalls a market move of over 8% in 1978.

**Figure 5.6: USD BoE ERI - Cumulative Returns - Late Signals**

As noted, the model says there is a finite probability that a bubble will continue, which makes it rational for some investors to remain in the market. Therefore we would expect the model to have below perfect success ratios. Indeed, if the model predicts the end of a true bubble too early we would expect large drawdowns as per the CHF BoE ERI alarm. Of course our choice of methodology regarding inspection date frequency, data granularity (daily), fitting algorithms, and robustness techniques conceived for equity markets may also be sub-optimal when applied to FX rates which will affect predictive performance.

### 5.3.1 Signal Strength

We have not considered the effect of alarm strength when backtesting the bubble and negative bubble alarms. This was both for simplicity and because the alarms were sparse in nature. As outlined above, **Signal strength** is simply the percentage of fits on a given inspection day which meet the 4 constraints. We regress the 30 day market returns on the *Signal* strength (in the case of a series of alarms, the strength on the last day of the series). Bubble *Signals* are given a negative sign on the x-axis while negative bubble *Signals* have a positive sign.

The regression results are illustrated in Figure 5.7 and would suggest that alarm strength does indeed have meaning, with a positive correlation found for both the ERI and currency pair signals. This may be due to the robustness effect of using a *multiscale* inspection approach or indeed may suggest a bubble (negative bubble) which is occurring on multiple scales will have
Figure 5.7: Signal Strength versus Market Returns - BoE ERI and Currency pair Signals

Figure 5.7 Above, we regress 30 day market returns on signal strength. The strength of bubble signals are given as a negative sign on the x-axis while the strength of negative bubble signals have a positive sign. The regression line is colored magenta.

a sharper correction when it crashes (rebounds).

5.4 USD Lattice Alarm

Currency pairs present an interesting problem compared to other assets as its is not clear, in the event a bubble is detected in the exchange rate time series, which currency is in a bubble or negative bubble. For example in the period prior to March 1985, some some researchers argued the USD was in a bubble while others argued the GBP was in a negative bubble. As a single currency can conceivable be in either a bubble or negative bubble, it is not clear which currency is driving the exchange rate during overly exuberant periods.

One solution is to simply rely on the alarm signals given for a BoE Exchange Rate Index as they capture an overall value of a currency. However it seems possible if not likely, that the construction of the index, which represents a weighted average of a number of exchange rates, distorts or destroys some of the log-periodic power law patterns which may precede a crash. For example, the USD may be in a bubble versus the GBP but not versus the CHF and it perhaps is not possible to capture this by relying on the USD BoE ERI alone when constructing an alarm.

Therefore, an alternative route is to examine multiple exchange rates of a given currency, and count the number of times a bubble Signal is given for that currency. For example, we have produced bubble Signals for 10 USD exchange rates, USD/GBP, USD/CHF, USD/JPY etc. Therefore on a given inspection date, we can count the number of times a bubble signal is given for the USD across the 10 exchange rates. If a bubble signal occurs in $N$ (e.g. $N = 2$) or more exchange rates it suggests, at that point in time, the USD is in a bubble and is the primary driver of an exchange rate. Likewise, if $N$ or more negative bubbles signal occurs in the 10 exchange rates, it suggests the USD is in a negative bubble and it, not the quote currency, is driving the exchange rate. In this way we construct a Lattice Alarm. We simply count the number of bubble and negative bubble Signals given for the USD on an inspection day. If $x \geq N$ signals are given, we assign the Lattice Alarm with a value $x$. Thus we aggregate information about the USD from multiple exchange rates. We have set $N = 2$, therefore the USD must be in a bubble or negative bubble in two cross rates to meet the Lattice alarm criteria.
Results from this approach suggest that monitoring a single exchange rate index for the presence of bubbles may not fully capture the bubble behaviour of a currency. Figure 5.8 below displays the Lattice Alarm. The USD BoE ERI is used to proxy the value of the USD. Note, while the individual USD exchange rates used to construct the Lattice Alarm form part of the BoE ERI index, the alarms are not given a weighting that reflects their weight in the BoE ERI index. We notice that the frequency of signals has increased from 11 bubble signals to 12 and from 13 negative bubble signals to 23 when compared to the USD BoE ERI signals (see Table 5.2 above). Furthermore, the cumulative returns are 46.75% over the entire backtest. This compares with 25.37% for the USD BoE ERI. Appendix D presents the cumulative return graph and Monte Carlo Permutation test for the Lattice Alarm. Indeed if a signal is informative we would expect the cumulative returns to increase as the number of independent signals increases. However, we also notice that the success ratios is higher than for the USD BoE ERI alarm. Bubble signal success ratios are 66% while negative bubble signals have a success ratio of 78%. The mean returns in the 30 days following a bubble signal are -1.26% and 1.72% following a negative bubble signal, compared with -1.36% and 0.78% for the USD BoE ERI alarm. We suggest the overall increased performance versus the BoE ERI alarm signals may come from a number of sources: false positives are reduced by requiring verification of bubbles across multiple markets, the USD BoE ERI index obscures a number of LPPL patterns due to its averaging construction, bubbles which grow in multiple exchange rates crash with greatly force.

**Figure 5.8: USD Lattice Alarm - Bubble & Negative Bubble alarms**

**Figure 5.8 The Lattice Alarm aggregates alarm signals from 10 USD exchange rates. Each exchange rate is organised with the USD as the base currency. At the end of each inspection day, the number of bubble and negative bubble signals $x$ are counted, provided this exceeds equals or exceeds 2, the alarm is assigned a value of $x$. Bubble alarms are colored red and negative bubble alarms are green. In effect, the strength of a bubble (negative bubble) alarm reflects the number of cross rates where the USD is in a bubble (negative bubble).**

### 5.5 Economic Backdrop to the Alarms

As indicated in Table 5.2 there are over 400 bubble or negative bubble signals in total across 21 time series. While it is not within the scope of this thesis to review each signal individually, in this section we briefly examine some of the prominent bubbles and negative bubbles identified by the LPPL model. Our primary method here was to search for major news stories which
Table 5.3: Economic context of Bubble & Negative Bubble Alarms

<table>
<thead>
<tr>
<th>Currency</th>
<th>Bubble Class</th>
<th>News Source</th>
<th>Date</th>
<th>News Event</th>
<th>BoE ERI Signal</th>
<th>Lattice Alarm</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD</td>
<td>Bubble</td>
<td>Bloomberg</td>
<td>26-Feb-85</td>
<td>5 European Central Banks sold $1bn, follows Fed Chair Volcker’s comments on strong dollar</td>
<td>26-Mar-85</td>
<td>12-Feb-85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>15-Jun-89</td>
<td>Trading stampede, Bank of Japan willing to spend $1bn/day to push dollar down</td>
<td>21-Jun-89</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>26-Aug-98</td>
<td>Russian Ruble Crisis, terms of national debt restructuring announced on Aug 25th</td>
<td>21-Aug-98</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>09-Mar-09</td>
<td>With threat of banking system collapse, Citi &amp; Bank of America announce profits</td>
<td>-</td>
<td>16-Mar-09</td>
</tr>
<tr>
<td></td>
<td>Negative Bubble</td>
<td>Reuters</td>
<td>30-Oct-78</td>
<td>A major dollar support program is introduced with wide support from Central Banks</td>
<td>24-Nov-78</td>
<td>13-Oct-78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>11-Feb-91</td>
<td>Desert Storm (Gulf War) starts 17th Jan, ends 28th Feb</td>
<td>-</td>
<td>15-Feb-91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reuters</td>
<td>09-Jan-04</td>
<td>Bank of Japan buys 1.76tn yen on Jan 9th, G10 countries discuss dollar decline on Jan 11th</td>
<td>-</td>
<td>05-Apr-04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>17-Mar-08</td>
<td>Bear Stearns bailed out Mar 14th, Fed lowers benchmark 75bps to 2.25 on Mar 19th</td>
<td>29-Jul-08</td>
<td>19-May-08</td>
</tr>
<tr>
<td>JPY</td>
<td>Bubble</td>
<td>Bloomberg</td>
<td>16-Aug-93</td>
<td>L. Summers, treasury Under Secretary, says yen rise dangerous, FED intervenes in market</td>
<td>-</td>
<td>05-Apr-93</td>
</tr>
<tr>
<td></td>
<td>Negative Bubble</td>
<td>Reuters</td>
<td>17-Apr-90</td>
<td>Nikkei peaks Jan ‘90, 4th round of monetary tightening March 20 (Nikkei looses 50% by Aug)</td>
<td>20-Mar-90</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>11-Aug-98</td>
<td>Russian Ruble crisis, Germany is the largest creditor and buys yen</td>
<td>21-Aug-98</td>
<td>-</td>
</tr>
<tr>
<td>GBP</td>
<td>Bubble</td>
<td>Bloomberg</td>
<td>24-Aug-90</td>
<td>Iraq invades Kuwait on Aug 2nd</td>
<td>28-Aug-90</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Negative Bubble</td>
<td>Bloomberg</td>
<td>28-Jan-85</td>
<td>Margaret Thatcher, PM, intervenes ending hands off approach with sharp interest rates rise</td>
<td>12-Feb-85</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FT</td>
<td>30-Dec-08</td>
<td>Mervyn King, Bank of England, says UK is entering recession first time in 16 years</td>
<td>02-Feb-09</td>
<td>-</td>
</tr>
<tr>
<td>CHF</td>
<td>Bubble</td>
<td>Reuters</td>
<td>25-Sep-78</td>
<td>Oct 78, Swiss National Bank sets floor on DM/CHF exchange rate</td>
<td>13-Jan-78</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bloomberg</td>
<td>10-Aug-11</td>
<td>Aug 11th, Swiss National Bank VP Thomas Jordan said a peg against Euro is legal</td>
<td>28-Sep-11</td>
<td>-</td>
</tr>
<tr>
<td>EUR</td>
<td>Negative Bubble</td>
<td>The Telegraph</td>
<td>25-Oct-00</td>
<td>Central Banks in Europe, US, &amp; Japan intervene to support Euro</td>
<td>20-Dec-00</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.3 News stories have been identified which coincide with prominent bubble and negative bubble signals which provide economic context to the signals. Stories have been sourced from the financial media with the approximate date of the article given. The USD Lattice Alarm verifies the BoE ERI alarms in the case of the USD.
coincide with and may offer some explanation for the bubbles. News articles were sourced from Bloomberg, Reuters, The Financial Times, and The Telegraph. We do not claim the articles were written in an unbiased way, or that they offer a definitive explanation for the exchange rate movements, however they provide an economic backdrop for a bubble signal. Table 5.3 presents the dates of signals given for a number of BoE ERIs. Additionally the Lattice Alarm verifies these alarm signals for the USD.

It is interesting to note that in 9 instances we can identify a central bank intervention in markets close to signal dates. Indeed in the case of the CHF, the Swiss National Bank set a floor on the DM/CHF exchange rate in 1978 and again set a floor in the EURO/CHF exchange rate in 2011. (Note the alarm was 8 months before the 1978 intervention). This would support the argument that the bubble alarms are indeed identifying bubble-type periods, where central banks perceived deemed intervention necessary. Alarms also coincided with times of financial stress. A negative bubble alarm was given for the USD in March 2008 as Bear Stearns was collapsing and it was followed by a positive bubble alarm in the depth of the financial crisis in March 2009. Furthermore, a negative bubble was signalled for the JPY during the collapse of the Nikkei 225 stock index and real estate market in early 1990. The alarms also suggests the USD entered a bubble during the Russian Ruble crisis in August 1998 while at the same time a negative bubble alarm was signalled for the JPY. In two instances, an alarm was signalled in close proximity to war times, a negative bubble for the USD during operation Desert Storm during the Gulf War in 1991 and a bubble for the GBP when Iraq invaded Kuwait in 1990.

5.6 Macroeconomic Bubble Drivers

As outlined in Section 4.3.6 we also investigate the economic factors which may affect investors’ willingness and ability to herd and expose themselves to crash risk. In this regard we consider the VIX index as a proxy for global risk appetite and funding availability and the TED spread as a proxy for credit risk and funding liquidity. We might expect increased bubble activity with lower values of these measure as such values would reflect easy funding availability and willingness to take risk.

To measure bubble activity across all currencies we consider the bubble and negative bubble signals discussed above. If taken as accurate, these signals indicate the termination of a bubble period. Therefore we can denote the preceding N days before a signal as a bubble or negative bubble period accordingly (these are not knowable \textit{ex ante}, the bubble signal is given at the end of a bubble). To create a single measure of bubble activity across multiple markets we aggregate the returns during bubble or negative bubble periods into a single Bubble Index. On a given day $t$ we sum the returns of those currencies in a bubble or negative bubble. The returns of a negative bubble are multiplied by -1. As we sum returns in case of 2 or more bubbles, the index will advance more rapidly if multiple bubble periods are overlapping. Furthermore, the index will advance more rapidly when the returns of any given bubble are higher, whether due to a more extreme bubble or the late stages of a bubble. In this way the Bubble Index broadly captures bubble activity across multiple markets.

We set $N = 252$ trading days to capture the returns 1 year prior to the termination of a bubble or negative bubble. We emphasise that this is not an investable index as we do not know the pre-bubble period \textit{ex ante}, we identify the 252 days based on bubble signals which only occur at the end of the bubble period. We only consider currency pairs when constructing the index as the risk factor portfolios we constructed using only currency pairs. Figure 5.9 illustrates the bubble index along with its first difference. We divide both the index and the first difference by their maximum value giving an upper limit of 1 for both measures. We see strong bubble activity prior to the 1985 Dollar and in the subsequent negative bubble period,
as the US Dollar was in a bubble against multiple currencies as indicated by the USD *Lattice* Alarm in *Figure 5.8*. Furthermore we note that bubble activity appears to subside in the wake of the recent financial crisis.

**Figure 5.9**: Global Currency Pair Bubble Index & Index First Difference

![Graph of Global Currency Pair Bubble Index and its First Difference](image)

**Figure 5.9** The Bubble Index reflects bubble activity across multiple currency pairs, identified as the 252 trading days prior to a bubble or negative signal. As the signal is given at the end of a bubble these 252 days are not knowable ex ante. Negative bubble returns are multiplied by -1 when constructing the index. The index advances more rapidly when bubbles are occurring in multiple markets or if bubble returns in a single market are stronger. The index first difference in the index is also displayed. We see strong bubble activity prior to the 1985 Dollar bubble. Bubble activity has diminished in the wake of the recent financial crisis.

**Figure 5.10**: VIX level & TED spread v ΔBubble Index

![Bar charts of VIX and TED spread against ΔBubble Index](image)

**Figure 5.10** We bucket the VIX and TED spread according to the prevailing level of bubble activity and average. A high (low) change in the Bubble Index is indicative of strong (weak) bubble activity. Results suggest funding availability and risk appetite as proxied by the VIX and TED spread have limited impact on bubble activity.

In order to study the effects that funding availability and risk appetite proxied by the VIX and credit risk and funding availability proxied by the TED spread we undertake a simple graphical analysis to visualise the relationship between these variables and the distribution of the Bubble Index first difference. We first identify the quartiles in the Bubble Index First Difference which are indicated in *Figure 5.9*. We then bucket VIX levels and TED spreads according to the prevailing Bubble activity. High (Low) values for Δ Bubble Index are observed in times of high (low) bubble activity where the bubble index is advancing rapidly (slowly).
As indicated in Figure 5.10 we observe little variation in the average VIX level across different levels of bubble activity and no clear pattern in the TED spread buckets. This suggests that our proxies for funding availability and risk appetite have limited impact on fueling bubble activity. Liquidity spirals theory suggests an asymmetric response to fundamental shocks, with negative shocks affecting funding and market liquidity negatively and positive shocks having limited impact. Our Bubble Index is constructed from pre-crash bubble periods, likely supported by positive shocks, which may explain these results.

Additionally, we consider how returns earned from currency risk factor portfolios relate to bubble activity. If economic agents are more likely to follow the crowd and herd into a bubble when their alternative return opportunities are depressed, we would expect average returns to these portfolios to be lower in times of high bubble activity. To analyse the relationship between currency risk portfolio returns and bubble activity we repeat the graphical analysis employed above. Results from Figure 5.11 would suggest that risk factor portfolios on average do not perform as well during periods of high bubble activity. Indeed we see across all 5 strategies average returns decrease with increasing bubble activity. This would support the argument that currency bubbles may be driven by currency investors seeking alternative return opportunities in times when common investment strategies are underperforming. We also notice an improvement in average returns during times of high bubble activity suggesting a richer relationship.

**Figure 5.11:** Currency Risk Portfolios Returns v ΔBubble Index

![Figure 5.11](image)

*Figure 5.11 Monthly returns of the currency portfolios are placed in 4 buckets according to the prevailing level of bubble activity. H (L) denotes high (low) levels of bubble activity given by the highest (lowest) quartile of changes in the Bubble Index. HML=Carry Risk, DOL=Dollar Risk, MOM=Momentum Portfolio, IML=Liquidity Risk, VOL=Volatility Risk. Results suggest the portfolio returns are depressed in times of high bubble activity supporting the argument that herding may increase in times when alternative returns are unattractive.*

5.6.1 Carry Trade and Currency Crashes

We explore bubbles and carry trade in a more granular fashion below. To do so we first decompose the returns to our carry trade strategy into two components: returns earned from the interest rate differential and returns earned from changes in the spot rate. Figure 5.12 illustrates these two building blocks along with the original HML Carry risk portfolio returns. We can see returns to the carry portfolio are primarily generated from the interest component.
Figure 5.12: HML Carry Risk Portfolio - Interest Rate and FX Spot Change Component

Figure 5.12 By decomposing the Carry Risk portfolio returns into those driven by interest rate differentials and those driven by changes in FX spot rates we can examine the relationship between currency bubbles and carry trade bubbles and crashes (See Table 5.4). Peaks in the spot rate components are colored red. This points follow periods of strong carry performance and precede periods of portfolio losses.

with the spot rate component increasing the volatility of the strategy and detracting from returns in aggregate. In our investigation we have focused only on the peaks in the spot component, highlighted in red. These points follow strong returns to the spot component and the carry portfolio and precede large drawdowns in both. We cross reference the holdings in the carry portfolio in the 12 months prior to and after the peaks with the bubble and negative bubble Signals created for the 10 currency pairs. As outlined in Section 4.3.6, the carry portfolio is comprised of 2 long positions and 2 short positions and rebalanced each month. Table 5.4 identifies a number of positive bubble signals for positions held within the carry portfolio which are in close proximity to the peak dates. In the case of the DKR and AUD, bubble signals warned against holding the positions close to the peak values of the carry portfolio. Both currencies subsequently greatly depreciated suggesting that bubble crashes may partly explain the unwinding and losses of carry trade strategies. While not prescient, a bubble signal for the JPY may indicate that part of the losses in late 1992 we driven by a bubble in the JPY (the carry portfolio was short the JPY) or indeed that carry trade unwinding drove enough capital flows back to Japan to instigate a bubble. While the SEK was not present in the carry portfolio following the 1992 peak, bubble signals would suggest that a bubble in the currency may have driven the carry trade portfolio to this maxima. Indeed this suggests that herding and bubble activity may offer an explanation for the Forward Premium puzzle - where high-interest-

Table 5.4: Carry Trade exposure to Currency Crashes & Bubbles

<table>
<thead>
<tr>
<th>Peak</th>
<th>CCY</th>
<th>Position</th>
<th>#months held</th>
<th>% return</th>
<th>BoE ERI Alarm</th>
<th>Currency Pair Alarm</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DKR</td>
<td>Long</td>
<td>10 of 12 post-peak</td>
<td>-23.5%</td>
<td>16-Sep-92</td>
<td>02-Sep-92</td>
<td>Carry Crash</td>
</tr>
<tr>
<td>1</td>
<td>JPY</td>
<td>Short</td>
<td>8 of 12 post-peak</td>
<td>16.1%</td>
<td>22-Nov-93</td>
<td>06-Dec-93</td>
<td>Carry Crash</td>
</tr>
<tr>
<td>1</td>
<td>SEK</td>
<td>Long</td>
<td>8 of 12 pre-peak</td>
<td>21.1%</td>
<td>16-Sep-92</td>
<td>02-Sep-92</td>
<td>Carry Bubble</td>
</tr>
<tr>
<td>2</td>
<td>AUD</td>
<td>Long</td>
<td>7 of 12 post-peak</td>
<td>-17.7%</td>
<td></td>
<td>12-Aug-08</td>
<td>Carry Crash</td>
</tr>
</tbody>
</table>

Table 5.4 Positions held in the HML Carry Risk portfolio are cross referenced with bubble alarms in order to explain carry trade bubbles and crashes. Position gives the exposure of the portfolio to the currency while % return gives the change in currency over the 12 month period following or preceding the peaks (JPY short resulted in loss). #months held refers to the number of months where the currency was 1 of the 4 in the Carry portfolio.
rate currencies tend to appreciate rather than depreciate in accordance with uncovered interest rate parity - as occurred in this case for the SEK. In Table 5.4 we label classify these events as *Carry Bubbles* or *Carry Crashes* accordingly.

### 5.7 Robustness & Economic Profitability

As noted in Section 5.3, the bubble signals we defined, which only tested the final day of an alarm in cases where alarms persisted for several consecutive inspection days, resulted in the most favourable performance for the model. Indeed, the final day in a series of consecutive alarms is not knowable *ex ante* and it may have been the final day exactly because large crashes or rebounds ensued and turned off the bubble alarm. As highlighted above, 55% of alarms lasted for more than 1 inspection day and were affected by this signal definition. Therefore we also consider an alternative signal (*ALT Signal*) which goes long or short on the day immediately following an alarm. This signal effectively only considers the first day an alarm was given in cases where the alarm lasted for several consecutive day and is knowable *ex ante*.

In order to compare the performance of this alternative signal specification with the existing definition, we aggregate the bubble signal returns into a single portfolio and calculate the non-compounding cumulative return. This portfolio goes long following a negative bubble signal and short following a bubble signal. When 2 or more signals indicated a position be taken, the portfolio was invested in equal amounts in each position. We examine holding periods of 10 and 30 days and the effect of transaction costs with a 0.25% cost per trade (0.5% to enter and exit a position). Figure 5.13 presents the results of this analysis for BoE ERIs and Currency Pairs.

Results suggest the the Currency Pair alarms had better predictive ability than the BoE ERI alarms with only the BoE ERI 30 day Alarm outperforming its Currency Pair equivalent (53% versus 40% cumulative returns). Furthermore the 10 day holding period portfolios outperforms their 30 day equivalent suggesting that the LPPL model has greater predictive performance at shorter horizons. We see that the *ALT Signal* does not perform as strongly as the signal definition adopted throughout this Chapter. However before cost performance continues to be positive supporting the predictive performance of the model. Cumulative returns may have been driven down when strongly performing signals no longer met the ALT Signal definition or because many signals now mistime the ending of a bubble by reacting too early. Although not presented graphically, we note that the USD *Lattice* alarm had cumulative returns of 30.77% based on this *ALT Signal* definition, which continues to outperform the USD BoE ERI alarm returns of 25.37% (which were based on the more favourably performing signal definition). Transaction costs eliminated profit opportunities for all portfolios with the exception of the Currency Pairs portfolio with 10 days holding period which returned 39% non-compounded. Compounded returns before costs for this portfolio reach reached 241% however after transaction costs, returns for comparable to the non-compounded returns at 40%.

It is interesting to note that the percentage of days the portfolios held a position ranged from 11.2% to 12.5% based on a 10 day holding period and from 34.34% to 40.5% for a 30 day holding period. The Currency Pairs portfolios were at the low end of the range possibly due to overlapping bubble signals relating the to USD.

Throughout this chapter we elected not to perform analysis on an in-sample out-of-sample basis as the the $\omega$ constraint is the only aspect of the alarm we would expect to change out-of-sample. This affects those fits qualified as a bubble or negative bubble. The remaining 3 constraints are based on the theory behind the model. The decision regarding the correct $\omega$ range was subjective in nature as the kernel densities for $\omega$ typically did not provide strong
Figure 5.13: Cumulative (Non-compounding) Returns Across All Alarms

**Figure 5.13** We consider all currencies when analysing the aggregate alarm performance by creating a portfolio which goes long following a negative bubble signal and short following a bubble signal and calculating the non-compounded cumulative returns. We also explore an Alternative Signal definition which effectively only considers the first day an alarm was given in cases where the alarm lasted for several consecutive days and is knowable ex ante. We also assess economic performance by accounting for transaction costs of 0.25% where AC denotes the after cost time series.

contrasts between the “pre-peak” and “non-pre-peak” fits. Therefore splitting the data into an in-sample and out-of-sample period would likely have made the decision less clear. However as indicated in Table 5.1, different $\omega$ ranges may have had only limited impact on the acceptance or rejection of a fit. The updated positive hazard rate constraint appeared to be most restrictive.

### 5.8 Summary

The results of this chapter can be summarised in a number of points:

1. The log frequency parameter $\omega$ was found to be lower for currency markets than the 6-13 range typical in other markets. Suggested acceptable ranges are 3.5-11 for currency pairs, and 4-13.5 for BoE Exchange Rate indices. This may suggest that currency bubbles are less affected by the tension between higher growth and crash expectations and that pure herding plays a larger role in the bubble.

2. At most, 48% of fits have $m$ exponent parameters between 0-1 suggesting that bounded optimisation (with bounds of 0-1) may increase the number of bubble alarms, and perhaps...
false positives.

3. The $t_c$ crash date parameter appears to have limited discriminative power in currency markets, as also seen in equity markets.

4. An updated positive hazard rate constraint, which is compatible with the new LPPL formulation, is the most restrictive constraint, potentially reducing false positives.

5. The LPPL model appears to both be applicable to and have good predictive ability within currency markets. Bubble Signals on average exhibited success ratios of 55-59%, with positive cumulative returns in 15 of 21 tested time series, and 24 of 42 alarms displaying at least 1 significant day.

6. Predictive performance appears better for USD currency pairs than BoE ERIs based on success ratios, mean returns, and cumulative returns. This may be driven by good USD predictive ability. G4 BoE ERI alarms had stronger performance than those for less liquid currencies.

7. There is evidence to suggest that stronger alarms (calculated as the percentage of fits meeting bubble conditions) predict larger market moves.

8. A Lattice Alarm may offer a novel method to aggregate information on a currency from across a number of different exchange rates and a means to distinguish bubbles from negative bubbles in a currency pair. Accuracy and frequency is increased when compared to an alarm calculated solely on a USD BoE ERI suggesting the Lattice Alarm may be more suited to monitoring the value of a currency than an alarm constructed on a currency index.

9. We find little evidence that funding availability contributes to bubble activity. However, we find that bubble activity is greater during times when the returns earned from currency risk factor portfolios are depressed. This supports the argument that herding behaviour is greater when economic agents face unattractive return alternatives.

10. We document the relationship between HML Carry Risk portfolio returns and currency bubbles. We find that currency bubbles may contribute to understanding the Forward Premium Puzzle and the drawdowns experienced by carry trade portfolios.

11. The LPPL model continues to exhibit predictive performance using a more robust definition of Bubble Signals (knowable ex ante). Evidence suggests the LPPL model has greater predictive performance at shorter horizons. After accounting for trading costs, some signal definitions continue to exhibit positive cumulative returns.
Conclusion

6.1 Summary & Conclusion

This thesis has explored the applicability of the Johansen-Ledoit-Sornette Log-Periodic Power Law model to currency markets as a means to detect the presence of bubbles and forecast their collapse. Previous research into the LPPL model concentrates on equity markets and the examination of currencies has been sparse to date. Further research into these markets was motivated on the following points: currencies pair exchange rates reflect the value of two separate currencies and as either can be in a bubble or negative bubble it is not trivial to identify the driving force behind an exchange rate change in bubble periods; the value of currencies are often gauged through a currency index and perhaps a bubble indicator is more suitably constructed on such indices than on cross rates; central banks play a key role in currency markets, often intervening to minimise unfavourable currency moves, and may extinguish bubbles early in their formation making the LPPL model less relevant for currencies for other markets.

Our results show that currency bubbles exhibit slightly different characteristics on average to those documented in other assets. Lower values for the angular log-frequency term \( \omega \) would suggest currency bubbles are defined more by power law growth. This may indicate that herding behaviour in currency bubbles, as compared to other assets, is a larger component of the price action than competing crash expectations.

We show that the LPPL model has slightly greater predictive performance for currency pairs than currency indices, potentially due to good USD predictive ability. The predictive performance of a novel Lattice alarm suggests that aggregating market information from multiple currency pairs may capture bubble information not reflected in currencies indices. Additionally it offers a means to distinguish bubbles in the base currency from negative bubbles in the quote currencies in the context of currency pairs.

While we find evidence of central bank market interventions close to bubble alarms, the frequency of the alarms would suggest that central banks do not extinguish all currency bubbles before they come into formation. Indeed, the bubble alarms appear to coincide or precede central bank intervention based on a cursory review of news articles in the financial media. Furthermore, the predictive performance of the bubble signals lends support to the applicability of the LPPL model to currency markets.
6.2 Future Research

While this thesis has broadly explored the application of the LPPL model to currency markets, it also opens room for further research in the area. We identify three topics for additional research:

Bubble Signal Generation
A key difficulty with the alarm system presented in this paper is the longevity of the bubble alarms. In many cases, the alarms persist for several consecutive inspection days and it is not clear how they should be interpreted. We define two different bubble signals which interpret the alarms and allow for performance assessment however both have shortcomings.

We test for bubbles every 10 days along the time series, but based on stronger predictive performance at shorter horizons (10 versus 30 days), a more granular inspection of currency time series may allow for greater accuracy regarding the termination of a bubble. Furthermore we notice in several instances that the bubble signals appear to mistime the termination of a bubble indicating that steps should be taken which attempt to eliminate false positives. A number of techniques and alternative model specifications have been suggested in Sornette et. al (2013).

Lattice Alarms
We introduce the Lattice alarm as a novel way to aggregate market information across a number of cross rates. Performance of this alarm suggests that monitoring a currency based solely on a currency index may not be optimal. However, we have only tested this concept for the USD. In order to verify this technique we suggest further exploring the Lattice alarm for non-USD currencies.

Carry Trade
The performance of Carry trade and the Forward Premium Puzzle is a topic often explored in academic literature. In this thesis we briefly consider the relationship between HML Carry Risk portfolio returns and the bubble signals we have generated. Results would suggest a relationship between the two, with currency bubbles contributing to carry trade returns and currency crashes deterring from returns. Furthermore, in the case of the JPY we suggest that carry trade flows and the unwinding of carry trade positions may instigate or contribute to a currency bubble. We suggest future research may explore this relationship in more detail.
Appendices
APPENDIX A

Bubble Alarms

A.1 BoE Exchange Rate Indices Alarms

Appendix A.1 presents the Bubble & Negative bubble alarms for the BoE Exchange Rate Indices. There is a separate alarm for Bubbles and for Negative Bubbles. Bubble alarms are given in red, Negative Bubble alarms are given in green. They are indicated on the respective y axes. The alarms are zero if no fits meet the constraints - no minimum threshold has been applied. To define the strength of the alarm we find the percentage of LPPL fits on each inspection day which meet all the 4 following constraints:

1. \( 0 < m < 1 \)
2. \( 4 < \omega < 13.5 \)
3. \( B < 0 \) (Bubbles) or \( B > 0 \) (Negative Bubbles)
4. \( \frac{\left|B\right|m}{\omega \sqrt{C_1^2 + C_2^2}} \geq 1 \)
A.2 Currency Pair Alarms

Appendix A.2 presents the Bubble & Negative bubble alarms for the Currency Pairs. There is a separate alarm for Bubbles and for Negative Bubbles. Bubble alarms are given in red, Negative Bubble alarms are given in green. They are indicated on the respective y axes. The alarms are zero if no fits meet the constraints - no minimum threshold has been applied. To define the strength of the alarm we find the percentage of LPPL fits on each inspection day which meet all the 4 following constraints:

1. $0 < m < 1$
2. $3.5 < \omega < 11$
3. $B < 0$ (Bubbles) or $B > 0$ (Negative Bubbles)
4. $\frac{|B|m}{\omega \sqrt{C_1^2 + C_2^2}} \geq 1$

USD / AUD Spot Rate − Bubble Alarm & Negative Bubble Alarm

USD / CAD Spot Rate − Bubble Alarm & Negative Bubble Alarm
Monte Carlo Permutation Significance Tests

B.1 BoE Exchange Rate Indices Monte Carlo Permutation Tests

Appendix B.1 presents the results of the Monte Carlo Permutation Tests for the BoE ERIs. The mean returns are colored blue. We intuitively expect negative returns from the bubble signal and likewise positive returns to follow a negative bubble signal. Significant days are marked in red. Significant quantiles are shaded in gray. Further details are given in Section 4.3.5.
NZD BoE ERI – Bubble Signal v Monte Carlo Returns
8 signals – 0 significant days

NZD BoE ERI – Negative Bubble Signal v Monte Carlo Returns
15 signals – 0 significant days

SEK BoE ERI – Bubble Signal v Monte Carlo Returns
4 signals – 12 significant days

SEK BoE ERI – Negative Bubble Signal v Monte Carlo Returns
8 signals – 0 significant days

USD BoE ERI – Bubble Signal v Monte Carlo Returns
11 signals – 50 significant days

USD BoE ERI – Negative Bubble Signal v Monte Carlo Returns
13 signals – 4 significant days

Mean Bubble Signal Returns
Monte Carlo Returns
B.2 Currency Pair Monte Carlo Permutation Tests

Appendix B presents the results of the Monte Carlo Permutation Tests for the Currency Pairs. The mean returns are colored blue. We intuitively expect negative returns from the bubble signal and likewise positive returns to follow a negative bubble signal. Significant days are marked in red. Significant quantiles are shaded in gray. Further details are given in Section 4.3.5.
USD / CHF Spot Rate – Bubble Signal v Monte Carlo Returns
8 signals – 5 significant days

USD / CHF Spot Rate – Negative Bubble Signal v Monte Carlo Returns
21 signals – 6 significant days

USD / DKR Spot Rate – Bubble Signal v Monte Carlo Returns
10 signals – 2 significant days

USD / DKR Spot Rate – Negative Bubble Signal v Monte Carlo Returns
17 signals – 3 significant days

USD / EUR Spot Rate – Bubble Signal v Monte Carlo Returns
9 signals – 2 significant days

USD / EUR Spot Rate – Negative Bubble Signal v Monte Carlo Returns
17 signals – 24 significant days
USD / NZD Spot Rate – Bubble Signal v Monte Carlo Returns
14 signals – 16 significant days

USD / NZD Spot Rate – Negative Bubble Signal v Monte Carlo Returns
17 signals – 0 significant days

USD / SEK Spot Rate – Bubble Signal v Monte Carlo Returns
13 signals – 13 significant days

USD / SEK Spot Rate – Negative Bubble Signal v Monte Carlo Returns
12 signals – 27 significant days
Alarm Signal Cumulative Returns

C.1 BoE Exchange Rate Indices Cumulative Returns

Non-compounded cumulative returns measures the cumulative directional accuracy of the Bubble and Negative Bubble Signals. Short (Long) positions are taken after Bubble (Negative Bubble) signals. Positions are held for 30 days. The cumulative returns are before costs.
C.2 Currency Pair Cumulative Returns

Non-compounded cumulative returns measures the cumulative directional accuracy of the Bubble and Negative Bubble Signals. Short (Long) positions are taken after Bubble (Negative Bubble) signals. Positions are held for 30 days. The cumulative returns are before costs.
To construct the USD *Lattice* Alarm we simply count the number of bubble and negative bubble Signals given for the USD across all cross rates on a given inspection day. All cross rates have been organised so that the USD is the base currency. If $x$ cross rates give signals that the USD is in a bubble we assign the Lattice Alarm with a value $x$. The alarm is illustrated below.

Non-compounded cumulative returns measures the cumulative directional accuracy of the USD *Lattice* Alarm Bubble and Negative Bubble Signals. Short (Long) positions are taken after Bubble (Negative Bubble) signals. Positions are held for 30 days. The cumulative returns are before costs. Cumulative Returns are given below.

The results of the Monte Carlo Permutation Tests for the USD *Lattice* Alarm are also presented below. The mean returns are colored blue. We intuitively expect negative returns from the bubble signal and likewise positive returns following a negative bubble signal. Significant days are marked in red. Significant quantiles are shaded in gray. Further details are given in Section 4.3.5.
USD Lattice Alarm – Cumulative Directional Accuracy – Positive & Negative Alarms Aggregate

USD Lattice Alarm – Bubble Signal v Monte Carlo Returns
12 signals – 12 significant days

Mean Bubble Signal Returns
Monte Carlo Returns
Significant Bubble Returns

USD Lattice Alarm – Negative Bubble Signal v Monte Carlo Returns
23 signals – 99 significant days

Mean Bubble Signal Returns
Monte Carlo Returns
Significant Bubble Returns

Cumulative Returns (Non-compounded & Before Costs)


FX Liquidity Risk Factor

Mancini, Ranaldo, and Wrampelmeyer (2012) introduce a tradable liquidity risk factor IML (illiquid minus liquid) given by a currency portfolio which is long in the two most illiquid currencies and short in the two most liquid currencies. Investors likely demand a premium for being exposed to liquidity risk as they suffer higher costs during long and unexpected illiquid environments. Therefore we expect this portfolio to earn a compensation for the liquidity risk it bears. We define unexpected illiquidity as the residuals of an AR(1) for global FX illiquidity in accordance with Menkhoff et. al (2012) in their construction of a FX volatility risk portfolio (Note: illiquidity = - liquidity). These residuals can be interpreted as the innovations, or the unexpected changes in liquidity as the AR(1) residuals are uncorrelated with their own lags.

In order to construct a global FX liquidity indicator we must first select the indicators which best capture liquidity, as many exist in the academic literature. Furthermore, we are constrained in that detailed high frequency market data, from which more accurate liquidity measures can be constructed, is not readily available far back in time. Dependent on the data provider, market data is typically limited to the closing price, mid-price, day high, day low, bid, and ask price.

Therefore we adopt the best low-frequency FX liquidity measures - defined as those which exhibit the highest correlation to high frequency liquidity measures, in accordance with Karnaukh, Ranaldo, and Soderlind (2014). After capturing accurate estimates of liquidity in the FX markets based on high frequency data, Karnaukh, Ranaldo, and Soderlind (2014) identify the high-low CS measure from Corwin and Schultz (2012), the Bayesian Gibbs sampler estimate of the effective cost of the Roll model (Hasbrouck, 2009), and an estimate of realized volatility, as the most reliable low frequency indicators of liquidity. Global FX liquidity is then measured as the first principle component across these indicators and across all currencies. We estimate each measure on a monthly basis.

The first low-frequency measure of liquidity is the CS measure outlined by Corwin and Schultz (2012). This is based on the high and low prices measured over a two day period. High prices are usually buyer initiated while low prices are usually seller initiated. As traders must cross the bid-ask spread to execute a trade, the ratio of high-to-low prices for a given day reflects both the volatility of the asset in question and the bid-ask spread. The variance of an asset grows proportionately with time however the bid-ask spread component does not. Thus the sum of the price ranges over 2 consecutive single days reflects 2 days’ volatility and twice the spread while the price range over one 2-day period reflects 2 days’ volatility but only 1 bid-ask spread. Corwin and Schultz (2012) derive a spread estimator which is a function of high-low ratios of 1-day and 2-day intervals. They show that this high-low spread estimate is
given by
\[ S = \frac{2(e^x - 1)}{1 + e^x} \tag{E.1} \]
with \( x \) given by
\[ x = \sqrt{\frac{2\beta - \sqrt{\beta}}{3 - 2\sqrt{2}}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}} \tag{E.2} \]
and \( \beta \) and \( \gamma \) given by
\[ \beta = \left( \ln \left( \frac{H_t}{L_t} \right) \right)^2 + \left( \ln \left( \frac{H_{t+1}}{L_{t+1}} \right) \right)^2, \quad \text{and} \quad \gamma = \left( \ln \left( \frac{H_{t,t+1}}{L_{t,t+1}} \right) \right)^2 \tag{E.3} \]
where \( H_t \) and \( L_t \) denote the observed high and low prices on day \( t \) (similarly for day \( t + 1 \)), while \( H_{t,t+1} \) and \( L_{t,t+1} \) are the high and low over two days (\( t \) to \( t + 1 \)). The method produces a negative estimate for transaction costs when the bid-ask spread is squeezing and in these cases we set the estimate to zero. Furthermore, we must correct for overnight returns because the ratio of high to low prices for a 2-day period reflects both the range of prices for those days and the overnight returns. If the day \( t + 1 \) low is above the day \( t \) close we assume that the price rose overnight from the close to the day \( t + 1 \) low and therefore decrease both the high and low for day \( t + 1 \) by the rise (close to low). Likewise, if the \( t + 1 \) high is below the \( t \) close, we assume the price fell overnight and increase the day \( t + 1 \) high and low by the fall (close to high). This measure closely approximates effective spreads. It may also capture transitory volatility over 2 days which may include temporary price pressure from large orders in addition to bid-ask spreads. It is computed for each currency for each 2-day period and then an average is calculated for each currency every month. A high \( CS \) value indicates lower liquidity.

The second liquidity measure is the effective cost of the Roll model Roll (1984) estimated using the Bayesian Gibbs approach of Hasbrouck (2009). The Matlab code for this method was obtained from the author’s website (See http://people.stern.nyu.edu/jhasbrou/). The Roll model suggests a model of security prices with transaction costs as follows
\[
\begin{cases}
  m_t = m_{t-1} + u_t \\
  p_t = m_t + cq_t
\end{cases} \tag{E.4}
\]
where \( m_t \) is the log midpoint quote prior to the \( t^{th} \) trade, \( p_t \) is the log trade price, and \( q_t \) is the indicator of trade direction which is +1 for a buy and -1 for a sell. Buys and sells occur with equal probability. \( u_t \) is the disturbance term which reflects new information and is assumed to be uncorrelated with the trade \( q_t \). The model implies that
\[ \Delta p_t = c \Delta q_t + u_t \tag{E.5} \]
where \( \Delta \) is the change operator. Under this model, the effective cost \( c \) is the square root of the auto-covariance of consecutive price changes given as
\[
\text{Roll} = \begin{cases} 
  2 \sqrt{-\text{Cov}(\Delta p_t, \Delta p_{t-1})}, & \text{when Cov} < 0 \\
  0, & \text{when Cov} > 0
\end{cases} \tag{E.6}
\]
where \( \Delta p_t \) is the change of the log midquote price between \( t \) and \( t - 1 \). The model is only feasible when the auto-covariance is negative and therefore the cost is set to zero when positive auto-covariance is observed. The higher the Roll spread the lower the liquidity.

Hasbrouck (2009) extends this model by assuming \( u_t \sim N(0, \sigma_u^2) \). By applying the Bayesian approach and Gibbs procedure, the unknown parameters of the model, the transaction cost \( c \),
the standard deviation of the disturbance $\sigma_{u^2}$, and the trade direction $q$ are estimated. We have used the code supplied by Joel Hasbrouck on his website setting the transaction cost prior to $\sqrt{\bar{p}^A - \bar{p}^B}$, where $\bar{p}^A$ and $\bar{p}^B$ are the monthly averages of the log ask and bid prices. The Gibbs measure is estimated at the end of each month for each currency based on that month’s data.

In order to measure the volatility of each currency, the third proxy for liquidity, we follow Menkhoff et. al (2012) and calculate the absolute daily log return $|r^k_{\tau}|$ for each currency $k$ on each day $\tau$ in the sample. By taking the absolute return rather than squared values, the impact of outliers is minimised. We then average daily values up to the monthly frequency. A higher value for volatility is indicative of lower liquidity.

Having estimated the three liquidity proxies for each currency, we then construct the global FX liquidity as the first principle component estimated on the 30 liquidity time series (3 per currency). We then take the negative value of this as each of the three measures above increase with illiquidity. In this way, a negative value of the global measure reflects worse liquidity. We define unexpected illiquidity shocks or innovations as the residuals of an AR(1) for global FX liquidity in accordance with Menkhoff et. al (2012).

Figure E.1 below illustrates global FX liquidity and liquidity shocks. A lower value is indicative of worse liquidity. Clearly visible is the large drop in global FX liquidity that occurred during the 2008/2009 financial crisis.

**Figure E.1:** Global FX Liquidity and Liquidity Shocks

![Graph](image)

**Figure E.1** Global FX liquidity is colored blue whereas liquidity shocks are given in red. Lower values indicate worse liquidity. Global FX liquidity is measured as the first principle component estimated across 10 currencies, with 3 liquidity measures for each currency. Shocks are measured as the residuals of an AR(1) for global FX liquidity.

In order to construct the IML (illiquid minus liquid) portfolio, we sort currencies based on their past betas to innovations in global FX liquidity using a rolling window of 36 months and go long the two most illiquid currencies and short the two most liquid, as measured by their betas. The portfolio is rebalanced monthly. This portfolio is presented in Figure 4.5 alongside the HML Carry Risk, DOL Dollar Risk, VOL Volatility Risk, and MOM Momentum portfolios.


