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DYNAMIC APPROACHES TO REAL ESTATE BUBBLES: METHODS AND EMPIRICAL STUDIES

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Abstract

This is a cumulative thesis i.e. a collection of five self-contained research papers, of which three are already published in peer-reviewed journals. The scientific contributions are mostly connected to the study of real estate bubbles, though tightly related topics are also treated insofar they enrich and extend the analysis.

Chapter two consolidates the results of studies that monitored the risks of bubble development in Switzerland's residential real estate market. The study uses the Log Periodic Power Law Singularity (LPPLS) model to analyze the development of asking prices of residential properties in all the Swiss districts between 2005-Q1 and 2016-Q2. The results follow the development of the market over a three years periods, and carefully document the transition of the Swiss market from a bubble regime to what seems, by the time in which this document is written, as a new slow growing phase. The material of this chapter is based on [Ardila et al., 2013a], where the methodology was described, and the biannual reports for the general audience that were published in collaboration with comparis.ch [Ardila et al., 2013b,c, 2014a,b, 2015b,c, Ahmed et al., 2016]. Therefore, this analysis - conducted and published incrementally between 2012-Q4 and 2016-Q2 - constitutes a hindsight-bias free diagnosis of the Swiss housing market. The content of this chapter serves to motivate the other contributions of this thesis.

Chapter three and four assess the applicability of the LPPLS model in the study of real estate bubbles. The two chapters provide supporting evidence for the dynamic approach to bubble diagnosis, employed in chapter two.

Chapter three takes a step away from the real estate market to substantiate one of the fundamental assumptions of the LPPLS model. We report strong evidence that changes of momentum, i.e. "acceleration", defined as the first difference of successive returns, is a novel effect complementing momentum of stock returns. The "acceleration" effect, which we argue is associated with procyclical mechanisms, help elucidate many previous reports of transient non-sustainable accelerating (upward or downward) log-prices as well as many anomalies associated with the momentum factor. This chapter is an edited version of [Ardila et al., 2015a].

Chapter four analyzes the ex-ante information content of the LPPLS and other bubble detection methods in the context of international real estate markets. We derive binary indicators from the causal application of five bubble statistical tests, and via logit regressions we assess the indicators' out-of-sample performance in the forecasting of tipping

points of housing bubbles and systemic financial crises. This chapter is based on [Ardila et al., 2016b].

Having put to the test the dynamic approach to diagnose real estate bubbles, chapters five and six seek to connect the bubble diagnosis with fundamental drivers of the property market, and the other phases of the financial cycle.

Chapter five presents a hybrid model for diagnosis and critical time forecasting of real estate bubbles. The model combines two elements: 1) the LPPLS model to describe endogenous price dynamics originated from positive feedback loops among economic agents; and 2) a diffusion index that creates a parsimonious representation of multiple macroeconomic variables. This structure allows us to analyze the interaction between a "bubble" and a "fundamental" component. We compare the model's in-sample and out-sample behavior on the housing price indices of 380 US metropolitan areas. This chapter is based on [Ardila et al., 2016c].

Chapter six is published in Ardila and Sornette [2016]. It proposes to date and analyze the financial cycle using the Maximum Overlap Discrete Wavelet Transform (MODWT). Our presentation points out limitations of the methods derived from the classical business cycle literature, while stressing their connection with wavelet analysis. The fundamental time-frequency uncertainty principle imposes replacing point estimates of turning points by interval estimates, which are themselves function of the scale of the analysis.

Finally, chapter seven presents the conclusions and suggests directions for future research. We discuss current gaps in the literature, and propose appropriate tools and building blocks to address them.

Zusammenfassung

Hierbei handelt es sich um eine kumulative Doktorarbeit, d.h. eine Sammlung von fünf abgeschlossenen Forschungsarbeiten, von denen drei bereits in wissenschaftlichen Zeitschriften veröffentlicht wurden. Die wissenschaftlichen Beiträge stehen hauptsächlich in Verbindung mit der Untersuchung von Immobilienblasen, aber es werden auch damit eng verbundene Themen behandelt, soweit sie die Analyse bereichern und erweitern. Im zweiten Kapitel werden die Ergebnisse von Studien konsolidiert, die die Risiken der Entwicklung von Blasen auf dem Schweizer Wohnimmobilienmarkt überprüft haben. Die Studie wendet das Log-Periodic-Power-Law-Singularity (LPPLS) Modell an, um die Entwicklung der Angebotspreise von Wohnimmobilien in allen Schweizer Gemeinden zwischen dem 1. Quartal 2005 und dem 2. Quartal 2016 zu analysieren. Die Ergebnisse spiegeln die Entwicklung des Markts über 3-Jahres-Zeiträume wider und dokumentieren sorgfältig die Transition des Schweizer Markts von einem Blasensystem hin zu einer scheinbar neuen Phase langsamen Wachstums, zumindest zu der Zeit, als die Arbeit geschrieben wurde. Das Material für dieses Kapitel basiert auf [Ardila et al., 2013a], wo die Methodologie beschrieben wurde, und auf den halbjährlichen Berichten für die breite öffentlichkeit, die in Zusammenarbeit mit comparis.ch [Ardila et al., 2013b,c, 2014a,b, 2015b,c, Ahmed et al., 2016] veröffentlicht wurden. Diese Analyse stellt darum eine rückblickende, unbefangene Diagnose des Schweizer Wohnungsmarkts dar. Der Inhalt dieses Kapitels diente als Anregung für die anderen Beiträge.

In Kapitel drei und vier wird beurteilt, ob das LPPLS-Modell bei der Untersuchung von Immobilienblasen anwendbar ist. Beide Kapitel bieten sachdienliche Beweise für die dynamische Herangehensweise zur Feststellung von Blasen, was in Kapitel zwei seine Anwendung findet. Kapitel drei nimmt zur Bekräftigung einer der grundlegenden Annahmen des LPPLS-Modells etwas Abstand vom Immobilienmarkt. Es gibt starke Anzeichen dafür, dass änderungen in der Dynamik, d.h. "Beschleunigung", die als erster Unterschied schrittweiser Erträge definiert ist, eine neuartige Auswirkung darstellt und die Dynamik der Aktienrendite ergänzt. Der "Beschleunigungseffekt" ist entsprechend der These mit prozyklischen Mechanismen verbunden und hilft dabei, viele vorhergehende Berichte zu flüchtiger, nicht nachhaltiger Beschleunigung (nach oben oder nach unten) von Log-Preisen sowie viele Anomalitäten im Zusammenhang mit dem Faktor Dynamik zu erläutern. Bei diesem Kapitel handelt es sich um eine überarbeitete Version von [Ardila et al., 2015a]. In Kapitel vier werden der Ex-ante-Informationsgehalt der LPPLS und

andere Methoden zur Ermittlung von Blasen im Kontext des internationalen Immobilienmarkts analysiert. Binäre Indikatoren werden von der kausalen Anwendung von fünf statistischen Prüfungsverfahren zu Blasen hergeleitet. über Logit-Regressionen wird die Out-of-Sample-Leistung der Indikatoren bei der Vorsage der Tipping Points von Immobilienblasen und systemischen Finanzkrisen beurteilt. Dieses Kapitel beruht auf [Ardila et al., 2016b].

Nach Erprobung der dynamischen Herangehensweise zur Feststellung von Immobilienblasen, geht es in Kapitel fünf und sechs darum, die Feststellung der Blase mit grundlegenden Immobilienmarkttreibern und den anderen Phasen des Finanzzyklus zu verbinden. In Kapitel fünf wird ein Hybridmodell zur Feststellung und kritischen zeitlichen Vorhersage von Immobilienblasen vorgestellt. Das Modell verbindet zwei Elemente: 1) das LPPLS-Modell zur Beschreibung endogener Preisdynamiken, die von positiven Feedbackschleifen unter Wirtschaftsakteuren stammen; und 2) ein Diffusionsindex, der eine sparsame Darstellung zahlreicher makroökonomischer Variablen schafft. Durch diese Struktur kann die Interaktion zwischen einer "Blase und einer "grundlegenden" Komponente analysiert werden. Das In-Sample-und Out-of-Sample-Verhalten des Modells wird anhand der Immobilienpreisindizes von 380 Metropolregionen in den USA verglichen. Dieses Kapitel beruht auf [Ardila et al., 2016c].

Kapitel sechs wird in Ardila and Sornette [2016] veröffentlicht. Es geht darum, den Finanzzyklus mithilfe des Maximum Overlap Discrete Wavelet Transform (MODWT) zu datieren und zu analysieren. Die Präsentation weist auf Grenzen der aus der klassischen Literatur zum Konjunkturzyklus stammenden Methoden hin und hebt deren Verbindung mit der Wavelet-Analyse hervor. Die zugrundeliegende Zeit-Frequenz-Unschärferelation verhängt Ersatzpunktschätzungen von Wendepunkten durch Intervallschätzungen, die selbst von der Analyseebene abhängen.

Im siebten Kapitel werden schließlich die Schlussfolgerung vorgestellt und Vorschläge für zukünftige Forschungsrichtungen gemacht. Diskutiert werden derzeitige Lücken in der Literatur, zudem werden angemessene Instrumente und Bausteine vorgeschlagen, um diese in Angriff zu nehmen.

Acknowledgments

Acknowledgments are supposed to be the easy part of a PhD thesis. After every chapter is written, all that is left is to look back, and recognize the positive role that friends and colleagues played during your doctoral studies. Nevertheless, I have found difficult, if not impossible, to express with words my gratitude towards all the people who made this journey feasible, meaningful, and pleasant.

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Contents

Chapter 1

Introduction

Much has changed since the last 10 years, when the so-called "great moderation" was regarded as the biggest success of modern economic theory, and research had found that the term "bubble" was not an attractive explanation for the lack of quantitative understanding of real estate prices [Leung, 2004]. The pivotal role of the real estate market to prepare the condition for the 2008 financial crisis and the systemic risks that the market carries has crystalized a number of important research questions, while obliging to revisit others issues from the past.

The existence and detection of bubbles in real estate markets is one of those fundamental questions in economics and finance. Empirical research on real estate bubbles is by no means an emergent field. The first influential studies on real estate bubbles date back to the 1980s and early 1990s when scholars such as Hamilton and Schwab [1985] and Case and Shiller [1990] developed parallels with stock markets to test the efficiency of real estate markets. Other studies have followed the same direction, rejecting the random walk hypothesis using data from different countries [Kuo, 1996, Clayton, 1997, Røed Larsen and Weum, 2008]. As consensus concerning market inefficiencies was built up, the empirical focus has moved towards detecting the presence of bubbles, the study of its consequences, and the predictability of their turning points.

A common practice has been to define a bubble as a period in which the price of an asset exceeds its fundamental value. Accordingly, the task of identifying housing bubbles has consisted of modeling the fundamental prices and comparing them with the actual prices. Large upward deviations of observed prices from fundamental values are interpreted as signs of overvaluations.

The diagnostic has been conducted based on simplified ratios analogous to valuation and affordability ratios, such as the price-to-income ratio (see e.g. Hamilton and Schwab, 1985; Campbell et al., 2009). These approaches have documented important deviations from fundamental factors, but they are prone to misspecification as they might not be able to fully capture the drivers of the market [Ghysels et al., 2012]. A case in point was the study conducted by McCarthy and Peach [2004], which concluded that the prices observed during the last US real estate bubble could be explained by a change of preferences in the population. Clearly, with the benefit of hindsight, this conclusion was proven to be incorrect.

Other works have modeled and estimated the fundamental value indirectly, via co-integration tests [Arshanapalli and Nelson, 2008], directly via fully specified supply and demand equations or expected value relations. Examples of supply and demand equations are: Kim and Min [2011], who tested for bubbles on Korean and Japanese markets and modeled explicitly rational expectations; and Goodman and Thibodeau [2008], who concluded that the price increases observed in the US market owed much to inelastic supplies of owner-occupied housing. Examples of diagnostics based on expected present values of future service flows (imputed rent) are the works of Himmelberg et al. [2005] and Fraser et al. [2008] who analyzed respectively the US and the New Zealand markets.

1.1 A critique to the classical approach

A common issue of the works discussed above, which complicates the job of scholars and policymakers, is the emergence of seemingly-plausible fundamental arguments that seek to justify the dramatic rise in asset prices [Jurgilas and Lansing, 2012]. In fact, an early work of Hamilton and Whiteman [1985] explains that one can always relax restrictions on the fundamentals to interpret what appears to be a speculative bubble as a consequence of some fundamental variable. An equally pressing issue for a practitioner or policy maker, especially after the sub-prime crisis, is that a deviation from the fundamentals does not necessarily inform about the proximity and intensity of the correction.

To illustrate the challenges faced by classical methodologies, we can take the life-cycle model employed in [Anundsen, 2015] to identify the econometric regime shift in the US market, which preceded the 2008 financial crisis. The methodology has as a departure point, equation 1.1. It states that the consumer's marginal willingness to pay for housing services in terms of other consumption goods should be equal to the cost in terms of forgone consumption.

$$\frac{U_H}{U_C} = PH\left[(1-\tau_y)(i+\tau_p) - \phi + \delta - \frac{\dot{PH}}{PH}\right] \tag{1.1}$$

where U_H/U_C is the marginal rate of substitution between the housing stock H, and a composite consumption good C, PH is a real house price index, and the term in brackets is known as the real user cost of housing UC. UC contains the nominal interest rate i, the property tax τ_p, tax deduction rate τ_y, the inflation rate of the overall price level ϕ, the housing depreciation rate δ, and the expected real housing price inflation $\frac{\dot{PH}}{PH}$. By the no arbitrage condition, the user cost of housing should be in equilibrium equal to the real imputed rent on housing services Q, which is unobservable,

$$Q = PH\left[(1-\tau_y)(i+\tau_p) - \phi + \delta - \frac{\dot{PH}}{PH}\right] \tag{1.2}$$

Using a semi-logarithmic approximation and proxying the unobservable Q with the observable rent R, we obtain a single expression that corresponds to the price to rent approach to understand the fundamental value of house prices,

$$ph = \gamma_r r + \gamma_{UC} UC \tag{1.3}$$

where lower-case letters denote variables that are on a logarithmic scale. The equilibrium correction representation of the price-to-rent model 1.3 can be expressed in the following way:

$$\begin{aligned} \Delta ph_t = \mu + \alpha_{ph}(ph - \gamma_r r - \gamma_{UC} UC)_{t-1} + \sum_{i=1}^{p} \rho_{ph,i} \Delta ph_{t-i} \\ + \sum_{i=0}^{p} \rho_{r,i} \Delta r_{t-i} + \sum_{i=0}^{p} \rho_{UC,i} \Delta UC_{t-i} + \epsilon_t \end{aligned} \tag{1.4}$$

Alternatively, we can avoid the $Q \approx R$ approximation, if we assume that the imputed rent is a function $f(Y, H)$ of income Y and the housing stock H. Replacing Q by $f(Y, H)$ in equation 1.2, and using again the semi-logarithmic approximation we obtain,

$$ph = \tilde{\gamma}_y y + \tilde{\gamma}_h h + \tilde{\gamma}_{UC} UC \tag{1.5}$$

Equation 1.5 is known as the inverted-demand model. Since the housing stock evolves slowly, it is assumed to be fixed in the short-run. This leads to the error correction representation in equation 1.6.

$$\begin{aligned} \Delta ph_t = \tilde{\mu} + \tilde{\alpha}_{ph}(ph - \tilde{\gamma}_y y - \tilde{\gamma}_h h - \tilde{\gamma}_{UC} UC)_{t-1} + \sum_{i=1}^{p} \tilde{\rho}_{ph,i} \Delta ph_{t-i} \\ + \sum_{i=0}^{p} \tilde{\rho}_{y,i} \Delta y_{t-i} + \sum_{i=0}^{p} \tilde{\rho}_{UC,i} \Delta UC_{t-i} + \epsilon_t \end{aligned} \tag{1.6}$$

From theory we expect that $ph - \gamma_r r - \gamma_{UC} UC$ and $ph - \tilde{\gamma}_y y - \tilde{\gamma}_h h - \tilde{\gamma}_{UC} UC$ are stationary, as prices should in the long run remain close to the fundamental value. Therefore, a bubble can be characterized as a period when prices are not co-integrated with the fundamental factors.

Anundsen [2015] proposes to test the null bubble hypothesis via the relations $H_{bubble} := \alpha_{ph} = 0$ and $\tilde{H}_{bubble} := \tilde{\alpha}_{ph} = 0$, against the alternatives of no bubble $H_{alt} := \alpha_{ph} < 0$ and $\tilde{H}_{alt} := \tilde{\alpha}_{ph} < 0$. No rejection of the null hypotheses should be interpreted as evidence of bubble. Using this methodology and a recursive estimation strategy, Anundsen [2015] identified an econometric regime shift - i.e. a bubble - in the US. Figure 1.1 shows the specific periods, as well as the evolution of the p-values for both null hypotheses $H_{bubble} := \alpha_{ph} = 0$ and $\tilde{H}_{bubble} := \tilde{\alpha}_{ph} = 0$. p-values above the dashed line correspond to bubble periods.

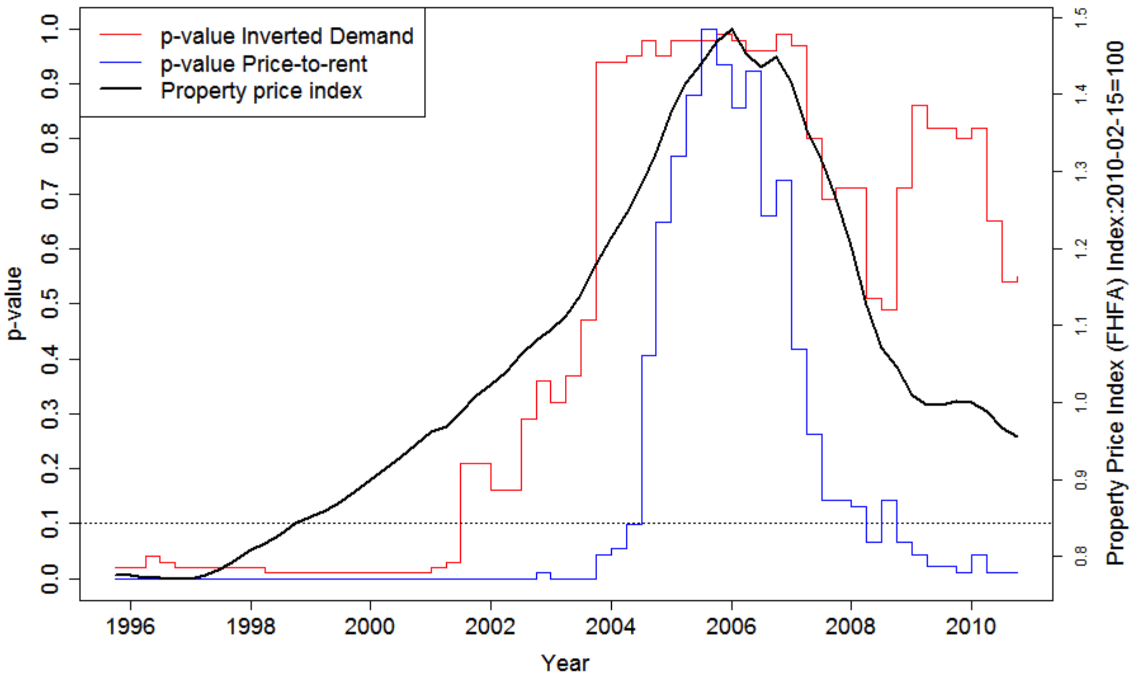
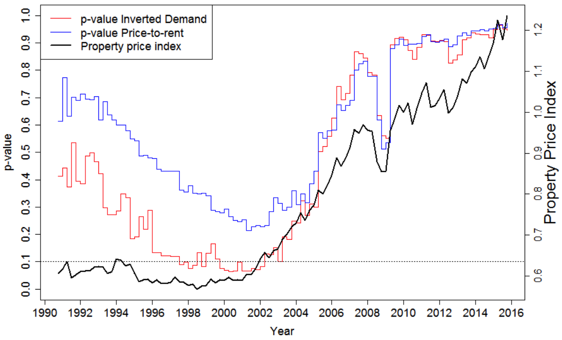


Figure 1.1: Anundsen [2015]'s bubble indicators for the US, according to the inverted-demand and price-to-rent models. Bubble periods correspond to p-values above the 0.1 threshold. The reader is directed to the original source for all the estimation details. Source: Kuert (2016).

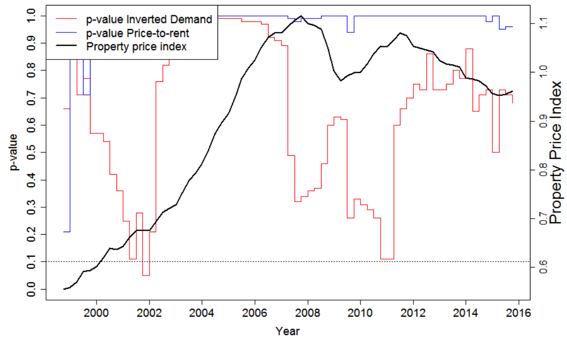
The methodology is well grounded in theory and contains the main econometric ideas employed not only to analyze bubbles, but more generally, the drivers of the housing market. However, despite this rigorous theoretical approach, several issues arise:

To begin with, as evident from figure 1.1, the inverted-demand and the price-to-rent model lead to very different results. The bubble period extends from 2000-Q4 to 2009-Q4 when using the inverted-demand approach, and from 2004-Q4 to 2007-Q4 when using the price-to-rent approach. That is, although the two theoretical models are closely connected and the treatment of fundamental factors is similar, the identified bubble periods differ substantially. In this case, the inverted-demand model tends to overestimate the beginning of the bubble period relative to the period determined by the price-to-rent model.

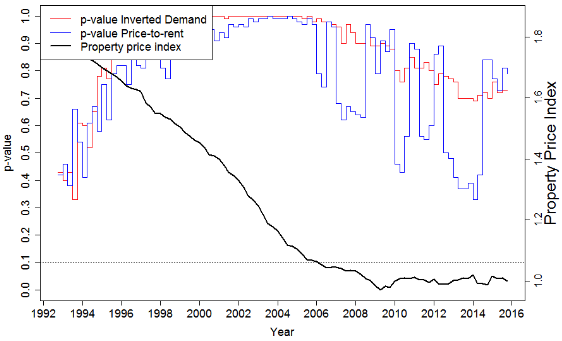
The methodology also seems highly sensitivity to the institutional environment in which the theory is applied, as well as to the regimes of the time series in which the models are estimated. Figure 1.2 shows [Anundsen, 2015]'s bubble indicators, computed for six OECD countries. According to the inverted-demand model, Canada, France, Japan, Netherlands, and Switzerland have continuously experienced a bubble regime since the beginning of the estimation period. The price-to-rent model leads to more conservative conclusions, and yet, the real estate markets of Canada, France, Japan, and Switzerland have consistently undergoing a bubble according to this model. Thus one has to conclude that the bubble tests are failing to timely reject the bubble hypothesis. This might be because of lacking statistical power, or absence of some essential fundamental factor.



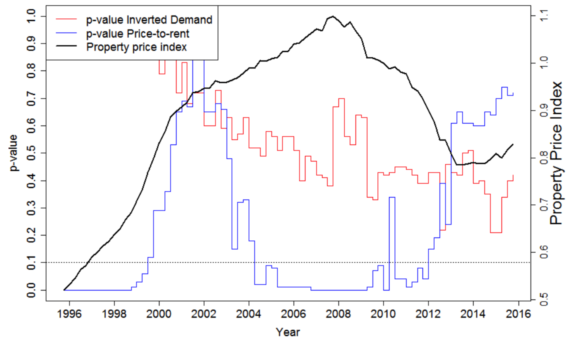
(a) Canada



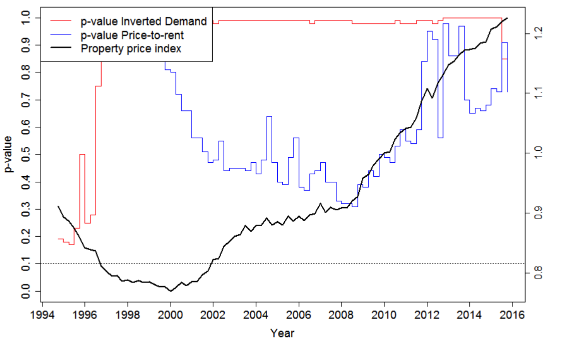
(b) France



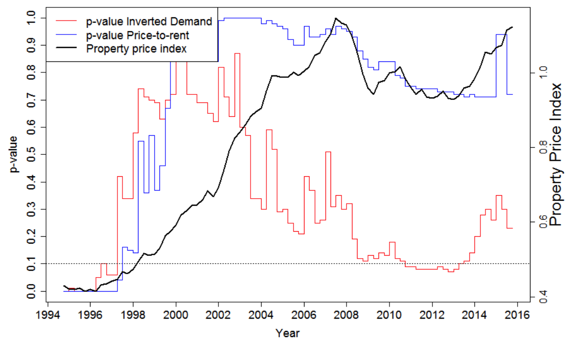
(c) Japan



(d) Netherlands



(e) Switzerland



(f) UK

Figure 1.2: Anundsen [2015]'s bubble indicators for the US, according to the inverted-demand and price-to-rent models. Bubble periods correspond to p-values above the 0.1 threshold. The reader is directed to the original source for all the estimation details. Source: Kuert (2016).

One can be wary of blindly applying a method without taking into account the institutional environment of the corresponding market and carefully adjusting the empirical strategy to each country. This is to prevent what Muellbauer et al. [2012] calls a sausage machine approach. However, even in the same institutional setup, the use of different proxies to operationalize the variables can lead to conflicting results. Figure 1.3 replicates figure 1.1 for the US, but using different time series from those of [Anundsen, 2015], to operationalize the fundamental variables.

In figure 1.3a, the US house price index was replaced by the respective index provided by the Bank of International Settlements BIS [1]. The BIS index is based on economic data by the federal reserve (national level), uses averaging as the main aggregation method, and does not adjust for seasonality. In comparison, the original series from the Federal Housing Finance Agency is based on a repeated sales method. They are 96% correlated in levels, although only 65% in log returns. In figure 1.3b, the housing stock time series was changed by that from Oxford Economics OE [2]. The OE time series also approximates the monetary value of the US housing stock, but there are little details concerning the estimation methodology. We observe that the bubble period is visibly affected in both cases relative to the original results. Changing the house price index leads to the under-rejection of the bubble hypothesis in the case of the price-to-rent approach, and a slightly erratic behavior in the case of the inverted-demand model. Changing the housing stock leads to consistently under-reject the bubble hypothesis.

More general issues can be discussed. A bubble identification strategy based on the careful analysis of the fundamental factors is feasible in countries such as the US and UK, with a long tradition of collecting robust and high quality data. However, macroeconomic data relevant for the housing market in other countries tend to be scantier and sparser. Switzerland, for example, has a rather recent history of collecting statistics, while highly informative surveys tend to be discontinued. More granular data, such as district-level data, is in general absence, complicating and limiting the scope of the econometric analysis.

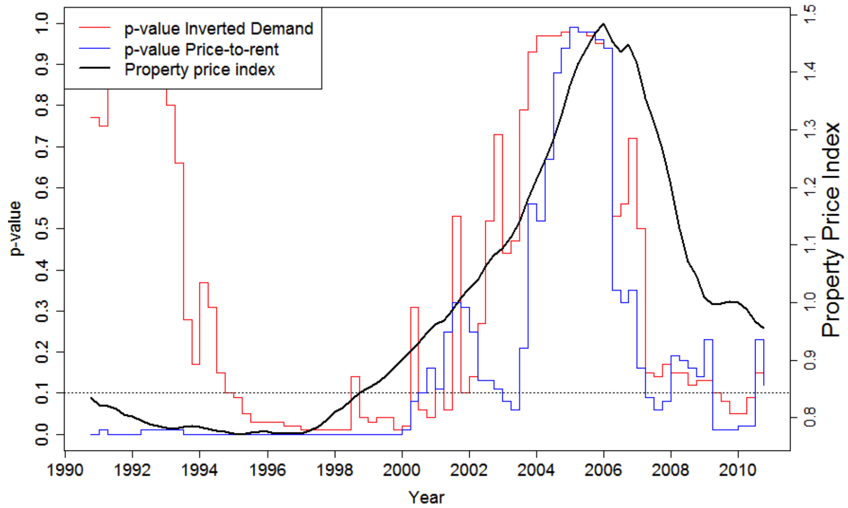
Given these limitations, we should not be surprised that the analysis of housing bubbles is still considered a difficult subject. The possibility is real that the resources and necessary information to understand the drivers of the housing market only become available after a price correction has happened. No wonder it has become common wisdom that housing bubbles can only be rationalized in hindsight.

1.2 Chapter contributions

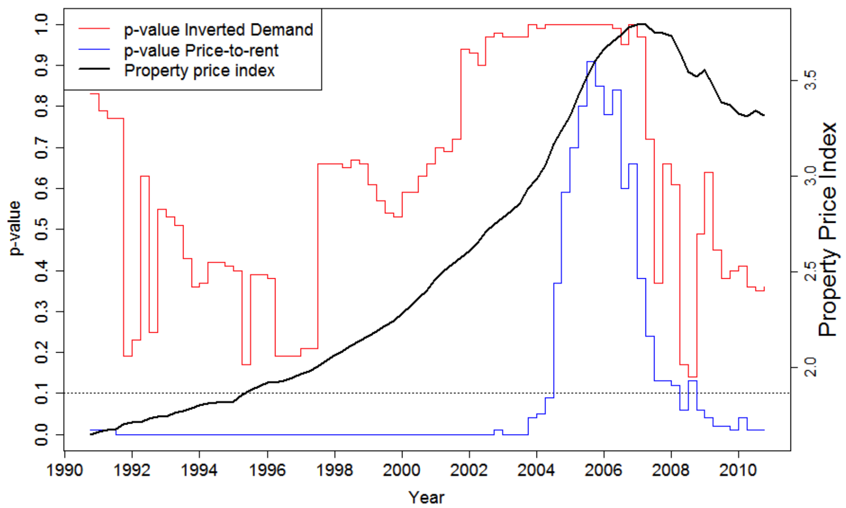
In order to circumvent these issues, we propose to identify and analyze real estate bubbles by studying the statistical properties of house price time series. As we argue in favor of this approach and discuss its limitations, we gradually move towards the examination of

[1] Bank of International Settlements. Residential property prices: selected series, `https://www.bis.org/statistics/pp/property_price_statistics.zip`.

[2] Available at Thompson Reuters.



(a) US - Alternative house index (Bank of International Settlements).



(b) US - Alternative housing stock time series (Oxford Economics).

Figure 1.3: Anundsen [2015]'s bubble indicators for the US with alternative time series. Bubble periods correspond to p-values above the 0.1 threshold. The reader is directed to the original source for the estimation details. Source: Kuert (2016).

the role of macroeconomic factors during bubble regimes.

The contributions of this cumulative thesis extend and are built upon the 20 years of research in financial economics that has sought to challenge the idea that the origin of crises is exogenous. On the contrary, the literature from which this thesis stems regards bubbles as the social analogues of so-called critical points studied in the statistical physics community in relation to magnetism, melting, and other phase transition of solids, liquids, gas and other phases of matter Sornette [2003]. According to this theory, a crash occurs because the market has entered an unstable phase and any small disturbance or process may have triggered the instability. Sornette [2003] eloquently describes these ideas in his ruler analogy:

Think of a ruler held up vertically on your finger: this very unstable position will lead eventually to its collapse, as a result of a small (or absence of adequate) motion of your hand or due to any tiny whiff of air. The collapse is fundamentally due to the unstable position; the instantaneous cause of the collapse is secondary. In the same vein, the growth of the sensitivity and the growing instability of the market close to such a critical point might explain why attempts to unravel the local origin of the crash have been so diverse. Essentially, anything would work once the system is ripe. In this view, a crash has fundamentally an endogenous or internal origin and exogenous or external shocks only serve as triggering factors.

As a consequence, the origin of crashes is much more subtle than often thought, as it is constructed progressively by the economy as a whole, as a self-organizing process. In this sense, the true cause of a crash could be termed a systemic instability. This thesis explores and apply these ideas to the real estate market. We focus on the implications that house price dynamics have for the stability of market, while extending bridges with the classical analysis of bubbles. More specifically, this thesis continues as follows:

Chapter two consolidates the results of studies that monitored the risks of bubble development in Switzerland's residential real estate market. The content of this chapter is presented as a case study that illustrates the use of the Log Periodic Power Law Singularity (LPPLS) model to analyze bubbles in real estate markets. The results follow the development of ask prices of residential properties in all the Swiss districts between 2005-Q1 and 2016-Q2, and carefully document the transition of the Swiss market from a bubble regime to what seems, by the time in which this document is written, as a new slow growing phase. The material of this chapter is based on [Ardila et al., 2013a], where the methodology was described, and the biannual reports for the general audience that were published in collaboration with comparis.ch [Ardila et al., 2013b,c, 2014a,b, 2015b,c, Ahmed et al., 2016]. Therefore, this analysis - conducted and published incrementally between 2012-Q4 and 2016-Q2 - constitutes a hindsight-bias free diagnosis of the Swiss housing market. The content of this chapter serves to motivate the other contributions of this thesis.

Chapter three and four assess the applicability of the LPPLS model in the study of real estate bubbles. The objective is to examine that assumptions underlying the analysis of chapter two.

In chapter three, we deviate from the real estate market to substantiate one of the fundamental assumptions of the LPPLS model. We report strong evidence that changes of momentum, i.e. "acceleration", defined as the first difference of successive returns, is an effect complementing momentum, which has manifested during bubble regimes. The "acceleration" effect, which we argue is associated with procyclical mechanisms, help elucidate many previous reports of transient non-sustainable accelerating (upward or downward) log-prices as well as many anomalies associated with the momentum factor. This chapter is an edited version of [Ardila et al., 2015a].

In chapter four, we examine the ex-ante information content of the LPPLS, and other bubble tests based on the analysis of price dynamics, in the context of international real estate markets. We directly study whether the tests can be employed to analyze bubble regimes previously documented in the literature of housing bubbles. In this sense, this chapter is an attempt to connect with the classical econometric literature of bubbles, as we are able to show that exuberant price dynamics can often be associated with deviations from the fundamental factors and have significant forecasting power related to the tipping point of bubbles. This chapter is based on [Ardila et al., 2016b].

Having put to the test the dynamic approach to diagnose real estate bubbles, chapters five and six seek to connect the bubble diagnosis with fundamental drivers of the property market, and the other phases of the financial cycle.

Chapter five presents a hybrid model for diagnosis and critical time forecasting of real estate bubbles. The model combines two elements: 1) the LPPLS model to describe endogenous price dynamics originated from positive feedback loops among economic agents; and 2) a diffusion index based on the $L1$-norm that creates a parsimonious representation of multiple macroeconomic variables. This structure allows us to analyze the interaction between a "bubble" and a "fundamental" component. This chapter is based on [Ardila et al., 2016c].

Chapter six is published in Ardila and Sornette [2016]. It proposes to date and analyze the financial cycle, using the Maximum Overlap Discrete Wavelet Transform. Our presentation points out limitations of the methods derived from the classical business cycle literature, while stressing their connection with wavelet analysis.

Finally, chapter seven presents the conclusions and suggests directions for future research. We discuss current gaps in the literature, and propose appropriate tools and building blocks to address them.

Chapter 2

Real estate bubbles: the case of Switzerland

The bubble identification strategies described in the introduction mostly rely on the use of the so called fundamental factors to assess the risk of a bubble in the market. In contrast, this thesis follows an identification strategy based purely on the asset's price dynamics. In the next chapters, we will examine to what extent this a legitimate approach to identify real estate bubbles, but for the time being, we present an empirical study that illustrates how the methodology can be employed to identify and analyze ex-ante the presence of a bubble in the market.

In this study, we have monitored the risks of development of bubbles in Switzerland's residential real estate market. The study uses the Log Periodic Power Law Singularity (LPPLS) bubble model to analyze the development of asking prices of residential properties in all Swiss districts between 2005-Q1 and 2016-Q2. The results follow the development of the market over a three years periods, and carefully documents the slow transition of the Swiss real estate market from a bubble regime to what seems by the time in which this document is written as a new plateau phase. The material of this section is based on [Ardila et al., 2013a], where the methodology was described, [Ardila et al., 2016a], a work in progress that studies the relationship between ask and transaction prices in Switzerland, and the biannual reports for the general audience that were published in collaboration with comparis.ch (Ardila et al. [2013b]; Ardila et al. [2013c]; Ardila et al. [2014a]; Ardila et al. [2014b]; Ardila et al. [2015b]; Ardila et al. [2015c]; Ahmed et al. [2016]). Therefore, this analysis – conducted and published incrementally between 2012-Q4 and 2016-Q2 – constitutes a hindsight-bias free diagnosis of the Swiss housing market.

The content of this chapter serves to motivate the other contributions of this thesis. We show that the diagnosis has been accurate, according to precision-like metrics, but also discuss the limitations of the analysis.

2.1 Introduction

The development of residential property prices over the past years in Switzerland has raised concerns about the existence of a bubble in this market. Key indicators, such as the ratio of home to rent prices, are deviating from the long-term equilibrium [UBS, 2013b], whereas the direct exposure of banks to real estate has grown enough to pose a threat for the stability of the financial sector [SNB, 2012]. The situation is of great importance, as real estate volatility on large scale and intensity can have long-lasting and destructive effects for an economy. This was directly illustrated by the aftershocks of the burst of real estate bubbles in the U.S., Spain, and Ireland [Allen and Carletti, 2010], and by the consequences of the bubble in Switzerland at the end of the 1980s. The Swiss real estate bubble, which was fueled by a decline in mortgage lending standards, caused a sharp drop in GDP of about 1.55 percent and resulted in severe price corrections, and widespread foreclosures [Bourassa et al., 2010]. A repetition of this crisis today could have similar repercussions, as real estate assets represent 43.6 percent of the Swiss households wealth according to data of the Swiss National Bank [SNB, 2011].

It has been argued that the recent development of prices is due to a mismatch between supply and demand (UBS [2013b]; Credit Suisse [2012]). On the one hand, the demand has benefited from three factors: historically low interest rates; a sustained rate of immigration; and increasing real wages. On the other hand, the supply has had problems to keep up with the strong demand as it suffers from lengthy production times [Credit Suisse, 2012]).

Nevertheless, this conclusion is based on the same type of fundamental analysis that failed to detect the U.S. real estate bubble in 2007. At that time, it was boldly argued that there was little ground for bubble concerns as home prices had - allegedly - moved in line with increases in family income and declines in nominal mortgage interest rates [McCarthy and Peach, 2004]. Yet, the bubble burst and we are still indirectly bearing the consequences of it. This stresses the importance of a dynamical approach, and represents a strong case for prudence, since the diagnosis of bubbles remains an controversial topic with elusive targets.

In this context, we have developed a collaboration with comparis.ch in 2012 in order to study the risks of a bubble in the residential Swiss real estate market. In this way, we received access to an exclusive set of data containing millions of records of ask prices giving us a unique view on the market, with a remarkable resolution in space. We have continuously analyzed the market twice a year using Log Periodic Power Law Singularity Model. This chapter compiles the results of the incremental analysis conducted for each of the 166 districts of Switzerland between 2005 and 2016.

As the comparis.ch database does not contain transactions, a significant part of this chapter is also devoted to study the relationship between ask and transaction prices, in order to argue that ask prices timely reflects the evolution of the market. To do so, we compared the comparis.ch database against the best available database of transactions for Switzerland, via density estimation techniques, state-of-art co-integration tests, and statistical methods to analyze Granger-causality in dynamic panel data models.

This chapter continues as follows. Section 2.2 describes the comparis.ch database and the database of transactions SRED, against which the former is evaluated. Section 2.3 presents the empirical strategy to analyze the two datasets, and to perform the bubble diagnostic. Section 2.4 describes the Swiss macroeconomic environment under which this study was conducted. Section 2.5 presents the results. Finally, section 2.6 concludes this chapter and closes the discussion.

2.2 Data

The data used in this analysis was collected incrementally by comparis.ch between January 2005 and July 2016. The property market division of comparis.ch gathers data from the 17 largest property portals in Switzerland, creating a rich view on the market, but also introducing a large and un-estimated number of duplicate ads (by 2016-Q2, 7 million records are present in the raw data). These duplicates advertise the same property, during the same period, and sometimes, with conflicting information. Within the scope of this study, the identification of the duplicates was crucial, as they could potentially affect the price indices.

We implemented a procedure based on the Support Vector Machine (SVM) algorithm Scholkopf and Smola [2001] and string distance measures Cohen et al. [2003] in order to identify the duplicate ads. The procedure determined, in a given zip code and a given quarter, the ads that represented the same residential property by analyzing the similarity between their different attributes (e.g. their title, description, and number of rooms). In this study, we have only included ads with positive price and living space, as this information was essential to develop the price indices. In addition, ads with different prices were considered different since this study did not intend to track the price changes of the properties on sale.

Although this database contains a timely and rich view of the Swiss real estate market, a valid concern is whether it appropriately reflects the developments of market prices. To examine this issue, we compared the comparis.ch database against the Swiss Real Estate Datapool (SRED) database for apartments in the 2005-Q1/2015-Q2 period (the overlapping period for which we had access to both databases). SRED is an association that aims to promote market efficiency and transparency in the Swiss housing market. Its database covers approximately 40% of all residential transactions in Switzerland, and it is arguably the highest quality data source available for the most liquid part of the market.

Table 2.1 gives summary statistics for this subset of the databases. As it can be observed, the two databases differ substantially in terms of volume. There is roughly a 5:1 ratio between their respective total number of observations (4.9 at district level and 5.7 at national level). The corresponding price developments, on the other hand, seem to behave alike. The ratio of average growth rate of prices per district per quarter is close to unity at the national and cantonal level, though it diverges significantly at the district level (with a ratio of 0.6).

Level	Source	Average #Obs.	Average #Obs. per quarter	Min. average #Obs. per quarter	Max average #Obs. per quarter	Average quarterly price growth rate	Std. deviation price growth rate
National	Tx	120000	2100	1100	3500	0.93	2.16
	Ask	680000	16200	4800	29100	1	1.59
	Ask/Tx	5.7	7.7	4.2	8.3	1.1	0.7
Cantonal	Tx	5000	81	35	146	0.91	12.65
	Ask	26000	624	186	1216	0.9	5.05
	Ask/Tx	5.5	7.7	5.4	8.3	1	0.4
District	Tx	850	14	4.9	32	1.25	20.36
	Ask	4000	98	27	203	0.76	10.96
	Ask/Tx	4.9	6.8	5.4	6.4	0.6	0.5

Table 2.1: Volume (#Obs) and price change aggregates of apartments for the databases of ask (Ask) and transaction (Tx) prices. Statistics correspond to the 2005-Q1/2015-Q2 period.

2.3 Empirical Strategy

2.3.1 Assessment of ask and transaction prices

In order to thoroughly study the relationship between ask and transaction prices we followed a twofold approach. First, we built quantile regressions and test for panel co-integration across different quantiles. Second, we estimated a dynamic panel data model to test for Granger causality between changes in ask and transaction prices. In the following, we briefly elaborate on the statistical tools that we employed to conduct this analysis.

Quantile Regressions

We start the analysis with the study of the time series properties of both databases across different quantiles. To do so, we used quantile regressions to compute district-level quarterly price indices corresponding to the τ-conditional quantile, allowing size and time effects to vary across quantiles. To simplify the estimations, we limit the analysis to ads and transactions in the 32 districts in which at least five transactions per quarter were observed.

A quantile regression estimates a conditional quantile function, in which a quantile of the response variable's conditional distribution is expressed as a function of the covariates. It allows its estimates to vary with the corresponding quantile, and thus it is useful when quantile effects might exist. In our context, the conditional quantile enables us to explore differences in the development of prices across different segments of the market, as housing characteristics might be valued differently at different points of the distribution. Each district-level index for district i has the form,

$$Q_\tau(\log p_{t,i}|size, \mathbf{T}) = \mathbf{X}\beta_{\mathbf{i}}^{\tau\prime} = [1, Size, \mathbf{T}]\left[\alpha_i^\tau, \beta_{size,i}^\tau, \beta_{\mathbf{T},\mathbf{i}}^\tau\right]' \tag{2.1}$$

where $Q_\tau(\bullet|\bullet)$ denotes the conditional quantile function for the τ-quantile that we want to estimate, $\mathbf{X}$ is the vector of co-variates, and $\beta_{\mathbf{i}}^\tau$ is the vector of corresponding coefficients, including an intercept α_i^τ. $\mathbf{X}$ contains the size of the property $Size$, and a vector of time dummy variables for each quarter, denoted as $\mathbf{T}$. $\beta_{\mathbf{i}}^\tau$ is obtained by solving,

$$\beta_{\mathbf{i}}^\tau = \underset{\alpha_{\mathbf{i}}^\tau}{\arg\min}\, \rho_\tau\left(\frac{1}{n}\sum_{j=1}^{n}\log p_{t,i,j} - \left(\alpha_{0,i} + \alpha_{i,size}Size_{i,j} + \sum_{i=1}^{T}\alpha_{i,T_i}T_{i,j}\right)\right) \tag{2.2}$$

being n the total number of observations, and ρ_τ the check function weighting the residual μ_j,

$$\rho_\tau(\mu_j) = \begin{cases} \tau\mu_j & \text{if } \mu_j \geq 0 \\ (1-\tau)\mu_j & \text{otherwise} \end{cases} \tag{2.3}$$

which is asymmetric when $\tau \neq 1/2$. For $\tau = 1/2$, this recovers the conditional median function, i.e. a calibration in the sense of the medians of the residuals. The resulting minimization problem is formulated as a linear function of parameters, and can be solved very efficiently by linear programming methods.

Co-integration

The use of co-integration techniques to test for the presence of long term relationship among integrated variables has enjoyed growing popularity. Given the low power of these techniques when applied to short time series, a natural extension has consisted of expanding them to panel data, while allowing as much as possible heterogeneity of the individual time series.

In order to formally test for co-integration between ask and transaction prices, we employed the sets of statistics proposed by [Westerlund, 2005] and [Pedroni, 2004]. They are designed to test the null hypothesis of no co-integration between time series x and y, containing N cross-sectional units, by inferring whether the residuals of a regression of y on x contain a unit root or not. Our motivation to employ multiple statistics was to examine the robustness of our findings. Specifically, consider the least square regression,

$$y_{i,t} = d_t\hat{\delta}_i + x_{i,t}\hat{\beta}_i + \hat{e}_{i,t} \tag{2.4}$$

where $i = 1..N$ denotes the cross-sectional units, and d_t is a vector of deterministic components with coefficients $\hat{\delta}_i$. The residuals of equation 2.4 are stationary when x and

y are co-integrated. Thus, testing the null hypothesis of no co-integration is equivalent to testing the regression residuals for a unit root. Equation 2.4 is able to accommodate individual specific short-run dynamics, individual specific fixed effects, as well as deterministic trends, and it does not constraint the slope coefficients to be the same across cross-sectional units.

Westerlund [2005]'s variance ratio tests might be regarded as panel data generalizations of [Breitung, 2002] and are based on the value taken by the autoregressive parameter ρ_i in equation 2.5.

$$\hat{e}_{ti} = \rho_i \hat{e}_{it} + u_{it} \tag{2.5}$$

Consider $\hat{E}_{it} = \sum_{j=1}^{t} \hat{e}_{ij}$ and $\hat{R}_i = \sum_{t=1}^{T} \hat{e}_{it}^2$, the first statistic,

$$VR_P = \sum_{i=1}^{N} \sum_{i=1}^{T} \hat{E}_{it}^2 (\sum_{i=1}^{N} \hat{R}_i^{-1}) \tag{2.6}$$

is constructed under the maintained assumption that the autoregressive parameter is the same for all the units. That is, the null and alternative hypotheses are formulated as $H_0 : \rho_i = 1$ for all i versus $H_1 : \rho_i = \rho$ and $\rho < 1$ for all i. Hence, rejection of the null hypothesis should be taken as evidence of co-integration for the entire panel. The second statistic,

$$VR_G = \sum_{i=1}^{N} \sum_{i=1}^{T} \hat{E}_{it}^2 \hat{R}_i^{-1} \tag{2.7}$$

is constructed under the maintained assumption that the autoregressive parameter may vary across units. Thus, the null and alternative hypotheses are formulated as $H_0 : \rho_i = 1$ for all i versus $H_1 : \rho_i < 1$ for $i = 1, ..., N_1$ and $\rho_i = 1$ for $N_1, ..., N$, where we require $N_1/N = \xi \in (0, 1]$ as N goes to infinity. Hence, rejection of the null hypothesis should be taken as evidence of co-integration for a non-vanishing fraction of the panel. The asymptotic distributions of 2.6 and 2.7 are,

$$T^{-2} N^{-1/2} VR_P - N^{1/2} \Theta_{w,3} \implies N(0, \Sigma_{w,33}) \tag{2.8}$$

$$T^{-2} N^{-1/2} VR_P - N^{1/2} \Theta_{w,1} \Theta_{w,2}^{-1} \implies N(0, \phi_w \tilde{\Sigma_w} \phi) \tag{2.9}$$

where $\tilde{\Sigma}$ denote the upper left 2×2 sub-matrix of Σ_w and $\phi_w = (\Theta_{w,2}^{-1}, -\Theta_{w,1} \Theta_{w,2}^{-2})$. $\Theta_{w,1}$, $\Theta_{w,2}$, Σ_w are moments of a vector Brownian motion functional, which Westerlund [2005] computes via MonteCarlo simulations. These values are constant and do not depend on the data. They only depend on whether equation 2.4 includes a trend or not.

Pedroni [2004] also proposes a set of statistics that supports panel and group alternative hypotheses. Let $\tilde{e}_{it} = (\Delta \hat{e}_{it}, \hat{e}_{it-1})'$ and $A_i = \sum_{t=1}^{T} \tilde{e}_{it} \tilde{e}_{it}'$. Then he defines the following test statistics for the null of no co-integration in heterogeneous panels,

$$Z_{\hat{\rho}_{NT}-1} = (\sum_{i=1}^{N} A_{22i})^{-1} \sum_{i=1}^{N} (A_{21i} - T\hat{\lambda}_i) \tag{2.10}$$

$$Z_{\hat{t}_{NT}} = (\tilde{\sigma}^2 \sum_{i=1}^{N} A_{22i})^{-1/2} \sum_{i=1}^{N} (A_{21i} - T\hat{\lambda}_i) \tag{2.11}$$

$$\tilde{Z}_{\hat{\rho}_{NT}-1} = \sum_{i=1}^{N} A_{22i})^{-1} (A_{21i} - T\hat{\lambda}_i) \tag{2.12}$$

$$\tilde{Z}_{\hat{t}_{NT}} = \sum_{i=1}^{N} (\tilde{\sigma_i}^2 A_{22i})^{-1/2} (A_{21i} - T\hat{\lambda}_i) \tag{2.13}$$

where $\hat{\mu}_{it} = \hat{e}_i t - \hat{\rho}_i \hat{e}_{it-1}$, $\hat{\lambda}_i = T^{-1} \sum_{s=1}^{K} w_{sK} \sum_{t=s+1}^{T} \hat{\mu}_{it} \hat{\mu}_{i,t-s}$ for some choice of lag window $w_s, K = 1 - 1/(1-K)$, $\hat{s}_i^2 = T^{-1} \sum_{t=2}^{T} \hat{\mu}_{it}^2$, $\hat{\sigma}_i^2 = \hat{s}_i^2 + 2\hat{\lambda}_i^2$, $\tilde{\sigma}_{NT}^2 = N^{-1} \sum i = 1^N \hat{\sigma}_i^2$ and $\hat{L}_{11}^2 = N^{-1} \sum_{i=1}^{N} \hat{L}_{11i}^2$, where $\hat{L}_{11i}^2 = \hat{\Omega}_{11i} - \hat{\Omega}_{21i}' \hat{\Omega}_{11i}^{-1} \hat{\Omega}_{21i}$ such that $\hat{\Omega}_i$ is a consistent estimator of Ω_i.

Similar to Westerlund [2005]'s statistics, rejection of the null hypothesis using $Z_{\hat{\rho}_{NT}-1}$ and $Z_{\hat{t}_{NT}}$ should be interpreted as evidence of co-integration for the whole panel, while rejection of the null using $\tilde{Z}_{\hat{\rho}_{NT}-1}$ and $\tilde{Z}_{\hat{t}_{NT}}$ should be interpreted as evidence of co-integration for a non-vanishing fraction of the panel. The asymptotic distributions of Pedroni [2004]'s statistics as $(T, N \to \inf)_{seq}$ are,

$$T\sqrt{N} Z_{\hat{\rho}_{NT}-1} - \Theta_2 \Theta_1^{-1} \sqrt{N} \implies N(0, \phi_{(2)}' \psi(2)' \phi_{(2)}) \tag{2.14}$$

$$Z_{\hat{t}_{NT}} - \Theta_2 (\Theta_1 (1 + \Theta_3))^{-1/2} \sqrt{N} \implies N(0, \phi_{(3)}' \psi(3)' \phi_{(3)}) \tag{2.15}$$

$$TN^{-1/2} \tilde{Z}_{\hat{\rho}_{NT}-1} - \tilde{\Theta}_1 \sqrt{N} \implies N(0, \tilde{\psi}_{11}) \tag{2.16}$$

$$N^{-1/2} \tilde{Z}_{\hat{t}_{NT}} - \tilde{\Theta}_2 \sqrt{N} \implies N(0, \tilde{\psi}_{22}) \tag{2.17}$$

where the values for ϕ_j are given by $\phi_{(1)}' = -\Theta_1^{-2}$, $\phi_{(2)}' = (-\Theta_2 \Theta_1^{-2}, \Theta_1^{-1})$ and $\phi_{(3)}' = (-\frac{1}{2} \Theta_2 \Theta_1^{-3/2} (1 + \Theta_3)^{-1/2}, \Theta_1^{-1/2} (1 + \Theta_3)^{-1/2}, -\frac{1}{2} \Theta_2 \Theta_1^{-1/2} (1 + \Theta_3)^{-3/2})$.

As in [Westerlund, 2005], Θ, $\tilde{\Theta}$, Ψ, and $\tilde{\Psi}$ are moments of functionals that do not depend on the data, but on whether the data generating process contains a trend. We do not include a trend for the calculation of any of the statistics, but time demean the indices to take into account that cross-sectional independence, an assumption of the statistics, is arguably violated across districts of the residential Swiss Market.

Causation

Lastly, we explored possible causal relationship between changes in ask and transaction prices. We considered regressions of the form,

$$\Delta \log p_{i,t}^{dv} = \alpha_{i,0} + \sum_{i,k}^{N} \alpha_k \Delta \log p_{i,t-k}^{dv} + \sum_{i,k}^{M} \delta_k \Delta \log p_{i,t-k}^{iv} + \epsilon_{i,t} \tag{2.18}$$

where DV and IV denote the dependent and independent variables. They correspond either to ask and transactions prices or transactions and ask prices respectively, depending on the direction of the causality that we study. The test of whether IV does not cause DV is simply a test of the joint hypothesis $\delta_k = 0, \forall k = 1..M$. This can be done using standard F-tests.

The estimation of equation 2.18 requires more care. It is common practice to first-difference the model in order to deal with the inconsistency introduced by the individual specific effects $\alpha_{i,0}$,

$$\tilde{\Delta} \log p_{i,t}^{dv} = \sum_{i,k}^{N} \alpha_k \tilde{\Delta} \log p_{i,t-k}^{dv} + \sum_{i,k}^{M} \delta_k \tilde{\Delta} \log p_{i,t-k}^{iv} + \epsilon_{i,t} - \epsilon_{i,t-1} \tag{2.19}$$

where $\tilde{\Delta}$ denotes the difference operation conducted to eliminate $\alpha_{i,0}$. However, OLS estimation of equation 2.19 is also inconsistent because the lagged dependent variables introduce correlation with the error term $\epsilon_{i,t} - \epsilon_{i,t-1}$. Therefore, we use the difference Generalized Method of Moments (GMM) estimator for dynamic models with panel data Arellano [2003]. The GMM panel data uses lagged dependent variables as valid instruments in equation 2.19. For example, with $N = 1$, $\Delta \log p_{i,t-2}^{dv}$ becomes available as an instrument for $\tilde{\Delta} \log p_{i,t-1}^{dv} = \Delta \log p_{i,t-1}^{dv} - \Delta \log p_{i,t-2}^{dv}$, since $\Delta \log p_{i,t-2}^{dv}$ is not correlated with $\epsilon_{i,t} - \epsilon_{i,t-1}$.

A more efficient estimator can be obtained by using additional lags of the dependent variable. For example, both $\Delta \log p_{i,t-2}^{dv}$ and $\Delta \log p_{i,t-3}^{dv}$ might be used as instruments for $\tilde{\Delta} \log p_{i,t-1}^{dv}$. Furthermore, the number of instruments available is highest for the dependent variable observed at time t closest to the final period: in period 3 there is only one available instrument, in period 4 there are two, and so on. Arellano and Bond [1991] proposes panel GMM estimation using these wider unbalanced instrument sets, which is known as the Arellano-Bond estimator. To simplify matters, we do not employ this more efficient estimator, but explore the use of different number of instruments to check for robustness in our results.

2.3.2 Real estate indices

We studied the development of prices in 166 Swiss districts[1]. In order to analyze the market, the ads in each district were categorized by type (i.e. apartment or house), and subsequently subdivided in three groups, according to the number of rooms, as described in table 2.2. The properties in each subgroup were aggregated quarterly using the median asking price and the median asking price per square meter for houses and apartments respectively.

Property Type	**Houses**		**Apartments**	
Measure	**Median Ask Price**		**Median Ask Price per Sqm**	
Size	**Min # Rooms**	**Max # Rooms**	**Min # Rooms**	**Max # Rooms**
Small	1	4.5	1	3.5
Medium	5	6.5	4	5.5
Large	7+		6+	

Table 2.2: Categorization of properties based on the number of rooms.

2.3.3 Bubble diagnostic

Our bubble analysis is based on the Log Periodic Power Law Singularity (LPPLS) model. The term "bubble" refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated [Case and Shiller, 2003]. Sornette and Woodard [2010] illustrate the concept of housing price bubble as follows:

"During a housing price bubble, homebuyers think that a home that they would normally consider too expensive for them is now an acceptable purchase because they will be compensated by significant further price increases. They will not need to save as much as they otherwise might, because they expect the increased value of their home to do the saving for them. First-time homebuyers may also worry during a housing bubble that if they do not buy now, they will not be able to afford a home later."

The LPPLS model states that a bubble is a transient, faster than exponential growth process, decorated with ever-increasing oscillations. In its microeconomic formulation, the model assumes a hierarchical organization of the market, comprised of two groups of agents: a group with rational expectations (the value investors), and a group of "noise" agents, who are irrational and exhibit herding behavior (the trend followers). Herding creates price-to-price or price-to-return positive feedback loops that yields an accelerated growth process. The tension and competition between the rational agents and the noise traders produces deviations around the growing prices that take the form of oscillations,

[1]The districts map provided by the Swiss Federal Statistical Office based on 2009 districts' divisions has been used as a basis for performing this study. The Swiss districts' borders regularly evolve (districts merge or split) and current districts name and borders might vary from the ones used in this study.

which increase in frequency as the time of the crash approaches. Intuitively, this can be compared with oscillations that can be observed right before a traffic jam.

In the LPPLS model, a crash signals a change of regime, in which the prices stop rising, and take a different dynamics. This can be a swift correction, like a crash, but also a slow deflation or stagnation. In fact, a less violent and slower end of bubbles is a better representative characteristic of real estate markets since properties are durable goods that people tend to hold whenever falling prices are observed. Moreover, a crash is never a certain event but is characterized by a probability distribution for its occurrence time. This is an essential ingredient for the bubble to exist as it is only rational for financial agents to continue investing when the risk of the crash to happen is compensated by the positive return generated by the financial bubble, and when there exists a small probability for the bubble to disappear smoothly. In other words, the bubble is only possible when the public opinion is not certain about its end [Demos et al., 2015].

Many examples of calibrations of financial bubbles with the LPPLS model are reported in [Jiang et al., 2010], [Johansen and Sornette, 2010], [Zhang et al., 2016], and [Sornette and Cauwels, 2015]. For example, the LPPLS has been successfully used to diagnose in advance the US real estate market bubble that burst in 2007, the oil bubble that crashed in 2008, and the Shanghai Composite index crashes in 2007 and 2009.

We employed the LPPLS model to study the possibility of a bubble in the market. Since the beginning of the study, and every 6 months, we calibrated the model to each of the 166 Swiss districts (see chapter 5 for calibration details). Every time we would obtain a valid calibration, the quality of the bubble signal was assessed, following a hybrid approach. On one hand, we applied the hard constraints described in [Filimonov and Sornette, 2013]. On the other hand, we also looked at each calibration individually to judge whether the calibration was a "strong" bubble signal. Among the criteria that we adopted to determine the signal's strength were the extent to which the fitted prices followed the original time series (i.e magnitude and distribution of the residuals), the smoothness or choppiness of the price index, the implied acceleration at the end of the calibrating window, and the geo-economic significance of the district in which a bubble signal was identified. The output of this process consisted of the classification scheme of figure 2.1, which was used to express the status of the districts based on the LPPLS analysis:

- Critical: a strong bubble signal from the LPPL analysis. This is an indication that a change of regime is imminent. The bracket of the expected time of the change of regime is only reported for this status.

- To Watch: a bubble signal from the LPPL analysis. However, the signal is not as strong as the "Critical" case.

- To Monitor: This status is only obtained after a district has been previously depicted as a "Critical" or "To Watch" district. The price could be increasing without

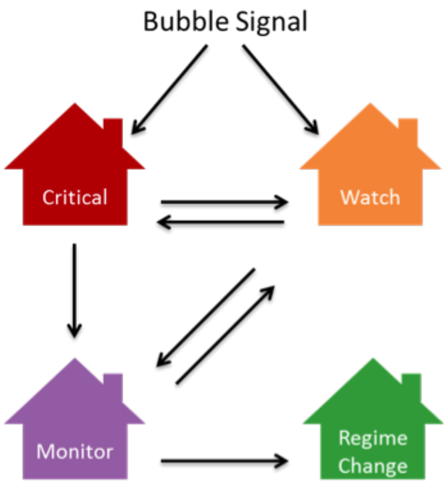


Figure 2.1: Classification of the districts

(anymore) a bubble signal or decreasing but there are not yet enough data points to declare a confirmation of a change of regime.

- Regime Change: This status is only obtained after a district has been previously depicted as a "To Monitor" district and the latest data points confirm a change of regime.

As the status of every districts could change with the arrival of new data, the following transitions were considered:

- A "Critical" district can downgrade into a "To Watch" (respectively a "To Monitor" district), reflecting a weakening of the presence/strength of the bubble signals (respectively a preliminary diagnostic of a change of regime).
- A "To Watch" district can become a "Critical" (respectively a "To Monitor" district) when the strength of the bubble indicators increases (respectively when there is evidence of an on-going change of regime).
- A "To Monitor" district can become a "To Watch" (respectively a "Regime Change" district) when the presence of bubble signals is more strongly confirmed (respectively when the price dynamics has validated the end of the bubble).

This categorization imposed a discipline that partially allows us to assess the quality of the ex-ante analysis. A 'Regime Change" or "To Monitor" district should seldom reappear in the future as "Critical" or "To Watch", as short term re-emergence of real estate bubbles is a rare phenomenon. Similarly, a "Critical" district should not remain in this status for long, as this could suggest overestimation of the bubble risk (i.e a false positive). Finally, formerly "To Watch" districts, once they become either "Critical" or 'To Monitor", were not expected to move back to the "To Watch" list, as this would imply an ambiguous risk assessment. This reasoning can be used to define the metrics described and computed in table 2.3, which we employed to examine the accuracy of the analysis. The discussion of their values is left for the results section.

Metric	**Description**	**Value**
"Critical"-precision	Number of districts that changed to "To Monitor" or "Regime Change" within 1.5 years, out of all "Critical" districts.	10 out of 12
"To Monitor"-NPV	Number of districts that did not ever change back to "To Watch" or "Critical", out of all "To Monitor" or "Regime Change" districts.	23 out of 25
"To Watch"-NPV	Number of districts that did not alternate between "To Monitor" and "Critical", out of all "To Watch" districts.	15 out of 17

Table 2.3: Information retrieval metrics to evaluate the ex-ante analysis. NPV stands for Negative Predictive Value, i.e. the proportion of negative results that are true negative results.

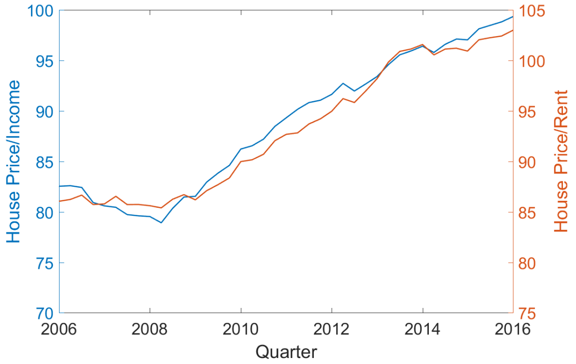
2.4 The Swiss macroeconomic environment

In this section, we provide some background information regarding the economic environment under which this study was conducted.

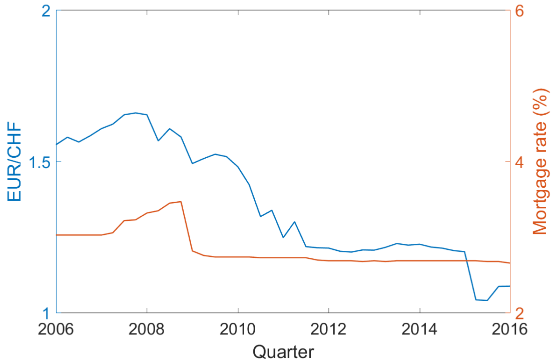
Classic economic measures suggest that house prices in Switzerland have increasingly misaligned from their fundamentals. First, figure 2.2e shows the diverging behavior of inflation and asking prices growth. Second, figure 2.2a shows that during the last years, residential Swiss real estate prices have gradually become less affordable, as evidenced by a positive trend in the price to disposable income ratio. Third, prices have also deviated steadily from the income that real estate can earn over time, as suggested by the increasing price to rent ratio (as shown in figure 2.2a).

Possible drivers of the housing demand in the country can be roughly summarized by two competing set of forces:

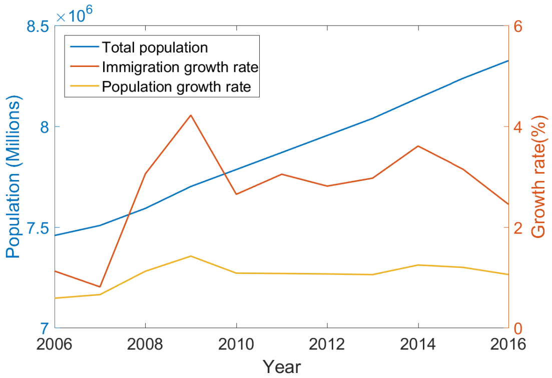
On the one hand, the low interest rate environment, exacerbated by the introduction of negative interest rates in 2015, has pushed the mortgage rates down. As shown in figure 2.2b, the Swiss average mortgage rate has remained down ever since the 2008 financial crisis, while the introduction of negative rates by the SNB to halt the overvaluation of the Swiss Franc after the removal of the CHF/EUR peg pushed the mortgage rate to even lower levels. This downward pressure, added to the housing demand from the increasing population and the slight increase in real incomes due to deflation, constitute a fundamentally positive push to the demand for real estate. Evidence is presented in figures 2.2c, 2.2d, and 2.2e. Figure 2.2c shows that over the past decade the population has been growing at a rate close to 1%. This is mainly driven by immigration, which has had a growth rate higher than 2% over the past decade. Figure 2.2d shows that the Swiss real



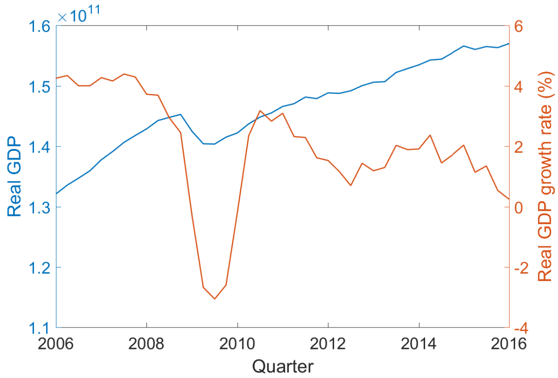
(a) Price to income and price to rent ratios.



(b) EUR/CHF and Swiss variable mortgage rate.

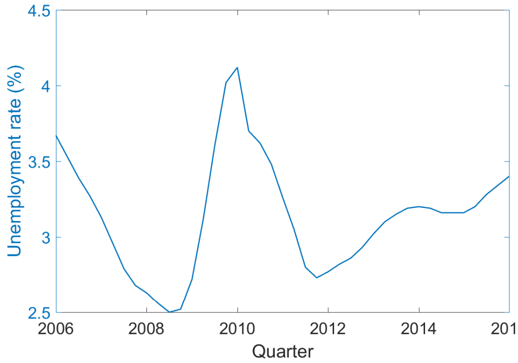


(c) Swiss population and immigration, year on year growth rates.



(d) Swiss Quarterly Real GDP (2010 prices, USD), year on year growth rate.

(e) Swiss inflation and housing asking price, year on year growth rates.



(f) Swiss unemployment rate.

Figure 2.2: Macroeconomic Indicators. Source: Thomson Reuters, OECD, SNB (2016)

GDP has been increasing since 2011, partially attributed to deflation. Figure 2.2e shows that Switzerland is in a deflationary environment since mid-2012.

On the other hand, the regulatory measures and the associated greater financial requirements regarding the purchase of owner-occupied housing required by the SNB, in addition to the increasing unemployment rate (which as depicted in figure 2.2f has been going up since mid-2011 - mainly attributed to the strong Franc), are exerting a major negative dampening effect. The SNB' measures include requiring Swiss banks to hold 2 percent extra capital against mortgage risk-weighted assets from June 30, 2014, up from the 1 percent they were required to hold previously. In addition, from July 2012, households must provide at least 10% of the house value as "hard" equity not taken from pension assets. Furthermore, new borrowers are required to reduce their long-to-value ratio to a maximum of two-thirds within 15 years, countering Swiss tax incentives to keep debt high as long as allowed by the mortgage contract.

These competing factors have made difficult the economic analysis of the Swiss housing market. It has remained uncleared which factors will eventually prevail, or whether new drivers will exogenously determined the future of the market. We do not make any claim in this regard, as our methodology is purely based on prices dynamics. Nevertheless, as we show in the following sections, as of 2016-Q2 and compared to 2012-Q4, the market has visibly cooled down. This might be interpreted as a success of the SNB macro-prudential policies, or as evidence that the challenging macroeconomic conditions attributed to a strong Swiss Franc have indeed restricted the internal demand for this kind of asset.

2.5 Results

2.5.1 Ask and transaction prices

Co-integration

In this section we formally test whether ask and transaction prices tend to move together. To do so, we create ask and transactions conditional quantile indices for 32 Swiss districts, and test for co-integration among them. We test each pair of indices individually, as well as the whole panel.

Visual inspection already suggests that transaction and ask prices are co-integrated. Figure 2.3 shows the development of ask and transaction prices of apartments at the national level. Each index corresponds to an average of median logarithmic prices, comprising the 32 districts in which at least five transactions per quarter were observed. It is interesting to notice the discrepancy between the indices starting in 2013. The price premium might be a consequence of the measures issued by the SNB to mitigate the bubble risk in the housing market. With prices expected to stop rising and demand remaining unassuaged, a gap between asking and transaction prices emerged in which transactions were (on average) conducted at higher prices than those originally advertised.

Table 2.4 reports the results for the individual district co-integration tests and for

District	Intercept	Slope	VR_G	VR_P	ρ_G	ρ_P	t_{NPP}	t_{NPG}
Affoltern	3.30	0.76	-1.19	-1.01	-3.67	-4.48	-3.15	-3.37
Bülach	-0.22	1.02	-1.21	-1.02	-5.47	-6.40	-4.72	-5.24
Dielsdorf	1.62	0.88	-1.23	-1.03	-5.06	-5.97	-5.81	-6.53
Hinwil	2.00	0.85	-1.14	-0.98	-4.15	-5.00	-4.13	-4.54
Horgen	-0.38	1.02	-1.24	-1.04	-4.63	-5.51	-5.03	-5.61
Meilen	1.16	0.91	-0.76	-0.76	-3.17	-3.95	-2.66	-2.79
Pfäffikon	0.61	0.96	-1.00	-0.90	-3.34	-4.12	-3.41	-3.68
Uster	1.67	0.87	-1.15	-0.99	-4.18	-5.02	-3.80	-4.14
Winterthur	0.88	0.93	-1.17	-1.00	-4.01	-4.84	-3.76	-4.09
Dietikon	2.76	0.79	-1.18	-1.01	-5.05	-5.96	-4.52	-5.00
Zurich	1.83	0.86	-0.93	-0.86	-5.54	-6.49	-4.03	-4.41
Zug	1.33	0.91	-1.12	-0.97	-6.31	-7.31	-4.96	-5.51
Basel-Stadt	0.48	0.96	-1.20	-1.01	-3.67	-4.48	-3.54	-3.83
Arlesheim	-0.04	1.00	-1.27	-1.05	-5.82	-6.79	-7.38	-8.40
Albula	-0.70	1.06	-1.03	-0.92	-5.73	-6.69	-4.73	-5.24
Prättigau-Davos	-0.46	1.03	-1.03	-0.92	-3.90	-4.73	-4.45	-4.91
Surselva	7.09	0.46	-1.19	-1.01	-4.48	-5.35	-4.31	-4.74
Baden	2.73	0.80	-1.14	-0.98	-4.06	-4.90	-3.37	-3.63
Bremgarten	2.68	0.80	-0.97	-0.89	-2.14	-2.84	-2.91	-3.09
Locarno	-0.92	1.05	-1.24	-1.04	-5.00	-5.91	-4.67	-5.17
Lugano	-5.25	1.37	-1.26	-1.05	-4.44	-5.31	-5.68	-6.37
Aigle	5.69	0.56	-1.01	-0.91	-4.32	-5.17	-3.51	-3.79
Lausanne	-0.85	1.05	-0.95	-0.87	-4.85	-5.75	-3.73	-4.06
Lavaux-Oron	1.83	0.86	-1.06	-0.93	-5.96	-6.94	-4.45	-4.92
Morges	-2.75	1.20	-0.71	-0.74	-6.24	-7.24	-4.22	-4.65
Nyon	-0.98	1.07	-1.00	-0.90	-2.36	-3.07	-2.21	-2.26
Riviera-Pays-d'Enhaut	0.10	0.98	-0.74	-0.75	-3.14	-3.91	-2.39	-2.47
Martigny	-2.73	1.20	-1.25	-1.04	-5.72	-6.68	-5.26	-5.87
Monthey	0.98	0.91	-1.10	-0.96	-5.25	-6.18	-3.95	-4.32
Sierre	4.29	0.66	-1.21	-1.02	-3.71	-4.52	-3.97	-4.35
Sion	-1.80	1.13	-1.19	-1.01	-5.23	-6.16	-5.28	-5.90
Geneva	-1.42	1.08	-0.67	-0.71	-4.30	-5.15	-3.35	-3.61
No time demeaned			-6.11	-5.31	-25.62	-31.67	-23.61	-25.89
Time demeaned			-3.74	-4.24	-24.58	-30.65	-22.99	-23.97

Table 2.4: Individual and panel co-integration test statistics between median ask and transaction property indices, for selected districts. The indices were built using median quantile regressions, as described in section 2.3.1. The standardized test statistics of equations 2.6-2.7 and 2.10-2.13, also explained in section 2.3.1, are asymptotically normal. Critical values are thus 1.645, 1.96, 2.575 for, respectively, the 0.9, 0.95, 0.99 confidence levels.

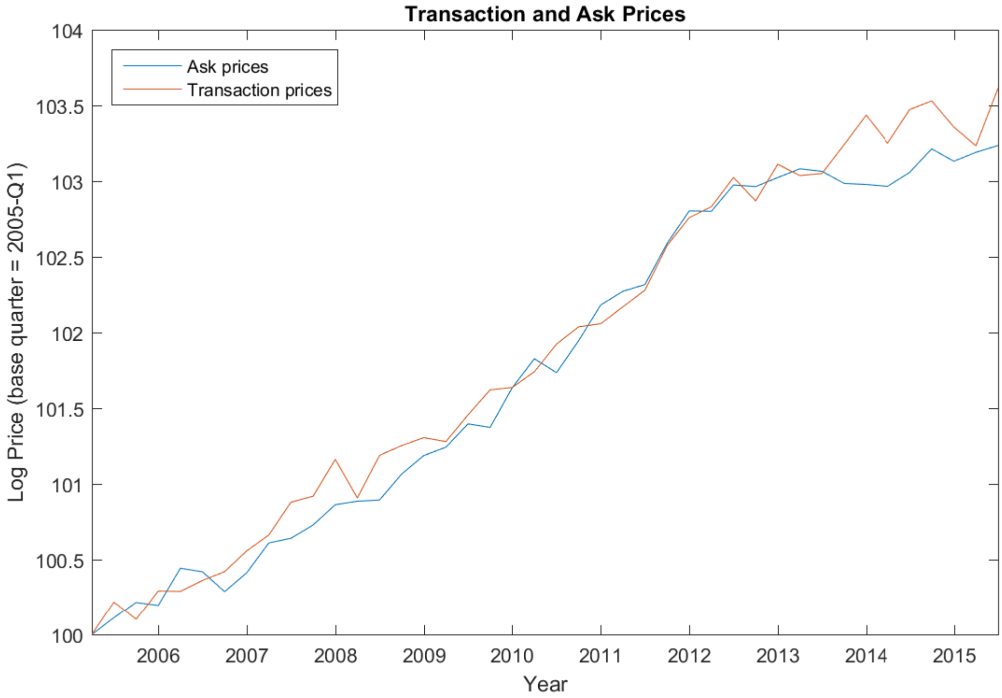


Figure 2.3: Transaction and asking prices for apartments (national aggregate).

the complete panel data, when using the median property indices. The Intercept and Slope columns correspond to the estimated values of equation 2.4, when the deterministic component does not include a trend. For all the statistics, large negative values should be interpreted as rejection of the null hypothesis of no co-integration.

The individual tests yields mostly evidence of co-integration. On the one hand, The statistics ρ_G, ρ_P, t_{NPP}, t_{NPG}, VR_P are all negative values, and significant at a 1% significance level. On the other hand, VR_G is mostly insignificant and we are unable to reject the null hypothesis for any of the districts. The tests applied on the whole panel suggest strong evidence of co-integration. All values are negative, and well below the critical values. This observation remains true even when demeaning the time series to control for dependence among districts. In this case, the absolute values of the statistics decrease slightly, but they remain strongly significant. We thus conclude that the median ask and transaction prices are co-integrated.

In table 2.5 we explore whether the co-integration conclusion extends to additional conditional quantiles. The examined quantiles cover the $0.1 - 0.9$ range, which correspond to the most representative segments of the market. The null hypothesis of no co-integration is rejected across all quantiles and by all tests. These results also extend to the demeaned time series, which control for the possible violation of the cross-sectional independence. Hence, there is strong evidence that ask prices reflect the dynamics of the transactions of the Swiss apartment market, at least in the districts studied. Unfortunately, we are unable

Type	Conditional Quantile τ	VR_G	VR_P	ρ_G	ρ_P	t_{NPP}	t_{NPG}
No time demeaned	0.1	-5.82	-5.06	-27.22	-35.33	-26.26	-27.8417
	0.25	-6.12	-5.25	-24.06	-29.14	-22.52	-25.29
	0.5	-6.10	-5.31	-25.62	-31.66	-23.61	-25.89
	0.75	-5.95	-5.30	-27.24	-32.27	-23.84	-26.28
	0.9	-5.39	-4.97	-26.49	-32.10	-23.22	-25.02
Time demeaned	0.1	-5.10	-5.10	-28.70	-36.31	-27.61	-27.69
	0.25	-4.12	-4.43	-24.49	-29.87	-22.08	-23.93
	0.5	-3.73	-4.23	-24.58	-30.64	-22.99	-23.96
	0.75	-3.50	-3.78	-24.98	-30.40	-22.15	-24.47
	0.9	-5.38	-5.00	-27.28	-32.30	-24.59	-26.98

Table 2.5: Panel co-integration test statistics between ask and transaction property indices. The panel contains the conditional τ-quantile indices for the districts listed in table 2.4. The indices were built using τ-quantile regressions, as described in section 2.3.1. The standardized test statistics of equations 2.6-2.7 and 2.10-2.13, also explained in section 2.3.1, are asymptotically normal. Critical values are thus 1.645, 1.96, 2.575 for, respectively, the 0.9, 0.95, 0.99 confidence levels.

to expand this analysis to the other districts, as there are simply not enough transactions. Nevertheless, in our understanding, nothing suggests that less liquid districts could behave differently.

Causality

We now examine causality between ask and transaction prices. For the bubble diagnosis, evidence of a positive causal relationship from ask to transaction prices would suggest an arbitrage opportunity, as agents could use this public information to anticipate price rises, which in turn could lead to further price rises. Alternatively, a causal relationship from transaction to ask prices would weaken a bubble diagnosis based on the later, as the identification of the bubble or its tipping point forecasting could be done after prices have already started to correct. In this sense, this would represent a strong drawback for the use of ask prices.

Table 2.6 presents the results. Estimates are based on the GMM estimator, described on section 2.3.1, which uses lag variables as instruments. Reported critical values correspond to bootstrapped estimates in order to control for small sample effects. With any number of lags and in any direction, there is no evidence of a causal relationship. The null hypothesis of no Granger-causality cannot be rejected in either of the cases. Hence, the co-integration between ask and transaction prices does not appear to originate from Granger causality. Exogenous factors or the same endogenous process with no time lag are more likely to explain the co-movement among these two variables. In other words, ask prices can be argued to be rather good proxies of the transaction prices. In addition, the lack of (line) predictability of returns, together with the co-movement of prices, suggests that a bubble analysis based on ask prices is indeed informative and a sound alternative to monitor the market, especially in light of the scarcity and sparsity of transactions taking

	DV = Median ask prices					
	4		6		8	
	DV	IV	DV	IV	DV	IV
1	-0.67	0.03	-0.84	0.02	-0.88	-0.04
2	-0.45	0.02	-0.76	0.03	-0.83	-0.09
3	-0.22	0.03	-0.64	0.04	-0.72	-0.14
4	-0.13	0.01	-0.58	0.07	-0.70	-0.13
5			-0.45	0.07	-0.62	-0.13
6			-0.37	0.17	-0.65	-0.04
7					-0.29	-0.20
F-statistic	0.36		0.30		0.25	
Critical Value	2.06		1.57		1.15	
	DV = Median transaction prices					
	DV	IV	DV	IV	DV	IV
1	-0.92	-0.15	-1.15	-0.09	-1.22	0.04
2	-0.66	-0.19	-1.16	-0.15	-1.26	0.09
3	-0.34	-0.22	-0.99	-0.22	-1.13	0.06
4	-0.15	-0.04	-0.82	-0.11	-0.98	-0.01
5			-0.54	-0.21	-0.73	0.01
6			-0.17	-0.26	-0.42	-0.08
7					-0.26	0.23
8					-0.18	0.21
F-tatistic	1.17		1.24		1.06	
Critical value	6.94		5.55		4.28	

Table 2.6: Granger causality tests. The table reports GMM estimations and Granger causality tests of the form $\Delta \log p_{i,t}^{dv} = \alpha_{i,0} + \sum_{i,k}^{N} \alpha_k \Delta \log p_{i,t-k}^{dv} + \sum_{i,k}^{M} \delta_k \Delta \log p_{i,t-k}^{iv} + \epsilon_{i,t}$, with $M = N = 4, 6, 8$. IV denotes the independent Variable. DV denotes the dependent variable. The $F - statistics$ tests whether $\delta_k = 0, \forall k = 1..M$. Estimations employ the median conditional indices described in section 2.3.1.

place in Switzerland.

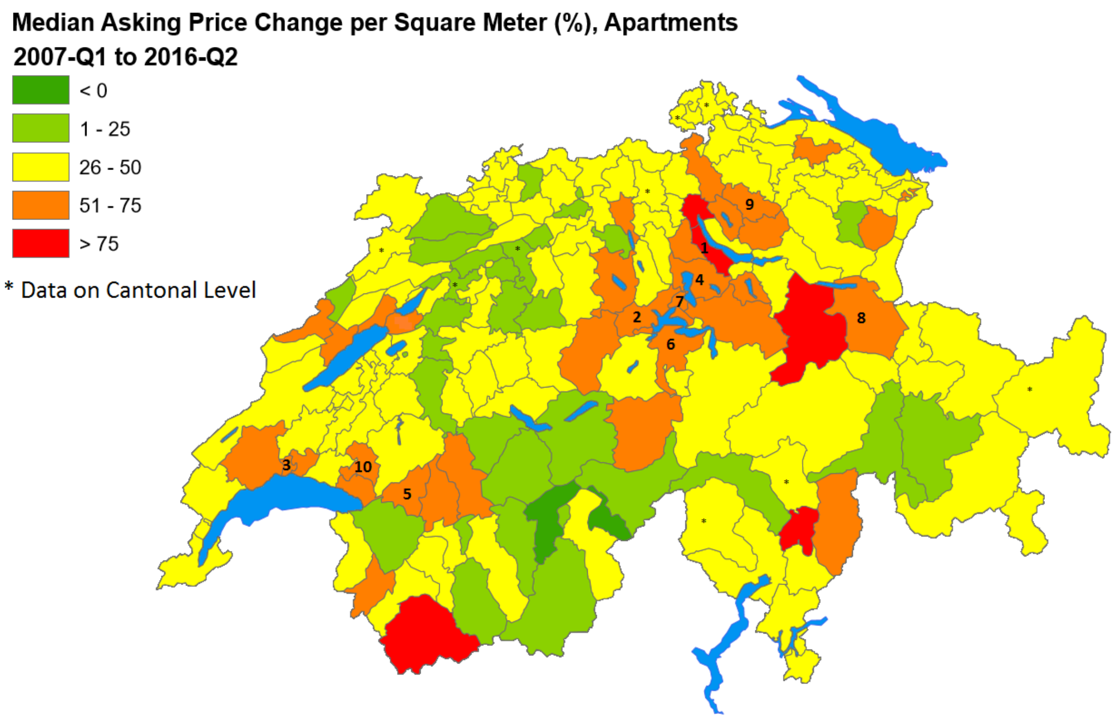
2.5.2 Real estate market in Switzerland

Figure 2.4 shows the change in median ask price per square meter between the first quarter of 2007 and the second quarter of 2016 for all apartments listed on comparis.ch. The regions marked with "*" represent the districts with not enough listings in either 2016-Q2 or 2007-Q1. The cantonal median price change per square meter values are shown for those districts. The top ten districts with the highest increase in the apartments' ask price per square meter between 2007-Q1 and 2016-Q2 appear labeled and listed. For these top ten districts, the median increase in ask price per square meter since last year is also reported (between 2015-Q2 and 2016-Q2). The district of Horgen, labeled 1, shows the highest price increase, where the median asking price of apartments per square meter has increased by 76% since the first quarter of 2007. The prices in the districts of Riviera, D'Entremont, Glarus, and Zurich, although marked in red, were based on either too few advertised properties or the data has been too noisy to be included in the top 10 districts with the highest ask price per square meter.

Additionally, figure 2.4 lists the ten districts with the lowest increase in the apartments' asking price per square meter between 2007-Q1 and 2016-Q2 with enough listings. For these bottom ten districts, the median increase in asking price per square meter since 2015-Q2 is also reported. The prices in the district of Unterklettgau, although marked in dark green in the figure, were based on too few advertised properties to be included in the bottom 10 districts with the lowest change in median ask price per square meter.

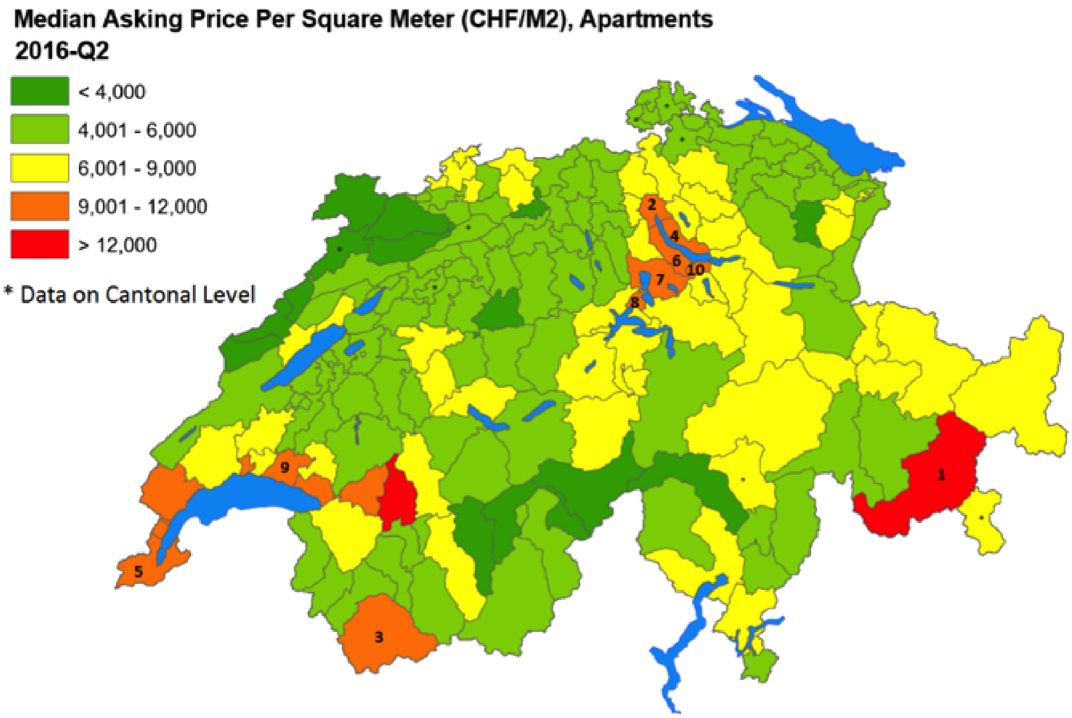
Figure 2.5 shows the median ask price per square meter for apartments as of 30 June 2016. The districts with "*" marks represent the districts with not enough listings in the second quarter of 2016. The cantonal median prices per square meter for apartments are shown for these districts. The top ten most expensive districts as of 30 June 2016 appear labeled and listed in the figure. The ten districts with the lowest median prices per square meter for apartments as of 30 June 2016 are listed in the table accompanying the figure. The prices in the district of Saanen, although marked in red, were based on too few advertised properties to be included in the top 10 districts with the highest asking price per square

The median ask prices for medium size houses (5 to 6.5 rooms) as of 2016-Q2 are shown in Figure 2.6 . Districts with "*" marks represent the districts with not enough listings in the second quarter of 2016. The cantonal median asking prices for medium size houses are shown for these districts. The top ten districts with currently most expensive medium size houses, as well as the ten districts with the lowest median prices for medium size houses as of 30 June 2016 are listed. The absence of districts such as the city of Zurich in this list does not necessarily mean that the ask prices in those districts were lower than the ones listed in figure 2.5, but that there was not enough medium size houses listed for sale during the second quarter of 2016 in those districts.



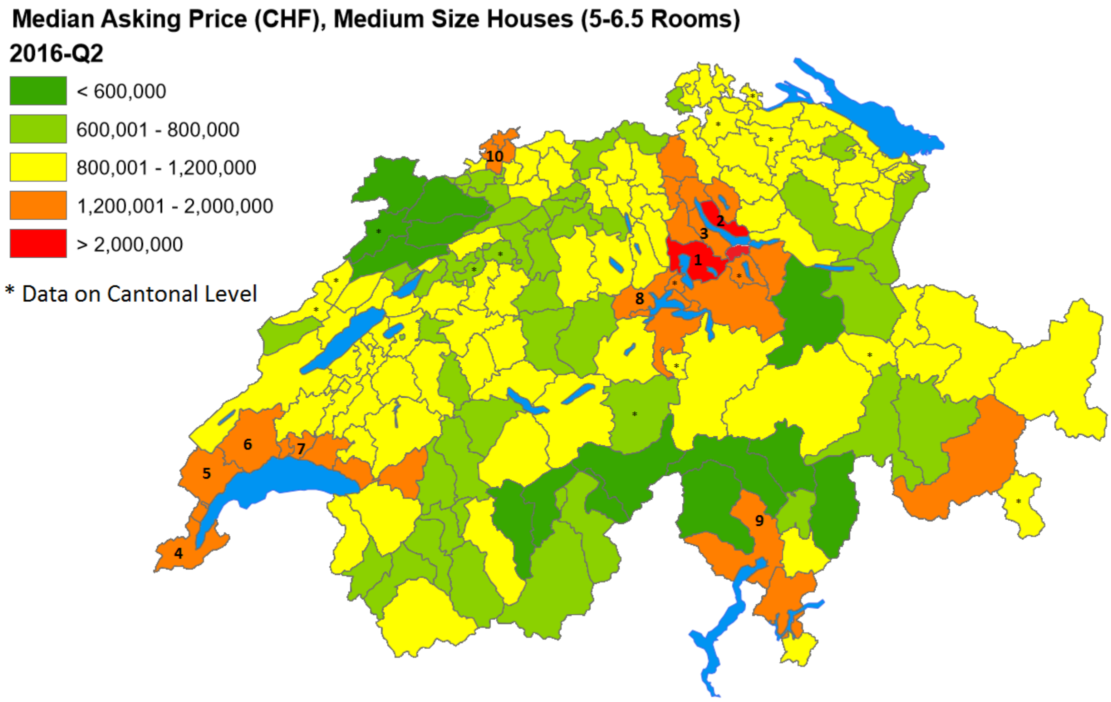
Top increase in median ask price per square meter				Bottom increase in median ask price per square meter			
Rank	**District Name**	**From 2007-Q1 to 2016-Q2**	**From 2015-Q2 to 2016-Q2**	**Rank**	**District Name**	**From 2007-Q1 to 2016-Q2**	**From 2015-Q2 to 2016-Q2**
1	Horgen	76%	10%	1	Raron	−10%	0%
2	Luzern	67%	−5%	2	Goms	11%	−4%
3	L'Ouest lausannois	65%	5%	3	Liestal	16%	−9%
4	Zug	65%	2%	4	D'Aigle	20%	3%
5	Riviera-Pays-d'Enhaut	62%	0%	5	Kulm	20%	−7%
6	Nidwalden	61%	−5%	9	D'Hérens	20%	−1%
7	Küssnacht (SZ)	59%	−5%	7	Lebern	21%	0%
8	Sarganserland	58%	14%	8	Leuk	22%	9%
9	Pfäffikon	58%	5%	9	Burgdorf	22%	0%
10	Veveyse	57%	0%	10	Büren	23%	5%

Figure 2.4: Change in median asking price per square meter for apartments in all Swiss districts between 2007-Q1 and 2016-Q2.



Highest ask price per square meter			Lowest ask price per square meter		
	District Name	CHF/m^2		**District Name**	$CHF/m2$
1	Maloja	13′000	1	Raron	3′000
2	Zürich	11′500	2	Leventina	3′000
3	Entremont	11′500	3	La Chaux-de-Fonds	3′500
4	Meilen	11′000	4	Delúmont	3′500
5	Géneva	11′000	5	Goms	4′000
6	Horgen	10′500	6	Trachselwald	4′000
7	Zug	10′000	7	Hinterland	4′000
8	Küssnacht (SZ)	10′000	8	Gösgen	4′000
9	Lavaux-Oron	10′000	9	Leuk	4′000
10	Höfe	9′500	10	Porrentruy	4′000

Figure 2.5: Median asking price per square meter for apartments in all Swiss districts as of 2016-Q2



Highest Ask Price			Lowest Ask Price		
	District Name	*CHF*		**District Name**	*CHF*
1	Zug	2′250′000	1	Porrentruy	450′000
2	Meilen	2′250′000	2	Blenio	450′000
3	Horgen	1′800′000	3	Leventina	450′000
4	Genève	1′600′000	4	Leuk	500′000
5	Nyon	1′600′000	5	Courtelary	550′000
6	Morges	1′400′000	6	Delúmont	600′000
7	L'Ouest lausannois	1′400′000	7	Thal	600′000
8	Luzern	1′400′000	8	Aarwangen	650′000
9	Locarno	1′400′000	9	Brig	650′000
10	Arlesheim	1′400′000	10	Wasseramt	650′000

Figure 2.6: Median asking price of medium size houses (5 to 6.5 rooms) in all Swiss districts as of 2016-Q2.

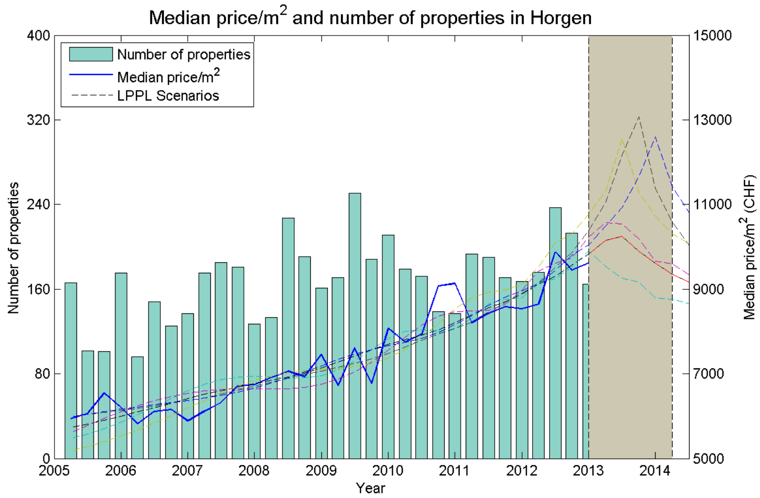
2.5.3 Bubble diagnostic

We applied the methodology to all subcategories of properties defined in table 2.2 over the period 2005-Q1 to 2016-Q2, starting in 2012-Q4, every six months, until 2016-Q2[2]. The diagnostic was conducted in real time and therefore it is free of hindsight bias[3] . An example of the bubble analysis on the development of the median ask price per square meter for all apartments in Horgen ("Critical" district) and the canton Zug ("To Watch' district) is shown in figure 2.7. Both regions exhibit the signals of the bubbles according to the LPPLS method: a super-exponential growth, accompanied by decorating oscillations. The gray area represents the 80 percent confidence interval of the critical time and the dotted lines represent possible LPPLS scenarios.

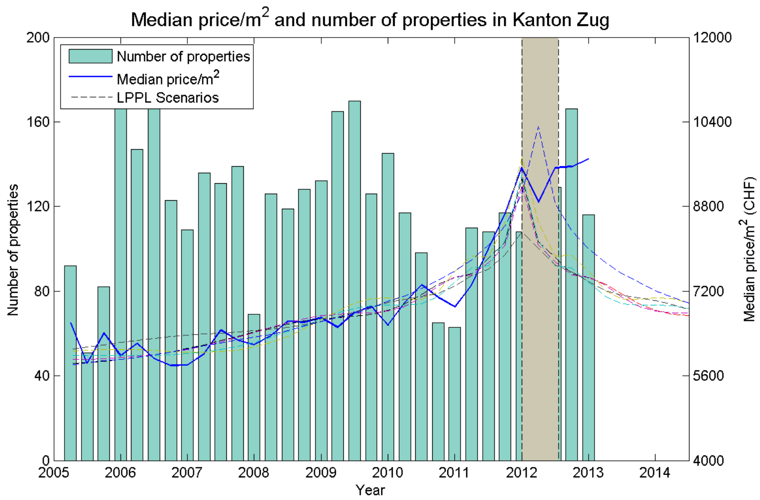
Figure 2.8 and 2.9 present the results as obtained in 2012-Q4 and 2016-Q2 respectively. In 2012-Q4, we identified 11 bubble regions, and 7 regions that should be watched. In 2016-Q2, the market had cooled down visibly, as no critical region was identified, while clear evidence of a regime change was observed in three districts; in addition, 13 districts fell in the "To Monitor" category, in which districts with former but no current bubble signals are classified.

[2]The exception is 2015-Q4, in which we did not monitor the market.

[3]The complete reports, with publication dates, are available at `http://www.er.ethz.ch/real-estate-observatory/publications/swiss-real-estate-market-reports.html`

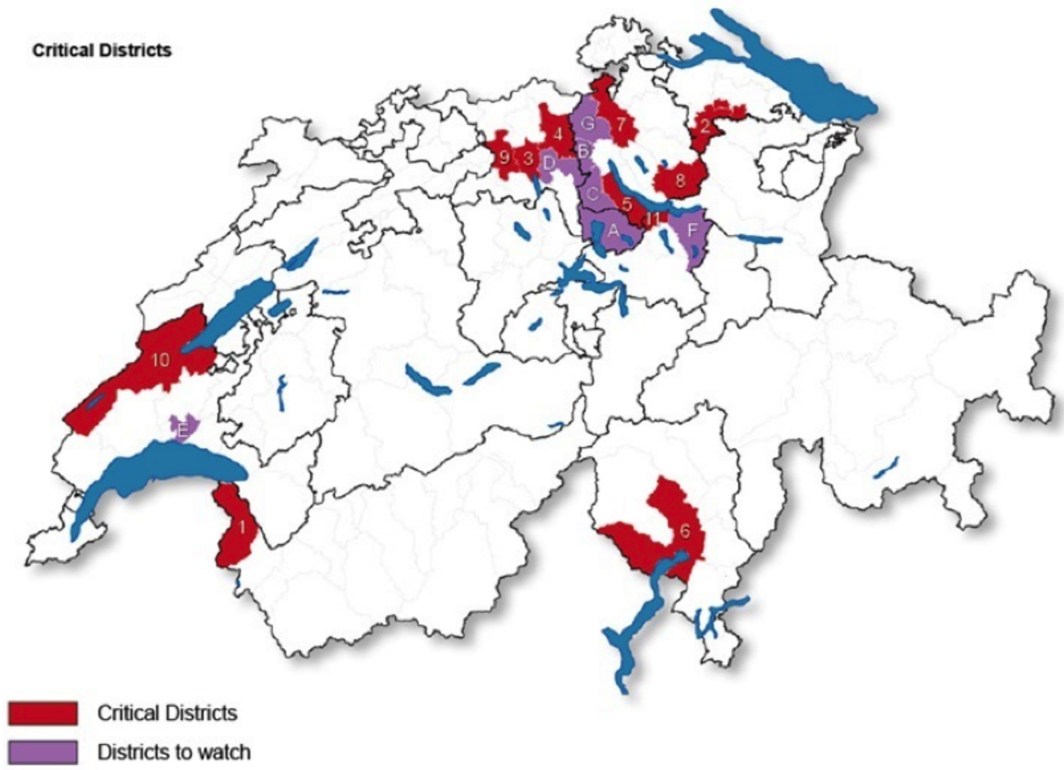


(a) Horgen, "Critical" district.



(b) Zug, "To Watch" district.

Figure 2.7: LPPLS calibration for Horgen and Zug. All apartments.



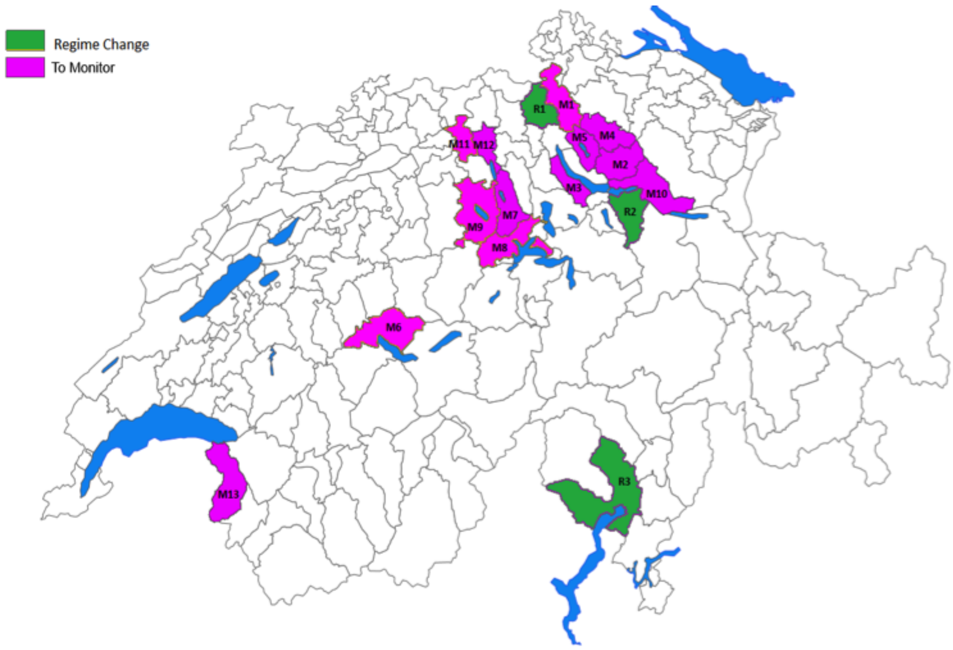
Critical Districts

ID	District	Property Type	Size	Critical Time
1	Monthey	Apartments	Medium	2013 Q1 – 2014 Q1
2	Münchwilen	Apartments	Medium	2013 Q1 – 2013 Q4
3	Lenzburg	Houses	Medium	2013 Q1 – 2014 Q1
4	Baden	Apartments	All	2013 Q1 – 2014 Q1
5	Horgen	Apartments	All	2013 Q1 – 2014 Q2
6	Locarno	Apartments	All	2013 Q1 – 2014 Q1
7	Bülach	Apartments	Medium	2013 Q1 – 2013 Q4
8	Hinwil	Houses	Medium	2013 Q1 – 2014 Q1
9	Aarau	Houses	Medium	2013 Q1 – 2014 Q1
10	Jura-Nord vaudois	Houses	Medium	2013 Q1 – 2014 Q1
11	Höfe	Apartments	Medium	2013 Q1 – 2014 Q1

Districts to watch

A	B	C	D	E	F	G
Zug	Dietikon	Affoltern	Bremgarten	Lausanne	March	Dielsdorf

Figure 2.8: Results of the LPPLS analysis as of 2012-Q4.

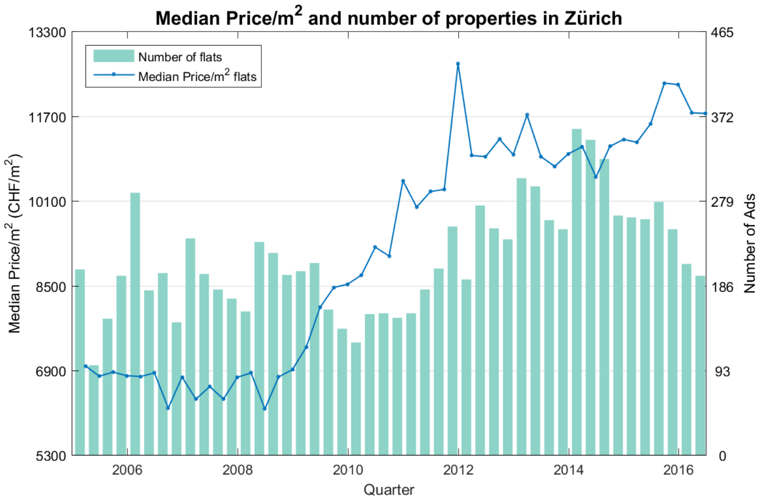


Label	District Name	Status	Property Type	Property Size
R1	Dielsdorf	Regime Change	Apartments	All
R2	March	Regime Change	Apartments	All
R3	Locarno	Regime Change	Apartments	All
M1	Bülach	To Monitor	Apartments	All
M2	Hinwil	To Monitor	Houses/Apartments	Medium/All
M3	Horgen	To Monitor	Apartments	All
M4	Pfäffikon	To Monitor	Apartments	All
M5	Uster	To Monitor	Apartments	All
M6	Thun	To Monitor	Apartments	All
M7	Hochdorf	To Monitor	Houses/Apartments	Medium/All
M8	Luzern	To Monitor	Apartments	All
M9	Sursee	To Monitor	Houses/Apartments	Medium/All
M10	See-Gaster	To Monitor	Apartments	All
M11	Aarau	To Monitor	Houses/Apartments	Medium/All
M12	Lenzburg	To Monitor	Houses/Apartments	Medium/All
M13	Monthey	To Monitor	Apartments	All

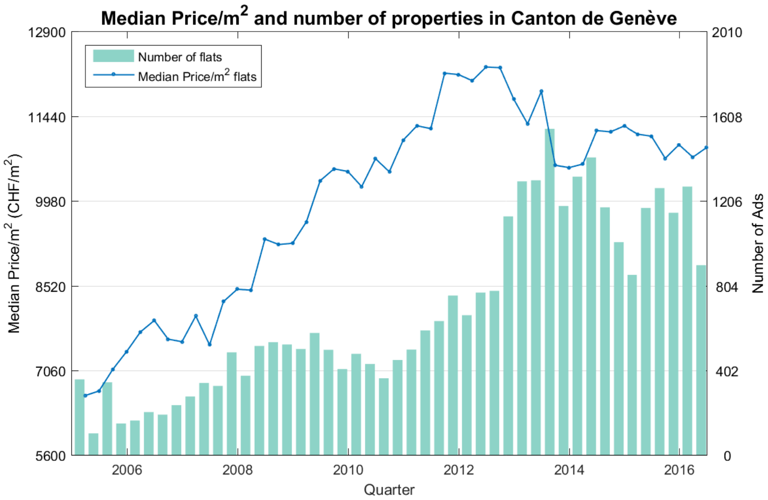
Figure 2.9: Results of the LPPLS analysis as of 2016-Q2.

District Name	2013-Q2			2013-Q4			2014-Q2			2014-Q4			2015-Q2			2016-Q2		
	Status	**Type**	**Size**	**Status**	**Type**	**Size**	**Status**	**Type**	**Size**	**Status**	**Type**	**Size**	**Status**	**Type**	**Size**	**Status**	**Type**	**Size**
Aarau	M	H/A	Med/All	M	H/A	Med /All	M	H/A	Med/All	M	H/A	Med/All	W	H/A	Med/All	M	H/A	Med/All
Affoltern	R	A	All	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Baden	C	A	All	M	A	All	R	A	All	-	-	-	-	-	-	-	-	-
Bremgarten	R	A	All	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Bülach	C	A	Med/S	C	A	All/Med/S	W	A	All/Med/S	W	A	All/S	W	A	All/S	M	A	All
Dielsdorf	C	A	All	W	A	All/Med	M	A	All/Med	M	A	All/Med	M	A	All	R	A	All
Dietikon	R	A	S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Hinwil	M	H/A	Med/All	M	H/A	Med/All	M	H/A	Med/All	M	H/A	Med/All	M	H/A	Med/All	M	H/A	Med/All
Hochdorf	-	-	-	W	A	Med/S	W	A	Med/S	M	H/A	Med/All	M	H/A	Med/All	M	H/A	Med/All
Höfe	M	A	Med	M	A	Med	M	A	Med	R	A	All	-	-	-	-	-	-
Horgen	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All
Jura-Nord Vaudois	M	H	Med	R	H	Med	-	-	-	-	-	-	-	-	-	-	-	-
Lausanne	M	A	All	M	A	All	R	A	All	-	-	-	-	-	-	-	-	-
Lenzburg	M	H	Med	M	H	Med	M	H	Med	M	H/A	Med /All	M	H/A	Med /All	M	H/A	Med /All
Locarno	M	A	All	W	A	All/S	W	A	All/S	M	A	All	M	A	All	R	A	All
Luzern	-	-	-	-	-	-	-	-	-	-	-	-	W	A	All	M	A	All
March	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All	R	A	All
Monthey	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All
Münchwilen	M	A/H	Med	M	H/A	Med /All	R	H/A	Med /All	-	-	-	-	-	-	-	-	-
Pfäffikon	W	A	Med	W	A	Med	W	A	Med	M	A	All	M	A	All	M	A	All
See-Gaster	-	-	-	W	A	All/Med	W	A	All/Med	M	A	All	M	A	All	M	A	All
Sursee	-	-	-	-	-	-	-	-	-	-	-	-	W	H/A	Med/All	M	H/A	Med/All
Thun	-	-	-	-	-	-	-	-	-	-	-	-	W	A	All	M	A	All
Uster	W	A	Med/S	M	A	All	M	A	All	M	A	All	M	A	All	M	A	All
Zug	R	A	All	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 2.7: Detailed 2013-Q2/2016-Q2 bubble diagnostic. Subcategories of Apartments (A) and Houses (H) can be “Critical” (C),“To Watch” (W), “To Monitor” (M), or “Regime Change” (R). *S* and *Med* stands for Small and Medium-size properties, as in to table 2.2.



(a) City of Zurich



(b) Canton of Geneva

Figure 2.10: Median ask price per square meter for apartments in Zurich and Geneva.

Interestingly, the bubble districts appeared clustered around the districts of Zurich and Geneva, but never included these districts. To understand the reason, the median ask price per square meter for apartments in these two geopolitically important districts are presented in figure 2.10 . Our assessment has found no bubble signatures in these regions despite the fact that these districts have gone through a significant price increase and tend to be consistently reported as risky regions by other studies (see section 2.5.4). The LPPLS model requires a faster than exponential price growth, prioritizing the price-dynamics rather than the absolute price value to diagnose a price development as a bubble (comparing a district to itself at previous times instead of comparing it to all its peers). The condition of a faster than exponential price growth is essential in our methodology, and is not fulfilled in the canton of Geneva or the city of Zurich. Therefore, the development of prices in these districts does not satisfy our definition of a bubble.

The detailed diagnostic (table 2.7), as conducted in 2013-Q2, 2013-Q4, 2014-Q2, 2014-Q4, 2015-Q2, and 2016-Q2, allows us to make two observations. First, the booming phase of the market reached its peak between 2012-Q4 and 2013-Q2, as no more than five districts entered the analysis afterwards, and none was directly classified as "Critical". This timeline coincided with the issuing of the SNB regulatory measures, whose effects by 2013-Q2 were uncertain. Second, we observed that the transition to new market regimes was slow, as districts typically remained in the "To Monitor" list for at least 1.5 years, while eight districts have kept this status between 2.5 and 3 years. This might be interpreted as a common trait of the real estate market, characterized by low frequency movements, or as a consequence of the strong demand factors that still push upwards the prices.

As for the accuracy of the forecasts, the performance is admittedly difficult to assess, as the classification of districts and the quality assessment of the LPPLS calibrations required subjective judgment. However, the classification scheme that we employed allowed us to compute the metrics presented in table 2.3. According to these quantities, the diagnosis has been remarkably accurate, as the "Critical"-precision reached 83%, while negative predictive values exceeded the 88% level. The rate of true negatives (specificity) and true positives (recall) is not possible to obtain, as the LPPLS methodology only suited to identify super-exponential-growth bubbles. In other words, we are unable to rule out whether other districts also experienced a phase of exuberance, different from a super-exponential growth. These are important questions to address, but exceed the scope of this study.

2.5.4 Comparison to other studies

We compared our results with the UBS Bubble Index, which is published quarterly [UBS, 2013a]. This index comprises six different sub-indices that track the relationship between purchase and rental prices, the relationship between house prices and household income, the relationship between house prices and inflation, the relationship between mortgage debt and income, the relationship between construction and gross domestic product (GDP), and the proportion of credit applications by UBS clients for residential property not intended for owner occupancy. The selection of exposed regions is further conducted using a multi-level process that considers the size of the regional population and the property price data. In its 2012Q4 report, the index rose to 1.11 from 1.02, and highlighted 17 exposed districts and nine monitored ones.

A one to one comparison with this index is not possible as the UBS index does not make any claim regarding the future development of prices, and even defines some regions differently. However, table 2.8 presents a simplified parallel, in which the districts depicted by the LPPLS model and the UBS Bubble Index are directly contrasted . The LPPLS model provides completely new information about nine districts (eight critical districts, and one to watch district), but does not report a critical situation in 19 districts, where

UBS points current exposure or need for monitoring.

	UBS exposed districts	UBS monitored districts	Not reported by UBS
LPPLS critical districts	Horgen (Zimmerberg) Höfe (March)	Bülach (partially in Glattal-Flurttal)	Monthey Müchwillen Lenzburg Baden Hinwil Locarno Aarau Jura-Nord valudois
LPPLS to watch districts	Dietikon(Limmatal) March Lausanne	Dielsdorf (partially in Glattal-Flurttal)	Bremgarten
Not reported by the LPPLS model	Prättigau-Davos Bernina (Oberengadin) Maloja (Oberengadin) Geneve Nyon Morges Lavaux-Oron (Vevey) Arlesheim (Unteres Basielbieg) Saanen(Saanen-Obersimmental) Zürich Meilen(Pfannestiel)	Basel Stadt Luzern Appenzell-Innerhoden Nidwalden Uster Gersau (Innerschvyz) Kussnacht (Innerschvyz) Schvyz (Innerschvyz)	

Table 2.8: Comparison between the LPPLS results and the UBS Bubble Index as of 2012-Q4

We looked closely at the exposed districts reported by the UBS Bubble Index that were not identified by the LPPLS model, as this could imply an overlook of latent threats. Among these discrepancies, arguably, the most prominent is the absence of the cantons of Geneva, Vaud, and Graubünden, consistently reported by the UBS Bubble Index as risky zones. Our assessment found moderate or no bubble signatures in these regions. The reason for this can be found both in the data as in the methodology. First of all, the method applied by UBS compares the values of a region with those of all the country to determine the extent of a regional exposure. As a result, overpopulated regions or those that have historically exhibited above-average prices will tend to be consistently reported in the exposed category. In contrast, the LPPLS model prioritizes the price-dynamics, requiring a faster than exponential growth to diagnose a price development as a bubble. The latter condition is essential, and is not fulfilled in the cantons of Geneva and Vaud, or in the district of Arlesheim. In these regions, the development of prices resembles a linear growth, which does not satisfy our definition of a bubble. For example, according to the comparis.ch database, the median asking price per square meter of apartments in the canton of Geneva has increased on average 156 CHF per quarter during the observed period. Thus, these cantons do not show the typical signals of the bubbles identified by the LPPLS model. A similar situation happened for the apartments of Zurich and Meilen.

The differences in Obersimmental and the two districts of Graubünden are bound to other reasons. The number of advertised properties that were available in the comparis.ch database for these districts over several quarters is small or zero. Hence, it is not possible to draw any statistical conclusion about their prices, and the results of this study should not undermine the alarms raised by the UBS index. Having mentioned this, it is also worth noting that the low number of properties might not be an issue specific to our dataset, but rather a predominant characteristic of these locations, which serve mainly as luxurious or touristic destinations. If this were the case, the consequences of a bubble in these regions would be marginal for the overall economy as very few people would be affected.

The exception in Graubünden is the apartments in the district of Davos, which have seen a very liquid market. There the situation needed further monitoring as the prices grew sharply during two years, starting from the beginning of 2010, and then stabilized. Although there were notable price increases, the acceleration occurred only during a very short period and therefore this region did not exhibit a signature of a bubble according to our model.

2.6 Conclusions and final discussion

This chapter has presented the consolidated results of the biannual reports that analyzed the residential Swiss real estate market. Despite the strong bubble signals that we found at the beginning of this study, we argued that the economic and political situation of Switzerland did not suggest a severe crash in the critical regions. A soft landing or stagnation of prices was a more probable scenario. The main reasons for this claim were as follows:

First, the rising property prices in Switzerland had not been accompanied by a boom in the construction sector as was the case for the bubbles in the U.S., Ireland and Spain. In these countries, high supply sensitivity introduced a construction boom that contributed to a stronger correction of high prices Allen and Carletti [2010]. On the contrary, the construction sector in Switzerland keeps moving slowly, and stays below historic averages. The vacancy rate in turn had stagnated at a low level, presenting a marginal increase of only one percent during 2012 year: from 38'420 empty apartments in June 2011 to 38920 empty s in June 2012[4].

Second, the SNB had already issued early and urgent measures to control the market. In February 2013, it ordered banks to hold a countercyclical capital buffer amounting to one percent of their risk weighted assets, backed by residential properties in Switzerland

[4]Leerwohnungsziffer stagniert auf tiefem Niveau (Swiss Federal Statistical Office, 2012). `http://www.bfs.admin.ch/bfs/portal/de/index/news/medienmitteilungen.html?pressID=8265` (visited on 02.22.2013)

[Basten and Koch, 2015] . With this policy, the central bank is directly aiming to reduce the exposure of banks to real estate, which has proved a key amplifier of previous crashes [Hilbers et al., 2001]. It was estimated that the new policy would impact as much as 25 percent of the country?s total mortgage volume[5], affecting especially Raiffeisen and regional banks, as most of their assets are mortgages.

Third, unlike the burst of the real-estate market bubble in Switzerland during the 1980s, which was fueled by a decline in mortgage lending standards [Westernhagen et al., 2004], Swiss banks were seeking to implement more conservative practices. The Swiss Financial Market Supervisory Authority FINMA approved a new set of minimum requirements for mortgage financing, drawn up by the Swiss Bankers Association (SBA). The new self-regulatory regime, which came into effect from July 2012 and was later updated in September 2014, for the first time requires a minimum 10 percent down payment from the own borrower's funds when purchasing a property and demands mortgages to be paid down to two thirds of the lending value within 15 years (SNB [2013]; SNB [2014]). This new scheme was intended to prevent households from taking greater risks, as they would be unable to overuse the money from their pension funds to make the down payment and would be pressed to reduce the burden of the debt.

Having said this, it is also important to keep in mind that the impact of the preventive measures was unclear. Not only there was no consensus concerning the role that central banks should play during bubble regimes (see Roubini [2006] and Posen [2006] for the main arguments), but also there was and there is still uncertainty regarding the strength and appropriate calibration of these measures. The fact that the Swiss monetary policy is anchored to the international milieu only makes it harder to maintain the stability of the market as interest rates cannot be revised upwards. In addition, the vigorous demand was an economic reality, so the moderation of price change increases in some districts was expected to be accompanied by increasing price pressures in their adjacent districts. This was plausible, not only because contagious effects have been observed in other housing market bubbles [Roehner, 1999], but also because immigrants, which represent an important driver of the current demand, are traditionally more flexible and willing to travel larger distances when looking for a place to live.

In hindsight, the output of these three years of analysis can be deemed accurate. A severe crash in the identified critical and to be watched districts was deemed unlikely, and a soft landing or stagnation of prices was considered a more probable scenario. By the time this thesis is written, this prognosis has indeed materialized. Nevertheless, we can also identify several limitations of the analysis. There was uncertainty regarding the appropriate methodology to use the LPPLS model as a tool to analyze the Swiss real estate market. Likewise, there was little understanding regarding the performance that we could expect, as well as a sound strategy to define suitable targets. For example, the 1.5 years

[5]Swiss Property Bubble Concern Seen Prompting Tightening (Bloomberg, 2013).
`http://www.bloomberg.com/news/2013-02-13/`
`swiss-property-bubble-concern-seen-prompting-tightening.html` (visited on 02.22.2013).

horizon that we chose to evaluate the precision of the forecast in the critical districts was a rather ad-hoc decision. Lastly, the macroeconomics of the real estate market only entered the study marginally, as fundamental factors do not play a role in the LPPLS model. We will seek to address some of these limitations in the following chapters.

Other issues will remain beyond the scope of this thesis. In particular, since the overall Swiss economic situation remains challenging, an exogenous shock cannot be discarded. Nonetheless, the results of this study extend only to endogenous crashes [Sornette et al., 2013]. Thus, the impact of possible shocks such as the adverse scenario contemplated by the Financial Stability Report of the SNB [SNB, 2012], which included a sharp escalation of the European debt crisis that leads to a deep recession in Switzerland, cannot be anticipated by the methods that we analyze.

Chapter 3

Characterization of bubble signatures: the acceleration effect

At the core of the methodology used in the previous chapter (i.e. the LPPLS model) lies the idea that acceleration in price dynamics is an essential feature of bubble regimes. In this chapter, we take a step away from the real estate market, in order to present evidence that substantiates this assumption.

We report strong evidence that changes of momentum, i.e. “acceleration”, defined as the first difference of successive returns, is a novel effect complementing momentum of stock returns. Γ-strategies based on the “acceleration” effect are on average quite profitable and beat momentum-based strategies in two-third of the cases among a large panel of parameterizations. We show that “acceleration” strategies profit from transient non-sustainable accelerating log-price dynamics, both on the long and short sides of the portfolios.

We argue that the “acceleration” effect is associated with procyclical mechanisms and psychological effects, such as desensitisation or habituation and the influence of the breakdown of the status quo. The “acceleration” effect and the Γ-strategies make more explicit and help elucidate many previous reports of transient non-sustainable accelerating (upward or downward) log-prices as well as many anomalies associated with the momentum factor. However, within standard asset pricing tests, Γ-pricing factors are poorer at explaining momentum-sorted portfolios than the reverse. This can be rationalised by the fact that the Γ effect represents the existence of transient positive feedbacks influencing the price formation process, which is only prevalent during special stock market regimes. Standard asset pricing tests are thus not the best suited to deal with such transient effects.

The content of this chapter is an edited version of [Ardila et al., 2015a], of which I am first author.

3.1 Introduction

Momentum, the tendency for rising asset prices to rise further, and falling prices to keep falling, enjoys a strong empirical support [Jegadeesh and Titman, 1993, Grinblatt et al., 1995, Jegadeesh and Titman, 2001, Grinblatt and Moskowitz, 2004] and provides an improved explanatory power in factor model regressions [Carhart, 1997, Fama and French, 2012]. Momentum has been documented in the US, Europe and Asia Pacific [Fama and French, 2012] as well as in different asset classes [Asness et al., 2013]. Additionally, it has withstood explanations based on industry effects, cross-correlation among assets, and data mining [Grundy and Martin, 2001, Jegadeesh and Titman, 2001]. Momentum is often attributed to investors' behavioral characteristics such as over-confidence, self-attribution and confirmation biases [Daniel et al., 1998], under-reaction and over-reaction [Barberis et al., 1998] as well as herding [Hoitash and Krishnan, 2008, Demirer et al., 2015]. It could also perhaps be accounted for by assuming efficient markets with rational investors in the presence of information noise [Crombez, 2001]. It is noteworthy that trend following strategies have a large number of followers among both individual and professional investors [Antonacci, 2014, Clenow, 2015].

In its simplest geometrical representation, momentum reflects the persistence of short-term returns, i.e. the existence of linear trends in the log-price processes, which can be referred to as "velocity" in the technical analysis language (see [Andersen et al., 2000] and references therein). Here, we report strong evidence that changes of momentum, i.e. "acceleration", defined specifically as the first difference of successive returns, constitutes an important source of momentum profits, in particular during the last 25 years. We dub the corresponding strategies Gamma (Γ), due to the curvature associated with the changes of trends that it captures, and in analogy with the option Gamma defined as the second-order derivative of the option price with respect to the underlying stock price. In asset pricing tests, we find that Γ-sorted portfolios can be mostly explained by the momentum-factor, but not the reverse. Thus, we conclude that acceleration can be considered a special manifestation of momentum, whose origins can be traced to broader phenomena.

We argue that the Γ-effect represents the existence of transient positive feedbacks influencing the price formation process. These procyclical mechanisms might include the market impact of option hedging, insurance portfolio strategies [Ho et al., 2010], market makers bid-ask spread in response to past volatility, learning of business networks, financing of firms by banks during boom compared to contracting times, algorithmic trading, asymmetric information on hedging strategies, stop-loss orders, portfolio execution optimization and order splitting, deregulation (e.g. the Gramm-Leach-Bliley act repelling the Glass-Steagall act), central banks easy monetary policies, as well as imitation, social influence and herding. When one or several of these mechanisms are at work, they tend to push the price further away from a sustainable fundamental price with constant return. The result is a finite lived accelerating upward swing or downward spiral, which is captured by Γ-strategies. While the price dynamics tends to mean reverse to a long term trend that is co-integrated with economic growth [El-Wassal, 2005, Sornette and Cauwels, 2014], at time scales up to a few years, prices do deviate by exhibiting finite-lived stochastic accelerating bursts, both upward and downward. A typical implication of a class of theoretical models of bubbles is that increasing rate of returns (positive acceleration) become necessary in order to sustain the bubble regime [Brunnermeier and Oehmke, 2013, Scherbina and Schlusche, 2014, Kaizoji et al., 2015]. This implication has started to be tested in empirical studies of financial bubbles [Phillips et al., 2011, Leiss et al., 2015].

At the individual investor level, there is also physiological and psychological evidence that there is a decrease in response to a constant stimulus, a phenomenon known as desensitisation or habituation [Rankin et al., 2009]. A variation (acceleration/deceleration) is needed to create a new perceived stimulus. Hence, investors might react inordinately to change of trends [Andersen et al., 2000]. Since acceleration amounts to a change of the momentum that embodies the previous prevailing trend, then from a psychological point of view, the Γ-effect amounts to a breakdown of the status quo [Samuelson and Zeckhauser, 1988, Kahneman et al., 1991]. In a sense, the Γ-effect can be thought of as a premium for the psychological costs of the heightened decision difficulties and the increased uncertainty associated with a deviation from the status quo [Fleming et al., 2010] that momentum represents.

This chapter is organized as follows. Section 2 describes our data. Section 3 presents our definition of "acceleration" and the performance of Γ-based portfolio investment strategies for a large panel of parametrizations. Section 4 compares in detail the most profitable Γ-strategies against their momentum equivalent. Section 5 dissects possible sources of the performance of the Γ-strategies. Section 6 presents a number of standard asset pricing tests pitting Γ-factors against the standard factors: market, size, book-to-value, and momentum. Section 7 concludes.

3.2 Data

The data used in this work are all the common stocks in the Center for Research in Security Prices (CRSP) database (share code 10 and 11) between 1928 and December 2015. Stocks with less than two years of existence (by time t) were discarded to control for survival bias. Following Fama and French [1992], the computation of the breakpoints of the portfolios and factors used in the asset pricing tests was done using stocks from the NYSE. The measures were then computed using stocks from all exchanges. Finally, to study the stock returns by industry, we employed the Fama-French 10 industry classification, which in turn is based on the stock's SIC code.

3.3 The returns of Acceleration portfolios

Given the f months return $r_{i,t}(f) := (p_{i,t}/p_{i,t-f}) - 1$ (discrete approximation of the differences in log-prices) of stock i observed at the end of time t (counted in months) over the time scale of f months, we define the intermediate variable $\Gamma_{i,t}(f)$ by

$$\Gamma_{i,t}(f) = r_{i,t}(f) - r_{i,t-f}(f) \ . \tag{3.1}$$

This expression for $\Gamma_{i,t}(f)$ is nothing but the first-difference of successive returns in time steps of f months, which provides a discrete approximation for log-price "acceleration". Based on equation 3.1 and in a direct analogy with the strategies examined by Lewellen [2002] to analyze momentum, we consider two types of strategies to study the acceleration in stock prices that we analyse in turn in the two following subsections.

3.3.1 Relative strength portfolios

The first type of strategy consists of relative strength portfolios. At the beginning of every month t, we create a portfolio in which the weight $w^{\Gamma}_{i,t}(f)$ of an asset i is determined by its Γ relative to the average Γ of the market at the end of month $t-1$. The $w^{\Gamma}_{i,t}(f)$ of an asset i is thus given by

$$\begin{aligned} w^{\Gamma}_{i,t}(f) &= \frac{1}{N}(\Gamma_{i,t-1}(f) - \Gamma_{m,t-1}(f)) \\ &= \frac{1}{N}\left[(r_{i,t-1}(f) - r_{i,t-1-f}(f)) - (r_{m,t-1}(f) - r_{m,t-1-f}(f))\right] \end{aligned} \tag{3.2}$$

where $\Gamma_{m,t-1}(f)$ and $r_{m,t-1}(f)$ correspond respectively to the Γ and returns of the equal-weighted index, and N is the total number of stocks (for notational convenience, we do not indicate the dependence of N with t). This strategy is long (short) on stocks with high (low) Γ. By construction, $\sum_{i=1}^{N} w^{\Gamma}_{i,t} = 0$. The profit π_{t+h-1} at the end of period $t+h-1$ of holding such portfolio for $h-$months is thus

$$\begin{aligned} \pi^{\Gamma}_{t+h-1}(f,h) &= \sum_{i=1}^{N} w^{\Gamma}_{i,t}(f) r_{i,t+h-1}(h) \\ &= \frac{1}{N}\sum_{i=1}^{N}(\Gamma_{i,t-1}(f) - \Gamma_{m,t-1}(f)) r_{i,t+h-1}(h) \\ &= \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1}(f) r_{i,t+h-1}(h)\right) - \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1-f}(f) r_{i,t+h-1}(h)\right) \\ &\quad - \left[r_{m,t-1}(f) - r_{m,t-1-f}(f)\right] r_{m,t+h-1}(h) \ . \end{aligned} \tag{3.3}$$

We compare the strategies defined by equations (3.2) and (3.3) with their Δ analogue, i.e., trading strategies in which the weight $w^{\Delta}_{i,t}(f)$ of an asset i in the portfolio created at the beginning of month t is determined by its momentum relative to the average momentum of the market at the end of time $t-1$. Thus, the weight of asset i in month t in a Δ-strategy is given by

$$w^{\Delta}_{i,t}(f) = \frac{1}{N}(r_{i,t-1}(f) - r_{m,t-1}(f)) \ . \tag{3.4}$$

The corresponding $h-$months profit at the end of period $t+h-1$ is

$$\begin{aligned}\pi^{\Delta}_{t+h-1}(f,h) &= \sum_{i=1}^{N} w^{\Delta}_{i,t}(f) r_{i,t+h-1}(h) \\ &= \frac{1}{N}\sum_{i=1}^{N}(r_{i,t-1}(f) - r_{m,t-1}(f)) r_{i,t+h-1}(h) \\ &= \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1}(f) r_{i,t+h-1}(h)\right) - r_{m,t-1}(f) r_{m,t+h-1}(h)\ .\end{aligned} \tag{3.5}$$

Table 3.1 presents the annualized profits for 36 different Γ and Δ-strategies. In addition to exploring the dependence of the results as a function of f and h, we introduce the additional parameter s to explore the sensitivity with respect to delays in implementing the positions in the portfolios. Specifically, a strategy is implemented at the beginning of time t, based on the estimation of the latest returns at time $t-1-s$. Expressions (3.2) and (3.4) correspond to $s=0$. For $s>0$, they are replaced respectively by $w^{\Gamma}_{i,t}(f) = \frac{1}{N}(\Gamma_{i,t-1-s}(f) - \Gamma_{m,t-1-s}(f))$ and $w^{\Delta}_{i,t}(f) = \frac{1}{N}(r_{i,t-1-s}(f) - r_{m,t-1-s}(f))$.

We find 25 Γ-strategies with positive returns and 20 of them are statistically significant. 8 out of the 11 strategies with negative profits correspond to a $f=3$ months formation period. The most successful Γ-strategy selects stocks based on their returns over the previous 12 months and then holds the portfolio for 3 months, with the value $s=0$ months of delay between return estimation and implementation; the second and third most profitable strategies are those based on the Γ of the previous 6 months and 12 months, with 6 and 0 months of delay in the implementation and 1 and 6 months as a holding period, respectively. In comparison, Δ-strategies exhibit positive profit for 19 out of the 36 examined strategies, 10 of them statistically significant. Their negative profits mostly correspond to a formation period $f=12$ months (7 cases). Comparing Γ and Δ-strategies, the profits and Sharpe ratios of Γ-strategies are higher respectively in 24 and 24 cases, i.e. for two-third of the 36 strategies.

3.3.2 Long/short portfolio strategies

Our second type of strategy are long/short portfolio strategies built on deciles. At the beginning of every month t, we rank stocks based on their $\Gamma_{i,t-1-s}(f)$. We then create 10 Γ-portfolios defined as the value-weighted portfolios of the stocks in each decile, and define the Γ-portfolio strategy as,

$$\begin{aligned}\Gamma_{s,f} = &\text{ long the top } \Gamma - \text{ranked decile stocks and} \\ &\text{short the bottom } \Gamma - \text{ranked decile stocks.}\end{aligned} \tag{3.6}$$

Each portfolio is held for h months following s months after the ranking month. Strategies described by equation 3.6 are another way to capture the common risk premium associated with acceleration in the log-prices of stocks. As before, we compare the long/short portfolio

Formation period f	Skipping months s	Holding period h $\mu[\pi(f,h)]$ $(\sigma[\pi(f,h)])$ Δ	Γ	$\frac{\mu[\pi(f,h)]}{\sigma[\pi(f,h)]}$ (t-stat) Δ	Γ
		$h=1$			
3	0	−1.638 (2.679)	−1.561 (2.986)	−0.177(−5.634)	−0.151(−4.816)
	3	−0.070 (2.142)	−0.563 (3.185)	−0.009(−0.303)	−0.051(−1.628)
	6	0.496 (2.360)	−0.147 (2.921)	0.061(1.938)	−0.014(−0.463)
6	0	−1.449 (3.552)	−2.503 (4.320)	−0.118(−3.758)	−0.167(−5.340)
	3	0.415 (3.104)	0.521 (4.007)	0.039(1.231)	0.038(1.199)
	6	1.065 (2.696)	2.090 (3.487)	0.114(3.638)	0.173(5.525)
12	0	−0.259 (4.632)	1.376 (5.789)	−0.016(−0.516)	0.069(2.191)
	3	0.258 (4.464)	1.753 (4.806)	0.017(0.533)	0.105(3.362)
	6	−0.088 (3.521)	1.356 (4.231)	−0.007(−0.231)	0.093(2.954)
		$h=3$			
3	0	−0.630 (2.560)	−0.758 (2.648)	−0.123(−3.927)	−0.143(−4.568)
	3	0.130 (1.998)	−0.517 (5.969)	0.033(1.041)	−0.043(−1.383)
	6	0.647 (4.791)	0.400 (5.361)	0.068(2.156)	0.037(1.190)
6	0	−0.283 (3.459)	−0.933 (4.868)	−0.041(−1.307)	−0.096(−3.060)
	3	0.663 (2.825)	1.124 (3.324)	0.117(3.744)	0.169(5.398)
	6	0.662 (3.114)	1.686 (3.897)	0.106(3.393)	0.216(6.904)
12	0	0.501 (4.377)	2.183 (4.719)	0.057(1.826)	0.231(7.385)
	3	0.188 (3.796)	1.743 (4.388)	0.025(0.791)	0.199(6.339)
	6	−0.476 (3.827)	0.908 (4.714)	−0.062(−1.987)	0.096(3.073)
		$h=6$			
3	0	−0.229 (2.151)	−0.599 (3.818)	−0.075(−2.399)	−0.111(−3.540)
	3	0.373 (2.755)	−0.100 (5.293)	0.096(3.058)	−0.013(−0.429)
	6	0.475 (4.078)	0.645 (4.536)	0.082(2.630)	0.101(3.212)
6	0	0.192 (2.798)	0.033 (4.303)	0.048(1.547)	0.005(0.173)
	3	0.722 (3.084)	1.470 (3.932)	0.166(5.284)	0.264(8.438)
	6	0.164 (3.096)	1.167 (3.753)	0.037(1.194)	0.220(7.016)
12	0	0.425 (3.852)	2.083 (4.681)	0.078(2.493)	0.315(10.046)
	3	−0.109 (3.658)	1.363 (4.697)	−0.021(−0.670)	0.205(6.550)
	6	−0.915 (3.946)	0.411 (4.858)	−0.164(−5.232)	0.060(1.909)
		$h=12$			
3	0	0.019 (2.099)	−0.041 (2.758)	0.009(0.289)	−0.015(−0.479)
	3	0.063 (2.265)	0.099 (4.229)	0.028(0.881)	0.023(0.745)
	6	−0.035 (3.288)	0.272 (3.660)	−0.011(−0.341)	0.074(2.374)
6	0	0.127 (3.274)	0.576 (4.175)	0.039(1.238)	0.138(4.407)
	3	−0.064 (3.112)	0.674 (3.417)	−0.020(−0.653)	0.197(6.294)
	6	−0.446 (2.961)	0.342 (3.399)	−0.151(−4.806)	0.101(3.215)
12	0	−0.290 (4.560)	1.189 (5.228)	−0.064(−2.032)	0.227(7.260)
	3	−0.857 (4.342)	0.507 (4.949)	−0.197(−6.303)	0.102(3.269)
	6	−1.313 (4.380)	−0.076 (4.881)	−0.300(−9.572)	−0.016(−0.496)

Table 3.1: The table reports annualized profits (in percentage) for momentum Δ and acceleration Γ-strategies based on different formation f, holding periods h, and skipping periods s. Γ-strategies invest at the beginning of every month t on asset i proportional to $w^{\Gamma}_{i,t} = \frac{1}{N}(\Gamma(f)_{i,t-1-s} - \Gamma(f)_{m,t-1-s})$, where $\sum_{i=0}^{N} w^{\Gamma}_{i,t} = 0$ and holds this portfolio for h months. Δ strategies invest at the beginning of every month t on asset i proportional to $w^{\Delta}_{i,t} = \frac{1}{N}(\Delta(f)_{i,t-1-s} - \Delta(f)_{m,t-1-s})$, where $\sum_{i=0}^{N} w^{\Delta}_{i,t} = 0$ and holds this portfolio for h months. Critical t-statistic values for all strategies are 1.97(95%), 2.24(97.5%), and 2.58(99%).

strategies against their momentum analogues. That is, at the beginning of month t we sort the stocks based on their previous $\Delta_{i,t-1-s}(f) = r_{i,t-1-s}(f)$ returns and create 10 decile value weighted portfolios. We then define the Δ-strategy as,

$$\begin{aligned} \Delta_{s,f} = &\text{ long the top } \Delta - \text{ranked decile stocks and} \\ &\text{short the bottom } \Delta - \text{ranked decile stocks.} \end{aligned} \tag{3.7}$$

Table 3.2 presents the annualized profits for the corresponding 36 different Γ and Δ-strategies, exploring the same set of parameters of table 3.1. Among long/short strategies, we observe 26 Γ-strategies with positive returns, and 23 of them are statistically significant. Similar to the relative strength portfolios, 8 out of the 10 strategies with negative profits correspond to a $f = 3$ months formation period. The most successful Γ-strategy selects stocks based on $\Gamma_{6,6}$, and then holds the portfolio for 1 month. This most successful Γ-strategy is however slightly below the equivalent $\Delta_{6,6}$-strategy, as the latter exhibit a 12.87% annualized profit against a 12.29% of the $\Gamma_{6,6}$-strategy. In comparison, Δ-strategies exhibit positive profit for 33 out of the 36 examined strategies, 27 of them statistically significant. Comparing Γ and Δ-strategies, the profits and Sharpe ratios of Γ-strategies are higher respectively in 15 and 18 cases.

Overall, we observe that Γ-strategies are more profitable when considering the whole set stocks, and not only the stocks with the most extreme returns (decile portfolios). Nevertheless, even using only the extreme deciles, risk-adjusted profits of Γ-strategies tend to be at least as high as those of momentum strategies. It is also evident from tables 3.1 and 3.2 that there is hardly any acceleration effect at short scale periods (3 months), and this explains 12 out 21 cases in which long/short acceleration strategies under-performs momentum. Finally, we observe that Γ-strategies seem to remain profitable for longer holding periods.

3.4 Acceleration versus momentum

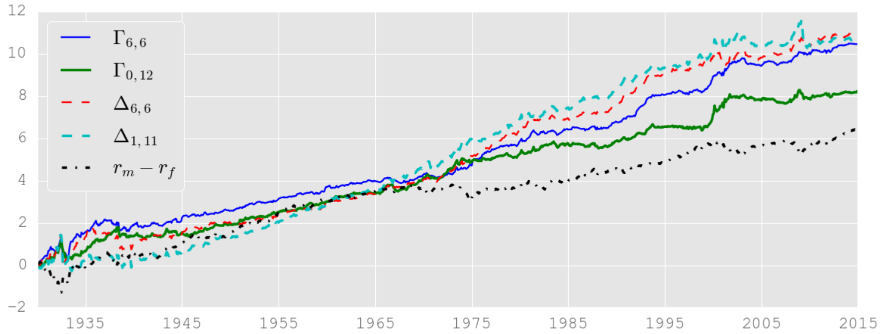
3.4.1 Comparison of the performance of acceleration and momentum based strategies

We now focus on the $\Gamma_{6,6}$ and $\Gamma_{0,12}$-strategies (defined by expression (3.6)), as they exhibited the strongest risk-adjusted profitability. We chose to study the strategies built on deciles and using a 1 month holding period, as they are better suited to be analyzed with standard asset pricing tools. Figure 3.1 presents the evolution of the cumulative profits for the two Γ-strategies and two analogous Δ-strategies. $\Delta_{1,11}$ corresponds to the yearly momentum typically studied in the literature. $\Delta_{6,6}$ is the intermediate horizon momentum analyzed by Novy-Marx [2012].

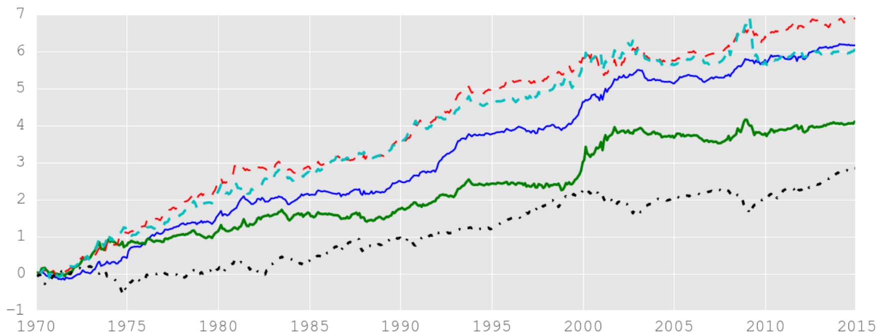
The cumulative profits of the different strategies grow steadily during the first 40 years, though at different average monthly rates. The exception is the first seven years of the 30s, in which $\Gamma_{6,6}$ and $\Gamma_{0,12}$-strategies exhibited a phase of very high growth, with average

Formation period f	Skipping months s	Holding period h			
		$\mu[\pi(f,h)]$ $(\sigma[\pi(f,h)])$		$\frac{\mu[\pi(f,h)]}{\sigma[\pi(f,h)]}$ (t-stat)	
		Δ	Γ	Δ	Γ
			$h=1$		
3	0	−5.315 (22.647)	−5.864 (18.343)	−0.068(−2.164)	−0.092(−2.947)
	3	5.365 (23.391)	−3.143 (18.477)	0.066(2.115)	−0.049(−1.568)
	6	7.230 (21.621)	−2.048 (17.569)	0.097(3.083)	−0.034(−1.075)
6	0	−2.548 (26.318)	−9.328 (17.600)	−0.028(−0.893)	−0.153(−4.886)
	3	9.344 (24.002)	3.090 (16.856)	0.112(3.589)	0.053(1.690)
	6	12.872 (22.996)	12.290 (17.347)	0.162(5.161)	0.205(6.532)
12	0	6.615 (28.117)	9.648 (18.289)	0.068(2.169)	0.152(4.864)
	3	7.811 (25.614)	7.736 (16.712)	0.088(2.812)	0.134(4.268)
	6	4.846 (24.081)	6.272 (16.350)	0.058(1.855)	0.111(3.537)
			$h=3$		
3	0	1.367 (26.918)	−2.448 (20.670)	0.025(0.811)	−0.059(−1.891)
	3	5.422 (24.549)	−1.641 (22.084)	0.110(3.527)	−0.037(−1.187)
	6	7.234 (24.573)	−0.263 (19.631)	0.147(4.701)	−0.007(−0.214)
6	0	2.909 (33.023)	−3.797 (20.155)	0.044(1.407)	−0.094(−3.008)
	3	9.302 (26.958)	5.857 (17.580)	0.173(5.510)	0.167(5.320)
	6	10.077 (24.394)	10.714 (19.207)	0.207(6.596)	0.279(8.908)
12	0	8.236 (33.116)	10.520 (18.400)	0.124(3.971)	0.286(9.130)
	3	6.840 (30.737)	7.775 (17.083)	0.111(3.554)	0.228(7.268)
	6	2.768 (25.358)	5.167 (16.297)	0.055(1.743)	0.159(5.063)
			$h=6$		
3	0	3.570 (25.977)	−2.016 (20.387)	0.097(3.104)	−0.070(−2.233)
	3	6.720 (22.996)	−1.262 (20.504)	0.207(6.600)	−0.044(−1.390)
	6	8.034 (20.968)	4.334 (19.371)	0.271(8.652)	0.158(5.053)
6	0	6.905 (28.008)	1.012 (18.899)	0.174(5.568)	0.038(1.210)
	3	10.524 (22.935)	8.781 (18.626)	0.324(10.362)	0.333(10.646)
	6	6.984 (19.860)	7.659 (17.475)	0.249(7.941)	0.310(9.898)
12	0	8.674 (27.530)	9.588 (18.179)	0.223(7.116)	0.373(11.911)
	3	5.863 (24.219)	6.759 (16.785)	0.171(5.467)	0.285(9.094)
	6	1.070 (21.320)	3.294 (17.035)	0.036(1.134)	0.137(4.367)
			$h=12$		
3	0	6.112 (25.534)	1.170 (23.060)	0.239(7.644)	0.051(1.621)
	3	4.622 (23.068)	1.340 (20.755)	0.200(6.398)	0.065(2.061)
	6	1.950 (23.209)	1.510 (19.675)	0.084(2.683)	0.077(2.451)
6	0	7.138 (28.849)	4.601 (21.296)	0.247(7.902)	0.216(6.900)
	3	4.908 (25.147)	5.028 (18.360)	0.195(6.234)	0.274(8.746)
	6	1.051 (23.447)	2.750 (19.377)	0.045(1.431)	0.142(4.532)
12	0	5.207 (28.193)	6.548 (20.062)	0.185(5.898)	0.326(10.424)
	3	1.600 (26.579)	3.386 (18.363)	0.060(1.922)	0.184(5.888)
	6	−1.731 (24.380)	1.111 (17.717)	−0.071(−2.267)	0.063(2.003)

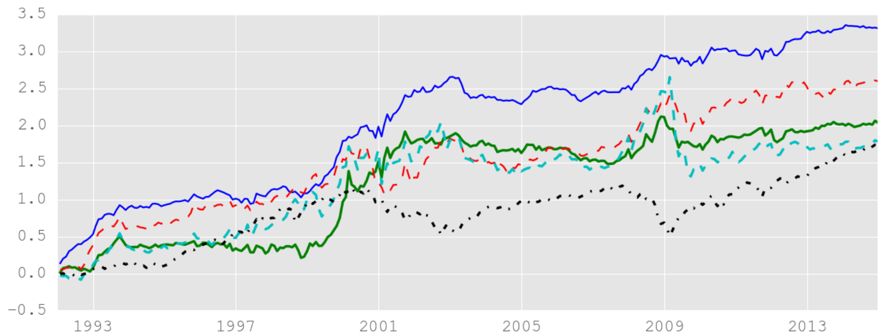
Table 3.2: The table reports annualized average profits (in percentage) for momentum Δ and acceleration Γ-strategies built on deciles, for different formation f, holding periods h, and skipping periods s. At the beginning of every month t , $\Gamma_{s,f}$-strategies rank stocks by $\Gamma(f)_{i,t-1-s}$, long the top $\Gamma_{s,f}$-ranked decile and short the bottom $\Gamma_{s,f}$-ranked decile, holding this portfolio for h months. At the beginning of every month t , $\Delta_{s,f}$ strategies rank stocks by $\Delta(f)_{i,t-1-s}$, long the top $\Delta s, f$-ranked decile and short the bottom $\Delta s, f$-ranked decile, holding this portfolio for h months. Critical t-statistic values for all strategies are 1.97(95%), 2.24(97.5%), and 2.58(99%).



(a) 1930-2015(exclusive)



(b) 1970-2015 (exclusive)



(c) 1992-2015(exclusive)

Figure 3.1: Cumulative monthly profits of $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ strategies, and the market portfolio (in excess of the 1-Month U.S. T-bill-rate). To construct the $\Gamma_{s,f}$-strategies, at the beginning of every month t, we compute decile breakpoints by sorting the NYSE stocks using $\Gamma_{t-1-s}(f)$, with s equals the number of months between the formation and holding period. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding decile according to their respective $\Gamma_{t-1-s}(f)$. The $\Gamma_{s,f}$-strategies result from longing the top decile portfolio and shorting the bottom decile portfolio, and holding the portfolio for one month. The $\Delta_{s,f}$ portfolios are constructed in a similar manner, but using the f months return $r_{t-1-s}(f)$ to compute the decile breakpoints.

monthly profits of 1.8% and 1.6% respectively. In the same years, $\Delta_{6,6}$ and $\Delta_{1,11}$-strategies displayed lower profits of 1.3% and 0.4%.

During the next 45 years, computing the cumulative returns of the strategies over a 5 years rolling window evidences that the profits from Γ-strategies peak over two periods, specifically, the half decades preceding 1994 and 2004[1]. The former period includes the volatile post-1987 crash period and the following strong recovery. The later period covers the dotcom crash, the following severe bearish market ending towards the end of 2002 and the strong rebound associated with the 1% rate policy of the Federal Reserve. These are indeed periods during which momentum is unstable while change of momentum (acceleration) is dominant.

The average monthly profits during both periods is respectively 2.35% and 1.95% for $\Gamma_{6,6}$ and $\Gamma_{0,12}$ (compared to 0.8% and 0.41% for the remaining years). Thus, 10 years of profits amount to about 46 and 58 percent of the cumulative gains. Δ-profits do not present this behavior; $\Delta_{6,6}$ and $\Delta_{1,11}$ exhibit average profit rates of 1.68% and 1.6% during 1989-1994 and 1999-2004 (exclusive), and somewhat similar rates of 1.16% and 0.98% during the other part of the half-sample.

The better performance of the Γ-portfolio over the momentum portfolio, which is mainly observed in the last quarter of the time series, can be understood from two contributions: (i) a larger return up to 2003 than even the momentum portfolio and (ii) much smaller volatility and drawdowns during the last quarter of the sample compared with the other portfolios (see section 3.4.2) . The first contribution is particularly interesting since it coincides with a period corresponding to a very strong super-exponential acceleration - the dot-com bubble - (Johansen and Sornette [2000]; Phillips et al. [2011]). In the latter case, the log-price exhibited strong deviations from a simple trend with marked convex and concave spells that can be associated with strong Γ signatures. This suggests that indeed Γ defined as the first-difference of r_t plays an important role in explaining asset returns during periods of exuberance and panic (bubbles).

	1930-2015(exclusive)				1930-1970 (exclusive)				1970-2015(exclusive)			
	$\Delta_{6,6}$	$\Delta_{1,11}$	$\Gamma_{6,6}$	$\Gamma_{0,12}$	$\Delta_{6,6}$	$\Delta_{1,11}$	$\Gamma_{6,6}$	$\Gamma_{0,12}$	$\Delta_{6,6}$	$\Delta_{1,11}$	$\Gamma_{6,6}$	$\Gamma_{0,12}$
$\Delta_{6,6}$	1.000	0.763	0.619	0.468	1.000	0.792	0.646	0.503	1.000	0.730	0.582	0.443
$\Delta_{1,11}$	0.763	1.000	0.409	0.626	0.792	1.000	0.393	0.609	0.730	1.000	0.430	0.645
$\Gamma_{6,6}$	0.619	0.409	1.000	0.567	0.646	0.393	1.000	0.550	0.582	0.430	1.000	0.601
$\Gamma_{0,12}$	0.468	0.626	0.567	1.000	0.503	0.609	0.550	1.000	0.443	0.645	0.601	1.000
$Mkt - r_f$	-0.233	-0.362	-0.052	-0.223	-0.395	-0.512	-0.110	-0.293	-0.004	-0.185	0.041	-0.159

Table 3.3: Pearson correlation of Γ and Δ-strategies for several time periods.

Γ and Δ-strategies exhibit positive cross-correlations, and negative correlation with the market (table 3.3). The estimates are only slightly affected by changing the analyzed period. In contrast, correlations with the market do exhibit dependence with the sample period, as they move closer to zero during the second half of the sample. Interestingly, all

[1]The specific period can vary slightly depending on what strategy is analyzed and what length of the rolling window is employed, but this does not affect the general argument.

values reported are visibly below those implied by i.i.d. Gaussian-distributed returns for which the correlation between $\Gamma_{i,t}(f)$ and $r_{i,t}(f)$ would be

$$\mathrm{Corr}(r_{i,t}(f), \Gamma_{i,t}(f)) = \frac{\sigma_{r_{i,t}(f)}}{\sigma_{r_{i,t}(f)-r_{i,t-f}(f)}} = \frac{1}{\sqrt{2}} \sim 0.71 \; . \tag{3.8}$$

In contrast, we observe correlation values that range from 0.58 to 0.65.

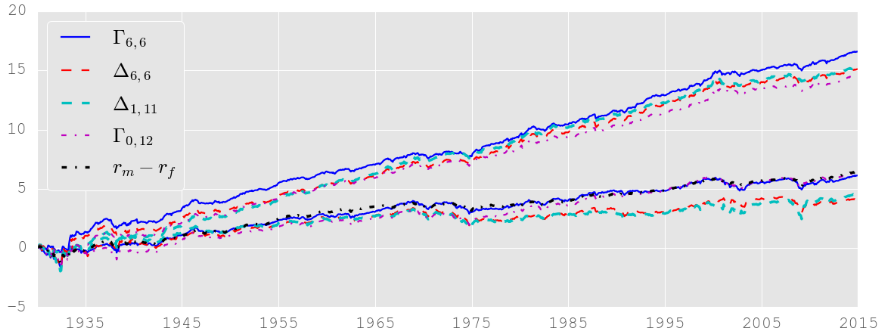
In figure 3.2, we de-aggregate the winners and losers portfolios to evidence another difference between the two strategies. The $\Gamma_{6,6}$-winner portfolio has the higher cumulative return over all the sample, and over both sub-samples; that is, stocks that accelerate clearly tend to yield positive returns in the future. The $\Gamma_{6,6}$-loser portfolios behave very different, and one might even be inclined to rule out any predictability of stock returns stemming from negative acceleration. Relative to the market, shorting Γ-losers significantly under-perform their momentum counter parts, and they clearly generate average negative returns. We will comment further on this insight in section 3.4.4.

3.4.2 Momentum crashes

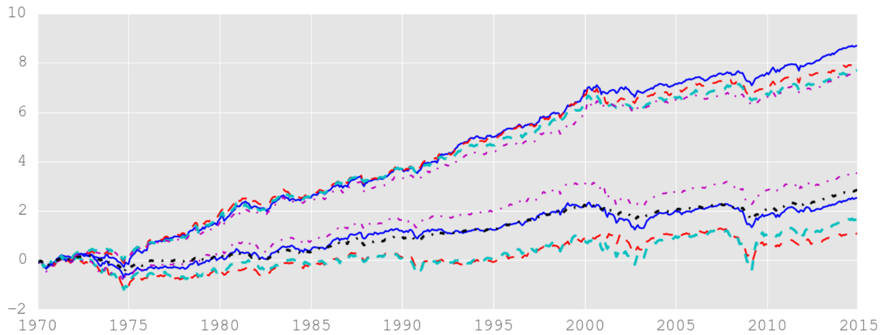
Date	$\Delta_{6,6}$	$\Delta_{1,11}$	$\Gamma_{6,6}$	$\Gamma_{0,12}$	$Mkt - r_f$
8/31/1932	-83.8	-78.0	-25.5	-21.5	37.1
7/29/1932	-67.8	-64.9	-43.3	-27.0	33.8
1/31/2001	-6.3	-46.6	-13.4	-22.2	3.1
4/30/2009	-10.0	-45.8	0.3	-12.4	10.2
9/29/1939	-22.7	-41.4	6.0	2.2	16.9
3/31/2009	-4.5	-39.0	0.4	0.2	9.0
6/30/1931	-22.6	-34.0	-13.2	-10.2	13.9
6/30/1938	-32.5	-33.3	-32.1	-34.3	23.9
5/31/1933	4.4	-31.2	24.8	0.3	21.4
4/28/1933	17.0	-26.0	33.6	-1.3	38.9
8/31/2009	-20.6	-25.4	0.3	0.9	3.3
5/29/2009	-11.6	-22.0	-7.8	-13.0	5.2
11/29/2002	-2.2	-20.2	2.7	-0.6	6.0
1/31/1975	-1.2	-19.3	19.0	7.3	13.7
4/30/1999	-11.3	-18.9	-2.3	-5.3	4.3

Table 3.4: Top 15 momentum ($\Delta_{1,11}$) monthly crashes. The table contains the 15 worst monthly returns of $\Delta_{1,11}$ strategies, as well as the returns of the other strategies during the corresponding month (all quantities in percentage).

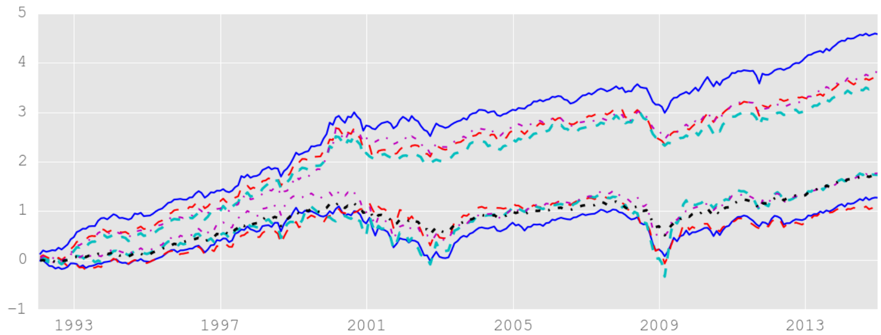
Daniel and Moskowitz [2014] documented that momentum strategies are negatively skewed, experiencing infrequent but strong and persistent strings of negative returns. Tables 3.4 and 3.5 show that Γ-strategies have been historically less susceptible to such crashes. Table 3.4 presents the lowest monthly negative profits of momentum, as well as the performance of the other strategies in the corresponding months. The two highest momentum losses take place in consecutive months and amount to -142.8%. These values are qualitatively similar to those reported by Daniel and Moskowitz [2014]. The intermediate momentum strategy, which generates higher profits, are no less susceptible to such strong periods of negative returns; the accumulated loss during the same months



(a) 1930-2015(exclusive)



(b) 1970-2015 (exclusive)



(c) 1992-2015(exclusive)

Figure 3.2: Cumulative monthly profits of the $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ top and bottom decile portfolios, as well as the market portfolio (in excess of the 1-Month U.S. T-bill-rate). To construct the-$\Gamma_(s, f)$ strategies, at the beginning of every month t, we compute decile breakpoints by sorting the NYSE stocks using $\Gamma_{t-1-s}(f)$, with s equals the number of months between the formation and holding period. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding decile according to their respective $\Gamma_{t-1-s}(f)$. The $\Delta_{s,f}$-portfolios are constructed in a similar manner, but using the f months return $r_{t-1-s}(f)$ to compute the decile breakpoints.

Date	$\Delta_{6,6}$	$\Delta_{1,11}$	$\Gamma_{6,6}$	$\Gamma_{0,12}$	$Mkt - r_f$
7/29/1932	-67.8	-64.9	-43.3	-27.0	33.8
6/30/1938	-32.5	-33.3	-32.1	-34.3	23.9
8/31/1932	-83.8	-78.0	-25.5	-21.5	37.1
2/28/1936	-8.5	-3.7	-15.6	0.4	2.5
9/30/1937	-17.7	-3.0	-15.3	6.7	-13.6
5/31/1940	-2.7	3.4	-15.0	-6.5	-22.0
6/30/1944	-12.4	-11.4	-14.5	-16.5	5.5
10/31/1966	-15.8	-9.0	-14.4	-2.6	3.9
4/29/1938	-16.1	-14.0	-13.6	-13.9	14.5
3/31/1939	2.8	-2.8	-13.6	-13.8	-12.0
3/31/1980	-14.1	-18.0	-13.4	-10.3	-12.9
1/31/2001	-6.3	-46.6	-13.4	-22.2	3.1
6/30/1931	-22.6	-34.0	-13.2	-10.2	13.9
7/31/1973	-13.3	-17.8	-12.2	-18.2	5.0
10/30/1987	-5.0	-2.3	-12.2	-12.6	-23.2

Table 3.5: Top 15 acceleration ($\Gamma_{6,6}$) monthly crashes. The table contains the 15 worst monthly returns of $\Gamma_{6,6}$ strategies, as well as the returns of the other strategies during the corresponding month (all quantities in percentage).

is -151.6%. Overall, the 15-worst performing months accumulate losses of 546.0% and 404.6% for $\Delta_{1,11}$ and $\Delta_{6,6}$ respectively. In contrast, Γ-strategies are much less prone to severe crashes, with cumulative losses of -278.0% and -267.4% for $\Gamma_{0,12}$ and $\Gamma_{6,6}$. Moreover, eight of the 15 largest $\Gamma_{6,6}$-losses are dominated by $\Delta_{1,11}$-losses, while none of the largest momentum losses is dominated by either of the acceleration strategies.

3.4.3 Spanning tests

In this section, we employ spanning tests to analyze the relationship between the strategies. The spanning tests regress the strategy returns on the returns of one or more explanatory strategies. If the information ratio (the intercept in the regression) is statistically significant, adding the test strategy (the dependent variable) to the investment opportunity set leads to an attainable Sharpe ratio that significantly exceeds that which can be achieved using only the explanatory strategies.

Table 3.6 displays the results for regressions with Γ-strategies as the dependent variables and Δ-strategies and the market portfolio as independent variables. Each specification is identified by an ID on the table. Specifications 1-2 and 5-6 are of the form $\Gamma_{s,f} - r_f = \alpha + \beta_\Delta \Delta_{s,f}$, where r_f is the risk free rate, $\Gamma_{s,f}$ either $\Gamma_{6,6}$ or $\Gamma_{0,12}$, and $\Delta_{s,f}$ either $\Delta_{6,6}$ or $\Delta_{1,11}$. Specifications 3-4 and 7-8 are of the form $\Gamma_{s,f} - r_f = \alpha + \beta_\Delta \Delta_{s,f} + \beta_{mkt}(Mkt - r_f)$, where Mkt is the market portfolio.

Specification 1 employs $\Delta_{6,6}$ as explanatory variable and $\Gamma_{6,6}$ as the dependent variable. This specification only yields significant information ratios for the last quarter of the sample, implying that the ex-post returns of $\Gamma_{6,6}$ can be mostly explained by $\Delta_{6,6}$. On the other, specification 2, which uses $\Delta_{1,10}$ as explanatory variable, yields significant information ratios for most periods; in this case we conclude that $\Delta_{1,11}$ cannot explain the returns of the intermediate horizon acceleration strategy. Regressions 5 and 6 of $\Gamma_{0,12}$, on $\Delta_{6,6}$

ID		Full sample	Half sample		Quarter sample			
		All	Early	Late	First	Second	Third	Fourth
		Dependent variable: $\Gamma_{6,6}$						
	alpha	0.002	0.004	0.002	0.006	0.000	−0.003	0.005
		(1.923)	(1.767)	(0.969)	(1.759)	(0.091)	(−1.412)	(2.334)
1	$\Delta_{6,6}$	0.467	0.477	0.453	0.467	0.546	0.494	0.434
		(25.159)	(18.452)	(16.476)	(13.146)	(12.733)	(11.278)	(12.295)
	$AdjR^2$	0.383	0.415	0.334	0.418	0.403	0.346	0.334
	alpha	0.005	0.005	0.004	0.009	0.000	0.000	0.008
		(3.276)	(2.100)	(2.621)	(2.043)	(0.038)	(0.143)	(3.352)
2	$\Delta_{1,11}$	0.253	0.255	0.251	0.248	0.333	0.259	0.251
		(14.142)	(9.295)	(10.917)	(6.549)	(7.283)	(6.000)	(9.228)
	$AdjR^2$	0.163	0.151	0.180	0.149	0.179	0.128	0.220
	alpha	0.002	0.002	0.001	0.004	0.001	−0.004	0.005
		(1.207)	(0.954)	(0.810)	(1.218)	(0.456)	(−1.574)	(2.406)
3	$Mkt - RF$	0.105	0.160	0.045	0.218	−0.078	0.114	−0.032
		(4.474)	(4.663)	(1.374)	(4.393)	(−1.691)	(2.601)	(−0.668)
	$\Delta_{6,6}$	0.488	0.527	0.453	0.548	0.553	0.486	0.432
		(25.709)	(19.157)	(16.495)	(14.095)	(12.886)	(11.183)	(12.217)
	$AdjR^2$	0.394	0.439	0.335	0.460	0.407	0.361	0.333
	alpha	0.004	0.004	0.004	0.008	0.000	−0.000	0.007
		(2.553)	(1.545)	(2.152)	(1.721)	(0.207)	(−0.181)	(3.099)
4	$Mkt - RF$	0.113	0.115	0.121	0.162	−0.044	0.172	0.064
		(3.956)	(2.565)	(3.276)	(2.376)	(−0.799)	(3.420)	(1.187)
	$\Delta_{1,11}$	0.282	0.297	0.265	0.315	0.335	0.268	0.260
		(14.686)	(9.344)	(11.431)	(6.708)	(7.311)	(6.335)	(9.215)
	$AdjR^2$	0.175	0.161	0.194	0.165	0.178	0.165	0.221
		Dependent variable: $\Gamma_{0,12}$						
	alpha	0.001	0.004	−0.002	0.005	0.004	−0.006	0.001
		(0.821)	(2.027)	(−0.919)	(1.262)	(2.056)	(−2.130)	(0.285)
5	$\Delta_{6,6}$	0.372	0.347	0.419	0.354	0.292	0.454	0.403
		(16.891)	(12.705)	(11.423)	(9.850)	(5.337)	(8.753)	(7.945)
	$AdjR^2$	0.218	0.251	0.194	0.287	0.103	0.240	0.172
	alpha	0.001	0.004	−0.002	0.006	0.001	−0.006	0.002
		(0.679)	(1.841)	(−0.972)	(1.654)	(0.289)	(−2.537)	(0.634)
6	$\Delta_{1,11}$	0.410	0.371	0.460	0.363	0.442	0.482	0.455
		(25.329)	(16.759)	(19.407)	(12.393)	(10.006)	(11.963)	(15.189)
	$AdjR^2$	0.386	0.369	0.411	0.390	0.293	0.373	0.434
	alpha	0.002	0.005	−0.001	0.006	0.005	−0.005	0.002
		(1.474)	(2.465)	(−0.478)	(1.499)	(2.359)	(−2.001)	(0.768)
7	$Mkt - RF$	−0.115	−0.096	−0.175	−0.094	−0.094	−0.140	−0.219
		(−4.145)	(−2.593)	(−4.003)	(−1.803)	(−1.592)	(−2.689)	(−3.185)
	$\Delta_{6,6}$	0.349	0.317	0.419	0.320	0.300	0.465	0.393
		(15.463)	(10.718)	(11.565)	(7.860)	(5.482)	(9.045)	(7.851)
	$AdjR^2$	0.230	0.260	0.216	0.293	0.109	0.260	0.197
	alpha	0.001	0.003	−0.001	0.005	0.001	−0.006	0.002
		(0.612)	(1.674)	(−0.804)	(1.516)	(0.661)	(−2.393)	(0.682)
8	$Mkt - RF$	0.008	0.023	−0.043	0.048	−0.093	−0.069	−0.020
		(0.317)	(0.623)	(−1.110)	(0.895)	(−1.777)	(−1.438)	(−0.337)
	$\Delta_{1,11}$	0.412	0.379	0.455	0.383	0.447	0.478	0.452
		(23.580)	(14.705)	(18.872)	(10.425)	(10.136)	(11.876)	(14.523)
	$AdjR^2$	0.385	0.368	0.411	0.389	0.300	0.376	0.433

Table 3.6: The table reports summary statistics for time series monthly regressions of the excess returns of $\Gamma_{6,6}$ and $\Gamma_{0,12}$ strategies on $\Delta_{1,11}$ and $\Delta_{6,6}$ strategies, spanning from January 1930 to December 2014. Reported are the intercept *alpha*, the corresponding regression coefficients, the t-statistics (in parentheses), and the adjusted R^2 coefficient.

ID		Full sample All	Half sample Early	Late	First	Quarter sample Second	Third	Fourth
					Dependent variable: $\Delta_{6,6}$			
1	*alpha*	−0.001	−0.001	0.000	−0.002	0.001	0.002	−0.001
		(−0.318)	(−0.268)	(0.066)	(−0.315)	(0.413)	(0.630)	(−0.387)
	$\Gamma_{6,6}$	0.823	0.874	0.745	0.898	0.747	0.704	0.773
		(25.280)	(18.496)	(16.562)	(13.126)	(12.872)	(11.172)	(12.273)
	$AdjR^2$	0.385	0.416	0.336	0.417	0.408	0.341	0.334
2	*alpha*	0.003	0.001	0.005	0.001	0.003	0.005	0.005
		(1.697)	(0.271)	(2.360)	(0.264)	(1.163)	(1.800)	(1.568)
	$\Gamma_{0,12}$	0.590	0.727	0.467	0.817	0.366	0.538	0.431
		(16.985)	(12.711)	(11.448)	(9.857)	(5.353)	(8.707)	(7.923)
	$AdjR^2$	0.220	0.251	0.194	0.287	0.104	0.238	0.171
3	*alpha*	0.001	0.003	0.000	0.002	−0.000	0.002	−0.001
		(0.801)	(1.118)	(0.134)	(0.554)	(−0.205)	(0.643)	(−0.312)
	$Mkt - RF$	−0.264	−0.408	−0.029	−0.510	0.134	−0.021	−0.029
		(−9.058)	(−10.285)	(−0.683)	(−9.395)	(2.522)	(−0.379)	(−0.446)
	$\Gamma_{6,6}$	0.808	0.825	0.746	0.830	0.751	0.708	0.771
		(25.766)	(19.174)	(16.568)	(14.065)	(13.090)	(11.050)	(12.195)
	$AdjR^2$	0.430	0.521	0.336	0.574	0.421	0.339	0.332
4	*alpha*	0.005	0.004	0.005	0.006	0.001	0.004	0.005
		(2.492)	(1.458)	(2.092)	(1.052)	(0.603)	(1.557)	(1.481)
	$Mkt - RF$	−0.179	−0.336	0.087	−0.419	0.139	0.144	0.027
		(−5.183)	(−6.828)	(1.840)	(−6.057)	(2.111)	(2.520)	(0.363)
	$\Gamma_{0,12}$	0.548	0.612	0.479	0.647	0.377	0.556	0.435
		(15.548)	(10.715)	(11.619)	(7.866)	(5.532)	(9.041)	(7.834)
	$AdjR^2$	0.239	0.316	0.198	0.380	0.117	0.255	0.169
					Dependent variable: $\Delta_{1,11}$			
5	*alpha*	0.001	0.003	−0.001	−0.001	0.008	0.004	−0.005
		(0.445)	(0.831)	(−0.431)	(−0.204)	(3.170)	(1.027)	(−1.202)
	$\Gamma_{6,6}$	0.651	0.601	0.730	0.614	0.552	0.505	0.886
		(14.244)	(9.335)	(11.065)	(6.530)	(7.372)	(5.950)	(9.297)
	$AdjR^2$	0.165	0.152	0.184	0.148	0.182	0.126	0.222
6	*alpha*	0.000	−0.000	0.000	−0.004	0.006	0.003	−0.002
		(0.049)	(−0.039)	(0.105)	(−0.594)	(2.425)	(1.088)	(−0.531)
	$\Gamma_{0,12}$	0.947	0.998	0.900	1.078	0.670	0.782	0.959
		(25.533)	(16.798)	(19.605)	(12.379)	(10.053)	(11.922)	(15.265)
	$AdjR^2$	0.390	0.370	0.416	0.389	0.295	0.371	0.437
7	*alpha*	0.005	0.009	0.000	0.005	0.007	0.004	−0.002
		(2.147)	(2.746)	(0.086)	(0.896)	(2.761)	(1.111)	(−0.489)
	$Mkt - RF$	−0.525	−0.669	−0.318	−0.816	0.091	−0.147	−0.451
		(−13.396)	(−13.034)	(−5.258)	(−11.746)	(1.320)	(−2.021)	(−4.769)
	$\Gamma_{6,6}$	0.622	0.522	0.744	0.504	0.555	0.535	0.855
		(14.733)	(9.366)	(11.541)	(6.681)	(7.422)	(6.247)	(9.275)
	$AdjR^2$	0.290	0.374	0.222	0.459	0.185	0.137	0.275
8	*alpha*	0.003	0.005	0.001	0.003	0.004	0.003	0.000
		(1.605)	(1.799)	(0.439)	(0.533)	(1.876)	(1.051)	(0.033)
	$Mkt - RF$	−0.361	−0.514	−0.130	−0.638	0.124	0.018	−0.278
		(−10.154)	(−10.669)	(−2.455)	(−9.631)	(1.926)	(0.290)	(−3.343)
	$\Gamma_{0,12}$	0.862	0.823	0.882	0.819	0.679	0.785	0.919
		(23.732)	(14.724)	(19.057)	(10.405)	(10.225)	(11.850)	(14.593)
	$AdjR^2$	0.445	0.490	0.421	0.559	0.303	0.369	0.456

Table 3.7: The table reports summary statistics for time series monthly regressions of the excess returns of $\Delta_{6,6}$ and $\Delta_{1,11}$ strategies on $\Gamma_{6,6}$ and $\Gamma_{0,12}$ strategies, spanning from January 1930 to December 2014. Reported are the intercept *alpha*, the corresponding regression coefficients, the t-statistics (in parentheses), and the adjusted R^2 coefficient.

and $\Delta_{1,11}$ respectively, behave somewhat similarly. The information ratios of regressions 5-6 are mostly insignificant, though in the early-half and second quarter of the sample the strategy generates significant and positive $\hat{\alpha}$'s when regressed against $\Delta_{6,6}$. In the third quarter of the sample and using specification 5, $\Gamma_{0,12}$ actually generates a significantly negative information ratio.

Adding the market as explanatory variable to the regressions does not alter the previous picture. In specification 3, $\Gamma_{6,6}$ still yields significant information ratios for the last quarter of the sample when compared against $\Delta_{6,6}$, and for the whole sample when compared against $\Delta_{1,11}$ (specification 4). The latter is admittedly driven by an over-performance in the last part of the sample; the t-statistic for the corresponding $\hat{\alpha}$ is well above the critical values. $\Gamma_{0,12}$ on the other hand produces a positive information ratio in the early-half and second quarter of the sample when compared against $\Delta_{6,6}$ (specification 7), but negative or not significant otherwise.

Table 3.7 inverts the specifications of table 3.6, by using Δ-strategies as the dependent variable and Γ-strategies and the market portfolio as the explanatory variables. Specifications 1-2 and 5-6 are now of the form $\Delta_{s,f} - r_f = \alpha + \beta_\Gamma \Gamma_{s,f}$. Specifications 3-4 and 7-8 are of the form $\Delta_{s,f} - r_f = \alpha + \beta_\Gamma \Gamma_{s,f} + \beta_{mkt}(Mkt - r_f)$.

Specifications 1 and 5 of $\Delta_{6,6}$ and $\Delta_{1,11}$ respectively on $\Gamma_{6,6}$ do not lead to significant information ratios, except for the second quarter of the sample, in which $\Gamma_{6,6}$ is unable to explain the returns of $\Delta_{1,11}$. Specifications 2 and 6 show that $\Gamma_{0,12}$ is able to explain the returns of $\Delta_{6,6}$ and $\Delta_{1,11}$, except for the second half for the former and the second quarter for the latter. Specifications 3 and 7 show that including the excess returns of the market portfolio as explanatory variable yields a significant information ratio over all the sample for $\Delta_{1,11}$ and insignificant ratios for $\Delta_{6,6}$. Specifications 4 and 8 reverse the previous results, as the information ratio over all the sample of $\Delta_{6,6}$ are now significant, while those of $\Delta_{1,11}$ are insignificant.

The spanning tests seem to be reflecting the correlation between Γ and Δ-strategies that derives from their construction, as they are partly built on the same quantity (i.e. a common previous return). Nevertheless, it is interesting to observe that $\Gamma_{6,6}$ and $\Gamma_{0,12}$ are also able to explain the returns of the momentum strategies despite their tendency to profit, at least during the second part of the sample, from very specific periods.

3.4.4 Acceleration evidenced as momentum of momentum

A natural question arising from the relatively high correlation between Γ and Δ-strategies, as well as from the results of the previous section, is whether the acceleration effect can be fully captured by momentum: that is, if once we account for momentum, the convexity implied by the acceleration effect plays no role in the predictability of stock prices. The spanning tests of section 3.4.3 yielded mixed results insofar information ratio estimates were only significant for particular periods. In order to better answer this question, we created 5x5-Δ nested portfolios based on $\Delta_{6,6}$ (the intermediate horizon momentum) and

	$\bar{r}$	$\sigma(r)$	SR	alpha	t-stat		$\bar{r}$	$\sigma(r)$	SR	alpha	t-stat
	25 $\Delta(6,6)$:$\Delta(12,6)$ portfolios						25 $\Delta(12,6)$:$\Delta(6,6)$ portfolios				
1: 1	0.576	9.481	0.031	−0.627	0.986	1: 1	0.569	9.613	0.030	−0.652	0.949
2	0.727	8.542	0.052	−0.405	1.661	2	0.940	8.085	0.081	−0.167	2.595
3	0.611	7.869	0.042	−0.469	1.332	3	1.225	7.472	0.126	0.177	4.025
4	0.562	8.036	0.035	−0.532	1.110	4	1.433	7.435	0.155	0.386	4.942
5	0.436	7.887	0.019	−0.642	0.620	5	1.595	7.760	0.169	0.572	5.400
2: 1	1.006	8.009	0.090	−0.095	2.882	2: 1	0.679	8.078	0.049	−0.412	1.565
2	0.870	7.123	0.082	−0.141	2.633	2	0.882	7.068	0.085	−0.128	2.707
3	0.874	6.925	0.085	−0.111	2.728	3	1.085	6.543	0.123	0.111	3.915
4	0.831	6.715	0.082	−0.140	2.604	4	1.101	6.159	0.133	0.169	4.242
5	0.769	6.588	0.074	−0.204	2.358	5	1.523	6.629	0.187	0.552	5.976
3: 1	1.066	6.884	0.114	0.077	3.632	3: 1	0.693	7.864	0.052	−0.381	1.663
2	1.039	6.391	0.118	0.080	3.778	2	0.960	7.172	0.094	−0.058	3.015
3	0.987	6.420	0.110	0.019	3.502	3	0.977	6.312	0.110	0.022	3.510
4	0.844	5.883	0.095	−0.066	3.043	4	1.000	5.860	0.122	0.090	3.905
5	0.882	6.198	0.097	−0.064	3.088	5	1.337	5.991	0.176	0.426	5.619
4: 1	1.333	6.846	0.153	0.340	4.898	4: 1	0.599	7.189	0.044	−0.430	1.403
2	1.151	5.921	0.147	0.237	4.682	2	0.822	6.182	0.087	−0.104	2.784
3	1.065	5.594	0.140	0.184	4.462	3	0.784	5.528	0.091	−0.085	2.894
4	1.032	5.540	0.135	0.163	4.319	4	1.028	5.473	0.136	0.168	4.349
5	0.963	6.190	0.110	0.020	3.507	5	1.406	6.193	0.181	0.476	5.790
5: 1	1.789	7.642	0.197	0.748	6.293	5: 1	0.474	7.866	0.024	−0.610	0.777
2	1.615	6.599	0.202	0.654	6.449	2	0.781	6.796	0.073	−0.201	2.342
3	1.436	6.492	0.178	0.491	5.671	3	0.771	6.298	0.078	−0.177	2.476
4	1.401	6.514	0.172	0.448	5.481	4	1.049	6.316	0.121	0.111	3.871
5	1.357	7.075	0.152	0.371	4.847	5	1.303	6.977	0.146	0.332	4.671

Table 3.8: The table contains summary statistics for the $\Delta_{12,6}$:$\Delta_{6,6}$ nested portfolios, alternating the sorting order. The 25 $\Delta_{6,6}$:$\Delta_{12,6}$ portfolios create first quintiles based on $\Delta_{6,6}$, and then within each $\Delta_{6,6}$ quintile, create quintiles based on $\Delta_{12,6}$. The 25 $\Delta_{12,6}$:$\Delta_{6,6}$ portfolios invert the previous process, by first creating quintiles based on $\Delta_{12,6}$. Reported are the mean and standard deviation of the raw monthly returns (in percentage), the Sharpe ratio SR, CAPM alphas (in percentage), and the t-statistic of the average excess return (of the 1-Month U.S. T-bill-rate).

$\Delta_{12,6}$ (the second quantity needed to build $\Gamma_{6,6}$). A nested portfolio is denoted as

$$NUMBER\_FIRST\_QUANTITY : NUMBER\_SECOND\_QUANTITY$$

For example, for the $\Delta_{6,6}$:$\Delta_{12,6}$ nested portfolios, 1:5 denotes the portfolio selected from the bottom quintile of $\Delta_{6,6}$ and then from the top quintile of $\Delta_{12,6}$. Absence of acceleration would imply no significant differences among the returns of the nested portfolios once we account for $\Delta_{6,6}$. Conversely, a sharp distinction between portfolios with high $\Delta_{6,6}$ and low $\Delta_{12,6}$ would support the acceleration hypothesis.

The left columns of table 3.8 displays the results. Consistent with the idea of acceleration, we observe a tendency of the profits within each $\Delta_{6,6}$-quintile to decrease as we move from the top sub-quintile to the bottom one; the exception is the bottom $\Delta_{6,6}$-quintile, as the average profit of the 1:1 portfolio is lower than that of the 1:2 portfolio. For instance, profits of the top $\Delta_{6,6}$-quintile vary from 1.8% to 1.3%, and profits of the second quintile vary from 0.77% to 1%. Furthermore, the portfolio that accelerated the most (5:1) exhibit the strongest profit, while the one that accelerated the least (1:5) yields the lowest value.

Importantly, these results are not affected by the order in which the portfolios are nested. The right columns of table 3.8 invert the nesting order; that is, first by constructing the quintile of $\Delta_{12,6}$ and then of $\Delta_{6,6}$. The bottom-top (top-bottom) portfolio contains now the stocks with the most (least) acceleration. Again, we observe profit patterns consistent with the presence of acceleration dynamics. The 1:5 portfolio has an average profit of 1.6%, while the 5:1 portfolio yields a 0.4% value. Other quintiles also exhibit within-quintile rising profits, implying momentum of momentum, i.e. acceleration.

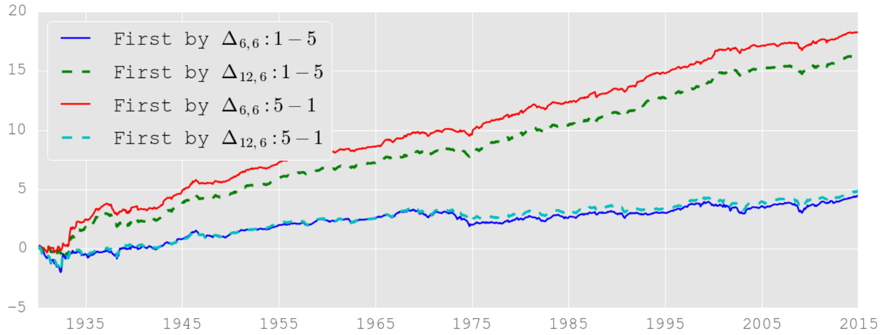
In figure 3.3, we take a closer look to the portfolios with the largest (resp. smallest) acceleration for each nesting order. The figure reveals a striking difference between the extreme portfolios, which suggests again that acceleration is a winner's phenomenon. As discussed in section 3.3, losers behave differently and in a way that appears to be consistent with behavioral explanations of momentum, such as prospect theory and the disposition effect [Hirshleifer, 2001, Grinblatt and Han, 2005]. Investors seem to be treating losers differently, sticking to them despite the fact that they have experienced accelerating negative returns.

3.5 Sources of acceleration profits

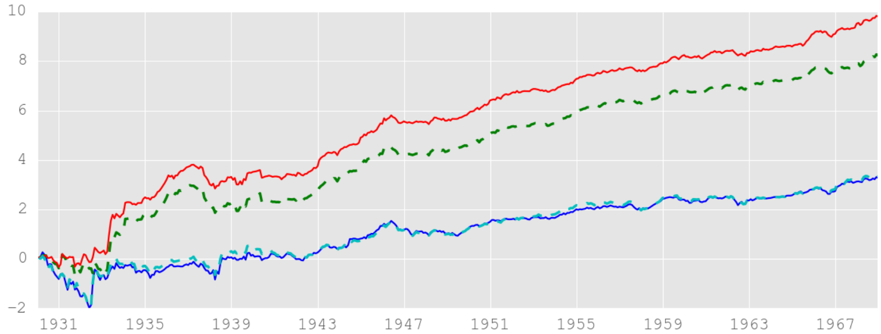
Having established that Γ-strategies are on average profitable and cannot be fully explained by pure momentum, we now investigate possible sources of profitability, and identify further differences and similarities with Δ-strategies.

3.5.1 Risk factors

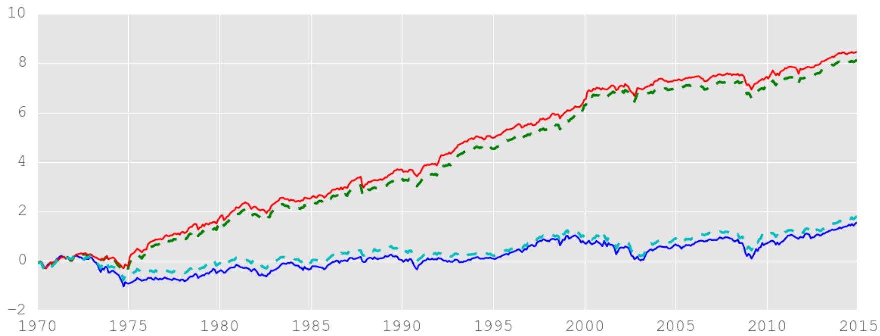
Table 3.9 compares the risks factors loadings of the Γ and Δ-strategies for time series regressions of the strategies on the market, the Fama-French book-to-value (HML) and



(a) 1930-2015(exclusive)



(b) 1930-1970 (exclusive)



(c) 1970-2015(exclusive)

Figure 3.3: Cumulative monthly profits of the portfolios that accelerated the most and the least among the 25 $\Delta_{6,6}$:$\Delta_{12,6}$ and 25 $\Delta_{12,6}$:$\Delta_{6,6}$ portfolios. The portfolios with the highest acceleration are 5:1 and 1:5 for $\Delta_{6,6}$:$\Delta_{12,6}$ and $\Delta_{12,6}$:$\Delta_{6,6}$ respectively. The portfolios with the lowest acceleration are 1:5 and 5:1 for $\Delta_{6,6}$:$\Delta_{12,6}$ and $\Delta_{12,6}$:$\Delta_{6,6}$ respectively. To construct the $\Delta_{6,6}$:$\Delta_{12,6}$ portfolios, at the beginning of every month t, we compute quintile breakpoints by sorting the NYSE stocks using $r_{t-1-6}(6)$. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding quintile according to their respective $r_{t-1-6}(6)$. We then repeat the same process within each quintile but using $r_{t-1-12}(6)$ to sort the stocks. The 25 $\Delta_{12,6}$:$\Delta_{6,6}$ portfolios portfolios are constructed in a similar manner, but using first $r_{t-1-12}(6)$ to create the quintiles.

	Alternative factor model specifications							
	Intermediate acceleration $\Gamma_{6,6}$					*Acceleration* $\Gamma_{0,12}$		
	1	2	3	4	5	6	7	8
alpha	0.7	0.8	0.7	0.2	0.5	0.7	0.6	−0.1
	(4.726)	(4.875)	(4.765)	(1.488)	(3.15)	(4.076)	(3.893)	(−1.018)
$Mkt - RF$		−0.045	−0.097	0.033		−0.222	−0.257	−0.071
		(−1.533)	(−3.127)	(1.147)		(−7.419)	(−7.915)	(−2.731)
SMB			0.304	0.315			0.155	0.171
			(5.992)	(7.023)			(2.928)	(4.213)
HML			−0.055	0.195			0.023	0.381
			(−1.24)	(4.633)			(0.492)	(10.029)
Mmt				0.549				0.786
				(16.889)				(26.783)
Adj R^2		0.001	0.034	0.246		0.05	0.057	0.447
	Intermediate momentum $\Delta_{6,6}$					*Momentum* $\Delta_{1,11}$		
	1	2	3	4	5	6	7	8
alpha	0.8	1.0	1.2	0.2	0.8	1.1	1.4	−0.1
	(3.802)	(4.837)	(6.311)	(1.584)	(3.078)	(4.796)	(6.149)	(−0.791)
$Mkt - RF$		−0.302	−0.201	0.036		−0.555	−0.408	−0.056
		(−8.091)	(−5.355)	(1.313)		(−12.872)	(−9.23)	(−2.694)
SMB			0.078	0.098			−0.15	−0.12
			(1.268)	(2.259)			(−2.088)	(−3.72)
HML			−0.718	−0.262			−0.716	−0.039
			(−13.353)	(−6.446)			(−11.318)	(−1.27)
Mmt				1.001				1.485
				(31.932)				(63.271)
Adj R^2		0.06	0.199	0.6		0.139	0.238	0.846

Table 3.9: The table reports summary statistics for time series monthly regressions of the excess returns of $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ strategies on CAPM ($Mkt - RF$), Small minus Big (SMB), High minus Low (HML), and Carhart's momentum (Mmt), spanning from January 1930 to December 2014. Specifications 1 and 5 correspond to the average excess returns (of the 1-Month U.S. T-bill-rate) for the respective strategy, $\overline{(r - RF)_t}$. Specifications 2 and 6 correspond to CAPM, $(r-RF)_t = \alpha+\beta(Mkt-RF)_t$. Specifications 3 and 7 correspond to the Fama-French FF three factors model, $(r-RF)_t = \alpha+\beta(Mkt-RF)_t + b_{SMB}SMB_t + b_{HML}HML_t$. Specifications 4 and 8 correspond to the FF model plus Carhart's momentum, $(r-RF)_t = \alpha+\beta(Mkt-RF)_t+b_{SMB}SMB_t+b_{HML}HML_t+b_{Mmt}Mmt_t$. Reported are the the intercept *alpha* (in percentage), the respective regression coefficients, the t-statistics (in parentheses), and adjusted R^2 coefficients.

size (SMB) factors, and Carhart's momentum (Mmt).

Specifications 2 and 6 show that $\Gamma_{6,6}$ and $\Gamma_{0,12}$-strategies have a zero and a negative $\hat{\beta}$s respectively. This contrasts with $\Delta_{6,6}$ and $\Delta_{0,12}$, whose $\hat{\beta}$s are both negative and larger in magnitude (-0.302 and -0.408). Specifications 3 and 7 show that just as momentum, acceleration strategies exhibit positive and significant alphas when regressed against the FF factors. However, contrary to momentum strategies, Γ-strategies do not load significantly on HML, as both coefficients are close to zero and not significant. The regressions also yield a positive relationship with size, which is different from the mixed sign-coefficients obtained for momentum. Specifications 4 and 8 reveal that Γ-strategies load highly and positively on Mmt, leading to non significant $\hat{\alpha}$'s. Finally, we notice that adjusted $\hat{R}^2$s tend to be smaller for Γ-strategies, suggesting that a lower fraction of the variance of the Γ returns is explained by either of the models. Such pattern might stem from the non-linear phenomenon that we are trying to capture, in which, as we commented in section 3.4, the returns of Γ-strategies have coincided with periods of strong super-exponential acceleration in stock price dynamics.

3.5.2 Size portfolios

We now study the role of size and shorting on the profitability of Γ-strategies. Hong et al. [2000] and Grinblatt and Moskowitz [2004] examined momentum returns across firm size and the long versus short side contributions to momentum profits. They concluded that momentum is stronger among small cap stocks and that two-thirds of the profits come from shorting. However, in a larger 86-year sample, Israel and Moskowitz [2013] find that momentum returns are largely unaffected by size and that the short side is no more profitable than the long one. In addition, they observed that, when risk adjusted, the short side of momentum becomes less important. As one can expect that shorting small cap stocks is more expensive and more difficult to implement, this would actually be a positive feature of the strategy.

To analyze the role of size and shorting, we examine the performance of Γ and Δ-strategies built within size quintiles. Table 3.10 presents the average monthly raw returns, CAPM alphas (in percentage), and t-statistics (in parentheses) of the quintile strategies (denoted by 5-1) and their long side (the top quintile portfolio). In addition, we report the ratio (as a percentage) of the returns of each long side to the respective strategy. Somewhat at odds with the findings of Israel and Moskowitz [2013], the returns of Δ-strategies do seem to be affected by size. Although there is no strict monotonic relationship in terms of returns, we observe a tendency of momentum returns to decline with size. $\Delta_{6,6}$ raw and risk-adjusted returns fall from 0.965% and 0.765% to 0.611% and 0.4% respectively. $\Delta_{1,11}$ raw and risk adjusted returns decline from 0.911% and 0.803% to 0.441% and 0.328% respectively. As size decreases and with risk adjustment, shorting becomes slightly less important for $\Delta_{6,6}$, but mildly more relevant for $\Delta_{1,11}$ (the percentage from the long side drops from 93.4% to 75.74%).

	Size 1	Size 2	Size 3	Size 4	Size 5
	Intermediary acceleration $\Gamma_{6,6}$				
Returns					
5-1	0.879	0.712	0.789	0.745	0.605
	(4.375)	(3.714)	(4.164)	(3.807)	(2.842)
Long side	1.868	1.617	1.607	1.474	1.221
	(5.057)	(5.139)	(5.590)	(5.830)	(5.213)
Percentage long side	212.550	227.142	203.703	197.922	201.769
Alphas					
5-1 spread	0.559	0.438	0.517	0.522	0.344
	(4.083)	(3.762)	(4.228)	(4.302)	(3.013)
Long side	0.650	0.493	0.524	0.493	0.308
	(3.385)	(3.719)	(5.013)	(5.774)	(4.719)
Long side/5-1	116.149	112.584	101.453	94.284	89.366
	Acceleration $\Gamma_{0,12}$				
Returns					
5-1	0.064	0.432	0.497	0.513	0.396
	(−1.281)	(1.153)	(1.706)	(1.765)	(0.922)
Long side (top portfolio)	1.409	1.393	1.403	1.332	1.123
	(3.923)	(4.577)	(5.002)	(5.360)	(4.938)
Long side/5-1	2202.290	322.749	282.537	259.664	283.359
Alphas					
5-1	−0.109	0.232	0.274	0.354	0.188
	(−0.647)	(1.815)	(2.197)	(2.784)	(1.539)
Long side (top portfolio)	0.292	0.327	0.369	0.385	0.255
	(1.594)	(2.620)	(3.644)	(4.541)	(3.705)
Long side/5-1	−266.821	141.200	134.768	108.858	135.363
	Intermediary momentum $\Delta_{6,6}$				
Returns					
5-1	0.965	0.729	0.900	0.708	0.611
	(4.171)	(2.955)	(3.764)	(2.578)	(2.219)
Long side (top portfolio)	1.782	1.581	1.680	1.462	1.224
	(5.226)	(5.244)	(6.058)	(5.750)	(5.203)
Long side/5-1	184.638	216.774	186.606	206.512	200.457
Alphas					
5-1 spread	0.765	0.583	0.740	0.570	0.400
	(4.696)	(3.957)	(4.576)	(3.534)	(2.711)
Long side (top portfolio)	0.629	0.498	0.631	0.491	0.319
	(3.687)	(3.915)	(5.831)	(5.274)	(4.349)
Long side/5-1	82.166	85.534	85.246	86.178	79.861
	Momentum $\Delta_{1,11}$				
Returns					
5-1	0.911	0.920	0.749	0.677	0.441
	(2.961)	(3.330)	(2.369)	(2.030)	(0.937)
Long side (top portfolio)	1.708	1.622	1.620	1.471	1.166
	(5.234)	(5.640)	(6.175)	(6.120)	(5.095)
Long side/5-1	187.470	176.270	216.380	217.081	264.147
Alphas					
5-1	0.803	0.868	0.725	0.646	0.328
	(3.868)	(4.767)	(3.920)	(3.515)	(1.994)
Long side/5-1	0.608	0.585	0.630	0.554	0.306
	(3.689)	(4.584)	(5.858)	(5.723)	(3.786)
Long side/5-1	75.742	67.396	86.847	85.823	93.405

Table 3.10: The table reports the performance of $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ strategies, built within size quintiles. To construct the $\Gamma_{s,f}$ strategies within each size quintile, at the beginning of every month t, we compute quintile breakpoints by sorting the NYSE stocks using $\Gamma_{t-1-s}(f)$. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding quintile according to their respective $\Gamma_{t-1-s}(f)$. The strategies long the top $\Gamma_{s,f}$-ranked quintile and short the bottom $\Gamma_{s,f}$-ranked quintile. The $\Delta_{s,f}$ strategies are constructed in a similar manner, but using the f months return $r_{t-1-s}(f)$ to compute the quintile breakpoints. Reported are average monthly raw returns, CAPM alphas (in percentage), and t-statistics (in parentheses) of the quintile strategies (denoted by 5-1) and their long side (the top quintile portfolio). In addition, we report the ratio (as a percentage) of the returns of each top quintile to the respective strategy.

$\Gamma_{6,6}$ strategies also seem to be affected by size, as similar returns patterns emerge across size-quintiles. Average raw and risk-adjusted returns fall from 0.879% and 0.559% to 0.605% and 0.344% respectively, and they remain significant. $\Gamma_{0,12}$ on the contrary do not generate significant returns among small and large caps (the latter if risk adjusted). This is in fact the opposite size-momentum interaction effect discussed in the literature, as momentum is thought to be stronger among small cap stocks.

As for the role of shorting, in all cases and contrary to momentum, acceleration strategies seem less sensitive to shorting. In terms of average raw returns, neither the percentage of the long side of the portfolio of $\Gamma_{6,6}$ nor that of $\Gamma_{0,12}$ appear to exhibit a monotonic relationship across quintiles. If the strategies are adjusted for risk, the long sides of the strategies do not exhibit a monotonic relationship either, and their contributions to total returns stay above 89% of the total returns, typically exceeding 100%. This observation is consistent with figure 3.2, in which the acceleration effect appears to be limited to the winners' portfolio, and therefore one would expect the short side to play a minor role.

3.5.3 January effect

It is generally known that momentum strategies under-perform in January. Additionally, Yao [2012] showed that a significant part of the superior performance of Novy-Marx [2012]'s intermediate horizon momentum can be attributed to seasonality. Table 3.11 shows that Γ-strategies are also dependent on the seasonal effect, though with the opposite direction. The table contains raw mean profits for January and non January months over the whole sample, and six different sub-periods.

In the 1930-2015 period, the $\Gamma_{6,6}$ strategy (10-1) exhibited raw mean profits of 1.57% in January against 0.97% in all the other months, meaning that $\Gamma_{6,6}$ has benefited from the January effect. $\Gamma_{0,12}$ in turn has tended to under-perform in January, but to a lesser extent when compared to momentum. $\Gamma_{0,12}$'s raw mean profits are close to zero in January, whereas average monthly profits of Δ-strategies range from -3.79% to -0.14. Thus, Γ-strategies have exhibited higher profits than those of Δ-strategies in January, which are clearly negative.

The magnitude of the $\Gamma_{6,6}$ and $\Gamma_{0,12}$ January's profits tend to vary strongly. During the periods 1930-1949 and 1970-1989, $\Gamma_{6,6}$ over-performed in January, with profits equal to 3.23% and 3.54% respectively, while it under-performed during the years 1950-1969 and 1990-2015, exhibiting profits of 0.06% and 0.12% respectively. Similarly, $\Gamma_{0,12}$ over-performed in January between 1970-1989, having profits of 2.16%, and underperformed in the rest of the sub-sample, bottoming in the 1990-2015 period with profits of -0.92%.

A partial explanation for the very strong and often positive seasonal effect of $\Gamma_{6,6}$ is that, by selecting winners from the previous 12-6 months period (and thereby excluding the last 6 months period), the strategy reduces its exposure to the seasonality. This is in line with Goyal and Wahal [2015] and Yao [2012]'s explanation of the Novy-Marx [2012]'s intermediate horizon momentum. In addition, both $\Gamma_{6,6}$ and $\Gamma_{0,12}$-strategies reduce further

	Int. acceleration $\Gamma_{6,6}$			Acceleration $\Gamma_{0,12}$			Int. momentum $\Delta_{6,6}$			Momentum $\Delta_{1,11}$		
	All	Jan	NonJan	All	Jan	NonJan	All	Jan	NonJan	All	Jan	NonJan
						1930-2015						
1	0.60	1.66	0.50	0.63	1.90	0.51	0.41	3.02	0.17	0.44	4.31	0.09
	(1.44)	(2.18)	(0.95)	(1.52)	(2.33)	(0.96)	(0.46)	(3.58)	(−0.40)	(0.52)	(4.24)	(−0.60)
10	1.63	3.23	1.48	1.43	1.89	1.39	1.48	2.24	1.41	1.50	1.60	1.49
	(6.16)	(3.73)	(5.29)	(5.73)	(2.32)	(5.29)	(5.63)	(2.50)	(5.11)	(6.04)	(1.83)	(5.76)
10-1	1.02	1.57	0.97	0.80	−0.00	0.88	1.07	−0.78	1.24	1.05	−2.70	1.40
	(4.73)	(2.17)	(4.26)	(3.15)	(−0.47)	(3.46)	(3.80)	(−1.63)	(4.39)	(3.07)	(−3.29)	(4.31)
						1930-1969						
1	0.75	1.99	0.74	0.59	2.30	0.55	0.64	3.52	0.46	0.60	4.76	0.33
	(1.67)	(2.08)	(1.55)	(1.28)	(2.17)	(1.11)	(1.06)	(2.99)	(0.65)	(0.90)	(3.94)	(0.36)
10	1.65	3.62	1.54	1.45	1.89	1.47	1.49	2.48	1.47	1.58	1.98	1.60
	(4.30)	(2.99)	(3.77)	(4.27)	(1.96)	(4.05)	(4.13)	(2.16)	(3.87)	(4.76)	(1.99)	(4.56)
10-1	0.89	1.63	0.79	0.86	−0.40	0.92	0.85	−1.05	1.01	0.98	−2.78	1.28
	(2.91)	(1.99)	(2.37)	(2.97)	(−0.92)	(3.02)	(2.02)	(−1.29)	(2.31)	(2.13)	(−2.99)	(2.67)
						1970-2015						
1	0.47	1.45	0.38	0.65	1.63	0.57	0.20	2.66	−0.02	0.30	4.07	−0.04
	(0.21)	(1.14)	(−0.13)	(0.85)	(1.25)	(0.51)	(−0.73)	(2.12)	(−1.45)	(−0.30)	(2.49)	(−1.24)
10	1.61	2.98	1.48	1.41	1.99	1.36	1.47	2.13	1.41	1.42	1.35	1.43
	(4.45)	(2.32)	(3.89)	(3.82)	(1.49)	(3.52)	(3.83)	(1.51)	(3.52)	(3.77)	(0.85)	(3.70)
10-1	1.14	1.53	1.10	0.75	0.36	0.79	1.27	−0.53	1.44	1.12	−2.73	1.47
	(3.91)	(1.22)	(3.74)	(1.52)	(−0.04)	(1.65)	(3.62)	(−0.99)	(4.19)	(2.23)	(−2.09)	(3.37)
						1930-1949						
1	0.64	1.76	0.47	0.46	2.77	0.16	0.53	3.78	0.13	0.66	6.50	0.03
	(0.89)	(1.31)	(0.57)	(0.65)	(1.71)	(0.17)	(0.54)	(1.88)	(0.09)	(0.64)	(3.22)	(−0.01)
10	1.72	4.99	1.39	1.27	2.22	1.11	1.37	3.64	1.15	1.22	2.71	1.03
	(2.64)	(2.33)	(1.94)	(2.27)	(1.41)	(1.78)	(2.33)	(1.91)	(1.78)	(2.31)	(1.82)	(1.76)
10-1	1.07	3.23	0.92	0.80	−0.55	0.95	0.84	−0.14	1.03	0.56	−3.79	1.00
	(2.17)	(2.33)	(1.66)	(1.76)	(−0.67)	(1.86)	(1.22)	(−0.11)	(1.34)	(0.69)	(−2.31)	(1.15)
						1950-1969						
1	0.86	2.28	0.95	0.73	1.84	0.86	0.76	3.14	0.72	0.54	2.96	0.53
	(2.13)	(1.56)	(2.55)	(1.64)	(1.22)	(2.17)	(1.82)	(2.32)	(1.75)	(1.00)	(2.16)	(1.04)
10	1.58	2.34	1.64	1.64	1.59	1.78	1.60	1.54	1.77	1.95	1.44	2.13
	(4.45)	(1.96)	(4.50)	(4.73)	(1.30)	(4.95)	(4.30)	(1.11)	(4.70)	(5.19)	(1.00)	(5.60)
10-1	0.71	0.06	0.69	0.91	−0.25	0.92	0.85	−1.60	1.05	1.41	−1.52	1.60
	(2.30)	(−0.33)	(2.08)	(3.14)	(−0.58)	(3.07)	(2.52)	(−2.31)	(3.25)	(4.41)	(−1.64)	(5.03)
						1970-1989						
1	0.50	1.06	0.44	0.68	1.68	0.58	0.05	3.76	−0.25	0.03	4.75	−0.35
	(−0.31)	(0.29)	(−0.44)	(0.16)	(0.63)	(−0.07)	(−1.34)	(1.54)	(−2.01)	(−1.27)	(1.73)	(−2.07)
10	1.54	4.60	1.17	1.41	3.84	1.10	1.57	3.00	1.35	1.54	2.44	1.38
	(2.23)	(2.01)	(1.30)	(2.05)	(1.77)	(1.20)	(2.27)	(1.18)	(1.67)	(2.30)	(0.94)	(1.82)
10-1	1.04	3.54	0.74	0.73	2.16	0.51	1.51	−0.75	1.60	1.51	−2.32	1.74
	(1.60)	(2.15)	(0.46)	(0.40)	(1.14)	(−0.32)	(2.82)	(−0.79)	(3.01)	(2.40)	(−1.36)	(3.03)
						1990-2015						
1	0.44	1.50	0.35	0.64	1.32	0.57	0.32	1.61	0.20	0.52	3.35	0.26
	(0.53)	(1.16)	(0.25)	(0.98)	(0.90)	(0.78)	(0.16)	(1.24)	(−0.13)	(0.51)	(1.59)	(0.02)
10	1.66	1.62	1.67	1.41	0.40	1.50	1.40	1.33	1.40	1.32	0.32	1.41
	(4.01)	(1.06)	(3.86)	(3.28)	(0.12)	(3.39)	(3.10)	(0.78)	(3.00)	(3.01)	(0.05)	(3.14)
10-1	1.22	0.12	1.32	0.78	−0.92	0.93	1.08	−0.29	1.21	0.80	−3.03	1.15
	(3.75)	(−0.11)	(4.13)	(1.57)	(−0.79)	(2.02)	(2.41)	(−0.48)	(2.65)	(1.14)	(−1.50)	(1.86)

Table 3.11: The table reports January and non January average raw returns for the top (10), bottom (1), and top minus bottom (10-1) decile portfolios based on $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ over several periods. To construct $\Gamma_{s,f}$ decile portfolios, at the beginning of every month t, we compute decile breakpoints by sorting the NYSE stocks using $\Gamma_{t-1-s}(f)$, with s equals the number of months between the formation and holding period. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding decile according to their respective $\Gamma_{t-1-s}(f)$. The $\Delta_{s,f}$ portfolios are constructed in a similar manner, but using the f months return $r_{t-1-s}(f)$ to compute the decile breakpoints. Reported are January and non January average monthly raw returns (in percentage) and corresponding t-statistics (in parentheses).

their exposure to the seasonality, as they will tend to select the losers among the winners of the prior 18-12 and 24-12 months respectively. These stocks will likely have high values of Γ during the formation period, due to the tendency of the losers to rebound in January. In other words, the strategies are betting in favor of the seasonality.

It is worth noting that these results do not weaken our previous observation about the link between the profitability of Γ-strategies and particular market regimes. The sub-period containing the highest profitable years of acceleration strategies (i.e. 1990-2015) actually exhibited negative or insignificant positive performance during January. As such, the seasonality cannot explain the positive results obtained during these years.

3.5.4 Acceleration and market states

In this section, we analyze the dependency of acceleration on the state of the market. Following Cooper et al. [2004], we examine whether conditioning on two market states is important to the profitability of acceleration strategies. An Up state is when the lagged three year market return is non-negative. A Down state is when the three year lagged market return is negative. Cooper et al. [2004] observed that the performance of momentum is dramatically different following Down states; momentum thus profits exclusively following Up states.

Table 3.12 shows the average excess returns, CAPM-adjusted returns and FF-adjusted average returns (i.e. alphas) of acceleration and momentum following Up and Down states. The table includes their corresponding t-statistics. Acceleration and momentum returns are statistically and economically significant following Up markets. Average excess returns, as well as CAPM-adjusted and FF-adjusted returns present values ranging from 0.7% to 0.13% and t-statistics are well above the critical values. On the contrary, as in [Cooper et al., 2004], there is no evidence of Δ and Γ positive returns following Down markets: returns are not statistically significant. In particular, we observe that although the returns of $\Delta_{6,6}$ and $\Gamma_{6,6}$ strategies are in fact slightly more positive in a Down state, their t-statistics remain below the critical values.

3.5.5 Serial and cross sectional correlation

Following Lewellen [2002], we know revisit the relative strength Γ-strategies introduced in section 3.3.1 to shed further light about its possible sources of profitability. The Γ-profit $\pi^{\Gamma}_{t+h-1}(f,h)$ can be expressed as

$$\pi^{\Gamma}_{t+h-1}(f,h) = \pi^{\Delta}_{t+h-1}(f,h) - \pi^{2\Delta}_{t+h-1}(f,h) \ , \tag{3.9}$$

	Int. acceleration $\Gamma_{6,6}$	*Acceleration* $\Gamma_{0,12}$	*Int. momentum* $\Delta_{6,6}$	*Acceleration* $\Delta_{1,11}$
	Average monthly returns following 36-months Up Markets			
Mean-return	0.7	0.6	0.8	1.0
	(4.768)	(3.701)	(4.14)	(4.779)
CAPM alpha	0.7	0.8	0.8	1.1
	(4.833)	(4.672)	(4.22)	(5.215)
Fama-French alpha	0.7	0.7	1.0	1.3
	(4.575)	(4.397)	(5.632)	(5.971)
	Average monthly return following 36-months Down Markets			
Mean-return	0.9	−0.0	1.0	−0.4
	(1.612)	(−0.035)	(1.141)	(−0.366)
CAPM alpha	1.0	0.1	1.4	0.4
	(1.687)	(0.238)	(1.918)	(0.445)
Fama-French alpha	0.5	−0.0	1.2	0.5
	(0.919)	(−0.049)	(1.824)	(0.697)
	Return difference between Up Markets and Down Markets			
Mean-returns	−0.2	0.6	−0.2	1.4
CAPM alpha	−0.3	0.7	−0.6	0.7
Fama-French alpha	0.2	0.8	−0.2	0.7

Table 3.12: The table reports average monthly excess returns, and alphas of time series monthly regressions of $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ strategies on CAPM and the Fama-French model, for months in Up and Down market states (quantities in percentage). In parentheses and below each quantity, the corresponding t-statistics. An Up state is when the lagged three year market return is non-negative. A Down state is when the three year lagged market return is negative. The regressions span from January 1930 to December 2014.

where $\pi^{\Delta}_{t+h-1}(f,h)$ is defined by (3.5) and

$$\begin{aligned}\pi^{2\Delta}_{t+h-1}(f,h) &:= \frac{1}{N}\sum_{i=1}^{N}(r_{i,t-1-f}(f) - r_{m,t-1-f}(f))r_{t+h-1,i}(h) \\ &= \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1-f}(f)r_{i,t+h-1}(h)\right) - r_{m,t-1-f}(f)r_{t+h-1,m}(h)\end{aligned} \tag{3.10}$$

It has the same structure as expression (3.5), but derives from it with $t-1$ replaced by $t-1-f$. The performance of Γ-strategies is thus directly related to that ($\pi^{\Delta}_{t+h-1}(f,h)$) of Δ-strategies. But there is another important term $\pi^{2\Delta}_{t+h-1}(f,h)$ that makes the story richer.

In order to unravel its meaning, assuming that the unconditional mean returns of individuals stocks are constants, making the holding period and formation period equal to 1 to simplify the analysis (i.e. $h = f = 1$), we can decompose the Γ profits into four components by taking the expectation of equation (3.3):

$$E\left[\pi^{\Gamma}_t\right] = O_1 - O_2 - C_1 + C_2 \ , \tag{3.11}$$

where

$$O_1 := \frac{N-1}{N^2}\sum_{i=1}^{N} Cov\left[r_{i,t-1}, r_{i,t}\right] \ , \tag{3.12}$$

$$O_2 := \frac{N-1}{N^2} \sum_{i=1}^{N} Cov\left[r_{i,t-1-f}, r_{i,t}\right] \ , \tag{3.13}$$

$$C_1 := Cov\left[r_{m,t-1}, r_{m,t}\right] - \frac{1}{N^2} \sum_{i=1}^{N} Cov\left[r_{i,t-1}, r_{i,t}\right] \ , \tag{3.14}$$

$$C_2 := Cov\left[r_{m,t-1-f}, r_{m,t}\right] - \frac{1}{N^2} \sum_{i=1}^{N} Cov\left[r_{i,t-1-f}, r_{i,t}\right] \ . \tag{3.15}$$

The first two terms O_1 and O_2 depend on the auto-covariances of the returns of the constituting stocks, respectively between $t-1$ and t and between $t-2$ and t. The last two terms C_1 and C_2 provide measures of the cross-sectional diversity of the covariance of the returns of the constituting stocks compared with that of the equally-weighted index, again between $t-1$ and t and between $t-2$ and t, respectively. Γ-strategies perform well when stocks are positively first-order auto-correlated and negatively second-order auto-correlated (over the time scale of $f-$months). This implies that, if firms with a positive return today have a positive return in the next period, but a lower or negative return two periods into the future, then Γ-strategies can exploit this. On the other hand, cross-serial covariances should be negative with respect to the last period but positive with respect to the period before the last one, for Γ-strategies to profit. In other words, the average one lag (resp. two lags) return covariance over all stocks should be larger (resp. smaller) than the equi-weighted index return covariance in order to contribute positively to the performance of Γ-strategies.

It is instructive to contrast this decomposition (3.11) with the equivalent representation for Δ-strategies:

$$E\left[\pi_t^{\Delta}\right] = O_1 - C_1 + \sigma^2(\mu) \ , \tag{3.16}$$

where O_1 and C_1 are given by (3.12) and (3.14) respectively and

$$\sigma^2(\mu) := \sum_{i=1}^{N} \left[\mu_i - \mu_m\right]^2 \ . \tag{3.17}$$

The meaning of the first term O_1 is obvious: a positive serial covariance of stock returns provides the simplest metric of return persistance, which can be directly exploited to obtain the optimal prediction for the next return based on the linear Wiener filter. The negative contribution of the second term C_1 means that the average one lag return covariance over all stocks should be larger than the equi-weighted index return covariance to contribute positively to the performance of Δ-strategies. The last term $\sigma^2(\mu)$ quantifies cross-sectional variation in mean returns. If stock prices follow random walks with vanishing first-order auto-covariances, then Δ-strategies can still profit from the cross-sectional variation in mean returns, in absence of any time series predictability [Conrad and Kaul, 1998].

Compared with Δ-strategies, Γ-strategies do not depend on the cross-sectional variation in mean returns that contributes to $E\left[\pi_t^{\Delta}\right]$. However, Γ-strategies can profit from the second order terms O_2 (3.13) and C_2 (3.15). Note that the second order terms contribute to the performance of Γ-strategies with the sign opposite to the first order terms. The existence of these contributions suggests that stock returns tend to be positively second order cross-serially correlated, implying that positive returns of others today will have a positive influence of the stock returns in the future. Moreover, stock returns tend to be negatively second order serially correlated, implying that individual winners (losers) today will tend to be losers (winners) two periods into the future.

From the evidence of positive performance of Γ-strategies, we have hinted at the fact that stock returns tend to be negatively second order serially correlated, implying that individual winners (losers) today will tend to be losers (winners) two periods into the future. Together with the positive correlation between first-order returns, this would imply a short-term persistence in returns and a reversal over longer time horizon during specific market regimes. This observation seems to be contradicting the short-term reversal effect of individual stock returns documented by Jegadeesh [1990], Lehmann [1990], Lo and McKinley [1990] and the return continuation for individual stocks in the medium-run [Jegadeesh and Titman, 1993] as well as their combination at the weekly time scales [Gutierrez and Kelley, 2008, Huehn and Scholz, 2015]. Our results are more in agreement with the evidence provided by Moskowitz et al. [2012] of persistence in returns for one to 12 months that partially reverses over longer horizons.

3.6 Asset pricing tests

3.6.1 Portfolio construction and summary statistics

The explanatory returns that we use in our asset pricing tests are the market factor, the Fama-French book-to-value (HML) and size (SMB) factors, and the four versions of momentum delta strategies considered above. The target portfolios that are supposed to be explained by these factors are the standard 25 book-to-value and size Fama and French (FF) portfolio, as well as the 10 industry portfolio based on the FF classification. We extracted both portfolios from Ken French's data library[2]. In addition, we test the pricing models on Δ-portfolios and Γ-portfolios, defined as the value-weighted portfolios of the stocks in each decile of their momentum-ranked and acceleration-ranked values at the end of each month.

Tables 3.13 and 3.14 report the summary statistics for the portfolios. Three observations can be made. The Δ, Γ, and 25 book-to-value and size portfolios exhibit considerable cross-sectional variation. Average excess monthly returns range from 0.135 to 1.218 for the $\Gamma_{6,6}$-portfolios, 0.154 to 1.032 for the $\Gamma_{0,12}$-portfolios, -0.031 to 1.060 for the $\Delta_{6,6}$-portfolios, -0.044 to 1.074 for the $\Delta_{1,11}$-portfolios, and 0.573 to 1.391 for the book-to-value

[2]French, K., 2012. Data library `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg. No.	Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg. No.
	10 $\Gamma_{6,6}$ portfolios						10 $\Delta_{6,6}$ portfolios				
Low	0.135	7.123	0.019	0.464	337	Low	−0.031	8.654	−0.004	−0.087	392
2	0.250	6.234	0.040	0.979	244	2	0.140	7.217	0.019	0.474	264
3	0.375	5.943	0.063	1.539	227	3	0.443	6.735	0.066	1.604	234
4	0.444	5.700	0.078	1.900	221	4	0.350	6.177	0.057	1.384	221
5	0.464	5.603	0.083	2.022	218	5	0.531	5.935	0.089	2.184	214
6	0.584	5.708	0.102	2.499	219	6	0.485	5.783	0.084	2.049	210
7	0.624	5.462	0.114	2.788	220	7	0.677	5.413	0.125	3.054	210
8	0.770	5.909	0.130	3.182	227	8	0.683	5.701	0.120	2.924	213
9	0.861	6.389	0.135	3.291	245	9	0.953	6.156	0.155	3.780	228
High	1.218	7.019	0.174	4.237	337	High	1.060	6.886	0.154	3.759	310
	10 $\Gamma_{0,12}$ portfolios						10 $\Delta_{1,11}$ portfolios				
Low	0.154	7.307	0.021	0.516	330	Low	−0.044	9.782	−0.004	−0.109	408
2	0.332	6.168	0.054	1.313	247	2	0.279	8.016	0.035	0.850	270
3	0.428	5.762	0.074	1.812	229	3	0.354	6.995	0.051	1.236	235
4	0.469	5.483	0.086	2.089	220	4	0.436	6.365	0.069	1.673	220
5	0.480	5.844	0.082	2.006	218	5	0.485	5.950	0.082	1.992	211
6	0.589	5.534	0.107	2.600	220	6	0.520	5.864	0.089	2.166	206
7	0.665	5.717	0.116	2.840	223	7	0.599	5.482	0.109	2.667	207
8	0.681	5.683	0.120	2.925	231	8	0.742	5.409	0.137	3.347	213
9	0.730	5.922	0.123	3.011	250	9	0.793	5.712	0.139	3.389	228
High	1.032	6.528	0.158	3.860	329	High	1.074	6.528	0.164	4.016	301

Table 3.13: The table reports summary statistics for the $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ decile portfolios over the period from January 1930 to December 2014. To construct $\Gamma_(s, f)$ portfolios, at the end of every month t, we compute decile breakpoints by sorting the NYSE stocks using $\Gamma_{t-s}(f)$, with s equals the number of months between the formation and holding period. Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding decile according to their respective $\Gamma_{t-s}(f)$. The $\Delta_{s,f}$ portfolios are constructed in a similar manner, but using the f months return $r_{t-s}(f)$ to compute the decile breakpoints. Reported are the average excess size weighted return (of the 1-Month U.S. T-bill-rate, in percentage), t-statistic of the average return, standard deviation of returns, and Sharpe ratio SR of each value.

Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat
10 Industry portfolios					25 Value-Size portfolios				
No Durables	0.689	4.660	0.148	4.598	Small: Low	0.599	12.374	0.048	1.505
Durables	0.793	7.712	0.103	3.197	2	0.756	9.933	0.076	2.367
Manufact.	0.698	6.289	0.111	3.453	3	1.096	9.122	0.120	3.737
Energy	0.770	6.107	0.126	3.922	4	1.229	8.409	0.146	4.546
Hi Tec	0.732	7.256	0.101	3.140	High	1.391	9.454	0.147	4.579
Telco	0.536	4.564	0.118	3.656	2: Low	0.656	8.083	0.081	2.523
Shops	0.725	5.832	0.124	3.868	2	0.923	7.546	0.122	3.804
Healthcare	0.789	5.625	0.140	4.363	3	1.041	7.355	0.142	4.405
Utilities	0.525	5.426	0.097	3.010	4	1.111	7.565	0.147	4.568
Other	0.609	6.555	0.093	2.889	High	1.252	8.815	0.142	4.421
					3: Low	0.739	7.495	0.099	3.067
					2	0.887	6.529	0.136	4.229
					3	0.931	6.600	0.141	4.388
					4	1.022	7.014	0.146	4.536
					High	1.157	8.557	0.135	4.207
					4: Low	0.689	6.222	0.111	3.444
					2	0.730	6.127	0.119	3.709
					3	0.876	6.557	0.134	4.155
					4	0.946	6.936	0.136	4.245
					High	1.032	8.790	0.117	3.655
					Big: Low	0.573	5.334	0.107	3.343
					2	0.586	5.295	0.111	3.443
					3	0.645	5.733	0.112	3.499
					4	0.664	6.605	0.101	3.129
					High	0.905	8.568	0.106	3.287

Table 3.14: The table contains summary statistics for the 25 B/M-size portfolio and the 10 industry portfolios over the period from January 1930 to December 2014. The portfolios are constructed according to the rules described by [Fama and French, 1996], and were extracted from Ken French's data library. Reported are the average excess size weighted return (of the 1-Month U.S. T-bill-rate, in percentage), t-statistic of the average return, standard deviation of returns, and Sharpe ratio SR of each value.

and size portfolios. Industry portfolios present somewhat lower cross-sectional variation, with monthly returns ranging from 0.525 (Utilities) to 0.793 (Energy). Second, excess returns are in general significant at a 5% confidence level, while exhibiting monthly Sharpe ratios typically around 0.1. Finally, Δ and Γ-portfolios are in general well diversified, with an average number of stocks in each portfolio typically above 200[3].

3.6.2 Results of asset pricing tests

We regress the Γ, Δ, book-to-value and size, and industry portfolios on the CAPM, the Fama-French 3-factor model (FF), and the FF model plus the four Δ and Γ-strategies that we have analyzed; we will refer to the latter four as the extended models. To determine if the models are able to explain the portfolios excess returns, we use the GRS statistic [Gibbons et al., 1989] with a 5% significance level. The GRS statistic tests the hypothesis that the intercepts ("alpha's") of our time series regressions on all portfolios are equal to 0. Rejecting the null hypothesis of $\alpha_i = 0, \forall i = 1..N$, with N the number of portfolios, implies that the model presents significant pricing errors and is unable to explain the returns of the portfolios. In addition, following the recommendation of Lewellen et al. [2010], we also report the Sharpe ratio $SR(a)$ of the intercepts of the models (the unexplained average returns), which is defined as $SR(a) = (a'S^{-1}a)^{1/2}$, where a is the column vector of all the intercepts, and S is the covariance matrix of regression residuals. As discussed by Fama and French [2012], the advantage of $SR(a)$ as a summary statistic is that it combines the regression intercepts with the covariance matrix of the regression residuals, which is an important determinant of the precision of the alpha's. However, as $SR(a)$ combines information about both the magnitude of the intercepts and their precision, it is still useful to have the information about the two pieces provided by the average absolute intercept, the average and median R^2, and the average standard error of the intercepts.

Table 3.15 summarizes the results of the regressions and associated test statistics. The $\Delta_{6,6}$-portfolios, the $\Delta_{1,11}$-based portfolios and the Γ-based portfolios cannot be explained neither by the CAPM, nor by the standard factor models. We examine in turn each of the extended models.

For the $\Delta_{6,6}$-portfolios, the GRS of all these models is well above the critical value, implying that the null hypothesis of all intercepts equal 0 can be soundly rejected. Only the $FF + \Delta_{6,6}$ model exhibits a good performance, having the lowest, highest, and lowest estimates of the GRS statistic, the adjusted R^2, and the $SR(a)$ metrics. Given the close relationship between the $\Delta_{6,6}$-factor and the $\Delta_{6,6}$-portfolios, this performance is not surprising: one can expect that any model will benefit from a direct relationship between the quantity used to construct the portfolios and one of its factors.

Similar to the $\Delta_{6,6}$-portfolios, we observe that the returns of the $\Delta_{1,11}$-portfolios are explained neither by the standard models nor by the Γ-based models. The momentum-based models exhibit a good performance, as their GRS statistics are below the critical

[3]Since we obtained the book-to-value and size and industry portfolios from the FF, we are unable to report the average number of firms on these portfolios.

Asset pricing model	GRS F-Stat	$p-value$	$SR(a)$	Avg. $\|\alpha\|$	$\sigma(\|\alpha\|)$	Avg. $AdjR^2$
25 Value and Size Portfolios						
$Mkt-r_f$(CAPM)	2.9563	0.0000	0.2772	0.19	0.0021	0.7774
FF	2.7358	0.0000	0.2679	0.12	0.0019	0.9119
$FF+\Delta_{6,6}$	2.4710	0.0001	0.2622	0.11	0.0017	0.9124
$FF+\Delta_{1,11}$	2.2518	0.0004	0.2496	0.10	0.0017	0.9124
$FF+\Gamma_{6,6}$	2.4812	0.0001	0.2606	0.11	0.0017	0.9125
$FF+\Gamma_{0,12}$	2.4908	0.0001	0.2596	0.12	0.0019	0.9121
10-industry portfolios						
$Mkt-r_f$(CAPM)	3.2087	0.0004	0.1804	0.12	0.0012	0.7364
FF	4.9893	0.0000	0.2260	0.15	0.0017	0.7547
$FF+\Delta_{6,6}$	4.5200	0.0000	0.2214	0.14	0.0015	0.7561
$FF+\Delta_{1,11}$	4.6777	0.0000	0.2246	0.14	0.0015	0.7572
$FF+\Gamma_{6,6}$	3.9730	0.0000	0.2059	0.13	0.0015	0.7551
$FF+\Gamma_{0,12}$	4.1239	0.0000	0.2086	0.13	0.0014	0.7560
$\Delta_{6,6}$ portfolios						
$Mkt-r_f$(CAPM)	7.8056	0.0000	0.2813	0.29	0.0039	0.8443
FF	8.7922	0.0000	0.2999	0.35	0.0045	0.8656
$FF+\Delta_{6,6}$	2.2929	0.0117	0.1577	0.0007	0.0009	0.9222
$FF+\Delta_{1,11}$	3.8837	0.0000	0.2047	0.12	0.0016	0.8980
$FF+\Gamma_{6,6}$	4.8021	0.0000	0.2264	0.14	0.0020	0.8984
$FF+\Gamma_{0,12}$	6.3114	0.0000	0.2581	0.21	0.0029	0.8836
$\Gamma_{6,6}$ portfolios						
$Mkt-r_f$(CAPM)	5.1587	0.0000	0.2287	0.23	0.0030	0.8707
FF	5.0176	0.0000	0.2266	0.22	0.0029	0.8799
$FF+\Delta_{6,6}$	0.8521	0.5783	0.0961	0.05	0.0006	0.9002
$FF+\Delta_{1,11}$	1.7909	0.0581	0.1390	0.11	0.0014	0.8887
$FF+\Gamma_{6,6}$	0.5879	0.8248	0.0792	0.04	0.0004	0.9182
$FF+\Gamma_{0,12}$	2.3131	0.0109	0.1563	0.11	0.0015	0.8944
$\Delta_{1,11}$ portfolios						
$Mkt-r_f$(CAPM)	5.1645	0.0000	0.2288	0.28	0.0039	0.8262
FF	5.9928	0.0000	0.2476	0.35	0.0046	0.8457
$FF+\Delta_{6,6}$	1.7263	0.0704	0.1368	0.09	0.0012	0.8883
$FF+\Delta_{1,11}$	0.5279	0.8712	0.0755	0.05	0.0005	0.9134
$FF+\Gamma_{6,6}$	3.3101	0.0003	0.1879	0.17	0.0027	0.8641
$FF+\Gamma_{0,12}$	3.0554	0.0008	0.1796	0.15	0.0023	0.8798
$\Gamma_{0,12}$ portfolios						
$Mkt-r_f$(CAPM)	4.3145	0.0000	0.2091	0.19	0.0025	0.8582
FF	3.8693	0.0000	0.1990	0.17	0.0024	0.8697
$FF+\Delta_{6,6}$	1.7693	0.0619	0.1385	0.07	0.0010	0.8835
$FF+\Delta_{1,11}$	1.3763	0.1859	0.1219	0.06	0.0008	0.8920
$FF+\Gamma_{6,6}$	1.5892	0.1046	0.1302	0.07	0.0008	0.8868
$FF+\Gamma_{0,12}$	0.7148	0.7112	0.0869	0.04	0.0005	0.9116

Table 3.15: The table reports summary statistics for time series monthly regressions of the excess returns of $\Gamma_{6,6}$, $\Gamma_{0,12}$, $\Delta_{6,6}$, and $\Delta_{1,11}$ decile portfolios, and 25 B/M-size and 10 industry portfolios on CAPM, FF, $FF+\Delta$, and $FF+\Gamma$. The regressions span from January 1930 to December 2014. Reported are the GRS statistic, the p-value of the GRS test, $SR(a)$, the average and standard deviation of $|\alpha|$ (in percentage), and the average adjusted R^2. The GRS statistic tests whether all intercepts are zero; critical GRS values for all four models are 1.97(90%), 2.41(95%), 2.84(97.5%), and 3.4(99%). The $SR(a)$ equals $(a'S^{-1}a)^{1/2}$, where a is the column vector of all the intercepts, and S is the covariance matrix of regression residuals; it corresponds to the Sharpe ratio for the intercepts (unexplained average returns) of a model.

values. The GRS statistics for the $FF{+}\Delta_{6,6}$ and the $FF{+}\Delta_{1,11}$ model are 1.726 and 0.871, implying p-values of 0.070 and 0.528 respectively. The Γ models clearly under-perform on the two portfolio sets based on momentum, with GRS statistics that are almost double those of the momentum-based models.

In contrast, the returns of the $\Gamma_{6,6}$-portfolios are appropriately described not only by the $\Gamma_{6,6}$-based model (as one would expect), but also by the Δ-based models. The lowest GRS statistic (0.588, with p-value of 0.825) is obtained for the $FF+\Gamma_{6,6}$ model, but it is followed closely by that of $FF+\Delta_{6,6}$ (0.852, with p-value of 0.578). Likewise, $SR(a)$ and $|\alpha|$ values are similar. The $FF+\Gamma_{6,6}$ model estimates for these two metrics are 0.079 and 0.04%, while $FF+\Delta_{6,6}$ model estimates equal 0.096 and 0.05%. The $FF+\Delta_{1,11}$ model's GRS statistic is 1.791, more than twice $\Gamma_{6,6}$ and $\Delta_{6,6}$'s estimates, but also below the critical value.

The $\Gamma_{0,12}$-portfolios are in general well explained by the extended models. As in the previous cases, we observe that the best performing model is the one that contains the quantity used to build the portfolios. The $FF+\Gamma_{0,12}$ model's estimates of the GRS statistic, the $SR(a)$, and average $|\alpha|$ equal 0.715, 0.087, and 0.04% respectively. The GRS statistics for the $FF+\Delta_{6,6}$, $FF+\Delta_{1,11}$, and $FF+\Gamma_{6,6}$ are correspondingly 1.769, 1.376, and 1.589.

As for explaining the book-to-value and size portfolios, the $FF+\Delta_{1,11}$ models comes first in terms of the GRS and $SR(a)$ values with 2.251 and 0.249, followed closely by the $FF+\Delta_{6,6}$ model with 2.471 and 0.261. However, one should not over-emphasise this difference because the performance of all the models except CAPM is similar. The GRS statistic is in all cases above the critical value, rejecting the null hypothesis, while the average $|\alpha|$ is actually very small (0.11%). Likewise, their average R^2s are all close to 0.9, much better than the 0.72 value of CAPM.

The statistics obtained for the industry portfolios are also less convincing. The null hypothesis of all alphas equal to 0 is rejected in all models, as the GRS statistics and the $SR(a)$ reach values well above 3 (except for CAPM) and 0.8 respectively. The average $|\alpha|$'s move above 0.13% and the average R^2's are all close to 0.75 (except for CAPM). Hence, the models are in general not as good at explaining the industry portfolios as they are for the portfolios built on quantities derived from prior returns.

Overall, we observe that Γ-based models are unable to explain Δ-based portfolios. These results, together with the evidence reported in the previous sections, suggest that, rather than an ever-present phenomenon in stock price dynamics, the acceleration effect takes place during special market regimes. Accordingly, when analysing the acceleration portfolios in the ill-suited framework of an ever present effect, Table 3.15 reports that the momentum factors are the best (or the least worst) to explain them. Reciprocally, building on the insight that the acceleration effects are transient, it is not surprising that they only partly account for the more perversive momentum effects.

3.7 Concluding remarks

We have analyzed the effect of “acceleration” of log-prices on the predictability of stock returns. By proposing a simple quantification of acceleration based on the first difference of returns, we have built Γ-strategies for portfolio investments based on the “acceleration” effect and compared them with standard momentum-based strategies. We found that Γ-strategies are on average profitable and beat momentum-based strategies in two out of three cases, for a large panel of parameterizations. Intuitively, “acceleration” corresponds to a change of momentum. Its high significance suggests deep relations with procyclical mechanisms and psychological effects, such as desensitisation or habituation and the influence of the breakdown of the status quo associated with heightened decision difficulties and increased uncertainty. The “acceleration” effect and the Γ-strategies make more explicit and help elucidate many previous reports of transient non-sustainable accelerating (upward or downward) log-prices associated with positive feedback mechanisms [Johansen et al., 2000, Sornette, 2003, Johansen and Sornette, 2010, Jiang et al., 2010, Corsi and Sornette, 2014, Leiss et al., 2015], which are now being used routinely for advanced bubble warning signals [Sornette and Cauwels, 2015].

We also constructed a new Γ-factor and compared its performance against that of momentum, via standard asset pricing tests. We find that momentum still explains better, when added to the three Fama-French factor model, the cross-section of stock returns. Combining all the provided evidence, we propose that the Γ-factor represents the existence of transient positive feedbacks influencing the price formation process, which is only prevalent during special regimes in stock market dynamics.

Chapter 4

Evaluation of price-based bubble tests

Having presented evidence that acceleration is indeed an important feature of stock prices, we inquiry to what extent the LPPLS and other price-based bubble tests can be used to study real estate bubbles. Although the content of chapter 2 hints in this direction, the evidence presented so far is insufficient, as it assumes rather than demonstrates the validity of the methodology. On top of that, the methodology employed during the Swiss real estate analysis was admittedly subjective, as we employed soft criteria to assess the quality of a bubble signal.

To directly address this question, we analyze the information content of statistical tests for bubble detection in the context of international real estate markets. We derive binary indicators from the causal application of five statistical tests to log house prices, and via logit regressions we assess the indicators' out-of-sample performance in the forecasting of tipping points of housing bubbles. Our sample comprises 17 countries, a time span of almost 40 years, and 19 bubble episodes identified directly from the literature of housing bubbles. In this sense, this chapter extends bridges to the classical econometric literature of bubbles, as we connect exuberant price dynamics with the peak of deviations from the fundamental value. In our assessment, three of the indicators - two based on the identification of super-exponential trends and one based on the scaled ratio of the sum of squared forecast errors - exhibit significant out-of-sample results. Combining the indicators via simple threshold-rules yields the most robust results.

This chapter is an edited version of [Ardila et al., 2016b], of which I am first author.

4.1 Introduction

With the goal of developing early warning signals of future crises, many economic indicators have been analyzed in the aftermath of the 2007-2009 financial crisis. The list includes measures such as credit to GDP growth, the debt service ratio, and gaps of property prices [Drehmann and Juselius, 2014]. Attempts to estimate large scale deviations from equilibrium relations in housing markets often rely on ad-hoc rules that follow the application of the Hodritt-Presscott filter [Jordà et al., 2015]. Contrastingly, the use of formal statistical tests based on precise definitions of what a large deviation (i.e. a bubble) is has been very limited. In addition, the studies that actually apply these techniques tend to presuppose rather than to examine the validity of the tests [Pavlidis et al., 2013]. Thus, the question of whether statistical tests for bubble detection can be used to practically identify bubbles in housing markets remains largely unexplored[1].

The assessment of these indicators is relevant not only within the context of early warning systems, but also for any attempt to empirically study the causes and consequences of bubble regimes. An inappropriate definition of bubbles or a flawed bubble identification strategy might rule out, even in hindsight, valid and alternative interpretations of the data. If crashes are for example used to identify bubbles, bubble regimes are likely to be associated with poor economic performance, as only episodes with crashes, and potentially very evident ones, will enter the sample of the study (a typical case of selection bias).

This paper analyzes the information content of statistical tests for bubble detection in the context of international real estate markets. We evaluate the ex-ante performance of five different indicators derived from three types of statistical tests for bubble detection: 1) two tests based on the identification of super-exponential trends; 2) one test based on the detection of mildly explosive regimes; 3) two tests based on the scaled ratio of forecasting errors. We mainly compare the indicators individually, though towards the end of the paper we also analyze simple rules to combine them.

Following the literature on early warning signals (Drehmann and Juselius [2014]; Edison [2003]), we study the performance of the bubble indicators in the forecasting of tipping points of real estate bubbles and systemic financial crises. We run logit regressions having the indicators as dependent variables, and evaluate the performance of the regressions via the statistical significance of the area of the receiver operating characteristic curve (AUROC statistic). Our dependent variable consists of a binary indicator for the dates of the bubbles' tipping points and systemic banking crises. The dates of the bubbles' tipping points were obtained from a literature review on drivers of house prices. In total, we reviewed 19 single-country, 3 bi-country, and 3 multi-country studies to identify the periods of extreme overvaluation. We argue that our dating approach is more objective than any other technical rule previously employed in the literature, as such rules do not take into account the particularities of each housing market and depend on strong (and often symmetric) corrections to identify the end of a bubble period. The dates of systemic banking crises are those reported by Laeven and Valencia [2008], Valencia and Laeven [2012] and Drehmann et al. [2012] for the countries in our sample.

We emphasize that, although we employ the methodologies of the literature on early warning signals, our aim, more generally, is to contribute to the understanding of the tools

[1] Although the Federal Reserve Bank of Dallas regularly applies the Phillips test to international markets, there are no formal assessments regarding the performance of this procedure in forecasting the tipping point of past bubbles [Mack et al., 2011].

available to identify and analyze bubble regimes. Thus, we are not working on an early warning signal system because this would require endogenizing the cost of false positives as well as false negatives, which necessarily depends on the applications and circumstances.

Overall, our results suggest that the statistical tests for bubble detection contain significant in-sample and out-of-sample forecasting power. All tests present significant in-sample performance, whereas three out of the five indicators present significant out-of-sample forecasting power. We contribute to the literature on early warning indicators by documenting and comparing the forecasting skill of these statistical methods for bubble detection. Our results also emphasize the importance of identifying and understanding the consequences of periods of explosive price development, as they directly link statistical exuberance to historic episodes of strong deviations from fundamental factors. In addition, the fact that the supLPPLS and supPL contain significant value relative to the other approaches highlights the need to further employ diagnosis that are sensitive to the super-exponential growth pattern, which is increasingly supported as being a key stylized fact of financial and housing bubbles (Hüsler et al. [2013]; Leiss et al. [2015]; Gjerstad and Smith [2014]). Finally, our results contrasts with those of Bourassa et al. [2016], who found that a price-to-rent approach is a reliable signal for a bubble. We find that the ex-ante power of the statistical tests, when applied to price-to-income and price-to-rent ratios might not in general be sufficient to ex-ante identify the bubble regime.

This paper is organized as follows: section 4.2 describes the sample set and the chronology of bubbles. Section 4.3 presents the bubble detection tests from which our indicators are derived. Section 4.4 explains the methodology to construct and assess the indicators. Section 4.5 discusses the results. Section 4.6 presents the conclusions.

4.2 Database and the chronology of bubbles

Our sample comprises 18 countries, with quarterly data between 1975Q1 and 2013Q3[2]. The economies were selected based on relevance and availability. In addition to the list of house prices from [Mack et al., 2011], we included real house prices from two countries from the Asia-pacific region: Thailand and South Korea. These two countries are important as Asian countries suffered a systemic financial crises between 1997 and 1998, which stopped the real estate booms taking place in these economies. Thus, their real estate cycles remained disconnected from the last housing boom of US and Europe, and significantly enrich the information in our sample. All series are in real terms (deflated by CPI), and normalized by their respective value in 2005Q1 to ensure comparability of the units.

4.2.1 Bubble tipping points

In order to identify the bubble tipping points, we surveyed the literature on drivers of house prices. Empirical detection of bubbles poses several challenges as there is no general

[2]Excluding Thailand, for which the time series starts in 1990Q4.

theoretical or empirical definition of a bubble [Case and Shiller, 2003]. Furthermore, hindsight might reveal or obscure the presence of bubble episodes. A crash might be regarded as evidence in favor of a bubble. On the contrary, absence of a strong correction following a protracted boom might lead scholars to incorrectly conclude that a period of exuberance was indeed justified by unrecognized fundamental factors.

Hence, to conduct this study and contrary to the vast majority of indicators used in the literature of macroeconomic crises (see e.g. Jordà et al. [2015]), we opt to identify bubbles by compiling the results from 25 different studies based on a fundamental analysis of prices (19 single-country, 3 bi-country, and 3 multi-country studies). These studies are typically based on inverted demand equations or life cycle consumptions models [Anundsen, 2015], and include variables such as interest rate, property tax, housing stock, disposable income, among others. We argue that our identification strategy provides us with a more objective benchmark against which to evaluate the performance of the indicators as fundamental-based studies take into consideration the internal macroeconomic conditions of the countries, especially those studies associated specifically to a single country.

Admittedly, our survey is far from perfect as bubbles remain a contentious concept. The identification strategy is based only on articles that interpret deviations from fundamentals as signs of bubbles. Their primary assumption - unless expectations are explicitly modeled - is that the lagged appreciation unexplained by the fundamental variables represents speculative pressures. However, it is fair to acknowledge that many studies identify such deviations without explicitly referring to speculative or exuberant periods. An alternative but not necessarily incompatible explanation is that real house prices have a tendency to overshoot following a shock to the market. As a result, observed house prices are likely to deviate from the long-run equilibrium level. Another issue typically raised is that the identification strategy might be subject to misspecification, as the econometrician has no way to know the "true" model [Flood and Hodrick, 1990].

The studies seldom report explicitly the peak of the bubble. Therefore, we chose the peak of the overvaluation period as the variable of analysis. In total, we identified 19 housing bubbles over the two long real estate cycles that the world has experienced plus the housing bubbles in Asia. Table 4.1 presents our complete list of tipping points matched to their original sources. The table includes the real log returns four years before and after the tipping point (Δ_{-4} and Δ_4 respectively), as well as the ratio of these two quantities ($\frac{\Delta_4}{\Delta_{-4}}$). The vertical lines in the subplots in the table represent the peaks of the bubbles.

A salient feature is the asymmetric behavior of booms and bursts, in which the former tend to be more pronounced than the latter. Only five countries, Ireland(2007), Japan, Sweden, Switzerland, and the US, exhibit symmetric boom-bust behaviors. Four bubbles, those in Canada, France, the Netherlands, and UK(2005), present almost constant price development four years after the peak when computed in real terms. House prices in Ireland(2002) quickly recovered after the bubble-burst episode. In the Netherlands and Denmark, prices continued rising after the peak of the overvaluation so that the peak does not coincide with a local maximum in the price level, illustrating the fact that adjustments

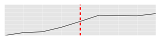
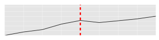
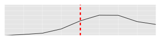
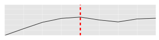
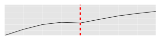
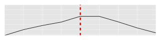
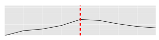
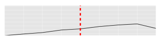
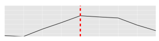
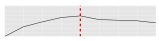
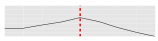
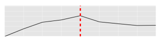
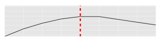
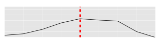
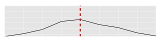
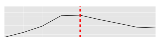
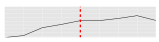
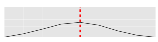
Country	Peak t_p	Δ_{-4}	Δ_4	$\frac{\Delta_4}{\Delta_{-4}}$	$\ln p_t$ $[t_p - 4, t_p + 4]$	Reference
Australia	2003	0.44	0.14	0.31		Glindro et al. [2011] Berry and Dalton [2004]
Canada	2008	0.31	0.1	0.29		Walks [2014]
Denmark	2006	0.33	-0.08	-0.25		Sørensen [2013] Dam et al. [2011]
France	2008	0.35	-0.02	-0.06		Antipa and Lecat [2010]
Ireland	2002	0.42	0.39	-0.93		Connor et al. [2012] Stevenson [2008] Hott and Monnin [2008]
	2007	0.39	-0.33	-0.87		
Japan	1991	0.3	-0.3	-0.97		Hott and Monnin [2008] Barsky [2009]
Netherlands	2006	0.08	0.01	0.075		Hott and Monnin [2008] Francke et al. [2009]
Norway	1989	0.43	-0.32	-0.75		Anundsen and Jansen [2011] Jacobsen and Naug [2005]
S. Africa	2008	0.47	-0.16	-0.34		Das et al. [2011]
S. Korea	1991	0.23	-0.41	-1.74		Glindro et al. [2011] Kim and Min [2011]
Spain	1992	0.42	-0.19	-0.46		Ayuso and Restoy [2006] Neal and García-Iglesias [2013] Antipa and Lecat [2010]
	2007	0.42	-0.18	-0.42		
Sweden	1990	0.29	-0.33	-1.13		Hort [1998] Andreas Claussen [2013] Sørensen [2013]
Switzerland	1990	0.30	-0.29	0.97		Hott and Monnin [2008] Hott [2012]
	2013	0.19	-	-		
UK	1990	0.5	-0.29	0.58		Muellbauer and Murphy [1997] Cameron et al. [2006] Black et al. [2006] Hott and Monnin [2008]
	2005	0.43	0.01	0.14		
US	2006	0.22	-0.22	-1		Hott and Monnin [2008] Mikhed and Zemčík [2009] Anundsen [2015]

Table 4.1: Summary of 19 bubble peaks in 15 countries according to studies surveyed. The table shows the years of the peaks, the real log returns four years before and after the peaks (Δ_{-4} and Δ_4 respectively), the real house price development around these years, and the ratio $\frac{\Delta_4}{\Delta_{-4}}$. The vertical lines in the small figures denote the bubble peaks. No peaks are reported for Germany, Italy, and Thailand as Nobili and Zollino [2012] and Glindro et al. [2011] concluded that prices in these countries have evolved (mostly) according to the fundamentals.

in fundamental factors, such as a drop in the interest rate, might also align prices with fundamentals and prevent a visible correction from happening. The existence of such patterns challenges identification strategies based on local maxima. Bubbles do not always suddenly burst, the end phase of the bubble can follow very different dynamics, and as Brunnermeier and Oehmke [2013] have remarked, we do not know enough yet about how and why prices evolve the way they do once the expansion phase of the real estate cycle has passed.

4.2.2 Crises

The dates of systemic crises that we take are the years with a beginning of a systemic banking crises, reported by Laeven and Valencia [2008], Valencia and Laeven [2012] and Drehmann et al. [2012] for the countries in our sample. In case of discrepancies, we choose the earliest reported year among these two sources. Table 4.2 presents the actual values. Canada and South Africa do not appear in the table as there are no crises reported for these two countries.

Country	**Year crisis**	**Country**	**Year crisis**
Australia	1989	Norway	1990
Denmark	2008	S. Korea	1997
Germany	2007	Spain	2008
France	2008	Sweden	1991, 2008
Ireland	2008	Switzerland	2008
Italy	2008	Thailand	1983, 1997
Japan	1997	UK	1990, 2007
Netherlands	2008	USA	1988, 2007

Table 4.2: Systemic banking crises. The table reports the years with a beginning of a systemic banking crises. For the countries in our sample, we consolidated the dates reported by Laeven and Valencia [2008], Valencia and Laeven [2012] and Drehmann et al. [2012]. In case of discrepancies, we chose the earliest reported year. Canada and South Africa do not appear with crisis events.

4.3 Bubble detection tests

In this section, we present several bubble detection tests proposed in the literature of asset bubbles. We are interested in identifying a bubble in a log prices time series $\ln p_t$ with $t = 1, ..., [\tau T], [\tau T]+1, ..., T$, $\tau \in (0,1)$ and $[\tau T]$ denoting the greatest integer smaller than or equal to τT. $[\tau T]$ corresponds to the starting period in which the bubble is detected.

The tests can be motivated within the theory of rational expectations bubbles (REB). The departure point of REB is the familiar no arbitrage condition for the price of an asset,

$$P_t = \frac{1}{1+R} E_t(P_{t+1} + D_{t+1}) \tag{4.1}$$

where P_t is the asset price at time t, D_t is the dividend received from the asset for ownership between $t-1$ and t (i.e. the inputed rent in a real estate context), $R > 0$ is the discount rate, and $E_t(\bullet)$ denotes the expectation conditional on the information at time t. A log linear approximation of equation 4.1 and forward iteration yields (Campbell and Shiller [1988]; Phillips et al. [2011]),

$$p_t = p_t^f + b_t \ , \tag{4.2}$$

where

$$p_t^f = \frac{\kappa - \gamma}{1-\rho} + (1-\rho)\sum_{t=1}^{\infty} \rho^i E_t(d_{t+1+i}) \tag{4.3}$$

$$b_t = \lim_{i \to \infty} \rho^i E_t(p_{t+i}) \tag{4.4}$$

$$E_t(b_{t+1}) = \frac{1}{\rho} b_t = (1 + \exp(\overline{d-p}))b_t \tag{4.5}$$

with $p_t = \ln P_t$, $d_t = \ln D_t$, $\gamma = \ln(1+R)$, $\rho = 1/(1+\exp(\overline{d-p}))$, $\overline{d-p}$ equals the average log dividend price ratio, and $\kappa = -\ln\rho - (1-\rho)\ln(\frac{1}{\rho} - 1)$. p_t^f is called the fundamental component of the price, and b_t the rational bubble component. In the absence of a bubble (i.e. $b_t = 0$), $p_t = p_t^f$ and p_t is solely determined by the dividends. Conversely, if $b_t \neq 0$, equation 4.5 implies the following process,

$$b_t = \frac{1}{\rho} b_{t-1} + \epsilon_{b,t} = (1+g)b_{t-1} + \epsilon_{b,t} \ , \qquad E_{t-1}(\epsilon_{b,t}) = 0 \tag{4.6}$$

where $g = \frac{1}{\rho} - 1 = exp(\overline{d-p})$ is the growth rate of the natural logarithm of the bubble and $\epsilon_{b,t}$ is a martingale difference. Since $g > 0$, equation 4.5 implies that b_t is a sub-martingale, and equation 4.6 states that bubbles, if they are present, should manifest explosive characteristic in log prices.

The rational expectations bubble theory has several drawbacks. Models based on REB provide conditions for an economic equilibrium with bubbles, but they do not explain the dynamics that lead to the boom. In other words, bubbles in these models do not emerge, but they exist or appear randomly and it is unclear when/under what circumstances a market moves from a stable to an unstable regime. Rational expectation bubbles, if they exist, must be positive, infinitely lived, and hence, require assets with infinite maturity [Blanchard and Watson, 1982b]. These properties are at odds with empirical evidence such as, for example, the bubble on Chinese call warrants discussed by Palan [2013]. Giglio et al. [2016] exclude infinitely lived bubbles on the housing markets of Singapore and the U.K [3]. Scheinkman and Xiong [2004] argue that models of rational bubbles are incapable of explaining the increase in trading volume that is typically observed in the historic bubble periods. Lux and Sornette [2002] demonstrate that exogenous rational bubbles are not compatible with some of the stylized facts of financial data at a very elementary level.

While these limitations are serious, there exists versions of REB that attempt to circumvent them, such as those developed in [Johansen et al., 2000] and [Lin and Sornette, 2013] for instance. Sornette [2002] proposed a resolution of the objections raised by Lux

[3]According to the authors, these markets give the best chances of detecting a bubble in the data.

and Sornette [2002]. We thus use below the REB theory as a common framework to illustrate the main differences among the bubble tests. At this point, it is worth stressing that the analysis of bubbles remains incomplete, and their research requires pragmatic choices as well as a constant dialogue between any proposed theory and the supporting empirical studies.

4.3.1 Super-exponential based procedure

The first type of bubble detection test is based on the identification of a transient super-exponential trend in the dynamics of the log prices. This kind of dynamics was first proposed on the basis of empirical observations in [Sornette et al., 1996, Feigenbaum and Freund, 1996]. It was later justified by Johansen et al. [2000] and Johansen et al. [1999] within the framework of rational expectation model of bubbles. From a theoretical view point, the authors argue that the no arbitrage condition, together with a hierarchical self-reinforcing organization of the market, and the need of investors to be compensated for the risk of the crash, generates a power law finite-time singular price dynamics as the bubble approaches its end. As a result of the positive feedbacks, such super-exponential dynamics is unsustainable as it ends in a finite time singularity, which signals a change of regime (the end of the bubble).

Statistically, a bubble can thus be identified via a non-nested hypothesis test of model selection [Davidson and MacKinnon, 1981]. If there is enough evidence to reject the null hypothesis of a non-explosive process in favor of the alternative super-exponential trend, the bubble hypothesis can be supported. Specifically, we compare a stationary $AR(1)$ process in the log returns against $\Delta \ln \hat{p}_{t sexp}$, the log returns predicted by a fitted super-exponential trend in the subsample between $[\tau T]$ and T:

$$\Delta \ln p_t = \rho \Delta \ln p_{t-1} + \alpha_{sexp} \Delta \ln \hat{p}_{t sexp} + \epsilon_t, \text{for } t = [\tau T], [\tau T] + 1, ..., T \tag{4.7}$$

where ϵ_t is a white noise process. The null hypothesis of no bubble after $[\tau T]$ period is rejected if the t-statistic $\hat{t}_{\alpha_{sexp}}$ for the estimate of α_{sexp} exceeds the corresponding critical value[4]. When the starting date of the super-exponential trend is not known, the statistic takes the following form:

$$\text{supSEXP}(\tau_0) = \sup_{\tau \in [0, \tau_0]} \hat{t}_{\alpha_{sexp}} \tag{4.8}$$

where τ_0 determines the interval in which the super exponential trend is tested.

We employ two different super-exponential specifications to obtain $\Delta \ln \hat{p}_{t sexp}$. First, the fitted values given by the log periodic power law singularity (LPPLS) model:

$$\ln p_{t sexp} = A + (t_c - t)^m [B + C \cos(\omega \ln (t_c - t) - \phi)] \tag{4.9}$$

where $0 < m < 1$, $B < 0$, $3 < \omega < 15$, and $|C| (\omega^2 + m^2)^{1/2} \leq |B| m$ [Filimonov and

[4] As a remark, $\rho \geq 1$ would also suggest super-exponential behavior, but we chose not to test this alternative to focus on the super-exponential trend.

Sornette, 2013]. t_c corresponds to the non-random time of the termination of the bubble. As calibration of equation 4.9 on quarterly data can be difficult due to the low frequency of the volatility of house prices and the relatively large number of parameters (7 in total, 3 nonlinear, 4 linear after the reformulation in [Filimonov and Sornette, 2013]), we also explore a simplification of the LPPLS model that excludes the log periodic oscillations (hereafter PL model):

$$\ln p_{tsexp} = A + B(t_c - t)^m \tag{4.10}$$

where as in equation 4.9, $0 < m < 1$ and $B < 0$. These last two conditions ensure that the instantaneous expected return diverges at t_c. In practice, it does not of course, but the hypothesis is that the average price trajectory can be approximated over a time interval until close to its turning point by such a process with increasing returns.

4.3.2 The Kim-Busetti-Taylor statistic

In the case where d_t is stationary (i.e. $I(0)$), as it is observed for the inputed rent of a real estate asset, absence of bubbles implies by equation 4.2 that Δp_t should also be stationary. Hence, rejection of the stationarity of Δp_t in favor of the non-stationarity hypothesis can be interpreted as evidence of bubbles. In this context, Homm and Breitung [2012] employed the tests of Kim [2000] and Busetti and Taylor [2004] for a bubble detection setup.

Kim [2000] and Busetti and Taylor [2004] proposed independently a statistic for testing the null hypothesis that a stationary time series switches to a process with higher persistence (e.g. from $I(0)$ to $I(1)$). Their focus is on the Gaussian unobserved components model,

$$y_t = \beta_t + \mu_t + \epsilon_t \quad \text{ for } t = 1, 2, ..., T \tag{4.11}$$

$$\mu_t = \mu_{t-1} + 1(t > [\tau T]\, \eta_t) \quad \text{ for } \tau \in (0, 1) \tag{4.12}$$

where $1(\bullet)$ is the indicator function, ϵ_t and η_t are mutually independent mean zero IID Gaussian processes with variance σ^2 and σ_n^2 respectively, and β_t is a deterministic component that we take as constant. Equations 4.11 and 4.12 describe a process that is stationary up to $[\tau T]$, but is $I(1)$ after the break if and only if $\sigma_n^2 > 0$. Consequently, a test for stationarity against the non-stationary hypothesis can be framed as testing the null hypothesis

$$H_0 : \sigma_n^2 = 0 \tag{4.13}$$

against the alternative,

$$H_1 : \sigma_n^2 > 0 \tag{4.14}$$

A test statistic that rejects (4.13) for large values is,

$$KBT_\tau = \frac{[(1-\tau)T]^{-2} \sum_{t=[\tau T]+1}^{T} (\sum_{i=[\tau T]+1}^{t} \hat{\epsilon}_{1,i})^2}{[\tau T]^{-2} \sum_{t=1}^{[\tau T]} (\sum_{i=1}^{t} \hat{\epsilon}_{0,i})^2} \tag{4.15}$$

where $\hat{\epsilon}_{0,i}$ are the OLS residuals from the regression of y_t on an intercept in $t = 1, ..., [\tau T]$, and $\hat{\epsilon}_{1,i}$ are the OLS residuals from the regression of y_t on an intercept in $t = [\tau T]+1, ..., T$. Equation 4.15 is a Chow-type test and can be interpreted as the scaled ratio of the sum of squared forecast errors.

Although Homm and Breitung [2012] adapted the KBT_τ statistic to be applicable directly on log prices, we opt to use the statistic given by equation 4.15 to test stationarity of the returns in order not to rely on the forecast associated with the random walk assumption. Thus, under the alternative hypothesis, the return process switches at $[\tau T]$ from a stationary process to a process with persistent returns. If the returns are persistent, the corresponding time series exhibits explosive behavior. When the breakpoint $[\tau T]$ is unknown, the statistic can be expressed as

$$\text{supKBT}(\tau_0) = \sup_{\tau \in [\tau_0, 1-\tau_0]} KBT_\tau \tag{4.16}$$

and rejects the null hypothesis of no bubble for large values.

4.3.3 The Busetti-Taylor statistic

A second test also adapted by Homm and Breitung [2012] for a bubble setup is the Busetti-Taylor statistic. The Busetti-Taylor statistic takes again the two cases 4.13 and 4.14 as the null and the alternative hypotheses respectively. When τ is known, the statistic corresponds to the locally best invariant test against changes in the order of integration under the assumption of Gaussianity. Namely, it has maximum local power close to the null hypothesis under invariant transformations.

We deviate again from Homm and Breitung [2012] and proceed as with the KBT statistic by implementing directly the test as originally proposed in [Busetti and Taylor, 2004],

$$BT_\tau = \hat{\sigma}^{-2}(T - [\tau T])^{-2} \sum_{t=[\tau T]+1}^{T} (\sum_{j=t}^{T} \hat{\epsilon}_j)^2 \tag{4.17}$$

where $\hat{\epsilon}_j$ are the OLS residuals from the regression of $\Delta \ln p_t$ on an intercept, and $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t^2$. When the breakpoint is not known, the Busetti-Taylor statistic takes the following form

$$\text{supBT}(\tau_0) = \sup_{\tau \in [0, 1-\tau_0]} BT_\tau \tag{4.18}$$

and, as in the previous statistics, rejects the null hypothesis of no bubble for large values.

4.3.4 The Phillips statistic

Phillips and Yu [2011] pointed out that, since $g > 0$ in equation 4.6, if $b_t \neq 0$, equation 4.2 implies that p_t will also be explosive irrespective of whether d_t is an integrated process or a stationary process. As a direct way to test for bubbles, they thus proposed to examine evidence of explosive behavior in p_t via a time-varying AR model and an econometric procedure based on recursive Dick Fuller (DF) t-statistics. The time-varying AR model reads

$$p_t = \rho p_{t-1} + \epsilon_t \ , \tag{4.19}$$

where ϵ_t is a white noise process with $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$, and $p_0 = c < \infty$. Under the null hypothesis, the process starts as a random walk (i.e. $\rho = 1$) but under the alternative, ρ becomes larger than 1 and the process changes to an explosive process at an unknown time $[\tau T]$. Thus, the procedure consists of testing directly for explosive behavior via right-tailed unit root test for certain sub-periods of the data. Let $\hat{\rho}_t$ denote the OLS estimator of ρ and $\sigma_{\rho,\tau}$ the usual estimator for the standard deviation of $\hat{\rho}_\tau$ using the subsample $\{p_1, ..., p_{[\tau T]}\}$. The forward recursive DF tests is given by,

$$\text{supDF}(\tau_0) = \sup_{\tau \in [\tau_0, 1]} DF_\tau \tag{4.20}$$

with $DF_\tau = \frac{\hat{\rho}_\tau - 1}{\hat{\sigma}_{\rho,\tau}}$. The test rejects the null hypothesis of no bubble for large values of supDF(τ_0)[5].

4.4 Ex-ante identification and evaluation of bubbles

4.4.1 Ex-ante identification strategy

We apply the procedures described in section 4.3 to test for bubbles in the log of real house prices for every country in our sample. The supBT, supKBT, and supDF tests are applied on rolling windows with length of 60 quarters (15 years), correcting for serial correlation in the residuals, and setting $\tau_0 = 0.2$. These values for τ are not far from the 0.1 value typically used in other studies, and ensure that all of our estimations are based on at least 12 quarters (3 years). The super-exponential-trend tests employ a rolling window with a maximum length of 60 quarters, setting $\tau_0 = 0.3$. Both types of nonlinear trends (the LPPLS and the PL) are estimated by minimizing the sum of squared residuals using the procedure described in [Filimonov and Sornette, 2013]. Given the relatively small sample size of our estimations, we use Monte Carlo simulations with $15,000$ replications to calculate the critical values of all tests. Results of these simulations are shown in table 4.3.

A bubble signal by test j is triggered at time t when the statistic is significant at a 5% α-level:

$$BI_t^j = \begin{cases} 1 & \text{p-value}_t^j < \alpha \text{ and } \Delta^y \ln p_t \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.21}$$

[5] Phillips and Yu [2011] also developed an asymptotic distribution theory for mildly explosive processes in order to date the emergence or collapse of the bubbles. As we focus on bubble detection, we refrain from presenting that part of their contributions.

Level	90%	95%	97.5%	99%	99.9%
supLPPLS	*	2.19	3.2	4.27	6.67
supPL	1.66	2.18	2.59	3.078	4.47
supKBT	11.54	15.58	20.39	26.75	51.5
supBT	1.17	1.46	1.76	2.11	3.00
supDF	1.49	1.91	2.32	2.89	5.52

Table 4.3: Critical values for the different bubble tests. The reported values correspond to Monte Carlo simulations for samples of size 60 over 15′000 repetitions. The * indicates that more than 90% of the estimations did not fulfill the LPPLS constraints, so critical values do not apply for this level.

where $\Delta^y \ln p_t > 0$ denotes the yearly log returns and are set to be positive to ensure that alarms are triggered only while prices are rising.

Figure 4.1 presents the resulting bubble signals for the housing markets of Australia and the UK[6]. For Australia (panel 4.1a), timely bubble signals were triggered prior to 2004 in three cases (supDF, supPL, supLPPLS), while alarms not associated with bubbles were also detected around 1990 in all but one of the indicators (supLPPLS). For UK (panel 4.1b), three out of the five indicators (supBT, supPL, supLPPLS) timely identify the bubble prior to the 1990 tipping point while four detected a bubble prior to the 2005 tipping point (supDF being the exception); false alarms were triggered during the second half of the 90s in two of the indicators (supDF and supKBT).

4.4.2 Evaluation methodology

Following Schularick and Taylor [2012], we analyze the relationship between the bubble indicators and our dependent variable using yearly logit regressions. We use the following specification:

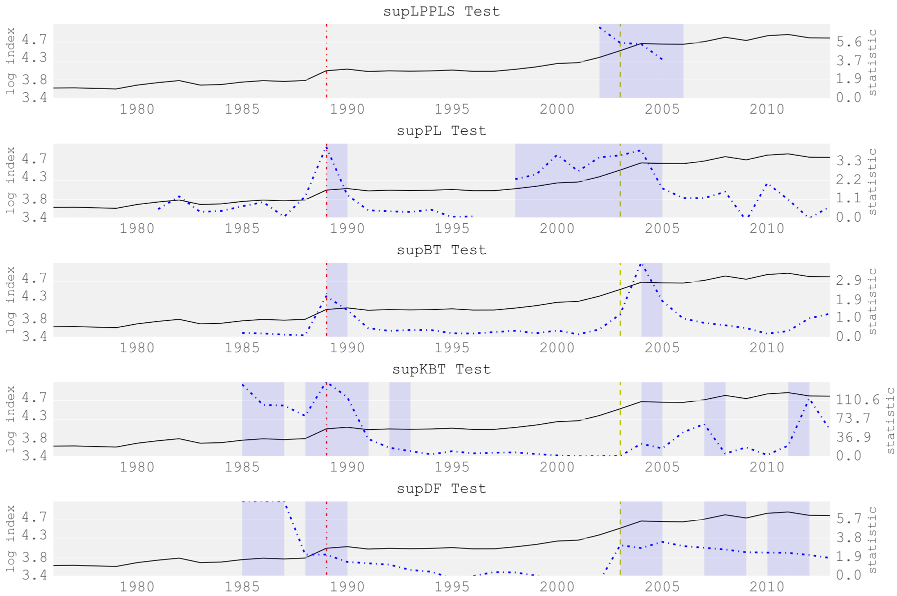
$$y_{i,t}^{bubble} = \Lambda\left(\sum_{k=0}^{MAXLAG} \beta_k BI_{i,t-k-1}^{j}\right) \tag{4.22}$$

where i, k, j denotes indices for the country, the time lag, and the bubble indicator respectively, $\dot{\Lambda()}$ **is the logit function** , $MAXLAG$[7] is set to 3, and $y_{i,t}^{bubble} = 1$ if there was a tipping point at time t in country i (and 0 otherwise). The quality of the models is assessed via the significance of the Area under the Receiver Operating Characteristics Curve (AUROC). The ROC curve characterizes the quality of a forecast system by describing the system's ability to anticipate correctly the occurrence or non-occurrence of predefined events [Mason and Graham, 2002]. The AUROC is increasing with the predictive power of the indicator and lies between 0 and 1.

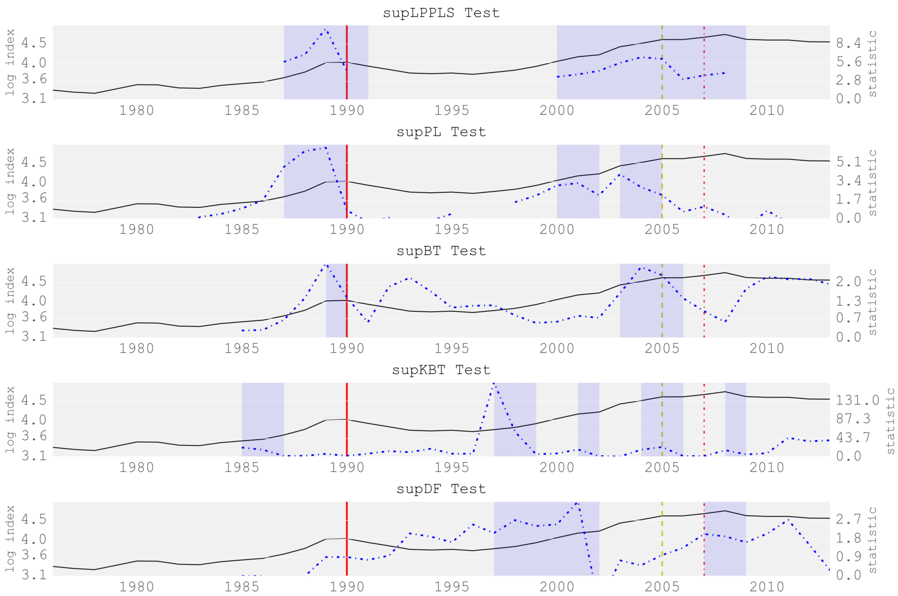
To obtain the ROC for a probabilistic forecast system, the probability at which a positive warning is issued is varied across a range of thresholds. For each threshold, the

[6] Signals for the other countries are available in the supplementary material of the paper.

[7] We explored with values between 2 and 5 and the results did not qualitatively differ.



(a) Australia



(b) UK

Figure 4.1: Bubble indicators for Australia and the UK. The indicators, active in the shaded areas, are triggered when the corresponding statistic exceeds the 95% critical values. The solid line depicts (left y-axis) the country's real log house index on which the tests are applied. The dash lines (right y-axis) correspond to the calculated statistics. The vertical dashed yellow and red lines denote the tipping dates of the bubbles and financial crises respectively, according to tables 4.1 and 4.2. The solid red line corresponds to a year with a crisis and a bubble peak.

correspondence between the forecast and the observation is determined. This correspondence is described by a two component vector defined by the rate of true positive and false negative. Each pair of rates gives a coordinate in the ROC space that defines the ROC curve. It can be shown that when the forecast system has skill, the area under the curve will exceed 0.5 (0.5 being the case of an informative indicator). Furthermore, the significance of the ROC area can be objectively assessed via a rescaled Mann-Whitney U statistic, which tests the significance of forecast event probabilities for cases where events actually occur. For large samples, the significance of these measure can be assessed using a normal-distribution approximation:

$$U = AUROC \cdot \frac{e'e}{2} \sim N(\mu = \frac{e'e}{2}, \sigma^2 = \frac{e'e(n+1)}{12}) \tag{4.23}$$

where n is the total number of forecasts, e the total number of events (i.e. bubble tipping points), and $e' = n-e$ the total number of non-events. The null hypothesis of no forecasting skill is rejected for large values of U. As discussed by Mason and Graham [2002], the errors of these approximations tend to be small for relatively large samples (a large number of forecasts).

We study the AUROC obtained in-sample for the different bubble indicators. In addition, we study two types of out-of-sample performance: by country and by period. Out-of-sample by country calibrates the model multiple times, each time leaving one country out of the calibration in order to use it to assess the models' performance. Out-of-sample by period dynamically calibrates the models using data up to the t_n period and tests the performance in year t_{n+1} for $t_n = 1998...2012$.

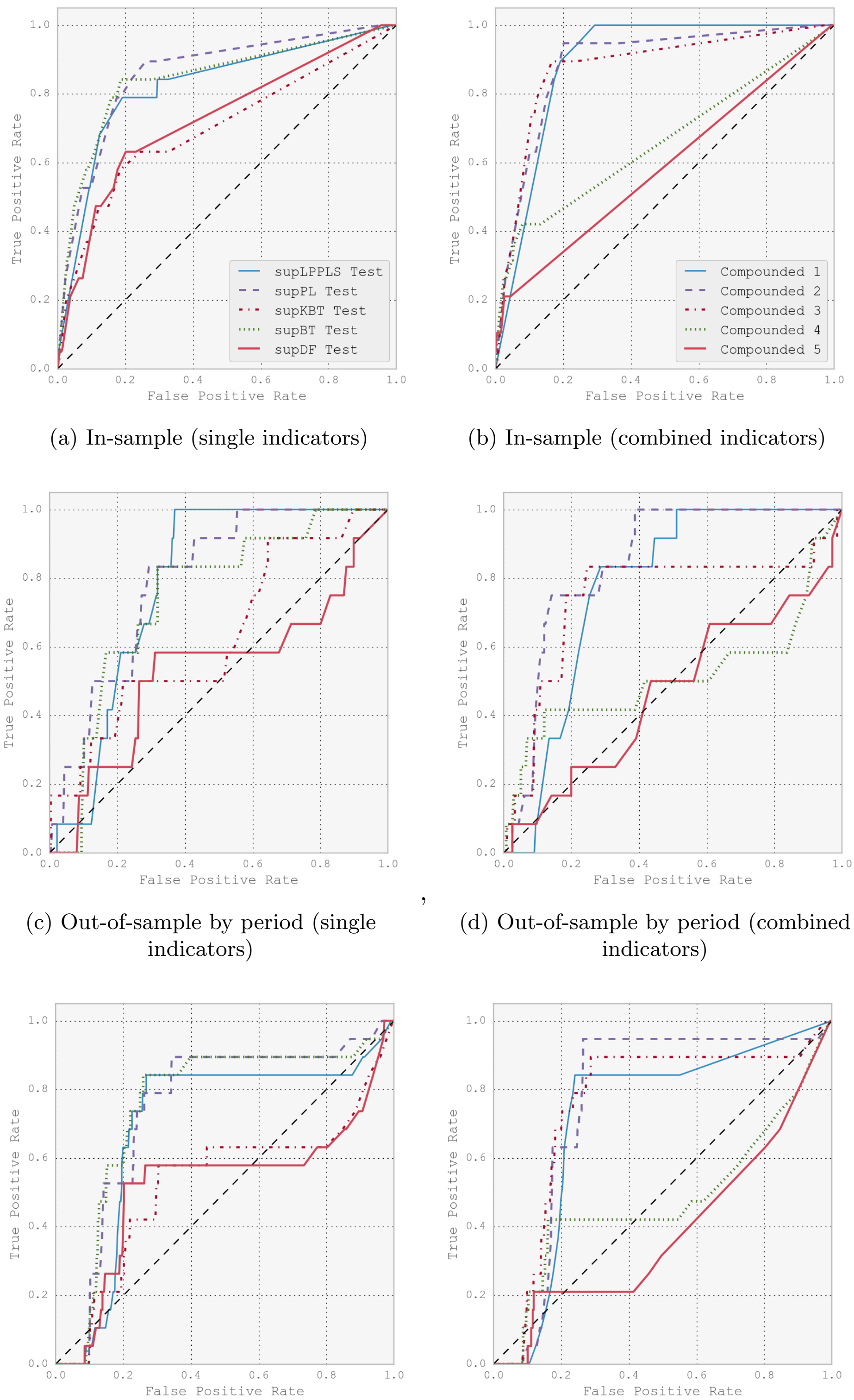
4.5 Empirical results

4.5.1 In-sample and out-of-sample performance

Figure 4.2 summarizes the main results, also reported numerically in table 4.4. AUROCs are expressed in percentage terms, while the respective U-statistic appears in parentheses.

The in-sample AUROC of the different indicators is large with high statistical significance. The first row of table 4.4 shows that all AUROCs are above 0.7, and significant at the 99% level. The supPL test exhibits the highest performance, followed by the supBT and supLPPLS tests. The performance of the other two indicators, supKBT and supDF, is somewhat lower but still significantly positive. In-sample results are thus encouraging and suggest that the indicators contain relevant information associated to the end of the bubble. In order to test whether they translate to out-of-sample performance, we move now to the out-of-sample analysis.

Panels 4.2c and 4.2e show the results for the out-of-sample by period and by country tests respectively. Not surprisingly, the performance of all regressions drops visibly, from an average of 0.79 to an average of 0.69 (out-of-sample by period) and 0.65 (one-country-out). Nevertheless, three of the indicators contain significant predictive power in the



(a) In-sample (single indicators)

(b) In-sample (combined indicators)

(c) Out-of-sample by period (single indicators)

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(d) Out-of-sample by period (combined indicators)

(e) Out-of-sample by country (single indicators)

(f) Out-of-sample by country (combined indicators)

Figure 4.2: Receiver operating characteristic (ROC) curves derived from bubble logit regressions (4.22). The independent variables are the bubble indicators derived from log prices (see equations (4.21) and (4.24)). The dependent variable $y_{i,t}^{bubble}$ is set to 1 if a bubble peaked at time t in country i.

Single bubble indicators (based on log prices)					
Test:	supLPPLS	supPL	supKBT	supBT	supDF
In-sample	82.380***	85.994***	70.290**	84.529***	72.831***
	(4.811)	(5.348)	(3.015)	(5.131)	(3.392)
Out-of-sample by period	78.185***	79.423***	62.908	74.306**	53.306
	(3.305)	(3.450)	(1.514)	(2.850)	(0.388)
Out-of-sample by country	69.561**	74.428***	52.727	75.192***	52.386
	(2.910)	(3.634)	(0.406)	(3.748)	(0.355)
Single bubble indicators (based on log price-to-income)					
In-sample	74.306***	78.536***	60.353	73.776***	53.481
	(3.608)	(4.236)	(1.537)	(3.529)	(0.517)
Out-of-sample by period	64.231*	64.615*	65.641*	69.455*	58.109
	(1.667)	(1.712)	(1.832)	(2.278)	(0.950)
Out-of-sample by country	58.868	64.203*	49.123	60.988	33.707
	(1.318)	(2.111)	(−0.130)	(1.633)	(−2.422)
Single bubble indicators (based on log price-to-rent)					
In-sample	82.867***	79.208***	60.138	74.210***	59.265
	(4.879)	(4.336)	(1.505)	(3.594)	(1.375)
Out-of-sample by period	77.484***	63.734	36.875	54.936	37.019
	(3.219)	(1.608)	(−1.537)	(0.578)	(−1.520)
Out-of-sample by country	63.641*	63.028*	36.057	61.093*	35.373
	(2.028)	(1.937)	(−2.073)	(1.649)	(−2.174)
Combined bubble indicators (based on log prices)					
Threshold:	1	2	3	4	5
In-sample	89.177***	88.763***	87.747***	66.095**	58.804
	(5.821)	(5.760)	(5.609)	(2.392)	(1.308)
Out-of-sample by period	77.249***	84.617***	74.245**	51.495	46.633
	(3.196)	(4.059)	(2.843)	(0.175)	(−0.395)
Out-of-sample by country	71.967***	76.822***	75.520***	47.899	38.900
	(3.268)	(3.991)	(3.797)	(−0.313)	(−1.651)

Table 4.4: In-sample and out-of-sample AUROCs and AUROC statistics. Each column presents the AUROC (in percentage terms) and the AUROC statistic (in parentheses) for the denoted indicator. Out-of-sample leaving one country out corresponds to excluding once each country from the dataset, estimating the model, and evaluating the model's performance for the country out of the sample. Out-of-sample by period calibrates the model up to year t and evaluates the model's performance at year $t+1$ for t between 1998 and 2012. Significance levels are marked with *, **, and *** for 95%, 99%, and 99.9% respectively.

out-of-sample by period test, exhibiting AUROC close to or above 0.75. The supLP test achieves the highest predictive power, followed by the supLPPLS (2nd) and the supBT tests (3rd).

As for the setup consisting in leaving one country out, the results are somewhat less favorable. On one hand, The AUROCs based on the supBT, the supPL, the supLPPLS tests are still significant, but the significance of the latter dropped, while supBT, showing the best performance, is still significant at the 99.9% level. On the other hand, the AUROCs of the indicators based on the supKBT and the supDF tests have decreased more (16.1% and 1.7% respectively). The performance of the indicators have thus decreased, evidencing the indicators' difficulties in transferring the learnings from a set of countries to an unseen market.

The differences between the results of the tests might be explained in several ways. The fact that the supBT statistic dominated the supKBT statistic, despite the fact that they assume the same data generating process, highlights the impact that a test's statistical power has on the bubble detection. The observed lacking forecasting skill of the supDF statistic, especially compared to those reported by Homm and Breitung [2012], might stem from our use of sliding windows of a maximum of 60 quarters to ensure that any alarm corresponds to a current bubble episode[8]. The satisfactory results of the supLPPLS and the supPL tests might be revealing key features of bubbles, since they presuppose bubbles as fundamentally different market regimes, driven by slow-maturation and transient non-linear dynamics with a necessary end associated with the finite-time singularity. Other readings are possible, but we decided not to inquire further, as surely our results fall short of disentangling the origin of the variations in the results of the tests. Admittedly, the bubble analysis that we have conducted face a joint-hypothesis problem, as the differences among the tests might arise either because of the theory on which the statistics are based or because of their statistical power.

Nevertheless, beyond the specific results of each of the indicators, we argue that the emerging picture is very positive. The bubble tests have forecasting skill associated to the end of the bubble, and they can complement the ex-ante identification based on the HP filter and similar approaches, often used in the literature. As the bubble tests rely on very few parameters inferred from the past, we believe they are likely to be more robust, and therefore more appropriate, to ex-ante study future housing boom episodes. Moreover, the fact that the supLPPLS and supPL contain significant value relative to the other approaches highlights the need to further employ diagnosis that are sensitive to the super-exponential growth pattern, which is increasingly supported as being a key stylized facts of financial and housing bubbles (Hüsler et al. [2013]; Leiss et al. [2015]; Gjerstad and Smith [2014]).

[8]Homm and Breitung [2012] employed the whole sample to date the bubbles, which is difficult in an ex-ante setup. Although they also proposed the CUSUM and FLUC tests to monitor time series, these statistics require a bubble-free period as a training sample, which would introduce further complications and make difficult a comparison between the tests.

4.5.2 Assessment based on Price-to-Income and Price-to-Rent ratios

A common question regarding the practical application of the statistical tests that we have analyzed is whether they should be applied directly on prices, or on some measure intended to reflect the misalignment of prices with the fundamental value, such as price-to-income and price-to-rent ratios. In this section, we seek to shed some light on this issue by analyzing the performance of the analogous bubbles indicators based on these two ratios. To conduct this assessment, we gathered the countries' (log) price-to-income and (log) price-to-rent time series from the OECD database[9]. Figure 4.3 summarizes the main results, also reported numerically in panels 2 and 3 of table 4.4.

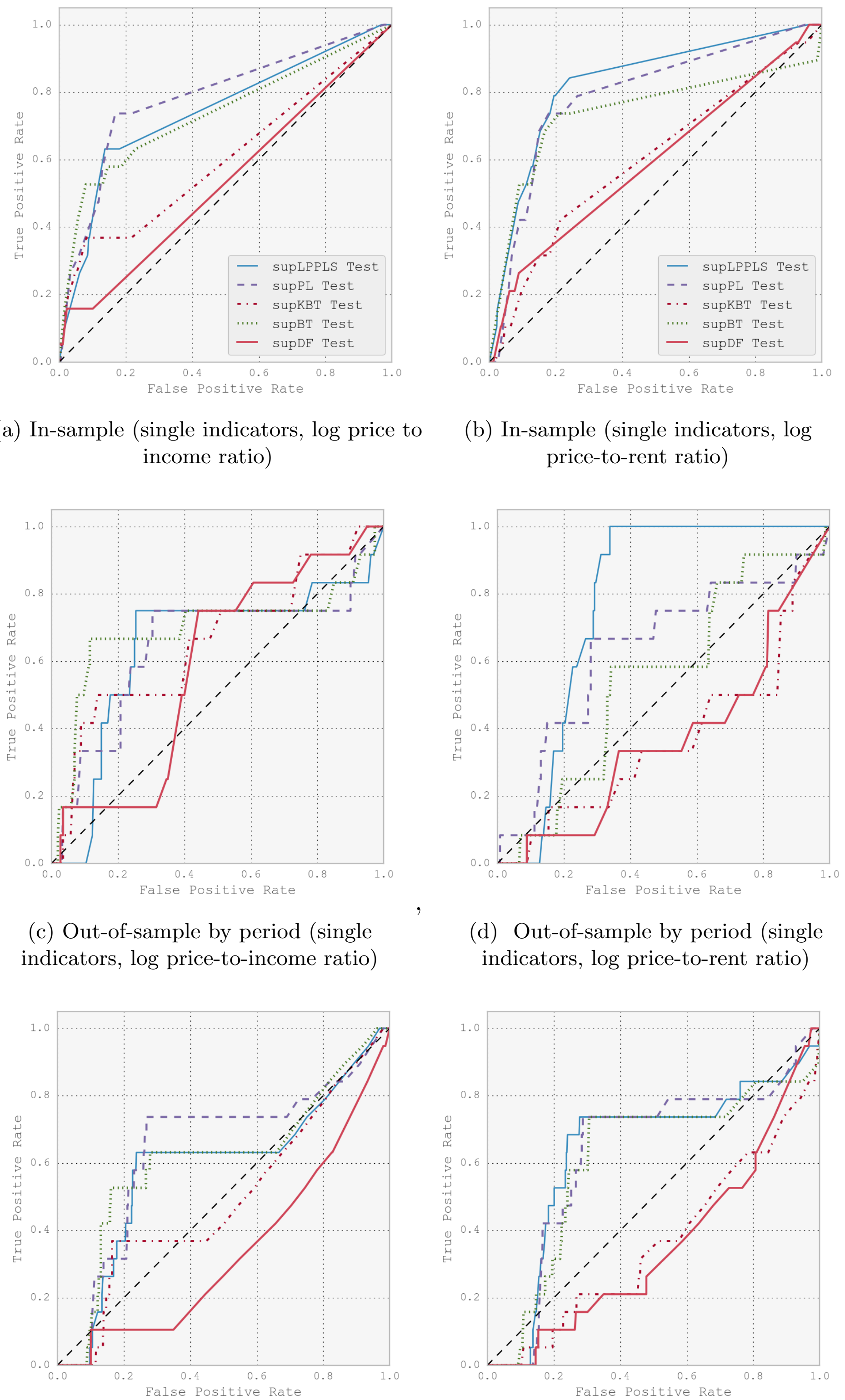
The performance of the ratio-based indicators drops substantially relative to that of the price-based indicators. In-sample, average AUROCs values decrease from 0.78 to 0.68 and 0.71 for the price-to-income and price-to-rent ratios respectively. Out-of-sample by period, the corresponding average values fall from 0.69 to 0.64 and 0.54. Out-of-sample by country, the average AUROCs values plunge to the relatively low levels of 0.53 and 0.51 for the price-to-income and price-to-rent ratios respectively.

The significance of the AUROC statistics also deteriorates. For both ratios, two of the indicators (supKBT and supDF) are not in-sample significant anymore; in contrast to the highly in-sample significance levels observed by the use of price levels. Only the supLPPLS-based indicator yields a significant out-of-sample by period AUROC value when derived from the price-to-rent ratios, while the significance level of all the indicators decreases in the equivalent quantities derived from the price-to-income ratios. Similarly, in the out-of-sample by country set-up, there is only one indicator derived from price-to-income ratios with significant results (supPL), and the significance level of the three significant indicators derived from price-to-rent ratios is lower than the equivalent quantities derived from prices.

Lastly, in figure 4.3 we observe that the maximum attainable rate of true positive decreases out-of-sample for most of the indicators, which differs from the perfect true positive rate generally attainable by the indicators derived from price levels. In the out-of-sample by country set-up, none of the indicators reaches a perfect rate of true positives, and only the supLPPLS indicator achieves the maximum rate in the out-of-sample by period set-up, when derived from the log price to rent ratio. In other words, the bubble tests when applied on ratios, have not exhibited sufficient power to ex-ante reject the null hypothesis of no bubble. As a consequence, there is no path for the corresponding indicators to timely trigger the bubble signal, for any admissible false positive rate.

In our view, the lower performance yielded by these two ratios not only points to their well-known limitations as poor proxies for house price deviations with the fundamental value, but also to the endogenous nature of housing bubbles, which often co-occur as part of a more broader macroeconomic phenomenon. To the extent that rent, disposable

[9]Prices: Analytical house price indicators `http://www.oecd-ilibrary.org/economics/data/prices/prices-analytical-house-price-indicators-edition-2016-1_6f6a769e-en`. We had to exclude Thailand for this set of tests, as this country is not part of the OECD.



(a) In-sample (single indicators, log price to income ratio)

(b) In-sample (single indicators, log price-to-rent ratio)

(c) Out-of-sample by period (single indicators, log price-to-income ratio)

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(d) Out-of-sample by period (single indicators, log price-to-rent ratio)

(e) Out-of-sample by country (single indicators, log price-to-income ratio)

(f) Out-of-sample by country (single indicators, log price-to-rent ratio)

Figure 4.3: Receiver operating characteristic (ROC) curves derived from bubble logit regressions (4.22). The independent variables are the bubble indicators derived from log price-to-income and price-to-rent ratios (see equations (4.21) and (4.24)). The dependent variable $y_{i,t}^{bubble}$ is set to 1 if a bubble peaked at time t in country i.

income, and possibly other macroeconomic factors might also be endogenous to the bubble regime - as it is arguably the case of housing and credit booms coinciding with economic booms -, ratio-based tests might lack the statistical power to detect the bubble episode. Prices might be considered in line with fundamentals, because the so-called fundamental factors might also be going through an unstable phase.

4.5.3 Combination of the indicators

We now explore whether there are gains in combining the different bubble indicators (BI). We create a composite indicator based on the number of simultaneous alarms at a given time period. Namely:

$$I_t^C(k) = \begin{cases} 1 & \sum_{j=1}^{5} BI_{j,t} \geq k \\ 0 & \text{otherwise} \end{cases} \tag{4.24}$$

where j denotes one of the five indicators and k corresponds to the necessary threshold to trigger the composite bubble signal. Panels 4.2b, 4.2f, and 4.2d of figure 4.2, and the second part of table 4.4 present the results.

We make two observations. First, combining the single indicators seems to yield more robust performance. Excluding the results for $k = 4, 5$, which correspond anyhow to very high thresholds, the in-sample and out-of-sample AUROCs are significant and above 0.87 (in-sample) and 0.72 (out-of-sample). Second, these results are positive enough to improve over the single top performing indicator (supPL or suptBT depending on the setup). The in-sample performance of the composite indicator is notably higher than the single indicator and the out-of-sample results are up to 6.5 percentage points higher. These two observations together suggest that, in the presence of model uncertainty, combining the indicators (i.e. using multiple tests for bubbles in a more general set-up) is a sensible decision. The resulting gains might derive not only in greater performance, but also in increasing robustness. Our results are thus consistent with the rather large literature, both in economics and more generally in pattern recognition, on the improved performance of compound predictors (Kittler et al. [1998]; Timmermann [2006]). The gains are remarkable in light of the simplicity of the rule that we have employed.

4.5.4 Bubbles and systemic financial crises

We have already presented supporting evidence that the bubble tests contain significant information to identify the end of the bubble period. We now contrast these results with those obtained by applying the same tests to forecast systemic financial crises. The goal is to further shed light on the phenomenon that the bubble tests are capturing. We use the regression

$$y_{i,t}^{crisis} = \Lambda\left(\sum_{k=0}^{MAXLAG} \beta_k BI_{i,t-k-1}^{j}\right) \tag{4.25}$$

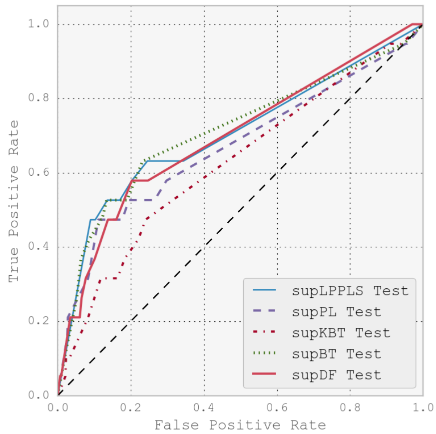
where i, k, j denotes indices for the country, the time lag, and the bubble indicator respectively, $\Lambda(\cdot)$ is the logit function, $MAXLAG$ is set to 3, and $y_{i,t}^{crisis} = 1$ if a crisis started at time t in country i (and 0 otherwise). As before, we discuss the AUROCs and AUROC statistics to analyze the indicators' performance.

Single bubble indicators (based on log prices)					
Test:	supLPPLS	supPL	supKBT	supBT	supDF
In-sample	70.325**	66.888**	62.598*	71.100***	69.653**
	(3.020)	(2.509)	(1.872)	(3.135)	(2.920)
Out-of-sample by period	55.530	55.087	40.729	58.697	43.338
	(0.622)	(0.572)	(−1.043)	(0.978)	(−0.749)
Out-of-sample by country	58.480	50.451	43.000	57.942	53.015
	(1.262)	(0.067)	(−1.041)	(1.182)	(0.449)
Combined bubble indicators (based on log prices)					
Threshold:	1	2	3	4	5
In-sample	73.460***	78.452***	70.411**	66.457**	57.369
	(3.486)	(4.228)	(3.033)	(2.445)	(1.095)
Out-of-sample by period	57.795	61.372	62.570	38.464	39.104
	(0.877)	(1.279)	(1.414)	(−1.298)	(−1.226)
Out-of-sample by country	59.102	65.386*	55.983	52.853	42.924
	(1.354)	(2.289)	(0.890)	(0.424)	(−1.053)

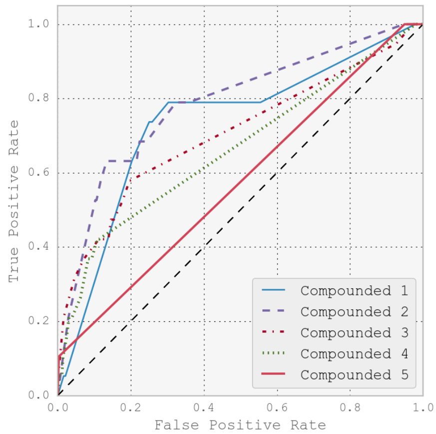
Table 4.5: In-sample and out-of-sample AUROCs and AUROC statistics from crisis logit regressions (4.25). Each column presents the AUROC (in percentage terms) and the AUROC statistic (in parentheses) for the denoted indicator. Out-of-sample leaving one country out corresponds to excluding once each country from the dataset, estimating the model, and evaluating the model's performance for the country out of the sample. Out-of-sample by period calibrates the model up to year t and evaluates the model's performance at year $t+1$ for t between 1998 and 2012. Significance levels are marked with *, **, and *** for 95%, 99%, and 99.9% respectively.

Figure 4.4 and table 4.5 summarize the results for the single and combined indicators. The differences in performance are appreciable. In general, only the AUROC of in-sample regressions are statistically significant. The performance of out-of-sample AUROCs of single and combined indicators drops visibly and no single indicator yields significant results. Among the out-of-sample tests, the maximum achieved AUROC slightly exceeds the 0.65 value (combined BI, with threshold 2) against the 0.77 value of the equivalent quantity reached by the bubble logit regressions of the previous section.

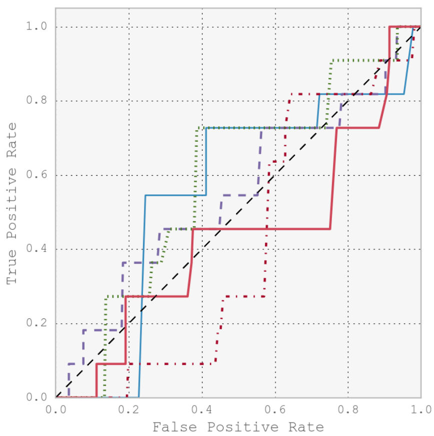
We interpret these results as clear evidence that bubbles and crises are different phenomena. This is not to deny the strong relationship that may exist among them, but to emphasize that they should be treated and analyzed differently. In addition, the poor performance of the indicators to predict crises may be interpreted as a sanity check for our results. The indicators have been able to capture well the events for which they are intended, and have underperformed when used to forecast a related, but different, type of event.



(a) In-sample (single indicators)



(b) In-sample (combined indicators)



(c) Out-of-sample by period (single indicators)

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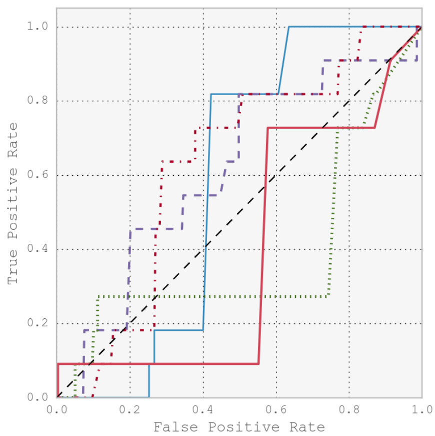
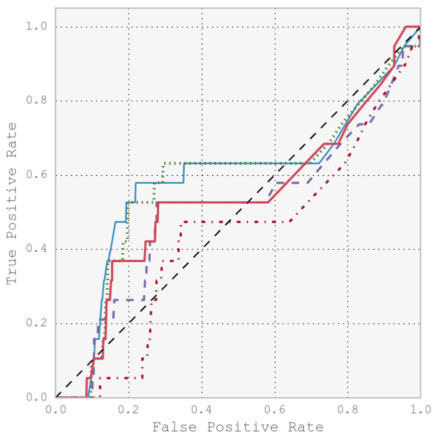
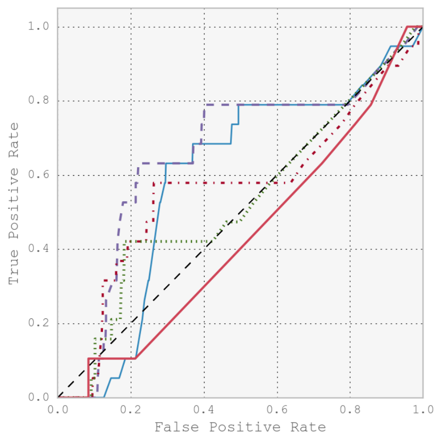


(d) Out-of-sample by period (combined indicators)



(e) Out-of-sample by country (single indicators)



(f) Out-of-sample by country (combined indicators)

Figure 4.4: Receiver operating characteristic (ROC) curves derived from crisis logit regressions (4.25). The independent variables are the bubble indicators derived from log prices (see equations (4.21) and (4.24)). The dependent variable $y_{i,t}^{crisis}$ is set to 1 if a crisis started at time t in country i.

4.5.5 Robustness checks

Finally, we assess the sensitivity of our results to the length of the rolling window employed by the bubble tests. In stationary time series, a longer a window length should increase the power of the tests. However, in non-stationary time series, such as real estate prices, exhibiting commonly several structural breaks, a longer sample can complicate the bubble identification, as the price dynamics can change with changing market conditions. Intuitively, the tests might be unable to distinguish between past and current bubble periods, or label recoveries followed by protracted busts, as new exuberance periods.

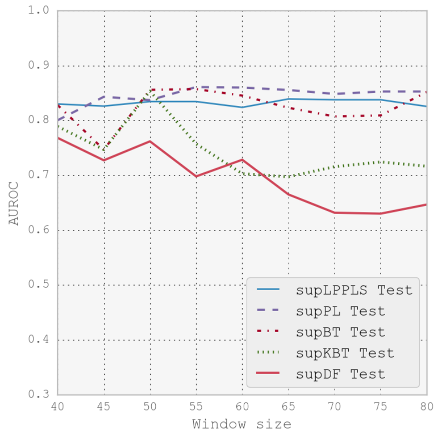
Concretely, using rolling window lengths varying between 40 and 80 quarters, with 5 quarters of increment (for a total of 9 different values), we re-calculated the bubble indicators and re-evaluated their performance for the in-sample and out-of-sample set-ups. Figure 4.5 presents the AUROC values of the individual and compounded indicators.

The supLPPLS, supPL, and supBT-based indicators present relatively low sensitivity to the window length. In-sample (panel 4.5a), the respective AUROCs remain above 0.8. Out-of-sample by period (panel 4.5c) and by country (panel 4.5e), AUROC values of the supLPPLS and supPL indicators vary only slightly, while supBT's performance peaks in the 50-70 length range and only deteriorates mildly (less than 5 percent relative to the peak) as the length becomes much shorter or larger. On the contrary, the supDF and the supKBT statistics do present substantial sensitivity to the window length. In-sample and out-of-sample, there is a visible negative and rather volatile trend as the window length increases. Their AUROC values peak at a length of 50 quarters, while bottom bellow 0.4 in the 65-70 range. Overall, these results are consistent with the relative performance of the indicators discussed in section 4.5.1.

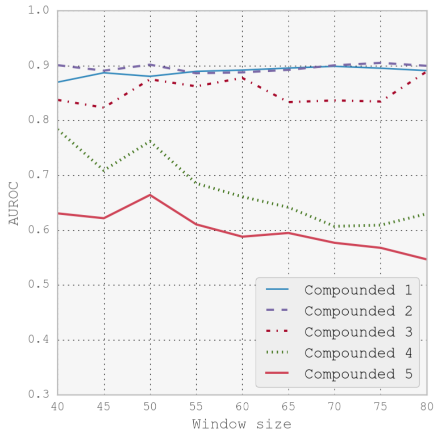
Also consistent with the previous discussions, combining the indicators generates higher AUROC values relative to the single indicators regardless of the window length. Using 1 or 2 indicators to trigger the bubble signal yields AUROC values close to 0.9 in-sample (panel 4.5b), and above 0.75 out-of sample (panels 4.5d and 4.5f), for most of the window lengths explored. Using 3 indicators yields somewhat more volatile results, though they remain fairly consistent in the 50-65 quarters range. Lastly, combining 4 and 5 indicators clearly leads to under-performing and highly volatile values. The latter should not be surprising, as two of the indicators (those based on the supDF and supKBT tests) yield results very sensitive to the window length.

4.6 Conclusion

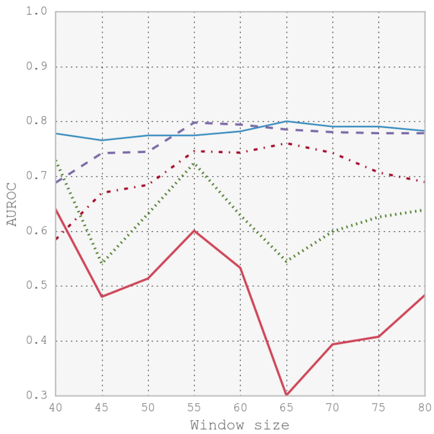
This paper analyzed the information content of statistical tests for bubble detection in the context of international real estate markets. We derived binary indicators from the causal application of selected statistical tests to log house prices, and via logit regressions we assessed the indicators' in-sample and out-of-sample performance in the forecasting of tipping points of housing bubbles and systemic financial crises. We have argued that our



(a) In-sample (single indicators)



(b) In-sample (combined indicators)



(c) Out-of-sample by period (single indicators)

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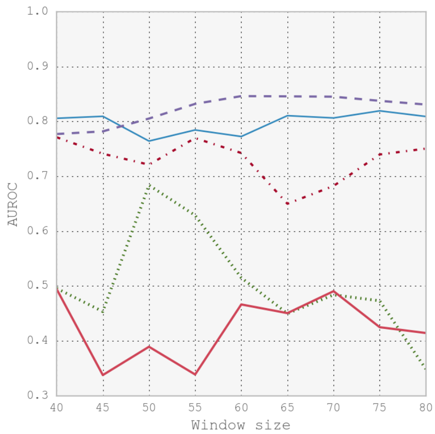
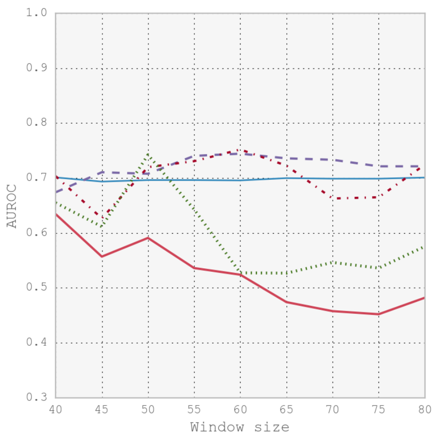
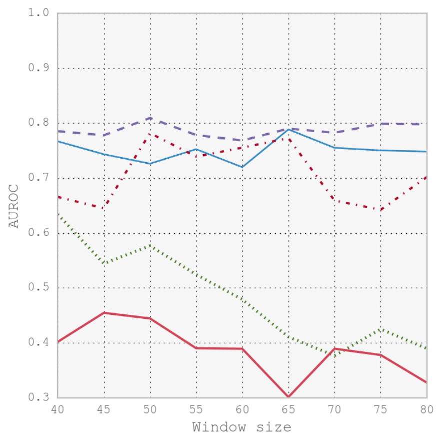


(d) Out-of-sample by period (combined indicators)



(e) Out-of-sample by country (single indicators)



(f) Out-of-sample by country (combined indicators)

Figure 4.5: Robustness checks to the length of the rolling window used by the bubble tests. Each plot shows the area under the receiver operating characteristic curves AUROCs of the denoted set of bubble indicators, when varying the window length.

benchmark is more robust and objective than any technical-rule-based benchmark, which typically identifies ex-post bubble episodes by sizing price expansions and corrections and then applying an arbitrary threshold.

Overall, our results suggest that the tests contain significant ex-ante information related to the end of the bubble period (with supPL and supBT being the tests that performed best), while combining the indicators yielded the highest performance. Robustness checks that vary the rolling window length employed by the bubble tests further support these conclusions. In addition, the application of the tests to price-to-income and rent-to-income ratios led to a substantial deterioration in the performance of the indicators.

The patterns that we have documented constitute a useful contribution to the analysis of real estate bubbles and to the literature on early warning signals. In addition, our results are also insightful because they stress the need for understanding the theoretical underpinnings behind explosiveness in price dynamics and the consequences of such phenomenon, as well as the importance of using diagnoses based on the super-exponential growth pattern to identify bubble regimes.

Chapter 5

A large scale factor model for real estate bubbles: a role for fundamental factors

If bubble tests based purely on price dynamics enable the identification of real estate bubbles and contain significant information to forecast their end, is there any role left for the so-called fundamental factors? Would macroeconomic factors facilitate the interpretation of the bubble signal or increase its forecasting power? If so, what factors are relevant? can they be identified ex-ante?

In this chapter we seek to address some these questions. We present a hybrid model for diagnosis and critical time forecasting of real estate bubbles. The model combines two elements: 1) the Log Periodic Power Law Singular (LPPLS) model to describe endogenous price dynamics originated from positive feedback loops among economic agents; and 2) a diffusion index that creates a parsimonious representation of multiple macroeconomic variables. We explicitly compare the in-sample and out-sample behavior of our model on the housing price indices of 380 US metropolitan areas. Empirical results suggest that the model is able to forecast the end of the bubbles and to identify variables that are highly relevant during the bubble regime. Such macroeconomic variables, when significantly contributing to the observed price dynamics, allow the identification, on average, of bubbles closer to their turning point. In other words and challenging common wisdom, high correlation between macroeconomic factors and exuberant prices seem to indicate the impending end of the bubble regime.

This chapter is an edited version of [Ardila et al., 2016c], of which I am first author.

5.1 Introduction

Real estate bubbles have a substantial impact on both economic growth and financial stability, whose bursts frequently result in severe and long recessions with high unemployment rates (Claessens et al., 2009). Detecting real estate bubbles sufficiently early and forecasting their end is thus extremely important. For example, this might guide central banks improve their monetary policy, assist banks and regulators develop better capital requirements and underwriting standards, warn households from speculative waves, and inform institutional investors about periods of upcoming high-volatility.

The real time analysis of the topic remains difficult due to the contemporary emergence of seemingly-plausible arguments that seek to justify the observed dramatic rise in asset prices by fundamental valuation arguments [Jurgilas and Lansing, 2012]. The complications are compounded by the low frequency at which real estate prices are observed, the segmentation and thinness of housing markets, and transitory or structural changes in the relationship of prices with their fundamentals that can occur during different regimes (Muellbauer and Murphy, 1997).

We seek to address the problem of detecting real estate bubbles and forecasting their critical time. To this end, we propose a factor model that combines the Log Periodic Power Law Singular (LPPLS) model for bubble detection, and a large number of macroeconomic variables. The LPPLS model [Johansen et al., 2000] is a nonlinear model that embodies the effect of positive feedback loops among economic agents, which may lead to unsustainable price developments with predictable critical times. The macroeconomic variables in turn are integrated via a diffusion index method[1] named Sparse Partial Least Squares [Chun and Keleş, 2010], which finds a sparse combination of variables highly correlated with the dependent variable. The combined model consists of a linear equation that combines both outputs, allowing us to study the interaction between the terms. The data requirements of our approach are longitudinally low (short length time series), though cross-sectionally high (many factors), as we aim to obtain a parsimonious representation of the price dynamics using only recent information about the economy, but exploiting the information content in a large number of macroeconomic time series.

We make two contributions to the literature of real estate bubbles. First, we discuss the interaction between a pure time-series approach, such as that of the LPPLS model, and the information provided by the short-term dynamics of the macroeconomic variables. By doing so, we highlight the empirical difficulties of analyzing bubbles in a high-dimensional set-up. Second, we describe how our approach can be employed to monitor real estate bubbles, improve the forecast of their critical time, i.e. the time of their highest peak preceding the crash or change of regime, and give initial guidance concerning the variables that might be driving - or might be slaved to - the bubble.

Our approach distinguishes itself from classic bubble diagnosis methodologies, as we jointly forecast the end of the bubble with its detection, and do not rely on a specific structural relationship to diagnose the bubble. We also deviate from most forecasting methodologies as we explicitly attempt to identify bubble regimes as well as to obtain insights about their underlying drivers. This is all done by linearly combining the LPPLS model, which models the endogenous dynamics of an unstable explosive regime, with a

[1] Here, we use the term "diffusion index" to mean a summary indicator of the common tendency of a set of statistics based on macroeconomic variables used to calculate economic turning points.

diffusion index based on a sparse composition of many macroeconomic time series during the window of instability. In addition, these elements facilitate the interpretation of the results, reduce the length of the required data, and also address two recurrent issues of macroeconomic crises: 1) the out-of-equilibrium dynamics likely happening during bubble regimes that makes the use of historical data difficult; and 2) the "this time is different syndrome" based on the fact that most new bubbles leading to crises are accompanied by the emergence of an innovation not captured by existing economic models.

We support our arguments with an empirical test on the housing price indices from 380 US metropolitan statistical areas (MSAs), and sets of 15, 35, and 90 macroeconomic time series. In the spirit of dynamic forecasting, we systematically scanned these housing indices searching for bubbles at every quarter between 2000Q1-2006Q1, and analyzed the performance of the forecast critical time. Our search methodology is carefully designed to respect causality (i.e, no look ahead). Empirical results suggest that macroeconomic variables can be used to single out bubbles closer to their turning point. Yet, once a bubble regime has been identified by the LPPLS model, the macroeconomic time series are not able to explain by themselves the observed prices, confirming the signs of endogenous overvaluation. In a more general sense, our results suggest that examining the correlation between explosive price dynamics and other macroeconomic time series may improve a forecasting model during periods of instabilities.

The rest of this chapter is organized as follows. Section 5.2 explains the building blocks of our model. Section 5.3 describes the data and details the setting of our empirical test. Section 5.4 presents and discusses our results. Finally, section 5.5 concludes this paper.

5.2 Methodology

5.2.1 General set-up

The hybrid model integrating the LPPLS model with macroeconomic variables is specified below in equations 5.1, 5.2, and 5.3. The model relies on two independent building blocks that are combined through a separate estimation procedure. In equation 5.1, house price returns $\Delta \ln p_t$, are written as a linear combination of two non-linear terms plus an additive random error u_t, exhibiting possibly serial correlation. The first term, resulting from equation 5.2, is the LPPLS model's contribution to the prices. This model describes the dynamics of positive feedback loops that induce unstable price developments, and ultimately, a change of regime that marks the end of the bubble. Subsection 5.2.2 elaborates on this model.

The second term, resulting from equations 5.3, is a diffusion index that summarizes the information of d macroeconomic variables $X \in \Re^{n \times d}$ to explain the vector of returns $\Delta \ln p$. Each row of X is a vector observation at a fixed time t, and each column corresponds to an uni-variate time series for one of the macroeconomic variables. Analogously to the standard Principal Component Analysis (PCA), the idea is to reduce the set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables. The specific method that we use to create this representation is named Sparse Partial Least Squares (SPLS) [Chun and Keleş, 2010] and is described in subsection 5.2.3. The key assumption, already evident in equation 5.3, is that the terms on the left hand side, i.e. the variables X and $\Delta \ln p$, can be decomposed into common latent factors $T \in \Re^{n\times K}$, matrices of coefficients $S \in \Re^{d\times K}$ and $Q \in \Re^{q\times K}$, and additive random errors $E \in \Re^{n\times d}$ and $F \in \Re^{n\times q}$, where in practice $K \ll d$, and $q = 1$ for the uni-variate response case.

$$\text{Combined model: } \Delta \ln p_t = \alpha_{LPPLS} \Delta \ln p_{tLPPLS} + \alpha_{mf} \Delta \ln p_{tmf} + u_t \tag{5.1}$$

$$\text{LPPLS comp:} \quad \ln p_{tLPPLS} = A + |t_c - t|^m [B + C \cos(\omega \ln |t_c - t| - \phi)] \tag{5.2}$$

$$\text{Macroeconomic comp: } \begin{cases} \Delta \ln p_{mf} = TQ^\top + F & \text{(5.3a)} \\ X = TS^\top + E & \text{(5.3b)} \end{cases}$$

5.2.2 Log periodic power law singularity model

To identify real estate bubbles and to forecast their critical time, we employ the log periodic power law singular (LPPLS) specification of the log-price dynamics derived from the Johansen-Ledoit-Sornette (JLS) model (Johansen et al., 2000). The derivation of the JLS model is briefly presented in appendix 5.6.1.

Starting from the rational expectation model of bubbles and crashes developed by Blanchard [1979], the LPPLS model considers the critical behaviour that may emerge in the self-organization of complex systems subjected to positive feedback processes. It identifies an endogenous bubble based on two distinctive "signatures". First, a transient, faster than exponential growth process ending in a finite time singularity, resulting from amplification mechanisms that take the form of price-to-return positive feedback (Sornette et al., 2003; Lin and Sornette, 2013; Corsi and Sornette, 2014). Laboratory experiments have recently provided explicit supporting evidence for this hypothesis [Hüsler et al., 2013]. Second, accelerating oscillations stemming from the existence of a discrete hierarchy in the organization of agents [Sornette, 1998, Zhou et al., 2005], or from the interplay between the inertia of transforming information into decision together with nonlinear momentum and price-reversal trading styles [Ide and Sornette, 2002]. According to the LPPLS model, these two elements constitute a signature of an unsustainable regime in the market, where any small disturbance or process can trigger a correction. The model has been previously applied in several contexts [Johansen and Sornette, 2010], including ex-ante diagnoses of the housing markets of US and UK at the national level [Zhou and Sornette, 2003], of US at the state level [Zhou and Sornette, 2006b], and of Switzerland at the district level [Ardila et al., 2013a]. It is important to stress that the LPPLS model represents the expected future log-price behavior that is anticipated by the rational investors [Sornette et al., 2013]. Stochastic generalisations of nonlinear positive feedbacks leading to stochastic finite time singularities have been introduced in (Sornette and Andersen, 2002;Andersen and Sornette, 2004; Lin and Sornette, 2013; Lin et al., 2014).

Equation 5.2 summarizes the LPPLS model for the expected log-price $\ln p_{tLPPLS}$, which has by definition no direct stochastic component. The term $|t_c - t|^m$ describes the faster than exponential growth dynamics due to positive feedback mechanisms. The exponent m captures the acceleration of the bubble. The critical time t_c corresponds to the most probable time for a regime change, at which the prices stop rising faster than exponentially, and crosses over to a different dynamics. The log periodic term $\cos(\omega \ln |t_c - t| - \phi)$ in turn specifies the accelerating oscillations of the log-price, in a sense describing an accelerating long-term volatility structure. The angular log-frequency ω quantifies the accelerating oscillations that obey the symmetry of discrete scale invariance, representing a discrete hierarchical pattern [Sornette, 1998]. It is not an inverse of time but rather quantifies the scaling ratio between the durations between successive local peaks of the log-price.

In its primitive form, an important characteristic of the LPPLS model is that the dynamics is specified only for times earlier than t_c. The model does not aim to describe the post-peak behavior. It reduces its ambition to solely describing the bubble dynamics and signaling the change of regime to *something else* (which remains unknown) once the bubble ends. Therefore, in general, we apply the LPPLS model at time before t_c so that the absolute value is not necessary. It is however possible to extend the model beyond t_c by using the absolute values $|t_c - t|$ in expression 5.2, which amounts to assuming that the price dynamics after the peak is a decelerating decay that is symmetric to the accelerated growth before the peak at t_c. This provides a parsimonious parameterisation, which turns out often to be sufficient.

The calibration of the LPPLS model, described in detail in [Filimonov and Sornette, 2013], minimizes the sum of squared residuals $\sum(\ln p_t - \ln \hat{p}_{tLPPLS})^2$ of a reformulated version of equation 5.2. The reformulation reduces the number of nonlinear parameters from 4 to 3 (t_c, m, ω). As this problem is still non-linear, the calibration is conducted by solving a nested optimization problem. The surface of the nonlinear parameters is traversed using a nonlinear optimization algorithm such as the unbounded Nelder-Mead Simplex search, or the bounded Tabu search. The cost function of the nonlinear problem takes the 3 nonlinear parameters as given and minimizes the sum of squared residuals to estimate the linear parameters. Since this procedure might converge to a local minimum, the algorithm is typically restarted multiples times using different starting points.

For the model to identify sensible bubble dynamics, the estimated parameters need to fulfill a set of constraints that have theoretical and empirical roots. They are commonly referred as the stylized facts of the LPPLS [Sornette et al., 2013]:

$$B < 0$$

$$0 < m < 1$$

$$|C|\sqrt{m^2+\omega^2} \leq -Bm$$

$$6 < \omega < 13$$

Conditions $B < 0$ and $0 < m < 1$ ensure a faster-than-exponential acceleration of the log-price with a vertical slope at the critical time t_c and a price finite even at t_c. Other cases can be easily excluded. For example, $m = 1$ is not valid as in this case the price follows the standard average exponential growth trajectory (with the coefficient B being negative) and t_c disappears in its function of the terminal time of the dynamics. Neither is $m < 0$ acceptable since in this case a divergence of the price occurs. The restriction $|C|\sqrt{m^2+\omega^2} \leq -Bm$ ensures that, in the framework of rational expectation bubbles, the hazard rate $h(t)$ remains always positive. Condition $6 < \omega < 13$ constrains the angular frequency of the log-periodic oscillations to be neither too fast (otherwise they would fit the random component of the data), nor too slow (otherwise they would provide a contribution to the trend). A calibration that fulfills these stylized facts is referred as a *qualified LPPLS fit*. Importantly, if a calibration does not fulfill the constraints, the model does not make any claim concerning other possible kind of dynamics that could also represent a bubble.

5.2.3 Diffusion index based on Sparse Partial Least Squares

We build a diffusion index to linearly combine the role of macroeconomic variables with the LPPLS model. Diffusion indices have become a key area of macroeconomic forecasting research as progress in information technology has increased the depth and breath of the data available. Diffusion indices address the problem of forecasting a single time series using a very large number of predictors, potentially many more predictors than dates at which the time series are observed [Stock and Watson, 1998]. They attempt to reduce the dimensionality of the predictors by identifying a set of dynamic latent (i.e. not observable) factors from which the variables can be generated.

We argue that diffusion indices are well suited for the kind of out-of-equilibrium analysis required during ex-ante bubble regimes studies, especially considering the particularities of real estate markets. Economic theory might provide clear guidelines on the long term drivers of housing prices, such as mortgage rates, land costs, and demographics. Yet, short-term dynamics might substantially deviate from long term equilibrium relationships, or might be highly dependent on the analyzed market. For example, whereas the recent bubbles of Spain and Ireland were primarily driven by housing demand and technology shocks (Mayer and Gareis, 2013; Aspachs-Bracons and Rabanal, 2010), the US bubble was mainly caused by a relaxation of mortgage down-payment constraints, accommodative monetary policy, and financial innovations [Aron et al., 2012]. Real estate housing models based on long-term relationships are thus prone to misspecification. On the contrary, a diffusion index assumes very little about the structure of the data, aiming to use several sources of information in order to explain the dynamics of the data. Admittedly, this might obscure the specific drivers of the bubble, but can generate early warning signals that can be further inspected to determine the possible causes and consequences of the overvaluation.

As a method to estimate the diffusion index, we used Sparse Partial Least Squares (SPLS) [Chun and Keleş, 2010], which, to our knowledge, has not been previously considered within the economics literature and has several important features that substantiate its application in the context of real estate time series. Similarly to Partial Least Squares (PLS) [Wold, 1978], SPLS is an iterative algorithm that attempts to find latent factors that are highly correlated with both the response and the independent variables. The variables selected by SPLS balance the most variable directions in the space of the regressors, with the most relevant variables for the independent variable at hand. This aspect is crucial for our application as booming real estate times series are short. Hence, employing unguided selection procedures, in which the dependent variable is not used to estimate the latent factors [Kelly and Pruitt, 2011], might not yield latent factors that are relevant to describe the development of the prices. In addition, and contrary to PLS and Principal Component Regression (PCR), SPLS finds sparse compositions of the dynamic factors, facilitating their interpretation and reducing the likelihood of over-fitting.

SPLS operates under the assumption of the basic latent decomposition expressed in equation 5.3. Accordingly, estimating the diffusion index consists of finding the terms on the right hand side of equation 5.3 based on the observed variables X and $\Delta \ln p$, namely, a decomposition of the prices and macroeconomic variables into common latent factors $T \in \Re^{n\times K}$, matrices of coefficients $S \in \Re^{d\times K}$ and $Q \in \Re^{q\times K}$, and additive random errors $E \in \Re^{n\times d}$ and $F \in \Re^{n\times q}$, where in practice $K \ll d$. This is the same factor structure underlying PLS regression. In addition, SPLS assumes that the latent factors can be constructed from a subset of active variables of X.

The departure point of SPLS is the following optimization problem:

$$\hat{w_k} = \arg\max_w \;\; w^\top P w \tag{5.4}$$

$$\text{subject to } w^\top w = 1, |w| \leq \lambda, j = 1, \ldots, k-1$$

where $P = X^\top Y Y^\top X$, and Y is the response variable. In our setting, X contains the m macroeconomic variables sampled at n time values, and Y equals the uni-variate time series of price returns $\Delta \ln p_t$.

The solution to this problem yields the first SPLS direction w_1, which in turn can be used to construct the first latent factor. As visible from the objective function, w_1 has high variance and high correlation with Y. The L_1 constraint $|w| \leq \lambda$ encourages a sparse composition of the direction and is also used by related methods such as LASSO [Tibshirani, 1996].

Formulation 5.4 however does not yield a sparse enough solution. Therefore, SPLS reformulates problem 5.4 into problem 5.5 below. To do so, SPLS generalizes the regression formulation of Sparse Principal Component Analysis [Zou et al., 2006], and imposes a L_1

penalty onto a surrogate of direction vector c instead of the original direction vector, while keeping the latter and c close to each other. The direction is renamed α (instead of w) to emphasize that the solution might be different. The first L_1 penalty ($|c|_1$) encourages sparsity on c, and the second L_2 penalty ($|c|_2$) takes care of a potential singularity in $P = X^\top YY^\top X$ when solving for c.

$$\min_{\alpha,c} -\kappa\alpha^\top P\alpha + (1-\kappa)(c-\alpha)^\top P(c-\alpha) + \lambda_1|c|_1 + \lambda_2|c|_2 \tag{5.5}$$

$$\text{subject to } \alpha^\top\alpha = 1$$

The formulation 5.5 seems to have four tuning parameters (κ, λ_1, λ_2, and K), but the SPLS regression has actually only two key tuning parameters. The thresholding parameter λ_1, which controls the sparsity of directions, and the number of hidden components K. Setting the λ_2 parameter to infinity yields the thresholded estimator that only depends on λ_1, and the solution does not depend on κ for an univariate response variable.

To obtain the other directions and correspondingly the other latent factors, SPLS embeds problem 5.5 into an iterative algorithm to determine directions that are orthogonal to one another. The algorithm is reproduced in appendix 5.6.2. In each iteration of the algorithm, problem 5.5 is first solved in order to identify variables correlated with the solution vector $\hat{\alpha}_k$. These variables, as well as all variables identified in previous iterations, constitute a set of active variables that are used as an input of a standard PLS regression. The residual of this PLS regression is used as the new response for the next iteration. The process is repeated K times, the number of specified latent factors.

5.2.4 Estimation Procedure

We estimate the model described in section 5.2.3 as a two-step procedure. First, we calibrate equations 5.2 and 5.3 according to the algorithms described in sections 5.2.2 and 5.2.3. Second, we estimate the coefficients of equation 5.1.

The diagnosis of the bubble has the LPPLS model as a backbone. By construction, the aim of this model is to identify periods where the returns accelerate. Consequently, bubble periods are prone to contain integrated returns and the estimation of equation 5.1 via Ordinary Least Squares (OLS) is not appropriate [Phillips, 1986]. If the diffusion index is not co-integrated with $\Delta \ln p_t$, the regression is subject to spurious correlation. If there is co-integration between the diffusion index and $\Delta \ln p_t$, contemporaneous and serial dependence of the regressors will render the distribution of t and Chi-square ratios non-standard.

To cope with these issues, we employed the Fully-Modified Ordinary Least Squares Estimator (FM-OLS). The FM-OLS makes two semi-parametric corrections to adjust for serial correlations and endogeneity within the residuals, and thereby to eliminate the "so called" second-order bias from the limiting distribution of the estimates. The estimator also allows conventional χ^2 to be used for inference. In particular, the null hypotheses H_0

and H_1 developed in section 5.3 can be tested by constructing a modified Wald statistic. Simulations performed by Phillips and Loretan [1991] suggests that the FM-OLS performs well even in small samples.

We note that the diffusion index is built using only data from the period in which a qualified LPPLS fit was identified. This treatment is justified by the economic literature suggesting that the relationship between real estate markets and economic changes is regime-dependent (see e.g. Nneji et al., 2013). As we only focus on the transient dynamics of the bubble regime, the memory of the model is limited to the period where an instability is detected by the LPPLS methodology. In this way, we extract the relative influence of the macro-economic variables only in bubble periods, avoiding contamination by other non-bubble regimes.

5.2.5 Degrees of freedom

Hypothesis tests and inference on the coefficients of equation 5.1 require knowledge of the degrees of freedom. In a standard linear regression framework $Y = X\beta + \epsilon$, the degrees of freedom correspond to the parameters of the model. However, we cannot extend this result to the model defined in section 5.2.1 since the regressors in equation 5.1 depend non-linearly on the independent variables by construction. In addition, the SPLS fitting procedure can be regarded as an embedded variable selection algorithm, whose flexibility to adapt to the data increases with the number of input variables. To see this, it is sufficient to notice that a macroeconomic index based on k uncorrelated variables is more flexible than one based on k highly correlated variables. Hence, a proper estimation of the degrees of freedom should take into account the complexity of the estimation process, which goes beyond the apparent simplicity of equation 5.1.

Therefore, we apply the algorithm proposed by Ye [1998] to estimate the degrees of freedom for an arbitrary complex model M. His definition captures the heuristic that, if the fitted values yielded by model M can adapt easily to perturbations in the data, the model is more flexible and should be penalized for its complexity to avoid overfitting. The algorithm is a Monte Carlo method that adds noise to the independent variable (i.e. $\Delta \ln p_t$) in order to estimate the sensitivity of the fitted vector with respect the observed values of $\Delta \ln p_t$. The procedure is non-asymptotic in nature and thus is free of the sample-size.

5.3 Empirical test

5.3.1 Formulation

We applied our model to the housing market of 380 US Metropolitan Statistical Areas (MSAs). Our aim was to examine the relationship between the dynamics modeled by the diffusion index and the LPPLS model. To do so, we investigated the in-sample and out-of-sample performance of the model.

For each MSA, we estimated equations 5.1, 5.2, and 5.3 at every end quarter between 2000Q1 and 2006Q1, varying the window length between 36 and 44 quarters (9 and 11 years). The idea was to emulate the dynamic forecasting that an analyst performs. Under these circumstances, at every quarter he would ex-ante analyze the possibility of a bubble in each market by using the last available observation, but he would not know the appropriate initial date to calibrate the model (i.e. the proper window length), since the starting date of the bubble is ex-ante unknown. If the analyst obtains a qualified LPPLS fit from the calibration, a forecast for the critical time is given by the estimated $\hat{t}_c$.

We chose to focus on the 2000Q1-2006Q1 as a forecasting period for three reasons. First, it is a well-known fact that this market suffered a real estate bubble of enormous proportions during the 2000s, allowing us to move from the sometimes debatable question "is there a bubble in the market?" to the question "when is the bubble going to end?". Second, the different MSAs provide a large number of samples from an event type that otherwise is not frequently observed. Third, the sample is not only large but also diverse as different MSAs exhibited different critical times, price trajectories, and crash reactions.

Every time we identified a qualified LPPLS fit in a MSA, and thus made a forecast about the peak of the bubble, we considered the following two hypotheses on equation 5.1:

1. $H_0 := \{\alpha_{LPPLS} = 1, \alpha_{mf} = 0\}$. If rejected, the macroeconomic factors, represented by the diffusion index term, are necessary to describe the prices.
2. $H_1 := \{\alpha_{LPPLS} = 0, \alpha_{mf} = 1\}$. If rejected, the LPPLS component is necessary to explain the development of prices.

The inference was conducted at a 5% significance level, estimating the degrees of freedom with the procedure of section 5.2.5. We grouped the calibrations with qualified LPPLS fits according to the hypotheses that were rejected (hereafter H_0-rejected and H_1-rejected fits), and subsequently studied the differences in the forecasting precision between these groups (see section 5.3.3). Since there could be more than one qualified LPPLS fit for each end quarter and such fits tend to cluster, we decided to compute the median of the corresponding χ^2 statistics, instead of treating these values independently. Similarly, if more than one qualified LPPLS fit was obtained, we employed the median $\hat{t}_c$ as the forecast for that period. In the following, we detail our setting, describing further the data set, the benchmark and metrics used to evaluate the forecasting performance, as well as the methodology applied to calibrate the model and make the forecasts.

5.3.2 Data set

The data set includes quarterly housing price indices from the metropolitan statistical areas (MSA), and several macroeconomic indices from the US economy[2]. The data covers the period from 1975Q1 to 2013Q1. The macroeconomic variables that we employed are

[2] All series and housing indices were downloaded from the St. Louis Fed's FRED database: http://research.stlouisfed.org/fred2/

a subset of the 122 variables used by Bork and Møller [2012] to forecast housing prices, transformed appropriately to ensure stationarity. The set comprises macroeconomic time series from different sectors such as Housing, Credit, Financial Markets, among others. To test the impact of the selected variables on the results, we opted to create three nested data sets with 15, 35, and 90 macroeconomic variables and perform the dynamic forecast with each of them. Appendix 5.6.3 contains the complete list of variables. To simplify the discussion, we only present the results for the sets of 15 and 35 variables, and refer to the set of 90 variables to discuss the interpretability of the factors. A possible criticism that could be raised is that some values that we use in our regression may not have been available in real time. To address this concern, we also run our tests excluding values from the last quarter of each time window, finding that the results are not visibly affected.

5.3.3 Evaluation of forecasting

To evaluate the forecasting skills of the H_i-rejected fits, we analyzed their aggregated forecasting precision:

$$\#\text{correct fits}_{H_i}/\#\text{qualified fits}_{H_i} \tag{5.6}$$

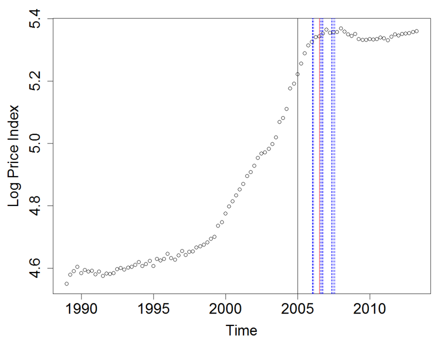
where #correct fits is the number of fits with correct forecast in the H_i-rejected group and $\#\text{qualified fits}_{H_i}$ is the total number of fits in the corresponding group.

The forecast of a qualified fit was considered correct if the estimated critical time $\hat{t}_c$ for the end of the bubble fell within a 3 years wide window centered around the turning point t_{sb}. This treatment is justified for two reasons. First, according to the rational expectation LPPLS bubble model, the occurrence of the real observed turning point of real estate prices has a probabilistic interpretation, in the sense that it has the largest probability to occur at the critical time t_c estimated by our fitting procedure, but it can occur at different times albeit with a decreasing probability. The fact that the turning point (also called the time of the change of regime) is distributed, and not deterministically located at the single critical time, is an essential condition within the rational expectation framework for the bubble to exist. Second, house prices are sticky downward and characterized by inertia. It is thus unlikely that an economic downturn would lead to a precipitous decline in home values [Case et al., 2000], and some time might be required before the actual change of regime can be observed. Additionally, we conducted the Welch's t test to evaluate whether the average $\overline{t_{sb} - \hat{t}_c}$ of the H_i-rejected sets and that of the whole set of fits are different. This test is applied to reject the null hypothesis that two population means are equal.

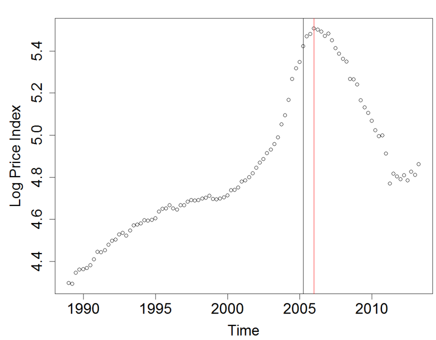
5.3.4 Development of a benchmark: dating corrections

The metrics described in section 5.3.3 require the date of the turning point t_{sb}, which is in reality unknown [Croce and Haurin, 2009]. It varies for each MSA as the sub-prime crisis started to unravel during 2006 and fully developed into the global financial crisis by late 2007. In some districts, a swift correction followed the tipping point whereas, in others, a slow deflation or stagnation characterized the new market regime. In fact, a less violent and slower end of bubbles is a better representative characteristic of real estate markets since properties are durable goods that people tend to hold whenever falling prices are observed [DiPasquale and Wheaton, 1996]. Moreover, only a subset of districts and states went through genuine bubbles, as documented in [Zhou and Sornette, 2006b].

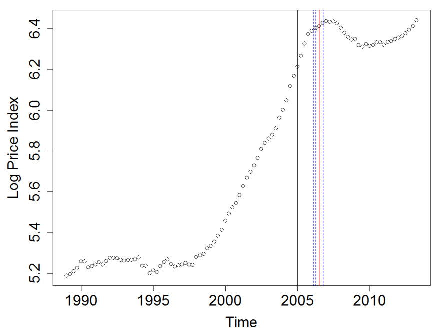
To date the turning point of each MSA, we employed a heuristic approach. Based on the complete house price time series (starting for most MSAs in 1975Q1, and ending in 2013Q1), we scanned every potential breakpoint time τ between 2003Q1 and 2009Q2 and used the Chow test to identify the date t_{ct} of the regime change that had the highest statistical significance. Breakpoints t_{ct} were adjusted to the quarter with the highest price within a 1.5 year window centered around t_{ct} to date the turning point; that is, the peak of the bubble. This adjusted value t_{sb} was the value employed as a benchmark. Figure 5.1 presents examples of the regime changes identified by this procedure.



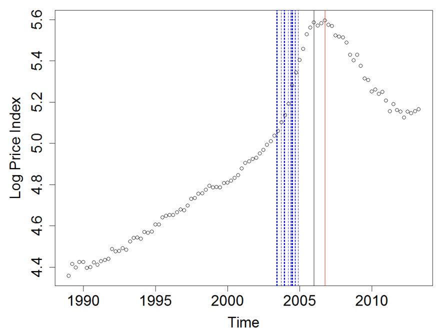
(a) House Price Index for Burlington South, Vermont



(b) House Price Index for Carson City, Nevada



(c) House Price Index for the district of Columbia



(d) House Price Index for Panama City, Florida

Figure 5.1: Examples of forecast critical times for four different MSAs. The black vertical lines represent the breakpoints as identified by the Chow tests, the thin (red, online) lines show the date of the turning point defined as a benchmark, and the (blue, online) dashed lines correspond to different estimations of the critical time using the LPPLS model. Note that the LPPLS model failed to identify a bubble for Carson City (i.e. no qualified LPPLS was found) as the accelerating phase only started after 2000 and our LPPLS calibration procedure makes use of windows of duration between 9 and 11 years.

5.3.5 Calibration of the model and testing methodology

The model was calibrated for different values of the parameters of the Sparse Partial Least Squares (SPLS) index in order to assess their impact. In particular, we examined a number $K \in \{2, 3, 4\}$ of reduced variables and the sparsity parameter $\lambda \in \{0.9, 0.95, 0.97, 0.98, 0.99\}$. These values were selected intentionally low for K, and high for λ as the window used to calibrate the index is short and the number of variables very high, making the model susceptible to overfitting. Although we report results for all these parametrizations to lessen concerns about data snooping, we observe that a cross-validation procedure can be employed to select the "best" model for each calibration. In our case, the model parameters of the best models, namely the most frequent optimal parametrization across all calibrated time windows, were $K = 2$ and $\lambda = 0.97$.

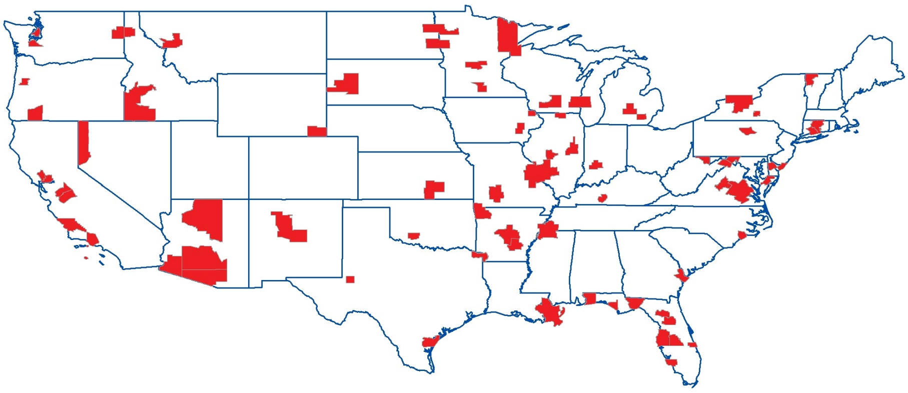
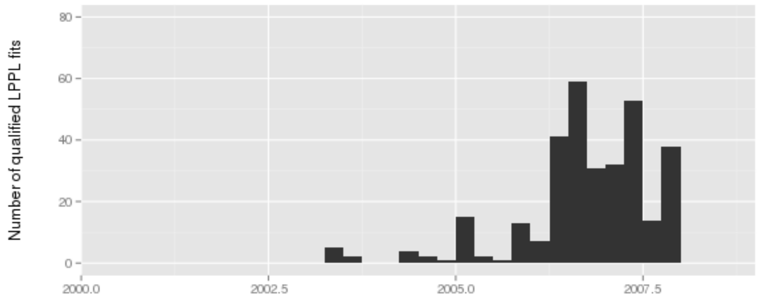
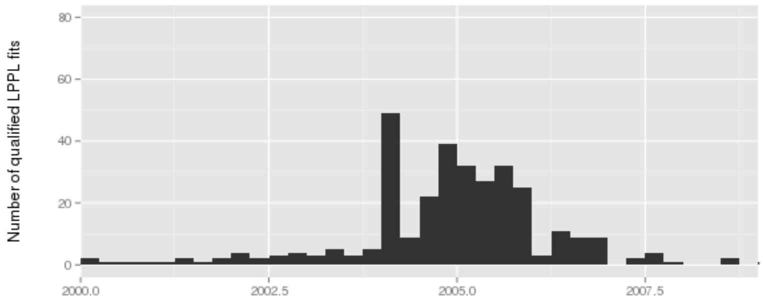


Figure 5.2: MSAs with qualified LPPLS fits (red). Hawaii and Alaska are not drawn here, but there were no qualified fits in these states. To calibrate the model, we used windows with end dates ranging from 2000Q1 to 2006Q1 and length varying between 36 and 44 quarters.

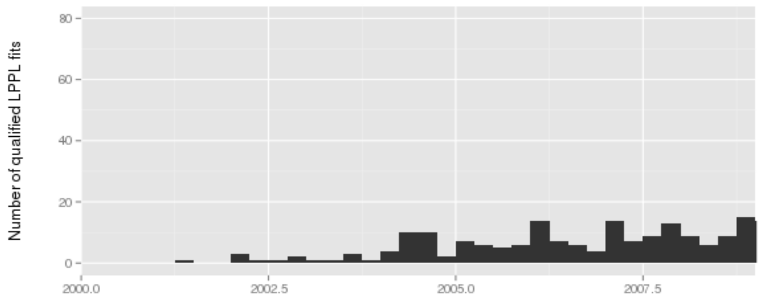
Overall, we obtained 320 qualified LPPLS fits, located in 85 different MSAs. Figure 5.2 presents the geographical location of these regions, whereas the distribution of the fits' critical times is presented in figure 5.3. The distributions of generalized degrees of freedom (GDF) according to the different data sets are presented in figure 5.4. Models with 15, and 35 variables exhibited respectively a median rounded-up GDF of 5, and 6. These values are substantially lower than the large number of time series employed in this analysis, and also lower than the 7 parameters of the LPPLS model, but larger than its 3 irreducible nonlinear parameters. Figure 5.5 presents the distribution of estimates $\hat{\alpha}_{LPPLS}$ and $\hat{\alpha}_{mf}$ (in equation 5.1) for MSAs with qualified LPPLS fits. The coefficient of the diffusion index $\hat{\alpha}_{mf}$ tends to be lower than that of the LPPLS factor $\hat{\alpha}_{LPPLS}$, with an overall mean (across all MSAs and set of macroeconomic variables) of 0.27 for $\hat{\alpha}_{mf}$, compared to a value of 0.72 for $\hat{\alpha}_{LPPLS}$. In addition, the figure suggests substantial variation in the within-factor coefficients, with a standard deviation of 0.2 for either of the factors, arguably consistent with the market segmentation that the US experienced during the bubble period.



(a) Distribution of critical times given by the (adjusted) Chow test

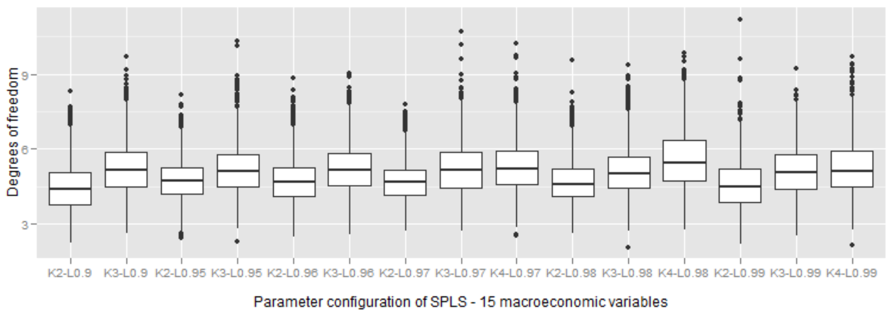


(b) Distribution of critical times given by the LPPLS model

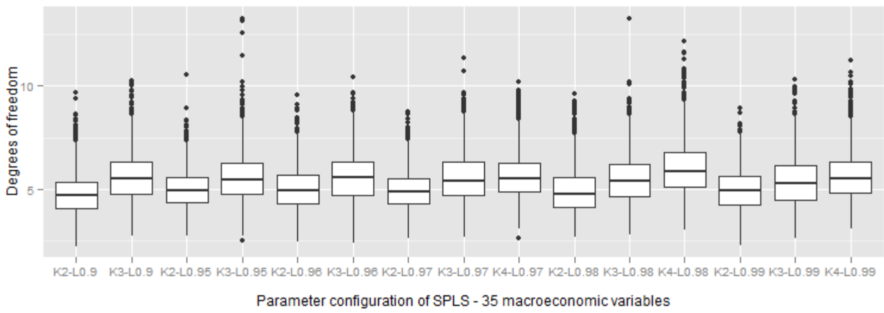


(c) Distribution of critical times given by the random estimation

Figure 5.3: Distribution of critical times by the different estimation procedures for the 320 qualified LPPLS fits. Conditioned on the identification of a qualified LPPLS fit, the random estimation samples uniformly a t_c from the interval $(t_n - 0.2bw, t_n + 0.2bw)$, with $bw = t_n - t_1$ and t_1 and t_n, respectively the first and the last point of the calibrating window. Note that the Chow test is not causal as it uses data after the critical time and it is carried out in order to provide an objective automatic determination of the change of regime. In contrast, the LPPLS procedure is causal and provides a genuine forecast of the critical time in the future of the window of analysis.

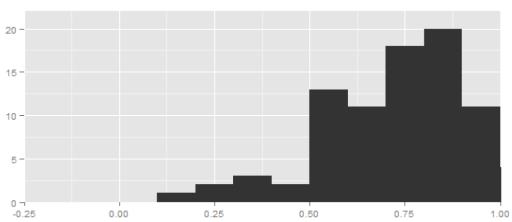


(a) Set of 15 macroeconomic variables

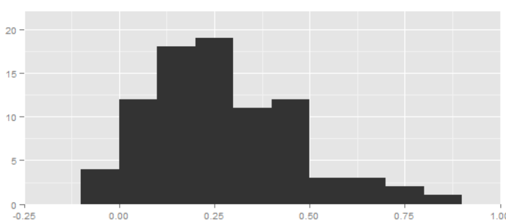


(b) Set of 35 macroeconomic variables

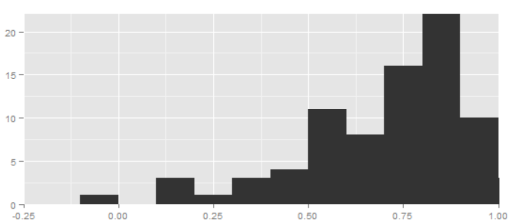
Figure 5.4: Distribution of degrees of freedom for the estimated models. Each bar corresponds to a different parametrization of the diffusion index, varying the number of factors K and the sparsity parameter λ. Thus, the 15 box plots correspond to 15 pairs (K, λ), as indicated on the abscissa. For instance, the first box plot on the left with K3-L0.9 corresponds to the models constructed with $(K = 3, \lambda = 0.9)$.



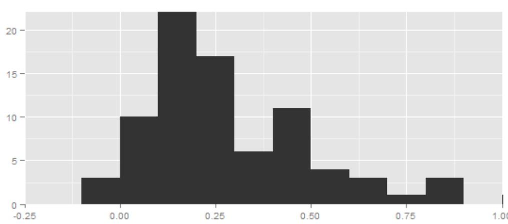
(a) Set of 15 macroeconomic variables - $\hat{\alpha}_{LPPLS}$



(b) Set of 15 macroeconomic variables - $\hat{\alpha}_{mf}$



(c) Set of 35 macroeconomic variables - $\hat{\alpha}_{LPPLS}$



(d) Set of 35 macroeconomic variables - $\hat{\alpha}_{mf}$

Figure 5.5: Distribution of $\hat{\alpha}_{LPPLS}$ and $\hat{\alpha}_{mf}$ for the US MSAs, with $K = 2$ and $\lambda = 0.98$. Each data point corresponds to the mean estimates of $\hat{\alpha}_{LPPLS}$ and $\hat{\alpha}_{mf}$ over all calibrating windows with qualified LPPLS fits of a single MSA.

5.4 Discussion and results

5.4.1 Set-up of the presentation of the results

Our main results are summarized in tables 5.1 and 5.2. For each of the parametrizations of the SPLS index mentioned above, the tables show the number and aggregated forecasting precision (defined by expression (5.6)) of the qualified LPPLS fits for which the hypotheses $H_0 := \{\alpha_{LPPLS} = 1, \alpha_{mf} = 0\}$ and $H_1 := \{\alpha_{LPPLS} = 0, \alpha_{mf} = 1\}$ were rejected. The results for the estimated critical times are contrasted against two values: 1) the aggregated precision of all qualified LPPLS fits; 2) the aggregated precision of a random estimation of the critical time. The random estimation consists of sampling uniformly a t_c from the interval $(t_n - 0.2bw, t_n + 0.2bw)$, with $bw = t_n - t_1$ and t_1 and t_n, respectively the first and the last point of the calibrating window. This interval bw corresponds to the duration of the same time window from which the LPPLS estimates t_c. Given that the minimum size of our calibrating windows is 36 quarters and the maximum size is 44, this implies that the random t_c is sampled from intervals with size varying between 3.5 and 4.3 years.

5.4.2 Significance of the LPPLS component

We first note that the number of qualified LPPLS fits for which the hypothesis H_1 is rejected is much larger than the number of qualified LPPLS fits for which the hypothesis H_0 is rejected, suggesting that the parsimonious representations of the price development given by the macroeconomic time series are not sufficient to describe the faster than exponential growth dynamics. The total number of qualified LPPLS fits where H_1 is rejected is at least 79% of the 320 fits that we originally identified. The values range from 255, for $K = 3$ and $\lambda = 0.9$ in table 5.2, to 300 fits, for $K = 2$ and $\lambda = 0.98$ in table 5.2, with a median across all sets and parameters of 276 (86% of the fits). The observation holds true regardless of the set of macroeconomic variables, and parameters K and λ used to create the index. This matches the idea that bubbles are, in essence, endogenous and transient phenomena, and that positive feedback loops played a predominant role during the subprime crisis.

The random estimation of the critical time, always the second row in the tables, performs fairly bad, having a precision that is 8.2% lower than that of the LPPLS model. The differences in performance are significant, with p-values from the Welch's t test so small that we report them as 0. The LPPLS model is thus mildly but consistently capturing the information necessary to determine the critical time of the bubble.

It is worthwhile emphasizing that we do not extend our analysis to unqualified LPPLS fits as the model has hardly a valid interpretation without the stylized facts mentioned in section 5.2.2 (though the relaxation of some constraints is subject of current research). Calibrations outside the valid range of parameters might reflect over-fitting. For instance, calibrations with $\hat{m} \geq 1$ or $\hat{B} > 0$ do not describe a super exponential trend in the data. Likewise, low values of ω (e.g. $\hat{\omega} \leq 3$) corresponds to a situation in which the log-periodic

Table 5.1: Summary statistics of the qualified LPPLS fits for the different parametrizations of the diffusion index - set of 15 macroeconomic variables - Forecasts of t_c made between 2000Q1 and 2006Q1. Values constitute statistics from the subset of qualified fits where hypothesis $H_i, i \in \{0, 1\}$ was rejected. The first two data rows are benchmark values against which the subsets are compared: 1) the statistics from all qualified fits; and 2) the statistics from the random estimation of t_c, as described in subsection 5.4.1. *Precision* is the aggregated precision of t_c, as defined in subsection 5.3.3. *Total* is the total number of qualified fits where the hypothesis was rejected. $\overline{t_{sb} - \hat{t}_c}$ is the average distance to the benchmark turning point t_{sb}, estimated as described in subsection 5.3.4. Finally, the reported p-value corresponds to testing $\overline{(t_{sb} - \hat{t}_c)}_{All} = \overline{(t_{sb} - \hat{t}_c)}_{H_i}$ using the Welch's t test.

Parameters		Rejection of $H_0 := \alpha_{LPPLS} = 1, \alpha_{mf} = 0$				Rejection of $H_1 := \alpha_{LPPLS} = 0, \alpha_{mf} = 1$			
K	λ	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ $(\sigma_{t_{sb}-\hat{t}_c})$	p-value	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ $(\sigma_{t_{sb}-\hat{t}_c})$	p-value
LPPLS		0.416	320	1.791 (1.501)	-	0.416	320	1.791 (1.5010)	-
Random		0.334	320	-1.685 (2.911)	0	0.334	320	-1.685 (2.911)	0
2	0.9	0.613	80	0.852 (1.625)	0.00462	0.382	283	1.763 (1.487)	0.1435
3	0.9	0.547	117	1.093 (1.819)	0.04936	0.387	269	1.754 (1.411)	0.11906
2	0.95	0.61	59	1.092 (1.597)	0.12071	0.395	286	1.759 (1.427)	0.16485
3	0.95	0.59	100	0.87 (1.788)	0.00077	0.398	274	1.75 (1.397)	0.14157
2	0.97	0.607	56	1.092 (1.605)	0.11102	0.392	288	1.752 (1.432)	0.15014
3	0.97	0.57	93	1.033 (1.824)	0.03276	0.393	280	1.75 (1.402)	0.17178
4	0.97	0.504	121	1.197 (1.788)	0.03278	0.393	272	1.759 (1.378)	0.13893
2	0.98	0.593	54	1.092 (1.689)	0.07073	0.399	296	1.744 (1.462)	0.08983
3	0.98	0.558	86	0.951 (1.815)	0.00136	0.394	282	1.75 (1.409)	0.18968
4	0.98	0.531	113	1.197 (1.819)	0.10297	0.378	270	1.772 (1.386)	0.19215
2	0.99	0.571	49	1.19 (1.665)	0.3039	0.4	300	1.733 (1.504)	0.07374
3	0.99	0.577	78	1.092 (1.673)	0.00355	0.396	285	1.75 (1.421)	0.13347
4	0.99	0.569	109	1.092 (1.761)	0.08747	0.391	276	1.75 (1.402)	0.08391

Table 5.2: Summary statistics of the qualified LPPLS fits for the different parametrizations of the diffusion index - set of 35 macroeconomic variables - Forecasts of t_c made between 2000Q1 and 2006Q1. Values constitute statistics from the subset of qualified fits where hypothesis $H_i, i \in \{0,1\}$ was rejected. The first two data rows are benchmark values against which the subsets are compared: 1) the statistics from all qualified fits; and 2) the statistics from the random estimation of t_c, as described in subsection 5.4.1. *Precision* is the aggregated precision of t_c, as defined in subsection 5.3.3. *Total* is the total number of qualified fits where the hypothesis was rejected. $\overline{t_{sb} - \hat{t}_c}$ is the average distance to the benchmark turning point t_{sb}, estimated as described in subsection 5.3.4. Finally, the reported p-value corresponds to testing $\overline{(t_{sb} - \hat{t}_c)}_{All} = \overline{(t_{sb} - \hat{t}_c)}_{H_i}$ using the Welch's t test.

Parameters		Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$				Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$			
K	λ	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ $(\sigma_{t_{sb}-\hat{t}_c})$	p-value	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ $(\sigma_{t_{sb}-\hat{t}_c})$	p-value
LPPLS		0.416	320	1.791 (1.501)	-	0.416	320	1.791 (1.501)	-
Random		0.334	320	-1.771 (2.861)	0	0.334	320	-1.771 (2.861)	0
2	0.9	0.527	93	1.092 (1.614)	0.00461	0.387	279	1.754 (1.496)	0.09211
3	0.9	0.522	136	1.309 (1.601)	0.01149	0.349	255	1.795 (1.515)	0.1816
2	0.95	0.557	70	1.074 (1.676)	0.07662	0.396	293	1.75 (1.446)	0.09268
3	0.95	0.57	100	1.159 (1.627)	0.00424	0.369	271	1.779 (1.519)	0.13911
2	0.97	0.61	59	1.092 (1.462)	0.09124	0.404	297	1.712 (1.454)	0.06891
3	0.97	0.577	97	1.092 (1.682)	0.02008	0.373	276	1.772 (1.464)	0.12211
4	0.97	0.53	115	1.19 (1.594)	0.00329	0.381	268	1.784 (1.453)	0.18008
2	0.98	0.611	54	1.093 (1.567)	0.06985	0.403	300	1.705 (1.472)	0.05908
3	0.98	0.556	90	1.266 (1.709)	0.01553	0.384	281	1.763 (1.46)	0.09996
4	0.98	0.569	102	1.075 (1.572)	0.00433	0.374	270	1.78 (1.513)	0.10853
2	0.99	0.618	55	1.056 (1.582)	0.12461	0.401	299	1.712 (1.473)	0.05532
3	0.99	0.56	84	1.159 (1.662)	0.01134	0.384	284	1.764 (1.481)	0.1243
4	0.99	0.535	99	1.197 (1.553)	0.01972	0.378	270	1.764 (1.5)	0.10279

term is contributing to the trend. Calibrations with these parameters are practically possible and can generate fits with very small residuals, rendering the statistical analysis invalid.

5.4.3 Identification of true positives

Interestingly, we observe that qualified LPPLS fits in which H_0 is rejected have a median precision about 0.12 higher than the complete set of fits, regardless of the parameters of the diffusion index. The precision in the surviving fits rises to a peak of 0.62 for $K = 2$ and $\lambda = 0.99$ in table 5.2, with a minimum of 0.53 for $K = 4$ and $\lambda = 0.97$ in table 5.2. Likewise, $\overline{\hat{t}_c - t_{sb}}$ in the H_0-rejected subset is lower, with values between 0.852, for $K = 2$ and $\lambda = 0.9$ in table 5.1, and 1.3, for $K = 3$ and $\lambda = 0.9$ also in table 5.2. The differences are mostly significant at a 0.05 level, as shown by the p-values from the Welch's t tests.

In contrast, rejection of H_1 does not improve significantly the precision. Aggregated values are very close to those of the whole set. For instance, precision values for the H_1-rejected sets in table 5.1 are between 0.37 and 0.39, which are not far from the 0.416 precision of the complete set of fits. The pattern is again robust to changes in parameters, and in the set of macroeconomic variables.

This behavior is counter-intuitive as common knowledge states that macroeconomic variables cannot in general explain speculative bubbles. Yet, the results point out in the opposite direction, as the diffusion index is helping to identify, on average, bubbles close to criticality. Equally puzzling, evidence in favor of the LPPLS component is not improving the forecasts when first considering the explanatory power of the macroeconomic variables.

We reconcile these facts by noticing that the usefulness of the macroeconomic variables, even during bubble regimes to describe the price dynamics, hold true *conditional* on the fact that a "bubble factor" is included. We further study this interpretation by examining the precision of critical time forecasts solely based on the diffusion index component. Following the methodology explained in section 5.3.1 to make dynamic forecasts, we estimated the diffusion index at every end quarter t between 2000Q1 and 2006Q1 using all available information up to time $t - h$ (in order to ensure causality), and made forecasts for $h \in \{4, 5, 6\}$ quarters ahead. Since the index does not explicitly predict a critical time, we interpreted a negative return over the forecasting horizon h as the prediction of a critical time h quarters ahead.

The results are presented in table 5.3. The table shows the forecasting precision for all 380 MSAs and for the 85 MSAs where we identified qualified LPPLS fits. For the set of 15 macroeconomic variables, the forecasting precision peaks at 0.28 (for $h = 5$ in the MSAs with qualified fits), and averages across all horizons 0.25 and 0.24 in all the MSAs and in the MSAs with qualified fits respectively. For the set of 35 macroeconomic variables, the forecasting precision peaks at 0.22, and averages 0.21 in both groups of MSAs. The performance of the critical time estimates obtained from the diffusion index component is in all cases visibly lower than that observed for our complete model, regardless of the

Table 5.3: Forecasting precision of critical time estimates from the diffusion index based on an h-periods forecast. Estimates are made at every end quarter t between 2000Q1 and 2006Q1 using all available information up to time $t-h$ (in order to ensure causality). We chose $k = 4$ and $\lambda = 0.95$ for the parameters of the diffusion index based on cross-validation over all MSAs.

Horizon(quarters)	15 Macroeconomic Variables		35 Macroeconomic Variables	
	All MSAs	MSAs with qualified fits	All MSAs	MSAs with qualified fits
4	0.23	0.20	0.21	0.22
5	0.27	0.28	0.21	0.22
6	0.26	0.24	0.22	0.18

forecasting horizon, the set of macroeconomic variables, and the subset of MSAs on which the precision is computed.

Thus, our findings do not support the conclusion that macroeconomic factors alone can account for the real estate price dynamics during bubble regimes. Once a significant component of the price is explained by the "bubble factor", residual structures in the price dynamics seem to require the information embedded in the macroeconomic factors. This may be due to the fact that the macroeconomic variables, when included together with the "bubble factor", are only reflecting short term dynamics, as only the sub-sample with a qualified LPPLS fit is employed for the calibration. In the opposite case, namely a calibration based on all the historical data, we would expect the macroeconomic variables to reveal the long term relationship between prices and fundamentals. The diffusion index might be signalling a broader imbalance in the economy, suggesting that detection of explosiveness in other variables might help to detect the instability period. This is an interesting direction that we intend to explore in the future. On the contrary, evidence in favor of the LPPLS component is not increasing the quality of the forecast, possibly because the t-tests are not adding extra information to that already embedded in the stylized facts of the LPPLS model, which are used to select the bubble periods.

5.4.4 Qualified fits distant from their turning points

One could argue that our finding of a higher average precision of all qualified fits as well as that of the H_0-rejected fits might be spurious, because we know that there is a turning point for each house price index and therefore we might introduce a hindsight bias. To address this possibility, we de-aggregated the fits for the $2000Q1 - 2003Q4$ window, where no structural break was detected (i.e. false positives). Tables 5.4 and 5.5 present the results. There are 64 qualified fits in this subset, with aggregated precision dropping from 0.41 to 0.14. As expected, the precision is lower than the value 0.5 exhibited by the random estimation.

Several further observations can be made. In Tables 5.4 and 5.5, one can observe that the random estimations give the highest precision value. This is coincidental as it reflects the overlap between the forecasting horizon (from which the t_c is randomly sampled) and the distribution of observed critical times. The average distance of the

Table 5.4: Summary statistics of the qualified LPPLS fits for the different parametrizations of the diffusion index - set of 15 macroeconomic variables - Forecasts of t_c made between 2000Q1 and 2003Q4. Values constitute statistics from the subset of qualified fits where hypothesis $H_i, i \in \{0, 1\}$ was rejected. The first two data rows are benchmark values against which the subsets are compared: 1) the statistics from all qualified fits; and 2) the statistics from the random estimation of t_c, as described in subsection 5.4.1. *Precision* is the aggregated precision of t_c, as defined in subsection 5.3.3. *Total* is the total number of qualified fits where the hypothesis was rejected. $\overline{t_{sb} - \hat{t}_c}$ is the average distance to the benchmark turning point t_{sb}, estimated as described in subsection 5.3.4. Finally, the reported p-value corresponds to testing $\overline{(t_{sb} - \hat{t}_c)}_{All} = \overline{(t_{sb} - \hat{t}_c)}_{H_i}$ using the Welch's t test.

Parameters		Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$				Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$			
K	λ	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ ($\sigma_{t_{sb}-\hat{t}_c}$)	p-value	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ ($\sigma_{t_{sb}-\hat{t}_c}$)	p-value
LPPLS		0.141	64	3.223 (1.257)	-	0.141	64	3.223 (1.257)	-
Random		0.5	64	-0.111 (2.469)	0	0.5	64	-0.111 (2.469)	0
2	0.9	0	19	3.147 (1.004)	0.29351	0.109	55	3.115 (1.242)	0.39807
3	0.9	0.08	25	3.389 (1.367)	0.17228	0.118	51	3.053 (1.189)	0.35137
2	0.95	0	14	3.547 (1.005)	0.05398	0.107	56	3.102 (1.202)	0.39314
3	0.95	0.091	22	3.307 (1.26)	0.18819	0.118	51	3.053 (1.218)	0.36753
2	0.97	0	14	3.547 (1.005)	0.02788	0.107	56	3.102 (1.202)	0.39324
3	0.97	0.091	22	3.307 (1.26)	0.25604	0.115	52	3.071 (1.261)	0.39175
4	0.97	0.067	30	3.375 (1.184)	0.15076	0.118	51	3.053 (1.169)	0.32888
2	0.98	0	14	3.547 (1.005)	0.02024	0.107	56	3.102 (1.202)	0.39331
3	0.98	0.095	21	3.252 (1.098)	0.30832	0.113	53	3.088 (1.26)	0.39687
4	0.98	0.065	31	3.361 (1.284)	0.17644	0.118	51	3.053 (1.216)	0.3525
2	0.99	0	14	3.547 (1.005)	0.05398	0.105	57	3.115 (1.24)	0.39767
3	0.99	0.1	20	3.199 (1.092)	0.33994	0.113	53	3.088 (1.207)	0.38293
4	0.99	0.069	29	3.361 (1.328)	0.18401	0.12	50	3.035 (1.165)	0.3013

Table 5.5: Summary statistics of the qualified LPPLS fits for the different parametrizations of the diffusion index - set of 35 macroeconomic variables - Forecasts of t_c made between 2000Q1 and 2003Q4. Values constitute statistics from the subset of qualified fits where hypothesis $H_i, i \in \{0,1\}$ was rejected. The first two data rows are benchmark values against which the subsets are compared: 1) the statistics from all qualified fits; and 2) the statistics from the random estimation of t_c, as described in subsection 5.4.1. *Precision* is the aggregated precision of t_c, as defined in subsection 5.3.3. *Total* is the total number of qualified fits where the hypothesis was rejected. $\overline{t_{sb} - \hat{t}_c}$ is the average distance to the benchmark turning point t_{sb}, estimated as described in subsection 5.3.4. Finally, the reported p-value corresponds to testing $\overline{(t_{sb} - \hat{t}_c)}_{All} = \overline{(t_{sb} - \hat{t}_c)}_{H_i}$ using the Welch's t test.

Parameters		Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$				Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$			
K	λ	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ ($\sigma_{t_{sb}-\hat{t}_c}$)	p-value	Precision	Total	$\overline{t_{sb} - \hat{t}_c}$ ($\sigma_{t_{sb}-\hat{t}_c}$)	p-value
LPPLS		0.141	64	3.223 (1.257)	-	0.141	64	3.223 (1.257)	-
Random		0.5	64	-0.111 (2.469)	0	0.5	64	-0.111 (2.469)	0
2	0.9	0	17	3.389 (0.925)	0.08149	0.107	56	3.102 (1.196)	0.38744
3	0.9	0.115	26	3.199 (1.117)	0.39449	0.109	55	3.053 (1.197)	0.36163
2	0.95	0	15	3.705 (0.979)	0.04523	0.138	58	3.035 (1.226)	0.33662
3	0.95	0.15	20	3.307 (1.16)	0.32595	0.109	55	3.053 (1.25)	0.3879
2	0.97	0	13	3.389 (1.004)	0.05231	0.14	57	3.053 (1.23)	0.35209
3	0.97	0.158	19	3.389 (1.228)	0.26937	0.113	53	3.053 (1.212)	0.37168
4	0.97	0.154	26	3.102 (1.154)	0.36775	0.113	53	3.053 (1.217)	0.38486
2	0.98	0	13	3.389 (1.004)	0.03602	0.14	57	3.053 (1.23)	0.35127
3	0.98	0.15	20	3.547 (1.211)	0.20016	0.113	53	3.053 (1.212)	0.37212
4	0.98	0.16	25	3.115 (1.133)	0.38089	0.115	52	3.035 (1.257)	0.38835
2	0.99	0	13	3.389 (1.004)	0.08865	0.14	57	3.053 (1.23)	0.35047
3	0.99	0.111	18	3.375 (1.162)	0.21106	0.109	55	3.088 (1.242)	0.39455
4	0.99	0.16	25	3.115 (1.157)	0.36992	0.12	50	3.066 (1.274)	0.39501

random estimation to the observed critical times is actually negative -0.11, meaning that the random estimations tend to lie after the actual date of the change of regime.

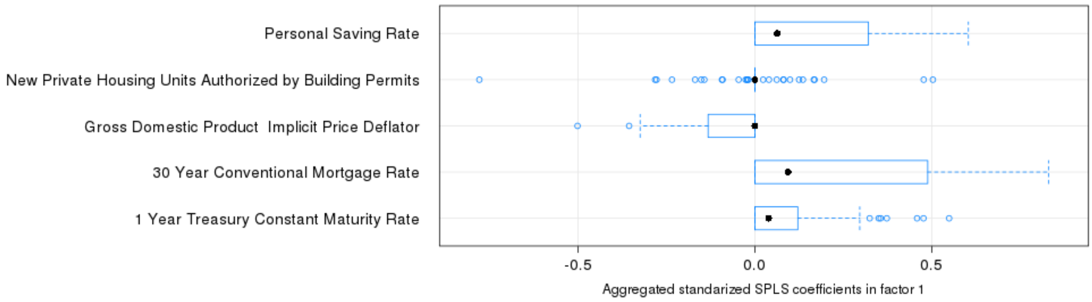
The presence of at least 20% of LPPLS fits that are qualified in the early time window $2000Q1 - 2003Q4$ is the main source of forecast errors, in the form of overestimating the proximity of the end of the bubble. This is supported by figure 5.3, in which most of the $\hat{t}_c$'s are very close to the upper end of the calibrating window. Another possible cause for these false positives is the fact that our method, like any other, has a finite prediction horizon, and therefore it is unable to estimate a turning point that is too far into the future.

Despite these negative results, there are also encouraging ones. First, in contrast with the results presented in Tables 1 and 2, most of the models do not reject H_0 or H_1, both for the sets of 15 and of 35 economic variables. For $K = 2$ and $\lambda \geq 0.97$, the number of rejected fits varies from only 13 to at most 16. Thus, evidence is inconclusive. One possible interpretation is that the information contained on the macroeconomic time series might be signaling that the turning points are still distant. Therefore, qualified LPPLS fits are not selected because there is no imminent regime change. And correspondingly, the precisions are not higher in the subsets for which either of the hypotheses H_0 and H_1 are rejected than over all LPPLS fits. Note also that the number of qualified fits in this time window $2000Q1-2003Q4$ is only 20% of the 320 identified fits over the full sample $2000Q1-2006Q1$, which is only 40% larger. If the fits were randomly distributed, they should amount to close to 60% of the total number (and not 20%), implying that most of the qualified fits are closer to the actual turning point (i.e. in the interval $2004Q1 - 2006Q1$). This suggests that the combination of these two indicators can provide improved skills for predicting the genuine structural breaks (the ends of real-estate bubbles): the large relative number of qualified LPPLS fits together with the significance of macroeconomic variables seem together to diagnose well the ends of bubbles.

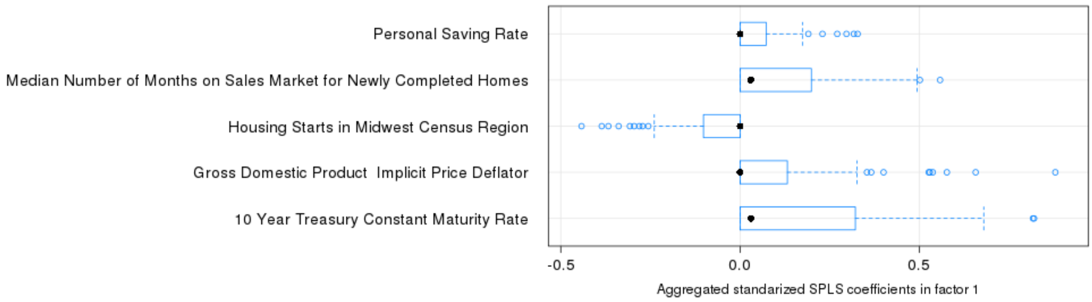
5.4.5 Interpretability of the diffusion index

Although the specific sets of variables play no significant role on the previous findings, they can impact the interpretability of the results. In this section, we discuss the set of 90 economic time series, in addition to the sets of 15 and 35 variables that we analyzed above. Figure 5.6 aggregates the standardized coefficients of the diffusion indices and present the factors with highest contribution for the different sets of time series for $K = 2$ and $\lambda = 0.98$. For the sake of simplicity, we have aggregated the lagged coefficients of the same variable.

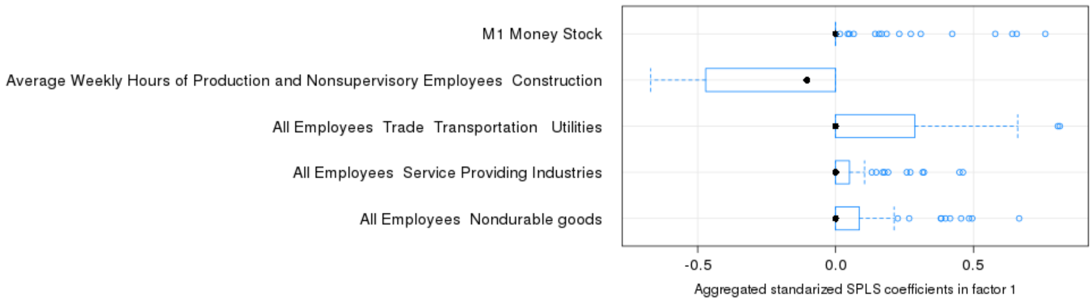
The top variables of the sets of 15 and 35 macroeconomic variables are informative and coincide with the ex-post analyses conducted on the US real estate bubble. Mortgage rates, saving rates, and short term interest rate have been recognized by most scholars and pundits as being important facilitating factors before the recent crisis [Aron et al., 2012]. In contrast, variables such as the number of employees in the sectors of non durable goods,



(a) Set of 15 macroeconomic time series



(b) Set of 35 macroeconomic time series



(c) Set of 90 macroeconomic time series

Figure 5.6: Selected macroeconomic time series for the US MSAs, with $K = 2$ and $\lambda = 0.98$. The values represent aggregated standardized coefficients over all qualified calibrations, where lagged values have also been collapsed.

transportation, and utilities, selected from the set of 90 time series, are harder to relate to the current consensus about the crisis. Hence, for the purpose of real time forecasting, using an unrestricted set of variables might not be necessarily justified. In addition, as we intentionally limit the length of the calibrating window to be equal to the duration of the booming period, the coefficients will tend to be biased, specially as the number of variables grows, and they are not meant to estimate elasticities or replace estimates from a structural econometric analysis. We only see this approach as a means to identify bubbles and screen variables that might be playing an important role during booming periods, and which can escape the analysis conducted during "normal" times.

5.4.6 Economic significance tested via trading strategies

Finally, we implement a trading strategy using the estimated critical times as signals to exit MSA markets with bubbles. Our objective here is not to conduct a rigorous trading exercise, but only to analyze the economic significance of our approach. Every time we obtain an estimate of a critical time in an MSA, we exit the corresponding market and invest the corresponding amount at the risk free rate. Statistically significant returns in excess of a buy and hold strategy would constitute supporting evidence of economic significance of the information provided by the model.

Concretely, given critical time estimates $\{\hat{t}_c^k\}$ in MSA_k, the corresponding weight $w_{k,t}$ at time t in a portfolio of $N = 380$ house indices is:

$$w_{k,t} = \frac{\max(0, 1 - \rho \sum_{\hat{t}_c^k < t} 1)}{N} \tag{5.7}$$

where $\rho \in [0, 1]$ is a smoothing parameter to allow for partial exits from the k market after a forecast critical time; the idea is to explore whether the benefits of riding the bubble by keeping a position in a super-exponential house market offset the risk of a correction.

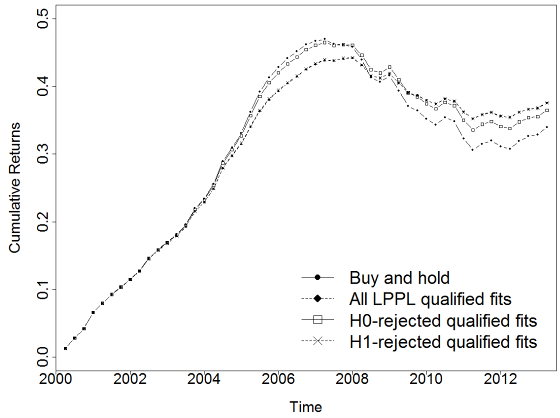
The one-period return of such portfolio is given by:

$$r_t = \sum_{k=1}^{N} (w_{k,t} r_{k,t} + (\frac{1}{N} - w_{k,t}) r_{f,t}) \tag{5.8}$$

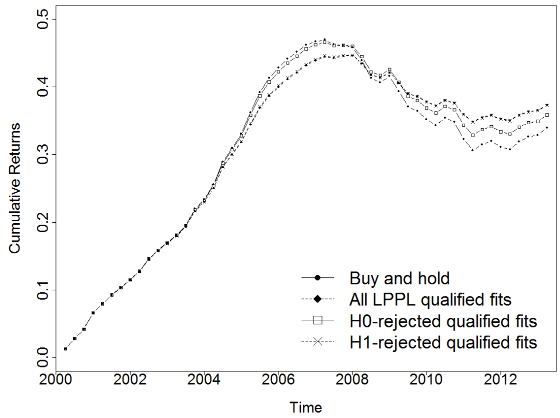
where $r_{k,t}$ is the one period return of MSA_k and $r_{f,t}$ is the risk free rate.

The performance of the strategy is studied over the holding period $t = 2000Q1 ... 2013Q1$ for $\rho \in \{0.25, 0.5, 1\}$, and using the critical time estimates from all qualified LPPLS fits, H_0-rejected fits, and H_1-rejected fits. Figure 5.7 presents the cumulative returns $\sum_{j<t} r_j$, and table 5.6 contains summary statistics of the excess quarterly returns of the strategy relative to a portfolio holding all MSAs.

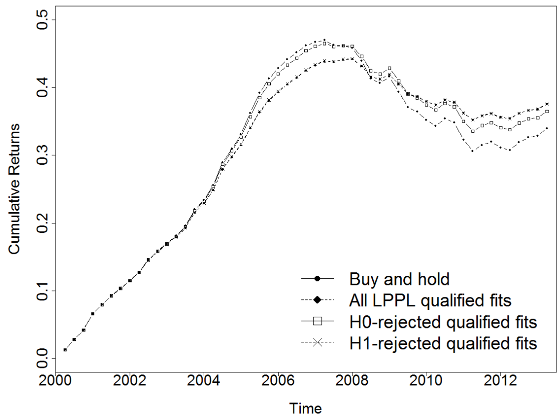
In figure 5.7, we observe that the total cumulative returns $\sum_{t=2000Q1}^{2013Q1} r_t$ are in all cases higher compared to the the buy and hold portfolio, though as expected, the latter outperforms the strategies at the peak of the bubble. By the end of the holding period (2013Q1), the portfolio using all qualified fits yields the highest total cumulative returns



(a) $\rho = 0.25$



(b) $\rho = 0.5$



(c) $\rho = 1$

Figure 5.7: Cumulative returns $\sum_{j<t} r_j$ of house index portfolios with holding period between 2000Q1 and 2013Q1. The weighted returns are obtained as indicated in equation 5.8, where the weight of each MSA_k is computed according to equation 5.7. We take the 3 months US treasury bill as the risk free rate.

Table 5.6: Average excess returns relative to a buy and hold strategy (in percentage terms) of house index portfolios with holding period between 2000Q1 and 2013Q1. The weighted returns are obtained as indicated in equation 5.8, where the weight of each MSA_k is computed according to equation 5.7. We take the 3 months US treasury bill as the risk free rate.

	Portfolio strategy	μ	σ	t-statistic	p-value
	All qualified LPPLS fits	0.051	0.002	1.552	0.0634
$\rho = 0.25$	Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$	0.020	0.001	2.207	0.0159
	Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$	0.048	0.002	1.557	0.0627
	All qualified LPPLS fits	0.064	0.003	1.403	0.0833
$\rho = 0.5$	Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$	0.036	0.001	2.323	0.0121
	Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$	0.063	0.003	1.432	0.0791
	All qualified LPPLS fits	0.068	0.004	1.285	0.1022
$\rho = 1$	Rejection of $H_0 := \alpha_{LPPLS} = 1$, $\alpha_{mf} = 0$	0.048	0.001	2.321	0.0121
	Rejection of $H_1 := \alpha_{LPPLS} = 0$, $\alpha_{mf} = 1$	0.068	0.004	1.313	0.0975

(0.38), reflecting the strong sequence of negative returns observed in the US housing market after 2008 and accordingly, the benefits of having left the market. On the other hand, we notice from table 5.6 that the average excess returns are significant at varying levels. For the H_0-rejected fits, the returns are significant at the 5% level. For all the qualified fits and for the H_1-rejected fits, returns are significant at the 10% level, exhibiting smaller p-values as ρ decreases. These results are consistent with the gains in precision in the H_0-rejected fits discussed in section 5.4.3, and the observed tendency of the model to overestimate the proximity of the crash (section 5.4.4).

5.5 Conclusion

We have presented a hybrid model for real time diagnosis of real estate bubbles, which combines the LPPLS model and a diffusion index. The application of the model to the housing price indices of the 380 US metropolitan statistical areas (MSAs) has allowed us to examine its properties. The results are summarized as follows:

1. We have confirmed the significance of the LPPLS component to describe the evolution of the house prices despite the information contained in the diffusion index about the macroeconomic variables. The forecast of t_c seems to be informative, though the proximity of the crash can be overestimated.

2. The information of the diffusion index, when significant, allows the identification, on average, of bubbles closer to their turning point. This insight does not depend on the parameters of the diffusion index nor the set of macroeconomic variables used for its construction.

3. The variables selected by the diffusion index are informative depending on the complete set of variables used to create the index. Hence, economic theory is still important to guide the selection of these variables, and the use of large scale data mining techniques does not replace intelligent human assessments, especially as we

intentionally limit the length of the time series used for the calibration in order to be relevant for transient bubbles.

These results suggest the model can assist the real-time diagnoses of bubbles and their interpretation. Diagnosis was already possible from the sole use of the LPPLS model. Yet, the addition of the diffusion index adds interpretability and a mechanism to classify housing bubbles according to testable hypotheses.

5.6 Appendices

5.6.1 Derivation of the LPPLS model

The LPPLS model we use was proposed by Johansen et al. [1999, 2000], and is usually referred to in the literature of financial bubbles as the JLS model. It starts from the rational expectation settings of Blanchard and Watson [1982a], where the observed price p_o of an asset can be written as

$$p_o = p^* + p \ , \tag{5.9}$$

where p^* and p represent respectively the fundamental value and the bubble component. Eq. (5.9) shows that the price is a linear combination of the fundamental value and the bubble component. The JLS model specifies the dynamics of the bubble component *independently* of the dynamics of the fundamental price. The later can be specified according to standard valuation models, for instance leading to the usual geometrical random walk benchmark. The JLS model adds to this featureless fundamental price the so-called log-periodic power law structure, which is used to diagnose the presence of bubbles. Lin et al. [2014] have considered a self-consistent mean-reverting process for p^* that makes consistent the calibration of the observed price p_o by the JLS model. Zhou and Sornette [2006a] have estimated the LPPLS model together with fundamental economic factors such as interest rates. Yan et al. [2012] have presented an extension of the JLS model to include the so-called "Zipf factor", which describes the diversification risk of the stock market portfolio. Keeping all the dynamical characteristics of a bubble described in the JLS model, the Zipf factor provides additional information about the concentration of stock gains over time.

The JLS model starts from the assumption that the dynamics of the bubble component of the price satisfies a simple stochastic differential equation with drift and jump:

$$\frac{dp}{p} = \mu(t)dt + \sigma dW - \kappa dj, \tag{5.10}$$

where p is the stock market bubble price, $\mu(t)$ is the drift (or trend) and dW is the increment of a Wiener process (with zero mean and unit variance). The term dj represents a discontinuous jump such that $j = 0$ before the crash and $j = 1$ after the crash occurs. The loss amplitude associated with the occurrence of a crash is determined by the parameter κ. Each successive crash corresponds to a jump of j by one unit. The dynamics of the jumps is governed by a crash hazard rate $h(t)$. Since $h(t)dt$ is the probability that the crash occurs between t and $t + dt$ conditional on the fact that it has not yet happened, we have $E_t[dj] = 1 \times h(t)dt + 0 \times (1 - h(t)dt)$ and therefore the expectation of dj is given by

$$E_t[dj] = h(t)dt. \tag{5.11}$$

Under the assumption of the JLS model, noise traders exhibit collective herding behaviors that may destabilize the market. The model assumes that the aggregate effect of noise traders can be accounted for by the following dynamics of the crash hazard rate:

$$h(t) = B'(t_c - t)^{m-1} + C'(t_c - t)^{m-1} \cos(\omega \ln(t_c - t) - \phi'). \tag{5.12}$$

The cosine part of the second term in the r.h.s. of (5.12) takes into account the existence of possible hierarchical cascades [Sornette and Johansen, 1998] of accelerating panic punctuating the growth of the bubble, resulting from a preexisting hierarchy in noise trader sizes [Zhou et al., 2005] and/or the interplay between market price impact inertia and nonlinear fundamental value investing [Ide and Sornette, 2002]. Expression (5.12) also contains a hyperbolic power law growth ending at a finite-time singularity, which embodies the positive feedbacks resulting from the technical and behavioral mechanisms summarized above in the introduction.

The no-arbitrage condition expresses that the unconditional expectation $E_t[dp]$ of the price increment must be 0, which leads to

$$\mu(t) \equiv \mathrm{E}\left[\frac{dp/dt}{p}\right]_{\text{no crash}} = \kappa h(t) \ , \tag{5.13}$$

by taking the expectation of (5.10). Note that $\mu(t)dt$ is the return $\frac{dp}{p}$ over the infinitesimal time interval dt in the absence of crash. Using this and substituting (5.12) in (5.13) and integrating yields the so-called log-periodic power law singular (LPPLS) equation:

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi) \tag{5.14}$$

where $B = -\kappa B'/m$ and $C = -\kappa C'/\sqrt{m^2 + \omega^2}$. Note that this expression (5.14) describes the average price dynamics only up to the end of the bubble. The JLS model does not specify what happens beyond t_c. This critical time t_c is the termination of the bubble regime and the transition time to another regime. The parameter t_c represents the non-random time of the termination of the bubble. However, its precise value is not known with absolute precision, and its estimation can be written as

$$t_c^{estimated} = t_c^{true} + \epsilon \ , \tag{5.15}$$

where ϵ is an error term distributed according to some distribution, while t_c^{true} is deterministic. Lin and Sornette [2013] have recently extended the model to include a stochastic mean reversal dynamics of the critical time t_c, thus capturing the uncertain anticipation of investors concerning the end of the bubble.

5.6.2 SPLS algorithm

In the following, we reproduce the SPLS algorithm as presented by Chun and Keleş [2010]. Take the structure defined in equation 5.3, using Y for the dependent variable instead of $\Delta \ln p_{mf}$, to describe the general setting. Denote X the matrix of co-variates, and define A to be an index set for active variables, K as the number of components, X_A as the matrix of co-variates contained in A, and β^{PLS} the coefficients of X_A, when expressing Y as a function of X in equation 5.3.

Result: Estimation of β^{PLS} by $\hat{\beta}^{PLS}$
Set $\hat{\beta}^{PLS} = 0$, $A = \{\}$, $k = 1$, and $Y_1 = Y$.
for $k = 1...K$ **do**
  Find $\hat{w}$ by solving problem 5.5 in section 5.2.3 with $P = X^\top Y_1 Y_1^\top X$.
  Update A as $\{i : \hat{w}_i \neq 0\} \cup \left\{i : \hat{\beta}^{PLS} \neq 0\right\}$.
  Fit PLS with X_A by using k number of latent components.
  Update $\hat{\beta}^{PLS}$ by using the new PLS estimates of the direction vectors.
  Update Y_1 and k through $Y_1 \leftarrow Y - X\hat{\beta}^{PLS}$.
end

Algorithm 1: SPLS algorithm.

One can use Y, X, and $\hat{\beta}^{PLS}$ to explicitly estimate the structure of equation 5.3. However, this is not required if one is only interested in the composition of β^{PLS} and the fitted response $\hat{Y}$. Once the set of active variables is determined, Wold's PLS algorithm is employed, and $\hat{Y}$ can be directly expressed as a linear function of X_A: $\hat{Y} = X_A \hat{\beta}^{PLS}(K)$.

5.6.3 List of macroeconomic time series used in the US MSAs analysis

The list of macroeconomic time series used to create the diffusion indices for the US MSA are presented in this appendix. The list comprises a set of 90 variables, which constitute a subset of the variables employed by Bork and Møller [2012] to forecast housing prices. Time series used in the subsets of 35 and 15 variables are labeled correspondingly with * and +. All series were downloaded from St. Louis Fed's FRED database.

Output and Income

- Disposable Personal Income
- Real personal income excluding current transfer receipts
- Gross domestic product (chain-type price index)*+
- Industrial Production Index*+
- Industrial Production: Consumer Goods
- Industrial Production: Durable Consumer Goods
- Industrial Production: Final Products (Market Group)
- Industrial Production: Nondurable Consumer Goods

- Industrial Production: Business Equipment
- Industrial Production: Materials
- Industrial Production: Durable Materials
- Industrial Production: nondurable Materials
- Industrial Production: Manufacturing (NAICS)
- ISM Manufacturing: Production Index*
- ISM Manufacturing: New Orders Index*
- ISM Manufacturing: Inventories Index*
- ISM Manufacturing: Supplier Deliveries Index*

Employment, Hours, and Earnings

- Capacity Utilization: Manufacturing (NAICS)*
- Civilian Labor Force
- Civilian Employment
- Unemployed
- Average (Mean) Duration of Unemployment*+
- Civilians Unemployed - 15 Weeks and Over
- Civilians Unemployed for 15-26 Weeks
- Civilians Unemployed for 27 Weeks and Over
- Civilians Unemployed for 5-14 Weeks
- Civilians Unemployed - Less Than 5 Weeks*
- All Employees: Total nonfarm
- All Employees: Total Private Industries
- All Employees: Goods-Producing Industries
- All Employees: Mining and logging
- All Employees: Construction
- All Employees: Manufacturing
- All Employees: Durable goods
- All Employees: Nondurable goods
- All Employees: Service-Providing Industries
- All Employees: Trade, Transportation and Utilities
- All Employees: Wholesale Trade
- All Employees: Retail Trade
- All Employees: Financial Activities
- All Employees: Government
- All Employees: Information Services
- All Employees: Professional and Business Services
- Average Weekly Hours of Production and Nonsupervisory Employees: Construction*
- Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing*

Housing

- Housing Starts: Total: New Privately Owned Housing Units Started*+
- Housing Starts in Midwest Census Region*
- Housing Starts in Northeast Census Region*
- Housing Starts in South Census Region*
- Housing Starts in West Census Region*
- New One Family Houses Sold: United States
- New Private Housing Units Authorized by Building Permits*

- New Privately-Owned Housing Units Under Construction: Total*+
- New Homes Sold in the United States*
- Median Number of Months on Sales Market for Newly Completed Homes*+

Money and Credit

- Commercial and Industrial Loans at All Commercial Banks
- Consumer Loans at All Commercial Banks
- Currency Component of M1
- M1 Money Stock
- M2 Money Stock*+
- Real Estate Loans at All Commercial Banks
- Personal Saving Rate*+
- Households and Nonprofit Organizations; Home Mortgages*+

Bond and Exchange Rates

- Effective Federal Funds Rate
- 3-Month Treasury Bill: Secondary Market Rate
- 6-Month Treasury Bill: Secondary Market Rate
- 1-Year Treasury Constant Maturity Rate*+
- 3-Month Treasury Constant Maturity Rate
- 5-Year Treasury Constant Maturity Rate*
- 7-Year Treasury Constant Maturity Rate*
- 10-Year Treasury Constant Maturity Rate*+
- 30-Year Conventional Mortgage Rate*+
- Moody's Seasoned Aaa Corporate Bond Yield*+
- Moody's Seasoned Baa Corporate Bond Yield*+
- Canada / U.S. Foreign Exchange Rate*
- U.S. / U.K. Foreign Exchange Rate
- Japan / U.S. Foreign Exchange Rate
- Switzerland / U.S. Foreign Exchange Rate *+

Prices

- Producer Price Index: Crude Materials for Further Processing
- Producer Price Index: Finished Consumer Foods
- Producer Price Index: Finished Consumer Goods
- Producer Price Index: Intermediate Materials: Supplies and Components
- Consumer Price Index: All Items for the United States
- Consumer Price Index for All Urban Consumers: Private transportation
- Consumer Price Index for All Urban Consumers: Nondurables less food
- Consumer Price Index for All Urban Consumers: Commodities less food*
- Personal Consumption Expenditures
- Gross Domestic Product: Implicit Price Deflator*

Stock Markets

- S&P 500 Stock Price Index
- Dow Jones Industrial Average

Chapter 6

Beyond real estate bubbles: the analysis of the financial cycle

Up to now, the contributions of this thesis have been directly related to the analysis of real estate bubbles. We have presented models and statistical tests that enable the identification and interpretation of real estate bubble regimes, and discussed evidence that substantiate the methods' assumptions and/or support their performance. However, real estate bubbles represent only a transient phase within the real estate cycle, which in turn is solely a component of what it is known in the literature as the financial cycle; the term denotes self-reinforcing interactions between perceptions of value and risk, attitudes towards risk and financing constraints, which translate into booms followed by busts. These interactions can amplify economic fluctuations and possibly lead to serious financial distress and economic dislocation [Borio, 2014].

If one intends to analyze the different phases of the financial cycle or put real estate bubbles into a broader macroeconomic context, the methods that we have presented are clearly insufficient. In this chapter, we seek, at least partly, to fill in this gap by proposing to date and analyze the financial cycle using the Maximum Overlap Discrete Wavelet Transform (MODWT). Our presentation points out limitations of the methods derived from the classical business cycle literature, while stressing their connection with wavelet analysis. The fundamental time-frequency uncertainty principle imposes replacing point estimates of turning points by interval estimates, which are themselves function of the scale of the analysis. We use financial time series from 19 OECD countries to illustrate the applicability of the tool. In addition, the uncertainty interval that we propose serve to justify the centered 2-3 years windows that we employed to evaluate the forecast on previous chapters, as they roughly correspond to the length of these windows.

This chapter is an edited version of [Ardila and Sornette, 2016], of which I am first author.

6.1 Introduction

Since the 2008 financial crisis, understanding the stylized empirical regularities of the financial cycle has emerged as an important research question [Borio, 2014]. Indeed, to comprehend the influence of financial upturns and downturns on macroeconomic cycles, we first need to identify these stylized facts.

Dating and analyzing financial cycles requires suitable tools. In this regard, scholars have borrowed substantially from the business cycle literature, mainly by employing the Bry-Boschan Quarterly (BBQ) algorithm for the detection of business cycle turning points, or one of its variations [Claessens et al., 2012]. Similarly, studies interested in medium term fluctuations have used band-pass filters in order to isolate the periodic components of financial time series ranging from 8 to 16 years [Drehmann et al., 2012]. Research has not, however, sufficiently explored the meaning of the outputs of the BBQ algorithm in the presence of the medium-term dynamics of financial time series. Most analyzes tend to either disregard the frequency properties of the cycles or to conduct time and frequency analysis independently. Failing to make an explicit connection between the time and frequency domains can cause great misunderstanding, as more than 50 years of research in business cycles has evidenced [Harding and Pagan, 2005]. Moreover, connecting time and frequency analyzes is especially relevant in the current state of the literature, when we have yet to understand the financial cycle and its properties.

Building upon the insights of Yogo [2008], Michis [2014], and Lera and Sornette [2015 (`http://ssrn.com/abstract=2703882`] who discuss respectively the applicability of wavelets in the analysis of business cycles (US GDP), the evaluation of economic forecasts, and the study of growth patterns in the US GDP, we use the Maximum Overlap Discrete Wavelet Transform (MODWT) to provide a scale dependent determination of turning points. This allows us to point out that the BBQ algorithm is deeply related to a time-frequency decomposition of the time series at a fixed scale. We show that, by construction, the MODWT combines time and frequency analyzes in an optimal way in the sense of saturating the bound of the time-frequency uncertainty principle. For our empirical application, we follow Drehmann et al. [2012] and use credit and house prices to characterize the financial cycle in 19 OECD countries. We thus contribute not only to the literature of business and financial cycles, but also to growing literature on the applicability of wavelet analysis to finance and economics [Gençay et al., 2001, Wu and Lee, 2015, Yazgan and Özkan, 2015].

6.2 The Maximum Overlap Discrete Wavelet Transform (MODWT)

6.2.1 General presentation

We follow Percival and Walden [2006] and Gençay et al. [2001] in their introduction of the MODWT. Let $\boldsymbol{x}$ be a column vector containing a sequence $x_0, x_1, ..., x_{N-1}$ of N observations of a real-valued time series. We assume that the observation x_t was collected at time $t\Delta t$, where Δt is the time interval between adjacent observations (e.g. quarterly). The MODWT of level J is a translation invariance transform of $\boldsymbol{x}$ defined by $\tilde{\boldsymbol{w}} = \widetilde{W}\boldsymbol{x}$, where $\tilde{\boldsymbol{w}}$ is a column vector of length $N(J+1)$, and $\widetilde{W}$ is a $(J+1)N \times N$ real-valued matrix defining the MODWT. The vector $\tilde{\boldsymbol{w}}$ contains the transform coefficients, and may be organized into $J+1$ vectors via

$$\tilde{\boldsymbol{w}} = [\tilde{\boldsymbol{w}}_1, \tilde{\boldsymbol{w}}_2, ..., \tilde{\boldsymbol{w}}_J, \tilde{\boldsymbol{v}}_J]^T \tag{6.1}$$

where $\tilde{\boldsymbol{w}}_j$ of length N and $\tilde{\boldsymbol{v}}_J$ of length N are called, respectively, vectors of wavelet and scaling coefficients. Qualitatively, if we let $\tau_j = 2^{j-1}$, each $\tilde{\boldsymbol{w}}_j$ is associated with changes on a scale $\tau_j\Delta t$ at a localized set of times; i.e. the wavelet coefficients tell us how much a weighted average changes from a particular time period of effective length $\tau_j\Delta t$ to the next. The coefficients in $\tilde{\boldsymbol{v}}_J$ are in turn associated with variations on scales $\tau_{J+1}\Delta t$ and higher; that is, a scaling coefficient is a weighted average on a scale of length $\tau_{J+1}\Delta t$.

The MODWT is an energy preserving transform in the sense that

$$\|x\|^2 = \sum_{j=1}^{J} \|\tilde{w}_j\|^2 + \|\tilde{v}_J\|^2 \tag{6.2}$$

In a wavelet analysis of variance, each individual wavelet coefficient is associated with a band of frequencies and a specific time scale, whereas Fourier coefficients are associated with a specific frequency only.

6.2.2 Construction of the MODWT

The MODWT matrix $\widetilde{W}$ is made up of $J+1$ submatrices, each of them of size $N \times N$,

$$\tilde{W} = \left[\tilde{W}_1, \tilde{W}_2, ..., \tilde{W}_J, \tilde{V}_J\right]^T . \tag{6.3}$$

and may be described in terms of linear filtering operations. Let $h_1 \equiv h_{1,0}, ..., h_{1,L_1}$ be a a wavelet filter of even length $L_1 \leq N$. h_1 may be selected from a Daubechies compactly supported wavelet family, such as the Haar wavelet filter $h_{1,0} = 1/\sqrt{2}, h_{1,1} = -1/\sqrt{2}$. By definition, h_1 must sum up to zero, have unit norm and be orthogonal to its even shifts:

$$\sum_{n=0}^{L_1-1-2l} h_{1,n}h_{1,n+2l} = \begin{cases} 1, & l=0 \\ 0, & l=1,2,...,(L_1-2)/2 \end{cases} \tag{6.4}$$

h_1 is associated with the scale $\tau_1\Delta t$ and works as an approximate high-pass filter with a pass-band given by the interval of frequencies $[1/(4\Delta t), 1/(2\Delta t)]$. h_1 has a low-pass (scaling) complement $g_1 \equiv g_{1,0},...,g_{1,L-1}$, defined via the quadrature mirror $g_{1,m} = (-1)^{m+1}h_{1,L-1-m}$. Also, let us define the wavelet filter h_j for higher scales $\tau_j\Delta t$ as the inverse discrete Fourier transform of

$$H_{j,k} = H_{1,2^{j-1}k \mod N} \prod_{l=0}^{j-2} G_{1,2^l k \mod N}, k=0,...,N-1 \tag{6.5}$$

where $H_{1,k} = \sum_{n=0}^{N-1} h_{1,n}e^{-i2\pi nk/N}, k=0,,...,N-1$ is the discrete Fourier Transform of the wavelet filter h_1 padded with $N-L_1$ zeros, and $G_{1,k}$ is defined similarly in terms of g_1. Elements $h_{j,L_j}, h_{j,L_j+1},...,h_{j,N-1}$ will be equal to zero when $L_j = (2^j-1)(L_1-1)+1 < N$ so that the resulting wavelet filter associated with scale τ_j has length $\min(N, L_j)$. Finally, define the scaling filter g_J for scale $2\tau_J$ as the inverse discrete Fourier Transform of

$$G_{J,k} = \prod_{l=0}^{J-1} G_{1,2^l k \mod N}, k=0,...,N-1 \ . \tag{6.6}$$

g_J works as an (approximate) low-pass filter with pass-band $\left[0, 1/(2^{J+1}\Delta t)\right]$. We are now ready to describe the construction of $\tilde{W}$ as in (6.3). For $\tilde{W}_1$, let us consider,

$$\boldsymbol{h}_1 = [h_{1,0}, h_{1,N-1}, h_{1,N-2}, .., h_{1,1}] \tag{6.7}$$

the vector of zero-padded unit scale wave filter coefficients in reverse order, and let us circularly shift the rescaled wavelet filter vector $\tilde{\boldsymbol{h}}_1^k = \boldsymbol{h}_1^k/2 = [h_{1,k},...,h_{1,1+k \mod N}]/2$ by integer units k to the right. Then, we have

$$\tilde{W}_1 = \left[\tilde{\boldsymbol{h}}_1^{(1)}, \tilde{\boldsymbol{h}}_1^{(2)},...,\tilde{\boldsymbol{h}}_1^{(N-2)}, \tilde{\boldsymbol{h}}_1^{(N-1)}, \tilde{\boldsymbol{h}}_1\right]^T \tag{6.8}$$

Similarly, matrix $\tilde{W}_j$ is constructed by circularly shifting $\tilde{\boldsymbol{h}}_j$ (the rescaled vector of zero-padded scale j wavelet filter coefficients) to the right by integer units. The vector of scaling coefficients $\tilde{V}_J$ is constructed analogously using g_J.

6.3 Dating the financial cycle

6.3.1 Correspondence between the conditions of the BBQ algorithm and wavelet coefficients

The nonparametric approach to date business cycles, known as the BBQ algorithm, consists of identifying local maxima and minima over a specific window in quarterly data. In addition, the algorithm contains censoring rules to guarantee a minimum length of the cycle, requires peaks and troughs to alternate, and a trough (peak) to be lower (higher) than the preceding peak (trough). Taking a five-quarter window centered at time t, the main rule of the BBQ algorithm for peaks (troughs) can be summarized as follows:

$$\{\Delta x_t > (<) 0, \Delta x_{t+1} < (>) 0\} \tag{6.9}$$

$$\{\Delta_2 x_t > (<) 0, \Delta_2 x_{t+2} < (>) 0\} \tag{6.10}$$

where $\Delta x_t = x_t - x_{t-1}$ and $\Delta_2 x_t = x_t - x_{t-2}$.

One can express condition (6.9) in terms of the wavelet coefficients generated by the Haar wavelet (which, as mentioned in section 6.2.2, is summarized by its high-pass filter coefficients $h_{1,0} = 1/\sqrt{2}, h_{1,1} = -1/\sqrt{2}$),

$$\{\Delta x_t = x_t - x_{t-1} \propto w_{t,1} > 0, \Delta x_{t+1} = x_{t+1} - x_t \propto w_{t+1,1} < 0\} \tag{6.11}$$

Condition (6.10) cannot be exactly mapped onto one acting on wavelet coefficients, but a close alternative is to consider the difference of the last two six-months period averages $\bar{x}_{t,t-1} := (x_t + x_{t-1})/2, \bar{x}_{t-2,t-3} := (x_{t-2} + x_{t-3})/2$,

$$\left\{\bar{x}_{t,t-1} - \bar{x}_{t-2,t-3} \propto w_{t,2} > 0, \bar{x}_{t,t-1} - \bar{x}_{t+2,t+3} \propto w_{t+2^2-1,2} < 0\right\} \tag{6.12}$$

Hence, there is a partial equivalence between the main BBQ conditions and the wavelet coefficients, allowing us to reinterpret the former as a condition on localized variances (see equation 6.2), and to link the BBQ algorithm to the frequency domain. To further support our argument, figure 6.1 compares the BBQ rules (6.9) and (6.10) against the wavelet conditions (6.11) and (6.12) when applied to the US GDP. The dates identified by the two methods coincide in 20 out of 23 cases, a very close match especially since we do not apply any censoring rule in either of the procedures. Two of the differences disappear once the proper censoring rules is applied to the BBQ dates. Hence, one could argue that the wavelet conditions (6.11) and (6.12) already contain part of the censoring logic applied to the BBQ algorithm. As for the 2001 recession, it is somewhat exceptional due to its very short duration[1]. Therefore, its identification based on quarterly data is complicated; one can see that the BBQ algorithm does not properly handle this period either, as the trough marking the end of the recession is missed by the algorithm.

[1] According to official NBER dates, it started in March 2001 and finished in November of the same year.

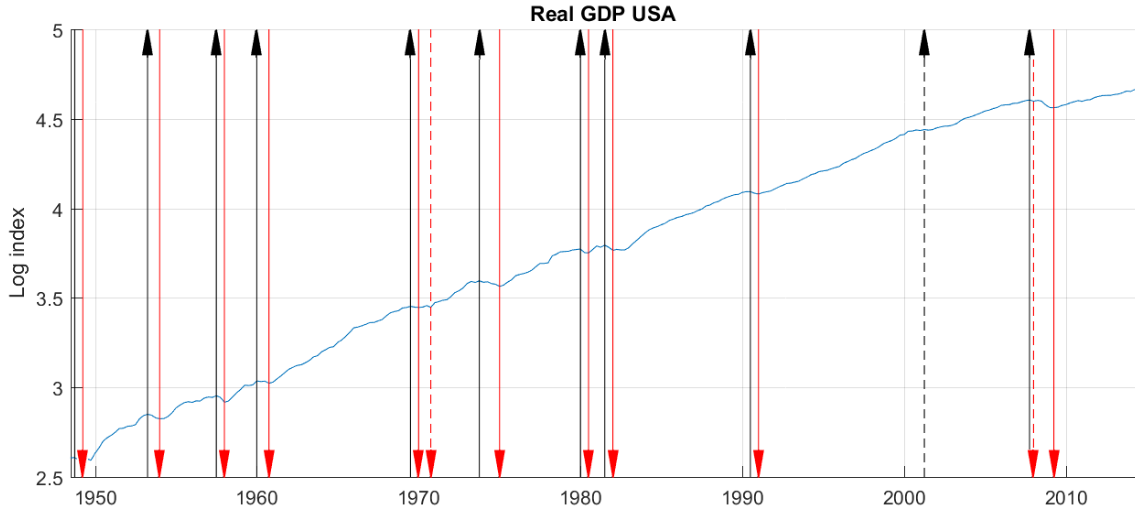


Figure 6.1: US GDP index. Solid black (red) arrows represent coinciding dates between the BBQ algorithm (6.9) and (6.10) and the wavelet rules (6.11) and (6.12) for peaks (troughs). Dashed arrows represent BBQ dates not reported by the wavelet rules. All dates reported by the wavelet rules are also reported by the BBQ algorithm.

6.3.2 Extension of the BBQ algorithm using the wavelet correspondence to capture cycles

From section 6.3.1, we can propose a natural way to extend the BBQ algorithm to capture medium term cycles. If short-term cycles are identified by coinciding change of signs in the wavelet coefficients of scales 1-2, an extension for medium-term cycles can be defined in terms of the wavelet coefficients of scales 1-4. Specifically, for peaks (troughs) we propose,

$$\left\{w_{t,j} > (<)0, w_{t+2^j-1,j} < (>)0 : j \in \{1, 2, 3, 4\}\right\} \tag{6.13}$$

Further extensions are possible. For example, longer cycles could be defined in terms of scales 1-6, but we do not proceed further in this direction as our focus lies on the financial cycle and higher scales would require longer time series not available in our sample.

Figure 6.2 shows the peaks and troughs found in the US house index (top panel) by applying condition (6.13) on the MODWT coefficients of scales 1 to 4 shown in the four bottom panels. The complete description of the content of this figure is postponed until after the definition of the interval estimates introduced in the next subsection.

6.3.3 The uncertainty principle

While the conditions (6.13) can be refined, they provide already a structured and objective path to analyze medium term fluctuations. The turning points are in addition characterized according to their frequency band. Moreover, the time-frequency decomposition highlights the boundaries imposed by the uncertainty principle of information

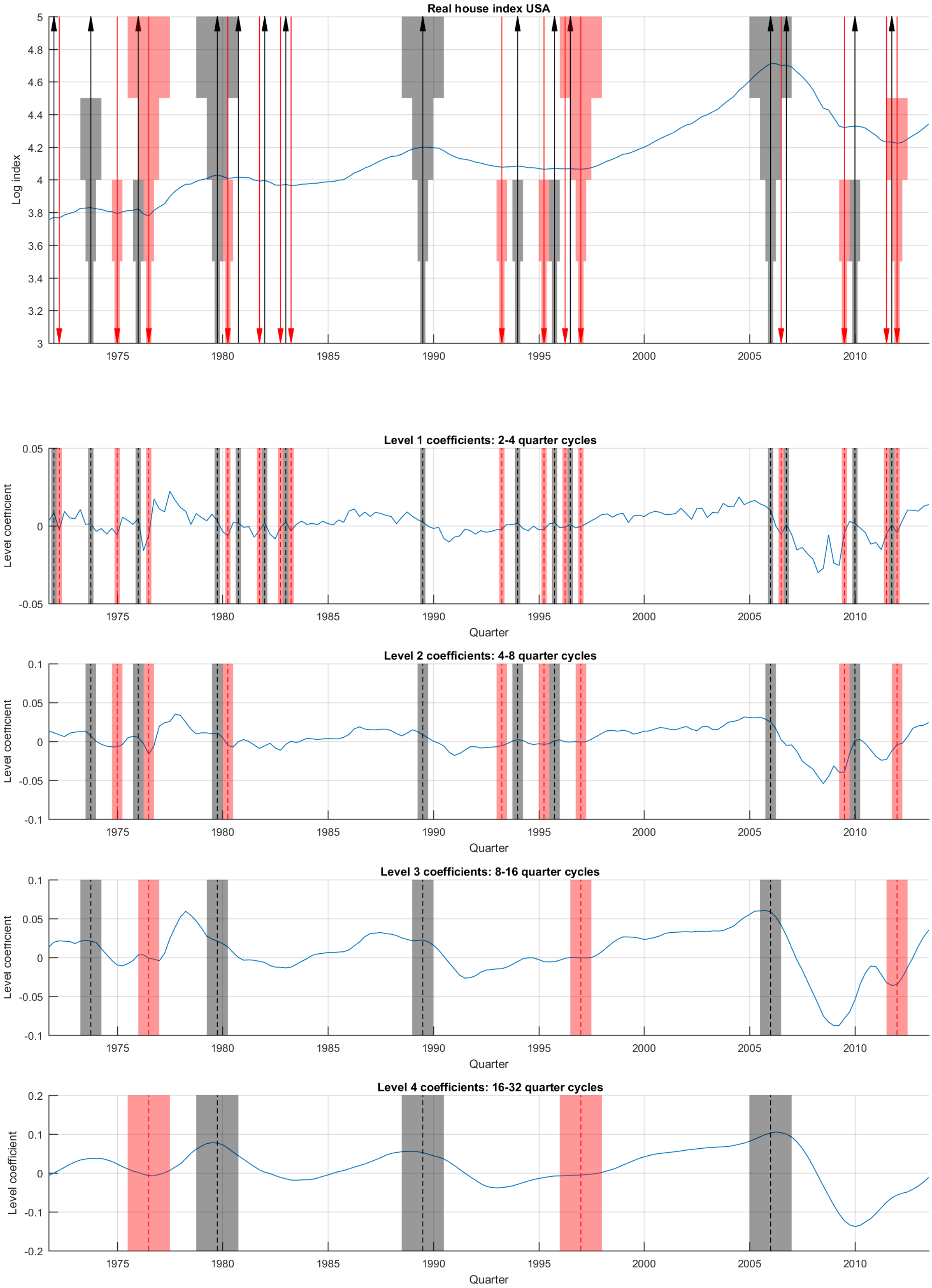


Figure 6.2: US house index (top panel) and MODWT coefficients of scales 1 to 4 (bottom panels). Dashes black (red) lines represent a peak (trough) based on the MODWT rule (for a single scale). Grey (red) shaded regions correspond to uncertainty intervals in peaks (troughs), computed according to equation (6.15) and centered around the corresponding peak (trough). Solid black (red) arrows in the top panel represent the output of the BBQ algorithm for peaks (troughs).

theory [Cohen, 1989, Gabor, 1991]

$$\Delta t' \Delta f' \geq \frac{1}{2} \, , \tag{6.14}$$

where $\Delta t'$ and $\Delta f'$ correspond respectively to time and frequency windows of analysis. The uncertainty principle (6.14) imposes that a precise dating comes at the cost of a large uncertainty on the frequency component. Reciprocally, a precise determination of the frequency component of a signal leads to a large uncertainty on its dating. Thus, it is impossible to determine the exact frequency composition of any time series, the bounds (time-wise) being broader and more relevant at lower frequencies.

Any dating procedure should therefore take into account this uncertainty principle, by replacing point estimates by intervals centered around the turning point. This amounts to move from point estimates of turning points to interval estimates. Model calibration, evaluation of forecasts, and more generally, research methodology should be designed accordingly: first, by dismissing the assumption that past dates are precisely known (contrary to the business cycle literature where NBER dates are taken as given); second, by recognizing the existing uncertainty during the analysis or evaluation (which is different from the forecast error). Concretely, we suggest the following interval estimate

$$t_p \pm \Delta t * 2^j / 4 \, , \tag{6.15}$$

where j is the maximum scale used for the dating, to replace the point estimate t_p.

We use the MODWT to study the US house prices[2] and apply rule (6.13) to identify peaks (troughs) at every scale j using the interval estimate provided by (6.15). Figure 6.2 reveals a rich characterization of turning points. The most striking feature is provided by level 4 (bottom panel), in which the four major visually obvious peaks and troughs in the top panel are nicely localized, together with their span. Lower levels, with higher resolutions, confirm them with narrowing them down with smaller interval estimates. The resolution increase obtained by the lower levels brings additional secondary peaks and troughs. Combining the four levels of resolution provides a hierarchy of peaks and troughs shown in the top panel.

It is also worth mentioning the absence of a clear trough before the 1990 peak, as the latter is immediately preceded by the 1979 peak. This challenge the idea of peak-trough-peak cycles. We observe that two consecutive peaks (in the appropriate frequency bands) might already capture a full financial cycle, and analogous troughs might not appear. Hence, peaks might not always alternate with troughs, and the financial cycle might be better characterized in terms of peak-to-peak distances. A possible explanation for this phenomenon is the determinant role that governments and central banks play in the development of credit and housing markets, as they are able to significantly change market conditions via public policies, such as expansionary monetary measures and fiscal

[2] Other decompositions for house prices, equity indices, and credit for OECD countries are available in the supplementary material.

stimulus.

Table 6.1: Integration tests of financial time series in OECD countries. Results are based on the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for a unit root in univariate time series. I(1) and I(2) indicate, respectively, rejection of the stationarity hypothesis in log levels and log returns. We employed a 0.05 α-level and selected the lag number according to the Schwarz criterion.

	Credit		House Index		Equity		GDP	
Country	1995	2015	1995	2015	1995	2015	1995	2015
AUS	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
BEL	I(2)	I(1)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
CAN	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
CHE	I(1)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
DEU	I(2)	I(2)	I(1)	I(2)	I(1)	I(1)	I(1)	I(1)
DNK	I(2)	I(2)	I(1)	I(2)	I(1)	I(1)	I(1)	I(1)
ESP	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
FIN	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(2)	I(1)
FRA	I(2)	I(2)	I(1)	I(2)	I(1)	I(1)	I(1)	I(1)
GBR	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
IRL	I(2)	I(2)	I(1)	I(2)	I(1)	I(1)	I(1)	I(2)
ITA	I(1)	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)
JPN	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
KOR	I(1)	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)
NLD	I(1)	I(2)	I(2)	I(2)	I(1)	I(2)	I(1)	I(1)
NOR	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)	I(1)
SWE	I(2)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
USA	I(1)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)
ZAF	I(1)	I(2)	I(2)	I(2)	I(1)	I(1)	I(1)	I(1)

6.3.4 Frequency analysis

Complementing the interval estimates of peaks and troughs, the analysis of business and financial cycles can be performed by using the band-pass filters provided by the MODWT. This provides a localized frequency decomposition that addresses crucial issues of lack of stationarity and implicit assumptions.

Indeed, the standard approach consisting in differentiating the data to ensure stationarity switches the analysis to the growth cycle, which leads to a common source of confusion within the business cycle literature [Harding and Pagan, 2002]. On the other hand, filters for stationary data often assume a non-unit root process, or constant variance across the frequency components (see e.g Corbae and Ouliaris [2006]). But, credit and house prices are highly non-stationary. Indeed, statistical tests suggest either a $I(1)$ or $I(2)$ dynamics, depending on the end point of the sample, as reported in table 6.1.

In using the MODWT, a concern is that the wavelet choice impacts variance and correlation estimates across the different scales. Nevertheless, this should be seen as a feature, as the trade-off between the frequency and the non-parametric analysis is made explicit. For instance, estimates based on the Haar wavelet will be consistent with non-parametric tipping points à la Bry and Boschan, but they are prone to frequency leakage.

Application of other wavelets will lead to better estimates, but the connection between the frequency components and the local minima and maxima in the original time series becomes less clear.

Table 6.2: Average variance estimates of financial time series in OECD economies. Values are based on the Haar and the DB3 wavelets.

Series	Haar wavelet			DB3 wavelet		
	$\sum_{j=1}^{4}\hat{\sigma}(\tau_j)^2$	$\hat{\sigma}(\tau_5)^2$	Ratio	$\sum_{j=1}^{4}\hat{\sigma}(\tau_j)^2$	$\hat{\sigma}(\tau_5)^2$	Ratio
Credit	0.614	1.315	2.142	0.098	0.323	3.306
House Index	0.527	0.763	1.449	0.182	0.473	2.595
Equity	3.251	2.931	0.902	2.482	2.788	1.123
GDP	0.199	0.410	2.062	0.046	0.051	1.089

To illustrate this point, we used wavelets to revisit some of the stylized facts of the financial cycle via a time-frequency decomposition. Table 6.2 presents the average ratio of the variance estimates at scale 5 divided by the variances estimates across scales 1-4. Consistent with Drehmann et al. [2012], the ratios for credit and house prices exhibit significant energy concentration at lower scales, supporting their high activity in medium term fluctuations. However, although the GDP variance estimate based on the Haar wavelet points to a similar direction, this pattern disappears when using the Daubechies wavelet DB3 with 3 vanishing moments.

6.4 Conclusions

We have presented dating rules for the financial cycle based on the Maximum Overlap Discrete Wavelet Transform (MODWT). Our analysis not only stresses the connection and involved trade-offs between the turning point identification and the frequency decomposition, but also introduces the need to consider uncertainty in the dating procedure. We thus propose to replace point estimates of turning points by interval estimates, which are themselves function of the scale of analysis.

As discussed, our proposal has implications for economic forecasting, and in particular turning point forecasting. In addition, it has implications for economic modeling, as the output of a model should be consistent with the time-frequency structure of financial time series revealed by the wavelet analysis. Finally, the rules that we propose are highly relevant for empirical research on financial crises, as they are suitable substitutes for the ad hoc time-threshold that scholars tend to choose to determine the role of economic variables in the vicinity of crises (see e.g. Gorton and Ordoñez [2016] and Jordà et al. [2015]).

Chapter 7

Future directions for research in real estate bubbles

The contributions of this thesis have mostly revolved around methods to ex-ante identify real estate bubbles regimes, based on house price dynamics. We illustrated the applicability of these techniques, showed that they have significant information content regarding the end of the bubble, and provided means to integrate the role of macroeconomic variables into the analysis. These methods can certainly be improved in terms of performance and interpretability. Their applicability to other markers can also be tested. However, the aftermath of the financial crisis made clear that merely identifying bubbles is not enough. A bigger and strongly needed research agenda would include alternatives to determine and control the impact (negative and positive effects) that bubbles poses to the economy, on top of methods to anticipate the probability for them to reach their turning point. In this final chapter, we discuss important gaps of the literature and suggest directions for them to be addressed.

7.1 Bubbles and systemic risk

Methods to ex-ante evaluate systemic risks have yet to mature. As Hansen [2012] concludes, the compendium of systemic risk measures identified in [Bisias et al., 2012] should be viewed merely as an interesting start. Approaches borrowed directly from risk management theory such as value-at-risk are agnostic by construction, whereas state-of-the-art Bayesian VAR methods are still unable to forecast tail events [Österholm, 2012]. As we have seen, bubble indicators attempting to identify explosive price dynamics such as those developed by Phillips and Yu [2011], Homm and Breitung [2012], and Sornette [2003] exhibit weak links to the factors fueling the bubble, possibly under-utilize other precursors for the bubbles turning point, and are not meant to quantify the impact of the bubble end. There is thus a strong case for a new generation of indicators tailored to the lesson of the last years.

In Hansen [2012]'s words, the need for sound theoretical underpinnings for producing policy relevant research identified by Koopmans many decades ago still applies to the quantification of systemic risk. Such quantification needs to be grounded on the theoretical literature on financial bubbles, which can be seen as attempts to formalize Minsky's characterization of bubbles. In this characterization, an initial displacement - for example a new technology - leads to expectations of increased profits and economic growth. This is followed by a run-up phase, and eventually a crisis where investors dump the assets.

Research on bubble has mostly focused on the understanding of the run-up phase of bubbles, leaving others aspects such as the emergence, propagation, and deflation basically unattended (Caballero and Krishnamurthy [2006]; Abreu and Brunnermeier [2003]; Scheinkman and Xiong [2004]; Doblas-Madrid [2012]; Santos and Woodford [1997]). For example, Scheinkman and Xiong [2004]'s model is based on the expectation of traders to take advantage from the future differences in opinions and existence of short-selling constraints. Abreu and Brunnermeier [2003] provide a setting in which news events, by enabling synchronization, can have a disproportionate impact relative to their intrinsic informational content. Doblas-Madrid [2012] shows that riding bubbles is optimal as long as the growth rate of the bubble and the probability of selling before the crash are high enough. All these bubble models posit that bubbles burst essentially instantaneously. How bubbles deflate over time, how they propagate and correlate with other assets, and how they emerge is not very well understood [Brunnermeier and Oehmke, 2013]. Similarly, the literature has yet to converge on the shared role that financial frictions and behavior factors play during a bubble regime.

7.1.1 Emergence and Deflation of Bubbles

In practice, real estate bubbles deflate over time exhibiting heterogeneous patterns. The issue is particularly puzzling within OECD economies since, after the wave of housing bubbles stopped, prices among the different countries have followed divergent paths [Hirata et al., 2012]. Part of the problem is that real estate bubbles have been mostly treated within the literature of business cycles (in the form of booms and busts), whereas theoretical models of bubbles, like those described above, are circumscribed to simplified types of securities suitable to study the properties of financial bubbles in general. As such, the mechanisms emphasized in these models do not fully capture the particularities of the real estate markets. Real estate is special not only because of the proven systemic risks that it induces, but also because of its frictions and the overwhelming presence of less sophisticated investors in the market.

The body of literature that conceptualized bubbles in a macroeconomic context has not tackled these gaps. The contributions stemming from this field analyze the equilibrium implications of bubbles by incorporating stylized models with households, entrepreneurs, and intermediaries facing different types of constraints [Brunnermeier and Oehmke, 2013]. For instance, Wang et al. [2015] propose a model where housing bubbles emerge because entrepreneurs face borrowing constraints and housing provide liquidity. Caballero and

Krishnamurthy [2006] show that financial under-development facilitates the existence of bubbles and leads agents to undervalue aggregated risk. Ventura [2012] presents a model where bubbles tend to appear and expand in countries where productivity is low relative to the rest of the world. Miao and Wang [2014] show that sectorial bubbles (including real estate) have a capital reallocation effect in the sense that bubbles in a sector attract more capital to be reallocated to that sector. Hence, according to the latter model, bubbles may misallocate resources across sectors and reduce welfare.

There are several limitations to these approaches. The bubbles analyzed by these methods are rational expectations bubbles. In a typical rational expectations bubble model, agents have identical rational expectations, but prices include an extra bubble component that is always expected to grow on average (taking into account the effect of the crash) at the rate equal to the risk free rate. Models of rational bubbles are incapable of explaining the increase in trading volume that is typically observed in the historic bubble periods, and have non-stationarity properties that are at odds with empirical observations [Scheinkman and Xiong, 2004]. Statistical tests conducted by Giglio et al. [2016] on the housing markets of Singapore and the U.K., which according to the authors give the best chances of detecting a bubble in the data, show that no infinitely lived bubble was present. These models provide conditions for an economic equilibrium with bubbles, but they do not explain the dynamics that lead to the boom. In other words, bubbles in these models do not emerge, but they exist or appear randomly and it is unclear when/under what circumstances a market moves from a stable to an unstable regime.

A related stream of literature comprises the set of dynamical stochastic general equilibrium (DSGE) models, which analyze the dynamics of booms and busts in the presence of financial frictions and attempt to explain the fluctuations in house prices resulting from economic shocks (Iacoviello [2005]; Iacoviello and Neri [2010]; Lambertini et al. [2013b]; Mendoza [2010]; Tomura [2010]; Nguyen [2013]). DSGE models are typically populated by rational agents and do not contemplate the possibility of bubbles as sources of excessive run-ups. DSGE models that exhibit nonlinear behavior are based on either one of two standard extensions to the New Keynesian framework that features financial frictions. In the first framework, households and firms face collateral constraints that limit the quantity of loans [Iacoviello, 2005]. The second one corresponds to the financial accelerated framework developed by Bernanke et al. [1999], where nonlinearities arise as a result of the cost for financial intermediaries of monitoring entrepreneurs.

During the last crisis, DSGE model underwent heavy attacks (Stiglitz [2011]; Caballero [2010]). Not only were these models unable to anticipate the turning point, but also they did not foresee the depth and breath of the recession. DSGE models tend to underestimate the collapse in asset prices and do not sufficiently capture nonlinear and asymmetric effects (Mendoza [2010]; Tomura [2010]). According to De Grauwe [2010], the problem of DSGE-models (and more generally of macroeconomic models based on rational expectations) is that they assume extraordinary cognitive capabilities of individual agents, and need a lot of ad-hoc assumptions to make them fit the data. DSGE models able to answer the above mentioned critiques have only recently been developed. Interestingly, they incorporate endogenous out-of-equilibrium dynamics, though they have yet to center the analysis on the housing market (Brunnermeier et al. [2012]; He and Krishnamurthy [2011]). For instance, Brunnermeier and Sannikov [2014] present a model where nonlinearity as amplification is completely absent near the steady state but becomes large away from it. The latter model also exhibits slow recovery, endogenous risks, and what the authors call the volatility paradox; that is, the idea that the system is prone to crisis even if exogenous risk is low. These insights resonate with other modeling approaches coming from econophysics [Sornette, 2014]. For example, Bonart et al. [2014] study a dynamical model of interconnected firms. In their model, the standard rational equilibrium is still formally a stationary solution of the dynamics, but this equilibrium becomes linearly unstable in a whole region of parameter space.

Fagiolo and Roventini [2012] have argued that, instead of trying to add additional frictions to models, economists should consider the economy as a complex evolving system, i.e. as an ecology populated by heterogeneous agents whose far-from-equilibrium interactions continuously change the structure of the system. In this regard, alternatives, such as agent-based computational economics ACE [LeBaron and Tesfatsion, 2008] have been proposed. However, the ACE methodology is still in its infancy and authors such as Gualdi et al. [2015] and Sornette [2014] have reasoned that ACE, tough promising, face significant methodological challenges as they have yet to overcome the curse of dimensionality and the Lucas critique. A promising contribution for the ABM literature was done by Geanakoplos et al. [2012], which by using mortgages payment data at the zip code level were able to forecast the US housing price index and gauge the impact of several macroeconomic variables on such prices. Unfortunately, their methodology requires extensive amounts of data that to our knowledge is unavailable at the international level.

These recent works (both ACE and dynamic models with non-linearities) however, are important because they shed light on the small shocks, large business cycles puzzle. Up to now, bubbles in macroeconomics have been analyzed under the lenses of deterministic models that require a single shock that moves the economy towards a bubble regime, represented as steady state in a high valuation equilibrium. As a result, they have left unanswered the question of how small and different shocks can collude to create bubbles. In simpler words, we are only beginning to formalize the story of the perfect storm. Duca et al. [2012] have gathered empirical evidence to explain the subprime crisis, but a formal framework focused on real estate is yet to appear.

The methodological tools used by Bonart et al. [2014] and the continuous stochastic approach used by Brunnermeier and Sannikov [2014] may be used to improve our understanding of a real estate bubbles, but it has to be done in a comprehensive framework, with an eye on the necessary empirical research. In this ideal framework, as we argue below, we need to include the current account and international real estate markets, the balance sheet of households, and move away from the representative agent model to include heterogeneous agents pertinent to the market and the context in which they make decisions. Finally, in order to understand how real estate bubbles deflate, we also need to gather insights from urban economics, as the elasticity of supply has proven to be a key factor in the correction of the prices after a booming period (Anundsen and Jansen [2013]; Caldera and Johansson [2013]; Glaeser et al. [2008]; Muellbauer and Murphy [2008]).

7.1.2 Propagation of Real Estate Bubbles and Linkages with Other Assets

As part of our still limited understanding concerning a real estate bubbles life cycle, we do not know enough about how real estate bubbles propagate internationally and among assets. Most real estate bubbles do not develop alone. Both, during the 90s boom and during the 2002-2008 boom, accelerated price developments manifested simultaneously in several countries and several assets. However, some countries tend not to be relevant in bubble models. What determined the extent to which bubbles arise in multiples countries? Why some countries are spared from the booming period? What can break the synchronization between markets? The linkage with capital flows and the current account with the real estate market is not new. However, the interpretation of the linkages between real estate prices and capital flows is subject to considerable debate, even in the most recent crisis. The question is relevant and ought to be analyzed from a behavioral and a macroeconomic point of view, especially as policy makers attempt to curb local price development when movements may stem from international common factors.

Among the hypotheses considered for the last crisis, a body of literature under the name of international imbalances has emerged (Caballero et al. [2006]; Basco [2014]). One leading explanation has been the existence of a global savings glut, in which developing economies have sought to store value in the developed world (Bernanke et al. [2005]; Caballero and Krishnamurthy [2006]). However, Laibson and Mollerstrom [2010] contended that the theory of a global savings glut does not explain the observed data. Jinjarak and Sheffrin [2011] suggest that the idea of a savings glut driving real estate booms is also misplaced: to the extent that current account surpluses affected real estate, they were mediated through financial markets. Hume and Sentence [2009] add that while the global savings glut may account for the cycle's initial phase, other factors - such as the conduct of monetary policy and perceptions of declining macroeconomic risk - were more important from the mid-2000s onwards. Likewise, In't Veld et al. [2011] estimate an open economy DSGE model with financial frictions for the US and the rest of the world to evaluate various competing explanations about the recent boom-bust cycle. They find that the savings glut hypothesis is insufficient for explaining all aspects of the boom in the US, and claim that bubbles in the stock and housing market are crucial.

Despite the apparent consensus on the importance of housing market movements for the real economy, our understanding of the sources of synchronization in housing markets is rather limited, as the findings, nature and interpretation of shocks vary significantly across empirical studies (Hirata et al. [2012]; Bracke [2013]). From the growing literature that analyzes the importance of various shocks in driving national and global house prices, this is good evidence that there are significant spillovers among different countries, but they might spill in a heterogeneous way [Ciccarelli et al., 2012]. Global, regional, and country-specific factors need to be taken into account in order to understand the origin and deflation of housing bubbles. While the great moderation was observed in most countries, the timing and magnitude of the decline of volatility differed substantially across countries [Stock and Watson, 2005]. The studies are supportive of both current account, and credit growth channels, and provide mixed evidence for monetary and interest rate channels. Some authors have mentioned the possibility of momentum channels playing an important role, whereas others emphasize country-specific house price shocks in the transmission and synchronization of house prices. On top of this, the consensus on the high degree of synchronization of housing market across countries needs to be taken with caution, as most of the research comprises only OECD economies, giving a biased picture. The few studies that encompass advanced and developing economies tend to confirm the existence of regional factors [Cesa-Bianchi, 2013]. Hence, more work concerning the diffusion of bubbles across different countries is needed.

Likewise, the linkages of housing prices with other assets are the source of constant debate. How housing booms and bubbles in other assets interact? On the one hand, real estate prices are considered important indicators for deep and long recessions [Claessens et al., 2012]. On the other hand, forecasting papers have found unstable forecasting power of financial markets to output growth and real estate prices (Galvão [2013]; Stock and Watson [2003]). At the same time, theoretical models predict that bubbles in different assets can coexist and interact with one another. Dynamic models exploring the possibility of running bubbles are scarce, as they do not allow for more than one risky asset (and thus one bubble). Different theories need to be empirically and theoretically explored. For example, can the difference in productivity rates explain different appreciation rates? Do bubbles feed from one another instead of cannibalizing each other? How has the dynamics of other asset markets affected housing bubbles? Can a succession of mini-crashes on different asset markets induce long-term correlations in the real estate markets that prolong the booming period and exacerbate the subsequent correction?

7.1.3 Financial Frictions and Behavioral Factors

Lambertini et al. [2013a] and Brunnermeier and Oehmke [2013], among others, have stressed the importance of considering the behavior of economic agents to understand boom and bust periods. However, there is also a lack of understanding of the interplay between behavioral aspects of real estate bubbles and the frictions that are conducive for them to happen. Behavioral explanations typically invoke animal spirit arguments of the sort described by Akerlof and Schiller [2009] whereas financial-friction models endogenize financial constraints and agency problems. Very few papers model explicitly heterogeneous agents along with macroeconomic constraints (other than the common differentiation between borrowers and lenders) and, if they do so, they sacrifice the modeling of macroeconomic frictions or they introduce a classification of agents more appropriate for financial markets (e.g. speculative and fundamental traders). Even less progress has been made in determining the role of speculators in the real estate booms, as many critics in the popular press point to speculators as fueling the 2000 2006 US housing bubble, but data suggest that investors represented a small proportion of total transactions [Mayer et al., 2009]. The problem is that the imposition of frictions, such as borrowing constraints, mimics the effect of behavioral models and it is difficult to distinguish empirically between behavioral and non-behavioral explanations [Driscoll and Holden, 2014].

An important lesson from the last crisis is that high prices paid during the boom and the low prices paid during the bust are typically compatible with reasonable models of housing valuation [Ely, 2013]. Thus, buyers do not seem to be irrational but rather cognitively limited investors. Lambertini et al. [2013a] provide evidence on the importance of news and consumers beliefs for housing-market dynamics and aggregate fluctuations. This has motivated a modeling approach in which households are classified between optimistic and pessimistic beliefs. Piazzesi and Schneider [2009] find that a small number of optimists can have a large impact on housing prices. However, in their model, heterogeneity in beliefs per se is not enough to generate protracted booms and busts of the sort observed in the data. Tomura [2013] constructs a model where the over-optimism of mortgage borrowers generates housing-market boom-bust cycles, if mortgage borrowers are credit-constrained and savers do not share their optimism. However, the model replicates stylized features of housing-market boom-bust cycles only in developed countries. Rebelo et al. [2012] present a model where agents with heterogeneous expectations can change their view because of social dynamics. The nature of boom-bust episodes is determined ex-post depending on which agents happened to be correct about the future. Dieci and Westerhoff [2013] show the destabilizing effect that speculators can have when their number is allowed to vary or when the speculators switch between rules.

Both explanations are complementary, and we need to determine the extent to which each of them impacts the market. Namely, to disentangle financial frictions from behavioral patterns in order to understand under what circumstances agents behaviors can become destabilizing factors. For instance, we need to explore the impact of and interplay between different types investor onto the markets, such as the value investor proposed by Graham [2006], and recently explored by Buchsteiner and Zavodov [2015] in order to understand asset bubbles in an international context.

7.1.4 Real estate bubbles and growth

There is now a growing consensus that credit-fueled bubbles lead to deep and long recessions. Accordingly, it is now believed that central banks and governments ought to pro-actively implement macroprudential policies. As these policies have the potential to stiffen growth, we need to better understand the positive benefits that a bubble might have, as well as being able to distinguish between bad and good booming periods. Important growth theories with bubbles have come from Olivier [2000], [Sornette, 2008], [Gisler et al., 2011], [Martin and Ventura, 2012], Hirano et al. [2015], and Matsuoka and Shibata [2012].

Olivier [2000] develops a model in the context of speculative bubbles and concluded that the real impact of a bubble depends on the type of asset that is the target of speculation. [Sornette, 2008] and [Gisler et al., 2011] posit the "social bubble hypothesis": bubbles can be very beneficial by making society at large take large risks, which would not otherwise face from a standard cost-benefit analysis; the large risks allow innovators to get funding to explore novel R&D paths, sometimes leading to great discoveries. Martin and Ventura [2012] construct a stylized overlapping-generation (OLG) model of economic growth with bubbles to show how bubbles can mitigate the effects of financial frictions. Hirano et al. [2015] examines whether bubbles are growth-enhancing or growth-impairing in the long run. They show that, when the quality of the financial system is relatively high, bubbles boost long-run growth, and the effect of the burst depends on the quality of the financial system. Matsuoka and Shibata [2012] introduce a bubbly asset with credit market imperfections and multiple technologies, and show that multiple bubbly steady states can exist, and bubbles may cause underdevelopment traps by preventing the adoption of high productivity technology.

However, these models are only meant to illustrate stylized facts, and we are far from understanding the empirical consequences that curbing a real estate bubble might have. Not surprisingly, Crowe et al. [2013] surveyed the literature of macro prudential policies to conclude that the correct policy response to real estate booms is deemed as an art more than a science. As a very familiar example, the impact of the multiple SNB regulatory measures that we discussed in chapter 2 has been unclear.

7.2 How to tackle these challenges?

Given the previous discussion, we argue that four main research questions should be addressed in order to move forward the research agenda on bubbles and systemic risks:

- Emergence and Deflation of Bubbles: Can we determine the common circumstances under which real estate bubbles emerge? Can we characterize the possible dynamics after a real estate bubbles turning point (e.g. soft landing or crash)? Why have real estate bubbles deflated differently in different countries? Why have they had divergent consequences on economic output? How do financial frictions impact the deflation dynamics of bubbles? How can we distinguish between real estate booms and real estate bubbles?

- Propagation of Real Estate Bubbles and Linkages with Other Assets: What is the relationship between international real estate bubbles and other assets? Are booms in other assets conducive or adverse to the emergence and growth of real estate bubbles? When and how do local, regional, and global factors matter? What transmission channels instigate and break the synchronization among international real estate booms?

- Financial Frictions and Behavioral Factors: How can behavioral factors and the underlying interacting structure among economic agents amplify or mitigate real estate bubbles? Can we identify the transmission channels that modulate the effect of agents with bounded rationality on real estate bubbles? How to better characterize behavioral biases and the structure of agents in a real estate context?

- Identification of Bubbles and Systemic Risk: Can we use the insights from the above mentioned questions to estimate the risk that a real estate bubble poses to the economy, and even forecast their potential consequences? How can we characterize the macroeconomic consequences of crisis level shocks that are very large but infrequent? How can we manage and mitigate the risk in such a way that economic growth is not stiffened?

Ideally, these questions should be tackled in a comprehensive framework that accounts for all phases of real estate bubbles. In general terms, we believe in the suitability of a Bayesian methodology, balancing existing methods with the application of state of the art techniques. This approach requires knowledge of economics and of economic data, stochastic processes, non-linear dynamic models, and skills in statistical learning theory.

In Bayesian inference, the information of a distribution, called the prior, is used to updated the probability estimate for a hypothesis as additional evidenced is acquired. This framework is nowadays widely recognized as an ideal framework to combine different sources of information that go beyond the sole sample. We envisage two possible uses for the priors. First, to impose loose restrictions associated with theoretical economic concepts. Second, to regularize the likelihood functions in areas of the parameter space in which the estimation becomes difficult and to facilitate the shrinkage of parameters; that is, the realm of statistical learning theory.

7.2.1 Theoretical economic concepts

Macroeconomic insights may be incorporated through priors coming from dynamic models with different financial frictions. As we focus on the housing sector, the collateral frictions introduced by Iacoviello [2005] and Iacoviello and Neri [2010], as well as their subsequent extensions (e.g. Favilukis et al. [2010]; Kiyotaki et al. [2011]) may be a sound start. They allow for an open economy with multiple risky assets, and may be extended based on the critiques raised by Muellbauer and Murphy [2008], the insights of [Mian and Sufi, 2011] about the role of households debt during the last crisis, and the relevant ideas in this regard that were discussed in section 7.1. The modeling approach used by Brunnermeier and Sannikov [2014] may be advantageous, since the concept of equilibrium introduced in their framework is more compatible with the idea of an economy that may develop endogenous risks when put out-of-equilibrium.

Similarly, new priors may be introduced in order to deal with the difficulty to observe extreme events in a finite sample, the goal to model endogenous risk, and the idea to consider richer structures between economic agents. This is a significant deviation from mainstream macroeconomic research, which is pervaded with priors that induce a stationary bias on the system and do not take into account the dependence structure among agents (e.g. the Minnesota prior). Such stationary priors may be justified in a long-term analysis, but they may also miss the dynamics of the economy during the transient phase associated with a bubble, which in our understanding lead to criticality and rather abrupt changes of regimes. Therefore, concepts and tools borrowed from the science of phase transitions and critical phenomena [Sornette, 2006] may be used to illuminate these issues, in an attempt to introduce priors that characterize the instability in a tractable and parsimonious fashion.

As an example, consider the Ising model (see the review in Sornette [2014]). Conventional macrodynamic models are particularly weak to represent relevant agent-interaction as well as the structure of these interactions. The standard objective function is one in which agents maximize time-separable utility that depends only on the agent's own consumption (often plus labor supply or leisure), but it is mostly independent of the environment [Turnovsky, 2011]. On the contrary, the Ising model is a good example that can be used to model social interaction behavior among agents. The Ising model, introduced initially as a mathematical model of ferromagnetism in statistical mechanics [Brush, 1967], is now part of the common culture of physics, as the simplest representation of interacting elements with a finite number of possible states. The model consists of a large number of magnetic moments (or spins) connected by links within a graph, network or grid. In the simplest version, the spins can only take two values (+-1), which represent the direction in which they point (up or down). Each spin interacts with its direct neighbors, tending to align together in a common direction, while the temperature tends to make the spin orientations random. Due to the fight between the ordering alignment interaction and the disordering temperature, the Ising model exhibits a non-trivial phase transition in systems at and above two spatial dimensions and in networks of sufficiently high connectivity. Beyond ferromagnetism, it has developed into different generalized forms that find interesting applications in the physics of ill-condensed matter such as spin-glasses Mézard et al. [1990] and in neurobiology [Hopfield, 1982]. Since its introduction, the Ising model and its variations have been extensively used in models of social cooperation and of collective behaviors (see Sornette [2014] for a review and references therein, as well as Huang [2015]). In the research of our group at ETH Zurich, we have already used variants of the Ising model to analyze bubbles and crashes. For example, Harras and Sornette [2011] studied a simple agent-based model of bubbles to clarify how their proximate triggering factor relates to their fundamental mechanism. The model offers a simple reconciliation of the two opposite (herding versus fundamental) proposals for the origin of bubbles and crashes within a single framework and justifies the existence of two populations in the distribution of returns, exemplifying the concept that crashes are qualitatively different from the rest of the price moves [Johansen and Sornette, 2010].

We propose the use of priors reflecting the dynamics of Ising-like Bubble to analyze real estate bubbles. Chen et al. [2014] scratch the surface on how this can be done by using a very stylized dynamic model. They compare the mesoscopic modeling of market sentiment using the Brock-Hommes adaptive belief system (ABS), with the microscopic modeling of market sentiment by applying the Ising model to different social networks. They find that it is very difficult to reproduce the dynamics generated by the Ising model using different parameterizations of the ABS machine, and therefore there are some gains. We stressed that this is only the tip of the iceberg as, if necessary, other tools such as random graphs, analysis of long-term correlations, among others [Sornette, 2006], can be useful in order to circumvent the current drawbacks of the representative agent, quantify the distance to a regime change, and estimate the fragility of the system.

7.2.2 Statistical Learning: Large Scale Models, Penalized Likelihood, and Mixed Frequency Data

Currently, there is a fundamental tension in empirical analysis within economics between (i) relying on prior beliefs based on theory and empirical evidence - encoded in classical hypotheses or in Bayesian priors - and (ii) letting data speak freely [Hansen, 2012]. We need to continue experimenting at the boundary between non-parametric methods and the structure imposed in traditional econometric analyses. In particular, the success of penalized likelihood methods and other forms of regularizations to forecast macroeconomic time series using a large number of predictors make the case for the exploitation of a large number of predictors, in which proxies and frictions to macroeconomic variables are discovered rather than imposed on the system. This may be done not only using standard (FA)VAR methodologies [Stock and Watson, 2012], but also, as in [Ardila et al., 2016c], with techniques that rely on penalized likelihood methods to select the variables highly correlated with our objective variables (possibly house prices, output growth, though we will also consider the possibility of treating the parameters associated to these time series as nuisance parameters to focus on the measurement of systemic risks). Typically, these methods introduce L1-like constraints in order to identify a sparse combination of the variables. There are several advantages of using penalized likelihood methods in a large-scale context. The interpretation of the factors is facilitated as only time series that provide significant information are selected. Low-dimensional structures can be easily explored among huge numbers of candidate models. Data can be used more efficiently as specific properties are emphasized, while nuisance features can be disregarded. Finally, noise accumulation, which is a common problem in high dimensional statistical learning, can be mitigated.

In addition, the scarcity of macroeconomic data, and the long-lasting repercussions that a real estate bubble might bring about, calls for more efficient use of the data, especially since most macroeconomic time series are only observed at monthly or quarterly frequencies. Recent important contributions in macroeconomic forecasting that use mixed frequency data, such as MIDAS and MS-MIDAS (Guerin and Marcelino, 2013), as well as factors models that allow richer correlation structures such as multi-level factor models (Moench and Ng [2011]; Moench et al. [2013]) should be incorporated into the empirical analysis of bubbles in order to improve the calibration of the models and their forecasting properties.

Finally, an increasing role should be given to the treatment of nonlinearities. Currently, theoretical models (including DSGE and other forms of dynamic models) commonly rely on nonlinearities to explain corrections of the sort observed during the subprime crisis. However, linear models have remained at the center of the empirical research in the different attempts to analyze the crisis. The problem is that in practice it is very difficult to distinguish linear dynamics from their nonlinear counterparts. As Ng and Wright [2013] points out, it seems that current methods have limited ability to achieve this endeavor. Essentially, there is an identification issue as different structural models can fit aggregate macroeconomic data about equally well. We need methods and frameworks that explicitly consider nonlinearities and systematically study the conclusions that they produce in comparison to more standard methodologies. The use of explicit nonlinear dynamic models and derived priors will help us mitigate these issues.

Admittedly, non-linear models face their own challenges (Transtrum et al. [2010]; Chen et al. [2011]). They are not easy to calibrate and are inherently more complex. In addition, advanced inference techniques for highly complex models might induce dynamics that are not genuine features of the real system [Mastromatteo and Marsili, 2011]. Recent advances in economics, physics, and biology can help to deal with the estimation of nonlinear stochastic models in a data rich environment. Examples of these methodologies are the frameworks developed by Boivin and Giannoni [2006], Bretó et al. [2009], Giordani et al. [2011], and Nishiyama et al. [2011]. Boivin and Giannoni [2006] construct a framework that provides an interpretation of all information contained in a large data through the lenses of a DSGE model. Bretó et al. [2009] develop a plug-play inference framework using an iterated filtering procedure that estimates implicit dynamic models. Giordani et al. [2011] present the particle filter as a general approach to analyze complex nonlinear state models. Nishiyama et al. [2011] propose a nonparametric test for possibly nonlinear causality. To tackle the complexity challenge, developments in Statistical Learning Theory to elucidate the complexity of macroeconomic forecasting models may be proved helpful, such as the estimation of the Rademacher complexity, the effective sample size, and the generalization error bounds in state-space models (see e.g. McDonald et al. [2012]).

7.3 Closing remarks

The presentation of these ideas has been on purpose wide, and they obviously require further development However, the case for deep understanding of real estate bubbles is clear, and important gaps in the literature have been hereby well documented. A successful response to the above mentioned questions would have practical relevance to areas such as forecasting and risk management. They would also contribute to a much needed-revamp of the theory and methods on economic systems. Beyond the direct scientific impact that addressing the gaps would have, this research would have a much deeper potential. If we are successful in combining interacting heterogeneous agents and other tools from statistical physics with neoclassical elements, this contribution could spill over other fields of finance and economics and be systematically adopted by other research groups, replacing the limited but pervasive approach of the representative agent while maintaining tractability in the models. Likewise, systemic risk indicators guided by theory and with significant forecasting power would contest the idea that systemic crises are unpredictable events for which we can only hope to react once they have happened.

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