Spurious switching processes in financial markets

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The random walk model of Bachelier (1900), later extended into the geometrical Brownian model (GBM), forms a reasonable first-order approximation of the dynamics of financial market prices. While the GBM is based on the generally verified absence of linear correlation of returns, many studies have show in addition such stylized facts as the existence of long-memory in the volatility, volatility clustering and multifractality, fat tails in the distributions of returns, correlation between volatility and volume, time-reversal asymmetry, the leverage effect, gain-loss asymmetries and many others.

Recently, Preis et al. [1, 2] have claimed the discovery of a new stylized fact in the form of universal power laws associated with so-called switching points. They find that local maxima of volatility and volume, and local minima of intertrade times, are reached and followed by power laws in the time to the extrema that are reminiscent of critical points in Physics. The power laws are found to hold from time scales ranging from milliseconds to years.

Here we claim that these power laws are also found in the minimal random walk. They derive from the statistical method used by Preis et al. to define the switching points, namely from conditioning of the statistics on the local trends between successive minima and maxima with respect to time windows of a certain size. Independently of any assumption on the underlying generating process, this definition imposes stringent conditions on the price increments before and after the extrema and lead to skewness of distributions of price changes before and after the peak with the conditional mean being larger than the unconditional mean. This skewness results in the presence of a peak in volatility that Preis et al. [1, 2] interpreted as the existence of emergent critical phase transitions in the dynamics of price and volume processes. We have reproduced exactly the same analysis as Preis et al. [1, 2] on the null model of the random walk and have been able to reproduce not only the peak itself, but also the asymmetry and perhaps even more surprising the power law decay (over one decade of scales) around the peak.

Our finding applies directly to the volatility. The peaks in volume and intertrade intervals and the dynamics around them are also reproduced by adding to the random walk a few simple realistic ingredients. For volumes, one just needs to take into account the well-known correlation between absolute price increments and volume. In contrast to the extremely weak correlation between signed price increments and volume, the correlation between absolute price increments and volume is very strong at the daily time scale and weaker but clearly pronounced at the tick-by-tick time scale. Due to this correlation, the structure of the volume dynamics reproduces qualitatively that of the absolute price increments, showing asymmetric peak with power law decay around it.

The negative peak in the intertrade intervals is a result of the format of transaction price data used in [1, 2]. In order-driven exchanges, large orders are not executed at one price, but trigger several transactions corresponding to "walking the book" where part of an order is executed at successively increasing best next prices until fully completed. Being triggered with a single order, all these transactions appear in the log-file with an identical time stamp, which corresponds to a zero intertrade interval. At the same time, such sequences of transactions are moving price significantly, especially if more than two levels in the order book are involved. To account for this "walk of the book", we have considered a toy model where price follows simple random walk and intertrade intervals τ are a mixture of (i) iid exponentially distributed random variables reflecting a Poisson process for the order flow and (ii) an atom at $\tau = 0$ with given probability mass, which accounts for the times when the price moved in the same direction as in the previous time step. This atom present in the conditional distribution is sufficient to translate the peak of volatility into the negative peak in intertrade times and recover the associated power laws.

While the minimal random walk model cannot reproduce precisely the exact exponents and ranges of the power laws reported by Preis et al. [1, 2], more elaborated models of the well-known financial stylized facts allow to match these values quantitatively. This statement is illustrated with the quasi-multifractal model [3] that accounts for the long-term memory and heavy tailed statistics of real price returns.

We are thus led to conclude that there is no new "switching" phenomenon, as the peaks and power laws are straightforward consequences of the selection of biased statistical subsets of realizations in otherwise featureless processes. In the switching phenomena reported by Preis et al. [1, 2], there is no more than statistical conditioning and some correlations.

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1. **Preis T., J.J. Schneider, and H.E. Stanley (2011) Switching processes in financial markets, PNAS 108 (19): 7674–7678.**
2. **Preis T. (2011) Econophysics: complex correlations and trend switchings in financial time series, The European Physical Journal Special Topics 194 (1): 5–86.**
3. **Saichev A, and V. Filimonov (2008) Numerical simulation of the realizations and spectra of a quasi-multifractal diffusion process, JETP Letters 87 (9): 506–510.**

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