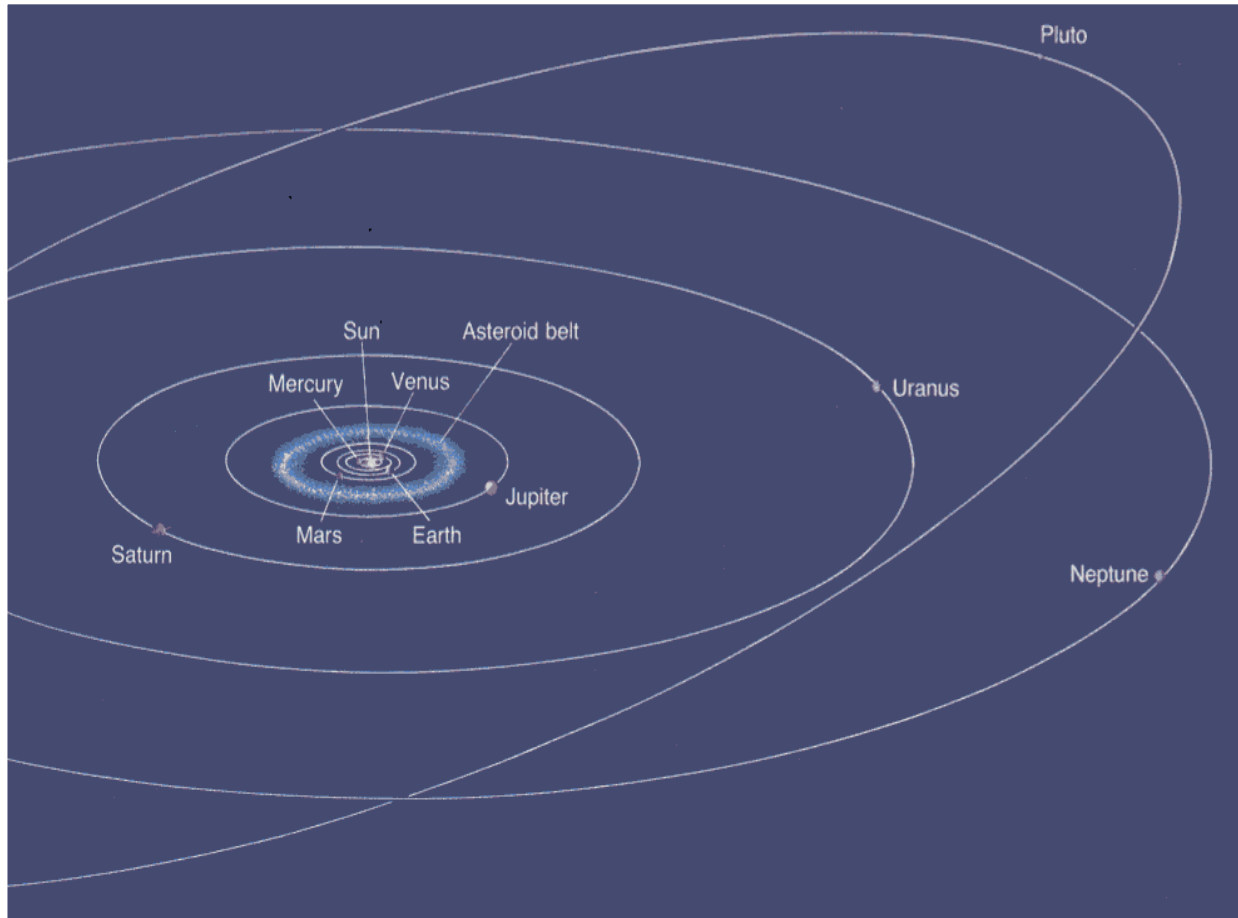


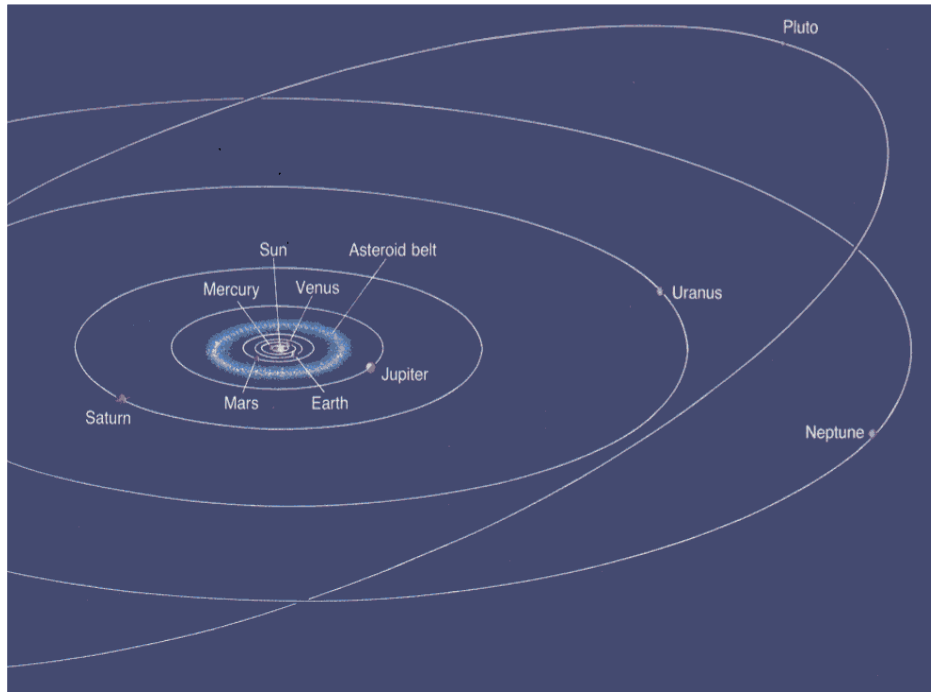
Chapter 2: Our Solar System

1. The formation of planetary systems, based on our own



99.9% of mass is in the Sun, so the gravitational field is simple $\propto r^{-2}$

- Orbits are ellipses with Sun at one focus (Kepler 1)
- Relation between period T and semi-major axis A : $T^2 = (4\pi^2/GM_{\odot}) A^3$ (Kepler 2)
- Constant “areal velocity” $\underline{\mathbf{v}} \times \underline{\mathbf{r}}$ (= conservation of angular momentum) (Kepler 3)



Object	ϵ
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009
Pluto	0.249
Sedna	0.855
Halley's Comet	0.967
Comet Hale-Bopp	0.995

- In our Solar System, the eccentricities of the planets are very low $\epsilon < 0.1$ (except for Mercury and Pluto), i.e. planets have almost circular orbits

$$r_{\text{peri}} = (1 - \epsilon)a \quad r_{\text{ap}} = (1 + \epsilon)a$$

So, variation in solar heating is modest, close encounters do not happen

- Aligned angular momentum vectors of Sun's spin, all planetary orbits (except Pluto), and (almost all) satellite orbits and most planetary spins

Disks are natural result of the conservation of angular momentum, since circular orbits have the highest L for a given energy. Easy to show for Keplerian orbits around central mass that:

$$E = -\frac{GMm}{2a}$$
$$L^2 = GMm^2 \frac{b^2}{a}$$
$$= 2m|E|b^2$$

L is maximised for a given E , or E is minimized for a given L , when $b = a$
(given that $b \leq a$)

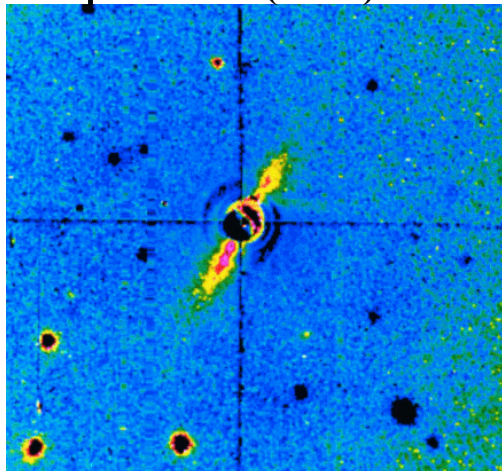
Gas that is losing energy but conserving angular momentum will naturally settle into a flattened spinning disk with circular orbits.

Circular orbits minimize collisions between gas. Collisions \rightarrow heating \rightarrow radiative cooling is how bulk kinetic energy of the gas is lost in the nebula

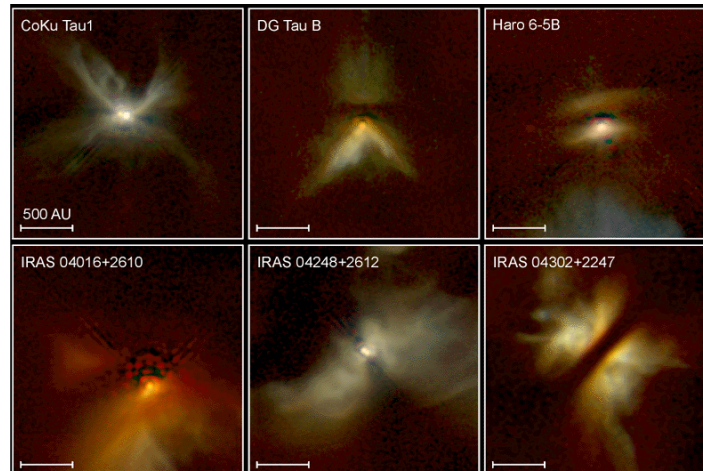
→ Formation of the Solar System out of a flattened rotating gas cloud that also produced the Sun at the center – the “**Solar Nebula**” **accretion disk**. Idea dates from ~1700’s (Kant, Laplace), but are they common?

Similar disks have now been seen ubiquitously associated with forming and recently formed stars (seen in reflected star-light, dust obscuration and thermal emission)

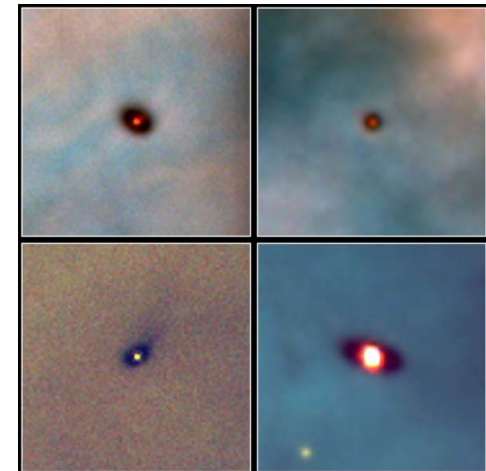
β Pic disk (1984)



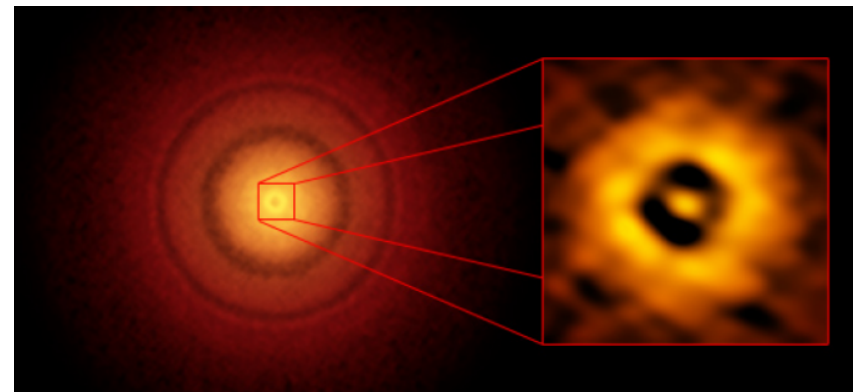
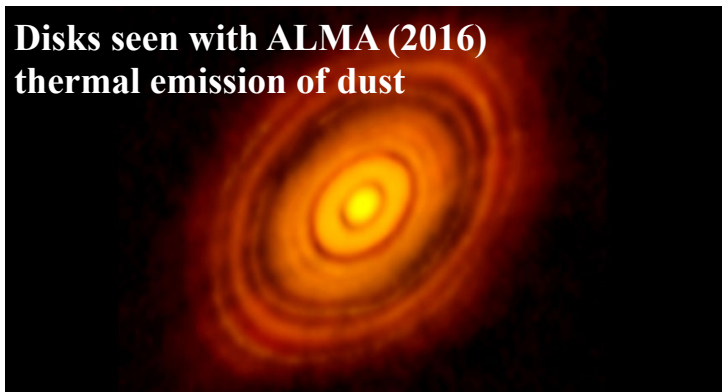
Disks seen edge on around young stars (1995)



Protoplanetary disks shadowing in Orion (1995)



Disks seen with ALMA (2016)
thermal emission of dust

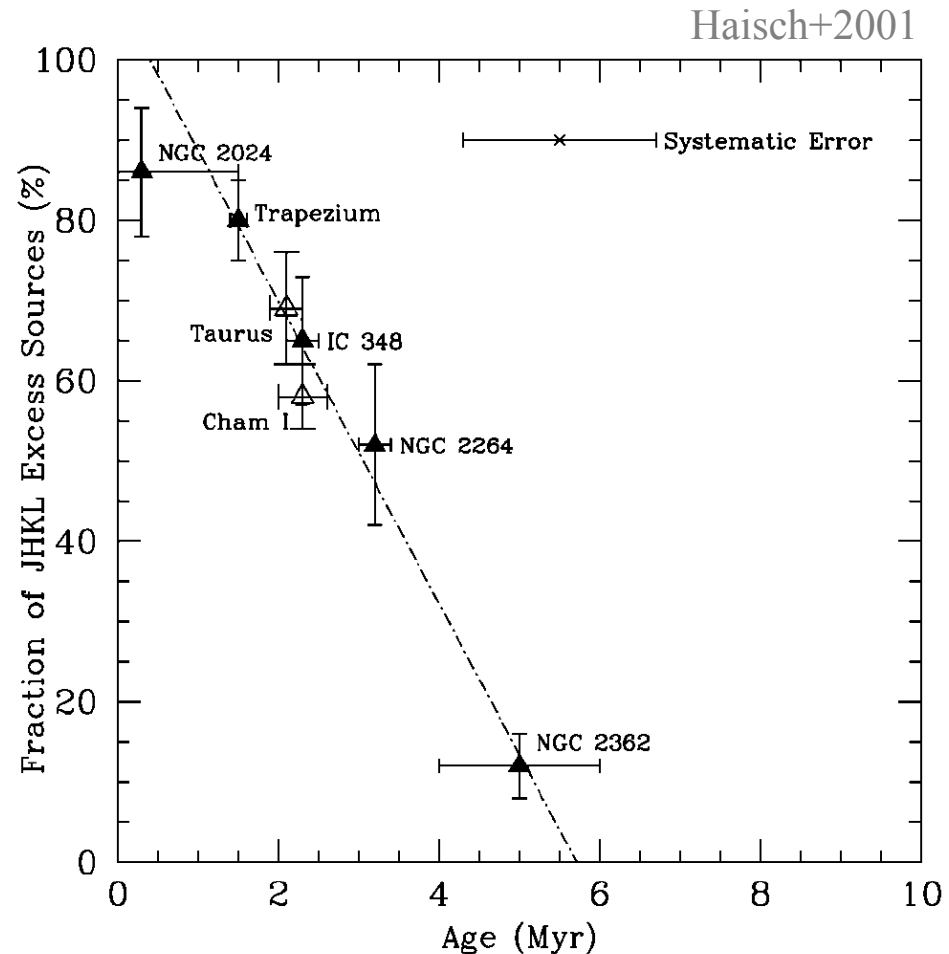


What is the timescale for planetary formation in disks?

The lifetime of pre-planetary dust+gas disks around young stars seems to be short (few million years).

The “JHKL Excess” is due to thermal emission from hot dust in the disks, which radiates in excess of the star at 1-4 μm .

The fraction of stars showing excess steadily decreases with the age of star clusters (stars formed at the same time)



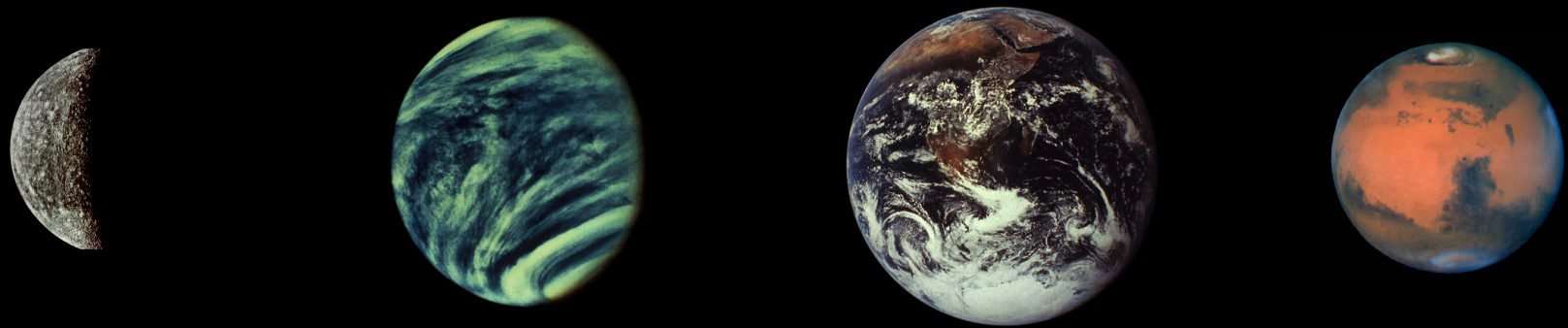
Angular momentum in the Solar System in spins and orbits

$$L_{spin} = \frac{2\pi I}{T_{spin}} \sim \frac{2\pi \cdot 0.1mr^2}{T_{spin}}; \quad L_{orb} = \frac{2\pi mR^2}{T_{orb}}$$

- Almost all of the angular momentum in Solar System is in the orbits of the planets (especially Jupiter) and not in the spin of the Sun.
- The specific angular momentum of planetary orbits is 10^5 larger than for the Sun's spin, and similar to that of a presumed progenitor gas cloud.

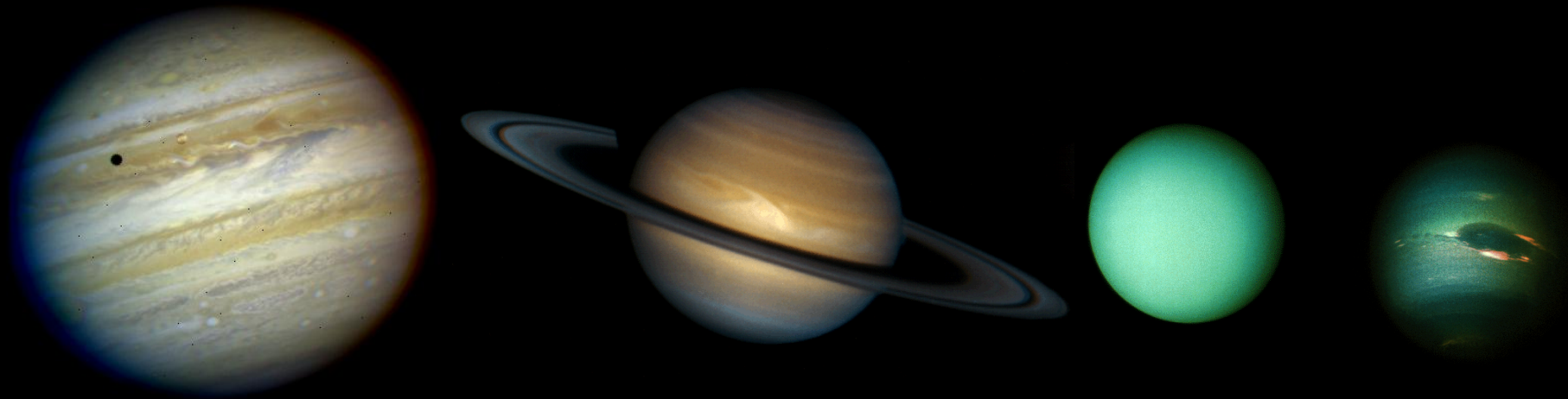
	m (kg)	r (m)	T (s)	L(kg m ² /s)	L/M m ² /s
Sun spin	$2 \cdot 10^{30}$	$6 \cdot 10^8$	$2.6 \cdot 10^6$	$1.7 \cdot 10^{41}$	$8 \cdot 10^{10}$
Earth orbit	$6 \cdot 10^{24}$	$1.5 \cdot 10^{11}$	$3.1 \cdot 10^7$	$2.7 \cdot 10^{40}$	$4.5 \cdot 10^{15}$
Jupiter orbit	$2 \cdot 10^{27}$	$7.8 \cdot 10^{11}$	$3.7 \cdot 10^8$	$2.1 \cdot 10^{43}$	$1.0 \cdot 10^{16}$
Gas Cloud	$2 \cdot 10^{30}$	$9.5 \cdot 10^{15}$ Light year	$3 \cdot 10^{15}$ 10^8 years	10^{47}	$5 \cdot 10^{16}$

- Conclusion: Material in the Sun must have lost almost all of its angular momentum during its formation. Not so for planet formation.



Properties of the planets:

- Inner “terrestrial” planets
- Outer “Jovian” or “Gas Giant” planets



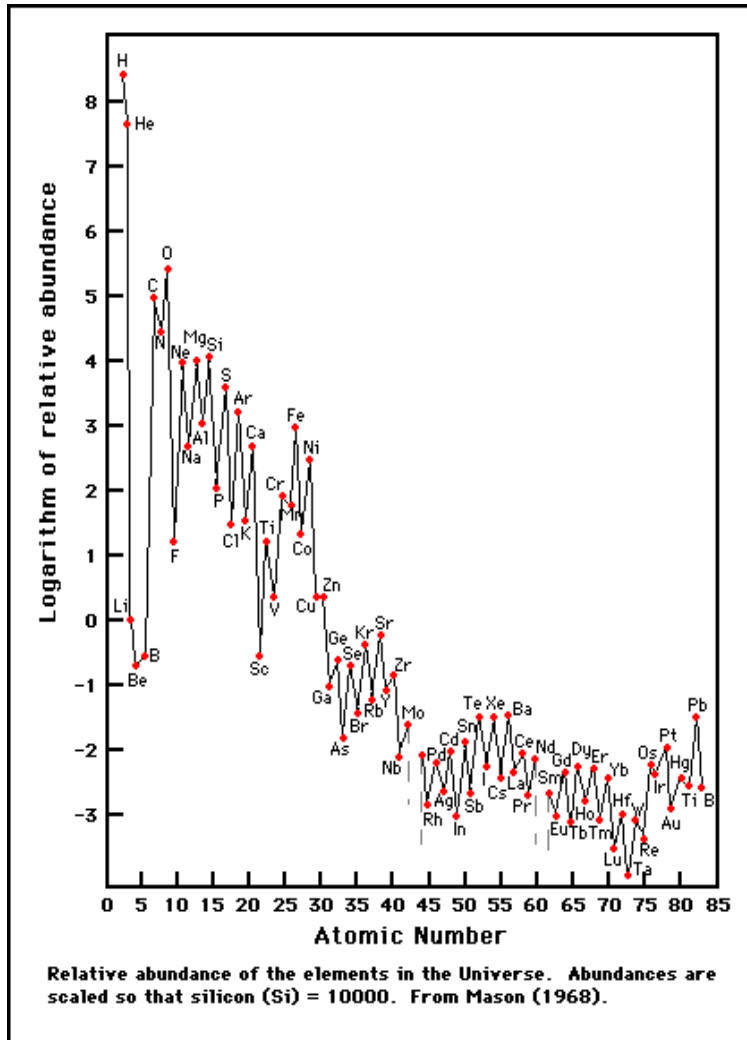
Bulk differences between terrestrial and Giant planets

	Terrestrial planets	Gas Giants
Basic form	Rocky	Primarily gas
Orbital distance ($R_{\text{earth}} = \text{AU}$)	0.39-1.52	5.2-30.1
"Surface" temperature (K)	200-750	75-170
Mass (M_{Earth})	0.055-1.0	14.5-320
Radius (r_{earth})	0.38-1.0	3.9-11.2
Mean density (gm cm^{-3})	3.95-5.52	0.7-1.64
Rotation period	24h - 243d	9.8h - 19.2h

Density differences reflect differences in bulk composition

Object	Distance(AU)	Density (g/cm ³)	Bulk composition, dominant species
Mercury	0.4	5.4	iron, nickel, silicates
Venus	0.7	5.4	silicates, iron, nickel
Earth	1.0	5.5	silicates, iron, nickel
Moon	1.0	3.3	silicates
Mars	1.4	3.9	silicates, iron, sulfur
Jupiter	5.2	1.3	H, He
Io		3.4	silicates
Europa		3.0	silicates, water, ice
Ganymede		1.9	water ice, silicates
Callisto		1.8	water ice, silicates
Saturn	9.6	0.7	H, He
Titan		1.8	water ice, silicates
Uranus	19.2	1.2	ices, H, He
Neptune	30.1	1.6	ices, H, He
Triton		2.1	silicates, ices
Pluto	39.4	2.1	silicates, ices

Remember: also big differences with abundances in the Universe as a whole



	Sun**	Earth	Human
H	70.537	0.000	9.271
He	27.505	0.000	0.000
O	0.967	29.293	63.222
C	0.307	0.000	19.149
Ne	0.171	0.000	0.000
N	0.109	0.000	5.106
Mg	0.074	12.303	0.000
Si	0.065	14.354	0.000
Fe	0.130	34.859	0.000
S	0.099	3.750	1.264
Ar	0.009	0.000	0.000
Al	0.006	1.088	0.000
Ca	0.006	1.084	1.398
Na	0.004	0.556	0.000
Ni	0.009	2.376	0.000
Cr	0.001	0.248	0.000
P	0.001	0.091	0.612

Abundances by mass

**Sun is typical of other stars, and gas in the Galaxy, and of the Universe as a whole

The formation of the central Sun and of the (dense) Solar System close to it, must have involved the loss of angular momentum (transport of angular momentum outwards and of mass inwards):

→ Accretion disk physics

The formation of the planets must have involved some process(es) that were *chemical* specific – i.e. not simply gravity (or most other astrophysical processes).

→ How to grow dust grains from μm size (ubiquitous in interstellar clouds) to 10^4+ km size of planets?

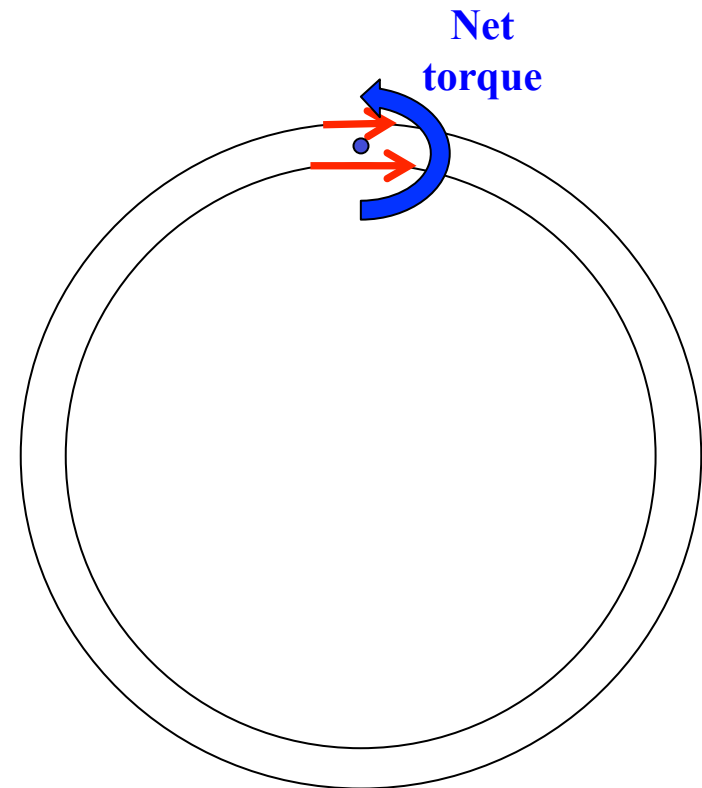
Angular momentum transport in accretion disks

A disk of material around (dominant) central object has differential rotation $\omega \propto r^{-3/2}$ or $v \propto r^{-1/2}$

Torques acting on material in the disk transfer angular momentum from fast rotating inner parts to slower rotating outer parts of disks: Torques arise from differential orbital velocities, via:

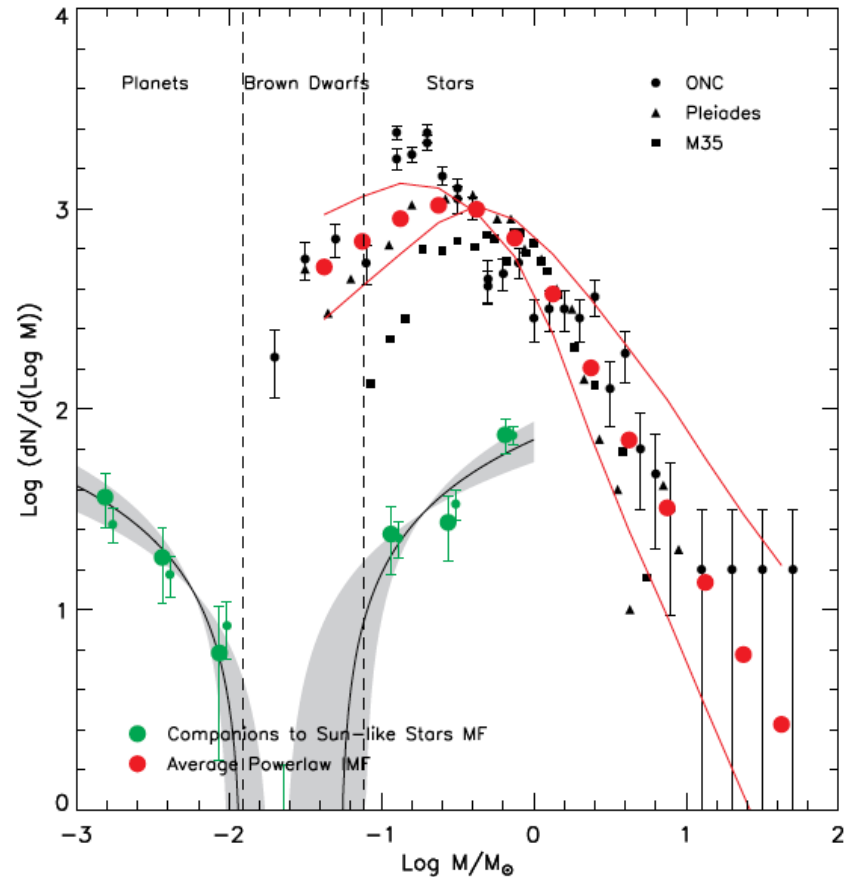
1. Magnetic fields anchored to ionized material.
2. Density inhomogeneities sheared to spiral waves, producing gravitational torques
3. Friction due to convective (vertical) motions in disk

Net effect is for matter to be transported inwards and angular momentum outwards. *This is an accretion disk.* They are often found in astrophysics (e.g. accretion onto a black hole)



Comment: Are we forming planets, stars or brown dwarfs?

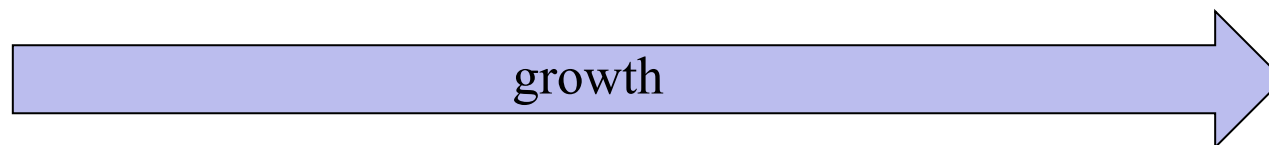
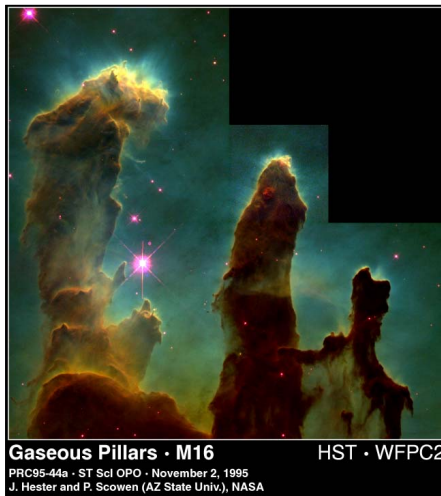
- More than half of all stars in the solar neighbourhood are in binary systems.
- “Stars” less than $0.08 M_{\odot}$ ($= 80 M_{\text{Jupiter}}$) never ignite H fusion and never become a star \rightarrow “brown dwarf”. They cool and fade
- What is the difference between making a brown dwarf and a planet?
 - There appears to be a dearth of “companions” formed with 0.01 to $0.1 M_{\odot}$. This likely reflects different formation processes.
 - above: bulk gravitational instability
 - below: growth of dust grains



μm dust grains are ubiquitous in gas in the galaxy. Typical $m_{\text{dust}}/m_{\text{gas}} \sim 1\%$. A significant fraction of elements above H and He are in dust for $T_{\text{gas}} < 1000 \text{ K}$.

Three phases during the formation of the bodies in the Solar System ($\times 10^4$!)

- Initial growth of dust grains (μm to cm)
- Formation of “planetessimals” (cm to km)
- Growth of planetessimals to make (small number) of large planets (km- 10^4 km)



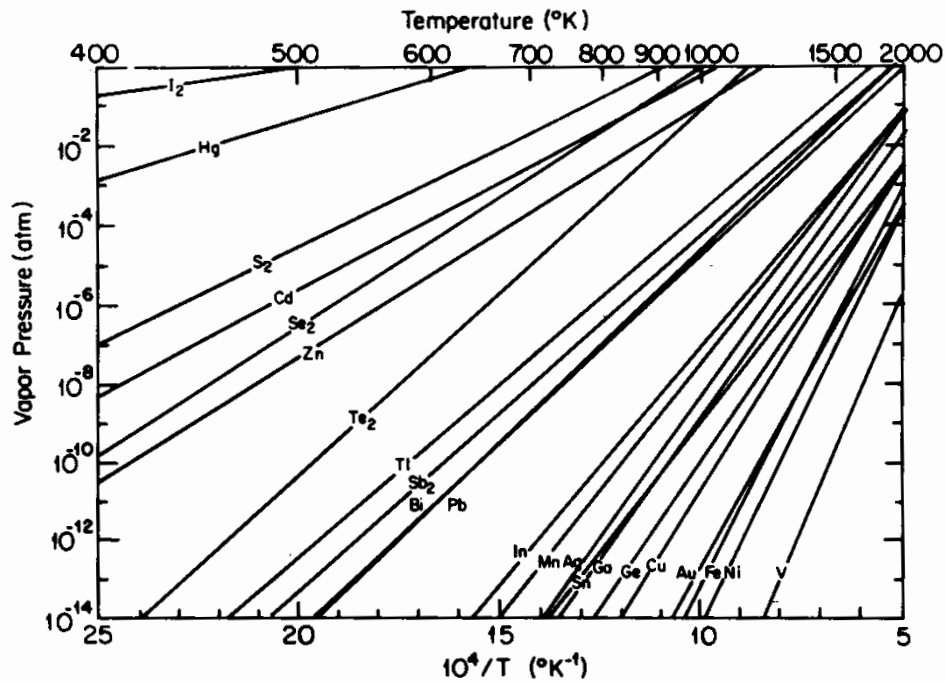
Step 1: Condensation and other non-gravitational effects

Different atomic/molecular species will condense out of the gaseous phase at different temperatures.

Antoine's Law

Vapor pressure P
above a surface drops
exponentially at
temperatures below
 $T \sim \lambda/R$

$$\ln P = \frac{-\lambda}{RT} + c$$



$P_r = 1 \text{ atm}$		$P_r = 6.6 \times 10^{-3} \text{ atm}$	
Compound or Element	$T(^{\circ}\text{K})$	Compound or Element	$T(^{\circ}\text{K})$
MgAl ₂ O ₄	2050	CaTiO ₃	1740
CaTiO ₃	2010	MgAl ₂ O ₄	1680
Al ₂ SiO ₅	1920	Al ₂ SiO ₅	1650
Ca ₂ SiO ₄	1900	Fe	1620
CaAl ₂ Si ₂ O ₈	1900	CaAl ₂ Si ₂ O ₈	1620
CaSiO ₃	1860	Ca ₂ SiO ₄	1600
Fe	1790	CaSiO ₃	1580
CaMgSi ₂ O ₆	1770	CaMgSi ₂ O ₆	1560
KAlSi ₃ O ₈	1720	KAlSi ₃ O ₈	1470
Ni	1690	MgSiO ₃	1470
MgSiO ₃	1670*	SiO ₂	1450
SiO ₂	1650	Ni	1440
Mg ₂ SiO ₄	1620*	Mg ₂ SiO ₄	1420
NaAlSi ₃ O ₈	1550	NaAlSi ₃ O ₈	1320
MnSiO ₃	1410	MnSiO ₃	1240
Na ₂ SiO ₃	1350	MnS	1160
K ₂ SiO ₃	1320	Na ₂ SiO ₃	1160
MnS	1300	K ₂ SiO ₃	1120
Cu	1260	Cu	1090
Ge	1150	Ge	970
Au	1100	Au	920
Ga	1015	Ga	880
Sn	940	Zn ₂ SiO ₄	820
Zn ₂ SiO ₄	930	Sn	806
Ag	880	Ag	788
ZnS	790	ZnS	730
FeS	680	FeS	680
Pb	655	Pb	570
CdS	625	CdS	570
Bi	620	PbCl ₂	535
PbCl ₂	570	Bi	530
Tl	540	Tl	475
In	400	Fe ₃ O ₄	400
Fe ₃ O ₄	400	In	360
H ₂ O	260	H ₂ O	210
Hg	196	Hg	181

The “Condensation Sequence” in the young Solar System

<u>Temperature</u>	<u>Condensate</u>	<u>Where</u>
1500	Metal Oxides	Mercury
1300	Iron and Nickel	
1200	Silicates	
1000	Feldspars	Venus
700	Troilite	Earth, Mars
<hr/>		
175	H ₂ O ice	Jupiter, Saturn
150	NH ₃ ice	Uranus, Neptune
120	CH ₄ ice	

“snow line”

Condensation is a chemical process, not a gravitational one, and it leads to chemical composition changes relative to the surrounding gas

Pressure effects in the disk will enhance grain growth

- Gas in an accretion disk feels (at a low level) a radial force from the pressure gradient in the disk, as well as from the dominant gravity
- Effect of the pressure relative to gravity is much smaller for large and/or dense grains relative to gas atoms/molecules

$$\frac{F_G}{F_P} \propto \frac{m}{r^2} \propto \rho r$$

Dust grains therefore migrate towards the center (i.e. gas orbits at slightly below Keplerian speed because of non-gravity force, less felt by dust).

Dust grains also sink towards the plane of the disk since they experience less vertical pressure in the disk.

Both effects enhance grain-grain collisions and sticking together of grains

Step 2: Gravitational effects

Both condensation and non-gravitational accretion (i.e. collisions between leading to sticking) will be surface effects (and therefore $\propto r^2$).

Purely gravitational effects will depend on the mass ($\propto r^3$) and will become more important as r increases.

How to determine this gravitational growth? (*Don't worry too much for non-physicists*)

Consider stability of spinning disk of material of surface density Σ that has a small density perturbation in the form of a wave-like disturbance:

$$\Delta = \frac{\delta\Sigma}{\Sigma} = \Delta_0 \exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t))$$

real $\omega \rightarrow$ sound-wave oscillations

imaginary $\omega \rightarrow$ exponential collapse

The dispersion relation for ω , depends on the wave number $k = 2\pi/\lambda$, the angular rotation speed Ω , the surface density Σ , and the sound speed c_s .

$$\Delta = \Delta_0 \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t))$$

$$\omega^2 = k^2 c_s^2 + \Omega^2 - 2\pi G \Sigma k$$

Oscillation or collapse depends on the sign of the RHS.
Growth requires imaginary ω , i.e. negative ω^2 .

(1) When the sound speed c_s is negligible, then collapse will occur on all scales up to some maximum size λ given by the surface density and rotation rate, producing objects of mass M

$$2\pi G \Sigma k \geq \Omega^2$$

$$\lambda \leq \lambda_{\max} \sim 4\pi^2 G \Sigma \Omega^{-2}$$

$$M_{\max} = 16\pi^4 G^2 \Sigma^3 \Omega^{-4}$$

Does it work? In the proto-Solar System, we expect $\lambda_{\max} \sim 10^4$ km from expected Σ (and Ω). This is about right for producing collapsed objects of about 10km size (planetessimals).

Aside: note that M_{\max} expected to vary as Σ^3 .

(2) Note that if the sound speed is not negligible, then the analysis reverts to the classic Jeans analysis - small scale fluctuations do not grow on interesting length scales: i.e. high c_s sound speed *stabilizes* the disk

$$2\pi G\Sigma k \geq k^2 c_s^2$$

$$\lambda \geq \frac{c_s^2}{G\Sigma} \sim 10^{18} m \sim 10^7 AU$$

The threshold c_s given by

$$k^2 c_s^2 \sim \Omega^2$$

When we get collapse on scales above

$$\lambda \sim c_s \frac{2\pi}{\Omega} \sim c_s T_{rot}$$

Distance pressure wave travels in rotation period

i.e. a gas composed of slow moving massive particles will be more gravitationally unstable than one of lower mass particles with higher speeds

Sound speed is given by the mass of the particles since collisions lead to equipartition of energy (e.g. 10^{-7} ms^{-1} for mg masses).

$$c_s \sim v = \left(\frac{3kT}{m} \right)^{1/2}$$

Conclusion: Growth of grains through condensation and non-gravitational sticking, and the associated reduction of sound speed c_s , is essential for allowing the material in the disk to become gravitationally unstable to produce 10km-sized bodies

Do the timescales work?

Growth time for collapse is given by $\tau \sim \omega^{-1}$

This is about 10^6 years for $\lambda \sim 10^4$ km and $c_s \sim 10^{-7}$ ms $^{-1}$

Again, this is just about OK given observed constraints on the lifetime of disks



Step 3: Clearing the Nebula

- Assembly of 10 km planetesimals into planets through collisions
- Removal of remaining planetesimals and removal of gas (ejection through close encounters with planets and solar pressure respectively)

The end result is likely to be rather stochastic and unpredictable

Note:

- large collisions at late epochs between “proto-planets” are likely
- Transport of volatile rich objects from beyond “snow-line” in outer Solar System into inner Solar System is possible.

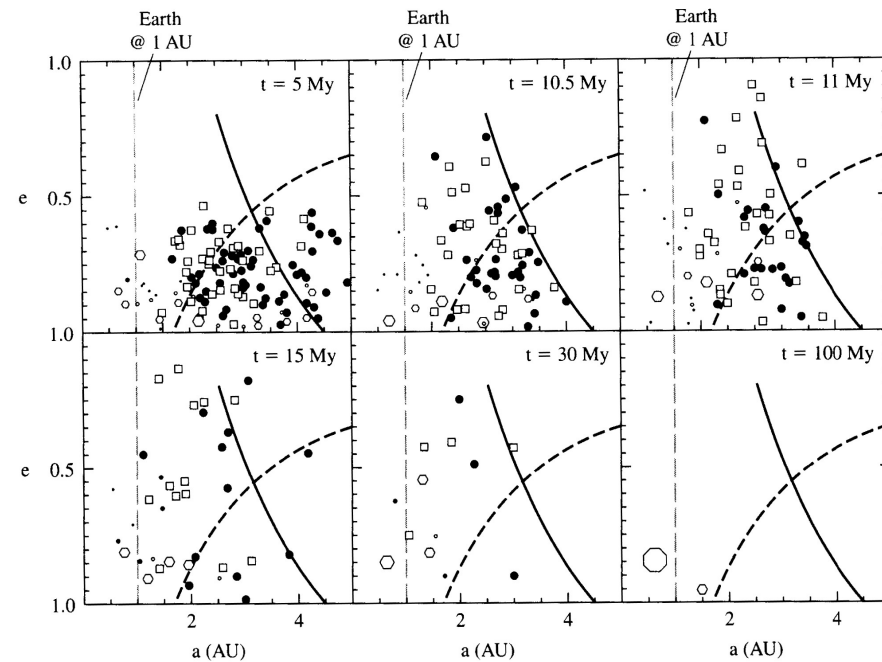
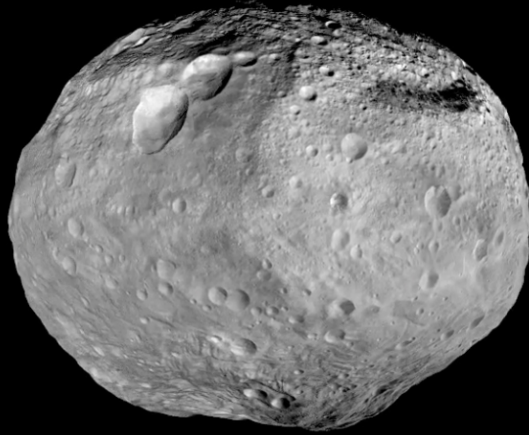


FIGURE 6.6 Simulation of the assembly of the terrestrial planets from lunar- to Mars-sized fragments. Each panel shows the distribution in orbital semimajor axis and eccentricity (Chapter 1), and the series of panels are a progression in time (labeled in millions of years). Planetesimals grow by collision, and their increasing mass is schematically shown by the size of the circles. The dotted line illustrates the realm (to the right of the line) where Jupiter’s gravity is so large that rapid ejection of planetesimals occurs. To simplify the calculation, Jupiter’s influence is “inserted” into the picture beginning in the second panel. The location of water-bearing planetesimals is shown as being at 2.5 AU and beyond. At the end of this simulation two terrestrial planets are formed.

Comets and asteroids are surviving planetessimals still in a relatively pristine state – being volatile-rich and less volatile respectively

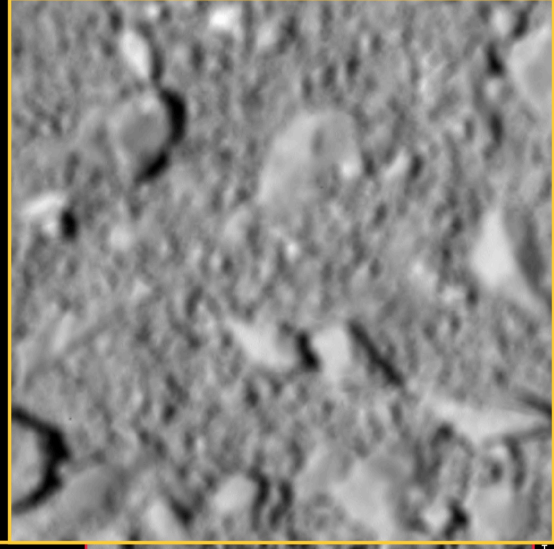
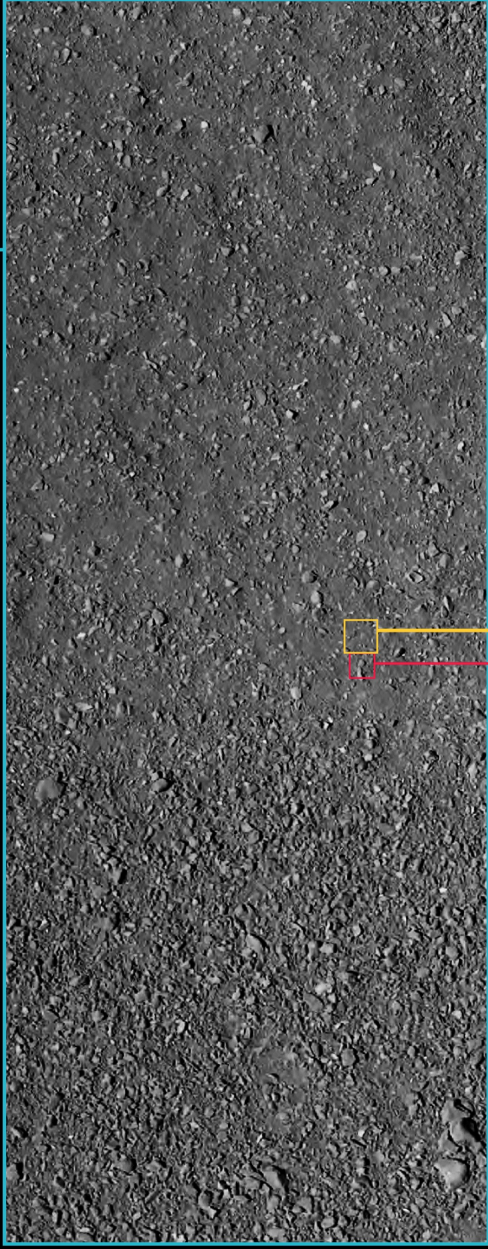
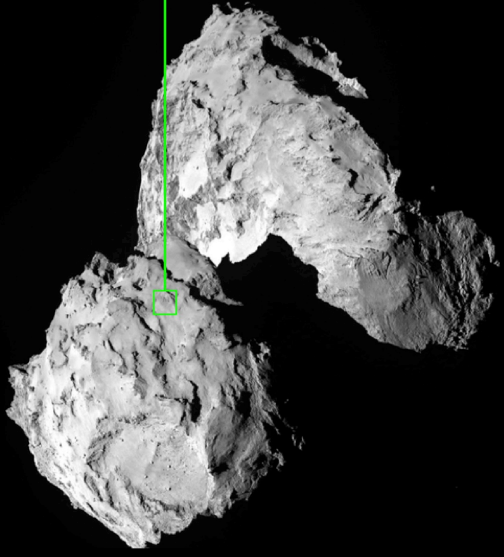
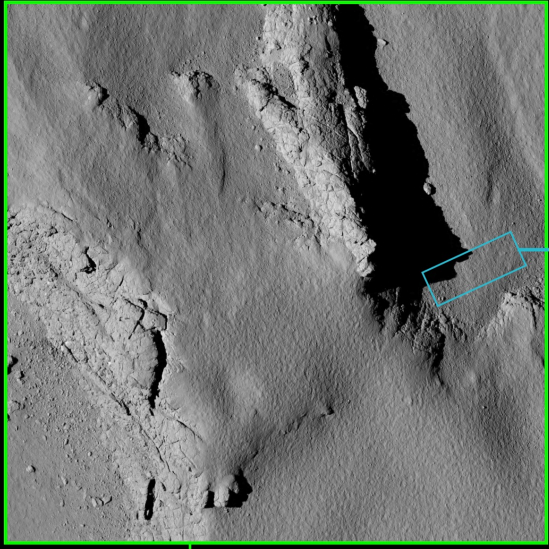


Asteroid Vesta



Comet 67P

225 m

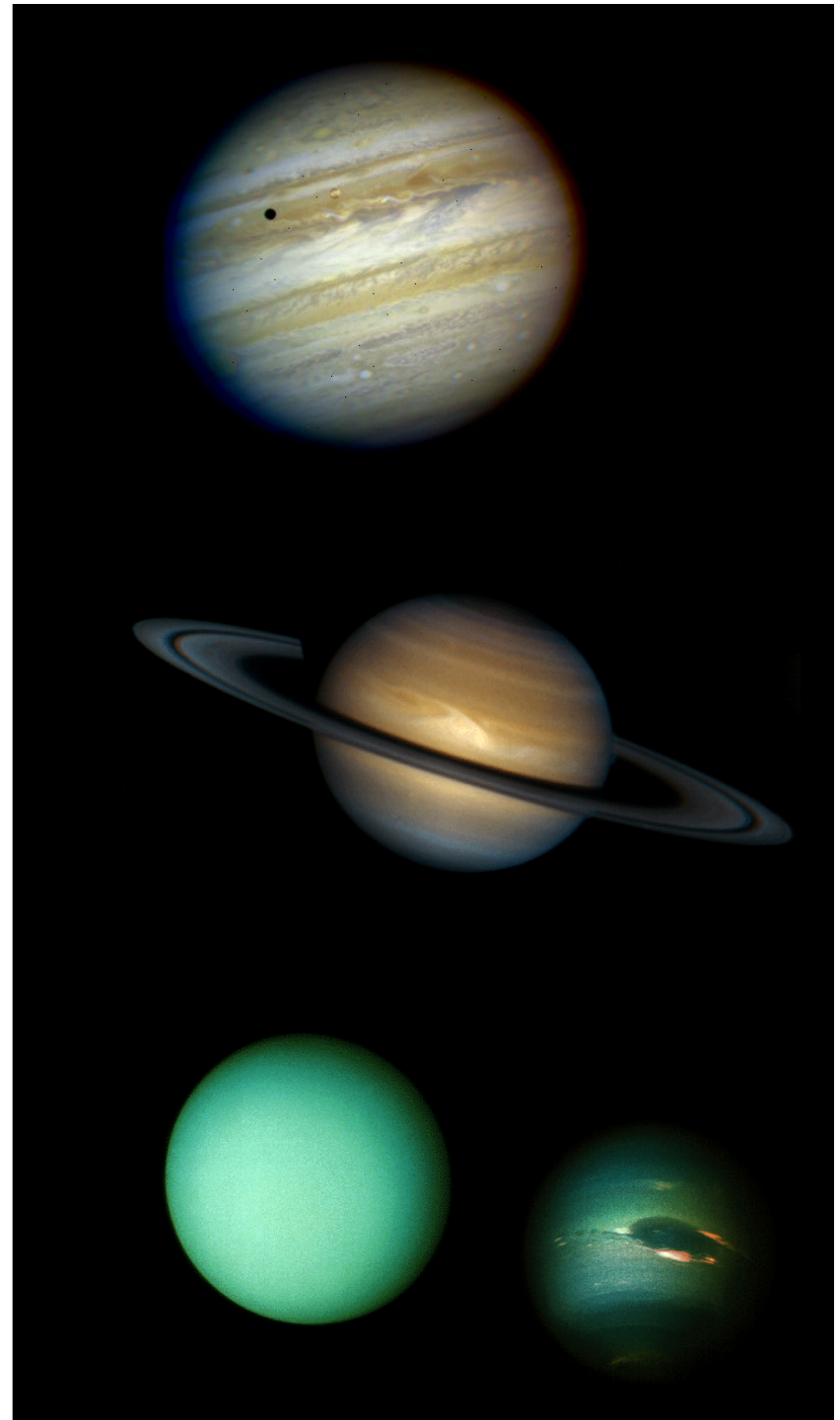


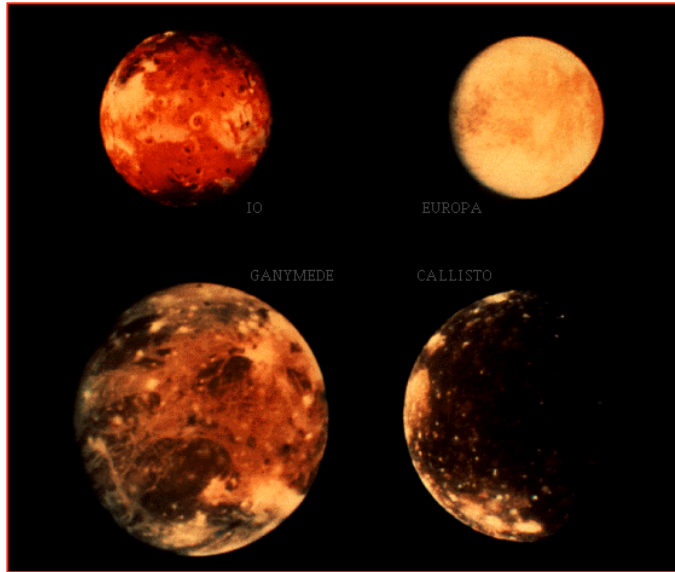
1 m

Giant planets were able to gravitationally attract substantial amounts of H and He gas from the Nebula. This requires a solid core of $>5 M_{\text{earth}}$.

Major moons of Jovian planets formed in situ out of spinning disk of gas around them (Solar System formation in miniature) and also display gradients of composition etc. But some were likely late captures.

Note: it is not clear that Jupiter and Saturn do actually possess a rocky core (esp Saturn). This plus concerns about timescale has led to alternative scenario of coherent collapse of gas cloud (but what about non-solar abundance?)





reflect differences in bulk composition

		Density (g/cm ³)	Bulk composition
		5.4	iron, nickel, silicates
		5.4	silicates, iron, nickel
		5.5	silicates, iron, nickel
		3.3	silicates
		3.9	silicates, iron, sulfur
Jupiter	5.2	1.3	H, He
	Io	3.4	silicates
	Europa	3.0	silicates, water, ice
	Ganymede	1.9	water ice, silicates
	Callisto	1.8	water ice, silicates
Saturn	9.6	0.7	H, He
	Titan	1.8	water ice, silicates
Uranus	19.2	1.2	ices, H, He
Neptune	30.1	1.6	ices, H, He
	Triton	2.1	silicates, ices
Pluto	39.4	2.1	silicates, ices

Formation of the Earth's Moon (with unusually large mass ratio 1:83)

Historical ideas:

The Fission Theory: The Moon was once part of the Earth and somehow separated from the Earth early in the history of the Solar System. The present Pacific Ocean basin was the most popular site for the part of the Earth from which the Moon came.

The Capture Theory: The Moon was formed somewhere else, and was later captured by the gravitational field of the Earth.

The Condensation Theory: The Moon and the Earth condensed together as a binary system from the original nebula that formed the Solar System.

The Colliding Planetesimals Theory: The interaction of earth-orbiting and Sun-orbiting planetesimals (very large chunks of rocks like asteroids) early in the history of the Solar System led to their breakup. The Moon condensed from this debris.

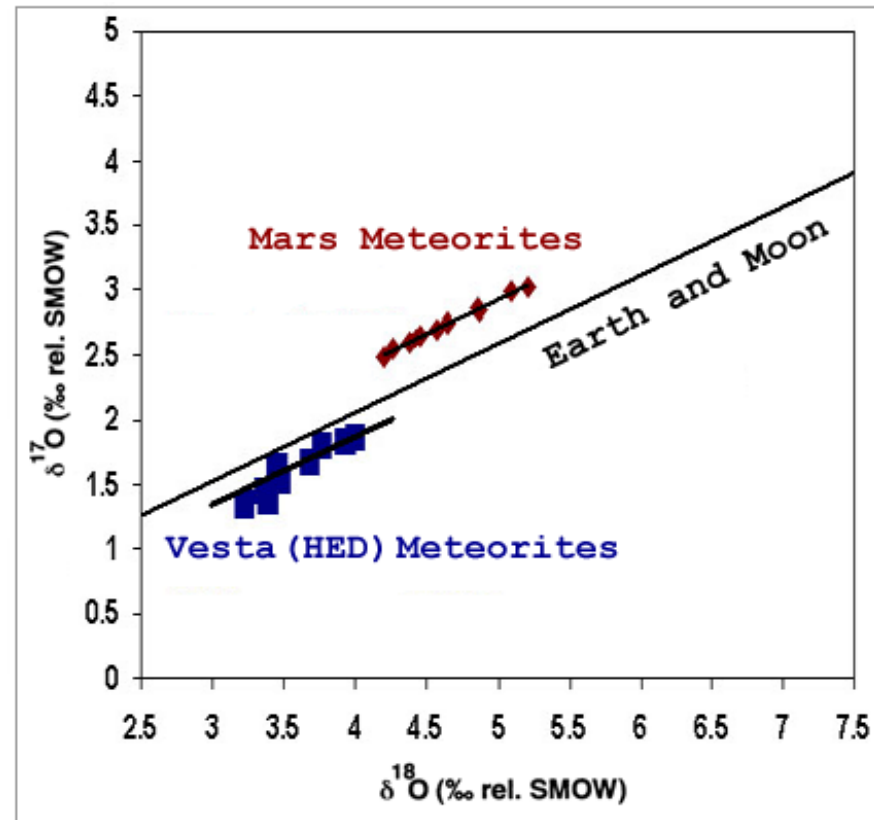
Now almost universally accepted:

The Ejected Ring Theory: A “planetesimal” the size of Mars struck the earth, ejecting large volumes of matter. A disk of orbiting material was formed, and this matter eventually condensed to form the Moon in orbit around the Earth.



- Oxygen isotopic ratios identical to Earth (c.f. Mars and Vesta). There is an imperfectly understood gradient in $^{16}\text{O}/^{17}\text{O}/^{18}\text{O}$ within the Solar System.
- But, the chemical composition is different from Earth as a whole, being more similar to just the Earth's crust without Fe, Ni that is in Earth's core

Currently accepted idea: impact with Mars-sized body that formed at same distance from the Sun (perhaps in L4/L5 point?)



PSRD graphic



Radioactive dating of rock formation

Radioactive decay producing
“daughter” iD from “parent” kP (and
no other source)

$$\frac{d{}^iD}{dt} = -\frac{d{}^kP}{dt} = \lambda {}^kP$$

Surviving kP :

$${}^kP = {}^kP_0 e^{-\lambda t}$$

Number of iD produced from
radioactivity

$${}^iD_r = {}^kP_0 (1 - e^{-\lambda t}) = {}^kP (e^{\lambda t} - 1)$$

Total number of iD present:

$${}^iD = {}^iD_0 + {}^iD_r = {}^iD_0 + {}^kP (e^{\lambda t} - 1)$$

Divide by another (stable and not-produced) isotope of D . If the P/D ratio varies within the rock due to initial chemical inhomogeneities, then the slope of the line gives the age (in terms of decay constant λ)

$$\frac{{}^iD}{{}^jD} = \frac{{}^iD_0}{{}^jD} + \frac{{}^kP}{{}^jD} (e^{\lambda t} - 1)$$

Half-lives:	$^{87}\text{Rb} \rightarrow ^{87}\text{Sr}$	4.99×10^{10} yrs
$\tau = 0.693\lambda^{-1}$	$^{232}\text{Th} \rightarrow ^{208}\text{Pb}$	1.39×10^{10} yrs
	$^{238}\text{U} \rightarrow ^{206}\text{Pb}$	4.50×10^9 yrs
	$^{235}\text{U} \rightarrow ^{207}\text{Pb}$	7.13×10^8 yrs
	$^{147}\text{Sm} \rightarrow ^{143}\text{Nd}$	10.6×10^{10} yrs

Note: the method relies on

(a) *presence* of chemical inhomogeneities;

(b) *absence* of initial isotopic inhomogeneities;

(c) atoms remaining in place \rightarrow it works only after solidification of rock, i.e. the “age” of the rock is the time since solidification

Meteorites: $4.55 \pm 0.01 \times 10^9$ yr

Moon (Apollo 17) 4.4×10^9 yr

Consistent with estimate of age of Sun (i.e. the solar lifetime $\sim 1.0 \times 10^{10}$ yr)

2 year old rock
(Hawaiian lava) and
4.5 billion year old
rock (meteorite)



Ages of meteorites

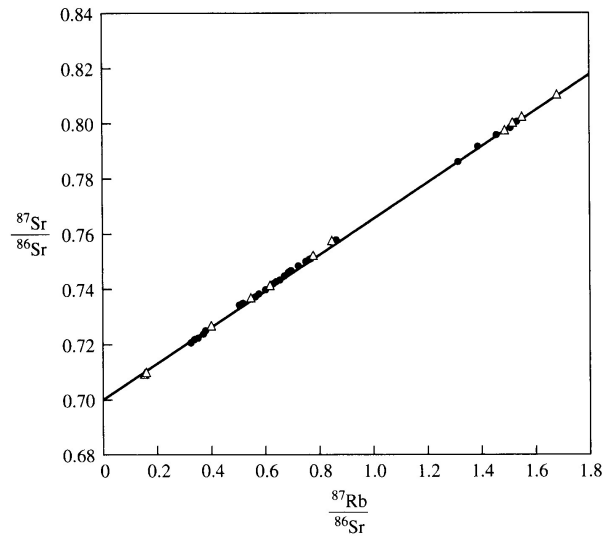
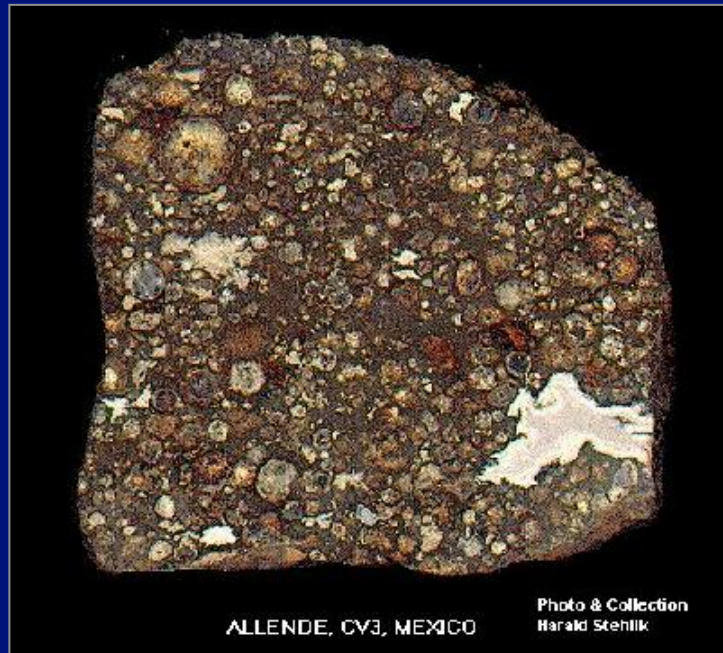
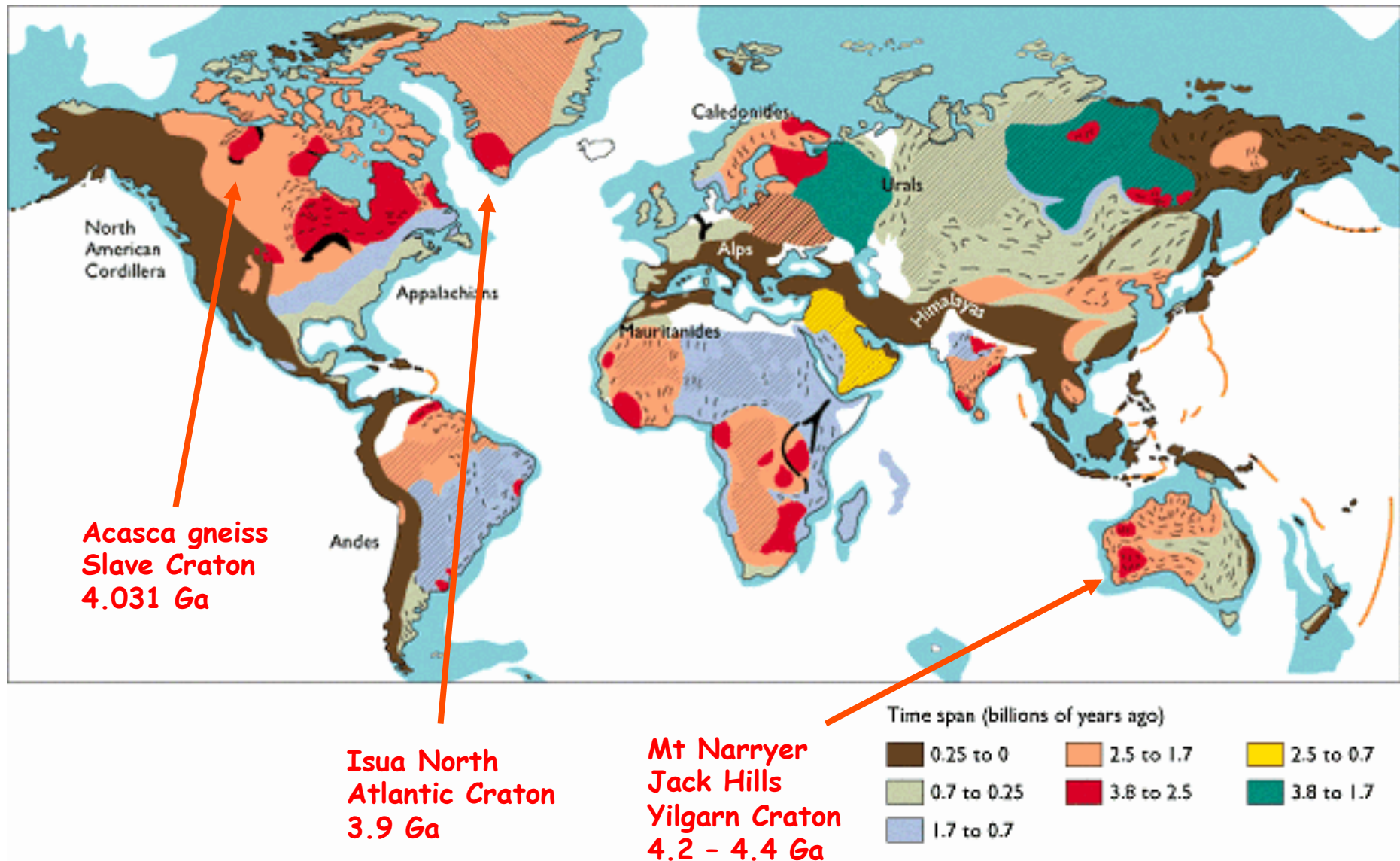


FIGURE 6.9 Rubidium–strontium diagram for a number of different chondritic samples of various types (labeled by different symbols), forming a beautiful straight line with an age of 4.55 ± 0.01 billion years.



Meteorite Name (of years) <i>Allende</i>	Material Dated	Method	Age (billions)
	whole rock	Ar-Ar	4.52 +/- 0.02
	whole rock	Ar-Ar	4.53 +/- 0.02
	whole rock	Ar-Ar	4.48 +/- 0.02
	whole rock	Ar-Ar	4.55 +/- 0.03
	whole rock	Ar-Ar	4.55 +/- 0.03
	whole rock	Ar-Ar	4.57 +/- 0.03
	whole rock	Ar-Ar	4.50 +/- 0.02
	whole rock	Ar-Ar	4.56 +/- 0.05
	whole rock	Ar-Ar	4.44 +/- 0.06
	13 isochron	Rb-Sr	4.46 +/- 0.08
<i>Guarena</i>	whole rock	Ar-Ar	4.43 +/- 0.06
<i>Shaw</i>	whole rock	Ar-Ar	4.40 +/- 0.06
	whole rock	Ar-Ar	4.29 +/- 0.06
<i>Olivenza</i>	18 isochron	Rb-Sr	4.53 +/- 0.16
	whole rock	Ar-Ar	4.49 +/- 0.06
<i>St. Severin</i>	4 isochron	Sm-Nd	4.55 +/- 0.33
	10 isochron	Rb-Sr	4.51 +/- 0.15
	whole rock	Ar-Ar	4.43 +/- 0.04
	whole rock	Ar-Ar	4.38 +/- 0.04
	whole rock	Ar-Ar	4.42 +/- 0.04
<i>Indarch</i>	9 isochron	Rb-Sr	4.46 +/- 0.08
	12 isochron	Rb-Sr	4.39 +/- 0.04
<i>Juvinas</i>	5 isochron	Sm-Nd	4.56 +/- 0.08
	5 isochron	Rb-Sr	4.50 +/- 0.07
<i>Moama</i>	3 isochron	Sm-Nd	4.46 +/- 0.03
	4 isochron	Sm-Nd	4.52 +/- 0.05
<i>Y-75011</i>	9 isochron	Rb-Sr	4.50 +/- 0.05
	7 isochron	Sm-Nd	4.52 +/- 0.16
	5 isochron	Rb-Sr	4.46 +/- 0.06
	4 isochron	Sm-Nd	4.52 +/- 0.33
<i>Angra dos Reis</i>	7 isochron	Sm-Nd	4.55 +/- 0.04
	3 isochron	Sm-Nd	4.56 +/- 0.04
<i>Mundrabilla</i>	silicates	Ar-Ar	4.57 +/- 0.06
	olivine	Ar-Ar	4.54 +/- 0.04
	plagioclase	Ar-Ar	4.50 +/- 0.04
<i>Weekeroo Station</i>	4 isochron	Rb-Sr	4.39 +/- 0.07
	silicates	Ar-Ar	4.54 +/- 0.03

Ages of oldest rocks at surface of the Earth



Continental rock on Earth is much older than oceanic basalt. On continents, the oldest rocks only rarely exposed on the surface

Chronology of formation of Earth and inner Solar System

Age	When	What
0 (Sun formed)	4.55 Gyr before present	First solids formed
5 million years		Gas and dust ejected from young Solar System
30 million years		Earth melts and differentiates
100 million years		Large impact formed the Moon
500 million years		Cratering declines sharply, Solar System more or less as today
700 million years	3.8 Gyr before present	First evidence for Life on Earth?

Aside on (quite different) radioactive carbon dating for biological material



The ratio of ${}^{14}\text{C}/{}^{12}\text{C}$ in the atmosphere is maintained at an equilibrium value (1.5×10^{-12}) by production of new ${}^{14}\text{C}$ in upper atmosphere from cosmic ray impacts with ${}^{12}\text{C}$

Living things continually exchange carbon with atmosphere. Dead things do not.

Once a living thing dies the ratio of ${}^{14}\text{C}/{}^{12}\text{C}$ declines due to the decay of ${}^{14}\text{C}$.

$$\frac{{}^{14}\text{C}}{{}^{12}\text{C}} = \frac{{}^{14}\text{C}}{\text{{}^{12}\text{C}}_{atmos}} e^{-\lambda t}$$

2. Geological evolution of terrestrial bodies: sources of heat

Clearing the nebula phase → impacts of planetessimals

Kinetic energy of impact:

$$KE = \frac{1}{2}mv^2$$

Which velocity is relevant?

$$v_{esc} = \sqrt{\frac{2GM}{r}} \sim 11 \text{ kms}^{-1}$$

Escape speed is speed of something dropped from infinity

$$v_{orb} = \sqrt{\frac{GM_{sun}}{R}} \sim 29 \text{ kms}^{-1}$$

Orbital speeds in Solar System (but these are not randomly oriented)

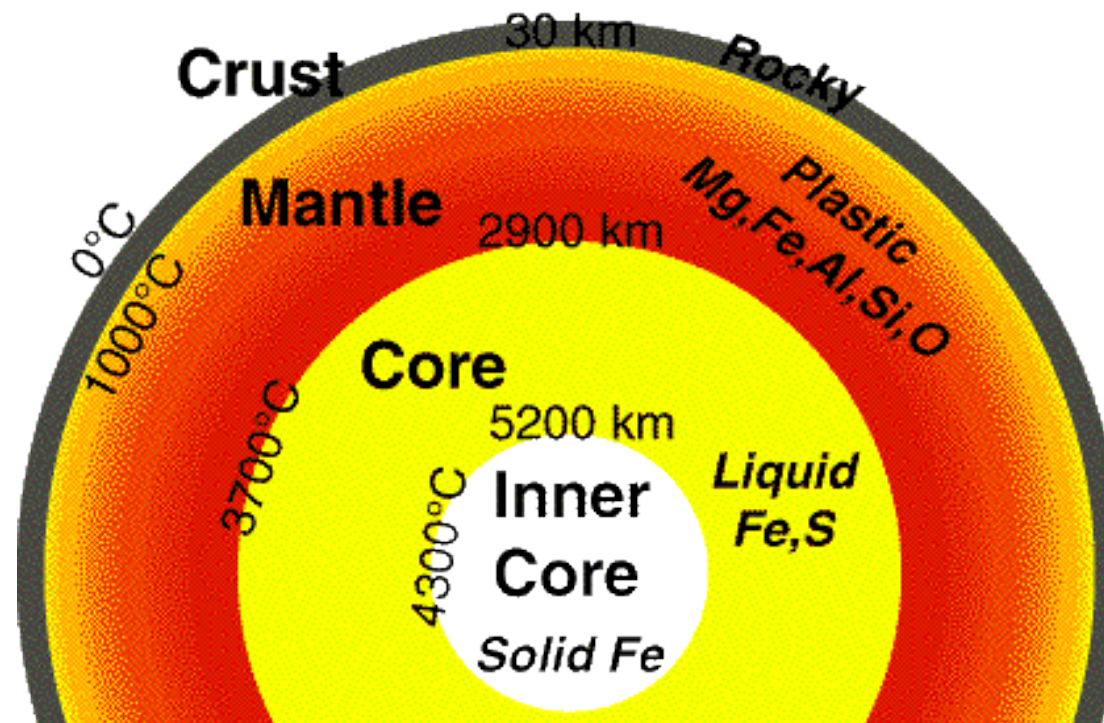
$$\frac{1}{2}v_{esc}^2 \sim 10^8 \text{ Jkg}^{-1}$$

Compare this with the specific heat capacity ($10^3 \text{ Jkg}^{-1} \text{ K}^{-1}$) and the latent heat of fusion ($4 \times 10^5 \text{ Jkg}^{-1}$) of Basalt rock

Conclusion: An impacting planetesimal can melt about 10^2 times its own mass

Melting leads to:

- Differentiation (dense substances sink to center)
- Outgassing of any volatile substances from interior



Cooling of hot planets

Heat losses \propto area $\propto r^2$

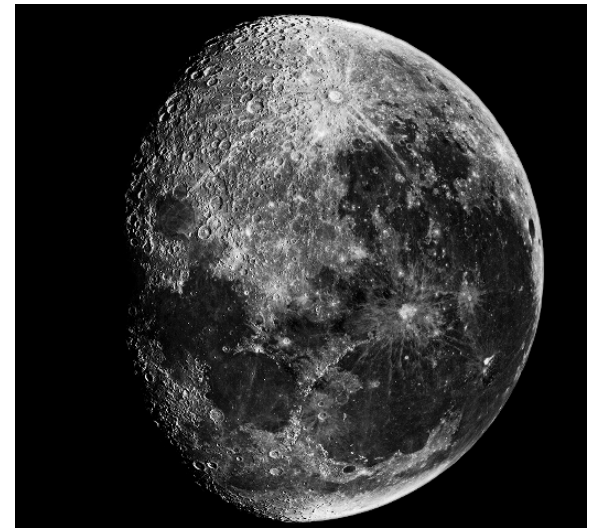
Heat production from gravity \propto -PE \propto mass²/ r $\propto r^5$

→ Small objects will cool quicker (e.g. Moon, Mars, Mercury) leading to early termination of geological activity(c.f. Venus and Earth)

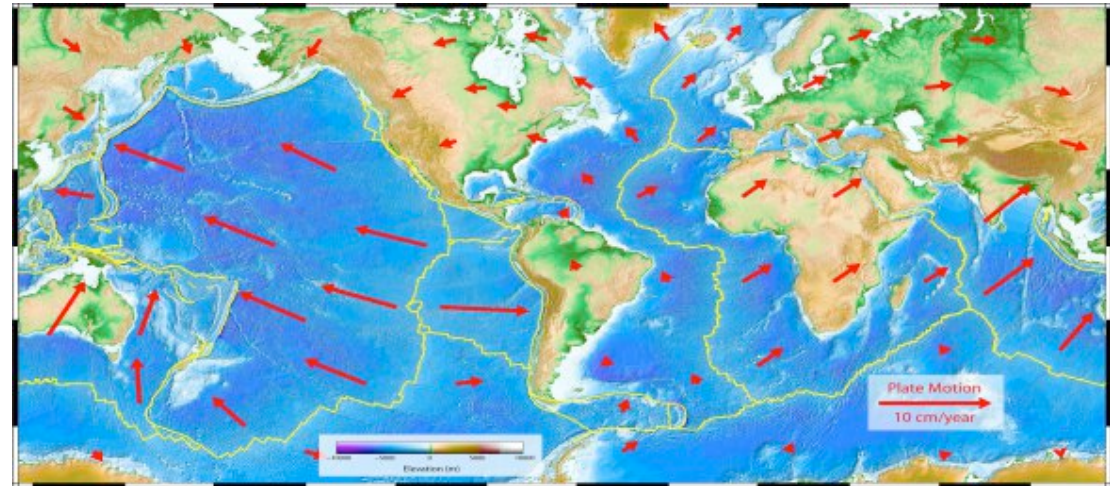
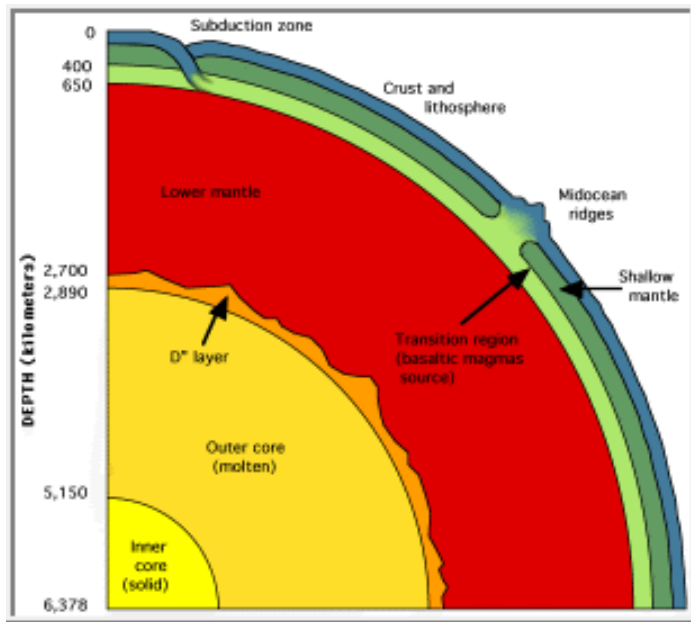
→ They show old (heavily cratered) surfaces because they solidified rapidly and have had little surface reprocessing by geological activity

Planetary interiors can be kept hot by any of the following additional heat sources

- Decay of radioactive nuclides within interior
- Tidal compression effects (e.g. Io)
- Gravitational Kelvin-Helmholz contraction (gas giants)

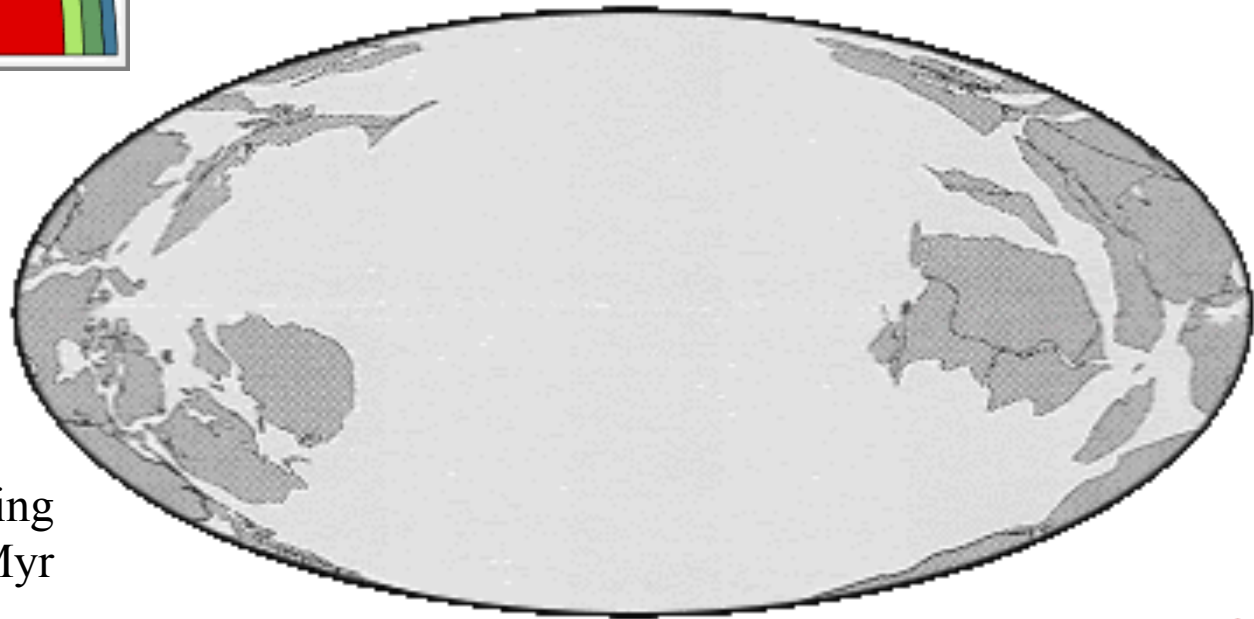


Non-rigid interiors: geological activity on Earth



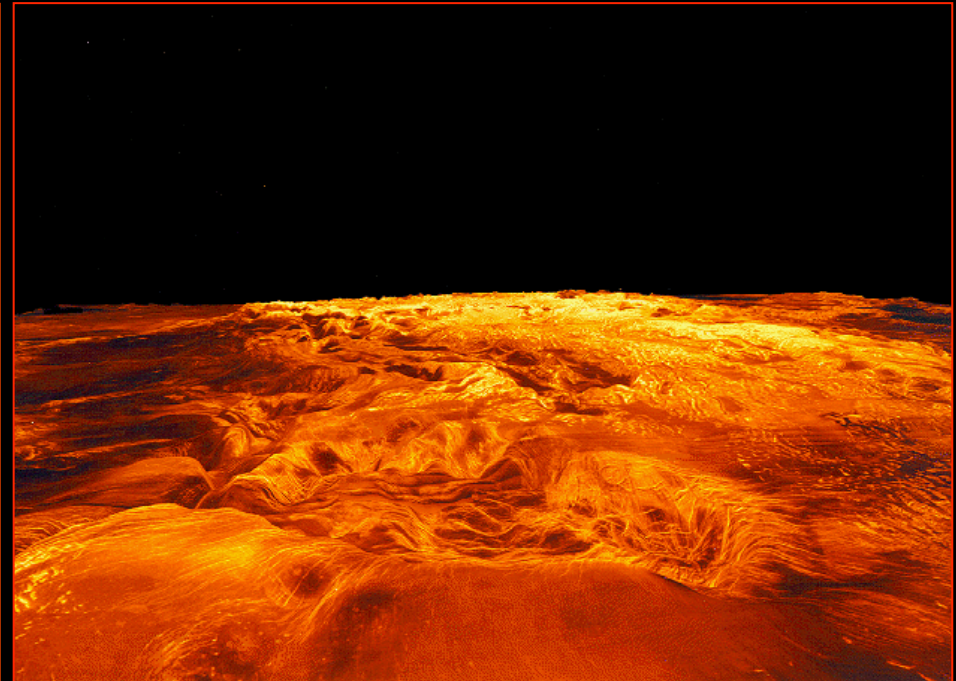
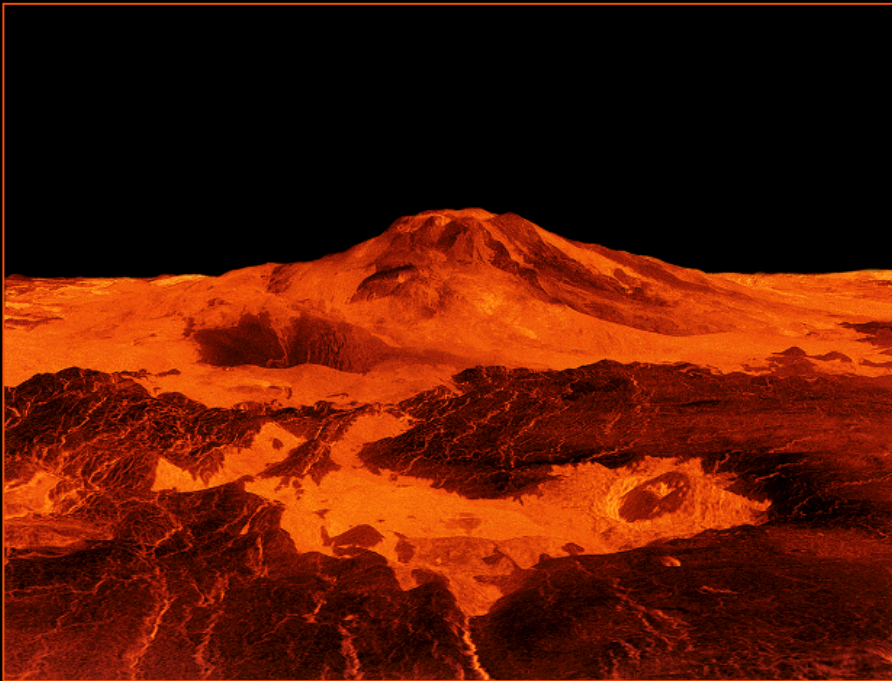
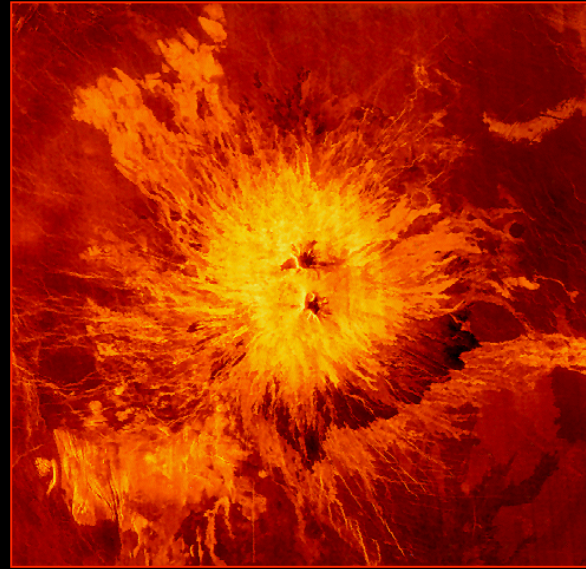
Average age of oceanic crust (< 100 Myr), much less than average age of continental crust (~ 2 Gyr)

Continental motions during the last 750 Myr



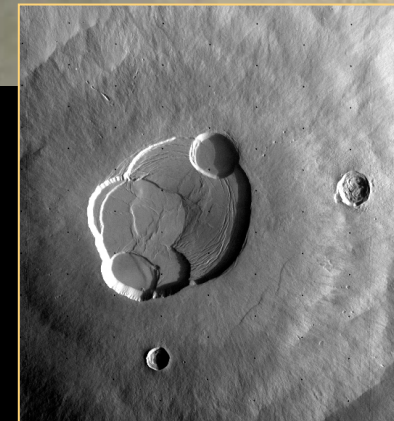
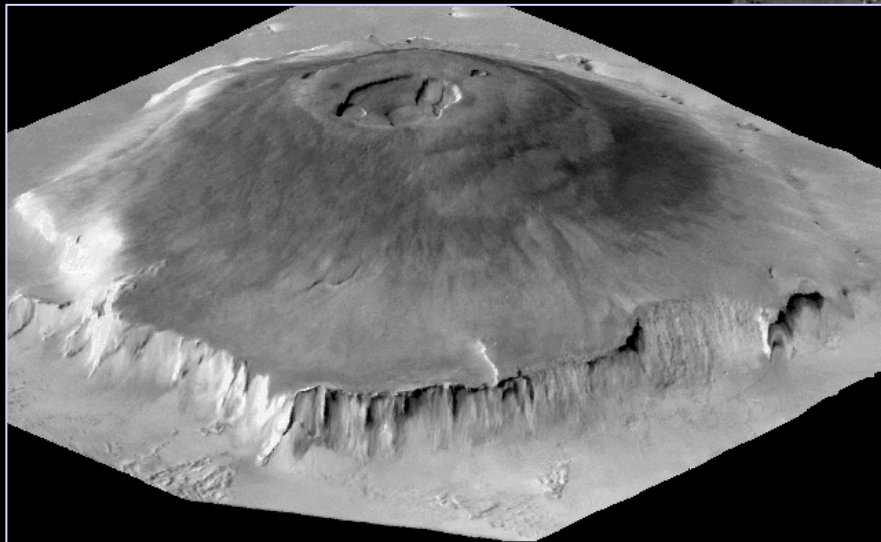
Venus

- Ongoing activity (young surface 300-600 Myr)
- No tectonic plates
- Difference with Earth reflects water content?



Mars does have (huge) volcanoes, but:

- No plate tectonics
- No ongoing volcanic activity (last major episode 500 Myr ago)



Tidal forces on objects

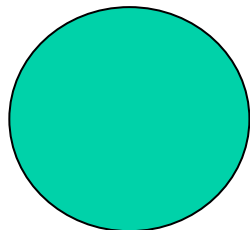
Consider a small object orbiting in the gravitational field of a larger one

$$g = -\frac{GM}{R^2} \quad \Delta a = \frac{d}{dR} \frac{GM}{R^2} \Delta R = 2 \frac{GM}{R^3} \Delta R$$

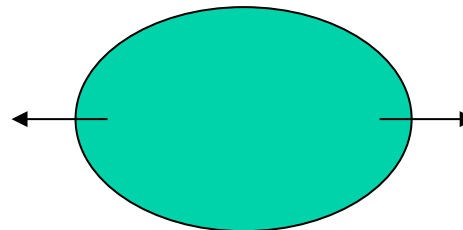
Differential acceleration

Distance to primary object

Size of secondary object



Uniform g



Varying g across an object produces local tidal forces distorting object (e.g. ocean tides on Earth)

Tidal effects have several interesting roles in our story ...

Tidal forces on objects

$$\Delta a = 2 \frac{GM}{R^3} r$$

- (1) *Roche limit*: disruption of satellite of radius r orbiting at distance R when the tidal forces exceed the satellite's own gravity holding it together

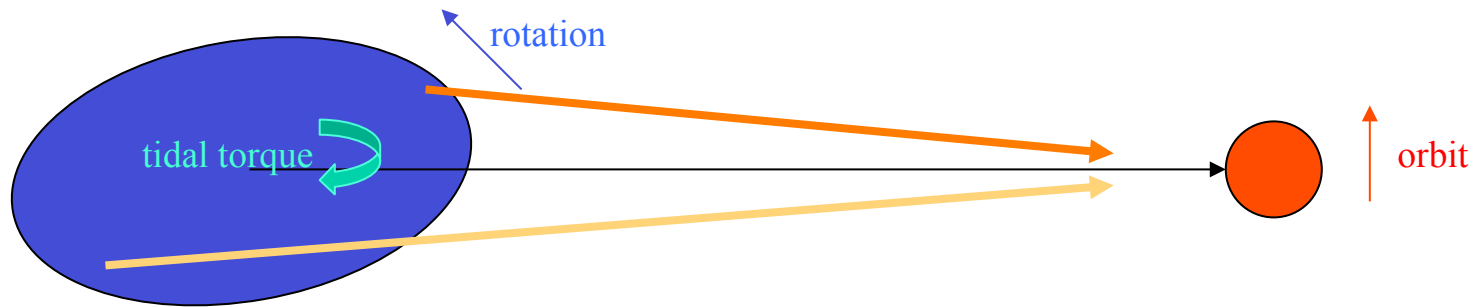
$$2 \frac{GM_{planet}}{R^3} r > \frac{GM_{moon}}{r^2}$$

$$R < 2^{1/3} \left(\frac{\bar{\rho}_{planet}}{\bar{\rho}_{moon}} \right)^{1/3} r_{planet}$$

So, if $\rho_{planet} \sim \rho_{satellite}$, the satellite is disrupted when the orbital radius is comparable to the primary radius (e.g. Saturn's rings)



(2) *Tidal locking*: Rotation at $\omega_{\text{rot}} \neq \omega_{\text{orbit}}$ produces misalignment of tidal bulge due to friction (whether solid or liquid body) and thus torques



These torques act to make $\omega_{\text{rot}} = \omega_{\text{orbit}}$

- Moon's spin already synchronised with orbit (keeps one face towards us)
- Earth's spin is slowing (our "day" is lengthening by 2ms per century)

Tidal torques transfer angular momentum between from the spins to the orbit

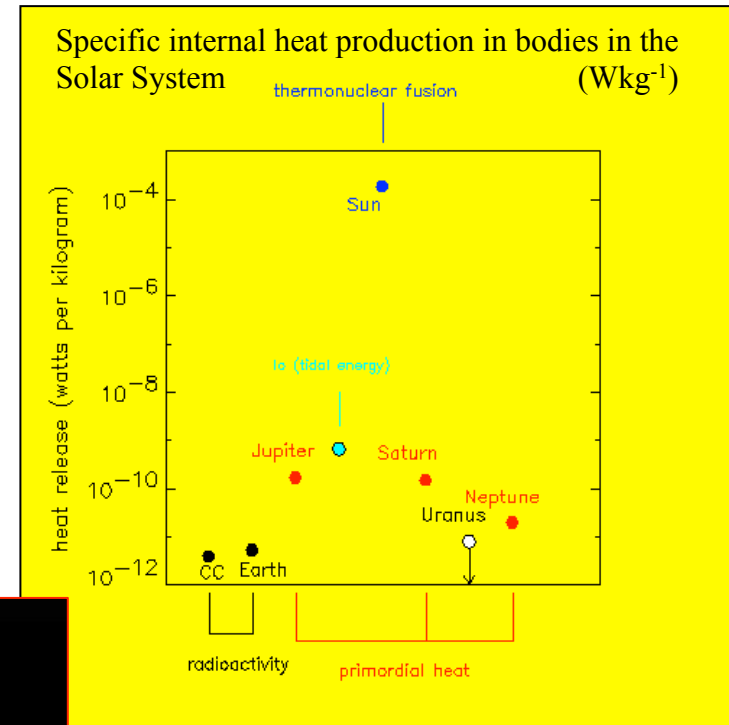
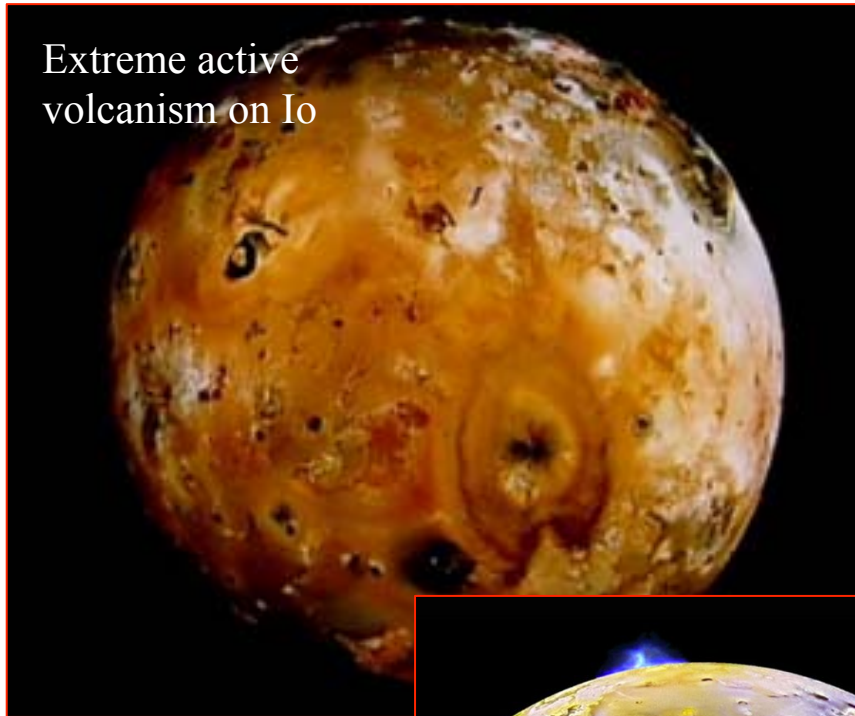
- Radius of the Moon's orbit is increasing as the angular momentum increases

Implications for Life? Tidal pools on shores of ocean? Tidal locking of planets with stars will produce extreme temperature variations across surface?

(3) *Internal heating*: friction associated with repeated tidal deformations produces internal heating, even in systems with tidally locked rotation if on eccentric orbit (note R^3 dependence)

$$\Delta a = 2 \frac{GM}{R^3} r$$

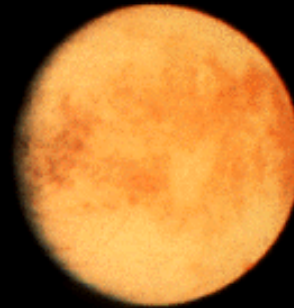
e.g. Io, innermost satellite of Jupiter, whose surface elevation changes by up to 100m during its 41 hour orbit!



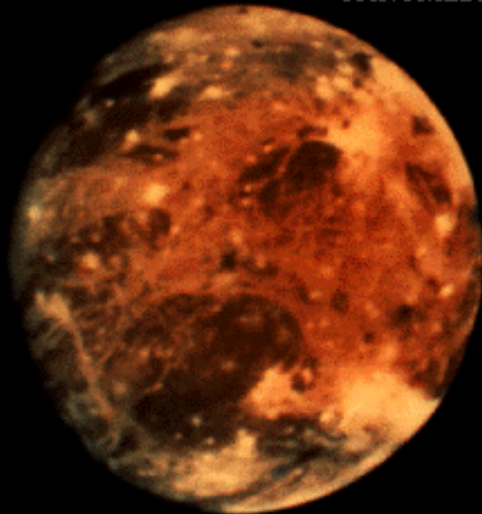
Moon	Period (days)	Diam (km)	Density (g/cc)	Eccentricity
Io	1.769	3642	3.529	0.0041
Europa	3.551	3120	3.018	0.0101
Ganymede	7.155	5268	1.936	0.0015
Callisto	16.689	4800	1.851	0.007



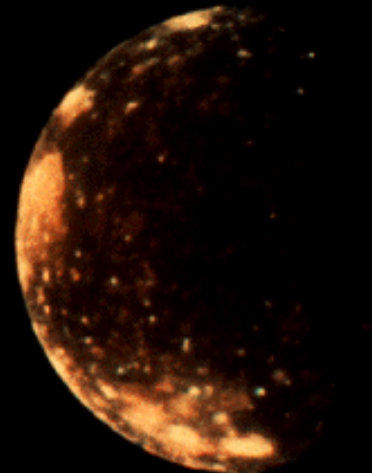
IO



EUROPA

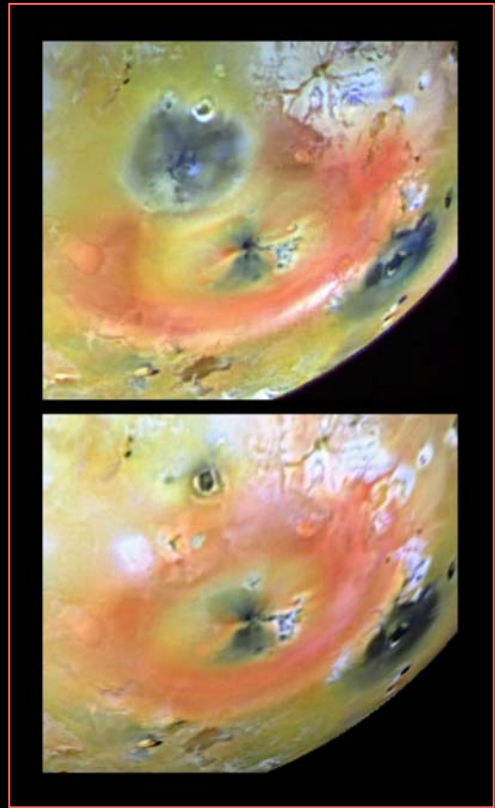
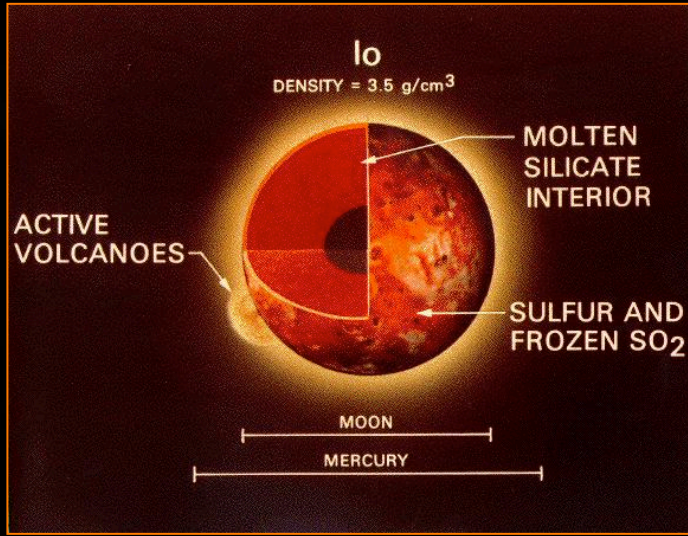
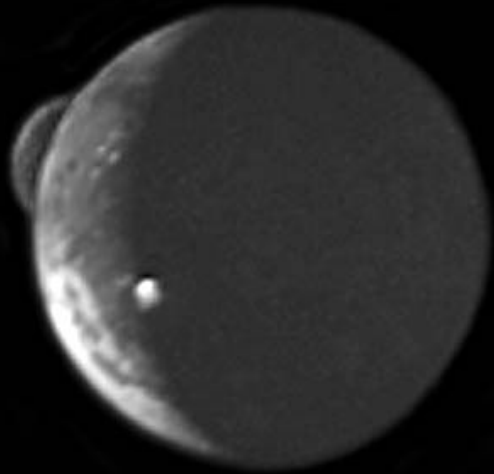


GANYMEDE

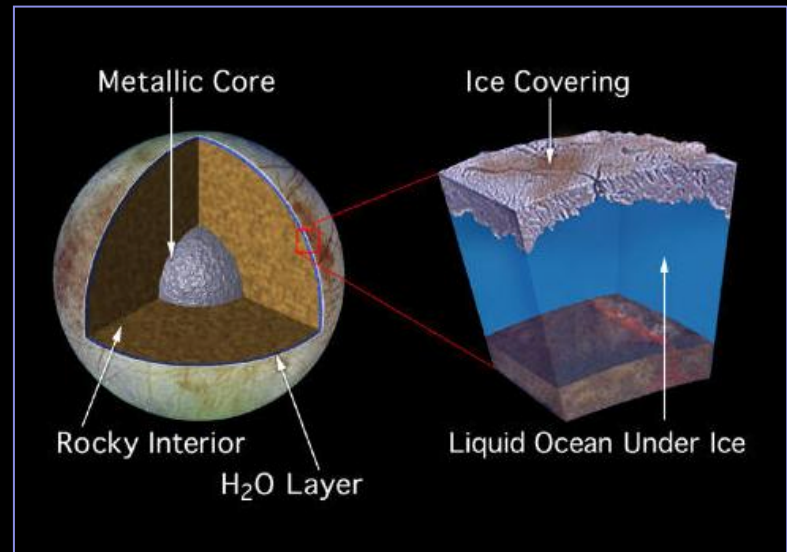


CALLISTO

Volcanoes on Io



H₂O Geysers on Europa



3. Origin and evolution of planetary and satellite atmospheres

Sources of planetary and satellite atmospheres

First, note that the atmosphere on Earth is a very small fraction (0.02%) of the total mass (even including the oceans as atmosphere)

- Solid material 6.0×10^{24} kg
- Water ocean 1.4×10^{21} kg
- Gas atmosphere 5×10^{18} kg

Possible sources of the atmospheres

- Capture of gases from original Solar Nebula (H, He) – only relevant for the massive Outer Planets
- Outgassing of volatile substances from the interior during molten phase
- *Most favoured*: Subsequent impacts by volatile rich planetessimals (perturbed from outer solar system) during the “clearing of the Nebula”. NB. Easy to get “water worlds”, completely covered by water (c.f. Earth: average depth of ocean 3.6 km, highest mountain above sea level = 8.8 km)

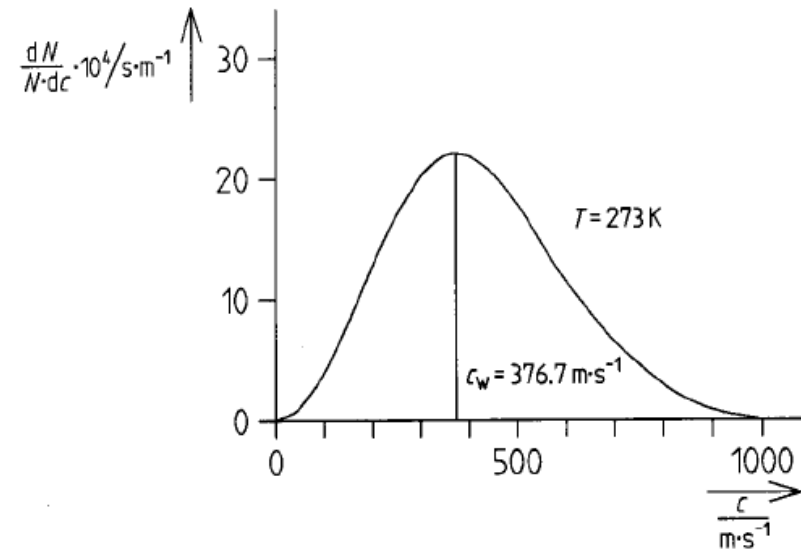
Loss of planetary and satellite atmospheres

- Thermal leakage to space
- (Stripping due to large impacts)

At some altitude (=“exosphere”), collisions between gas particles become negligible, and they move on ballistic trajectories determined only by gravity, and those with $v > v_{\text{esc}}$ can leave the planet.

Repartition of thermal energy at lower altitudes means the atmosphere can continually “leak” through this high velocity tail, even if $v_{\text{rms}} \ll v_{\text{esc}}$

Good rule of thumb in Solar System: loss of atmosphere results if $v_{\text{rms}} > \frac{1}{6} v_{\text{esc}}$



Maxwellian velocity distribution:

$$n_v dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp \left\{ \frac{-mv^2}{2kT} \right\} 4\pi v^2 dv$$

$$v_{rms} > \frac{1}{6} v_{esc}$$

v_{rms} for different
gas species

$$v_{rms} = \sqrt{\frac{3kT}{m_{gas}}}$$

v_{esc} for
each planet

$$v_{esc} = \sqrt{\frac{2GM_{planet}}{R_{planet}}} \rightarrow$$

$$T_{esc} \geq \frac{1}{54} \frac{GM_{planet} m_{gas}}{kR_{planet}}$$

Gives the critical temperature for
each species on each planet

e.g. for N_2 :

$T_{esc}(\text{Earth}) \sim 3900\text{K}$:

N_2 stays ($T_{exo} \sim 1000\text{K}$)

$T_{esc}(\text{Moon}) \sim 180\text{K}$:

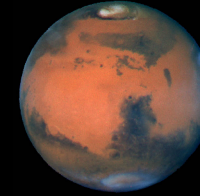
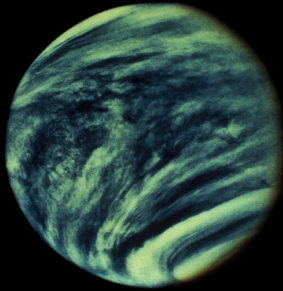
N_2 lost

$T_{esc}(\text{Mars}) \sim 700\text{K}$:

marginal

Note: Photodissociation by solar ultraviolet radiation of volatile species like CH_4 , NH_3 , (H_2O) produces $H_{(2)}$ which is almost always quickly lost.

Comparing the atmospheres of three terrestrial planets



	mass 10^{24} kg	radius km	v_{esc} kms^{-1}	Solar dist. A.U.	Density gm/cm^3	Atm. pressure (bar)	Surface Temp. K	Primary atmos. compon.
Venus	4.9	6000	10.4	0.72	5.24	90	750	CO_2 (96%) N_2 (3.5%)
Earth	6.0	6400	11.2	1.00	5.52	1	300	N_2 (78%) O_2 (21%) H_2O (~1%) Ar (1%) (+300 bar in liquid H_2O)
Mars	0.6	3400	4.8	1.52	3.9	< 0.01	180-290	CO_2 (95%) N_2 (3%) (+ten times more CO_2 in polar ice caps)

Greenhouse effects in atmospheres:

A planet receives solar radiation at visible wavelengths ($T_{\text{eff}} \sim 5800\text{K}$) and itself radiates at infrared wavelengths ($T_{\text{eff}} \sim 300\text{K}$).

What happens if the atmosphere is transparent at one wavelength but not at the other?

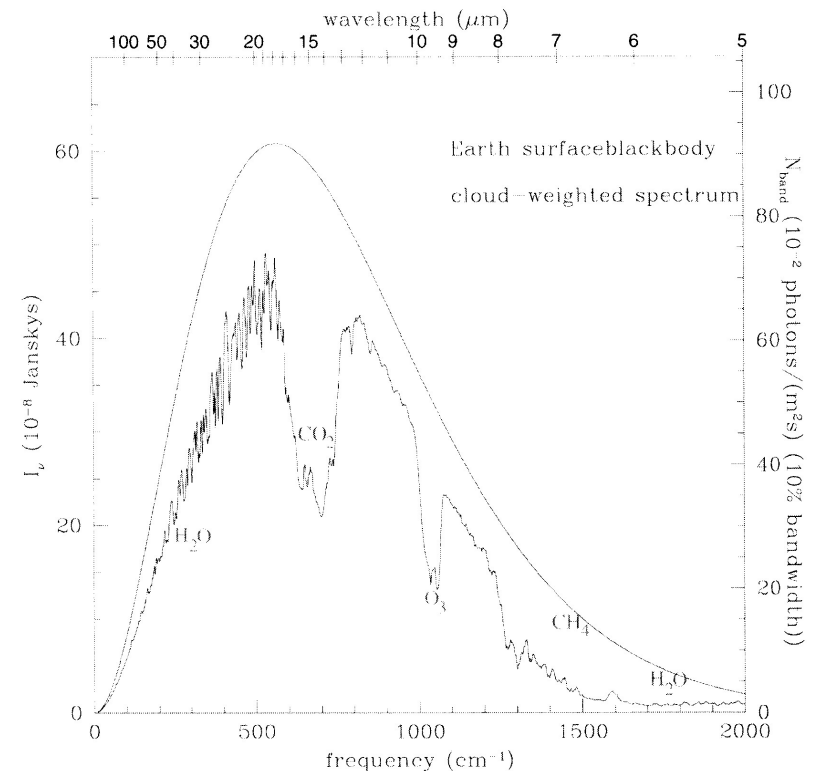
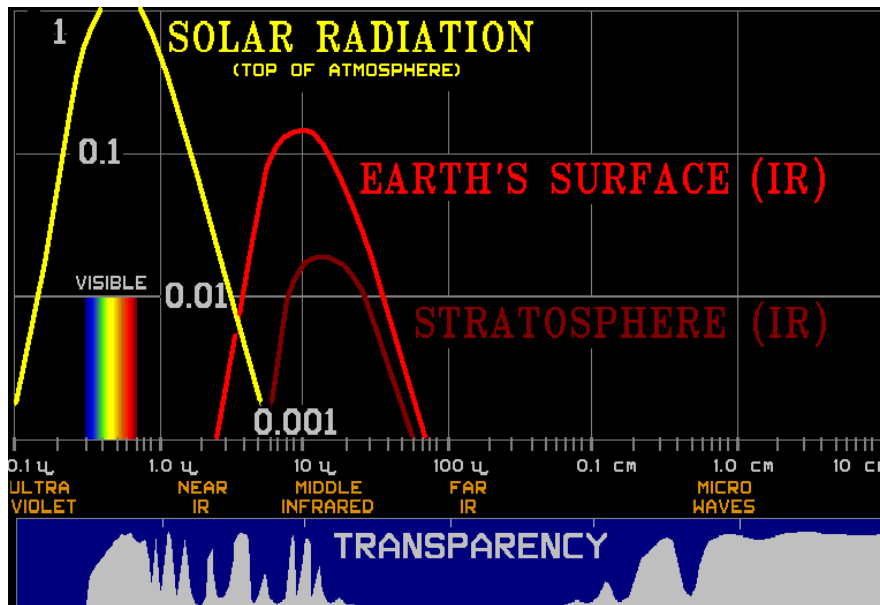


FIGURE 11.4 Spectrum of the Earth's atmosphere is plotted as intensity of light (arbitrary, linear units) versus wavelength in microns (10^{-6} meters). The spectrum shows absorption at near-infrared and infrared wavelengths caused by water vapor, carbon dioxide, and methane.

Equilibrium temperature in the absence of any atmosphere:

The Sun has temperature, T_S , radius R_S , and lies at a distance D

The planet has radius r , and reflects a fraction a of the incoming light (the “albedo”)

This will set up an equilibrium temperature T_E

$$4\pi r^2 \sigma T_E^4 = (1 - a) 4\pi R_S^2 \sigma T_S^4 \frac{2\pi r^2}{4\pi D^2}$$

$$T_E = T_S (1 - a)^{1/4} \sqrt{\frac{R_S}{2D}}$$

Note: T_E is independent the radius r of the planet



Now add a partially transparent/opaque atmosphere

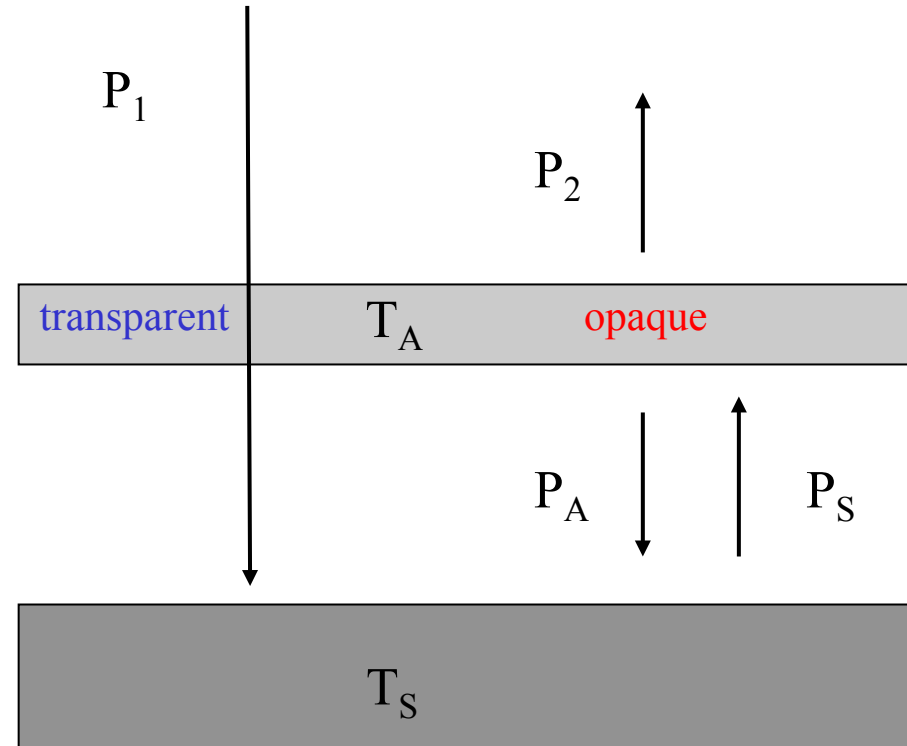
Set up some simultaneous equations involving energy flows

$$P_1 = P_2 = P_A = 0.5\sigma T_A^4$$

$$P_S = \sigma T_S^4 = P_1 + P_A = 2P_1$$

$$\sigma T_E^4 = P_1 \quad (\text{equilibrium temperature in absence of an atmosphere})$$

$$T_S = \sqrt[4]{2} T_E \quad \text{About a 20\% effect for this simple case}$$



Note for more complex situations bigger effects:

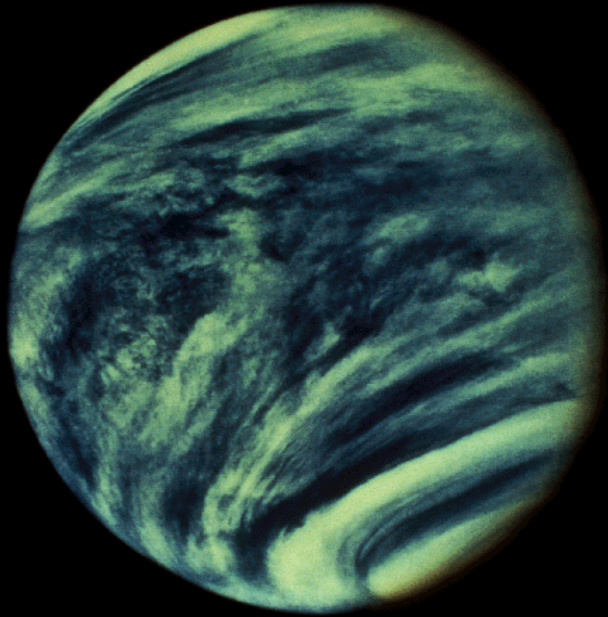
(a) For incomplete opacity in atmosphere

$$P_S = \sigma T_S^4 = P_1 + P_A = (2 - f) P_1$$

(b) For multiple n (or continuous τ) layers in the atmosphere

$$T_S = \sqrt[4]{(1 + n)} T_E = \sqrt[4]{(1 + \tau)} T_E$$

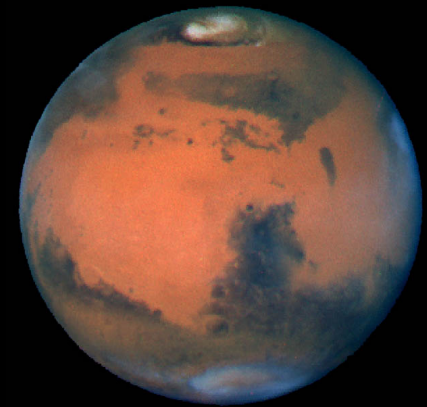
Greenhouse effects on inner planets



Venus
 $T \sim 2T_e$



Earth
 $T \sim 1.1T_e$



Mars
 $T \sim 1.02T_e$

Now let's go back and look at the early evolution of our three planets:

- All with have an initial atmosphere rich in volatiles brought in by “comets” – H_2O , NH_3 and CH_4
(Note: O,C and N are cosmically the most abundant elements – ignoring He and Ne – with O:C:N ratio $\sim 10:3:1$)
- Photodissociation of these and the loss of the H will convert the initial H-rich reducing atmosphere to a H-poor atmosphere of CO , CO_2 , N_2 and H_2O and small amounts of free O_2 (N.B. the amount of H loss is indicated by the D/H ratio since D less easily lost than H)
- The outer planets remain rich in H, He and hydrogenated gases

Evolution of the Earth (1)

- This is dominated by the fact that H₂O condensed out and formed Earth's oceans (from a very early time).
- CO₂ is highly soluble in water: rain (produced by the “water cycle”) scrubbed the CO₂ from atmosphere → producing solution of H₂CO₃ in water
- Reactions with metal ions in oceans → e.g. CaCO₃ (rock) (marine life helps but is not essential, most CaCO₃ is not biological)
- Very small quantities of CO₂ and H₂O that remained in the Earth's atmosphere produce a modest greenhouse effect (boost of +35K)

Note that the CO₂ currently “locked” in near-surface rocks would be sufficient to make 70 bar atmosphere of CO₂, i.e. similar to that seen on Venus

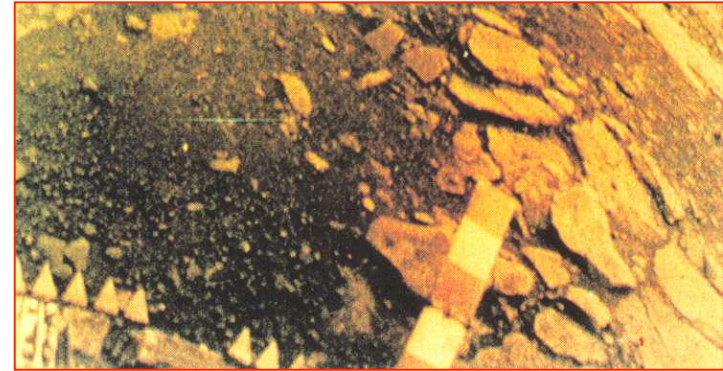
Evolution of the Earth (2)

- Subsequent biological activity (photosynthesis) produced free O₂ out of CO₂.
- This initially oxidized the CO → CO₂, plus oxidized Fe, S in surface rocks.
- After these were saturated, continued production of O₂ raised the atmospheric O₂ level to 20% (about 2.5 billion years ago) and maintained it there.

O₂ abundance in atmosphere is far from equilibrium and is a strong signature of Life that could be detected from far away. Especially the simultaneous presence of O₂ and trace CH₄ is a strong biosignature.

What happened to Venus?

H₂O did not condense and did not scrub out the CO₂. Why?? ($T_e \sim 330\text{K}$)



The initial greenhouse effect was strong enough to vaporise water in a runaway effect:

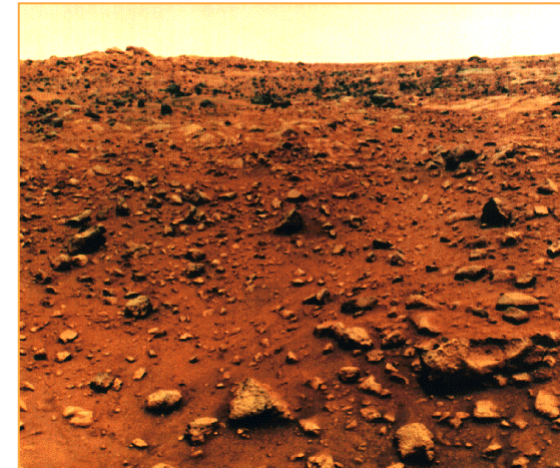
higher $T \rightarrow$ more evaporation \rightarrow more greenhouse effect \rightarrow higher $T \rightarrow$ more evaporation etc.

The water in the atmosphere was then almost entirely destroyed through photo-dissociation, leaving CO₂ to dominate the atmosphere: [D/H is 100 times higher on Venus than Earth, suggesting $> 99.9\%$ of the H was lost from Venus.

As noted above, the current CO₂ content of Venus' atmosphere is comparable to that in Earth's rocks, and the atmospheric N₂ content is similar

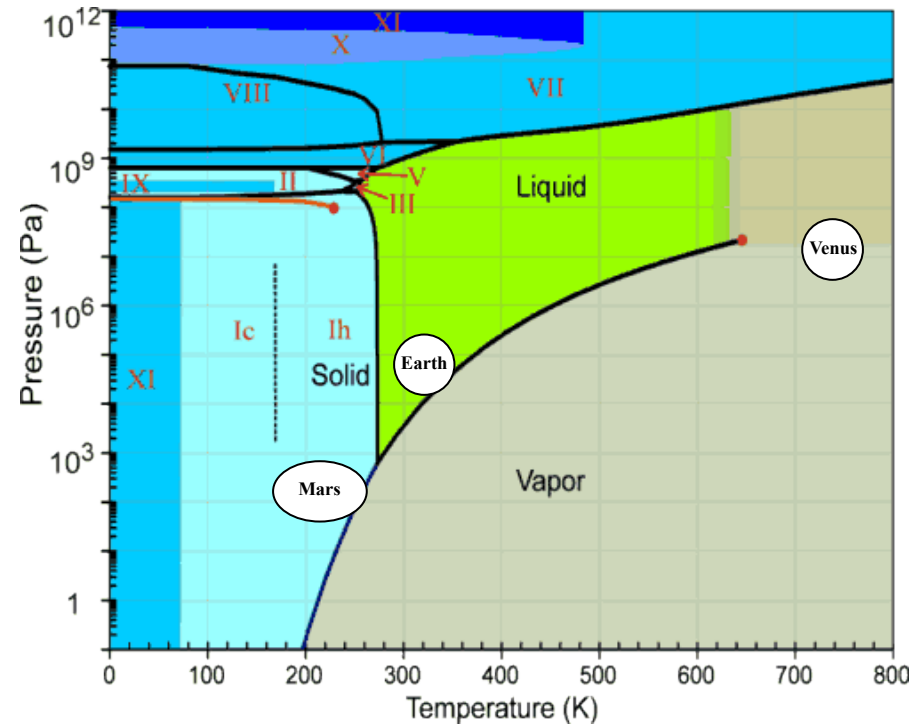
What happened to Mars?

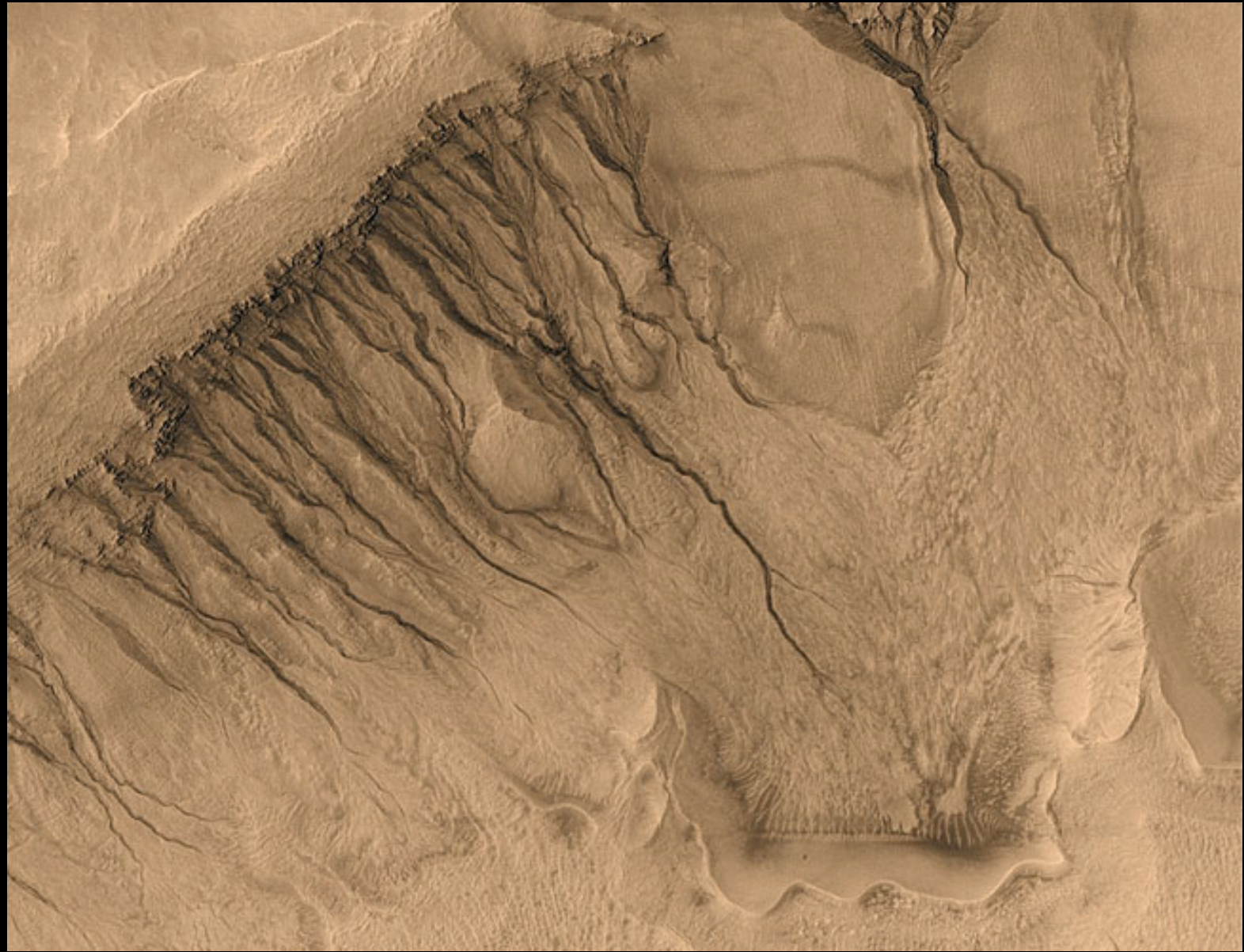
Retention of the atmosphere is marginal.
Impacts thought to have stripped away much of the atmosphere (e.g. Ar ratio → suggests a factor of 100?).

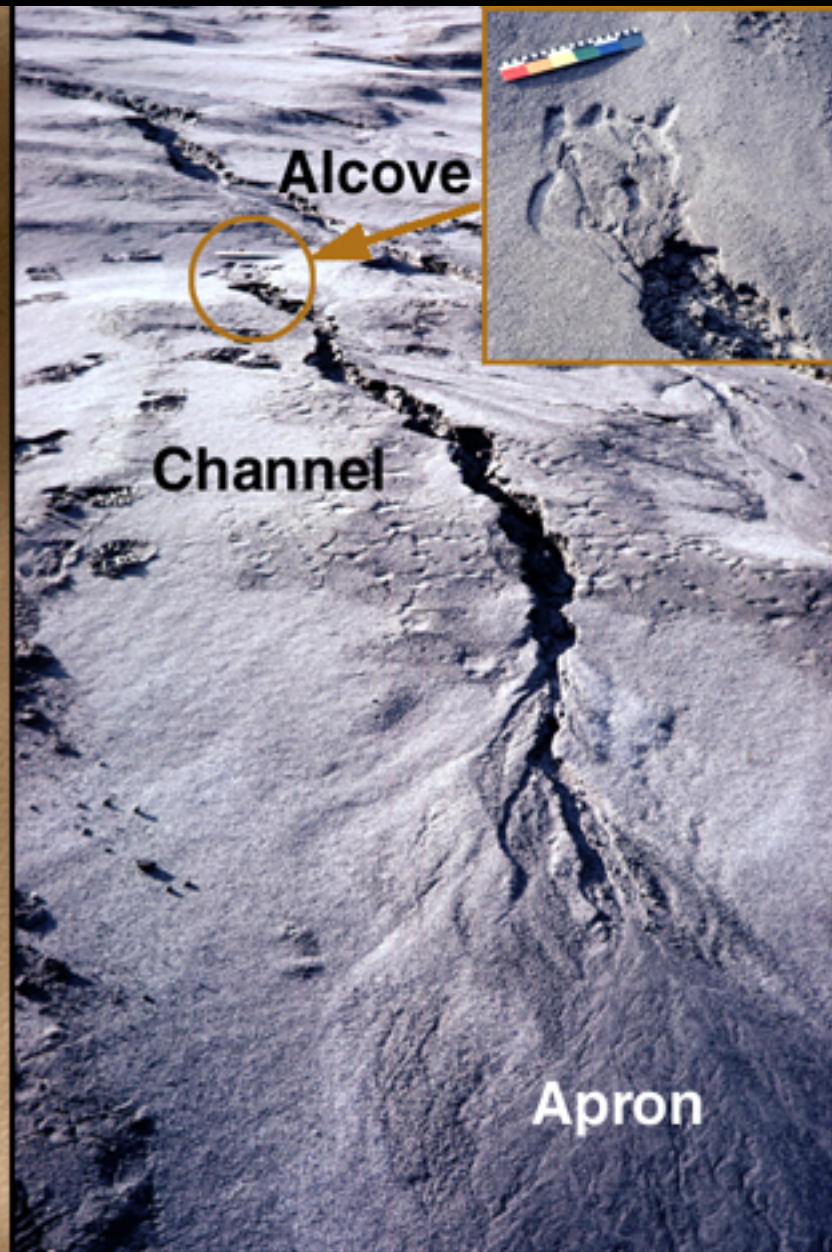
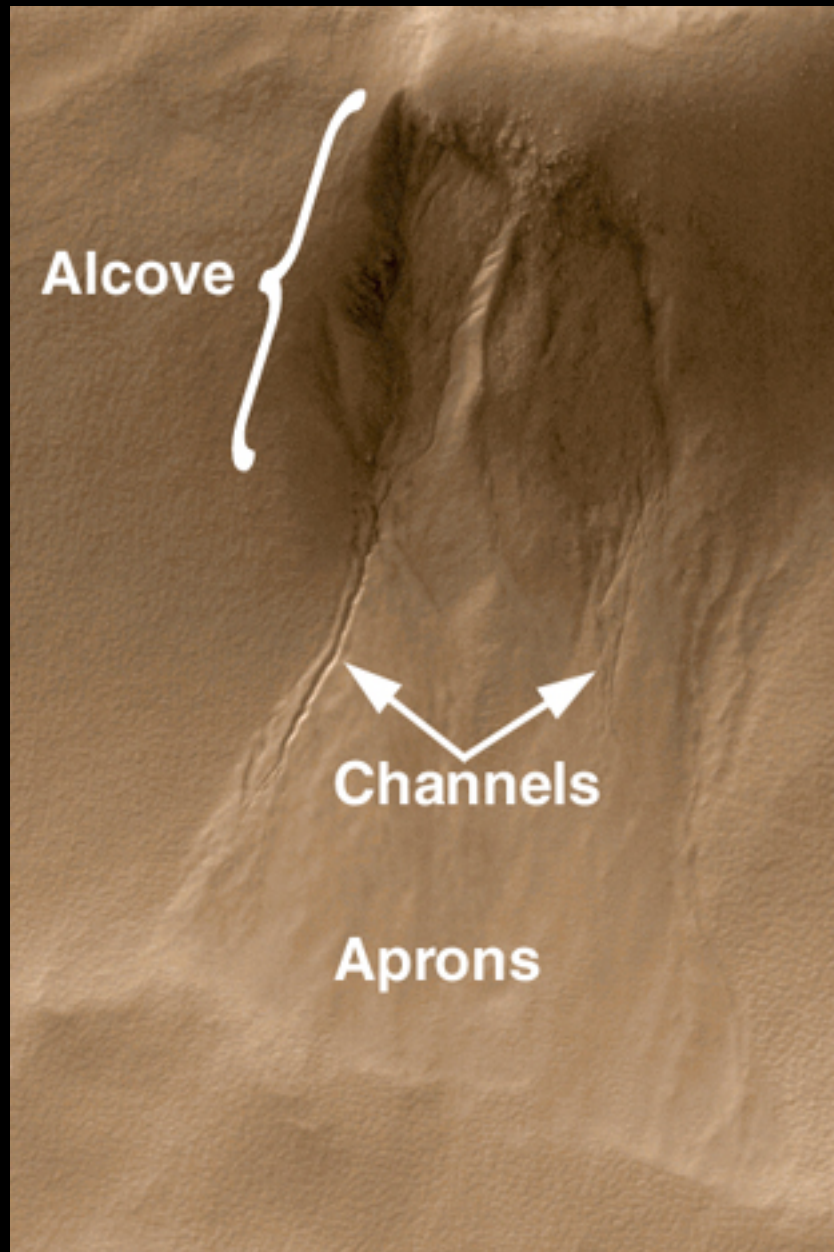


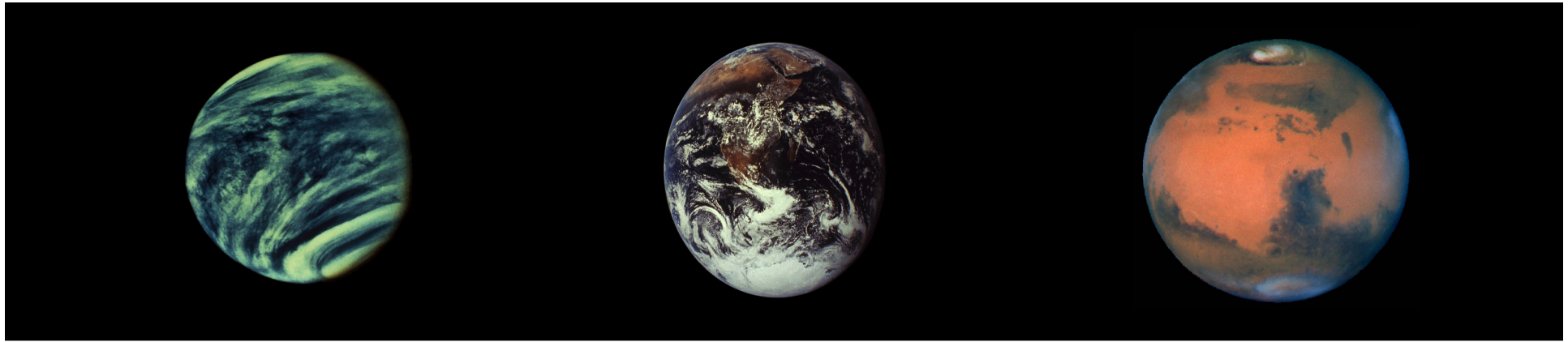
The weakness of the greenhouse effect (+5K) leads to condensation of CO₂ and H₂O on polar ice caps (removing ~ 90% of available CO₂ → runaway “freezeout”

The present atmosphere is unable to support liquid water (temperature is below the triple point), yet there is surface evidence for “flows”?









So let's return to our comparison:

	Apparent atmospheric compositions			
	Pressure	CO₂	H₂O	N₂
Venus	90	0.96	-	0.04
Earth	1	-	0.01	0.78** +0.21 O₂
Mars	0.01	0.95	-	0.03
	After taking into account oceans and rocks, and dividing by the mass of the planet			
Venus	...	9.6×10⁻⁵	>2×10⁻⁵	2×10⁻⁶
Earth	...	16×10⁻⁵	2.8×10⁻⁴	2×10⁻⁶
Mars	...	>3.5×10⁻⁸	>5×10⁻⁶	4×10⁻⁸

These three rather similar planets had three very different histories reflecting rather small differences in their mass and distance from the Sun

Key ideas: Formation and evolution of the Solar System

- A proto-planetary disk of material (gas+dust) is a natural consequence of the star-formation process. *Would expect planetary systems to be common (and they are!)*
- Planet formation involves chemical differentiation because of the condensation process on grain surfaces. *2% atomic diversity is concentrated up to to ~ 100%*
- The difficult step of growth is from cm to km sized bodies. Stochastic evolution of later stages building planets. *Don't be surprised by a diversity of planetary systems?*
- The heating of massive bodies during collapse leads to chemical differentiation
- Potential importance of other sources of heat (e.g. tides) on small bodies.
Other possible local heat sources, e.g. sub-surface water ocean on Europa
- Atmospheres come from impacts of volatile rich planetessimals during the last “clearing of the Nebula” phase. *Provided favourable conditions for Life on Earth...*
- Variety of subsequent evolution due to “amplification” of small initial differences. ...
but the conditions for Life may not be met everywhere.