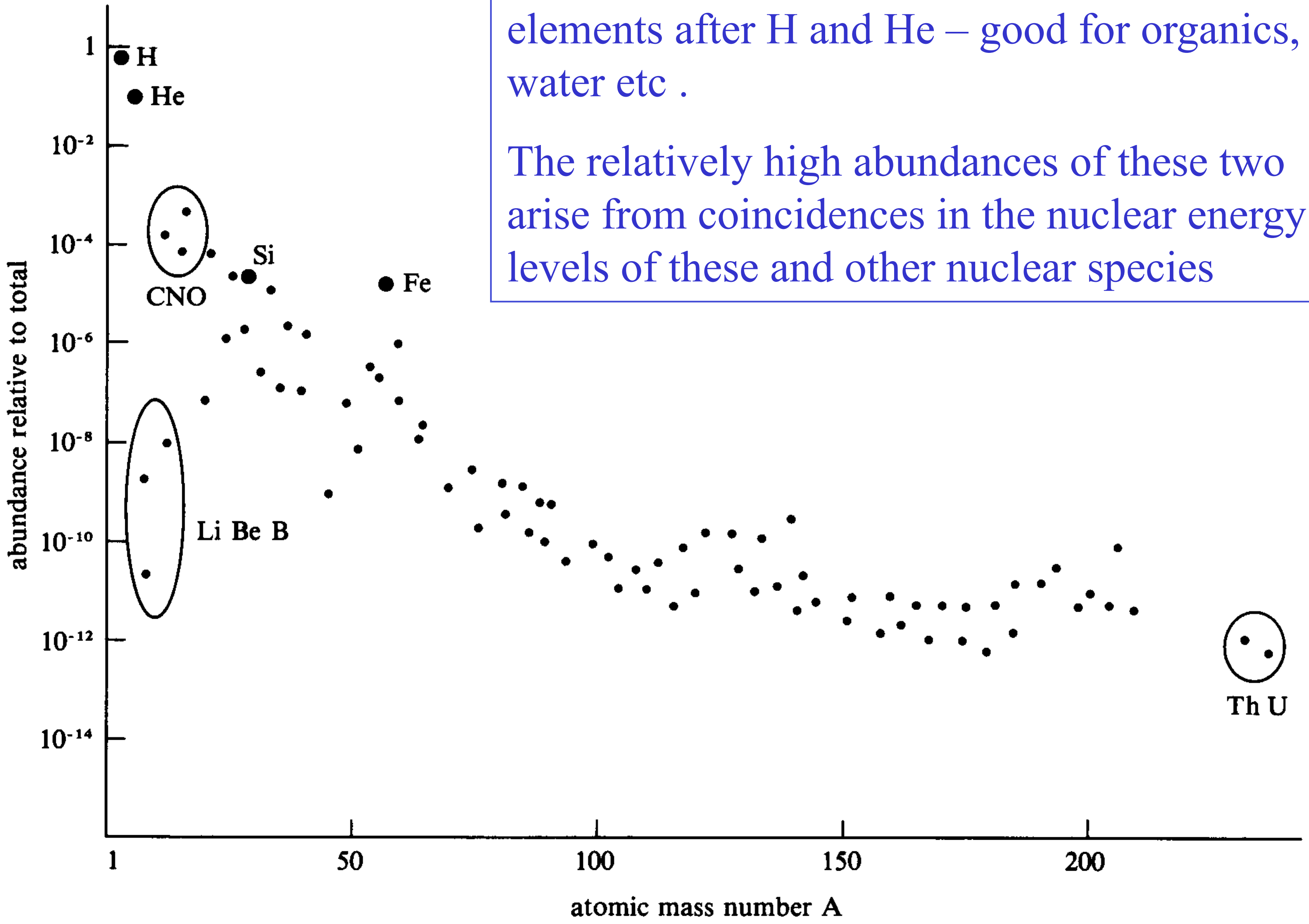
Origin of the chemical elements
(Part II)



Consequence: O and C are the most abundant elements after H and He – good for organics, water etc .

The relatively high abundances of these two arise from coincidences in the nuclear energy levels of these and other nuclear species

Pressure from degenerate matter

R. H. Fowler (1926): application of Pauli exclusion principle.
In a dense gas all the lower energy levels become filled with electrons and this results into a pressure which resists the gravitational force

$$P = \frac{\pi^2 \hbar^2}{5 m_e m_H^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\rho}{\mu_e}\right)^{5/3},$$

Can get order of magnitude using Heisenberg Uncertainty Principle: $\Delta x \Delta p \sim h/2\pi$
Consider object with density n particles per unit volume, the available volume per particle is n^{-1}, so $\Delta \mathrm{x} \sim n^{-1/3}$ and $p \sim (h/2\pi) \,.\, n^{1/3}$

Pressure will be product of the number density and mean kinetic energy $\sim n \,.\, E$

$$\boxed{P \sim \frac{\hbar^2}{2m_e} n_e^{5/3}}$$

Non-relativistic: $E = p^2/2m$

Relativistic $E = cp$

$$\boxed{P \sim \hbar c \; n_e^{4/3}}$$

Aside: Mass-radius relation and White Dwarf stability

Hydrostatic support requires $$\frac{dP}{dM} \sim \frac{GM}{4\pi r^4}$$

For non-relativistic case, we have $$\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{5/3} \qquad \Rightarrow \quad P \propto \frac{M^{5/3}}{R^5}$$

Applying hydrostatic support, gives a mass-radius relation $$\frac{M^{2/3}}{R^5} \propto \frac{M}{R^4} \qquad \Rightarrow \quad R \propto M^{-1/3}$$

Adding more mass to an object supported by degenerate electrons will cause the radius to shrink, the density to go up and the pressure to increase. Likewise, a self-gravitating object with non-relativistic degenerate matter is stable. But note the E per particle increases as $n^{2/3}$

For relativistic case, we have $$\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{4/3} \qquad \Rightarrow \quad P \propto \frac{M^{4/3}}{R^4}$$

Resulting in a mass-radius relation $$\frac{M^{1/3}}{R^4} \propto \frac{M}{R^4} \qquad \Rightarrow$$ Supported M is independent of R and object is unstable

Collapse beyond Chandrasekhar mass limit

The maximum mass that can be supported by degenerate electrons is 1.4 $M_\odot$ (Chandrasekhar 1931). This is the observed maximum mass of White Dwarfs.

Can neutron degeneracy stabilize collapsed objects?

Recall $P \sim m^{-1}n^{5/3}$ in non-relativistic case: After collapse by factor X, the density n increases by X^3, so

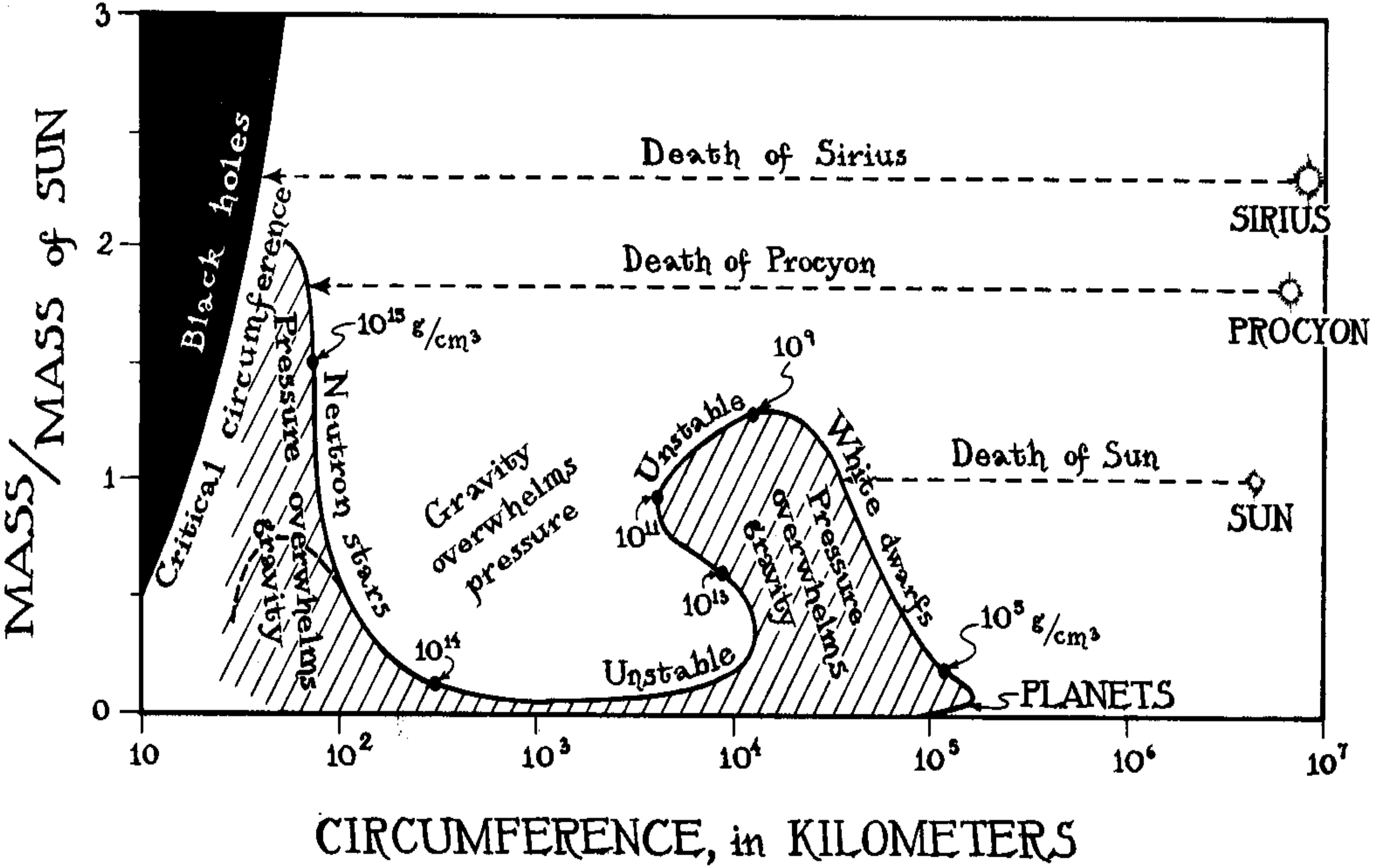
- pressure × area increases as X^3/m
- gravitational forces increases as X^2

so we would expect stabilization when the radius has shrunk by factor equal to the mass ratio of the particles: $X \sim m_n/m_e \sim 1000$, i.e. for radii of order 10 km c.f. 10^4 km for White Dwarfs.

Collapse of stellar core > 1.4 $M_\odot$ causes $e^- + p^+ \rightarrow n$ because n has lower energy state than p + e (because of enormous energy of e)

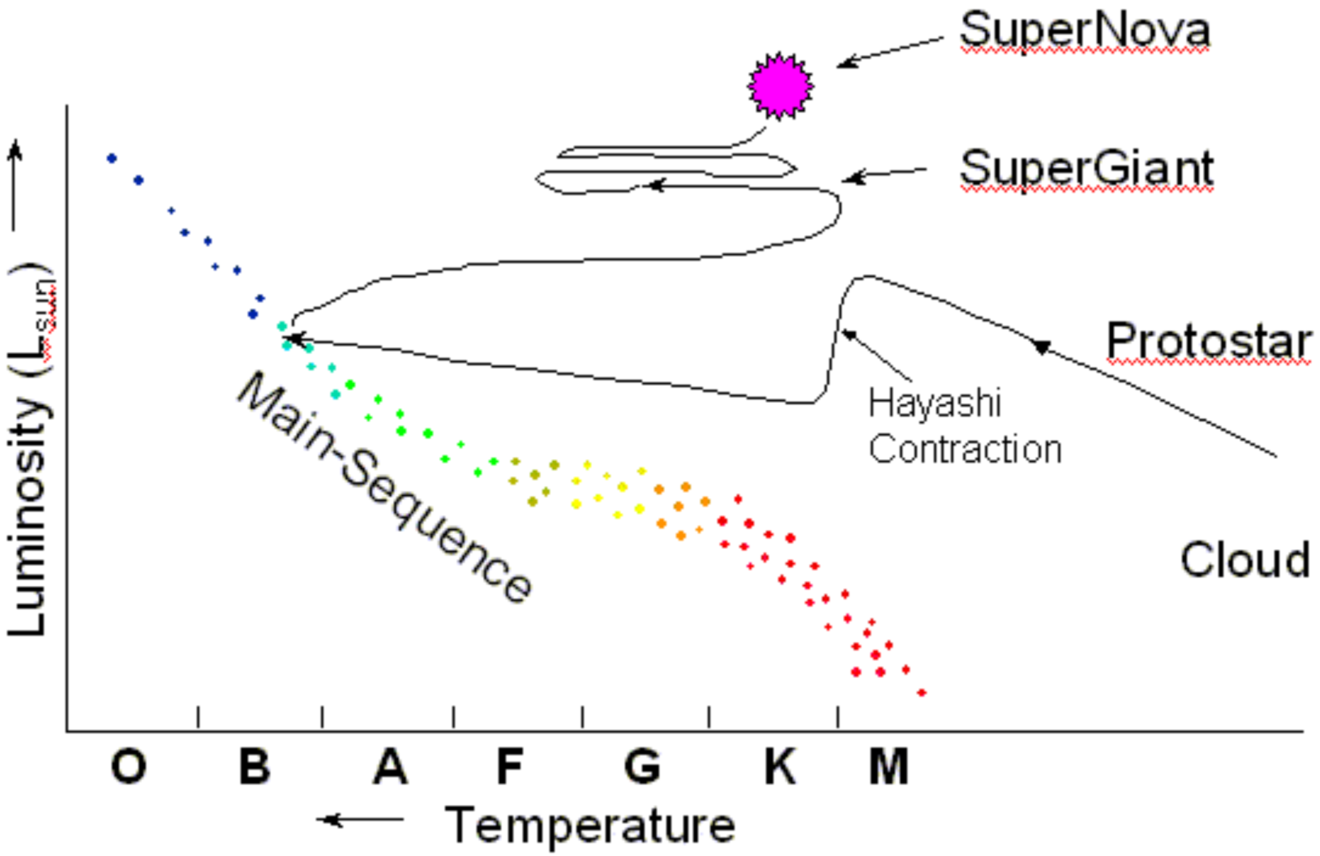
i.e. electrons are "squeezed out of existence"

Overview: collapsed objects of stellar masses in the Universe



From Kip Thorne "Black Holes & Time Warps"

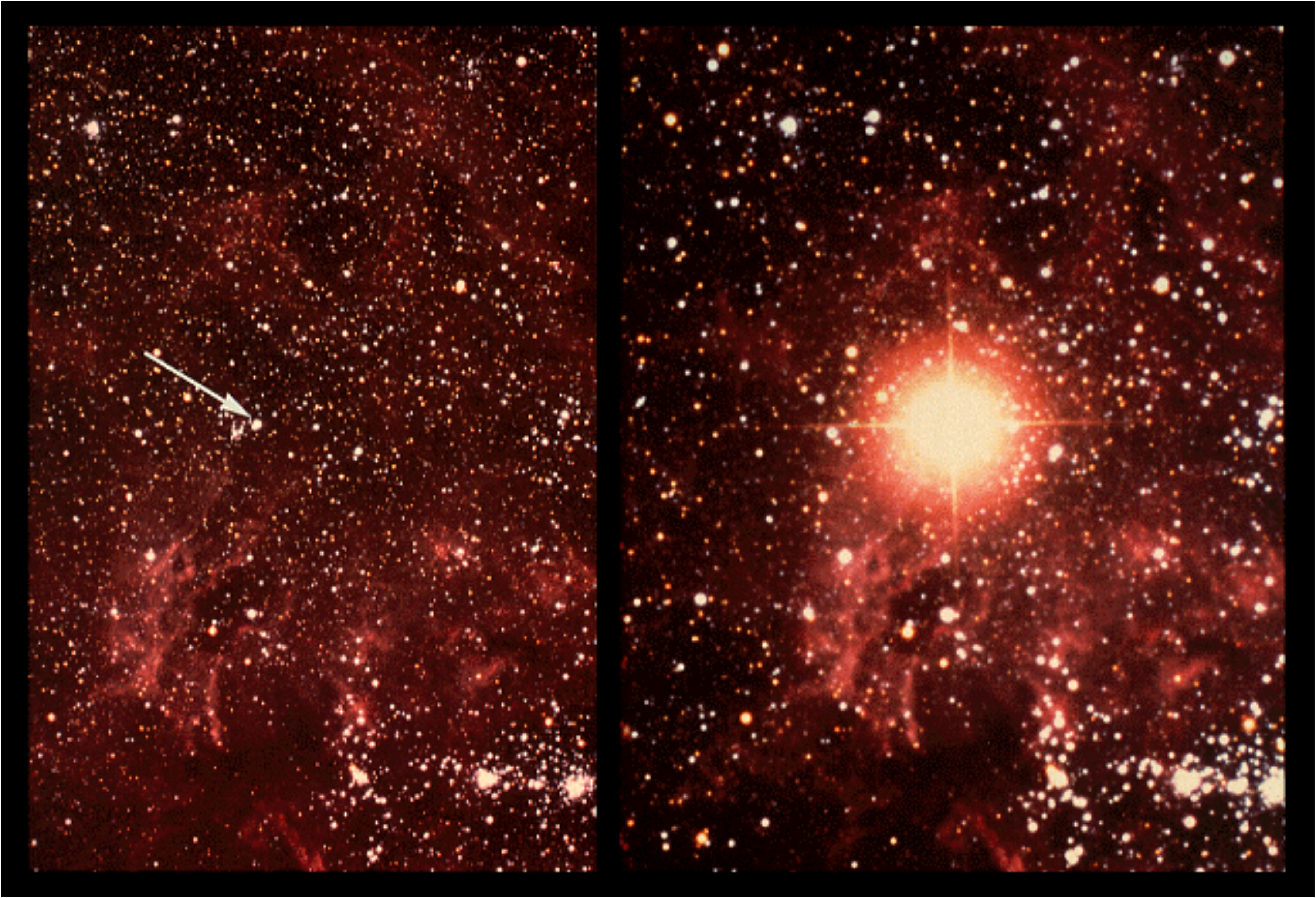
Evolution of 20 $M_\odot$ star



Type II Supernovae

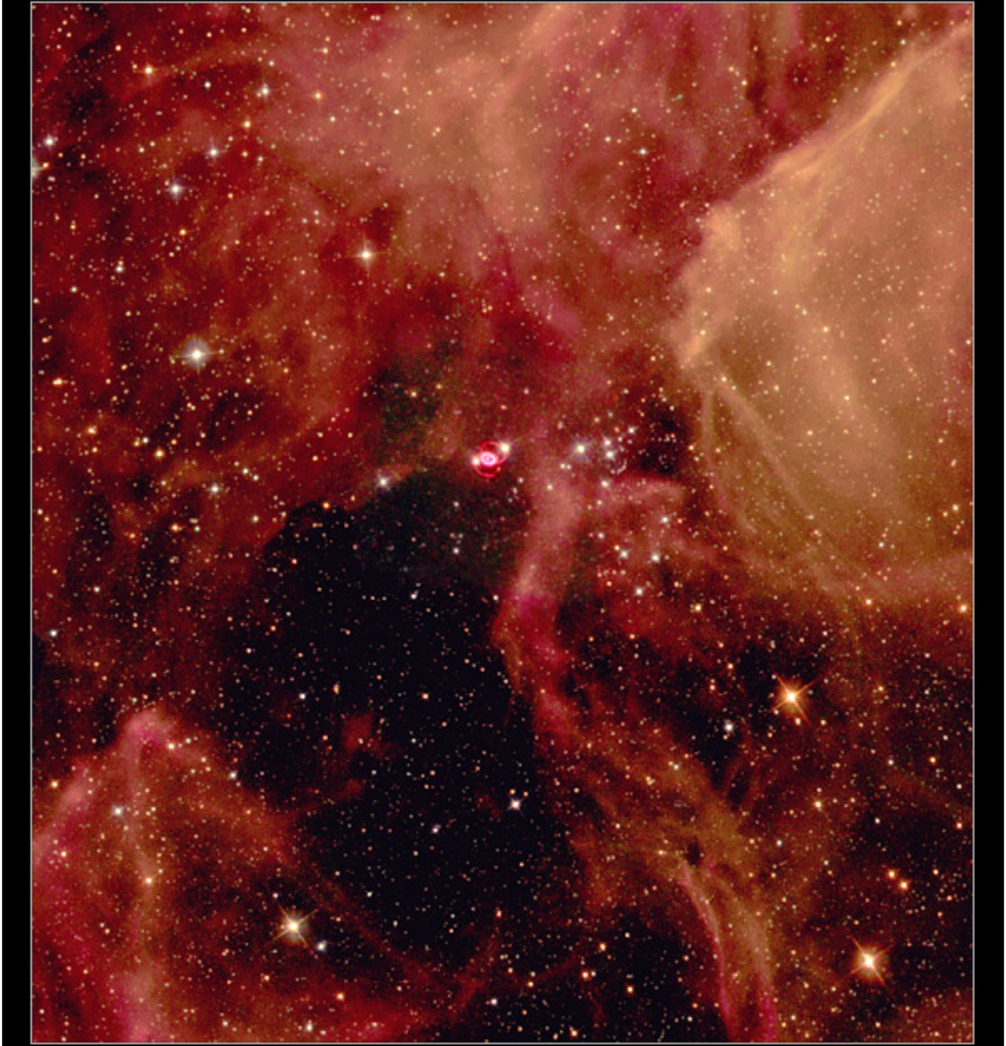
- Catastrophic collapse of cores of stars with initial $M > 8\ M_\odot$ during ~ 1 second
- No further nuclear production of energy is possible from the iron-rich core which starts contracting and heating up catastrophically. Core passes through degenerate electrons as $m_{\rm core} > 1.4\ M_\odot$.
- Every channel for the loss of energy causes the core to contract more and more.
  - o unbound particles start being energetically favoured.
  - o ^{56}Fe begins to photo-disintegrate into α-particles and neutrons subtracting energy from the core
  - o α-particles themselves photo-disintegrate and the core begins to fall freely under its own self-gravity.
  - o Almost all the free electrons are "squeezed" into the protons to form neutrons at nuclear densities
- Copious numbers of neutrinos (one per proton-electron combination) are produced, and stream from core, further subtracting energy
- The pressure from neutrino pulse is believed to be responsible for exploding away the stars' outer layers ("believed" because no model yet really works in 3d)
- Pulse of neutrinos was detected shortly before visual sighting of SN 1987a in Large Magellanic Cloud (first visual detection of SN for ~ 400 years)
- Core collapses to ~10 km where it is stabilized by degenerate neutrons (or → BH?)

Neutron stars and supernovae



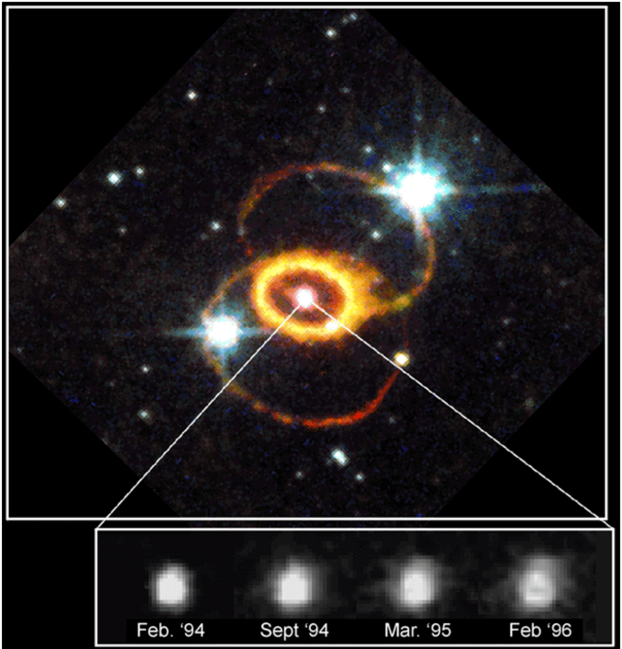
SN 1987A

Supernova 1987A



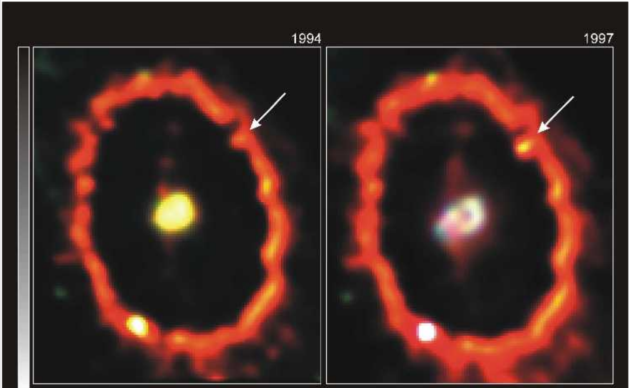
Hubble Heritage

PRC99-04 • Space Telescope Science Institute • Hubble Heritage Team (AURA/STScI/NASA)



Supernova 1987A HST • WFPC2

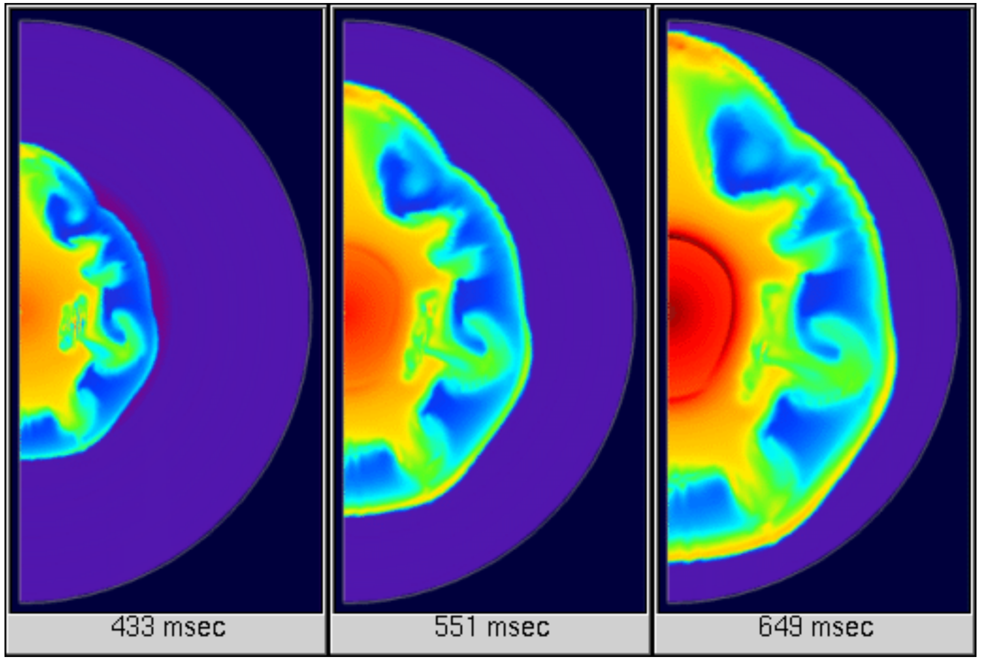
PRC97-03 • ST ScI OPO • January 14, 1997
J. Pun (NASA/GSFC), R. Kirshner (CfA) and NASA

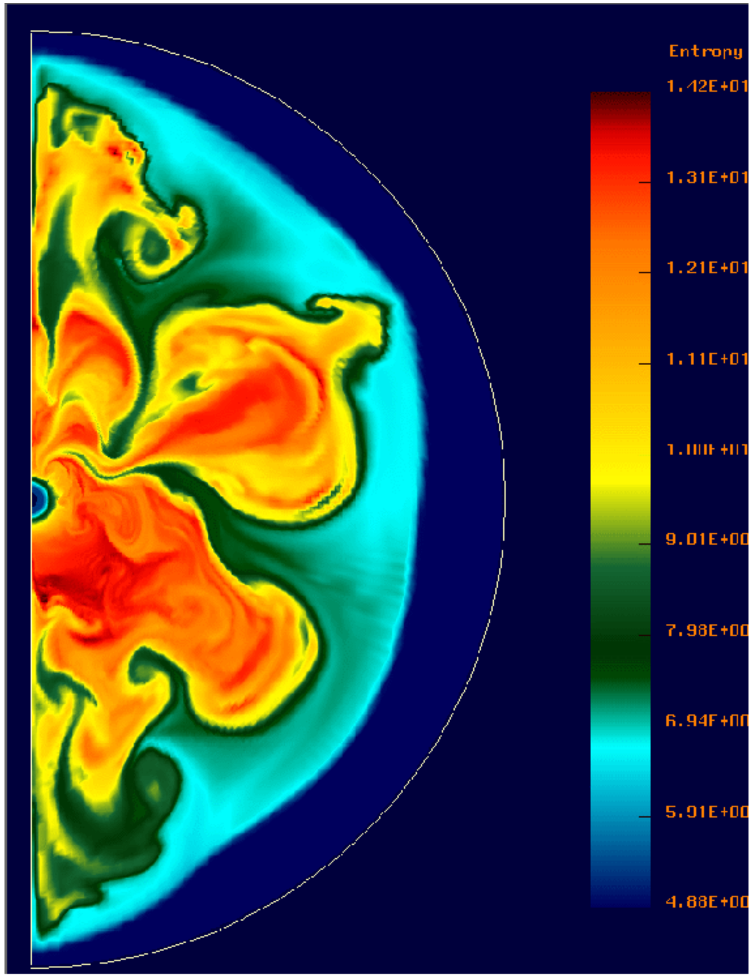


Supernova 1987A Ring
Hubble Space Telescope WFPC2

PRC98-08b • February 10, 1998 • ST ScI OPO • P. Garnavich (Harvard-Smithsonian Center for Astrophysics) and NASA

Neutrino driven explosion





Energetics of Supernovae

We saw how current thermal energy of a thermal-pressure supported star will always be comparable to its (negative) gravitational potential energy, via the virial condition

$$K = -\frac{U}{2}$$

Gravitational Potential Energy is proportional to R^{-1}.

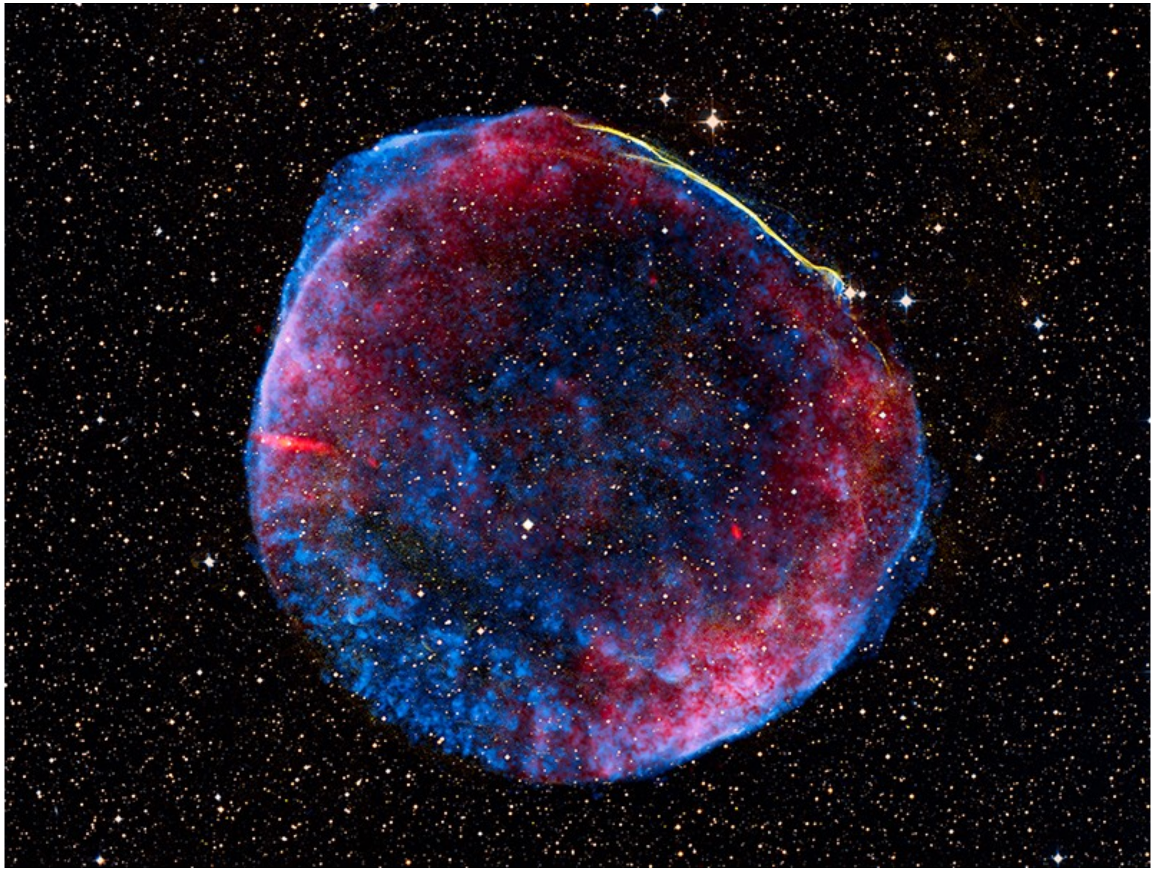
$$U \sim -\frac{GM^2}{R}$$

Key point: when a star collapses from $O(10^6)$ km to $O(10)$km in a core-collapse supernova, you release 10^5 times the potential energy (and thermal energy) that the object had as a star.

Ejection of debris from supernovae

Why do you get explosion and ejection of material from *collapse*?

- Total energy released is ~ gravitational binding energy of all material.
- Escape speed from surface of the neutron star ~ 0.3c.
- If not all the material is ejected, then some will have E > binding energy, so the ejection speed at infinity can easily be 0.1c.



Remnant of SN 1006

Neutron capture (1): The s-process

Question: How are elements beyond the iron peak formed?

- definitely not by energetically favoured fusion of smaller nuclei.

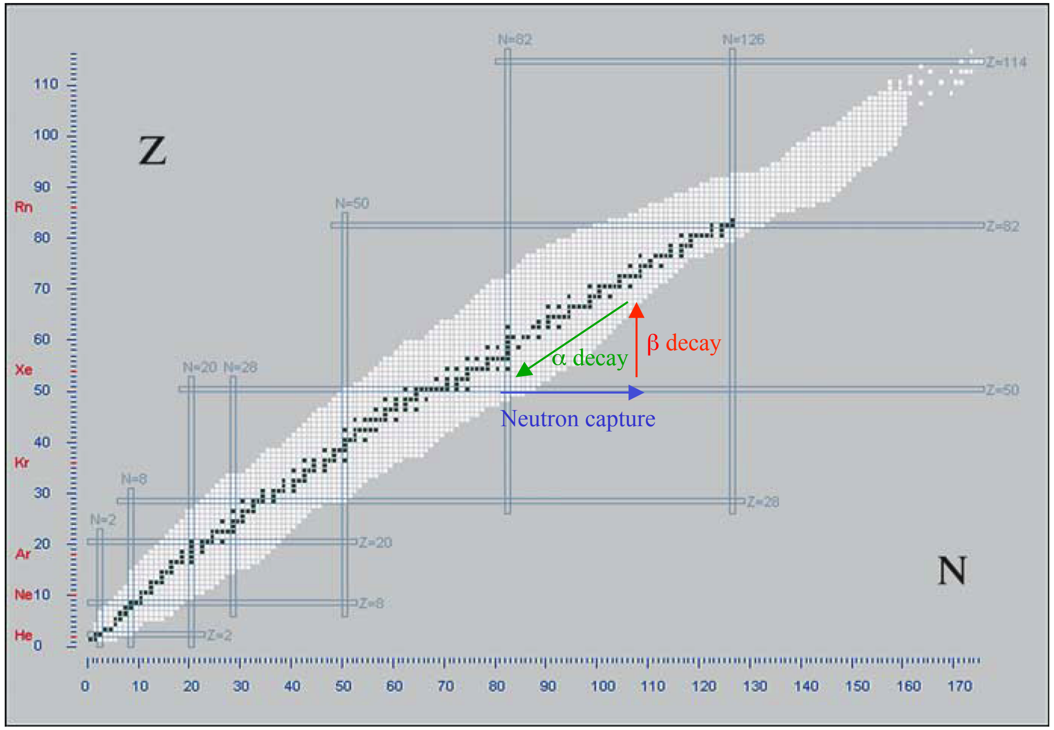
Answer: By neutron capture

Since a neutron is electrically neutral, even a slow neutron can be absorbed by an atomic nucleus, which might then be unstable to β-decay, increasing Z by one.

If the rate of production of free neutrons is sufficiently slow, all the nuclides formed by neutron capture that are unstable will have a chance to β-decay before a second neutron is absorbed.

The elements formed this way are called *s*-process elements (*s* stands for "slow"). This occurs during last ~10^2 years of massive star's life when conditions in cores produce some free neutrons.

Neutron capture (2) : r-process



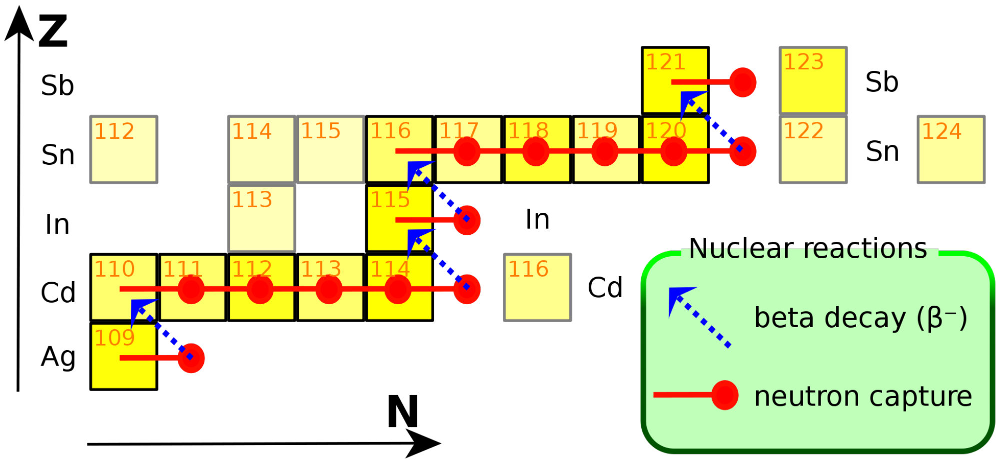
In a core-collapse SNa event, the rate of production of free neutrons becomes very rapid (flux ~10^{22} cm^{-2} s^{-1}; T~ 10^{10} K); as a consequence the elements formed by neutron capture will not have a chance to β-decay before the capture of another neutron.

Eventually, after the period of neutron production, the newly formed very heavy elements will have a chance to α and β-decay.

The sequence of elements formed this way - called the r-process elements (r stands for “rapid”) - generally differs from the sequence formed by the *s*-processes.

s-process does one neutron-adding step at a time via reasonably stable nuclei

r-process adds many neutrons at once followed by various decay routes



Different elements/isotopes are primarily produced either by s- or r-process:

s-process

$^{56}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{57}\mathrm{Fe}$

$^{57}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{58}\mathrm{Fe}$

$^{58}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{59}\mathrm{Fe}$

$^{59}\mathrm{Fe} \rightarrow {}^{59}\mathrm{Co}$

r-process

$^{59}\mathrm{Fe} + 2\mathrm{n} \rightarrow {}^{61}\mathrm{Fe} \rightarrow {}^{61}\mathrm{Co}$

most Co on Earth is $^{59}\mathrm{Co}$

s-process is expected to occur in last few hundred years of massive star's life, produces elements up to Bi (Z = 83)

Evidence: ^{99}Te is unstable $t_{1/2}$~ 200,000 yrs yet seen in atmospheres of AGB stars (convected up from core).

s-process stops at Bi because of cycle:

$^{209}\mathrm{Bi} + \mathrm{n} \rightarrow {}^{210}\mathrm{Bi}$

$^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + \mathrm{e}^-$

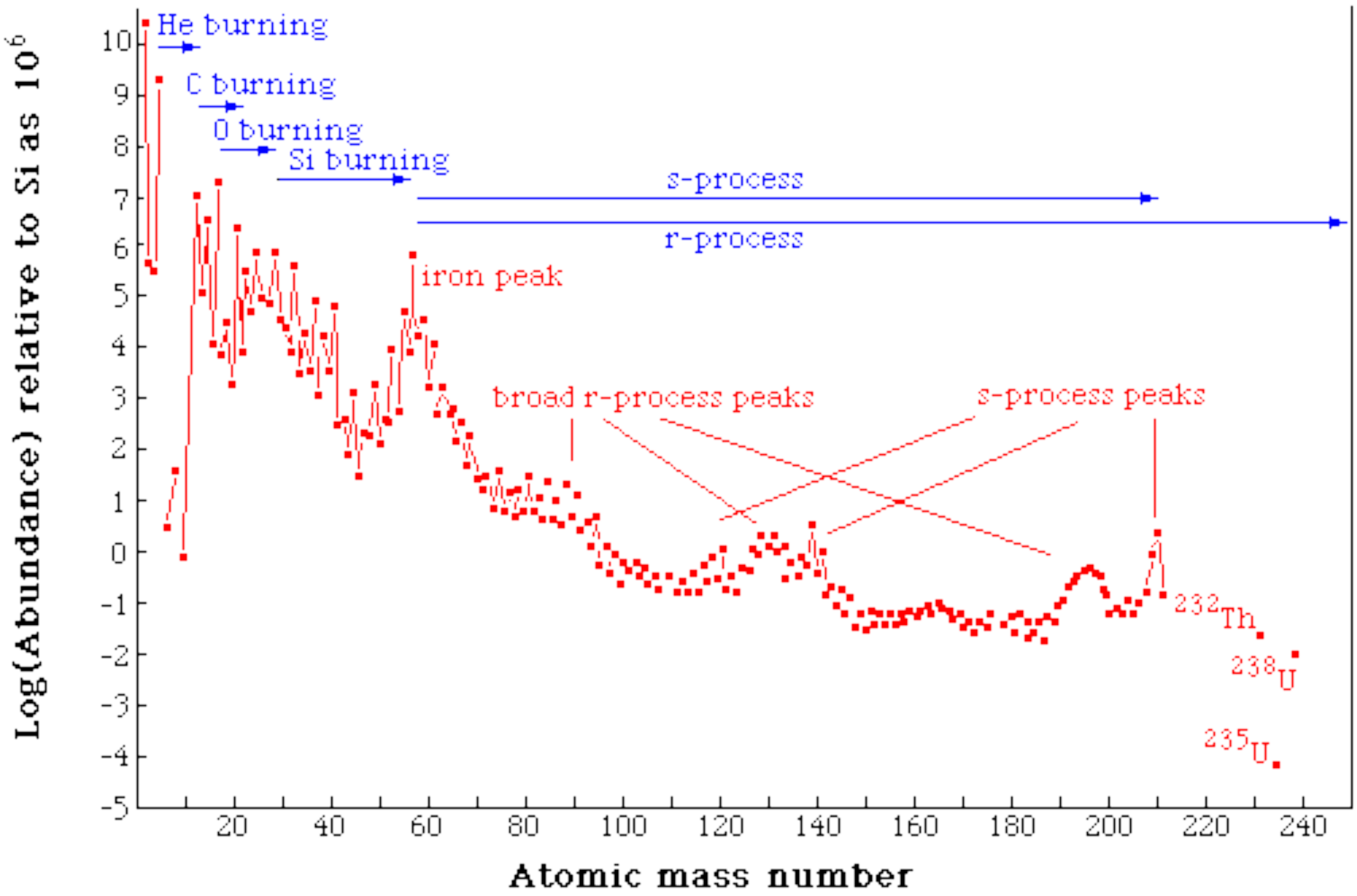
$^{210}\mathrm{Po} \rightarrow {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He}$

$^{206}\mathrm{Pb} + 3\mathrm{n} \rightarrow {}^{209}\mathrm{Pb}$

$^{209}\mathrm{Pb} \rightarrow {}^{209}\mathrm{Bi} + \mathrm{e}^-$

r-process only happens in ~ 1 second of a supernova collapse and is the only channel for production of elements above Bismuth (Polonium to Uranium), and is dominant for many other elements including Gold, Silver etc.

Solar system abundances



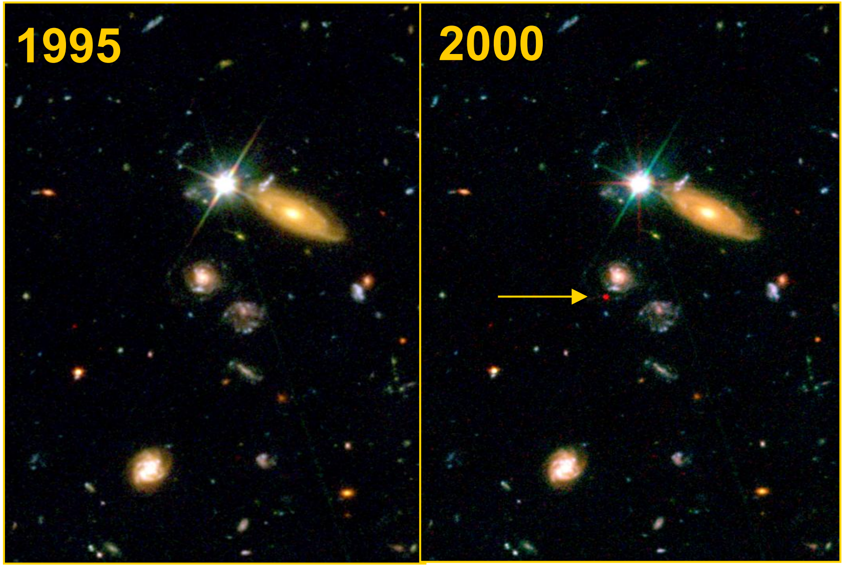
Also: Supernovae Type 1a

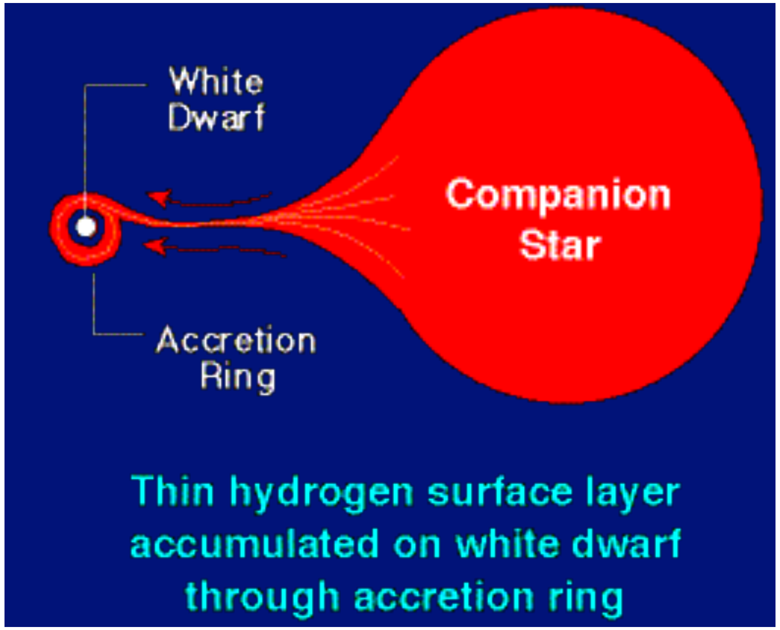
White dwarf is made of Carbon nuclei.

If mass is added to a White Dwarf (e.g. by accretion in a binary system) then at a point near the Chandrasekhar mass, the temperature and density increase enough to cause thermonuclear ignition of the Carbon nuclei.

This causes the WD to explode and produce a SN1a (identified by absence of H in spectrum). Of interest for two reasons:

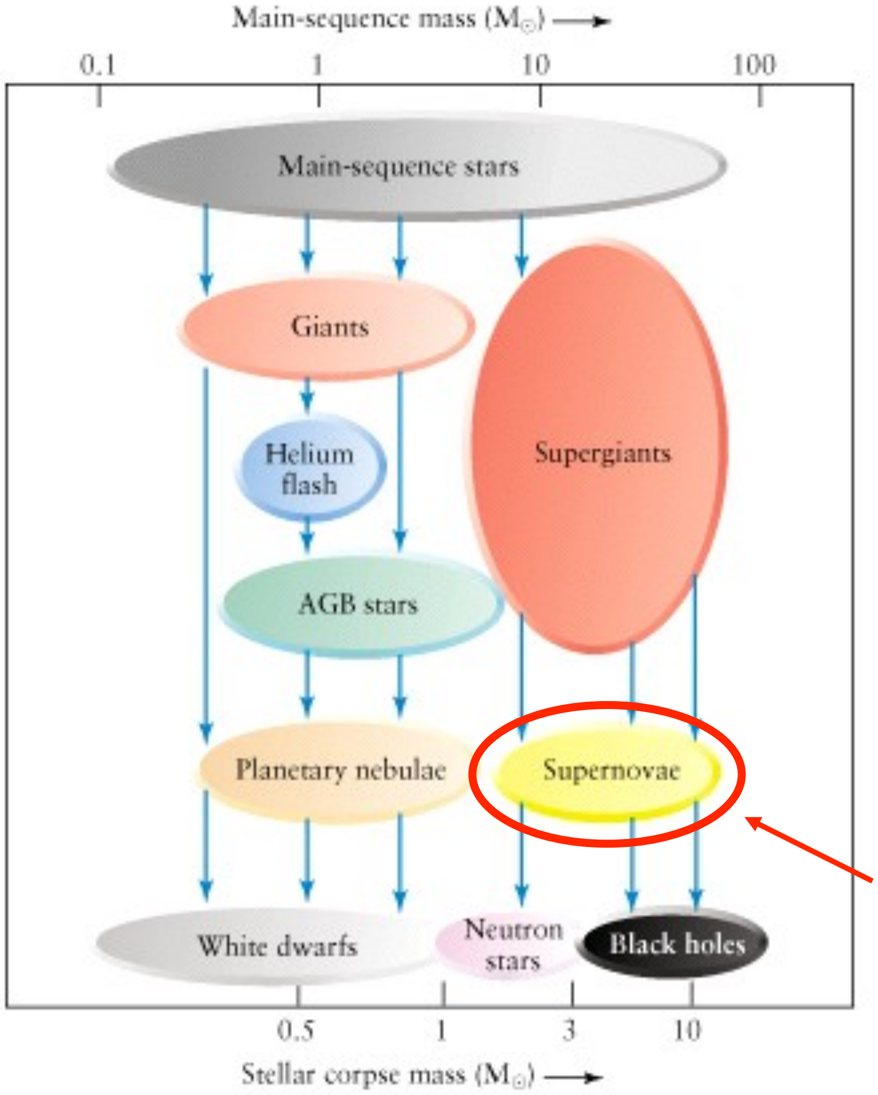
- Efficient production of Iron c.f. α-elements (^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S etc)
- "Standard candles" for cosmology





Thin hydrogen surface layer accumulated on white dwarf through accretion ring

Stellar Evolution Summary



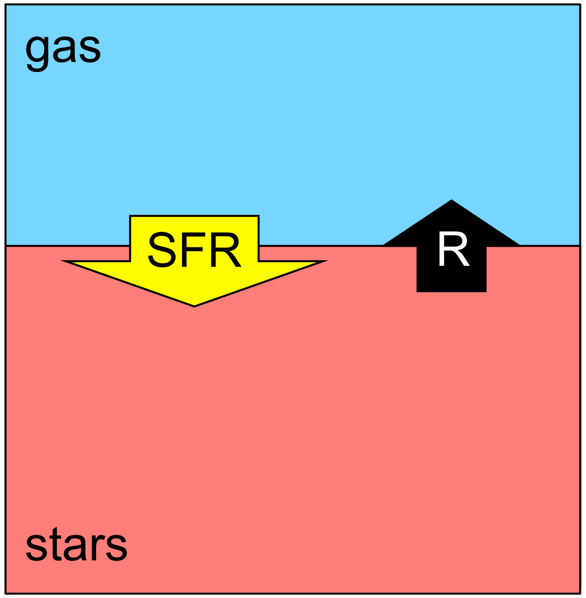
Supernovae and Life

SN: return chemical elements to surrounding ISM, available for subsequent star/planet formation and Life

“Chemical evolution” of the interstellar gas in galaxies

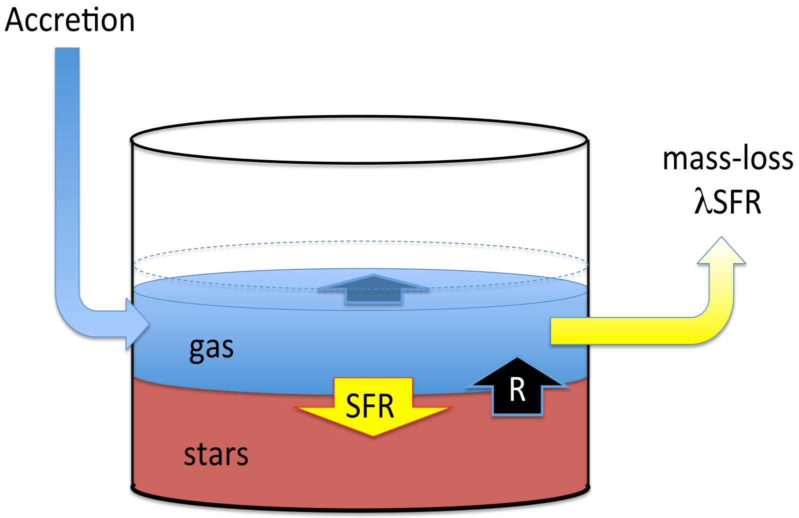
The yield y is defined as the mass of heavy elements (> ^{4}He) returned to the interstellar gas by SN per unit mass of material that is formed into a stars. For a standard mass distribution of stars, $y \sim 0.01$. Gas metallicity will be O(y).

Simple “closed box” evolution



$$Z = -y \times \ln\left({m_{gas}}/{m_{tot}} \right)$$

Flow-through evolution regulated by gas mass



$$Z = \frac{y}{1 + \lambda(1-R) + \left({m_{gas}}/{m_{star}} \right)}$$

Chapter 4 – key ideas

- Fusion in stellar cores provides long-term source of energy. Required high temperatures established by gravitational collapse.

- Fusion synthesizes elements, in principle up to ^{56}Fe if temperatures are hot enough, i.e. in the most massive stars.

- Ignition of H in collapsing proto-stars is largely inevitable unless the mass is too low ($< 0.08\ M_{sun}$).

- Velocity tail + quantum tunnelling produces sharp "Gamow Peak" in energy of reactants and a strong T dependence. The latter, together with negative heat capacity, stabilizes stars.

- Interaction of well-defined energy of reaction + Gamow Peak with excitation levels in nuclei has large effect on reaction rates.
    - formation of Carbon is less impossible than you would think (3-body reaction) due to favorable excited state
    - destruction of Carbon is hindered

Chapter 4 – key ideas

- Elements above ^{56}Fe require energy to be created via neutron capture, via the *s*-process (in last stages of star) and in *r*-process in supernovae.

- Degenerate electrons (temperature independent) can support cores up to 1.4 M_{sun}. Relativistic degenerate matter is unstable to collapse.

- Collapse to neutron star releases vast amount of potential energy, and ejects enriched material into surrounding space at velocities up to 0.1*c*.

- The yield y is the mass of heavy elements (> ^{4}He) returned to the interstellar medium per unit mass of material formed into a set of stars. For standard mass distribution of stars, $y \sim 0.01$. The abundance of heavy elements in interstellar gas will be of this order.