Cosmology – Part II

Origin of structure in the Universe
The future of the Universe

Initial density fluctuations

Gravitational instability causes density fluctuations to grow

Given Fourier mode has fixed comoving wavelength as Universe expands (therefore physical wavelength increases). Some physical effects (e.g. damping) will often be driven by physical wavelength.

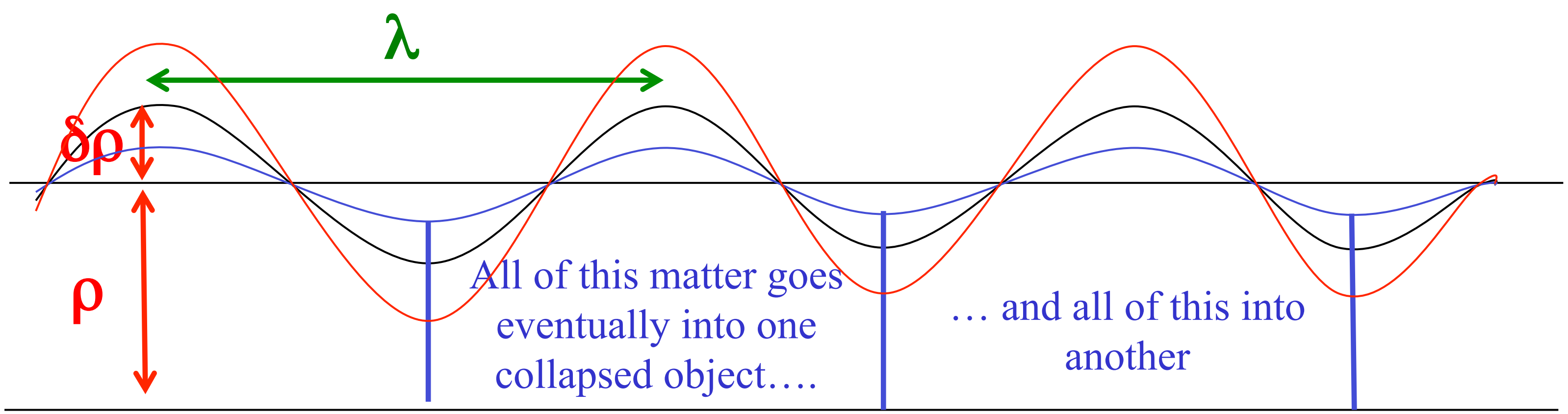
Conventional to think in terms of comoving wavenumber k and physical wavelength λ

$$\lambda = R(\tau)\, 2\pi\, k^{-1}$$

Note that the mass associated with a density perturbation of wavelength λ or wave number k is simply proportional to k^{-3}.

$$M \sim \rho_m \lambda^3 \propto k^{-3}$$

The amplitude of the density fluctuation is given by $\Delta_k = \delta\rho/\rho$.



Actual density field of the Universe $\Delta(x)$ is a Fourier sum of different modes with a power spectrum described by a power-law in comoving wave number

$$\Delta(\bar{x}) = \frac{1}{(2\pi)^3}\int \Delta_k e^{-i\bar{k}\bar{x}} d^3\bar{k}$$

$$\Delta_k^2 \propto k^n$$

Some interesting values of n:

- All $n > -3$ have homogeneity on the largest scales.
- A completely random mass distribution has $n = 0$.
- The $n = 1$ spectrum has many interesting properties: Not least, at all epochs the amplitude of those fluctuations *that are on the scale of the horizon (c/H)* are always the same even though the scale of the horizon changes. It is called the Harrison-Peebles-Zeldovich spectrum. It is naturally produced from Inflation − see next slide). Introduce $Q = \Delta$ on scale of the horizon.
- $n = 4$ arises from a purely local perturbation of matter

Our Universe appears to have created a primordial $n = 1$ spectrum, which was subsequently modified on small scales (see later). $Q \sim 10^{-5}$.

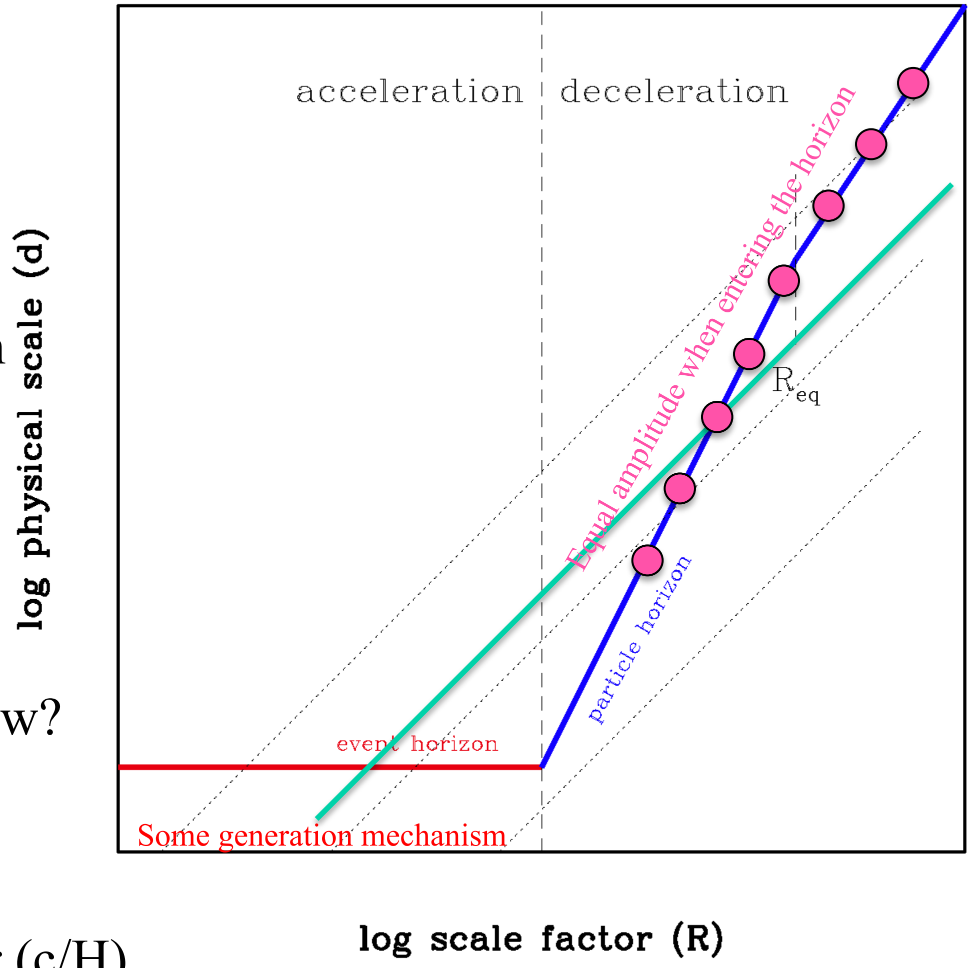
Aside for enthusiasts: The origin of n = 1 fluctuation spectrum

$$\omega_{H,particle} = \int_{\tau_1}^{now} \frac{c}{R(\tau)} d\tau$$

Particle horizon: have we received information on an event in the past? Decelerating universes have an expanding (comoving) particle horizon

$$\omega_{H,event} = \int_{now}^{\tau_2} \frac{c}{R(\tau)} d\tau$$

Event horizon: will we ever receive information on an event happening now? Accelerating universes have a contracting (comoving) event horizon



- The horizon scale is always of order (c/H).
- A given fluctuation scale may leave the horizon (through event-horizon) and then re-enter the horizon (through particle-horizon), with the same Δ_k when it does so **(why?...)**

The amplitude growth of different Fourier components

Classic analysis of Jeans (static medium) further developed by Lifshitz for an expanding medium

$$\frac{d^2\Delta_k}{dt^2} + 2\frac{\dot{R}}{R}\frac{d\Delta_k}{dt} = \left\{ 4\pi G\bar{\rho} - k^2 c_s^2 \right\}\Delta_k$$

All solutions depend on sign of the right hand term, i.e. whether k is greater or less than a critical Jeans wave-number k_J:

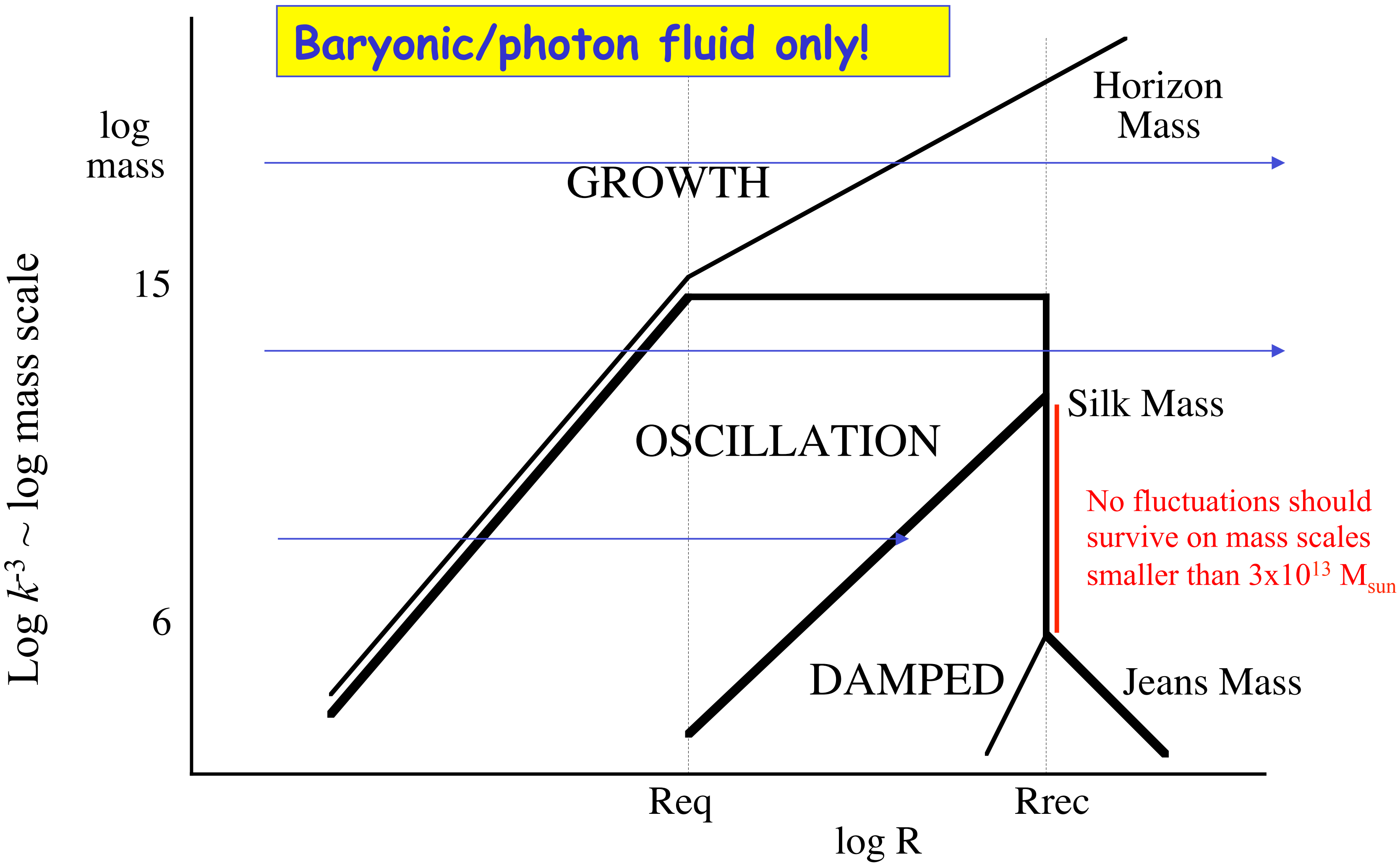
$$k_J^2 = \frac{4\pi G\bar{\rho}}{c_s^2}$$

On large scales $k << k_J$ (i.e. $\lambda >> \lambda_J$) we have potential for collapse (i.e. growth of Δ_k):

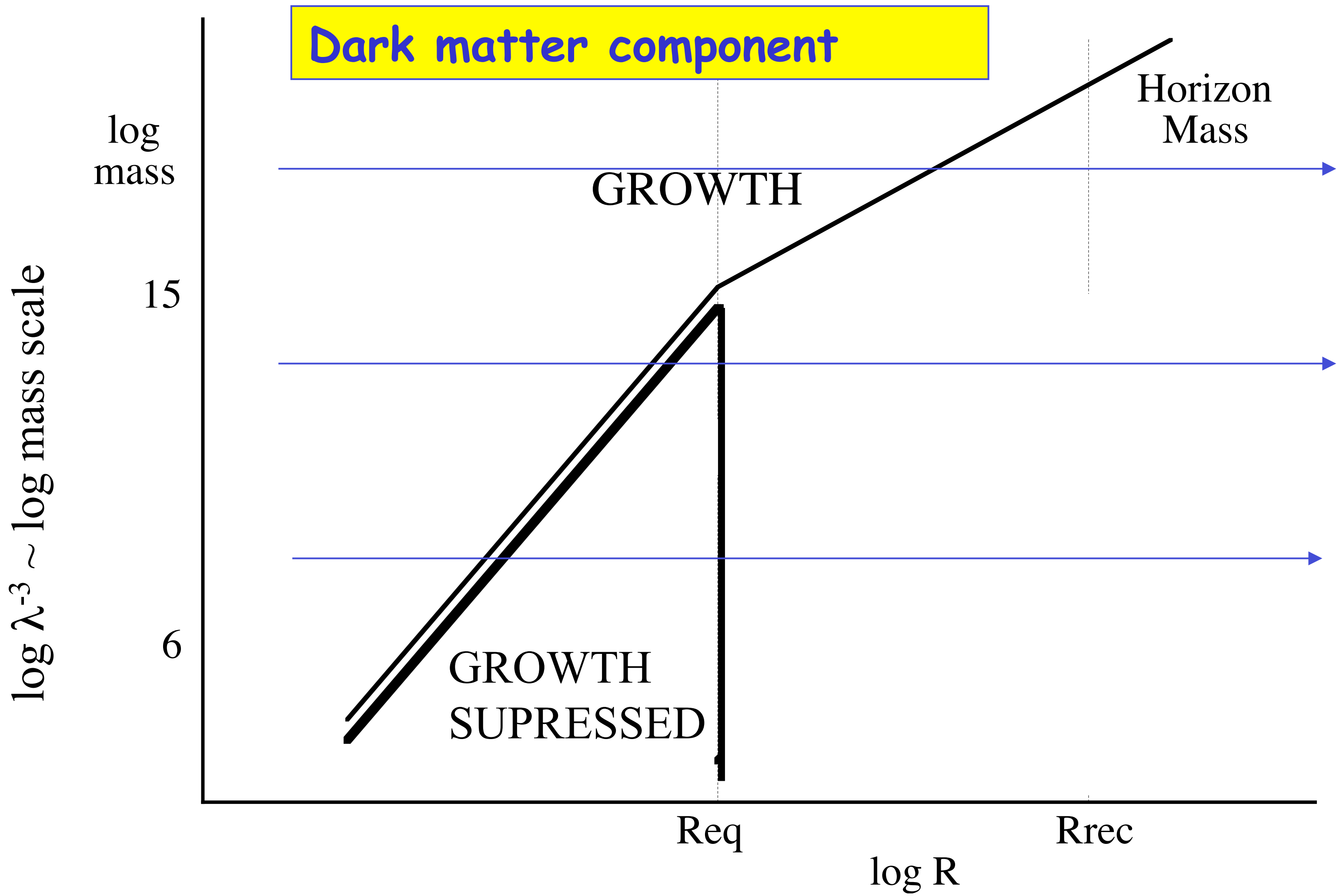
- the solutions will also depend on the R-dot term.
- *exponential* growth for static medium (with R-dot = 0)
- at best *linear* growth $\Delta \propto R$, for decelerating expanding medium and *no growth at all* if the expansion of the Universe is not significantly decelerating or is accelerating

On small scales, $\lambda << \lambda_\vartheta$, we have *oscillating* solutions when pressure (given by c_s^2) is significant. Especially important when Universe is ionized plasma. *This Jeans length is comparable to the horizon when pressure comes from the CMB radiation*.

Note: Dark matter *does not interact* and therefore has zero sound speed and $\lambda_J = 0$, and it never oscillates (i.e. there is no pressure)



Baryonic fluctuations will oscillate when their length scale is below the Jeans length. They may eventually damp out (due to diffusion effects) on scales upto the Silk Mass (>> mass of galaxy). This is a potentially rather serious problem for baryon-only scenarios for galaxy formation: galaxies should not exist!

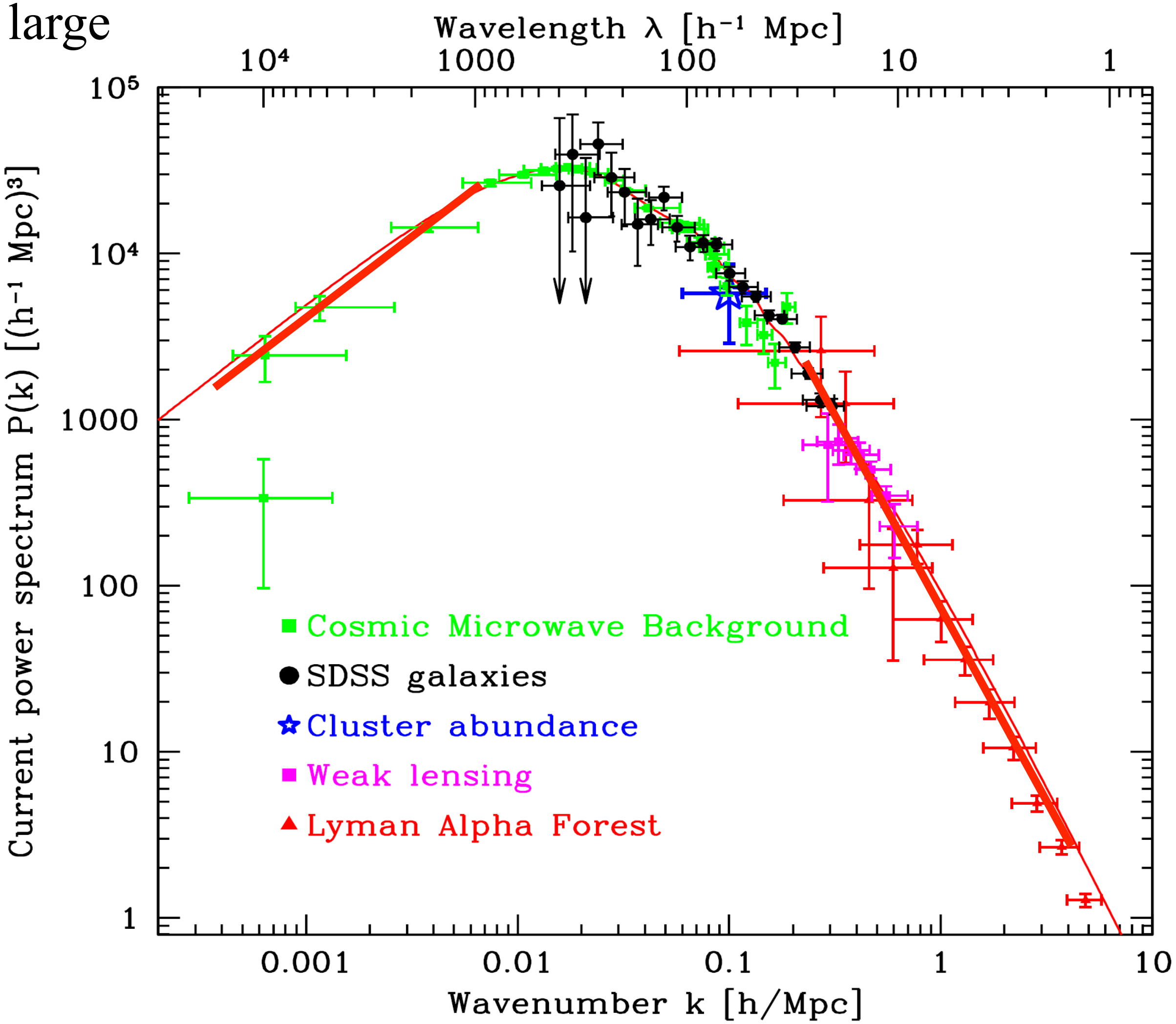


But, dark matter fluctuations are able to continue growing and are not damped, but growth is suppressed below horizon scale for as long as the Universe is still dominated by radiation, modifying primordial $n = 1$ spectrum on small scales.

Existence of dark matter is why fluctuations on galaxy scales "survive".

Cold Dark Matter (CDM) spectrum matches the Universe well

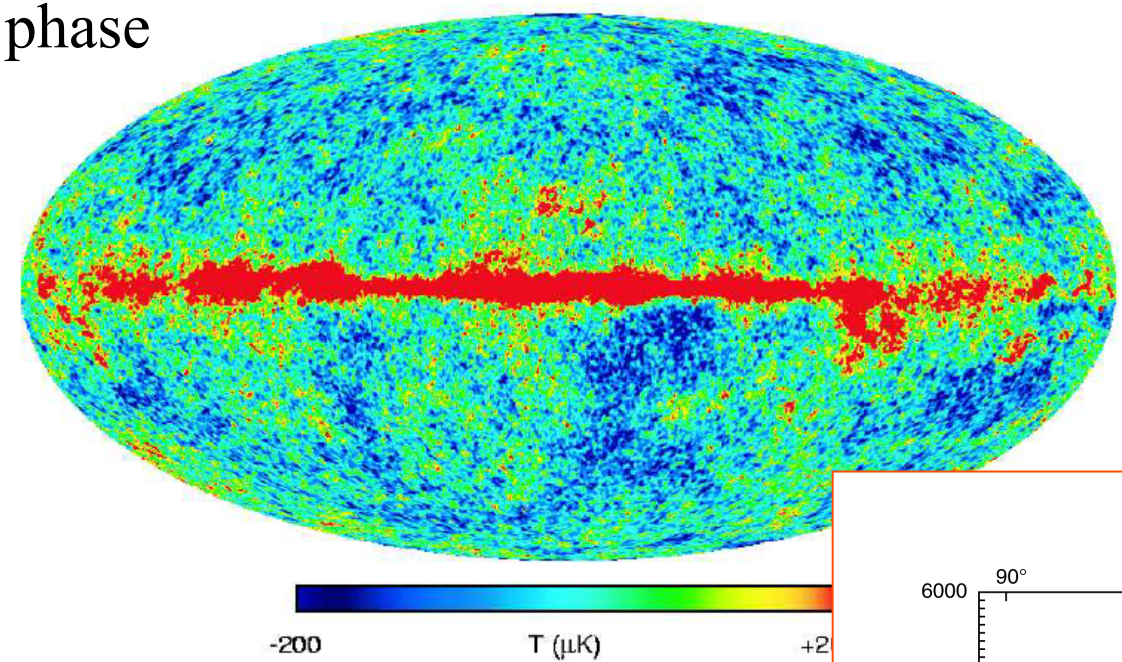
n = 1 on very large scales from primordial fluctuations



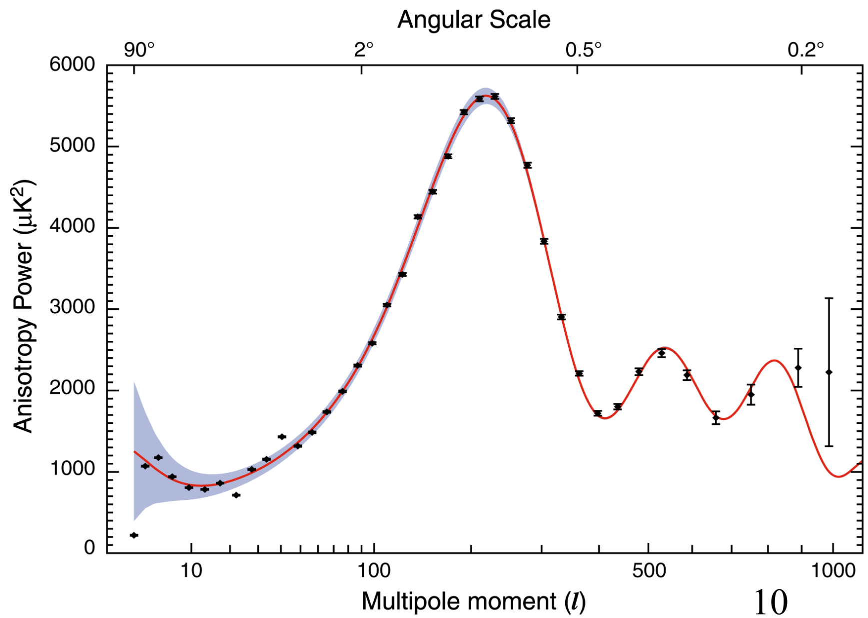
n = -3 on small scales due to suppression on small scales while Universe radiation dominated

P(k) observations

The last scattering surface of the CMB is established when the Universe recombines to neutral atoms, at the end of the oscillating phase



Detailed calculations of expected density fluctuations beautifully reproduce the observed spectrum of fluctuations on the CMB LSS.



Summary: origin of density fluctuations in the Universe

- An $n = 1$ primordial spectrum, probably produced by Inflation, with characteristic amplitude on horizon scale (always) of $\delta\rho/\rho \sim 10^{-5}$.

- This spectrum of fluctuations modified by suppression on small scales to $n \sim -3$.

- Because the density is dominated by (non-interacting) dark matter, density fluctuations do survive through to when the Universe forms neutral atoms (and pressure turns off).

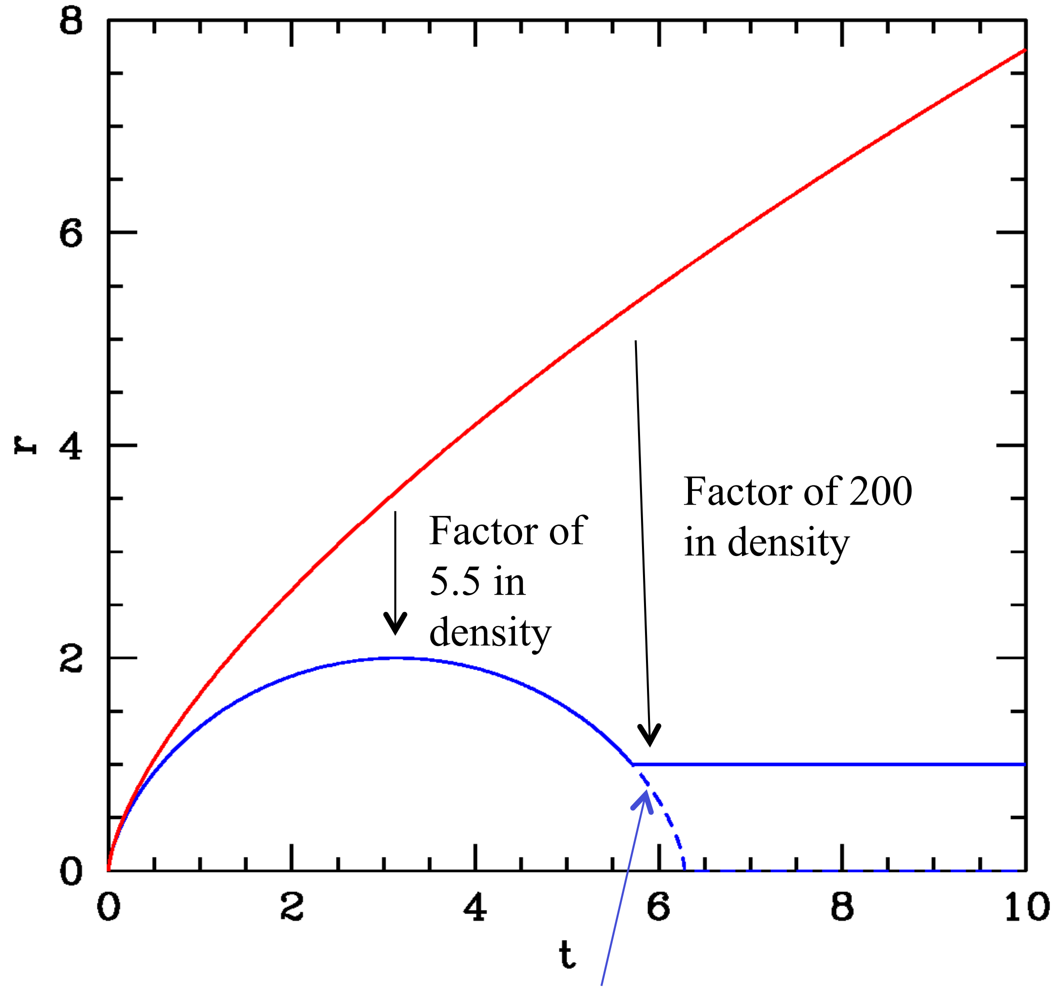
- Growth of fluctuations is at best linear with R, c.f. exponential in static medium.

  - All beautifully verified by observations of the CMB Last Scattering Surface at t = 380,000 yrs after Big Bang.

Formation of gravitationally bound objects

- Gravitational instability continues to linearly increase density contrast $\delta\rho/\rho$.
- As $\delta\rho/\rho$ approaches unity, non-linear collapse begins. Object breaks away from general expansion and collapses to form a self-gravitating object

Idealized collapse of “top-hat” density distribution

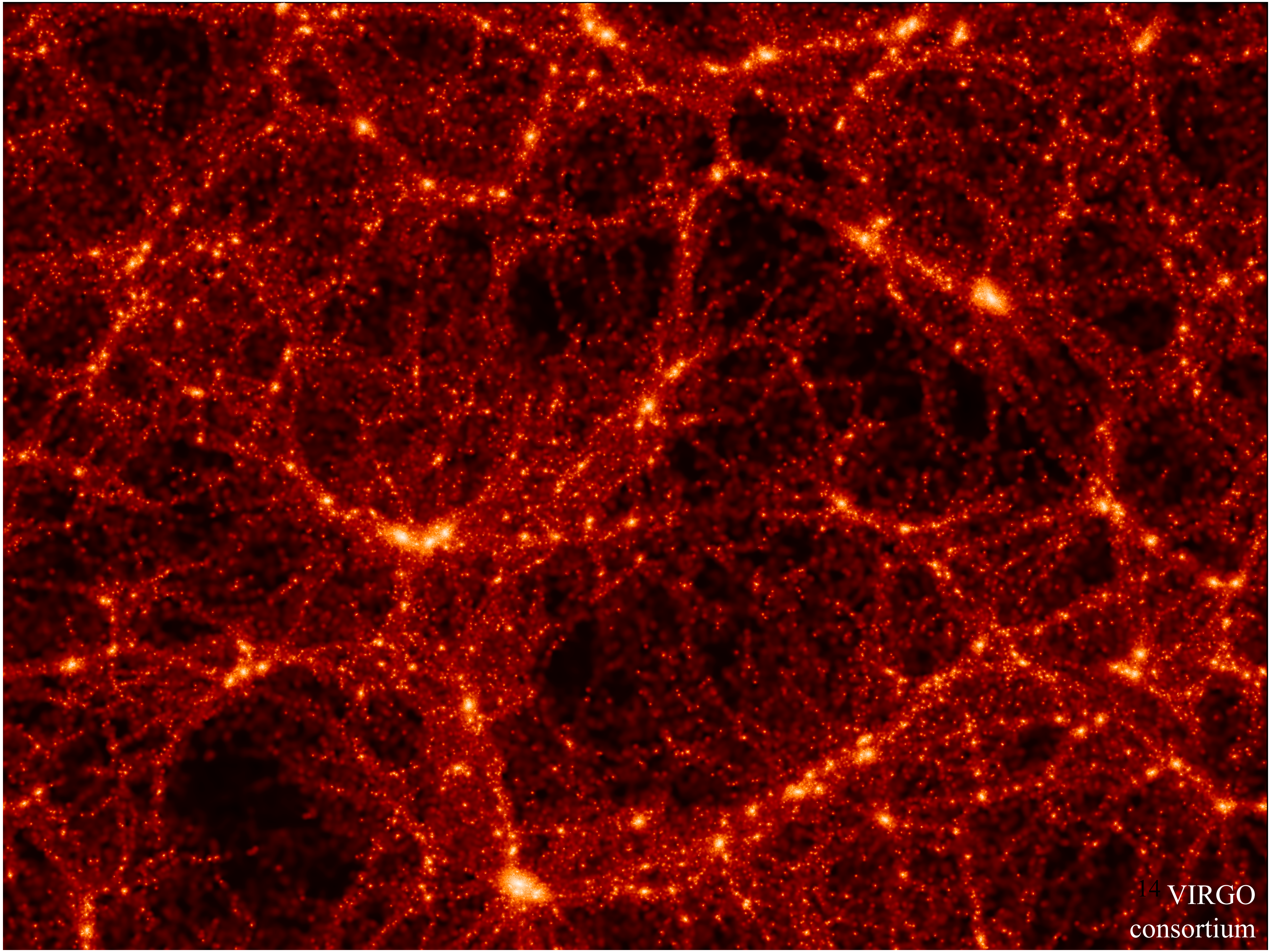


Virial condition stabilizes collapse after a factor of 2 decrease in size

The collapsed, self-gravitating, virialized dark matter structures are called (oddly) dark matter halos

Gravitational collapse alone only produces a change of 200 in the density (at the time of collapse)

But, if we see a structure with an overdensity of > 200, it must have virialised.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What happens to the baryons in a dark matter halo?

Galaxy: Most baryons in stars, concentrated at the center of a dark matter halo

Large scale structures – **groups** or **clusters**

Baryons in a hot gas distributed like dark matter (plus ~ 20% in stars in galaxies)

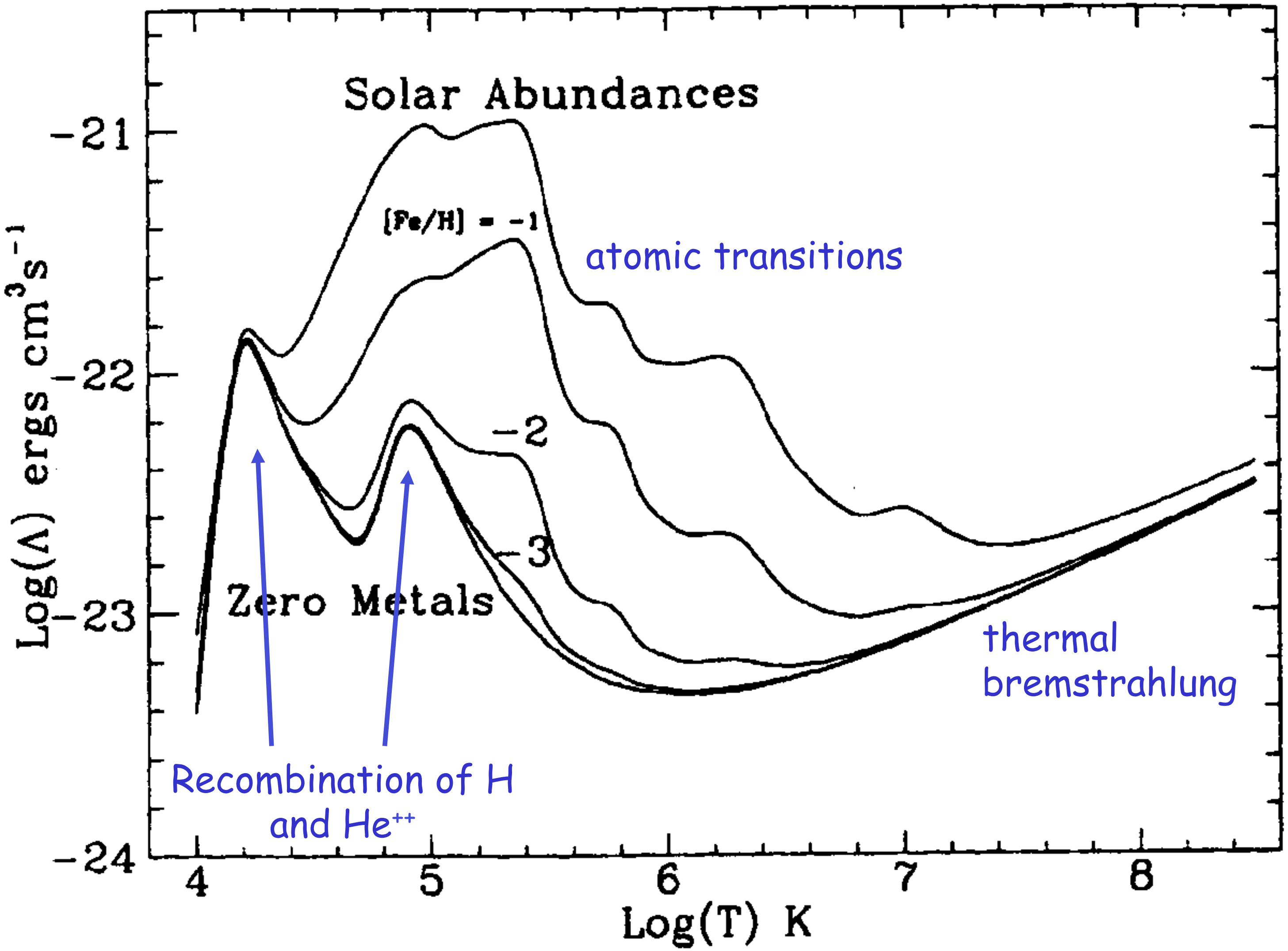
Formation of galaxies

- How does baryonic material separate from dark matter
- Why are clusters not giant galaxies?

Key question: will the baryonic gas in a halo cool and lose energy on a short timescale?

Radiative cooling rate of a gas depends on square of density n^2

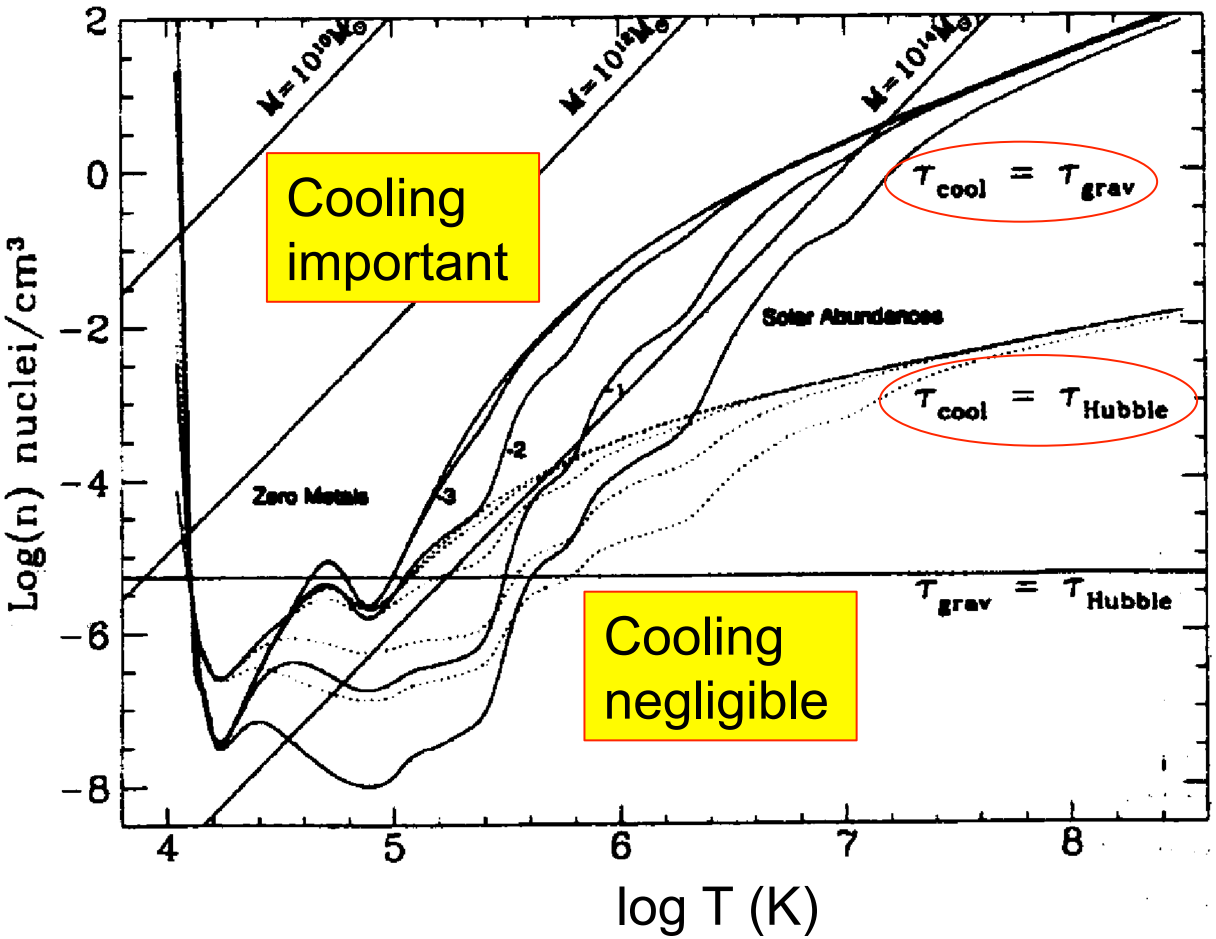
$$\frac{dE}{dT} = -n^2 \Lambda(T)$$



Cooling function

Is cooling important for a gas?

Comparison of cooling and gravitational collapse timescales depends on number density *n* and temperature T, as well (to some degree) on the metallicity (composition) of gas



$$t_{cool} \sim \frac{E}{dE/dt} \sim \frac{3kT}{2n\Lambda(T)}$$

$$t_{collapse} \sim (4\pi G\rho)^{-1/2}$$

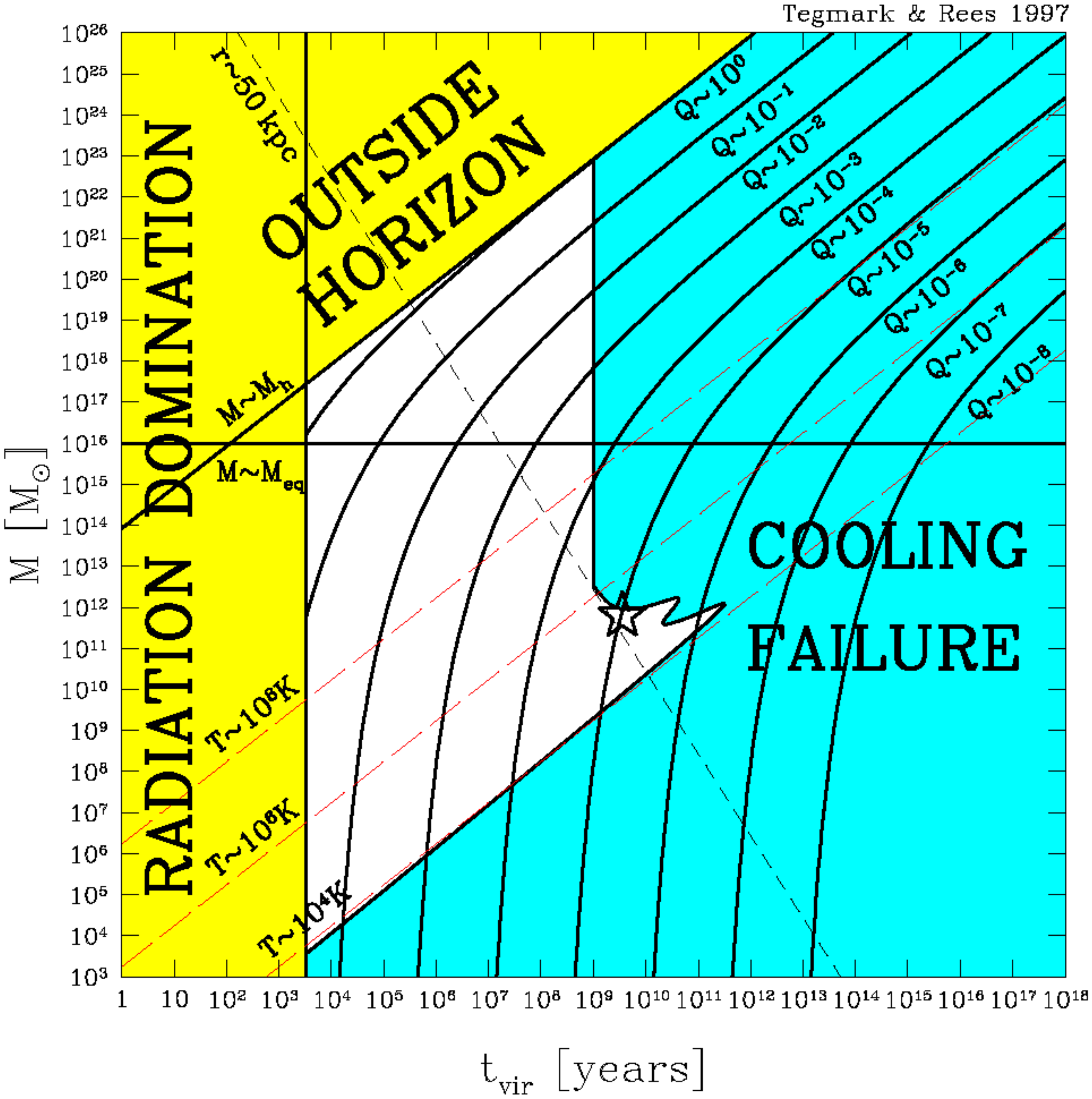
Note: density in a halo will depend on when it formed (200× cosmic density) and therefore on Δ.

So, a "galaxy" is found in a halo where the baryonic material was dense enough (and at right temperature) to be able to dissipationally cool, losing energy and sinking towards the center of the (uncoolable) dark matter halo, thereby increasing in density and separating spatially from the dark matter.

What halts dissipation?

- Matter becomes dissipationless (e.g. forms into stars)
- *Most important*: Angular momentum rotationally supports cold disk of gas. Disk is a natural outcome of conservation of J in contracting system

The angular momentum was acquired by the protogalaxy through torques from surrounding asymmetric density field.
The amount of angular momentum gives collapse factors of $\times 10$, i.e. density increases $\times 10^3$



Constraints on Q

Anthropic cosmological considerations:

A. Need to set $\Omega \sim 1$ at early epochs to high precision to produce a long-lived large Universe, i.e. to avoid Ω_k ($=|1-\Omega|$) dominating Friedmann equation and producing either recollapse ($\Omega_k \sim -1$) or an empty Universe ($\Omega_k \sim +1$, $\Omega \sim 0$) after small amount of expansion. "Solved" with concept of *inflation*

B. Best to have Q (= $\delta\rho/\rho$ on the scale of the horizon) $\sim 10^{-5 \pm 1}$

- $Q \ll 10^{-6}$: structure form very late with low density and baryonic cooling does not lead to dissipation and further concentration of baryons in galaxy formation
- $Q \gg 10^{-4}$: structure forms early with very high densities, star-star interactions disrupt planetary systems

Q comes from amplitude of inflaton $V(\phi)$ – presently unconstrained by any known Physics

C. Need to have $\Omega_M/\Omega_\Lambda > 10^{-5}$??
If Dark Energy dominates the Universe too early, then structure will stop forming.
$(1+z_\Lambda)^3 \sim \Omega_\Lambda/\Omega_M$ Value of Ω_Λ is completely not understood

D. Need baryogenesis etc. for non-zero Ω_B (see previous discussion on Baryogenesis)

Can view these as either: This is the (largely unknown) Physics that made Life possible in the Universe *OR* perhaps this is the only type of Universe that Life could ever observe?

The future of matter in the Universe

Let us assume that the geometry/equation of state is such that the Universe continues expanding *indefinitely*. What happens to the structures (complexity) in the Universe that we see at the present time $\tau \sim 10^{10}$ yr ?

A. Classical timescales (up to 10^{25} years):

Stellar lifetimes

End of Sun's Life (relevant for us): In 5×10^9 yr Sun will have exhausted H in core and cease to be a Main Sequence → Earth consumed in expanded Red Giant Phase. 10^9 yr later, Sun will be a White Dwarf.

Maximum lifetime of low mass stars (0.08 $M_\odot$) is about 10^{12} yr

Star-formation rate in the Galaxy is declining ~ exponentially with $\tau_{1/2} \sim 5\times10^9$ yr.

Unlikely that there is continuing fusion after ~ **10^{13} yr**

→ **end of the "stellar epoch" in the Universe (which started few 10^8 yr after the Big Bang)**

Disruption of planetary systems

Timescale for disruption of a given planetary system due to close encounter with another star, from population of number density n, moving with velocity v $\tau = (nv\sigma)^{-1}$

$$\sigma = 2\pi r^2$$

What is r? It must be of order the orbital radius of the planet.

$$n \sim 3\times10^{-41}\ \mathrm{km}^{-3}$$

$$\sigma \sim 2\times10^{16}\ \mathrm{km}^2$$

$$v \sim 50\ \mathrm{km\,s}^{-1}$$

$$\Rightarrow \quad \tau \sim 10^{15}\ \mathrm{yr}$$

On time scales of 10^{15} yr, planets are removed from stellar remnants by dynamical interactions

Evaporation of galaxies

Energy exchange between stars will lead to some stars being lost from galaxy ($v > v_{esc}$) while the remainder sink to the bottom of potential well ("evaporation").

System of N stars of mass m in self-gravitating system of radius R: Virial theorem gives

$$v^2 \sim \frac{GNm}{R}$$

What is now σ? For significant energy exchange, it must be of order the separation at which the potential energy of the interaction is comparable to the typical kinetic energy of stars.

$$r \sim \frac{Gm}{v^2}$$

$$\sigma \sim \left(\frac{Gm}{v^2}\right)^2 \sim \left(\frac{R}{N}\right)^2$$

$$\tau \sim (n\sigma v)^{-1} \sim \left(\frac{NR^{-3}}{Gm}\right)^{1/2}$$

$$N \sim 10^{11}$$

$$R \sim 3\times10^{17} \text{ km}$$

$$m \sim 10^{30} \text{ kg}$$

$$\Rightarrow \quad \tau \sim 10^{19} \text{ yr}$$

On time scales of **10^{19} yr**, galaxies themselves “evaporate”, leaving central super-compact galaxies and with many stellar remnants (blackholes, neutron stars, black dwarfs and planets) moving through intergalactic space.

What about gravitational radiation?

Energy loss from orbiting objects due to gravitational radiation (as seen in binary pulsar system) is given in terms of energy, E, period P, and velocity v

$$\frac{dE}{dt} \sim \left(\frac{v}{c}\right)^5 \frac{E}{P} \quad \Rightarrow \quad \tau \sim \left(\frac{v}{c}\right)^{-5} P$$

Earth orbiting the Sun: $\tau \sim 10^{20}$ yr (longer than disruption timescale, so irrelevant)

Generally for a self-gravitating object of total mass M and radius R

$$\frac{GM}{R} \sim v^2 \sim \left(\frac{R}{P}\right)^2 \quad \Rightarrow \quad \tau \sim G^{-3} R^4 M^{-3} c^5$$

$M \sim 10^{11}\ M_\odot$, $R \sim 1$ kpc, $\rightarrow$ $\tau \sim 10^{20}$ yr

Conclusion: Final state of "non-evaporated" remnants must be supermassive blackholes on timescales of order 10^{20} yr.

B. Greater than 10^{25} years

Decays of structures through quantum effects

Decay of the proton:

Expected under GUT theories t ~ $10^{33.5+}$ years
(also: required for primordial baryogenesis to work)

$$p \rightarrow e^{+} + \nu + \bar{\nu} + \gamma$$
$$n \rightarrow e^{+} + e^{-} + \nu + \bar{\nu} + \gamma$$

In compact objects, almost all the rest-mass is liberated (1 GeV per baryon) through e+e- annihilation. Any free protons form positron-electron plasma

Overall number of protons will decay exponentially $N_p(\tau) = N_0 \exp(-\tau/\tau_p)$

Aside: energy released from proton decay keeps stellar remnants at significant temperatures long after they would otherwise have cooled: neutron stars @ 100K, black dwarfs @ 5K

Even if Life increases the available volume at speed of light (i.e. controls all of the particle horizon), the total number of atoms decreases exponentially after ~ 1000 t_p

Decay of the proton

Evaporation of blackholes through Hawking radiation:

Lifetime of BH: $$\tau \sim 10^{66}\left(\frac{M_{BH}}{M_{sun}}\right)^3 \text{yr}$$

t ~ 10^{99} for galactic masses, 10^{117} for supercluster masses (probably the largest that had been formed)

Positronium annihilation of diffuse gas:

Excited Positronium P_n formed by e^+ and e^-, on timescales of 10^{73} yrs through three-body reactions

$$e^+ + e^- + e^\pm \Rightarrow P_n + e^\pm$$

Typically quantum level $n \sim 10^{22}$, $r \sim 10^{12}$ Mpc etc…

Decay to ground-state (and rapid annihilation thereafter) with time scale of 10^{117} yr

What will we actually see? – assume that the Universe is entering a period of exponential accelerated expansion, due to $\Omega_\Lambda \sim 0.75$

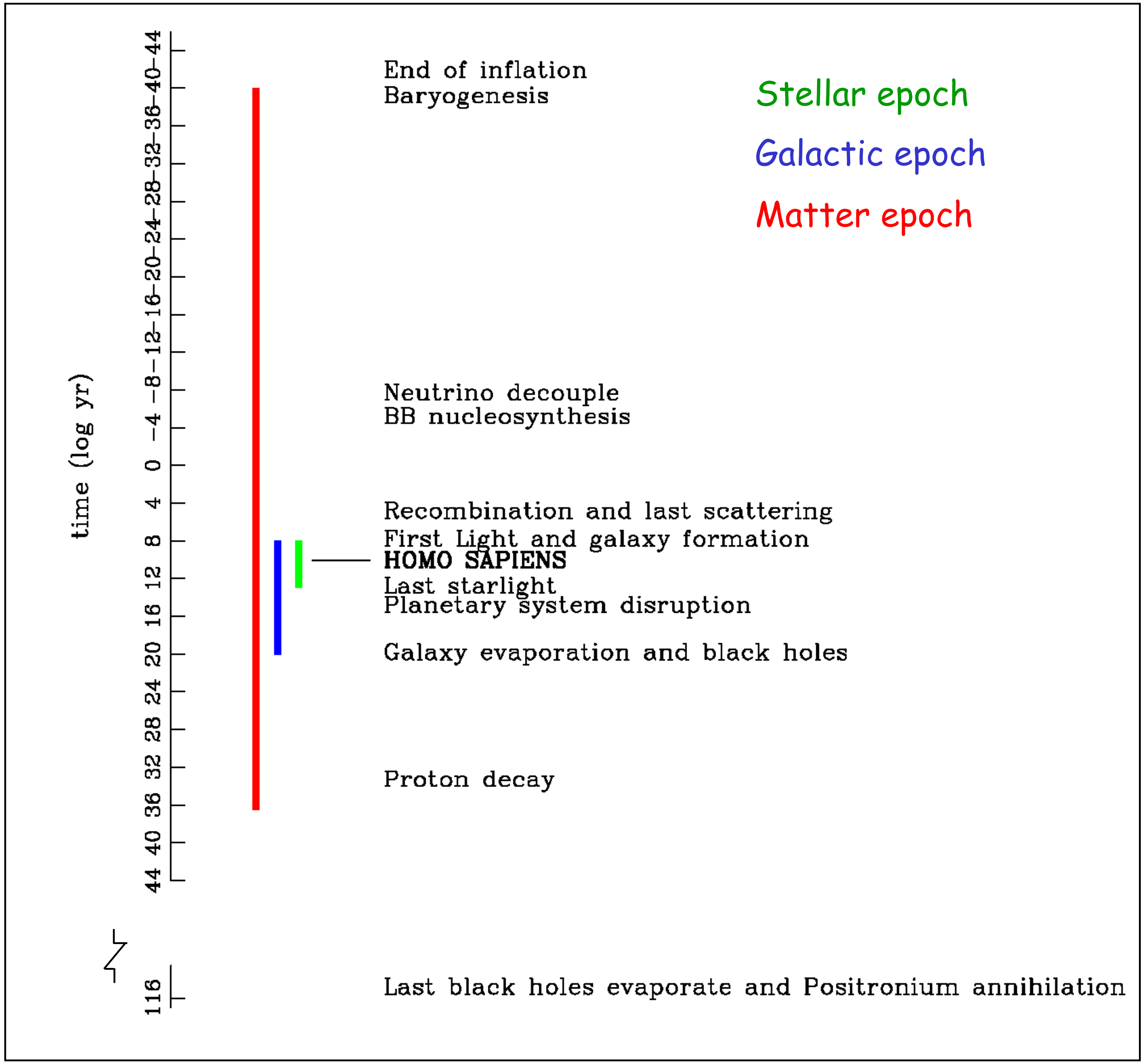
Our current particle horizon at radius ~ c/H will change into an event horizon, through which matter will "disappear". In comoving space the horizon will come towards us with an exponential timescale of ~ H^{-1}, i.e. 13 Gyr as more and more of comoving space is expanded out of the horizon (redshifts tending to infinity)

Present horizon distance c/H ~ 4300 Mpc: Even the nearest galaxies with distances of ~ 1 Mpc will disappear from the sky after about 8 e-folds, i.e. after about 100 billion years.

Time is running out for extragalactic astronomy!

Will the horizon encroach within our galaxy? - depends on $\rho_\Lambda(R)$

If r_L starts to be comparable to densities within gravitationally bound structures then dynamics will be governed by ρ_Λ and bound objects will fly apart…..



Ephemeral phases of the Universe