Stars: sources of energy and the origin of the chemical elements in the universe

What powers the Sun?

First idea: Is it simply gravity? (Kelvin & Helmholtz, late 1800's)

A sphere of uniform density (underestimate?) has negative gravitational potential energy:

$$U_g = -\frac{3}{5}\frac{GM^2}{R}$$

Decrease in radius as Sun shrinks liberates potential energy, but 50% of this (from Virial Theorem) must go into thermal energy to maintain the gas pressure support (i.e. to heat gas to higher temperature):

$$\text{Available energy} = 0.5\, U_g \sim 10^{41}\, J$$

$$L_{Sun} \sim 4\times10^{26}\, W$$

Timescale to maintain solar luminosity:

$$t_{KH} \sim \frac{0.5\, U_g}{L_{sun}} \sim 10^7\, yr$$

Note that every factor of 2 collapse would release as much energy again

Until the radioactive dating of age of Earth rocks (~1900) this was thought to give a sufficient lifetime.

What about “reactions” involving the material in the Sun ?

Number of particles $$n \sim \frac{M_{sun}}{m_p} \sim 10^{57}$$

Energies of chemical reactions (changing electron orbits) are typically eV (10^{-19} J) per particle, so of order 10^{38} J energy is available. Gives lifetime of order only 10^4 years! → this problem led to Kelvin’s gravitational idea

Energies of nuclear reactions (changing structures of atomic nuclei) are typically MeV (10^{-13} J) per particle, so 10^{44} J is available. This gives lifetime of order 10^{10} years, i.e. what we need for the age of the Solar System.

Before leaving gravity, note one interesting effect:

The Virial Condition for the equilibrium state of any self-gravitating system is:

$$2K + W = 0, \quad \text{i.e.} \quad E = W+K = -K = 0.5W$$

K = kinetic energy = 3/2 NkT
W = potential energy
E = total energy

Note the curious behaviour of a self-gravitating body under its own pressure support: it has negative heat capacity:

$$E = -K = -\frac{3}{2} NkT$$

$$\frac{dE}{dT} = -\frac{3}{2} Nk$$

So a self-gravitating system increases it's temperature when it loses thermal energy (this is the same effect as when an orbiting satellite speeds up under frictional drag forces).

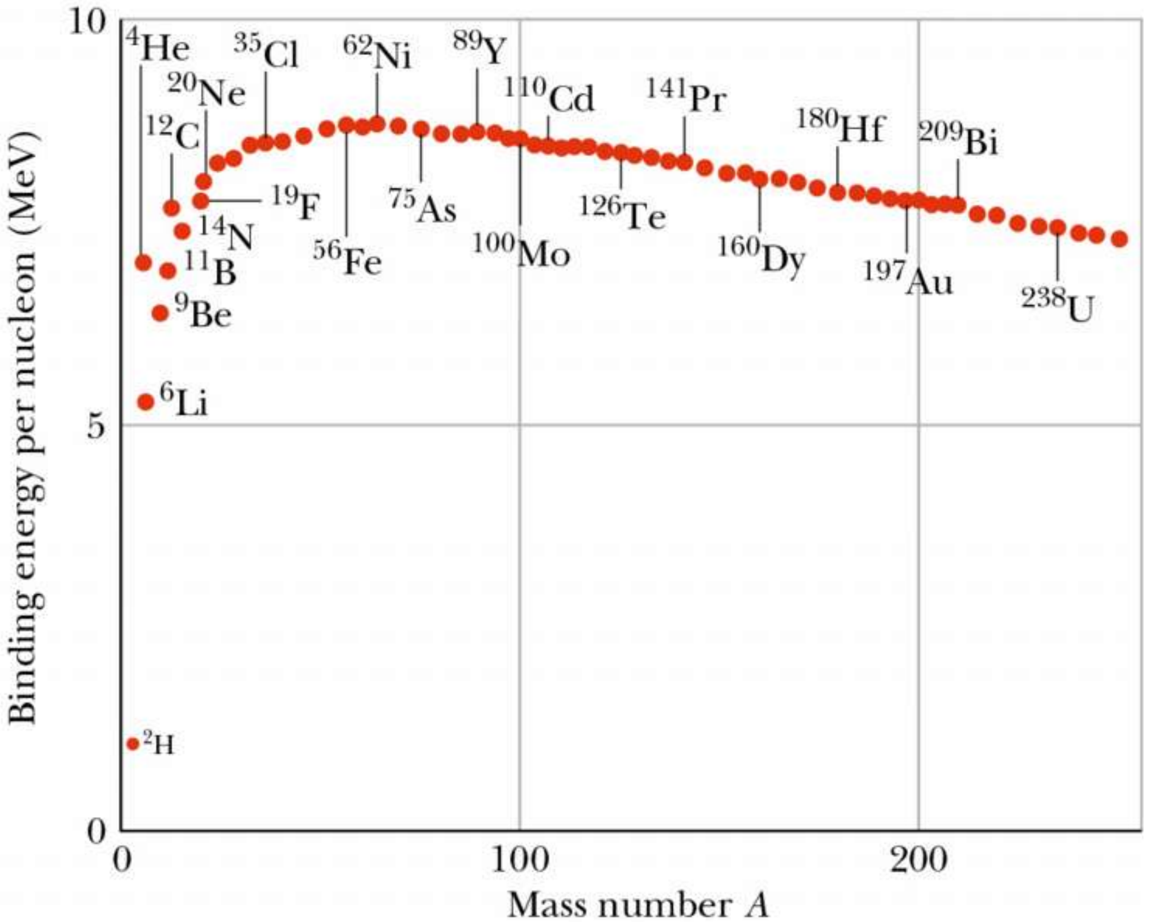
Nuclear binding energies

$m_p = 1.6726 \times 10^{-27}$ kg = 1.0073 a.m.u
$m_n = 1.6749 \times 10^{-27}$ kg = 1.0086 a.m.u.
$2m_p+2m_n = 4.0318$ a.m.u.
$m_{He} = 4.0026$ a.m.u.
a.m.u.= 1.6605×10^{-27} kg (= 931 MeV)

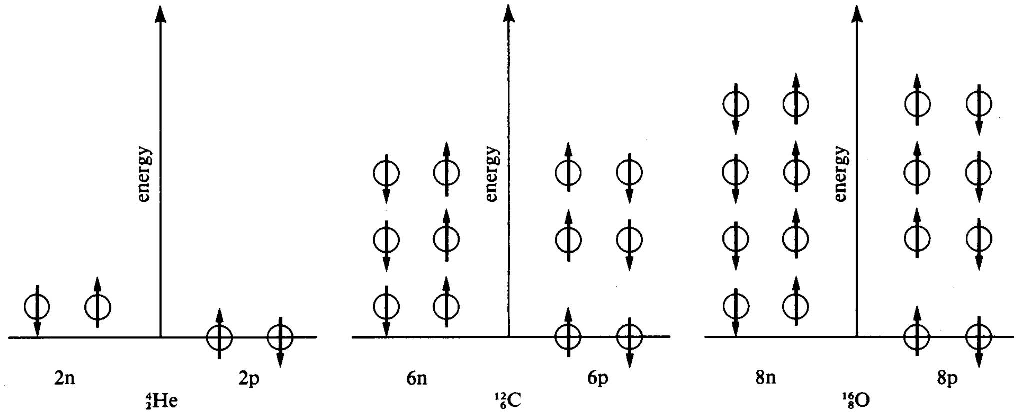
Making ^{4}He out of 2n + 2p results in 0.7% of the mass (i.e. 26 MeV, or 6.5 MeV per particle) being lost, due to the increased (negative) binding energy of the particles in the ^{4}He nucleus.

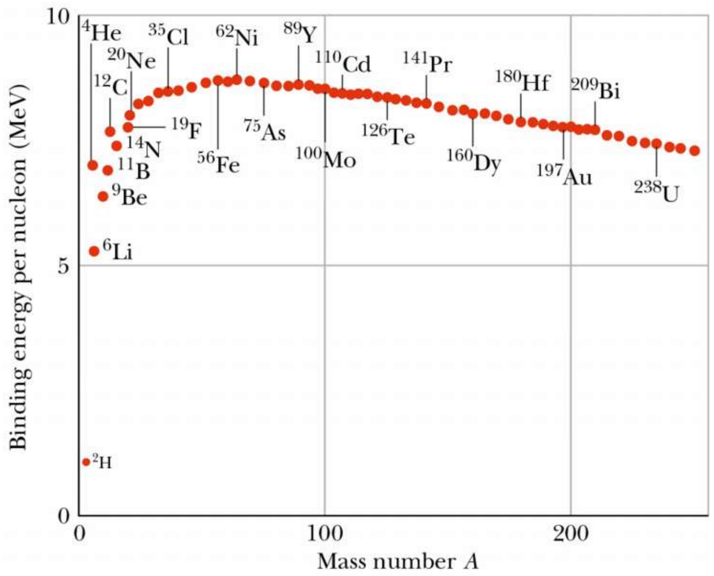
Assembly of nuclei up to ^{56}Fe releases binding energy

These "fusion" reactions
(a) provide the long-lived energy source in stars (Bethe 1939) and
(b) produce the diversity of chemical elements out of initial H(+^{4}He) that was produced in the Big Bang (B2FH 1957)



Nuclear energy levels



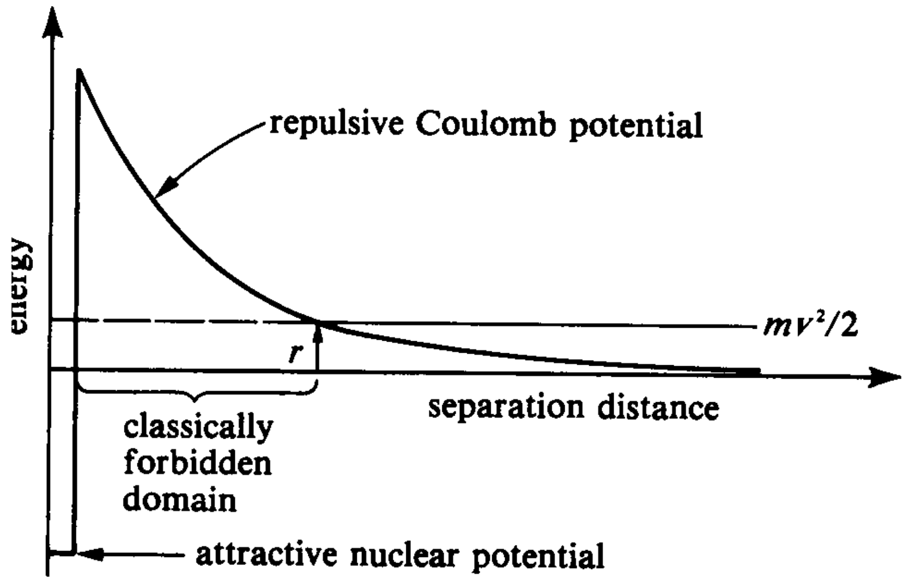


- The Pauli exclusion principle ensures that $n_p \sim n_n$
- Proton levels are slightly more widely spaced because of electrostatic repulsion

- Gross shape of the binding energy curve is a result of the finite range of the Strong Force (the strong attraction saturates as the number of nucleons increases too far).
- Creation of nuclei beyond ^{56}Fe *requires* energy

Fusion reaction rates:

Difficulty of fusion arises from the electrostatic (Coulomb) repulsion between positively charged nuclei



Electrostatic potential energy of two charges separated by distance *r*

$$U_e = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Point of closest approach if the initial velocity of approach is *v*

$$r_{close} = \frac{2 q_1 q_2}{4\pi\varepsilon_0 m v^2}$$

Naively setting $r \sim 10^{-15}$ m and $mv^2 = 3kT$ would require $T \sim 10^{10}$K $\gg T_{sun}$

We've forgotten two effects:

- quantum tunneling through potential barrier
- At a given temperature, there is a distribution of velocities of the nuclei

Velocity in center of mass frame of two particles of reduced $m = m_1 m_2/(m_1+m_2)$

$$f(v)dv \propto \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

At a given T, the number of particles at high velocity v drops exponentially with v,

Quantum tunneling probability through a distance *r is* given by de Broglie λ*:*

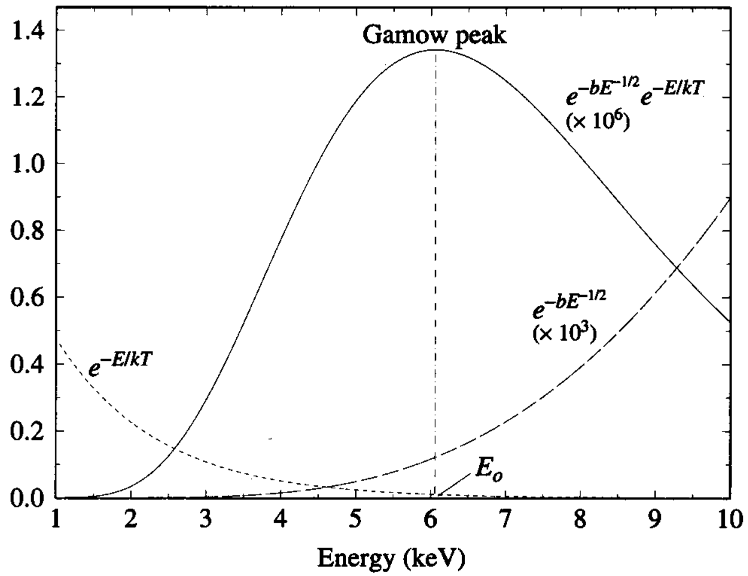
$$P \propto \exp\left(-\frac{2\pi^2 r_{close}}{\lambda}\right) \propto \exp\left(-\frac{4\pi^2 q_1 q_2}{4\pi\varepsilon_0 hv}\right)$$

Probability of quantum tunneling *<u>increases</u> exponentially with v*

Probability of a fusion reaction happening to a given pair of particles will be proportional to the product of these two terms

$$dN \propto n_1 n_2 \sigma\, v \exp\left(-\frac{mv^2}{2kT} - \frac{\pi q_1 q_2}{\varepsilon_0 h v}\right) dv\, dt$$

$$\propto \exp\left(-\alpha E - \beta E^{-1/2}\right) dE dt$$



This is a highly peaked function with a maximum (dN/dE) = 0 when the two terms are equal (plus a factor of 2 from differentiating)

Velocity of peak $$v_{max} = \left(\frac{\pi q_1 q_2 kT}{\varepsilon_0 h m}\right)^{1/3}$$

→ This means that most of the reactions will occur with kinetic energies close to this so-called **Gamow Peak**.

→ Furthermore, the overall rate of reactions *strongly increases* with temperature

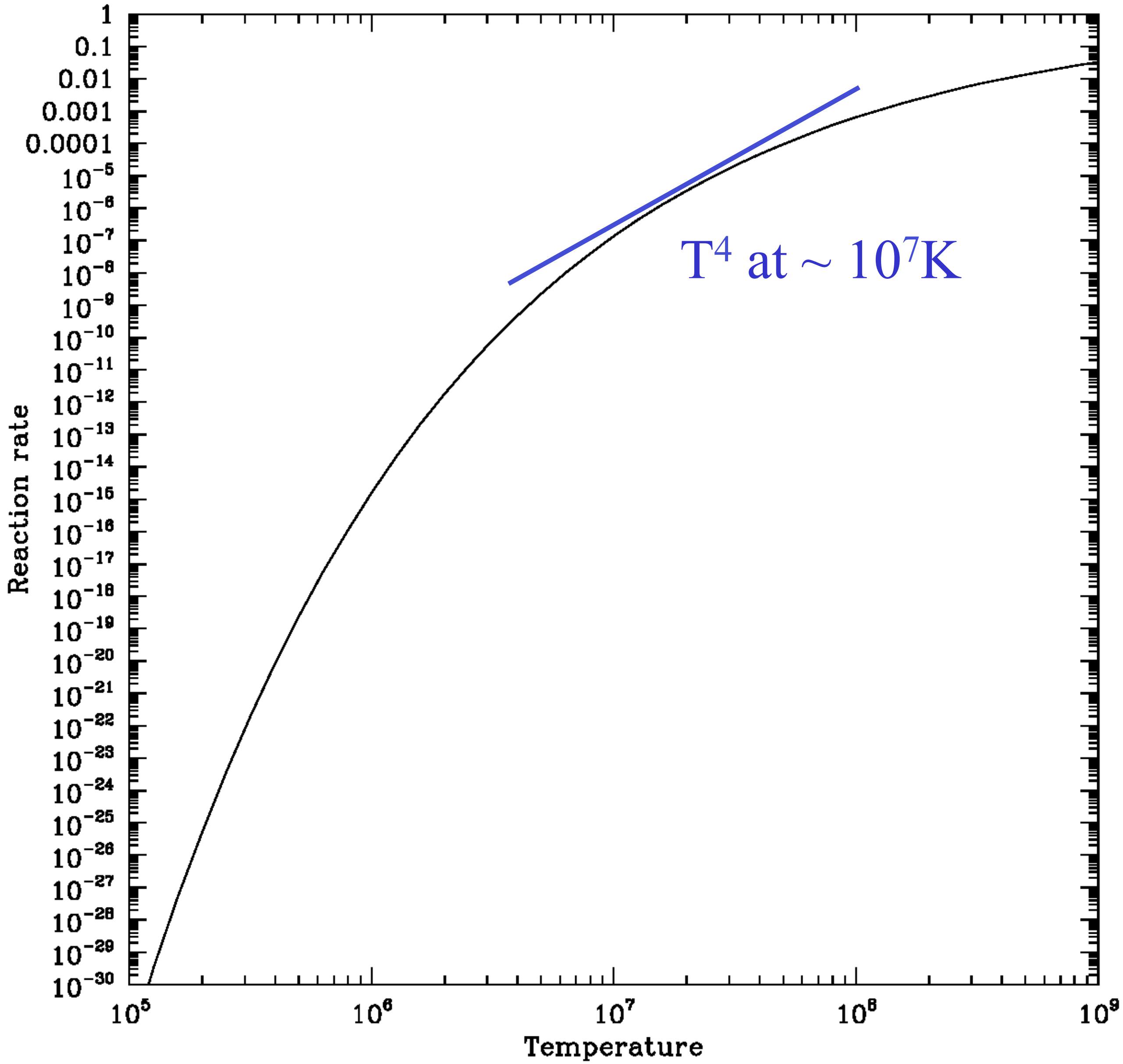
$$dN/dT \propto \exp\left(-\frac{3}{2}\left(\frac{\pi q_1 q_2}{h\varepsilon_0}\right)^{2/3}\left(\frac{m}{kT}\right)^{1/3}\right) \propto \exp -\left(\frac{T_0}{T}\right)^{1/3}$$

$$T_0 = \left(\frac{3}{2}\right)^3 \left(\frac{\pi q_1 q_2}{h\varepsilon_0}\right)^2 \left(\frac{m}{k}\right) \sim 4\times 10^{10} K$$

Note: the very steep temperature dependence of the rate of fusion reactions.

This coupled with the negative heat capacity of self-gravitating gas provides a natural "thermostat" to keep star stable at more or less constant temperature

This is why stars like the Sun do not explode like H-bombs.

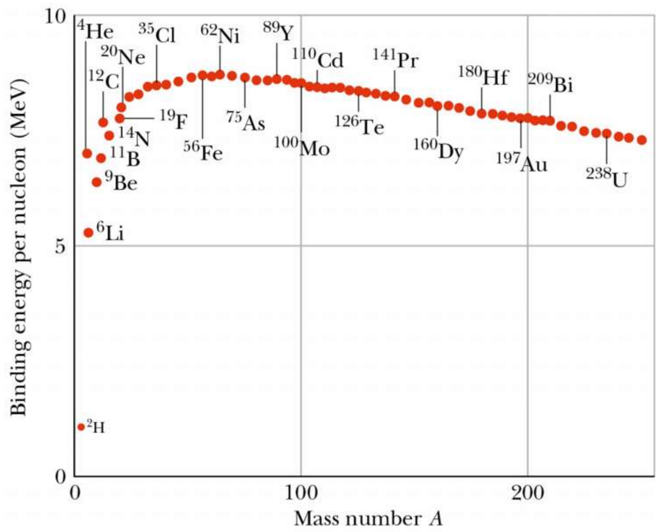


Fusion of more massive nuclei

- This will require higher temperatures because of larger +ve nuclear charges

H to He	1×10^{7} K
He to C, O	1×10^{8} K
C to O, Ne, Na, Mg	5×10^{8} K
Ne to O, Mg	1×10^{8} K
O to Mg – S	2×10^{8} K
Si to around Fe	3×10^{9} K

- Also, the flattening of the binding energy curve means that less energy is released per reaction at higher nuclear masses



Therefore, expect basic evolution of a star as follows:

- gravitational collapse heats up interior
- then, first fuse H→He at "low temperatures" until the H in core consumed;
- gravitational collapse of core to raise T,
- then, fuse He → C at higher temperature, until most He in core is consumed (shorter time),
- gravitational collapse to raise T,
- fuse C → O, and so on

We need to ask, what sources of pressure can support a star against it's own self-gravity?

X mass fraction of H
Y mass fraction of He
Z mass fraction of metals

Available sources of pressure to resist collapse

- Normal non-degenerate thermal pressure: $P = nkT \sim (\rho kT/m_H)(2X+0.75Y+0.5Z)$
- Radiation pressure: $P=aT^4/3$
- Degenerate plasma (non-relativistic): $P \propto [0.5(1+X)\rho]^{5/3}$
- Degenerate plasma (relativistic): $P \propto [0.5(1+X)\rho]^{4/3}$
- Atomic and molecular interactions in cold material (e.g. you and me, Earth)
- Quark and hadronic interactions
- Rotation and magnetic fields

Check: are the interiors of stars hot enough for fusion?
(c.f. surface temperature which is only 6000 K for the Sun)

Quite general arguments indicate yes, independent of energy source, if the pressure source is indeed normal thermal pressure

$$U_{grav} = -f\frac{GM^2}{R} \quad \text{with } f \text{ of order unity}$$

Hydrostatic support at radius r

$$\frac{dP}{dr} = -\frac{Gm_r}{r^2}\rho \quad \Rightarrow \quad \langle P \rangle_V = -\frac{1}{3}\frac{U_{grav}}{V}$$

Thermal pressure $\quad \langle P \rangle \sim \frac{\langle \rho \rangle}{\bar{m}} kT_{\text{int}}$

$$T_{\text{int}} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^6\,K \qquad \text{for Sun}$$

If it is supported by thermal gas pressure, the internal temperature of the Sun must be about 3 million K, i.e. sufficient to support fusion.

N.B. ***Fusion occurs because the Sun is hot, not the other way around.***
It is hot because it is a thermal-pressure supported self-gravitating system of given mass and radius.

Are stars inevitable?

$$T_{int} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^6 K \qquad \text{for Sun}$$

- Proto-star will slowly collapse in quasi-hydrostatic equilibrium with $T_{interior}$ rising as R shrinks as it loses energy.
- The negative heat capacity (from the virial condition) causes the proto-star to increase in temperature as it slowly shrinks because energy is lost through radiation to the exterior. The virial condition implies ½ of the "liberated" U_{grav} goes into internal heat energy, the rest is radiated by the proto-star.
- When $T_{interior}$ reaches the point where the rate of internal fusion can balance the heat loss at the surface, a static stable structure (= "star") is formed, with constant size. Radiation loses are replaced by fusion energy input.
- This H-fusion ignition point will *inevitably* be reached by the protostar at some R unless another source of pressure (e.g. degenerate electron pressure, which is independent of T) can support the object before T gets high enough for fusion.
- For the Sun, the temperature would have to rise to 10^7K before the density became high enough for degenerate pressure to be important, somewhat below the temperature at which H-fusion occurs.
- For proto-stars with $M < 0.08\ M_{sun}$ the temperature never gets high enough for fusion ("brown dwarfs" = failed stars). But H-fusion in all more massive objects is inevitable.

Mass-luminosity relation

Details need not concern us

$$T_{\text{int}} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^{6}\,K \qquad \text{for Sun}$$

Again, very general arguments indicate that the surface luminosity of a star will be related to it's mass as:

$$L = (4\pi)^{2}\,\frac{4}{3^{4}}\,\frac{caG^{4}}{k^{4}}\,\frac{1}{\kappa}\,M^{3}$$

a Stefan(-Boltzmann) constant

k Boltzmann constant

κ opacity of the gas

Ends up as $L \propto M^{3.5}$

Main point: since the H fuel supply will be roughly proportional to the mass of the star, it implies that the lifetime of a star in the stable H→ He fusion phase (= "Main Sequence") will be a strong function of mass.

$$t_{life} \propto \frac{L}{M} \propto M^{2.5}$$

0.1 $M_\odot$ star	lifetime ~ 2.10^{12} year
Our Sun	lifetime ~ 10 billion years
25 $M_\odot$ star	lifetime ~ 7 million years

Basic point: self-gravity vs. thermodynamics

- Ordinary stars (like our Sun) are self-gravitating and therefore have to be hot inside to sustain the thermal pressure that balances the inward pull of the self-gravity
- The surrounding space, however, is typically much colder and energy therefore flows continuously from the star to the surroundings.
- No thermodynamic equilibrium is possible under this circumstances: losing energy, the star will be forced to slowly contract, liberating gravitational potential energy, and to get hotter and hotter.
- Thermonuclear production of replacement energy keeps the star stable at ~ constant size, for some amount of time. When the star runs out of fuel a major structural re-assessment has to happen through further contraction.

Brown Dwarfs

- A brown dwarf is a self-gravitating gaseous object composed mainly of hydrogen and helium, whose mass is too small to ignite stable thermonuclear hydrogen fusion in its interior (but they do produce some helium isotopes by fusing protons with deuterium, lithium and berillium).

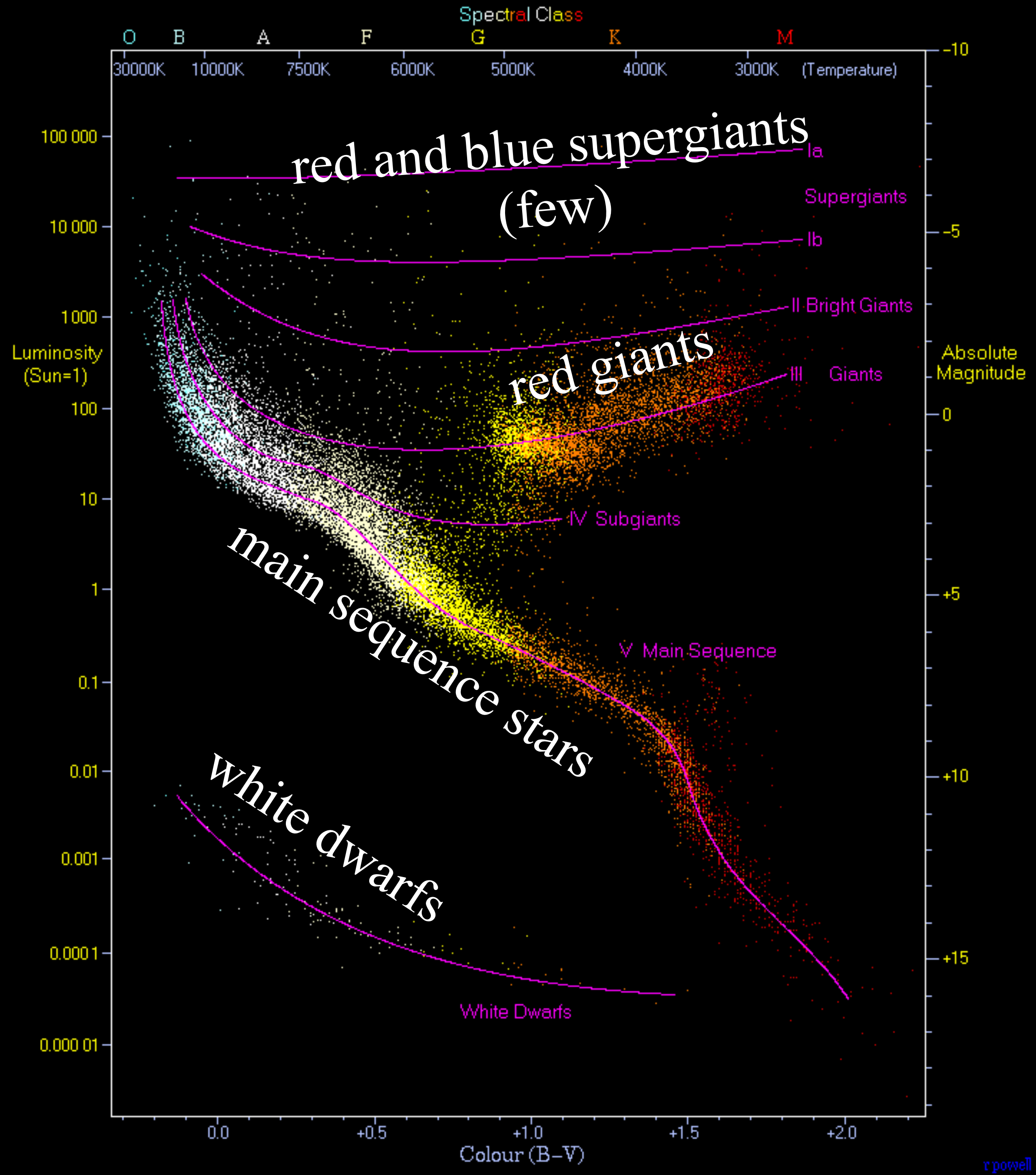
- Theoretical expectations: $M < 0.08\ M_{\odot}$.

- Incapable of generating substantial amounts of nuclear energy, the gravitational contraction of a brown dwarf takes place until the pressure of the degenerate electrons in its interior stops the collapse at an interior temperature that is below the fusion temperature of H.

- Young BDs can release substantial amounts of gravitational energy (comparable to protostars and low mass stars) through Kelvin-Helmhotz contraction, but older BDs, once supported by degenerate electron pressure, simply radiate remnant internal heat only. Effective radiative lifetime is short, i.e. only of order 10 million years (c.f. 10^{12} years for stars above $0.08\ M_{\odot}$).

The Hertzsprung-Russell diagram

luminosity vs. surface temperature



Main-sequence stars

Objects with $M > 0.08\ M_\odot$ ignite stable thermonuclear reactions in their centres and become "stars". In the initial phase of their lives they are characterized by chemical homogeneity and hydrogen burning in their cores.

Stars with $M < 2\ M_\odot$ fuse Hydrogen into Helium by the so-called "p-p chain", while more massive stars do it by the "CNO cycle" (see next slide)

In the Hertzsprung-Russell diagram MS stars populate an approximately one-dimensional locus along which positions are mainly determined by the stellar mass. *Vogt-Russell Theorem: The mass (and chemical composition) of a star uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution.*

Observed scaling relations for Main Sequence stars well matched by theoretical models: Rough scaling relations are as follows

$$L \propto M^{3.5} \qquad t_{life} \propto M^{-2.5} \qquad R \propto M \qquad T_{surf} \propto M^{0.5}$$

Reaction pathways for H $\rightarrow$ ^{4}He fusion

(N.B. must include weak interaction to convert p to n)

Extremely slow due to weak interaction ($p \rightarrow n$)
$\tau \sim 10^{10}$ yr in Sun

p-p chain

$^1H + {}^1H \rightarrow {}^2H + e^- + \nu$

$^2H + {}^1H \rightarrow {}^3He + \gamma$

$^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$

Slow, but less so than above due to spontaneous decay of unstable nuclei

^{13}N ($\tau_{1/2} \sim 600s$) and
^{15}O ($\tau_{1/2} \sim 120s$)

But, requires higher T because of higher +ve charge on nuclei

CNO cycle

$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma$

$^{13}N \rightarrow {}^{13}C + e- + \nu$

$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma$

$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma$

$^{15}O \rightarrow {}^{15}N + e^- + \nu$

$^{15}N + {}^1H \rightarrow {}^{12}C + {}^4He$

Later stages of stellar evolution: What happens when Hydrogen fuel is exhausted in the core?

Inert He core, surrounded by a shell that is still burning H to He.

Changes structure of star: ×1000+ increase in L, plus 100x increase in size (with therefore decrease in temperature) → Red Giant Star

(N.B. Sun will consume Earth when it is Red Giant)

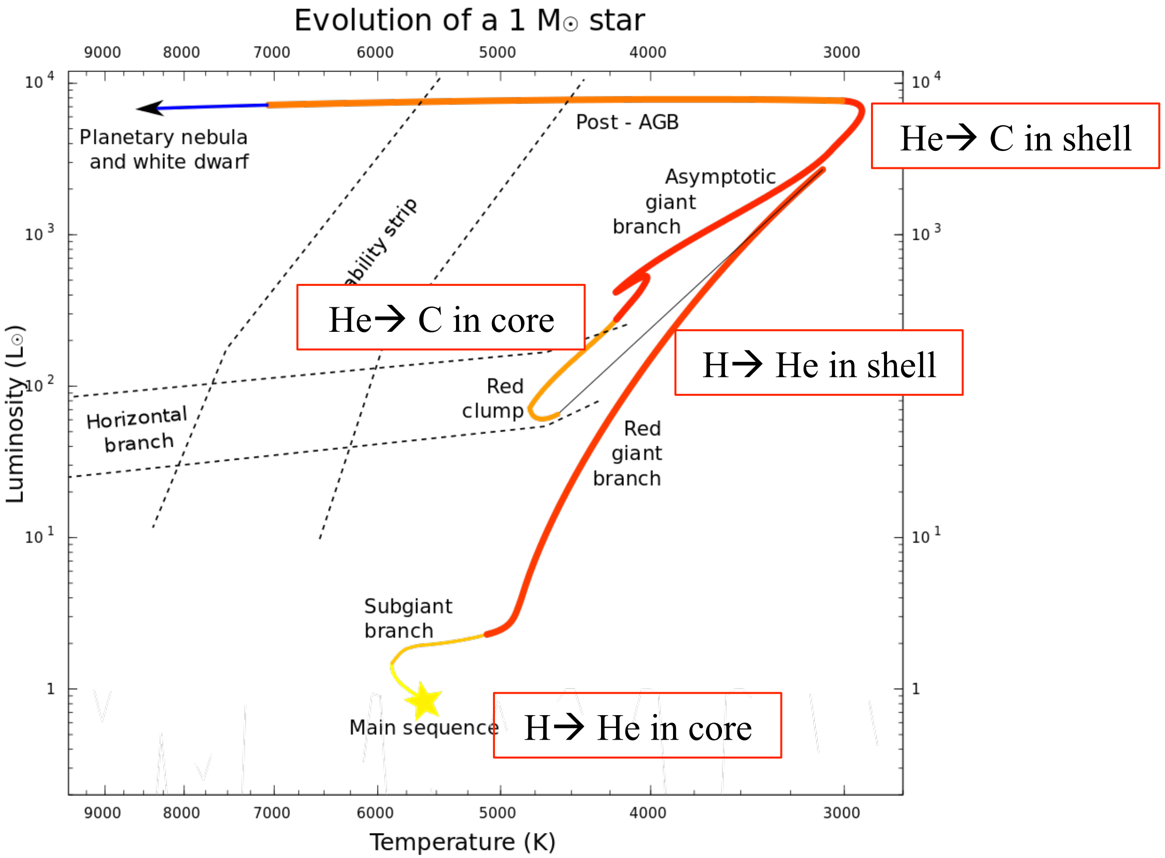
Stars with $M < 0.4\ M_{\odot}$ will not become red giants because they do not become hot enough for He burning, but this has never happened so far because their MS lifetimes are longer than the present age of the Universe.

Temperature rise in core may be enough to ignite He to C fusion → Horizontal Branch Star

Followed by He shell burning and so on → Asymptotic Giant Branch Star

If the stellar core ever gets dense enough to be supported by degenerate electron pressure (which is independent of temperature), then it will cease contracting and stop increasing its temperature.

Therefore no new fusion reactions will occur. As example, our Sun will never fuse C → O because it will stabilize its core while still below the temperature required to fuse C → O.



White Dwarfs (from intermediate mass stars)

When the $^4He \rightarrow C$ "triple alpha" process in a $M < 4\ M_{\odot g}$Red Giant star is complete, the star core starts contracting. The temperature stars too low to ignite the carbon fusion process, thus the collapse keeps going until it is halted by the pressure arising from electron degeneracy.

The the star shed its extended envelope (making a so-called Planetary Nebula), revealing the the very hot and small (relative to the Sun) central core, which is called a White Dwarf.

Typical white dwarf specs:

- mass of about 1 $M_{\odot}$ (on average)
- size of about 1 Earth diameter
- ~ 4% of solar luminosity
- surface temperature of ~ 25,000 K

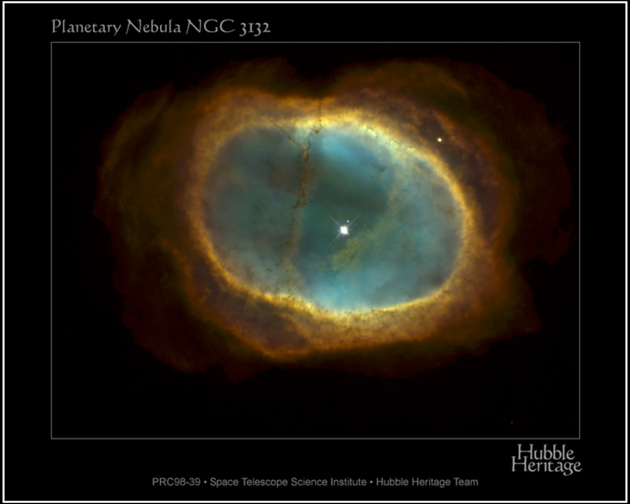
What about Life?

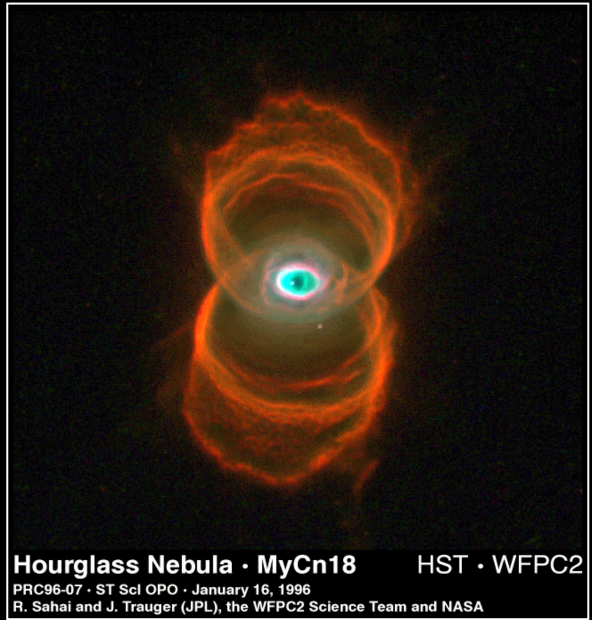
Good news: Because they are so small (and therefore have a very low luminosity) they will shine for a very long time, even though they are no longer producing more energy from fusion.

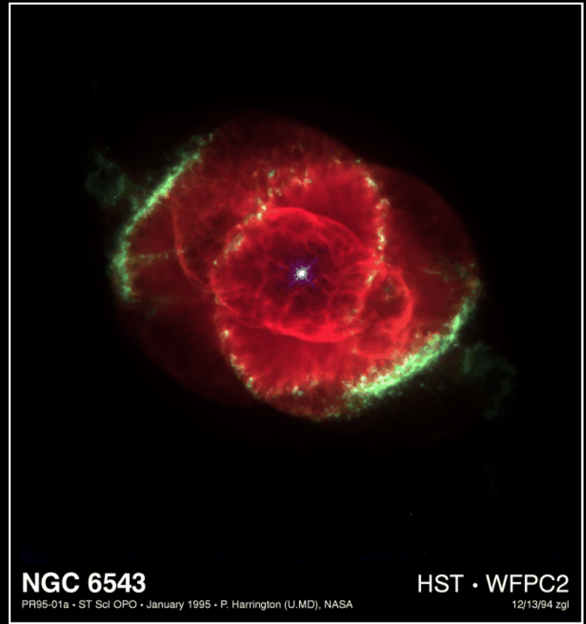
Dyson spheres??

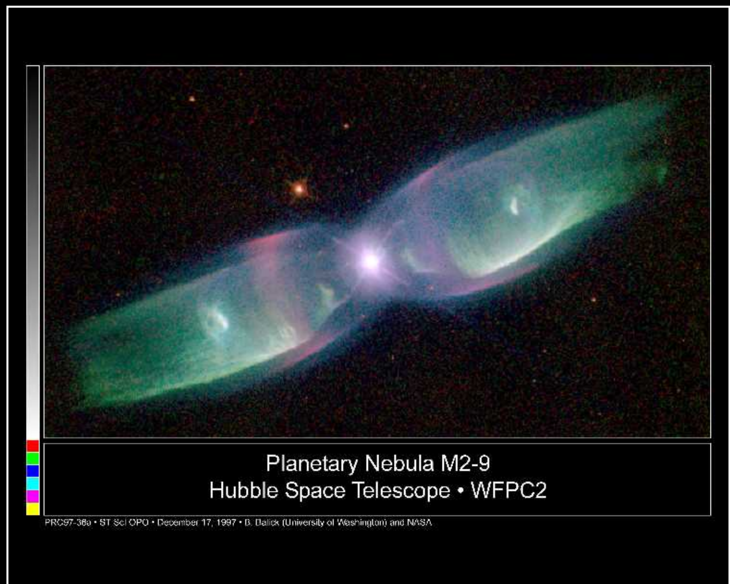
Bad news: All the products of fusion are still locked up in the interiors. Ejected material is largely unprocessed.

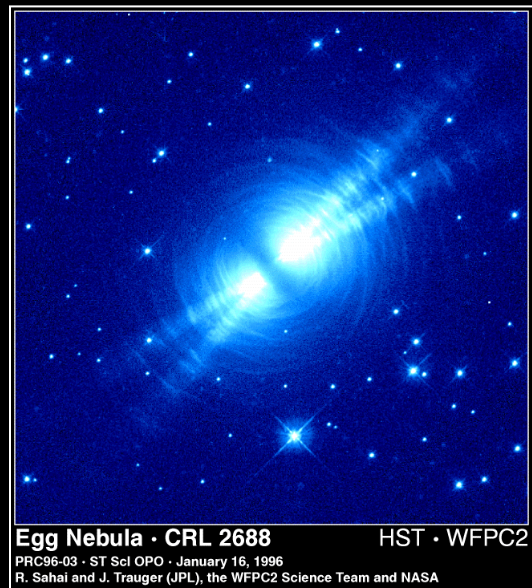
Young White Dwarfs at centers of Planetary Nebulae (nothing to do with planets)

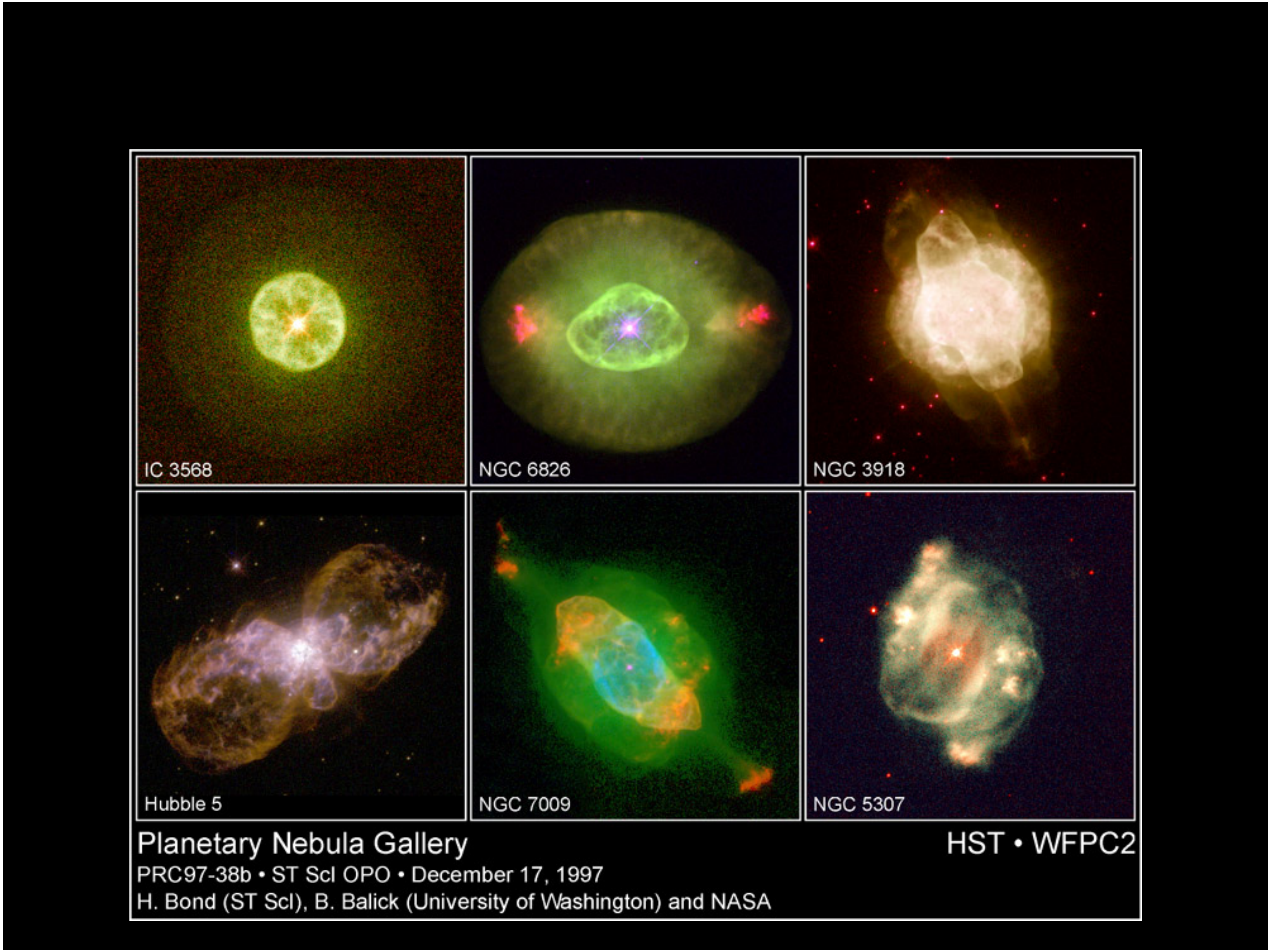











IC 3568
NGC 6826
NGC 3918
Hubble 5
NGC 7009
NGC 5307
Planetary Nebula Gallery
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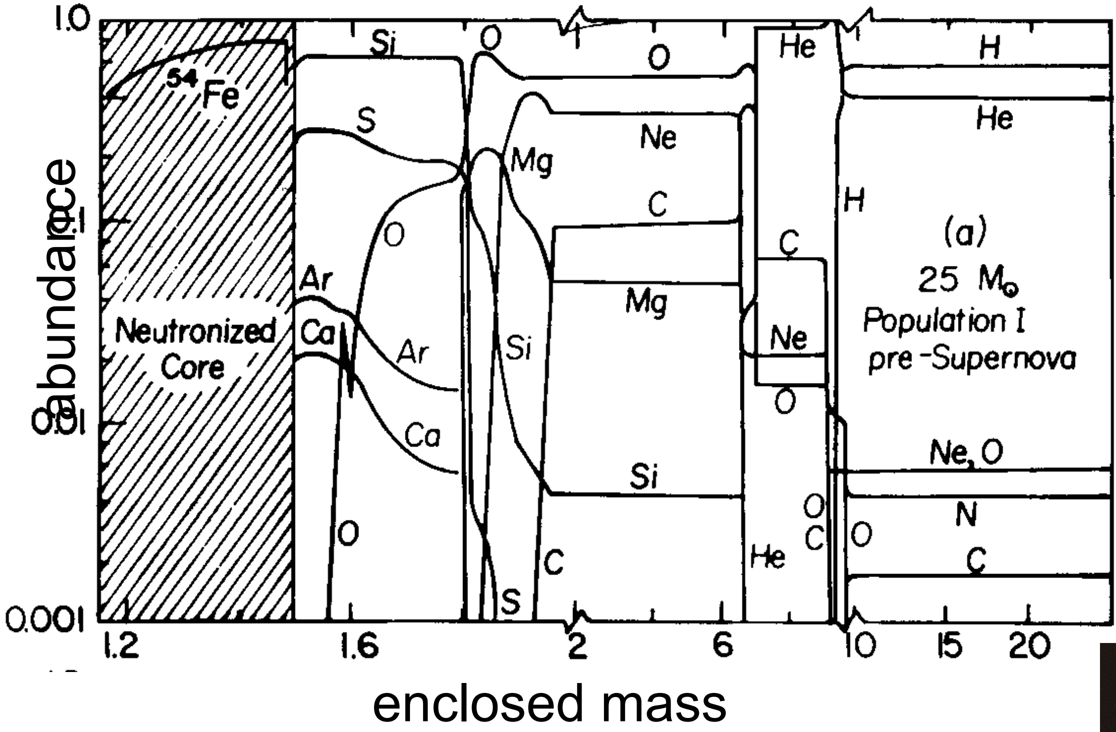
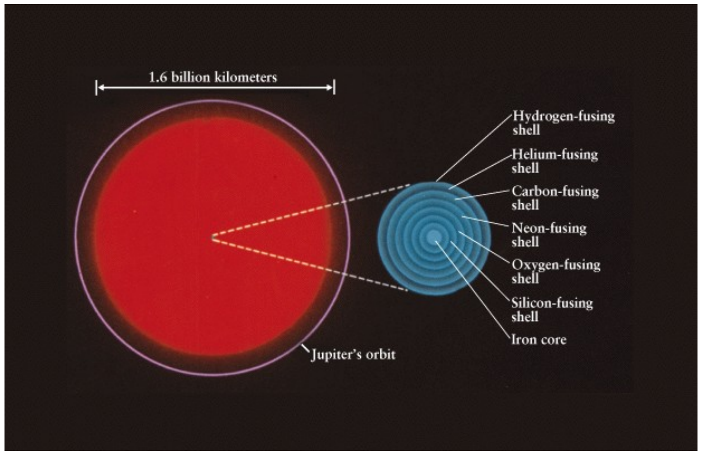
But, in more massive stars temperatures are high enough to go all the way to an Fe-rich core.



Final structure of a 25 M_{sun} star



Onion-like structure with successive shells burning different fusion reactions

But time is running out:

The energy per reaction is dropping as the masses of nuclei increase, but the required temperature, and therefore the luminosity (= energy loss rate) is increasing.

For a 25 $M_{\odot}$ star:

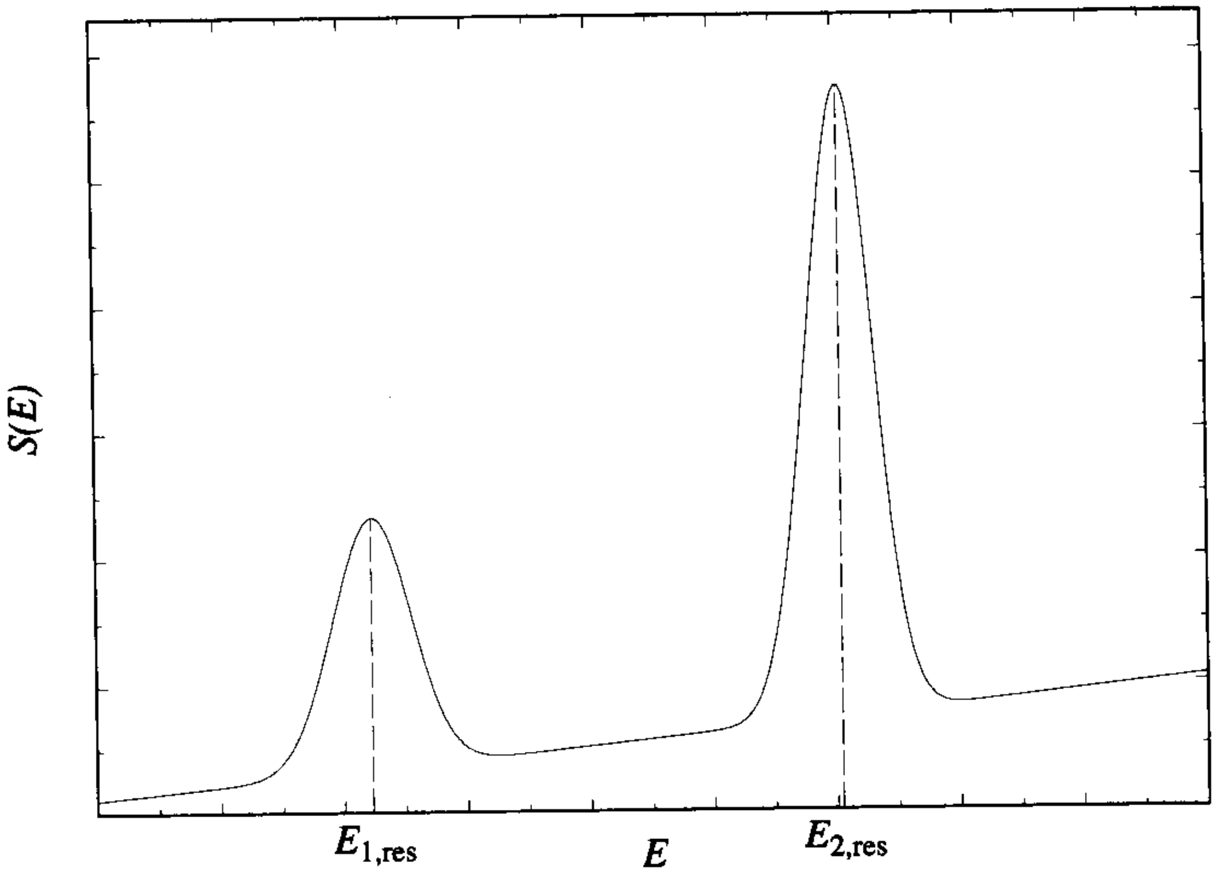
- Hydrogen burns in the core for 7 million years
- Helium burns in the core in the core for 500,000 years
- Carbon burns in the core for 600 years
- Oxygen burns in the core for 6 months
- Silicon burns in the core for only 1 day

So, what happens next?
We'll return to this next week

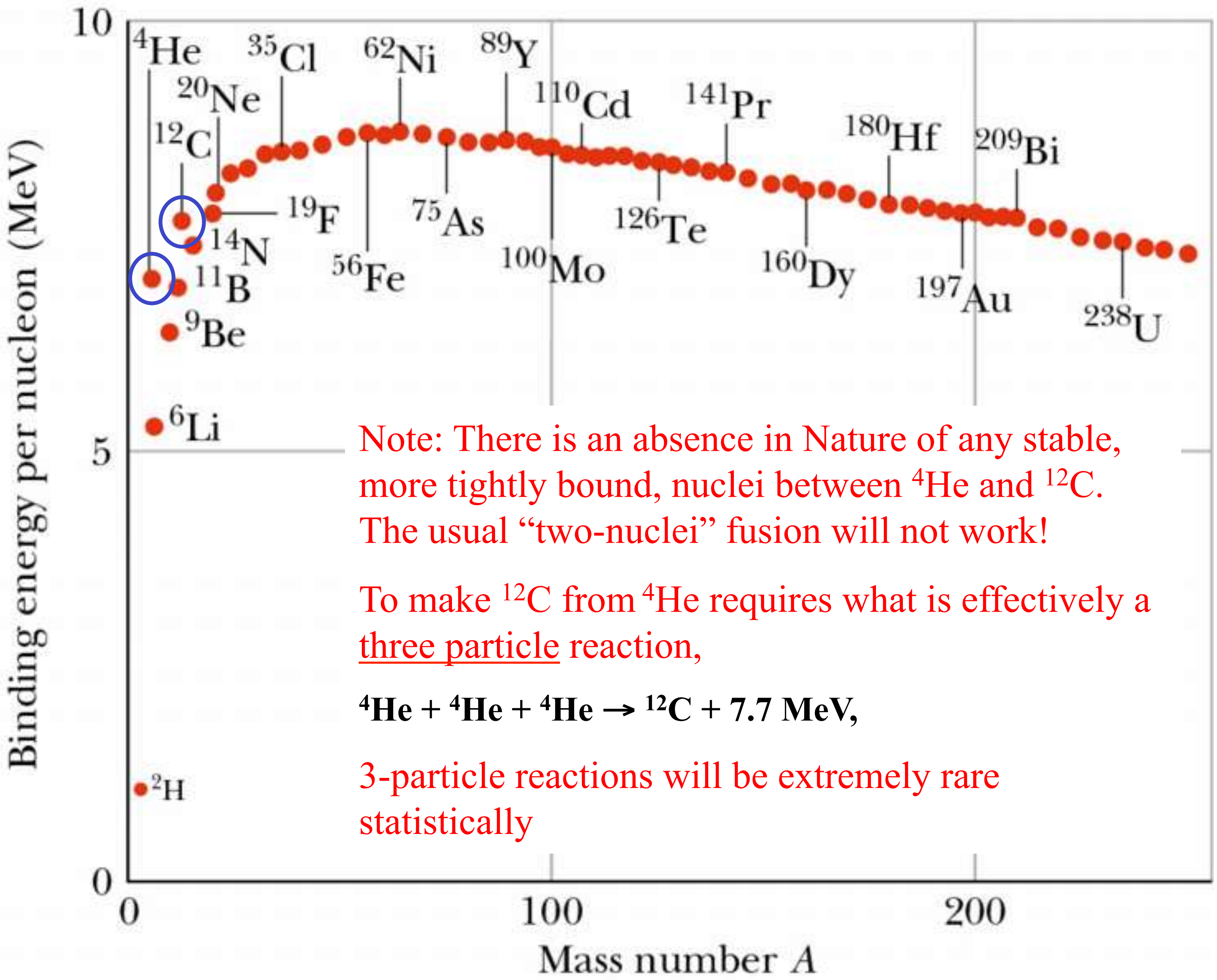
But first, back to that Gamow Peak and the problem of Carbon

Key point: Gamow peak means that most reactions occur with the same K.E., and therefore produce products at the same "energy"… i.e. the Δmc^2 of the reaction in question plus the K.E. of the reacting particles as given by the Gamow peak.

One consequence: The existence of well-placed energy levels in the product nucleus leads to resonances, i.e. greatly increased cross section σ at particular product energies, leading to wide variation of reaction rates for different reactions.



Now let's look at the situation with carbon production



Note: There is an absence in Nature of any stable, more tightly bound, nuclei between ^{4}He and ^{12}C. The usual “two-nuclei” fusion will not work!

To make ^{12}C from ^{4}He requires what is effectively a <u>three particle</u> reaction,

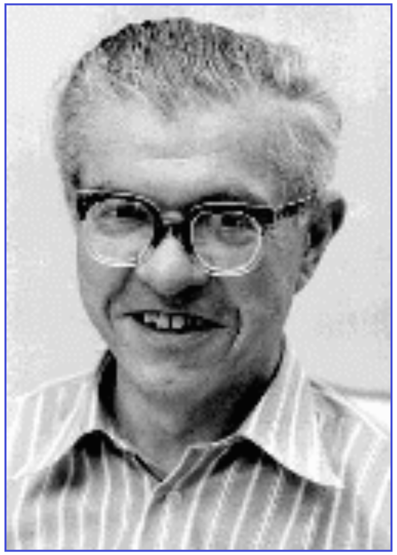
$^{4}He + {}^{4}He + {}^{4}He \rightarrow {}^{12}C + 7.7$ MeV,

3-particle reactions will be extremely rare statistically

- Production of ^{8}Be from two ^{4}He requires 0.1 MeV of energy.
- And the ^{8}Be is then unstable with half life of 10^{-17} s (!)

But, while it survives, ^{8}Be can join with another ^{4}He to make an excited state of ^{12}C.

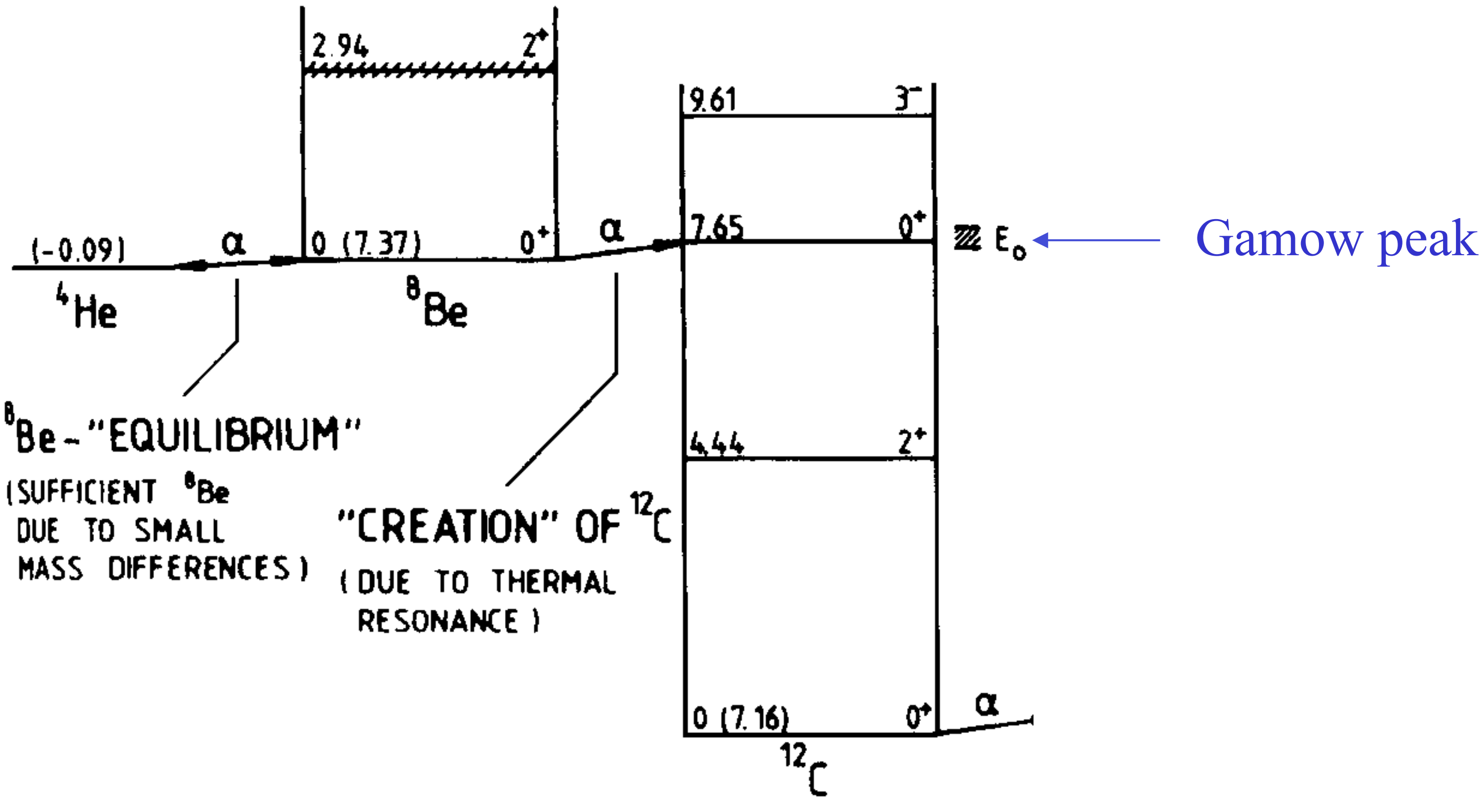
The ^{8}Be +^{4}He has an energy 7.33 MeV above the ground state of ^{12}C. The excited ^{12}C nucleus will be just above this because of the kinetic energy in Gamow peak.



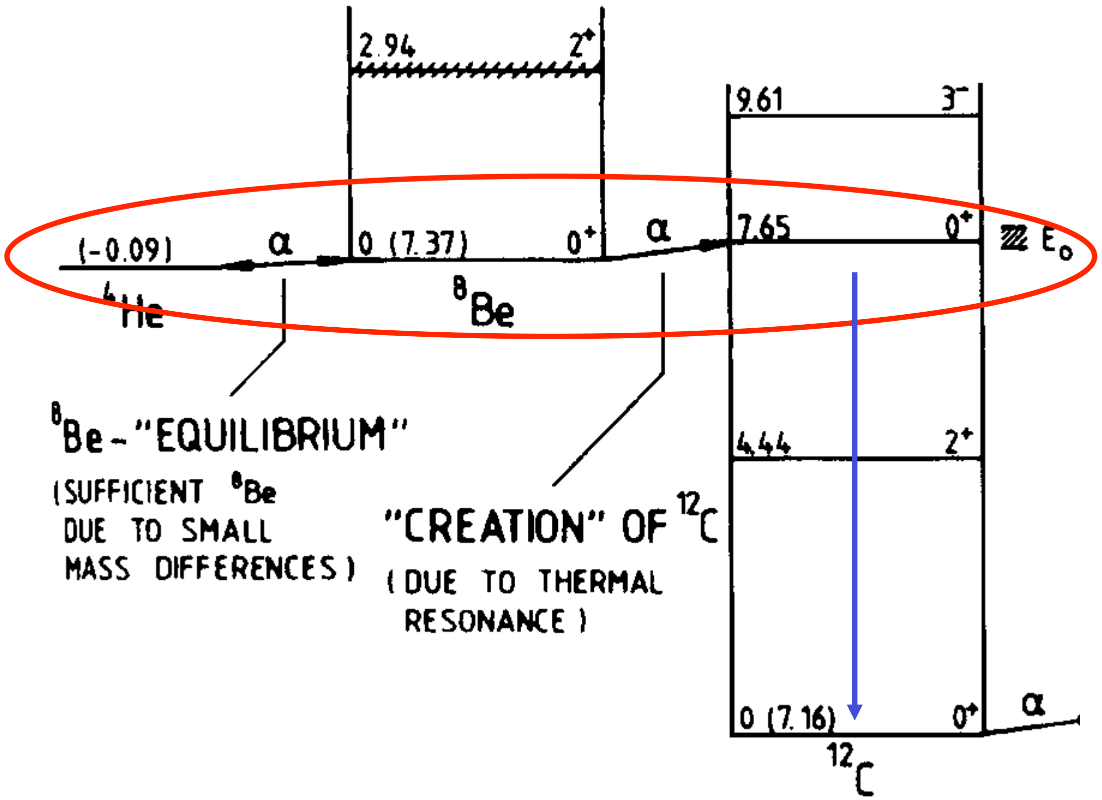
Hoyle (1952) correctly predicted that ^{12}C must have a resonant state just above 7.33 MeV for this reaction to occur often enough to produce significant amounts of ^{12}C in the Universe.

A major resonant state of ^{12}C was subsequently found at 7.65 MeV – a triumph of theoretical astrophysics!

Most ^{12}C* falls apart back to ^{8}Be + ^{4}He, but some de-excites to the ^{12}C ground-state through emission of γ-ray photons.
We are made of these lucky de-excitations.



C Resonance level



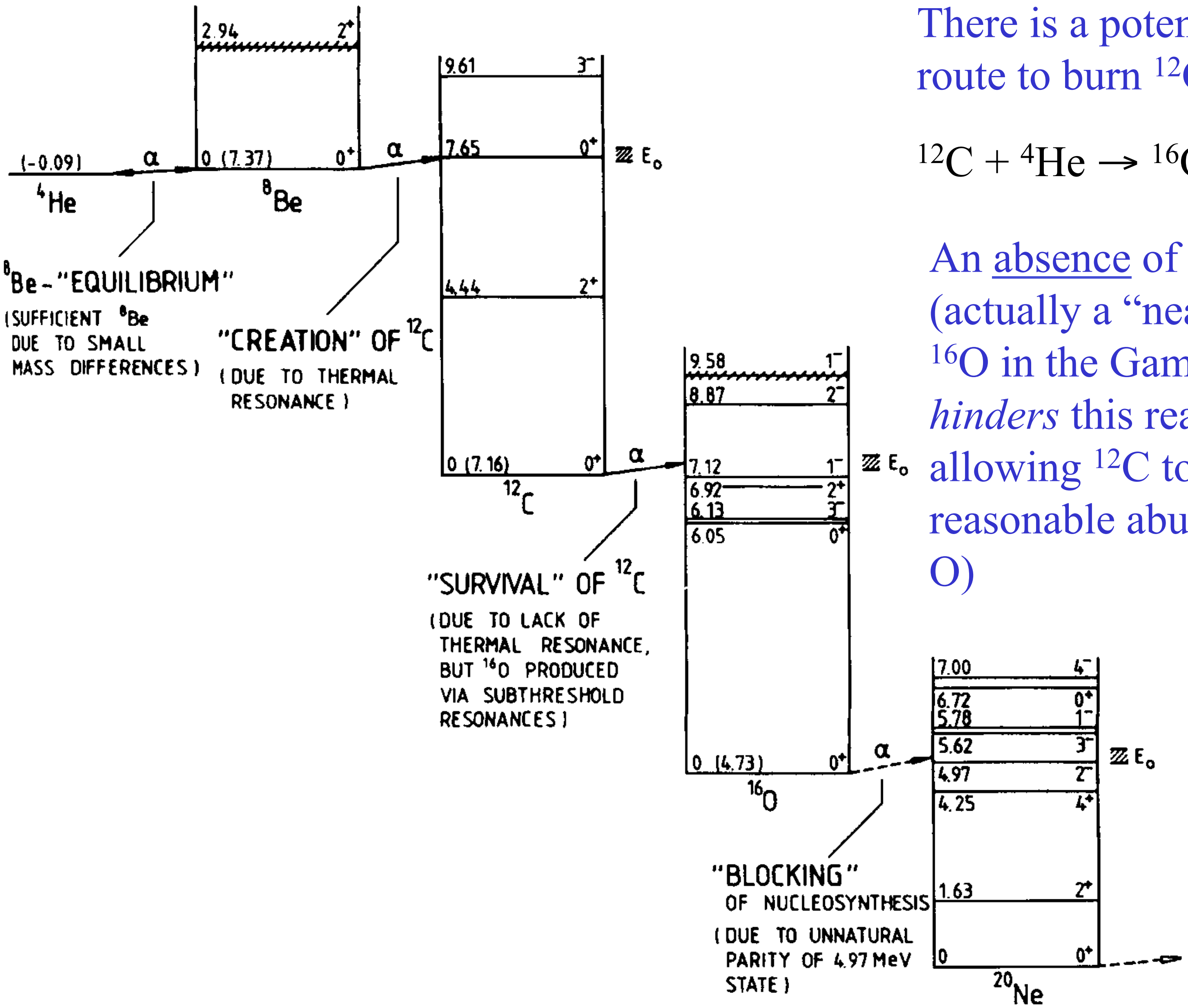
Three states of ^{4}He, ^{8}Be and ^{12}C* are so similar in energy that these three species will be in equilibrium, described by Saha equation.

Production of ^{12}C is effectively "leakage" out of equilibrium from ^{12}C* excited state

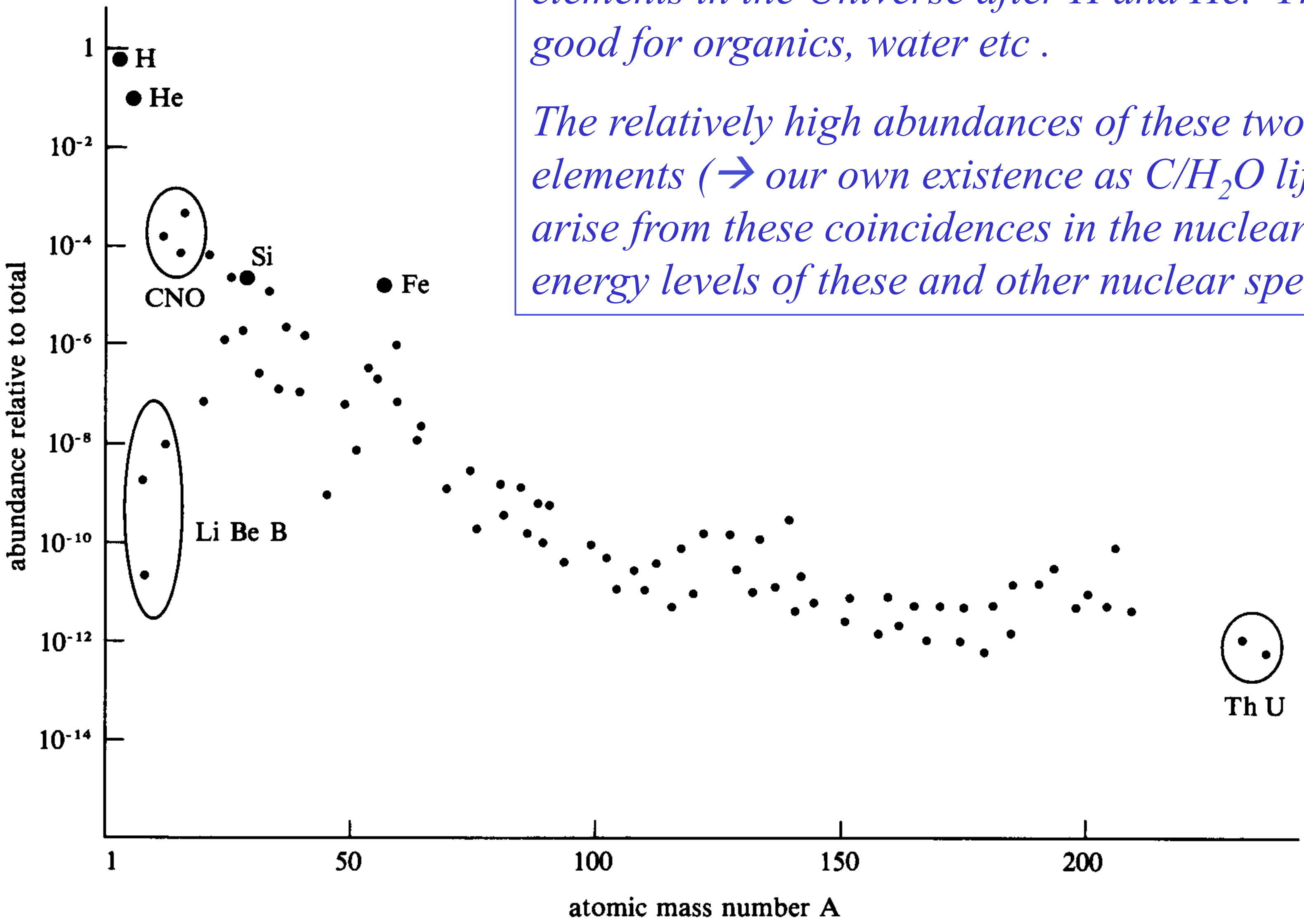
There is a potentially easy route to burn ^{12}C:

$^{12}C + ^{4}He \rightarrow ^{16}O + 7.16$ MeV

An <u>absence</u> of a resonance (actually a "near-miss") for ^{16}O in the Gamow band *hinders* this reaction, allowing ^{12}C to survive in reasonable abundance (~ 1/3 O)



Destruction of ^{16}O also hindered



Consequence: O and C are the most abundant elements in the Universe after H and He. This is good for organics, water etc .

The relatively high abundances of these two elements (→ our own existence as C/H_2O life?) arise from these coincidences in the nuclear energy levels of these and other nuclear species

Key ideas:

- Hot stars are inevitable from gravitational collapse, gravitational potential energy and the virial condition.
- Fusion is also inevitable (for $M > 0.08\ M_{\odot}$) because the interior temperature will eventually get hot enough.
- Fusion will produce a long-lived star at (roughly) constant size for as long as there is fuel. The negative heat capacity stabilizes the star and prevents an explosion.
- Timescales and outcomes of stellar evolution will depend on the mass of the star − an interplay of temperature, fuel, sources of pressure etc.
- The details of fusion depend on the details of nuclear structure: e.g. the existence of Gamow peaks and the interesting case of production of ^{12}C and ^{16}O that are the basis of (our) Life.