Origin of the chemical elements
(Part II)

Pressure from degenerate matter

R. H. Fowler (1926): application of Pauli exclusion principle.
In a dense gas all the lower energy levels become filled with electrons and this results into a pressure which resists the gravitational force

$$P = \frac{\pi^2 \hbar^2}{5 m_e m_H^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\rho}{\mu_e}\right)^{5/3},$$

Can get order of magnitude estimate simply using Heisenberg Uncertainty Principle: $\Delta x \Delta p \sim h/2\pi$. Consider object with density n particles per unit volume, the available volume per particle is n^{-1}, so $\Delta x \sim n^{-1/3}$ and $p \sim (h/2\pi) \,.\, n^{1/3}$

The pressure will be product of the number density and the mean kinetic energy $\sim n \,.\, E$

Non-relativistic case: $E = p^2/2m$

$$\boxed{P_{non-rel} \sim \frac{\hbar^2}{2m_e} n_e^{5/3}}$$

Relativistic case $E = cp$

$$\boxed{P_{rel} \sim \hbar c \, n_e^{4/3}}$$

Mass-radius relation and White Dwarf stability

We had before (part 1, slide 13),that hydrostatic support requires

$$P \sim -\frac{1}{3}\frac{U_{grav}}{V} \sim \frac{GM^2}{4\pi R^4}$$

For the <u>non-relativistic</u> case of degenerate pressure, we have

$$\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{5/3} \quad \Rightarrow \quad P \propto \frac{M^{5/3}}{R^5}$$

Putting these together, gives a mass-radius relation

$$\frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4} \quad \Rightarrow \quad \boxed{R \propto M^{-1/3}}$$

Therefore, adding more mass to an object that is supported by degenerate electrons will cause the radius to <u>shrink</u>, the density to go up and the pressure to increase. This makes a self-gravitating object supported by non-relativistic degenerate matter stable.

But, note that the E per particle increases as $n^{2/3}$, so eventually the non-relativistic matter will become relativistic. What then happens?

Mass-radius relation and White Dwarf stability

We had before (part 1, slide 13),that hydrostatic support requires

$$P \sim -\frac{1}{3}\frac{U_{grav}}{V} \sim \frac{GM^2}{4\pi R^4}$$

For the relativistic case, we have

$$\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{4/3} \qquad \Rightarrow \quad P \propto \frac{M^{4/3}}{R^4}$$

Now we have that the mass that can be supported is <u>independent</u> of R. In other words, the object is now unstable to collapse.

$$\frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4} \qquad \Rightarrow M^{-1/3} \propto R^0$$

This means that there is a maximum mass that can be supported. Adding further mass, once the particles are relativistic,therefore causes collapse.

For degenerate electrons, this is the Chandrasekhar mass = 1.44 $M_{\odot}$

Collapse beyond Chandrasekhar mass limit

The maximum mass that can be supported by degenerate <u>electrons</u> is 1.44 $M_\odot$ (Chandrasekhar 1931). This is indeed the observed maximum mass of White Dwarfs.

Obvious question: Can degenerate neutrons then stabilize a collapsed object?

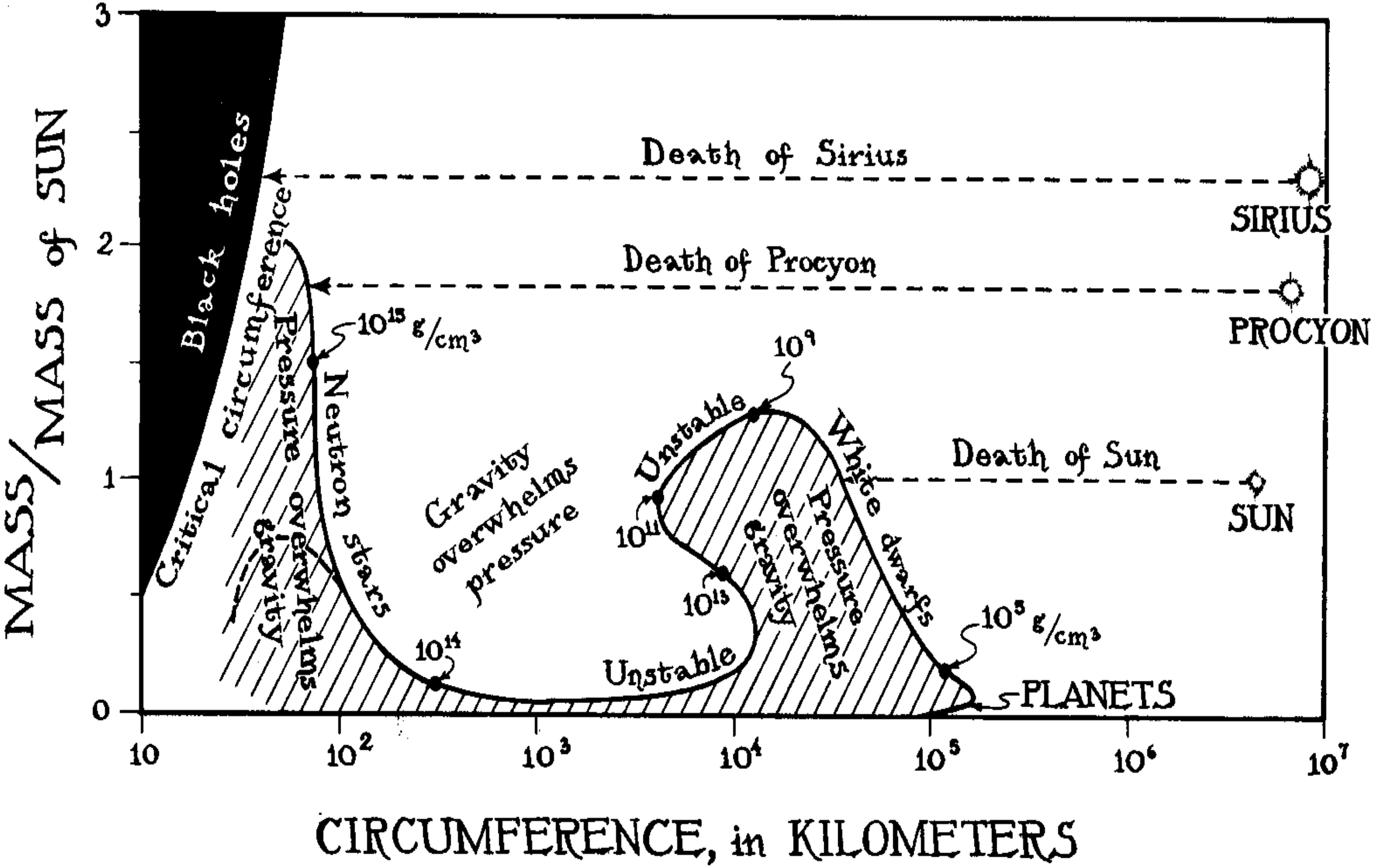
Recall $P \sim m^{-1} n^{5/3}$ in the non-relativistic case. So, neutrons exert $m_e/m_n \sim 10^{-3}$ of the pressure of electrons (which is why we ignored it earlier).

But, collapsing by factor X, the density n increases by X^3, so the pressure increases as X^5. But note the required pressure also increases as X^4.

Net effect, we would expect stabilization by degenerate neutrons when the radius has shrunk by factor equal to the mass ratio of the particles: $X \sim m_n/m_e \sim 1000$, i.e. at a radius of order 10 km , compared with 10^4 km for White Dwarfs.

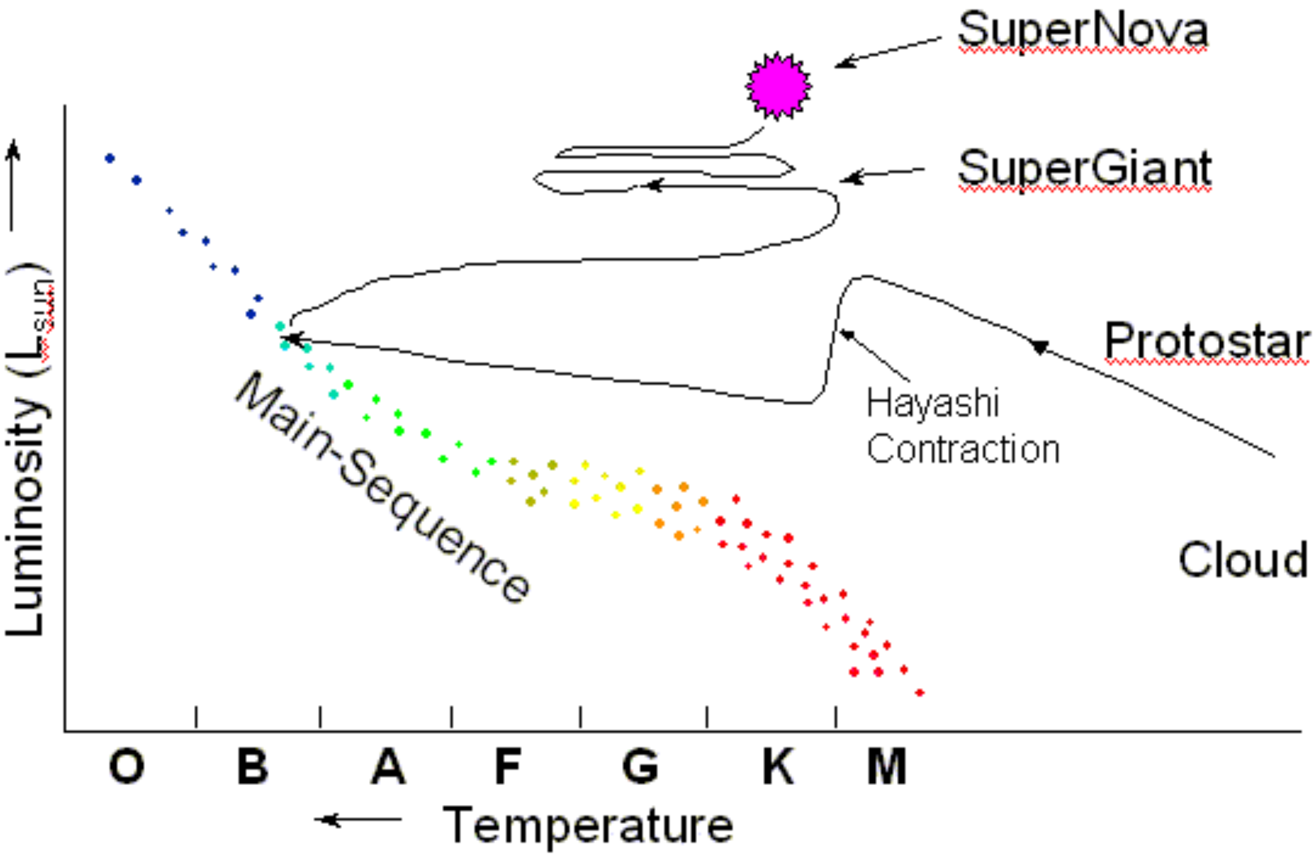
Collapse of stellar core > 1.4 $M_\odot$ causes $e^- + p^+ \rightarrow n$ because n has lower energy state than $p + e$ (because of enormous relativistic energy of e). i.e. the electrons are "squeezed out of existence" by gravity.

Overview: stellar mass collapsed objects in the Universe



From Kip Thorne "Black Holes & Time Warps"

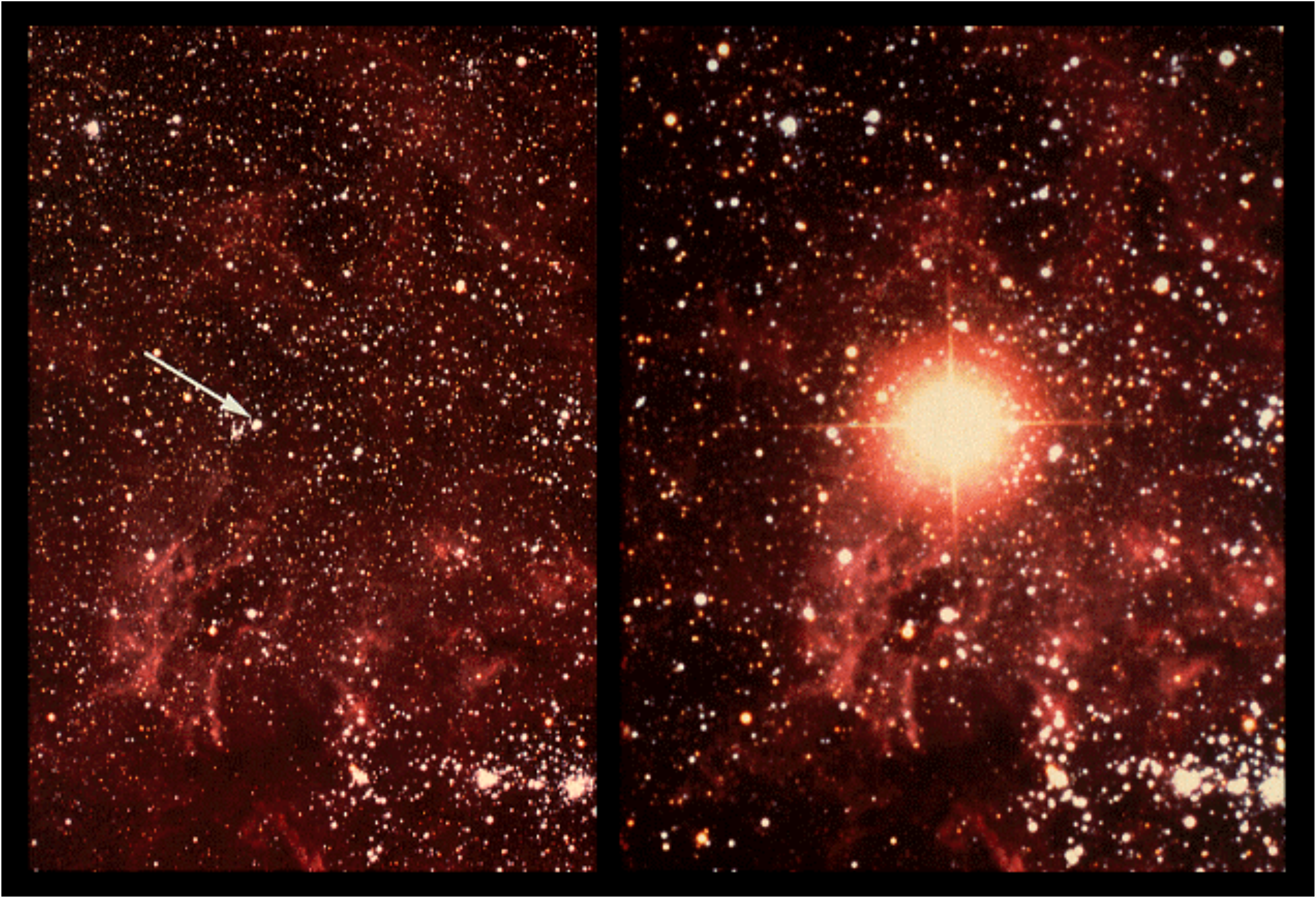
Evolution of 20 $M_\odot$ star



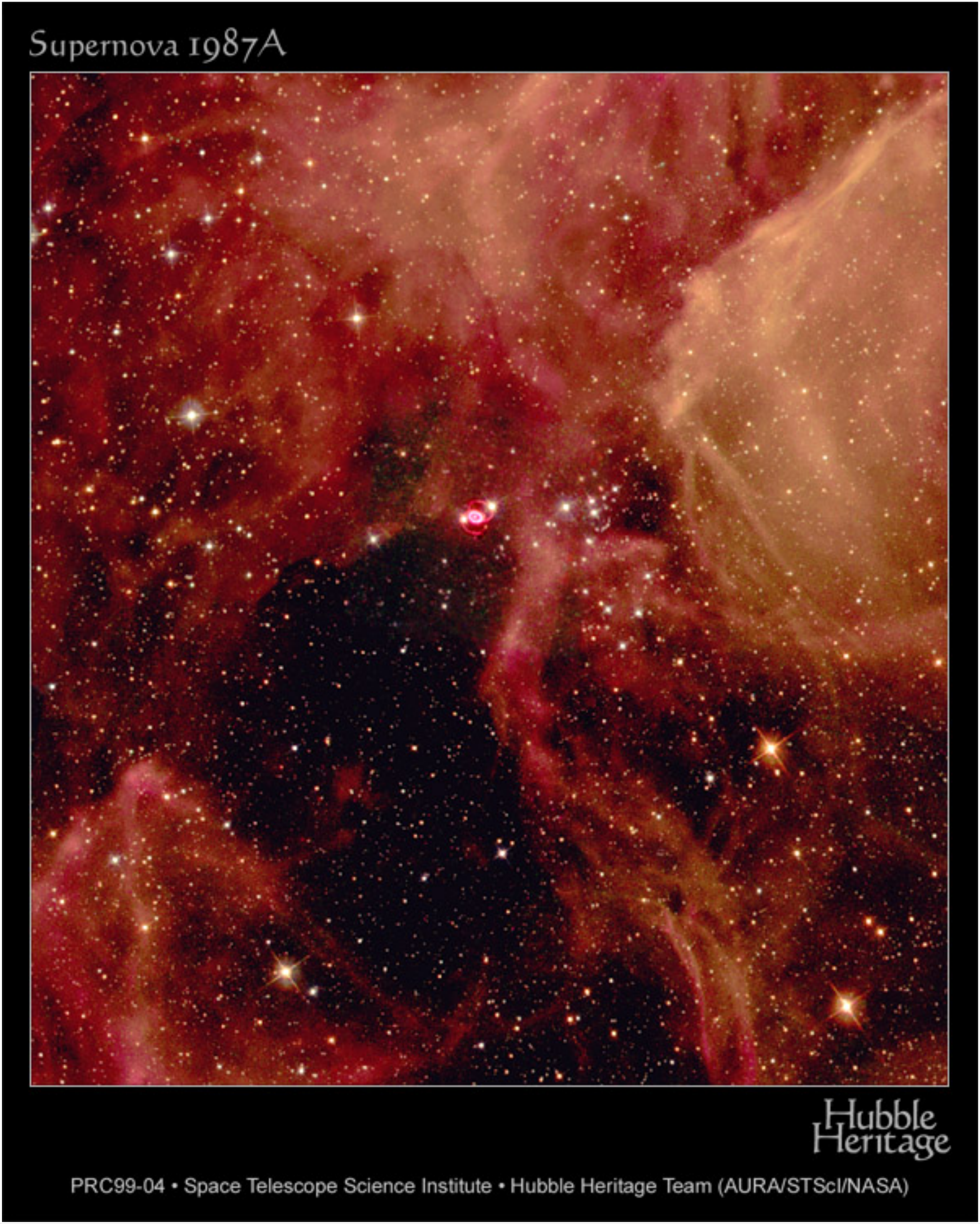
Type II Supernovae

- Catastrophic collapse (duration 1 sec) of cores of stars that had initial $M > 8\ M_\odot$
- No further nuclear production of energy is possible from the iron-rich core which starts contracting and heating up. The core exceeds the Chandrasekhar limit and cannot therefore be supported by degenerate electrons.
- There are also a number of channels for taking energy out of the core, causing it to contract more and more.
  - ○ ^{56}Fe begins to photo-disintegrate into α-particles and neutrons absorbing energy from the core.
  - ○ α-particles themselves photo-disintegrate and the core begins to fall freely under its own self-gravity.
  - ○ Almost all the free electrons combine with protons to form neutrons at nuclear densities.
- Copious numbers of neutrinos (one for every proton-electron combination) are produced, and stream out of the core, further removing energy
- The pressure from this neutrino pulse is believed** to be responsible for exploding away the stars' outer layers (** because no model yet really works in 3d)
- Pulse of neutrinos was detected shortly before visual sighting of SN 1987a in Large Magellanic Cloud (first visual detection of SN for ~ 400 years)
- Core collapses to ~10 km where it is stabilized by degenerate neutrons (or → BH?)

Neutron stars and supernovae

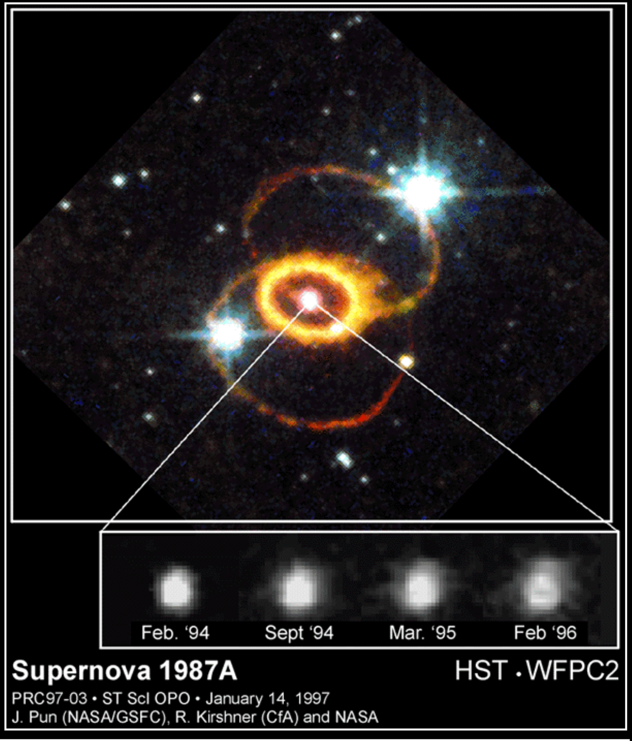


SN 1987A



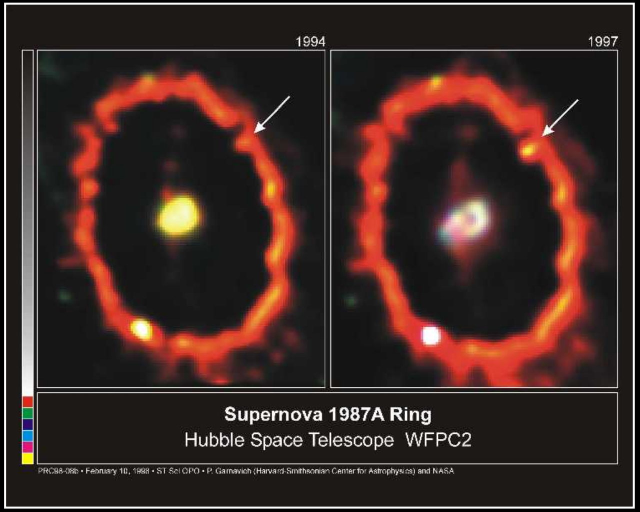
Supernova 1987A

PRC99-04 • Space Telescope Science Institute • Hubble Heritage Team (AURA/STScI/NASA)



Supernova 1987A HST • WFPC2

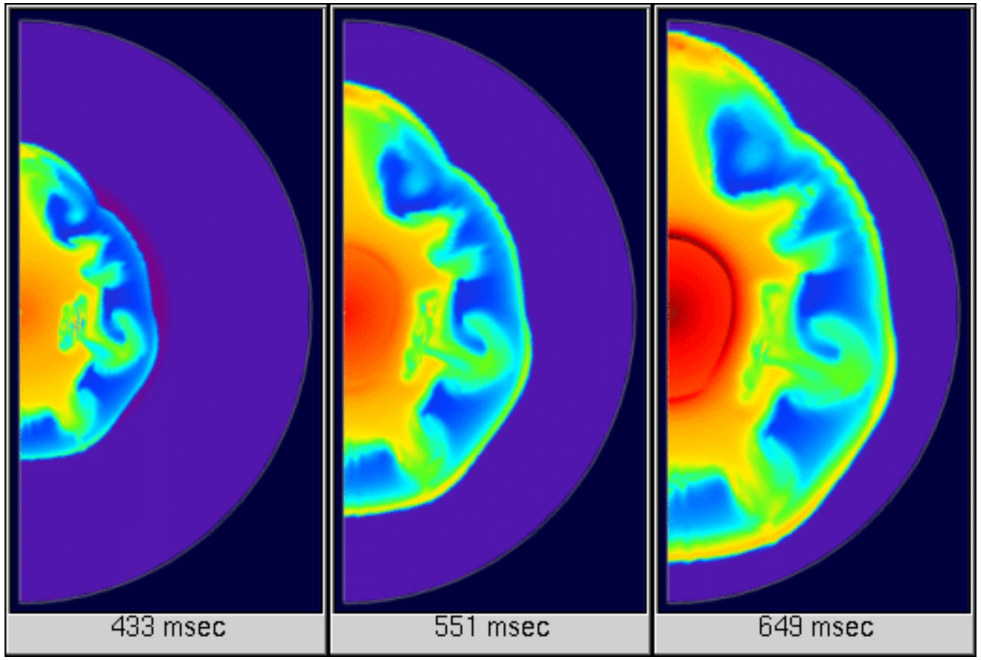
PRC97-03 • ST ScI OPO • January 14, 1997
J. Pun (NASA/GSFC), R. Kirshner (CfA) and NASA

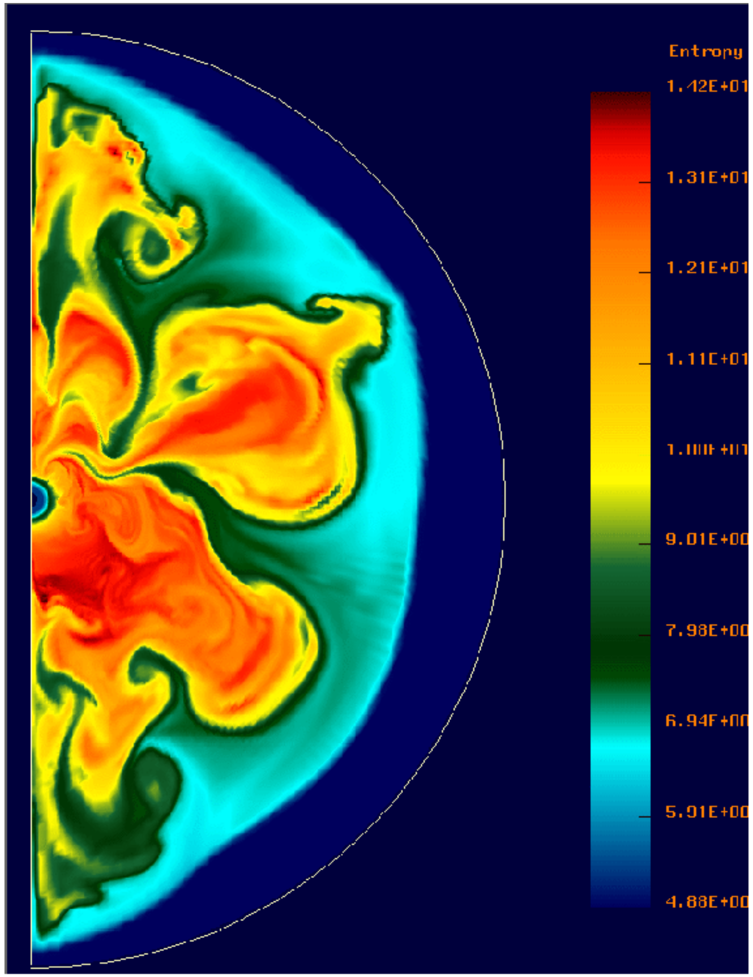


Supernova 1987A Ring
Hubble Space Telescope WFPC2

PRC98-08b • February 10, 1998 • ST ScI OPO • P. Garnavich (Harvard-Smithsonian Center for Astrophysics) and NASA

Neutrino driven explosion





Energetics of Supernovae

We saw how current thermal energy of a thermal-pressure supported star will always be comparable to its (negative) gravitational potential energy, via the virial condition

$$K = -\frac{U}{2}$$

The gravitational potential energy is proportional to R^{-1}.

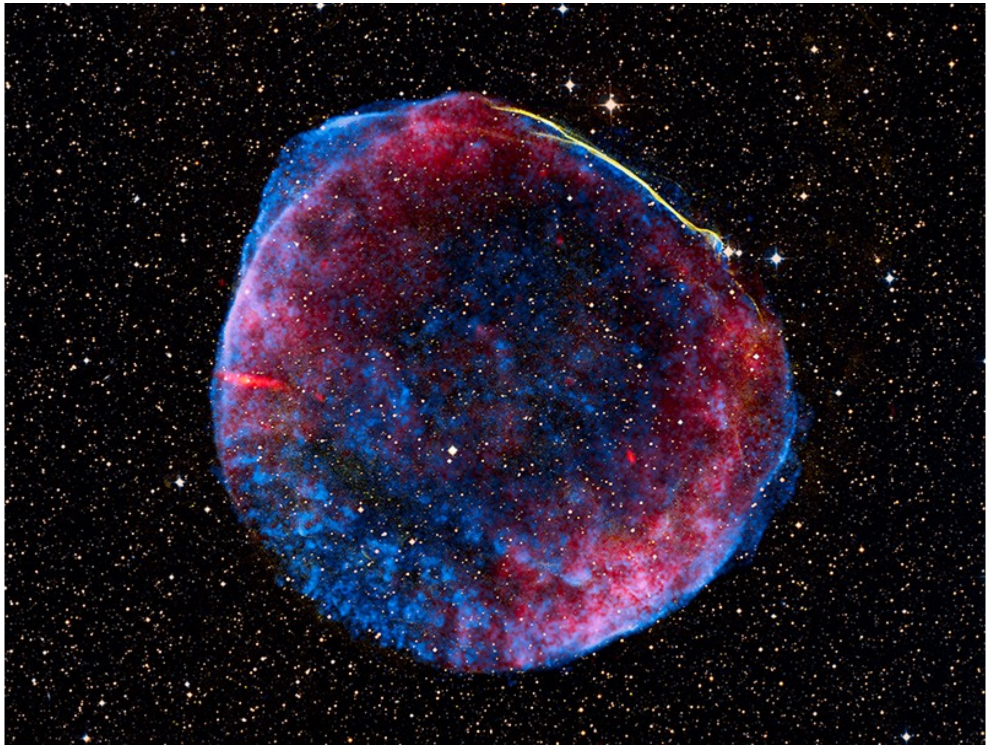
$$U \sim -\frac{GM^2}{R}$$

Key point: when a star collapses from $O(10^6)$ km to $O(10)$km in a core-collapse supernova, the energy released is about 10^5 times the potential energy (and thermal energy) that the object had as a normal star.

Ejection of debris from supernovae

Why do you get explosion and ejection of material from *collapse*?

- Total energy released is of order the gravitational binding energy of all the material (about 10^{46} J).
- Most of this is in the neutrinos. Of order 1% is reabsorbed by outer material.
- Since the "escape speed" from the surface of the neutron star ~ 0.3c, if only some of the material is ejected this can have E >> binding energy, so the asymptotic ejection speed (even "at infinity") can easily be 0.1c.



Remnant of SN 1006

Neutron capture (1): The s-process

Question: How are elements beyond the iron peak formed?

- definitely not by energetically favourable fusion of smaller nuclei!

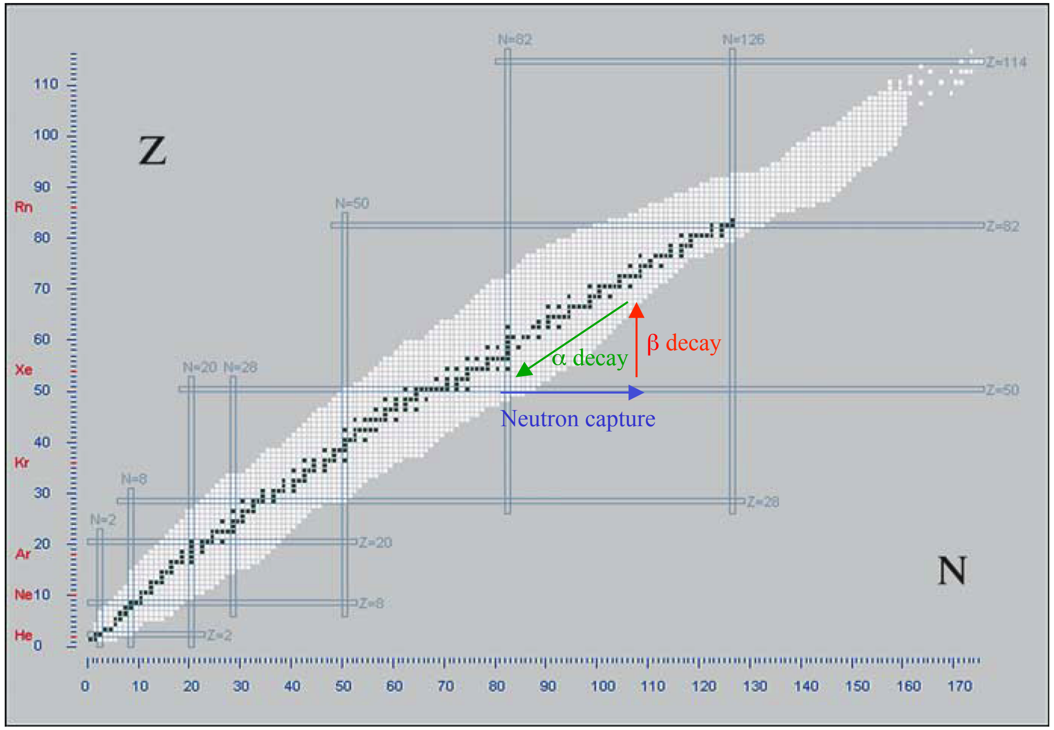
Answer: By neutron capture

Since a neutron is electrically neutral, even a slow neutron can be absorbed by an atomic nucleus, which might then be unstable to β-decay, increasing Z by one.

If the rate of capture of free neutrons is sufficiently slow, all the nuclides formed by neutron capture that are unstable will have a chance to β-decay before another neutron is absorbed.

The elements formed this way are called *s*-process elements (*s* stands for "slow"). This occurs during last $\sim 10^2$ years of massive star's life when the conditions in stellar cores produce some free neutrons.

Neutron capture (2) : r-process



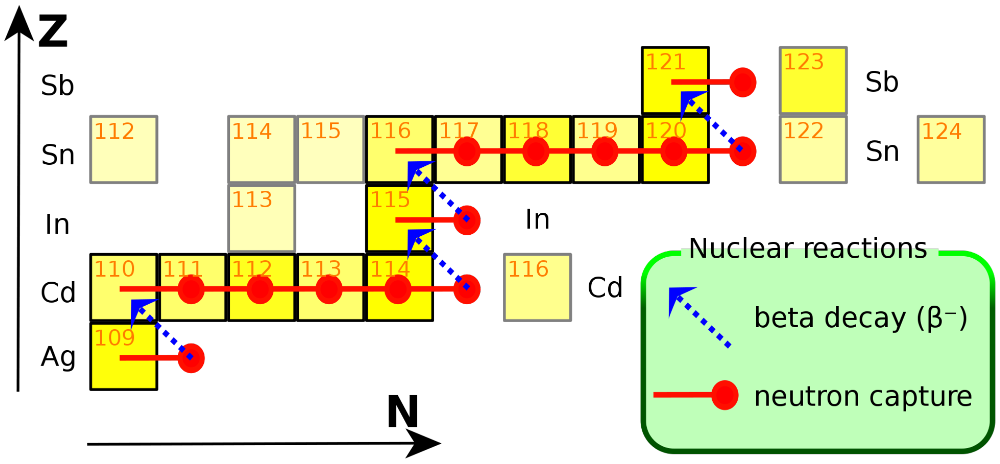
In a core-collapse SNa event, the rate of production of free neutrons becomes very rapid (flux ~10^{22} cm^{-2} s^{-1}; T~ 10^{10} K); as a consequence the elements formed by neutron capture will not have a chance to β-decay before the capture of another neutron.

Eventually, after the period of neutron production, the newly formed very heavy elements will have a chance to α and β-decay.

The sequence of elements formed this way - called the *r*-process elements (*r* stands for "rapid") - generally differs from the sequence formed by the *s*-processes.

s-process does one neutron-adding step at a time via reasonably stable nuclei

r-process adds many neutrons at once followed by various decay routes



Different elements/isotopes are primarily produced either by the *s*- or the *r*-process: Example: production of different isotopes of Cobalt.

s-process

$^{56}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{57}\mathrm{Fe}$

$^{57}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{58}\mathrm{Fe}$

$^{58}\mathrm{Fe} + \mathrm{n} \rightarrow {}^{59}\mathrm{Fe}$

$^{59}\mathrm{Fe} \rightarrow \mathbf{{}^{59}Co}$

r-process

$^{59}\mathrm{Fe} + 2\mathrm{n} \rightarrow {}^{61}\mathrm{Fe} \rightarrow \mathbf{{}^{61}Co}$

most Co on Earth is *s*-process produced $^{59}\mathrm{Co}$

The *s*-process is expected to occur during the last few hundred years of a massive star's life, and produces elements up to Bismuth (Z = 83)

Evidence: ^{99}Te *(Z=43) is unstable* $t_{1/2}$*~ 200,000 yrs and yet is seen in small quantities the atmospheres of AGB stars (having been convected up from the core).*

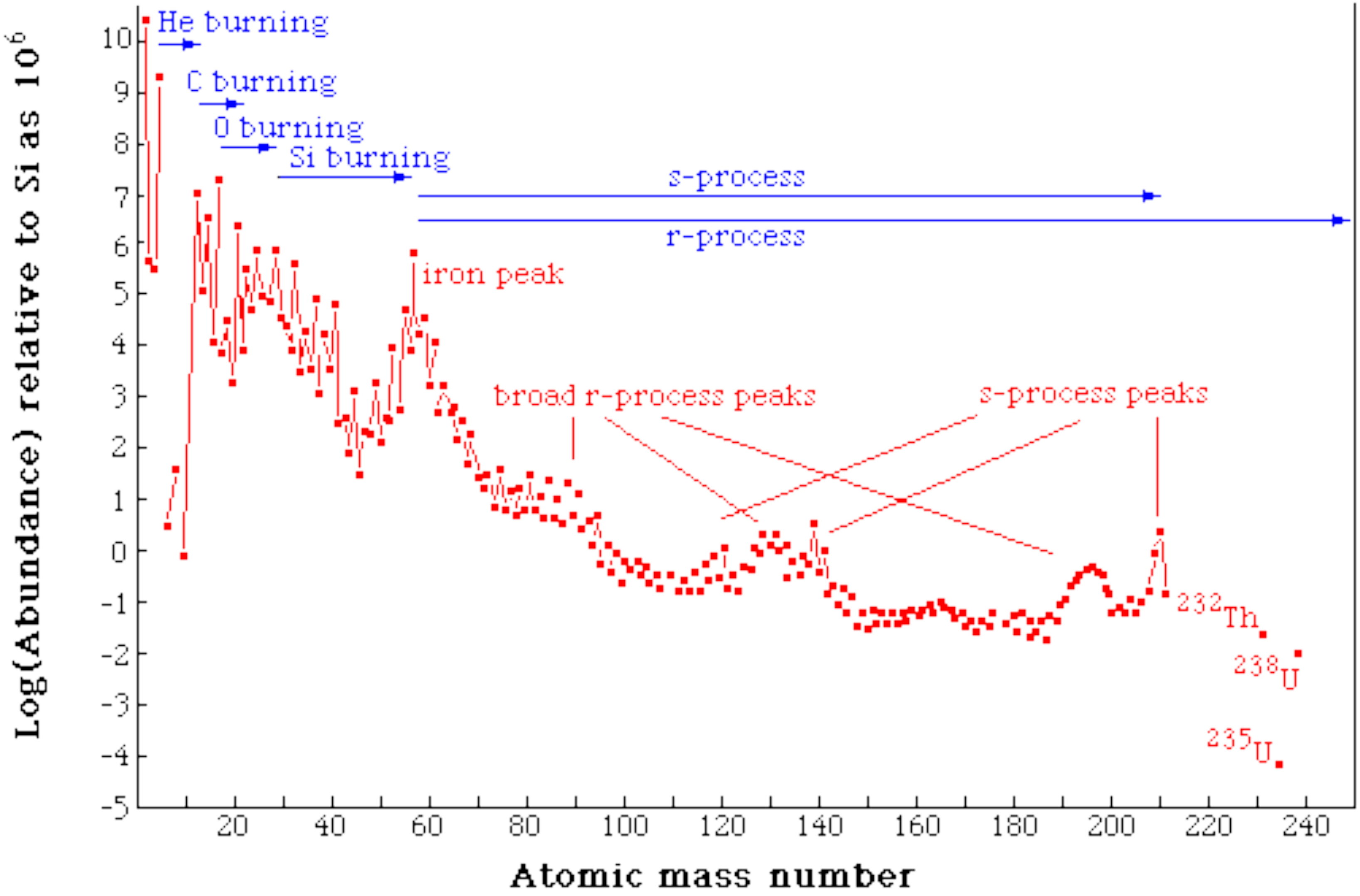
The *s*-process stops at Bismuth because of a cycle:

$$^{209}\mathrm{Bi} + n \rightarrow {}^{210}\mathrm{Bi}$$
$$^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + e^-$$
$$^{210}\mathrm{Po} \rightarrow {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He}$$
$$^{206}\mathrm{Pb} + 3\mathrm{n} \rightarrow {}^{209}\mathrm{Pb}$$
$$^{209}\mathrm{Pb} \rightarrow {}^{209}\mathrm{Bi} + e^-$$

Effectively, four captured neutrons are converted into ^{4}He plus 2 *e*-, instead of making heavier nuclei.

The *r*-process only happens in the one second or so of a supernova collapse and is the only channel for production of elements above Bismuth (Polonium Z = 84 to Uranium z = 92), and is the dominant for many other elements including Gold (79), Silver (47) etc.

Solar system abundances



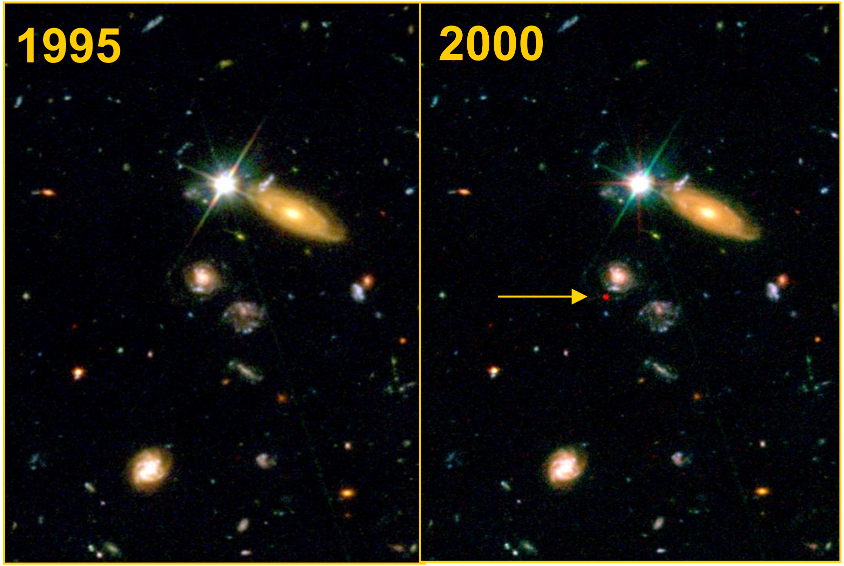
Also: Supernovae Type 1a

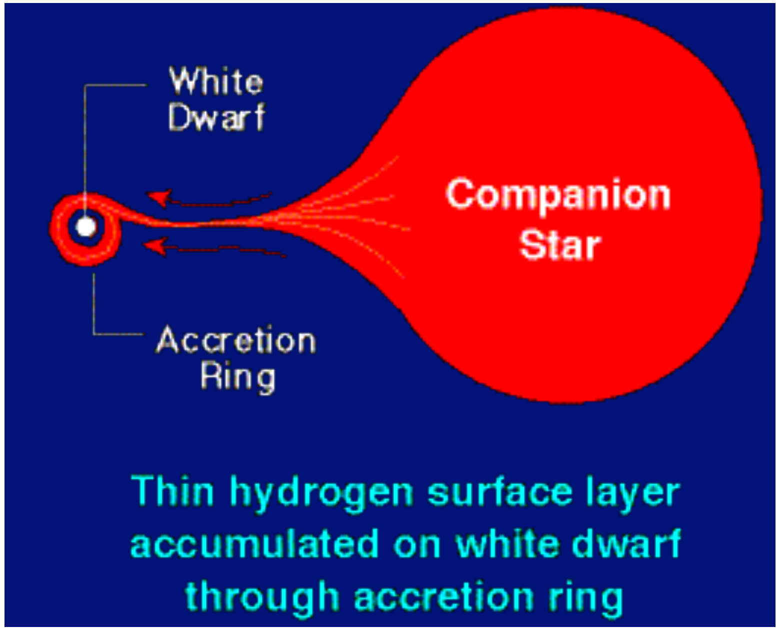
White dwarf is made of Carbon nuclei.

If mass is added to a White Dwarf (e.g. by accretion in a binary system) then at a point near to the Chandrasekhar mass, the temperature and density increase enough to cause thermonuclear ignition of the Carbon nuclei.

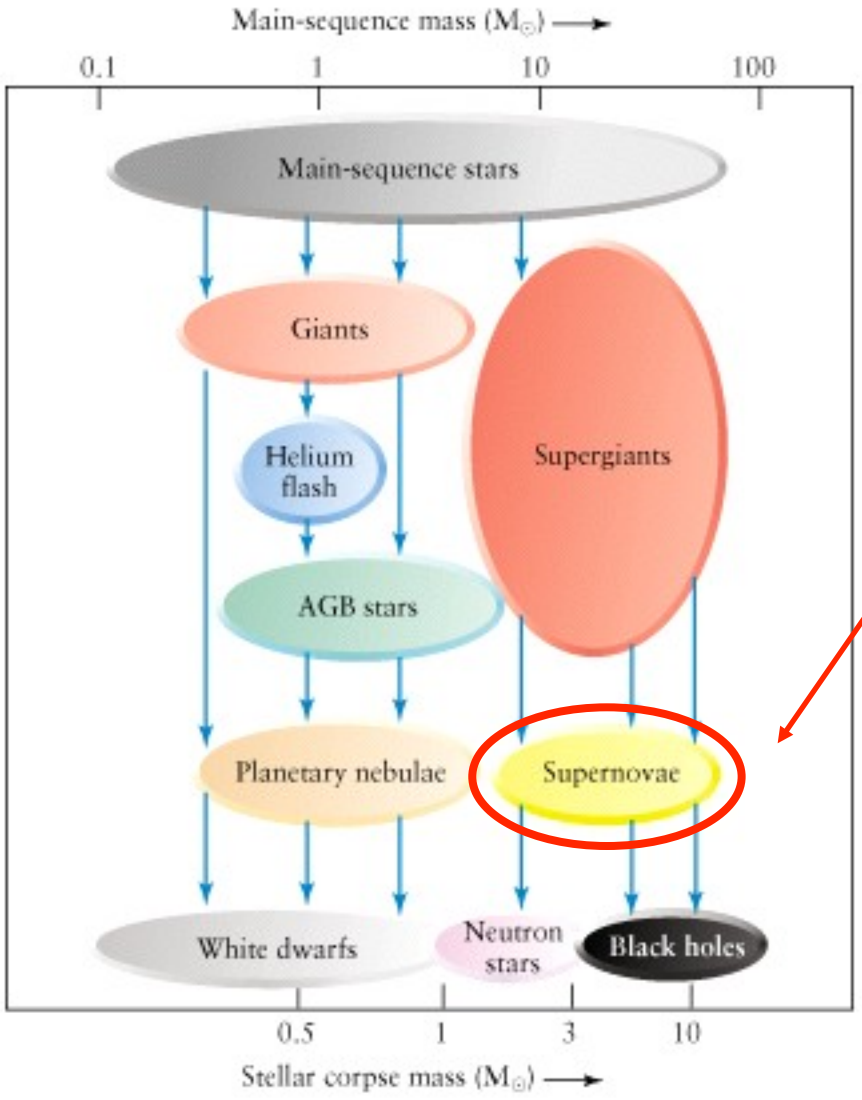
This causes the WD to explode and produce a SN1a (identified by absence of H in spectrum). Of interest for two reasons:

- Efficient production of Iron relative to the so-called α-elements (^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S etc)
- "Standard candles" for cosmology





Stellar Evolution Summary



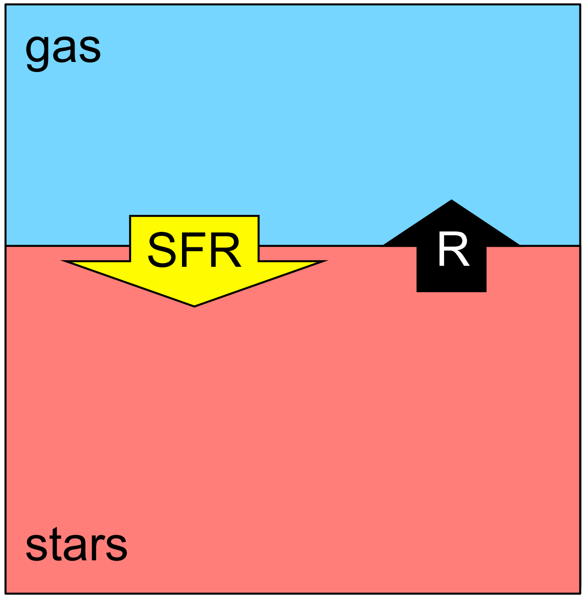
Supernovae and Life

Supernovae return chemical elements to surrounding ISM, which are then available for subsequent star/ planet formation and Life

"Chemical evolution" of the interstellar gas in galaxies

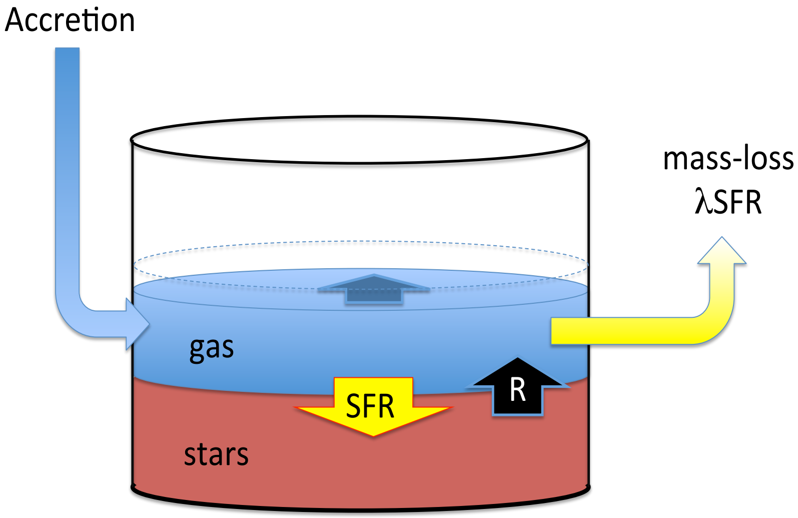
The yield y is defined as the mass of heavy elements (> ^{4}He) returned to the interstellar gas by SN per unit mass of material that is formed into a stars. For a standard mass distribution of stars, $y \sim 0.01$. Gas metallicity will be O(y).

Simple "closed box" evolution



$$\boxed{Z = -y \times \ln\left({}^{m_{gas}}\!/_{m_{tot}} \right)}$$

Flow-through evolution regulated by gas mass



$$\boxed{Z = \frac{y}{1 + \lambda(1 - R) + \left({}^{m_{gas}}\!/_{m_{star}} \right)}}$$

Part 4 – key ideas

- Fusion in stellar cores provides long-term source of energy. Required high temperatures established by gravitational collapse.
- Fusion synthesizes elements, in principle up to ^{56}Fe if temperatures are hot enough, i.e. in the most massive stars.
- Ignition of H in collapsing proto-stars is largely inevitable unless the mass is too low ($< 0.08\ M_{sun}$).
- Velocity tail + quantum tunnelling produces sharp "Gamow Peak" in energy of reactants and a strong T dependence. The latter, together with negative heat capacity, stabilizes stars.
- Interaction of well-defined energy of reaction + Gamow Peak with excitation levels in nuclei has large effect on reaction rates.
    - formation of Carbon is less impossible than you would think (3-body reaction) due to favorable excited state
    - destruction of Carbon is hindered

Chapter 4 – key ideas

- Elements above ^{56}Fe require energy to be created via neutron capture, via the *s*-process (in last stages of star) and in *r*-process in supernovae.
- Degenerate electrons (temperature independent) can support cores up to 1.4 M_{sun}. Relativistic degenerate matter is unstable to collapse.
- Collapse to neutron star releases vast amount of potential energy, and ejects enriched material into surrounding space at velocities up to 0.1*c*.
- The yield y is the mass of heavy elements (> ^{4}He) returned to the interstellar medium per unit mass of material formed into a set of stars. For standard mass distribution of stars, $y \sim 0.01$. The abundance of heavy elements in interstellar gas will be of this same order. Available to make e.g. planets and Life.