Cosmology – Part II

Origin of structure in the Universe
The future of the Universe

Initial density fluctuations

Gravitational instability causes density fluctuations to grow

Given Fourier mode has fixed comoving wavelength as Universe expands (therefore physical wavelength increases). Some physical effects (e.g. damping) will often be driven by physical wavelength.

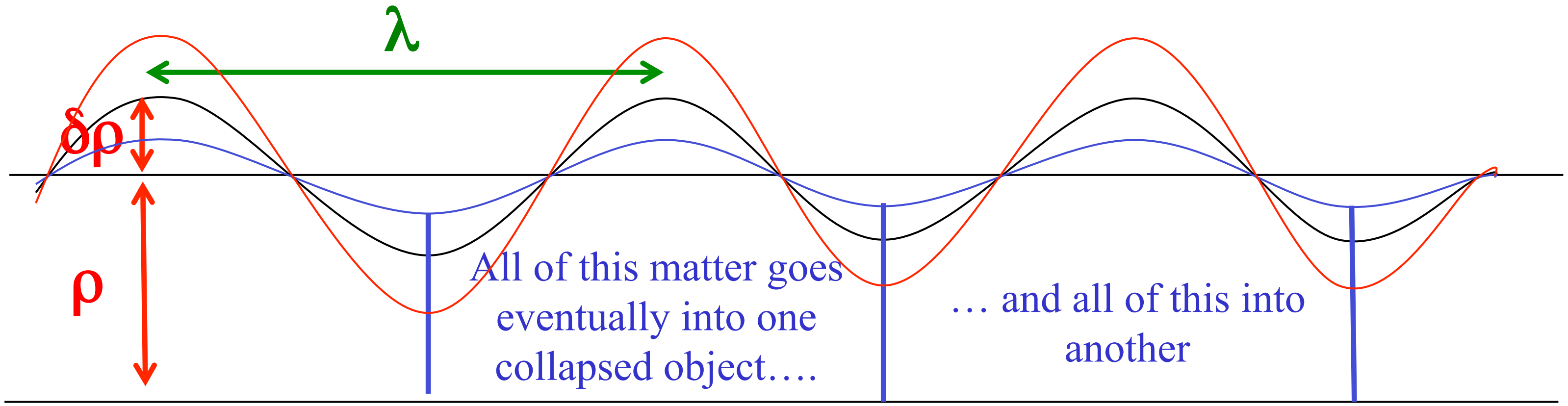
Conventional to think in terms of comoving wavenumber *k* and physical wavelength λ

$$\boxed{\lambda = R(\tau)\, 2\pi\, k^{-1}}$$

Note that the mass associated with a density perturbation of wavelength λ or wave number *k* is simply proportional to k^{-3}.

$$\boxed{M \sim \rho_m \lambda^3 \propto k^{-3}}$$

The amplitude of this density fluctuation is given by $\Delta_k = \delta\rho/\rho$.



All of this matter goes eventually into one collapsed object….

… and all of this into another

Actual density field of the Universe $\Delta(x)$ is a Fourier sum of different modes with a power spectrum described by a power-law in comoving wave number k

$$\boxed{\Delta(\bar{x}) = \frac{1}{(2\pi)^3}\int \Delta_k e^{-i\bar{k}\bar{x}} d^3\bar{k}} \qquad \boxed{\Delta_k^2 \propto k^n}$$

Some interesting values of n:

- All $n > -3$ will have homogeneity on the largest scales, as required.
- A completely random mass distribution has $n = 0$.
- $n = +4$ arises from a purely local perturbation of the matter distribution.
- The $n = +1$ spectrum has many interesting properties: Not least, at all epochs the amplitude of those fluctuations *that are on the scale of the horizon (c/H)* is the same, independent of epoch, even though the scale of the horizon changes. Can then introduce $Q = \Delta$ on the scale of the current horizon to characterize absolute amplitude. It is called the Harrison-Peebles-Zeldovich spectrum. It is naturally produced from Inflation (see next slide).

Our Universe appears to have created a primordial $n = 1$ spectrum, which was subsequently modified on small scales (see later). $Q \sim 10^{-5}$.

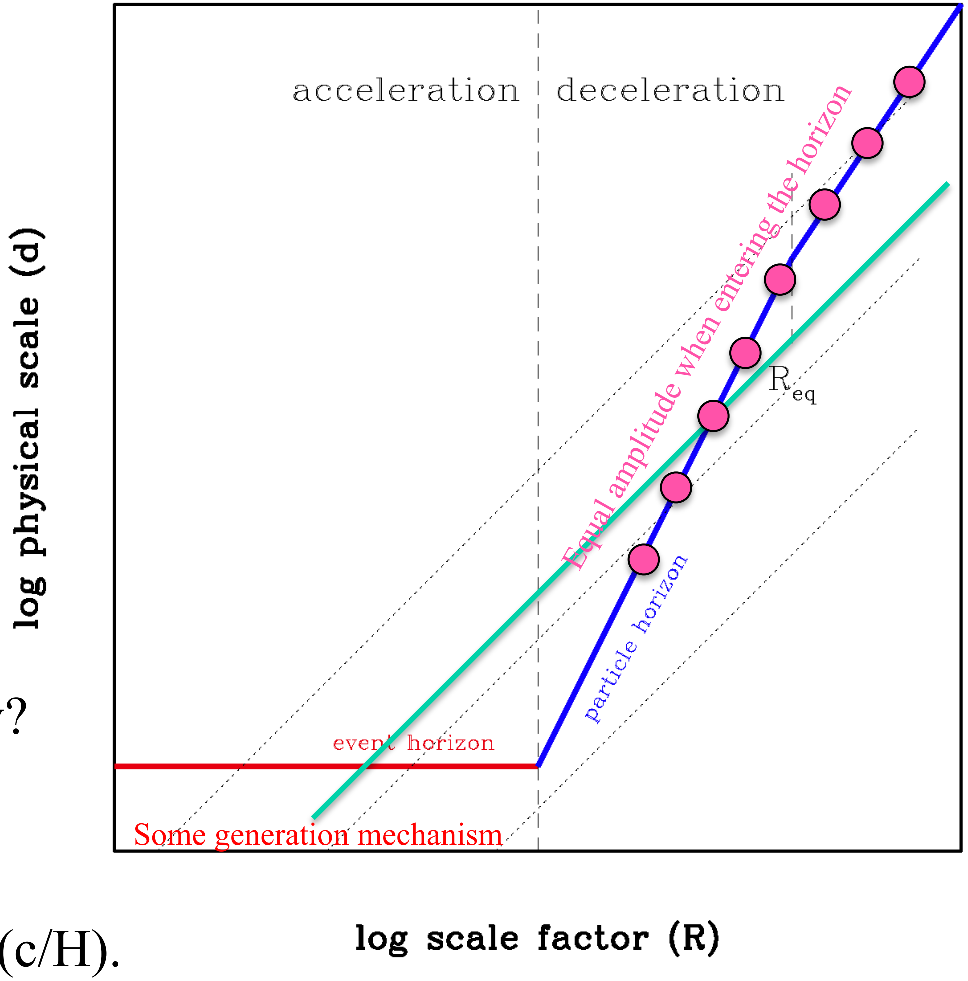
Aside for enthusiasts: the origin of n = 1 fluctuation spectrum

$$\omega_{H,particle} = \int_{\tau_1}^{now} \frac{c}{R(\tau)} d\tau$$

Particle horizon: have we received information on an event in the past? Decelerating universes have an expanding (comoving) particle horizon

$$\omega_{H,event} = \int_{now}^{\tau_2} \frac{c}{R(\tau)} d\tau$$

Event horizon: will we ever receive information on an event happening now? Accelerating universes have a contracting (comoving) event horizon



- The horizon scale is always of order (c/H).
- A given fluctuation scale may leave the horizon (through event-horizon) and then re-enter the horizon (through particle-horizon), with the same Δ_k when it does so **(why?...)**

The amplitude growth of different Fourier components

Classic analysis of Jeans (static medium) further developed by Lifshitz for an expanding medium

$$\boxed{\frac{d^2\Delta_k}{dt^2}+2\frac{\dot{R}}{R}\frac{d\Delta_k}{dt}=\left\{\underline{4\pi G\bar{\rho}-k^2c_s^2}\right\}\Delta_k}$$

All solutions depend on sign of the right hand term, i.e. whether *k* is greater or less than a critical Jeans wave-number k_J:

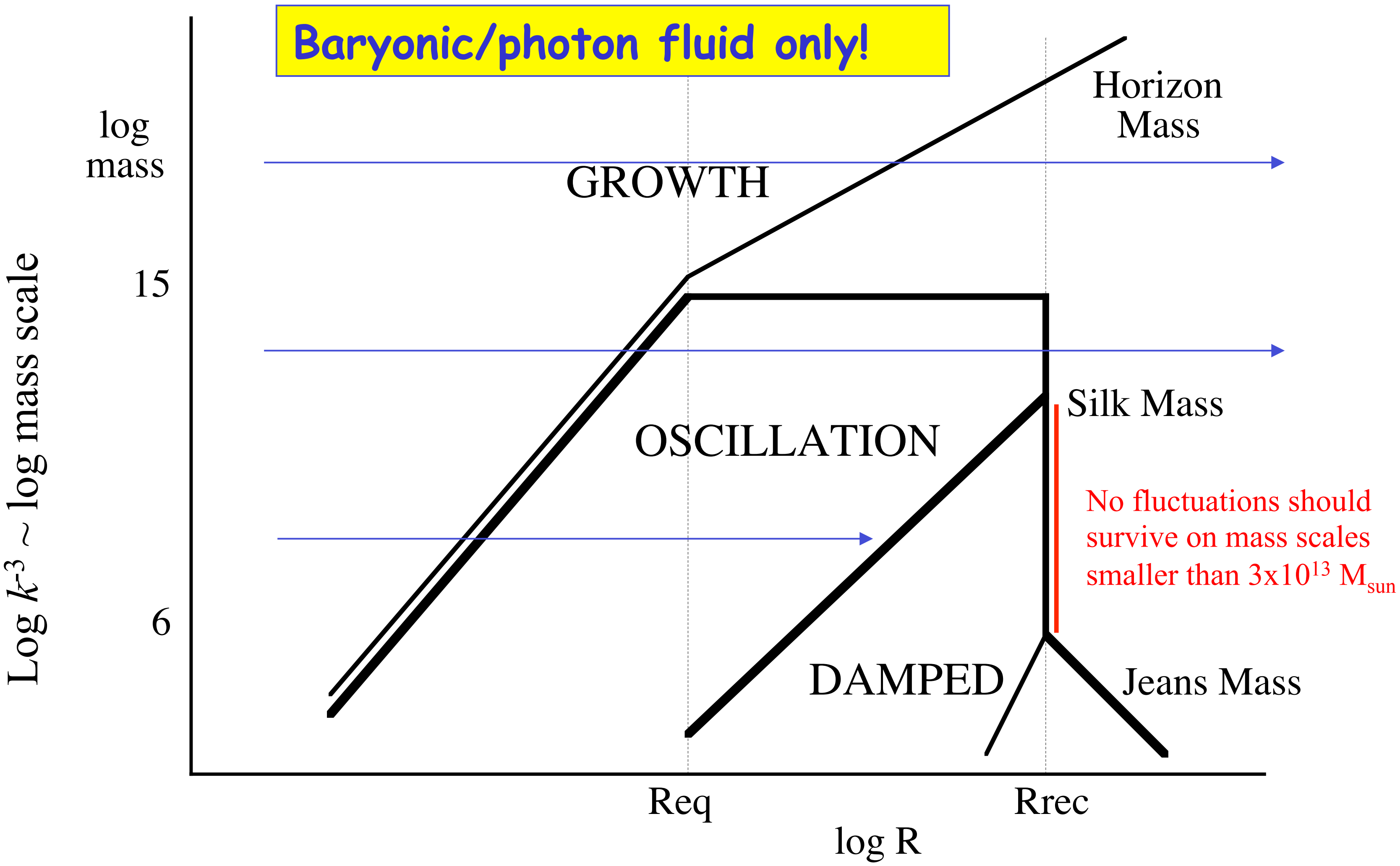
$$k_J^2=\frac{4\pi G\bar{\rho}}{c_s^2}$$

On large scales $k \ll k_J$ (i.e. $\lambda \gg \lambda_J$) we have potential for growth of Δ_k:

- the solutions will also depend on the *R*-dot term.
- *exponential* growth for static medium (with R-dot = 0)
- at best *linear* growth $\Delta \propto R$, for decelerating expanding medium and *no growth at all* if the expansion of the Universe is not significantly decelerating or is accelerating

On small scales, $\lambda \ll \lambda_{\vartheta}$, we have *oscillating* solutions since the pressure (given by c_s^2) is significant. Especially important when Universe is an ionized plasma. *When the pressure comes from the relic radiation, the Jeans length is comparable to the horizon*.

Note: Dark matter *does not interact* and therefore has zero sound speed and $\lambda_J = 0$, and it never oscillates (because there is no pressure)

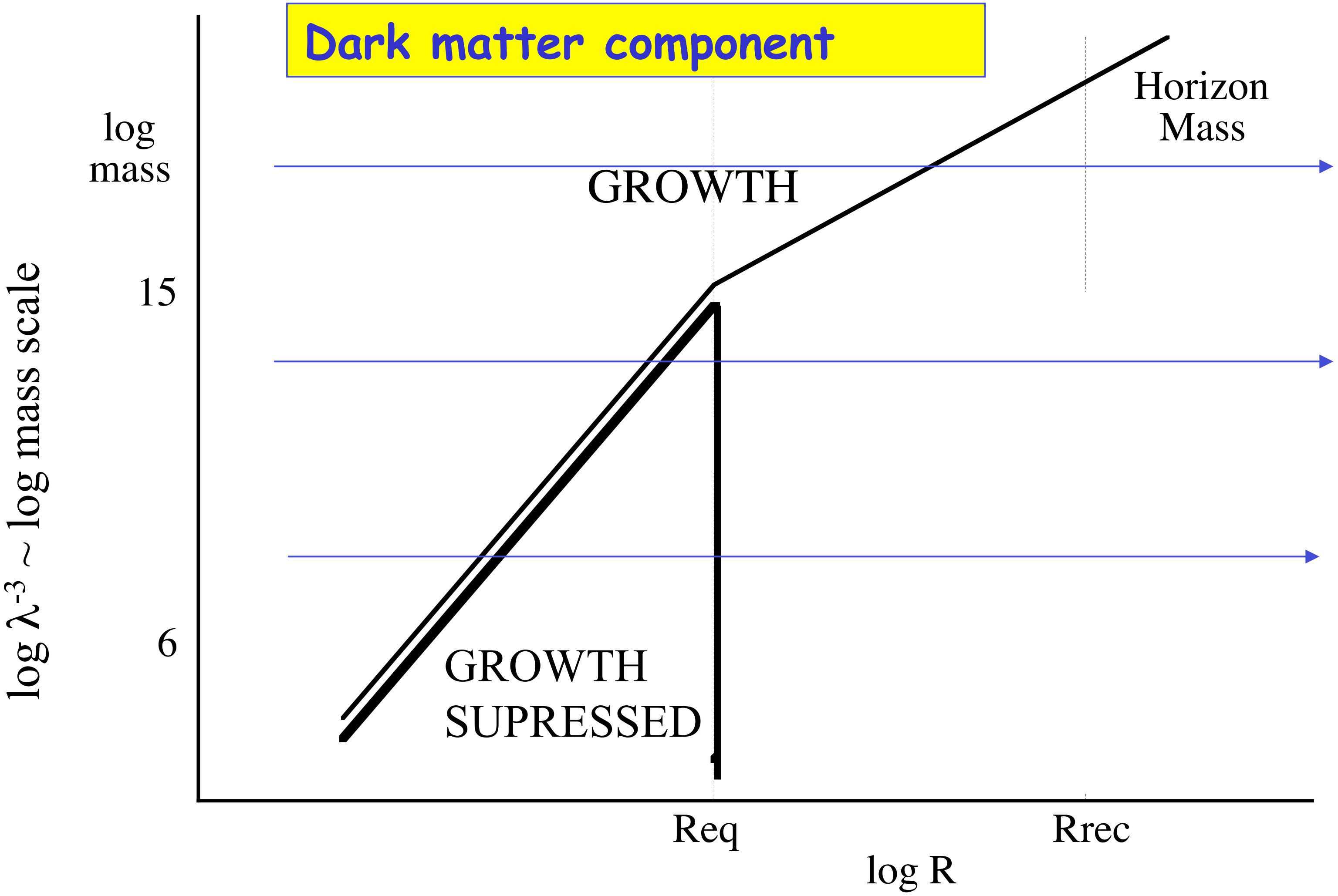


Baryonic/photon fluid only!

No fluctuations should survive on mass scales smaller than $3\text{x}10^{13}$ M_{sun}

Baryonic fluctuations will oscillate when their length scale is below the Jeans length. They may damp out (due to photon diffusion effects) on scales up to the "Silk Mass" (>> mass of galaxy). This is a potentially rather serious problem for baryon-only scenarios for galaxy formation: galaxies should not exist!

Dark matter component

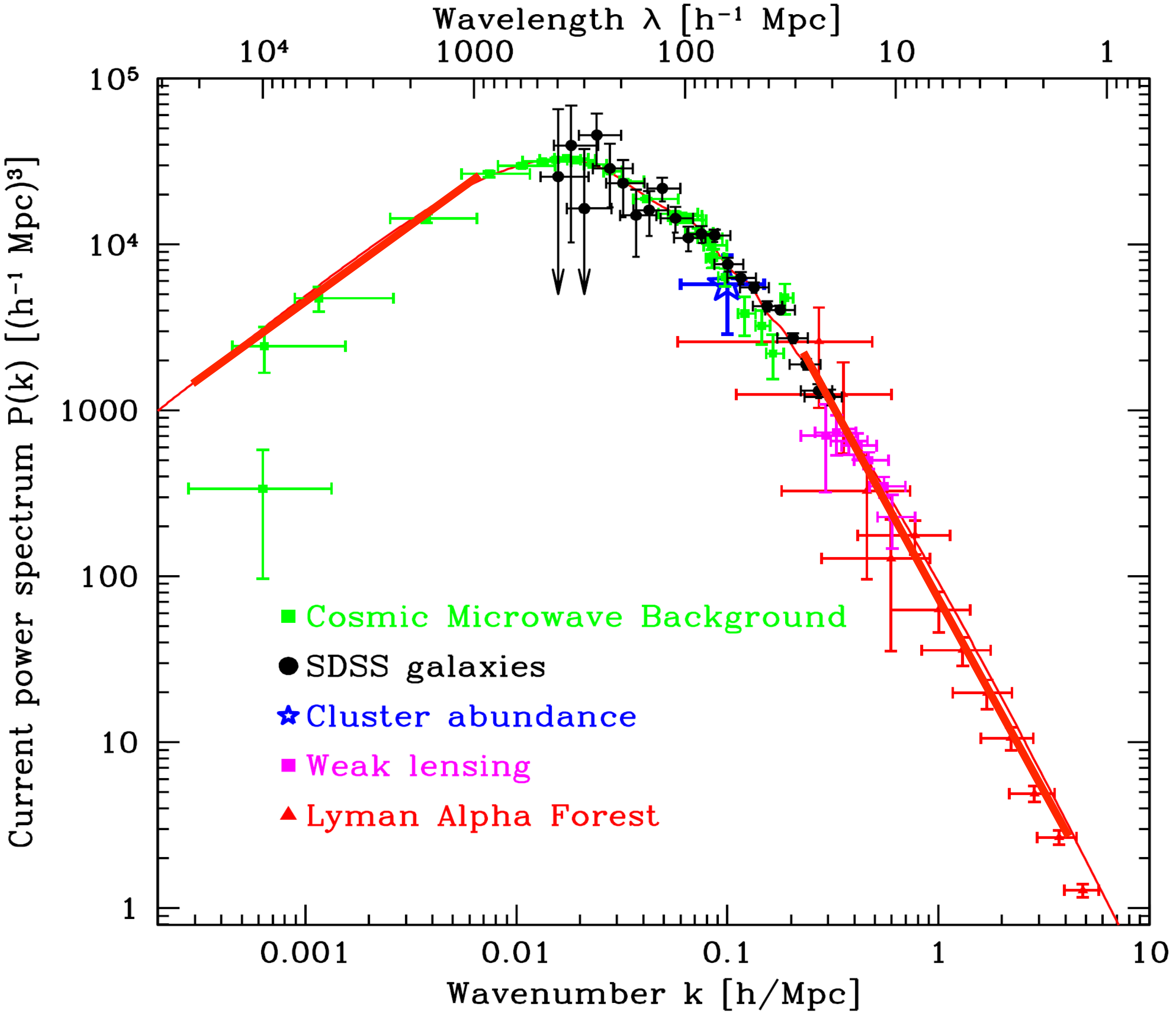


But, dark matter fluctuations are able to <u>continue growing and are not damped</u>, but their growth is infact halted below the horizon scale for as long as the Universe is still dominated by radiation. This modifies the primordial $n = 1$ spectrum on small scales.

Existence of dark matter is why fluctuations on galaxy scales "survive".

Prediction for Cold Dark Matter (CDM) spectrum matches the observed Universe well

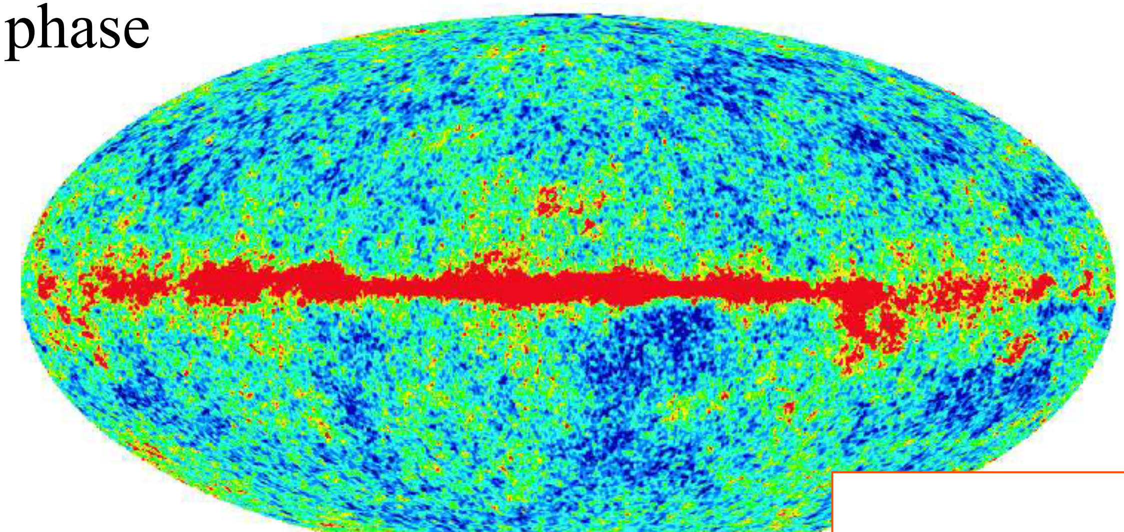
$n = +1$ on very large scales from primordial fluctuations



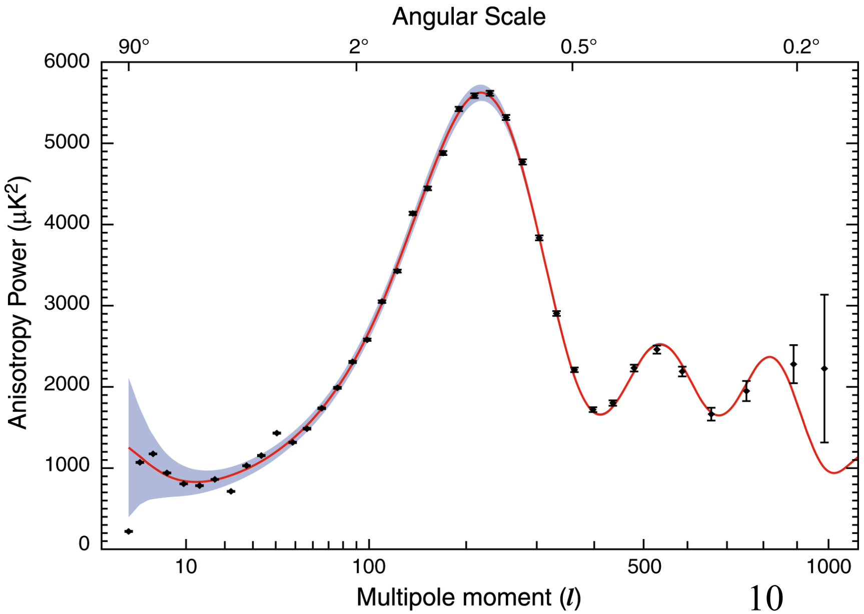
$n = -3$ on small scales due to the suppression of growth on small (sub-horizon) scales while the Universe is radiation dominated

P(k) observations

The last scattering surface of the CMB is established when the Universe recombines to neutral atoms, at the end of the oscillating phase



Detailed calculations of the expected density (temperature) fluctuations beautifully reproduces the observed angular spectrum of fluctuations on the CMB Last Scattering Surface.



Summary: origin of density fluctuations in the Universe

- An $n = +1$ primordial spectrum, probably produced by Inflation, with characteristic amplitude on horizon scale (at all epochs) of $\delta\rho/\rho \sim 10^{-5}$.

- This spectrum of fluctuations is modified on small scales to $n \sim -3$ because of the suppression of growth sub-horizon when Universe radiation dominated.

- Because the density is dominated by (non-interacting) dark matter, density fluctuations are able to survive through the sound-oscillation phase to when the Universe forms neutral atoms (and pressure turns off).

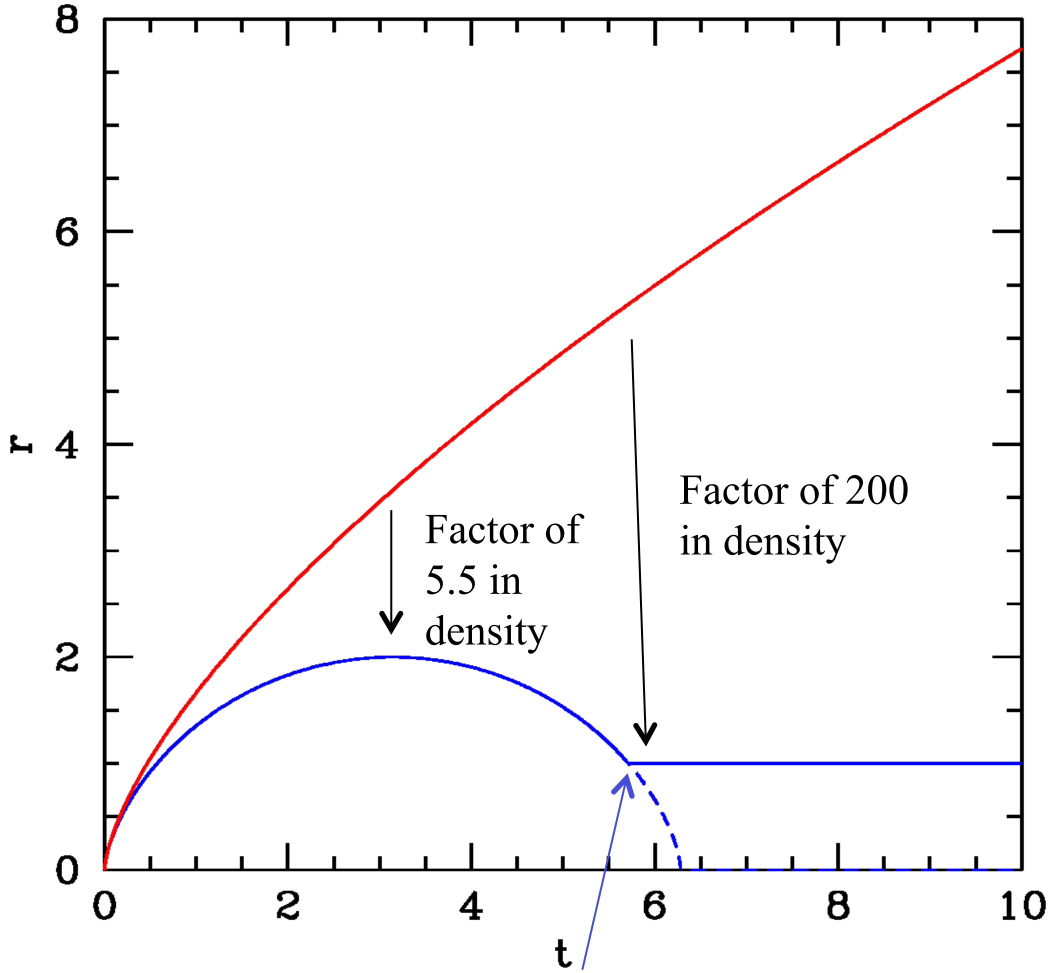
- Growth of the amplitude of fluctuations is at best linear with R, c.f. it is exponential in a static medium.

- All this is beautifully verified by observations of the CMB Last Scattering Surface that is established 380,000 years after the Big Bang.

Formation of gravitationally bound objects

- Gravitational instability continues to linearly increase the density contrast $\delta\rho/\rho$.
- As $\delta\rho/\rho$ approaches unity, non-linear collapse begins. Over-dense regions break away from the general expansion and collapse to form self-gravitating objects.

Idealized collapse of "top-hat" density distribution



Factor of 5.5 in density

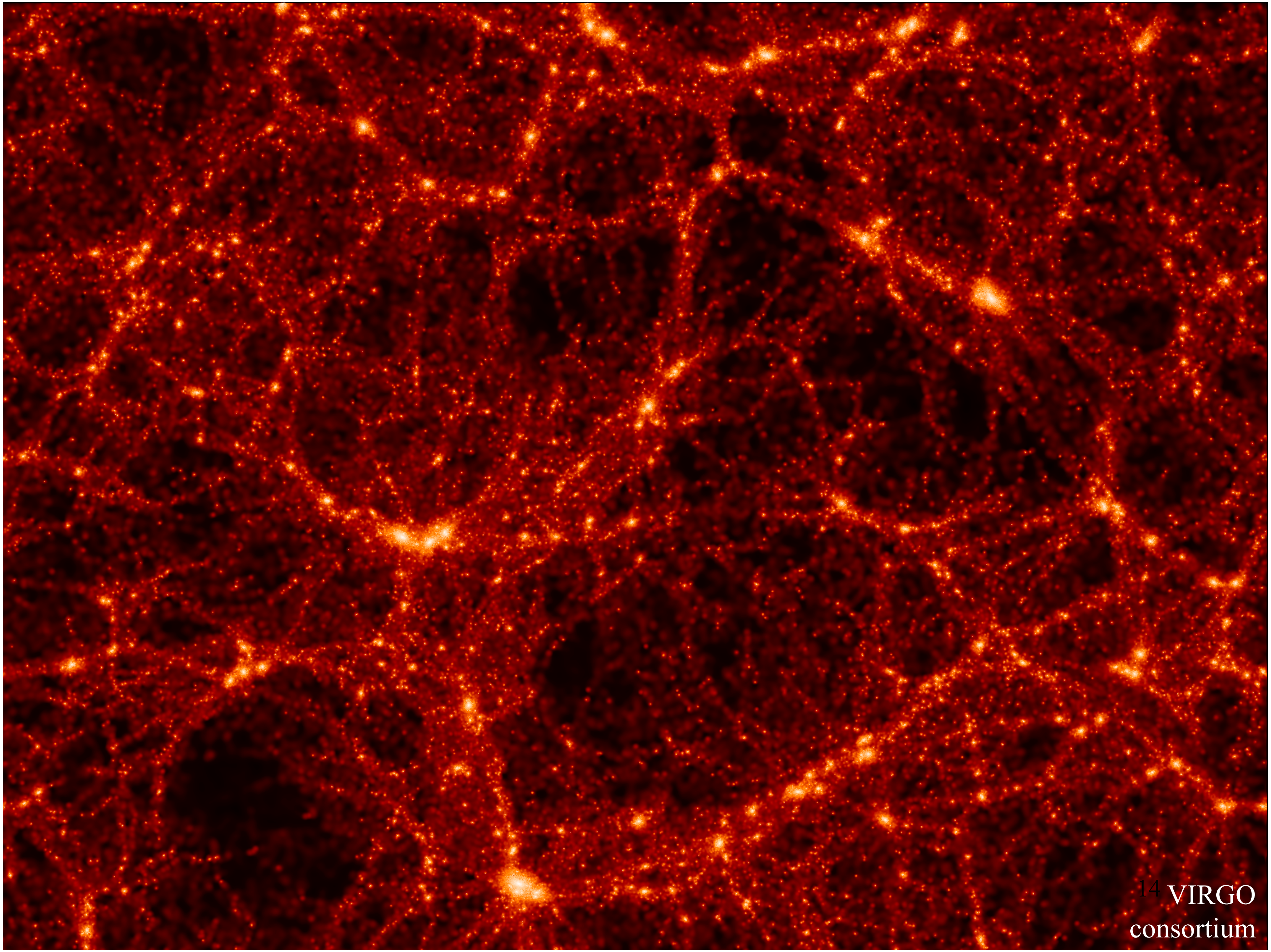
Factor of 200 in density

Virial condition stabilizes collapse after a factor of 2 decrease in size

The collapsed, self-gravitating, virialized dark matter structures are called (oddly?) dark matter halos

Gravitational collapse only produces a change of 200 in the density (at the time of collapse).

If we see a structure in the Universe with an overdensity of 200 or more, it must have virialised.



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What happens to the baryons in a dark matter halo?

Galaxies: Most baryons in stars, concentrated at the center of a dark matter halo

Larger scale structures – **groups** or **clusters:** Baryons in a hot gas distributed like dark matter (plus ~ 20% in stars in galaxies)

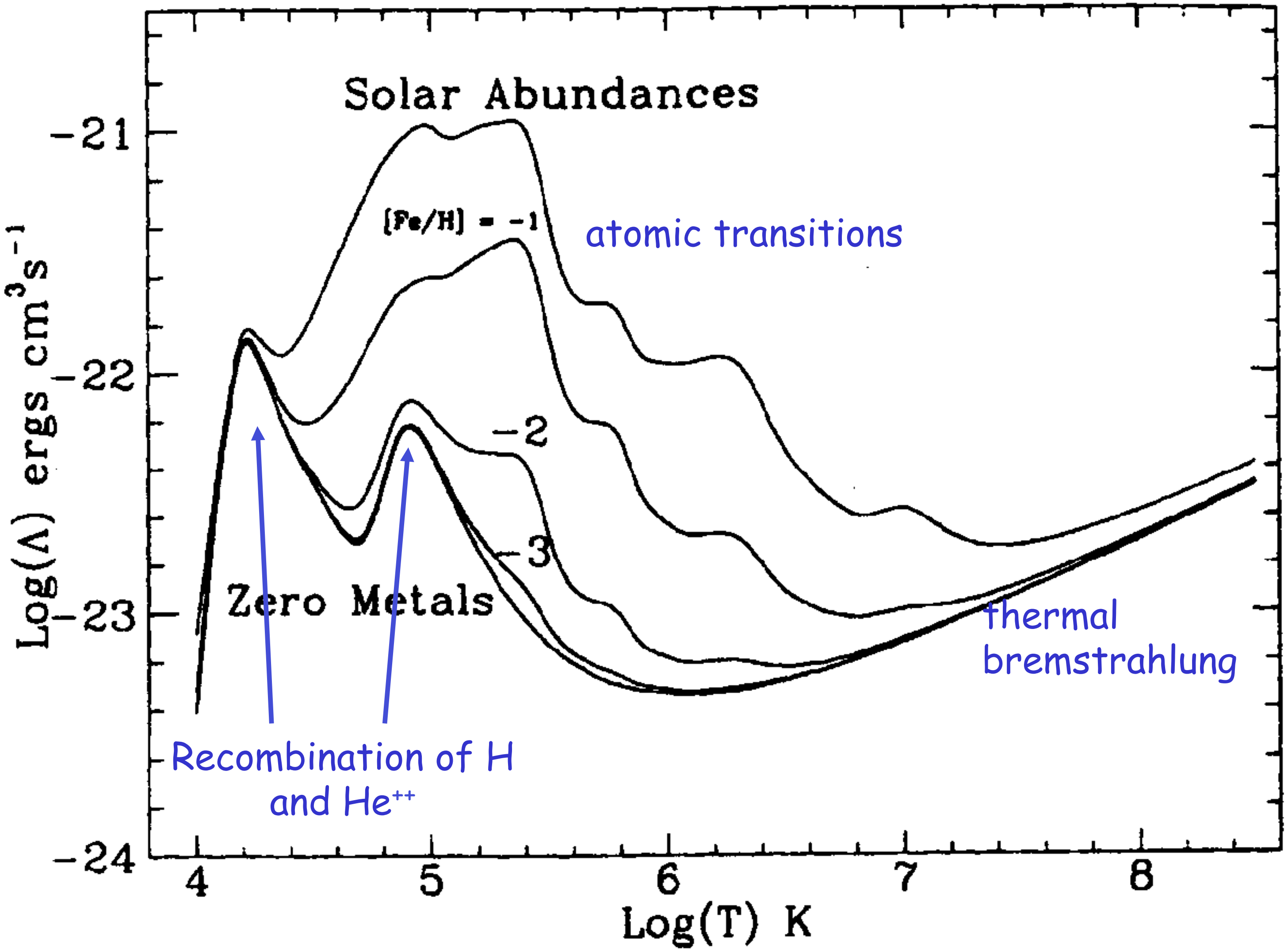
Formation of galaxies

- How does baryonic material separate from the dark matter
- Why are clusters not giant galaxies?

Key question: will the baryonic gas in a halo cool and lose energy on a short timescale?

Radiative cooling rate of a gas depends on the temperature, the atomic composition, and the square of the density n^2 **(why?)**

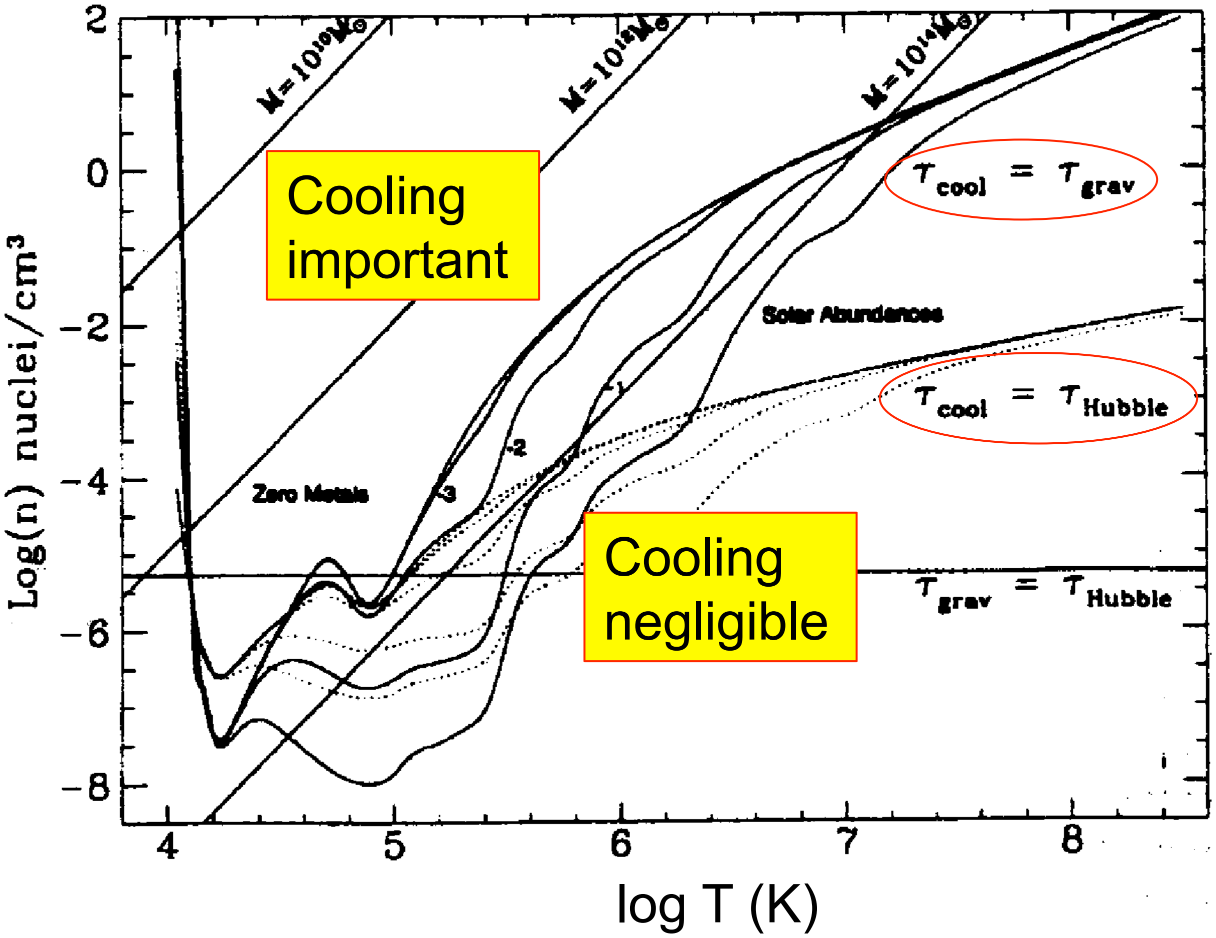
$$\frac{dE}{dT} = -n^2\Lambda(T)$$



Cooling function

Is cooling important for a gas? Comparison of t_{cool} and t_{collapse}

Gravitational collapse time also depends on density. So the relative cooling and gravitational collapse timescales depends on the number density *n* and temperature T, as well as on the metallicity (composition) of the gas



$$t_{cool} \sim \frac{E}{dE/dt} \sim \frac{3kT}{2n\Lambda(T)}$$

$$t_{collapse} \sim (4\pi G\rho)^{-1/2}$$

Remember: the density in a halo will depend on when it formed, because it is 200× cosmic density, and therefore on the relative amplitude of the initial Δ.

So, a “galaxy” is found in a DM halo in which the baryonic material was dense enough (and at the right temperature) to be able to cool effectively, losing energy and sinking towards the center of the (uncoolable) dark matter halo. The baryons increase in density and separate spatially from the dark matter.

What halts dissipation?

- Matter becomes dissipationless (e.g. forms into stars)
- *Most important*: Angular momentum rotationally supports cold disk of gas. Flat rotating disk is a natural outcome of the conservation of J in contracting system.

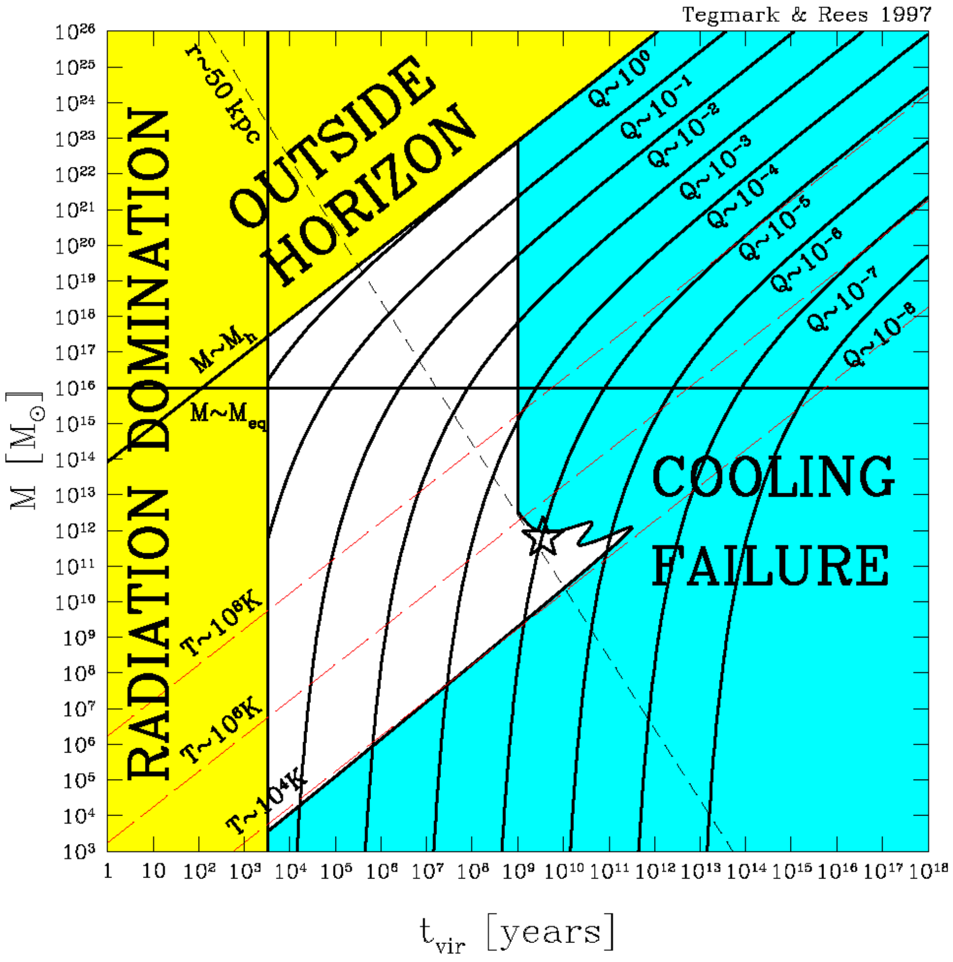
The angular momentum was acquired by the proto-galaxy through torques from surrounding (asymmetric) density field.

The typical amount of angular momentum gained gives collapse factors of ×10, i.e. density increases $\times 10^3$

The value of Q will determine when and with what density and temperature typical objects of a given (DM) mass will form.

High Q > 10^{-4} means objects form early with high density.

The density may be too high for planetary systems to survive?



Low Q < 10^{-4} means objects form late with low density and inefficient cooling

Q ~ 10^{-5} produces "galaxies" only in a fairly narrow range of mass i.e. DM halo masses around $10^9 - 10^{12}\ M_\odot$

Anthropic cosmological considerations:

A. Need to set $\Omega \sim 1$ at early epochs to high precision to produce a long-lived large Universe, i.e. to avoid either recollapse or an empty Universe ($\Omega_m \sim 0$) after small amount of expansion. "Solved" with concept of *inflation* due to a postulated inflation field of unknown origin.

B. It is best to have Q (= $\delta\rho/\rho$ on the scale of the horizon) $\sim 10^{-5 \pm 1}$

- $Q << 10^{-6}$: structure form very late with low density and baryonic cooling does not lead to dissipation and further concentration of baryons in galaxy formation
- $Q >> 10^{-4}$: structure forms early with very high densities, star-star interactions disrupt planetary systems

Q comes from amplitude of inflaton field $V(\phi)$ – presently unconstrained by any known physics

C. Need to have $\Omega_M/\Omega_\Lambda > 10^{-5}$
If Dark Energy dominated the late-epoch Universe too early, then structure will stop forming. $(1+z_\Lambda)^3 \sim \Omega_\Lambda/\Omega_M$ The value of Ω_Λ is completely not understood

D. Need baryogenesis for non-zero Ω_B (see last week discussion on Baryogenesis)

Can view these as either: (A) this is the (still largely unknown) Physics that made our Life possible in the Universe *OR* (B) perhaps this is the only type of Universe that any Life could ever observe?

Epilogue: The future of matter in the Universe

Let us assume that the geometry/equation of state is such that the Universe continues expanding *indefinitely*. What happens to the structures (complexity) in the Universe that we see at the present time $\tau \sim 10^{10}$ yr ?

A. Classical timescales (up to 10^{25} years):

1. Stellar lifetimes

End of Sun's Life (relevant for us): In 5×10^9 yr Sun will have exhausted H in core and cease to be a Main Sequence → Earth consumed in expanded Red Giant Phase. 10^9 yr later, Sun will be a White Dwarf.

Maximum lifetime of low mass stars ($0.08\ M_\odot$) is about 10^{12} yr

Star-formation rate in the Galaxy is declining ~ exponentially with $\tau_{1/2} \sim 5\times10^9$ yr.

Unlikely that there is continuing fusion after ~ **10^{13} yr**

→ **end of the "stellar epoch" in the Universe (which started few 10^8 yr after the Big Bang)**

No more fusion power from stars.

2. Disruption of planetary systems ?

Timescale for disruption of a given planetary system due to close encounter with another star, from population of number density n, moving with velocity v $\tau = (nv\sigma)^{-1}$

$$\sigma = 2\pi r^2$$

What is r? It must be of order the orbital radius of the planet.

$$n \sim 3\times10^{-41}\ \mathrm{km}^{-3}$$

$$\sigma \sim 2\times10^{16}\ \mathrm{km}^2$$

$$v \sim 50\ \mathrm{km\,s}^{-1}$$

$$\Rightarrow \quad \tau \sim 10^{15}\,\mathrm{yr}$$

On time scales of 10^{15} yr, planets are removed from their host stars (= stellar remnants, WD, NS etc) by dynamical interactions

3. Evaporation of galaxies

Random energy exchange between stars through encounters will lead to some stars being lost from galaxy ($v > v_{esc}$) while the remainder will lose energy and sink to the bottom of the potential well. This is called "evaporation".

System of N stars of mass m in self-gravitating system of radius R: Virial theorem gives

$$v^2 \sim \frac{GNm}{R}$$

What is now σ? For significant energy exchange, the passage must occur with separation at which the potential energy of the interaction is comparable to the typical kinetic energy of stars, which can be written in terms of the R and N

$$\sigma \sim \left(\frac{Gm}{v^2}\right)^2 \sim \left(\frac{R}{N}\right)^2$$

$$\tau \sim (n\sigma v)^{-1} \sim \left(\frac{NR^{-3}}{Gm}\right)^{1/2} \sim 10^{19}\ \text{yr}$$

On timescales of order 10^{19} years, galaxies evaporate: central dense star system plus planets and stellar remnants dispersed in intergalactic space

4. What about gravitational radiation?

Energy loss from orbiting objects due to gravitational radiation (as seen in binary pulsar system) is given in terms of energy, E, period P, and velocity v

$$\frac{dE}{dt} \sim \left(\frac{v}{c}\right)^5 \frac{E}{P} \quad \Rightarrow \quad \tau \sim \left(\frac{v}{c}\right)^{-5} P$$

Earth orbiting the Sun: $\tau \sim 10^{20}$ yr (longer than disruption timescale, so irrelevant)

Generally for a self-gravitating object of total mass M and radius R

$$\frac{GM}{R} \sim v^2 \sim \left(\frac{R}{P}\right)^2 \quad \Rightarrow \quad \tau \sim G^{-3} R^4 M^{-3} c^5$$

$M \sim 10^{11}\ \mathrm{M}_\odot$, $R \sim 1$ kpc, $\rightarrow$ $\tau \sim 10^{20}$ yr

Conclusion: Final state of "non-evaporated" galactic remnants must be supermassive blackholes, forming on timescales of order 10^{20} yr.

B. Greater than 10^{25} years

Decays of structures through quantum effects (!)

1. Decay of the proton:

Expected under GUT theories $t_{p.1/2} \sim 10^{33.5+}$ years (also: required for primordial baryogenesis to work)

$$p \rightarrow e^{+} + \nu + \bar{\nu} + \gamma$$
$$n \rightarrow e^{+} + e^{-} + \nu + \bar{\nu} + \gamma$$

In compact objects, almost all the rest-mass is liberated (1 GeV per baryon) through e+e- annihilation. Any free protons form positron-electron plasma

Overall number of protons will decay exponentially $N_p(\tau) = N_0 \exp(-\tau / \tau_p)$

Aside: energy released from proton decay keeps stellar remnants at significant temperatures long after they would otherwise have cooled: neutron stars @ 100K, cold white dwarfs @ 5K

Even if Life increases the "available volume" at the speed of light (i.e. if Life controls all of the particle horizon), the total number of atoms available to it decreases exponentially after ~ 1000 $t_{p,1/2}$

2. Evaporation of blackholes through Hawking radiation:

Lifetime of BH: $\tau \sim 10^{66}\left(\frac{M_{BH}}{M_{sun}}\right)^{3}$ yr

$t \sim 10^{99}$ for galactic masses, 10^{117} for supercluster masses (probably the largest that had been formed)

3. Positronium annihilation of diffuse gas:

Excited Positronium P_n formed by e^+ and e^-, on timescales of 10^{73} yrs through three-body reactions

$$e^{+} + e^{-} + e^{\pm} \Rightarrow P_{n} + e^{\pm}$$

Typically quantum level $n \sim 10^{22}$, $r \sim 10^{12}$ Mpc etc…

Decay to ground-state (and rapid annihilation thereafter) with time scale of 10^{117} yr

But what will we actually see of all this?

We'll assume that the Universe is now entering a period of exponential accelerated expansion, due to our observing $\Omega_\Lambda \sim 0.75$.

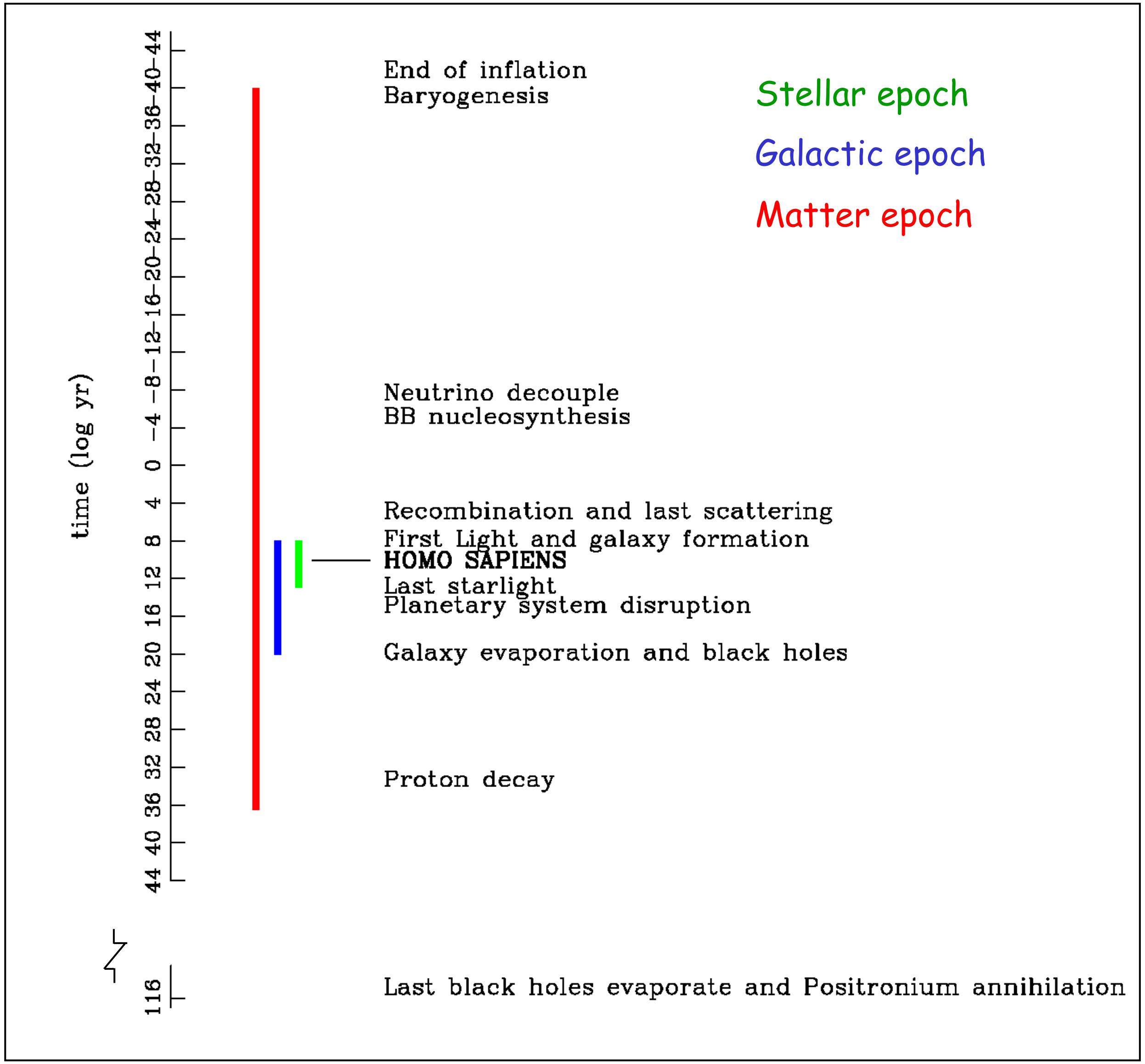
Our current particle horizon of radius $\sim c/H$ will change into an event horizon, through which matter will progressively "disappear". In other words, in *comoving* space the horizon will come towards us with an exponential timescale of $\sim H^{-1}$, i.e. 13 Gyr, as more and more of comoving space is expanded out of the horizon (i.e. redshifts tending to infinity).

The present horizon distance $c/H \sim 4300$ Mpc: Even the nearest galaxies with distances of ~ 1 Mpc will disappear from the sky after about 8 e-folds of expansion, i.e. after something like 100 billion years.

Indeed, time is running out for extragalactic astronomy!

Will the horizon ever encroach within our galaxy? That depends on $\rho_\Lambda(R)$.

If ρ_Λ increases (who knows?) and becomes comparable to the densities within gravitationally bound structures, then their dynamics will be governed by ρ_Λ and bound objects will fall apart…..



Ephemeral phases of the Universe

The last slide: The seven astrophysical processes that led to you and me

1. **Inflation** produced a long-lived Universe with $\Omega \cong 1$. It also produced density fluctuations ($Q \sim 10^{-5}$) of the right amplitude and scale to make "galaxies".
2. **Baryogenesis** produced a non-zero Baryon number, resulting in having some matter left over after matter-antimatter annihilation.
3. Dark matter enabled density fluctuations on galactic scales to survive the acoustic oscillations while the Universe still a plasma and to grow via **gravitational instability**.
4. **Cooling of the baryonic gas** then separated the baryonic from the dark matter in galaxy-scale haloes, concentrating baryons up to a density 10^6 that of the universe as a whole.
5. **Gravitational collapse** of gas clouds heats up the interiors of "stars", enabling **nuclear fusion** reactions which (a) stabilize the (shining) star against collapse for long periods and (b) produce atomic diversity up to iron, including C,O etc. The catastrophic gravitational collapse at the end of the life of a (massive) star (c) produces the remaining elements and (d) violently ejects the enriched material back into the surrounding space in supernovae.
6. **Condensation** concentrates these heavy elements onto dust grains (the 2% impurities become dominant), producing both rocky and volatile material, and producing, through further gravitational growth, dense proto-planets and ultimately planets…
   … and in these dense, warm, atomically-diverse environments, familiar chemical-based Life could develop!

Last thought: We were all there. Every atom in our body went through 1-6