Chapter 4: Stars

Part 1: Stars as sources of energy

What powers the Sun?

First idea: Is it simply gravity? (Kelvin & Helmholtz, late 1800's)

A sphere of uniform density has negative gravitational potential energy (more negative if density higher at center). Is the Sun simply converting gravitational potential energy to radiation?

$$U_g = -\frac{3}{5}\frac{GM^2}{R}$$

Any decrease in radius as the Sun shrinks will <u>liberate</u> potential energy. But 50% of this (from Virial Theorem) must go into increasing the thermal energy to maintain the gas pressure support at a higher temperature):

$$\text{Available energy} = 0.5\,U_g \sim 10^{41}\,J$$

$$L_{Sun} \sim 4\times 10^{26}\,W$$

So, we get a timescale for the Sun to maintain the observed solar luminosity:

$$t_{KH} \sim \frac{0.5\,U_g}{L_{sun}} \sim 10^{7}\,yr$$

Note that every factor of 2 in collapse releases as much energy again!

Until the radioactive dating of age of Earth rocks (~1900) this was thought to give a sufficient lifetime for the Sun.

What about "reactions" involving the material in the Sun ?

Let's do a rough order of magnitude calculation

Total number of particles in the Sun:

$$n \sim \frac{M_{sun}}{m_p} \sim 10^{57}$$

Typical energies of <u>chemical</u> reactions (involving changed in electron orbits) are of order eV (10^{-19} J) per particle, so of order 10^{38} J of energy is available from the particles in the Sun. This gives only a very short lifetime, of order only 10^4 years! This is what led Kelvin and Helmholtz to develop the idea of gravity as the (much more effective) energy source.

In contrast, the typical energies of <u>nuclear</u> reactions (changing structure of atomic nuclei) are typically MeV (10^{-13} J) per particle, so of order 10^{44} J is available. This gives a lifetime of order 10^{10} years, i.e. what we need for the age of the Solar System.

Before leaving gravity, note one interesting and important effect:

The Virial Condition for the equilibrium state of any self-gravitating** system is

$$2K + W = 0, \quad \text{i.e.} \quad E = W + K = -K = 0.5W$$

K = sum of kinetic energy = 3/2 NkT if thermal gas
W = gravitational potential energy (negative)
E = total energy
** this applies to any dynamical system dominated by it's own gravity. Kinetic energy may be due to velocities of objects (stars in a globular cluster) or gas particles (thermal pressure in a star).

Note the curious behaviour of a self-gravitating body under its own pressure support: it has negative heat capacity:

$$E = -K = -\frac{3}{2}NkT$$

$$\frac{dE}{dT} = -\frac{3}{2}Nk$$

So any self-gravitating system increases it's temperature when it loses thermal energy (and vice versa)! This is the same effect that causes an orbiting satellite to "speed up" under the retarding frictional drag forces from the atmosphere!

Fusion: Nuclear binding energies

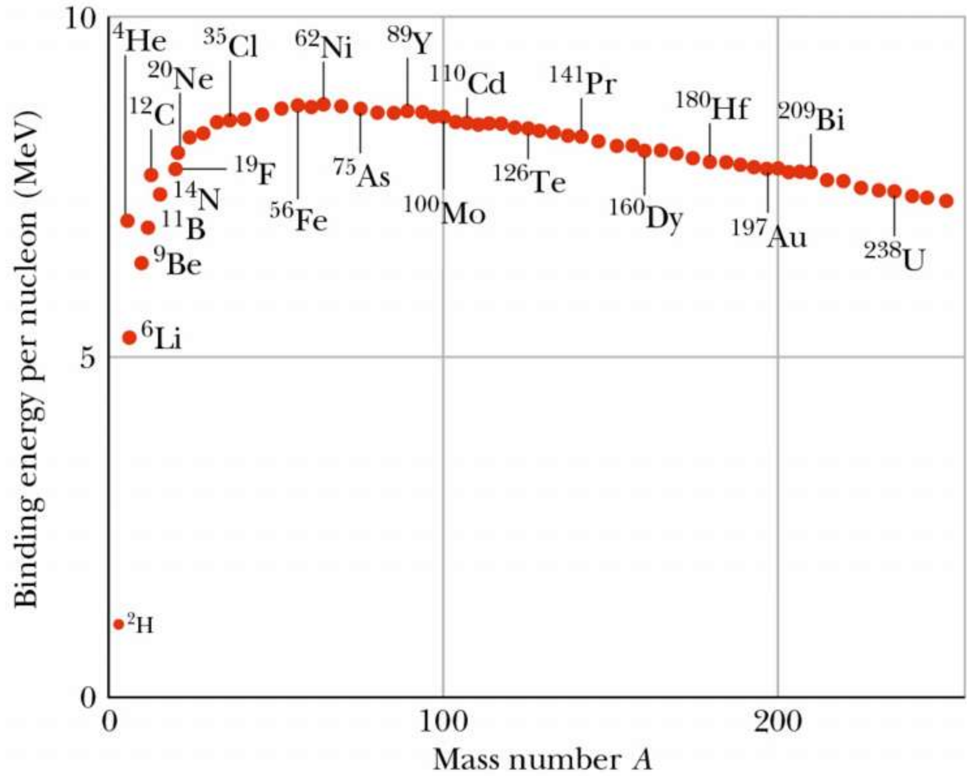
$m_p = 1.6726 \times 10^{-27}$ kg = 1.0073 a.m.u
$m_n = 1.6749 \times 10^{-27}$ kg = 1.0086 a.m.u.
$2m_p + 2m_n = 4.0318$ a.m.u.
$m_{He} = 4.0026$ a.m.u.
a.m.u.= 1.6605 $\times 10^{-27}$ kg (= 931 MeV)

A ^{4}He nucleus has a lower rest mass than the $2n + 2p$ that it is made out of. This is because of the (negative) binding energy of the *n* and *p* in the nucleus, due to the strong attraction of the "Strong Force" that holds the nucleus together against the electrostatic repulsion of the protons.

If we are able to make a ^{4}He nucleus out of $2n + 2p$, this binding energy is released. The binding energy of ^{4}He is 0.7% of the rest mass (26 MeV, or 6.5 MeV per particle).

Other more massive nuclei have even larger (negative) binding energies per particle.

Binding energy per nucleon for different elements



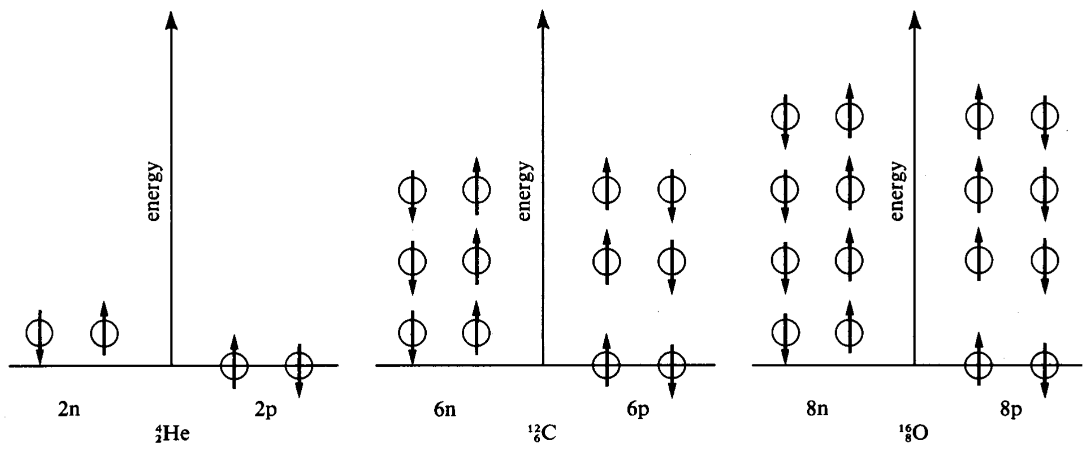
The binding energy per nucleon increases for atomic nuclei up to ^{56}Fe. So, making these nuclei out of *n*, *p*, ^{4}He or other intermediate nuclei, releases energy.

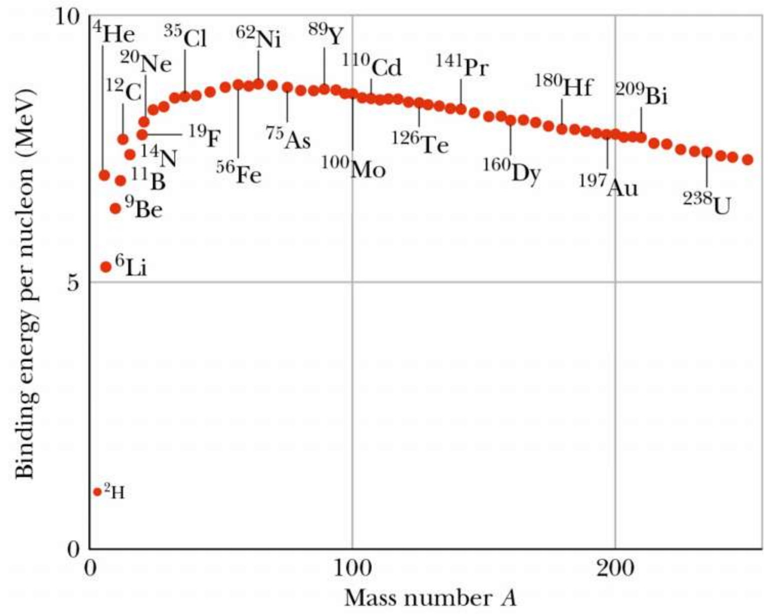
Nuclear "fusion" reactions making progressively more bound nuclei:

(a) provide the long-lived energy source in stars (Bethe 1939) and

(b) produce the diversity of chemical elements out of initial H (+^{4}He) that was produced in the Big Bang (classic paper: B2FH 1957)

Why does the binding energy curve have a peak?



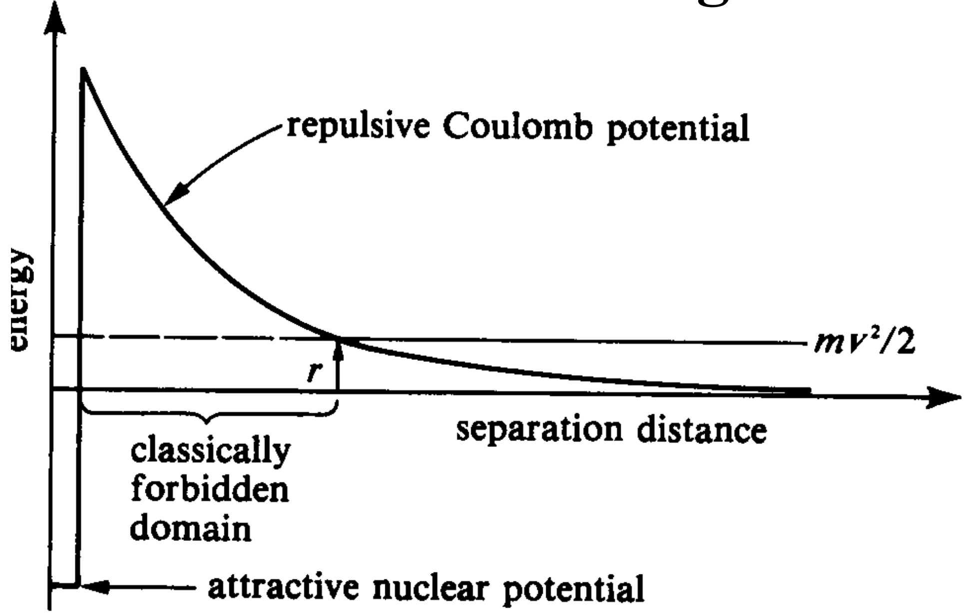


- Nuclear structure is dominated by the Pauli exclusion principle (particles cannot be in the same quantum state).
- This ensures that the minimum energy will be obtained for $n_p \sim n_n$.
- But, since the proton levels are slightly more widely spaced because of electrostatic repulsion, $n_n > n_p$ is preferred as the atomic mass increases.

- The peak (at ^{56}Fe) is then the result of the finite range of the Strong Force: the attraction between particles no longer increases with the number of nucleons once they are more than 10^{-15}m apart.
- Creation of nuclei beyond ^{56}Fe therefore *requires* energy

Now let's look at the rate of fusion reactions in a gas:

The difficulty of fusion arises from the (long-range) r^{-2} electrostatic repulsion between positively charged nuclei (Coulomb repulsion).



Electrostatic potential energy of two charges separated by distance r

$$U_e = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

We can (classically) calculate the point of closest approach if the initial velocity of approach is v

$$r_{close} = \frac{2q_1 q_2}{4\pi\varepsilon_0 m v^2}$$

Naively setting $r \sim 10^{-15}$ m (the range of the Strong Force) and $mv^2 = 3kT$ would then require $T \sim 10^{10}$K to get the particles close enough to fuse. This is about 1000 times the interior temperature of the Sun. What's wrong?

We've forgotten two crucial effects:

1. The broad distribution of velocities of the nuclei at a given T

   The velocities in the center of mass frame of two particles with reduced $m = m_1 m_2/(m_1+m_2)$ will be given by Maxwell distribution:

$$f(v)dv \propto \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

   Note that, at a given T, the number of particles at high velocity v drops *exponentially* with v^2.

2. Quantum tunneling through a potential barrier

   The probability of quantum tunneling through a distance r *is* given in terms of the de Broglie wavelength λ:

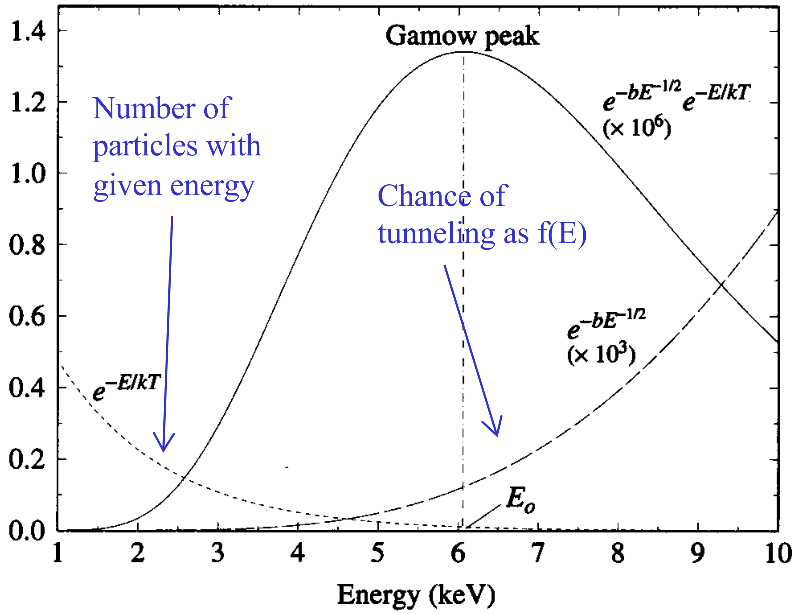
$$P \propto \exp\left(-\frac{2\pi^2 r_{close}}{\lambda}\right) \propto \exp\left(-\frac{4\pi^2 q_1 q_2}{4\pi\varepsilon_0 hv}\right)$$

   Note that the probability of quantum tunneling therefore *increases exponentially* with v as $e^{-1/v}$

The probability of a fusion reaction happening to a given pair of particles with a certain v will therefore be proportional to the product of these two competing terms

$$dN \propto n_1 n_2 \sigma \; v \exp\left(-\frac{mv^2}{2kT} - \frac{\pi q_1 q_2}{\varepsilon_0 h v} \right) dv\,dt$$

$$\propto \exp\left(-\alpha E - \beta E^{-1/2} \right) dE\,dt$$



This is a highly peaked function with a maximum $(dN/dE) = 0$ when the two terms are equal (plus a factor of 2 from differentiating)

Velocity (energy) of the peak in dN/dt

$$v_{\max} = \left(\frac{\pi q_1 q_2 k T}{\varepsilon_0 h m} \right)^{1/3}$$

And, the overall rate of reactions depends strongly on temperature:

$$dN/dT \propto \exp\left(-\frac{3}{2} \left(\frac{\pi q_1 q_2}{h \varepsilon_0} \right)^{2/3} \left(\frac{m}{kT} \right)^{1/3} \right) \propto \exp - \left(\frac{T_0}{T} \right)^{1/3}$$

$$T_0 = \left(\frac{3}{2} \right)^3 \left(\frac{\pi q_1 q_2}{h \varepsilon_0} \right)^2 \left(\frac{m}{k} \right) \sim 4 \times 10^{10}\,K$$

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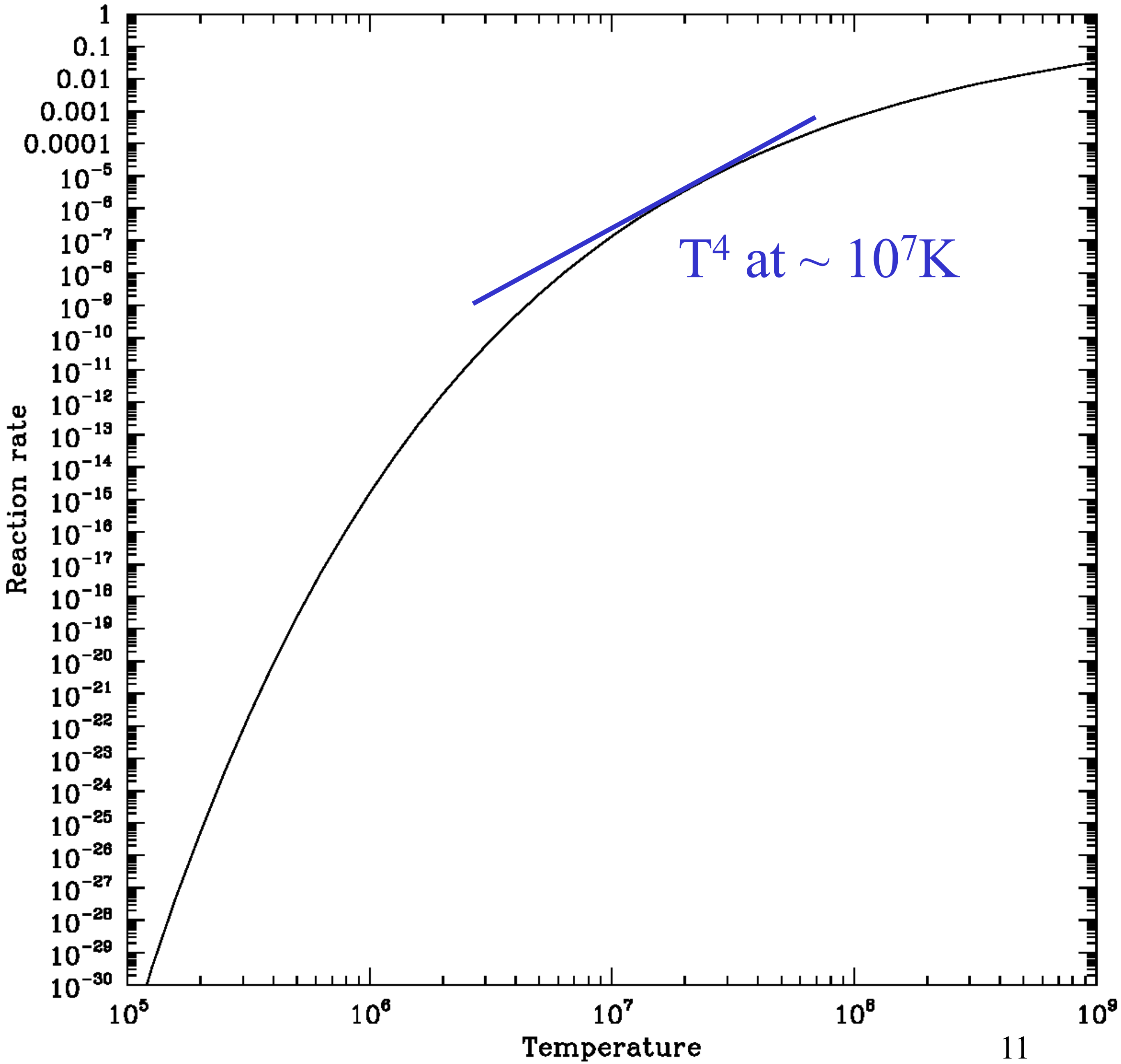
Two key points:

- most of the reactions will occur with kinetic energies close to this so-called **Gamow Peak** (see later).
- the overall rate of reactions *strongly increases* with temperature

Note: the very steep temperature dependence of the rate of fusion reactions.

This, coupled with the negative heat capacity of a self-gravitating gas, provides a natural "thermostat" to keep a star stable at more or less constant temperature for very long periods of time.

This is why stars like the Sun do not explode like H-bombs.



Fusion of more massive nuclei

- This will require higher temperatures because of larger +ve nuclear charges produce a higher Coulomb barrier

H to He	1×10^7 K
He to C, O	1×10^8 K
C to O, Ne, Na, Mg	5×10^8 K
Ne to O, Mg	1×10^8 K
O to Mg – S	2×10^8 K
Si to around Fe	3×10^9 K

- Also, the flattening of the binding energy curve *per nucleon* means that less energy is released per reaction at higher nuclear masses as we approach ^{56}Fe.

Therefore, we'd expect the basic evolution of a star to be as follows:

- gravitational collapse heats up interior as in Kelvin + negative heat capacity;
- first fuse H→He at "low temperatures" (thermostat maintains $T \sim 10^7$ K) until the H in the stellar core is all consumed;
- thermal energy loss will then raise T through negative heat capacity, i.e. from further gravitational collapse of core;
- when T rises high enough, fuse He → C at higher temperature, until most He in core is consumed (shorter time);
- gravitational collapse to raise T to the next threshold for fusion;
- fuse C → O, and so on….

We now need to ask, what sources of internal pressure can support a star against it's own self-gravity?

Some nomenclature:
X = mass fraction of H
Y = mass fraction of He
Z = mass fraction of "metals"

Available sources of pressure to resist collapse

- Normal non-degenerate thermal pressure of plasma: $P = nkT \sim (\rho kT/m_H)(2X+0.75Y+0.5Z)$
  (N.B. including the electrons)
- Radiation pressure: $P = aT^4/3$ a = Stefan-Boltzmann
- Degenerate* pressure (non-relativistic): $P \propto [0.5\ (1+X)\ \rho]^{5/3}$ independent of T
- Degenerate* pressure (relativistic): $P \propto [0.5\ (1+X)\ \rho]^{4/3}$ independent of T

And in general, but not relevant for stars:

- Atomic and molecular interactions in cold material (e.g. you and me, Earth)
- Quark and hadronic interactions
- Rotation and magnetic fields

**Degenerate pressure is produced when the Pauli Exclusion Principle forces particles into high energy states because they are being confined within very small volumes (i.e. at very high densities) – see later

Check: are the interiors of stars hot enough for fusion?
(c.f. surface temperature which is only 6000 K for the Sun)

Quite general arguments indicate yes, independent of energy source, if the pressure source is indeed normal thermal pressure

As before, we have: $$U_{grav} = -f\frac{GM^2}{R}$$ with f of order unity

Hydrostatic support at radius r requires

$$\frac{dP}{dR} = -\frac{Gm_r}{r^2}\rho \Longrightarrow \langle P \rangle_V \approx -\frac{1}{3}\frac{U_{grav}}{V}$$

And we know that thermal pressure

$$\langle P \rangle \sim \frac{\langle \rho \rangle}{\bar{m}} kT_{\text{int}}$$

Putting these together gives:

$$T_{\text{int}} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^6 K \qquad \text{for Sun}$$

i.e. if it is supported by thermal gas pressure, then the internal temperature of the Sun must be about 3 million K, i.e. sufficient to support fusion.

Important point: *Fusion occurs because the Sun is hot, not the other way around. The Sun is hot because it is a thermal-pressure-supported self-gravitating system of a certain mass and radius. Once formed with a given mass, it will stabilize at a certain radius when it is hot enough for fusion.*

So, are stars inevitable?

$$T_{int} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^6 K \qquad \text{for Sun}$$

- A proto-star formed with mass M will slowly collapse in quasi-hydrostatic equilibrium with $T_{interior}$ rising as R shrinks as it loses energy (through radiation to the exterior) because of the negative heat capacity (from the virial condition). The virial condition implies ½ of the "liberated" U_{grav} is radiated by the proto-star, the rest goes into internal heat energy to maintain pressure support.
- When $T_{interior}$ reaches the point where the rate of internal fusion can balance the radiative energy loss at the surface, a static stable structure (= "star") is formed, with constant size. The continuous radiation loses are replaced by fusion energy input in the core.
- This H-fusion ignition point will *inevitably* be reached by a protostar at some R unless another source of pressure (e.g. degenerate electron pressure, which is independent of T) can support the object before the T gets high enough for fusion.
- For the Sun, the temperature would have to rise to $>10^7$K before the density became high enough for degenerate pressure to be important, i.e. above the temperature at which H-fusion occurs. So the Sun became a star.
- But for proto-stars with $M < 0.08$ M_{sun} the temperature never gets high enough for fusion. These are called "brown dwarfs" (= failed stars). But H-fusion in all more massive objects is "inevitable" (at least with our physical constants).

Mass-luminosity(-lifetime) relation

Details need not concern us

$$T_{\text{int}} \sim \frac{GM\bar{m}}{3kR} \sim 3\times10^{6}\,K \qquad \text{for Sun}$$

Again, very general arguments indicate that the surface luminosity of a star will be related to it's mass as:

$$L = (4\pi)^2 \frac{4}{3^4} \frac{caG^4}{k^4} \frac{1}{\kappa} M^3$$

a Stefan(-Boltzmann) constant

k Boltzmann constant

κ opacity of the gas

Done properly, this ends up as $L \propto M^{3.5}$

Main point: since the H fuel supply will be roughly proportional to the mass of the star, it implies that the lifetime of a star in the stable H→ He fusion phase (= "Main Sequence" star) will be a strong function of mass.

$$t_{life} \propto \frac{L}{M} \propto M^{2.5}$$

0.1 M⊙ star	lifetime ~ 2.10[12] year
Our Sun	lifetime ~ 10 billion years
25 M⊙ star	lifetime ~ 7 million years

Basic point: self-gravity vs. thermodynamics

- Ordinary stars (like our Sun) are self-gravitating and therefore have to be hot inside to sustain the thermal pressure that balances the inward pull of their self-gravity.

- The surrounding space, however, is typically much colder and energy therefore flows continuously from the star to the surroundings.

- No thermodynamic equilibrium is possible under this circumstances: losing energy, the star will be forced to slowly contract, liberating gravitational potential energy, and to get hotter and hotter.

- Thermonuclear production of replacement energy keeps the star stable at ~ constant size, for some periods of time at the various temperature thresholds associated with successive fusion reactions. When the star runs out of a particular fuel a major structural re-assessment has to happen through further contraction.

This produces a managerie of oddly named beasts….

Brown Dwarfs

- A brown dwarf is a self-gravitating gaseous object composed mainly of hydrogen and helium, whose mass is too small to ignite stable thermonuclear hydrogen fusion in its interior (but they do produce some Helium isotopes by fusing protons with Deuterium ^2H, Lithium and Berillium).

- Theoretical expectations: $M < 0.08\ M_\odot$.

- Incapable of generating substantial amounts of nuclear energy through fusion, the gravitational contraction of a brown dwarf continues until the pressure of the degenerate electrons in its core halts the collapse at an interior temperature that is below the fusion temperature of H.

- Young Brown Dwarfs can release substantial amounts of gravitational energy (comparable to protostars and low mass stars) through Kelvin-Helmhotz contraction, but older Brown Dwarfs, once supported by degenerate electron pressure, simply radiate their remnant internal heat only. The effective radiative lifetime for a Brown Dwarf is short, i.e. only of order the Kelvin-Helmholtz lifetime of 10^7 years (c.f. the 10^{12} years for stars just above $0.08\ M_\odot$).

Main-sequence stars

Objects with $M > 0.08\ M_\odot$ ignite stable thermonuclear reactions in their centres and become "stars". In the initial phase of their lives they are characterized by chemical homogeneity and hydrogen burning <u>in their cores</u>.

Stars with $M < 2\ M_\odot$ fuse Hydrogen into Helium by the so-called "p-p chain", while more massive stars do it by the "CNO cycle" (see next slides)

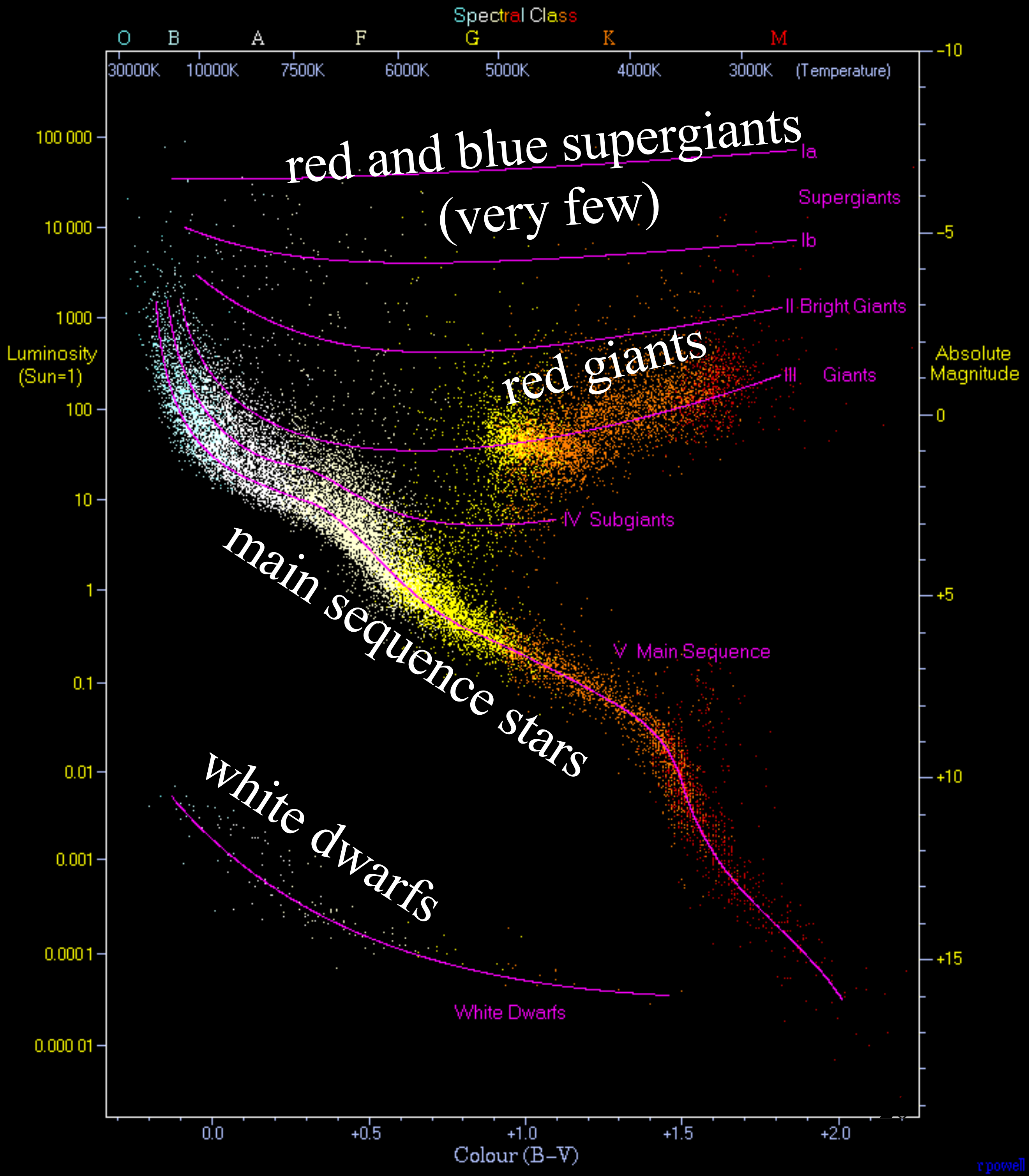
In the luminosity-temperature (Hertzsprung-Russell) diagram Main Sequence stars populate an approximately one-dimensional locus along which positions are mainly determined by the <u>stellar mass</u>. *Vogt-Russell Theorem: The mass (and chemical composition) of a star uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution over time.*

The observed scaling relations for Main Sequence stars are well matched by (detailed) theoretical models: Rough scaling relations are as follows

$$L \propto M^{3.5} \qquad t_{\rm life} \propto \mathrm{M}^{-2.5} \qquad R \propto M \qquad T_{\rm surf} \propto M^{0.5}$$

The Hertzsprung-Russell diagram

luminosity vs. surface temperature



Reaction pathways for H $\rightarrow$ ^{4}He fusion

(N.B. must include a weak interaction to convert p^+ to n)

Extremely slow due to weak interaction (p→n)
$\tau \sim 10^{10}$ yr in Sun

p-p chain

$^1H + {}^1H \rightarrow {}^2H + e^- + \nu$

$^2H + {}^1H \rightarrow {}^3He + \gamma$

$^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$

Slow, but less so than above due to spontaneous decay of unstable nuclei

^{13}N ($\tau_{1/2} \sim$ 600s) and
^{15}O ($\tau_{1/2} \sim$ 120s)

But, requires a higher *T* because of the higher +ve charge on C, N nuclei

CNO cycle

$^{12}C + {}^1H \rightarrow {}^{13}N + \gamma$

$^{13}N \rightarrow {}^{13}C + e- + \nu$

$^{13}C + {}^1H \rightarrow {}^{14}N + \gamma$

$^{14}N + {}^1H \rightarrow {}^{15}O + \gamma$

$^{15}O \rightarrow {}^{15}N + e^- + \nu$

$^{15}N + {}^1H \rightarrow {}^{12}C + {}^4He$

Later stages of stellar evolution: What happens when Hydrogen fuel is exhausted in the core?

First, we'll have an inert He core, surrounded by a shell that is still burning H to He. This changes the structure of the star: $\times$ 1000+ increase in L, with 100x increase in size (with therefore decrease in surface temperature)

→ **Red Giants**

→ N.B. Sun will consume Earth in outer envelope when it becomes a Red Giant.

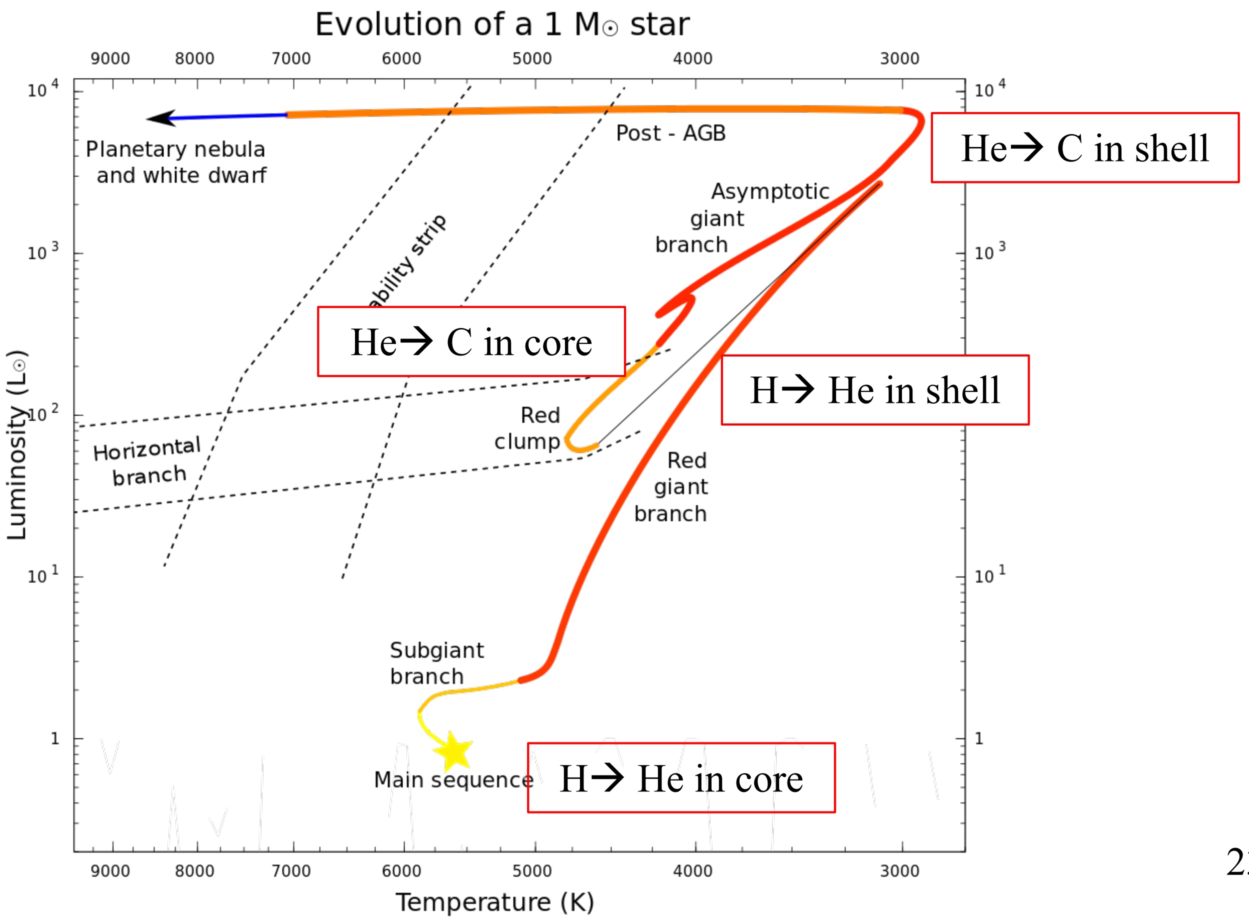
Note that stars with $M < 0.4\ M_{\odot}$ will not become red giants because they do not ever become hot enough for He burning (before support from degenerate electrons), but this has never happened (so far) because their Main Sequence lifetimes are *much longer* than the present age of the Universe.

For massive stars, subsequent temperature rise in the core may be enough to ignite He to C fusion in the core → **Horizontal Branch Star**

Followed by He shell burning and so on → **Asymptotic Giant Branch Star**

If the stellar core ever gets dense enough to be supported by degenerate electron pressure (which remember is independent of temperature), then the core will cease contracting and stop increasing its temperature.

Therefore no new fusion reactions will occur. As example, our Sun will never fuse C → O because it will stabilize its core while still below the temperature required to fuse C → O.



"White Dwarfs" (from intermediate mass stars)

When the $^4He \rightarrow C$ "triple alpha" process in a $M < 4\ M_{\odot g}$Red Giant star is complete, the stellar core starts contracting. The temperature is too low to ignite the carbon fusion process, thus the collapse keeps going until it is halted by the pressure arising from electron degeneracy.

The star sheds its extended envelope revealing the the very hot and small (relative to the Sun) central core, which is called a White Dwarf, which illuminates the surrounding envelope (making a so-called "Planetary Nebula" – probably nothing to do with planets!).

Typical white dwarfs:

- $M \sim 1\ M_\odot$ (on average)
- $R \sim$ Earth
- $L \sim 4\%$ of solar luminosity
- $T_{surf} \sim 25{,}000$ K

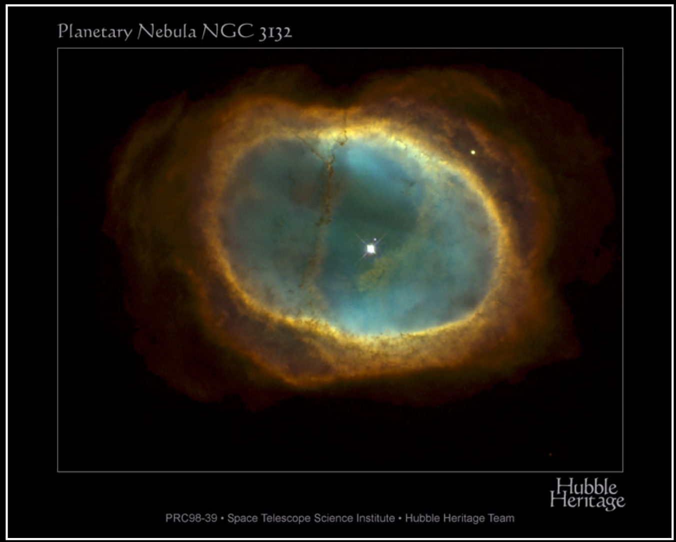
What about Life?

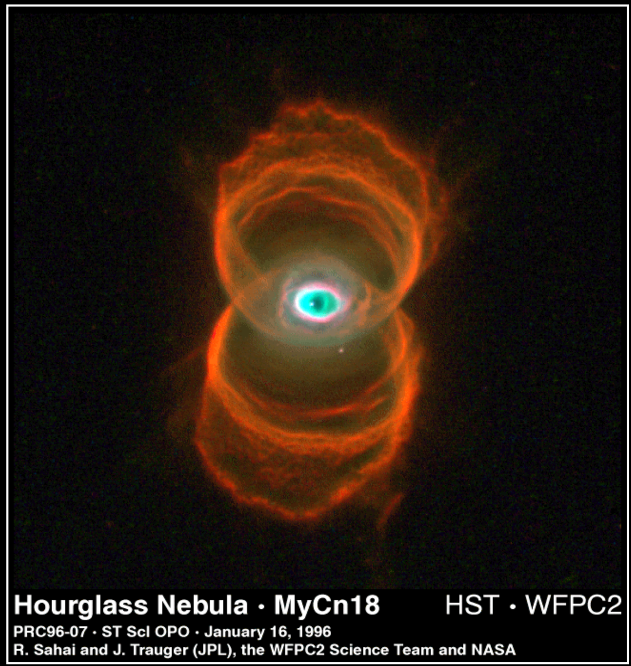
Good news: Because they are so small (and therefore have a very low luminosity) they will shine for a very long time, even though they are no longer producing any energy from fusion.

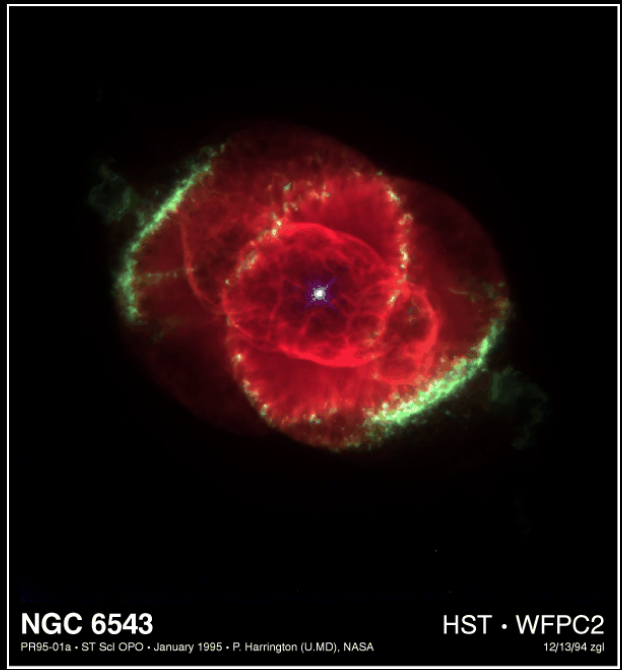
Dyson spheres??

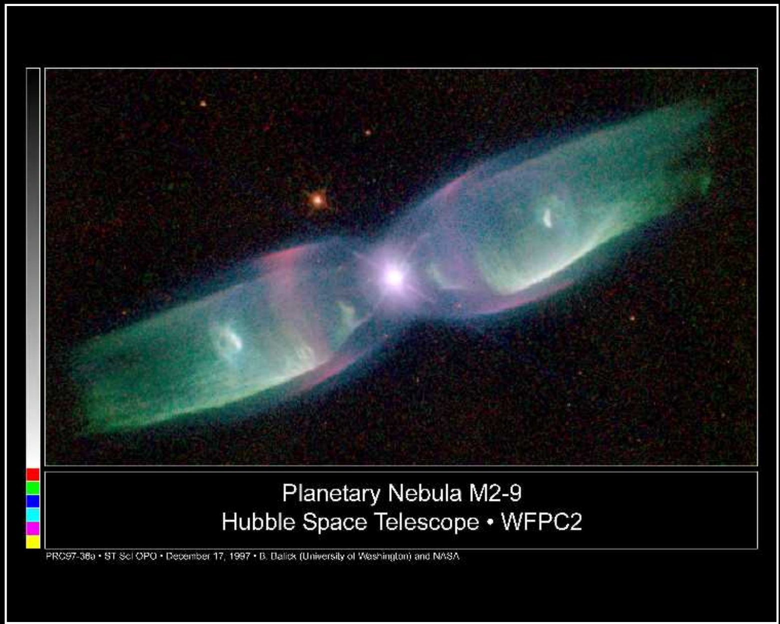
Bad news: All the products of fusion are still locked up in the interiors. Ejected material is largely unprocessed.

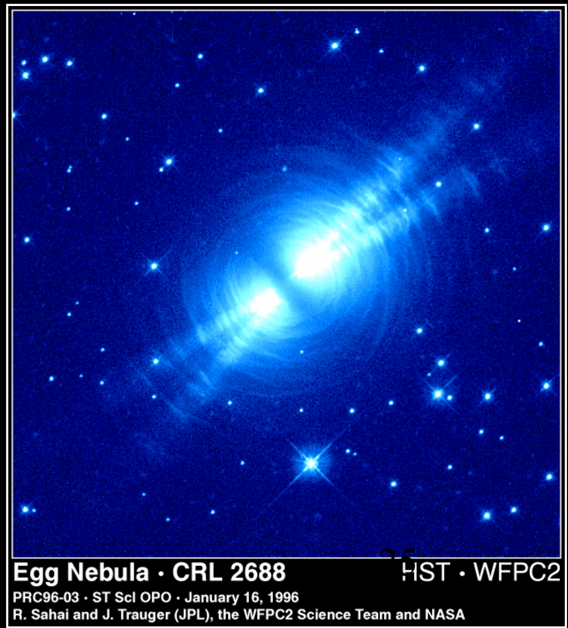
Young White Dwarfs at centers of Planetary Nebulae (nothing to do with planets)

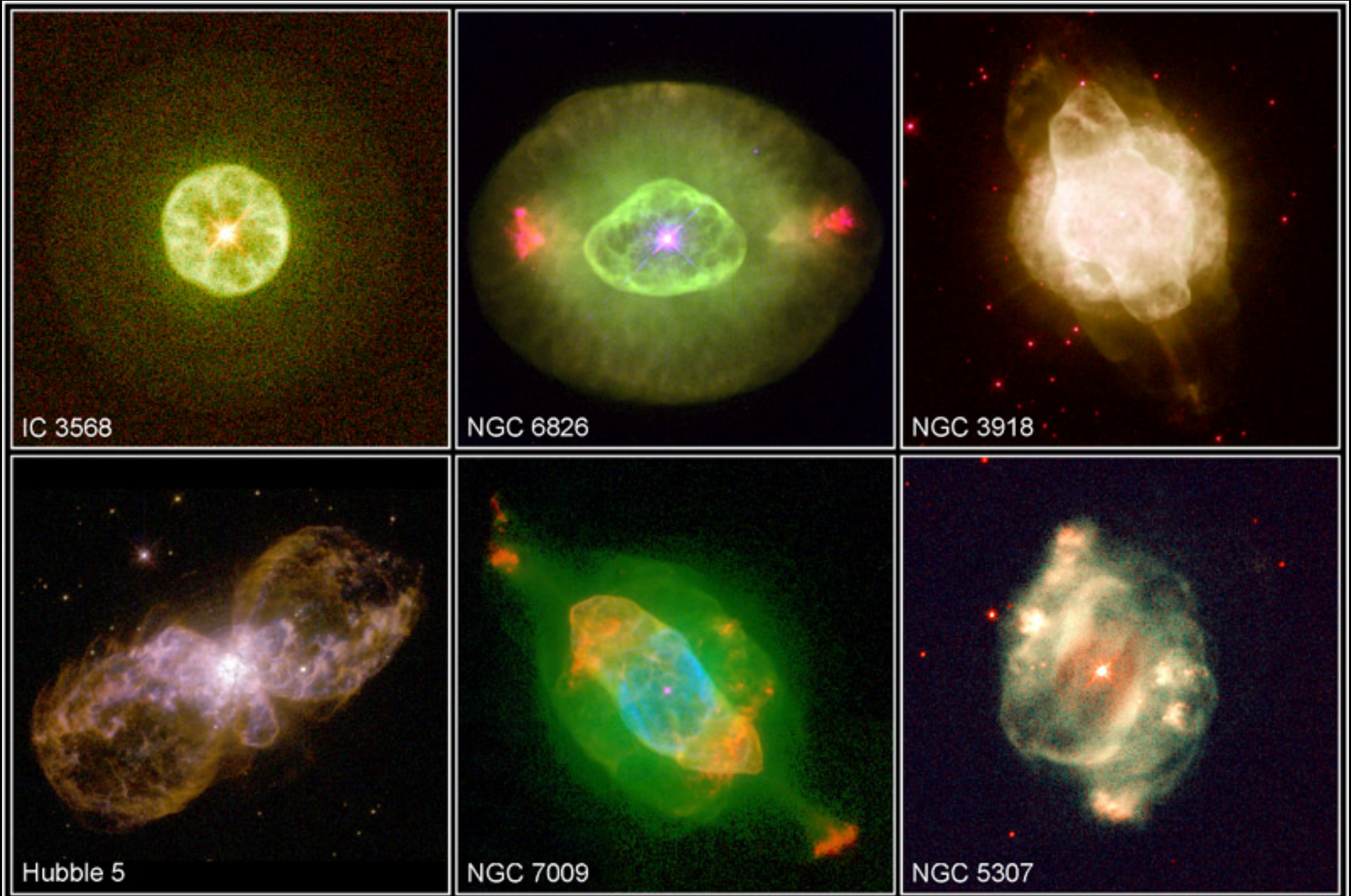












Planetary Nebula Gallery — HST • WFPC2

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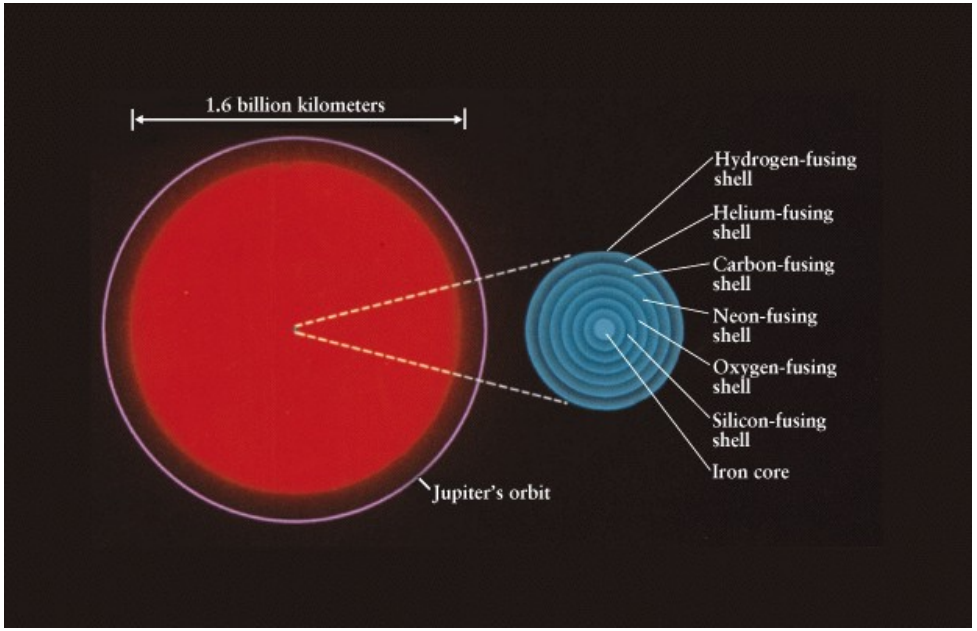
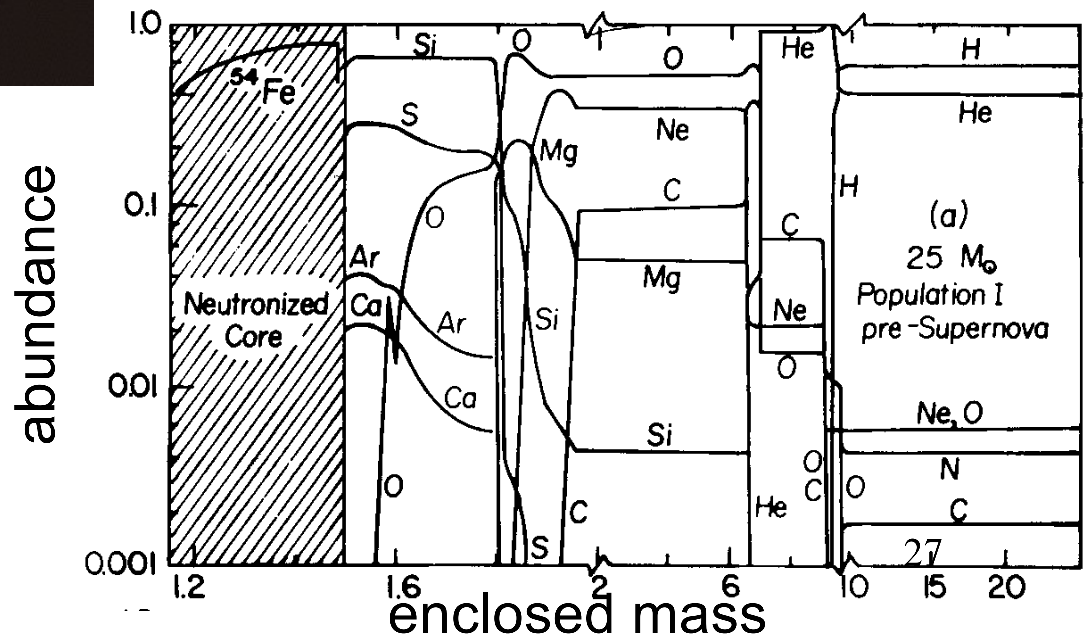
H. Bond (ST Scl), B. Balick (University of Washington) and NASA

The most massive stars…

But, in more massive stars, the temperatures are high enough for higher fusion reactions, possibly all the way up to having an Fe-rich core.



Onion-like core structure with successively cooler shells burning earlier fusion reactions



Final structure of a 25 M_{sun} star

But time is running out…

The energy per fusion reaction is dropping as the masses of nuclei increase (because of the flattening of the binding energy curve), but the required core temperature is increasing, and therefore the surface luminosity (= energy loss rate), is also increasing, so the required rate of energy production in the core is also greater.

For a 25 $M_{\odot}$ star:

- Hydrogen burns in the core for 7 million years
- Helium burns in the core in the core for 500,000 years
- Carbon burns in the core for 600 years
- Oxygen burns in the core for 6 months
- Silicon burns in the core for only 1 day

So, what happens next?
We'll return to this in Part 3

Key ideas from Part 1

- Hot stars are inevitable in the Universe from gravitational collapse, gravitational potential energy and the virial condition.

- Fusion is also inevitable (for objects with $M > 0.08\ M_{\odot}$ assuming our "physics") because the interior temperature will eventually get hot enough for fusion of H.

- Fusion will produce a long-lived star at (roughly) constant size for as long as there is fuel. The negative heat capacity stabilizes the star and prevents an explosion.

- Timescales and outcomes of stellar evolution will depend on the mass of the star – an interplay of temperature, fuel, sources of pressure etc.

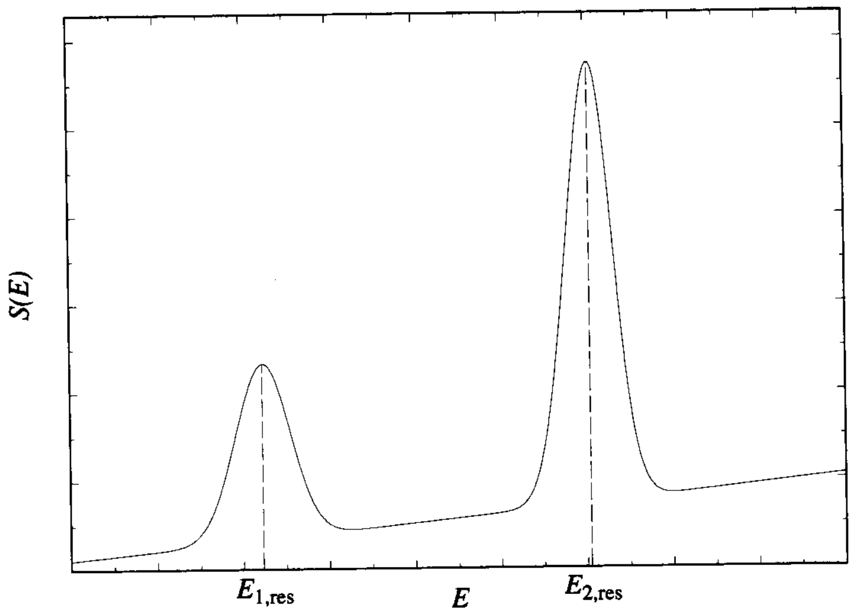
Part 2: The interesting case of Carbon

Back to that Gamow Peak and the problem of Carbon

Look again at what actually happens in a fusion reaction. The collision between the two nuclei produces an excited combined nucleus, whose excitation energy is set by the sum of the Δmc^2 binding energy of the fusion reaction in question plus the K.E. of the reacting particles. This excited state then de-excites, usually emitting a γ-ray, or other particle(s), which carry the energy away.

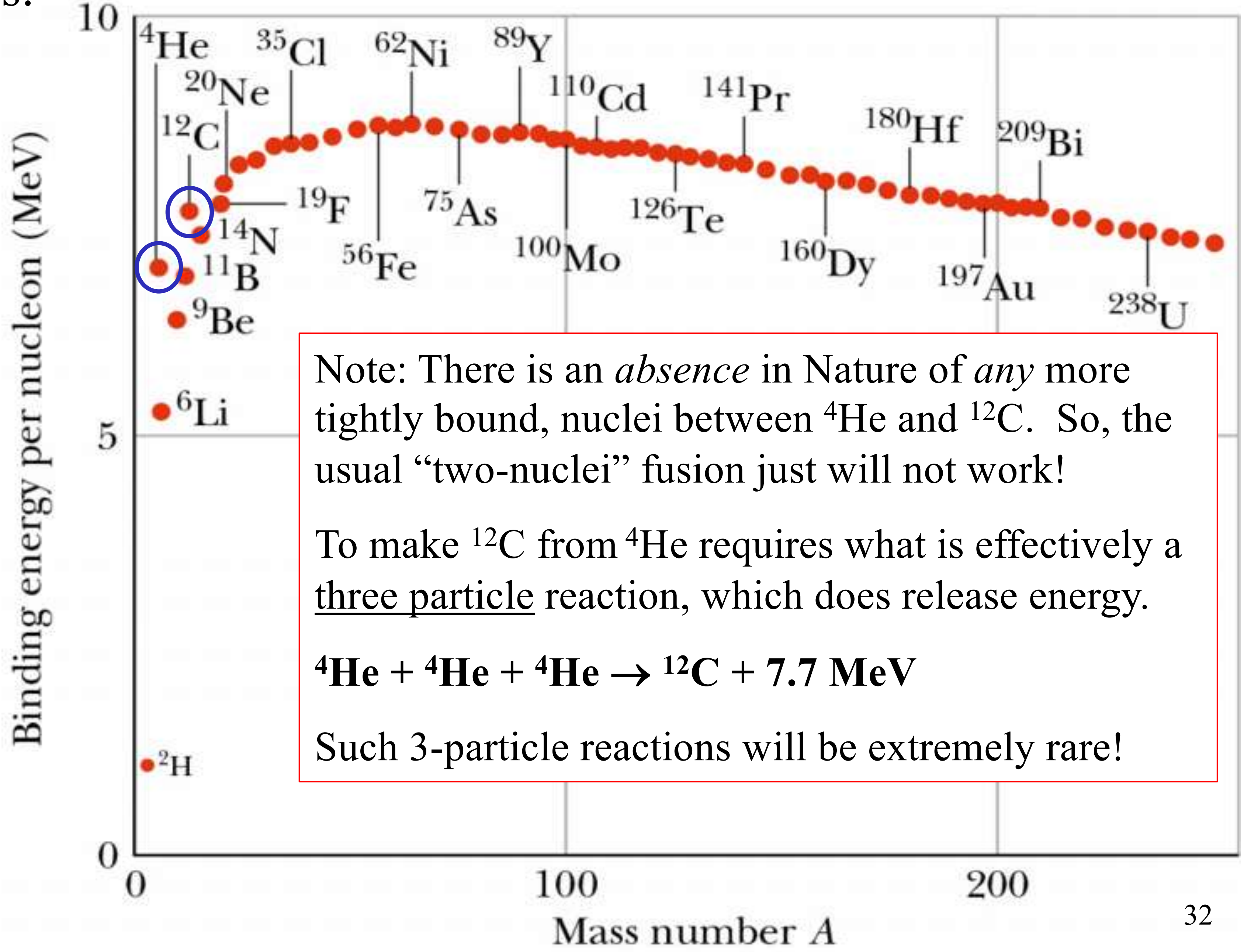
<u>Key point</u>: The narrowness of the Gamow peak means that most of these excited states reactions have very similar energy, since the K.E. of the reacting particles has a narrow range.

One consequence: If this typical excited energy of the product nucleus corresponds to a quantum excited state of the product nucleus then the reaction is more likely to happen, i.e. greatly increased cross section σ at particular product energies, leading to wide variation of reaction rates for different reactions.



Now let's look at the situation with Carbon production

Look again at the binding energy (per nucleon) curve at low masses:



Note: There is an *absence* in Nature of *any* more tightly bound, nuclei between ^{4}He and ^{12}C. So, the usual "two-nuclei" fusion just will not work!

To make ^{12}C from ^{4}He requires what is effectively a <u>three particle</u> reaction, which does release energy.

^{4}He + ^{4}He + ^{4}He → ^{12}C + 7.7 MeV

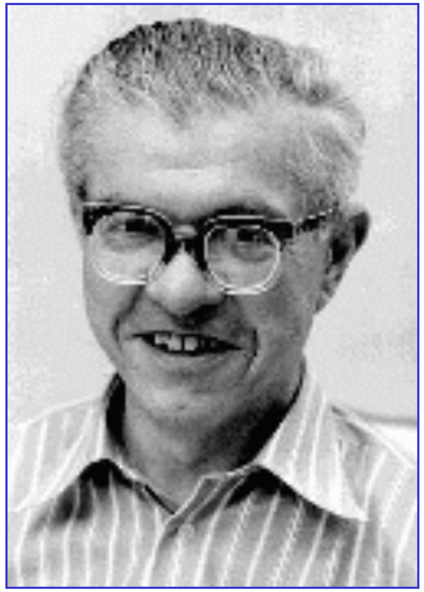
Such 3-particle reactions will be extremely rare!

How does this work? First, note that:

- Production of ^{8}Be from two ^{4}He requires 0.1 MeV of energy.
- The resulting ^{8}Be is then highly unstable with a half life of 10^{-17} s (!)

But, while it survives, ^{8}Be can join with another ^{4}He to make an excited state of ^{12}C.

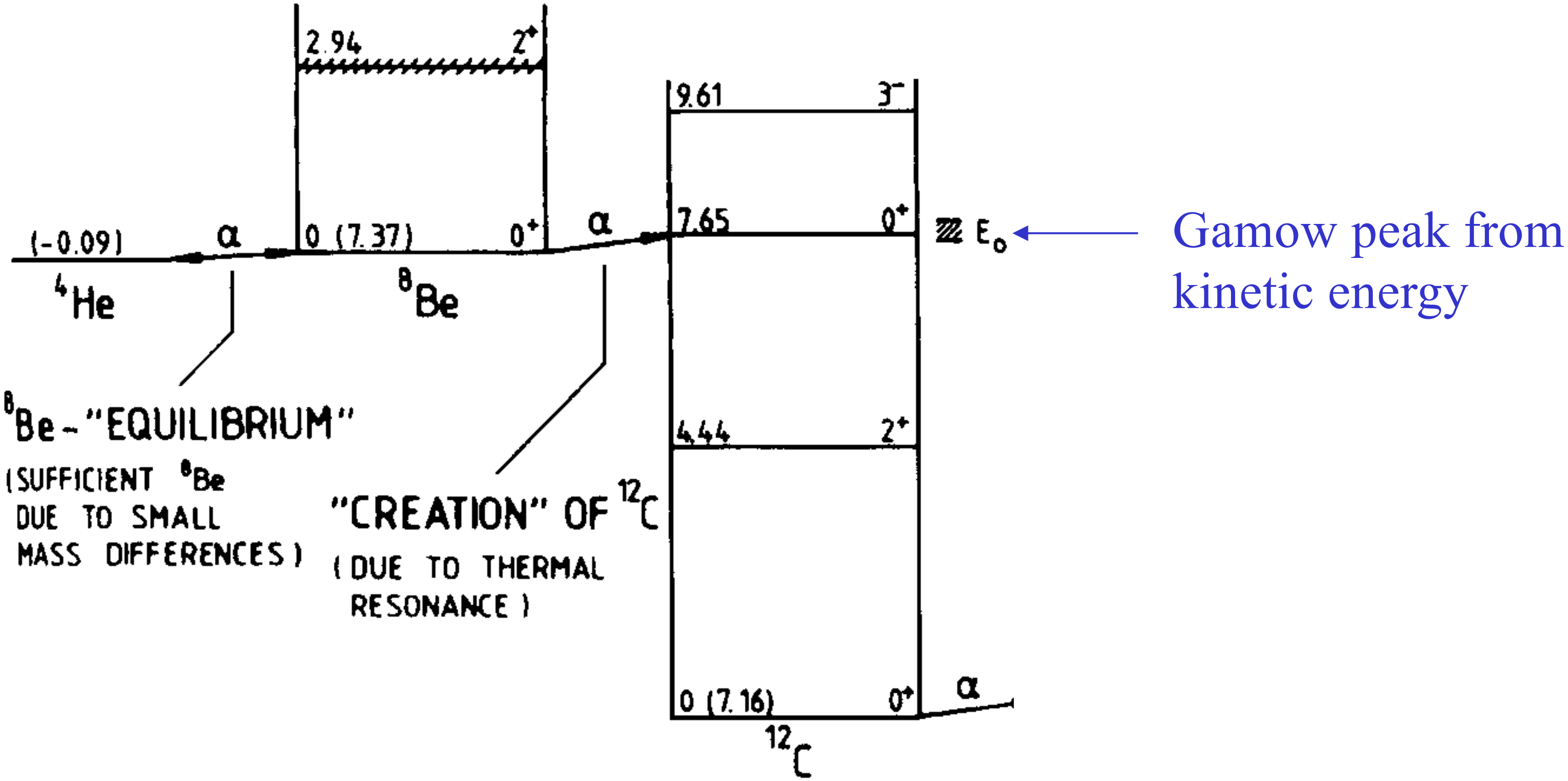
The binding energy of ^{12}C relative to that of ^{8}Be + ^{4}He is 7.33 MeV and the excited ^{12}C nucleus will be just above this because of the kinetic energy of the ^{8}Be and ^{4}He in the Gamow peak.



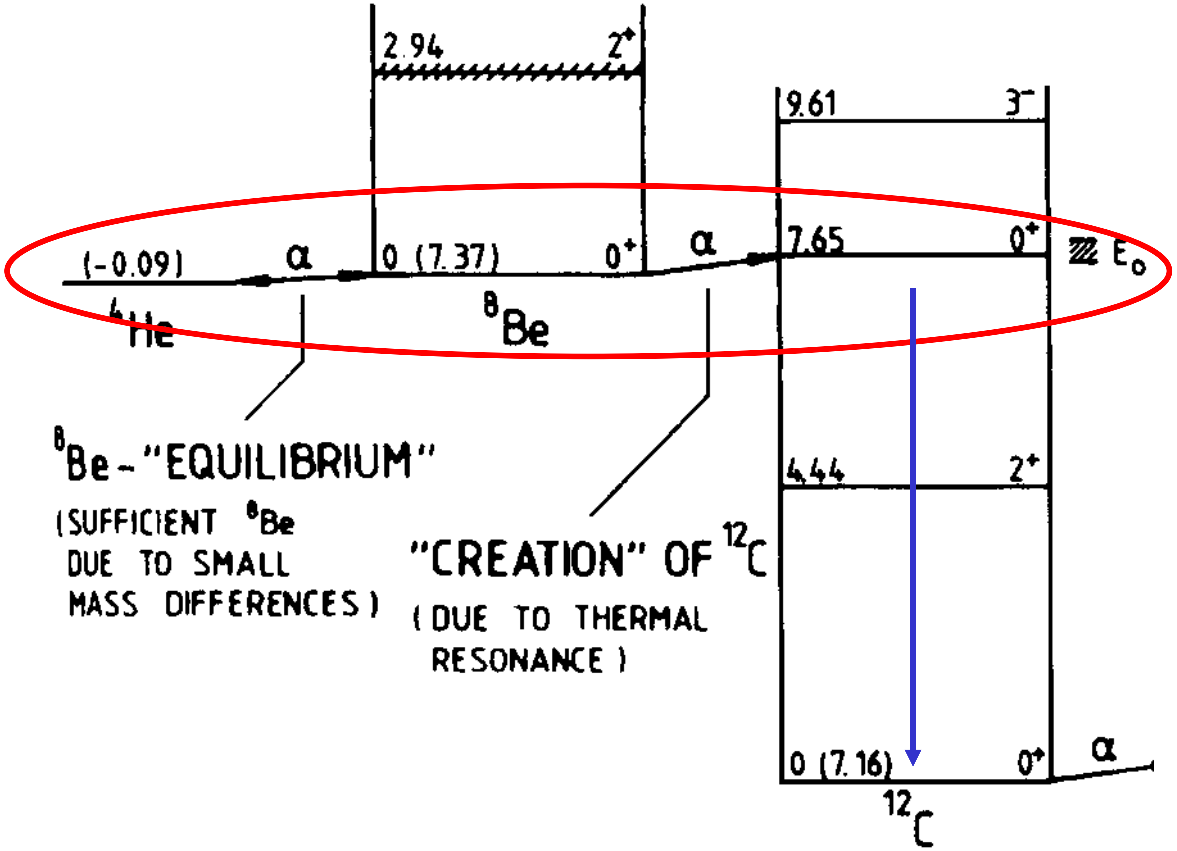
In 1952, Hoyle correctly predicted that ^{12}C must have a resonant nuclear excited state just above 7.33 MeV otherwise this reaction would not occur often enough to produce the observed amounts of ^{12}C in the Univers.

A major resonant state of ^{12}C nucleus was subsequently found at 7.65 MeV – a triumph of theoretical astrophysics!

Most ^{12}C* falls apart to ^{8}Be + ^{4}He, but some de-excites to the ^{12}C ground-state by emitting a γ-ray photon. *We are made of these lucky de-excitations.*

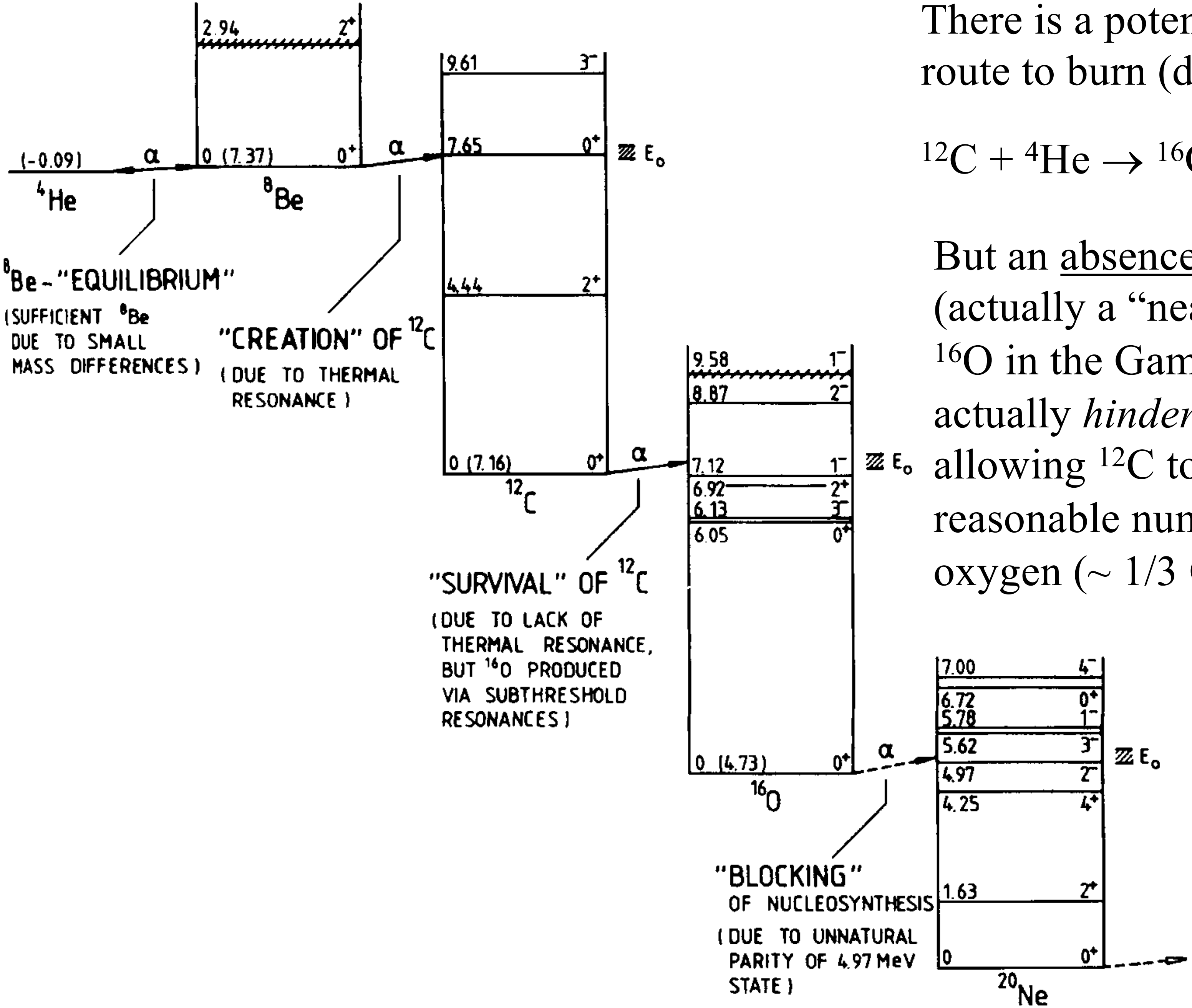


C Resonance level



Three states of ^4He, ^8Be and $^{12}C^*$ are so similar in energy that these three species will effectively be in equilibrium, described by Saha equation.

Production of ^{12}C is effectively "leakage" out of equilibrium from $^{12}C^*$ excited state

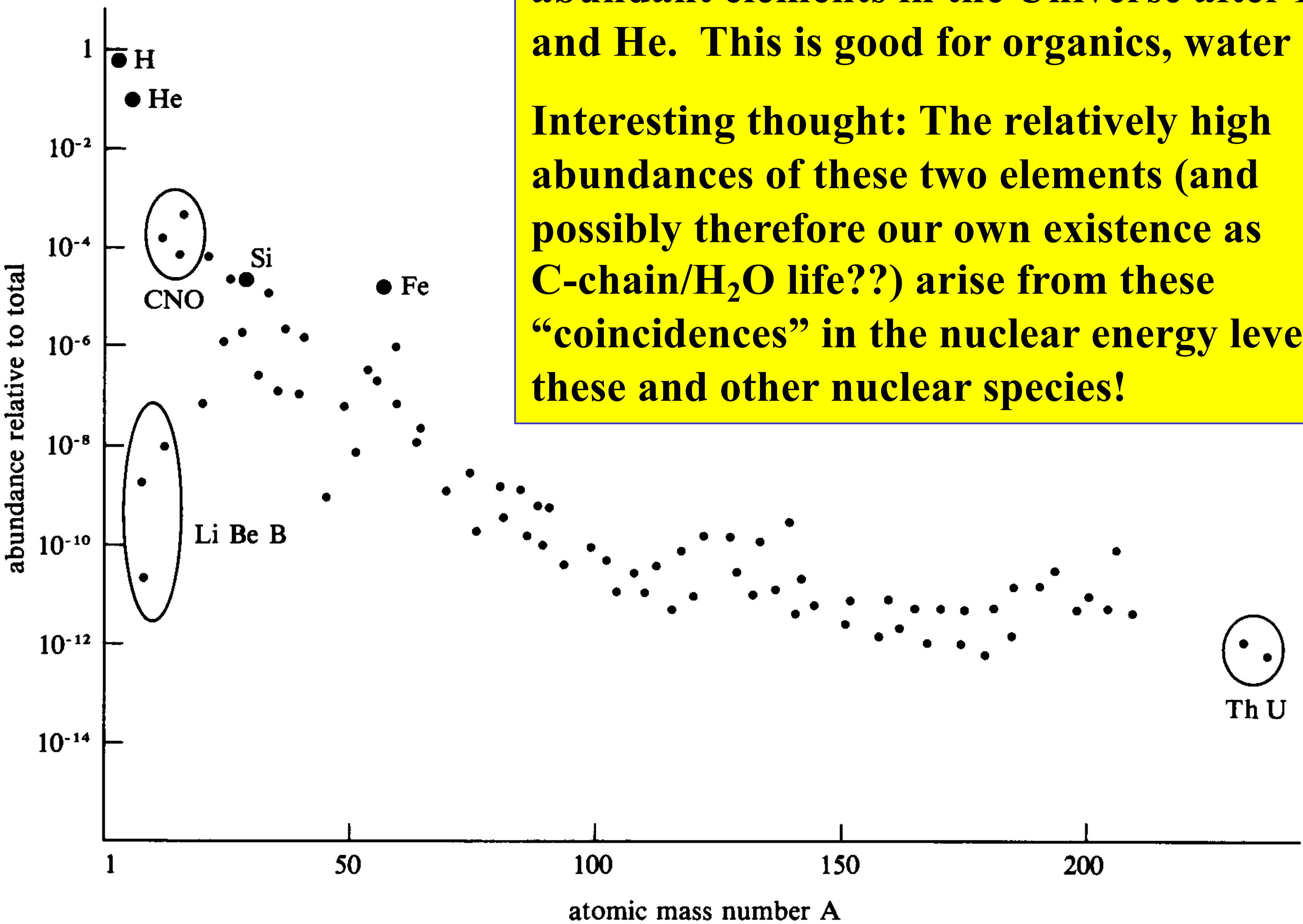


There is a potentially easy route to burn (destroy) ^{12}C:

$^{12}C + {}^{4}He \rightarrow {}^{16}O + 7.16$ MeV

But an absence of a resonance (actually a "near-miss") for ^{16}O in the Gamow band actually *hinders* this reaction, allowing ^{12}C to survive in reasonable numbers relative to oxygen (~ 1/3 O)

Finally,it turns out that the destruction of ^{16}O to ^{20}Ne is also hindered!



Consequence: O and C are the two most abundant elements in the Universe after H and He. This is good for organics, water etc .

Interesting thought: The relatively high abundances of these two elements (and possibly therefore our own existence as C-chain/H_2O life??) arise from these "coincidences" in the nuclear energy levels of these and other nuclear species!

Part 3: Origin of the chemical elements in the Galaxy

In the first part of this Chapter, we saw how elements up to ^{56}Fe could be produced in the cores of stars through fusion reactions, starting with H -> ^{4}He and then building up (in at least the most massive stars) to ^{56}Fe.

- What about all the other elements above ^{56}Fe?
- Also, especially relevant for Life, what good are these fusion products if they remain locked in the cores of the stars where they were produced? How do they get out into the surroundings in order to be available for future use?

Look again at pressure from degenerate matter

R. H. Fowler (1926): application of Pauli exclusion principle.
In a (very!) dense gas all the lower energy states are filled with electrons, forcing electrons into high energy levels if it is further compressed, and this results in a pressure which resists the gravitational force. A proper derivation gives:

$$P = \frac{\pi^2 \hbar^2}{5 m_e m_H^{5/3}} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\rho}{\mu_e}\right)^{5/3}$$

We can get order of magnitude estimate simply by using the Heisenberg Uncertainty Principle: $\Delta x \Delta p \sim h/2\pi$. Consider an object with density n particles per unit volume, the available volume per particle is $\sim n^{-1}$, so $\Delta x \sim n^{-1/3}$ and $p \sim (h/2\pi) \,.\, n^{1/3}$

The pressure will be given by the product of the number density n and the mean kinetic energy E, which differs for relativistic and non-relativistic cases:

Non-relativistic particles: $E = p^2/2m$

Relativistic particles $E = cp$

$$P_{non-rel} \sim \frac{\hbar^2}{2m_e} n_e^{5/3}$$

$$P_{rel} \sim \hbar c \, n_e^{4/3}$$

Mass-radius relation and White Dwarf stability

In Part 1, we had that hydrostatic support requires $P \sim -\frac{1}{3}\frac{U_{grav}}{V} \sim \frac{GM^2}{4\pi R^4}$

For the non-relativistic case of degenerate pressure, we have $\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{5/3} \quad \Rightarrow \quad P \propto \frac{M^{5/3}}{R^5}$

Putting these two pressures together, gives a mass-radius relation $\frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4} \quad \Rightarrow \quad \boxed{R \propto M^{-1/3}}$

Therefore, adding more mass to an object that is supported by degenerate electrons will cause actually cause the radius to shrink*, the density to go up and the degenerate pressure to increase enough to meet the increased required pressure. This makes a self-gravitating object that is supported by non-relativistic degenerate matter stable.

But, note that the E per particle increases as $n^{2/3}$, so as the density increases eventually the non-relativistic matter will become relativistic! What then happens?

** *Note, Jupiter is quite close to this regime, c.f. the Earth.*

Mass-radius relation and White Dwarf stability

Now, repeating for the relativistic case, we have:

$$P \sim -\frac{1}{3}\frac{U_{grav}}{V} \sim \frac{GM^2}{4\pi R^4}$$

Now we find that the mass that can be supported by relativistic degeneracy pressure is actually <u>independent</u> of *R*, i.e. the change in pressure with *R* is balanced by the change in the required pressure. An object is now unstable to collapse because the pressure does not increase as *R* is reduced.

$$\rho \propto \frac{M}{R^3}; \quad P \propto \rho^{4/3} \quad \Rightarrow \quad P \propto \frac{M^{4/3}}{R^4}$$

$$\frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4} \quad \Rightarrow M^{-1/3} \propto R^0$$

This means that there is a maximum mass that can be supported by relativistic degenerate pressure*. Adding further mass, once the particles are relativistic, causes run-away collapse. For degenerate electrons, this limit is called the Chandrasekhar mass, and has a value 1.44 $M_\odot$

* Note that thermal pressure of hot gas can of course support more massive objects, but only if the high temperatures are maintained by injecting energy (e.g. fusion) to replace that lost by cooling.

Collapse beyond Chandrasekhar mass limit

The maximum mass that can be supported by degenerate electron pressure is therefore 1.44 $M_\odot$ (Chandrasekhar 1931). This is indeed the observed maximum mass of White Dwarfs.

Obvious question: Can degenerate neutrons support a collapsed object?

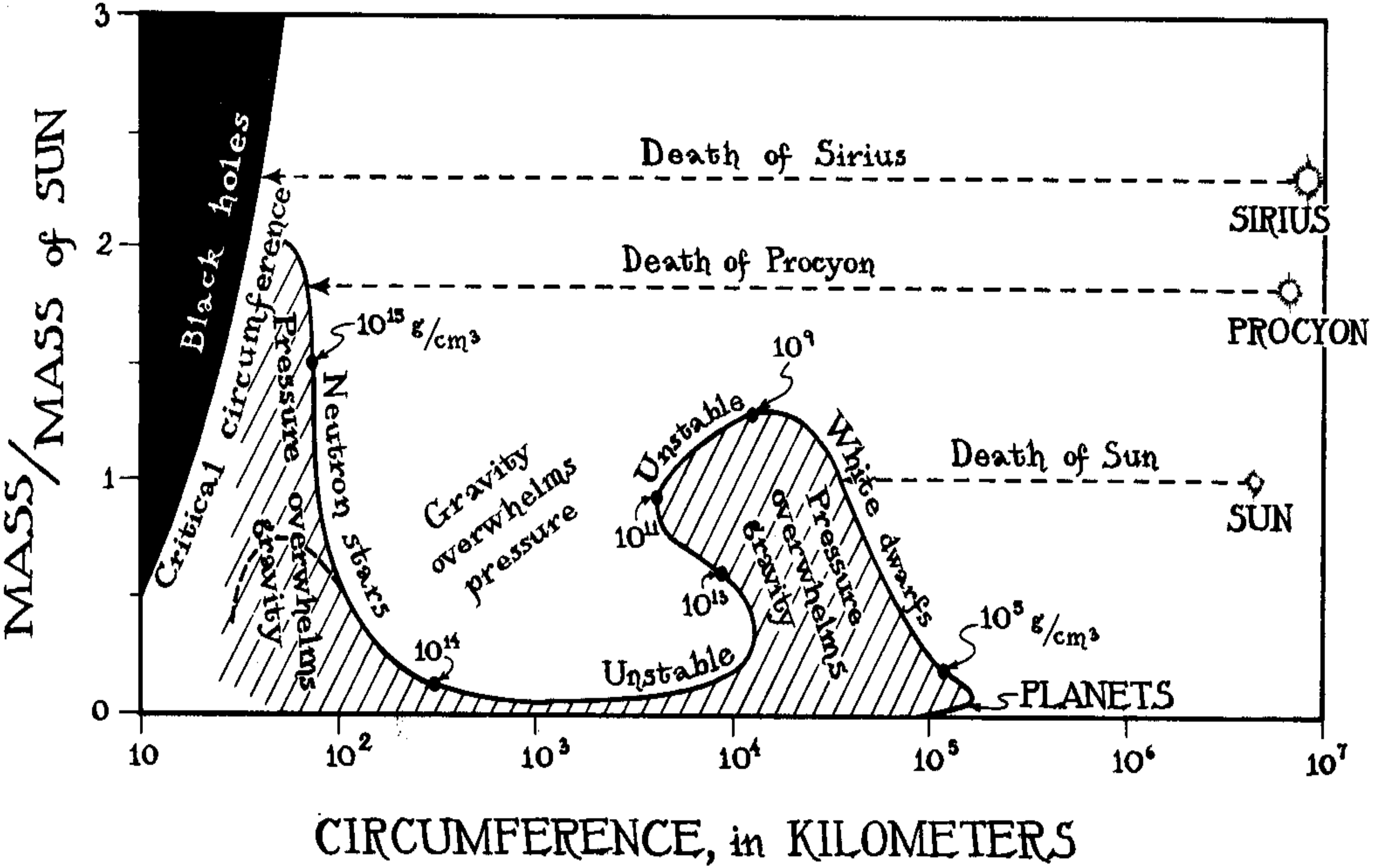
Recall $P \sim m^{-1} n^{5/3}$ in the non-relativistic case. So, neutrons exert $m_e/m_n \sim 10^{-3}$ of the pressure of electrons (which is why we ignored it earlier).

But, collapsing by factor X, the density n increases by X^3, so the pressure increases as X^5. But note the required pressure also increases as X^4!

Net effect, we would expect stabilization by degenerate neutrons when the radius has shrunk by factor equal to the mass ratio of the particles: $m_n/m_e \sim 1000$, i.e. at a radius of order 10 km , compared with 10^4 km for White Dwarfs.

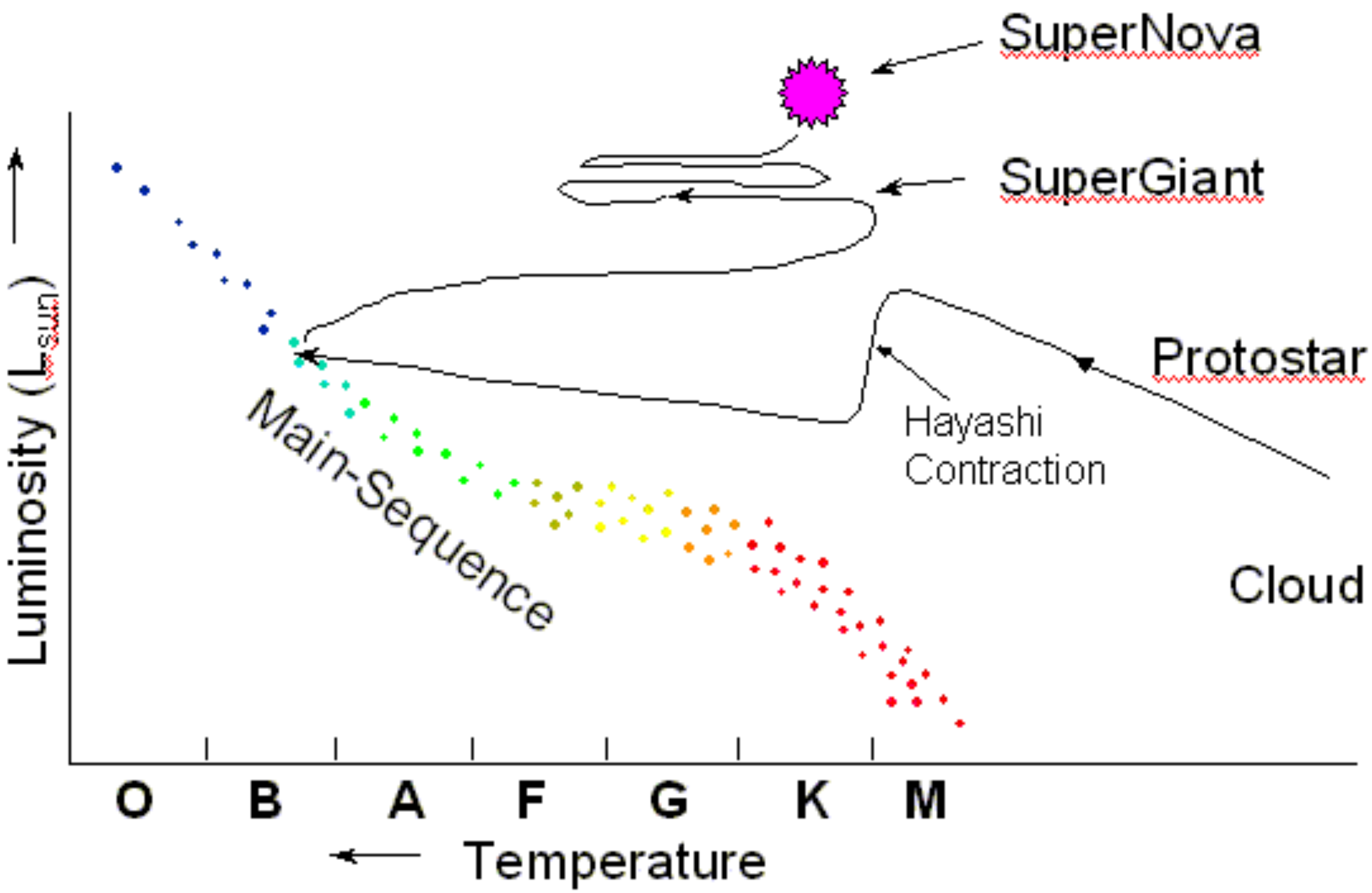
Collapse of stellar core > 1.4 $M_\odot$ causes $e^- + p^+ \rightarrow n$ because the n will have lower energy state than $p + e$ (because of the enormous relativistic energy of the e). i.e. the electrons are literally "squeezed out of existence" by gravity!

Overview: support of stellar-mass objects in the Universe



From Kip Thorne “Black Holes & Time Warps”

What happens at the end of the life of a massive star?
Evolution of 20 $M_\odot$ star

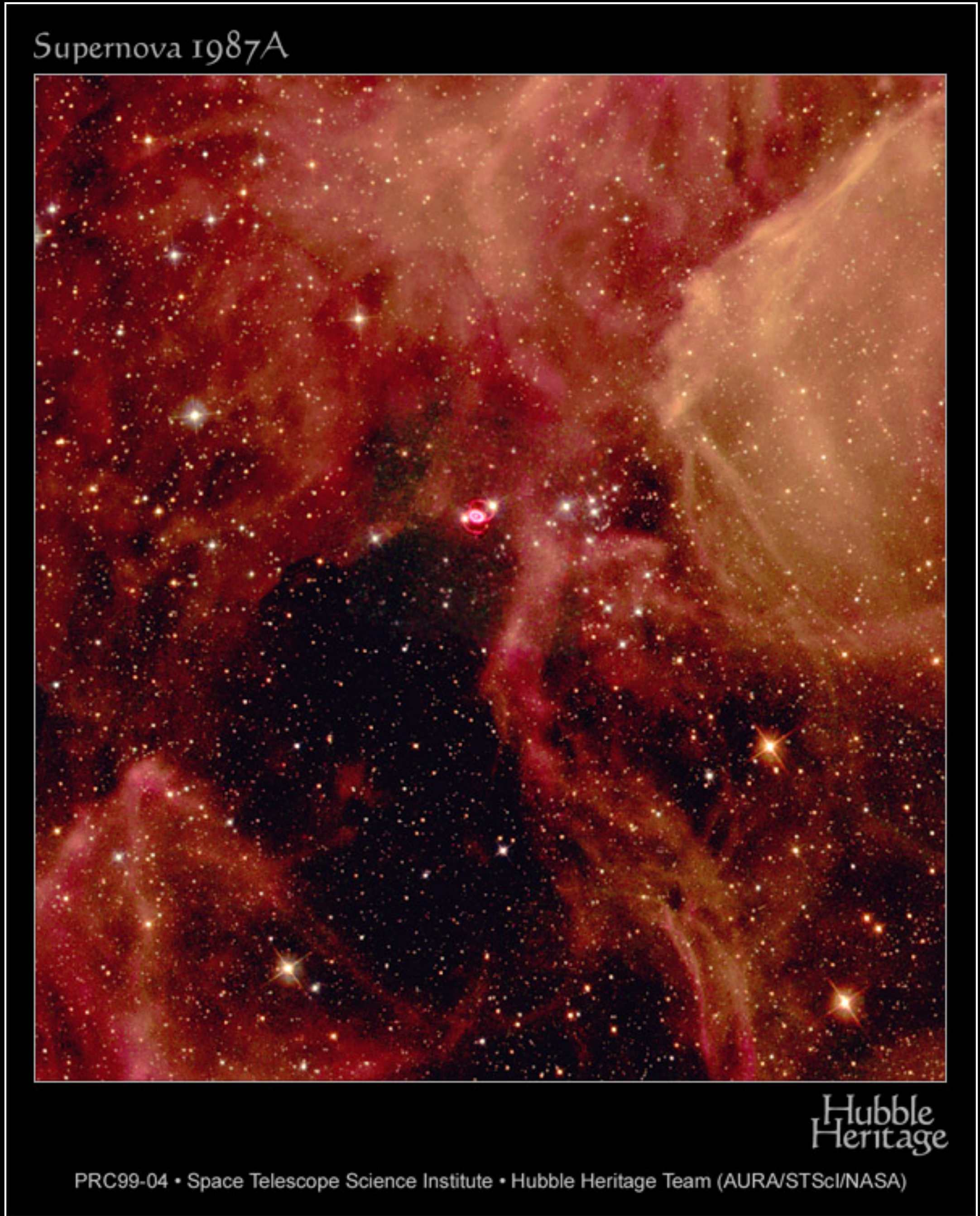


Core collapse (Type II) Supernovae

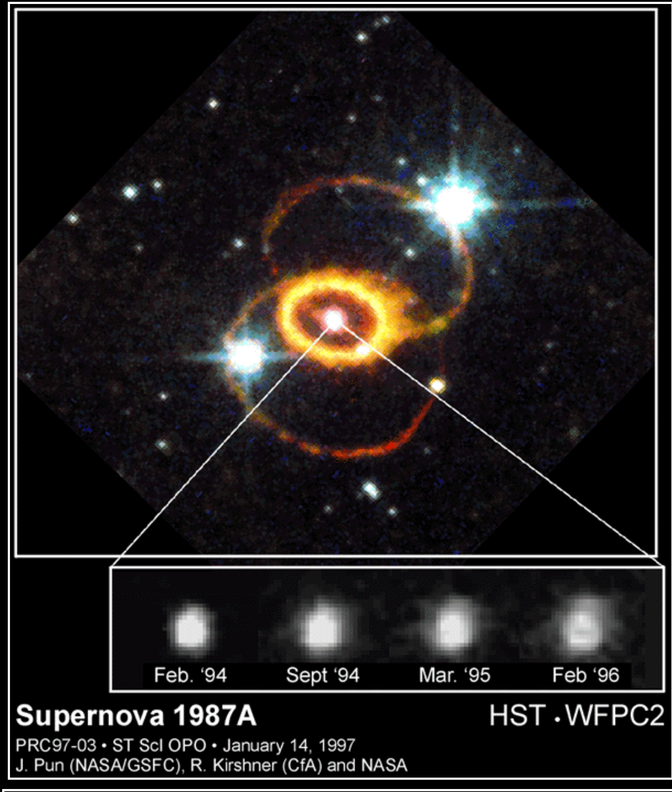
- Catastrophic collapse (duration 1 sec) of Fe-cores of stars that had initial $M > 8\ M_{\odot}$
- No further nuclear production of energy is possible from the Fe-rich core which starts contracting and heating up. The core exceeds the Chandrasekhar limit and certainly cannot be supported by degenerate electrons.
- There are also a number of channels for taking energy *out* of the core, causing it to contract more and more.
    - ^{56}Fe begins to photo-disintegrate into α-particles and neutrons absorbing energy from the core (the reverse of fusion).
    - α-particles themselves photo-disintegrate.
    - Almost all the free electrons combine with protons to form neutrons at nuclear densities.
- Copious numbers of neutrinos (one for every proton-electron combination) are therefore produced, and stream out of the core, further removing energy
- The pressure from this neutrino pulse is believed to be responsible for exploding away the stars' outer layers ("believed" because no model yet really works in 3d)
- Pulse of neutrinos was detected shortly before visual sighting of SN 1987a in Large Magellanic Cloud (first visual detection of SN for ~ 400 years)
- Core collapses to ~10 km where it is stabilized by degenerate neutrons (or continues to collapse to form a black hole?)

Neutron stars and supernovae

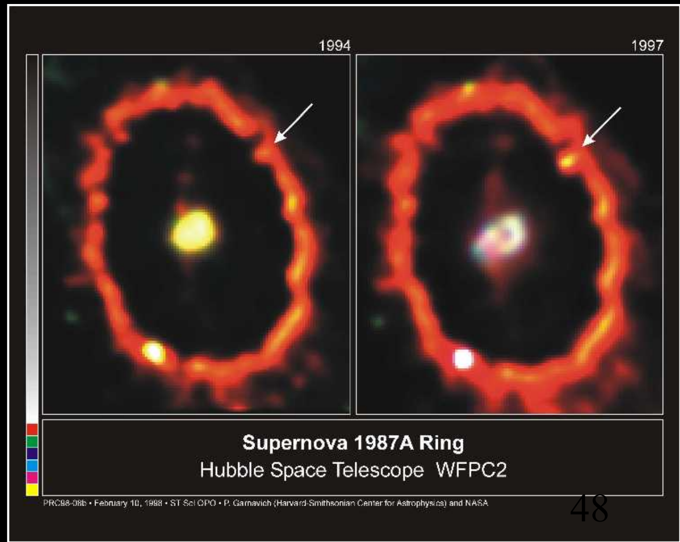
SN 1987A



Supernova 1987A
Hubble Heritage
PRC99-04 • Space Telescope Science Institute • Hubble Heritage Team (AURA/STScI/NASA)

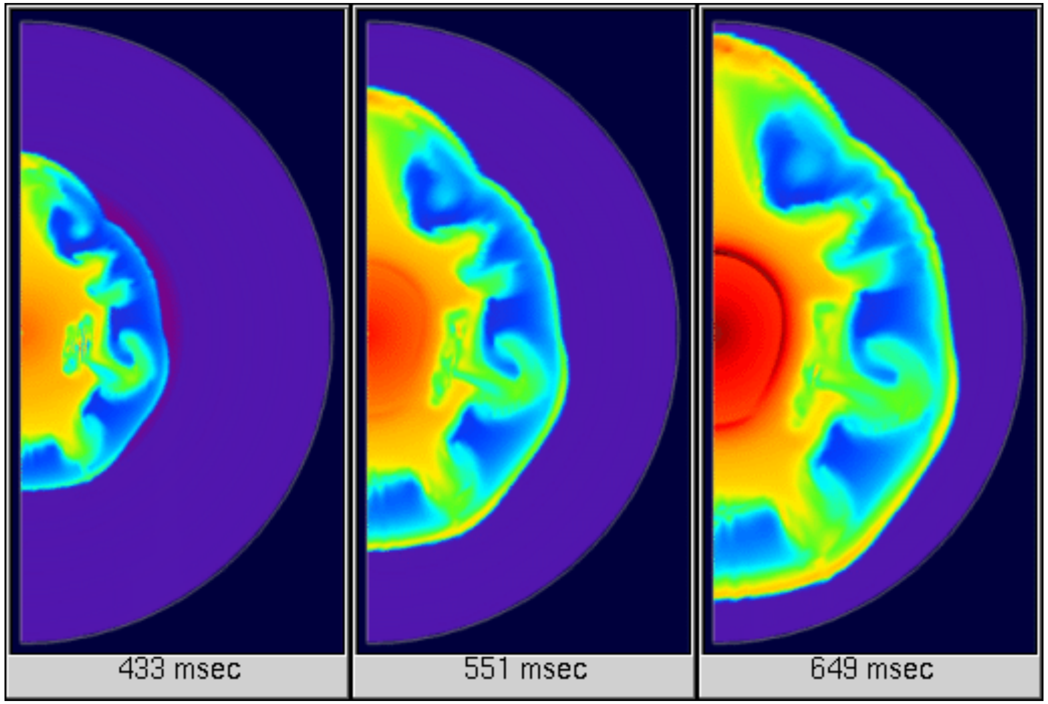


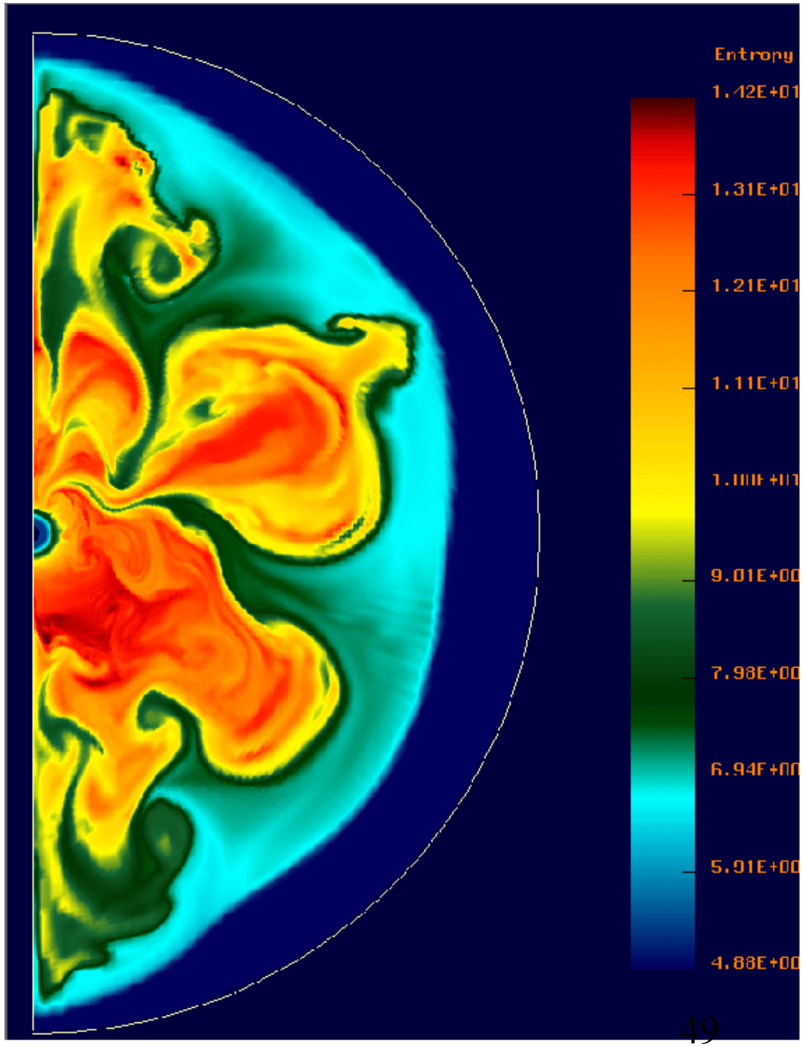
Feb. '94
Sept '94
Mar. '95
Feb '96
Supernova 1987A
HST • WFPC2
PRC97-03 • ST ScI OPO • January 14, 1997
J. Pun (NASA/GSFC), R. Kirshner (CfA) and NASA



1994
1997
Supernova 1987A Ring
Hubble Space Telescope WFPC2

Neutrino driven explosion





Energetics of Supernovae

We saw how current thermal energy of a normal thermal-pressure supported star will always be comparable to its (negative) gravitational potential energy, via the virial condition

$$K = -\frac{U}{2}$$

We know the gravitational potential energy is proportional to $-R^{-1}$.

$$U \sim -\frac{GM^2}{R}$$

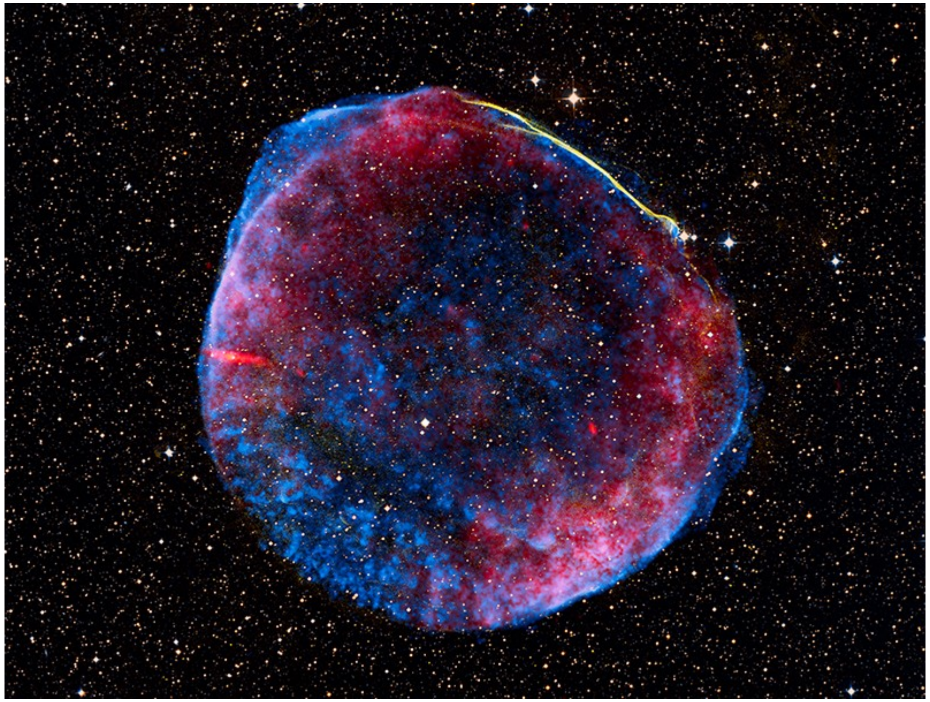
Therefore, a key point: when a star collapses from $O(10^6)$ km to $O(10)$ km in a core-collapse supernova, the energy released, in about a second, is about 10^5 times the potential energy (and thermal energy) that the object had as a normal star, i.e. of order 10^{46} J.

Furthermore, from the Kelvin-Helmhotz argument, we know that this thermal energy was enough to keep the star shining for something like 10^7 years, i.e. the rate of energy release in a supernova is $\sim 10^{19}$ that of the normal star!

Ejection of debris from supernovae

So, why do you get an explosion and *ejection* of material from *collapse*?

- Total energy released is of order the gravitational binding energy of all the material in the neutron star (about 10^{46} J).
- Most of this energy is in the pulse of neutrinos. Of order 1% is reabsorbed by outer envelope of star.
- Since the "escape speed" from the surface of the neutron star ~ 0.3c, if only some of the material is ejected this can have E >> binding energy, so the asymptotic ejection speed (even "at infinity") can easily be 0.1c, i.e. 30,000 km/s.
- *This is how enriched material is returned to the surrounding galaxy.*



Remnant of SN 1006

The formation of the elements above ^{56}Fe

Question: How are elements beyond the iron peak formed?

- definitely not by energetically favourable fusion of smaller nuclei!

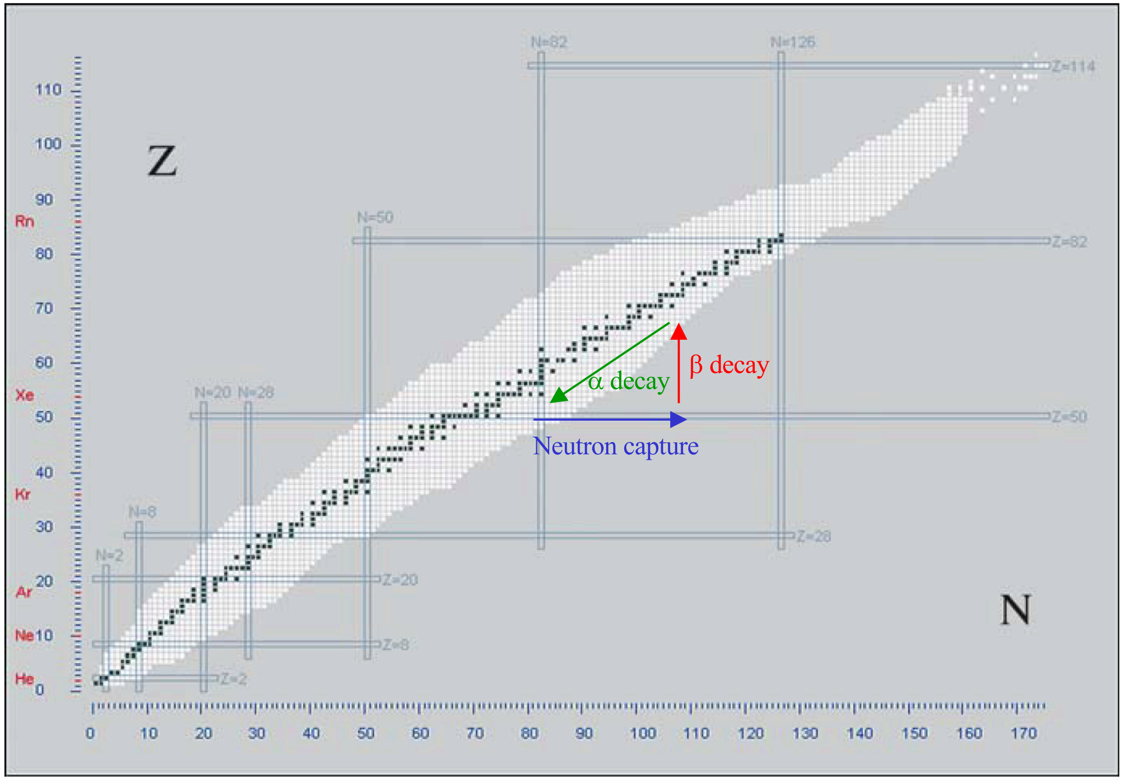
Answer: By neutron capture

Since a neutron is electrically neutral, even a slow neutron can be absorbed by an atomic nucleus, which might then be unstable to β-decay, increasing the atomic number Z by one.

If the rate of capture of free neutrons is sufficiently slow, all the nuclides that are formed by neutron capture that are unstable will have a chance to β-decay before another neutron is absorbed.

The elements formed this way are called *s*-process elements (*s* stands for "slow"). This occurs during last $\sim 10^2$ years of massive star's life when the conditions in stellar cores produce some free neutrons.

Neutron capture: the r-process



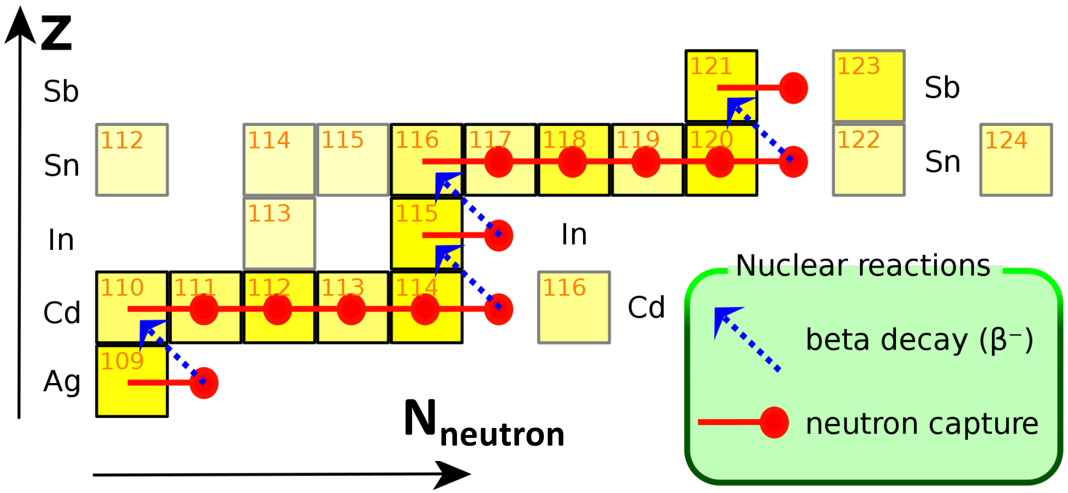
In a core-collapse SNa event, the rate of production of free neutrons becomes very rapid (flux ~10^{22} cm^{-2} s^{-1}; T~ 10^{10} K); as a consequence, the nuclides formed by neutron capture will not have a chance to β-decay before the capture of another neutron.

Eventually, after the period of neutron production, the newly formed very heavy elements will have a chance to α and β-decay back to the stability locus.

The sequence of elements formed this way - called the *r*-process elements (*r* stands for “rapid”) - generally differs from the sequence formed by the *s*-processes.

s-process does one neutron-adding step at a time via reasonably stable nuclei

r-process adds many neutrons at once followed by various decay routes



Different elements/isotopes are primarily produced either by the *s*- or the *r*-process: Example: production of different isotopes of Cobalt.

s-process

$^{56}Fe + n \rightarrow {}^{57}Fe$

$^{57}Fe + n \rightarrow {}^{58}Fe$

$^{58}Fe + n \rightarrow {}^{59}Fe$

$^{59}Fe \rightarrow$ $\mathbf{^{59}Co}$

r-process

$^{59}Fe + 2n \rightarrow {}^{61}Fe \rightarrow$ $\mathbf{^{61}Co}$

most of the Co on Earth is *s*-process produced ^{59}Co

The *s*-process is expected to occur during the last few hundred years of a massive star's life, and produces elements up to Bismuth (Z = 83)

Evidence: ^{99}*Te (Z=43) is unstable with half-life* $t_{1/2} \sim$ *200,000 yrs and yet it is seen in very small quantities in the atmospheres of AGB stars (having been convected up from the core).*

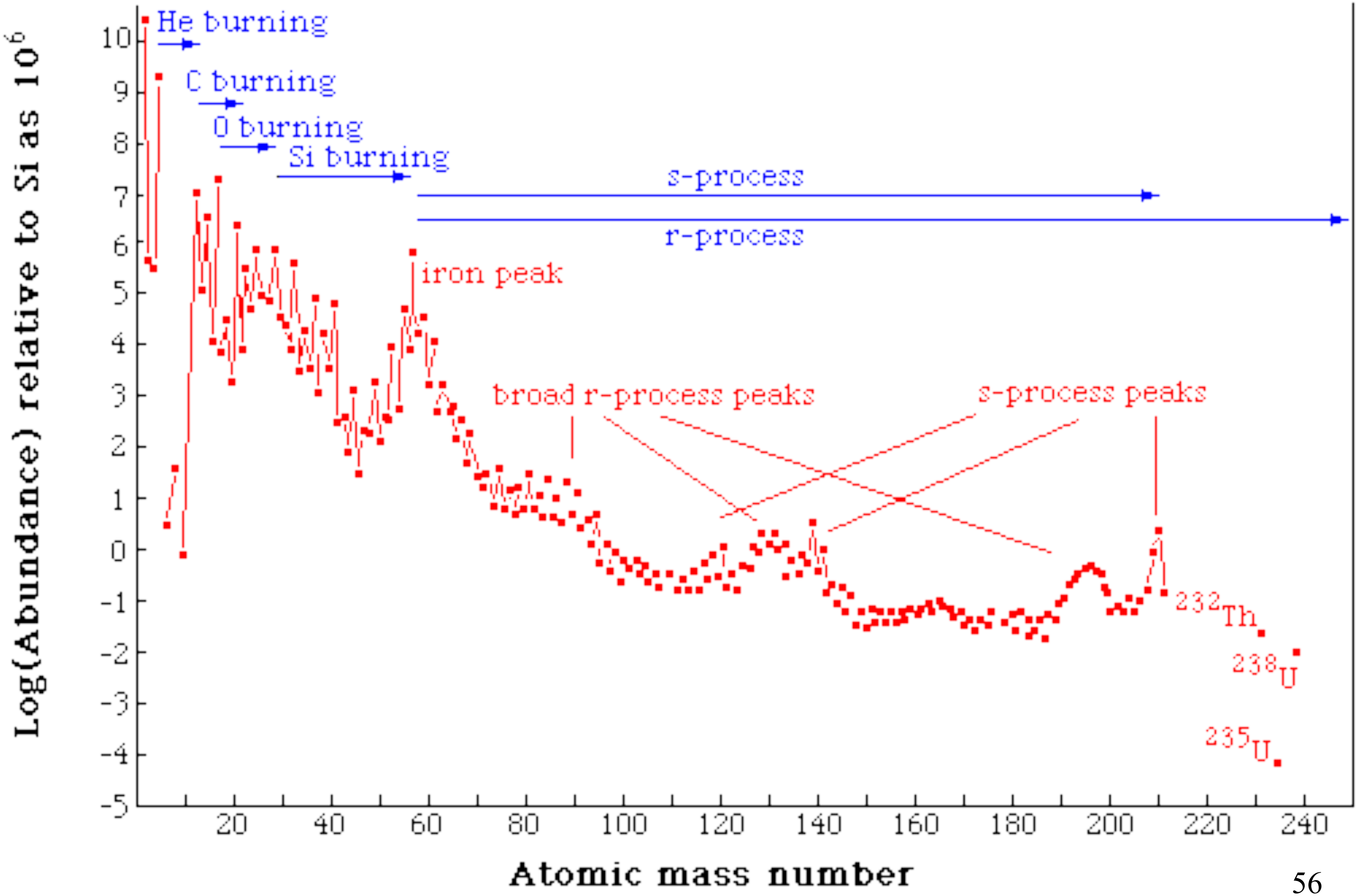
The *s*-process stops at Bismuth because of a cycle:

$$^{209}\mathrm{Bi} + n \rightarrow {}^{210}\mathrm{Bi}$$
$$^{210}\mathrm{Bi} \rightarrow {}^{210}\mathrm{Po} + e^-$$
$$^{210}\mathrm{Po} \rightarrow {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He}$$
$$^{206}\mathrm{Pb} + 3\mathrm{n} \rightarrow {}^{209}\mathrm{Pb}$$
$$^{209}\mathrm{Pb} \rightarrow {}^{209}\mathrm{Bi} + e^-$$

Effectively, four captured neutrons are converted into ^{4}He plus 2 *e*-, instead of making heavier nuclei.

The *r*-process only happens in the one second or so of a supernova collapse and is the <u>only</u> channel for production of elements above Bismuth (Polonium Z = 84 to Uranium Z = 92), and is the <u>dominant</u> production mechanism for many other elements including Gold (Z=79), Silver (Z=47) etc.

Solar system abundances



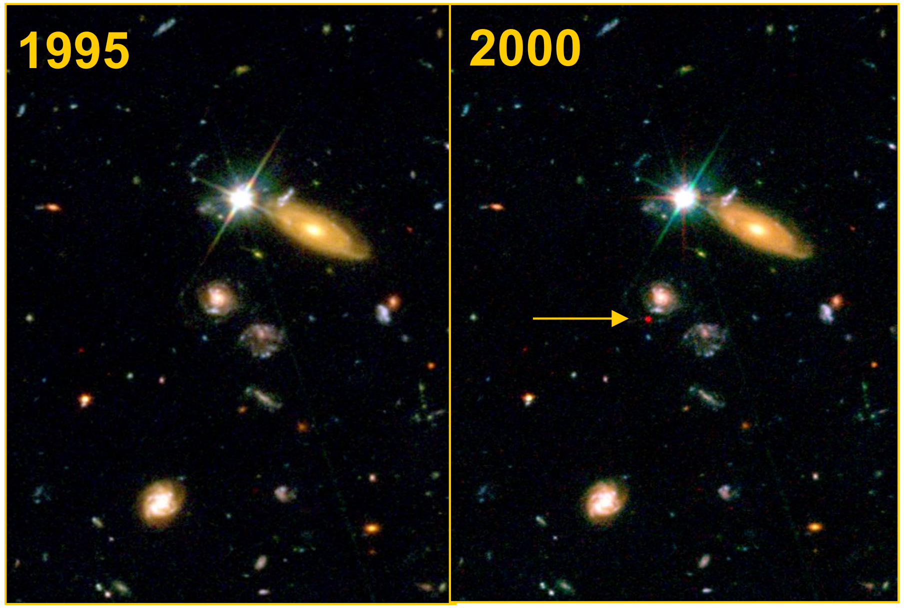
Also: Supernovae Type 1a

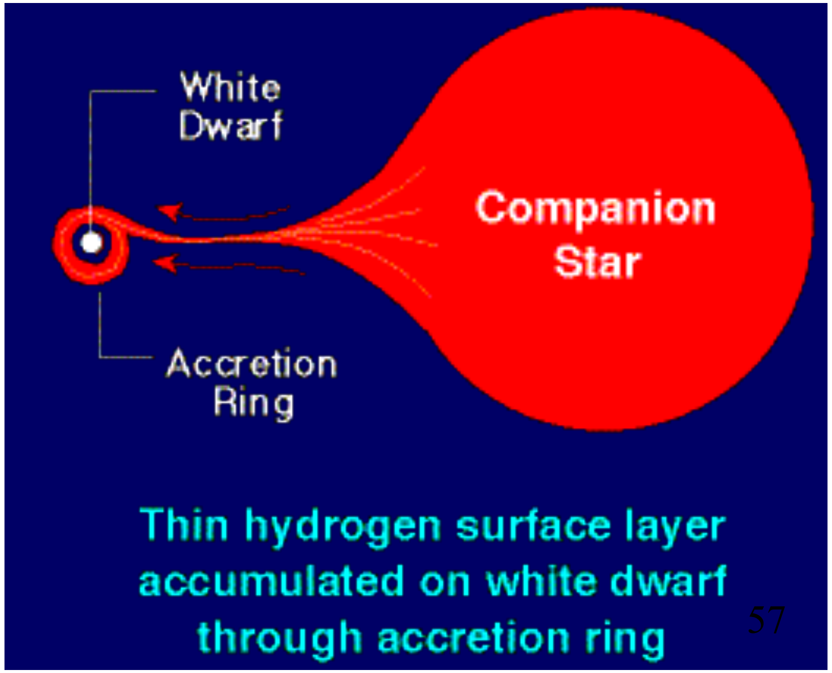
White dwarf is made of Carbon nuclei.

If mass is added to a White Dwarf (e.g. by accretion in a binary system) then at a point near to the Chandrasekhar mass, the temperature and density increase enough to cause thermonuclear ignition of the Carbon nuclei.

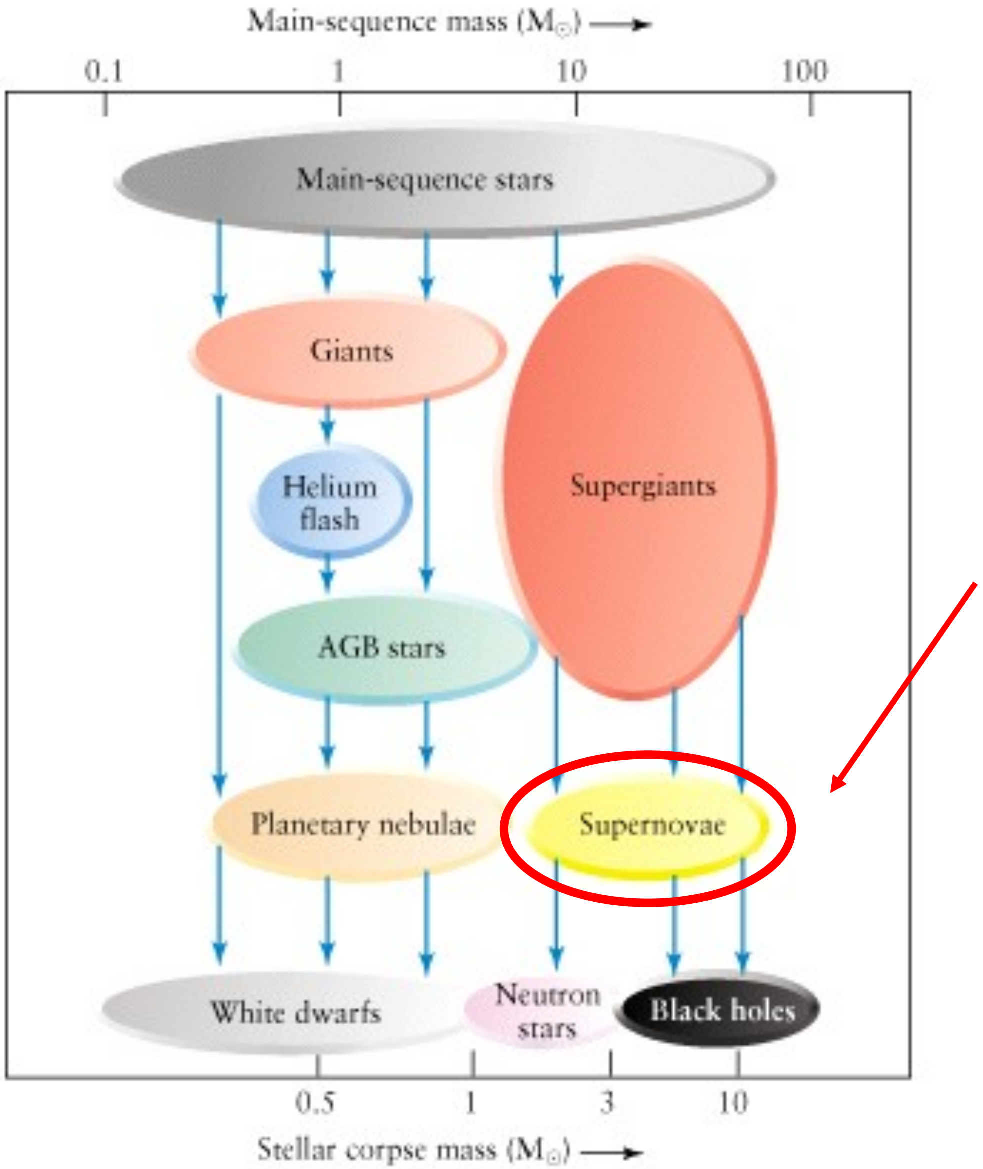
This causes the WD to explode and produce a SN1a (identified by absence of H in spectrum). Of interest for two reasons:

- Efficient production of Iron relative to the so-called α-elements (^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S etc)
- "Standard candles" for cosmology





Stellar Evolution Summary



Significance of supernovae for Life:

It is the explosive supernovae that return all the chemical elements to the surrounding ISM, which are then available for subsequent star/planet formation and Life.

"Chemical evolution" of the interstellar gas in galaxies

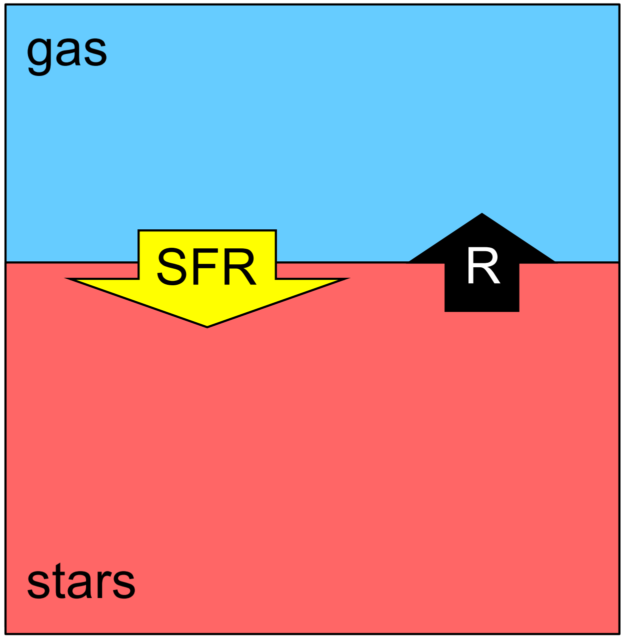
Core collapse supernovae come from massive stars with short Main Sequence lifetimes, ~ 10^7 yr – so short that we can often assume an "instantaneous recycling". SN1a progenitors (binary stars?) likely appear later (10^8-10^9 yr).

We define the yield *y* to be the mass of heavy elements (> ^{4}He) that are returned to the interstellar gas by supernovae per unit mass of material that is formed into stars. It will depend on the ratio of high to low mass stars (the "initial mass function" or i.m.f.), but for a standard mass distribution of stars, we get *y* ~ 0.01.

Successive generations of stars therefore enrich the gas in galaxies, as characterized by the mass fraction of heavy elements (> ^{4}He), called the metallicity *Z*.

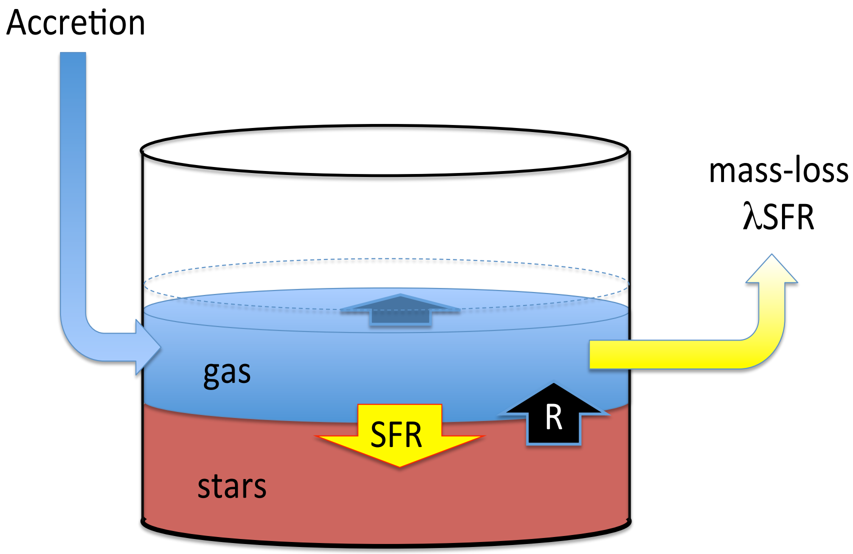
One can make arbitrarily complex models for the development of $Z(t)$ of a galaxy. Two very simple cases are shown below for (a) a "closed box" of gas being progressively made into stars, and (b) for a steady-state flow-through system in which gas flows into the galaxy and is formed into stars or is ejected. Both of these have $Z \sim y$.

Simple "closed box" evolution



$$Z = -y \times \ln\left({m_{gas}}/{m_{tot}} \right)$$

Constant flow-through evolution



$$Z = \frac{y}{1 + \lambda(1 - R)}$$

Chapter 4 – key ideas

- Fusion in stellar cores provides long-term source of energy. Required high temperatures established by gravitational collapse.

- Fusion synthesizes elements, in principle up to ^{56}Fe if temperatures are hot enough, i.e. in the most massive stars.

- Ignition of H in collapsing proto-stars is largely inevitable unless the mass is too low ($< 0.08\ M_{sun}$).

- Velocity tail + quantum tunnelling produces sharp "Gamow Peak" in energy of reactants and a strong T dependence. The latter, together with negative heat capacity, stabilizes stars.

- Interaction of well-defined energy of reaction + Gamow Peak with excitation levels in nuclei has large effect on reaction rates.
    - formation of Carbon is less impossible than you would think (3-body reaction) due to favorable excited state
    - destruction of Carbon is hindered

Chapter 4 – key ideas

- Elements above ^{56}Fe require energy to be created via neutron capture, via the *s*-process (in last stages of star) and in *r*-process in supernovae.

- Degenerate electrons (temperature independent) can support cores only up to 1.4 M_{sun}. Relativistic degenerate matter is unstable to collapse.

- Collapse of core to neutron star releases vast amount of potential energy, and ejects enriched material into surrounding space at velocities up to $0.1c$.

- The yield y is the mass of heavy elements (> ^4He) returned to the interstellar medium per unit mass of material formed into a set of stars. For standard mass distribution of stars, $y \sim 0.01$. The abundance of heavy elements in interstellar gas in the Galaxy will be of this same order.

- This material is then available to make e.g. planets and Life.