Cosmology – Part II

Origin of structure in the Universe
The future of the Universe

Initial density fluctuations

Gravitational instability causes density fluctuations to grow

Given Fourier mode has fixed comoving wavelength as Universe expands (therefore physical wavelength increases). Some physical effects (e.g. damping) will often be driven by physical wavelength.

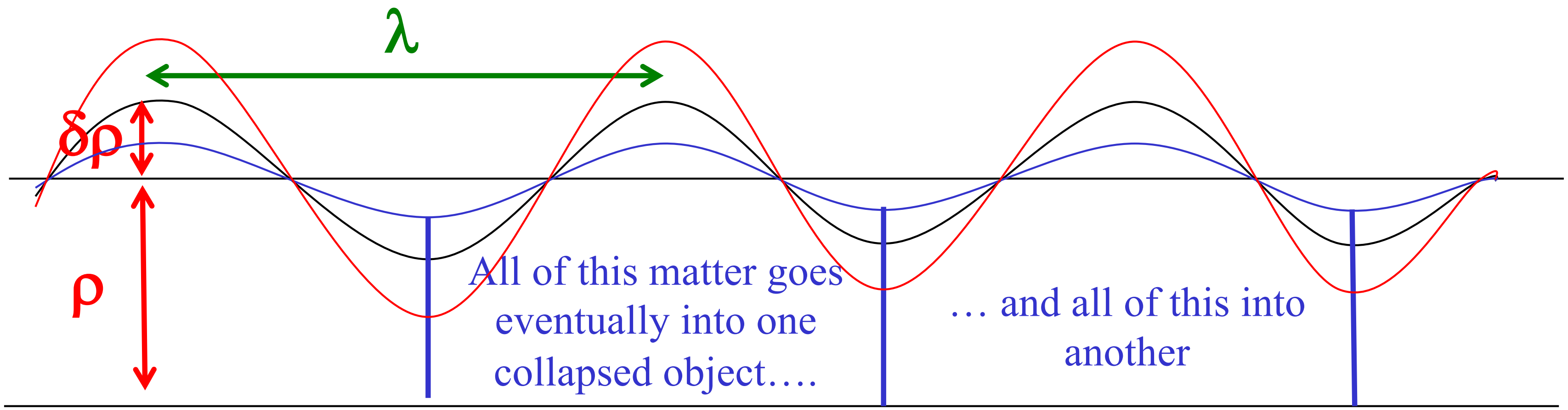
Conventional to think in terms of comoving wavenumber *k* and physical wavelength λ

$$\lambda = R(\tau)\, 2\pi\, k^{-1}$$

Note that the mass associated with a density perturbation of wavelength λ or wave number *k* is simply proportional to *k -3*.

$$M \sim \rho_m \lambda^3 \propto k^{-3}$$

The amplitude of this density fluctuation is given by $\Delta_k = \delta\rho/\rho$.



Actual density field of the Universe $\Delta(x)$ is a Fourier sum of different modes with a power spectrum described by a power-law in comoving wave number k

$$\Delta(\bar{x}) = \frac{1}{(2\pi)^3} \int \Delta_k e^{-i\bar{k}\bar{x}} d^3\bar{k}$$

$$\Delta_k^2 \propto k^n$$

Some interesting values of n:

- All $n > -3$ will have homogeneity on the largest scales, as required.
- A completely random mass distribution has $n = 0$.
- $n = +4$ arises from a purely local perturbation of the matter distribution.
- The $n = +1$ spectrum has many interesting properties: Not least, at all epochs the amplitude of those fluctuations *that are on the scale of the horizon (c/H)* is the same, independent of epoch, even though the scale of the horizon changes. We can then introduce $Q = \Delta$ on the scale of the current horizon to characterize absolute amplitude. It is called the Harrison-Peebles-Zeldovich spectrum. It is naturally produced from Inflation (see next slide).

Our Universe appears to have created a primordial $n = 1$ spectrum, which was subsequently modified on small scales (see later). Our $Q \sim 10^{-5}$.

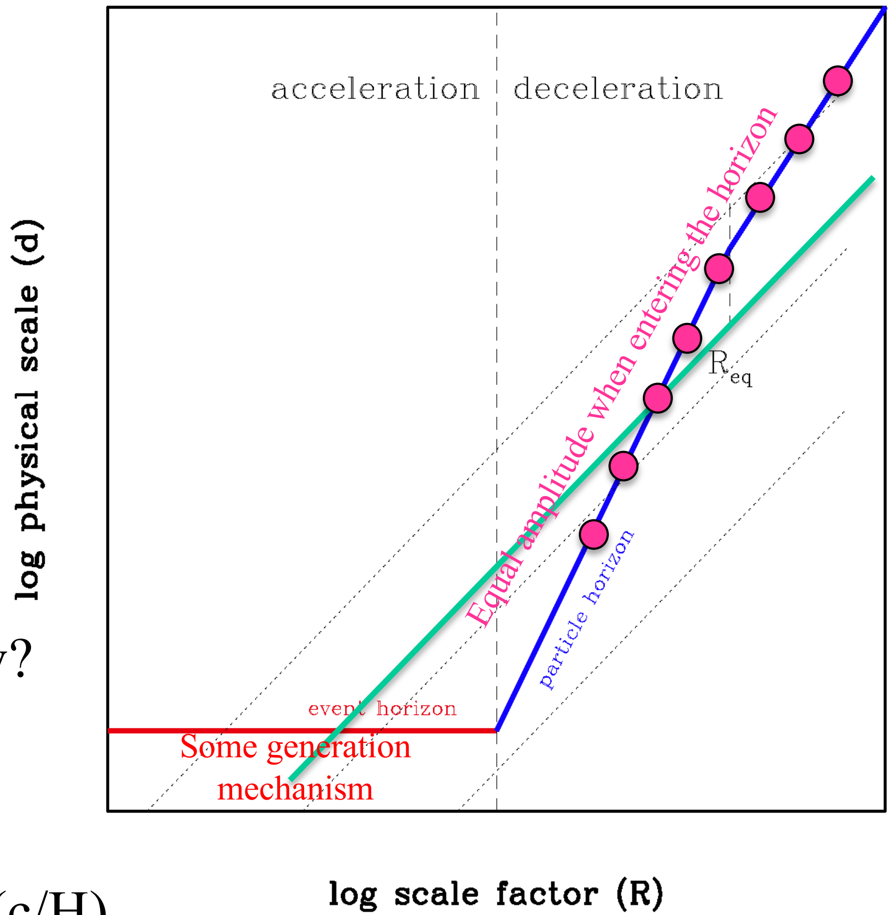
Aside for enthusiasts: the origin of n = 1 fluctuation spectrum

$$\omega_{H,particle} = \int_{\tau_1}^{now} \frac{c}{R(\tau)} d\tau$$

Particle horizon: have we received information on an event in the past? *Decelerating* universes have an expanding (comoving) particle horizon

$$\omega_{H,event} = \int_{now}^{\tau_2} \frac{c}{R(\tau)} d\tau$$

Event horizon: will we ever receive information on an event happening now? *Accelerating* universes have a contracting (comoving) event horizon



- The horizon scale is always of order (c/H).
- A given fluctuation scale may leave the horizon during inflation (through the event-horizon) and then re-enter the horizon (through the particle-horizon) when the universe is decelerating, with the same Δ_k when it does so **(why?...)**
- In this way, inflation naturally produces an n = 1 spectrum.

The amplitude growth of different Fourier components

The classic analysis of Jeans (static medium) was further developed by Lifshitz for an expanding medium

$$\frac{d^2\Delta_k}{dt^2} + 2\frac{\dot{R}}{R}\frac{d\Delta_k}{dt} = \left\{ \underline{4\pi G\bar{\rho} - k^2 c_s^2} \right\} \Delta_k$$

All solutions depend on sign of the right hand term, i.e. whether k is greater or less than a critical Jeans wave-number k_J:

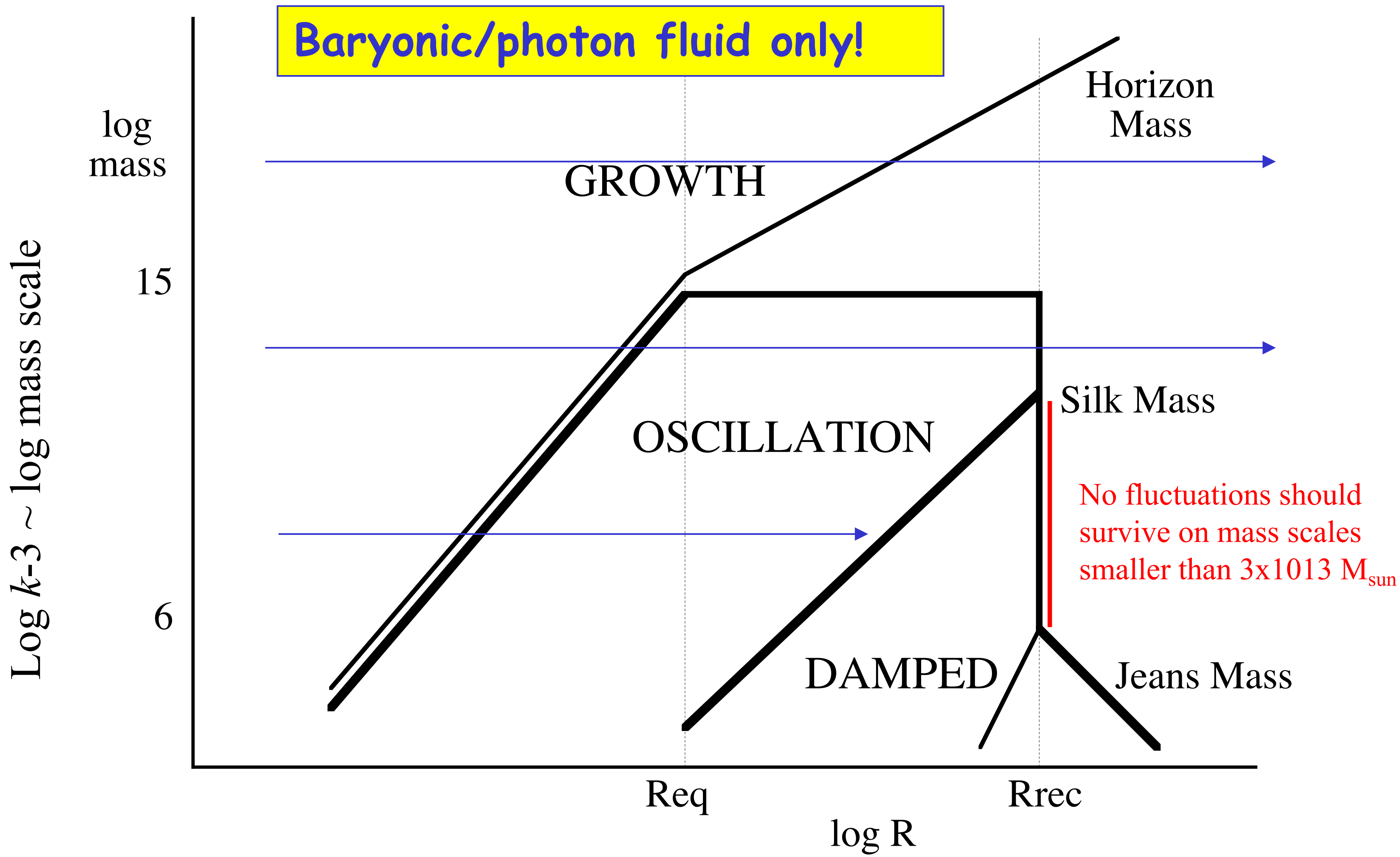
$$k_J^2 = \frac{4\pi G\bar{\rho}}{c_s^2}$$

On large scales $k << k_J$ (i.e. $\lambda >> \lambda_J$) we have potential for growth of Δ_k:

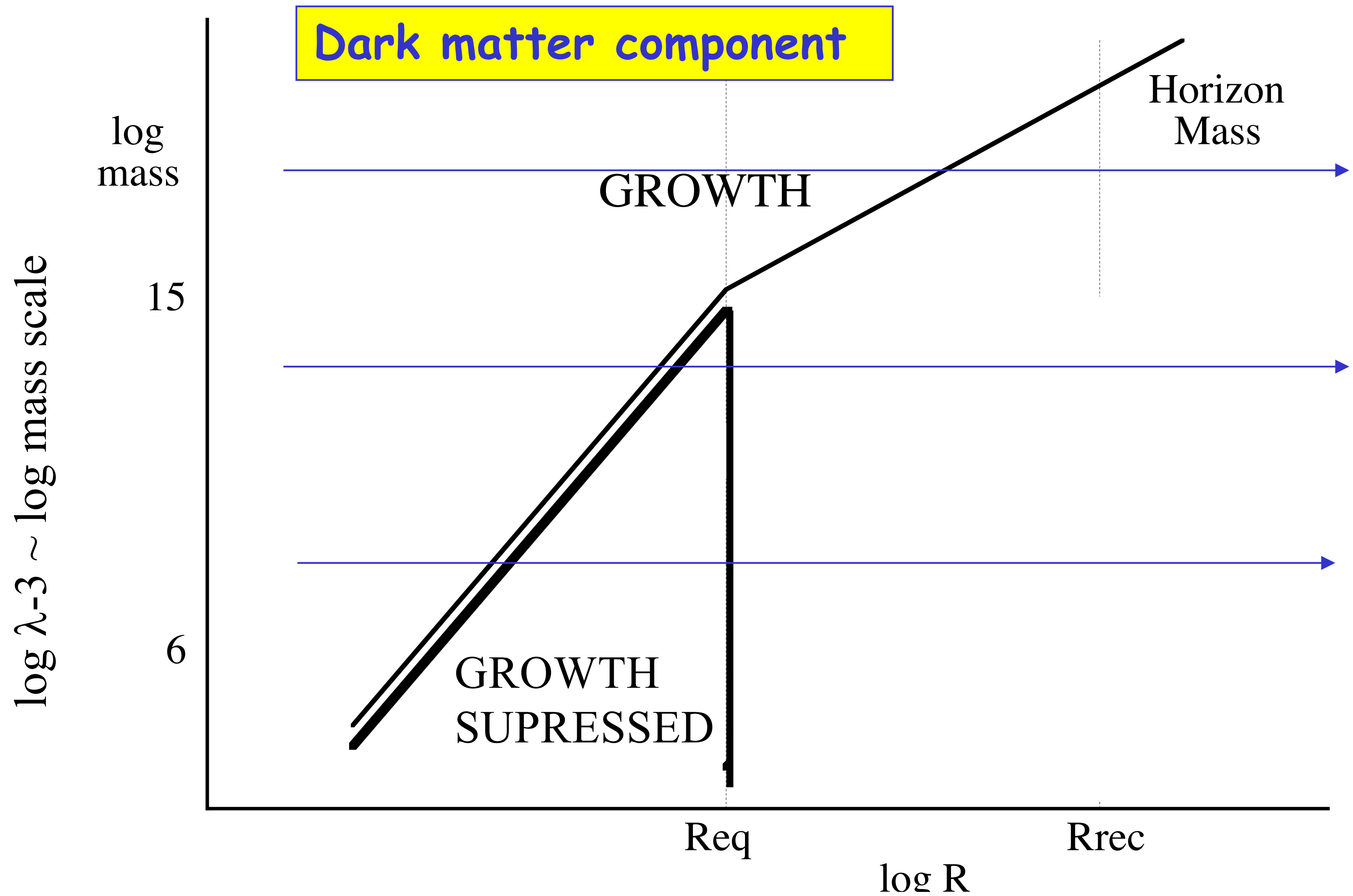
- the solutions will depend on the *R*-dot term.
- We get *exponential* growth for a static medium (with R-dot = 0)
- We get at best *linear* growth $\Delta \propto R$, for a decelerating expanding medium and *no growth at all* if the expansion of the Universe is not significantly decelerating or is accelerating

On small scales, $\lambda << \lambda_\vartheta$, we have *oscillating* solutions since the pressure (given by $c_s 2$) is significant. This is especially important when Universe is an ionized plasma. *When the pressure comes from the relic radiation, the Jeans length is comparable to the horizon*.

Note: Dark matter *does not interact* and therefore has zero sound speed and $\lambda_J = 0$, and it never oscillates (because there is no pressure).



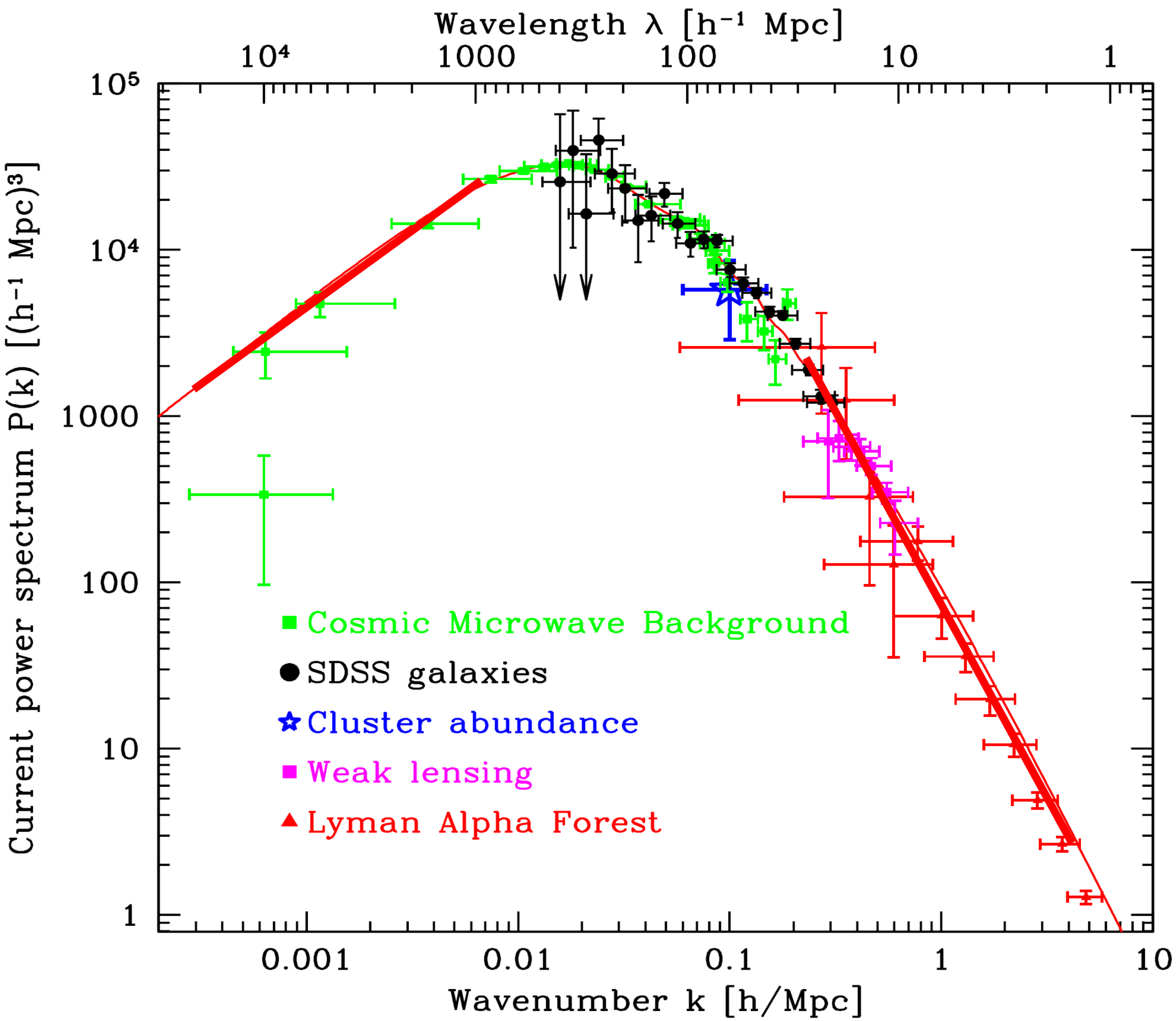
Baryonic fluctuations will oscillate when their length scale is below the Jeans length. They may damp out (due to photon diffusion effects) on scales up to the "Silk Mass" (>> mass of galaxy). This Silk damping is a potentially rather serious problem for <u>baryon-only</u> scenarios for galaxy formation: galaxies should not exist!



But, dark matter fluctuations are able to continue growing and are not damped, but their growth is in fact suppressed below the horizon scale for as long as the Universe is still dominated by radiation. This suppresses the primordial $n = 1$ spectrum on small scales, producing the characteristic "CDM spectrum". *Note also, the existence of dark matter is why density fluctuations on galaxy scales "survive" Silk damping.*

Prediction for Cold Dark Matter (CDM) spectrum matches the observed Universe well

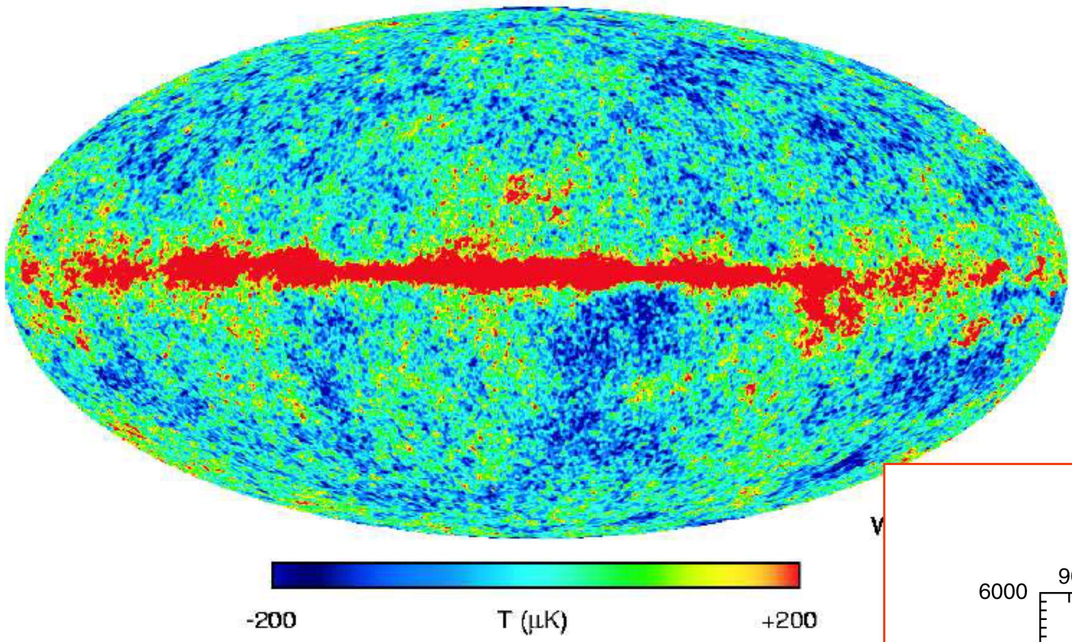
$n = +1$ on very large scales from primordial fluctuations

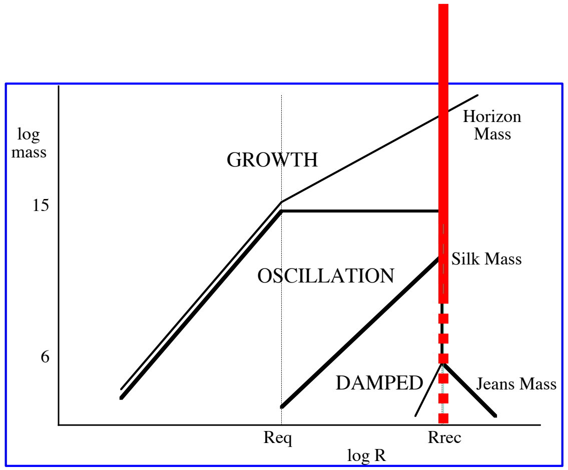


$n = -3$ on galactic scales due to the suppression of growth on small (sub-horizon) scales while the Universe was radiation dominated

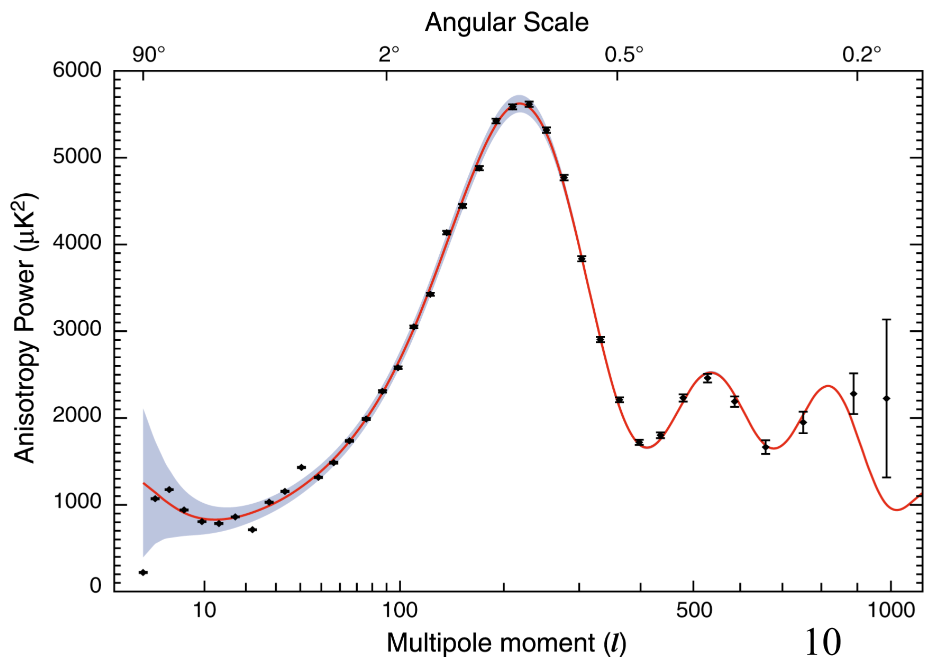
P(k) observations

The last scattering surface (LSS) of the CMB is established when the Universe recombines to neutral atoms, i.e. when pressure “turns off” at the end of the oscillation phase.





Detailed calculations of the expected density (temperature) fluctuations beautifully reproduces the observed angular spectrum of fluctuations on the CMB Last Scattering Surface. The bumps and wiggles are a relic of the oscillating phases.



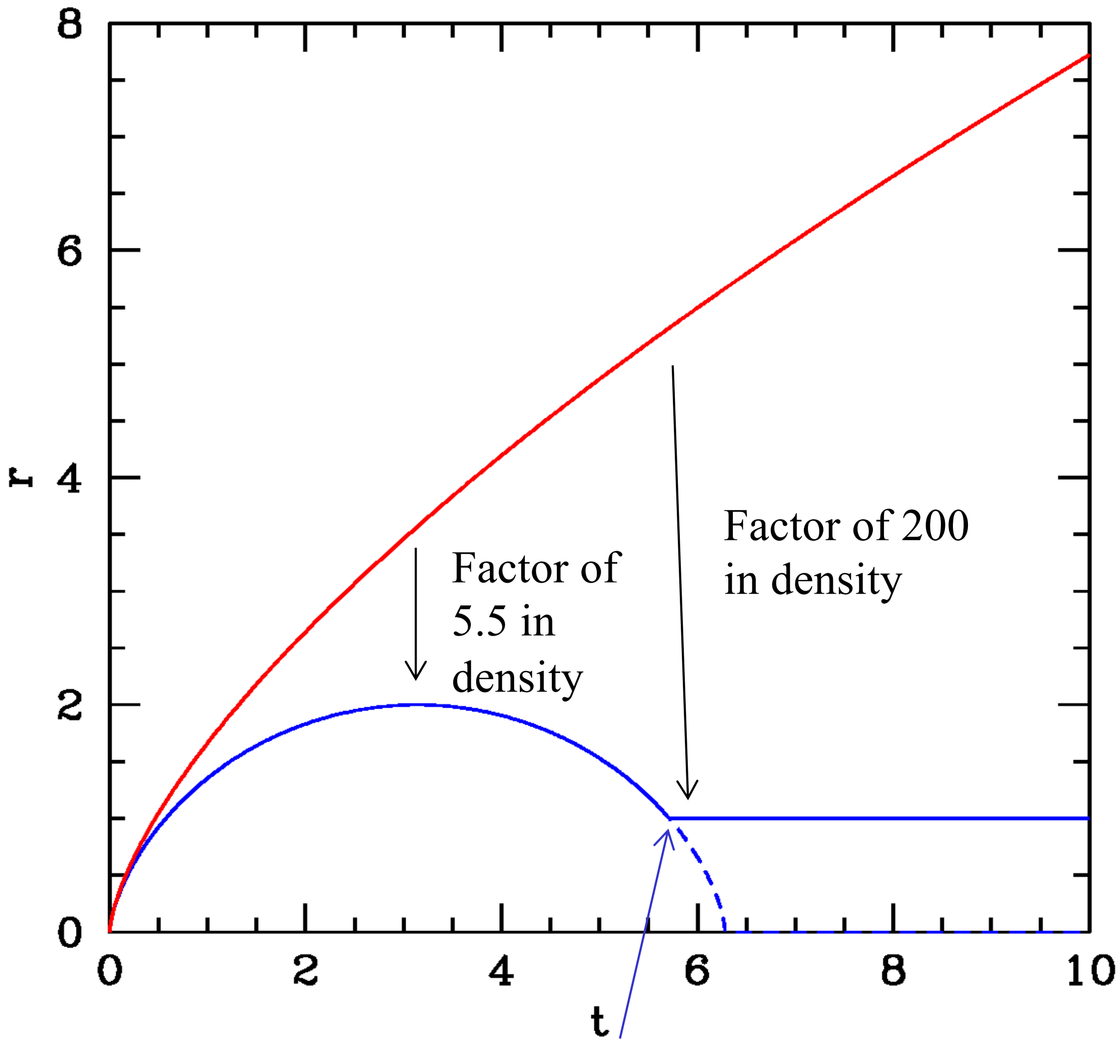
Summary: origin of density fluctuations in the Universe

- An $n = +1$ primordial spectrum was (probably) produced by Inflation, with characteristic amplitude on horizon scale (at all epochs) of $Q \sim \delta\rho/\rho|_H \sim 10^{-5}$.
- This spectrum of fluctuations is modified on small scales to $n \sim -3$ because of the suppression of growth sub-horizon when Universe was radiation dominated.
- Because the density is dominated by (non-interacting) dark matter, the density fluctuations on galactic scales are in fact able to survive through the sound-oscillation phase through to when the Universe forms neutral atoms (and pressure turns off).
- Growth of the amplitude of fluctuations is at best linear with R, c.f. it is exponential in a static medium.
- All this is beautifully verified by observations of the CMB Last Scattering Surface that is established 380,000 years after the Big Bang.

Formation of gravitationally bound objects

- Gravitational instability continues to linearly increase the density contrast $\delta\rho/\rho$.
- As $\delta\rho/\rho$ approaches unity, non-linear collapse begins. Over-dense regions break away from the general expansion and collapse to form self-gravitating objects.

Idealized collapse of "top-hat" density distribution

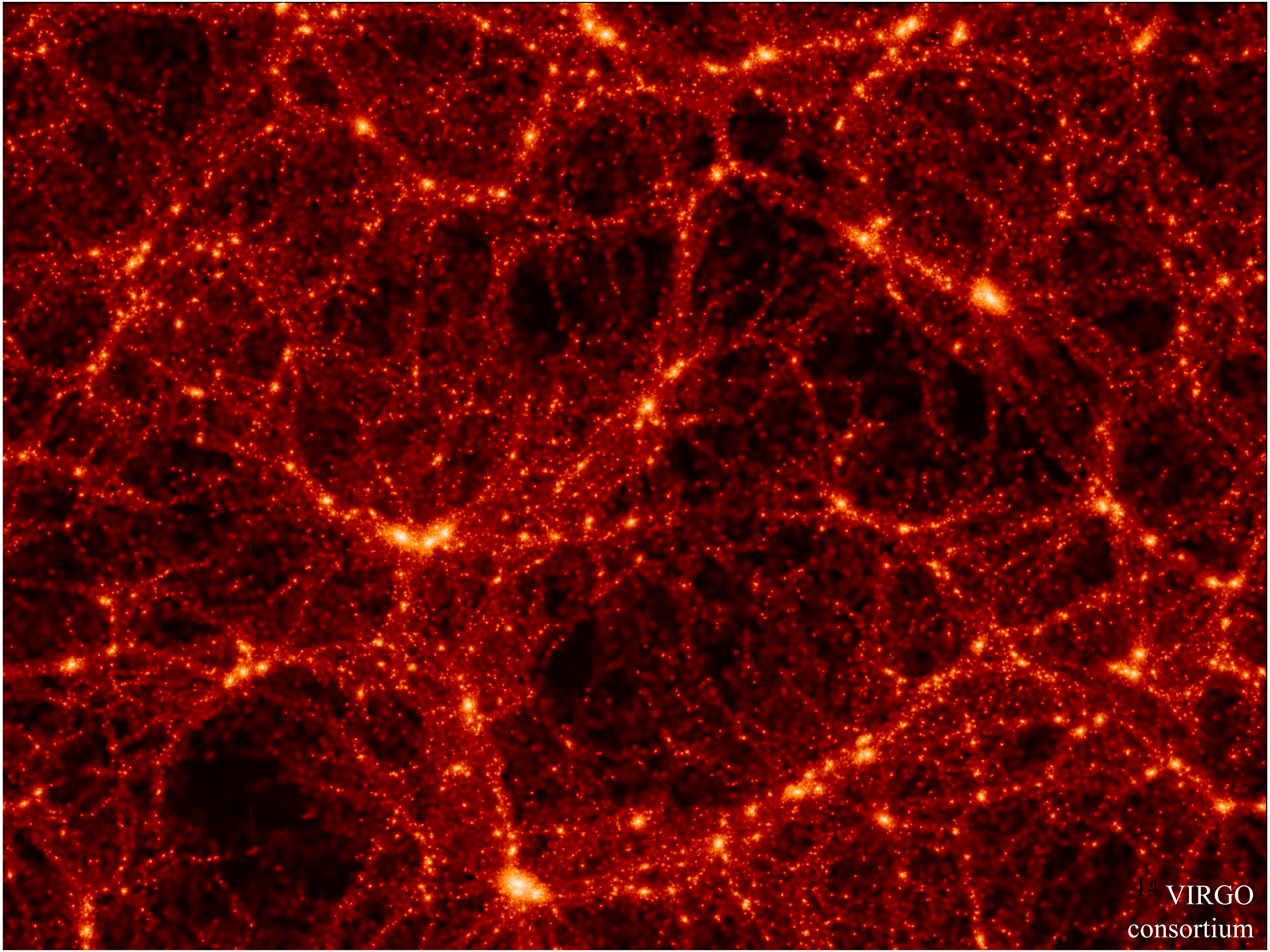


Virial condition stabilizes collapse after a factor of 2 decrease in size

The collapsed, self-gravitating, virialized dark matter structures are called (oddly?) dark matter "halos"

Gravitational collapse only produces a change of 200 in the density (at the time of collapse).

If we see a structure in the Universe with an overdensity of 200 or more, it must have virialised.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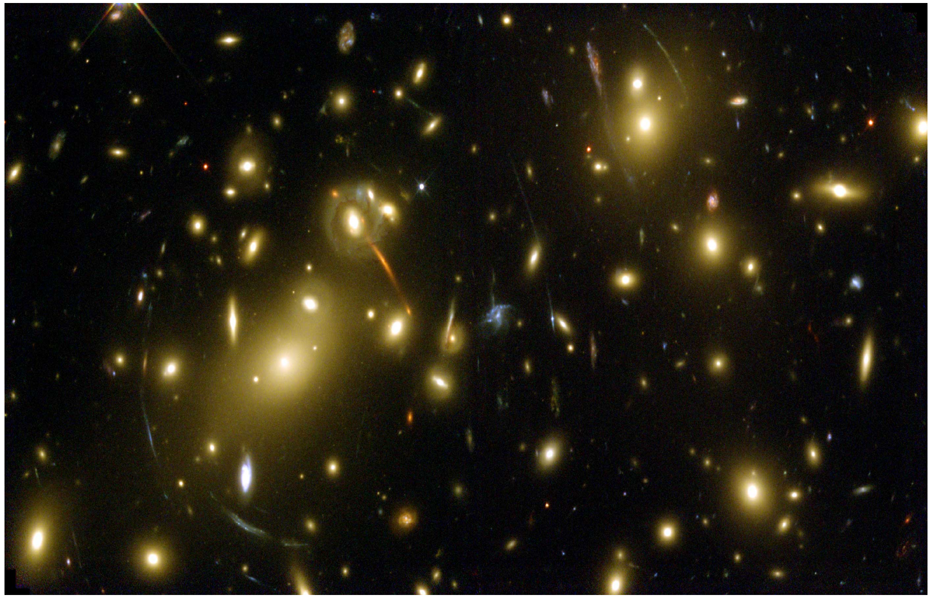
What happens to the baryons in a dark matter halo?

Galaxies: Most baryons are in stars, concentrated at the center of a dark matter halo



Larger scale structures – **groups** or **clusters:** The baryons are in a hot gas distributed like dark matter (plus ~ 20% in stars in galaxies)

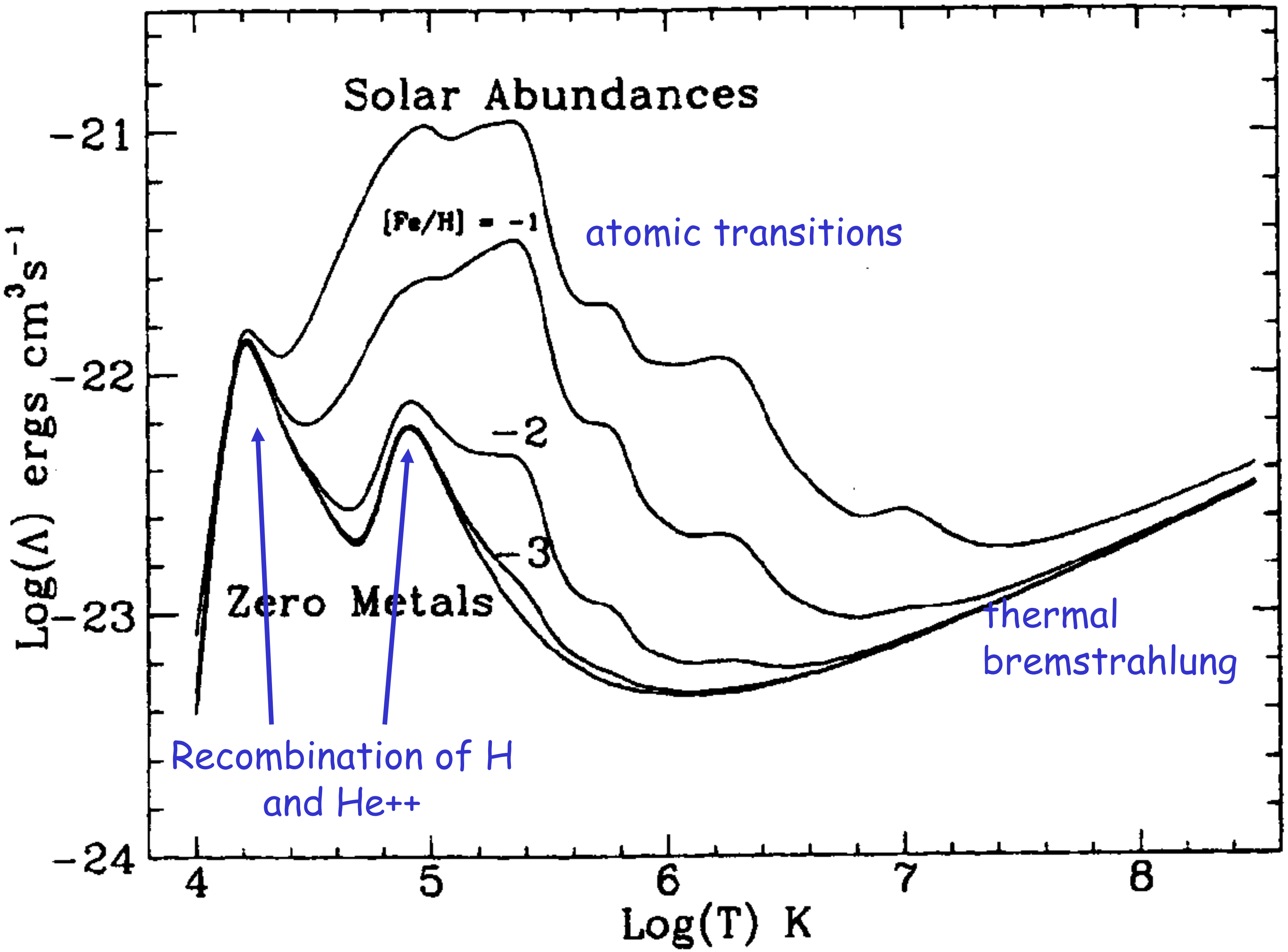
Formation of "galaxies"

- How does the baryonic material separate from the dark matter in "galaxies"? And why does it not do so on larger scales?

Key question: will the baryonic gas in a halo cool and lose energy on a short timescale?

Radiative cooling rate of a gas depends on the temperature, the atomic composition, and the square of the density n^2 (why?)

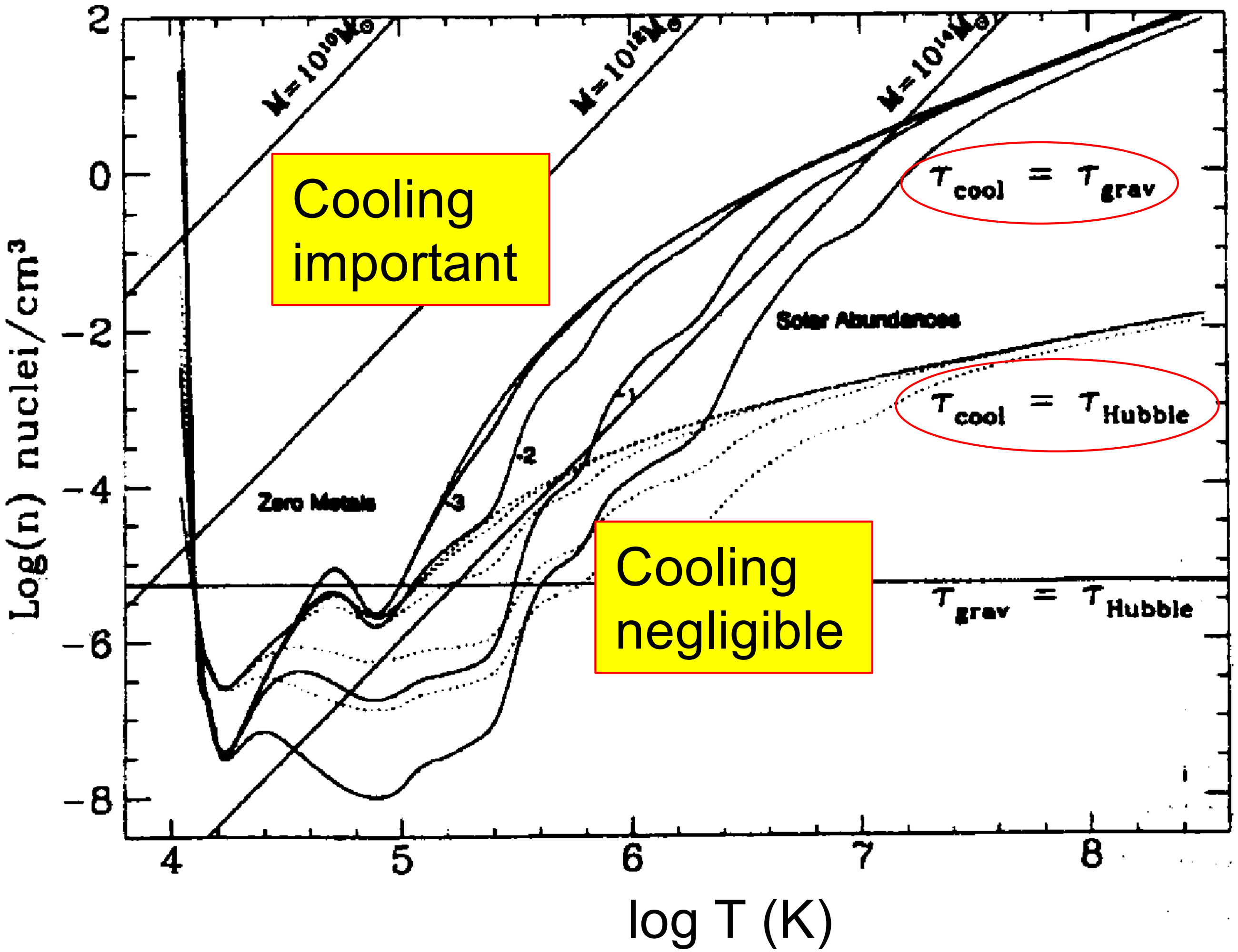
$$\frac{dE}{dT} = -n^2 \Lambda(T)$$



Cooling function

Is cooling important for a gas? Comparison of t_{cool} and $t_{collapse}$

Gravitational collapse time also depends on density. So the relative cooling and gravitational collapse timescales depends on the number density n and temperature T of the gas (as well as on the metallicity/composition of the gas)



$$t_{cool} \sim \frac{E}{dE/dt} \sim \frac{3kT}{2n\Lambda(T)}$$

$$t_{collapse} \sim (4\pi G\rho)^{-1/2}$$

Remember: the density in a halo will depend on when it formed, because it is 200 × cosmic density, and therefore on the relative amplitude of the initial Δ.

So, a "galaxy" is formed in a DM halo when the baryonic material was dense enough (and at the right temperature) to be able to cool effectively, losing energy and sinking towards the center of the (uncoolable) dark matter halo. The baryons increase in density and separate spatially from the dark matter.

What halts dissipation?

- Matter becomes dissipationless (e.g. forms into stars)
- *Most important*: Angular momentum rotationally supports a cold disk of gas. A flat rotating disk is a natural outcome of the conservation of J in contracting system (as on much smaller scales in a protoplanetary disk!)

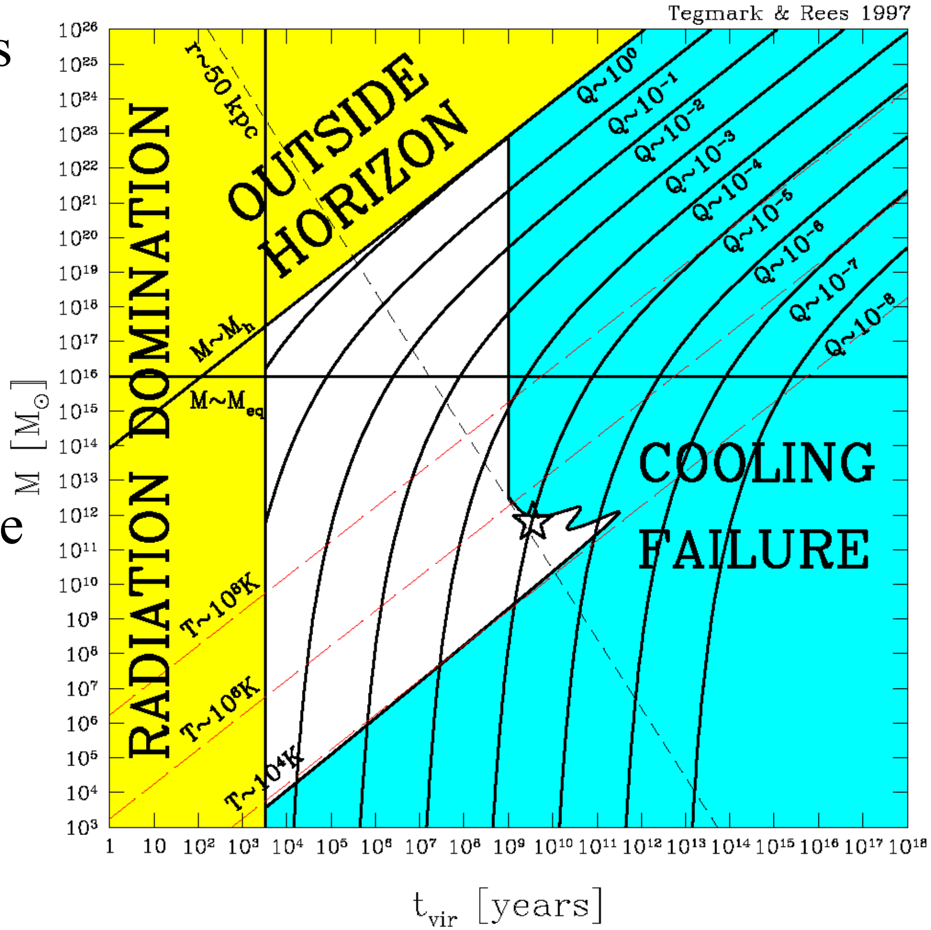
The angular momentum was initially acquired by the proto-galaxy through torques from surrounding (asymmetric) density field.

The typical amount of angular momentum that is gained gives collapse factors of $\times 10$, i.e. density increases $\times 10^3$.

Recap: The value of Q will determine *when* and therefore with what *density* and *temperature* typical dark matter halos of a given mass will form. The density and temperature (and composition) then determines whether cooling of the baryons will be effective in forming a galaxy, or not.

High Q > 10^{-4} means objects form early with high density.

OK for Life? Possibly. But the density within the galaxy may in fact be too high for planetary systems to survive encounters between stars.



Low Q < 10^{-4} means halos form late with a low density and inefficient cooling. *Galaxies would not form.*

Our observed $Q \sim 10^{-5}$ produces "galaxies" only in a fairly narrow range of mass, i.e. DM halo masses around $10^9 - 10^{12}\ M_\odot$. We do not understand well what sets the value of Q during Inflation. **Is $Q \sim 10^{-5}$ somehow optimal for Life?**

Anthropic cosmological considerations:

A. Need to set $\Omega \sim 1$ at early epochs to high precision to produce a long-lived large Universe, i.e. to avoid either recollapse or an empty Universe ($\Omega_m \sim 0$) after small amount of expansion. "Solved" with the concept of *inflation* due to a postulated inflation field of unknown origin. [Inflation also produces n=+1 density fluctuations].

B. It is best to have $Q \sim 10^{-5 \pm 1}$ (= $\delta\rho/\rho$ on the scale of the horizon in $n = 1$)

- $Q << 10^{-6}$: structure form very late with low density and baryonic cooling does not lead to dissipation and further concentration of baryons in galaxy formation
- $Q >> 10^{-4}$: structure forms early with very high densities, star-star interactions disrupt planetary systems

Q comes from amplitude of inflaton field $V(\phi)$ – presently unconstrained by any known physics

C. Need to have $\Omega_M/\Omega_\Lambda > 10^{-5}$
If Dark Energy dominated the late-epoch Universe too early, then structure will stop forming. $(1+z_\Lambda)^3 \sim \Omega_\Lambda/\Omega_M$ The value of Ω_Λ is completely not understood.

D. Need baryogenesis for non-zero Ω_B (understood in concept, not in detail)

Can view these as either: (A) this is the (still largely unknown) Physics that made our Life possible in the Universe *OR* (B) perhaps this is the only type of Universe that any Life could ever observe?

We will skip this last section in the lectures this year.
Please read for your interest and amusement.

Epilogue: The future of matter in the Universe

Let us assume that the geometry/equation of state is such that the Universe continues to expand *indefinitely*. What happens to the structures (complexity) in the Universe that we see at the present time $\tau \sim 10^{10}$ yr ?

A. Classical timescales (up to $\tau \sim 10^{25}$ years):

1. Stellar lifetimes (upto 10^{12} yr).

End of the Sun's Life (relevant for us, at least): In 5×10^{9} yr the Sun will have exhausted H in its core and cease to be a Main Sequence star → Earth will be consumed in expanded Red Giant Phase. 10^{9} yr later, the Sun will become a White Dwarf stellar remnant.

Maximum lifetime of low mass stars (0.08 $M_{\odot}$) is about 10^{12} yr.

Star-formation rate in the Galaxy is declining ~ exponentially with $\tau_{1/2}$ ~5 Gyr. So, there is unlikely that there is continuing fusion after **$\tau \sim 10^{13}$ yr.**

→ **end of the "stellar epoch" in the Universe (which started a few 10^{8} yr after the Big Bang). No more fusion power from stars.**

2. Disruption of planetary systems ?

Timescale for disruption of a given planetary system due to close encounter with another star, from population of number density n, moving with velocity v

$$\tau = (nv\sigma)^{-1}$$
$$\sigma = 2\pi r^2$$

What is r? It must be of order the orbital radius of the planet.

$$n \sim 3\times10^{-41}\ \mathrm{km}^{-3}$$
$$\sigma \sim 2\times10^{16}\ \mathrm{km}^{2}$$
$$v \sim 50\ \mathrm{km\,s}^{-1}$$
$$\Rightarrow \quad \tau \sim 10^{15}\ \mathrm{yr}$$

On time scales of 10^{15} yr, planets are removed from their host stars (= stellar remnants, WD, NS etc) by dynamical interactions

3. Evaporation of galaxies

Random energy exchange between stars through encounters will lead to some stars being lost from a galaxy ($v > v_{esc}$) while the remainder will lose energy and sink to the bottom of the potential well. This is called "evaporation".

A system of N stars of mass m in self-gravitating system of radius R: Virial theorem gives

$$v^2 \sim \frac{GNm}{R}$$

What is now σ? For significant energy exchange, the passage must occur with separation at which the potential energy of the interaction is comparable to the typical kinetic energy of stars, which can be written in terms of the R and N

$$\sigma \sim \left(\frac{Gm}{v^2}\right)^2 \sim \left(\frac{R}{N}\right)^2$$

$$\tau \sim (n\sigma v)^{-1} \sim \left(\frac{NR^{-3}}{Gm}\right)^{1/2} \sim 10^{19} \text{ yr}$$

On timescales of order 10^{19} years, galaxies will "evaporate", leaving central dense systems plus planets and stellar remnants dispersed through intergalactic space

4. What about gravitational radiation?

Energy loss from orbiting objects due to gravitational radiation (as seen in binary pulsar system) is given in terms of energy, *E*, period *P*, and velocity *v*

$$\frac{dE}{dt} \sim \left(\frac{v}{c}\right)^5 \frac{E}{P} \quad \Rightarrow \quad \tau \sim \left(\frac{v}{c}\right)^{-5} P$$

Earth orbiting the Sun: $\tau \sim 10^{20}$ yr (longer than the planetary system disruption timescale, so irrelevant)

Generally for a self-gravitating object of total mass *M* and radius *R*

$$\frac{GM}{R} \sim v^2 \sim \left(\frac{R}{P}\right)^2 \quad \Rightarrow \quad \tau \sim G^{-3} R^4 M^{-3} c^5$$

$M \sim 10^{11}\, M_\odot$, $R \sim 1$ kpc, $\rightarrow$ $\tau \sim 10^{20}$ yr

Conclusion: The final state of the "non-evaporated" galactic remnants must be supermassive blackholes, which will form on timescales of order 10^{20} yr.

B. Greater than 10^{25} years

Decays of structures through quantum effects (!)

1. Decay of the proton:

Expected under GUT theories $t_{p.1/2} \sim 10^{33.5+}$ years

(also: required for primordial baryogenesis to work)

$$p \rightarrow e^{+} + \nu + \bar{\nu} + \gamma$$

$$n \rightarrow e^{+} + e^{-} + \nu + \bar{\nu} + \gamma$$

In compact objects, almost all the rest-mass is liberated (1 GeV per baryon) through e+e- annihilation. Any free protons in space form an electron-positron plasma.

The overall number of protons will decline exponentially

$$N_p(\tau) = N_0 \exp(-\tau / \tau_p)$$

Aside: energy released from proton decay keeps stellar remnants at significant temperatures long after they would otherwise have cooled: neutron stars @ 100K, cold white dwarfs @ 5K

Even if Life increases its "available resource volume" at the speed of light (i.e. if Life controls all of the particle horizon), the total number of atoms available to it will decreases exponentially after ~ 1000 $t_{p,1/2}$.

2. Evaporation of blackholes through Hawking radiation:

Lifetime of BH: $$\tau \sim 10^{66} \left(\frac{M_{BH}}{M_{sun}} \right)^3 \text{yr}$$

t ~ 10^{99} for galactic masses, 10^{117} for supercluster masses (probably the largest that had been formed)

3. Positronium annihilation of diffuse gas:

Excited Positronium P_n formed by e+ and e-, on timescales of 10^{73} yrs through three-body reactions

$$e^+ + e^- + e^\pm \Rightarrow P_n + e^\pm$$

Typically quantum level $n \sim 10^{22}$, $r \sim 10^{12}$ Mpc etc…

Decay to ground-state (and rapid annihilation thereafter) with time scale of 10^{117} yr

But what will we actually see of all this?

Let us assume that the Universe is now entering a period of exponential accelerated expansion, due to our observation of $\Omega_\Lambda \sim 0.75$.

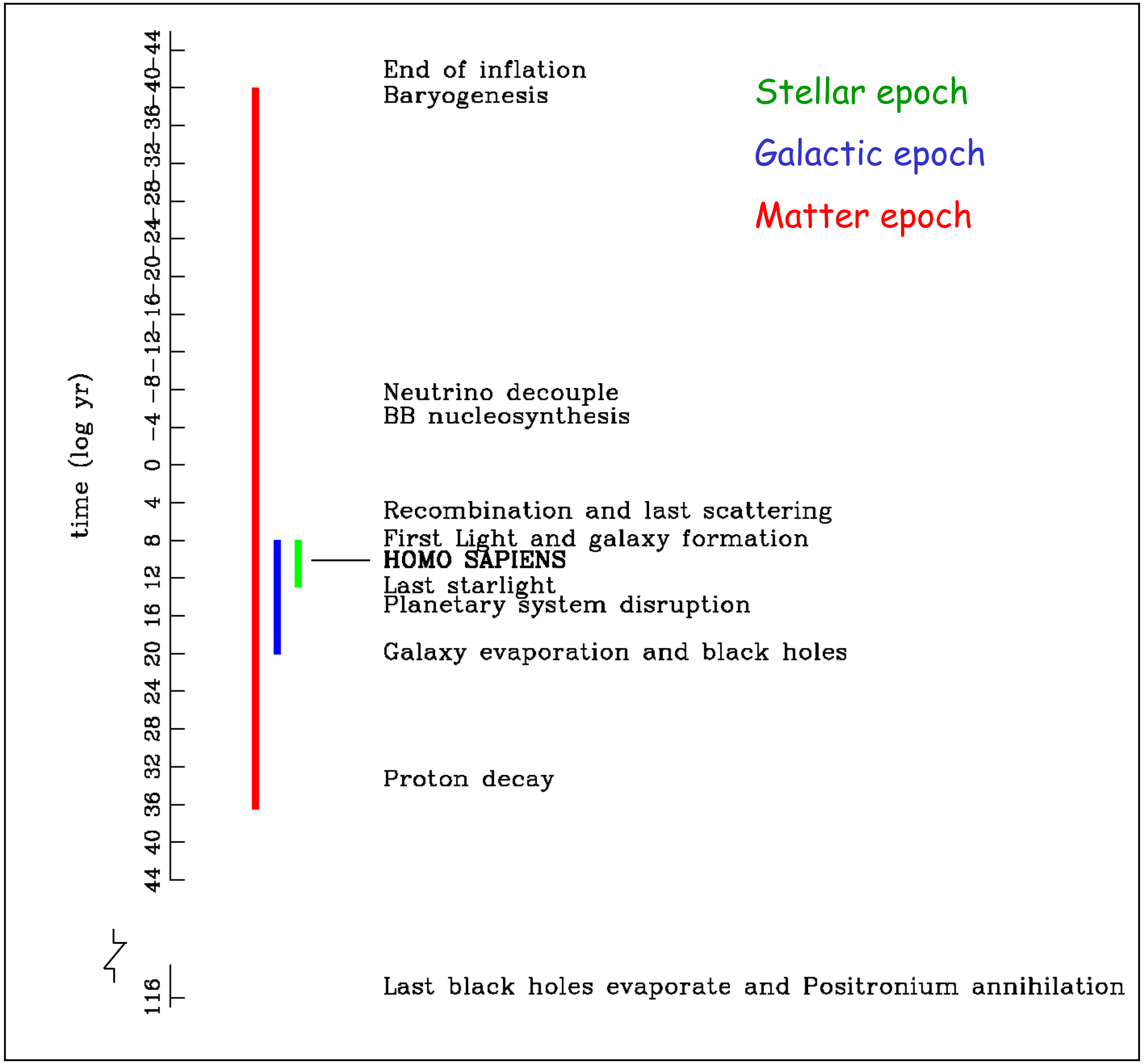
Our current particle horizon of radius ~ *c/H* will change into an event horizon, through which matter will progressively "disappear". In other words, in *comoving* space the horizon will come towards us with an exponential timescale of ~ *H*-1, i.e. 13 Gyr, as more and more of comoving space is expanded out of the horizon (i.e. redshifts tending to infinity).

The present horizon distance *c*/*H* ~ 4300 Mpc: Even the nearest galaxies with distances of ~ 1 Mpc will therefore disappear from the sky after about 8 e-folds of expansion, i.e. after only or order 100 billion years.

Indeed, time is running out for extragalactic astronomy!

Will the horizon ever encroach within our galaxy? That depends on $\rho_\Lambda(R)$.

If ρ_Λ increases (who knows?) and becomes comparable to the densities within gravitationally bound structures, then their dynamics will be governed by ρ_Λ and bound objects will be pulled apart…..



Ephemeral phases of the Universe

The last slide: The six astrophysical processes that led to you and me

1. **Inflation** produced a long-lived Universe with $\Omega \cong 1$. It also produced density fluctuations ($Q \sim 10^{-5}$) of the right amplitude and scale to make "galaxies".
2. **Baryogenesis** produced a non-zero Baryon number, resulting in having some matter left over after matter-antimatter annihilation.
3. Dark matter enabled density fluctuations on galactic scales to survive the acoustic oscillations while the Universe still a plasma and to grow via **(3) gravitational instability**.
4. **Cooling of the baryonic gas** then separated the baryonic from the dark matter in galaxy-scale haloes, concentrating baryons up to a density 10^6 that of the universe as a whole.
5. **Gravitational collapse** of gas clouds in galaxies heats up the interiors of "stars", enabling (5) **nuclear fusion** reactions which both (a) stabilize the (shining) star against collapse for long periods and (b) produce atomic diversity up to iron, including C,O etc. The catastrophic **gravitational collapse** at the end of the life of a (massive) star (c) produces the remaining elements and (d) violently ejects the enriched material back into the surrounding space in supernovae.
6. **Condensation** concentrates these heavy elements onto dust grains (the 2% impurities become dominant), producing both rocky and volatile material, and producing, through further **gravitational growth**, dense proto-planets and ultimately planets…
   … and in these dense, warm, atomically-diverse environments, familiar chemical-based Life could develop!

Last thought: We were all there. Every atom in our body went through 1-6