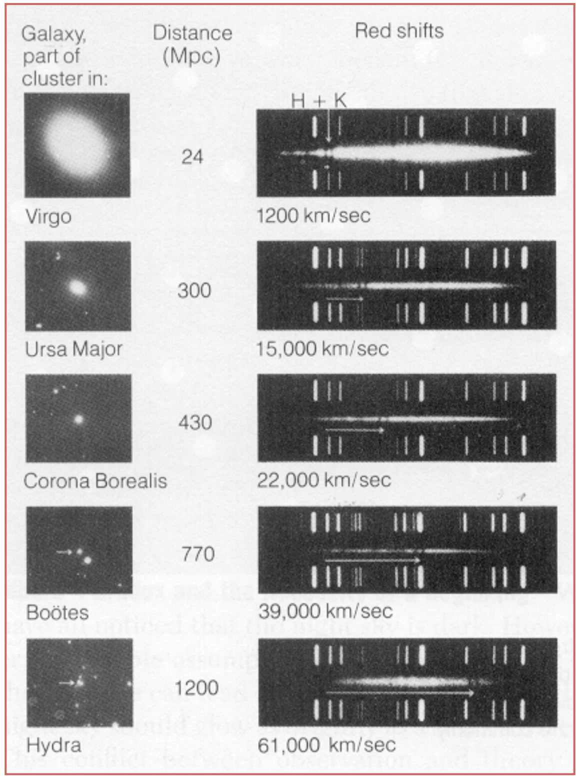
Introduction to Big Bang Cosmology

The Universe is expanding

Hubble (1929) redshift-distance relation (– but also Lemaitre!)

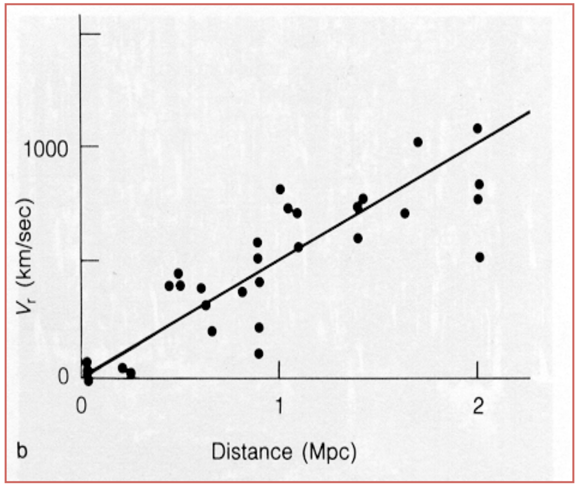
The "redshift" is the shift of spectral features to longer wavelengths (or lower frequencies) in the spectra of distant galaxies

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{\nu_{em} - \nu_{obs}}{\nu_{obs}}$$



Relativistic Doppler effect (but see later)

$$z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1 \quad \Rightarrow \quad z = \frac{v}{c} \quad \text{for} \quad z << 1$$



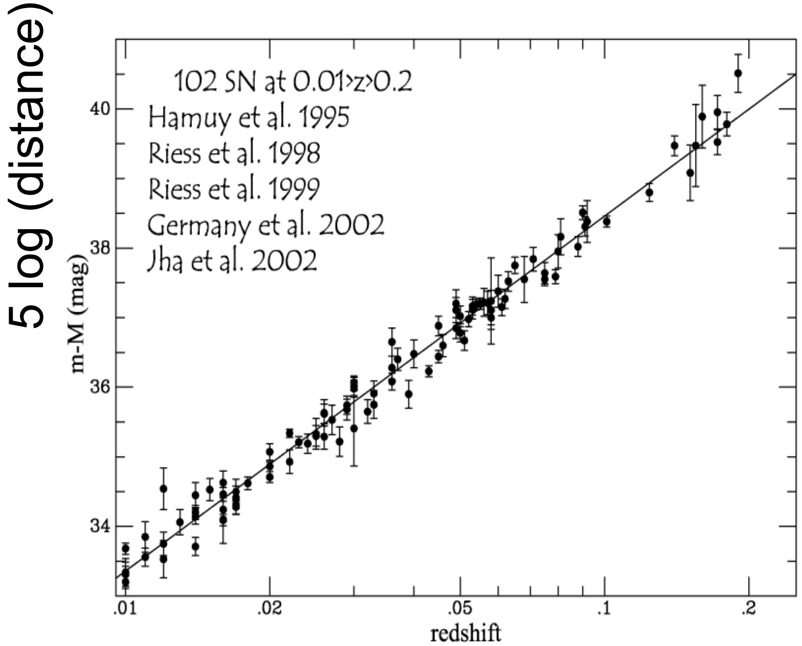
Hubble's "Law"	$z = \frac{H_0}{c} d$	$v = H_0 d$

V $\propto$ d is a signature of an expanding "space"

Although the redshift can be interpreted as a Doppler effect in our inertial frame, defining an inertial frame over large distances becomes problematic.

There is a better way of thinking about the redshift in an expanding Universe (see next slides)

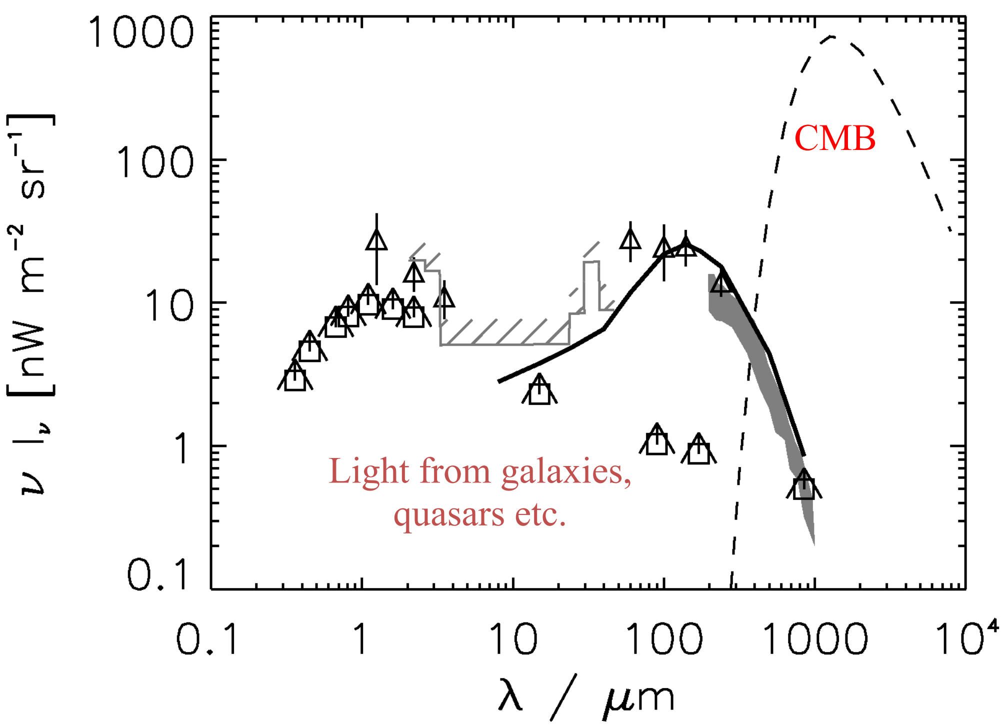
SN1a "standard candles"

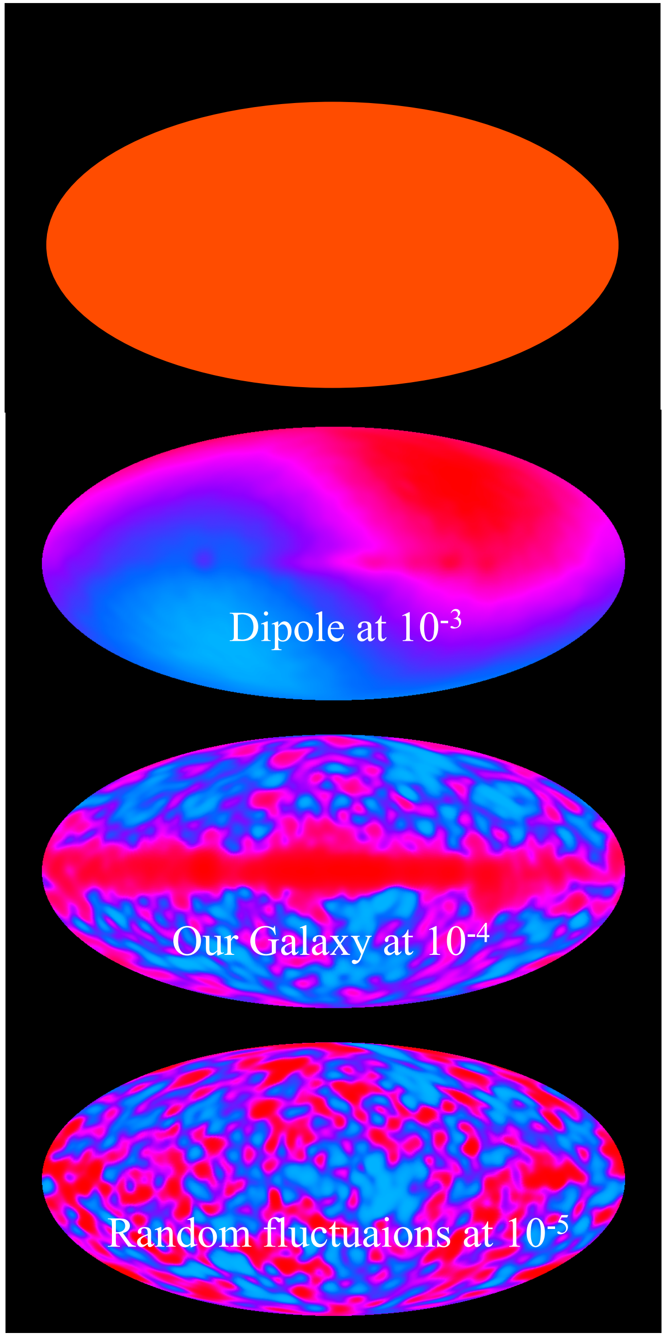


The Universe appears to us (on large scales) to be isotropic and thus homogeneous

Best evidence for this:

- Cosmic Microwave Background (about 97% of radiant energy in Universe)
- Also, galaxy counts etc.





An isotropic and homogeneous expanding Universe is described by the Robertson-Walker metric

$$ds^2 = c^2 d\tau^2 - R^2(\tau)(d\omega^2 + S_k^2(\omega)(d\theta^2 + \sin^2\theta\, d\varphi^2))$$

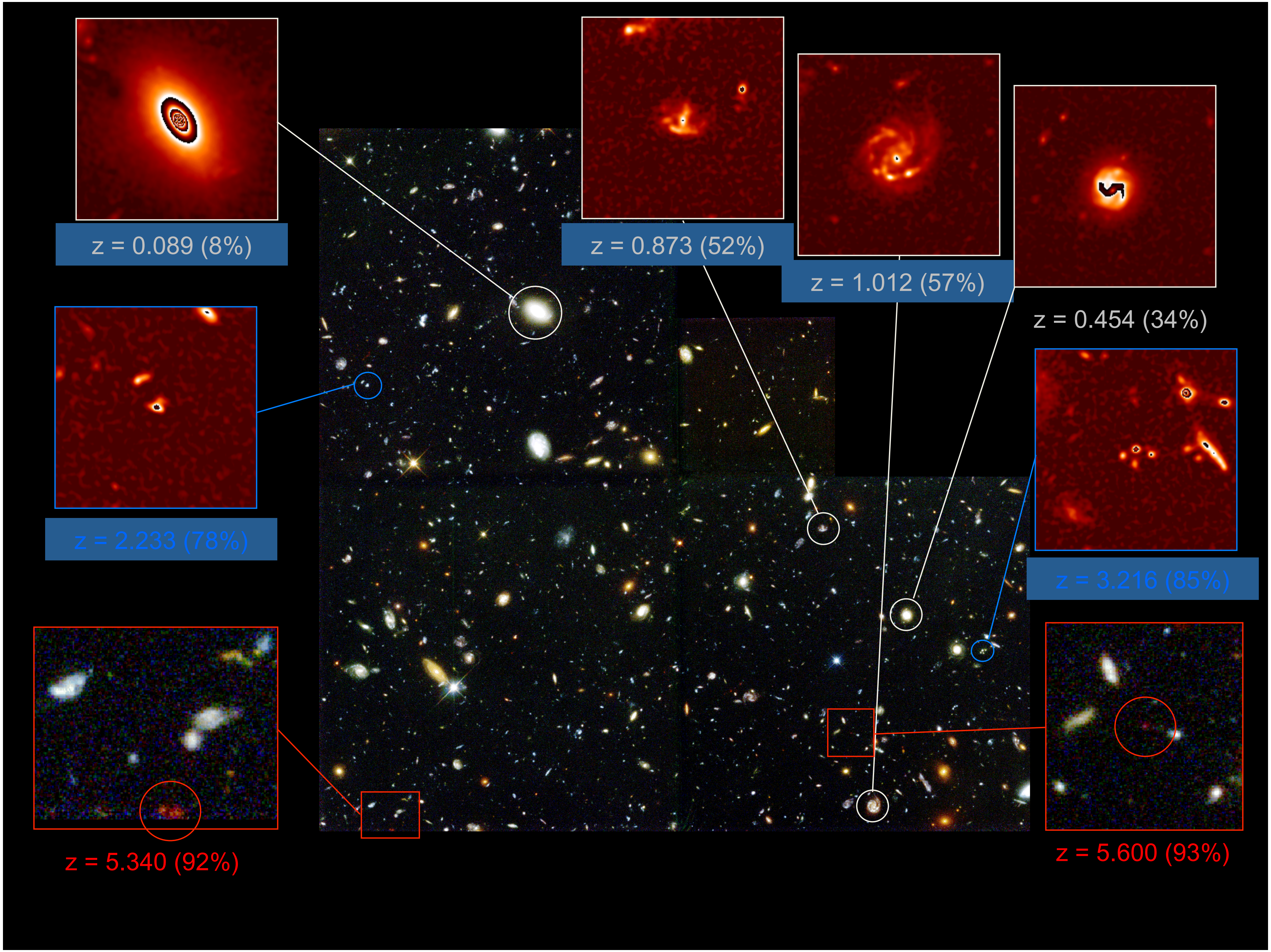
$R(\tau)$ is a cosmic scale factor describing the expansion, and τ is a cosmic time, or epoch.

ω, θ and ϕ are a "comoving" polar coordinate system, such that a given galaxy has fixed position even as the Universe expands. $S_k(w)$ represents effect of curvature of space.

Most important for us: the more useful concept of redshift in an expanding Universe. It is easy to show that if we observe a source of light with redshift z, then this simply tells as the cosmic scale factor when the light was emitted.

$$(1 + z) = \frac{R(now)}{R(\tau_{emit})}$$

Also, Hubble's parameter H is just $H = \frac{\dot{R}}{R}$


z = 0.089 (8%)
z = 0.873 (52%)
z = 1.012 (57%)
z = 0.454 (34%)
z = 2.233 (78%)
z = 3.216 (85%)
z = 5.340 (92%)
z = 5.600 (93%)

Consequences of the Robertson-Walker metric

Principle of Equivalence: Freely falling reference frame must reduce *locally* to the familiar Minkowski form of the metric.

$$ds^2 = c^2 dt^2 - dl^2$$

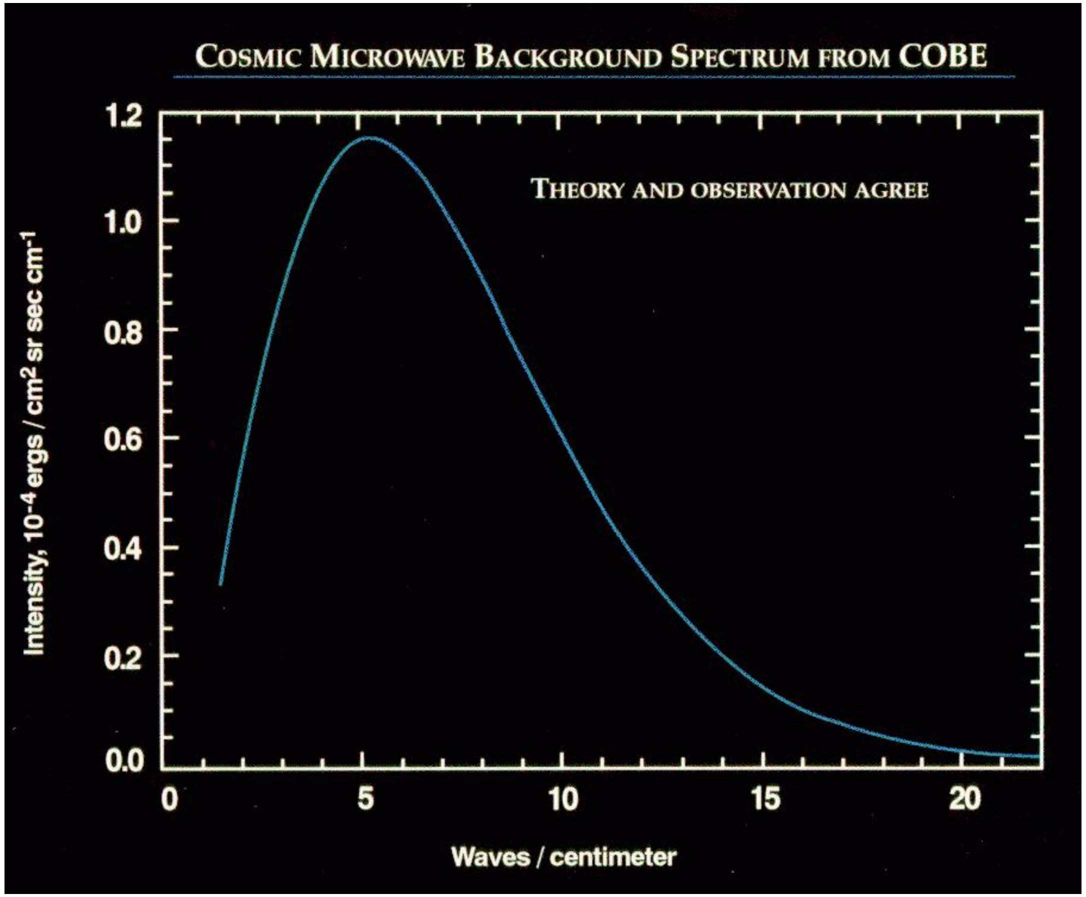
The Robertson-Walker metric therefore allows us to do physics "over there" and transform to observables in our own local frame via invariant ds^2.

Some effects you'd need to know if you were working in this field, but won't concern us very much:

- Clocks appear to run slow (time dilation) by factor $(1+z)$.
- Things appear bigger than you might expect by $(1+z)$, and a fixed rod eventually gets bigger in angular size as the distance increases.
- Photons lose energy as $(1+z)$ (by definition).
- The above lead to surface brightness being dimmed by $(1+z)^4$, and ...
- ... a blackbody appears as a blackbody but with a temperature reduced by $(1+z)$

Black body radiation

The Cosmic Microwave Background (CMB) is a perfect black-body spectrum in shape *and* intensity.
T = 2.736 K



RW metric "explains" the BB spectrum and very low temperature of the CMB. The Universe is manifestly *not* in thermal equilibrium today. But this radiation field was once very much hotter when there would have been thermal equilibrium between matter and radiation. As the Universe cooled, this thermal equilibrium was lost. The CMB today is the *relic* radiation of a hot Universe.

The Friedmann equation for $R(\tau)$

To get $R(\tau)$ we need to reduce the GR field equations for a homogenous isotropic Universe to give the Friedmann equation linking the dynamics, density, and curvature of the Universe.

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - \frac{kc^2}{A^2}$$

The solutions are determined by how the density changes as the Universe expands, $\rho(R)$, the "equation of state". In what form is the gravitating mass at a particular time?

- Normal "cold" matter — $\rho \propto R^{-3}$ — $R \propto \tau^{2/3}$ decelerated
- Radiation or relativistic particles — $\rho \propto R^{-4}$ — $R \propto \tau^{1/2}$ decelerated
- False vacuum? — $\rho \sim$ constant — $R \propto e^{H\tau}$ accelerated
- Empty (curvature dominates) — $\rho \sim 0$ — $R \propto \tau$ undecelerated

Different components may dominate at different times, and may even transmute from one form into another (e.g. relativistic particles cool).

The parameter Ω

Introduce ratio of density in each component i to a critical density defined in terms of H (= $R^{-1}\ dR/d\tau$)

$$\Omega_i = \frac{\rho_i}{\rho_c} \quad \text{with} \quad \rho_c = \frac{3H^2}{8\pi G}$$

$$\left|\left(\sum \Omega_i - 1\right)\right| = \frac{c^2}{A^2}\frac{1}{\dot{R}^2}$$

Friedmann equation again

It can be useful to think of an effective "Ω_k" coming from curvature.

$$\Omega_k = \frac{c^2}{(A\dot{R})^2}$$

Then, the sum of all Ω_i = unity. This is a check of the validity of the Friedmann equation, i.e. of GR itself.

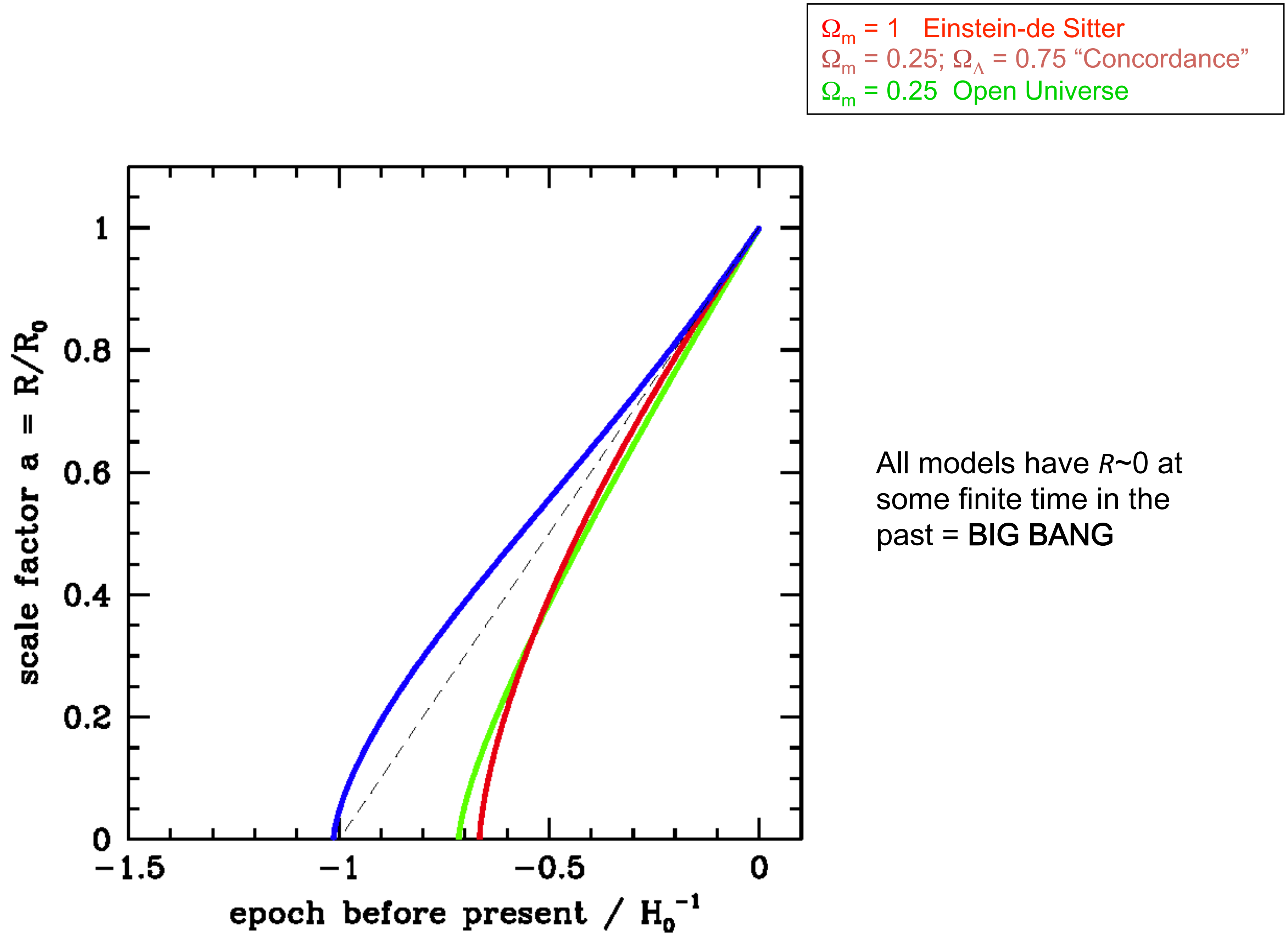
$$\sum \Omega_i + \Omega_k = 1$$

The Figures which follow compare various quantities for three cosmological models. Fifteen years ago, these would have been regarded as equally plausible possibilities.

$\Omega_k = 0$; $\Omega_m = 1$ (flat, no Dark Energy) "Einstein-de Sitter model"

$\Omega_{kv} = 0$; $\Omega_{\Lambda,0} = 0.75$; $\Omega_{m,0} = 0.25$ "Concordance"

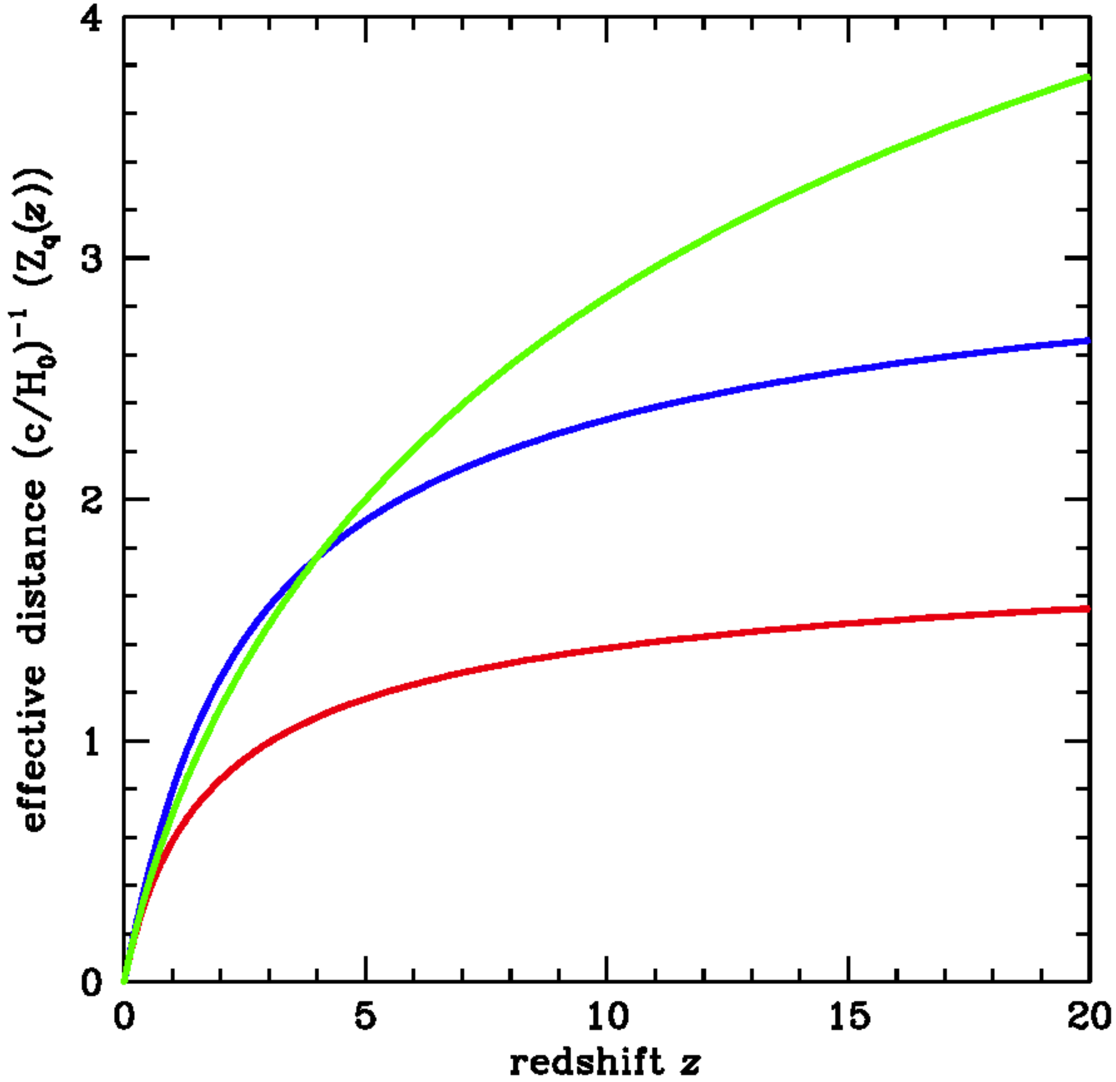
$\Omega_k = 0.75$; $\Omega_{m,0} = 0.25$ (low density, no Dark Energy) "Open Universe"



All models have R~0 at some finite time in the past = **BIG BANG**

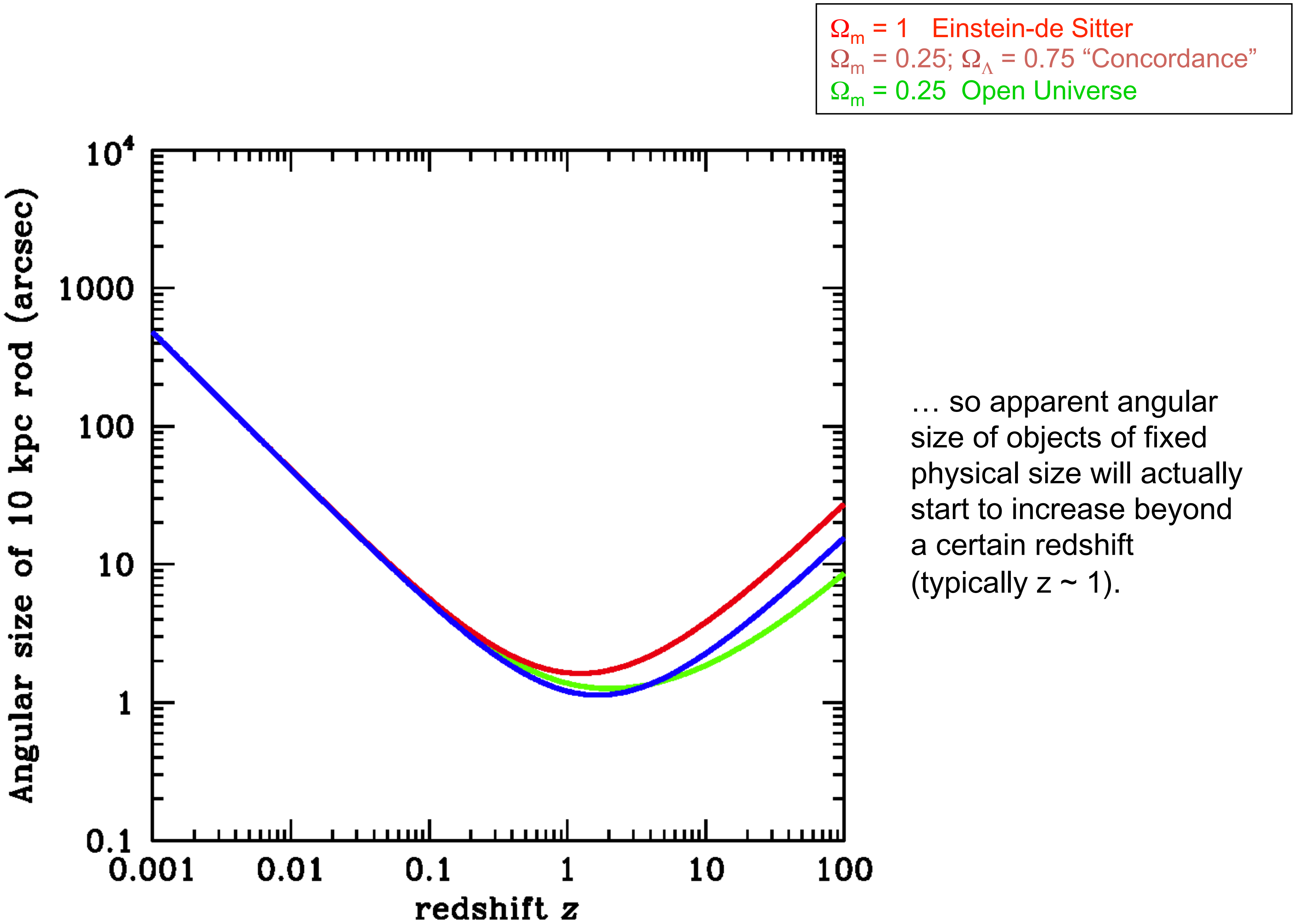
R(τ) and the Big Bang

Ω_m = 1 Einstein-de Sitter
Ω_m = 0.25; Ω_Λ = 0.75 "Concordance"
Ω_m = 0.25 Open Universe



Note how two Ω = 1 models track each other at high *z* – same geometry and same matter-dominated dynamics at high *z*

D(z)

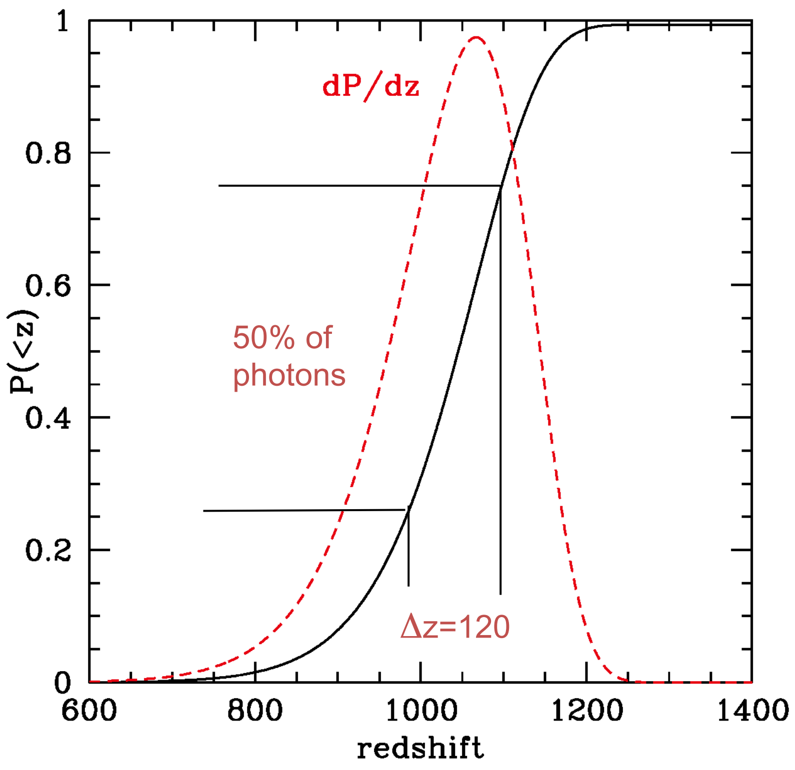


… so apparent angular size of objects of fixed physical size will actually start to increase beyond a certain redshift (typically z ~ 1).

Angular size of 10 kpc rod

The Universe is full of CMB photons, travelling in straight lines.

We can ask of those photons we detect: when did these photons last scatter off of an electron, i.e. scatter onto their present trajectory towards us?

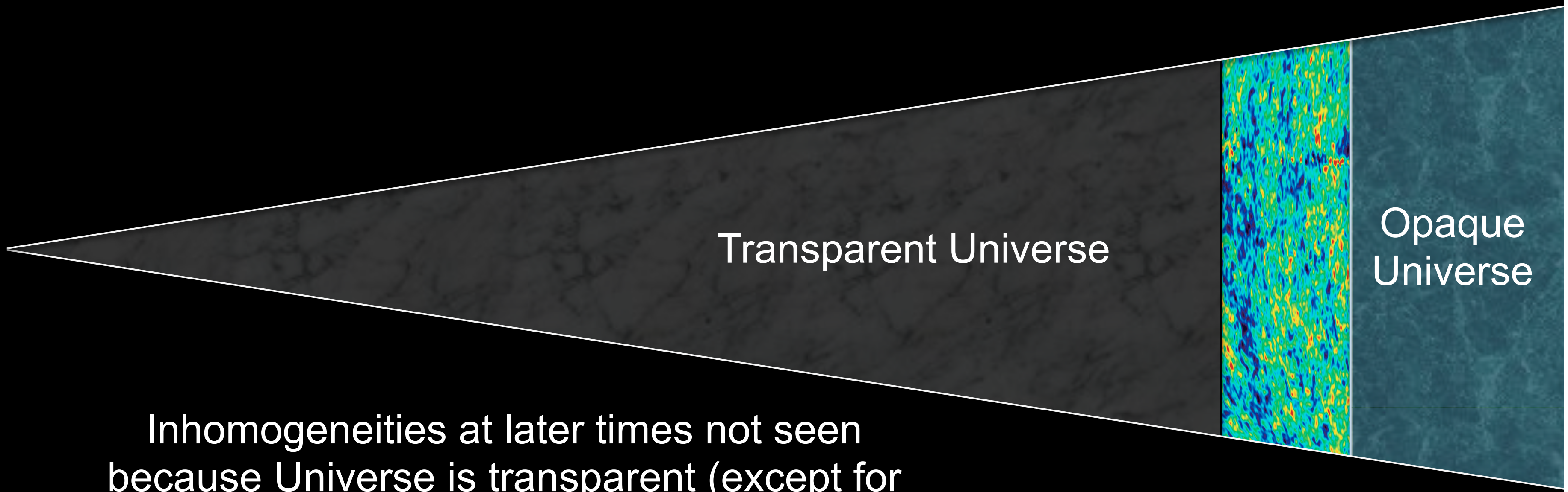


We can calculate the density of free electrons (which increases rapidly at z > 1000 because H is ionized)

The probability distribution of *where the photons were last scattered is dP(z)/dz* is roughly Gaussian with a mean *z* of 1065 and σ ~ 80.

The Last Scattering Surface (LSS) of the CMB is well-defined with a relatively narrow width in redshift space (of order 10%).

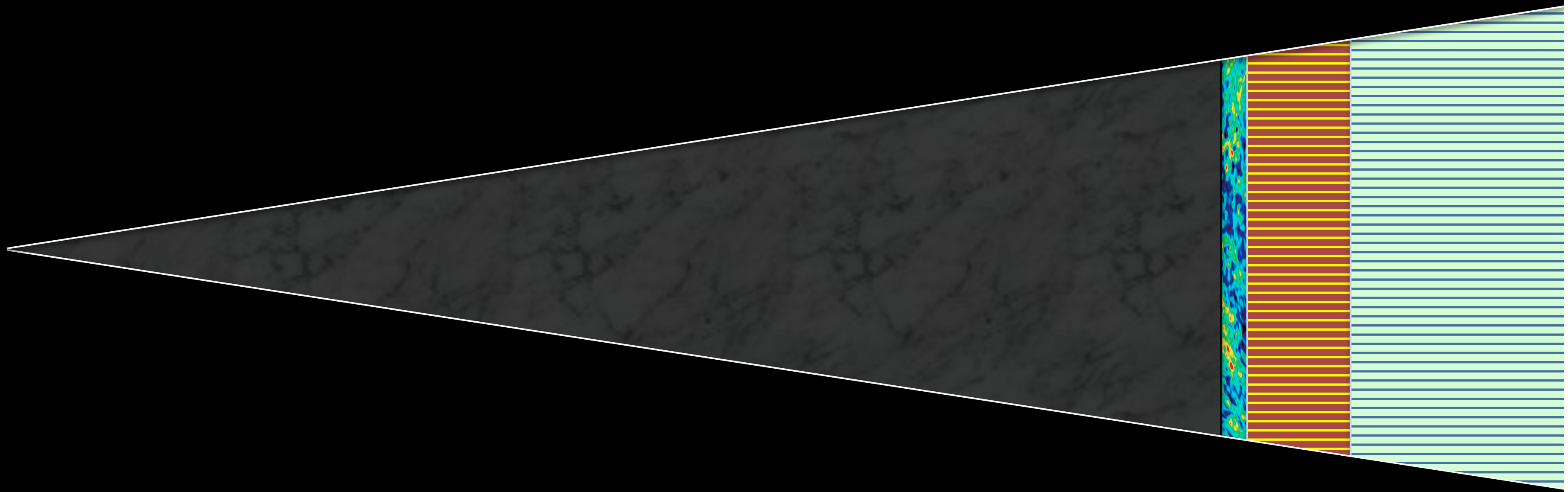
The Last Scattering Surface is the transition between opaque and transparent Universe



Inhomogeneities at later times not seen because Universe is transparent (except for e.g. small gravitational lensing effects)

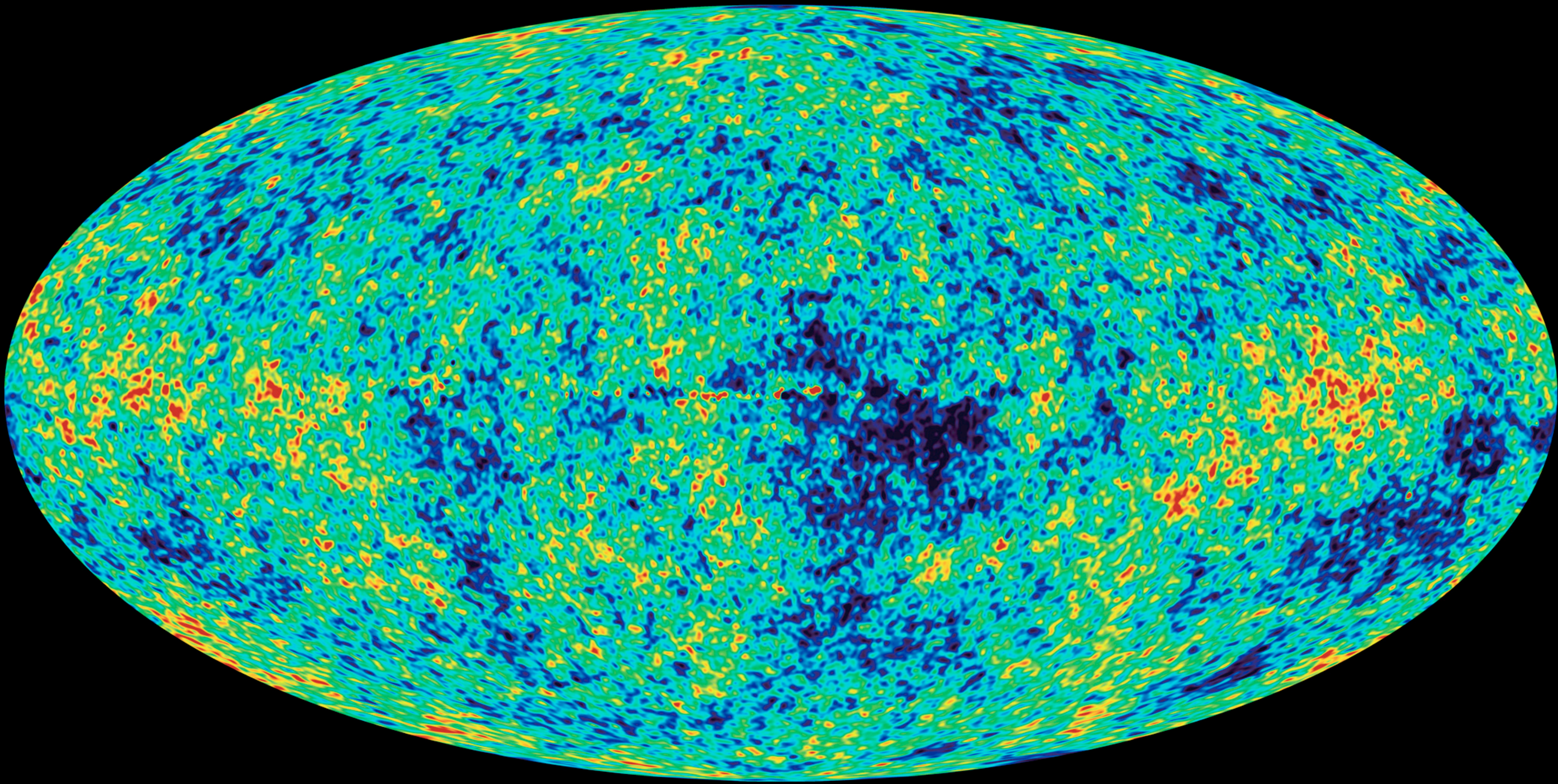
Inhomogeneities at earlier times not seen because all positional information is washed out by multiple scattering

Small anisotropies in the CMB distribution reflect inhomogeneities in the Universe on the Last Scattering Surface that are imprinted as small temperature variations



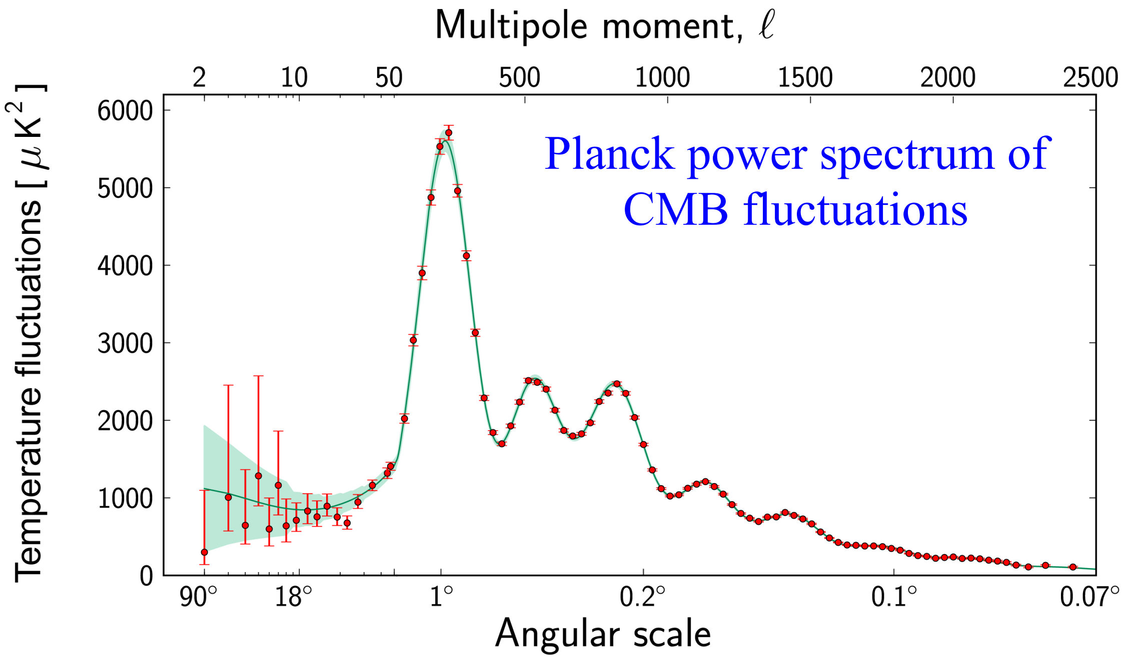
Each Fourier mode in the density distribution of the Universe at LSS will correspond to a Fourier mode in the brightness distribution of the CMB

Small anisotropies in the CMB distribution reflect inhomogeneities in the Universe on the (fuzzy) Last Scattering Surface



Rather than Fourier modes, it is convenient to characterise CMB brightness variations in terms of spherical harmonics

$$\frac{\Delta T}{T}(\theta,\varphi) = \sum_{l,m} a_{l,m} \, Y_{l,m}(\theta,\varphi)$$

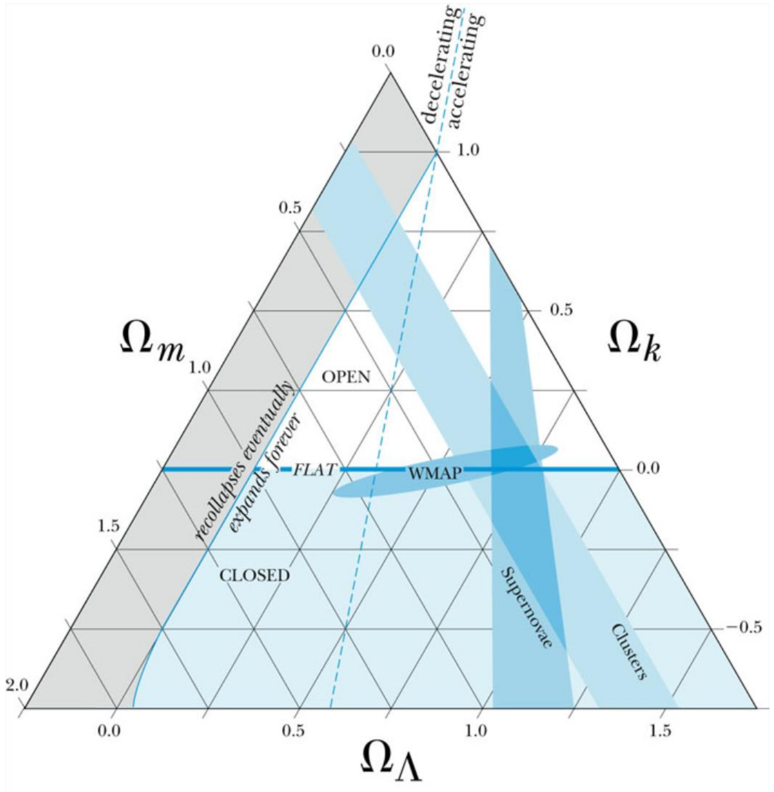


- The Universe was very homogeneous at $z \sim 1000$ (note μK^2 scale)
- We understand the (early-ish) Universe very well (380,000 yr after BB)
- CMB fluctuation spectrum is a Rosetta stone for cosmological parameters

Observational determination of cosmological parameters

- CMB fluctuations
- Estimates of matter density from dynamics
- Big Bang Nucleosynthesis (first three minutes H → ^4He, ^3He, ^2H, ^7Li etc)
- SN1a distance brightness relation

⋯ are all <u>entirely</u> consistent with a universe described by ~ 6 numbers, the so-called Concordance Cosmology



What do we mean by the Concordance Cosmology?

Effectively six numbers:

Spatially flat universe, with 5x more dark matter than baryonic matter, and now dominated by False Vacuum (Dark Energy)

$\Omega_\Lambda = 0.69 \pm 0.02$

$\Omega_M = 0.30 \pm 0.02$ of which $\Omega_B = 0.049 \pm 0.01$ and the remainder in Cold Dark Matter

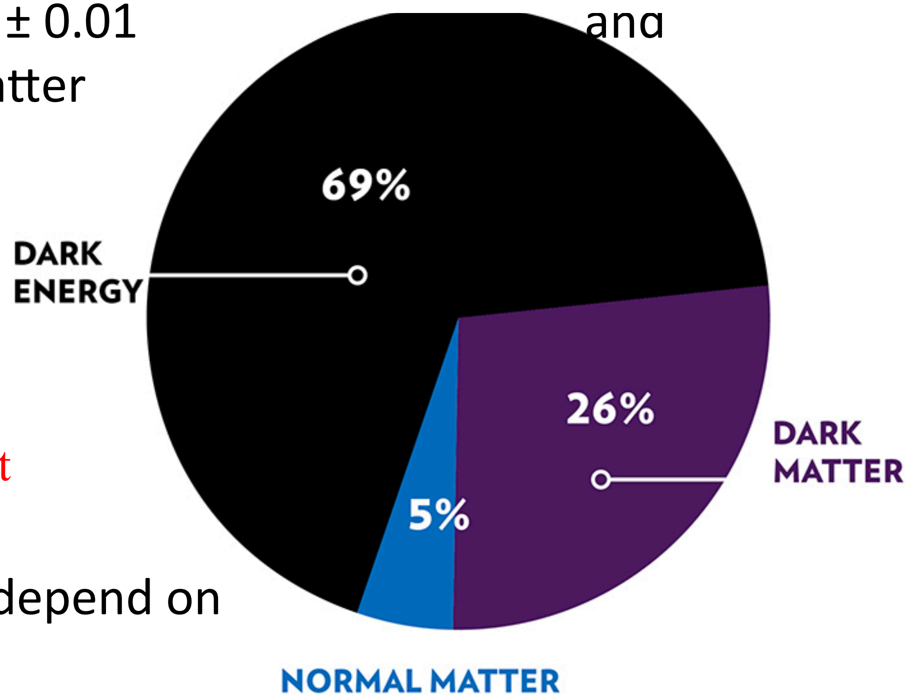
$|\Omega_k| < 0.01$

$H_0 = 68 \pm 3$ kms^{-1} Mpc^{-1}

$\sigma_8 = 0.81 \pm 0.03$

$n = 0.97 \pm 0.01$

Describing dark matter density fluctuations (next lectures)



The precise values and uncertainties depend on

- exactly which data sets are used
- which priors are adopted

Above values and uncertainties should be taken as indicative