Chapter 4

Physics of the interstellar medium

Components of the interstellar medium

The description of the interstellar medium requires the consideration of several physical components: different forms of baryonic matter, the magnetic fields, and the radiation fields.

baryonic matter	gas	molecular gas	
		atomic gas	
		ionized gas	
	dust	small, solid particles	$\lesssim 1\,\mu$m (smoke)
	cosmic rays	relativistic particles	
radiation field			
magnetic field			

Thus, the interstellar medium is a complicated physical system with properties that depend on the:

– mutual interaction of the different components of the interstellar medium,

– interaction of the interstellar medium with stars.

4.1 Gas

4.1.1 Description of a gas in thermodynamic equilibrium

The following formula are valid for a gas in thermodynamic equilibrium.

Temperature and kinetic motion of the particles. The mean kinetic energy of gas particles is given by the temperature of the gas according to:

$$\langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} kT \tag{4.1}$$

For different particles there is (equipartition): $\langle m_1 v_1^2/2 \rangle = \langle m_2 v_2^2/2 \rangle$

The Maxwell-Boltzmann velocity distribution (Fig. 4.1) for the particles is :

$$f_v(T) = \frac{n(v)\,dv}{n} = \Big(\frac{m}{2\pi kT}\Big)^{3/2} e^{-mv^2/2kT}\,4\pi v^2\,dv \tag{4.2}$$

Maximum of $n(v)$: $v_T = \sqrt{2kT/m}$; mean value: $\sqrt{\langle v^2 \rangle} = \sqrt{3kT/m}$
Examples: $v_T(\mathrm{H}, 10^4\,\mathrm{K}) = 12.9$ km/s, $v_T(\mathrm{e}^-, 10^4\,\mathrm{K}) = 550$ km/s.

Figure 4.1: Maxwell-Boltzmann velocity distribution.

Boltzmann equation for the level population of atoms and molecules:

$$\frac{N_i(X^n)}{N_1(X^{+n})} = \frac{g_i}{g_1}\, e^{-E_i/kT}\,.$$

In equilibrium the level population of atoms and molecules have a “Boltzmann” distribution, expressed here as population of level i with statistical weight g_i and excitation energy E_i of an ion X^n relative to the population of the ground state $i = 1$ of that ion.

Saha equation for the ionization degree: The Saha equation describes the gas or plasma ionization degree in thermodynamic equilibrium

$$\frac{N_e N_1(X^{n+1})}{N_1(X^n)} = 2\frac{g_1(X^{n+1})}{g_1(X^n)}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}\,.$$

The Saha equation is given for the ground states of two consecutive ionization states X^n and X^{n+1} with statistical weight $g_1(X^n)$ and $g_1(X^{n+1})$. χ is the energy required to ionize X^n from the ground state.

Planck function for the radiation field: The radiation intensity in a volume element in thermodynamic equilibrium can be described by the Planck equation for the intensity distribution of a perfect black body

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT} - 1}\,. \tag{4.3}$$

Detailed balance: For a gas in thermodynamic equilibrium there exists a detailed balance of microscopic processes, in the sense that the rates for a given process are equal to the rates for the inverse process. Examples for microscopic processes and inverse processes for neutral or ionized atoms are:

collisional excitation $\longleftrightarrow$ collisional de-excitation

line absorption $\longleftrightarrow$ spontaneous and stimulated line emission
collisional ionization $\longleftrightarrow$ 3-body recombination
photo-ionization $\longleftrightarrow$ radiative recombination

4.1.2 Description of the diffuse gas

The gas in the interstellar medium is far from a thermodynamic equilibrium. Therefore the gas properties cannot be simply described by the temperature T. For this reason the temperature equilibrium has to be evaluated considering individual heating and cooling processes, which depend for example on the radiation field, the gas temperature, the level excitation, and the ionization degree.

Radiation field. Essentially everywhere in the Universe the gas temperature $T_{\rm gas}$ is higher than the temperature $T_{\rm rad}$ of the black-body radiation from the 3 K micro-wave background which dominates the global radiation field. On the other hand there are various types of other radiation sources (e.g. thermal radiation of dust, stars, galaxies, ...) which can be important locally. The radiation from these discrete sources is usually strongly diluted and the energy distribution may depart strongly from a black-body curve. Thus, there is essentially everywhere:

$$T_{\rm rad} \neq T_{\rm gas} \,. \tag{4.4}$$

The radiation field may be described by a diluted Planck-function

$$F_\nu = W \cdot B_\nu(T_{\rm rad}) \quad \text{with e.g.} \quad W < 10^{-10} \,,$$

and for many application the radiation field can even be neglected.

Particle densities. In the disk of spiral galaxies, a rough average of the mean proton or mean baryon density is of the order

$$n_b \approx 1\,{\rm cm}^{-3} \,,$$

while a dense interstellar cloud may reach a density of $n_b \approx 10^{+6}\,{\rm cm}^{-3}$. The density is only $n_b \approx 10^{-3}\,{\rm cm}^{-3}$ in hot bubbles and in the galactic halo the value approaches the mean density of baryons in the universe, which is $n_b \approx 10^{-7}\,{\rm cm}^{-3}$.

Thus, there exist the following dominant density regimes for baryons:

stars	$n_p \gtrsim 10^{+20}\,{\rm cm}^{-3}$
diffuse matter in the galactic disk	$n_p \approx 10^{-3} - 10^{+7}\,{\rm cm}^{-3}$
galactic halo	$n_p < 10^{-3}\,{\rm cm}^{-3}$

The Universe is made up, except for an extremely small fraction ($\epsilon \ll 10^{-24}$), of space filled with diffuse matter having a very low baryon density. The density and pressure of the interstellar medium is typically lower than what is reachable in the best vacuum chambers in the laboratory.

Velocity distribution of particles. Fortunately, the Maxwell velocity distribution $f_v(T_{\rm gas})$ is still a good approximation for the diffuse gas. This simplifies very much all calculations, because the kinetic motion of the particles is defined at a given point by a single parameter; the gas temperature $T_{\rm gas}$ or for ionized gas the electron temperature T_e.

Why is the Maxwell velocity distribution valid?
This is not obvious when considering the mean free path length of a particle $\langle \ell \rangle$ and the mean time $\langle t_\ell \rangle$ between two collisions:

$$\langle \ell \rangle \approx 1/n\sigma \qquad \langle t_\ell \rangle \approx \langle \ell \rangle / v_T \tag{4.5}$$

Atomic cross sections are of the order of $\sigma = \pi r_B^2$, where $r_B = 0.53$ Å is the Bohr radius; thus $\sigma \approx 1\text{Å}^2$ (Å $= 10^{-8}$cm).

Example: The mean free path and the mean time between two collisions for an electron in the diffuse interstellar gas in the Milky Way (particle density of $n = 1\,\mathrm{cm}^{-3}$ and temperature $T_{\rm gas} = 10000$ K or $v_e = 550$ km/s) are on the order:

$$\langle \ell \rangle \approx 10^{16}\,\mathrm{cm} = 670 \text{ AU} \qquad \langle t_\ell \rangle \approx 2 \cdot 10^8\,\mathrm{s} = 6\,\mathrm{yr}$$

Note that the kinetic velocity v_T of protons and ions is much lower, and therefore they undergo much less frequently interactions with other particles, apart from many interaction with electrons.

The Maxwell velocity distribution is only valid, because:

- the typical structures (clouds) are larger than $\langle \ell \rangle$,
- the typical time scale for temporal variations is longer than $\langle t_\ell \rangle$,
- the predominant processes for the interstellar gas are the collisions between electrons and electrons, electrons and protons, and electrons and hydrogen or helium atoms, which are (essentially) all elastic collisions. Therefore the kinetic energy is well exchanged and randomized between the particles.

Level population for atoms and molecules: In general the Boltzmann equation for the energy level population is not valid in the interstellar medium because the radiation field is strongly diluted and the radiative transition rates are far from a detailed balance.

The Boltzmann equation for the level population may still be valid for cases where the collisional transitions rates are much higher than all radiative transitions rates (Fig. 4.2). This occurs in high density clouds (many transitions) and for low lying levels of many atoms, ions, and molecules with only slow downward (spontaneous) transitions. Important examples are:

- fine-structure levels of the ground state in many atoms and ions,
- hyperfine-structure level of H I,
- rotational levels of the ground state of H_2,
- rotational levels of molecules in dense molecular clouds.

For these cases, the level population is defined by the gas temperature $T_{\rm gas}$.

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Figure 4.2: Illustration of the dominant processes for the population of energy level of atoms and molecules.

Apart from these special cases the level population of an atom has to be calculated from equilibrium equations which take the individual transition processes (collisional and radiative) into account.

4.1.3 Ionization

The Saha equation is not valid for the gas in the interstellar medium and it must be distinguished between two ionization regimes, photoionization and collisional ionization.

Photoionization equilibrium. Hot stars emit a lot of energetic photons ($h\nu > 13.6$ eV) which are capable to ionize the surrounding hydrogen gas. The ionization degree at a given location can be described by the following equilibrium (rates per volume element and time interval):

number of photo-ionizations = number of radiative recombinations.

For hydrogen this can be written as:

$$N_{\mathrm{H}^0} \int_{\nu_0}^{\infty} \frac{4\pi I_\nu}{h\nu}\, a_\nu(\mathrm{H})\, d\nu = N_e\, N_p\, \alpha(\mathrm{H}, T)\,. \tag{4.6}$$

The number of ionization depends on

$$\Gamma_\nu = \int_{\nu_0}^{\infty} \frac{4\pi I_\nu}{h\nu}\, d\nu\,, \tag{4.7}$$

which is the flux of ionizing photons $\nu > \nu_0$ in [photons/cm^{2}s] ($\nu_0 = 3.3 \cdot 10^{15}$ Hz, equivalent to a photon energy of 13.6eV). Γ_ν dilutes with distance d from the photon source like $\propto 1/d^2$ and may be further reduced by absorptions.

$a_\nu(H)$ is the photoionization cross section for hydrogen, given for $\nu > \nu_0$ by

$$a_\nu(H) \approx 6.3 \cdot 10^{-18}\, \mathrm{cm}^2 \cdot (\nu_0/\nu)^3\,.$$

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Figure 4.3: Photoionization cross section for H^0, He^0 and He^+.

The number of radiative recombinations is described by the densities of electrons and protons and the recombination coefficient for hydrogen:

$$\alpha_B(\mathrm{H},T) = 2.6 \cdot 10^{-13}(T/10^4\mathrm{K})^{-0.7}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$$

Photo-ionized nebulae have always a temperature on the order 10'000 K for reasons which will be discussed later in connection with the cooling curve.

For rough estimates on the ionization degree, the number of ionization can be simplified to $\approx N_{\mathrm{H}^0}\,\bar{a}(\mathrm{H})\,\Gamma$, using $\bar{a}(H) = 2.6 \cdot 10^{-18}\,\mathrm{cm}^2$ and $\alpha_B(\mathrm{H}, 10'000\,\mathrm{K})$. This yields the following approximation for the ionization degree

$$\frac{N_p}{N_{\mathrm{H}^0}} \approx 10^{-5}\frac{\Gamma}{N_e}\,,\,[\mathrm{cm}^{-1}\,\mathrm{s}] \qquad (4.8)$$

which is given by the ratio between the flux of ionizing photons and the electron density N_e. The term Γ/N_e is called the ionization parameter.

Equilibrium for collisional ionization. If diffuse gas is hot enough for collisional ionization then the ionization degree is given by the rates of the two following processes:

number of collisional ionizations = number of radiative recombinations.

This is equivalent to the rate equation for the ions X^m and X^{m+1}:

$$N_e\,N(\mathrm{X}^m)\,\gamma_e(\mathrm{X}^m,T) = N_e\,N(\mathrm{X}^{m+1})\alpha(\mathrm{X}^m,T)$$

$\gamma_e(\mathrm{X}^m,T)$: ionization coefficient for the ionization by electrons
$\alpha(\mathrm{X}^m,T)$: recombination coefficient

Collisional ionization and radiative recombination are both proportional to the electron density and the equilibrium depends only on the ionization and recombination coefficients. Therefore, one can assume for a gas in an equilibrium state (not rapidly changing with

time):
The ionization degree of collisionally ionized gas is a function of temperature:

$$\frac{N(\mathrm{X}^{m+1})}{N(\mathrm{X}^m)} = \mathrm{funct}(T) = \frac{\gamma_e(T)}{\alpha(T)}$$

For example the ionization degree of hydrogen is $N(\mathrm{H}^+)/N(\mathrm{H}) \approx 0.003$, 0.09, 1.2, 14, and 83 for electron temperatures of $T_e = 10'000$ K, 12'500 K, 15'800 K, 20'000 K and 25'100 K respectively. A good diagnostic tool for the determination of the ionization degree of a hot, collisionally ionized gas are the emission lines from different ionization states of Fe (see Slide 4–1).

Recombination time scale: The equilibrium for collisional ionization or the photo-ionization equilibrium requires a constant input of energy. For collisionally ionized regions this energy is provided usually by shock fronts due to gas moving supersonically. In photo-ionized regions this is the ionizing radiation. If this energy sources stops then the gas will recombine within a typical time scale of:

$$t_{\mathrm{rec}} \approx \frac{N_p}{N_e\, N_p\, \alpha(\mathrm{H}, T)} \approx \frac{4 \cdot 10^{12}\mathrm{sec}}{N_e\, [\mathrm{cm}^{-3}]}$$

4.1.4 H II-regions

We assume that all ionizing photons from a hot star are absorbed by a surrounding nebula. Such an ionized nebula is called radiation bounded.

For such a nebula we can formulate a **"global" photo-ionization equilibrium**, where the emission rate of ionizing photons $\nu > \nu_0$ is equal to the number of recombinations in the entire nebula. For a spherically symmetric, homogeneous nebula this can be written as:

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\, d\nu = Q(\mathrm{H}^0) = \frac{4\pi}{3}\, r_s^3\, N_e N_p \alpha_B\,. \tag{4.9}$$

$Q(H^0)$: emitted, ionizing photons [photons/s]
r_s: radius of the ionized nebula
$N_H = N_e = N_p$: electron density (= proton density for a pure hydrogen nebula)
α_B: recombination coefficient for all recombinations into excited levels of H I; radiative recombinations to the ground state produce again an ionizing photon and are not counted for the ionization equilibrium.

The radial extension of the nebula is called the Strömgren-radius r_s, which follows from the previous equation:

$$r_s = \left(\frac{3}{4\pi\, \alpha_B}\, \frac{Q(\mathrm{H}^0)}{N_e^2}\right)^{1/3}\,. \tag{4.10}$$

The mass of the ionized matter in the ionized hydrogen nebula is (m_p = proton mass):

$$m_s = \frac{m_p}{\alpha_B}\, \frac{Q(\mathrm{H}^0)}{N_e}$$

Table 4.1 lists Strömgren-radii for different types of hot and massive main sequence stars, adopting a density of $N_p = N_e = 100\,\mathrm{cm}^{-3}$ and a temperature of $T = 7500$ K.

Table 4.1: Parameters for a spherical Strömgren nebula ($N_e = 100$ cm^{-3}, $T_e = 7500$ K.

star	M$_V$	T^* [K]	log Q(H^0)	r_s [pc]
O5	-5.6	48000	49.67	5.0
O7	-5.4	35000	48.84	2.6
B0	-4.4	30000	47.67	1.1

Photoionization of helium. Essentially the same formalism as for the hydrogen applies also for helium. If the light source emits sufficiently hard photons, then a He^{+2}-zone forms close to the source until the photons with energy > 54eV are absorbed. Then follows a surrounding He$^+$ region by the ionization of He^0 by photons with energy > 24.6eV and further out the He^0-region (see Slide 4–2).

The H I-Strömgren radius remains practically unchanged when He-ionization is included, because each photon that ionizes He I or He II will produce also at least one recombination photon which is capable to ionize hydrogen.

Photo-ionization of heavy elements. For the calculation of the ionization structure of the heavy elements additional processes (e.g. charge exchange) and the diffuse radiation field has to be considered. Numerical calculations are required for accurate estimates. Thereby the ionization structure depends quite importantly on the ionization structure of helium, because the location of the He^{+2}, He$^+$, and He^0-zones defines the radiation spectrum in the nebula, which is strongly changed by the photoionization thresholds and the diffuse emission produced by the recombination of helium atoms and ions.

Qualitatively, there results for the heavy elements always an ionization stratification, with the more highly ionized species close to the radiation source and lower ionized species further out (Slide 4–2). The highest ionization stage present in the nebula gives a qualitative indication of the spectral distribution (≈ radiation temperature) of the radiation from the ionizing source.

4.2 Dust

Interstellar dust is made of small solid particles with radius $a < 1$ μm similar in size to cigarette smoke particles (nano-particles). Interstellar dust can be observed in different ways:

- dark clouds (e.g. the coal-sack region),
- extinction and reddening, mainly in the galactic disk,
- light polarization of back-ground sources,
- strong IR emission,
- scattering of light.

4.2.1 Extinction, reddening and interstellar polarization

Extinction and reddening. The extinction (absorption and scattering) of light from "background" sources depends strongly on wavelength. Short wavelengths (UV-light) are very strongly absorbed and scattered (see extinction curve). For this reason the extinction causes a reddening of the colors of "background" sources in the visual band. The extinction curve has a strong maximum around 220 nm. This can produce in the far-UV continuum of stars a strong absorption minimum. In the visual the extinction curve is smooth and it is approximately a straight line in the extinction A_λ vs. $1/\lambda$ plot (Fig. 4.4). This is equivalent to an extinction cross section which behaves like $\kappa(\lambda) \approx 1/\lambda$.

Figure 4.4: Mean extinction curve $\mathrm{A}_\lambda/\mathrm{E}_{\mathrm{B-V}}$; the normalization in the visual is $A_{\mathrm{V}} = 3.1 \cdot E_{\mathrm{B-V}}$.

The interstellar reddening of an object by dust along the line of sight is often described by the color excess [units in magnitudes] for the filters B (blue) and V (visual = green/yellow):

$$E_{\mathrm{B-V}} = A_B - A_V = (\mathrm{B}-\mathrm{V}) - (\mathrm{B}-\mathrm{V})_0, \tag{4.11}$$

which is equivalent to the difference between the measured color (B − V) and the initial (intrinsic) color $(\mathrm{B}-\mathrm{V})_0$ of an object. The intrinsic color of a star can for example be determined from its spectral type.

The relation between the extinction (reduction of the brightness of an object) and the color excess is:

$$A_{\mathrm{V}} \approx 3.1 \cdot E_{\mathrm{B-V}}. \tag{4.12}$$

This relation holds for the dust in most regions of the Milky Way disk. For some special star forming regions clear deviations from this relation are observed. This points to the fact that the properties of the dust particles are there different from "normal".

Polarization. The absorption by dust particles introduces a linear polarization of the light from the "background" source. The polarization curve $p(\lambda)$ has a broad maximum around 5500 Å with half the maximum value around 12000 Å and 2600 Å (Fig. 4.5). Typically, the polarization p is several % for a reddening of $E_{\mathrm{B-V}} = 1$ mag ($p \leq 9 \cdot E_{\mathrm{B-V}}$ %/mag).

Figure 4.5: Wavelength dependence of the interstellar polarization.

The polarization is due to a preferred orientation of the anisotropic (oblate and prolate) dust particles in the Galactic magnetic field. The elongated dust particles are forced by magnetic torques to rotate with their rotation axis parallel to the magnetic field lines. Thus, the orientation of the particles is predominantly perpendicular to the magnetic field lines. For light with wavelengths on similar scales as the particle dimensions the absorption will be stronger for waves with an E-vector oriented parallel to the elongated particle. It results a polarization $p_Q = (I_\perp - I_\parallel)/(I_\perp + I_\parallel)$ parallel to the magnetic field. The fact, that the polarization is strongest in the visual region indicates that particles with a size (diameter) of about 500 nm are most efficient for producing the interstellar polarization.

The measurements of the interstellar polarization to many stars in Milky Way reveal that the galactic magnetic field is aligned with the galactic disk in roughly azimuthal orientation (Slide 4–3). Such measurements of the interstellar polarization direction are very important for the investigation of the large and small scale magnetic field structure in the Galaxy.

4.2.2 Particle properties

Particle size. The extinction and polarization properties of the interstellar dust provide important information on the size of dust particles (see Fig. 4.6):

- Large particles with radii $a \gg \lambda$ absorb and scatter the light (UV-vis-IR domain) in a wavelength-independent way. Thus, the extinction is proportional to the cross section of the particle $\kappa(\lambda \ll a) \approx \pi a^2$.
- Very small particles $a \ll \lambda$ scatter light according to the Rayleigh-scattering laws with a cross section proportional to $\kappa(\lambda \gg a) \propto \lambda^{-4}$.
- The extinction curve is compatible with an average absorption cross section proportional to $\kappa(\lambda) \propto \lambda^{-1}$. This indicates that there exists a broad distribution of particle sizes in the range $a \approx 0.01 - 1$ μm with a power law of roughly $n_S(a) \propto a^{-3}$ (detailed fits yield a power law index of -3.5) for the size distribution of the interstellar particles).
- A large fraction of particles with sizes $a \approx 0.3$ μm must be anisotropic and well aligned perpendicular to the interstellar magnetic field in order to produce the observed maximum around 0.55 μm in the interstellar polarization curve.

Figure 4.6: Dust extinction for different particles sizes.

Dust particle density. The average density of the interstellar dust particles in the Milky Way disk can be estimated from the observed mean extinction. This extinction is roughly 1 mag/kpc (V-band) or an optical depth of about $\tau \approx 1$/kpc. The most efficient absorbers in the V-band are the particles with diameters $2a \approx \lambda = 0.55$ μm. We can use for the particles along the line of sight the cross section: $\pi a^2 \approx 2 \cdot 10^{-9}\,\mathrm{cm}^{-2}$. It follows from

$$\tau = \pi a^2 \cdot n_S \cdot \mathrm{kpc} \approx 1 \tag{4.13}$$

the density of particles with sizes around $a = 0.2$ to 0.3 μm of about $n_S \approx 1.5 \cdot 10^{-13}\,\mathrm{cm}^{-3}$ (this corresponds to 150 particles per km^3). This is a very small dust particle density when compared to hydrogen $n_{\mathrm{H}} \approx 1\,\mathrm{cm}^{-3}$. Despite this, these dust particles are dominating the extinction in the visual region.

The average gas to dust mass ratio is about 160 (± 60) in the Milky Way disk. Thus, about one third of the mass of the heavy elements is bound in dust particles (assuming a metallicity of $Z = 0.02$). Dust and gas are quite well mixed in the interstellar medium and therefore there exists an empirical relation between extinction and hydrogen column density:

$$N_{\mathrm{H}} \approx 6 \cdot 10^{21}\, E_{\mathrm{B-V}}\, \mathrm{mag}^{-1}\, \mathrm{cm}^{-2}. \tag{4.14}$$

Composition. The main components in interstellar space, H and He, form no solid particles for the existing temperatures (> 5 K). For this reason, the main components of the dust are heavy elements. For the composition of dust particles one has to distinguish between two types of elements:

- elements, which easily condense in dust particles (refractory elements), e.g.: Al, Si, Mg, Ca, Cr, Ti, Fe and Ni.
- elements, which are not (noble gases) or not easily bound in dust particles, e.g.: Ne, Ar, N, O, S, Zn.

The abundance of different dust particle types can be inferred from spectroscopic signatures. The observations indicate for the Milky Way disk:

mass	particle type	examples
60 %	silicates	quartz SiO_2, silicates (Mg,Fe)[SiO_4]
20 %	organic molecules	carbon-polymers
12 %	graphite	
4 %	amorphous carbon	
1 %	"PAHs"	poly-aromatic-hydrocarbons, e.g. benzol

Ices of different kinds, e.g. from water H_2O, methane CH_4, and ammonium NH_3 may condense in dense and cold molecular clouds as mantle around a dust nucleus.

Examples for spectroscopic signatures from dust particles are emission or absorption band at the following wavelengths (see also Slide 4–4):

silicates:	9.7, 18 μm	graphite:	2200 Å (?)
ice (H_2O):	3.1 μm,	PAH:	3.3, 6.2, 7.7, 8.7, 11.3 μm

4.2.3 Temperature and emission of the dust particles

The temperature of the dust particles depends strongly on the radiation field, and therefore on the environment. The dust emission from the galactic discs has typically a spectral energy distribution corresponding to a black body radiation temperature of $10 - 30$ K.

The dust temperature can also be significantly higher $\approx 100 - 1000$ K, for example in regions with a strong UV-visual radiation field as expected near bright stars. Above 1000 K the dust particle sublimate. The ice-mantles sublimate already for dust particle temperatures of about 100 K.

The dust particles absorb very efficiently visual and UV-radiation because the particle sizes are comparable to these wavelengths (see above). The emission of radiation in the far-IR (around 100 μm) is not efficient, because the emitting particle is much smaller than the emitted wavelength (a dipole antenna with a length l is also not efficient in emitting radiation with $\lambda \gg l$. For this reason, each absorbed UV-visual photon enhances immediately the temperature of the absorbing particle. Then it takes some time ($\sim$ seconds) until the particle has cooled down again by the emission of many far-IR photons (Fig. 4.7). The absorption of an energetic photon by a small particles yields a high particle temperature because the heat capacity is small. Thereafter, the cooling time is relatively long, because of the small size (inefficient emission) of the particle. Thus, the ice mantels of small particles sublimate first. The spectral energy distribution of the "thermal" emission of a large volume of dust particles corresponds to the black-body radiation with a temperature corresponding to the mean temperature of the dust particles.

Figure 4.7: Temperature as function of time for large (left) and small (right) dust particles in the radiation field of a hot star

IR-galaxies. In many galaxies the interstellar dust hides large regions with embedded sources, like clusters of young, massive stars or an active galactic nucleus. These sources emit a lot of radiation in the visual and UV wavelength range which is first absorbed by the surrounding dust and then re-radiated by the dust as black-body radiation in the far-IR spectral region (Slide 4–4). For this reason these galaxies emit most of their energy around $50 - 100$ μm. Galaxies, which emit much more radiation in the IR than in the UV-visual are called IR-galaxies. The brightest galaxies of this type (so-called ULIGs = ultra-luminous infrared galaxies) emit more than $10^{12}L_\odot$ in the IR. They belong to the brightest galaxies in the Universe.

4.2.4 Evolution of the interstellar dust

Dust particles form in slow and dense stellar winds. They may also form and grow in dense clouds. Various processes erode, modify and destroy the dust particles and this "processing" homogenizes the particle properties in a galaxy.

Condensation and grows. There are two main regimes where dust particles may form or grow:

– stellar winds from cool stars,
– dense molecular clouds.

The gas temperature in dense stellar winds from cool stars drops rapidly with distance from the star. As soon as the temperature is below the dust condensation temperature $T < T_{\rm cond} \approx 1000$ K dust particles will form. Because of the formation of dust, the stellar wind becomes optically thick in the visual (and UV) range so that momentum from the radiation field is transferred to momentum of the optical thick gas/dust (radiation pressure). The chemical composition of the dust particles formed depends on the chemical abundances in the stellar wind. In cool stars the most abundant molecule in the atmosphere besides H_2 is CO. For oxygen-rich stars (O > C; M-type stars) all carbon is blocked in CO and therefore the most abundant dust particles will be silicates $(Fe,Mg)SiO_x$. In carbon-rich stars (C > O; C-stars) all oxygen is blocked in CO and the carbon rich particles like SiC, amorphous carbon, graphite, PAHs, etc. are formed.

Dust particles can also form and evolve in molecular clouds if the density is high enough. In particular the dust particles are reprocessed and therefore homogenized. Small particles can grow, and if the temperature is low enough then ice-mantels (H_2O, NH_3 or CO_2, CH_4) may condense around the dust

Erosion and destruction. The most important processes for the erosion and destruction of dust particles are:

- sublimation,
- absorption of high energy radiation,
- collision with fast moving (thermal) gas particles,
- collision with other dust particles.

Dust particles erode via evaporation of single atoms or molecules. This process is gradual and starts to be significant for temperatures of $T \gtrsim 30$ K. The evaporation is also enhanced if the particle is in a strong UV-visual radiation field. Single, energetic photons may be able to evaporate small particles, because of the relatively strong temperature rise after a photon absorption.

The absorption of an UV photon ($\lambda \lesssim 2000$ Å) may excite an atom or molecule of the dust particle, followed by the ejection of a component.

Collisions with thermal ions can strip off single or several atoms or molecules from the particle. This process is certainly very important and efficient in regions with high gas temperatures $T \gtrsim 10^5$ K. For this reasons the dust particles cannot survive in dense collisionally ionized gas, like supernova remnants.

Collisions between dust particles with large relative velocities $\gtrsim 1$ km/s can lead to the melting and evaporation of both particles. This process is of importance in dense clouds.

4.3 Magnetic fields

Signatures from galactic magnetic fields, e.g. synchrotron emission, interstellar polarization, or Faraday-rotation, can be observed in the Milky Way and many other galaxies.

The large scale magnetic fields in disc-galaxies are aligned with the disc and follow often the spiral structure (Slide 4–5). The origin of the magnetic fields in the Universe is unclear. If there are seed fields present, then they can be enhanced in disc galaxies due to the differential rotation.

Small scale structures of the magnetic field observed in the Milky Way are often connected with high density regions (molecular clouds) or strong dynamic effects, e.g. connected to H II regions and supernova remnants. The magnetic fields are important for the gas motion, because charged particles can essentially only move along the magnetic field lines and not perpendicular to them. In addition, the magnetic fields determine the motion of the relativistic particles (electrons and cosmic rays) in the interstellar medium.

The average magnetic field in the Milky Way has a strength of about 2 μG (1 G $= 10^{-4}$ T; magnetic flux density). The field strengths in H II-region can be about 10 times stronger and in molecular clouds even 100 times stronger.

Charge drift velocities in the magnetic field: The magnetic fields in the Milky Way require, that there exists a differential motion between the charged particles (electrons and ions), i.e. there must exist electric currents. For a field strength of $\approx \mu$G and a ionization of 1 % (fraction of charged particles) one can write according to the first Maxwell law $\mathrm{rot}\vec{B} = \mu_0 \vec{j}$ the following relation between the typical length scale $L = 100$ pc and the drift velocity v:

$$B/L \approx \mu_0 \cdot n_\mathrm{p} \cdot \mathrm{e} \cdot v\,.$$

($B = 1\ \mu$G, $n_\mathrm{p} = 0.01\,n_\mathrm{H}$, $\mathrm{e} = 1.6 \cdot 10^{-19}$ C and $\mu_0 = 1.26 \cdot 10^{-6}\,\mathrm{A\,s\,V^{-1}m^{-1}}$). This yields a differential drift velocity of the charges on the order $v = 10^{-6}$ cm/s. It seems obvious that such drift velocities are possible in the interstellar medium.

Temporal evolution of magnetic fields. Existing magnetic field have a very long life time. The magnetic field can be reduced, if the charges collide so that their relative drift velocities are reduced. Thus, the currents $\vec{j}$ decay if there exists an electric resistivity η in the medium. However, η is extremely small in the interstellar medium, and $\eta = 0$ is a very good approximation. The result is that **the magnetic fields are frozen in** for the interstellar plasma and the magnetic fields behave in the following way:

- the fields move with the plasma,
- the field strength is roughly proportional to the plasma density,
- the magnetic fields pressure $\propto B^2$ acts like a gas pressure,
- the B-field is stabilizing dense clouds against collapse.

The magnetic field may drift out of the highly neutral gas in molecular clouds through a process that is called ambipolar diffusion. Neutral clouds without magnetic fields can easily collapse and form stars.

The differential rotation $\Omega(R)$ in disc galaxies enhances the magnetic field in azimuthal direction. The fields in radial direction are enhanced by the α-effect in small-scale turbulences. Small scale field motions get due to the Coriolis force in the rotating disk a predominant rotation direction, so that the radial field components are enhanced. After a few disc rotations the field strength saturates because the enhancement by the $\Omega - \alpha$-dynamo is compensated by the field dissipation in magnetic reconnections.

4.4 Radiation field

The radiation field in the interstellar and intergalactic medium depends strongly on the location. Often there exists a bright star which dominates the radiation field. Dust may attenuate strongly for some places the visual, UV and soft ($E < 1$ keV) X-ray radiation. Important is also the distribution of neutral hydrogen, because H I blocks efficiently the ionizing far-UV radiation.

It is difficult to define an average radiation field for the interstellar gas. One possibility is to take the radiation field at the position of the sun. This is quite a reasonable approach, because the sun is not in a particular region of the Milky Way. Important components of the diffuse radiation are the cosmic micro-wave background, the diffuse thermal IR radiation from the dust in the galactic disc and the stellar light from the stars in the Milky Way. Slide 4–6 shows the spectral distribution of the diffuse radiation field in the solar neighborhood.

The radiation field has an important effect on the properties of the interstellar gas. The absorption of (energetic) photons can cause the following changes:

- heating of dust particles, the evaporation of the ice mantles and the dust cores,
- photo-dissociation of molecules,
- photo-ionization of atoms and ions as discussed in Sect. 4.1.3.

The gas is heated and additional charged particles are created in all these processes so that also the gas pressure is enhanced. Most important is the ionization of H I, because the optical depths for the ionizing far UV radiation and the electron density depend critically on the $N(\mathrm{H}^+)/N(\mathrm{H}^0)$-ratio.

4.5 Cosmic rays

Cosmic rays are high energy (relativistic) particles in the interstellar medium with

$$E \gg m_0 c^2 \tag{4.15}$$

(m_0c^2: 0.51 MeV for e^-; 928 MeV for p^+).

4.5.1 Properties of the cosmic rays

Energy distribution. The energy distribution of the ions (p^+, α, atomic nuclei) can be described by a power law:

$$J(>E) \sim E^{-q} \quad \text{with} \quad q \approx 1.7 - 2.1\,, \tag{4.16}$$

where $J(>E)$ is the flux of all particles with energy $> E$ (Slide 4–7). The highest measured energies are about $3 \cdot 10^{20}$ eV (≈ 50 Joule). However, such events are very rare $\approx 1\,\mathrm{km}^{-2}\,\mathrm{yr}^{-1}$. Note, that the particle energies reached at CERN LHC are of the order 10 TeV, that is 10^{13} eV. Of course the LHC produces these energies in large quantities.

Observations of cosmic rays. Cosmic rays are detected with particle detectors and Cherenkov-telescopes, which measure essentially the products of a collision with a particle in the Earth atmosphere. For the determination of the ion abundances of the initial particles, one has to bring detectors into space or at least into the stratosphere.

The highly relativistic particles which penetrate into the Earth atmosphere produce in collisions with N and O nuclei a particle shower, a cascade of hadronic particles, mainly pi-mesons ($\pi^{\pm}, \pi^0$), but also nucleons (p,n), anti-nucleons ($\bar{\mathrm{p}}, \bar{\mathrm{n}}$), kaons and hyperons, which collide again with N- and O-nuclei. The unstable particles decay via weak interaction and they produce electrons, positrons, myons, neutrinos, and photons. The photons can also produce matter-antimatter pairs.

The particles in the shower are often relativistic and move faster than the speed of light in air $v > c/n$ (n: refractive index of air) and they produce therefore a Cherenkov-light-cone. Cherenkov telescopes on the ground (Slide 4–8) and particle detectors on the ground (hadrons and charged particles) or underground (e.g. neutrinos) provide then information on the direction and energy of the initial cosmic ray particle.

Elemental abundances. The elemental abundance of the cosmic rays is rather similar to the solar abundance with two important differences (Slide 4–9). The light elements Li, Be, B, which are very rare in the sun and the rather rare heavy elements Sc, Ti, V have a strongly enhanced abundance, similar to the abundance of the next elements in the periodic system of elements. The explanation is that the the abundance minima are filled in by the spallation or fission of heavy elements, in particular of C and Fe. Because the particles travel with relativistic speed through interstellar space, they encounter on their path H and He nuclei, mainly in dense molecular clouds, and they lose in collisions protons and α-particles in a kind of erosion process. These collisions produce also π-mesons and other particles. Observationally important is the following decay of π^0-mesons which produces γ-rays with > 100 MeV which can be observed with detectors on satellites.

Cosmic rays are an important heating source for cold (≈ 30 K), dense molecular clouds, which are dust-shielded from the radiation of stars. Similar to the Earth atmospheres, a shower of energetic particles are created by an interaction with a cosmic ray particle. This leads then to a temperature enhancement in the cloud.

Relativistic electrons. There exists also a component of relativistic electrons in the cosmic rays which is however much weaker. The flux is about 100-times less than for protons. The observed energy regime for electrons is in the range 2 MeV – 1000 GeV. The electrons produce due to their relativistic motion in the galactic magnetic field **Synchrotron-radiation**, which can be observed easily with radio telescope.

4.5.2 Motion in the magnetic field

The motion of the charged cosmic ray particles depends on the terrestrial, interplanetary, and interstellar magnetic field. For the velocity component $v_\perp$ perpendicular to the magnetic field B the motion is controlled by the equilibrium of Lorentz force F_L and centrifugal force F_Z:

$$F_L = \frac{e}{c}\, v_\perp \cdot B = F_Z = m\omega^2 r_c = \frac{m v_\perp^2}{r_c}\,. \tag{4.17}$$

Thus, the particle move along circles with the cyclotron radius r_c (momentum: $p = mv_\perp$)

$$r_c = p\,\frac{c}{eB}\,. \tag{4.18}$$

For relativistic particles there is $p = \gamma m v$, $E = \gamma m c^2$ with the Lorentz factor $\gamma = (1 - (v/c)^2)^{-1/2}$. This yields the relativistic cyclotron radius

$$r_c = \frac{E}{eB} \quad \text{oder} \quad r_c[\mathrm{pc}] = 1.08 \cdot 10^{-6}\,\frac{E[\mathrm{GeV}]}{B[\mu\mathrm{G}]} \tag{4.19}$$

($e = 4.8 \cdot 10^{-10}\,\mathrm{g}^{1/2}\,\mathrm{cm}^{3/2}\,\mathrm{s}^{-1}$ and $1\,\mathrm{G} = \mathrm{g}^{1/2}\,\mathrm{cm}^{-1/2}\,\mathrm{s}^{-1}$).

The distribution of the directions of the cosmic rays is essentially isotropic due to the deviation of the particle motion in the galactic magnetic field (for small energies also the interplanetary and terrestrial magnetic fields are important). The cyclotron radius is of the order of the Galaxy $r_c \approx 10^5$ pc for very high energies $E \approx 10^{11}$ GeV. Particles with such energies move along a straight line and their direction of origin can be determined. On the other hand they can also escape easily from the galactic magnetic field into the intergalactic medium.

4.5.3 The origin of the cosmic rays

The decay of the π^0-mesons, which are created by collisions of the cosmic rays with interstellar matter, can be measured as diffuse γ-radiation in the galactic disc tracing the dense molecular clouds. This indicates that the cosmic rays are not a local phenomenon, but that they exist throughout the entire galactic disc.

The observed spallation (e.g. the overabundance of Li, Be, B) requires, that the relativistic particles pass typically through a column density of matter of about $1/\sigma \approx 5\ \mathrm{g\,cm^{-2}}$ before they reach us. Based on this, the following estimates can be made:

– mean density (ISM)	$n_\mathrm{H} = 1\,\mathrm{cm}^{-3} \rightarrow \rho = n_\mathrm{H} m_\mathrm{H} = 1.7 \cdot 10^{-24}\,\mathrm{g\,cm^{-3}}$
– travel distance	$3 \cdot 10^{24}\,\mathrm{cm} = 1\,\mathrm{Mpc} \approx 3 \cdot 10^6\,\mathrm{Lyr}$
– travel time	$\approx 3 \cdot 10^6$ years

The very high particle energies of the cosmic rays are most likely produced in magnetized shock-fronts. The particles are in these shocks mirrored back and forth (in and out) of a fast moving gas flows having a speed of ($\Delta v \approx 10000$ km/s). Each time the particle is mirrored it is accelerated by Δv. Such shock fronts are produced by supernova explosions and pulsar winds. The highest energy particles may originate from extra-galactic sources, for example quasars where shock fronts in relativistic jets are responsible for the acceleration.

4.6 Radiation processes

In astronomy the most important source of information is the observations of the electro-magnetic radiation. The flux of the observed radiation

$$\vec{F}(x, y, \lambda, t)$$

can be determined as function of the following parameters:

– coordinates: x, y (right ascension, declination) $\rightarrow$ intensity images,
– wavelength $\lambda \rightarrow$ spectral energy distribution,
– time $t \rightarrow$ light curves,
– polarization, which is the orientation of the electric vector of the electro-magnetic wave; thus the flux is a vector quantity $\vec{F}$.

Physical properties of astronomical objects can be derived from this information. However, this requires a good knowledge of the physics of the radiations processes which take place in the interstellar medium. Important radiation process are discussed in this chapter.

4.6.1 Radiation transport

The **radiative transfer equation** for a sight line describes the change of the radiation energy dI (or intensity) along the optical path ds by contributions from emission processes and the weakening of the intensity by absorption processes:

$$dI_\nu = \epsilon_\nu\, ds - \kappa_\nu I_\nu\, ds \tag{4.20}$$

$I_\nu =$ spectral intensity	$I(\vec{r}, \vec{n}, \nu, t)$	$\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}\,sr^{-1}}$
$\epsilon_\nu =$ emission coefficient	$\epsilon(\vec{r}, \vec{n}, \nu, t)$	$\mathrm{erg\,cm^{-3}\,s^{-1}\,Hz^{-1}\,sr^{-1}}$
$\kappa_\nu =$ absorption coefficient	$\kappa(\vec{r}, \nu, t)$	$\mathrm{cm^{-1}}$

The absorption coefficient $\kappa = \sigma \cdot n$ includes the cross section per particle $\sigma\,[\mathrm{cm}^2]$ and the particle density $n\,[\mathrm{cm}^{-3}]$. The geometric dilution $\sim 1/d^2$ of the radiation energy coming from a source is taken into account by the solid angle dependence $[\mathrm{sr}^{-1}]$.
The radiation transfer equation is a first order differential equation:

$$\frac{dI_\nu}{ds} = \epsilon_\nu - \kappa_\nu I_\nu\,. \tag{4.21}$$

Optical depth and source function. The transfer equation takes a particularly simple form if we use (except for $\kappa_\nu \approx 0$) the so-called optical depth $d\tau_\nu = \kappa_\nu ds$ and the source function $S_\nu = \epsilon_\nu/\kappa_\nu$. The source function is often a more convenient physical quantity than the emission coefficient, especially if the emission at a given point depends strongly on the absorption.

The optical depth is the absorption coefficient integrated along the optical path from x_0 to x:

$$\tau_\nu(x) = \int_{x_0}^{x} \kappa_\nu(s)\, ds\,. \tag{4.22}$$

The point x_0 is arbitrary (e.g. the location of the source or the observer), and it sets the zero point for the optical depth scale. A medium is called to be:

– optically thick or opaque for $\tau_\nu > 1$,
– optically thin or transparent for $\tau_\nu < 1$.

The photon **mean free path** ℓ_ν is defined by:

$$\langle \tau_\nu \rangle = \kappa_\nu \ell_\nu = 1 \quad \text{or} \quad \ell_\nu = \frac{1}{\kappa_\nu} = \frac{1}{N\sigma_\nu}\,. \tag{4.23}$$

The mean free path is just the reciprocal of the absorption coefficient for a homogeneous medium.

When using optical depth and source function then the transfer equation can be written as follows:

$$\frac{dI_\nu}{d\tau_\nu} = \frac{\epsilon_\nu}{\kappa_\nu} - I_\nu = S_\nu - I_\nu\,. \tag{4.24}$$

Integration gives the **formal solution**,

$$I_\nu\, e^{-\tau_\nu} = \int S_\nu\, e^{-\tau_\nu}\, d\tau \tag{4.25}$$

or expressed with the start and end values for the optical depths:

$$I_\nu(\tau_2) = I_\nu(\tau_1)\, e^{-(\tau_2-\tau_1)} + \int_{\tau_1}^{\tau_2} S_\nu(\tau)\, e^{-(\tau-\tau_1)}\, d\tau\,.$$

Often one can adopt $\tau_1 = 0$ and $I_\nu(\tau_1) = I_\nu(0)$:

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau)\, e^{-\tau}\, d\tau \tag{4.26}$$

Simple, but **very important special cases** for the description of the interstellar medium are:

- **only emission** of optically thin, diffuse gas without a background source ($\kappa_\nu = 0; I_\nu(0) = 0$):
$$I_\nu = \int \epsilon_\nu ds \tag{4.27}$$
- **only absorption** ($\epsilon_\nu = 0$) of radiation of a background source $I_\nu(0)$:
$$I_\nu = I_\nu(0)\, e^{-\tau_\nu} \tag{4.28}$$

4.7 Spectral lines: bound-bound radiation processes

The line emissivity ϵ^ℓ of an atom or molecule for a radiative decay of an upper level n to a lower level m is described by:

$$\epsilon^\ell = \int \epsilon^\ell_\nu \, d\nu = \frac{1}{4\pi} \, h\nu_{nm} \, A_{nm} \, N_n \quad \text{with} \quad \epsilon^\ell_\nu = \epsilon^\ell \, \Psi(\nu) \tag{4.29}$$

- N_n: density of particles in level n [cm^{-3}]
- A_{nm}: decay rate or transition probability for this transition [s^{-1}] (Einstein A-coefficient)
- $h\nu_{nm}$: energy for the radiated photon
- $1/4\pi$: per steradian
- $\Psi(\nu)$: normalized line profile function

$\Psi(\nu)$ describes the strong frequency dependence of the emission coefficient ϵ^ℓ_ν (and absorption coefficient κ^ℓ_ν). This includes the line profile due to the intrinsic line width or the natural line profile, the Doppler-broadening due to the kinetic motion of the particles (Gauss-function), and the Doppler-structure of the line due to large scale motions of the emitting gas.

The line absorption depends on the intensity:

$$\int \kappa^\ell_\nu \, I_\nu \, d\nu = \frac{1}{4\pi} \, h\nu_{nm} \, I(\nu_{mn}) \, (N_m B_{mn} - N_n B_{nm}) \tag{4.30}$$

- $g_m \, B_{mn} = g_n \, B_{nm}$: Einstein B-coefficients
- g_m, g_n: statistical weights for the levels N_m,N_n (there is $g_m = (2J_m + 1)$)

This gives the line integrated absorption coefficient:

$$\kappa^\ell = \frac{1}{4\pi} \, h\nu_{nm} \, (N_m B_{mn} - N_n B_{nm}) = \frac{1}{4\pi} \, h\nu_{nm} \, N_m B_{mn} \Big(1 - \frac{g_m}{g_n} \, \frac{N_n}{N_m}\Big) \,, \tag{4.31}$$

where the frequency dependence is again described by the normalized profile function $\kappa^\ell_\nu = \kappa^\ell \, \Psi(\nu)$. If $N_n/g_n > N_m/g_m$, which is equivalent to an inversion of the level population (over-population with respect to the Boltzmann-distribution), then the line absorption coefficient κ^ℓ_ν becomes negative, and the radiation is amplified by **stimulated emission** like in a laser.

The relations for the Einstein coefficients A_{nm} and B_{nm} follow from the requirement of detailed balance in thermodynamic equilibrium:

$$A_{nm} = \frac{2h\nu^3}{c^2} \, B_{nm} \quad \text{and} \quad B_{nm} = \frac{g_m}{g_n} B_{mn} \tag{4.32}$$

Detailed balance requires that the transition rates for radiative processes between two levels (say 1 and 2) are equal:

$$N_1 B_{12} B_{\nu_{12}}(T) = N_2 A_{21} + N_2 B_{21} B_{\nu_{12}}(T) \,.$$

where we have the processes: absorption equals spontaneous and induced emission. Solving for the Planck function and using the Boltzmann equation $N_1/N_2 = (g_1/g_2) \, e^{h\nu_{12}/kT}$ gives:

$$B_{\nu_{12}}(T) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21}/B_{21}}{(N_1/N_2)(B_{12}/B_{21}) - 1} = \frac{A_{21}/B_{21}}{g_1 B_{12}/g_2 B_{21} (e^{h\nu_{12}/kT})} \,.$$

This gives only the Planck function with the relations for the Einstein coefficients given above.

Important case: optically thin emission line:

$$I_\nu = \int \epsilon_\nu \, ds = \frac{1}{4\pi} \, h\nu_{nm} \, \Psi(\nu) \, A_{nm} \int N_n \, ds \tag{4.33}$$

The measured column density $\int N_n \, ds$ is always an average value for the observed solid angle. An accurate determination of the column density requires that the internal structure of the emission region is spatially resolved.

4.7.1 Rate equations for the level population

N_n is the population of level n which is defined by all the transition rates which populate this level $\sum_m N_m(R_{mn} + C_{mn})$ and the rates which depopulate this level $N_n \sum_m (R_{nm} + C_{nm})$. In an equilibrium state there is $dN/dt = 0$:

$$\frac{dN_n}{dt} = \sum_m N_m \, (R_{mn} + C_{mn}) - N_n \sum_m (R_{nm} + C_{nm}) = 0 \tag{4.34}$$

Rates for radiative transitions are given by R_{nm} and R_{mn} per time interval [s^{-1}] ($E_n > E_m$):

– $R_{nm} = A_{nm} + B_{nm} I_\nu$ spontaneous and induced line emission
– $R_{mn} = B_{mn} I_\nu$ line absorption

The transition rates for spontaneous emission depends on the type of transition. In astronomy it is distinguished between allowed transitions, inter-combination or semi-forbidden transitions, and forbidden transitions. In atomic physics the terms, electric dipole transitions, magnetic dipole transitions, and multipole (usually quadrupole) transitions are used.

Typical transition rates A are:
– $A \approx 10^8\,\mathrm{s}^{-1}$: allowed transitions (electric dipole)
– $A \approx 10^2\,\mathrm{s}^{-1}$: semi-forbidden transitions (electric dipole with spin-flip)
– $A \approx 10^{-2}\,\mathrm{s}^{-1}$: forbidden transitions (magn. dipole and electric quadrupole)
– $A \approx 10^{-5}\,\mathrm{s}^{-1}$: forbidden fine structure transitions
– $A \approx 10^{-15}\,\mathrm{s}^{-1}$: forbidden hyperfine structure transitions (e.g. H I)

Selection rules for dipole transitions:
For one electron atoms the selection rules are:
– $\Delta l = \pm 1$ (this includes a parity change for one electron systems)
– $\Delta m = 0, \pm 1$.

For many electron systems the selection rules are:
– parity change
– $\Delta S = 0$
– $\Delta L = 0, \pm 1$
– $\Delta J = 0, \pm 1$ except $J = 0$ to $J = 0$

In higher multipole transitions the spin may change (semi-forbidden transitions) or no parity change may be required (magnetic dipole or electric quadrupole transitions). There is no parity change in all transitions between states with the same electron orbit configurations like transitions between fine-structure levels or hyperfine-structure levels.

Rates for collisional transitions are described by C_{nm} and C_{mn} (in [s^{-1}]). It has to be distinguished between collisional deexcitation $n \to m$ and collisional excitation $m \to n$ ($E_n > E_m$):

$$C_{nm} = N_s\, Q_{nm} \quad \text{und} \quad C_{mn} = N_s\, Q_{mn} \tag{4.35}$$

where N_s is the density of the colliding particles (often e^-, p^+, H, H_2, etc.), and Q_{nm}, Q_{mn} are the collision rates [$\mathrm{cm}^3\,\mathrm{s}^{-1}$].

For collisional transitions in atoms by e^- between level n and m ($\Delta E_{nm} = h\nu_{nm} = E_n - E_m,\ n > m$) there is

$$C_{nm} = N_e\, \frac{1}{g_n}\, \frac{8.63 \cdot 10^{-6}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\sqrt{T\,[\mathrm{K}]}}\, \Omega_{nm} \qquad \text{collisional deexcitation} \tag{4.36}$$

$$C_{mn} = N_e\, \frac{1}{g_m}\, \frac{8.63 \cdot 10^{-6}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\sqrt{T\,[\mathrm{K}]}}\, \Omega_{mn} e^{-\Delta E_{nm}/kT} \qquad \text{collisional excitation} \tag{4.37}$$

$\Omega_{nm} = \Omega_{mn}$ are the collision strengths. Typical values for the collision strengths are of the order $\approx 0.1 - 10$, with usually only a small temperature dependence. It follows the following relation for the opposite collisional processes between level m and n:

$$C_{mn} = \frac{g_n}{g_m} C_{nm} e^{-\Delta E_{nm}/kT}\,. \tag{4.38}$$

4.7.2 Collisionally excited lines

2-level atom. The rate equation for a 2-level atom (or molecule) is, if we consider only collisional processes due to free electrons e^- (reasonable assumption for an ionized gas):

$$N_1\,(B_{12}U_\nu + N_e Q_{12}) = N_2\,(B_{21}U_\nu + A_{21} + N_e Q_{21}) \tag{4.39}$$

This equation becomes even more simplified if we neglect absorption and stimulated emission. This is a good approximation for the interstellar medium because of the weak radiation field. Typical dilution factors are $W \ll 10^{-10}$, and there is $B_{12}U_\nu \ll N_e Q_{12}$ and $B_{21}U_\nu \ll A_{21} + N_e Q_{21}$. This gives the following simple but very useful rate equation for a 2-level atom:

$$N_1\, N_e Q_{12} = N_2\,(A_{21} + N_e Q_{21}), \tag{4.40}$$

and with $Q_{12} = Q_{21}\,(g_2/g_1) e^{-h\nu/kT}$ ($h\nu = \Delta E_{21}$) follows the **level population ratio for a 2-level atom**:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\, \frac{N_e Q_{21}\, e^{-h\nu/kT}}{A_{21} + N_e Q_{21}} = \frac{g_2}{g_1}\, \frac{e^{-h\nu/kT}}{A_{21}/N_e Q_{21} + 1} \tag{4.41}$$

Low density regime: The spontaneous emission is much faster than the collisional deexcitation $N_e Q_{21} \ll A_{21}$ and it follows for the level population and the line emissivity ($\epsilon^\ell = (1/4\pi) h\nu_{nm} A_{nm} N_n$):

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\, \frac{N_e\, Q_{21}}{A_{21}}\, e^{-h\nu/kT} \quad \text{and} \quad \epsilon_{21} = \frac{1}{4\pi} h\nu_{21}\, N_e N_1\, \frac{g_2}{g_1} Q_{21} e^{-h\nu_{21}/kT} \tag{4.42}$$

The line emission coefficient is then proportional to the particle density squared $\epsilon^\ell \propto N_1 N_e$ (Fig. 4.8).

High density regime: Collisions are frequent and the collisional de-excitation is much faster than the spontaneous emission $N_e Q_{21} \gg A_{21}$. In this case the level population depends only on the collisions and we obtain the Boltzmann level distribution

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-h\nu/kT} \quad \text{and} \quad \epsilon_{21} = \frac{1}{4\pi} h\nu_{21} N_1 \frac{g_2}{g_1} A_{21} e^{-h\nu_{21}/kT} \tag{4.43}$$

The line emission is proportional to the density $\epsilon^\ell \propto N_1$ (Fig. 4.8).

Figure 4.8: Schematic behavior for emissivity per electron for a collisionally excited line as function of the density (N_k=critical density).

The **critical density** $N_k = A_{21}/Q_{21}$ for a line transition defines the border line between the high and low density regimes. When considering "real" multi-level atoms, then the critical density is a quantity of a particular level x, for which one has to evaluate the ratio between all line transitions and all de-populating collisional transitions

$$N_{x,k} = \frac{\sum_n A_{xn}}{\sum_n Q_{xn}} \,. \tag{4.44}$$

Collisional rates due to electrons in warm (photoionized) gas $T \approx 10^4$ K are on the order $Q_{21} \approx 10^{-7}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$. Rough estimates for the critical densities N_k for different types of line transitions in a photoionized nebula ($T \approx 10^4$ K, electron collisions dominate) are as follows

– allowed transitions	$A \approx 10^8\,\mathrm{s}^{-1}$	$N_k \approx 10^{15}\,\mathrm{cm}^{-3}$
– inter-combination lines	$A \approx 10^2\,\mathrm{s}^{-1}$	$N_k \approx 10^9\,\mathrm{cm}^{-3}$
– forbidden transitions	$A \approx 10^{-2}\,\mathrm{s}^{-1}$	$N_k \approx 10^5\,\mathrm{cm}^{-3}$
– forbidden fine structure lines	$A \approx 10^{-5}\,\mathrm{s}^{-1}$	$N_k \approx 10^2\,\mathrm{cm}^{-3}$

Important example: H I – 21 cm line (1420 MHz)

The ground level of atomic hydrogen H I has due to the non-zero nuclear spin a hyperfine splitting, between the parallel and anti-parallel configurations of the spins of the proton and the electron.
The parallel configuration (quantum number for the hyperfine-structure f=1, statistical weight g=2f+1=3) is energetically slightly higher ($\Delta E = h\nu \approx 10^{-5}\,\mathrm{eV}$) than the anti-parallel configuration (f=0, g=1).

energy level diagram H I
$1\,^2\mathrm{S}$

The transition from $f = 1$ to $f = 0$ has the following properties:

- the transition rate for spontaneous emission is extremely small $A_{21} = 3 \cdot 10^{-15}\,\mathrm{s}^{-1}$ (decay time 10^7 year!),
- The collision frequency with other particles in the cold (T = 100 K), diffuse $N_\mathrm{H} \approx 1\,\mathrm{cm}^{-3}$, partly neutral interstellar medium (electron collision dominant) is on the order $N_\mathrm{H}\,Q_{12} \approx 10^{-9}\,\mathrm{s}^{-1}$ (an H-atom gets a kick about every 60 years).

→ **collisions define the level population** and therefore the H I hyperfine structure level population is essentially always and everywhere in the interstellar medium in the high density regime:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\, e^{-h\nu/kT} \quad \text{where} \quad h\nu/kT < 10^{-5} \quad \text{for} \quad T > 10\ \mathrm{K} \tag{4.45}$$

$$\text{result}: \quad N_2 = 3\,N_1 = \frac{3}{4} N_{\mathrm{H}^0} \quad \text{and} \quad \epsilon_{21\,\mathrm{cm}} = \frac{1}{4\pi}\, h\nu\, A_{21} \frac{3}{4} N_{\mathrm{H}^0} \tag{4.46}$$

Thus, 75 % of the atomic hydrogen in the Universe will be in the excited state $f = 1$ of the two hyperfine structure levels. Because H is so abundant it is possible to observe the decay f=1 → f=0, despite the very long lifetime (small transition rate) of the excited level. In practice, the H I 21 cm line observations belong to the most important diagnostic tool for the investigation of cool, diffuse gas in the Milky Way and other galaxies. Thereby the measured surface brightness yields directly the column density along the line of sight $\int N_{\mathrm{H}^0}\, ds$.

Temperature and density determinations. We consider a 3-level atom (or molecule) with the following simplifications:

- no absorption or stimulated emission (induced radiation transitions),
- no transitions between level 2 and 3 ($N_1 \gg N_2$, N_3).

Then there is:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}\,\frac{e^{-h\nu_{21}/kT}}{A_{21}/N_e Q_{21} + 1} \qquad \frac{N_3}{N_1} = \frac{g_3}{g_1}\,\frac{e^{-h\nu_{31}/kT}}{A_{31}/N_e Q_{31} + 1} \tag{4.47}$$

$$\rightarrow \frac{N_3}{N_2} = \frac{g_3}{g_2}\,\frac{A_{21}/N_e Q_{21} + 1}{A_{31}/N_e Q_{31} + 1}\, e^{-h(\nu_{31}-\nu_{21})/kT} \tag{4.48}$$

If the line emissivities of two collisional excited lines differ in their temperature or density dependence, then they can be used for determining T_e and N_e in the emitting gas. For this we use the relation between level population ratio N_3/N_2 and line emissivity ratio (with $\epsilon_{nm} = (1/4\pi)\, h\nu_{nm}\, A_{nm}\, N_n$):

$$\frac{\epsilon^{\ell}_{31}}{\epsilon^{\ell}_{21}} = \frac{\nu_{31}\, A_{31}}{\nu_{21}\, A_{21}}\, \frac{N_3}{N_2} \tag{4.49}$$

Density determination:

- ideal for $\nu_{31} - \nu_{21} \approx 0$, $\rightarrow T_e$-dependence small,
- gas density N_e between $N_{k,2}$ and $N_{k,3}$,
- critical densities for the two transitions differ $N_{k,2} \neq N_{k,3}$,
- low and high density regimes: (with simplification $e^{-h(\nu_{31}-\nu_{21})/kT} = 1$)
  - low density: $N_e \ll N_{k,2}$, $N_{k,3}$ (or: $A_{nm}/N_e Q_{nm} \gg 1$)

$$\frac{N_3}{N_2} = \frac{g_3}{g_2}\, \frac{Q_{31}/A_{31}}{Q_{21}/A_{21}} \quad \rightarrow \quad \frac{\epsilon^{\ell}_{3}}{\epsilon^{\ell}_{2}} = \frac{g_3 Q_{31}}{g_2 Q_{21}} = \frac{\Omega_{13}}{\Omega_{12}} \tag{4.50}$$

  - high density: $N_e \gg N_{k,2}$, $N_{k,3}$ (or: $A_{nm}/N_e Q_{nm} \ll 1$)

$$\frac{N_3}{N_2} = \frac{g_3}{g_2} \quad \rightarrow \quad \frac{\epsilon^{\ell}_{3}}{\epsilon^{\ell}_{2}} = \frac{g_3 A_{31}}{g_2 A_{21}} \tag{4.51}$$

- Slide 4–10 illustrates the density determination using the [O II] and [S II] lines.

Figure 4.9: Schematic illustration for the density determination (left) using two emission lines with essentially identical excitation energy (right) but with different critical density

Temperature determination:

- ideal for the temperature determination is a 3-level atom with a large difference in the excitation energies $\nu_{31} - \nu_{21}$, so that the temperature dependence for the line ratio becomes large,
- the gas density should be for both transitions either in the low or high density regime, $N_e \ll N_{k,2}$, $N_{k,3}$ or the high density regime $N_e \gg N_{k,2}$, $N_{k,3}$, so that the line ratio is not strongly density dependent. For these conditions, there is:

$$\frac{N_3}{N_2} = const \cdot e^{-h(\nu_{31}-\nu_{21})/kT}. \tag{4.52}$$

  - The constant *const.* is for the low density regime:

$$\text{for} \quad N_e \rightarrow 0 \quad const = \frac{g_3}{g_2} \frac{Q_{31}/A_{31}}{Q_{21}/A_{21}}\,, \tag{4.53}$$

  - and for the high density regime

$$\text{for} \quad N_e \rightarrow \infty \quad const = \frac{g_3}{g_2} \tag{4.54}$$

Figure 4.10: Schematic illustration for the temperature determination (left) using two transitions with strongly different excitation states (right) and similar critical densities.

Slide 4–11 illustrates the temperature determination for the [O III] lines. This is an ideal case because level $n = 3$ decays to $n = 2$ and produces a line in the same wavelength range as the decay $2 \rightarrow 1$. For "real" emission line ratios there exists often a simultaneous dependence on temperature and density. In addition, the derived values are only valid for one ion and they represent some sort of average for the observed (usually inhomogeous) emission line region of that line. For this reason, the T_e and N_e determination for a nebula should be based on all diagnostic lines available. Slide 4–12 shows the emission line spectrum for the Orion nebula, a typical H II region.

4.7.3 Collisionally excited molecular lines

We consider here only the very important molecules H_2 and CO as examples for the excitation of molecules in the interstellar medium. There exist the following transitions between different energy states of molecules:

- rotational transitions: $J' - J''$
  energy levels of a rotator: $E_J \propto J(J+1)$; $\Delta E_J \approx 0.01$ eV
- (ro)-vibrational transitions: $\nu' - \nu''$
  energy levels of a harmonic oscillator: $E_\nu \propto \nu + 1/2$; $\Delta E_\nu \approx 0.3$ eV
- electronic transitions: $n'L' - n''L''$
  energy levels $E_n \propto -1/n^2$; $\Delta E_n \approx 10$ eV

Selection rules for the angular momentum change of a molecule due to allowed (dipole) transitions are:
$\Delta J = \pm 1$ or $\Delta L = \pm 1, 0$ (but not $L' = 0 - L'' = 0$).

schematic energy level diagram for a 2-atomic molecule:

transition	H_2	CO
rotational transitions		
dipole-transitions		
$J = 1 \to 0$	—	2.60 mm, $A = 7.2 \cdot 10^{-8}\mathrm{s}^{-1}$
$J = 2 \to 1$	—	1.30 mm, $A = 6.9 \cdot 10^{-7}\mathrm{s}^{-1}$
$J = 3 \to 2$	—	0.65 mm, $A = 2.5 \cdot 10^{-6}\mathrm{s}^{-1}$
quadrupole transitions		
$J = 2 \to 0$	28.2 μm, $A = 2.9 \cdot 10^{-11}\mathrm{s}^{-1}$	—
$J = 3 \to 1$	17.6 μm, $A = 4.8 \cdot 10^{-10}\mathrm{s}^{-1}$	—
ro-vibrational transitions		
dipole transitions		
$\nu = 1 \to 0,\ J = 1 \to 0$	—	4.7 μm,
$\nu = 2 \to 0,\ J = 1 \to 0$	—	2.3 μm,
quadrupole transitions		
$\nu = 1 \to 0,\ J = 2 \to 0$	2.12 μm,	
$\nu = 2 \to 0,\ J = 2 \to 0$	μm,	
electronic transitions		
e.g. ${}^1\Sigma_u^+ \to {}^1\Sigma_g^+$	UV (≈ 100 nm) $A \approx 10^7\mathrm{s}^{-1}$	UV (≈ 100 nm) $A \approx 10^7\mathrm{s}^{-1}$

4.7.4 Recombination lines: excitation through recombination

A recombination process involves the collision of a free electron with an ion X^{i+1} forming together an ion X^i. After this process, the ion (electron) may be in the ground state X^i_g or in the excited state X^i_n. The excited state can then decay to lower states by an emission of a line photon. Alternatively it may also be possible, but very unlikely for the low density in the interstellar medium, that the excited state is either re-ionized or deexcited by a collision, before a recombination line photon is emitted. Slides 4–13 and 4–14 show the levels and transitions of H I and He I, which are excited by recombination. The exited states X^i_n and corresponding lines of an ion, which are populated by recombination, can usually be distinguished from collisional excited lines.

There are two main recombination processes:

radiative recombination: $X^{i+1} + e^- \rightarrow X^i + h\nu$
This is the predominant process in the interstellar medium.

3-body recombination: $X^{i+1} + e^- + e^- \rightarrow X^i + +e^-$
This process is only important in high density gas, like stellar atmospheres, and can be neglected for the interstellar gas, because there are two electrons involved.

Emissivity for recombination lines: The emissivity for recombination lines depends on the recombination rate for radiative recombination, which can be described by

$$\epsilon^\ell = \frac{1}{4\pi}\, h\nu_{nm}\, \alpha^{\text{eff}}_{nm}\, N_e N(X^{+i+1})\,. \tag{4.55}$$

The emissivity per volume element is proportional to the density squared. The effective recombination coefficient for a recombination line $\alpha^{\text{eff}}_{nm}(T_e, N_e)$ (units [cm^3/s]) considers the population of the upper level n through the following processes:

- recombination directly into the level n
- cascades into level n from higher levels, which are populated by recombinations
- collisional transitions into level n from other level, which were also populated by recombinations.

The **temperature dependence** of the recombination lines behaves roughly like

$$\alpha^{\text{eff}}_{nm}(T_e) \propto 1/T\,. \tag{4.56}$$

This can be explained by the fact, that slow electrons have a higher chance to be captured by an ion.

Hydrogen and helium recombination lines The strongest recombination lines from diffuse gas regions are the lines from H I. In the visual range are the Balmer-transitions (transitions $n-2$), in the near-IR the Paschen ($n-3$), e.g. visible in the Orion spectrum in Slide 4–12, and Brackett lines ($n-4$) and in the far-UV the Lyman lines ($n-1$).

The He I and He II recombination lines are significantly weaker than the H I lines. One important factor is the abundance of helium which is typically 10 times lower. In addition, the He I energy levels are not degenerate for levels with different orbital angular momentum and further there are different levels for the singlet (electron spins anti-parallel) and the triplet states (spins parallel). Due to this there are many more but rather weak He I-lines in the spectrum (Slides 4–13 and 4–14).

Recombination lines from heavy elements are in astronomical objects again much weaker than the H I lines mainly due to the much lower abundance of these elements.

4.7.5 Absorption lines

Absorption lines are a very important source of information for the investigation of the interstellar medium. The atoms and ions in the diffuse gas are predominately in the ground state, because of the low density in the ground state. For this reason the interstellar gas produces essentially only absorptions from resonance lines. These are the absorptions by allowed transitions from the ground states of atoms and ions.

the strongest absorption lines are the resonance lines of atomic and molecular hydrogen H I and H_2 in the far-UV between 912Å and 1215Å (see Slides 4–15 to 4–17).
For the heavy elements the strongest lines are often the doublet-transitions ^{2}S$-$^{2}P of the isoelectronic sequences of Li, Na, and K. Some important absorption lines are:

Li-sequence	Na-sequence	K-sequence
C IV $\lambda\lambda$1548,1551	Na I $\lambda\lambda$5990,5996	Ca II $\lambda\lambda$3934,3968
N V $\lambda\lambda$1239,1243	Mg II $\lambda\lambda$2786,2803	
O VI $\lambda\lambda$1032,1038	Al III $\lambda\lambda$1855,1863	
	Si IV $\lambda\lambda$1394,1403	

Line strength. The strength of the line absorption coefficient κ^ℓ is defined by atomic parameters and the volume density of the absorbing atom (or ion) N_m in state m:

$$\kappa^\ell = \int \kappa^\ell_\nu \, d\nu = \frac{1}{c}\, h\nu_{nm}\, N_m B_{mn} \,. \tag{4.57}$$

The contribution from the stimulated emission ($-N_n B_{nm}$) can be neglected for most cases. The important point in this equation is the fact that the strength of the line absorption coefficient is proportional to the density of the absorbing particle.

The absorption coefficient is often expressed with oscillator strength f_{mn}, a description which comes from classical electrodynamics:

$$\kappa^\ell = \frac{\pi\, e^2}{m_e\, c}\, f_{mn} N_m \,. \tag{4.58}$$

The relation between oscillator strength and Einstein B coefficient is:
$f_{mn} = (m_e\, h\nu/\pi\, e^2)\, B_{mn}$.

The total line absorption follows through the integration of all particles along the line of sight and considering the frequency dependence of the line profile:

$$\int\int I_\nu \kappa^\ell_\nu(N_m)\, d\nu\, ds \tag{4.59}$$

It is often difficult to determine from the observed line absorption the column density $\int N_m\, ds$. This problem exists because the absorptions saturate, so that the absorption line depths in the spectrum is far from a linear relationship to the column density. For a meaningful interpretation a detailed analysis of the line structure is required.

Line profile structure
The line absorption depends usually strongly on frequency (or wavelength). Different effects play a role for the line structure:

The natural line profile. The natural line profile describes the line structure of a transition with frequency ν_0 of an atom, ion or molecule at rest. The line profile can be described by the Lorentz profile:

$$\psi_{\rm L}(\nu) = \frac{1}{\pi}\,\frac{\Gamma/4\pi}{(\nu-\nu_0)^2 + (\Gamma/4\pi)^2} \tag{4.60}$$

Γ is the transition rate, $1/\Gamma$ the life time of the two levels. The natural line width for allowed transitions is of the order $\Delta\lambda_n \approx 10^{-4}$Å (wavelength independent). The natural line width is extremely small when compared to the Doppler effect caused by the kinetic and dynamic motion of the gas. For this reason a pure Lorentz profile is rarely used in astrophysics.

The Doppler profile. The Doppler profile is used for absorption lines which are weak or have an intermediate strength. The Doppler profiles takes the kinetic motion of the absorbing particles in the gas cloud into account:

$$\psi_{\rm D}(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\,e^{-(\nu-\nu_0)^2/\Delta\nu_D^2} \tag{4.61}$$

The structure of the Doppler profile is a Gauss curve. $\Delta\nu_D$ is the Doppler width which follows from the velocity dispersion σ_v of the absorbing particle

$$\Delta\nu_D = \frac{\sigma_v}{c}\nu_0 \tag{4.62}$$

due to the kinetic velocity as defined by the Maxwell-Boltzmann velocity distribution. For a given temperature the Doppler width is:

$$\sigma_v = \left(\frac{2kT}{m}\right)^{1/2} = 12.9\,\left(\frac{T[{\rm K}]/10^4}{A}\right)^{1/2}\ {\rm km/s}\,. \tag{4.63}$$

Sometimes, the turbulent motion of the gas is included in this Doppler profiles.

The Voigt profile. The Voigt profile must be used for very strong absorption lines. This profile considers the fact, that the line wings defined by $(\nu-\nu_0) > \Delta\nu_D$ decrease faster for the Doppler profile than for the Lorentz profile. The Doppler profile falls off exponentially, but only quadratically for the Lorentz-profile. The Voigt profile is simply a more general line profile description which folds together the Lorentz and the Doppler profile:

$$\psi_V(\nu) = \frac{1}{\Delta\nu_D\sqrt{\pi}}\,\frac{\Gamma}{4\pi^2}\int_{-\infty}^{\infty}\frac{e^{-(\Delta\nu)^2/\Delta\nu_D^2}}{(\nu-\nu_0-\Delta\nu)^2 + (\Gamma/4\pi)^2}\,d(\Delta\nu) \tag{4.64}$$

Multiple components. Often different components of a line absorption are observed which are displaced in the spectrum. This can be due to clouds located at different distances and having different radial velocities. It is often hard to find out from absorption line observations which component is closer to the observer.

Spectroscopic observations:
A high spectral resolution is required to measure with sufficient accuracy the structure of interstellar absorption lines. The high spectral resolution requires that the background source is bright and has intrinsically no narrow lines. Thus it is only possible to probe with an absorption line analysis only certain line of sights towards well suited background objects, which are:

- bright, hot stars with broad lines (fast rotators) for interstellar absorptions,
- bright quasars for intergalactic absorptions.

Line equivalent width. A very basic quantity for the characterization of an absorption line is the equivalent width W_λ. W_λ measures the strength of a line absorption in the spectrum I_λ. W_λ measures the area in the spectrum between the normalized flux $I^n_\lambda = I_\lambda/I_{\rm cont}$ and the normalized continuum flux $I_{\rm cont} = 1$ (area in units or Å or nm). Expressed as mathematical formula:

$$W_\lambda = \int_{\rm Linie} (1 - I^n_\lambda)\, d\lambda\,. \tag{4.65}$$

The absorption depth at a given wavelength (or frequency) is defined by the optical depth τ:

$$W_\lambda = \int (1 - I^n_\lambda)\, d\lambda = \frac{\lambda^2}{c} \int (1 - e^{-\tau_\nu})\, d\nu \tag{4.66}$$

while the optical depth is the absorption coefficient integrated along the line of sight:

$$\tau_\nu = \int \kappa^\ell_\nu \, ds \tag{4.67}$$

Curve of growth

Weak lines. For weak absorption lines $I^n_\lambda \gtrsim 0.8$ the approximation $1 - e^{-\tau} \approx \tau$ is applicable and the equivalent width is:

$$W_\lambda \approx \frac{\lambda^2}{c} \int\int \kappa^\ell_\nu \, d\nu\, ds = \frac{\lambda^2}{c}\, \frac{\pi\, e^2}{m_e\, c} f_{mn} \int N_m \, ds\,. \tag{4.68}$$

The curve of growth is for weak absorption lines in the "linear regime". This means that each contribution to the line absorption (column density) produces an enhancement of the equivalent width independent of the wavelength of the absorption. This is equivalent to the statement that the equivalent width is proportional to the column density for weak lines (Fig. 4.11). The following relationship for λ and W_λ expressed in Å can be used:

$$\int N_m \, ds = \frac{1.13 \cdot 10^{20}}{\lambda^2[\text{Å}]\, f_{nm}} \cdot W_\lambda[\text{Å}] \tag{4.69}$$

Saturated lines. Stronger lines saturate in the Doppler core and the line can only grow in the Doppler wings if there are more absorbing particles. The equivalent width changes not much if the column density is enhanced because of the exponential decrease of the absorption coefficient in the line wings. The equivalent width approaches a limiting value which is proportional to the line width of the Doppler profile.

In this regime, one has to consider in the line analysis, that a displaced wavelength component, e.g. due to a cloud with different radial velocity, can produce a significant contribution to the equivalent width, while an additional component in the line center has no effect on the line profile.

Figure 4.11: Schematic illustration of the curve of growth for absorption lines.

Damped absorption lines. In very strong absorption lines the damping wings become visible which grow with increasing column density. These lines are called damped absorption lines. The damped profiles are due to the natural line profile for which the absorption coefficient decreases only quadratically with distance from the line center. Although the line wings of the natural line profile are very weak, they are in the far line wing still stronger than the exponentially decreasing Doppler wings and become visible for very strong absorption lines. Lines in which the damping wings dominate are in the regime where the equivalent width is proportional to the square of the column density:

$$\int N_m \, ds \propto W_\lambda^2 \tag{4.70}$$

Example: H I Lyα:
$\lambda = 1215$ Å, $f_{\mathrm{Ly}\alpha} = 0.41$
– for unsaturated (optically thin) lines ($W_\lambda \ll 0.3$ Å) there is:

$$\int N(\mathrm{H\,I}) \, ds = 1.8 \cdot 10^{14}\,\mathrm{cm}^{-2} \cdot W_\lambda[\text{Å}]\,, \tag{4.71}$$

– for damped absorptions ($W_\lambda \gg 1$ Å) there is:

$$\int N(\mathrm{H\,I}) \, ds = 1.9 \cdot 10^{18}\,\mathrm{cm}^{-2} \cdot (W_\lambda[\text{Å}])^2 \tag{4.72}$$

Example: H_2-molecular absorptions
H_2 is a special case: Only the singlet states $^1\Sigma_g$ with anti-parallel spins for the electrons are stable. In additions the Pauli principle requires that the quantum states of the H_2 systems are anti-symmetric (non-exchangeable).

$\rightarrow$ there are two types of molecular hydrogen H_2 depending on the relative orientation of the nuclear spins (see Fig. 4.12):

- **para-H_2**: nuclear spin antiparallel, J even, statistical weight $J(J+1)$
- **ortho-H_2**: nuclear spin parallel, J odd, statistical weight $3\,J(J+1)$

For this reason there are no allowed dipole-transitions between the different rotation and vibrational states of the ground level. A dipole-transition requires $\Delta J = \pm 1$, but such a transition would also require a nuclear spin flip which is not possible.

Figure 4.12: Energy level diagram for molecular hydrogen.

Electronic transitions from the ground state are possible, because the symmetry requirement (Pauli principle) does not apply if the principle quantum numbers of the two electrons are different. e.g. $^1\Sigma_g -^1\Sigma_u$ with $\nu'(=0) - \nu''$ und $J' - J''(= J' \pm 1)$
These electronic transitions produce very strong H_2 Lyman- und Werner bands in the far UV.

Temperature determination using H_2 absorption lines. Collisional processes dominate the level population for the lowest states of H_2, because the radiative transitions are forbidden between the rotational states. The population of the level $N_{J,\nu} = 0$ are therefore given by the Boltzmann-equation:

$$N_J \propto g_J\, e^{-E_J/kT} \tag{4.73}$$

The observations of the strength of H_2 far-UV lines yields therefore a good estimate for the temperature in molecular clouds.

4.8 Free-bound and free-free radiation processes

4.8.1 Recombination continuum

The electron, which is captured in a radiative recombination process, emits a photon in particular wavelength regions, which are characteristic for the recombining atom or ion. This radiation is well visible for the H I recombination. The recombination continuum may also be seen for He I and He II in high quality spectra.

The energy of the emitted photon is defined by the kinetic energy (relative to the recombining ion) of the capture electron and the energy difference between the ionization energy (usually set to zero) and the (negative) energy of the bound state into which the electron is captured initially:

$$h\nu = \frac{1}{2} m_e v_e^2 - \chi_n \tag{4.74}$$

For hydrogen this is $\chi_n = -Ry/n^2$. Recombination into level n produce according to this equation photons with an energy of at least $-\chi_n$ or more. This produces characteristic discontinuities in the spectrum of photoionized regions. The strongest case is the Balmer jump at 3648 Å.

Figure 4.13: Wavelength dependence of the Recombination continua for a hot and a cold emission nebula.

The emissivity for the recombination continuum can be calculated from the following formula:

$$j_\nu = \frac{1}{4\pi} N_e N(\mathrm{X}^{+m+1}) \sum_{n,L} \int_{v_e} v_e \, \sigma_{n,L}(\mathrm{X}^{m-1}, v_e) \, f(v_e, T_e) \, h\nu(v_e) \, dv_e \, . \tag{4.75}$$

The meaning of the different terms are:

- $1/4\pi$: considers emission in all directions
- $N_e N(\mathrm{X}^{+m+1})$: density of the particles involved
- $\sum_{n,L}$: summation over all levels
- $\int_{v_e} f(v_e, T_e) dv_e$: integration for a Maxwell distribution of electrons
- v_e: number of interaction is proportional to the electron velocity
- $\sigma_{n,L}(\mathrm{X}^{+m}, v_e)$: cross section for the recombination into level n, L.
  In general σ is large for small v_e, thus slow electrons are more frequently captured.
- $h\nu(v_e)$: energy of the emitted photon

Temperature dependence: The intensity jump and the gradient of the recombination continuum depend on the gas temperature. For low temperature the average kinetic energy of the captured electron is lower and more photons are emitted with an energy just above the "jump" energy. A high temperature gas emits more photons significantly above the jump, so that intensity jump and the gradient are smaller (Fig. 4.13).

low temperature → steep continuum and relatively strong jump
high temperature→ flat continuum and relatively small jump

4.8.2 Photoionization or photo-electric absorption

In a photo-ionization process a photon "is pulling out" and electron from an atom or ion:

$$\mathrm{X}^{+m} + h\nu \rightarrow \mathrm{X}^{+m+1} + e^- \tag{4.76}$$

The liberated electron has after the process a kinetic energy which is equal to that part of the photon energy, which was beyond the ionization energy $\chi_{\rm ion}$:

$$e^-(E_{\rm kin}) = h\nu - \chi_{\rm ion}\,. \tag{4.77}$$

This extra energy is in photo-ionized regions the most important energy source for the heating of the gas.

Absorption cross section. The photo-ionization cross section a_ν is zero $a_\nu = 0$ for photon energies below the ionization energy of a given atomic state. The cross section has a maximum value at the ionization energy and for higher energies the cross section decreases typically like (see Fig. 4.3):

$$a_\nu \propto \nu^{-3}\,. \tag{4.78}$$

The H I ionization edge. The H I ionization edge at 912 Å, or 13.6 eV is most important for the interaction between the radiation field and the ISM. If radiation above 13.6 eV is present, then the gas can be ionized and become transparent for ionizing radiation. If no radiation $E > 13.6\ eV$ is present then the gas becomes neutral and opaque for the ionizing radiation.

The H I ionization edge defines further two types of neutral elements. Elements which have an ionization edge below 13.6 eV and can be ionized by UV-photons with $h\nu < 13.6$ eV. These elements are also in the neutral H I-regions often ionized, e.g.:

– C II, Mg II, Si II, Ca II, Fe II.

Atoms with $\chi_{\rm ion} \geq 13.6$ eV are neutral when hydrogen is neutral, e.g.:

– He I, N I, O I, Ne I.

The absorption cross section of hydrogen and helium are small for high photon energies $h\nu > 100$ eV, in the soft (= low energy) X-ray range. For these energies one has to consider also photoelectric absorptions from heavy element despite their low abundance. In many electron atoms, X-ray photons can be absorbed efficiently by inner shell electrons (K- or L-shell). This produces discontinuities in the photoelectric absorption cross sections. The more abundant heavy elements dominate the interstellar absorption in the soft X-ray range due to K- and L-shell electron absorptions (Slide 4–18).

The averaged photo-electric absorption in the soft X-ray range is quite universal for **neutral** interstellar gas. The strength of the X-ray absorptions is essentially identical for atomic or molecular gas, or for neutral gas with dust particles.

The photo-ionization cross section is of course strongly reduced for highly ionized gas. Due to this, soft X-rays can also be observed for extra-galactic sources for sight lines perpendicular to the Milky Way disk.

4.9 Free-free radiation processes or bremsstrahlung

4.9.1 Radiation from accelerated charges

Whenever a charged particle is accelerated or decelerated it emits electromagnetic radiation. If this radiation is created by the interaction of fast electrons with atomic nuclei then it is called bremsstrahlung. In atomic physics this process is called free-free emission because the radiation corresponds to transitions between unbound states in the field of a nucleus.

The following scheme illustrates the origin of the radiation from an accelerated charged particle (from M.S. Longair, High Energy Astrophysics). The field lines are shown for a particle that suffers a small acceleration Δv. The electric field lines inside a sphere with radius $r = ct$ already "know" that the charge has moved, while the field lines outside this sphere have still the configuration from before the kick. In a shell with thickness $c\Delta t$ there must be an electric field component in i_ϕ or tangential direction. This "pulse" of transverse electromagnetic field propagates away from the charge with speed of light and represents the radiation from the accelerated charge (Slide 4–19).
The total power emitted from a single accelerated charge q is given by Larmor's formula:

$$P = \frac{2q^2|\dot{\vec{v}}|^2}{3c^3}\,. \tag{4.79}$$

This formula is valid for any form of acceleration (including charges moving in magnetic fields).

The emitted radiation from an accelerated particle has the following properties:

- the radiated energy is proportional to $P \propto q^2|\dot{\vec{v}}|^2$,
- the radiated energy has an angle dependence like a dipole $dP/d\Omega \propto \sin^2\theta$ where θ is the polar angle with respect to the acceleration vector ,
- the radiation is polarized with electric field vector parallel to the acceleration vector

Radiation spectrum. The spectrum of the emitted radiation depends on the time variation of the electric field. A regularly oscillating field (e.g. from a bound electron, or from a rotating or vibrating molecule) produces a line at a given wavelength or frequency. The frequency spread of this line, or the natural line width, is defined by the energy uncertainty principle $\Delta E \Delta t > h/2\pi$ or if we insert the energy of the emitted photon $E = h\nu$:

$$\Delta\nu\,\Delta t > \frac{1}{2\pi}\,. \tag{4.80}$$

If a charge, say an electron, is accelerated in the electric field of an ion then the radiation pulse is extremely short. Lets assume an electron with a relative speed of $v_T(10\ \mathrm{K}) =$ 550 km/s is accelerated by an an atom with a dimension of 1 Å, then the pulse duration is on the order 10^{-16} sec. This implies that the frequency (or energy) of the emitted photon is essentially unconstrained. For this reason, the free-free radiation is essentially frequency independent.

4.9.2 Thermal bremsstrahlung

Bremsstrahlung or free-free emission is produced by Coulomb collisions of electrons e^- with ions (p^+, He^+, etc.). In these collisions, charged particles are strongly accelerated so that radiation pulses are created. Most efficient is the acceleration of electrons in the field of ions. The frequency spectrum of a short pulse (Fourier transformation of a delta-function) is broad band. Thus the resulting spectrum I_ν is essentially flat for low frequencies ν and it has an exponential cut-off at the high frequency end. The exponential cut-off is defined by the kinetic energy distribution of the electron, which is for a thermal gas defined by the Maxwell velocity distribution, which has also an exponential cut-off. Essentially, there can be no photons emitted with an energy higher than the kinetic energy of the accelerated particle.

The emission coefficient has the following temperature and density dependence:

$$j_\nu \propto \frac{N_i\, N_e}{\sqrt{T}}\, e^{-h\nu/kT}\,. \tag{4.81}$$

The exact formula is:

$$j_\nu = 5.44 \cdot 10^{-39}\, g_{\rm ff}\, z_i^2 \frac{N_i\, N_e}{\sqrt{T}}\, e^{-h\nu/kT} {\rm erg cm^{-3}\, s^{-1} Hz^{-1}}\,, \tag{4.82}$$

where $g_{\rm ff}(T_e, z_i, \nu) \approx 1-2$ is the Gaunt-factor, a quantum-mechanical correction factor to the classical formula, z_i is the charge for the ion ($=1$ for a hydrogen nebula).
The characteristic energy or wavelength for the exponential cut-off is given by $-h\nu/kT = 1$. It is:

- for $T = 10^4$ K at $\lambda = 1.4\,\mu$m (warm photo-ionized gas)
- for $T = 10^7$ K at $\lambda = 14\,$Å $\approx 0.9\,$keV (hot collisionally ionized gas)

In total the energy radiated by bremsstrahlung is obtained through integration over all frequencies:

$$\epsilon_{\rm ff} \propto N_i\, N_e \sqrt{T}\,, \tag{4.83}$$

or exactly: $\epsilon_{\rm ff} = 1.43 \cdot 10^{-27}\, z_i^2\, \langle g_{\rm ff}\rangle N_i\, N_e \sqrt{T}\, {\rm erg\, cm^{-3}\, s^{-1}}$ where N_i, N_e are particle densities per cm^3 and T_e the electron temperature. Free-free emission dominates the cooling of collisionally ionized gas if $T_e > 10^6$ K.

Free-free absorption coefficient
A gas which emits free-free radiation becomes for frequencies low enough $\nu < \nu_0$ optically thick. This fact follows from the Kirchhoff law, which defines the Planck radiation $B_\nu(T)$ as the maximum source function for a thermal gas with temperature T:

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \leq B_\nu(T) \tag{4.84}$$

The emissivity j_ν for the free-free radiation is for low frequencies essentially frequency-independent, while the Planck radiation decreases for $\nu \to 0$ (with $e^{h\nu/kT} \to 1 + h\nu/kT$) like:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \to \frac{2\nu^2 kT}{c^2} \tag{4.85}$$

According to the Kirchhoff law $\kappa_\nu = j_\nu / B_\nu$ the free-free absorption coefficient is:

$$\kappa_\nu = j_\nu \frac{c^2}{2\nu^2 kT} \propto \frac{N_i\, N_e}{T^{3/2}} \frac{1}{\nu^2} \tag{4.86}$$

Thus, for high frequencies $\nu \to \infty$ the free-free absorption becomes rapidly very small.

Figure 4.14: Wavelength dependence of the radio continuum from an ionized nebula.

Result (Fig 4.14):

– for high frequencies the emission is optically thin and the observed radiation flux is:

$$I(\nu) \propto \int j_\nu ds \propto \int N_i\, N_e \frac{e^{-h\nu/kT}}{\sqrt{T}}\, ds \tag{4.87}$$

– for low frequencies the free-free emission is optically thick and the radiation flux is defined by the black-body radiation with a temperature of T, thus:

$$I(\nu) \propto \nu^2\, T\,. \tag{4.88}$$

Approximation for relativistic bremsstrahlung. In collisionally ionized hot gas $T > 10^8$ K, as observed in rich clusters of galaxies, the thermal electrons may reach relativistic velocities $v_T = 55'000$ km/s. For such gas a relativistic correction is necessary. A simple formulation of this effect for the total bremsstrahlung emission is:

$$\epsilon_{\rm ff} = 1.43 \cdot 10^{-27}\, z_i^2\, \langle g_{\rm ff} \rangle N_i\, N_e \sqrt{T}\, (1 + 4.4 \cdot 10^{-10}\;\, T[K])\, {\rm erg\, cm^{-3}\, s^{-1}} \tag{4.89}$$

The term in brackets is the relativistic correction which is only relevant for very high temperatures.

4.10 Compton and Thomson scattering

In Compton scattering energy and momentum of a photon is transferred to the scattering particle, usually an electron. This process is for the electron gas not so important, because the hard radiation field is too weak in the interstellar space to produce any additional gas heating via this process.
More important is the inverse effect, **inverse Compton scattering**. In this process a moving electron transfers kinetic energy to the scattered photon so that the energy distribution of the radiation is changed. This process provides an important diagnostic tool for the investigation of the electron energies (or gas temperature) in hot gas. This is particularly important for fully ionized gas, where no or hardly any atomic emission lines are emitted.

Thomson scattering: For low energies, $h\nu \ll m_e c^2$ (or $E_\gamma \ll 511$ keV) one can use the classical Thomson scattering cross section for photon scattering. In this case the scattering is essentially elastic in the frame of the electron.

The total cross section is:

$$\sigma_e = \frac{8\pi}{3} r_0^2 = 5.56 \cdot 10^{-25}\,\mathrm{cm}^2 \tag{4.90}$$

where $r_0 = e^2/m_e c^2$ is the classical electron radius.

- The scattering cross section has a angle dependence like $d\sigma/d\Omega = r_0^2/2\,(1+\sin^2\theta)$. Thus the scattering favors forward and backward scatterings.
- The scattered radiation is linearly polarized even for unpolarized incoming radiation. The polarization is 100 % for right angle scatterings, with an orientation perpendicular to the scattering plane. The polarization degree is given by:

$$\Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta} \tag{4.91}$$

Thomson scattering is a dipole-type scattering process and the polarization can be understood like the emission of an oscillating particle which was disturbed (accelerated) by an oscillating radiation field.

Thomson scattering can form a significant wavelength independent opacity source for astrophysical plasmas.

Compton scattering: In Compton scattering (photon energy $h\nu \approx m_e c^2$) energy and momentum is transferred from the photon to the electron (assumed to be at rest). The wavelength change $\lambda_2 - \lambda_1$ for the photon in a Compton scattering (e.g. Tipler) follows from the conservation of energy and momentum in an inelastic collision.

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1-\cos\theta)\,. \tag{4.92}$$

This is equivalent to a relative photon energy loss of:

$$\frac{\epsilon_1 - \epsilon_2}{\epsilon_2} = \frac{\epsilon_2}{m_e c^2}(1-\cos\theta) \tag{4.93}$$

When averages are taken over the scattering angle θ then the net loss for the photon field, or the energy increase for the electron gas is:

$$\langle\frac{\Delta\epsilon}{\epsilon}\rangle = \frac{h\nu}{m_e c^2} \tag{4.94}$$

Inverse Compton scattering: Electrons with kinetic motion can also transfer energy to photons via the Doppler effect. However, for a cold gas with slowly moving electrons, there are equal rates for "positive" and "negative" Doppler shifts. In a hot gas, where the electrons move fast (relativistically) there exists a second order effect (the fast moving electrons see more photons in the direction of motion), which leads to a enhancement of the average photon energy via electron scattering (inverse Compton scattering).

Without going into details the mean amplification of photon energies per scattering is

$$\langle\frac{\Delta\epsilon}{\epsilon}\rangle = \frac{4}{3}(\frac{v}{c})^2 = \frac{4kT_e}{m_e c^2}, \tag{4.95}$$

where $\langle m_e v^2\rangle/2 = 3kT_e/2$ was used for the kinetic motion of the electrons.

Comptonization: As a result we get the equation which describes the energy exchange between the radiation field and the electron gas through Compton collisions. The energy change of the radiation field is:

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{h\nu}{m_e c^2} + \frac{4kT_e}{m_e c^2} \tag{4.96}$$

This equation defines the conditions under which energy is transferred to and from the photon field:

– if $h\nu = 4kT_e$, then there is no energy transfer
– if $h\nu > 4kT_e$, then energy is transferred from a hard radiation field to the cool gas
– if $h\nu < 4kT_e$, then energy is transferred from hot gas to the radiation field.

Due to the large distance to high energy sources (AGN, X-ray binary stars) the hard radiation field is always strongly diluted. Therefore the first and second cases are not important for the interstellar medium.

4.11 Temperature equilibrium

The gas in the interstellar space is far from a thermodynamic equilibrium. For this reason the equilibrium temperature at a given position depends on various **heating** and **cooling** processes. Detailed computations are required to estimate the equilibrium temperature.

4.11.1 Heating function H for neutral and photo-ionized gas

Important processes for the gas heating in molecular clouds are collisions by cosmic rays (relativistic particles) and the photo-dissociation of molecules. Photoionization by radiation from stars and other UV sources is the dominant heating process for the atomic, diffuse gas and the photo-ionized gas.

Shocks due to supersonic gas motions can in addition heat the gas to high temperatures. In shocks the dynamic energy of a gas cloud is converted into kinetic (internal) energy of the gas. Supersonic gas flows are produced by stellar winds and supernova explosions. The heating by shocks depends strongly on time and the location and is therefore difficult to describe accurately.

Important heating processes are:

- photo-ionization: $h\nu + \mathrm{X}^m \rightarrow \mathrm{X}^{m+1} + e^-(E_{\mathrm{kin}})$.
  The heating per volume element is given by the number of ionizations multiplied by the extra photon energy above the ionization threshold:

$$H = N_{\mathrm{H}^0} \int_{\nu_0}^{\infty} \Gamma_\nu \, h(\nu - \nu_0) \, a_\nu d\nu \, . \tag{4.97}$$

- photo-dissociation: $h\nu + \mathrm{XY} \rightarrow \mathrm{X}(E_{\mathrm{kin}}) + \mathrm{Y}(E'_{\mathrm{kin}})$
- photo-electric absorption by dust: $h\nu + \mathrm{dust} \rightarrow \mathrm{dust}' + e^-(E_{\mathrm{kin}})$
- collisions with cosmic ray particles: $P_{\mathrm{cr}} + X \rightarrow P_{\mathrm{cr}} + Y_1(E_{\mathrm{kin}}) + \ldots + e^-(E_{\mathrm{kin}}) + \ldots$

A heating process produces particles with a kinetic energy ($\gg 3kT/2$) and therefore it contributes to the heating of the gas. In each microscopic heating process, one particle takes part. The energy originates from remote sources (e.g. stellar radiation or relativistic particles), which is converted into kinetic energy of the gas:
the heating per unit volume (cm^3) is proportional to the particle density n:

$$\mathrm{heating} = n \cdot H \tag{4.98}$$

As a first approximation the heating H does not depend on gas parameters, like T or n. However, H depends on the intensity of the radiation field or of the cosmic rays.

4.11.2 Cooling of the gas

The cooling of the gas is mainly due to line emission. Bremsstrahlung (free-free radiation) is the dominant cooling process for gas with very high temperatures $T > 10^6$ K. Important cooling processes are:

– collisionally excited lines: $e^-(E_{\rm kin}) + X_g^m \rightarrow X_i^m \rightarrow X_g^m + h\nu$
– Bremsstrahlung: $e^-(E_{\rm kin}) + X^m \rightarrow e^-(E'_{\rm kin}) + h\nu$

Less important gas cooling processes are the thermal emission by dust particles (contributes in particular in molecular clouds to the cooling) and thermal conduction (in regions with strong temperature gradients like shocks).

The basic process for the cooling by **line emission** is, that an atom or molecule is put into an excited state by a collision with another gas particle (e.g. by an electron), from where it returns to the ground state through the emission of a photon.

kinetic energy of the gas $\rightarrow$ inner energy of the particle
$\rightarrow$ emission of a line photon (hν)

Bremsstrahlung is emitted by charged gas particles which are accelerated or decelerated by collisions with other gas particles. Also in this process kinetic energy of the gas is transformed into radiation energy.

Radiation energy is produced in all important gas cooling processes by the collision of **two** particles, thus:
the cooling is proportional to the particle density squared n^2:

$$\text{cooling} = n^2 \cdot \Lambda(T) \tag{4.99}$$

4.11.3 The cooling function $\Lambda(T)$

The cooling of the gas depends strongly on the temperature of the gas. For this reason the efficiency of the gas cooling is described by the cooling function $\Lambda(T)$. In addition there exists also some dependence of the cooling on the elemental abundances which are important in special cases (e.g. early universe or supernova remnants). Since the elemental abundances are rather homogeneous in the Universe the abundance effects can often be neglected.

An efficient cooling requires:

– an abundant particle (e.g. hydrogen H, C, N, O, CO),
– with an excited state having an excitation energy χ within the range of the kinetic energy of the gas particles, thus $\chi \approx kT$,
– and an excited state with a decay time shorter than the typical time interval to the next collision which may de-excite collisionally the particle (this would convert the excitation energy back to kinetic energy).

Order of magnitude values for collisional rates γ are:

– $\gamma \approx 10^{-11}$cm^{3}s^{-1} for collisions between neutral particles
– $\gamma \approx 10^{-9}$cm^{3}s^{-1} for collisions between a charged and a neutral particle
– $\gamma \approx 10^{-7}$cm^{3}s^{-1} for collisions between charged particles

The collisions per second [s^{-1}] and particle are $n \cdot \gamma$.

Important emission lines for the gas cooling.

T < 1'000 K

The most abundant particle in molecular clouds is H_2. But the gas cooling by H_2 is very inefficient due to the symmetry of this particle and there exists no fast decay from excited rotational levels of H_2 ($A_{2\to 0} = 3 \cdot 10^{-11}\mathrm{s}^{-1}$). For this reason the main process for the cooling in molecular clouds is the emission of photons by rotational transitions of CO (see Slide 4–20). The lowest transitions are:

CO 2.6 mm, $J = 1 \to 0$,	$A \approx 10^{-2}\,\mathrm{s}^{-1}$
CO 1.3 mm, $J = 2 \to 1$	$A \approx 10^{-2}\,\mathrm{s}^{-1}$

The cooling of cold, atomic gas is mainly due to lines from fine structure transitions emitted in the far IR, e.g.:

O I 63.2 μm, ^{3}P, $J = 1 \to 2$,	$A = 9 \cdot 10^{-5}\,\mathrm{s}^{-1}$
O I 145.5 μm, ^{3}P, $J = 0 \to 1$,	$A = 2 \cdot 10^{-5}\,\mathrm{s}^{-1}$
C II 157.7 μm, ^{2}P, $J = 3/2 \to 1/2$,	$A = 2 \cdot 10^{-6}\,\mathrm{s}^{-1}$

T = 1'000 – 30'000 K

Gas with neutral hydrogen H I can cool through the excitation of H I and the emission of Lyman lines e.g.:

H I Lyα λ1215Å, $n = 2 \to 1$,	$A = 5 \cdot 10^{8}\,\mathrm{s}^{-1}$

This process is only efficient for gas with high temperature because the excitation energy is rather high for the first excited state $n = 2$: $\chi = 10.6\mathrm{eV} = 1.7 \cdot 10^{-11}\mathrm{erg} \quad \to \quad e^{-\chi/kT} \approx e^{-10^5\mathrm{K}/T}$.

Often, hydrogen is highly ionized and therefore the cooling trough neutral hydrogen can be very in-efficient. Efficient for the cooling are different nebular lines from ions which are abundant in ionized nebulae (Slide 4–21). Dependent on the ionization degree of the gas the following lines are important coolants:

C III]	[1907],1909 Å, ^{3}P^o →^{1}S,	$A \approx [0.01], 100\,\mathrm{s}^{-1}$
C IV	1548,1551 Å, ^{2}P^o →^{2}S,	$A \approx 3 \cdot 10^{8}\,\mathrm{s}^{-1}$
[N II]	6548,6583 Å, ^{3}S→^{1}D,	$A \approx 10^{-3}\,\mathrm{s}^{-1}$
[O II]	3726,3728 Å, ^{4}S→^{2}D,	$A \approx 10^{-4}\,\mathrm{s}^{-1}$
[O III]	4959,5007 Å, ^{3}S→^{1}D,	$A \approx 10^{-2}\,\mathrm{s}^{-1}$
O VI	1032,1038 Å, ^{2}P^o →^{2}S,	$A \approx 4 \cdot 10^{8}\,\mathrm{s}^{-1}$
[S II]	6716,6731 Å, ^{4}S→^{2}D,	$A \approx 10^{-3}\,\mathrm{s}^{-1}$

T = 30'000 – 10^7 K

Hot (collisionally ionized) gas emits many lines from different, highly ionized atoms. Strong lines are e.g. from ions of the H and He iso-electronic sequences (Slide 4–22), like O VII and O VIII or from the many ionization states of iron (Fe X – Fe XXVI).

T > 10^6 K

Bremsstrahlung contributes always to the cooling of an ionized gas. At very high temperature, essentially all atoms are fully ionized and line radiation is no more possible. Bremsstrahlung is for this case the dominating gas cooling process. The cooling, equivalent to the radiation emitted by Bremsstrahlung is proportional to $\sqrt{T}$. For fully ionized gas with solar abundances the emitted luminosity per volume element is given by

$$L_{\mathrm{BS}} = 2 \cdot 10^{-27} n_e^2 \sqrt{T}\,\mathrm{erg cm^{-3} s^{-1}}\,. \tag{4.100}$$

Table 4.2: Summary of the most important heating and cooling processes.

gas type	heating	cooling
molecular clouds	cosmic rays	molecular lines, CO, H_2O
cold, neutral gas	UV radiation (stars)	fine structure lines, C II, O I
warm, neutral gas	UV radiation (stars, AGN)	Lyα, nebular lines, [O I], [S II]
photo-ionized gas	UV radiation (stars, AGN)	Lyα, [O II], [N II], [O III]
collisionally ionized gas	shocks	X-ray lines, bremsstrahlung

Strongly simplified, it can be said that the temperature equilibrium for the diffuse gas in the interstellar medium is determined by:

$$n^2 \cdot \Lambda(T) = n \cdot H \quad \text{and} \quad n \cdot \Lambda(T) = H \approx \text{const.} \tag{4.101}$$

The "cosmic" cooling curve. The gas cooling processes are always the same for diffuse gas and one can describe the cooling with an universal cooling curve (Fig. 4.15). This curve illustrates the energy loss by gas cooling processes and it is given in units of [energy cm^3/s]. The cooling curves has a major maximum around 10^5 K where the cooling by atomic lines is most efficient and a smaller bump around 300-1000 K where molecules and atomic fine structure lines are efficient. There is a minimum around 10^7 K where all atoms are fully ionized so that no line emission is possible.

Figure 4.15: Schematic illustration of the cooling curve.

Table 4.3: Characteristic cooling time scales.

gas type	$n\,[\mathrm{cm}^{-3}]$	T [K]	$\Lambda\,[\mathrm{erg\,cm^3\,s^{-1}}]$	τ_{th}
molecular gas	10^5	50	10^{-26}	$7 \cdot 10^6\,\mathrm{s} \approx 80$ d
diffuse, ionized gas	10^{-1}	10^4	10^{-24}	$1.4 \cdot 10^{13}\,\mathrm{s} \approx 4 \cdot 10^5\,\mathrm{J}$
H II-region	10^3	10^4	10^{-24}	$1.4 \cdot 10^9\,\mathrm{s} \approx 40\,\mathrm{J}$
SN-remnant	10	10^6	$2 \cdot 10^{-23}$	$7 \cdot 10^{11}\,\mathrm{s} \approx 2 \cdot 10^4\,\mathrm{J}$
coll.ionized gas	10^{-3}	10^7	10^{-23}	see exercise

4.11.4 Cooling time scale

The cooling time scale for the diffuse gas, which is the time required for the cooling of a gas cloud if the heating is switched off, can be roughly estimated from the cooling function according to:

$$\tau_{\mathrm{th}} = \frac{U}{n^2\Lambda(T)} \approx \frac{nkT}{n^2\Lambda(T)} = \frac{kT}{n\Lambda(T)} \tag{4.102}$$

(U: kinetic energy (inner energy) of the gas in cm^{-3}). As first approximation one may approximate $\Lambda(T) \propto T$. Thus the cooling time scale behaves (very roughly) like $\tau_{\mathrm{th}} \approx 1/n$. Thus:
high density gas cools rapidly, while diffuse, low density gas cools slowly.

Figure 4.16: The spezific cooling curve $\Lambda(T)/T$.

4.11.5 Equilibrium temperatures.

For the Milky Way disk it can be assumed that there exists a very rough pressure equilibrium for the diffuse Gas. Thus a hydrostatic stratification of the gas can be assumed in the direction perpendicular to the disk. In addition we can adopt the (simplified) law for the temperature equilibrium: $n \cdot \Lambda(T) = H \approx$ const. Based on this we obtain the following, very rough relation between gas pressure, temperature and cooling function:

$$p = nkT = kT\frac{H}{\Lambda(T)} = \mathrm{const.} \tag{4.103}$$

The "specific" cooling function $\Lambda(T)/T \propto 1/p$ is drawn in Fig. 4.16. There exist for a given gas pressure different intersections with the "specific" cooling curve $\Lambda(T)/T$. At these intersections the gas temperatur could be in an equilibrium state. However, the temperature equilibrium is only stable for intersections where the gradient of the "specific" cooling curve is positive. For intersections with curve sections having a negative gradient the equilibrium is not stable.

stable equilibrium: $(\mathrm{d}(\Lambda(T)/T)/\mathrm{d}T > 0)$,
if the temperature is slightly disturbed then the temperature will go back to the equilibrium point:

– increase of $T \rightarrow$ increase of $\Lambda(T)/T \rightarrow$ more cooling
– decrease of $T \rightarrow$ decrease of $\Lambda(T)/T \rightarrow$ less cooling

unstable equilibrium: $(\mathrm{d}(\Lambda(T)/T)/\mathrm{d}T < 0)$,
after a small temperature disturbance the temperature T will drift away from the equilibrium point:

– increase of $T \rightarrow$ decrease of $\Lambda(T)/T \rightarrow$ less cooling
– decrease of $T \rightarrow$ increase of $\Lambda(T)/T \rightarrow$ additional cooling

The "specific" cooling curve has two stable temperature regimes with a positive gradient. Due to this, there exist two predominant temperatures for the interstellar gas:
cold gas $\mathbf{T < 100}$ K and warm gas $\mathbf{T \approx 10000}$ K

Hot $T > 10^5$ K gas cannot exist in a stable temperature equilibrium (theoretically). But the cooling time scale for hot gas is often so long (because of the low density), that it can survive for a very long time. Diffuse, hot gas $T > 10^6$ K is therefore the third type of interstellar gas which is frequently present.

The observed parameters of the dominant interstellar components in the Milky Way can be plotted in a density-temperature diagram (Fig. 4.17).

Figure 4.17: Parameters for dominant interstellar components.

The diagram in Fig. 4.17 illustrates the following:

- the gas exists predominately in 3 temperature regimes (cold, warm and hot)
- there exists, very roughly, a pressure equilibrium ($n \cdot T \approx 1000\mathrm{K/cm}^3$) for the diffuse gas in the Milky Way

4.12 Dynamics of the interstellar gas

The interstellar matter is not static but moves always and everywhere around. This gas motions is caused by various processes which induce typical gas velocities as follows:

- galactic rotation $v \approx 200$ km/s
- peculiar motion of galaxies $v \approx 500$ km/s
- speed of stellar winds $v \approx 20 - 3000$ km/s
- ejection velocity of supernova explosions $v \approx 10000$ km/s,
- outflows (broad absorption lines) from active galactic nuclei $v \approx 10000$ km/s.

The gas velocities are very often much larger than the sound velocity of the gas $v \gg v_s$. For this reason many important hydrodynamical processes in the interstellar medium are due to **supersonic flows** which produce non-linear effects, in particular **shocks**.

4.12.1 Basic equations for the gas dynamics

For a simple description of gas-dynamical processes in the interstellar medium one can often start for first useful estimates with strongly simplified equations based on the following assumptions:

- $\vec{B} = 0$ no magnetic field :
  Neglecting magnetic fields simplifies the treatment of hydrodynamic processes enormously. However, neglecting magnetic fields can be a very critical choice because many hydrodynamic problems may not be understood without magnetic fields. Sometimes it is useful to assume at least that the magnetic field moves with the gas (it is frozen in) and that the field adds just another pressure term $\sim B^2/8\pi$.

- $\vec{E} = 0$ no electric fields :
  This is a very good approximation because charged particles e^-, p^+ are abundant and they can move freely.

- viscosity $\eta = 0$:
  A very good approximation due to the low density.

- no hydrodynamic coupling between matter and radiation field (no radiation pressure)

The equation of motion:

$$\rho \frac{d\vec{v}}{dt} = \rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\,\mathrm{grad}\vec{v} \right) = -\vec{\mathrm{grad}}p - \rho\,\vec{\mathrm{grad}}\Phi\,, \tag{4.104}$$

and the equation for the conservation of mass:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v}\,\vec{\mathrm{grad}}\rho = -\rho\,\mathrm{div}\vec{v} \tag{4.105}$$

where $\vec{v}$ is the gas velocity, p the pressure, ρ the density and Φ the gravitational potential. Further there is:

- d/dt: the time-derivative in the co-moving coordinate system
  $\rightarrow$ **Lagrange-system**,

– $\partial/\partial t$: the partial time-derivative for a fixed point in space
  $\rightarrow$ **Euler-system**.

The variables are:

velocity field $\vec{v}(x_i, t)$	density $\rho(x_i, t)$
gas pressure $p(x_i, t)$	gravitational potential $\Phi(x_i, t)$

These equations alone cannot be solved because there are more variables than equations. This means that some of the functions must be known, for example the gravitational potential Φ or the local energy balance which requires knowledge on the heating and cooling processes.

Energy conservation:

The energy conservation can in general not be expressed as local differential equation, because the heating depends on the interaction with the distant surroundings, e.g the heating by the absorption of ionizing UV-radiation from stars or the heating by collisions with relativistic cosmic ray particles. Two useful simplifications for first order estimates are:

– **Adiabatic hydrodynamics**: It is assumed, that the energy is conserved locally. This means, that the heating and cooling of the gas is neglected. This is a quite reasonable simplifying assumption for regions with very low densities and long time scales for cooling (e.g. hot, collisionally ionized, diffuse gas).

– **Isothermal hydrodynamics**: In this case a constant temperature is adopted for the gas, e.g. 10'000 K for photo-ionized gas or 100 K for neutral gas. In this approach it is assumed, that the heating due to compression or the cooling due to expansion is compensated immediately by enhanced or reduced radiative cooling. Thus, there is no local energy conservation in this case. This approximation is useful for regions with high gas density where the radiative cooling is very efficient.

Gravitation

For the gravitation the Poisson equation is used:

$$\Delta\Phi = 4\pi G\,\rho(x_i, t) \tag{4.106}$$

The solution of this equation depends very much on the geometric scale for the distribution of the mass with respect to the size of the gas structure studied.

relatively simple:	The gas-dynamics is solved in a pre-defined and constant gravitational potential, e.g. the motion of the gas in the potential of a galaxy.
very difficult:	The gas-dynamics for a "self-gravitating gas" is very delicate, because gradΦ has a dynamic (non-linear) component and small inhomogeneities in the density distribution can grow to large gravitational instabilities for the gas. An important example for this non-linear behavior is the collapse of a gas cloud in a star forming region.

4.12.2 Shocks

The sound velocity v_s defines the propagation velocity of a pressure waves. If there exist supersonic flows $v > v_s$, then hydrodynamic effects occur at a given location without a "preceding warning". This will produce strong discontinuities in the gas parameters, so-called **shocks**.

Sound velocity. The square of the sound velocity v_s is defined in compressible media by the density derivative $d/d\rho$ of the pressure p:

$$v_s^2 = \frac{dp}{d\rho} \tag{4.107}$$

The adiabatic sound velocity for adiabatic gas $p = (p_0/\rho_0) \cdot \rho^\gamma$ is given by (for an ideal gas):

$$v_s^2 = \frac{dp}{d\rho} = \gamma \, \frac{p_0}{\rho_0} \, \rho^{\gamma-1} = \gamma \, \frac{p}{\rho} = \frac{5}{3} \frac{kT}{m_T} \tag{4.108}$$

$\gamma = 5/3$ for ionized and atomic gas; $\gamma < 5/3$ else
$m_T =$ mean particle mass (e.g. $m_T \approx 0.5 \, m_p$ for ionized H-gas)
The isothermal sound velocity for isothermal gas $p = \rho \cdot kT/m_T$:

$$v_s^2 = \frac{dp}{d\rho} = \frac{p}{\rho} = \frac{kT}{m_T} \tag{4.109}$$

The sound velocity is of the same order as the mean (mass weighted) kinetic velocity of the particles ($\sim$ protons in ionized gas). Rough estimates for the sound velocity are: $v_s \approx 1$, 10, and 100 km/s for temperature of $T = 100$, 10^4, and 10^6 K, respectively.

Conservation laws for idealized shocks. The hydrodynamic conservation laws can be used for the description of basic properties of shocks, without studying the complicating processes taking place at the shock fronts. For this we consider one-dimensional flows with a shock front. The parameters p_1, ρ_1, and v_1, stand for the pressure, density, and gas velocity in front of the shock front p_2, ρ_2, and v_2 after the shock front. The gas velocities v_1 and v_2 are expressed relative to the velocity of the shock front which is set equal to zero.

Figure 4.18: Illustration of shock parameters.

The so-called **Rankine-Hugoniot conditions** are:

– based on the mass conservation: $\rho_1 v_1 = \rho_2 v_2$

– based on the equation of motion: $p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2$

In addition we have to consider also the energy budget. The energy budget depends on the treatment for the energy loss due to radiative cooling.

Isothermal shocks. The isothermal shock is a simple model case, in which one assumes that the temperature is identical before and after the shock:

$$T_1 = T_2 \tag{4.110}$$

This assumption requires an extremely efficient cooling, in order to radiate away (instead of heating up the gas) all the energy produced by the work due to the shock. The isothermal shock can be a useful approximation for shocks in high density gas, where the cooling is very efficient. It is also necessary that the gas is optically thin so that the energy can be radiated away.

With the isothermal sound velocity $v_s^2 = p/\rho$ the equations can be solved with the following algebra:

$$\underbrace{p_1}_{\rho_1 v_s^2} + \rho_1 v_1^2 = \underbrace{p_2}_{\rho_2 v_s^2} + \rho_2 v_2^2 \tag{4.111}$$

and:

$$v_s^2 (\rho_1 - \rho_2) = \underbrace{v_2^2 \rho_2}_{v_1^2 \rho_1^2/\rho_2} - v_1^2 \rho_1 = v_1^2 \frac{\rho 1}{\rho_2} (\rho_1 - \rho_2) \, . \tag{4.112}$$

The result is:

$$\frac{\rho_2}{\rho_1} = \frac{v_1^2}{v_s^2} = M^2 \, , \tag{4.113}$$

where M is the Mach number for the gas flow in front of the shock (in the coordinate system of the shock). For an outside observer this is the Mach number for the shock velocity in the pre-shock medium. The compression or **the density jump in an isothermal shock is proportional to the square of the Mach number**. The Mach number for shocks in the interstellar medium is often very high, e.g. $M \approx 100 - 1000$ for supernovae. The compression is under isothermal conditions very high, on the order $\rho_2/\rho_1 = 10^4 - 10^6$.

Further there is:

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{v_1^2}{v_s^2} \quad \rightarrow \quad v_s^2 = v_1 \cdot v_2 \, , \tag{4.114}$$

which is equivalent to the statement, that the velocity of the post-shock gas is smaller than the sound velocity in the coordinate system of the shock front.

An illustrative description for an isothermal shock is a snow-plough, which piles all material up and carries it away.

Adiabatic shocks. In an adiabatic shock the energy is conserved locally and the work done by the shock front is put into the heating of the post shock gas. The adiabatic shock is a good approximation for very thin gas, where the cooling is not efficient.

For adiabatic shocks we have another equation, besides the Rankine-Hugoniot conditions, which describes the total energy flow through the shock front:

$$v_2\left(\frac{1}{2}\rho_2 v_2^2 + U_2\right) - v_1\left(\frac{1}{2}\rho_1 v_1^2 + U_1\right) = v_1 p_1 - v_2 p_2 \tag{4.115}$$

where $\rho v^2/2$ is the kinetic energy of the gas, $U = p/(\gamma - 1)$ the inner energy of the gas, and the term on the right side is the work due to the pressure change at the shock front $d/dt(p \cdot A\Delta x) = d/dt(F_A \Delta x)$.

With the Rankine-Hugoniot conditions and a lot of algebra for the case $M \gg 1$ there is:

$$\frac{\rho_2}{\rho_1} \approx \frac{\gamma+1}{\gamma-1} \rightarrow \qquad \frac{\rho_2}{\rho_1} \approx 4 \qquad \text{for} \qquad \gamma = \frac{5}{3} \tag{4.116}$$

The density jump is a factor 4 for an adiabatic shock of an ideal gas.

The temperature after the shock follows from the gas equation and the equation of motion:

$$T_2 = \frac{m_\mathrm{H}}{k}\,\frac{p_2}{\rho_2} \qquad \text{and} \qquad \frac{p_2}{\rho_2} = \underbrace{\frac{p_1}{\rho_2}}_{=0\,\text{für}\,p_1 \ll p_2} + \underbrace{\frac{\rho_1}{\rho_2}}_{1/4}\, v_1^2 - \underbrace{v_2^2}_{v_1^2\,\rho_1^2/\rho_2^2 = v_1^2/16} \tag{4.117}$$

This gives the result:

$$T_2 \approx \frac{3}{16}\,\frac{m_\mathrm{H}}{k}\,v_1^2 = 1.4 \cdot 10^7\ \mathrm{K}\left(\frac{v_1}{100\,\mathrm{km/s}}\right)^2 \tag{4.118}$$

Thus the temperature of the post-shock gas in an adiabatic shock is typically on the order $10^6 - 10^8$ K.

More realistic shocks. Observations of shocks show a combination of both cases, the adiabatic and the isothermal shock. The temperature can reach near the shock front a very high temperature and the adiabatic approximation is not bad. Correspondingly one has then a density jump which is not far from the factor 4. Further away from the front in the post-shock region the gas has sufficient time to cool and it approaches more the parameters for gas in an isothermal shock. This means that the gas cools down and becomes quite dense and may be visible as so-called "radiative shock".

Magnetic fields may also play a role in shocks. Especially, the B-field may be responsible for a magnetic pressure term which can be significant or even a dominant contribution to the total pressure in shocks which are dense and behave like isothermal shocks. Thus, the compression for shocks in dense gas with magnetic fields may be significantly smaller than in isothermal shocks due to the magnetic pressure.

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Figure 4.19: Schematic structure for a more realistic shock model.

4.12.3 Example: supernova shells

The velocity and the kinetic energy of a supernova shell is immediately after the explosion enormous:

$$v_{\rm SN} \approx 15000 \text{ km/s} \quad \text{und} \quad E_{\rm kin} \approx 4 \cdot 10^{50}\,\text{erg} \tag{4.119}$$

This corresponds to the radiation energy which is delivered by our Sun in $3.5 \cdot 10^9$ years.

first phase: free expansion

The first phase is characterized by:

– essentially a gas motion in free space (vacuum) $\rightarrow$ free expansion,
– last until the swept-up mass of the interstellar medium is comparable to the mass of the supernova shell.

An estimate on the swept-up mass may be based on the mean mass density in the Milky Way disc $\rho = 1.6 \cdot 10^{-24}\,\text{g}\,\text{cm}^{-3}$ (corresponds to a particle density of $n_{\rm H} = 1\,\text{cm}^{-3}$). A sphere of diffuse gas with a mass comparable to the supernova shell with a mass of $M_{\rm SN} \approx 1\ \text{M}_\odot$ has a radius of:

$$\frac{4\pi\, r^3}{3}\,\rho = M_{\rm SN} \quad \rightarrow \quad r = 2\,\text{pc} \approx 6 \text{ Lj}\,. \tag{4.120}$$

The free expansion phase last with an expansion velocity of 15000 km/s $= c/20$ about:

$$t = \frac{r}{v} = 120 \text{ years}\,. \tag{4.121}$$

second phase: adiabatic shock

The density of the supernova-shell is in this phase still relatively small, because not much ISM has been swept-up. Due to the low density the radiative cooling is relatively unimportant and the shock can be approximated by adiabatic conditions:

– The velocity of the pre-shock gas is for a distant observer equal to zero. However, the velocity is relative to the shock front on the order 10'000 km/s.

– The post-shock gas moves with a velocity of 2500 km/s relative to the shock. For an outside observer the velocity of the post-shock gas is 7500 km/s.
– The density inside the shock is about 4 times higher than the density of the ISM, but the temperature is extremely high $> 10^7$ K. Thus in this phase a hot, tenuous bubble is formed.
– This phase lasts about 100-1000 years and the shell radius grows to $r = 1 - 10$ pc

Figure 4.20: Structure of a spherical, adiabatic shock from a supernova shell.

Third phase: isothermal shock

This last phase is characterized by the fact that the supernova shell has accumulated a lot of interstellar material and there will be an isothermal or radiative shock.

– the shock velocity has decreased to $\sim 100 - 3000$ km/s
– there is a huge density jump $\rho_2/\rho_1 = M^2 \approx 10^4 - 10^6$ like for a snow-plough.

The velocity evolution, or the deceleration of the supernova shell can be roughly described by the momentum conservations, considering a spherical shell of gas moving into a thin surrounding gas.

$$M_{\rm SN}\, v_{\rm SN} \approx (M_{\rm SN} + m_{\rm ism})\, v_{\rm shock} \quad \text{because} \quad m_{\rm ism} \propto r^3 \quad \text{thus} \quad v_{\rm shock} \quad \propto \frac{1}{r^3}\,. \tag{4.122}$$

This equation describes very roughly the shock velocity $v_{\rm shock}$ and the radius of the supernova shell r which can both be used to make estimates on the age of the supernova shell.