## 4.4 Atmospheres of solar system planets

For spectroscopic studies of planets one needs to understand the net emission of the radiation from the surface or the atmosphere. Radiative transfer in planetary atmosphere is therefore a very important topic for the analysis of solar system objects but also for direct observations of extra-solar planets. In this section we discuss some basic properties of planetary atmospheres.

#### 4.4.1 Hydrostatic structure of atmospheres

The planet structure equation from Section 3.2 apply also for planetary atmospheres. One can often make the following simplifications:

- the atmosphere can be calculated in a plane-parallel geometry considering only a vertical or height dependence z,
- the vertical dependence of the gravitational acceleration can often be neglected for the pressure range 10 bar – 0.01 bar and one can just use  $g(z) = g(z = 0) = g(R) = g = GM_P/R_P^2$ .
- the equation of state can be described by the ideal gas law

$$P(\rho) = \frac{\rho kT}{\mu} \quad \text{or} \quad \rho(P) = \frac{\mu P}{kT} \,,$$

where  $\mu$  is the mean particle mass (in [kg] or [g]),

- a mean particle mass which is constant with height  $\mu(z) = \mu$  can often be used in a first approximation,
- a temperature which is constant with height T(z) = T can often be used as first approximation.

**Pressure structure.** The differential equation for the pressure gradient is:

$$\frac{dP(z)}{dz} = -g(z)\rho(z) = -g(z)\frac{\mu(z)P(z)}{kT(z)}$$

which yields the general solution:

$$P(z) = P_0 e^{-\int_0^z 1/H_P(z)dz}$$
 with  $H_P(z) = \frac{kT(z)}{g(z)\mu(z)}$ 

For a homogeneous, isothermal atmosphere and g(z) = g,  $\mu(z) = \mu$  a simple exponential pressure law is obtained

$$P(z) = P_0 e^{-z/H_P}$$
 with  $H_P = \frac{kT}{g\mu}$ , (4.18)

where  $H_P$  is the pressure scale height. This is the vertical length scale over which the pressure decreases by a factor  $e^{-1} = 0.368$ .

**Density structure.** Very similar equations apply for the density structure and one obtains for an homogeneous, isothermal atmosphere a density structure equivalent to the pressure:

$$\rho(z) = \rho_0 e^{-z/H_{\rho}} \quad \text{with} \quad H_{\rho} = \frac{kT}{g\mu}.$$
(4.19)

where  $H_{\rho}$  is the density scale height and  $\rho_0$  the density at the reference point  $z_0 = 0$ . In the simple case described here the pressure and density scale heights are identical:

$$H_P = H_\rho = H$$
.

object	$T_{\rm ground}$	$P_{\text{ground}}$ [bar]	$T_{\rm eff}$	$\mu/\mu_{ m H}$	$g$ $[m s^{-2}]$	H [km]	$dT/dz _{\rm ad}$ [K/km]	$\frac{1}{[\mathrm{kms}^{-1}]}$
Mercury		$10^{-14}$	448 K		3.7			4.4
Venus	$730~{ m K}$	92	$328 \mathrm{~K}$	44	8.9	6.9		10.4
Earth	$288~{\rm K}$	1.01	$263~{\rm K}$	28	9.8	8.4		11.2
Mars	$215~{\rm K}$	0.006	$227~{\rm K}$	44	3.7	11.		5.0
Jupiter			$124~{\rm K}$	2.3	23.1	19.	1.9	59.5
Saturn			$95~{ m K}$	2.3	9.0	38.	0.84	35.5
Uranus			$59~{ m K}$	2.3	8.7	24.	0.85	21.3
Neptune			$59~{ m K}$	2.3	11.0	19.	0.86	23.5
Titan	93 K	1.46	80 K	28	1.4	17.		2.6

Table 4.3: Basic atmospheric parameters for planets with atmospheres and Titan.

Scale heights for planets. The equation for the scale height indicate the following relationships:

- The scale height depends on the atmospheric properties. For a planet with given radius  $R_p$  and bulk density  $\bar{\rho}$  (or mass) the scale height is proportional to the atmospheric temperature  $H \propto T$  and inverse proportional to the mean particle mass  $H \propto 1/\mu$ ,
- The scale height depends for given atmospheric temperature and composition on the planet properties. The scale height is inverse proportional to the surface gravity  $H \propto 1/g = R_P^2/GM_P \propto 1/R_p\bar{\rho}$ .

The scale height can be particularly large for hot planets, with a hydrogen atmosphere and a small gravitational acceleration (large radius and low mean density).

For solar system planets the scale heights are given in Table 4.3. H was calculated with the indicated  $T_{\rm eff}$  and mean particle mass  $\mu/\mu_{\rm H}$ . The scale heights are in a narrow range of 5 – 40 km. From the composition, one would expect much smaller scale heights for the terrestrial planets when compared to giant planets because of the much larger particle mass ( $\approx 30$  in terrestrial planets but only 2.3 in giant planets). But this effect is compensated by the higher atmosphere temperature and lower gravity for the terrestrial planets. **Column density.** The column density  $\Sigma(z_0)$  gives the total density of gas per unit area above a certain height  $z_0$  (e.g. defined as  $z_0 = 0$ ).  $\Sigma(z_0)$  is an important quantity for the calculation of the optical depth. Since the density drop-off with height is exponential  $\Sigma(z_0)$  for is proportional to the density at  $\rho(z_0) = \rho_0$ 

$$\Sigma(z_0) = \rho_0 \int_0^\infty e^{-z/H} dz = -\rho_0 H e^{-z/H} |_0^\infty = \rho_0 H.$$

This can be directly linked to the pressure

$$\Sigma(P_0) = \frac{\mu P_0}{kT} H = \frac{P_0}{g} \,. \tag{4.20}$$

All solar system planets have a gravitational acceleration at the surface of the order  $g \approx 10 \,\mathrm{m\,s^{-2}}$ . Thus for order of magnitude estimates one can use a surface density of  $\Sigma(1\mathrm{bar}) \approx 1\mathrm{kg\,cm^{-2}}$ .

**Chemical composition for atmospheres of solar system planets** An important input parameter for the analysis of atmospheres is their composition which is given in Table 4.4. We will discuss later the interpretation of these abundances.

Table 4.4: Abundances by mass of the most important chemical spezies for solar system objects.

object	dominant molecule	secondary constituents	minor constituents
Venus	96.5 % $CO_2$	$3.5 \% N_2$	$0.01 \% \mathrm{SO}_2$
Earth	$78.1 \% N_2$	$20.1 \% O_2$	$0.93~\%$ Ar, $0.03~\%$ CO $_2$
Mars	$95.3 \% CO_2$	$2.7 \% N_2$	1.6~% Ar, $0.27~%$ N <sub>2</sub>
Jupiter	$85 \% H_2$	$15~\%~{\rm He}$	$0.24~\%~{ m CH_4}$
Saturn	$94 \% H_2$	6 % He	$0.3~\%~\mathrm{CH}_4$
Uranus	$85~\%~\mathrm{H_2}$	$15~\%~{\rm He}$	$1 \% \mathrm{CH}_4$
Neptune	$85~\%~{ m H}_2$	$15~\%~{\rm He}$	$1 \% \mathrm{CH}_4$
Titan	92 % N $_2$	4~% CH <sub>4</sub> , $4~%$ Ar	

## 4.4.2 Thermal structure of planetary atmospheres

The vertical structure of planetary atmospheres can be characterized by their thermal structure which depends on the heating processes and energy transport mechanisms. For our discussion of these processes we take the Earth atmosphere as a guideline. The atmospheric temperature profile is used as basis to distinguish different atmospheric layers. The vertical structure of Earth atmosphere is illustrated in Figure 4.5 and described in Table 4.5.

Figure 4.5: Vertical structure of Earth atmosphere.

Heating processes. There are the following important heating processes for planetary atmospheres. Starting from the top to the bottom of the atmosphere these are:

- Ionization. Neutral atoms in the high atmosphere absorb easily the solar far-UV radiation. Each ionization by a photon with energy  $h\nu$  above the ionization energy  $h\nu_0$  of an atom will produce an energetic electron with the "excess energy"  $\Delta E = h(\nu - \nu_0)$ . Ionization is only important in the uppermost thermosphere, because further down there will be no ionizing photons left.
- Particle radiation and plasma processes related to the planet magnetosphere can contribute to the heating of the upper atmosphere. For Earth, the impact of these effects depends a lot on the solar activity cycle and the associated enhancement of solar mass ejections and magnetic storms.
- Photodissociation of molecules by UV-photons is an important heating process in the stratosphere. In the Earth atmosphere the main processes are the dissociation of  $O_3$  and  $O_2$ . Below the stratosphere there are no UV-photons left.
- Light absorption. The optical and near-IR light gets absorbed in the troposphere at pressure levels around 1 10 bar. At these pressures collision induced absorption sets in, and the density of absorbing molecules becomes high enough for significant

layer or boundary	z [km]	T [K]	P [bar]	comment
troposphere	0–12	290-215	1-0.1	heated by the surface, with a decreasing temperature gradient due to convection
tropopause	12	215	0.1	vertical temperature minimum and upper limit of the convection layer
stratosphere	12–50	215-270	$0.1 - 10^{-3}$	temperature increases due to absorption of UV radiation by $O_3$
stratopause	50	270	$10^{-4}$	intermediate temperature maxi- mum
mesosphere	50-85	270–190	$10^{-4} - 10^{-6}$	decrease in temperature due to the lack of heating processes
mesopause	85	190	$10^{-7}$	absolute temperature minimum in the atmosphere
turbopause	100	200	$10^{-8}$	below this level the composition is quite homogeneous, above it the particles are stratified ac- cording to their weight
thermosphere	85-500	190-1000	$10^{-7} - 10^{-10}$	the gas is heated due to ioniza- tion by solar far-UV photons
exobase	$\sim 500$	1000	$\approx 10^{-11}$	above this limit particles can es- cape, the height changes with so- lar activity
exosphere	$\gtrsim 500$	$\gtrsim 1000$	$< 10^{-11}$	composed mainly of H and He particles which escape to space

Table 4.5: Parameters and boundaries of the different atmospheric layers in the Earth atmosphere

absorption of optical/near-IR light. The absorbed photon energy is converted first into intrinsic rotational or vibrational energy of the molecule, which is then transferred to thermal motion of the gas.

- **Surface heating.** The optical and near-IR light of the star can also be absorbed by the ground surface which is heated up. This provides a hot bottom for terrestrial objects.
- Internal energy. Gas giants are still contracting and therefore they have a steady upward energy flow which heats the troposphere from below.

**Energy transport processes.** The three major energy transport mechanisms for planetary atmosphere are radiation, convection and conduction. Usually one process dominates for the definition of the temperature structure in the atmospheres.

- Convection is the energy transport by vertical gas flows. It only sets in if the conditions for convection are favorable. There must be a dense gas and a fast temperature decrease with height for convection. All solar system planets show convection in their troposphere because the upper tropospheric layers cool efficiently by radiation, while the low troposphere is strongly heated by internal energy or irradiation. Convection is discussed in detail in the following section.
- Radiation energy transport is important for optically thin atmospheric layers. Optical light from the sun (central star) is efficiently deposited in the lower troposphere while UV light is absorbed in the upper atmosphere. Thermal radiation emitted in the IR wavelength band can easily escape from the upper troposphere and all layer above causing always a cooling. Deep in the troposphere the emission of IR-light is not efficient, because the gas is optically thick in the IR range and the radiation energy is essentially "trapped".
- Conduction is the energy transport by collision between particles. Conduction is the mechanism which transfers the energy from a hot surface to the gas because the surface particles have a high kinetic motion. Conduction is also the process which transports the energy in the thermosphere and exosphere. Because of the large mean free path length between collisions ( $\Delta s \gtrsim H$ ) these outermost layers are homogeneous in temperature.

### Temperature structure for solar system planets.

- The giant planets show also a troposphere and a tropopause with a temperature between 50 and 100 K. The stratosphere reaches a temperature of 150 K. The heating is due to photochemical absorption by haze (photo-chemical smog), while the cooling is mainly due to emission lines of  $C_2H_2$  (acetylene) and  $C_2H_6$  (ethane). Above comes the thermosphere where the temperature reaches about 800 1200 K. There is no well defined mesosphere for the giant planets.
- Venus has a troposphere which extends up to the tropopause at 70 km where the pressure is about 0.1 bar and which marks also the top of the cloud layers. Above this follows a constant temperature region up to about 100 km. Further above there exists a strong difference between a 300 K thermosphere on the day side and a much colder, only 100 K so-called cryosphere, on the night side. The temperature in the thermosphere is relatively low because  $CO_2$  is a molecule which can efficiently cool the higher atmosphere.
- Mars has only a very thin atmosphere, lacking the density of "normal tropospheres and stratospheres". Thus, the temperature decreases from the surface temperature of about 220 K to 120 K above 50 km. The temperature increases then above 120 km to about 160 K. Mars has like Venus "no really hot" thermosphere because of the efficient  $CO_2$  cooling.

### 4.4.3 Tropospheric Convection

Convection is the energy transport by gas flows and it is a dominant energy transport process in the troposphere. Convection will occur if the following conditions are fulfilled

- a gas parcel, which is slightly hotter, and therefore slightly less dense and lighter than its surroundings will start to rise,
- the ambient pressure decreases and the parcel expands, and cools adiabatically (heat transfer to the surroundings can be neglected),
- if the parcel is, after some upwards motion and adiabatic expansion (and cooling), still hotter and less dense than the surroundings then it will continue to rise in a convective flow.

Convection stops if the parcel is after some upward motion colder and denser than the surroundings. The condition for convection is determined by the relation of two temperature gradients:

- the adiabatic temperature gradient  $dT/dz|_{ad}$ , which follows from the first law of thermodynamics (see below),
- the surrounding atmospheric temperature gradient  $dT/dz|_{\text{atmos}}$  which is determined by all heating, cooling and energy transport processes.

If the adiabatic temperature gradient is shallower than the atmospheric temperature gradient

$$-\frac{dT}{dz}\Big|_{\rm ad} < -\frac{dT}{dz}\Big|_{\rm atmos} \tag{4.21}$$

then the atmosphere is unstable with respect to convection and convection will set in. The above relation is often also given with a different sign  $dT/dz|_{\rm ad} > dT/dz|_{\rm atmos}$ . Equation 4.21 is equivalent to the following statements:

- convection may occur if the temperature decreases fast with height,
- convection does not occur if the temperature is almost constant with height or when the temperature rises with height.

**Derivation of the adiabatic lapse rate.** The adiabatic temperature gradient or adiabatic lapse rate follows from the first law of thermodynamics which describes energy conservation:

$$dU = dQ - dW.$$

The change in internal energy dU of a gas is equal to the change in thermal energy (added or lost) of the gas dQ and the work done by or put into the system dW. The following relations are valid:

- dQ = 0: there is not heat exchange to the surroundings in an adiabatic expansion or compression process,
- $dU = mc_V dT = m(c_P R_s) dT = mc_p dT mR_s dT$  describes the change in internal energy, where m is the mass of the gas parcel,  $c_V$  and  $c_P$  the specific heat capacities at constant volume and constant pressure, and R is the specific gas constant,
- $dW = P \, dV = mR_s \, dT (m/\rho) \, dP$  is the work put into the gas by compression or done by the gas by expansion. In addition there is  $P \, dV = mR_s \, dT - (m/\rho) \, dP$ , which follows from  $P \, dV + dP \, V = mR_s \, dT$ , the total derivative of the ideal gas law  $PV = mR_s T$ , and  $V = m/\rho$ .

Now we can rewrite the energy equation

$$dU = mc_p dT - mR_s dT = -mR_s dT + (m/\rho) dP = -dW$$

and obtain the adiabatic temperature-pressure gradient

$$\frac{dT}{dP} = \frac{1}{\rho c_p}$$

which yields with the hydrostatic pressure law  $dP = -g\rho dz$  the **adiabatic lapse rate** as final result

$$\frac{dT}{dz} = -\frac{g}{c_p} \,. \tag{4.22}$$

This lapse rate is valid for a dry atmosphere where no condensation occurs.

Lapse rates for Earth atmosphere. For the Earth the adiabatic lapse rate is about -10 K/km. If condensation occurs then the "moist" adiabatic lapse rate should be used which is slightly different with a value of -5 K/km. The average atmospheric lapse rate is -6.5 K/km. This means:

- the Earth atmosphere is stable against convection, if no condensation occurs and the dry adiabatic lapse rate applies,
- the Earth atmosphere is unstable if condensation occurs and convection will take place as soon as the moist lapse rate is appropriate,
- cloud formation and condensation are closely connected to convection.

**Convection in other solar system planets.** All solar system planets with a substantial atmosphere have a troposphere where convection dominates. Convection is a very efficient mechanism for transporting energy whenever the temperature gradient is superadiabatic (temperature decreases faster than the adiabatic temperature gradient). This places a firm limit how fast the temperature increases with depth in planetary atmospheres.

The troposphere extends in all solar system planets from > 1 bar to a pressure level of about 0.1 bar. This is the range where most of the thermal radiation is escaping from the planetary atmosphere. The main heat source due to the absorption of stellar radiation (and the internal heat for the giant planets) is below the troposphere. Thus, the troposphere is characterized by a strong heat source at the bottom and strong radiation losses at the top which leads naturally to the observed "convective" temperature structure.

#### 4.4.4 Atmospheric escape

Particle escape from an atmosphere involves three steps. First a gas particles must be transported from the lower to the upper atmosphere. Then the particles must be transformed from an atmospheric gas particle, usually molecules, to neutral or ionized atoms which can then in a third step be accelerated to high speed and escape.

The basic process for escape is thermal or hydrostatic escape, where particles in the high atmosphere have thermal velocities which are large enough for escape. In addition the density must be low enough that the particle does not collide on its escape trajectory.

The escape velocity  $v_{esc}$  for a particle is reached if its kinetic energy is equal to its potential energy (the energy required to leave the planet):

$$\frac{1}{2}mv_{\rm esc}^2 = \frac{mM_PG}{R_P} \tag{4.23}$$

which yields:

$$v_{\rm esc} = \sqrt{\frac{2GM_P}{R_P}} \,. \tag{4.24}$$

Figure 4.7: Schematic shape of the Maxwell-Boltzmann velocity distribution function.

The particle velocity for a gas in thermal equilibrium can be described by the Maxwell-Boltzmann velocity distribution which gives the number of particles per velocity bin dv

$$n(v)dv = \frac{4n}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv \,,$$

where m is the particle mass, and n the total number density of particles and T the temperature of the gas.

We may use the most likely velocity  $\bar{v}$  of this distribution for an estimate on the particle velocity:

$$\bar{v} = \sqrt{\frac{2kT}{m}} \tag{4.25}$$

For T one should use the temperature of the thermosphere which is for giant planets and Earth around  $T \approx 1000$  K. The velocity distribution for large velocities decays exponentially. This means that in a gas there are always a small fraction of particles with velocities which are a factor of a few higher than  $\bar{v}$ . If always a small fraction of particles of a certain kind can escape then after some time (millions or billions of years) a substantial amount of particles may escape.

Considering the formulas for  $v_{\rm esc}$  and  $\bar{v}$  one can easily derive the following dependencies:

- light particles, in particular hydrogen, escape much easier than heavy particles such as C, N, or O. This explains why Venus, Earth and Mars have essentially no H gas but still CO<sub>2</sub>, N<sub>2</sub>, O<sub>2</sub> gas made of the elements C, N, or O.
- a planet with high escape velocity (essentially a planet with a large mass) can keep much better an atmosphere. This explains why the Earth has an atmosphere and the moon has none.
- A planet with a cold exosphere will have lower thermal velocities and keep more easily an atmosphere. This may explain why the strongly irradiated planet Mercury has no atmosphere while Titan has one.

#### 4.4.5 Evolution of the chemical composition of planetary atmospheres

The giant planets have a composition which is close to the solar composition. Hydrogen and helium dominate strongly. Atmospheres with substantial amounts of hydrogen are called "reducing atmospheres". They contain a lot of methane  $CH_4$ , ammonia  $NH_3$ , and also water vapor  $H_2O$  and hydrogen sulfide  $H_2S$  are expected. But essentially only  $CH_4$ and  $NH_3$  were detected while the other constituents are expected to be trapped in the deeper layers. Not much evolution in composition seems to have been taken place for the solar system giants since their initial formation. The situation is very different for the atmosphere of terrestrial planets.

**Primitive and secondary atmospheres of terrestrial planets.** The atmospheres of the terrestrial planets evolved strongly in the past. If Earth ever had a H and He rich atmosphere then it was lost completely. Also Ne is strongly underabundant indicating that it never existed in large quantities in Earth's atmosphere or that it was lost during a very early epoch. Therefore, it is assumed that all volatile elements in the Earth atmosphere originate from volcanic processes and outgassing. This provides, as observed, the elements

H, C, N, O, and S but not the noble gases He and Ne. Ar is quite abundant in the atmosphere of Earth because of the radioactive decay of  $^{40}$ K.

For a given elemental composition we may expect abundances as expected from chemical equilibrium. The following reactions are important for the composition of the terrestrial planets:

$$CH_4 + H_2O \iff CO + 3H_2$$

$$2NH_3 \iff N_2 + 3H_2$$

$$CO + H_2O \iff CO_2 + H_2$$

$$(4.26)$$

The escape of hydrogen shifts the equilibrium to the right and this can explain the predominance of  $CO_2$  and  $N_2$  in Venus, Mars and Titan.

In the Earth atmosphere there is a significant lack of  $CO_2$ . Carbon dioxide is dissolved in the water oceans and blocked as calcium carbonate  $CaCO_3$  as rock. Volcanic outgassing brings then partly the  $CO_2$  back into the atmospheres.

The abundance of  $O_2$  in the Earth atmosphere is mainly due to photosynthesis of green plants. According to chemical equilibrium  $O_2$  would disappear rapidly (on geological atmospheres) through oxidation from the atmosphere. This non-equilibrium chemistry is an important signature of life on Earth.

# 4.5 Spectra of substellar objects

#### 4.5.1 New spectral types

M-type main sequence stars were for about a century the lowest mass objects known outside the solar system. The first substellar object, GD 165B, was detected in 1988. At that time it was a "strange" companion to a white dwarf star. The revolution came around 1997 after the development of instruments with large and sensitive IR array detector. Near-IR sky surveys, like 2MASS (2-micron all sky survey), found nearby very cool objects. Many of the new objects have a spectrum like GD 165B which changed its status from a strange object to a prototype of the new class of brown dwarfs. New spectral classes based on red and near-IR spectra had to be introduced for extending the standard spectral sequence of stellar objects. Dominant absorptions are molecular bands from  $H_2O$  water vapor and  $CH_4$  methane, which are key features for the definition of the new spectral types L, T. Up to now (2013) many hundred L and T dwarfs were detected. The introduction of another class for even cooler objects, the Y dwarfs, is currently discussed.

We provide here a brief characterization of the spectral classes for low mass objects. One of the slides shows the transistion of spectral features from M- to L- and T-type objects in the 700 - 900 nm range and in the near-IR respectively. Table 4.6 summarizes some key properties of these systems.

**Spectral class M:** Many bright stars are of spectral type M, like e.g.  $\alpha$  Sco or  $\alpha$  Ori, but these are all evolved high or intermediate mass giant stars. M-stars are red objects indicating that they must be cool < 4000 K. No M-star on the main-sequence is visible to the naked eye, because they are too faint and emit their radiation mainly in the near-IR range.

Their spectral classification was traditionally based in the visual wavelength range. The red spectrum of M-stars is dominated by molecular bands of TiO. These absorption become stronger for cooler objects providing a well established scheme for the association of spectral types to surface temperatures. The TiO-band at 705 nm is an important spectral feature for the classification of M-stars. Other absorptions, like CaH or VO, are used to refine the classification.

**Spectral type L:** The spectral type L describes objects with strong  $H_2O$  absorptions in the near-IR, strong resonance lines of the alkali atoms NaI, KI, RbI and CsI, absorptions form metal-hydrates CrH and FeH. The warmer (early) L dwarfs show still the TiO like the M-stars but their strength decreases rapidely with decreasing photospheric temperature. The water vapour absorptions of L-stars are difficult to observe with ground-based observations because the same absorptions are present in the Earth atmosphere and they often prevent accurate measurements in the corresponding spectral bands. Typical temperatures for spectral type L are in the range 1300 to 2000 K.

**Spectral type T:** T-dwarfs have a surface temperature of about 700 to 1300 K and are characterized by strong absorptions of methane in the near-IR. They emit a lot of light in a few band between 1 and 1.6  $\mu$ m, because the CH<sub>4</sub>-bands block the emission of radiation from the photosphere efficiently around 1.1 and 1.4  $\mu$ m and in the range between 1.7 - 3.5  $\mu$ m. T-dwarf are therefore blue objects in the colors J - H, J - K, and J - M. This is one of the reasons why they were initially not found because according to their IR-colors they looked like "uninteresting" blue background stars.

spectral type	V–K	J–K	T [K]	spectral features
M0 M5 M9	$3.9 \\ 6.0 \\ 7.5$	$0.8 \\ 0.9 \\ 0.9$	$3800 \\ 2800 \\ 2400$	weak TiO (e.g. 705 nm), CaH, strong TiO, strong TiO, FeH (870 nm), weak H <sub>2</sub> O(near-IR)
L2 L8		$1.3 \\ 1.8$	$2200 \\ 1500$	strong $H_2O$ (near-IR), KI, CrH, weak TiO strong $H_2O$ , FeH, CrH,
T2 T6		$\begin{array}{c} 0.8 \\ -0.2 \end{array}$	$\begin{array}{c} 1200\\ 900 \end{array}$	$CH_4, H_2O, \dots$ strong $CH_4$
Y			<700	

Table 4.6: Characteristics of the spectral types for low mass objects.

according to Kirkpatrick, 2005, Annu.Rev.Astron.Astrophys.,43,195