Astronomical Imaging and Spectroscopy with CCD detectors

Short summary on basic principles for observations with the ETH student telescope. This text is based on the lecture notes "Astronomical Observations" of H.M. Schmid from 2012.

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Chapter 1

Detectors

1.1 Types of detectors

There are two basic detector principles for the measurement of electromagnetic radiation:

- electromagnetic wave detection or coherent detection at long wavelengths > 300 μm in the radio domain. These detectors must be fed with radiation of the same phase.
- photon detection or incoherent detection at shorter wavelengths $\lambda < 300$ μm. Photons excite or liberate one or many electrons and this can then produce an electronic signal or they excite phonons raising the temperature in a detector.

Detectors may also be characterized by other properties. For example one can distinguish according to their spatial multiplexing capabilities:

- single channel detector,
- imaging or array detector,

or their temporal resolution:

- photon counting detector and coherent detection,
- integrating detectors,

or whether they can distinguish between different photon energies / wavelengths:

- energy sensitive (spectrometric),
- broad-band detectors without spectral resolution.

Theoretically, the ideal photon detector is an imaging array which registers for each detector element the photon arrival times, the photon polarizations, and photon energies. The ideal electromagnetic wave detector measures the wavelengths, phases, polarizations, amplitude, and directions of the arriving electromagnetic waves. Such detectors are registering all the information $\mathbf{I}(x, y, \lambda, t)$ contained in the arriving electromagnetic radiation.

Usually such detectors are not available, but there are many tricks which allow to obtain more parameters with simple detectors. Such tricks are:

- single channel detector scanning over a spatial area or a spectral range,
- integrating detectors with short integrations for better temporal resolution,
- spectral dispersion of the radiation onto a broad-band, 2-dimensional detector array,
- and many other solutions.

1.2 Signal and noise

In all measurements of a target the total signal includes:

- S_t, the signal of the target,
- S_b, the "background" signal of the sky and near-by sources (objects in the side-lobes of radio telescopes),
- S_i, the "instrumental" signal from the telescope and instrument, including electronic noise from the detector.

The goal of a measurement is a reliable evaluation of the target signal S_t where the measuring errors are negligible (for a given science case) or as small as possible. The quality of a measurement is often expressed as a signal to noise ratio:

$$\frac{\text{target signal}}{\text{noise}} = \frac{\text{S}}{\text{N}}\,.$$

In a first step one has to distinguish the target signal S_t from the background signal S_b and the instrumental signal S_i. Then one has to evaluate the different contributions to the noise N, the noise in the target signal N_t, the noise in the background signal N_b, and the noise due to the instrument N_i. It is important to understand these noise sources in order to find strategies to reduce them.

Detector noise. Often the detectors are an important contributor to the "noise" signal of the instrument. One can distinguish between two general types of instrumental noise signal N_i:

- additive noise, which is independent of the target signal
- multiplicative noise, which depends on the target signal $N_i = a \cdot S_t$

For the discussion of the importance of detector noise we consider the simplification that the the detector noise N_d dominates the instrumental noise $N_i \approx N_d$.

Photon shot noise. Shot noise is the statistical noise when a finite number counts c of photons (or electrons in an electronic circuit) are measured. The measured number of photons is distributed around "an average value" according to the *Poisson distribution*. The Poisson distribution approaches for large photon number counts the *normal distribution*. The standard deviation of the photon noise – the photon shot noise – is then equal to the square root of the average number of photons:

$$N = \sqrt{c}\,. \tag{1.1}$$

Usually the target signal $S_t = c_t$ and the background signal $S_b = c_b$ behave according to Poisson statistics. Two detector noise sources, the read-out noise and the dark current, behave also according to the Poisson statistics.

Simple examples. We can assume that the uncertainty associated with the target signal and background signal behave like shot noise

$$N_{t+b} = \sqrt{N_t^2 + N_b^2} = \sqrt{S_t + S_b}\,. \tag{1.2}$$

We can distinguish between the following simple cases:

- *detector noise negligible:* the detector noise is smaller than the shot noise from the target and background $N_i < N_{t+b}$. In this case the detector is not contributing significantly to the measuring uncertainty. The S/N is then independent from the detector/instrument noise:

$$\frac{\mathrm{S}}{\mathrm{N}} \approx \frac{S_t}{\sqrt{S_t + S_b}} \tag{1.3}$$

- *detector noise dominant:* the detector noise is stronger than the shot noise from the target and background $N_i > N_{t+b}$. In this case the reduction of the detector noise is important.
  - The detector noise is just additive and does not depend on the target signal.

$$\frac{\mathrm{S}}{\mathrm{N}} \approx \frac{S_t}{N_i} \tag{1.4}$$

    One way to improve the S/N ratio is an enhancement of the target signal without enhancing the detector noise (e.g. longer integrations) or changing the measuring setup to reduce the detector noise (e.g. better focused signal on the imaging detector). Reducing the additive detector noise is often a most important issue for faint sources.
  - The detector noise is multiplicative and is a fraction of the target signal $N_i = a \cdot S_t$.

$$\frac{\mathrm{S}}{\mathrm{N}} \approx \frac{S_t}{a \cdot S_t} = \frac{1}{a} \tag{1.5}$$

    Longer integrations do not help. The disturbing effect must be reduced with better detector calibrations (flat-field calibration), averaging procedures, or a better detector design. Reducing multiplicative detector noise is often important for high S/N observations of bright targets.

1.3 CCD arrays

Charge coupled devices are now used extensively as detectors in astronomy. They are sensitive from the X-ray region to about 1 μm. The device is made of an array of individual picture elements (pixels) arranged in rows and columns.

In order to obtain pictures the photo-charges must be collected within one pixel with an electrostatic field capable of holding many charges. This can be obtained with metal electrodes on a silicon semiconductor separated by a thin (0.1 μm) SiO_2 layer as insulator. The structure behaves like a capacitor which can store charges. If the silicon is doped then an applied voltage will produce a p-n junction which can separate electrons and holes and attract electrons near the positively charged electrodes.

At the end of an exposure the charge packages collected in each pixel are transferred along the columns to one end of the array into a separate read-out row or serial register. In 3 phase CCDs, each row has three electrodes in column direction which can move the charges by applying consecutively voltages to these three rows. The charges in all rows are shifted synchronously along the columns in discrete row-shift steps. In the serial register the charges are transfered along the row to the read-out amplifier. This read-out happens between two row-shift steps so that always one row is read-out before all charges on the CCD are shifted by another row towards the read-out register.
CCD arrays exist in many different sizes and in many different variations which are not discussed here in detail. Typical pixel dimensions are $10 - 30$ μm squared, Often used CCDs array have 1000 x 1000 pixels (or 1k x 1k), 2k x 2k, or 2k x 4k pixels. In some

instruments 2, 4, or more such devices are put together to form a mosaic of detectors in order to form even larger focal plane arrays. In these mosaics there is a gap of the order $1 - 10$ mm between the individual arrays.

Some important properties of the CCD arrays:

- very high quantum efficiency,
- the sensitivity of each pixel can be calibrated,
- the same output amplifier is used for up to 10^6 pixels,
- read-out noise and other disturbing detector effects are often very low.

A more detailed characterization of CCD properties is given in the next section.

Pixel binning. A vertical (along the column) read-out pixel binning is possible by transferring two (or more) pixel rows into the read-out register before read-out. Adding up the charges of two (or more) pixels from the read-out register in the output amplifier gate produces a horizontal pixel binning. Pixel binning can be very useful because there are more photons in a binned pixel with the same or not much enhanced read-out noise as a single pixel.

1.3.1 Important CCD parameters for observations

Sometimes one has the possibility to choose for a telescope/instrument configuration the CCD detector, or one can choose for a given CCD between different read-out modes. This often allows to optimize significantly the system efficiency depending on the needs.
Important considerations for a detector selection are:

- which detector provides the better spatial resolution and area coverage (or spectral resolution and coverage for a spectrograph),
- the quantum efficiency can differ easily by 30 % overall or for certain wavelengths,
- the read-out overhead for bright targets can be significant and therefore one should study the different read-out modes available,
- one detector may have much less disturbing detector effects,
- flat-field calibration may be much more accurate for certain detectors (no fringing).

Considering all the options, one should also not forget that the standard mode of operation of a CCD is always less risky than trying some special features for just a slight improvement.

1.3.2 Parameters for CCD detectors

For an assessment of different detectors or different detector modes one needs specifications for detector parameters.

Characteristic parameters for CCDs:

parameter	explanation	typical values for CCDs
QE quantum efficiency	ratio between the detected photons and incident photons	e.g. $QE_{peak} > 80$ %
spectral response	dependence of the QE with wavelength	e.g. 30 % at 300 nm, 80 % from 400 nm to 800nm, 30 % at 1 μm
full well capacity	number of charges (e.g. e^-/pixel) that can be stored before saturation	typically 10^5
read-out gain	the conversion of electron charges into analogue detector units (ADU) or counts	typical value 1-3 e^-/ADU
non-linearity	deviation from linearity of the registered photons with increasing illumination	< 3 %
cosmetics	description of the defect pixels or pixel columns	$\ll 1$ % typically
DC, dark current	thermal current per pixel for an unilluminated detector	$< 1\,e^-/(hr\,pixel)$ at 100 K
RON, read out noise	noise introduced by the read-out process	$< 10e^-$/pixel
CTE	the charge transfer efficiency for shifting by one pixel	often > 0.99999
dynamic range	ratio of the maximum exposure level before saturation to the systematic noise level due to dark current and read-out	> 1000
read out time	time required to read out a detector after an exposure	typical range 1 sec to 1 min

1.4 Basic detector data reduction steps

Electronic imaging devices such as CCDs and infrared arrays require different data reduction and calibration steps for quantitative measurements in e.g. photometry and spectroscopy. For the data reduction it is important to understand the properties of the detectors used.
The following detector effects must be corrected:

- bias subtraction (including eventually an electronic fixed pattern noise),
- dark current subtraction,
- bad pixel correction,
- removal of cosmic ray events,
- flat-fielding (quantum efficiency variations across the array).

1.4.1 Bias subtraction

Bias frames are un-illuminated frames with zero integration. Bias frames contain a bias level introduced by the electronics, the read-noise, and possibly also a fixed (read-out) pattern from the electronics

Bias level: CCDs introduce an offset charge on each pixel which has to be subtracted.

- if the bias level is constant over the CCD then a scalar value should be used for the bias subtraction,
- if this bias level is not constant over the CCD then a fixed pattern noise frame has to be determined from many bias frames and subtracted.

It should be considered that the subtraction of the bias level (e.g. just subtracting a bias frame) would introduce noise to each science frame at the level of the read-out noise. Therefore it is better to subtract a scalar value $q_{\rm bias}$ if possible. A slight uncertainty in $q_{\rm bias}$ is often of no concern because the sky background subtraction can account for this.

Random read-out noise RON. The read-out process introduces a random "Gaussian" noise. The level of the random read-out noise follows from the bias frame (eventually after the subtraction of the fixed pattern noise → corrected bias).

The read-out noise per pixel $N_{\rm pix}$ ($= N_{x,y,i}$ for a given pixel x, y on the detector and integration i) is the standard deviation (rms: root mean squares) from the mean of the registered electrons per pixels, or detector counts ADU, in the bias frame. Of course, pixel defects or cosmic ray event should be excluded from the determination of $N_{\rm pix}$.
We consider the S/N of an observations if the read-out noise $N_{\rm ron}$ is the dominant noise source besides the photon shot noise:

$$\frac{\rm S}{\rm N} = \frac{S_t}{\sqrt{S_t + N_{\rm ron}^2}}\,. \tag{1.6}$$

Depending on the observation one must consider for the read-out noise $N_{\rm ron}$ the number of pixels $n_{x,y}$ and the number of integrations n_i involved in the measurements. This is in the case of Gaussian random noise:

$$N_{\rm ron}^2 = \sum_{x,y,i} N_{x,y,i}^2 = N_{\rm pix}^2 (n_{x,y} + n_i)\,. \tag{1.7}$$

This equation indicates that there are two ways to reduce the read-out noise:

- reduce the number of pixels, or the size of the aperture, for the measurement (e.g. by pixel binning),
- reduce the number of integrations.

For averaged bias frames the random read-out noise should decrease like the square-root of the number of the frames averaged. This simple test is useful as check whether really random read-out noise is measured. The bias level and read-out noise can evolve and it should be monitored during the night (e.g. for CCDs a few frames at the beginning of the night, in the middle of the night, and at the end of the night).

Fixed pattern noise. A number of bias frames can be averaged in order to investigate the presence of a fixed pattern introduced by the read-out process. A well defined fixed pattern noise image can be subtracted from all frames. This must be done carefully in order to avoid that the residuals from the fixed pattern determination produce spurious features in the science data.

The impact of a weak, uncorrected fixed pattern can be harmful if many integrations n_i are added up, because the fix pattern noise increases with the number of integrations. For a mathematical formulation one has to split up the pixel noise $N_{\rm pix}$ into the random noise contribution $N_{\rm rand}$ and the fix pattern contribution $N_{\rm fix}$. These two contribution behave roughly like:

$$N^2_{{\rm ron},x,y} = N^2_{\rm rand} n_i + N^2_{\rm fix} n_i^2 \,. \tag{1.8}$$

It is clear that the fixed pattern noise becomes the dominant read-out noise component for a large numbers of integrations. **Image dithering** is one way to average down a fixed pattern noise. Image dithering is achieved by moving the image to n_{dith} slightly offset detector positions so that a detector pattern is averaged down. The impact of image dithering depends very much on the nature of the fixed pattern noise (kind of random features, periodic pattern etc.). For a first estimate, one may assume that $n_{\rm dith}$ image dithering reduces the fixed pattern noise according to the following formula:

$$N^2_{{\rm ron},x,y} = N^2_{\rm rand} n_i + N^2_{\rm fix} \frac{n_i^2}{n_{\rm dith}} \,. \tag{1.9}$$

Image dithering is often used in imaging. Dithering is also useful for reducing the effects of bad detector pixels or the uncertainties in the flat field calibrations (see below).

1.4.2 Dark current subtraction:

A dark current can be created in the detector by the thermal motion of the electrons. Therefore, the dark current depends strongly on the detector temperature. The dark current is determined with un-illuminated frames with long integration times. The bias frame has to be subtracted and the remaining signal is the dark current. The dark-current should increase linearly with the integration time $S_{\rm dark} \propto t$

If significant then the dark-current is subtracted from all science frames. For this correction one should use scalar values or smoothed dark frames for the same reason as for the bias subtraction.

Some remarks:

- The dark current for CCDs operated at about 80 K (liquid N_2-cooling) is typically very low (< few counts/pixel and hour) and can be neglected. In this case dark frames can be taken as a check during day-time calibrations.

- For warm CCDs (0°C in e.g. amateur telescopes) the dark-current is often very high and the subtraction is critical. In this case it can be useful to take a dark frame with the same integration time after each observation and subtract this from the science frame (smoothing could reduce the noise).
- If the dark current is unexpectedly high then the detector cooling may not operate properly.

For estimates on the noise introduced by the dark current $N_{\rm dark} = \sqrt{S_{\rm dark}}$, essentially the same formulation as for the read-out noise applies:

$$\frac{\rm S}{\rm N} = \frac{S_t}{\sqrt{S_t + S_{\rm dark}}} = \frac{S_t}{\sqrt{S_t + N^2_{\rm dark}}}\,. \tag{1.10}$$

If the dark current is the dominant noise source $S_{\rm dark} > S_t$ then the S/N will improve like ${\rm S/N} \propto \sqrt{t}$ with exposure time.

1.4.3 Bad-pixel correction

There are many different types of bad pixels. The most important are:

- dead (black) pixels with no or strongly reduced pixel efficiency,
- hot (white) pixels which accumulate charges unrelated to the illumination,
- in CCDs, there can be bad pixel rows due to the strongly reduced charge transfer efficiency of one pixel.

Bad pixels can be identified by a series of flat-field exposures with different exposure times. Pixels which do not show a count level correlating (linearly) with exposure time are bad pixels.
Bad pixels can be corrected with an interpolation over the bad pixel using the adjacent pixels. This procedure may use a pre-determined bad pixel map. Of course at the position of the bad pixel the scientific information is lost.

Dithering. Image dithering, moving the image from the sky on the detector in consecutive integrations is a powerful way to overcome bad pixel effects. For example one can take three dithered images and re-align the images in the data reduction process. Bad pixels can then be eliminated statistically ($> 5\sigma$ outliers in single frames with no counterpart in the dithered frame, median filtering etc.).

Bad pixels can be very harmful if they are not considered in the observing strategy with appropriate measures (e.g. dithering). The effect must be studied on a case by case basis because the impact depends very much on the science case and the used detector. The realigment of dithered images is done after detector flat-fielding.

1.4.4 Removal of cosmic ray events

Cosmic ray events produce point like signals (single pixel signals) with up to several thousand photo-electrons. These events can be identified easily with multiple exposures down to the level of several times the random fluctuations (e.g. $\approx 5\sigma$) in the surrounding pixels. In certain applications, they can be hard to identify because they look like a real signal, e.g. like a star. The number of cosmic ray events increases roughly proportional to the integration time. Cosmic rays are more frequent for instruments at high altitude sites and in particular for space instruments.

Removal of cosmic ray events and the correction for bad pixels can use to some extend quite similar reduction algorithms.

1.4.5 Flat-fielding

With a flat-field image the pixel-to-pixel sensitivity differences for the imaging detector are calibrated. This accounts also for interference effects in the detector substrate, and localized transmission structures due to dust and inhomogeneities on optical components.

For a good flat-field many exposures of equally illuminated frames are averaged. Before averaging, the individual flat field exposures must be bias (and dark) subtracted and corrected for bad pixels and cosmics.
Flat-fields can be obtained by several methods:

- *Screen flatfields* are exposures of a diffusely illuminated screen in the telescope dome onto which the telescope can be pointed. In this approach the light goes through the telescope and the whole instrument. Despite this rays from an illuminated screen may behave differently when compared to a real sky observation and cause problems

in critical applications. Further difficulties could be that the lamps are (often) too faint for calibrations in the blue / near-UV spectral region. Screen flatfields may not be useful when taken during the day because of the contamination by daylight (solar spectrum).

- *Internal flatfields* are exposures of an uniform illumination of the focal plane with a "flat-field lamp" in the instrument. As with the screen flatfields the light passes not exactly the same way through the instrument as for the science observations. Again, flatfielding in the blue / UV might be difficult. The big advantage of internal flatfields is that they can be carried out during the day without a degradation of the quality and for large telescope they provide a smoother illumination than a dome screen.
- *Twilight flatfields* are sky exposures taken during twilight. This is for many application the preferred method because the light path is identical to a science application. Sky flatfields require a thorough preparation because the sky brightness changes rapidly during twilight.
- *Sky flatfields* are constructed from a series of sparsely populated sky observations; this method is the standard method in mid-IR applications where the flux of the sky is high. Sky flatfields are often done with the science target in the field. Dithered images are then taken with a displacement which is larger than the diameter of the target.

The flat-field frame should be normalized to unity so that the photo-electron count numbers c are essentially preserved:

$$\mathrm{FF} = \frac{S_{x,y,i}(\mathrm{FF})}{\langle S_{x,y,i}(\mathrm{FF})\rangle}$$

All science frames (bias and dark subtracted and corrected for bad pixels and cosmics) are divided by the flat-field.

Because the flatfielding is a multiplicative correction to the data it can be critical for many applications. For this reason the determination of the flatfields correction often needs much attention.
Things to consider are:

- Flat-field calibrations are usually only valid for one specific instrument mode. For example, each filter requires its own set of flat-field calibration exposures.
- Errors of x % in the flat-field correction (division) produce errors of the same x % in the flux measurements. This applies also for the random noise in the flat-field.
- The flat-fielding precision can be affected by various not easily recognizable effects:
  - differential illumination of the detector between flat-field exposures and science exposure (e.g. due to the different light path or due to mechanical flexures)
  - colour differences between sky target and the flat-field source

Fringing. Flat-fielding is particularly difficult if there exists an interference patterns introduced by thinned, back-illuminated CCDs in narrow-band red/near-IR filter observations. Slight changes in the colour of the illumination source or small instrument flexures might produce effects which are too large for a precise flat-field calibration.

1.4.6 Summary of data reduction steps

A simple recipe for the data reduction for the detector effects has the following form:

$$\text{reduced frame} = \frac{\text{object frame} - \text{bias/dark frame}}{\text{flat field} - \text{bias/dark frame}} \,. \tag{1.11}$$

This includes careful checking and consideration of cosmic ray hits, fix pattern noise, and bad pixels as described in the previous sections.

Chapter 2

Photometry

The original photometric system is the UBV (ultraviolet, blue, visual) system introduced around 1950 by Johnson and later extended with the R and I (red and infrared) bands. Classical photoelectric photometers employed photomultiplier tubes. Many different photometric systems were introduced later, particularly important was the extension to the near-IR and mid-IR bands with infrared detectors, and the introduction of radio flux measurements. Numerous new photometric systems were then introduced by the satellites covering the wavelength regime from the far-IR to the hard γ-rays.

2.1 Science considerations

The photometry, or the measurement of the spectral irradiance, is often a most important measurement for the characterization of an astrophysical target. The following parameters and deduced quantities can be derived from photometry:

- the detection proves the presence of a source,
- the apparent brightness of a source can be used for determining its luminosity,
- the morphology of the sources characterizes the type of object (extended / point like object; symmetric / asymmetric source, etc.)
- the spectral energy distribution (SED) yields the total apparent luminosity of the source and allows a characterization of all important radiation components in a system,
- the color of a system can already provide a good characterization of the system (e.g. temperature of a star, stellar population in a galaxy),
- the temporal variability of the source can be investigated with repeated observations.

Photometry measures a lot of the observable parameters provided by the electromagnetic radiation $\mathbf{I}(x, y, \lambda, t)$ from a target.

2.2 Photometric signal extraction

The first step in the photometric measurement is the extraction of the detector signal. We consider two methods: aperture photometry and PSF fitting.

2.2.1 Aperture photometry

Aperture photometry measures the light from a sources contained within an aperture in the focal plane. The aperture is usually circular and has a certain angular diameter.

With **photoelectric aperture photometers** the aperture was a hardware mask with a hole in the focal plane of the telescope. The photometric measurement consists then of:

- measurements of the flux for the target and the sky background $S_t + S_b$
- measurements of the sky background S_b only.

Subtraction of the sky measurement from the target plus sky measurement yields then the target signal:

$$S_t = (S_t + S_b) - S_b$$

This measurements for target and sky can be taken consecutively or simultaneously using multiple aperture instruments.

With **imaging detectors** one can select software apertures for the target and the sky. Usually a round aperture is chosen for the target with an annular ring just outside for the background. Software apertures can be optimized in various ways for improving the photometric measurements:

- accurate centering on the target,
- the size and the shape of the aperture can be adapted to each individual object,
- the aperture size can be reduced in order to reduce the disturbing background,
- for background subtraction a clean (object free) sky region can be used, which represents best the background at the position of the target,
- the impact of the selected apertures (target and background) on the photometric measurement can be investigated by varying the aperture geometry,
- possible uncertainties due to other sources or instrumental effects can be recognized in the image and considered in the analysis.

All these possibilities offered by imaging detectors should be considered in the signal extraction procedure in order to optimize the results.

2.2.2 PSF fitting

Point spread function (PSF) fitting or profile fitting uses model signals and fits them to the data. Usually the PSF has the shape of point sources, e.g. stars, in the analyzed image. The source model is varied until a good match is obtained for a source, or for all sources in the image. Software packages, e.g. DAOPHOT, ROMAPHOT, and others are available for this procedure. The flux of a target is then defined by the fitted model curve. The stellar images are usually fitted with a radial Gaussian intensity profile

$$I(r) = I_0 \mathrm{e}^{-r^2/\sigma^2} , \tag{2.1}$$

where I_0 is the peak intensity, and r the radial distance from the center of the profile. The quantity σ measures the width of the flux distribution:

- $\pm 1\sigma$, a circle with $r = \sigma$, contains 68 % of the signal,
- $\pm 2.5\sigma$ contains 98.7 % of the signal,
- 1.665 σ corresponds to the full width at half maximum (FWHM) of the Gaussian peak.

The fitted PSF can be subtracted from the image. If significant residuals remain then there might be fainter stars close to the fitted source.
Double or multiple stars can be disentangled with a multi-PSF fitting to the blended sources. In this way the signal of all sources in a blend can be obtained.
Extended sources, like galaxies, can be fitted with more complicated flux distribution models. Such an approach provides then the flux contributions of different morphological components, like bulge, bar, disk, halo, etc.

2.3 Calibration of photometric data

Photometric calibration is the transformation of the reduced detector signal (number of photo-electrons) into an irradiance (or radiation flux) from the source outside of Earth's atmosphere. This procedure requires the determination of

- the instrument efficiency and wavelength dependence in the passband,
- the atmospheric transmission for ground-based observations.

The normal procedure is that one measures the signal of a target and of a well known photometric standard object with exactly the same equipment. The comparison of the two signals yields then the photometric flux of the target.

2.3.1 Magnitude system

The magnitude system is the photometric system used in the ground-based visual and infrared work. By definition the magnitude difference of two objects is:

$$\Delta m = m_1 - m_2 = -2.5 \log_{10} \frac{I_1}{I_2} \,. \tag{2.2}$$

The zero point for the magnitude system is defined by the standard star Vega, which is a 0-mag star in all wavelength bands.

$$\text{Vega}: \quad m_U = m_B = m_V = ... = m_N = 0.0^m$$

This means that also all colors of Vega are zero:

$$\text{Vega}: \quad B - V = m_B - m_V = m_X - m_Y = 0.0^m$$

Standard filters. Widely used standard filters for the measurements of stellar magnitudes are the Johnson filters.

Characteristic values for the Johnson filters are:

band	λ_{central} [nm]	width $\Delta\lambda$ [nm]	photon flux[a] 0^{mag}
U	360	70	$6.0 \cdot 10^7$
B	440	90	$1.5 \cdot 10^8$
V	550	90	$9.6 \cdot 10^7$
R	700	220	$5.5 \cdot 10^7$
I	880	240	$4.0 \cdot 10^7$
J	1250	380	$2.2 \cdot 10^7$
H	1650	310	$9.6 \cdot 10^6$
K	2200	480	$4.5 \cdot 10^6$
L	3400	700	$1.2 \cdot 10^6$
M	4900	300	$5.6 \cdot 10^5$
N	10200	5000	$5.7 \cdot 10^4$
Q	20000	5000	$7.3 \cdot 10^3$

[a]: photons $\text{s}^{-1}\text{m}^{-2}\text{nm}^{-1}$

There are many different types of photometric filter systems and the transformation of magnitudes from one system to another requires careful considerations. The problem is that even identical filters used in different instruments with a different spectral dependence of the efficiency produce different magnitudes.

Example. Two instruments use the same type of B-band filters with central wavelength 450 nm and a width of 80 nm, but instrument A has a blue sensitive CCD with essentially a constant quantum efficiency over the filter pass band, while instrument B has a red-sensitive CCD for which the quantum efficiency QE behaves like QE $\propto \lambda$ (40 % at 400 nm and 60 % at 500 nm) in this wavelength range. A red star will then have a higher flux (lower magnitude) when measured with instrument B.

Due to the dependence of the measured magnitudes on the instrument efficiency it is very helpful to use filters and aim for instrument efficiency curves which are similar or equal to a widely used standard photometric system. In this way one can use the existing magnitude measurements and color indices for the objects without the need to determine a new set of photometric standard stars or to define the correction values for the conversion of magnitudes of objects between the own system and a standard photometric system. This magnitude conversion depends essentially on the color of the measured objects.

2.3.2 Standard stars

Vega is far too bright to serve as calibration star for instruments built for galaxies and other faint objects. For this reason there exist many lists of faint photometric standard stars for visual and near-IR observations. For the mid-IR one uses bright red giants for the calibration because of the strong sky background at these wavelengths. Good photometric standard stars have the following properties:

- photometrically constant object at the 0.1 % to 1.0 % level as proven by long term monitoring,
- object with a normal continuum spectrum, in particular without narrow spectroscopic features like strong emission lines,
- single source without potentially disturbing other sources nearby.

A popular standard star list is given in: Landold A.U., 1992, Astronomical Journal Vol. 104, page 304.

2.3.3 Atmospheric transmission

In ground-based observations the flux from a source is attenuated by the Earth atmosphere. Even for clear atmospheric "windows" the atmospheric extinction is about 10 % and often significantly higher. For this reason the atmospheric transmission must be taken into account for photometric work. Various effects of the atmosphere have to be considered:

- strongly variable transmission due to clouds,
- airmass,
- attenuation per unit airmass,
- photometric scintillation due to atmospheric turbulence.

Clouds. Clouds are very harmful for photometric measurements. Even a very thin, hardly recognizable layer of cirrus clouds can easily introduce transmission variations at a level of 20 % or more. For this reason one has to monitor carefully the clear sky conditions during the night and one should only use data taken under good ("photometric") atmospheric conditions. Nights with some signs of clouds should not be used for photometric work since it is often hard to clarify afterwards in the data reduction process whether unexpected and therefore particularly interesting measurements are real or just due to passing clouds. For this reason one should plan absolutely calibrated photometric

measurements preferentially for sites where the atmospheric conditions are often stable and reliable. Zurich is certainly not such a site.

Transmission of the "clear" atmosphere. The transmission T or attenuation $1-T$ of the clear (cloud free) atmosphere depends on the wavelength dependent vertical optical depth $\tau(\lambda)$ of the atmosphere and the airmass.

$$T = \frac{I}{I_0} = \mathrm{e}^{-\mathrm{airmass}\cdot\tau(\lambda)} \tag{2.3}$$

The airmass accounts essentially for the zenith angle z dependent path length of the light through the atmosphere:

$$\mathrm{airmass} = \frac{1}{\cos z} \quad (= \sec z)\,. \tag{2.4}$$

For large zenith angles $z > 60^\circ$ one should use a more accurate formula. However, photometric measurements for large zenith angles are difficult and should be avoided.

The extinction $\tau(\lambda)$ is mainly determined by three principle atmospheric components:

- Rayleigh scattering by molecules
- aerosol particles
- molecular absorptions

Rayleigh scattering. Rayleigh scattering by molecules such as N_2 and O_2 depends on wavelength like $\tau \propto \lambda^{-4}$. It is very strong ($20-60$ %) in the blue / near-UV spectral region. Of course the Rayleigh scattering decreases with altitude like the air pressure.

Aerosol particles. Aerosols consists of fine dust, water droplets, ice crystals, and pollution. Aerosols are mainly a problem at low sites and much reduced at high mountain sites. The extinction by aerosols is rather weak, 5-10 % in the visual, and grey, like $\tau \propto \lambda^{-1}$.

Molecular absorption. Molecules produce discrete absorption lines and bands. The most important absorbers are ozone, water vapor, and oxygen.

- *Ozone:* O_3 is responsible for the UV transmission cutoff at 320 nm and a broad shallow extinction of 5 % between 500-700 nm.
- *Water vapor:* H_2O produces very strong line absorption bands in the infrared spectral region, which block at certain wavelengths essentially all the incoming light. The IR photometric bands are just located in the spectral bands between the strongly absorbing H_2O bands. Water vapor absorptions are also strongly attenuating the sub-mm to mm wavelength band in the radio domain. The mm-radio telescopes are therefore located at dry, high altitude sites like Mauna Kea on Hawaii (4000m) or Chajnantor in Chile (5000m).
- *Oxygen:* Molecular oxygen O_2 produces absorption bands in the visual, e.g. at 762 nm (A-band), 687 nm (B-band), and at 628 nm. These are narrow bands and their impact on photometric measurements can usually be neglected. A strong O_2 band is also present at 60 GHz in the radio range.

Photometric scintillation. The atmospheric turbulence cells with typical diameter of about 20 cm produce a time dependent focussing and defocusing of the incoming wavefronts resulting in a photometric scintillation for point sources: the stars twinkle. Planets twinkle much less because they are extended and therefore the small scale phenomenon

(coherence angle ≈ 5 arcsec) is averaged out. The flux variations are at a level of 10 %, on timescales of less than 10 ms, and for telescope apertures less than 10 cm.
The effect is strongly reduced for telescopes with large apertures $\gg 10$ cm and long integration times $\gg 10$ ms. On a 1 m telescope with integrations of > 1 s the photometric scintillation is below 0.1 %.

At the level of $1.0 - 0.1$ % other instrumental effects become important. For example for imaging detectors the jitter of the star image on the detector pixels is an important source of error because the efficiency of the pixels needs to be calibrated with very high precision for an accurate measurement.

2.3.4 Summary on photometric calibration

For the photometric measurement one needs the following steps:

- a flux measurement for the target,
- flux measurement for photometric standard stars with the same instrument,
- correction for the atmospheric extinction for the target star and photometric standard star,
- flux ratio conversion into a magnitude difference,
- calculation of the magnitude of the target, using the magnitude of the standard star and the derived magnitude difference,
- consideration of possible color effects due to deviations of the instrument efficiency from the standard filter pass band.

The atmospheric extinction correction is not necessary if the target is measured in the same image (simultaneously) with the calibration source.

2.3.5 Relative photometry

The attenuation by clouds is roughly grey. For this reason one can still carry out "relative" photometric imaging when the sky conditions are not perfect. In this case one can measure the brightness of targets with respect to other objects in the same image, preferentially objects with known brightness. Different types of such science programs are possible, for example:

- measurements of the object magnitude on a relative scale, preferentially with respect to a well calibrated source in the field,
- measurements of the colors of an object, again relative to known sources in the field,
- search of objects with special colors or strong spectral features which can be recognized with special filters (e.g. Hα line filter),
- monitoring of the brightness variability with respect to non-variable objects in the field.

When planing "absolute" photometry then one should always consider a backup program for "relative" photometry, which can be carried out under non-photometric atmospheric conditions. Relative photometry can always be "upgraded" to an absolutely calibrated measurement with later "absolute" calibrations of the brightest, non-variable targets in the image.

2.4 Detection of faint sources

For the detection of faint sources one has to push the S/N ratio by enhancing the signal as much as possible and reducing the noise as much as possible. The accurate calibration of the data is not such an issue since the detection will have a low S/N, say S/N $\approx 5-10$ and the calibration must then also be only at this level.
Important points to be considered for an optimization of a detection are:

- optimization of the sky transmission,
- reduction of the sky emission,
- optimization of the instrument efficiency,
- the dependence on seeing.

Sky transmission. The sky transmission was discussed in detail in the previous section. The sky transmission can be optimized by observations near the meridian (minimum air mass for given telescope latitude and object declination). This is particularly important for wavelength regions where the atmospheric absorption is significant. For example when asking for ESO service time, one can specify conditions for the airmass.

For wavelength bands where absorption by water vapor is important one should try to carry out the observations during "dry" nights, if possible.

2.4.1 Sky emission

A dark sky is an important precondition for the detection of faint sources. There are several components which can contribute to the sky emission:

- artificial lights,
- twilight,
- moon light,
- scattered light from interplanetary dust,
- airglow and auroral emission,
- thermal emission from the atmosphere and the telescope.

Artificial lights. The modern civilization produces a lot of artificial light which is very harmful for astronomical observations from the ground. For this reason it is important to protect observatories from disturbing light sources. Faint sources (galaxies) cannot be studied at sites with significant light pollution.

We should also reduce the "light pollution" in populated areas in order to protect good dark sky view to the stars and the Milky Way for the general public.

Twilight. The average time from sunset to sunrise is 12 hours. However there is quite some time after sunset and before sunrise where the sky brightness is still very high due to the twilight. Different phases of twilight are distinguished depending on the angle θ_s of the sun below the astronomical horizon.

- civil twilight: $\theta_s = 0^\circ$ to -6°,
- nautical twilight: $\theta_s = -6^\circ$ to -12°,
- astronomical twilight: $\theta_s = -12^\circ$ to -18°.

For observations of faint sources in the visual region one should avoid astronomical twilight. For near-IR work and bright targets (standard stars) one can use also observations taken during nautical twilight while mid-IR observations are not much affected by the twilight. The sun position moves 15° per hour so that the duration for astronomical observations during one night is roughly the time between astronomical sunset and sunrise minus about 2 hours.

Moon light. Solar light, reflected from the moon is scattered in the Earth atmosphere and contributes to the sky emission. The dominant process is Rayleigh scattering with some contributions due to scattering by aerosols and clouds. Moon light is particularly strong in the near-UV and blue (Rayleigh scattering) and decreases towards longer wavelengths. The effect of the moon light can be neglected in the near-IR and mid-IR.

The scattered moon light depends strongly on the moon phase and changes the sky brightness dramatically (more than a factor of 30) in the visual. For this reason the moon phase is an important constraint for observations of faint sources. Observatories distinguish for their observing time scheduling between:

- dark time: new moon phase ± 3-4 days,
- grey time: half moon phase ± 3-4 days,
- bright time: full moon ± 3-4 days,

The following table gives an overview of the sky brightness in mag arcsec^{-2} as function of the moon phase.

days from new moon	U	B	V	R	I	z
14	17.0	19.5	20.0	19.9	19.2	18.1
10	18.5	20.7	20.7	20.3	19.5	18.3
7	19.9	21.6	21.4	20.6	19.7	18.6
0	22.0	22.7	21.8	20.9	19.9	18.8
sky brightness for new moon without atmospheric emission lines						
0	22.0	22.7	22.0	21.0	20.5	20.5

Airglow and auroral lines. The whole red and near-IR region is strongly contaminated by numerous narrow (line width < 0.05 nm) emission lines due to the OH molecules. These lines are a major problem for the detection of faint sources. For example, for high redshift objects the Lyα cutoff is at $(z + 1) \cdot 121$ nm, or at 847 nm for $z \approx 6$. This means that the emission of high redshift objects has to be searched in the spectral region where the OH emission dominates (and which is also affected by H_2O absorption lines). The OH emission is induced in a thin layer at an altitude of 90 km by solar UV radiation. The strength of the emission can vary by a factor of 2 or more within one hour due to large scale wave motions near the mesopause.
The OH lines are blended for small spectral resolution of $R < 500$. For a higher resolution one can investigate the region between the lines and search for faint sources. This possibility is exploited with massive 3-D spectrometers with spectral resolution > 1000.

In the visual regions there are only a few strong auroral lines, for example due to atomic oxygen; O I at 556 nm, 630nm and 636 nm. They are strongly variable in time and from location to location.

Scattered light from interplanetary dust. The reflection of solar light by interplanetary dust produces the zodiacal light and the gegenschein. These two components dominate the sky emission in the 350-750 nm region at a dark site (without moon). The gegenschein is a feature due to the enhanced back-scattering of dust particles and it is therefore located around the "anti-sun" direction.

Thermal radiation. The thermal radiation of the atmosphere and telescope starts at about 2.2 μm and it increases strongly towards longer wavelengths.
Since the atmosphere and the telescope optics have a temperature T they emit a thermal blackbody radiation

$$S_\lambda = \epsilon_\lambda \cdot B_\lambda(T)\,, \tag{2.5}$$

with an emissivity of ϵ_λ. According to the Kirchhoff law the emissivity is related to the absorption κ_λ of the atmosphere or an optical component

$$\epsilon_\lambda = 1 - \kappa_\lambda \tag{2.6}$$

Atmosphere: The absorption of the atmosphere is dominated by the water vapor layers and the corresponding temperature is given by the average temperature of these layers. The emissivity is:

- $\epsilon = 1$ at wavelengths where the atmosphere is opaque,
- $\epsilon \approx 0.1$ in good spectral windows with low absorption.

Telescope and instrument: The telescope is typically $\approx 20°$K warmer than the atmosphere (water vapor). Therefore the telescope emission is higher at wavelength < 13 μm. A typical telescope mirror reflects $R = 0.95 - 0.99$ of the light and the transmission through a lens is also at the $T = 0.95 - 0.99$ level. Thus the total emissivity of the telescope and instrument is the summed up emissivity contribution of all warm components. In addition one has to consider the opaque dust on the components which emit like blackbodies with $\epsilon = 1$. In particular the primary mirror of astronomical telescopes can be pretty dirty with dust covering several % of the surface. The dust on dirty components can add a dominant contribution to the thermal emission budget. The instrument emission is reduced by putting most components into a cold dewar.

2.4.2 Optimize instrument efficiency

For faint sources it is important to understand well the instrumental limitations on throughput and sensitivity. The goal is to have as much signal as possible and to reduce the noise signal as much as possible.

The design of a telescope or an instrument must be optimized for high throughput. The transmission of an instrument can be enhanced by using a minimum number of optical components with high efficiency, because mirrors do not reflect all light and transmission optics does not transmit all light. In the final efficiency budget the efficiency of each component must be multiplied.

A strongly simplified example for a transmission budget of an imager:
Typically a telescope has two aluminum mirrors which reflect each about 90 % of the light. A collimating lens system and a camera lens system transmit each about 95 %, the transmission of a broad band filter in the passband is about 80 %, and a detector can have a quantum efficiency of 80 %. The total efficiency is then given by:

$$0.90^2 \cdot 0.95^2 \cdot 0.80 \cdot 0.80 = 0.47$$

This simple example shows that a significant fraction of the photons are "lost" in the instrument.

Often one has to work with a given instrument. Despite this one can still optimize the detection efficiency with a good observing strategy.

The signal can be enhanced by selecting an optimum instrument configuration:

- select a pass-band where the signal from the target is strongest,
- use the instrument configuration with the highest quantum efficiency in the selected pass-band,
- enhance as much as possible the width of the pass-band,
- optimize the integration time by reducing overheads.

On the other side one has to reduce as much as possible the noise sources with good choices for the instrument configuration:

- select a passband which minimizes the sky background (the thermal emission),
- select detector modes with the lowest detector noise level,
- optimize the spatial resolution of the detector to the required spatial resolution of the observations. As a rough rule the full width at half maximum of the source (in seeing limited or diffraction limited observations) should match about 2 detector pixels.

Often it is not possible to optimize the throughput without enhancing also the background. Therefore one should carry out a trade-off study and compare the different options.

2.4.3 Dependence on seeing

The spatial resolution is very important for the detection of faint, point-like sources in background noise limited observations. If a point target S_t can be focussed onto a smaller detector area then the S/N is strongly enhanced, because the sampled background signal decreases like the area of the focussed target image.

$$S_b \propto \text{area} \approx \text{seeing}^2 \,.$$

Thus, the S/N-ratio behaves like

$$\frac{\mathbf{S}}{\mathbf{N}} = \frac{S_t}{\sqrt{S_t + S_b}} \overset{S_B \geq S_t}{\approx} \frac{S_t}{\sqrt{S_b}} \propto \frac{1}{\text{seeing}} \,. \tag{2.7}$$

The seeing is therefore a most important parameter for the detection of faint sources. For a good site and for an excellent telescope like the VLT the seeing can be as small as 0.4 arcsec, with a median value of about 0.8 arcsec.

Achieving an enhanced spatial resolution has therefore not only the goal to get sharper images but also a deeper detection limit. This explains the huge efforts made for achieving a better spatial resolution with adaptive optics at ground-based telescopes.

Chapter 3

Spectroscopy

Astronomical spectroscopy started with the detection of the absorption lines in the solar spectrum by Frauenhofer at the beginning of the 19th century. Without understanding the nature of these lines he named the strongest of them with letters. Only about 50 years later it was recognized by Kirchhoff and Bunsen that some of these lines are due to atoms in the solar atmosphere. Frauenhofer's letters survived into the modern scientific jargon:

	A	B	C	a	D	E	b	F	G	H
ident.	O_2^a	O_2^a	Hα	O_2^a	Na I	Fe I	Mg I	Hβ	Hγ	Ca II
λ[nm]	759	687	656	628	589	527	517	481	434	397

^{a}: absorptions from the Earth's atmosphere

Spectroscopy often allows a detailed physical characterization of the observed stars and astrophysical plasmas. It is a long and interesting history from the initial analysis of stellar and nebular spectra to modern astronomical spectroscopy. Most important applications of spectroscopy are:

- determination of temperature, surface gravity, abundances, and many other parameters of stars. Most basic are the stellar spectral types

  $$\mathrm{O} - \mathrm{B} - \mathrm{A} - \mathrm{F} - \mathrm{G} - \mathrm{K} - \mathrm{M}$$

  which define a temperature sequence from hot to cold stars.

- the measurement of Doppler shift for spectral lines due to the relative motion of the target or / and for distant objects due to the cosmological expansion

  $$\frac{\Delta\lambda}{\lambda} = \frac{v_{\mathrm{rad}}}{c} \quad \text{for} \quad v_{\mathrm{rad}} \ll c \quad \text{or/and} \quad \frac{\Delta\lambda}{\lambda} = z$$

  respectively,

- the determination of temperatures, particle densities, abundances and other parameters of gaseous nebula from emission lines,

- the determination of column densities, abundances, temperatures and other parameters of the absorbing gas located between a light source and the observer.

Spectroscopy is a cornerstone for astrophysical studies providing detailed information on essentially all targets. Often a target is only well studied if its spectral properties are known.

3.1 Science requirements

Spectroscopy provides the wavelength dependence of the intensity $I(\lambda)$ of the target. Thereby the key parameters are:

- the spectral resolution $R = \lambda/\Delta\lambda$,
- the spectral coverage $[\lambda_{\min}, \lambda_{\max}]$.

The spectral resolution and coverage should be optimized for each particular science case. The resolution should be good enough to measure accurately a spectral feature. This means that the investigated spectral feature can be separated from other structures in the spectrum from the source, the background or the instrument so that its strengths or shape can be determined with sufficient precision. The spectral coverage should be optimized so that all important spectral features can be covered within a minimum of telescope time. Depending on the science needs one can combine the spectral information with spatial information $I(x, \lambda)$ or $I(x, y, \lambda)$. Another possibility is to measure the temporal variability of the spectrum $I(\lambda, t)$.

The science goal defines which spectral quantities of a target have to be measured:

- *Rough spectral characterization;*
  in many cases even a coarse spectrum allows a quite good characterization of the object type.
- *Spectral dependence of the flux;*
  this requires a broad wavelength coverage and an accurate calibration of the spectral efficiency of the instrument.
- *Equivalent width;*
  equivalent width measurements are relative measurements of the attenuation of a spectral continuum by an absorption providing information on the abundance of the absorbing species.
- *Emission line strengths;*
  emission line diagnostic is based on the strength of the lines which should be measured absolutely, or relative to another well-known emission like H I lines.
- *Line profile structures;*
  useful for studies on the radial velocity structure or radiative transfer effects in an object. Depending on the scientific aim one has to select the appropriate spectral resolution to resolve sufficiently the line profile.
- *Doppler shift measurements*;
  requires an appropriate precision of the radial velocity calibration and suitable spectral features to measure radial velocity shifts.
- *Spatially resolved spectroscopy;*
  this combines spectroscopic measurements with spatial information and is most useful for the investigation of the structure of extended objects or to disentangle blended objects.
- *Time resolved spectroscopy;*
  for the investigation of the temporal variability of spectral features for the study of variable objects.

3.2 Spectrographs

Only X-ray and γ-ray detectors have sufficient energy resolution to provide a spectrum of the incoming radiation directly. At longer wavelengths a spectrograph is necessary. Spectrographs which disperse the light are based on the following concept:

- an entrance aperture at the focal plane of the telescope,
- a dispersive element (e.g. grating, prism, etc.) in a collimated beam section,
- a camera optics which focusses the dispersed beam onto the detector so that the different wavelengths from a point are imaged along a line.

A different type of spectrometer is the Fourier transform spectrometer which measures the interference pattern of an interferometer, where the path length of one arm is variable. The observed constructive and destructive interferences can be converted to a wavelength spectrum using Fourier transformation. This type of spectrometer has only special application in Astronomy.

3.2.1 Basic equations for grating spectrographs

A grating oriented perpendicular to the incoming beam produces for a given wavelength λ constructive interferences for the angles θ according to:

$$\sin\theta_m = \frac{m \cdot \lambda}{a}\,, \tag{3.1}$$

where a is the periodic separation between the grating lines and m an integer number for the interference order. One should be aware that an overlap of the different orders ($m-1$, m, $m+1$, etc.) can occur.

The angular dispersion $d\theta/d\lambda$ of the grating follows from differentiation (determine first $d\lambda/d\theta$):

$$\frac{d\theta}{d\lambda} = \frac{m}{a\cos\theta}\,.$$

The angular width W_θ of a monochromatic interference peak is broad for few grating lines and it becomes narrower as the number of illuminated grating lines N increases like

$$W_\theta = \frac{\lambda}{Na\cos\theta}\,.$$

This width can also be expressed in a wavelength width $\Delta\lambda$

$$\Delta\lambda = W_\theta \frac{d\lambda}{d\theta} = \frac{\lambda}{Na\cos\theta} \cdot \frac{a\,\cos\theta}{m} = \frac{\lambda}{N\,m}\,,$$

or a resolving power R for a more general characterization of the diffraction limited resolution of the grating

$$R = \frac{\lambda}{\Delta\lambda} = N\,m\,. \tag{3.2}$$

This formula indicates that the resolving power depends only on the number of illuminated grating lines and the dispersion order.

Gratings, in particular reflective gratings, are often inclined with respect to the incoming beam by an angle i which is the angle of the grating normal to the incoming beam. The angle θ is then defined by the interference order m and the zero order. In this case Equation (3.1) includes the term $\sin i$ for the grating inclination:

$$\sin\theta = \frac{m \cdot \lambda}{a} - \sin i\,. \tag{3.3}$$

This is called the grating equation. The resolving power is larger for inclined gratings because more lines are illuminated for a given beam diameter

$$R = \frac{\lambda}{\Delta\lambda} = \frac{N\,m}{\cos i}\,. \tag{3.4}$$

The grating equation describes also how one can change the central wavelength and the wavelength range for a given deflection angle θ by changing the tilt angle i. The following list gives the dependence of spectrum parameters on grating properties:

- the resolving power R depends only on the number of illuminated lines and the diffraction order
- R increases if the number of lines per mm of the grating are enhanced for a given beam (=pupil) diameter
- R can be enhanced for a given grating by a larger illuminating beam (larger pupil) or by tilting the grating so that the illuminated lines increase like $N_i = N_{i=0}/\cos i$
- R is substantially larger for higher diffraction orders $R \propto m$ (there is the restriction that overlap of grating orders occur)
- the wavelength region for a given deflection angle can be selected by changing the inclination i of the grating.

3.2.2 Different types of gratings

Simple gratings. Typical grating have about $100 - 1000$ rulings/mm. This yields for the first order diffraction spectrum and a collimated beam diameter (pupil diameter) of 1 cm a grating resolving power of $R = 1000 - 10000$. The first order spectrum can be contaminated by the second order spectrum with $\lambda_{m=2} = \lambda_{m=1}/2$ or higher order spectra $\lambda_{m\geq 3}$. For ground based optical spectroscopy this happens for $\lambda_{m=1} > 660$ nm, when second order light from above the UV-cutoff $\lambda_{m=2} > 330$ nm sets in. The second order can be suppressed with a short wavelength cutoff filter. For example a BG430 filter cuts all light shortwards of about 430 nm, allowing first order spectroscopy from 430 nm to 860 nm without second order contamination.
The same grating can also be used in second order with twice the resolution of the first order. In this case one has to select for a given wavelength range the correct pass-band filter to avoid the contamination by other orders.

Blazed gratings. Simple gratings are not very efficient since the light is distributed to several grating orders. The efficiency of reflecting gratings can be improved by an optimized inclination of the reflecting surfaces so that they reflect the light preferentially in the direction of the aimed interference order. Thus the grating efficiency is optimized for one particular wavelength or diffraction angle θ_b, the so-called blaze angle.

Echelle gratings. An extreme case of the blazed grating is the echelle grating. It is strongly inclined with respect to incoming beam and more importantly it is optimized (blazed) for high order diffraction directions, say $m = 10 - 100$. With this type of grating the resolving power can be strongly enhanced even if the grating is quite coarse. For example a beam of 2 cm diameter illuminating an echelle grating with 20 lines / mm, inclined by $i = 60^\circ$ ($1/\cos i = 2$) will see effectively 800 grating lines, which produce for $m \approx 50$ a spectral resolving power of $R = 40'000$. Of course for such a grating the free spectral range, without overlap by neighboring pixels, is only small and of the order $\Delta\lambda \approx \lambda/m$. Narrow band filters are required to select one particular order.
A more elegant solution is a **cross dispersion** with a second low order grating or a prism perpendicular to the dispersion of the echelle grating. In this way the individual orders are displaced with respect to each other and many orders of the echelle grating can be placed on a rectangular imaging detector.

Transmission gratings and grisms. The principles of transmission gratings are essentially the same as for reflecting gratings. One interesting variation of the transmission grating is the grisms. Grisms are a combination of a prism with a grating so that the spectrum is produced in the straight-through direction. The advantage of this concept is that they can be inserted in the collimated beam of an imaging camera. The same instrument serves then as imager and spectrograph.

3.2.3 Spectrograph apertures

The spectrograph aperture is typically matched to either the seeing limited resolution or the diffraction limited resolution of the telescope.

Slit aperture. A widely used aperture type for spectrographs is the slit aperture. The slit orientation is perpendicular to the spectrograph dispersion direction. Thus the slit is imaged onto the detector and any narrow wavelength dependence of the intensity produce a dark or bright line in the resulting spectrum. Therefore one speaks of emission and absorption lines when quasi-monochromatic absorptions or emissions of atoms or molecules are observed.
Slit width: Usually the slit width can be changed but then the spectral resolution or the resulting width of unresolved lines will change accordingly. For seeing limited observations the following choices can usually be made for the slit width:

- Narrow slits for high spectral resolution; they are usually matched to the "best seeing" conditions expected which is about $0.5 - 1.0$ arcsec.
- Wide slits for spectrophotometric observations; they are wide enough ($5 - 10$ arcsec) so that essentially no light is lost on the aperture.
- intermediate slit widths which allow an adjustment to the seeing conditions. The selected slit width is often $1 - 1.5$ times the seeing for a good compromise between spectral resolution, throughput, and background rejection.

Slit orientation: The orientation of the slit is another parameter which can in some case be chosen. Changing the slit orientation is usually achieved with a rotation of the entire spectrograph, since the slit orientation should remain perpendicular to the spectrograph dispersion direction. The rotation of an instrument may introduce mechanical flexures in the spectrograph which require a spectroscopic re-calibration in the same spectrograph position.
Standard slit orientations and consideration for the selection of a slit orientation are:

- The slit along the line through zenith and target (paralactic angle = 0°) is an obvious orientation for Alt-Az telescopes because then the instrument is fixed with respect to the telescope. An additional advantage is that the atmospheric dispersion is along the slit and the danger that one section of the spectrum is less well transmitted through the slit is minimized.
- A North-South slit direction is useful for equatorial telescopes, because the spectrograph is then fixed with respect to the telescope. For observations near the meridian the North-South direction coincides with the zenith direction so that slit loses due to atmospheric dispersion are minimized.
- A particular slit orientation may be chosen due to particular reasons:
  - a slit orientation in order to avoid light of a nearby bright source "falling" into the spectrograph,
  - a slit orientation which allows to place a second target onto the slit for simultaneous spectroscopy of two sources,
  - a slit orientation best suited for simultaneous measurement of the sky spectrum for the background correction,
  - the orientation best suited for long slit spectroscopy of an extended target where all points along the slit can be investigated spectroscopically.

No aperture. Having no aperture in the focal plane provides spectroscopy of each point in the aperture. The problem is of course that the spectra of neighboring points (in the dispersion direction) overlap. Also the sky background from the whole aperture is passing through the system and is dispersed causing a high background which is harmful for weak targets. Good applications for "aperture-free spectroscopy" are:

- Low resolution survey of bright targets in uncrowded fields; if the spectra are short and the number of bright targets small then the spectra of the different objects do not overlap.
- High resolution spectral imaging of targets with narrow emission lines: if the emission line target is compact and the emission lines well separated spectrally, then one can obtain "monochromatic" images for various emission lines simultaneously.

Entrance lens for a fiber. Often the entrance aperture of a spectrograph is a lens in the focal plane of the telescope which focusses the light into an optical fiber. The spectrograph can then be mounted in a laboratory near the telescope. Very stable fiber spectrographs of this kind can be built which are suited for high precision radial velocity measurements.

If multiple fibers are placed in the focal plane then one can obtain multi-object spectroscopy. The fiber outputs are placed in a line configuration in the entrance aperture of a spectrograph (like a slit with multiple sources.)

Custom made masks. Science programs may require very specific apertures. For example for the spectroscopy of a strongly distorted lensed galaxy one may use an arc-like aperture in the focal plane in order to investigate the object spectroscopically. Such an aperture can be made with a laser cutting machine. Some observatories offer the possibility to cut special spectrograph apertures using the detector images taken with the imaging mode of the same instrument.

Using custom made masks with multiple apertures is another way to take multi-object spectroscopy.

3.2.4 Spectroscopic data reduction

Many aspects of the spectroscopic data reduction are identical to the photometric data reduction. An important difference is that the wavelength scale has to be determined for the final data product.

The standard steps for the reduction of spectroscopic data are:

- cosmic ray event rejection,
- bias or dark subtraction,
- bad pixel correction,
- flat-fielding,
- target and sky spectrum extraction,
- wavelength calibration,
- sky background subtraction and corrections for atmospheric absorptions,
- spectrophotometric calibration.

Detector data reduction. The correction for detector effects are almost identical to the case of photometric imaging data.

Cosmic ray events: Cosmic ray events are quite easy to recognize in spectroscopy since the structure of the spectroscopic signal on the detector is predictable.

Bad pixels: Dithering for bad pixel correction is often not feasible or too complicated in spectroscopy. But for bad pixels the same applies as for the cosmic ray events - the expected target signal can be inferred from adjacent pixel rows (same spectrum) or pixel columns (same Gaussian intensity profile). Thus it is often possible to interpolate over bad pixels without risk that one introduces a spurious signal.

Read out bias or dark subtraction: The same applies as for the reduction of imaging photometry.

Flat-fielding: Spectroscopic flatfields often show a strong wavelength dependence in the intensity because the efficiency of the spectrograph and the illuminating lamp can depend strongly on wavelength. Thus, one may obtain flatfields where one half of the CCD where the blue part of the spectrum is registered, has 5000 counts, and the other ("red") half 50'000 counts. In this case one should normalize the flat-field locally and consider in the analysis that the flat-fielding noise depends on the wavelength.

Target and sky spectrum extraction. The spectrum extraction converts the 2-dimensional data into 1-dimensional data. All registered photoelectrons corresponding to a given wavelength should be added to the same bin in the 1-dimensional spectrum.

The location of the spectrum can be determined from a spectrum of a bright point source, while a spectral lamp illumination yields the wavelengths of different locations in the aperture.

The extraction of the target spectrum is rather easy if the spectrum is well aligned with the pixel rows (or column), so that several pixels located perpendicular to the spectrum can just be added up for one spectral bin. One can (and should) check with an extraction of the lamp spectrum whether the spectral resolution is degraded by the extraction.

If the spectrum is not aligned with the pixels and is even curved then one needs a careful spectrum extraction. However this problem has often been solved and one should be able to find well tested software for this task.

The width of the spectrum extraction depends on the science goals:

- a wide target extraction, including essentially 100 % of the target flux, for relative or absolute spectrophotometry,
- a narrow target extraction if the target is weak and one aims for the maximum S/N in the spectrum.

A more sophisticated extraction method weights the pixels according to the expected flux as determined from the averaged intensity profile. Since the method is more complicated also more care is required to verify the quality of the resulting spectra.

Sky extraction: The goal of the sky extraction is to obtain a sky spectrum which is well suited for the correction of the sky emission in the target spectrum. Therefore the sky spectrum should not be contaminated by light from the target. The sky spectrum has a higher S/N if a wider sky region is extracted. Of course one needs to rescale thereafter the strength of the sky spectrum to the strength in the target spectrum.

Wavelength calibration. A lamp line spectrum should be extracted in exactly the same way as the target spectrum and as the sky spectrum for the wavelength calibration. A wavelength calibration is only valid if nothing has been changed in the instrument setup between the calibration and the target observation. Usually the sky spectra need a separate wavelength calibration.

The extracted lamp spectrum is then used to define the conversion of pixel number into wavelength. This requires:

- an accurate determination of the centers of the spectral lines,
- a fit function $\lambda_i(\mathrm{pix}_i)$ using a higher order polynomial function or another suitable function so that one can convert the pixel spectrum $f(\mathrm{pix}_i)$ into a wavelength spectrum $f(\lambda_i)$,
- a λ-rebinning into a spectrum with equal wavelength steps $\Delta\lambda$ or $\Delta\log\lambda$.

There exist software packages which perform these calibration steps. Still a careful analysis of the resulting spectra is required. Important is an assessment of the precision of the wavelength calibration. The precision achieved routinely for the wavelength calibration is about a tenth of the spectral resolution. Achieving a higher precision in the wavelength calibration is possible but requires quite some investigations for optimizing the measuring and the data reduction process. Telluric absorption and emission lines often provide a good check for the wavelength calibration.

Sky background subtraction and correction for telluric absorptions. Subtraction of the sky background is straight forward if the sky spectrum is much weaker than the target. Different cases of sky subtraction are:

- weak sky spectrum continuum with a few strong, narrow lines; one may just ignore the lines or interpolate over the wavelength region affected by the line.
- quite a strong background continuum from scattered moon light; a narrow or optimum extraction may help because the effect of the sky spectrum is reduced and the subtraction yields a relatively good target spectrum. For flux calibration purposes one may then use a wide extraction.
- for the subtraction of strong, narrow sky lines one often needs to achieve a very good match of the sky lines in the target spectrum and the sky spectrum to obtain a corrected target spectrum without strong, narrow residuals. It can be helpful to extract a very narrow target spectrum and use the regions just beside the spectrum for the sky subtraction. Best results are obtained if the problem was already considered in the observing and calibration strategy.

– in the thermal infrared the background spectrum is often much stronger than the target spectrum; The sky spectrum should then be extracted from the same detector region as the target spectrum in order to take also the detector pixel noise into account. This is achieved by shifting the target spectrum on the detector between consecutive exposures. An alternative, although one which requires substantial telescope time, is a blank sky observation before or after the target observation.

Correction for telluric absorptions. For the correction of telluric absorptions one takes a flat comparison spectrum $F_{\rm obs}$ (e.g. a fast rotating B star spectrum) and constructs an absorption free spectrum $F_{\rm corr}$ by interpolating this spectrum for regions where telluric absorptions reduce the flux (taking intrinsic absorptions of the source into account). The normalized spectrum of the telluric absorptions is then obtained by the ratio $F_{\rm tell} = F_{\rm obs}({\rm B\,star})/F_{\rm corr}({\rm B\,star})$. A target spectrum corrected for telluric absorptions is then obtained by

$$F_{\rm corr} = F_{\rm obs}/F_{\rm tell}\,.$$

Flux calibration. The spectroscopic flux calibration must follow the same basic rules as a flux calibration of imaging data. The most important points are:

– a wide spectroscopic extraction so that all light is included in the final data from a measurement taken under photometric conditions for the determination of the spectroscopic efficiency curve of the instrument (telescope + spectrograph + detector),
– a correction of the spectrophotometric standard star for the sky transmission taking the airmass of the measurements into account. The sky transmission spectrum can be taken from the literature or be measured with two observations of the same standard star, one with small zenith angle or airmass ≈ 1 and one with large airmass ≈ 2.
– a wide extraction of the target star taken with a wide slit under photometric conditions,
– a correction for the atmospheric extinction for the target star,
– from the measured flux ratio (target star)/(standard star) the absolute flux spectrum of the target can be derived.

Sky transmission: Accurate knowledge of the sky transmission is particularly important in the blue, near-UV spectral region where the atmospheric transmission is quite low. For the other wavelength the sky transmission shows little spectral dependence.

Absolute spectrophotometric calibration and color calibration: An absolute spectropolarimetric calibration requires photometric conditions during the observations. Spectroscopy is often taken for single targets without simultaneous measurements of other nearby targets. In this case one cannot check the photometric calibration with measurements of nearby stars.

Filter convolution: It is always useful to derive the flux in broad-band filters X with an integration of the reduced spectrum $F(\lambda)$ weighted with the filter function $T_x(\lambda)$ for a magnitude determination:

$$F_X = \int F(\lambda)\, T_X(\lambda)\, {\rm d}\lambda\,.$$

Evaluating the same quantity for the spectrophotometric standard star can then be used as a check. The derived filter magnitude enables a comparison of the spectrophotometry to available imaging photometry.

This procedure is also very useful if "only" color calibrated spectroscopy was possible (e.g. due to clouds), because then the obtained spectra can be flux calibrated using filter photometry.

3.3 Measurements of spectral features

The relative or absolute flux in spectral features can be measured in the reduced spectrum by just integrating over the area enclosed. For this one needs to distinguish between different spectral features like:

- continuum of the target,
- spectral lines from the target,
- interstellar absorptions,
- uncorrected absorptions from the Earth atmosphere.

In order to disentangle accurately the different components it may be necessary to use a model of the emissions and absorptions.

Flux modelling: The best analysis of the measured spectrum can be achieved by modelling the spectrum, taking all the known properties of the source into account. Depending on the science goals the modelling can be simple or very complicated:

- *Simple:* a fit of a linear function, a power law, or Planck function for the slope of the continuum and Gaussian fits to spectral lines,
- *Sophisticated:* fits of detailed model spectra to the observed spectrum using a multi-dimensional optimization of the model parameters.

The first approach is certainly useful for a first characterization of the target. The second approach can provide detailed information on the target but it is often very time consuming and one needs to assess the scientific goals and merits of a more sophisticated analysis carefully.

3.3.1 Line center and line strength

For spectral lines the most basic characterization are the line center and the line strength. The line center can be measured in the following way:

- the wavelength of the flux maximum or flux minimum,
- the wavelength of the center of a Gaussian fit to the line,
- the wavelength of the median flux of the feature or any other well defined procedure.

One should determine barycentric wavelengths for measurements more accurate than $|\pm \Delta\lambda/\lambda| < \pm 10^{-4}$, corresponding to a radial velocity precision of RV < 30 km/s.

Radial velocity: If the spectral feature is identified and the rest wavelength known then one can use the radial velocity RV to characterize the line center. However, first one needs to specify the zero point of the RV scale: examples are barycentric RV, or RV with respect to RV zero-point defined for the target.

Multiple components: Lines may consist of several components each with a different wavelength or RV. Depending on the science case one can then derive the center of each component or the center of the entire feature.

Emission line strength: The emission line flux is the measured flux in the spectrum which can be attributed to a particular line. Often emission lines are blended with other lines and one needs to disentangle the flux of different components. This can be achieved with a multiple line fit to the data. The emission line strength should be given as absolute flux or as relative flux with respect to a prominent standard line (e.g. Hβ = 100 for visual spectra of photoionized nebulae). The precision of such a line measurement depends

- for weak lines on the uncertainties in the determination of the continuum level and the contamination by other lines,
- for strong lines on the photometric calibration (absolute or color calibration) of the spectrum.

Equivalent width: The equivalent width EW is the flux in an emission line or the strength of the attenuation of an absorption line with respect to the continuum level. The equivalent width gives the absorption line strength or emission line flux as wavelength width for the normalized continuum spectrum:

$$\mathrm{EW} = \int \frac{I(\lambda)}{I_{\mathrm{cont}}(\lambda)}\, d\lambda\,.$$

For absorption spectra the EW is given usually as a positive value with the units Å. Equivalent widths of absorptions are very useful units since they provide the strength of an absorption feature from continuum normalized spectra (without flux calibration). Equivalent widths do not depend in principle on the spectral resolution, except for the fact that the continuum definition is less accurate for low resolution spectra, and the contamination by other lines is much more difficult to take properly into account.

3.3.2 Line structures

If the spectral resolution is high enough then the structure of lines can be resolved and characterized. The most basic parameter for the line structure is the line width; usually one indicates the full width a half maximum FWHM. The measured FWHM must consider the spectral resolution of the spectrograph, which can be determined from the width of the lines in the wavelength calibration spectrum, unresolved lines in an astrophysical target, sky emission lines or telluric absorption lines. For line widths not much larger than the spectrograph resolution one should use the relation:

$$(\mathrm{FWHM}_{\mathrm{line}})^2 = (\mathrm{FWHM}_{\mathrm{measured}})^2 - (\mathrm{FWHM}_{\mathrm{spectrograph}})^2$$

The spectrograph resolution is not critical for lines much wider, say $> 4\times$, than the spectrograph resolution.

If there are complicated, multi-component line structures then one can try to decompose the profile into several Gaussian components characterized each by the central wavelength or radial velocity and the width of the Gaussian fit.

Generic line profiles. For certain physical phenomena generic types of spectral lines are produced and the line structure yields already useful parameters for the interpretation. Examples are:

- for absorption lines in rapidly rotating stars the line width is a good measure of the projected rotational velocity $v \sin i$,
- for emission lines from rotating disks the separation of the emission peaks measures the line of sight velocity of the disk rotation,
- the width of flat top emission lines from an optically thin emission from a spherically symmetric, expanding shell indicates the expansion velocity,
- in P-Cygni profiles the maximum velocity $RV_{\max}$ for the blue end of the absorption describes the outflow velocity,
- inverse P-Cygni profiles are produced by gas accreted by gravitating body.

3.3.3 Radial velocities

Spectra with many narrow absorption lines are well suited for high precision radial velocity measurements. The most accurate method is a cross-correlation of the target spectrum with a high S/N template spectrum of a RV standard object with the same or a similar spectrum. Since the same atomic transitions are present in all cool stars a spectrum from a solar type standard star is a good template spectrum for all cool stars of spectral type F, G, and K and even early type (elliptical) galaxies.
For the cross-correlation analysis it is important to use normalized spectra, and to introduce a logarithmic step width for the wavelength axis because a radial velocity shift corresponds to a multiplicative shift in the wavelength spectrum. One should not forget to discard all telluric absorption lines in the spectra in order to avoid erroneous results.