Ultrafast Laser Physics

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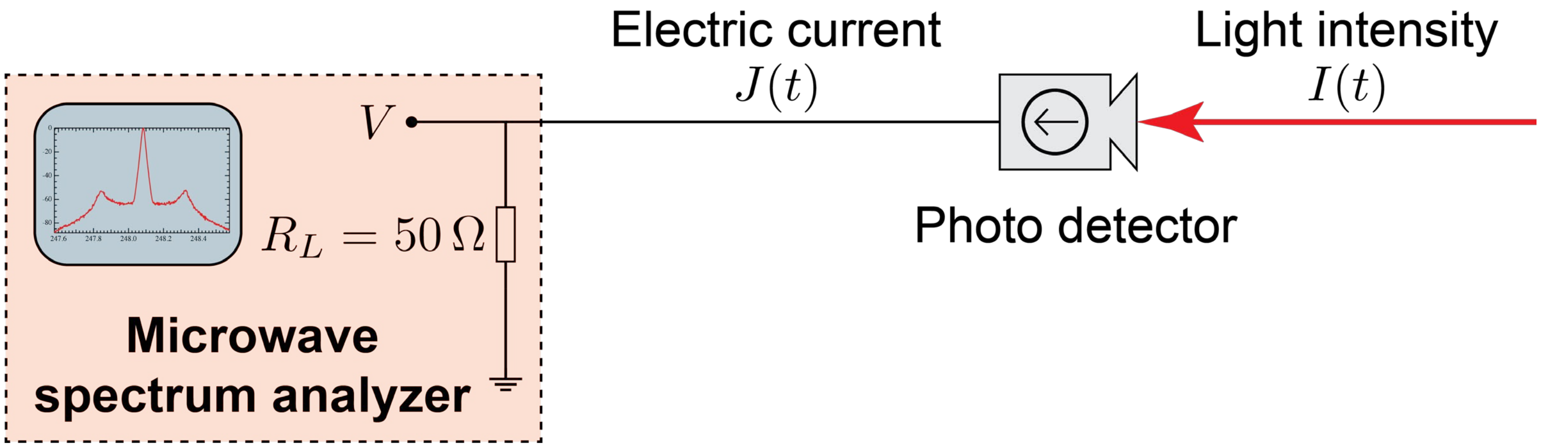
Chapter 10: Noise

Measurement of laser noise



The microwave spectrum analyzer measures the **power spectral density**

$$S_I(\omega) \propto \left|\tilde{I}(\omega)\right|^2 = F\left\{\mathrm{corr}\left(I(t), I(t)\right)\right\} = F\left\{R(\tau)\right\}$$

autocorrelation function: $R(-\tau) = R(\tau)$

$$R(-\tau) \equiv \int I(t)I(t-\tau)dt = \mathrm{corr}\left(I(t), I(t)\right) \overset{F}{\Longrightarrow} \tilde{I}\,(\omega)\tilde{I}^*(\omega)$$

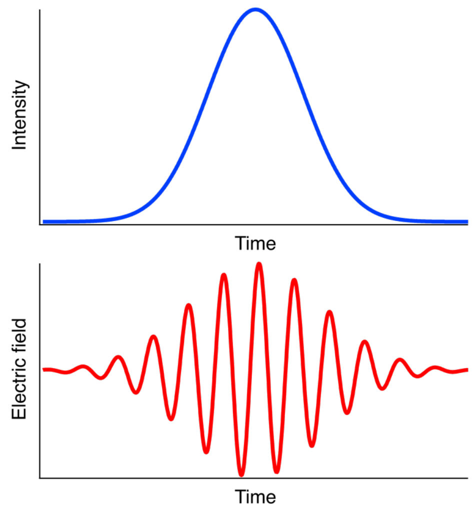
Important: mind the orders of magnitude

$$\tilde{I}(\omega) = F\{I(t)\} \propto F\{|E(t)|^2\}$$

slowly varying

$$\neq |E(\omega)|^2 = |F\{E(t)\}|^2$$

fast oscillating



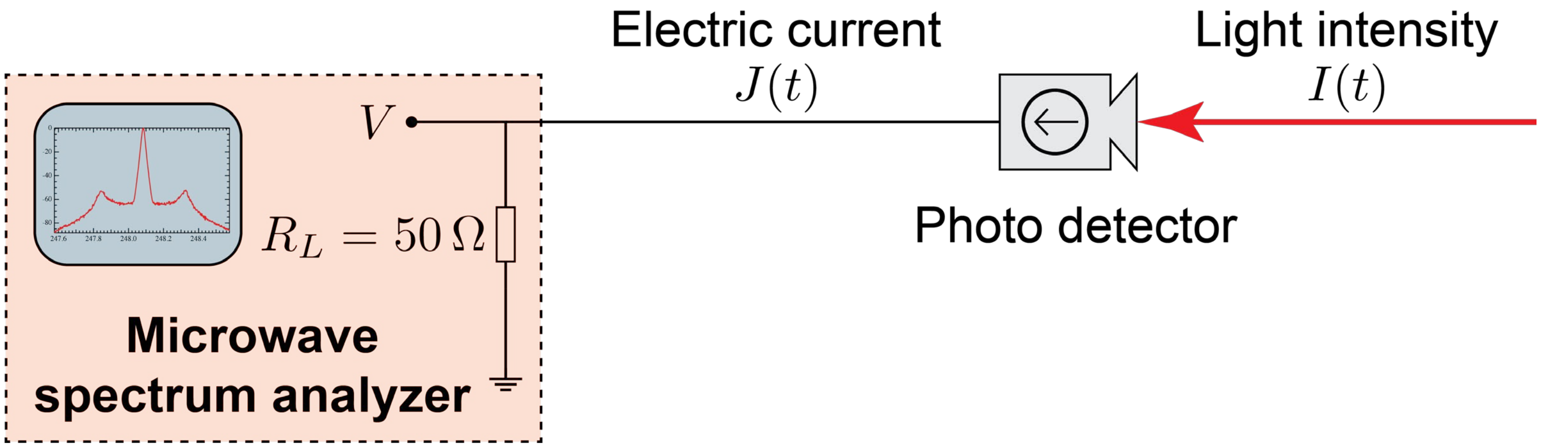
$$S_I(\omega) = \left|\tilde{I}(\omega)\right|^2 \propto \left|F\{|E(t)|^2\}\right|^2$$

$$\left|\tilde{I}(\omega)\right|^2 = \tilde{I}(\omega)\tilde{I}^*(\omega) = F\left\{\mathrm{corr}(I(t), I(t))\right\}$$

Wiener-Khinchin theorem

This is the autocorrelation function for the entire pulse train, not just a single pulse (in contrast to the intensity autocorrelation technique). I.e., time variable spans from –infinity to +infinity.

Measurement of laser noise



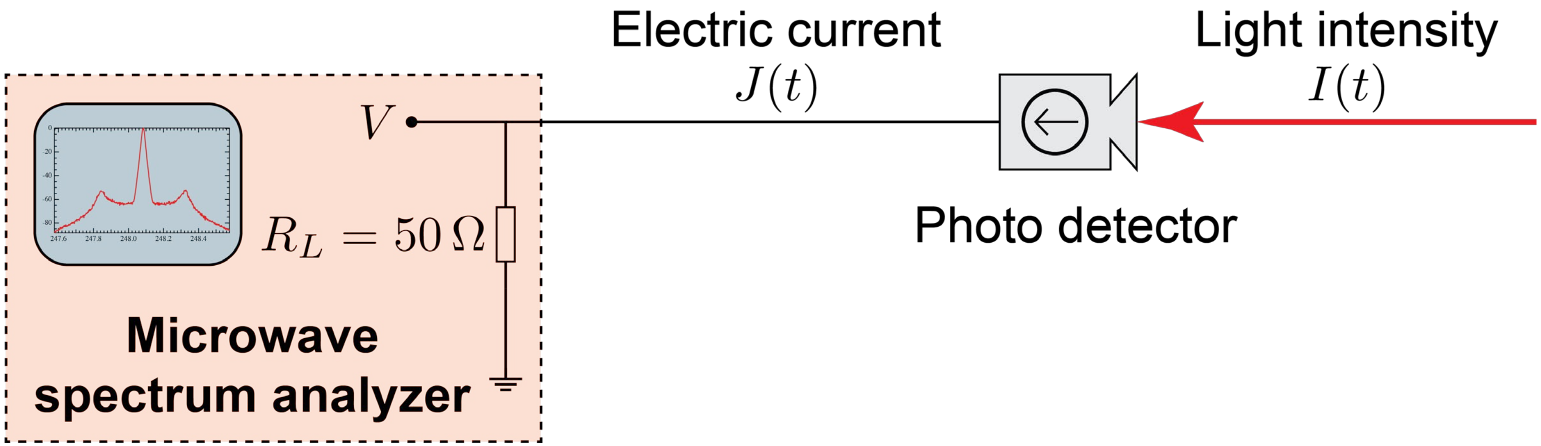
Modelocked laser without any noise

Pulses like delta-functions

$$I(t) = I_0 T \sum_{n=-\infty}^{+\infty} \delta(t - nT) \quad \longrightarrow \quad \tilde{I}(\omega) = F\{I(t)\} = 2\pi I_0 \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_T)$$

For more details see: U. Keller et al., *IEEE J. Quantum.Electron.*, **25**, 280 (1989)

Measurement of laser noise

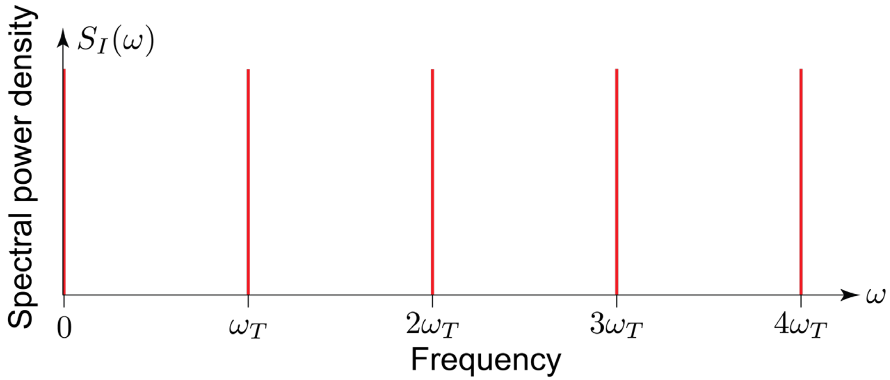


We measure the
power spectral density

$$S_I(\omega) \propto \left|\tilde{I}(\omega)\right|^2$$

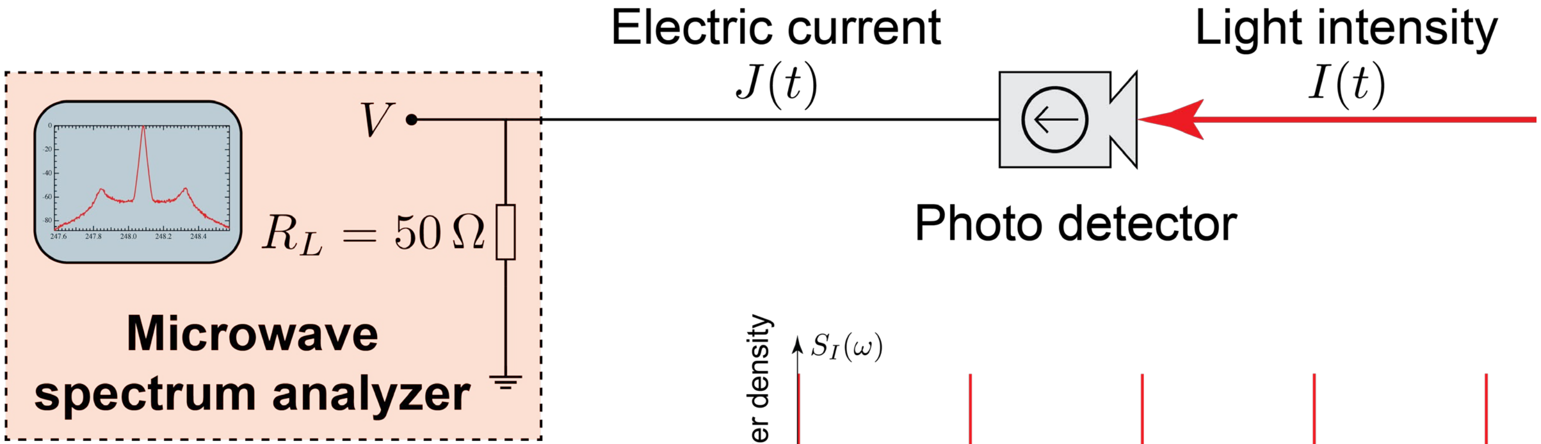
$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{m=-\infty}^{+\infty} \delta(\omega - m\omega_T)$$

$$\omega_T = \frac{2\pi}{T}$$

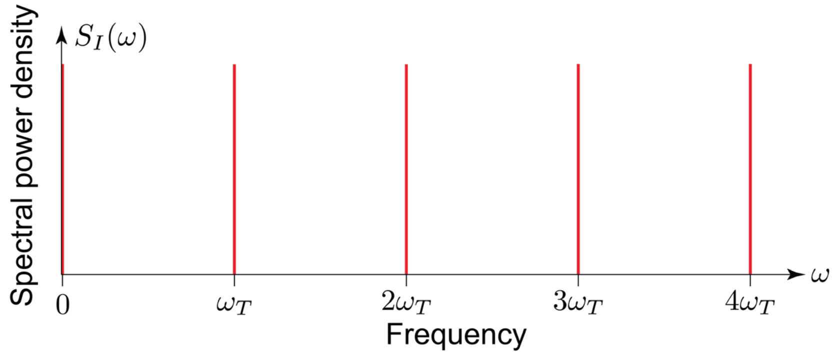


modelocked laser without any noise and any bandwidth limitations of microwave spectrum analyzer

Measurement of laser noise



$$I(t) = I_0 T \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$



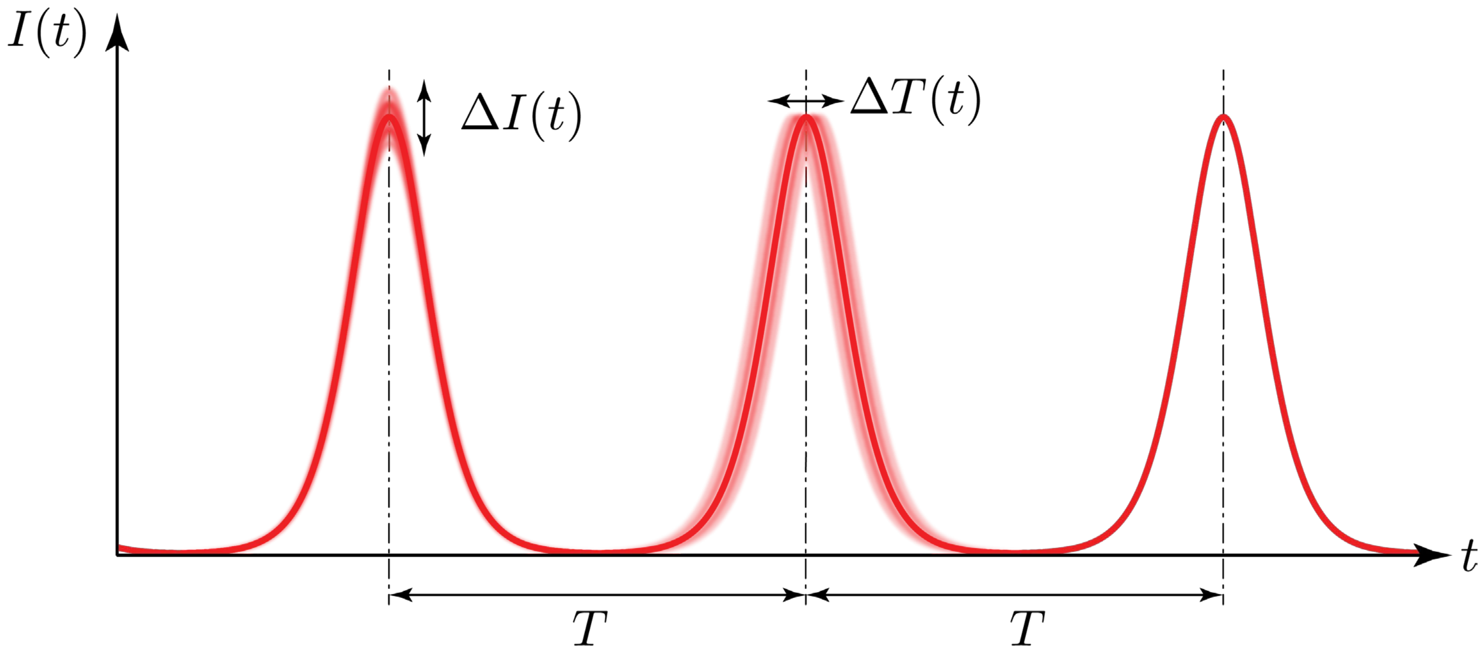
$$I(t) = I_0 T \left(1 + N(t)\right) \sum_{n=-\infty}^{+\infty} \delta\left(t - nT - \Delta T(t)\right)$$

$N(t)$: normalized intensity noise

$\Delta T(t)$: timing jitter

For more details see: U. Keller et al., *IEEE J. Quantum.Electron.*, **25**, 280 (1989)

Intensity noise and timing jitter

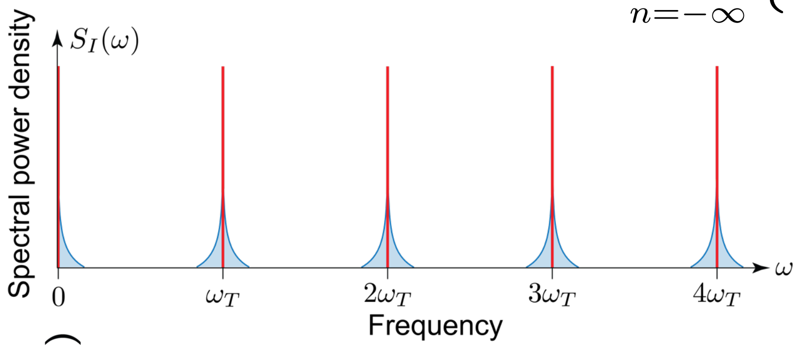


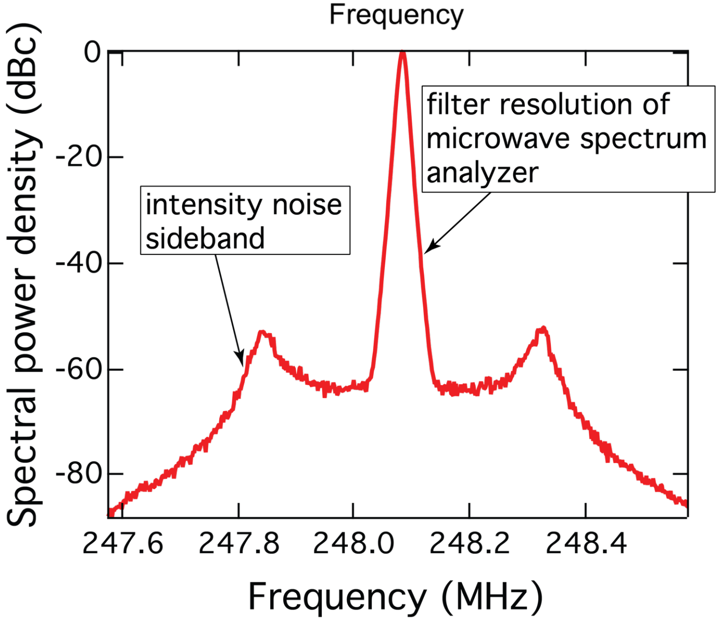
$$\Delta I(t) = I_0 T \cdot N(t)$$

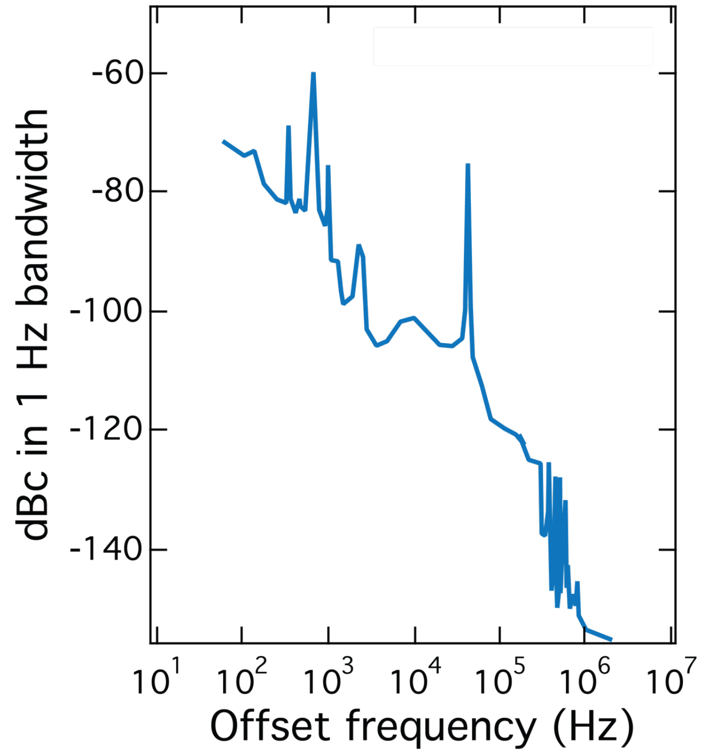
Intensity noise of mode-locked lasers

$$S_I(\omega) = 4\pi I_0^2 \sum_{n=-\infty}^{+\infty} \left\{ \delta(\omega - n\omega_T) + \underline{\left[\tilde{N}(\omega - n\omega_T)\right]^2} \right\}$$

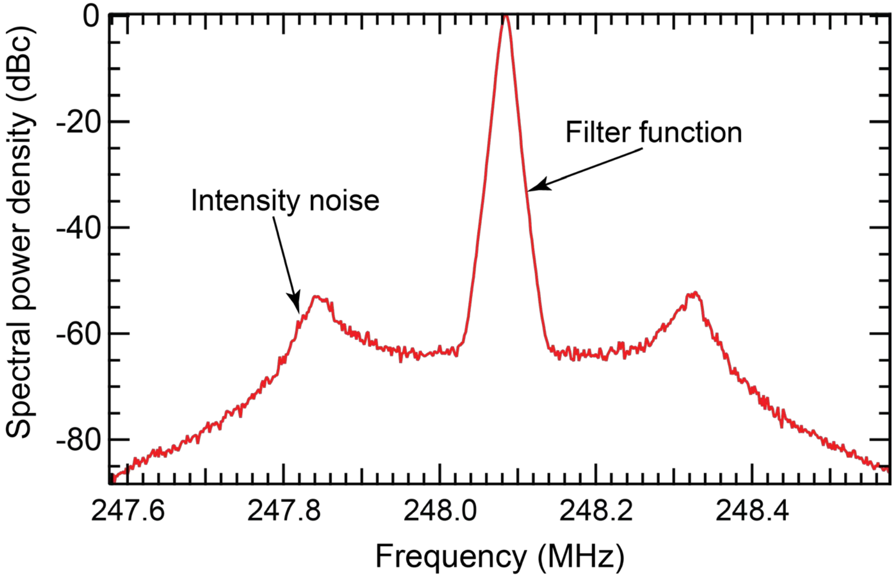
noise sidebands







Units: dB, dBm, dBc

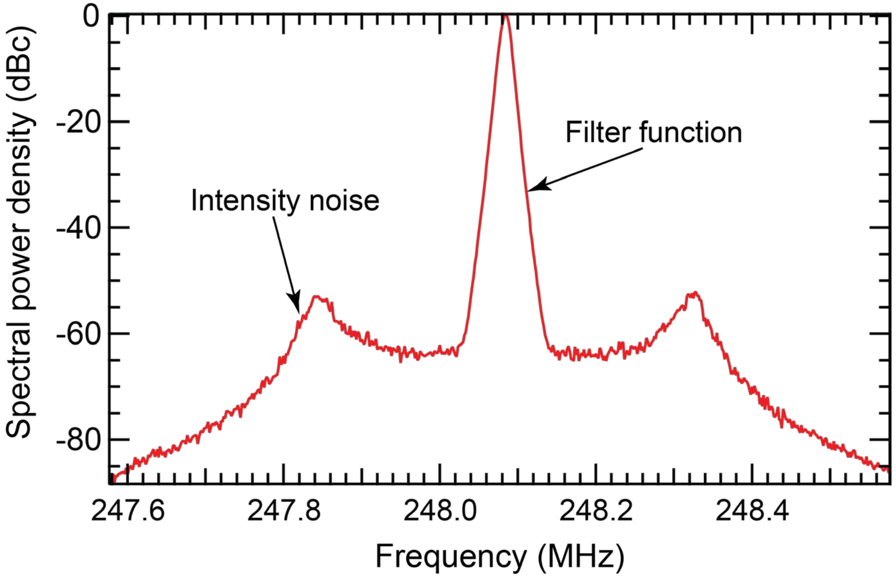


$$\mathrm{dB} \equiv 10 \log \frac{P_2}{P_1}$$

$$\mathrm{dBm} \equiv 10 \log \frac{P}{1\,\mathrm{mW}}$$

mW	dBm
0.1 mW	-10 dBm
1 mW	0 dBm
10 mW	10 dBm
100 mW	20 dBm
1000 mW	30 dBm
x 2	+ 3 dBm
20 mW	13 dBm

Units: dB, dBm, dBc



dBc
"how many dB below the carrier"

carrier is the peak of the harmonic signal

$$\mathrm{dB} \equiv 10 \log \frac{P_2}{P_1}$$

$$\mathrm{dBm} \equiv 10 \log \frac{P}{1\,\mathrm{mW}}$$

mW	dBm
0.1 mW	-10 dBm
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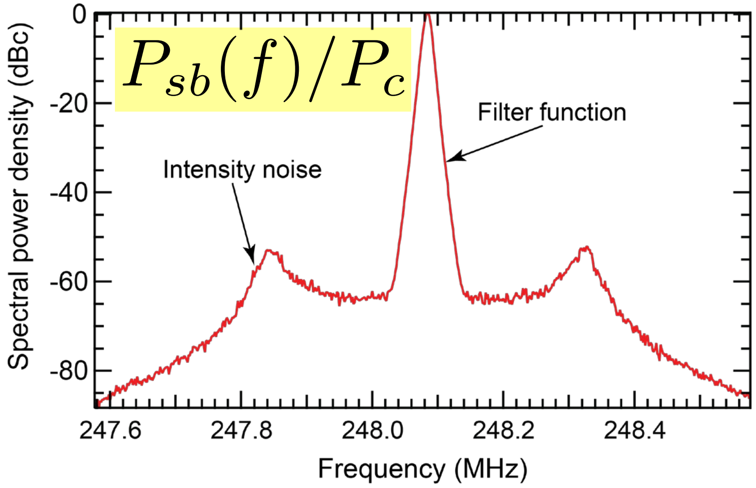
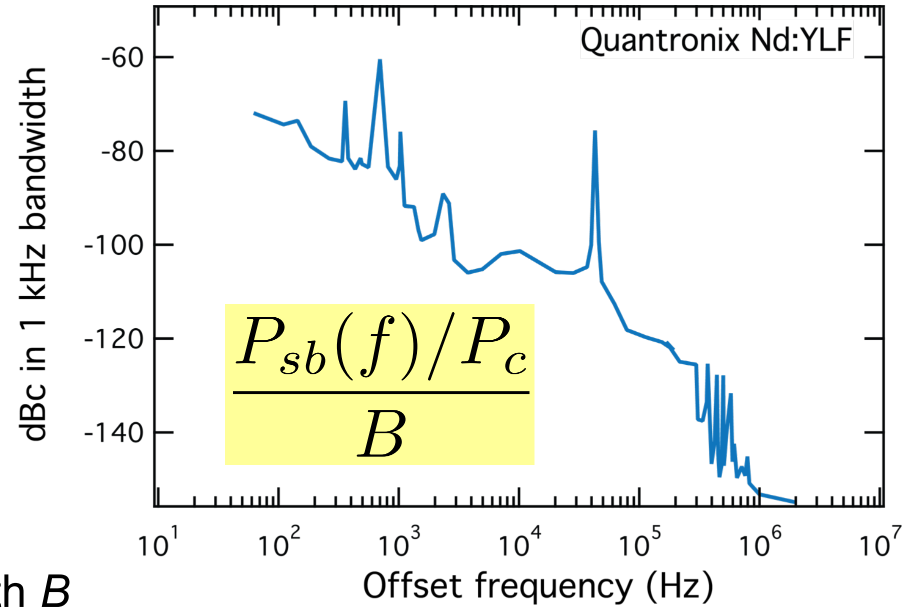
x 2	+ 3 dBm
20 mW	13 dBm

Power in noise sidebands





Measurement with a certain bandwidth *B*

P_{sb}: power in the intensity sidebands ("two-sided" around harmonics)

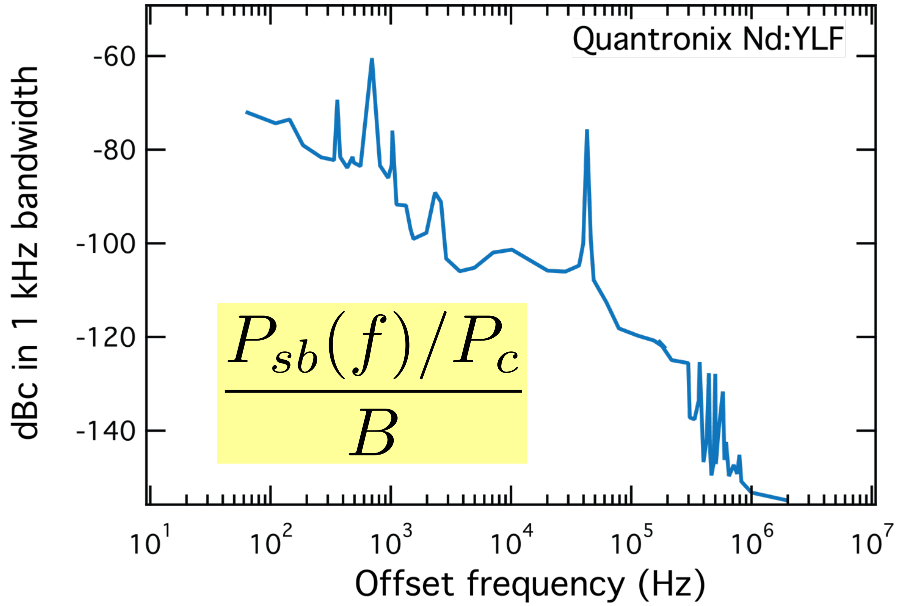
P_c : peak power of carrier

f : offset frequency or noise frequency (i.e. deviation from laser harmonics)

$$\frac{P_{sb}}{P_c} = 2\int_{nf_T+f_1}^{nf_T+f_2} \frac{P_{sb}(f)/P_c}{B} df$$

factor 2 because "two-sided" spectral density!

rms intensity noise



$$\frac{P_{sb}(f)/P_c}{B}$$

rms intensity noise or variance of intensity noise:

$$\sigma_N\left[\omega_1,\omega_2\right] = \sqrt{\langle N^2(t)\rangle}$$

$$= \sqrt{\frac{1}{\pi}\int_{\omega_1}^{\omega_2} S_N(\omega)d\omega}$$

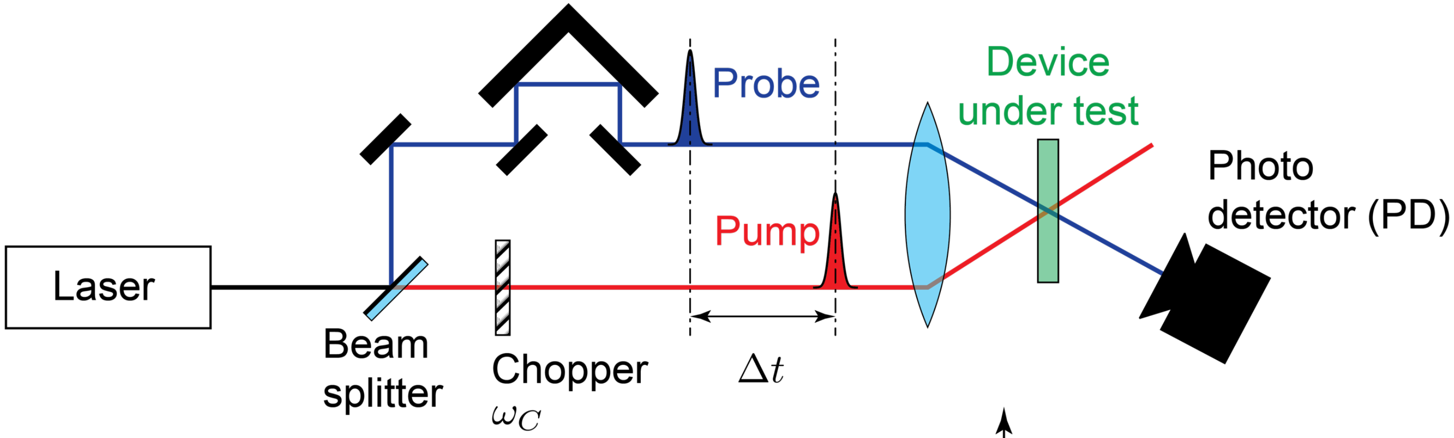
P_{sb}: power in the intensity sidebands
P_c : peak power of carrier

$$\frac{P_{sb}}{P_c} = 2\int_{nf_T+f_1}^{nf_T+f_2} \frac{P_{sb}(f)/P_c}{B}df$$

$$\sigma_N\left[f_1,f_2\right] = \sqrt{\frac{P_{sb}}{P_c}}$$

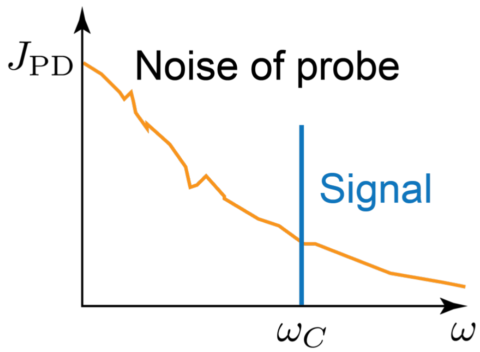
Ultrafast pump-probe techniques

Differential transmission spectroscopy



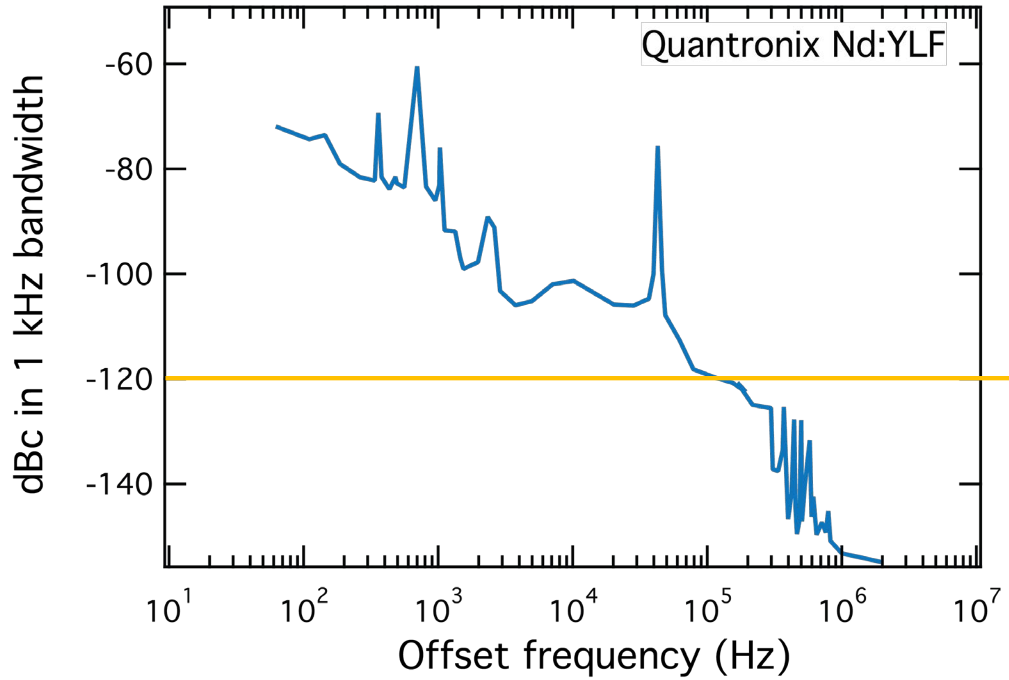
$$\text{signal} = \left[T(\Delta t, I_{\text{pump}}) - T(I_{\text{pump}} = 0)\right] I_{\text{probe}}$$

Transmission of device under test with
pump on pump off



Ultrafast measurements need some kind of nonlinearities in the measurement system (i.e. intensity dependent transmission)

Example: signal-to-noise 10^{-6}:1



Note: Microwave spectrum analyzer measures **power** of photocurrent!

$$20 \log \frac{S}{N} = 20 \log 10^{-6} = -120\,\mathrm{dBc}$$

Need a chop frequency of > 100 kHz

Timing jitter

$$I(t) = I_0 T \sum_{n=-\infty}^{+\infty} \delta\left(t - nT - \Delta T(t)\right)$$

↓

$$\tilde{I}_{\Delta T}(\omega) = F\left\{I_{\Delta T}(t)\right\} = -I_0 \sum_{n=-\infty}^{+\infty} in\omega_T \Delta\tilde{T}(\omega - n\omega_T)$$

$$S_{I_{\Delta T}}(\omega) = \tilde{I}_{\Delta T}(\omega) \cdot \tilde{I}^*_{\Delta T}(\omega) = I_0^2(-i)\cdot(+i) \sum_{n=-\infty}^{+\infty} n^2\omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T)\right]^2$$

↓

$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{n=-\infty}^{+\infty} \left\{\delta(\omega - n\omega_T) + \frac{1}{4\pi^2} n^2\omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T)\right]^2\right\}$$

Timing jitter of mode-locked laser

$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{n=-\infty}^{+\infty} \left\{ \delta(\omega - n\omega_T) + \frac{1}{4\pi^2} n^2 \omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T) \right]^2 \right\}$$

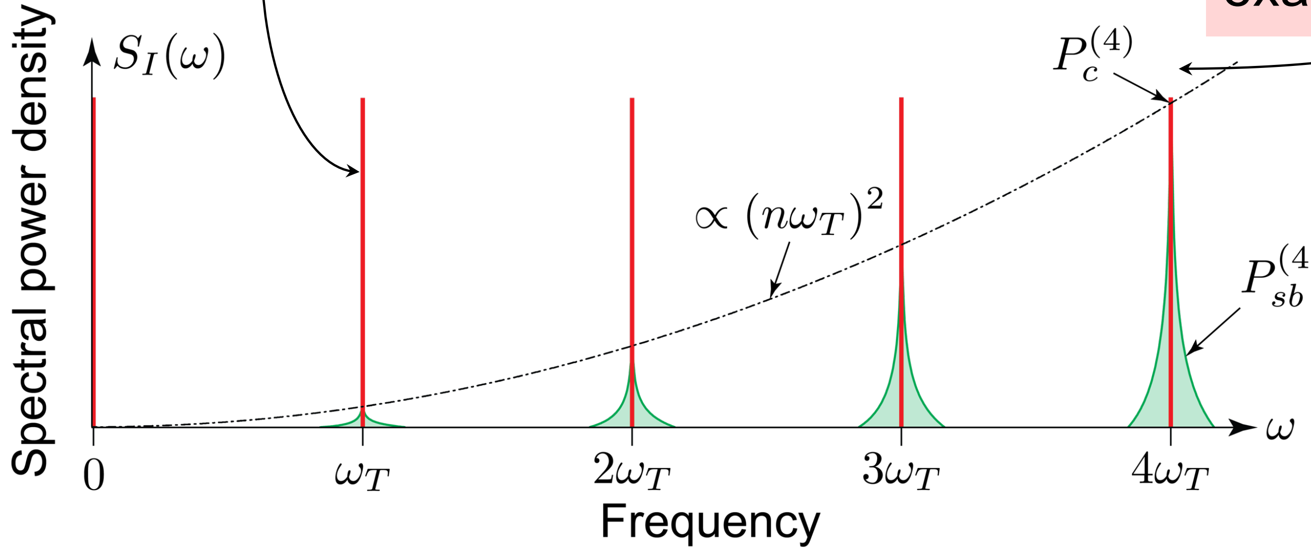
noise sidebands

normalized power density
(two-sided around harmonics)

$$L_1(f) = \frac{P_{sb}^{(1)}}{P_c^{(1)}} = \frac{1}{n^2} L_n(f)$$

$$L_n(f) = \frac{P_{sb}^{(n)}}{P_c^{(n)}}$$

example: n = 4

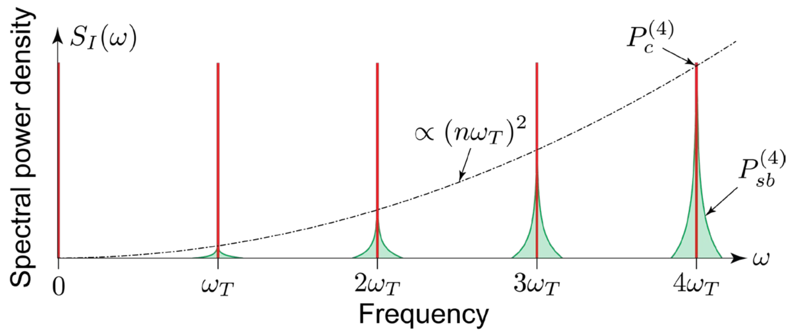


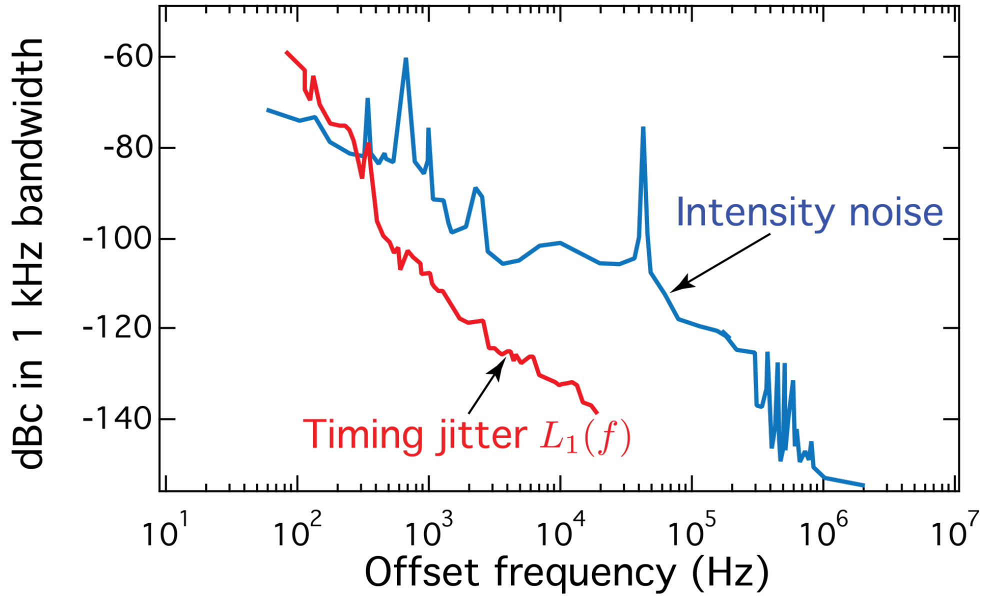
$\omega_T = 2\pi f_{\mathrm{rep}}$

Timing jitter of mode-locked laser

$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{n=-\infty}^{+\infty} \left\{ \delta(\omega - n\omega_T) + \frac{1}{4\pi^2} n^2 \omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T) \right]^2 \right\}$$

noise sidebands





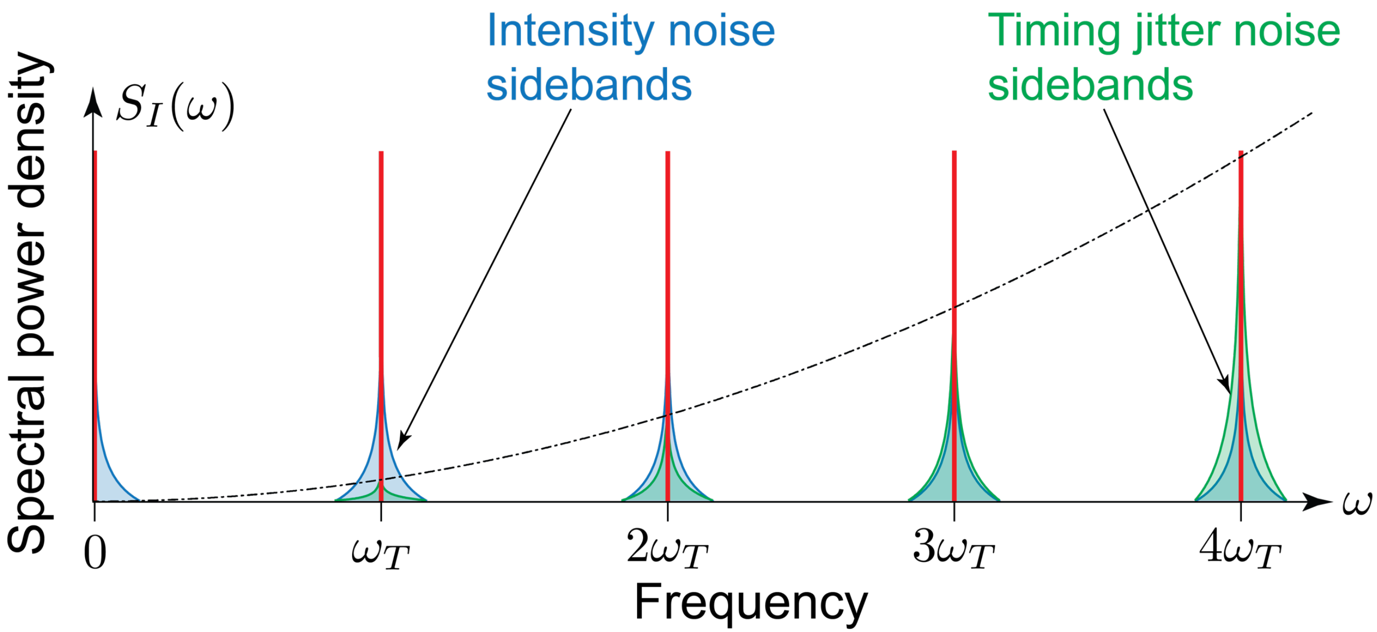
$$L_1(f) = \frac{P_{sb}^{(1)}}{P_c^{(1)}} = \frac{1}{n^2} L_n(f)$$

Measurement takes place at the n-th harmonic

Timing jitter sidebands stronger than intensity noise sidebands

Double check: $\propto n^2$
small noise approximation

Intensity noise and timing jitter



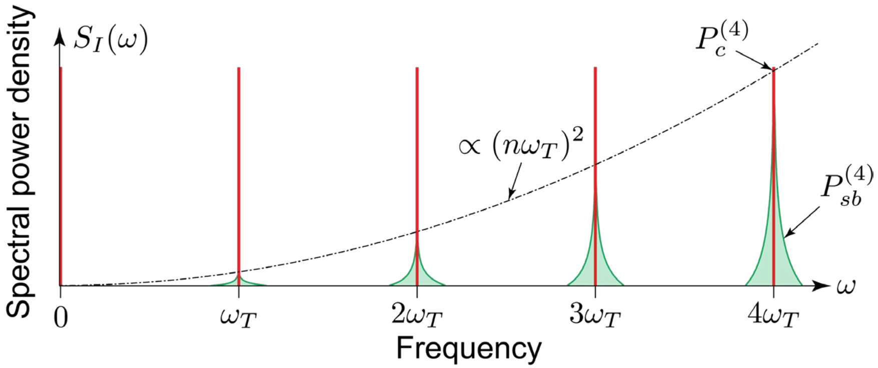
$$I(t) = I_0 T\,(1 + N(t)) \sum_{n=-\infty}^{+\infty} \delta\,(t - nT - \Delta T(t))$$

$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{n=-\infty}^{+\infty} \left\{ \delta(\omega - n\omega_T) + \left[\tilde{N}(\omega - n\omega_T)\right]^2 + \frac{1}{4\pi^2} n^2 \omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T)\right]^2 \right\}$$

rms timing jitter

$$S_I(\omega) = 4\pi^2 I_0^2 \sum_{n=-\infty}^{+\infty} \left\{ \delta(\omega - n\omega_T) + \frac{1}{4\pi^2} n^2 \omega_T^2 \left[\Delta\tilde{T}(\omega - n\omega_T) \right]^2 \right\}$$

noise sidebands



$$L_1(f) = \frac{1}{n^2} L_n(f) = \frac{1}{n^2} \frac{P_{sb}^{(n)}}{P_c^{(n)}}$$

$$\sigma_{\Delta T}[f_1, f_2] = \sqrt{\langle \Delta T(t)^2 \rangle} = \frac{1}{2\pi f_T} \sqrt{2 \int_{f_1}^{f_2} L_1(f) df} = \frac{1}{2\pi n f_T} \sqrt{2 \int_{f_1}^{f_2} L_n(f) df}$$

factor of 2
because $L_1(f)$ is "two-sided"

normalized power density
(two-sided around harmonics)

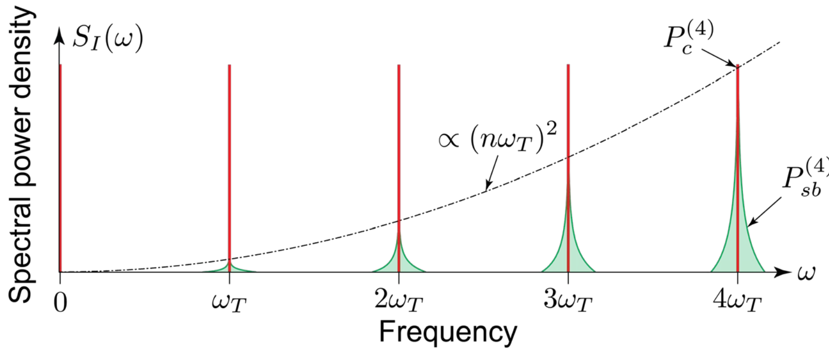
rms and FWHM timing jitter in ps?

rms timing jitter:

$$\sigma_{\Delta T}[f_1, f_2] = \sqrt{\langle \Delta T(t)^2 \rangle} = \frac{1}{2\pi f_T}\sqrt{2\int_{f_1}^{f_2} L_1(f)df} = \frac{1}{2\pi n f_T}\sqrt{2\int_{f_1}^{f_2} L_n(f)df}$$

$$L_1(f) = \frac{P_{sb}^{(1)}(f)}{P_c^{(1)}}$$

$$L_n(f) = \frac{P_{sb}^{(n)}(f)}{P_c^{(n)}}$$



FWHM timing jitter:
(Gaussian noise statistics)

$$\Delta T(t) = \frac{1}{\sqrt{2\pi\sigma_{\Delta T}}}\exp\left(\frac{-t^2}{2\sigma_{\Delta T}^2}\right) = \exp\left(-4\ln 2\left(\frac{t}{\tau_{\Delta T}}\right)^2\right)$$

$$\tau_{\Delta T} = \sqrt{8\ln 2}\,\sigma_{\Delta T} = 2.355\cdot\sigma_{\Delta T}$$

Example:

$$\sigma_{\Delta T}[130\,\text{Hz}, 20\,\text{kHz}] \approx \sigma_{\Delta T}[130\,\text{Hz}, \infty] = 9\,\text{ps} \;\Rightarrow\; \approx 21\,\text{ps FWHM}$$