# Ultrafast Laser Physics

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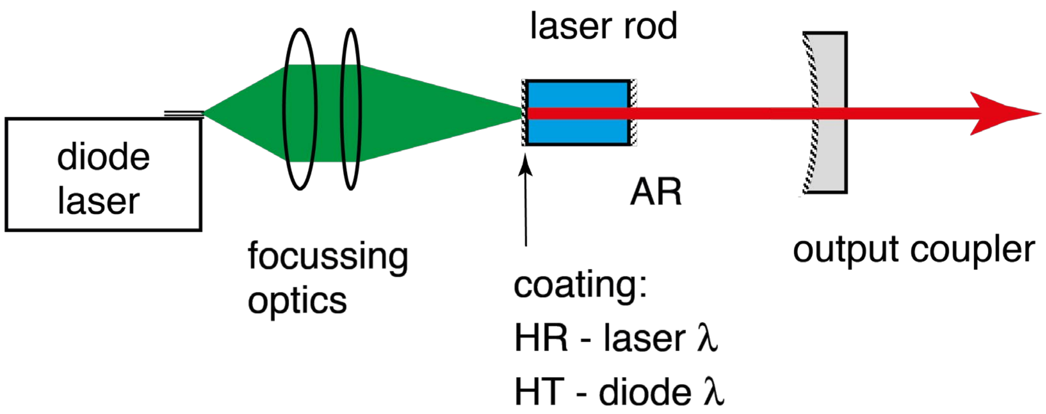
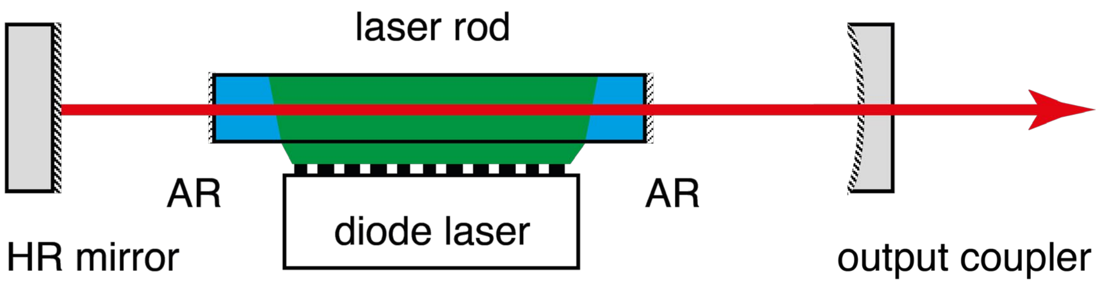
## *Chapter 5: Relaxation oscillations in lasers*

# Diode-pumped solid-state lasers

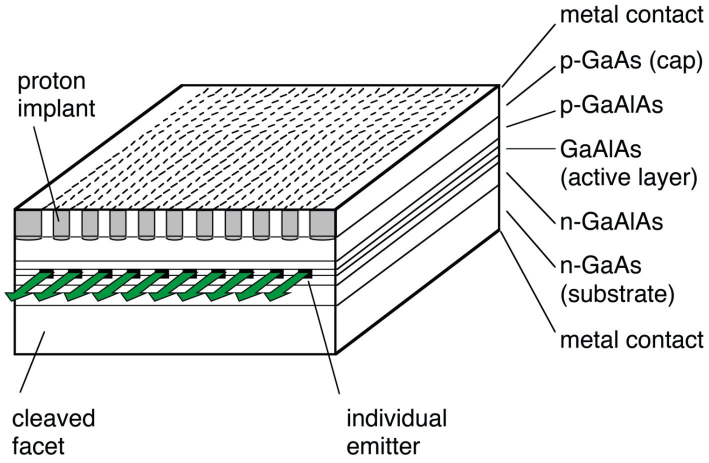
## Longitudinal pumping

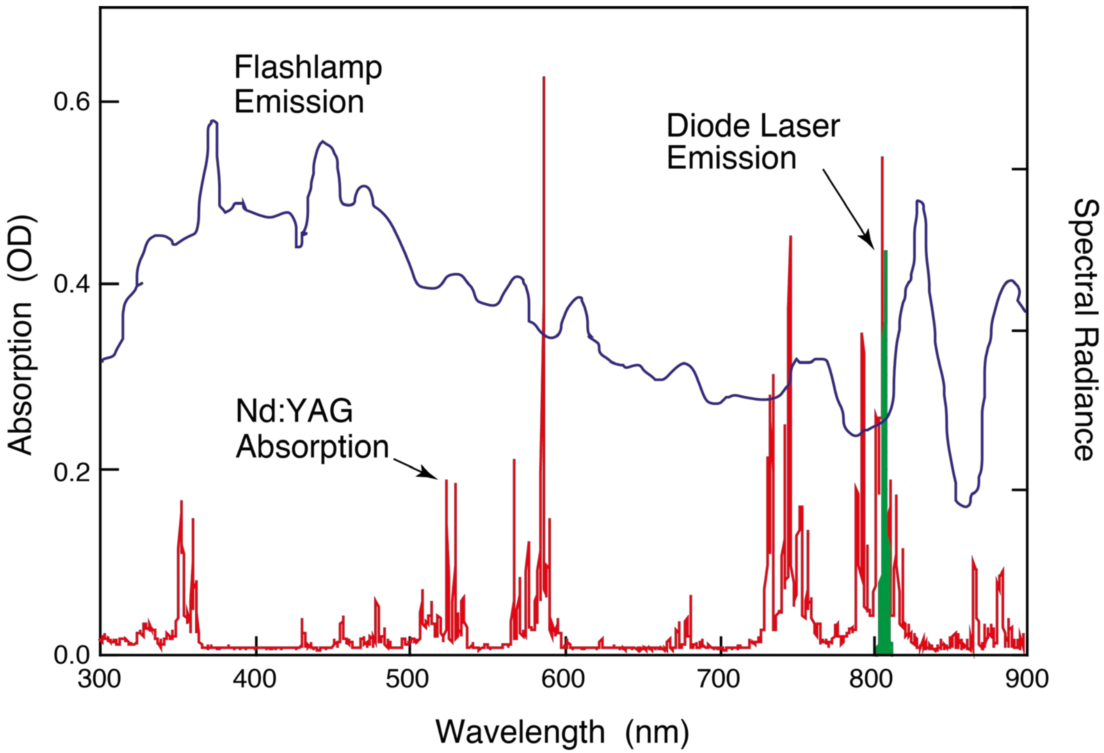



## Transversal pumping



# Diode-pumped solid-state lasers

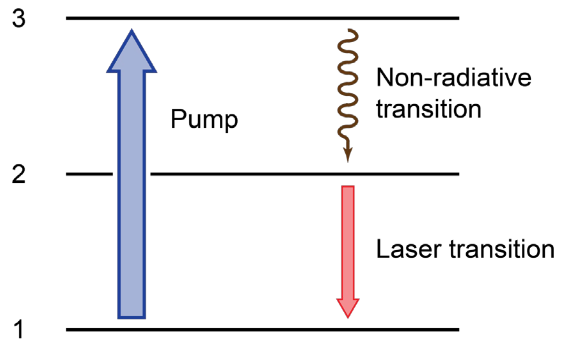


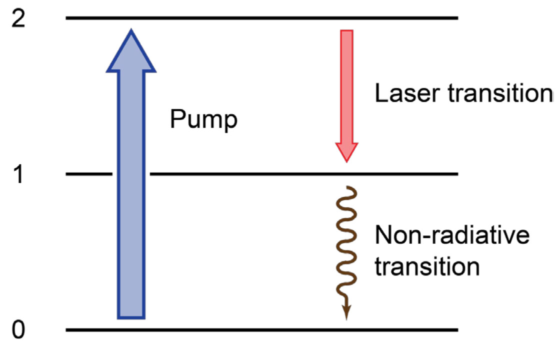



# Diode-pumped solid-state lasers

## Three-level system

Yb-doped

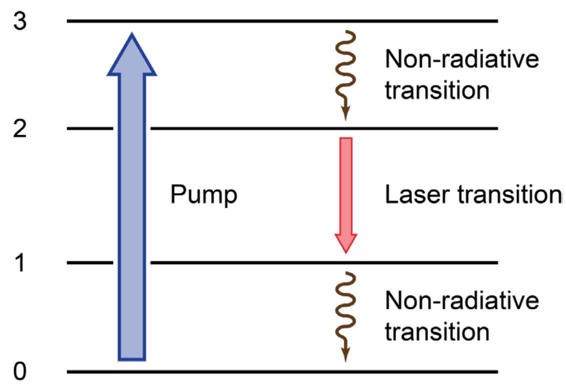


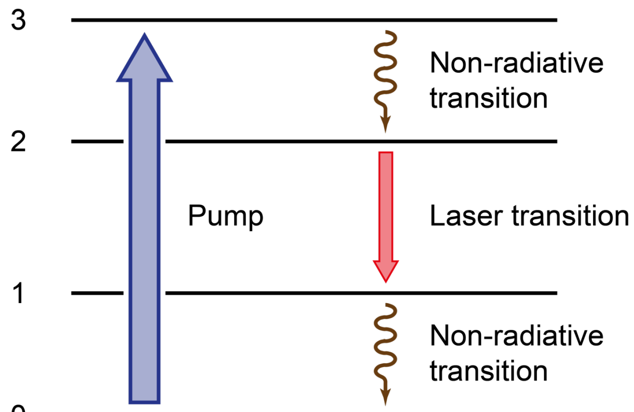

## Four-level system

Nd-doped

Ti:sapphire



# Laser rate equation




$$\frac{dn}{dt} = K(n+1)N - \gamma_c n$$

$$\frac{dN}{dt} = R_p - KnN - \gamma_L N$$

- ideal four-level system
- single mode laser
- homogeneous broadened gain material

$N$ inversion $N = N_2 - N_1$, $N_1 \approx 0$

$n$ photon number (intracavity)

$W^{stim} \equiv W_{12} = W_{21} = Kn$

$KN$ spontaneous emission rate

$KnN$ stimulated emission rate

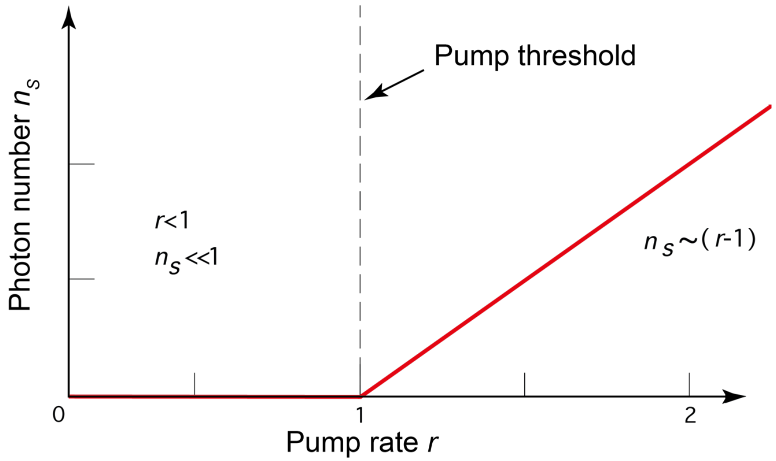
$\gamma_c$ cavity photon decay rate

$\gamma_L$ spontaneous decay rate of level 2

# Steady-state solution

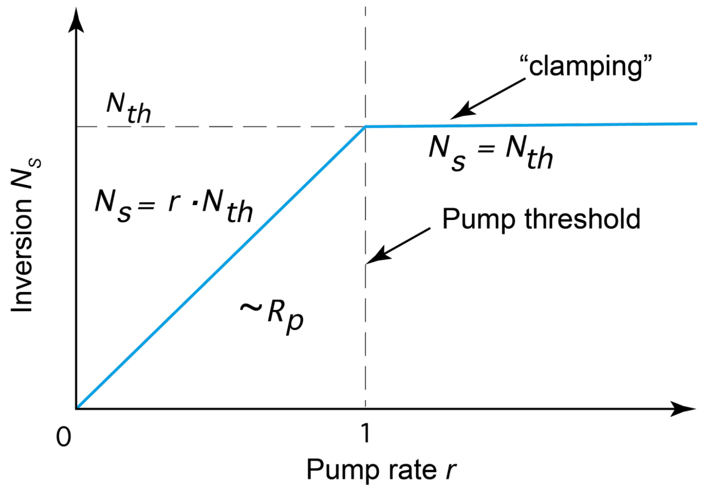
Above threshold, i.e. $r > 1$

$$\Leftrightarrow \begin{cases} n_s \approx \frac{\tau_c}{\tau_L} N_{th}(r-1) = \frac{\gamma_L}{K}(r-1) \\ N_s \approx N_{th} = \frac{\gamma_c}{K} \end{cases}$$




Below threshold, i.e. $r < 1$

$$\Leftrightarrow \begin{cases} n_s = \frac{r}{1-r} \\ N_s \approx r N_{th} \end{cases}$$



# Linearized rate equations

$$n(t) = n_s + n_1(t), \qquad n_1(t) \ll n_s$$
$$N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s$$

$$\frac{dn_s}{dt} = 0, \qquad \frac{dN_s}{dt} = 0$$

$$\boxed{\frac{dn_1(t)}{dt} = \gamma_L(r-1)N_1(t)}$$

$$\boxed{\frac{dN_1(t)}{dt} = -\gamma_L r N_1(t) - \gamma_c n_1(t)}$$

Solution: Ansatz

$$n_1(t) \propto e^{st}, \qquad N_1(t) \propto e^{st}$$

$$s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L\gamma_c(r-1)}$$

# Over-critical damping (no relaxation oscillations)

$$n(t) = n_s + n_1(t), \qquad n_1(t) \ll n_s$$
$$N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s$$

Solution: Ansatz

$$n_1(t) \propto e^{st}, \qquad N_1(t) \propto e^{st}$$

$$s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L\gamma_c(r-1)}$$

s is real and negative: $\left(\frac{r\gamma_L}{2}\right)^2 > \gamma_L\gamma_c(r-1)$

two solutions: two over-critically damped relaxation constants:

$$r\gamma_L \gg \gamma_c \quad \Rightarrow \quad \begin{cases} s_1 = -r\gamma_L & \text{Stimulated decay rate of excited atoms} \\ s_2 = -\gamma_c\frac{r-1}{r} & \text{Photon decay rate inside laser cavity} \end{cases}$$

$$s_2 = -\gamma_c\frac{r-1}{r} \quad \Rightarrow \quad s_2 \approx -\gamma_c, \quad \text{for} \quad r \gg 1$$

# Over-critical damping (no relaxation oscillations)

Example HeNe Laser (p. 200)

$\lambda$ = 632.8 nm, $\tau_L \approx$ 100 ns, $2l$ = 0.02 (2% output coupler), $T_R$ = 2 ns

$$\Rightarrow \quad \tau_c = \frac{T_R}{2l} \approx 100\,\mathrm{ns} \quad \Rightarrow \quad \tau_L \approx \tau_c, \quad \text{or} \quad \gamma_L \approx \gamma_c$$

For $r$ >>1 (when HeNe is pumped sufficiently far above threshold)

condition for over-critical damping is fulfilled:

$$r\gamma_L \gg \gamma_c$$

HeNe laser falls back into steady state after about 100 ns

# Under-critical damping (relaxation oscillations)

$$n(t) = n_s + n_1(t), \qquad n_1(t) \ll n_s$$
$$N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s$$

Solution: Ansatz

$$n_1(t) \propto e^{st}, \qquad N_1(t) \propto e^{st}$$

$$s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L \gamma_c (r-1)}$$

s is complex: $\left(\frac{r\gamma_L}{2}\right)^2 < \gamma_L \gamma_c (r-1)$

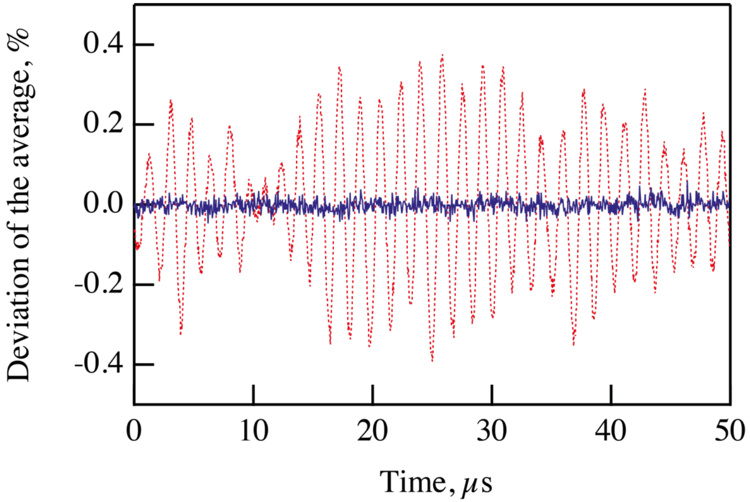
relaxation oscillations with an attenuation factor:

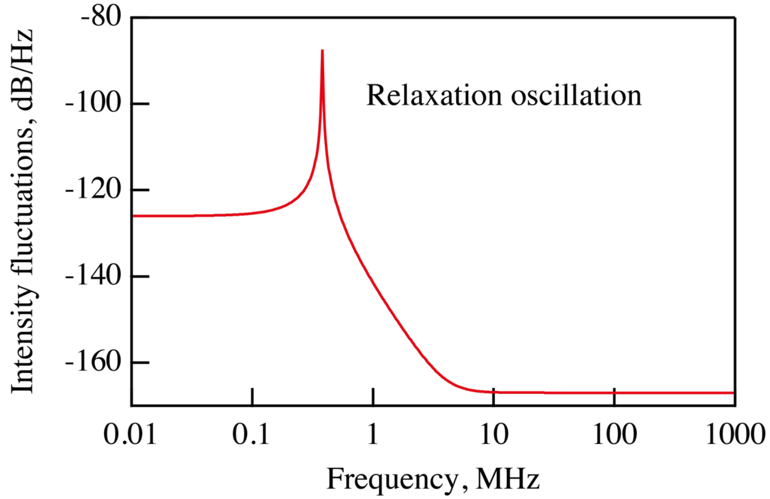
$$n(t) = n_s + n_1 e^{-\gamma_{relax} t} \cos(\omega_{relax} t)$$

$$\gamma_{relax} = \frac{r\gamma_L}{2}$$

$$\gamma_c \gg r\gamma_L : \quad \omega_{relax} \approx \sqrt{\frac{r-1}{\tau_L}\frac{1}{\tau_c}} = \sqrt{\frac{1}{\tau_{stim}}\frac{1}{\tau_c}}$$

# Relaxation oscillations




Relaxation oscillations in the time domain and frequency domain (microwave spectrum analyzer - not optical frequencies!)

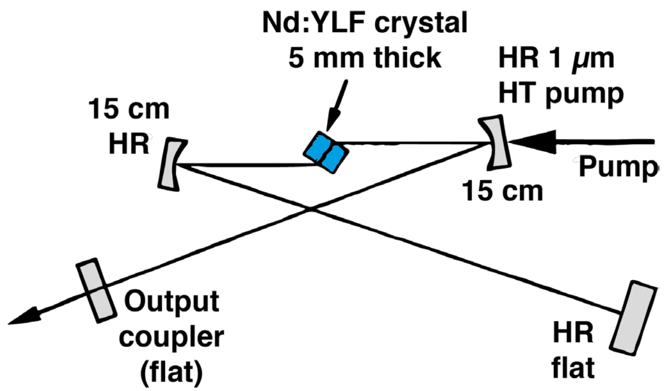
$$n(t) = n_s + n_1 e^{-\gamma_{relax} t} \cos(\omega_{relax} t)$$

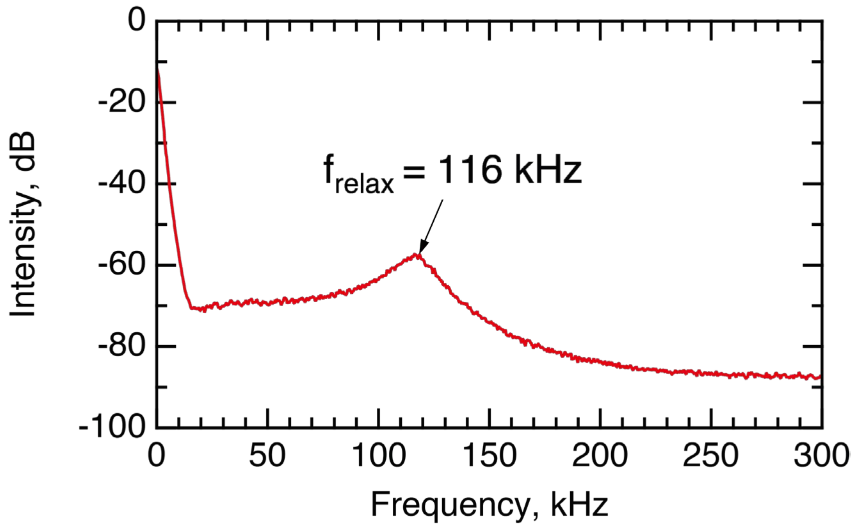
$$\gamma_{relax} = \frac{r\gamma_L}{2}$$

$$\gamma_c \gg r\gamma_L : \quad \omega_{relax} \approx \sqrt{\frac{r-1}{\tau_L}\frac{1}{\tau_c}} = \sqrt{\frac{1}{\tau_{stim}}\frac{1}{\tau_c}}$$

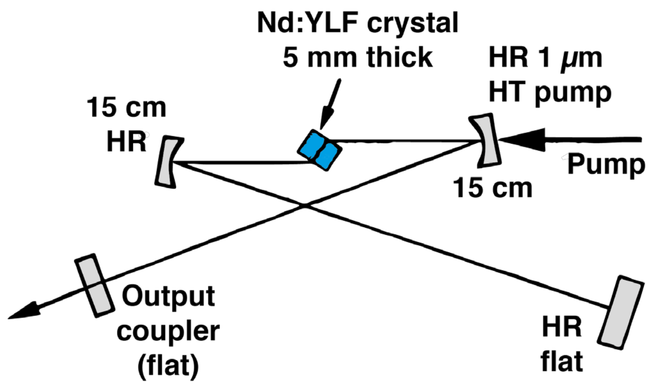
# Relaxation oscillations

Example:
diode-pumped cw Nd:YLF laser



# Measurement of small signal gain




$$g = \frac{g_0}{1 + 2I/I_{sat}}$$

$$f_{relax} \approx \frac{1}{2\pi}\sqrt{\gamma_L \gamma_c (r-1)} = \frac{1}{2\pi}\sqrt{\frac{r-1}{\tau_L \tau_c}} = \frac{1}{2\pi}\sqrt{\frac{\frac{g_0}{l}-1}{\tau_L \frac{T_R}{2l}}} = \frac{1}{2\pi}\sqrt{\frac{2g_0 - 2l}{\tau_L T_R}}$$

$$\tau_c = \frac{T_R}{2l}$$

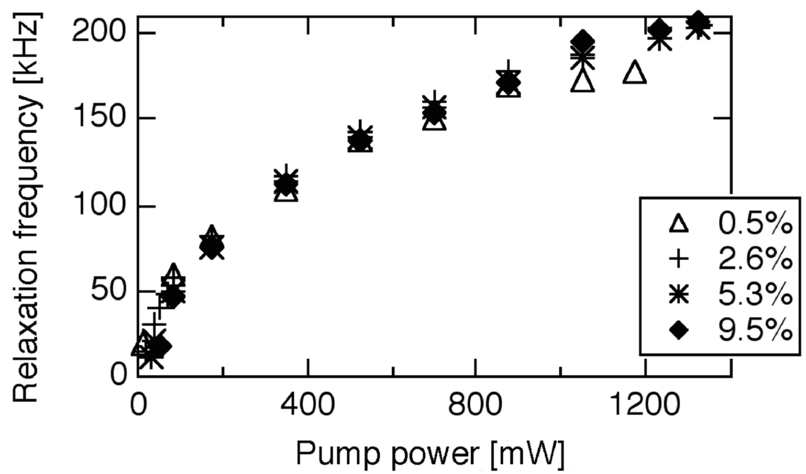
$$2g_0 \approx 4\pi^2 \tau_L T_R f_{relax}^2 + 2l$$

$$r = \frac{g_0}{l}$$

$$G_0 \approx e^{2g_0} = \exp\left(4\pi^2 \tau_L T_R f_{relax}^2 + 2l\right)$$

# Measurement of small signal gain

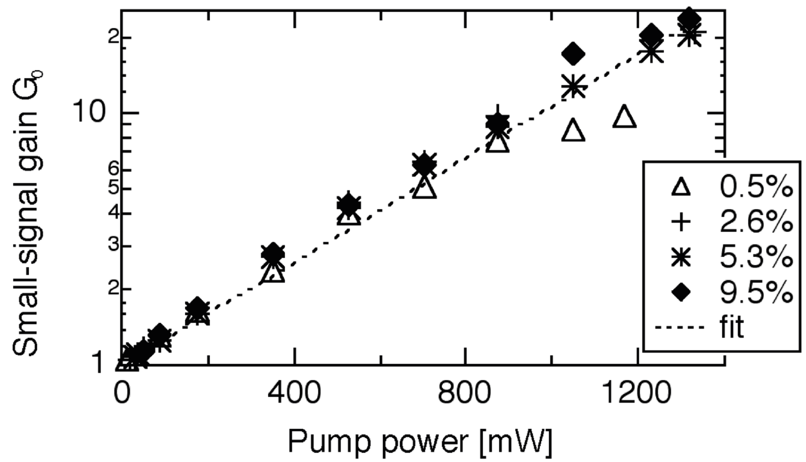
Measured relaxation oscillationfrequency for different outputcouplers, for $g_0 \gg l$




$$f_{relax} \approx \frac{1}{2\pi}\sqrt{\frac{2g_0 - 2l}{\tau_L T_R}} \xrightarrow{g_0 \gg l} \approx \frac{1}{2\pi}\sqrt{\frac{2g_0}{\tau_L T_R}}$$

relaxation oscillations independent of output coupler

$$2g_0 \approx 4\pi^2 \tau_L T_R f_{relax}^2 + 2l$$




$$G_0 \approx e^{2g_0} = \exp\left(4\pi^2 \tau_L T_R f_{relax}^2 + 2l\right)$$

Example: diode-pumped Nd:YLF laser