Eidgenössische Technische Hochschule Zürich
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Numerical simulation of fast Mølmer-Sørensen gate driven by optical standing waves

Semester Thesis

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Chapter 1

Introduction

In the emerging field of quantum computation, perfoming single and two qubit gates with high fidelities is fundamental, since a combination of these gates is sufficient to approximate arbitrary unitary evolution[1]. One of the first proposal for the experimental realisation of a two qubits entangling gate is the Cirac-Zoller gate[2] that allows the operation of a Control-NOT between two ions. However this gate had many drawbacks, for example it requires having both ions in the lowest vibrational state, a condition experimentally difficult to realize. For this reason many alternative schemes have been explored and particularly interesting ones are the phase gates. In this type of gates a mode of the two-ion system is displaced around a closed path in the phase space depending on the internal states of the ions. In this way the two ion state obtains a geometric phase proportional to the area enclosed in the phase space. This process does not depend from the vibrational state and is robust against changes in the vibrational motion during the operations.
In this work we have theoretically analysed the phase gate in the xy basis proposed by Mølmer and Sørensen in 1999 [3], commonly known as MS gate. We have studied a new configuration, based on standing waves, with which is possible to remove the carrier term and increase considerably the speed of the gate. Moreover we have shown how using pulse-shaping technique we can handle the remaining off-resonant terms and obtaint high fidelities. It is important to notice that the configuration presented in this work is particularly suited for an experimental implementation with the integrated optics used in [4], while can be hard to achieve with classical optical schemes.

Chapter 2

The Ms Gate

The MS gate is a multi-ions entangling gate that displace the motional state of the ions through the application of two fields, one detuned from the carrier frequency by $\Delta\omega_{blue} = \omega_{CM} + \delta$ where ω_{CM} is the centre of mass(mode in which the ions oscillate in phase) frequency and the other detuned by $\Delta\omega_{red} = -\omega_{CM} - \delta$ as we can see in fig 2.1. As a starting point we study the Hamiltonian describing the coupling between a single laser and one motional state of the ion. We will then generalize this description including two lasers and motional modes, for now we will generically write $\Delta\omega$ since the analysis is the same for blue and red detuning. The Hamiltonian describing the system is given by:

$$\begin{aligned} H_I =& H_0 + H_{int} \\ H_0 =& \hbar\omega_{CM}(a^\dagger a + \frac{1}{2}) + \hbar\omega_0\frac{\sigma_z}{2} \\ H_{int} =& \frac{\hbar\Omega}{2}(\sigma_+ e^{i[\eta(a+a^\dagger)-w_L t]} + h.c) \end{aligned} \tag{2.1}$$

We then go in the interaction picture with respect to H_0 by doing $H_I = e^{\frac{iH_0 t}{\hbar}} H_{int} e^{\frac{-iH_0 t}{\hbar}}$ and we have:

$$H_I = \hbar\sigma_+ e^{-i(\Delta\omega t - \varphi)} e^{i[\eta(a^\dagger e^{i\omega_{CM}t} + a e^{-i\omega_{CM}t})]} + h.c \tag{2.2}$$

where ω_{CM} is the centre of mass mode motional frequency, $\Delta w = \pm(w_{CM} + \delta) = w_L - w_0$ is the detuning of the laser frequency w_L from the carrier term frequency w_0, σ_+ is the raising operator for the spin in the z basis, such that $\sigma_+ |\downarrow\rangle_z = |\uparrow\rangle_z$, φ is the phase and Ω is the Rabi frequency of the laser. The Lamb-Dicke parameter η is defined as $\eta = k\cos\theta\sqrt{\frac{\hbar}{2M\omega_{CM}}}$, where k is the wavevector, θ is the angle between k and the direction in which the ion moves, and M is the ion mass. The Lamb-Dicke parameter represents the strength of the coupling between the field and the normal mode. In the Lamb-Dicke regime $\eta\sqrt{\langle n\rangle+1} \ll 1$, where $\langle n\rangle$ is the average photon number, we can expand the exponential:

$$H_I \simeq \hbar\sigma_+ e^{-i(\Delta wt-\varphi)}[1+i\eta(a^\dagger e^{iw_{CM}t}+ae^{-iw_{CM}t})]+h.c \tag{2.3}$$

Following [5] we can generalize the Hamiltonian to the case in which we consider two lasers with frequencies $\omega_b=\omega_0+\omega_{CM}+\delta$ and $\omega_r=\omega_0-\omega_{CM}-\delta$. We can define $\varphi_s = \frac{\varphi_b+\varphi_r}{2}$ and $\varphi_m = \frac{\varphi_b-\varphi_r}{2}$ where φ_r and φ_b are the phase of the lasers (respectively red and blue detuned). Using the formula $\sigma_\varphi = (\cos(\varphi)\sigma_x+\sin(\varphi)\sigma_y)$ (rotated basis) and $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ we can write the Hamiltonian for using two lasers:

$$\begin{aligned} H_I =& 2\hbar\Omega\cos(\Delta\omega t-\varphi_m)\sigma_{-\varphi_s} \\ &- \hbar\eta\Omega\sigma_{\frac{\pi}{2}-\varphi_s}(a^\dagger e^{i[(\omega_{CM}-\Delta\omega)t+\varphi_m]}+ae^{-i[(\omega_{CM}-\Delta\omega)t+\varphi_m]} \\ &- \hbar\eta\Omega\sigma_{\frac{\pi}{2}-\varphi_s}(a^\dagger e^{i[(\omega_{CM}+\Delta\omega)t+\varphi_m]}+ae^{-i[(\omega_{CM}+\Delta\omega)t+\varphi_m]} \end{aligned} \tag{2.4}$$

In the rest of our analysis we will fix $\varphi_r = -\varphi_b$, thus we will have $\varphi_s = 0$ and $\varphi_m = \varphi$, the Hamiltonian will be:

$$\begin{aligned} H_I =& 2\hbar\Omega\cos(\Delta\omega t-\varphi_m)\sigma_x \\ &- \hbar\eta\Omega\sigma_y(a^\dagger e^{i[(\omega_{CM}-\Delta\omega)t+\varphi_m]}+ae^{-i[(\omega_{CM}-\Delta\omega)t+\varphi_m]}) \\ &- \hbar\eta\Omega\sigma_y(a^\dagger e^{i[(\omega_{CM}+\Delta\omega)t+\varphi_m]}+ae^{-i[(\omega_{CM}+\Delta\omega)t+\varphi_m]}) \end{aligned} \tag{2.5}$$

Looking at our Hamiltonian we can notice that the first term, namely the carrier term, is not in the same basis as the other two, therefore does not commute. We can also notice that this term is not multiplied for η and thus its resonant coupling is $\frac{1}{\eta}$ stronger than the others. At this point we do the Rotating Wave Approximation(RWA), discarding terms in the Hamiltionian whose frequency is high(namely those with $\omega = \pm(\omega_{CM}+\Delta\omega)$). Notice that to do RWA δ has to be small such that $\Delta\omega \simeq \omega_{CM}$. As a matter of fact, if we want to achieve fast gates for which $\omega_{CM}t_g = \frac{2\pi\omega_{CM}}{\delta} = 1$, where t_g is the gate time, RWA breaks down and we cannot neglect the carrier term anymore. Using phase gate in the z basis, this term can be handled using pulse shaping, as done in [6]. This is not possible in the MS gate and, since it does not commute, we would like to delete the carrier contribution. To do so, we can create a standing wave using two laser beam and put our ions in the null of this wave. These beams will have the same frequency ω_L, what we call laser in our Hamiltonian, will be made by these two beams.
To create the standing wave the two beam will need to have opposite k vectors: $e^{ik_zz}-e^{-ik_zz} = 2i\sin(k_zz)$ where $k_zz = \eta(a^\dagger e^{iw_zt}+ae^{-iw_zt})$. Notice that we have assumed that the ion moves in the z direction and $\omega_z = \omega_{CM}$ and $k_z = |k|\cos\theta$.

Now Hamiltonian (2.2) describing the interaction of a single laser with one ion is given by:

$$H_I = \hbar\Omega\sigma_+ e^{-i(\Delta\omega t)}\sin([\eta(a^\dagger e^{iw_{CM}t}+ae^{-iw_{CM}t})])+h.c \tag{2.6}$$

We can now expand this expression and in the first order we have:

$$H_I = \hbar\Omega\sigma_+ e^{-i(\Delta\omega t)}[\eta(a^\dagger e^{iw_{CM}t} + ae^{-iw_{CM}t})] + h.c \tag{2.7}$$

Thus we have eliminated the carrier term. It is important to notice that this method is quite challenging to realize experimentally without using the system described in [4]. The optical scheme normally used are sensitive to variation in optical path length, due to air current or vibrations, therefore precisely fixing the ions in the null is not usually possible.
This is the starting point of our calculation and we will then generalize this expression by looking at both the center of mass and stretch mode of the two ions. It's important to notice that while in Hamiltonian 2.2 the expansion of the exponential leads to odd and even terms that don't commute, when we look at the expansion of the sin we just have odd terms that commute.

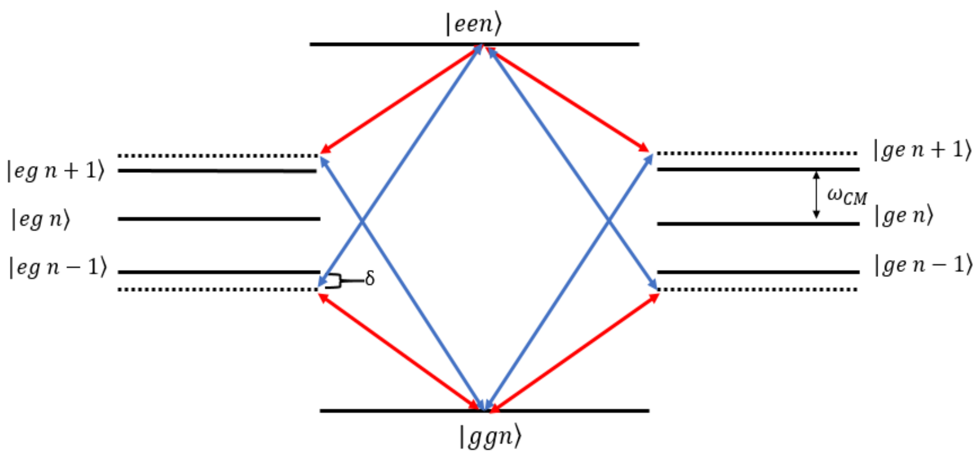

Figure 2.1: Energy levels of the two-ions system and visualization of the applied red and blue fields. ω_{CM} is the vibrational frequency of the centre of mass mode, while δ is the detuning from the red and blue sidebands. $|g\rangle$ and $|e\rangle$ are in the z basis, at the end of the gate the populations of the ions is changed i.e if initially they where in $|gg\rangle$ they end up in $|ee\rangle$. In the xy basis the population is left unchanged, but the state acquires a phase. Notice however that the motional state of the ions is unchanged at the end of the gate.

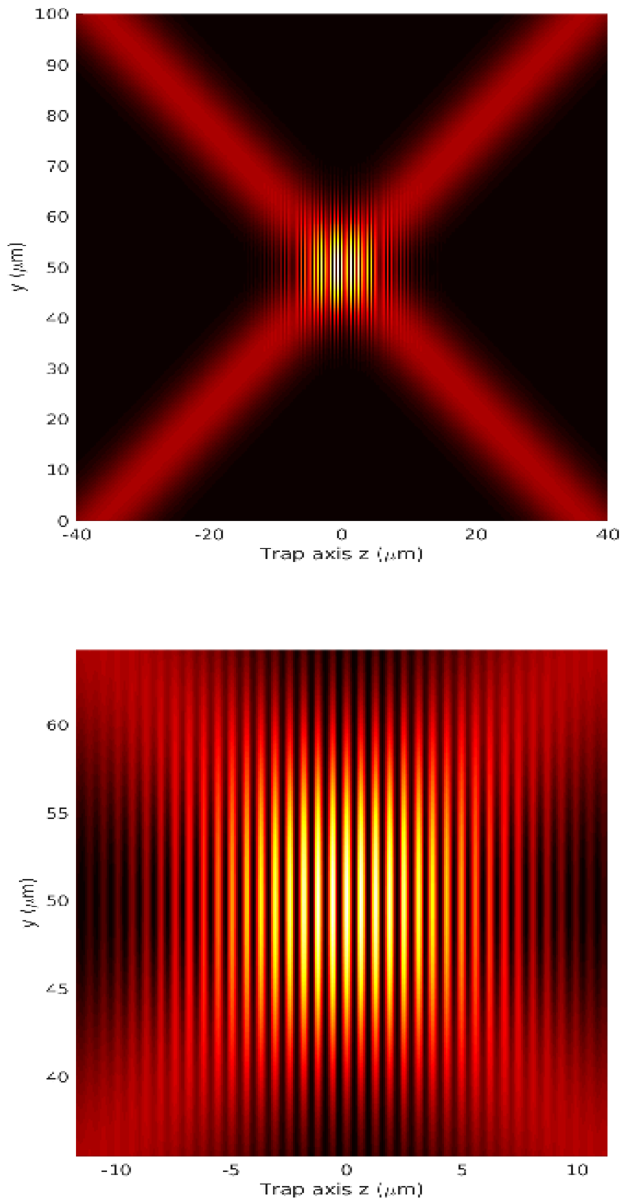

Figure 2.2: Using the system in [4] is possible to create a standing wave and confine the ions in the null of the field. In the first figure we can see the two beams applied to the ions, while the second image is a zoom in, showing the standing wave. The beam in the figure has a waist of $8\mu m$ and comes up of the chip at an angle of 36° from the vertical. The colors in the figure represents the intensities of the laser, with black and yellow being the minimum and maximum. Integrated optics is fundamental to achieve this goal, because the optical schemes generally used are too complex and introduce inevitable errors in the phase of the lasers.

Chapter 3

MS Gate Mechanism

The MS gate is a phase gate, namely its action is to displace the state of the system in the phase space. The propagator that gives the evolution of the state $\psi(t) = U(t)\psi(0)$ is given by:

$$U(t) = exp\{\frac{-i}{\hbar}(\int_0^t H_I(t')dt' + \frac{1}{2}\int_0^t dt' \int_0^{t'} [H_I(t'), H_I(t'')]dt'' + ..)\} \quad (3.1)$$

Considering only the first term in the expansion above, this operator is equivalent to the displacement operator defined as:

$$D(\alpha) = e^{\alpha a^\dagger + \alpha^* a} \quad (3.2)$$

Following [5] we have that α is given by:

$$\alpha(t) = 2i\eta\Omega(\sigma_y^{(1)} + \sigma_y^{(2)})e^{i\varphi_m}\int_0^t e^{i\delta t'}dt' \quad (3.3)$$

In this case we are considering a time-independent pulse Ω, we will see that changing this condition will help us reaching higher fidelities. The displacement operator translates the quantum state in phase space without changing its shape. Solving the integral we have:

$$\alpha(t) = 2i\eta\Omega(\sigma_y^{(1)} + \sigma_y^{(2)})e^{i\varphi_m}(1 - e^{i\delta t}) \quad (3.4)$$

We can see that the direction of the displacement depends from the qubit state in the σ_y basis and that in any case after $t = \frac{2\pi}{\delta}$ the state returns to its original position. If the spins are in different states then $\sigma_y^{(1)} + \sigma_y^{(2)} = 0$ therefore we don't have a displacement. However in this approximation we have not considered the higher order terms in (3.1) that are caused by the non-commutativity of the Hamiltonian with itself at different times. To solve this problem we can look at the displacement operator for each small interval of time dt and use the property $D(\alpha)D(\beta) = D(\alpha+\beta)e^{iIm(\alpha\beta^*)}$. In this way we can write equation (3.1) as $U(t) = D(\alpha(t))e^{i\varphi(t)}$ The phase is given by:

$$\varphi(t) = Im\int_0^t \alpha(t')d\alpha(t') = \frac{4(\sigma_y^{(1)}+\sigma_y^{(2)})^2\eta^2\Omega^2 t}{\delta\hbar^2} \tag{3.5}$$

Following [7] we can use the massless definition of position and momentum operators $x = \frac{a+a^\dagger}{\sqrt{2}}$, $p = \frac{a-a^\dagger}{i\sqrt{2}}$ to relate the total geometric phase to the Area in phase space.

$$\varphi(t) = Im\int_0^t \alpha(t')d\alpha(t') = \int_A \frac{dxdp}{\hbar} = \frac{A}{\hbar} \tag{3.6}$$

and again following [5] we fix the parameters such that $\frac{4\eta^2\Omega^2 t}{\delta} = \frac{\pi}{2}$. Notice that this condition implies that, to close the loop in the phase space at the end of the gate, when $t = \frac{2\pi}{\delta}$, we have to set $\Omega = \frac{\delta}{4\eta}$. Therefore the evolution operator at the end of the gate is :

$$U_{CM} = e^{i\frac{\pi}{2}\frac{(\sigma_y^{(1)}+\sigma_y^{(2)})^2}{\hbar^2}} \tag{3.7}$$

Applying this operator on the two-ion state, we have:

$$\begin{aligned} U_{CM}\left|\uparrow\uparrow\right\rangle_y &= \left|\uparrow\uparrow\right\rangle_y e^{i\frac{\pi}{2}} \\ U_{CM}\left|\uparrow\downarrow\right\rangle_y &= \left|\uparrow\downarrow\right\rangle_y \\ U_{CM}\left|\downarrow\uparrow\right\rangle_y &= \left|\downarrow\uparrow\right\rangle_y \\ U_{CM}\left|\downarrow\uparrow\right\rangle_y &= \left|\downarrow\uparrow\right\rangle_y e^{i\frac{\pi}{2}} \end{aligned} \tag{3.8}$$

Notice that in this case we are driving the Centre of Mass mode of the two-ion system, in which we have a displacement in phase space if both the Ions are in the same spin state. We can also choose to drive the stretch mode, in which we have displacement in phase space if the ions are in opposite spin state. In this case the evolution operator at the end of the gate will be given by:

$$U_S = e^{i\frac{\pi}{2}\frac{\left(\sigma_y^{(1)}-\sigma_y^{(2)}\right)^2}{\hbar}} \tag{3.9}$$

and the action on the two-qubit state can be seen as:

$$\begin{aligned} U_S\left|\uparrow\uparrow\right\rangle_y &= \left|\uparrow\uparrow\right\rangle_y \\ U_S\left|\uparrow\downarrow\right\rangle_y &= \left|\uparrow\downarrow\right\rangle_y e^{i\frac{\pi}{2}} \\ U_S\left|\downarrow\uparrow\right\rangle_y &= \left|\downarrow\uparrow\right\rangle_y e^{i\frac{\pi}{2}} \\ U_S\left|\downarrow\uparrow\right\rangle_y &= \left|\downarrow\uparrow\right\rangle_y \end{aligned} \tag{3.10}$$

A phase gate implementation that uses the stretch mode was done in [7], Notice that in this case the phase gate was in the z basis not in the xy.

For slow gates we can drive a single mode frequency, this is possible because using RWA we consider only the resonant terms. However, as we have seen before, for fast gates($\omega_{CM} t_g = 1$) RWA does no hold and thus driving just one mode will be impossible, thus to achieve high fidelity we have to change our implementation.

Chapter 4

Simulations

In our programme we have simulated the system, computing the density matrix starting with both spin in the ground state of the z basis $\rho = |\downarrow\downarrow\rangle_z \langle\downarrow\downarrow|_z \otimes \rho_{vib}$. We computed the density matrix at different times solving the Heisenberg equation:

$$\frac{d\rho(t)}{dt} = -i[H(t), \rho(t)] \tag{4.1}$$

We have used a medium order method (ode45 in Matlab) base on an explicit Rung-Kutta formula, the Dormand-Prince pair [9][10]. We then traced out the vibrational degree of freedom to obtain the spin density matrix: $\rho_{spin}(t) = Tr_{vib}[\rho(t)]$ and we could therefore look at the population of the different levels at each step as plotted in (4.1). In the simulations we have normalized every parameter with the common mode frequency, that we fixed as $\omega_{CM} = 1$. We are considering optical transition with $^{40}Ca^{+}$ therefore the carrier frequency is $\omega_0 = 411THZ$. The frequency of the centre of mass mode is around $2-5MHz$ and the ion are addressed with beams forming an angle of $\theta = 30°$, the detuning in figure (2.1) is $\delta = 0.025$, the pulse is constant $\Omega = 0.1786$ and the Lamb-Dicke parameter is $\eta = 0.0707$, which corresponds to 0.7 MHz for an angle $\theta = 30°$ and an ion of Ca(40 atomic mass) $m = 40 \cdot 1.67 \cdot 10^{-27}kg$. We have also checked the Fidelity of our gate at each time. In the z basis the action of the gate on the state $|\downarrow\downarrow\rangle_z$ is [8]

$$|\downarrow\downarrow\rangle_z \rightarrow \frac{1}{\sqrt{2}}\{|\uparrow\uparrow\rangle_z + ie^{i(\varphi_{s1}+\varphi_{s2})}|\downarrow\downarrow\rangle_z\} = |\psi_{expected}\rangle_z \tag{4.2}$$

Therefore the fidelity at each time is given by :

$$F(t) = \langle\psi_{expected}|\,\rho_{spin}(t)\,|\psi_{expected}\rangle \tag{4.3}$$

Generally we will look at the infidelity at the end of the gate, which is simply defined as $IF = 1 - F$. Using the parameters of (4.1), including only the common mode and without the carrier term we achieved infidelities below

$IF = 1 \cdot 10^{-5}$. Due to intrinsic numerical error of the computer we cannot trust results that are below $IF = 1 \cdot 10^{-5}$, therefore we fix this as the best achievable result . Another thing that we can do to check that our code works properly is to fix the detuning from the carrier as zero $\Delta\omega = 0$. In this case(if we include the carrier term in the Hamiltonian) we see Rabi flopping between the states $|\uparrow\uparrow\rangle_z$ and $|\downarrow\downarrow\rangle_z$ as expected.
We have also verified that, as expected by equation 2.4, having the carrier term strongly affects the Fidelity that can be reached. In figure 4.2 we have plotted the fidelities that we found for different gate time (expressed as number of Common mode period) and δ with and without the carrier term for a single pulse. Noticeably, while the fidelity without the carrier and with only the common mode remains almost constant, when we include the carrier the infidelities are higher for small values of α(number of COM mode period) and are never below 0.35 for $\alpha < 5$. Therefore to reach high values of the fidelity for fast gate the elimination of the carrier term is central.

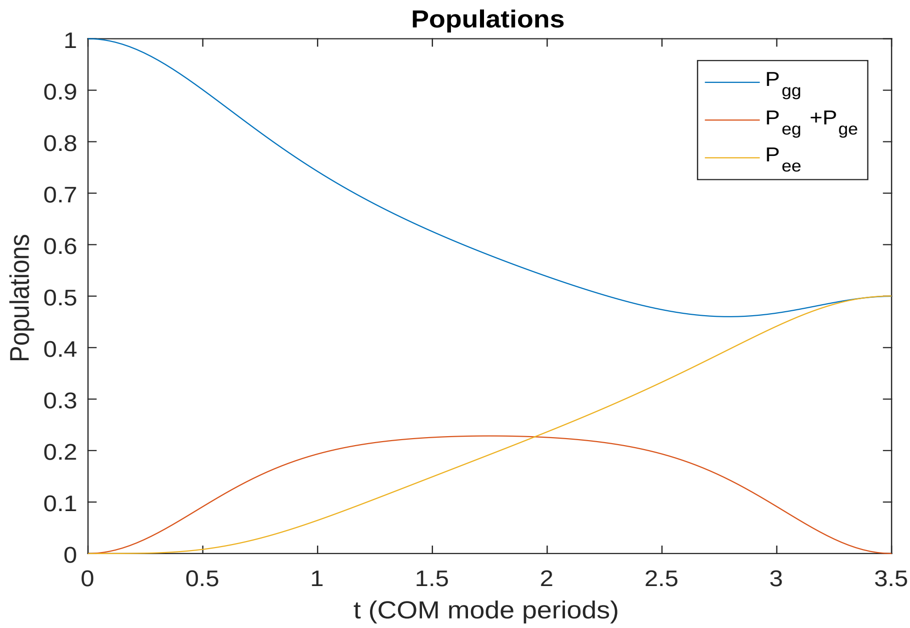

Figure 4.1: Populations as a function of time. The system is initially in $|gg\rangle$, the detuning is $\delta = 0.025$, Omega is $\Omega = 0.1769$ and the Lamb-Dicke parameter $\eta = 0.0707$. Notice that each parameter is normalized by the CM mode frequency.

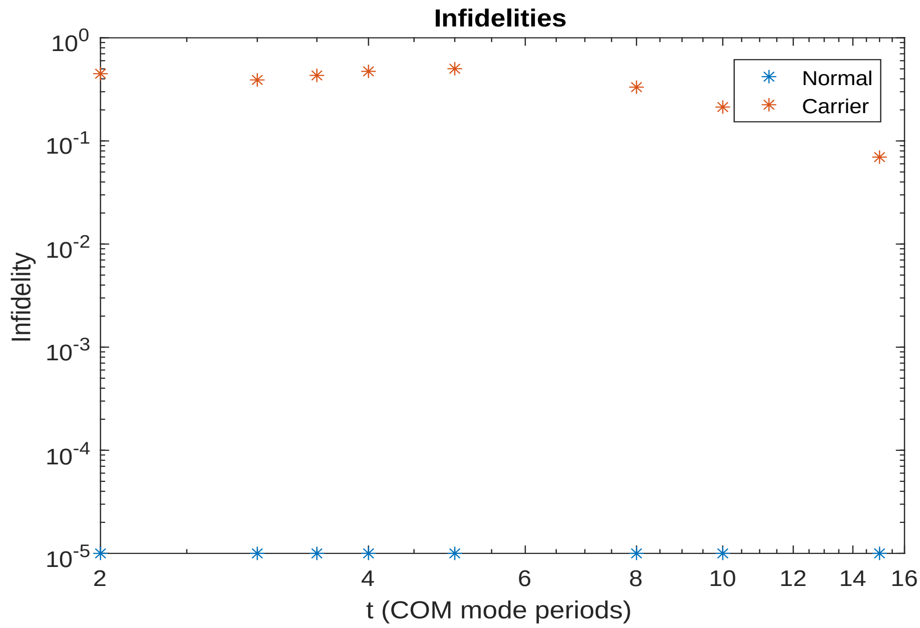

Figure 4.2: Infidelities as a function of gate time with and without the carrier term. We can see that when we consider the carrier term we cannot reach good infidelities, because this term does not commute with the rest of the Hamiltonian. We fix $1 \cdot 10^{-5}$ as the best achievable infidelities due to intrinsic numerical error of the computer.

4.1 Inclusion of Stretch Mode and Time Dependent Pulse

Having eliminated the carrier contributions, the next most important off-resonant term is the stretch mode. As we will see this term is quite important when we want to have fast gates in which $\omega_{CM} t_G = 1$ where t_G is the gate time, as a matter of fact we cannot reach high fidelities even without the carrier term if we don't handle properly the stretch mode. Notice that in our simulation we have fixed $\omega_{CM} = 1$ and the frequency of the stretch mode is $\omega_{stretch} = \sqrt{3}\omega_{CM} = \sqrt{3}$. The Lamb-Dicke parameter for the stretch mode is given by $\eta_{stretch} = \frac{\eta_{CM}}{3^{\frac{1}{4}}}$[8]. As we can see from figure (4.1) the inclusion of this mode affects the fidelity that we can reach for fast gate if we consider a constant Ω. To understand why this is the case we have to look back to the displacement in the phase space. The condition that we asked for closing the loop was that at the end of the gate $\alpha(t_g) = 0$. We can view the application of our lasers as the action of a time-dependent force that acts on an harmonic oscillator with frequency equal to the CM mode frequency. We can write this force as $f(t) = \Omega(t)cos(\omega t + \varphi)$ and rewrite equation 3.3 as:

$$\alpha(t) = \int_0^t f(t')e^{i\omega_{CM}t'}dt' = \int_0^t \Omega(t')cos(\omega t' + \varphi)e^{i\omega_{CM}t'}dt' \tag{4.1}$$

where $f(t)$ is the state-dependent force applied to our system by the lasers. In this case the condition corresponds to having a null at ω_{CM} in the fourier space. Now the problem is that, since the detuning δ scales as the inverse of the gate time $t_G = \frac{2\pi}{\delta}$, it will be high when we consider fast gates and therefore our lasers can excite not only the CM mode but also the stretch mode, because $\omega_L - \omega_0 - \omega_{CM} \simeq \omega_L - \omega_0 - \omega_{stretch}$ where ω_L is the frequency of the applied laser, $\omega_L = \omega_0 + \omega_{CM} + \delta$. Notice anyway that for fast gate the dependence is not completely linear. The fact that we excite two modes means that we have two integral of the form (4.4) that we want to put equal to zero. This means that we want to have the fourier transform of $f(t)cos(\omega t)$ equal to zero in the points ω_{CM} and $\omega_{stretch}$. However till now we had $\Omega = constant$ and the fourier transform of a rectangular signal is a sinc function which has nulls only at integer values. Since $\omega_{stretch} = \sqrt{3}\omega_{CM}$ it is impossible to close both loops in phase space. To overcome this problem we can take a time-dependent $\Omega(t)$.

We used symmetric sequences of pulses as in fig (4.2),(4.3), varying their intensities and spacing. To optimize the pulses we have used the fminsearch function in Matlab, which is based on the simplex search method of Lagarias et al.[11]. The parameters of the optimization where the length of the single pulses, their amplitude, the spacing between them, the total gate time and the detuning, in total we had (n+2) parameters for a symmetric n-segment pulse. We have tried combination with different numbers of sequences and look at how this affects the achievable fidelities. In fig (4.4) we have plotted the results obtained using symmetric sequences of 2, 3 and 9 pulses. Each point in the graph is the result of an optimization over the parameters of the pulses, the detuning and the gate time. We can see that even with series of three pulses we achieve high fidelities and fast gates. In figure (4.5) we can see the action of the gate in phase space using a pulse of three series, as we can see that loops corresponding to both modes are closed.

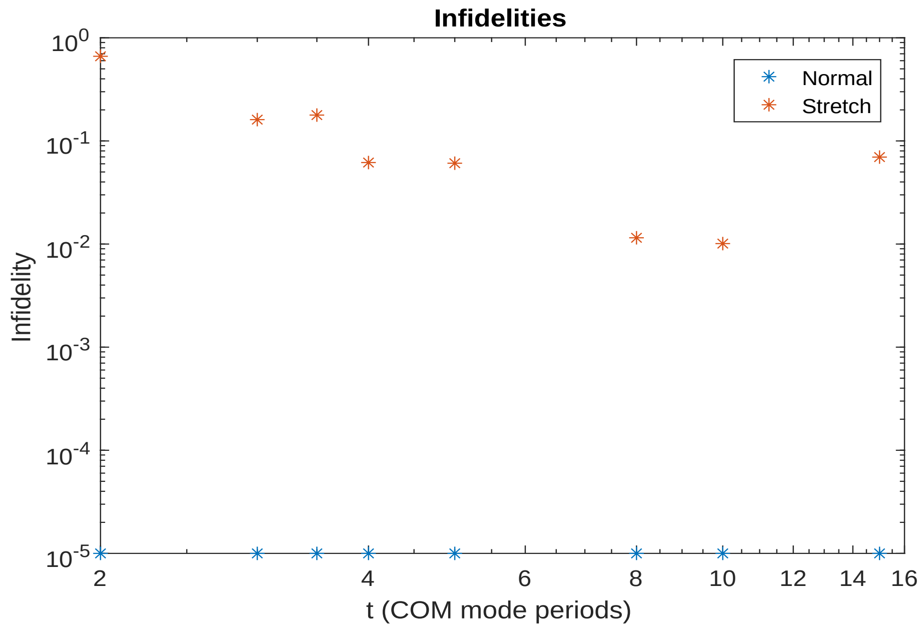

Figure 4.1: Infidelities achievable considering only the CM mode(blue) and including also the stretch mode(red), in both cases we are not considering the carrier term. We can see that for fast gate ($\omega_{CM}\frac{2\pi}{\delta} \simeq 1$) the effect of the Stretch mode becomes relevant and we cannot achieve infidelities lower than $1 \cdot 10^{-2}$, whereas in the ideal normal case we have infidelities of $1 \cdot 10^{-5}$ because there is no off-resonant coupling with the carrier. This figure refers to a single pulse sequence i.e the value of the pulse was constant for the all duration of the gate.

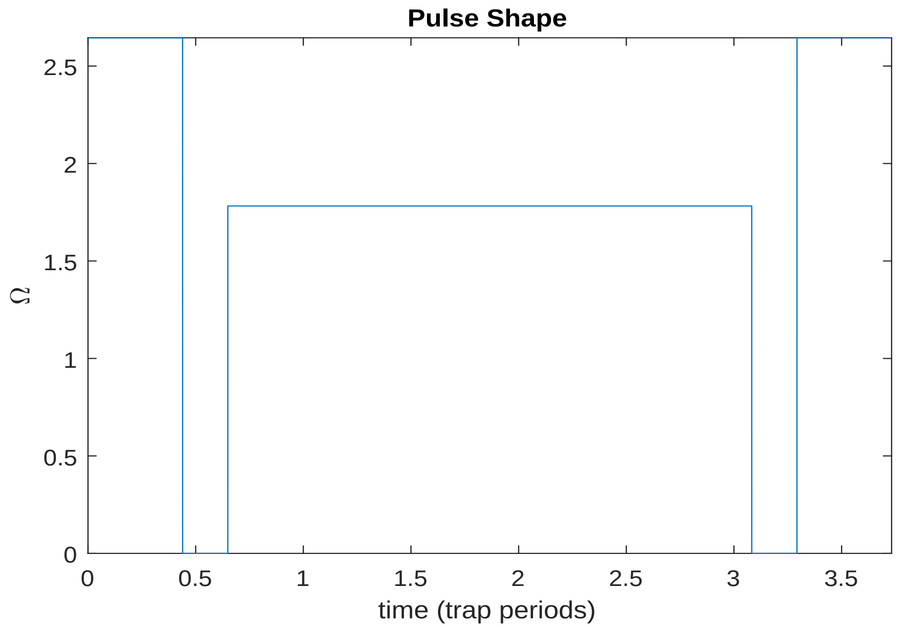

Figure 4.2: Symmetric series of three pulses. On the x axis we have the time of the gate express in COM mode periods.

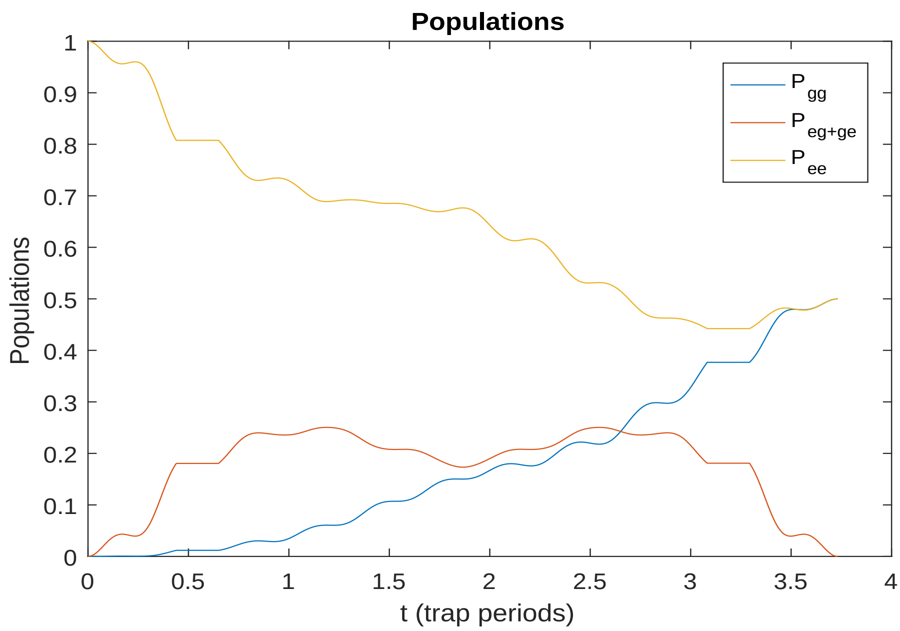

Figure 4.3: Population for a symmetric series of three pulses as shown in (4.2)

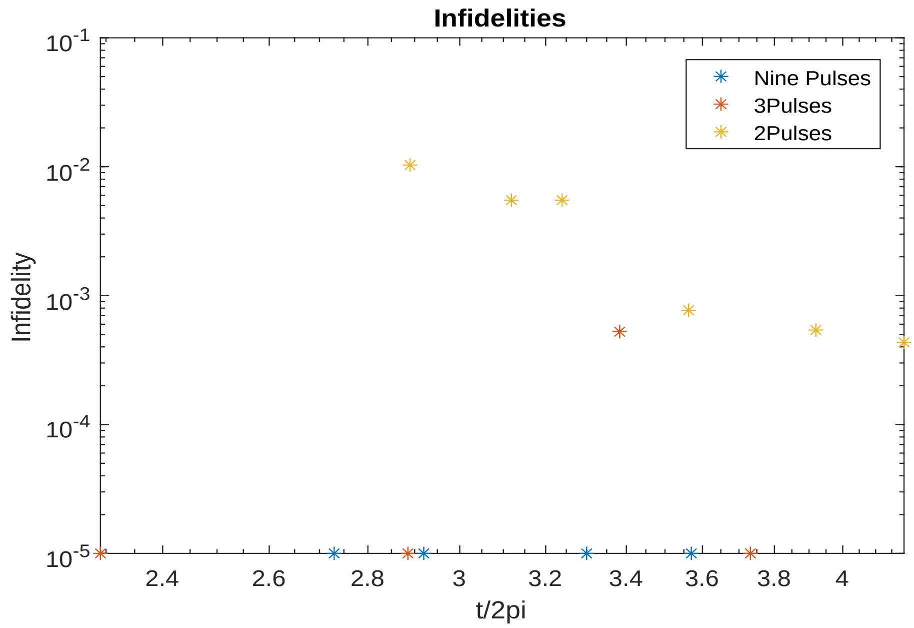

Figure 4.4: We have compared the fidelity that can be achieved using sequences made by a different number of pulses when we include at the first order both centre of mass and stretch mode, without the carrier. As we can see, three pulses are enough to achieve $IF = 1 \cdot 10^{-5}$.

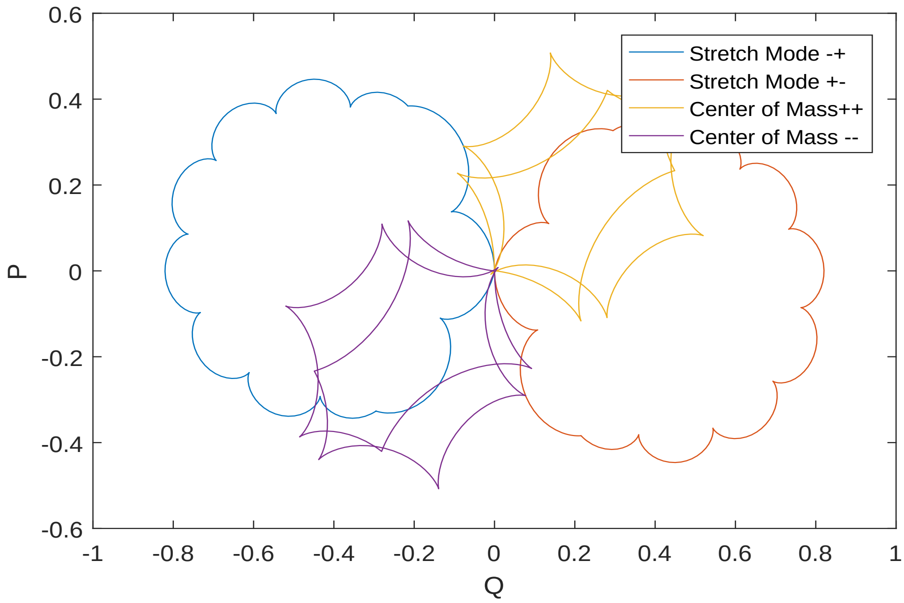

Figure 4.5: Effect of the gate in the phase space. For fast gates we excite both the stretch and COM mode. We can see that we have a state-dependent displacement and at the end of the gate the state goes back in the center. The figure refers to a symmetric sequence of three pulses with no carrier.

4.2 Other sources of error

Until now we have studied the two main contributions to the infidelities, the carrier term and the stretch mode, however others mechanism can affect our precision. Here we have studied two of them, namely third order terms in the expansion of $\sin([\eta(a^\dagger e^{iw_z t} + ae^{-iw_z t})])$ and residual carrier effects. As a matter of fact, our method is based on keeping the ion in the null trough a standing wave, we have looked at the effect of a small drift of the ions due to an imbalance in the lasers intensities of 5% that gives a small carrier contribution. We have also looked at how, considering the carrier zero to the first order but not to the second(like taking expansion of exponential without first term), affects our results and we found that we loose two order of magnitude in precision. The results for the case of a three pulses sequence are plotted in (4.1). Notice that the values of infidelity including the third order expansion and imbalance in the intensities are not optimized. This is because it's difficult to know the precise experimental error that we will have and thus doing an optimization does not make much sense. As we can see in the figure (4.1), if we don't have a perfect standing wave, the resulting small carrier term can strongly influence our achievable infidelities, with a loss of 3 order of magnitude in their values. However we must say that imbalance much lower than 5% should be possible using [4], therefore our method should guarantee high precision.

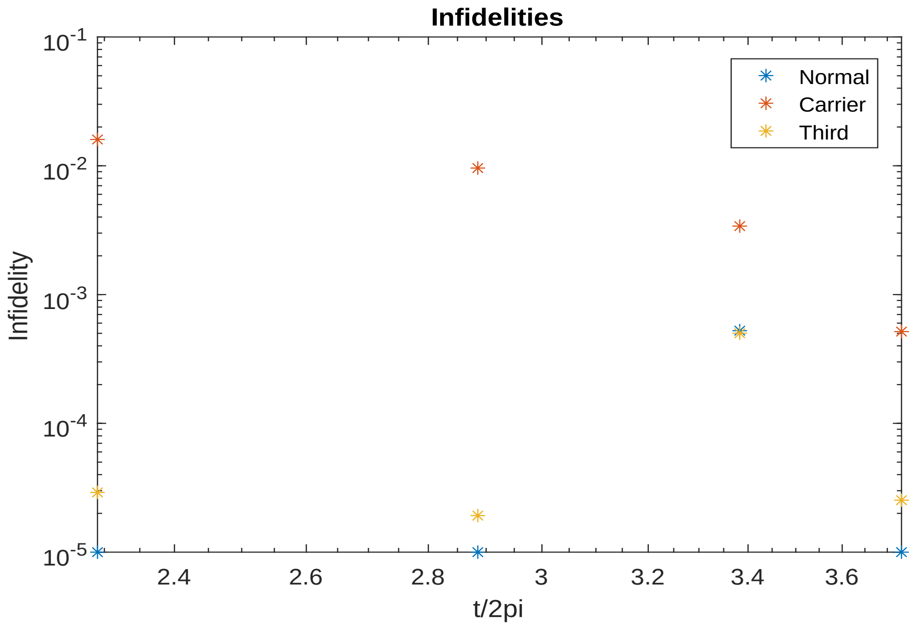

Figure 4.1: We have analysed the effect that a small carrier term(red) and the third order terms(yellow) can have on the achievable fidelities of our gate. In particular, we can see that the small carrier term caused by an imbalance in the laser beam, can considerably influence our precision, increasing the infidelity of 3 orders of magnitude.

Chapter 5

Conclusion

We have studied the basic working principle of the MS gate and analysed how, using a standing wave, we can strongly improve both fidelity and gate time, through the elimination of the carrier term. We have also studied how, for fast gate, we excite both the common and stretch mode and how this affects the achievable fidelity. To solve this problem we have simulated the performance of the gate using series of pulses instead of a single one and we have found that infidelities of the order of 10^{-5} should be possible for series of 3 pulses. We have then looked at how imperfection in the experimental apparatus, like imbalance in the laser intensities and consequent non complete elimination of the carrier, can impact our implementations and found that we can anyway reach infidelities of around 10^{-3} for 5% imbalance(lower values should be experimentally possible). Moreover we have also looked at the effect of third order terms, finding that their contribution can lower the fidelity of one order of magnitude maximum. Further analysis can be done considering how the radial modes affect the achievable infidelity.
In conclusion we have shown that fast(3 COM period), low infidelities(around 10^{-3}) and robust MS gate can be theoretically achieved using series of three pulses and standing wave elimination of the carrier term.

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