Objective Lenses for Trapped Ion Experiments with Beryllium

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Abstract

We present two diffraction limited objective lenses designed for trapped beryllium ion experiments. They are optimized for a working wavelength of 313 nm. The objectives are built entirely from commercially available spherical singlet lenses. This makes them cost effective. Nonetheless, they are low in spherical aberrations, both having a geometric spot size of less than 1 µm. Additionally, they are able to correct aberrations due to the view port of the vacuum chamber. The numerical apertures are 0.17 and 0.35, the effective focal lengths 91 mm and 54 mm respectively. The imaging systems have been simulated using the commercial ray tracing software Zemax.

Contents

1 Introduction

Trapped ion systems are one of the leading technologies for quantum information processing. Different ion species such as calcium, magnesium or beryllium can serve as quasi 2-level system to implement a qubit. In what's following, we will restrict ourselves to beryllium based qubits. A crucial part of information processing is the readout process, which is based on state dependent fluorescence. Using lasers, one of the qubit states (called the bright state) is driven into a fast decaying electronic state. This leads to an emission of photons at a wavelength of 313 nm. The other qubit state (called the dark state) couples only weakly to the laser and thus does not scatter any photons. The role of the objective lens is to collect the scattered light of the beryllium ions.
A typical set up of a trapped ion experiment is shown in figure 1. For ease of use, we'd like to be able to operate objective lens outside the vacuum chamber. This comes at the price of having to deal with aberrations due to light passing through the view port as well as being limited on how close one can put the objective lens to the ion trap. This matters because it puts a limit on the numerical aperture (NA) one can achieve. The imaging system should satisfy the following criteria:

- The photons emitted by the beryllium ions are distributed isotropically. To collect as many photons as possible, the lens must have a high NA
- To perform experiments with multiple ions, we need to be able to resolve each ion individually. The trapped ions are typically around 5-10 µm apart. To achieve such a high resolution, the lens must operate near the diffraction limit.
- Working with multiple ions, it is beneficial to have a diffraction limited field of view in the order of 100 µm. A large field of view has the additional advantage of being less susceptible to imperfect alignment.
- Aberration due to the view port need to be corrected
- Photons are detected with a CMOS sensor with a 2.4×2.4 µm pixel size. This sets a lower bound on the magnification of the objective lens.
- To keep the price low, the lens is built entirely from commercially available singlet lenses.

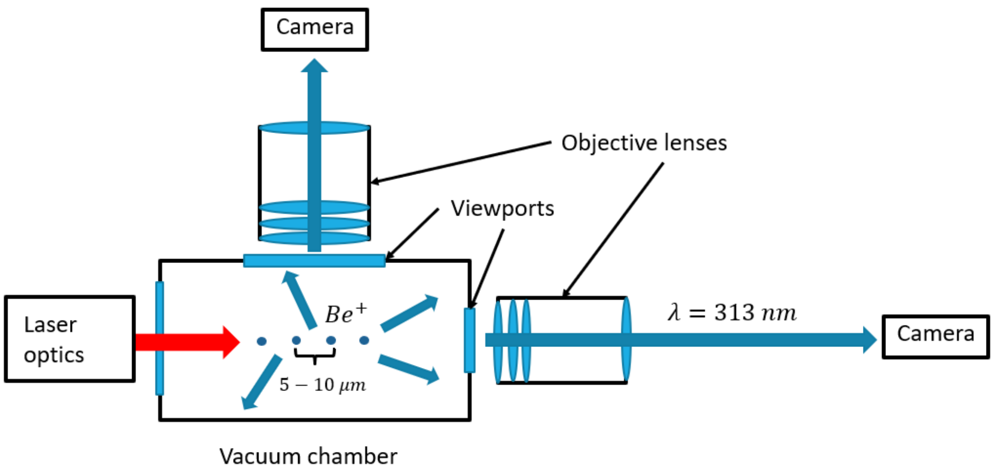

Figure 1: Typical trapped ion set up. The job of the lens is to collect the light emitted by the beryllium atoms and focus it onto a camera.

Our chamber (figure 1) has two view ports that can be used for imaging. They differ in size and window thickness. The window of the large view port has a thickness of 6.4 mm and diameter of 98 mm. The smaller view port has a window thickness of 3.3 mm and a diameter of 35.6 mm. The view ports are made from fused silica. The distance between the center of the vacuum chamber and the small view port is 97.6 mm while the distance between the center of the chamber and the large view port is 48 mm. Due to the geometry of the vacuum chamber, 2" (50.8 mm) optics is used to achieve a high NA. For the small view port, the maximally achievable NA is 0.17 and is limited by the window diameter. For the large view port, the diameter of the lens is the limiting factor of the NA. If the lens was in direct contact with the large view port window, one could in principle achieve an NA of 0.42. However, such a set up would be impractical and thus a small gap between window and lens is introduced at the cost of a slightly lower NA.

2 Aberrations and the Diffraction Limit

The behaviour of real lenses usually differs from ideal ones. Those deviations are called aberrations. They negatively affect the image quality of the lens. A perfect point source, for example, is generally not mapped to another point source. Although due to the wave nature of light, this is physically impossible anyway, aberrations usually contribute much more to this blurring effect than diffraction phenomena. Only if one carefully designs a lens to minimize aberrations, diffraction becomes the limiting factor of the image quality of the lens.

2.1 Diffraction Limit

Suppose for a moment we are dealing with a aberration free imaging system. Such a lens would map a point source to a diffraction pattern consisting of peaks and valleys in intensity. A common way to characterize the size of this diffraction pattern is the so called airy disk radius. This is the distance between the principal maxima of the interference pattern to its closest minima. The resolution of the imaging system can be related to the size of this disk, and is approximately given by the Abbé criterion

$$d \approx \frac{\lambda}{2NA} \tag{1}$$

where λ is the wavelength of light and NA the numerical aperture of the lens. In our set up, we are dealing with a wavelength of 313 nm and at least an NA of 0.17. Thus if we manage to reduce aberrations so far that their effects on image quality is small compared to effects from diffraction, we would expect to achieve resolution of 0.9 µm. This is enough to resolve individual ions in an experiment. So let's discuss some criteria to check whether a lens is working in the diffraction limited regime.

2.1.1 Spot Size

To decide whether a lens is working in the diffraction limited regime, we have to quantify the severity of aberrations in the system. A simple and intuitive way to achieve this is by determining the geometric spot size of lens. On a more technical level, the spot size essentially is a measure of transverse ray

aberrations. The interested reader can find a derivation of the relationship between transverse ray aberrations and the curvature of a (spherical) lens in the appendix. For this section, let's discuss how the spot size is computed numerically.
There are different way to characterize the geometric spot size of a lens. A useful one is the root mean square (rms) radius. To compute the rms-radius, one typically traces a finite number of rays (all emerging from the same point source) through the lens and calculates their intersection point with the image plane. For an on axis point source in an optical system with rotational symmetry around the optical axis, the rms-radius is then given by [4]

$$r_{RMS} = \sqrt{\sum_i r_i^2} \tag{2}$$

where r_i is the distance between the intersection point of the i-th ray with the image plane and the optical axis. We can than compare the rms-radius to the size of the airy disk. If the rms-radius is smaller than the airy disk radius, we consider the system to be diffraction limited [4].
Note that this method is based purely on geometrical optics. This is convenient because geometric rays can be efficiently traced numerically. Next, we discuss a method that considers some aspects of the wave nature of light.

2.1.2 Optical Path Length

The interference pattern in the image plane can be explained by wave front errors. We can think of the interference pattern as a superposition of plane waves that follow the geometric rays (this is an approximation though, linking wave optics to geometric optics. see eikonal equation). Destructive interference occurs when two rays differ in optical path length by an odd multiple of $\lambda/2$. Those differences in optical path length between different geometric rays are called wave front errors. To prevent destructive interference in the first place, one has to make sure that any two rays passing through the lens differ in optical path length by less than $\lambda/4$ [4] (the factor 1/4 is a bit arbitrary. this is a rough reference point, not a hard rule).
Like the spot size, optical path differences can be computed efficiently. Although it might seem that quantifying aberration by wave front errors is superior to spot size, it can be shown that wave front and transverse ray

aberrations differ only by a multiplicative factor (for spherical aberrations)[3, p. 115]. So one method is not necessarily better than the other.

2.1.3 Strehl Ratio

Finally, the Strehl ratio is a common way to express the quality of an imaging system. We will not give a technical definition here. However, we want to mention that is not based on approximations made by geometric optics. The Strehl ratio is a number ranging from 0 to 1. A Strehl ratio of 1 means that the imaging system is truly diffraction limited, i.e. has the highest physically possible image quality (given its size and working wavelength). A lens with a Strehl ratio larger than 0.8 is considered to be diffraction limited.
Although it is possible to optimize the lens by maximizing the Strehl ratio directly, it is computationally much more expensive than using methods based on geometric ray tracing (it involves computing an integral with a rapidly oscillating integrand [4]).

2.1.4 Field of View

All the above criteria depend on the position of the point source in the object plane. A lens might operate in the diffraction limit as long as the point source is sitting directly on the optical axis. Away from the optical axis however, the diffraction limit may no longer be satisfied. Thus the quality of the lens is also determined by the size of the area in the object plane, over which the diffraction limit is reached. The size of this area is called the field of view. For rotationally symmetric lenses, the field of view is circular and can be characterized by the diameter of the circle.
A large field of view is desirable because the object doesn't have to be placed precisely on the optical axis. This makes the set up easier. It also allows for larger (or more) objects to be imaged. Now that we have learned how to quantify aberrations, let's discuss how to minimize them.

2.2 Spherical Aberrations

Spherical lenses are inexpensive and available in a huge variety. Their drawback is that they exhibit spherical aberrations due to their non ideal surface shape. In this section, we discuss how to minimize spherical aberrations.

2.2.1 Lens Splitting

For a single lens, spherical aberrations are small, when the ratio of the aperture of the lens and its focal length is small (see appendix A). The focal length of a spherical lens scales linearly in the radius R of the surface curvature of the lens. Transverse ray aberrations (spot size) on the other hand scale with $1/R^2$. This means that splitting a single lens of given focal length into multiple lenses (while keeping the focal length constant) reduces the amount of aberrations in the system. In practice however, applying this technique alone requires to many lenses to reach the diffraction limit. There is a better way to do it.

2.2.2 Combining Convex and Concave Lenses

A more elegant way to minimize aberrations is the use of a combination of concave and convex lenses [1]. A convex (or converging) lens focusses light (coming from infinitely far away) while a concave lens makes light rays diverge. A convex lens is thicker in the center and gets slimmer towards the edges while a concave lens is thin in the center and gets thicker towards the edges. Due to their opposite geometry, combinations of convex and concave lenses can thus be used to minimize wave front errors. Combining this strategy with lens splitting, we found that we can design an objective low in aberration using fewer lenses than with the lens splitting strategy only. The disadvantage of this method is that concave lenses increase the focal length of the lens.
The essence of designing a diffraction limited lens from spherical ones is thus to pick a set of lenses that balances optical path lengths of rays passing through the lens. This can be challenging if the number of available lenses is small.

2.3 Window Aberrations

In addition to spherical aberrations, we also have to deal with aberrations due to the view port of the vacuum chamber. If we were to collimate the light emitted by the ions inside the vacuum chamber, this would not be an issue. This is because all light rays would hit the window at the same angle, which means that the extra optical path picked up by the rays due to the higher refractive index of the window is identical for each ray. In our case however, we collect the photons after they have passed the window. Depending on the angle a ray hits the glass with, it will pick up a different optical path length. Suppose a ray hits the window surface at an angle θ. Then the optical path travelled inside the window is given by

$$OPL = \frac{nd}{\cos(\arcsin(\frac{\sin(\theta)}{n}))} \tag{3}$$

where n is the refractive index of the glass and d the thickness of the window. For a 3 mm thick window made out of fused silica ($n = 1.46$), the optical path difference of a ray passing straight through the glass and one that hits the glass at an angle of 10 deg (corresponds to an NA = 0.17) is 31 µm. So even in the case of the small NA lens, the optical path difference due to the window is large compared to the working wavelength $\lambda = 313$ nm. This needs to be compensated.

Note that the view port acts similar to a concave lens, i.e. rays close to the optical axis pick up less optical path than marginal rays. Those wave front errors can be corrected using a suitable convex lens.

3 Lens Design

In this section we discus how we found the design of our objectives and minimized aberrations. Luckily, we didn't have to start from scratch because we found two promising designs we used as a starting point.

3.1 Existing Designs

The publication discussing the designs we were interested in are [1] and [2]. These objectives have been designed with single atom experiments in mind. They can be divided into a lens stack followed by a concave lens. The Alt lens [1] is based on a 3-lens lens stack while the other publication [2] includes a modified design based on a 4-lens lens stack. This allowed them to increase the NA of the objective.
Although these design seemed promising, they needed to be tweaked to work with our set up. First of all, those objectives have been optimized for a working wavelength of 852 nm. In this range of the electromagnetic spectrum, B7K glass is the most widely used material. At 313 nm however, lenses made from fused silica are more suitable due to the low transmission coefficient of B7K at this wavelength. Another factor when working with short wavelengths is that the diffraction limit is harder to reach. This is because the airy disk radius is proportional to the wavelength of the light and the geometric spot size should be smaller than the airy disk. Furthermore, due to the geometry of the vacuum chamber in existing designs, the lenses could be placed closer to the source. This allowed them to base their objective on 1" optics.
So overall we have two challenges. One is due to the shorter working wavelength and the other due to the fact that we have to work with 2" UV optics. Theoretically, this is a drawback because aberrations scale with the diameter of the lens cubed ([3, p. 115] and appendix A) and practically because we are more limited by the lens selection due to the market for 2" UV optics being smaller than for 1" B7K lenses.

3.2 Optimization

For the optimization process of our lens we used the commercial ray tracing software Zemax. The small view port objective is based on the Alt lens design, but using 2" fused silica optics instead of 1" B7K. Since the radii of

curvature and distances between lenses from were not directly transferable to our application, we needed to optimize them ourselves. In this section, we will give a rough overview of the optimization procedure. More Zemax specific details can be found in the appendix.
In a first step, we defined the variable parameters of the objectives. This included all the lens radii, the distance between window and lens stack, the distance between lens stack and concave lens as well as the distance between the concave lens and the image plane (see figure 2). To determine the magnification and maximize the field of view, we set up two fields (point sources of geometric rays): one on the optical axis, the other one 0.1 mm off axis. The optimization is based on Zemax's default wave front error minimization protocol. To ensure that the optimization yields designs that are compatible with our set up, we added the following constraints to the optimization process.

- Constrain the lens curvature to the interval $[-\frac{1}{R_{min}}, \frac{1}{R_{min}}]$ with $R_{min} =$ 30 mm. There are hardly any 2" lenses with larger curvature on the market.

- put a lower limit on the distance between first lens and the view port. This way, the objective lens can not directly touch the view port.

- put an upper limit on both the gap between the lens stack and the convex lens as well as the distance between the convex lens and the image plane. This ensures that the total length of the set up is kept reasonably short.

- set a target NA

- set a target magnification

We want to point out that some of the targets can be replaced by different ones without significantly altering the outcome of the optimization. Instead of magnification, one could also target a specific focal length. Instead of targeting a specific NA, one could also pick a set of variables that can not alter the NA. We have experimented with different sets of constraints and got good results with many of them.
To translate the optimized designs into the real world, we needed to find commercially available stock lenses that match the ideal lens shapes as closely

as possible. We achieved this by successively fixing one lens after another to the closest catalogue value we could find and re-optimized the remaining variables. We started with the lens type that had the smallest selection available. Proceeding until all lens shapes were fixed, we finally optimized the distances between the lenses. It took a few iterations to end up with the final designs. In each iteration we switched up the order of fixing the lens to catalogue value or tried out second closest lens matches.
The reader might wonder why we didn't optimize the distances between the lenses in the lens stack itself. In fact, we tried and came to the conclusion that there exist many local optima that yield practically the identical performance. We decided to keep those distances fixed and only fine tune them in the very last step when all lens shapes are already fixed.

4 Results

4.1 NA = 0.17 Objective

This objective lens (figure 2) is supposed to be used with the smaller view port. It consists of a convex three-lens stack and one concave lens to balance wave front aberrations. The distances and radii of the lenses are shown in table 1. The lens has an NA of 0.17, which is limited by the diameter of the view port. Without the view port, one could potentially achieve an NA of 0.21 (focus at infinity). Its airy disk radius is 8.2 µm, while the rms spot radius is 0.3 µm. The maximal optical path difference over the whole entrance pupil lies below $\lambda/100$ and its on axis Strehl ratio is 1.0. Thus, the lens satisfies all three diffraction limit criteria discussed above. The field of view is 0.57 mm (figure 4). That should be large enough for multi ion experiments (ion distances of 5-10 µm).
The focal length of the objective is 91 mm. In our set up, this leads to a magnification of -7.3 (the minus sign suggests that the image is flipped). Assuming the ions are separated by a distance of 5 µm, the spots on the CMOS sensor are separated by 36.5 µm. With a pixel size of 2.4 x 2.4 µm, two spots will be separated (peak to peak) by 15 pixels, while the spot size will cover around $9 = 3 \times 3$ to $16 = 4 \times 4$ pixels. This means that we should be able to resolve individual ions on our camera. A image simulation (pixel size is taken into account) is shown in figure 5.
Although this lens was optimized for a window thickness of 3.3 mm, the design can be adapted to work with various window thickness ranging from 1 mm up to 12 mm by simply changing the distance between the lens stack and the plano-concave lens.

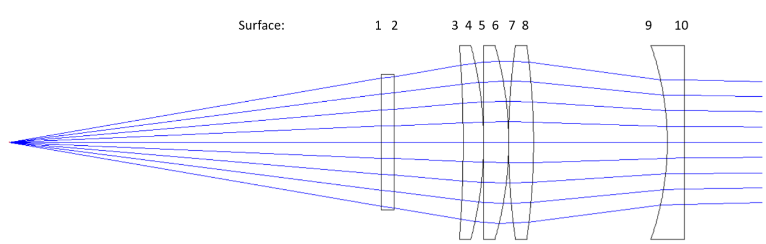

Figure 2: Objective lens for the smaller view port

4.2 NA = 0.35 Objective

This objective lens is supposed to be used with the larger view port. Here, the diameter of the window is no longer a limiting factor. Since this view port is closer to the trap center ($\approx$ 5 cm), we require a shorter focal length compared to the small view port lens. Adapting the NA = 0.17 lens design was not successful because we we're not able to decrease its focal length while keeping aberrations low enough. Adding an additional lens to the lens stack solved the issue. In the configuration shown in table 2, the lens has an NA of 0.35. Its airy disk radius is 4.1 μm, while its rms spot size is 0.9 μm. The optical path differences are below $\lambda/10$ and its on axis Strehl ratio is 0.99. As with the small view port lens, all three diffraction limit criteria are satisfied. It's field of view is 0.42 mm 4, which also should be enough for multi ion experiments.
The effective focal length of the lens is 53.7 mm. With the focus at infinity, the NA is 0.38. The magnification in our set up is -7.6. This is enough to resolve multiple ions on our CMOS camera (figure 5).
By adjusting the distance between the lens stack and the last lens, the objective can be tuned to a window thickness ranging from 1 to 12 mm.

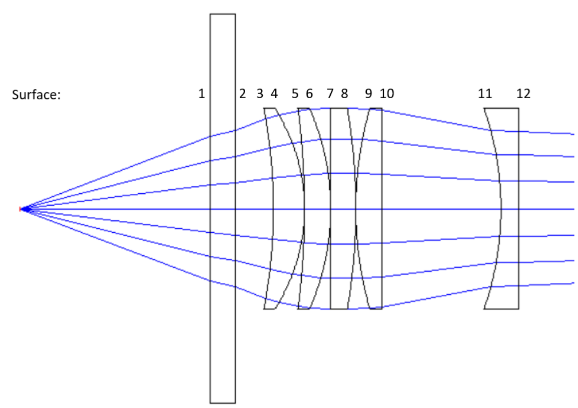

Figure 3: Objective lens for the large view port

4.3 Cost and Assembly

The lens cost of the objectives are approximately 1'200 EUR (NA = 0.17) and 1'400 EUR (NA = 0.35). The lenses are mounted in a lens tube from

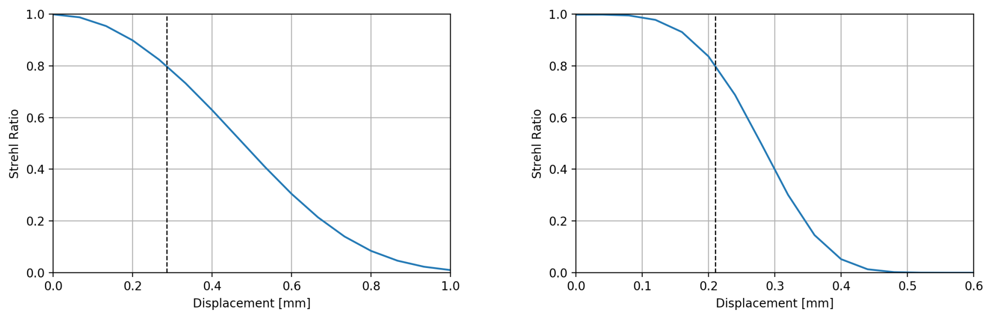

Figure 4: Strehl ratio vs point source off axis displacement. The diffraction limited field of view (Strehl ratio > 0.8) is 0.57 mm (left: NA = 0.17 lens) and 0.42 mm (right: NA = 0.35 lens)

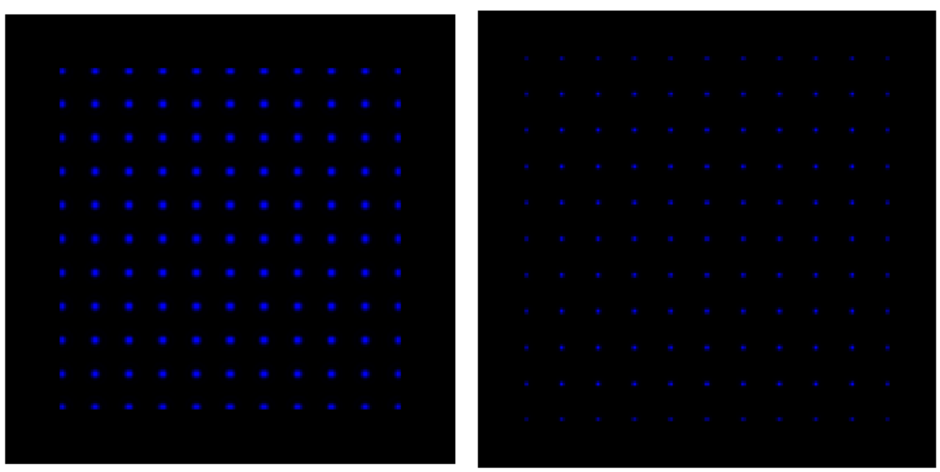

Figure 5: Image simulation of grid of dots (left: NA = 0.17 lens, right: NA = 0.35 lens). Nearest neighbours distance (in object plane) is 5 µm, similar to trapped ion experiments. The image size is 200 x 200 pixels with a pixel size of 2.4 x 2.4 µm

Thorlabs. Our workshop manufactured custom made spacing rings that keep the lenses in place. The parts have at least a manufacturing tolerance of 0.1 mm. Based on simulations, this should be sufficient.

Surface	Radius (mm)	Thickness (mm)	Material	Lens	Distributor
trap center	-	97.6	vacuum		
1	∞	3.3	fused silica	window	
2	∞	18.2	air		
3	-327.6	5.4	fused silica	LE4984-UV	Thorlabs
4	-97.6	0.0	air		
5	∞	6.6	fused silica	LA4984-UV	Thorlabs
6	-92.0	0.0	air		
7	183.0	6.5	fused silica	LB4842-UV	Thorlabs
8	-183.0	35.1	air		
9	-77.3	4.5	fused silica	PLCC-50.8-77.3-UV-248-355	CVI Laser Optics
10	∞	699.5	air		

Table 1: Lens data of NA = 0.17 objective

Surface	Curvature (mm)	Thickness (mm)	Material	Lens	Distributor
trap center	-	48.0	vacuum		
1	∞	6.4	fused silica	window	
2	∞	9.6	air		
3	-135.3	7.8	fused silica	LE4125-UV	Thorlabs
4	-46.5	0.0	air		
5	-193.4	6.6	fused silica	LE4560-UV	Thorlabs
6	-63.0	0.0	air		
7	∞	6.4	fused silica	PLCX-50.8-	CVI Laser Optics
8	-149.9	0.0	air	149.9-UV-248-355	
9	92.0	6.6	fused silica	LA4984-UV	Thorlabs
10	∞	30.0	air		
11	-77.3	4.5	fused silica	PLCC-50.8-	CVI Laser Optics
12	∞	420.3	air	77.3-UV-248-355	

Table 2: Lens data of NA = 0.35 objective

5 Discussion

We successfully designed two diffraction limited objective lenses built entirely from commercially available spherical singlet lenses. The respective prices of the objectives are 1'200 EUR (NA=0.17) and 1'400 EUR (NA=0.35). Based on the results from simulations, we conclude that the performance of our objective lenses is comparable to the performance of existing designs that already have been tested in single atom experiments. Due to the ongoing global pandemic however, we were not able to test the objectives in the lab yet.

We are aware that the distance between lens and image is rather large (70 cm for the smaller, and 42 cm for the larger NA lens). Decreasing this distance requires decreasing the focal length, which in turn requires using higher curvature lenses. This makes it challenging to keep aberrations low. If one doesn't have enough space for such a set up, one can use a mirror to deflect the light beam without drop in performance.

A practical challenge might be that imperfect alignment of the lens can negatively affect the performance of the lens. Based on simulations, the lens does still work in a diffraction limited regime when a small tilt of around 0.5 deg is present, but combining a tilt with a displacement from the optical axis can quickly degrade image quality. However, once the objectives are aligned close enough to their ideal orientation, one should be able to fine tune the alignment by using the shape of the dots on the CMOS sensor as a guide.

The biggest hurdle in designing a high NA, diffraction limited objective lens built from stock lenses is finding a well balanced set of lenses when the number of options available on the market is limited. If possible, we advise to use 1" optics for the design of a objective, since the market for 2" optics is significantly smaller. Alternatively, one might be successful by splitting up the imaging system into a collimating part used inside the vacuum chamber and a focussing part in front of the camera sensor. The flip side of this configuration is that it is less convenient to set up. In any case, we recommend browsing the lens catalogues of multiple manufacturers to find the closest match one needs.

A Spherical Aberrations

In this section we'd like to argue that (transverse) spherical aberrations are of order $O(R^{-2})$. Together with the fact that the focal length scales linearly in R, this explains why lens splitting reduces aberrations.
Suppose we are trying to focus a collimated beam of light using a plano convex lens (the planar side of the lens is facing the incoming light rays, see figure 6). To compute the transverse ray aberrations ϵ, we must first find the paraxial focal plane.

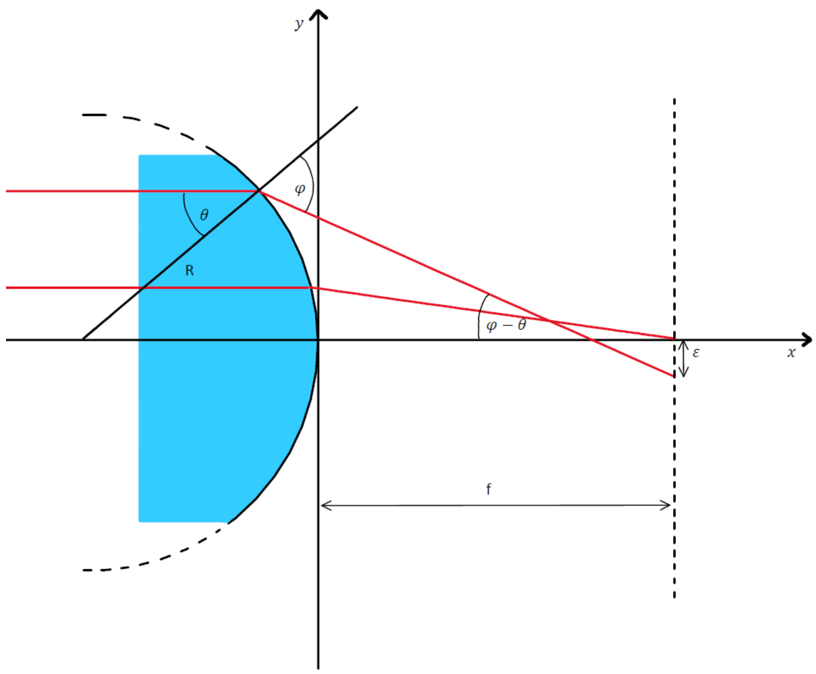

Figure 6: Transverse ray aberrations due to spherical lens surface

From Snell's law, we know that

$$\sin(\phi) = n\sin(\theta) = \frac{nh}{R} \tag{4}$$

where n is the refractive index of the glass (we assumed a refractive index of 1 outside the lens) and h the height at which the ray is hitting the spherical surface. Now let's determine the equation of the line segment $y = mx + q$

after the ray has left the lens. Let's also assume that $h/R \ll 1$. Snell's law then implies that $\theta, \phi \ll 1$. The slope of the line is given by

$$m = \tan(\theta - \phi) = \theta - \phi + O((h/R)^3) = -\frac{h}{R}(n-1) + O((h/R)^3) \qquad (5)$$

Next we need to determine the off set q of the line. The shape of the lens is given by $y^2 + (x+R)^2 = R^2$. For $y = h$, we find that

$$x = -R \pm \sqrt{R^2 - h^2} = -R \pm (R - \frac{h^2}{2R} + O((h/R)^3) \qquad (6)$$

We are interested in the intersection point closer to the y-axis, which thus lies approximately at $(-\frac{h^2}{2R}, h)$. Inserting this point and the previously found slope into the linear equation yields

$$h = -\frac{h(n-1)}{R}\left(-\frac{h^2}{2R}\right) + q \implies q = h - \frac{h^3(n-1)}{2R^2} \qquad (7)$$

Thus the line segment reads

$$y(x) = -\frac{h(n-1)}{R}x + h - \frac{h^3(1-n)}{2R^2} + O((h/R)^3) \qquad (8)$$

The paraxial focal length is defined by $y(x = f) = 0$ in the limit $h/R \to 0$, which yields $f = R/(n-1)$. This is consistent with the thin lens formula for $R_2 \to \infty$. Going back to $h/R \neq 0$, we find that the transverse ray aberrations are

$$\epsilon = y(x = f) = -\frac{h^3(n-1)}{2R^2} \in O(R^{-2}) \qquad (9)$$

which is what we wanted to show. Another important takeaway is that transverse aberrations scale with h^3/R^2. This means that increasing the diameter of the lens increases the geometric spot size even when keeping the ratio h/R constant.

B Optimization in Zemax

B.1 The Merit Function

Optimization in Zemax means minimizing the so called Merit function. The Merit function takes a set of variables (lens curvatures, distances between lenses, ...) and returns a real, non-negative number. The smaller the number returned by the merit function, the better the systems meets the targets. Finding a suitable lens design depends to a large part on how well the merit function is set up. Let's have a closer look at how the merit function is constructed.

B.2 Operands

The merit function is built from a list of operands. Each operand maps the chosen set of variables to a value V_i, characteristic for this operand. V_i might be the effective focal length, the distance between surface 1 and 2, the curvature of surface x, the optical path difference between the marginal ray and the center ray, etc ... . For each operand, one has to specify a target value T_i. The merit function is the weighted sum of the squared differences between current value V_i and target value T_i. Explicitly, we have

$$MF = \sqrt{\frac{\sum w_i (V_i - T_i)^2}{\sum w_i}} \tag{10}$$

where w_i is the weight of the operand.

Luckily, one doesn't have to build this list of operands from scratch. Zemax offers a variety of default merit function, which provide a good starting point for optimization. The two merit function we want to mention specifically are the "wavefront" and the "spot size" merit function. The "wavefront" merit function contains a list of operands for minimizing optical path differences while the "spot size" merit function minimizes transverse ray aberrations.

None of the default merit functions take properties like magnification, focal length or NA into account. Since those are often very relevant in practice, we strongly recommend adding operands to the default merit function. All the available operands can be found in Zemax's manual [4]). The complete list of operands can be a bit overwhelming. That's why we provide a compilation of the operands we found the most useful. For more details on how to work with these operands, consult the Zemax manual.

- MNCA/MXCA: minimal/maximal center air thickness. useful for limiting the allowed range of distances between individual lenses
- MNCG/MXCG: minimal/maximal center glass thickness. useful to limit the thickness of the lenses
- MNEG: minimal edge thickness in glass. useful if lens curvature is variable
- MNCV/MXCV: minimal/maximal curvature. useful to prevent high lens curvatures. for example set $MXCV = -MNCV = 1/|R_{min}|$
- TOTR: total track length. useful if one wants to keep the total length of the objective constant, but individual distances between lenses don't matter
- EFFL: effective focal length
- OBSN: object space numerical aperture. make sure that aperture type is set to "entrance pupil diameter" ("General -¿ Aperture") and set aperture value to the diameter of your objective. mark one of the surface from the entrance pupil with "make surface stop".
- PMAG/AMAG: paraxial/angular magnification. only works if you have at least two fields
- STRH: Strehl ratio, slows down optimization substantially due to high computational cost

  Certain operands only work properly when the system is set up accordingly (eg. OBSN and PMAG/AMAG). When adding operands, one has to make sure they don't conflict with one another. For example the magnification of the lens is a function of the focal length and the distance between object and lens. Depending on how one defines the targets of these operands, it might not be possible to achieve all of them simultaneously. A hint for conflicting operands is when the merit function consistently returns very high values. If this happens one can find the problematic operand by looking at the right most column in the merit function editor, which displays how much each operand contributes to the output of the merit function. One can then reconsider the choice of operands or change the weights of the operands.

B.3 Optimization algorithms

There are 3 different optimization algorithms available. We briefly mention how we used them. For more details we refer to the manual.

B.3.1 Local Optimization

This should be the most commonly used optimization algorithm. It is designed to find the next closest local minima of the merit function given an initial configuration. In the right column of the optimization window, one can see the initial merit function value. One row below, the current merit function value is displayed. We recommend using the option "infinite cycles" and have an eye on the current merit function value while the algorithm is running. Once the current merit function value has converged, stop the optimization and check the result. It can be helpful to use this algorithm with different initial configurations.

B.3.2 Hammer Optimization

This algorithm can be used to escape a local minimum of the merit function. It typically makes only minor adjustments to the design, so one should mainly use it if one already has a promising design, but want to fine tune it. It may require many optimization cycles until the algorithm finds a better solution. We had success running this over night.

B.3.3 Global Optimization

This algorithm can be used for creating a design idea from scratch. One simply needs to define the number of surfaces of the lens and make sure that the initial configuration properly traces the rays. After defining the variable of the system, the global optimization algorithm will create 10 new Zemax files, each representing a design suggestion. The merit function score of each design is displayed in the right column on the right. It can be difficult to get good designs with this algorithm. One way to increase the success of this method is to abort the algorithm before the designs reach very low merit function values.

References

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