Semester Project

Comparison of point spread functions of unobscured and obscured circular apertures

by

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Abstract

This semester project investigates the effect of partially obscuring a circular aperture by a circular obstruction at its center. The impulse response, also called point spread function, of the two systems is compared theoretically as well as experimentally.

Date of Submission: 26th of April 2018

ETH Zurich — Trapped Ion and Quantum Information

1 Introduction

For the purpose of isolating a small number of ions and selectively imaging them, the particles are usually confined in quadrupole ion traps, also called Paul traps. Unfortunately, the dynamic electromagnetic fields of such Paul traps result inevitably in some undesirable interactions between the electric fields of the trap and the charge of the ions. One consequence of this interaction is the so called radio-frequency (RF) driven motion [1].
Alternatively, trapping the ions using the standing electromagnetic wave of a laser combines the advantages of optical traps, like their scalable trapping geometries, with the benefits of trapping charged particles, like electronic addressability and coherent control.
In order to collect the light emitted by the ions inside the optical trap an aspherical lens is inserted into the cavity. A cylindrical hole must be cut into the lens along the axis of rotational symmetry in order to not interfere with the laser beam passing through it, see Figure 1.
This report aims to investigate the effect of using a cored lens for imaging purposes in contrast to an uncored lens, as dictated by the geometry of the experimental setup. Numerical calculations as well as experimental observations are taken into account to compare the two systems.

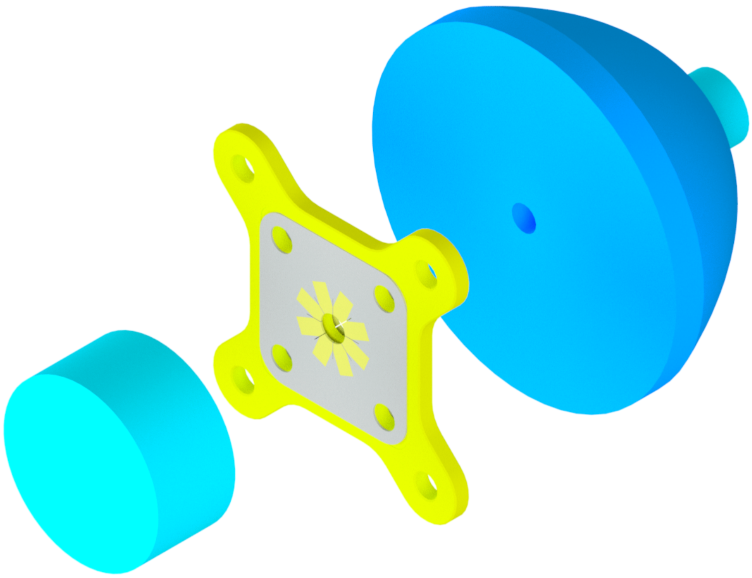

Figure 1: Illustration of the optical trap setup: Optical cavity in light blue, cored aspherical lens in darker blue and electronic component for ionic confinement in yellow. Lens manufactured by Edmund Optic #67-270 with plateau of 6 mm diameter on its convex side, a 2 mm wide hole through its center, at a distance of 15 mm to the ion trap.

2 Mathematical Description

Any imaging system can be described by an object-plane, an image-plane and an impulse response function, which uniquely defines the relationship between object- and image-plane. Due to the linearity property of optical imaging systems, the image of an object can be computed by splitting the object-plane into discrete point objects of varying intensity and then convolving these points with the system's point spread function, see Figure 2 for an illustration. Mathematically, with the optical axis of the system along the z-direction, object-plane described by $f(x,y)$ and point spread function by $h(x,y)$, the image-plane $g(x,y)$ if given by [2]:

$$g(x,y) = h(x,y) * f(x,y) \tag{1}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x-x', y-y') f(x', y') dx' dy' \tag{2}$$

For a uniformly illuminated circular aperture, the point spread function is also known as the Airy disk, depicted in the case of a real laser beam in Figure 2. The intensity pattern of the Airy disk can be described by the squared modulus of the Fourier transform of the circular aperture using the properties of the Bessel functions, here J_1 is the Bessel function of first kind and of order one:

$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 = I_0 \left(\frac{2J_1(x)}{x} \right)^2 \tag{3}$$

where I_0 is the maximum intensity, $k = 2\pi/\lambda$ is the wave number, a is the radius of the aperture and θ is the angle of observation with respect to the optical axis.

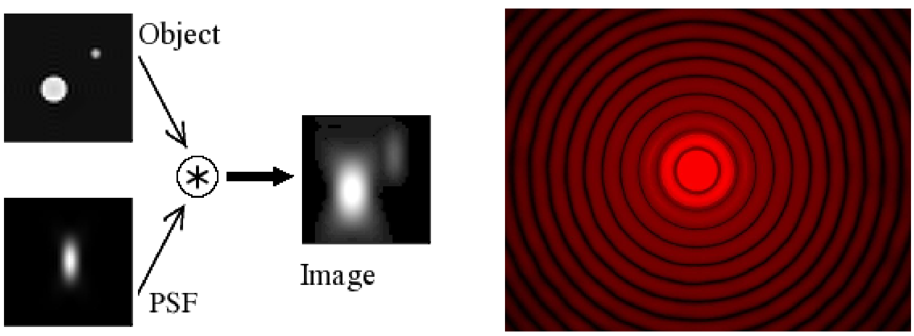

Figure 2: On the left, qualitative illustration of the resulting image created by convolution of the object plane with a PSF. On the right, real Airy disk created by a red laser beam.

2.1 Obscured Airy Disk

Similarly to the derivation of the unobscured Airy disk, the obscured point spread function is found by means of a Fourier transform which excludes the obscured region. With $0 \leq \epsilon \leq 1$ the obscuration ratio, that is the ratio of the diameter of the obscuring disk and the diameter of the aperture, the obscured Airy pattern is given by [3]:

$$I(x) = I_0 \left(\frac{2J_1(x)}{x} - \frac{2\epsilon J_1(\epsilon x)}{x} \right)^2 \tag{4}$$

where again I_0 is the maximum intensity, $x = ka \sin \theta$ represents a spacial coordinate, k is the wavenumber, a is the radius of the aperture and θ is the angle of observation with respect to the optical axis.

By obscuring a circular region at the center of the aperture the point spread function becomes shallower, while the ring pattern becomes slightly more pronounced, as can be seen in Figure 3. In other words, for an obscured aperture the resulting image is expected to be slightly dimmer and slightly less sharp compared to the unobscured aperture. The effect can also be thought of as a low pass optical filter, in opposition to a high pass filter, which is a commonly applied image sharpening technique in image processing tools.

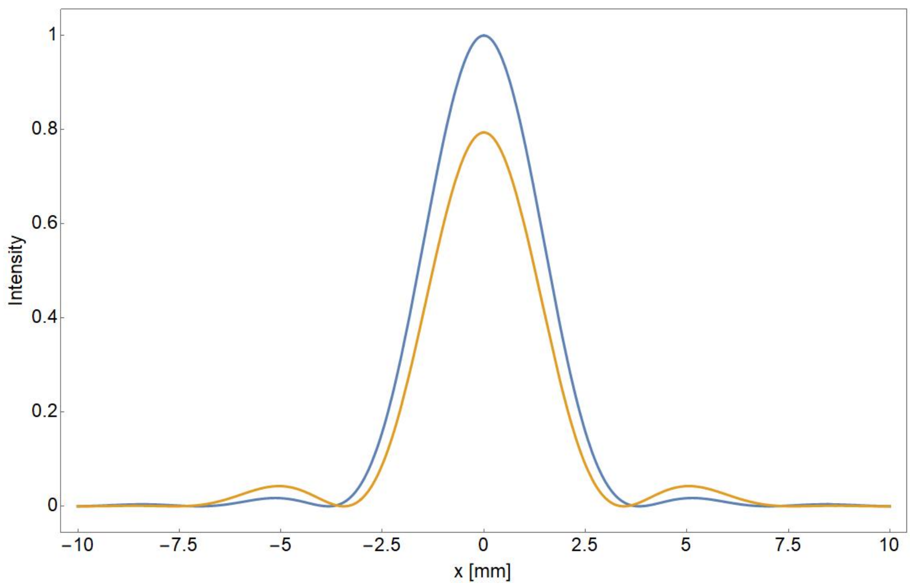

Figure 3: Graphical plot of the intensity pattern of the unobscured Airy disk (blue) and the obscured pattern (orange) for obscuration ratio of $\epsilon = 0.33$

2.2 Numerical Simulation

The uncored as well as the cored lens have been simulated using the ray tracing software OpticStudio by Zemax [4]. Cross-sectional point spread functions were calculated for both systems and the results have been plotted in Figure 4. The parameters used for the simulation can be found in Table 1.
In contrast to the theoretical prediction, which described quickly decaying rings, one observes a more irregular ring pattern. This was to be expected since the theoretical prediction is based on an idealized lens. Furthermore, the point spread function of the cored lens appears to not confirm the theoretical prediction of a more pronounced ring pattern in comparison to the PSF of the uncored lens. Overall the difference between the two systems is quite small, therefore it doesn't provide strong support against the hypothesis of a less sharp image.

Substrate	Fused Silica
Coating	UV (250-450nm)
Geometry	Aspherical
Diameter [mm]	25.0
Effective Focal Length [mm]	20.0
Back Focal Length [mm]	10.4
Center Thickness [mm]	14.00
f-number	0.8

Table 1: Lens parameters used in Zemax Software to calculate cross-sectional point spread functions for aspherical lenses.

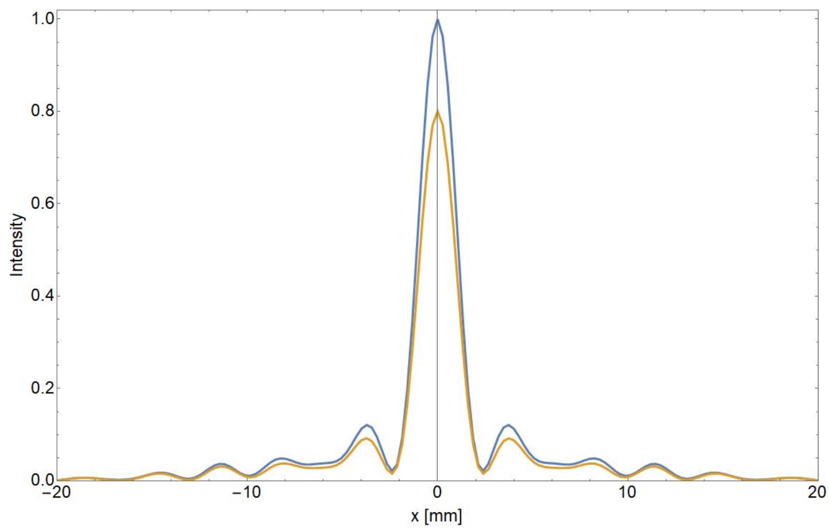

Figure 4: Zemax computation of cross-sectional point spread functions for aspherical uncored lens in blue and for the cored lens in orange.

3 Experimental Verification

In order to compare between the two imaging systems a USAF resolution target, see Figure 6, was set up according to the schematics in Figure 5. The mirrors M1 through M4 were introduced to better control the beam path, while the target was positioned at or close to the focal point of the lens.
For both the cored as well as the uncored lens several images of the resolution target have been taken, a selection of which can be found in Figure 7. Qualitatively, one can clearly observe how the cored lens produces images that are less sharp and dimmer compared to the uncored lens.
These observations are in agreement with the theoretical prediction as well as the numerical simulation.

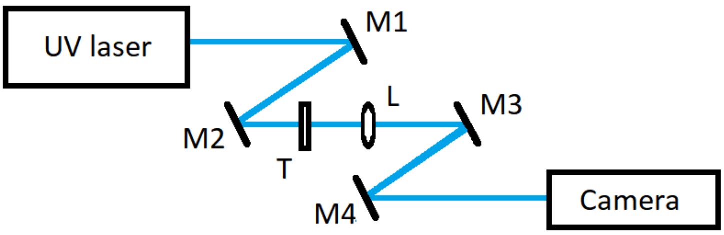

Figure 5: Imaging Setup: Mirrors marked with M, lens with L and target with T.

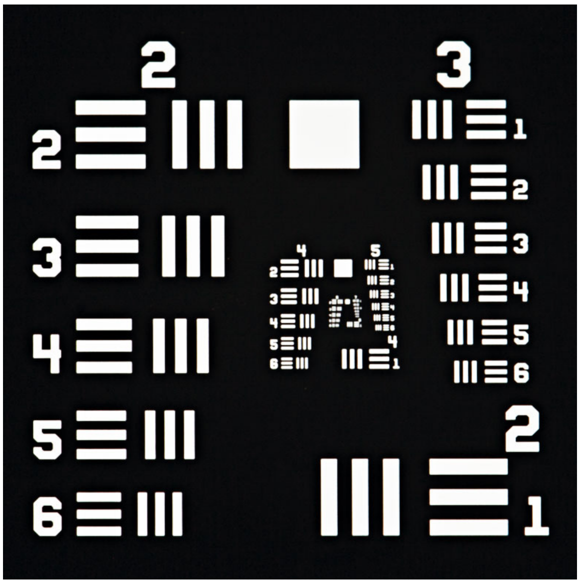

Figure 6: US Air Force resolution target used for comparison of the imaging characteristics between cored and uncored lenses.

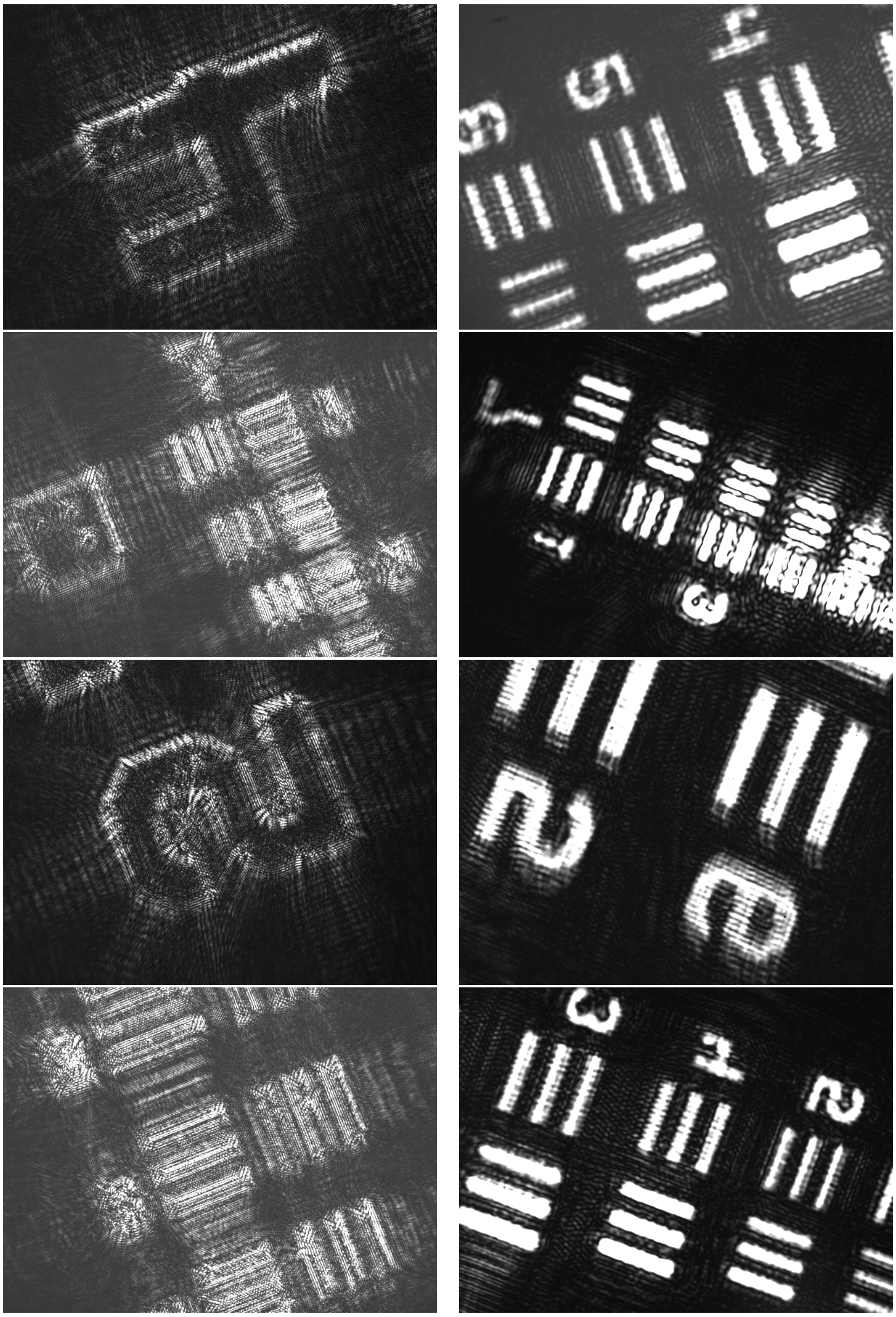

Figure 7: Images taken using a cored lens (left side) compared to images taken using an uncored lens (right side). Qualitatively, the images from the cored lens are blurred, while the edges remain clearly defined. This can be understood as a low pass filter.

4 Conclusions

The cored and uncored lenses have been examined and compared using numerical simulations with a ray-tracing program as well as experimentally by imaging a resolution target. The numerical simulations demonstrate that the point spread functions differ in some regards between the mathematical aperture and the real lenses, e.g. the off-center maxima of the Airy pattern, as seen in Figure 2, were predicted to decay more quickly according to the naive mathematical model. The central prediction for the obscured system, which states that the off-center maxima of the point spread function increase relative to the intensity at the center of the PSF, was not confirmed by numerical simulations, as seen in Figure 4. On the other hand, the predicted effect of blurring the image as a result of obscuring the central region of the lens was observed experimentally by imaging a standard resolution target. As described in the theoretical description, this effect can be understood as a low pass filter and observed by noticing the presence of sharp edges in the images of the cored lens and the absence of light from the larger structures, for example the centers of the numbers 4 and 2 in Figure 7 which appear hollow.
In regard to the imaging of ions, the low pass filter effect should not negatively impact the lenses ability to collect light from the minuscule point source that is an ion.

References

[1] H Landa et al. Modes of oscillation in radiofrequency Paul traps. *New Journal of Physics*, September 2012.

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[4] Zemax. Opticstudio. `https://www.zemax.com/opticstudio`.