High-Field Laser Physics

ETH Zürich

Spring Semester 2010

H. R. Reiss

OUTLINE

I. STRONG-FIELD ENVIRONMENT
   A. Atomic units
   B. The intensity-frequency diagram
   C. Inadequacy of the electric field as an intensity measure
   D. Failure of perturbation theory and its consequences
   E. Measures of intensity
   F. Upper and lower frequency limits on the dipole approximation

II. S MATRIX
   A. Physically motivated derivation of the S matrix
   B. Perturbation theory from the S matrix
   C. Nonperturbative analytical approximations
   D. The $A^2(t)$ term; classical-quantum difference

III. NONPERTURBATIVE METHODS
   A. Tunneling
   B. Strong-field approximation

IV. BASIC ELECTRODYNAMICS
   A. Fundamental Lorentz invariants: essential field identifiers
   B. Longitudinal and transverse fields
   C. Lorentz force
   D. Gauges and their special importance in strong fields

OUTLINE (continued)

V. APPLICATIONS
   A. Rates
   B. Spectra
   C. Momentum distributions
   D. Angular distributions
   E. Higher harmonic generation (HHG)
   F. Short pulses
VI. EFFECTS OF POLARIZATION
VII. STABILIZATION
VIII. RELATIVISTIC EFFECTS
   A. Relativistic notation
   B. Dirac equation and Dirac matrices
   C. Negative energy solutions
   D. Relativistic quantum mechanics (RQM) vs. relativistic quantum field theory (RQFT)
   E. Relativistic rates and spectra
IX. OVERVIEW
   A. Very low frequencies
   B. Compendium of misconceptions

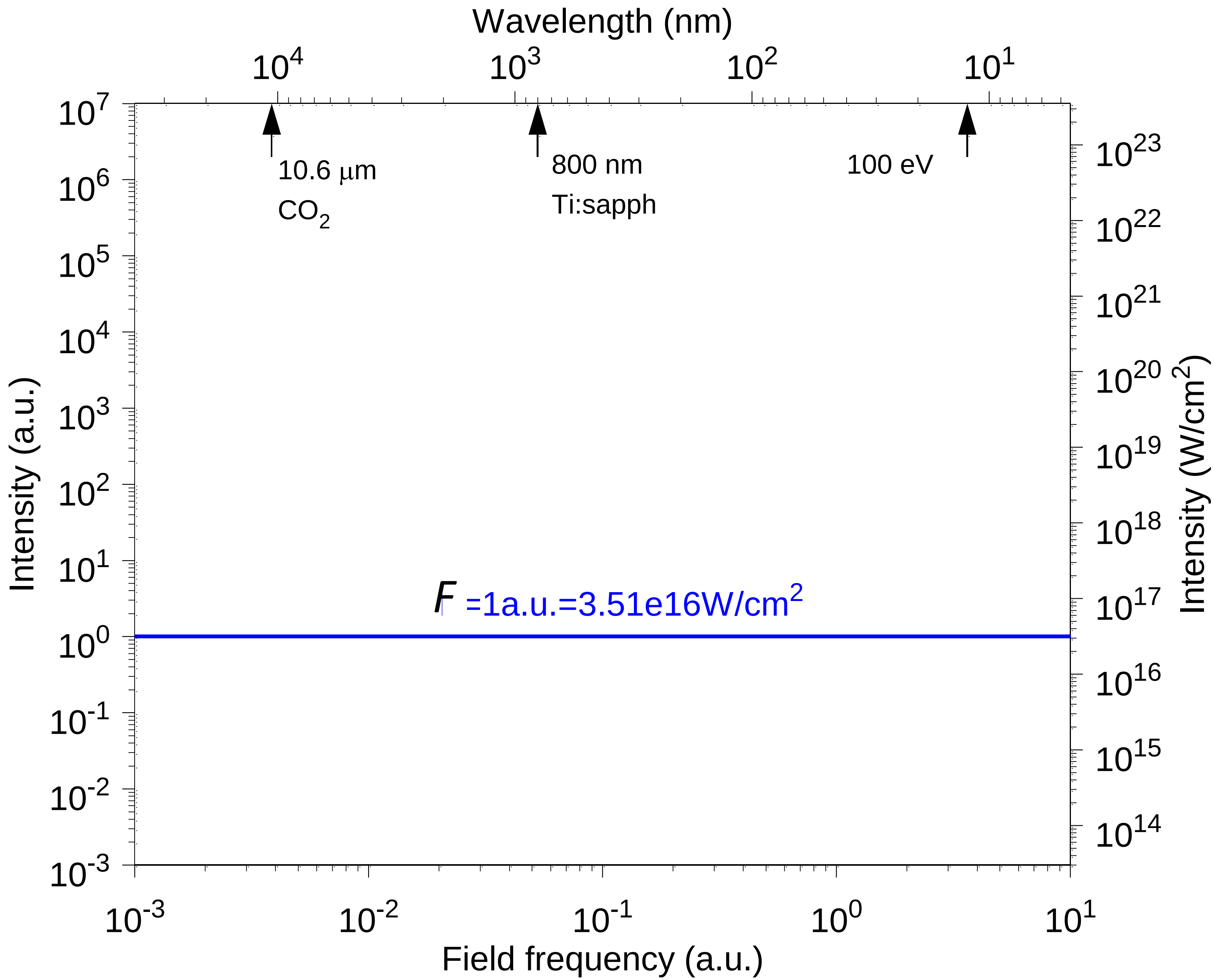
INTENSITY-FREQUENCY DIAGRAM

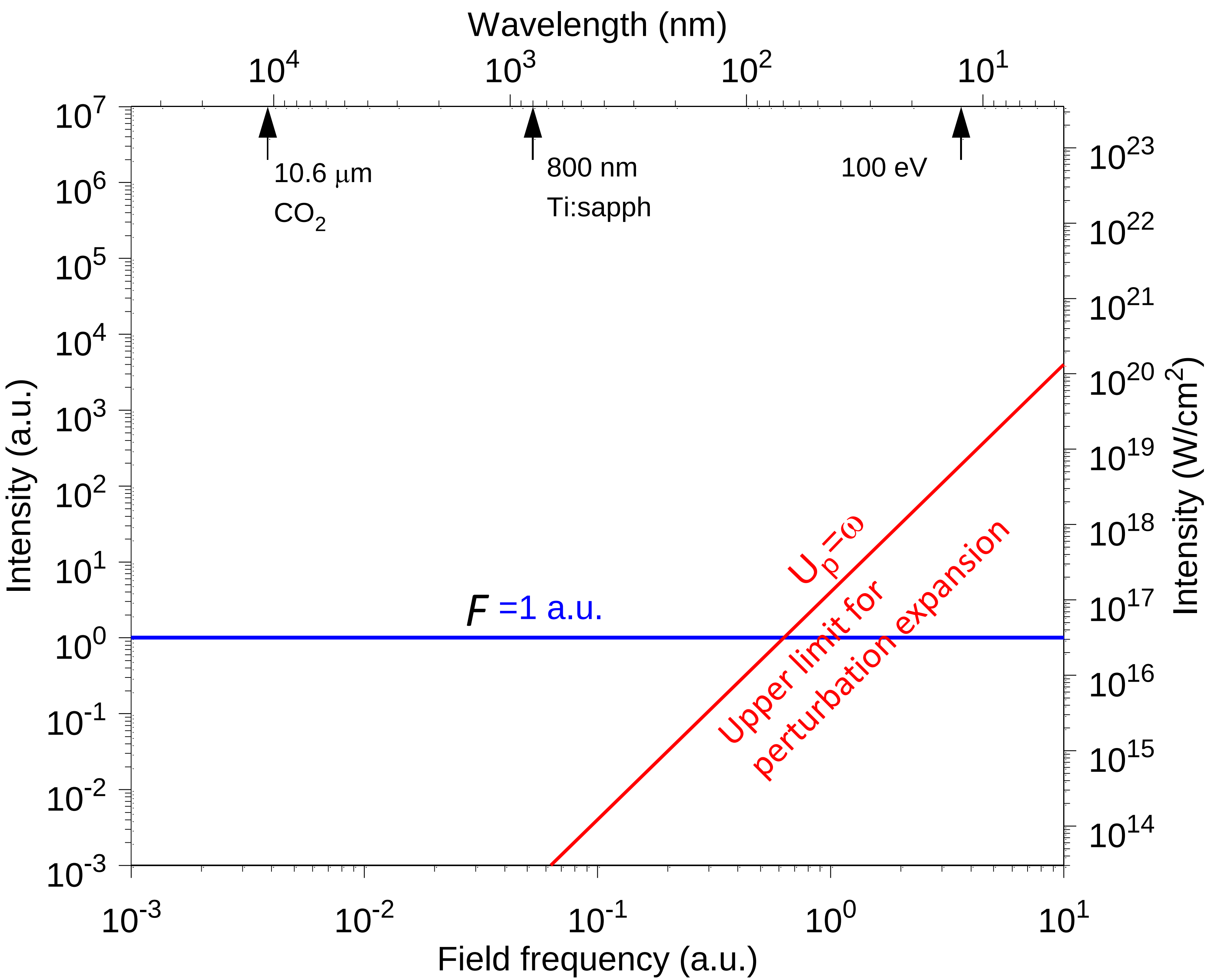
An intensity-frequency diagram is very informative; it will be used repeatedly.

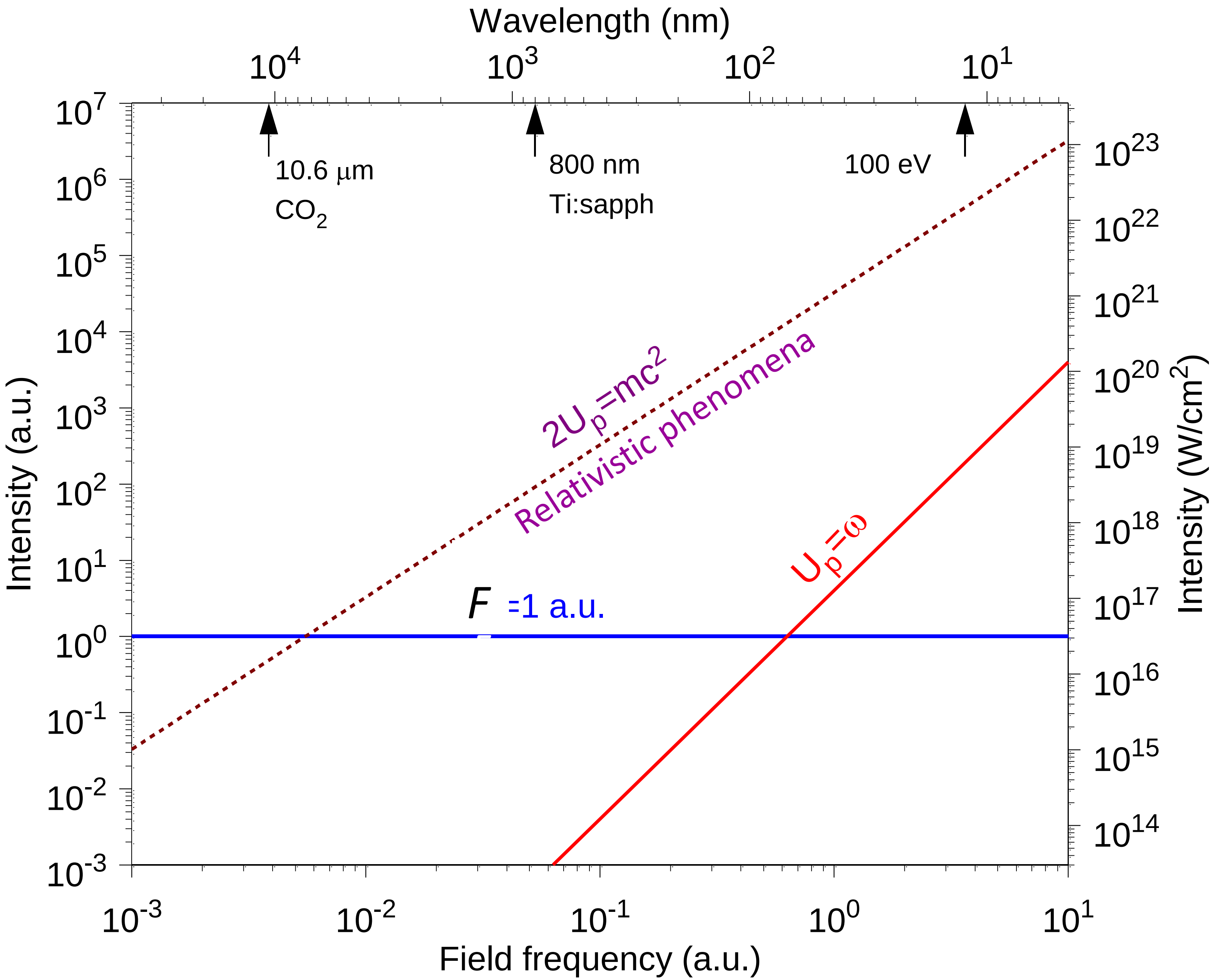
Some of the axis units are in a.u.
More complete details on atomic units will follow.
For now:

- 1 a.u. intensity = 3.51×10^{16} W/cm^2 $\Rightarrow$ $|\boldsymbol{F}| = 1$ a.u. $\Rightarrow$ applied electric field = Coulomb field at 1 a.u. from a Coulomb center.
- 1 a.u. length = a_0 = Bohr radius
- 1 a.u. frequency = 1 a.u. energy = $2R_\infty$ = 27.2 eV $\Rightarrow$ λ = 45.56 nm

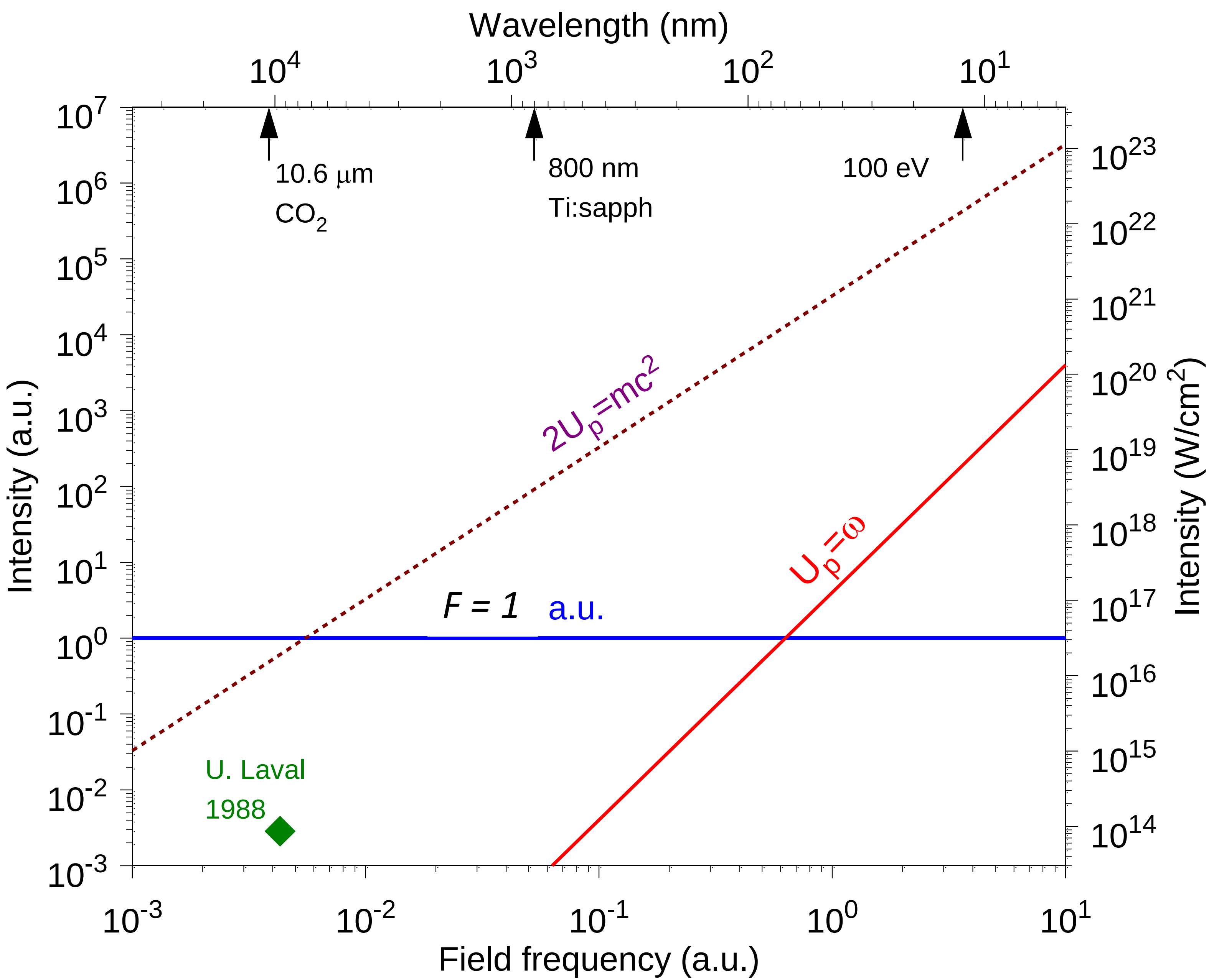
NOTE: The electric field will be designated by ***F*** rather than ***E*** to avoid confusion with the energy *E*.


Wavelength (nm)
10^4
10^3
10^2
10^1
10.6 μm
CO_2
800 nm
Ti:sapph
100 eV
Intensity (a.u.)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
Intensity (W/cm^2)
10^{23}
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
F =1a.u.=3.51e16W/cm^2
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
Field frequency (a.u.)


Wavelength (nm)
10.6 μm
CO2
800 nm
Ti:sapph
100 eV
F =1 a.u.
Up=ω
Upper limit for
perturbation expansion
Intensity (a.u.)
Intensity (W/cm2)
Field frequency (a.u.)



Wavelength (nm)
10^4
10^3
10^2
10^1
10.6 μm
CO_2
800 nm
Ti:sapph
100 eV
$2U_p=mc^2$
Relativistic phenomena
$U_p=\omega$
F =1 a.u.
Intensity (a.u.)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
Intensity (W/cm^2)
10^{23}
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
Field frequency (a.u.)


Wavelength (nm)
10^4
10^3
10^2
10^1
10.6 μm
CO2
800 nm
Ti:sapph
100 eV
2Up=mc2
Up=ω
F = 1 a.u.
U. Laval
1988
Intensity (a.u.)
Intensity (W/cm2)
Field frequency (a.u.)
10^-3
10^-2
10^-1
10^0
10^1

THE ELECTRIC FIELD IS NOT A GOOD INTENSITY MEASURE

The experiments at Université Laval (Canada) in 1988 used a CO_2 laser at 10.6 μm to ionize Xe at a peak intensity of 10^{14} W/cm^2.

9000 photons were required just to supply the ponderomotive energy U_p.

This was a very-strong-field experiment.

The maximum electric field was only $|\boldsymbol{F}| = 0.05$ a.u.

Notice also that relativistic effects can be reached at low frequencies and small electric fields.

WHY IS THE INTENSITY LIMIT ON PERTURBATION THEORY SO IMPORTANT?

When using any power series expansion, it may not be obvious from qualitative behavior that the radius of convergence has been surpassed.

However, when the expansion is used beyond the radius of convergence, the result may be drastically in error.

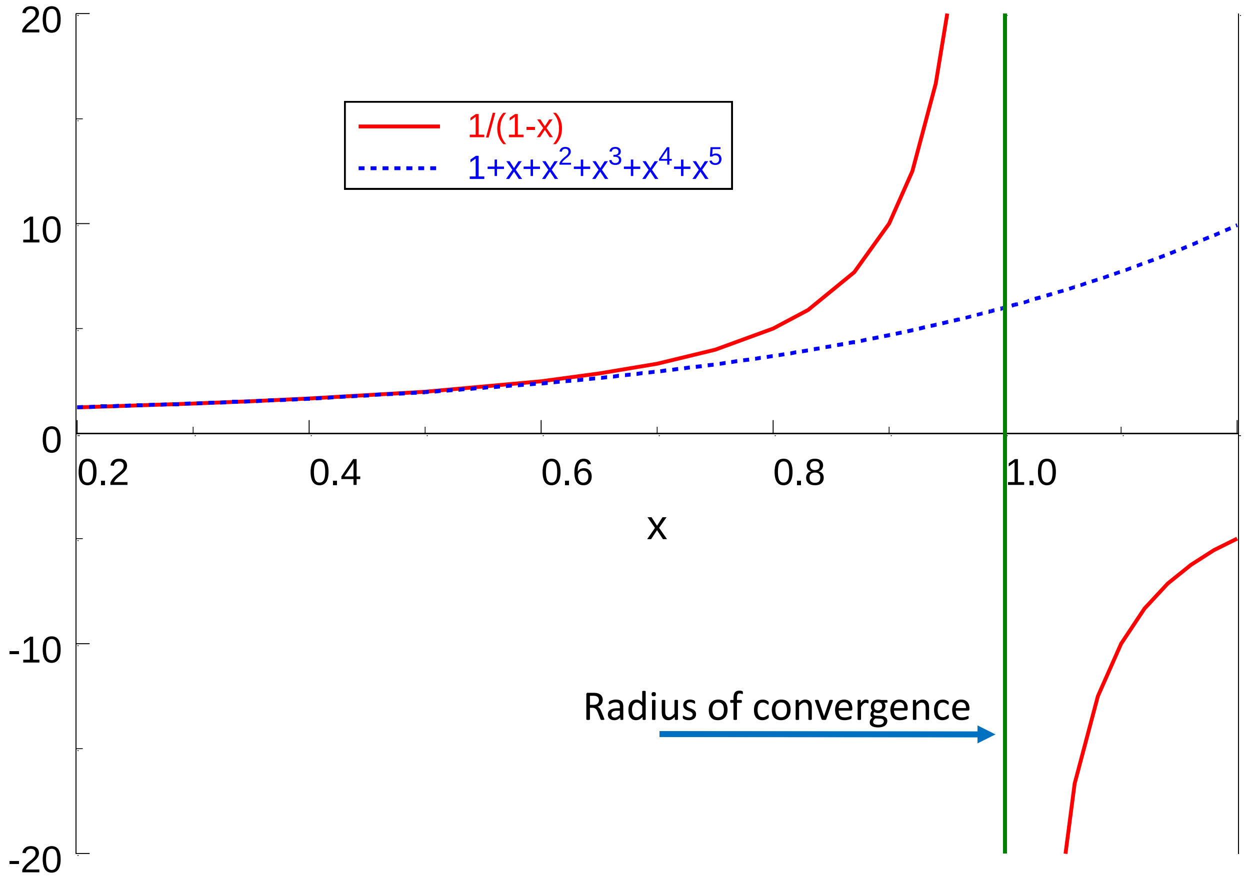
A simple example can illustrate this.
Consider the geometric series:

$$F(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$
$$|x| < 1$$

To simplify matters, suppose x is positive: x> 0.

1/(1-x) changes sign when x > 1, but the expansion is a sum of positive terms!

Perturbation theory (PT) in quantum mechanics does not fail so spectacularly, but PT can be completely misleading.


20
10
0
-10
-20
0.2
0.4
0.6
0.8
1.0
x
1/(1-x)
1+x+x2+x3+x4+x5
Radius of convergence

ATOMIC UNITS

Define 3 basic quantities to be of unit value. This will set values for e^2, $\hbar$, m.

<u>*Unit of length:*</u> $$a_0 = \frac{\hbar^2}{me^2} = 1\,a.u. = 5.29177249 \times 10^{-11}\,m$$

<u>*Unit of velocity:*</u> Set $$\frac{mv_0^2}{2} = R_\infty = \frac{me^4}{2\hbar^2} \Rightarrow v_0 = \frac{e^2}{\hbar} = 1\,a.u. = 2.18769142 \times 10^6\,\frac{m}{s}$$

<u>*Unit of time:*</u> Set $$\tau_0 = \frac{a_0}{v_0} = \frac{\hbar^2}{me^2} \cdot \frac{\hbar}{e^2} = \frac{\hbar^3}{me^4} = 1\,a.u. = 2.41888433 \times 10^{-17}\,s$$

These 3 definitions give: $\hbar^2 = me^2$, $\hbar = e^2$, $\hbar^3 = me^4$

The only solution is: $\hbar = 1$, $m = 1$, $e^2 = 1$

(Note: some of the choices, such as $mv_0^2/2 = R_\infty$ were selected to give $\hbar = 1$, $m = 1$, $e^2 = 1$.)

Derived quantities:

Unit of energy:

$$E_0 = 2R_\infty = \frac{me^4}{\hbar^2} = 1\ a.u. = 27.2113962\ eV$$

Unit of frequency:

$$\nu_0 = \frac{1}{\tau_0} = 1\ a.u. = \frac{me^4}{\hbar^3} = 4.13413732 \times 10^{16}\ Hz$$

Unit of electric field strength:

$$F_0 = \frac{2R_\infty}{ea_0} = \frac{me^4}{\hbar^2}\cdot\frac{1}{e}\cdot\frac{mc^2}{\hbar^2} = \left(\frac{e^2}{\hbar c}\right)^3\cdot\frac{mc^2}{e}\cdot\frac{mc}{\hbar}$$

$$= (dimensionless)^3\left(\frac{electron\ volts}{e}\right)\cdot\left(\frac{1}{length}\right) = \frac{volts}{distance}$$

$$= 1\ a.u. = 5.14220826 \times 10^{11}\ V/m$$

Unit of energy flux:

$$I_0 = \frac{c}{8\pi}F_0^2 = \frac{\alpha^5}{8\pi}\cdot\frac{(mc^2)^2}{\hbar}\cdot\frac{1}{(\lambda_e/2\pi)^2}$$

$$since\ F_0 \equiv 1\ a.u., \quad then\ I_0 = \frac{c}{8\pi}\ a.u.$$

$$I_{a.u.} = \frac{I}{I_0} = 8\pi\alpha I \Rightarrow I = \frac{I_{a.u.}}{8\pi\alpha}$$

ATOMIC UNITS, continued 2

Convenient conversions:

Wavelength to frequency: $$\omega(a.u.) = \frac{45.5633526}{\lambda(nm)}$$

example: $800\ nm \Rightarrow \omega\ (a.u.) = 45.5633526 / 800 = 0.0570\ a.u.$

Atomic units and *mc^2 units:*

$$1\ a.u. = 5.32513625 \times 10^{-5}\ mc^2$$

$$1\ mc^2\ unit = 1.87788622 \times 10^{4}\ a.u.$$

PONDEROMOTIVE ENERGY

The electric field strength is not a useful indicator of the strength of a plane-wave field. (However, it is the only strength indicator for a quasistatic electric (QSE) field. Much more to be said later.)

The limit for perturbation theory: $U_p < \omega$ $\qquad$ *U_p = ponderomotive energy*
Index for relativistic behavior: $U_p = O(mc^2)$

Ponderomotive energy is a useful guide to the coupling between an electron and a plane-wave field.

An electron in a plane-wave field oscillates ("quivers", "wiggles", "jitters") in consequence of the electromagnetic coupling between the field and the electron. *The energy of this interaction in the frame of reference in which it is minimal is the ponderomotive energy.*

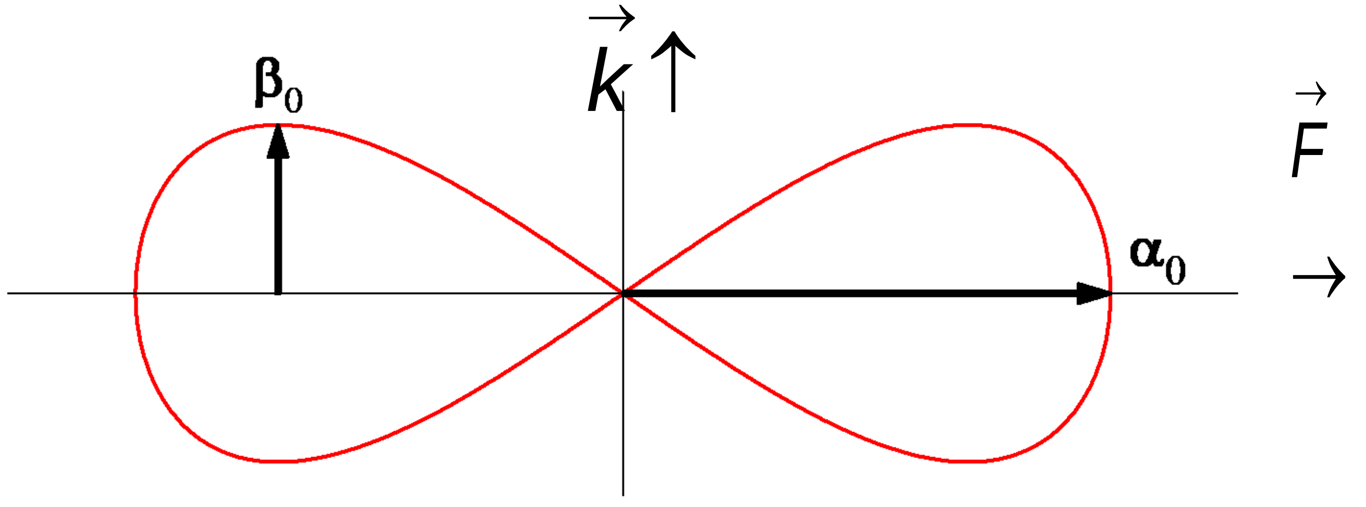
In a.u.:

$$U_p = \frac{I}{(2\omega)^2} = \frac{\langle \vec{F}^2 \rangle}{2\omega^2}$$

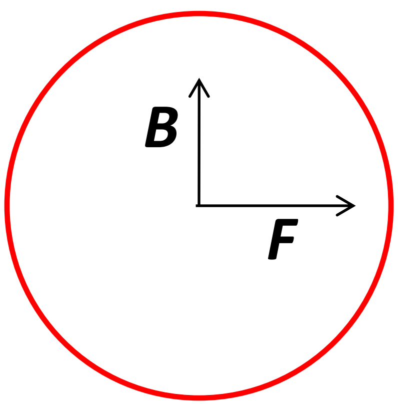
$$= \left(\frac{F_0}{2\omega}\right)^2 \quad \textit{linear polarization}$$

$$= \frac{1}{2}\left(\frac{F_0}{\omega}\right)^2 \quad \textit{circular polarization}$$

FIGURE-8 MOTION OF A FREE ELECTRON IN A LINEARLY POLARIZED PLANE-WAVE FIELD



CIRCULAR MOTION OF A FREE ELECTRON IN A CIRCULARLY POLARIZED PLANE-WAVE FIELD



MEASURES OF INTENSITY

Dimensionless measures are always to be preferred.

U_p is basic, but it can be compared to 3 other relevant energies in strong-field problems:

- ω energy of a single photon of the field
- E_B binding energy of an electron in a bound system
- mc^2 electron rest energy (actually, just c^2 in a.u.; the m is retained because of its familiarity)

This leads to 3 dimensionless measures of intensity:

- $z = U_p / \omega$ non-perturbative intensity parameter (note that $z \sim 1/\omega^3$)
- $z_1 = 2\, U_p / E_B$ bound-state intensity parameter ($z \sim 1/\omega^2$)
- $z_f = 2\, U_p / mc^2$ free-electron intensity parameter ($z \sim 1/\omega^2$)

A commonly used measure is the Keldysh parameter $\gamma_K = 1/ z_1^{1/2}$

γ_K is the ratio of tunneling time to wave period; a measure that implies a tunneling process, which is not appropriate to ionization by a plane wave. It is generally the ***only*** parameter employed, which is not sufficient.

MEASURES OF INTENSITY, continued

$z = U_p / \omega$

This is called the non-perturbative intensity parameter because a *channel closing* is non-analytical behavior that limits the radius of convergence of a perturbation expansion. An ionized electron must have at least the energy $U_p + E_B$ to be a physical particle. If $U_p \geq \omega$, then at least one additional photon must participate to satisfy energy conservation. If the minimum number of photons n_0 that must participate is increased from n_0 to $n_0 +1$, this is called a channel closing, and perturbation theory fails.

$z_1 = 2\, U_p / E_B$

This ratio compares the interaction energy of an electron with the field to the interaction energy of an electron with a binding potential. If $U_p > E_B$ the field is dominant, if $U_p < E_B$ then the binding potential is dominant. Validity of the Strong-Field Approximation (SFA) requires that the field should be dominant.

$z_f = 2\, U_p / mc^2$

This is the primary intensity parameter for free electrons.