MORE NOTES ON THE LENGTH GAUGE
8 March 2010

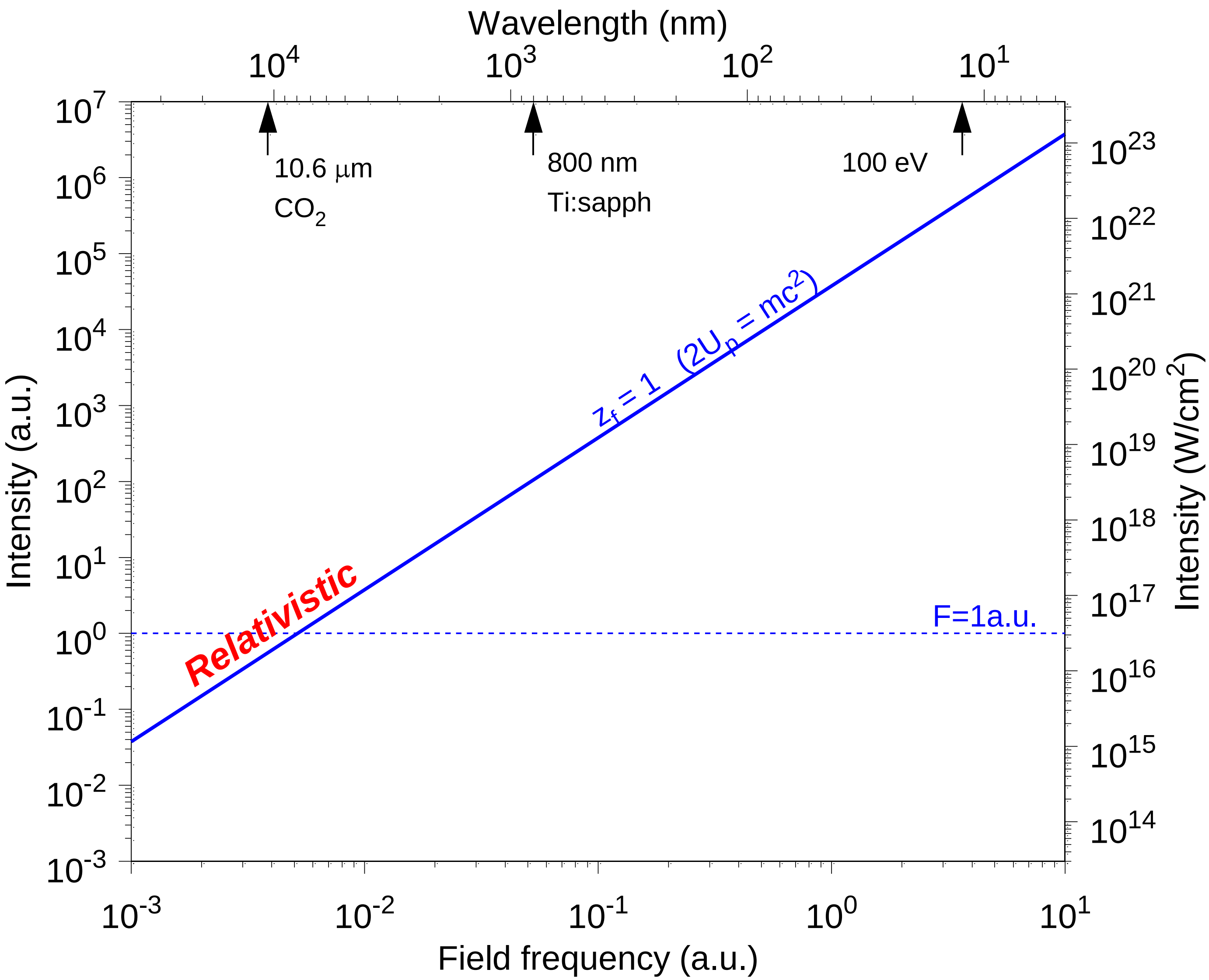
The atomic-ionization Schrödinger equation in the single-active-electron (SAE) model, in the length gauge (LG) and in a.u. is:

$$i\partial_t \Psi = \left[\frac{1}{2}(-i\vec{\nabla})^2 + V(r) + \vec{r} \cdot \vec{F} \right] \Psi$$

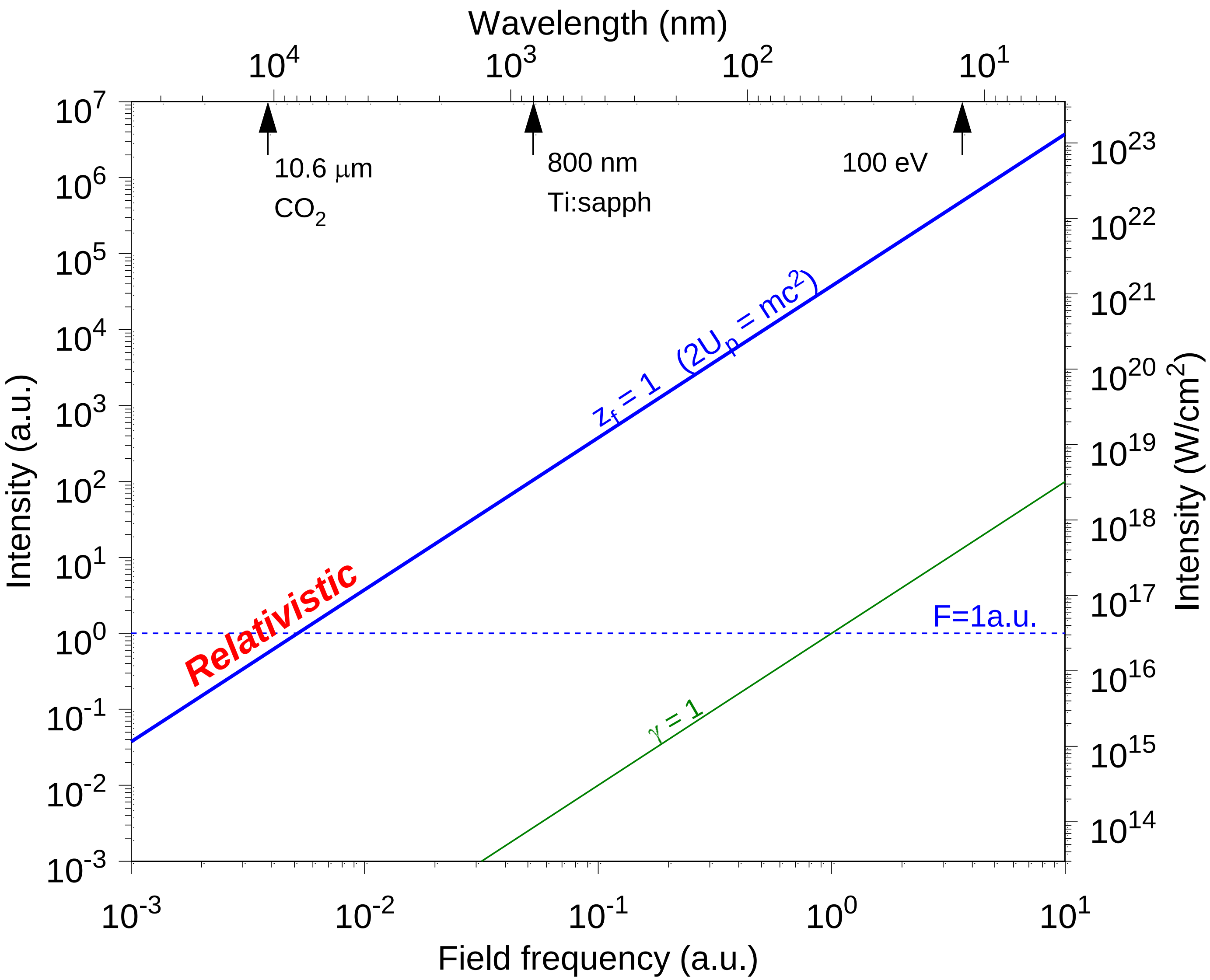
The only dependence on the "laser" field is the direct appearance of the electric field.

This leads to the difficulty already seen.

Furthermore, the elementary appearance of the "laser" and Coulomb fields in the Schrödinger equation is responsible for the existence of only a single parameter – the Keldysh γ -- when two parameters are needed to fix a point in the Intensity – Frequency diagram.

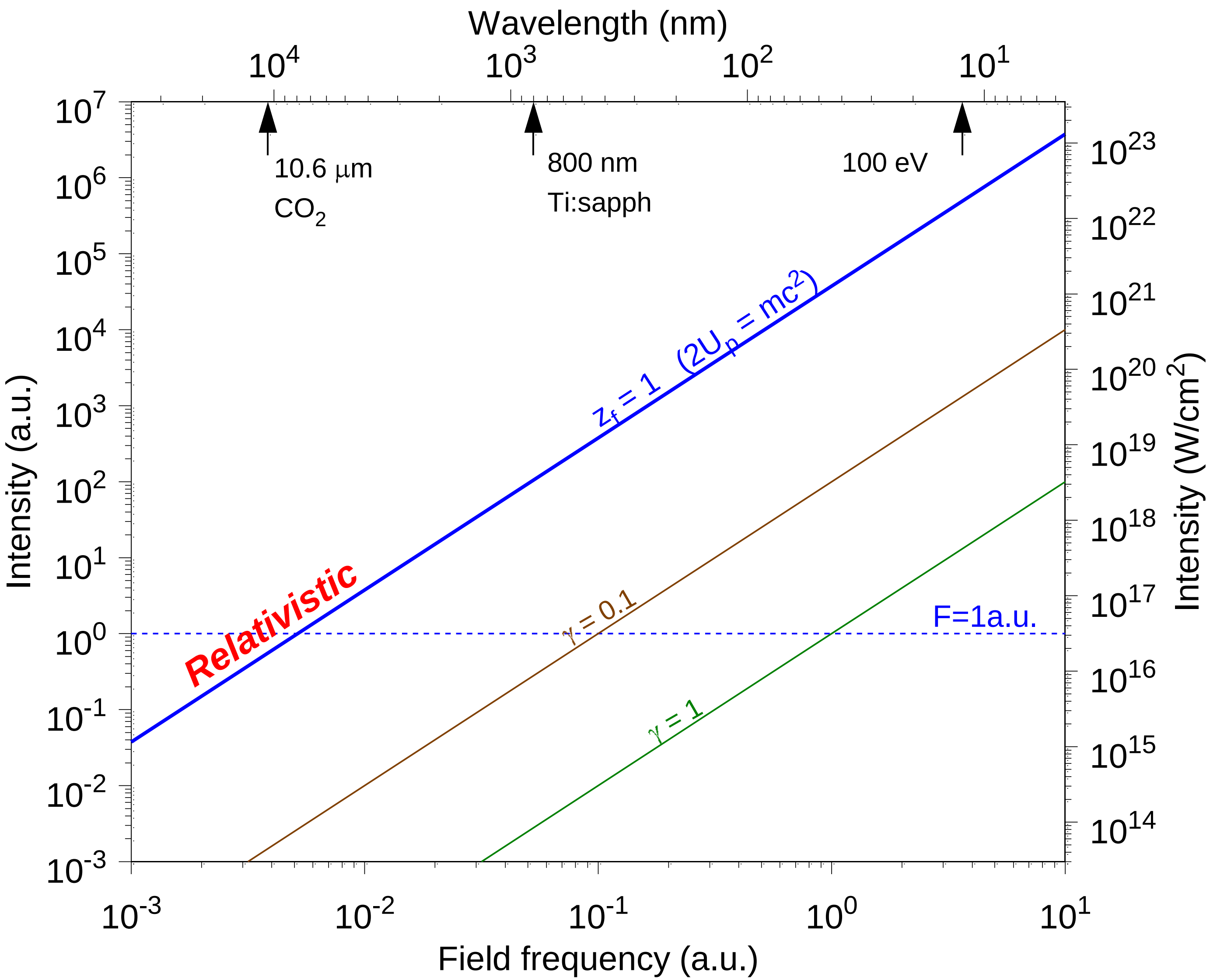


F is not a plausible index: *F=1* is relativistic at $\omega = 10^{-3}$ *and nonrelativistic at* $\omega = 10^{1}$

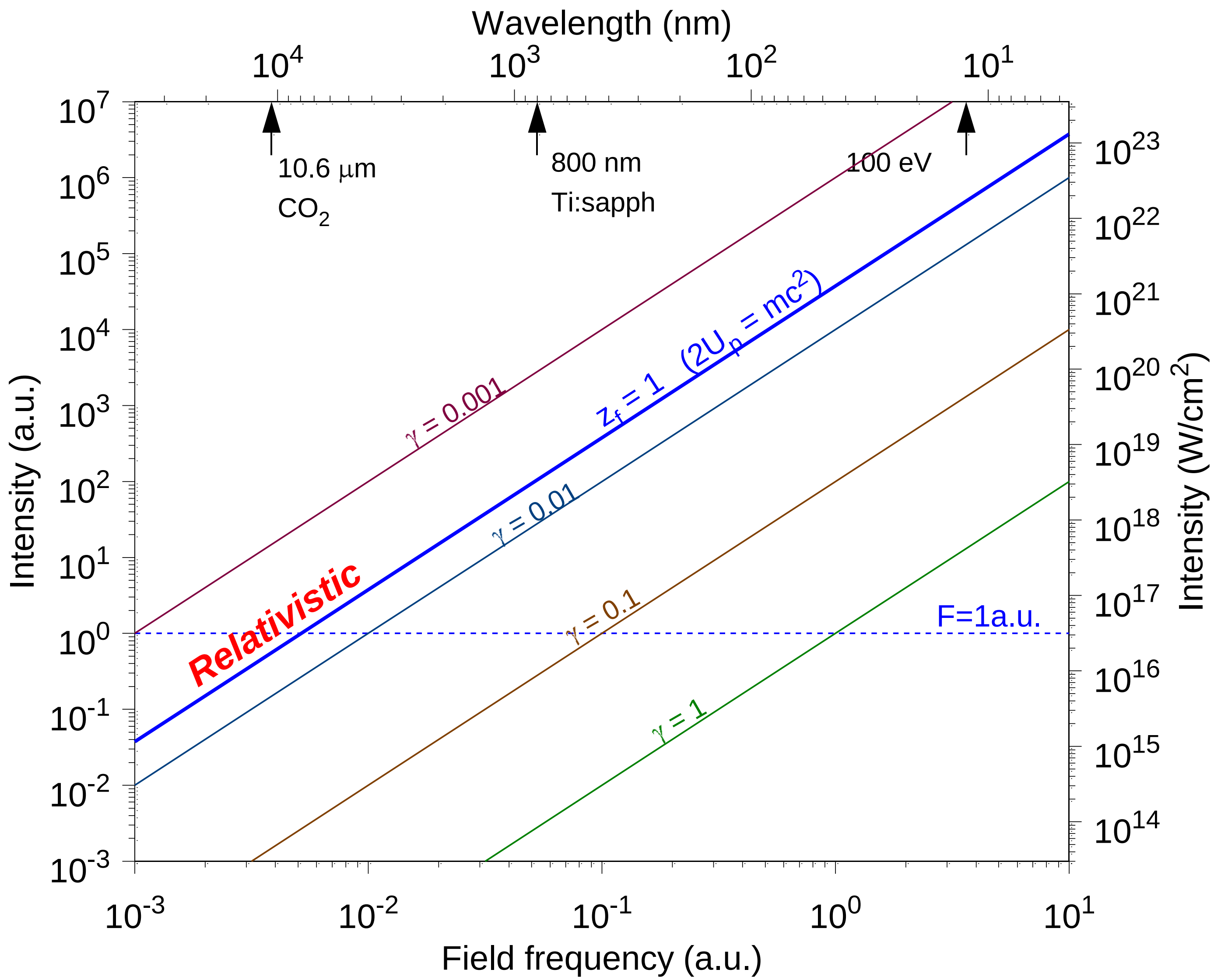


Keldysh γ does not define a point, it defines a line identifying very different types of fields.

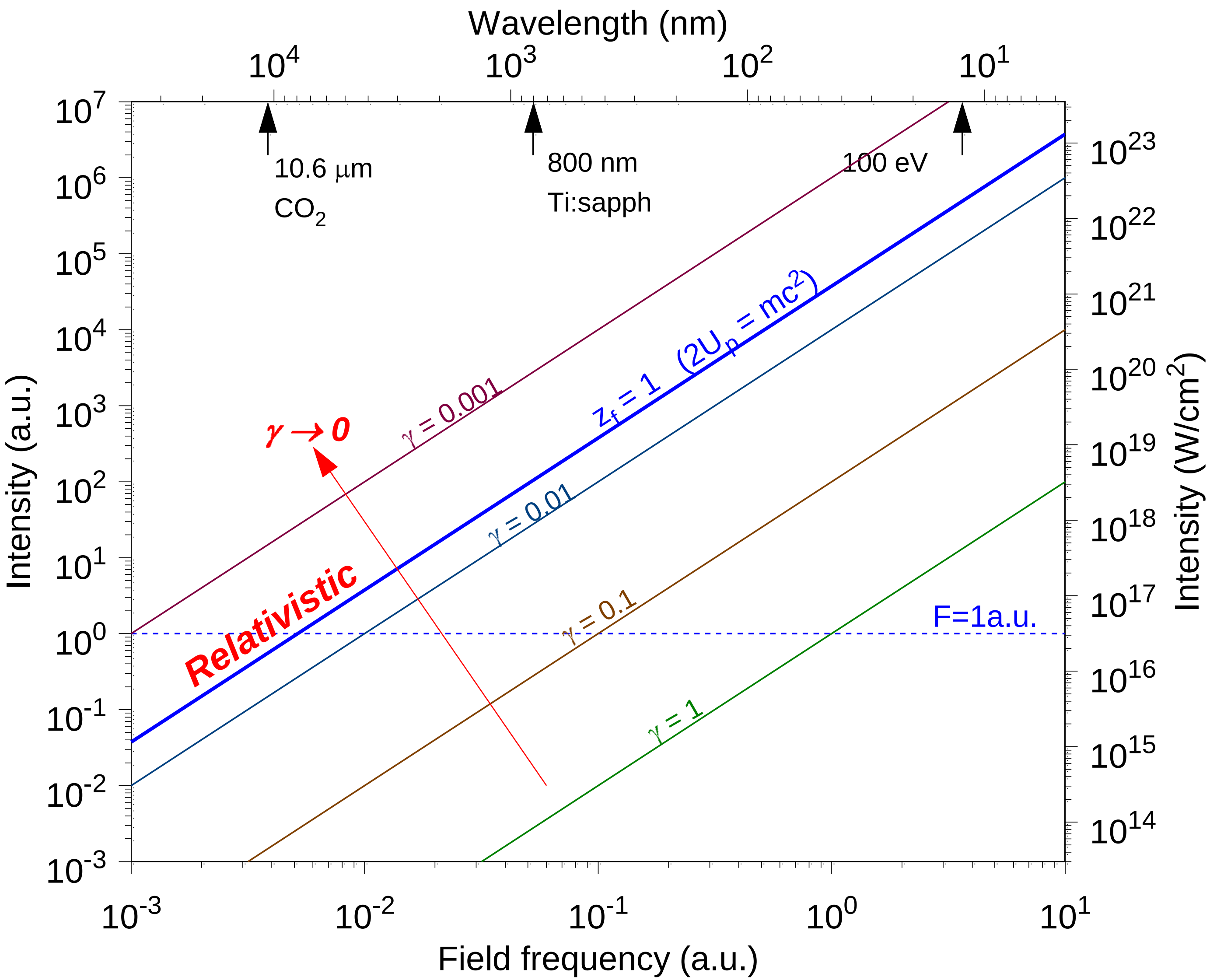
The figure is drawn for E_B = ½ *a.u.*



Different values of γ only serve to emphasize the problem.



Continuing the patter of decreasing γ leads inevitably into the relativistic domain.



The so-called "tunneling limit" is not a classical limit at all. It is a relativistic limit.

This problem of having only one parameter available when two are necessary to fix a point in the Intensity – Frequency plot does not occur in the Coulomb gauge (or velocity gauge (VG).

The VG has $z = U_p / \omega$ and $z_1 = 2\, U_p / E_B$ $(z_1 = 1/\gamma^2)$