MORE RESULTS FROM THE FIRST PPT PAPER

There is a section on a one-dimensional model that will be skipped apart from qualitative remarks.

PPT assume a turn-on of the "alternating" field at a specific time t_0. This requires a non-standard form for how the Green's function is employed. They later let $t_0 \to -\infty$, which restores the usual expression for the Green's function. There is no evident reason for this other than it is often done this way in textbooks using perturbation theory.

PPT then introduce a momentum representation for their Green's function, using the LG momentum p. However, they then suddenly express everything in terms of the VG canonical (or generalized) momentum π, where

$$\pi(t) = p - A(t) .$$

IMPORTANT:

In the LG, there is no vector potential $\Longrightarrow$ no difference between the canonical momentum π and the kinetic momentum p. In the VG, the canonical momentum is the *total momentum*, including the momentum of the electromagnetic field

$$\mathbf{p}_{field} = \frac{q}{c}\mathbf{A},$$

in full, Gaussian units, where q is the charge of a particle in the field. The kinetic momentum of the particle is

$$\mathbf{p}_{kinetic} = \mathbf{p}_{total} - \mathbf{p}_{field} = \pi - \frac{q}{c}\mathbf{A}.$$

In a.u., where q for an electron is $q = -1$, and re-ordering terms,

$$\pi = \mathbf{p}_{kinetic} + \frac{q}{c}\mathbf{A} = \mathbf{p}_{kinetic} - \frac{1}{c}\mathbf{A}.$$

PPT use electromagnetic units without the $1/c$ factor.

IMPLICATION:

By doing a Fourier transform in terms of p (which is both canonical and kinetic momentum in the LG) and then introducing π, PPT have effectively introduced a factor

$$\exp\left(i\mathbf{r}\cdot\mathbf{A}\right),$$

which is the gauge transformation factor between the LG and the VG.

PPT are now in the velocity gauge.

They introduce a specific form for the field

$$f\left(t\right) = F\cos\omega t \quad \Longrightarrow$$
$$A\left(t\right) = -F\frac{\sin\omega t}{\omega},$$

(reverting to one dimension and the PPT electromagnetic conventions).

ASSUMPTIONS:

* PPT regard the turn-on time $t_0 \to -\infty$ as an assumption. There is no harm in this.

* The initial-state wave function is taken to be a non-interacting bound-state wave function. This is widely regarded (i.e. by many others besides PPT) as an assumption, but the rigorous S-matrix derivation makes it clear that this is a necessity if the time-reversed form is used - which appears to be the case here.

* *PPT state that the mean time of ionization is assumed to be much larger than atomic times.*

THIS LAST REMARK IS OF SPECIAL INTEREST HERE. START A DISCUSSION OF THIS MATTER NOW.

HOW LONG DOES IT TAKE TO IONIZE AN ATOM?

The Heisenberg Uncertainty Principle relates to canonically conjugate variables. In full units:

$$\Delta x \Delta p \geq \hbar/2,$$
$$\Delta t \Delta E \geq \hbar/2.$$

Suppose

$$\Delta E = E_B,$$

the binding energy of an electron in an atom. *Then the corresponding Δt is the time during which it is uncertain whether or not ionization has taken place. This can be regarded as the time required to ionize the atom.*

$$\Delta t \approx \frac{\hbar}{2E_B}.$$

To be specific, suppose E_B is the binding energy of ground-state hydrogen, and now express quantities in a.u. That is, $E_B = 1/2$ a.u., $\hbar = 1$.

$$\Delta t \approx \frac{1}{2 \times (1/2)} = 1 a.u. = 2.4 \times 10^{-17}\,\mathrm{sec}$$

HOW LONG DOES IT TAKE TO TUNNEL THROUGH A BARRIER?

A definition of the Keldysh gamma parameter is

$$\gamma_K = \frac{\text{tunneling time}}{\text{wave period}}$$

Suppose $\gamma_K = 1$, and $\lambda = 800nm$.

$$\text{wave period} = \tau = \frac{\lambda}{c} = \frac{8 \times 10^{-7} m}{3 \times 10^{8} m/s} = 2.7 \times 10^{-15}\,\text{sec} \approx 100 a.u.$$

$$\text{tunneling time} = \gamma_K \times \text{wave period} \approx 100 a.u.$$

CONCLUSION:

TIME TO IONIZE = 1 a.u.

TUNNELING TIME = LIFE TIME = 100 a.u.

$$\frac{\text{TUNNELING TIME}}{\text{TIME TO IONIZE}} = 100$$

NUCLEAR ANALOG

Decay of uranium-238 by tunneling through a Coulomb barrier:

$$^{238}_{92}U_{146} \rightarrow ^{234}_{90}Th_{144} + ^{4}_{2}He_{2}$$

Half life $= 4.47 \times 10^{9} years = 1.4 \times 10^{17}\,\mathrm{sec} = 5.8 \times 10^{33} a.u.$

Binding energy $\approx 5\ MeV \approx 1.8 \times 10^{5} a.u. \Longrightarrow$ time to decay $= 1/\left(2E_B\right) = 2.7x10^{-6} a.u.$

TIME TO DECAY $= \mathbf{3} \times 10^{-6}$ **a.u.**

TUNNELING TIME = LIFE TIME $= 6 \times 10^{33}$ **a.u.**

$$\frac{\text{TUNNELING TIME}}{\text{TIME TO DECAY}} = 2 \times 10^{39}$$

In other words, an alpha decay occurs only once in 10^{17} sec, but when the decay happens, it occurs within 10^{-22} sec.

G. Gamow, Z. Phys. **51**, 204 (1928); **52**, 495 (1928).

IONIZATION TIME AND TUNNELING TIME ARE TWO DIFFERENT CONCEPTS

Ionization occurs on a quantum time scale.

If the period of a laser field can be considered a classical time, then atomic **tunneling occurs on a classical time scale.**

(Quantum vs. classical is unambiguous in the alpha-decay example.)

PPT recognize this distinction. Most authors do not.

See HRR, PRA **75**, 013413 (2007).

MORE PPT

STILL IN THE ONE-DIMENSIONAL PROBLEM

Ionization probabilities are arrived at by using a quantum current at large times.

$$j(x,t) = \frac{i}{2}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right).$$

They are still in the VG. They achieve a form in which the probability of ionization can be viewed as the sum of probabilities with the absorption of n photons, where the lowest order in the sum is

$$n_0 \geq \frac{\varkappa^2}{2\omega}\left(1+\frac{1}{2\gamma_K^2}\right) = \frac{E_B}{\omega}\left(1+\frac{1}{2}\frac{2U_p}{E_B}\right) = \frac{1}{\omega}\left(E_B+U_p\right).$$

This is identical to the threshold condition in the SFA: The field has to supply an energy greater than or equal to the sum of the binding energy and the ponderomotive energy.

To proceed further, they assume

$$\omega \ll E_B,$$

(already assumed earlier), use a saddle-point approximation, and find a result for a transition rate

$$w(F,\omega) = \exp\left[-\frac{2}{3}\frac{F_0}{F}g(\gamma)\right]R(\omega,\gamma),$$

where $g(\gamma)$ and $R(\omega,\gamma)$ are complicated functions, with $R(\omega,\gamma)$ containing the sum over photon orders.

The SFA in the VG does *NOT* have the tunneling exponential.

THREE-DIMENSIONAL CASE

PHOTODETACHMENT FROM A NEGATIVE ION

WHERE THE ELECTRON IS BOUND IN A SHORT-RANGE POTENTIAL

$$\Psi\left(\mathbf{r},t\right)=-i\int_{-\infty}^{t}dt'\int d^{3}\mathbf{r}G\left(\mathbf{r},t;\mathbf{r}',t'\right)H_{I}\left(\mathbf{r}'\right)\Psi\left(\mathbf{r}',t'\right),$$

where $G\left(\mathbf{r},t;\mathbf{r}',t'\right)$ is the Volkov propagator as transformed from the VG to the LG. This amounts to the insertion of the gauge-transformation factor

$$\exp\left[i\mathbf{r}\cdot\mathbf{A}\left(t\right)\right]$$

into the VG Volkov solution. The Volkov propagator (Green's function) in the LG is

$$G\left(\mathbf{r},t;\mathbf{r}',t'\right)=\frac{\theta\left(t-t'\right)}{\left(2\pi\right)^{3}}\int d^{3}p\exp\left\{i\left[\pi\left(t\right)\cdot\mathbf{r}-\pi\left(t'\right)\cdot\mathbf{r}'\right]-\frac{1}{2}\int_{t'}^{t}d\tau\pi^{2}\left(\tau\right)\right\}$$

$$\pi\left(t\right)=p-\mathbf{A}\left(t\right)$$

$$\mathbf{A}\left(t\right)=-\int_{-\infty}^{t}dt'\mathbf{F}\left(t'\right)$$

ESSENTIAL PROPERTIES OF THE VOLKOV SOLUTION

Properly speaking, the Volkov solution exists only in a relativistic form.

- A plane-wave field describes a wave propagating at the speed of light; hence relativity is necessary.
- The Volkov solution describes a free particle; there are no bounds on its motion; the dipole approximation is not applicable.

The Volkov solution exists naturally in the Coulomb gauge:

$$\phi = 0$$

$$\mathbf{A}\left(\mathbf{r},t\right) = A_0 \epsilon \cos\left(\omega t - \mathbf{k}\cdot\mathbf{r}\right)$$

$$\epsilon \perp \mathbf{k}$$

The last condition establishes the identity of a

PW (plane wave) as a **transverse field**:

THE ELECTRIC (AND MAGNETIC) FIELDS ARE PERPENDICULAR TO

THE DIRECTION OF PROPAGATION

DIPOLE APPROXIMATION VOLKOV SOLUTION

IN THE LENGTH GAUGE

In the LG:

$$H_I = \mathbf{r} \cdot \mathbf{F}.$$

- There is only one non-trivial spatial direction in the problem: the direction of $\mathbf{F}$.

- Because $\omega t - \mathbf{k} \cdot \mathbf{r} \to \omega t$ in the dipole approximation, there is no propagation direction $\mathbf{k}$.

- The only solution of the Schrödinger equation for an oscillatory field in the LG is the solution for a QSE field.

THE ONLY WAY TO INTRODUCE EVEN A DIPOLE-APPROXIMATION VOLKOV SOLUTION

INTO THE LG SE IS TO GAUGE-TRANSFORM THE VOLKOV SOLUTION FROM THE VG.

THEN THE VOLKOV SOLUTION IS IN A LG SE IN TERMS OF A VECTOR POTENTIAL.

THERE IS NO VECTOR POTENTIAL IN THE LG.

A BASIC DILEMMA

- There is no Volkov solution in the LG.
- A *DIPOLE APPROXIMATION* Volkov solution can be gauge-transformed from the VG.
- This Volkov solution is in terms of a vector potential *from a different gauge*.
- The vector potential must be removed to avoid a meaningless mixed-gauge representation.

$$\mathbf{A}(t) = -\int_{-\infty}^{t} d\tau \mathbf{F}(\tau).$$

The vector potential has been replaced by an integral of the electric field over all earlier times.

THE VOLKOV SOLUTION IN THE LG CAN BE FOUND

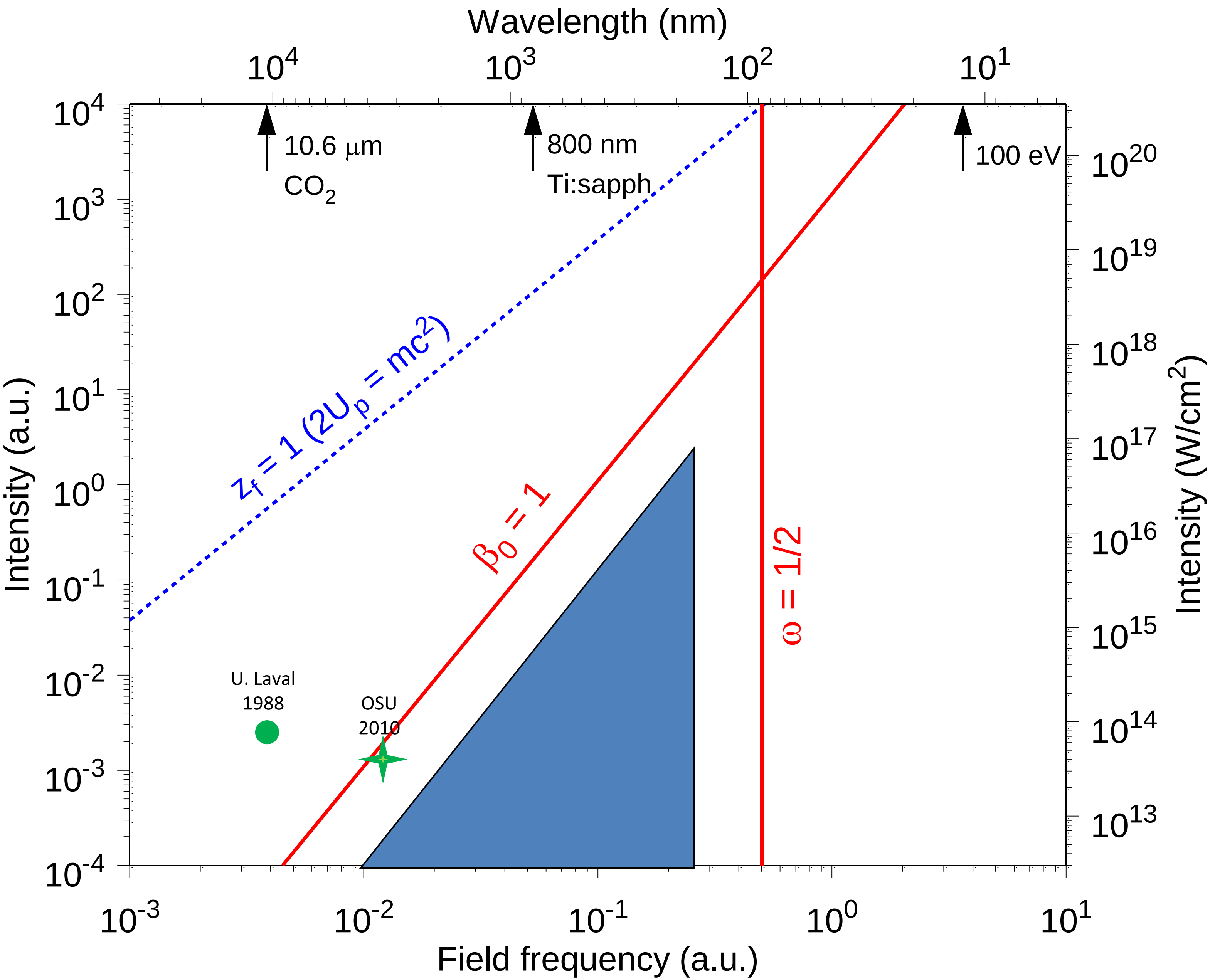
ONLY AS A NONLOCAL SOLUTION.

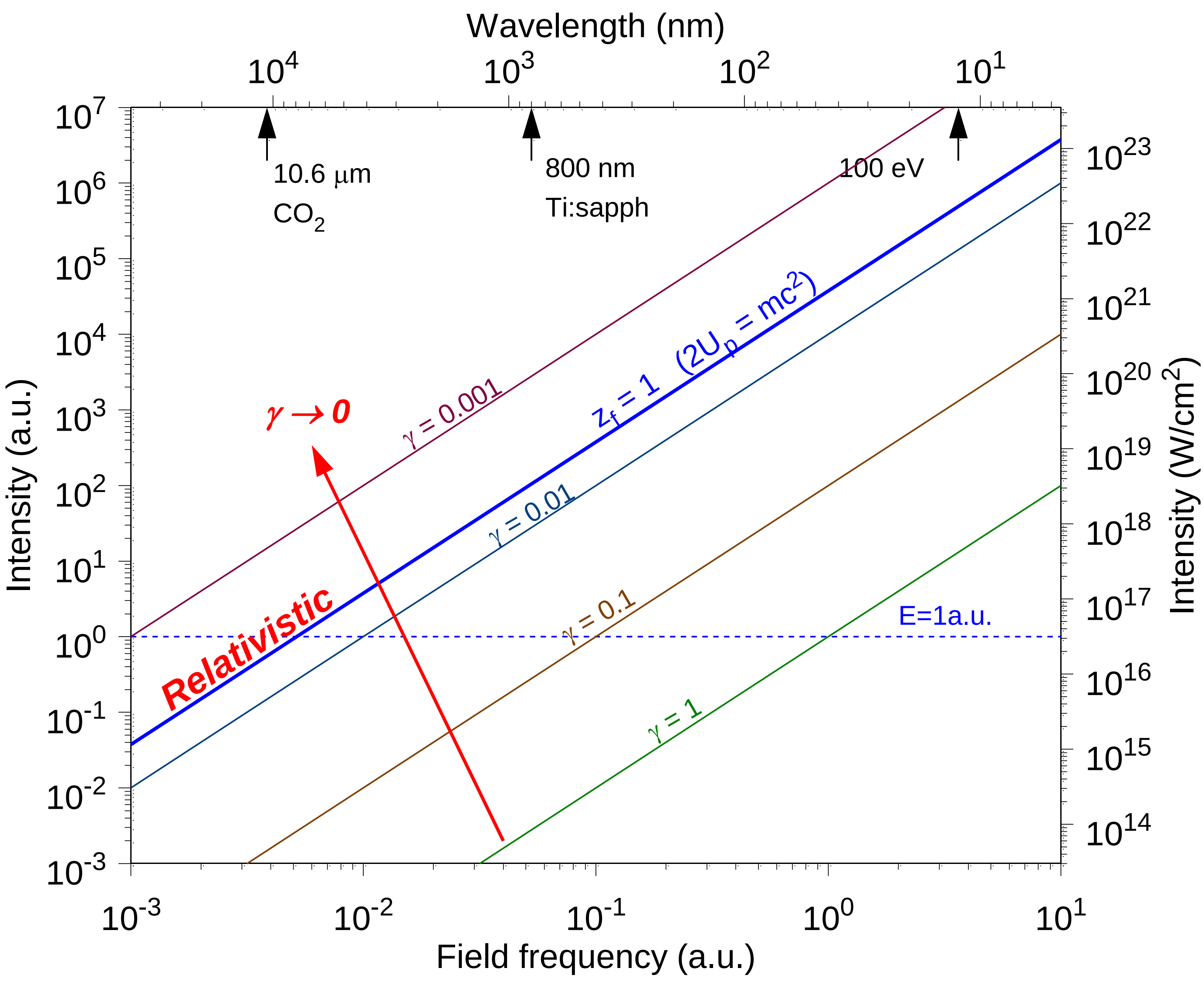
CONSEQUENCES

- No gauge equivalence of any kind exists if the dipole approximation is not satisfied.
- Even within the dipole approximation, other approximations do not have the same meaning in the two gauges.
- The tunneling exponential persists in the LG; it is not present in the VG.
- The above result has not been generally recognized, which leads to hopeless confusion.
- An example: the literature abounds with articles and reviews that make no distinction between the SFA and tunneling, or see no difference in the fundamentally distinct K, F, R methods.
- One (of many) major results of this failure to distinguish methods is that a standard of excellence widely employed is whether the $\omega \rightarrow 0$ goes to a static result; **THIS IS A FALSE CRITERION.**

The last-named problem afflicts *most* investigators in the field.

TUNNELING THEORIES FOR LASER-INDUCED PROCESSES ARE LIMITED TO THE SHADED AREA





Wavelength (nm)
10^4
10^3
10^2
10^1
10.6 μm
CO2
800 nm
Ti:sapph
100 eV
γ → 0
γ = 0.001
z_f = 1 (2U_p = mc^2)
γ = 0.01
Relativistic
γ = 0.1
E=1a.u.
γ = 1
Intensity (a.u.)
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
Intensity (W/cm^2)
10^23
10^22
10^21
10^20
10^19
10^18
10^17
10^16
10^15
10^14
10^-3
10^-2
10^-1
10^0
10^1
Field frequency (a.u.)

L. V. Keldysh, JETP **47**, 1945 (1964) [Sov. Phys. JETP **20**, 1307 (1965)].

Keldysh is famous in solid state physics. This is his *only* paper on strong-field phenomena.

The early (1964 and on) Soviet efforts in strong fields were relativistic, and all used the Dirac-Volkov solution, following HRR (1962).

LG and the dipole approximation are commonplace in solid-state physics, so Keldysh used both while retaining the Volkov solution.

Starting matrix element:

$$M_{fi} = \int d^3r \Psi_f^{*Volk}(\mathbf{r},t)\, \mathbf{r}\cdot\mathbf{F}\Phi_i(\mathbf{r})$$
$$= \left(\Psi_f^{Volk}, H_I\Phi_i\right).$$

This is exactly like the SFA matrix element, but done in the LG.

Keldysh was unaware of S-matrix methods, and made two mistaken interpretations:

* He describes this matrix element as a "direct" transition to the continuum (a perturbation-theorists attitude).

* He regards the use of an unperturbed initial state as an approximation, assuming that he has neglected a Stark shift.